



# Padalsalai's Telegram Groups!

( தலைப்பிற்கு கீழே உள்ள லிங்கை கிளிக் செய்து குழுவில் இணையவும்! )

- Padalsalai's NEWS - Group  
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- Padalsalai's Channel - Group  
<https://t.me/padasalaichannel>
- Lesson Plan - Group  
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- 12th Standard - Group  
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- 11th Standard - Group  
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## CHAPTER-11 PROBABILITY DISTRIBUTION.

Random Variable:

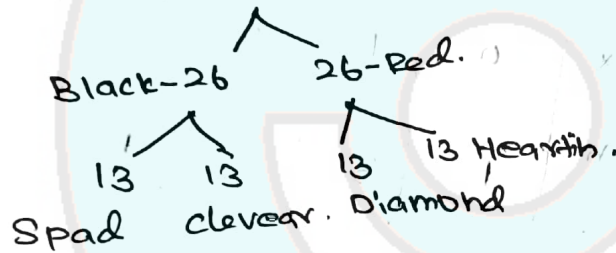
A random variable  $x$  is a function defined on the Sample Space  $S$  into the Real no. on  $R$ , such that the inverse of the point (or) subset (or) interval  $R$  is an event in  $S$ .

Ex: 11

Coin  $\rightarrow \{H, T\}$  possible event.

dice  $\rightarrow \{1, 2, 3, 4, 5, 6\}$  possible event.

cards  $\rightarrow 52$



Card 13  $\rightarrow \{2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A\}$

statement:  $(\text{face})^n$ , no. of events =  $n$

win:  $(2)^n$ , no. of wins

dice:  $(6)^n$ , no. of dices.

possible event  $x \rightarrow$  random variable.

collection of random variable  
(or)  
possible events } sample space.  
 $n(s)$



EXE 11.1

① let  $x$  be the random variable denoted has no. of tails.

$$x = \{0, 1, 2, 3\}$$

$$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$$

$$|n(S)| = 8$$

$$x(\text{no tail}) \Rightarrow x(HHH) = 0$$

$$x(\text{one tail}) \Rightarrow x(HHT) = x(HTH) = x(THH) = 1$$

$$x(\text{two tail}) \Rightarrow x(HTT) = x(THT) = x(TTH) = 2$$

$$x(3 \text{ tail}) \Rightarrow x(TTT) = 3$$

| Random variable $x$              | 0 | 1 | 2 | 3 | Total |
|----------------------------------|---|---|---|---|-------|
| No. of element in Inverse image. | 1 | 3 | 3 | 1 | 8     |

②

let  $x$  be the random variable denoted has no. of Black cards.

$$x = \{0, 1, 2\}$$

$$n(S) = {}^{52}C_2 = \frac{52 \times 51}{1 \times 2} = 1326$$

$$= {}^{52}C_2 = \frac{52 \times 51}{1 \times 2} = 26 \times 51 = 1326$$

$$n(S) = 1326$$

| Black | Other | Total |
|-------|-------|-------|
| 26    | 26    | 52    |

$$x(\text{no Black}) = {}^{26}C_0 \times {}^{26}C_2$$

$$n(S) = 1 = 1 \times \frac{26 \times 25}{1 \times 2}$$

$$= 325$$

$$x(\text{one Black}) = {}^{26}C_1 \times {}^{26}C_1$$

$$= 26 \times 26$$

$$= 676$$

$$x(2 \text{ Black}) = {}^{26}C_2 \times {}^{26}C_0$$

$$= \frac{26 \times 25}{1 \times 2} \times 1$$

$$= 325$$

| Random variable x                | 0   | 1   | 2   | Total |
|----------------------------------|-----|-----|-----|-------|
| No. of element in Inverse image. | 325 | 676 | 325 | 1326  |

3).

$$n(s) = {}^9C_3 = \frac{9 \times 8 \times 7}{1 \times 2 \times 3}$$

$$n(s) = 84$$

| mango | Apple | Total |
|-------|-------|-------|
| 5     | 4     | 9     |

Let x be the random variable denoted has taken apple.

$$x = \{0, 1, 2, 3\}$$

$$x(0) = x(\text{no apple}) = {}^5C_3 \times {}^4C_0$$

$$= \frac{5 \times 4 \times 3}{1 \times 2 \times 3} \times 1$$

$$= 10$$

$$\begin{aligned}
 x(1) &= x(\text{one apple}) = {}^5C_2 \times {}^4C_1 \\
 &= \frac{5 \times 4^2}{1 \times 2} \times 4 \\
 &= 40
 \end{aligned}$$

$$\begin{aligned}
 x(2) &= x(\text{two apple}) = {}^5C_1 \times {}^4C_2 \\
 &= 5 \times \frac{4 \times 3}{1 \times 2} \\
 &= 30
 \end{aligned}$$

$$\begin{aligned}
 x(3) &= x(3 \text{ apple}) = {}^5C_0 \times {}^4C_3 \\
 &= 1 \times 4 \\
 &= 4
 \end{aligned}$$

$$\overline{{}^4C_3 = {}^4C_1 = 4}$$

$$n C_0 = 1$$

$$n C_1 = n$$

| Random variable x               | 0  | 1  | 2  | 3 | Total. |
|---------------------------------|----|----|----|---|--------|
| No. of element in inverse image | 10 | 40 | 30 | 4 | 84     |

④

Let  $x$  be the random variable denoted has winning amount.

$$x = \{0, 1, 2\}$$

$$n(s) = {}^{14}C_2 = \frac{14 \times 13}{1 \times 2}$$

$$n(s) = 91$$

$$x(BB) = -10 - 10 = -20$$

$$x(BR) = -10 + 15 = 5$$

$$x(RR) = +15 + 15 = 30$$

| Red | Black | Total |
|-----|-------|-------|
| 6   | 8     | 14    |

Cumulative distribution function.

|        |      |      |      |      |   |
|--------|------|------|------|------|---|
| $x$    | -1   | 0    | 1    | 2    | 3 |
| $F(x)$ | 0.15 | 0.35 | 0.60 | 0.85 | 1 |

(i) probability mass function.

|        |      |      |      |      |      |
|--------|------|------|------|------|------|
| $x$    | -1   | 0    | 1    | 2    | 3    |
| $f(x)$ | 0.15 | 0.20 | 0.25 | 0.25 | 0.15 |

$$f(-1) = F(-1) - F(x < -1) = 0.15 - 0 = 0.15$$

$$f(0) = F(0) - F(-1) = 0.35 - 0.15 = 0.20$$

$$f(1) = F(1) - F(0) = 0.60 - 0.35 = 0.25$$

$$f(2) = F(2) - F(1) = 0.85 - 0.60 = 0.25$$

$$f(3) = F(3) - F(2) = 1 - 0.85 = 0.15$$

$$\begin{aligned} \text{(ii)} \quad P(x < 1) &= f(0) + f(-1) \\ &= 0.20 + 0.15 \\ &= 0.35 \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad P(x \geq 2) &= f(2) + f(3) \\ &= 0.25 + 0.15 \\ &= 0.40 \end{aligned}$$

cumulative distribution function.

|        |               |               |               |                |   |
|--------|---------------|---------------|---------------|----------------|---|
| $x$    | 0             | 1             | 2             | 3              | 4 |
| $F(x)$ | $\frac{1}{2}$ | $\frac{3}{5}$ | $\frac{4}{5}$ | $\frac{9}{10}$ | 1 |

(i) probability mass function.

|        |               |                |               |                |                |
|--------|---------------|----------------|---------------|----------------|----------------|
| $x$    | 0             | 1              | 2             | 3              | 4              |
| $f(x)$ | $\frac{1}{2}$ | $\frac{1}{10}$ | $\frac{1}{5}$ | $\frac{1}{10}$ | $\frac{1}{10}$ |

$$\begin{aligned}
 f(0) &= F(0) - F(x < 0) = \frac{1}{2} - 0 = \frac{1}{2} \\
 f(1) &= F(1) - F(0) = \frac{3}{5} - \frac{1}{2} = \frac{6-5}{10} = \frac{1}{10} \\
 f(2) &= F(2) - F(1) = \frac{4}{5} - \frac{3}{5} = \frac{4-3}{5} = \frac{1}{5} \\
 f(3) &= F(3) - F(2) = \frac{9}{10} - \frac{4}{5} = \frac{9-8}{10} = \frac{1}{10} \\
 f(4) &= F(4) - F(3) = 1 - \frac{9}{10} = \frac{10-9}{10} = \frac{1}{10}
 \end{aligned}$$

(ii)  $P(x < 3) = f(0) + f(1) + f(2)$

$$= \frac{1}{2} + \frac{1}{10} + \frac{1}{5}$$

$$= \frac{5+1+2}{10}$$

$$= \frac{8}{10}$$

$$= \frac{4}{5}$$

(iii)

$$P(x \geq 2) = f(2) + f(3) + f(4)$$

$$= \frac{1}{5} + \frac{1}{10} + \frac{1}{10}$$

$$= \frac{2+1+1}{10}$$

$$= \frac{4}{10}$$

$$= \frac{2}{5}$$

# CHAPTER 11 PROBABILITY DISTRIBUTION

Ex 11.2

6. probability mass function.

|        |       |        |        |      |      |
|--------|-------|--------|--------|------|------|
| $x$    | 1     | 2      | 3      | 4    | 5    |
| $f(x)$ | $k^2$ | $2k^2$ | $3k^2$ | $2k$ | $3k$ |

(i) Total probability = 1.  $\rightarrow \sum p(x_i) = 1$ .

$$f(1) + f(2) + f(3) + f(4) + f(5) = 1$$

$$k^2 + 2k^2 + 3k^2 + 2k + 3k = 1$$

$$6k^2 + 5k = 1$$

$$6k^2 + 5k - 1 = 0$$

$$(k+1)(k - \frac{1}{6}) = 0$$

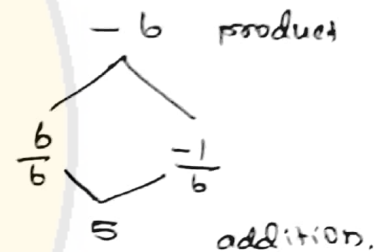
$$k+1=0, \quad k - \frac{1}{6} = 0$$

$$k = -1$$

not possible.

$$k = \frac{1}{6}$$

✓



probability values.

$$[0 \text{ to } 2]$$

(ii)

$$P(2 \leq x < 5) = f(2) + f(3) + f(4)$$

$$= 2k^2 + 3k^2 + 2k$$

$$= 5k^2 + 2k$$

$$= 5\left(\frac{1}{6}\right)^2 + 2\left(\frac{1}{6}\right)$$

$$= \frac{5}{36} + \frac{1}{3}$$

$$= \frac{5+12}{36}$$

$$= \frac{17}{36}$$

$$\text{ii) } P(3 < x) = f(4) + f(5)$$

$$= 2k + 3k$$

$$= 5k$$

$$= 5\left(\frac{1}{6}\right)$$

$$= \frac{5}{6}$$

$$k = \frac{1}{6}$$



Example 11.9.

Cumulative distribution function.

|        |      |      |      |   |  |
|--------|------|------|------|---|--|
| $x$    | -2   | -1   | 0    | 1 |  |
| $F(x)$ | 0.25 | 0.60 | 0.90 | 1 |  |

(i) probability mass function.

|        |      |      |      |      |  |
|--------|------|------|------|------|--|
| $x$    | -2   | -1   | 0    | 1    |  |
| $f(x)$ | 0.25 | 0.35 | 0.30 | 0.10 |  |

$$f(-2) = F(-2) - F(-2^-) = 0.25 - 0 = 0.25$$

$$f(-1) = F(-1) - F(-2) = 0.60 - 0.25 = 0.35$$

$$f(0) = F(0) - F(-1) = 0.90 - 0.60 = 0.30$$

$$f(1) = F(1) - F(0) = 1 - 0.90 = 0.10.$$

$$(ii) P(x < 0) = f(-2) + f(-1)$$

$$= 0.25 + 0.35$$

$$P(x < 0) = 0.60 \checkmark$$

$$(iii) P(x \geq -1) = f(-1) + f(0) + f(1)$$

$$= 0.35 + 0.30 + 0.10.$$

$$P(x \geq -1) = 0.75 \checkmark$$



# CHAPTER - II PROBABILITY DISTRIBUTION

⑥. Probability mass function:

|      |       |        |        |      |      |
|------|-------|--------|--------|------|------|
| x    | 1     | 2      | 3      | 4    | 5    |
| f(x) | $k^2$ | $2k^2$ | $3k^2$ | $2k$ | $3k$ |

Total probability = 1.

(i)

$$\sum p(x_i) = 1.$$

$$f(1) + f(2) + f(3) + f(4) + f(5) = 1.$$

$$k^2 + 2k^2 + 3k^2 + 2k + 3k = 1.$$

$$6k^2 + 5k = 1.$$

$$6k^2 + 5k - 1 = 0.$$

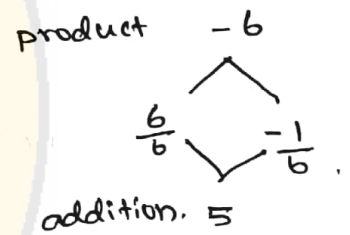
$$(k+1)(k-\frac{1}{6}) = 0.$$

$$k+1=0, \quad k-\frac{1}{6}=0.$$

$$\boxed{k = -1}$$

not possible.

$k = \frac{1}{6}$ . probability value to 1



(ii)

$$P(2 \leq x < 5) = f(2) + f(3) + f(4).$$

$$= 2k^2 + 3k^2 + 2k$$

$$= 5k^2 + 2k$$

$$= 5\left(\frac{1}{6}\right)^2 + 2\left(\frac{1}{6}\right)$$

$$= \frac{5}{36} + \frac{1}{3}.$$

$$= \frac{5 + 10 \times 12}{36} = \frac{5+12}{36} = \frac{17}{36}$$

$$(iii) P(3 < x) = f(4) + f(5).$$

$$= 2k + 3k$$

$$= 5k$$

$$= 5\left(\frac{1}{6}\right)$$

$$P(3 < x) = \frac{5}{6}.$$

5

cumulative distribution function.

$$F(x) = \begin{cases} 0 & -\infty < x < -1 \\ 0.15 & -1 \leq x < 0 \\ 0.35 & 0 \leq x < 1 \\ 0.60 & 1 \leq x < 2 \\ 0.85 & 2 \leq x < 3 \\ 1 & 3 \leq x < \infty \end{cases}$$

cumulative distribution function.

| $x$    | -1   | 0    | 1    | 2    | 3 |
|--------|------|------|------|------|---|
| $F(x)$ | 0.15 | 0.35 | 0.60 | 0.85 | 1 |

probability mass function.

| $x$    | -1   | 0    | 1    | 2    | 3    |
|--------|------|------|------|------|------|
| $f(x)$ | 0.15 | 0.20 | 0.25 | 0.25 | 0.15 |

$$f(0) = F(0) - F(-1)$$

difference.  
(second value) - (first value)

$$\begin{aligned}
 \text{(ii)} \quad P(X < 1) &= f(-1) + f(0) \\
 &= 0.15 + 0.20 \\
 &= 0.35
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad P(X \geq 2) &= f(2) + f(3) \\
 &= 0.25 + 0.15 \\
 &= 0.40.
 \end{aligned}$$

⑦ Cumulative distribution function.

|        |               |               |               |                |   |
|--------|---------------|---------------|---------------|----------------|---|
| $x$    | 0             | 1             | 2             | 3              | 4 |
| $F(x)$ | $\frac{1}{2}$ | $\frac{3}{5}$ | $\frac{4}{5}$ | $\frac{9}{10}$ | 1 |

probability mass function.

|        |               |                |               |                |                |
|--------|---------------|----------------|---------------|----------------|----------------|
| $x$    | 0             | 1              | 2             | 3              | 4              |
| $f(x)$ | $\frac{1}{2}$ | $\frac{1}{10}$ | $\frac{1}{5}$ | $\frac{1}{10}$ | $\frac{1}{10}$ |

→ difference  
second value - first value

rough column.

$$f(1) = f(1) - F(0) = \frac{3}{5} - \frac{1}{2} = \frac{6-5}{10} = \frac{1}{10}$$

$$f(2) = F(2) - F(1) = \frac{4}{5} - \frac{3}{5} = \frac{4-3}{5} = \frac{1}{5}$$

$$f(3) = F(3) - F(2) = \frac{9}{10} - \frac{4}{5} = \frac{9-8}{10} = \frac{1}{10}$$

$$f(4) = F(4) - F(3) = 1 - \frac{9}{10} = \frac{10-9}{10} = \frac{1}{10}$$

$$(ii) P(X < 3) = f(0) + f(1) + f(2)$$

$$= \frac{1}{2} + \frac{1}{10} + \frac{1}{5}$$

$$= \frac{5+1+2}{10}$$

$$= \frac{8}{10}$$

$$P(X < 3) = \frac{4}{5}$$

$$(iii) P(X \geq 2) = f(2) + f(3) + f(4)$$

$$= \frac{1}{5} + \frac{1}{10} + \frac{1}{10}$$

$$= \frac{2+1+1}{10}$$

$$= \frac{4}{10}$$

$$P(X \geq 2) = \frac{2}{5}$$

Padasalai.Net

Continuous Random Variable:-

Let  $S$  be a sample space and let a random variable  $X: S \rightarrow \mathbb{R}$  that takes on any value in a set  $I$  of  $\mathbb{R}$ . Then  $X$  is called a Continuous Random Variable.

$$\therefore P(X=x) = 0 \quad \forall x \in I.$$

Probability density function:

A non-negative real valued function  $f(x)$  is said to be probability density function, if for each possible outcome  $x$ ,

$x \in [a, b]$  of a continuous random variable  $X$  having the

$$P(a \leq X \leq b) = \int_a^b f(x) dx$$

$\xrightarrow{\text{Integrand}}$   $\xrightarrow{\text{Integrator}}$   $\xrightarrow{\text{property}}$   
 $\xrightarrow{\text{Symbols}}$   $\xrightarrow{\text{limit}}$

$$P(a \leq X \leq a) = \int_a^a f(x) dx = 0.$$

$$x \in [a, b]$$

Real line

$-a$     $\boxed{x=a}$     $a$

Theorem 11.2

A function  $f(\cdot)$  is a probability density function for some continuous random variable  $X$ , if and only if it satisfies the following properties.

- (i)  $f(x) \geq 0$  for every  $x$   $\rightarrow p(0 \leq x \leq 1)$
- (ii)  $\int_{-\infty}^{\infty} f(x) dx = 1$  (Total probability)

Cumulative Distribution function

The distribution function (or) cumulative distribution function,

$F(x)$  of a continuous random variable value  $x$  with probability density

$f(x)$  is ✓

$$\boxed{F(x)} = P(X \leq x) = \int_{-\infty}^x f(u) du, \quad -\infty < u < \infty$$

$$\int_a^b f(x) dx = \int_a^b f(u) du$$

Remarks:-

- 1) (i) Discrete :  $f(a) = P(X=a)$  is the probability that  $X$  takes the value of  $a$ .
- (ii) Continuous :  $f(a) \neq P(X=a)$  is the not probability that  $X$  takes the value of  $a$ .
- $P(X=a) = 0, \forall a \in R.$

2.

$$P(a < X \leq b) = P(a \leq X < b) = P(a < X \leq b) = P(a \leq X \leq b)$$

PROPERTIES OF DISTRIBUTION FUNCTION:-

- (i)  $0 \leq F(x) \leq 1$ .
- (ii)  $F(x)$  is a real valued non-decreasing.  
If  $x < y$ , then  $F(x) \leq F(y)$ .
- (iii)  $F(x)$  is continuous everywhere.
- (iv)  $\lim_{x \rightarrow -\infty} F(x) = F(-\infty) = 0$   
 $\lim_{x \rightarrow \infty} F(x) = F(\infty) = 1$ .
- (v)  $P(X > x) = 1 - P(X \leq x) = 1 - F(x)$ .
- (vi)  $P(a < X < b) = F(b) - F(a)$ . (between value).

Note: probability density function.

$$f(x) = \begin{cases} 1, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases} \quad (\text{or}) \quad \begin{cases} 1, & 0 \leq x < 1 \\ 0, & \text{otherwise} \end{cases} \quad (\text{or})$$

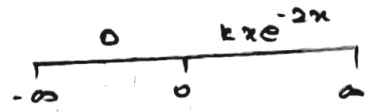
$$\begin{cases} 1, & 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$$



EXE 11.3.

① Total probability  $\int_{-\infty}^{\infty} f(x) dx = 1$ .

$$\int_{-\infty}^0 f(x) dx + \int_0^{\infty} f(x) dx = 1.$$



$$\int_{-\infty}^0 kxe^{2x} dx + \int_0^{\infty} kxe^{-2x} dx = 1$$

$$0 + k \int_0^{\infty} xe^{-2x} dx = 1$$

Gamma integration:-

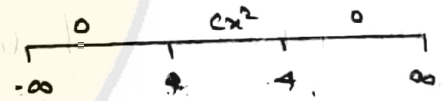
$$\int_0^{\infty} e^{-ax} x^n dx = \frac{n!}{a^{n+1}}$$

$$k \left[ \frac{1!}{(2)^2} \right] = 1$$

$$\boxed{k = 4}$$

Example 11.11

Total probability  $\int_{-\infty}^{\infty} f(x) dx = 1$ .



$$\int_{-\infty}^{-4} f(x) dx + \int_{-4}^4 f(x) dx + \int_4^{\infty} f(x) dx = 1$$

$$0 + \int_{-4}^4 cx^2 dx + 0 = 1$$

$$c \int_{-4}^4 x^2 dx = 1$$

$$c \left[ \frac{x^3}{3} \right]_{-4}^4 = 1$$

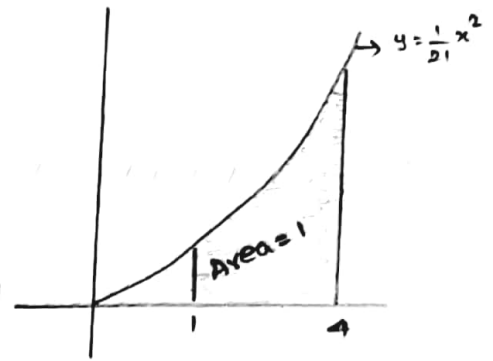
$$c \left[ \frac{64}{3} - \frac{-1}{3} \right] = 1$$

$$c = \frac{3}{63}$$

$$\boxed{c = \frac{1}{21}}$$

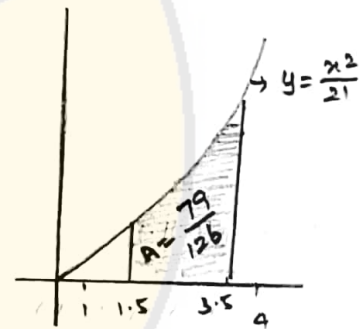


$$\therefore f(x) = \begin{cases} \frac{1}{21} x^2, & 1 < x < 4 \\ 0, & \text{otherwise.} \end{cases}$$

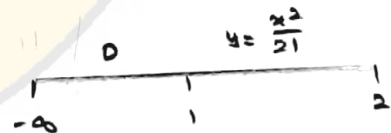


(ii)  $P(1.5 < x < 3.5) = P(1.5 \leq x \leq 3.5)$   
from the remarks.

$$\begin{aligned} \int_{1.5}^{3.5} f(x) dx &= \int_{1.5}^{3.5} \frac{x^2}{21} dx \\ &= \frac{1}{21} \left[ \frac{x^3}{3} \right]_{1.5}^{3.5} \\ &= \frac{1}{21 \times 3} [(3.5)^3 - (1.5)^3] \\ &= \frac{1}{63} [42.875 - 3.375] \\ &= \frac{1}{63} [39.5] \\ \int_{1.5}^{3.5} f(x) dx &= \frac{79}{126} \end{aligned}$$



(iii)  $P(x \leq 2) = \int_{-\infty}^2 f(x) dx$   
 $= \int_{-\infty}^1 f(x) dx + \int_1^2 f(x) dx$

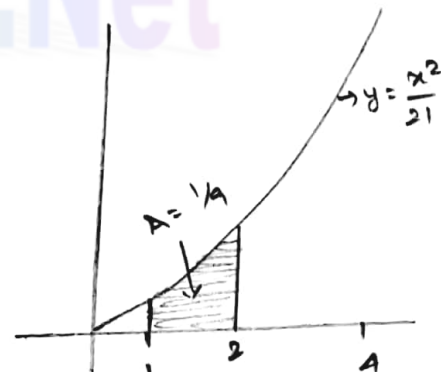


$$\begin{aligned} &= 0 + \int_1^2 \frac{x^2}{21} dx \\ &= \frac{1}{21} \left[ \frac{x^3}{3} \right]_1^2 \end{aligned}$$

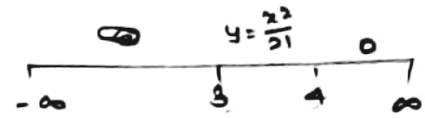
$$= \frac{1}{21} \left[ \frac{8}{3} - \frac{1}{3} \right]$$

$$= \frac{1}{21} \left[ \frac{7}{3} \right]$$

$$P(x \leq 2) = \frac{1}{9}$$



$$\begin{aligned}
 (iii) \quad P(3 < x) &= \int_3^{\infty} f(x) dx \\
 &= \int_3^4 f(x) dx + \int_4^{\infty} f(x) dx \\
 &= \int_3^4 \frac{x^2}{21} dx + 0
 \end{aligned}$$

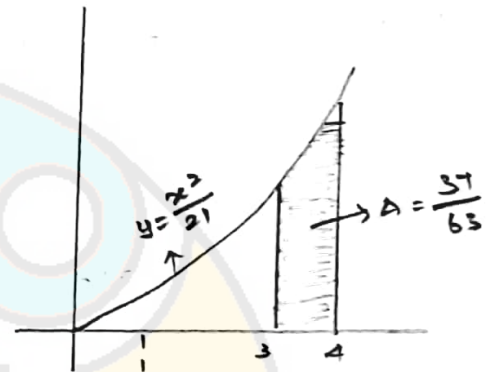


$$= \frac{1}{21} \left[ \frac{x^3}{3} \right]_3^4$$

$$= \frac{1}{21 \times 3} [4^3 - 3^3]$$

$$= \frac{1}{63} [64 - 27]$$

$$P(3 < x) = \frac{37}{63}$$



### Example 11.12

probability density function:  $f(x) = \begin{cases} x-1 & , 1 \leq x < 2 \\ -x+3 & , 2 \leq x < 3 \\ 0 & , \text{otherwise} \end{cases}$

By definition  $F(x) = P(X \leq x) = \int_{-\infty}^x f(u) du$

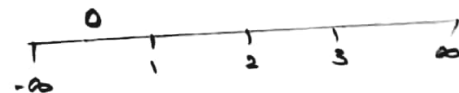
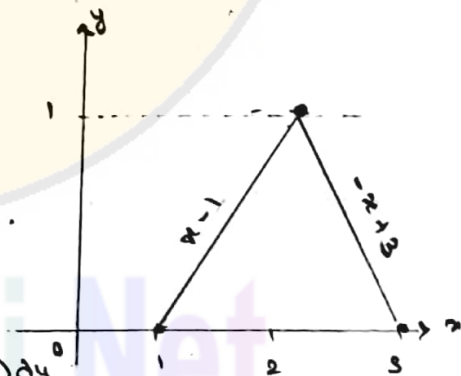
when  $x = 1$   $F(x) = P(X \leq x) = \int_{-\infty}^1 0 du = 0$

when  $1 \leq x < 2$

$$F(x) = P(X \leq x) = \int_{-\infty}^1 0 du + \int_1^x (u-1) du$$

$$= \left[ \frac{(u-1)^2}{2} \right]_1^x$$

$$= \frac{(x-1)^2}{2}$$



when  $2 \leq x < 3$ ,  $F(x) = P(X \leq x) = \int_{-\infty}^1 0 du + \int_1^2 (u-1) du + \int_2^x (3-u) du$ ,

$$= 0 + \left[ \frac{(u-1)^2}{2} \right]_1^2 + \left[ \frac{(3-u)^2}{-2} \right]_2^x$$

$$= 1 - \frac{(3-x)^2}{2}$$

when  $x \geq 3$ ,  $F(x) = P(X \leq x) = \int_{-\infty}^1 0 du + \int_1^2 (u-1) du + \int_2^3 (3-u) du + \int_3^x 0 du$

$$= 0 + \left[ \frac{(u-1)^2}{2} \right]_1^2 + \left[ \frac{(3-u)^2}{-2} \right]_2^3 + 0$$

$$= \left( \frac{1}{2} - 0 \right) + \left( +\frac{1}{2} + 0 \right)$$

$$= 1$$

(i) distribution function  $F(x)$

|        |                   |                     |                         |                     |
|--------|-------------------|---------------------|-------------------------|---------------------|
| $x$    | $-\infty < x < 1$ | $1 \leq x < 2$      | $2 \leq x < 3$          | $3 \leq x < \infty$ |
| $F(x)$ | 0                 | $\frac{(x-1)^2}{2}$ | $1 - \frac{(3-x)^2}{2}$ | 1                   |

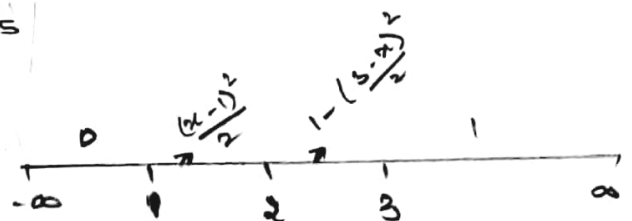
(ii)  $P(1.5 \leq x \leq 2.5) = F(2.5) - F(1.5)$

$$= \left[ 1 - \frac{(3-2.5)^2}{2} \right] - \left[ \frac{(1.5-1)^2}{2} \right]$$

$$= \frac{1.75 - 0.25}{2}$$

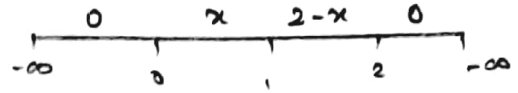
$$= \frac{1.50}{2}$$

$$= 0.75$$



③.

$$\begin{aligned}
 \text{(i)} \quad P(0.2 \leq x \leq 0.6) &= \int_{0.2}^{0.6} f(x) dx \\
 &= \int_{0.2}^{0.6} x dx \\
 &= \left[ \frac{x^2}{2} \right]_{0.2}^{0.6}
 \end{aligned}$$



$$\begin{aligned}
 &= \frac{1}{2} [(0.6)^2 - (0.2)^2] \\
 &= \frac{1}{2} [(0.6 + 0.2)(0.6 - 0.2)] \\
 &= \frac{1}{2} [(0.8)(0.4)] \\
 &= \frac{0.32}{2}
 \end{aligned}$$

$$P(0.2 \leq x \leq 0.6) = 0.16.$$

$$\begin{aligned}
 \text{(ii)} \quad P(1.2 \leq x < 1.8) &= \int_{1.2}^{1.8} f(x) dx \\
 &= \int_{1.2}^{1.8} (2 - x) dx
 \end{aligned}$$

$$\begin{aligned}
 &= \left[ 2x - \frac{x^2}{2} \right]_{1.2}^{1.8} \\
 &= \left[ 2(1.8) - \frac{(1.8)^2}{2} \right] - \left[ 2(1.2) - \frac{(1.2)^2}{2} \right]
 \end{aligned}$$

$$= 3.6 - \frac{3.24}{2} - 2.4 + \frac{1.44}{2}$$

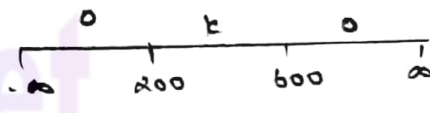
$$= 1.2 - \frac{1.8}{2}$$

$$= 1.2 - 0.9$$

$$P(1.2 \leq x < 1.8) = 0.3.$$

$$\begin{aligned}
 (\text{iii}) \quad P(0.5 \leq x < 1.5) &= \int_{0.5}^1 f(x) dx + \int_1^{1.5} f(x) dx \\
 &= \int_{0.5}^1 x dx + \int_1^{1.5} (2-x) dx \\
 &= \left[ \frac{x^2}{2} \right]_{0.5}^1 + \left[ 2x - \frac{x^2}{2} \right]_1^{1.5} \\
 &= \frac{1}{2} [1 - (0.5)^2] + \left[ \left\{ 2(1.5) - \frac{(1.5)^2}{2} \right\} - \left\{ 2(1) - \frac{1^2}{2} \right\} \right] \\
 &= \frac{1}{2} [0.75] + 3 - \frac{2.25}{2} - 2 + \frac{1}{2} \\
 &= 1 + \frac{1}{2} + \frac{1}{2} \left[ \frac{0.75 - 2.25}{2} \right] \\
 &= \frac{3}{2} + \frac{(-1.50)}{2} \\
 &= \frac{3 - 1.50}{2} = \frac{1.50}{2} \\
 P(0.5 \leq x < 1.5) &= 0.75
 \end{aligned}$$

③. Total probability  $\int_{-\infty}^{\infty} f(x) dx = 1$ .



$$\int_{-\infty}^{200} f(x) dx + \int_{200}^{600} f(x) dx + \int_{600}^{\infty} f(x) dx = 1$$

$$0 + \int_{200}^{600} k dx + 0 = 1$$

$$k [x]_{200}^{600} = 1$$

$$k (600 - 200) = 1$$

$k = \frac{1}{400}$

$$f(x) = \begin{cases} \frac{1}{400}, & 200 \leq x \leq 600 \\ 0, & \text{otherwise.} \end{cases}$$

(i) when  $-\infty < x < 200$

$$F(x) = P(X \leq x) = \int_{-\infty}^x f(u) du = \int_{-\infty}^x 0 du = 0.$$

(ii) when  $200 \leq x \leq 600$

$$\begin{aligned} F(x) = P(X \leq x) &= \int_{-\infty}^{200} f(u) du + \int_{200}^x f(u) du \\ &= 0 + \int_{200}^x \frac{1}{400} du \\ &= \frac{1}{400} [u]_{200}^x \\ &= \frac{1}{400} (x - 200) \end{aligned}$$

(iii) when  $600 \leq x < \infty$

$$F(x) = P(X \leq x) = \int_{-\infty}^{200} f(u) du + \int_{200}^{600} f(u) du + \int_{600}^{\infty} f(u) du$$

$$= 0 + \int_{200}^{600} \frac{1}{400} du + 0$$

$$= \frac{1}{400} [u]_{200}^{600}$$

$$= \frac{1}{400} [600 - 200]$$

$$= \frac{400}{400}$$

$$= 1.$$



$$F(x) = \begin{cases} 0 & , -\infty < x < 200 \\ \frac{1}{400}(x-200) & , 200 \leq x \leq 600 \\ 1 & , 600 \leq x < \infty \end{cases}$$

(iii) The probability fall between 300 litres and 500 litres,

$$F(500) - F(300) = \frac{1}{400}(500-200) - \frac{1}{400}(300-200)$$

$$= \frac{1}{400}(300-100)$$

$$= \frac{200}{400}$$

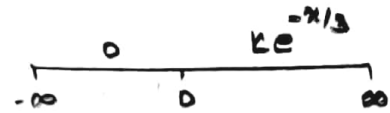
$$= \frac{1}{2}$$

$$F(500) - F(300) = 0.5$$

Padasalai.Net

4.

Total probability  $\int_{-\infty}^{\infty} f(x) dx = 1.$



$$(i) \int_{-\infty}^0 0 dx + \int_0^{\infty} k e^{-x/3} dx = 1$$

$$k \left[ \frac{e^{-x/3}}{-1/3} \right]_0^{\infty} = 1.$$

$$-3k \left[ e^{-x/3} \right]_0^{\infty} = 1.$$

$$-3k [e^{\infty} - e^0] = 1.$$

$$-3k [0 - 1] = 1$$

$$3k = 1$$

$$k = 1/3$$

$$\therefore f(x) = \begin{cases} \frac{e^{-x/3}}{3}, & \text{for } x > 0 \\ 0, & \text{for } x \leq 0 \end{cases}$$

(ii) The distribution function.

$$F(x) = P(X \leq x) = \int_{-\infty}^x f(u) du = 0.$$

when  $x \leq 0$

$$F(x) = P(X \leq x) = \int_{-\infty}^x f(u) du = \int_{-\infty}^x 0 du = 0.$$

when  $x > 0$

$$F(x) = P(X \leq x) = \int_{-\infty}^0 f(u) du + \int_0^x f(u) du$$

$$= 0 + \int_0^x \frac{e^{-u/3}}{3} du$$

$$= \frac{1}{3} \left[ \frac{e^{-u/3}}{-1/3} \right]_0^x$$

$$= - \left[ e^{-x/3} - e^0 \right] = 1 - e^{-x/3}$$

$$F(x) = \begin{cases} 0 & , -\infty < x < 0 \\ 1 - e^{-x/3} & , 0 \leq x < \infty \end{cases}$$

$$\begin{aligned} \text{iii) } P(x < 3) &= F(3) \\ &= 1 - e^{-3/3} \\ &= 1 - e^{-1} \end{aligned}$$

$$\begin{aligned} \text{iv) } P(5 \leq x) &= 1 - P(x < 5) \\ &= 1 - [1 - e^{-5/3}] \\ &= e^{-5/3} \end{aligned}$$

$$\begin{aligned} \text{v) } P(x \leq 4) &= F(4) \\ &= 1 - e^{-4/3} \end{aligned}$$

5.

i) The distribution function:-

$$F(x) = P(X \leq x) = \int_{-\infty}^x f(u) du$$

when  $-\infty < x < -1$ 

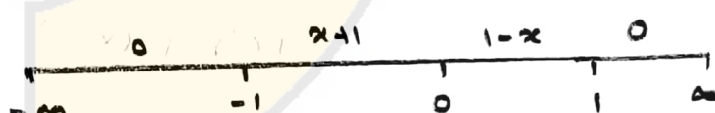
$$F(x) = P(X \leq x) = \int_{-\infty}^x f(u) du = \int_{-\infty}^x 0 du = 0$$

when  $-1 \leq x < 0$ .

$$F(x) = P(X \leq x) = \int_{-\infty}^{-1} f(u) du + \int_{-1}^x f(u) du$$

$$= 0 + \int_{-1}^x (u+1) du$$

$$= \left[ \frac{(u+1)^2}{2} + u \right]_{-1}^x$$



$$= \left[ \frac{x^2}{2} + x \right] - \left( \frac{1}{2} - 1 \right)$$

$$= \frac{x^2}{2} + x + \frac{1}{2},$$

when  $0 \leq x < 1$

$$F(x) = P(X \leq x) = \int_{-\infty}^x f(u) du = \int_{-\infty}^{-1} f(u) du + \int_{-1}^0 f(u) du + \int_0^x f(u) du$$

$$= 0 + \int_{-1}^0 (u+1) du + \int_0^x (1-u) du$$

$$= \left[ \frac{u^2}{2} + u \right]_{-1}^0 + \left[ u - \frac{u^2}{2} \right]_0^x$$

$$= \left[ 0 - \left( \frac{1}{2} - 1 \right) \right] + \left[ \left( x - \frac{x^2}{2} \right) - 0 \right]$$

$$= \frac{1}{2} + x - \frac{x^2}{2},$$

when  $1 \leq x < \infty$

$$F(x) = P(X \leq x) = \int_{-\infty}^x f(u) du = \int_{-\infty}^{-1} f(u) du + \int_{-1}^0 f(u) du + \int_0^1 f(u) du + \int_1^x f(u) du$$

$$= 0 + \int_{-1}^0 (u+1) du + \int_0^1 (1-u) du + 0$$

$$= \left[ \frac{u^2}{2} + u \right]_{-1}^0 + \left[ u - \frac{u^2}{2} \right]_0^1$$

$$= \left[ 0 - \left( \frac{1}{2} - 1 \right) \right] + \left[ \left( 1 - \frac{1}{2} \right) - \left( 0 - 0 \right) \right]$$

$$= \frac{1}{2} + \left[ \frac{1}{2} + 0 \right]$$

$$= \frac{1}{2} + \frac{1}{2}$$

$$= \frac{2}{2} = 1$$

$$F(x) = \begin{cases} 0 & , -\infty < x < -1 \\ \frac{x^2}{2} + x + \frac{1}{2} & , -1 \leq x < 0 \\ -\frac{x^2}{2} + x + \frac{1}{2} & , 0 \leq x < 1 \\ 1 & , 1 \leq x < \infty \end{cases}$$

$$(ii) P(-0.5 \leq x \leq 0.5) = F(0.5) - F(-0.5)$$

$$= \left[ -\frac{(0.5)^2}{2} + 0.5 + \frac{1}{2} \right] - \left[ \frac{(0.5)^2}{2} - 0.5 + \frac{1}{2} \right]$$

$$= -\frac{0.25}{2} + 0.5 + \frac{1}{2} - \left[ \frac{0.25}{2} - 0.5 + \frac{1}{2} \right]$$

$$= -\frac{0.5}{2} + 1$$

$$= -0.25 + 1$$

$$P(-0.5 \leq x \leq 0.5) = 0.75$$

6. Cumulative distribution function.

$$F(x) = \begin{cases} 0 & , x < 0 \\ \frac{1}{2}(x^2 + x) & , 0 \leq x < 1 \\ 1 & , x \geq 1 \end{cases}$$

$$f(x) = F'(x) = \begin{cases} 0 & , x < 0 \\ \frac{1}{2}(2x+1) & , 0 \leq x < 1 \\ 0 & , \text{otherwise.} \end{cases}$$

$$ii) F(0.6) - F(0.3) = P(0.3 \leq x \leq 0.6).$$

$$P(0.3 \leq x \leq 0.6) = F(0.6) - F(0.3)$$

$$= \frac{1}{2} [(0.6)^2 + 0.6] - \frac{1}{2} [(0.3)^2 + 0.3]$$

$$= \frac{1}{2} [0.36 + 0.6 - 0.09 - 0.3]$$

$$= \frac{1}{2} [0.06 + 0.6 - 0.09]$$

$$= \frac{1}{2} [0.6 - 0.03]$$

$$= \frac{1}{2} [0.57]$$

$$P(0.3 \leq x \leq 0.6) = 0.285.$$

Example 11.13

$$F(x) = \begin{cases} 0, & x < 0 \\ x, & 0 \leq x < 1 \\ 1, & 1 \leq x. \end{cases}$$

$$(i) f(x) = F'(x) = \begin{cases} 0, & x < 0 \\ 1, & 0 \leq x < 1 \\ 0, & 1 \leq x \end{cases}$$

$$f(x) = \begin{cases} 1, & 0 \leq x < 1 \\ 0, & \text{otherwise} \end{cases}$$

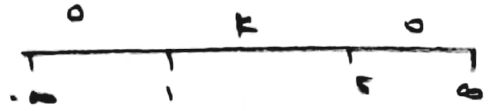
$$ii) P(0.2 \leq x < 0.7) = F(0.7) - F(0.2)$$

$$= 0.7 - 0.2$$

$$= 0.5$$

Example 11.14

$$f(x) = \begin{cases} k & , 1 \leq x < 5 \\ 0 & , \text{otherwise} \end{cases}$$



Total probability  $\int_{-\infty}^{\infty} f(x) dx = 1$

$$\int_{-\infty}^1 f(x) dx + \int_1^5 f(x) dx + \int_5^{\infty} f(x) dx = 1$$

$$\int_{-\infty}^1 0 dx + \int_1^5 k dx + \int_5^{\infty} 0 dx = 1$$

$$k [x]_1^5 = 1$$

$$k [5-1] = 1$$

$$k(4) = 1$$

$$\boxed{k = \frac{1}{4}}$$

$$(i) \therefore f(x) = \begin{cases} 1/4 & , 1 \leq x < 5 \\ 0 & , \text{otherwise} \end{cases}$$

(i) probability distribution function.

$$F(x) = P(X \leq x) = \int_{-\infty}^x f(u) du$$

when  $x \leq 1$ ,

$$F(x) = P(X \leq x) = \int_{-\infty}^x 0 dx = 0$$

when  $1 \leq x < 5$

$$F(x) = P(X \leq x) = \int_{-\infty}^1 0 du + \int_1^x f(u) du$$

$$= 0 + \int_1^x 1/4 du$$

$$= \frac{1}{4} [u]_1^x$$

$$= \frac{1}{4} [x-1]$$



when  $5 \leq x$

$$\begin{aligned}
 F(x) = P(X \leq x) &= \int_{-\infty}^1 f(u) du + \int_1^5 f(u) du + \int_5^{\infty} f(u) du \\
 &= 0 + \int_1^5 \frac{1}{4} du + 0 \\
 &= \frac{1}{4} [u]_1^5 \\
 &= \frac{1}{4} [5-1] \\
 &= \frac{1}{4} (4) \\
 &= 1.
 \end{aligned}$$

$$F(x) = \begin{cases} 0, & x < 1 \\ \frac{1}{4}(x-1), & 1 \leq x < 5 \\ 1, & 5 \leq x \end{cases}$$

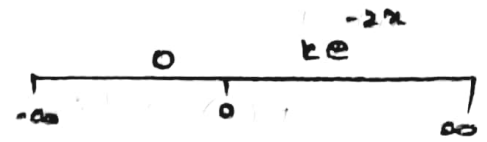
$$\begin{aligned}
 \text{(ii)} \quad P(X < 3) &= P(X \leq 3) = F(3) \\
 &= \frac{1}{4}(3-1) \\
 &= \frac{1}{4}(2) \\
 &= \frac{1}{2}.
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad P(2 < X < 4) &= F(4) - F(2) \\
 &= \frac{1}{4}(4-1) - \frac{1}{4}(2-1) \\
 &= \frac{1}{4}[3-1] \\
 &= \frac{1}{4}(2) \\
 &= \frac{1}{2}.
 \end{aligned}$$

$$\begin{aligned}
 \text{(iv)} \quad P(3 \leq x) &= 1 - P(X < 3) \\
 &= 1 - \frac{1}{2} \\
 &= \frac{1}{2}.
 \end{aligned}$$

Example 11.15  $f(x) = \begin{cases} k e^{-2x}, & x > 0 \\ 0, & x \leq 0 \end{cases}$

Total probability  $\int_{-\infty}^{\infty} f(x) dx = 1$



$$\int_{-\infty}^0 f(x) dx + \int_0^{\infty} f(x) dx = 1$$

$$\int_{-\infty}^0 0 dx + \int_0^{\infty} k e^{-2x} dx = 1$$

$$k \left[ \frac{e^{-2x}}{-2} \right]_0^{\infty} = 1$$

$$\frac{k}{-2} [e^{-\infty} - e^0] = 1$$

$$-\frac{k}{2} [0 - 1] = 1$$

$$\frac{k}{2} = 1$$

$$\boxed{k = 2}$$

(i)  $f(x) = \begin{cases} 2 e^{-2x}, & x > 0 \\ 0, & x \leq 0 \end{cases}$

(ii) the distribution function:-

$$F(x) = P(X \leq x) = \int_{-\infty}^x f(u) du.$$

when  $x \leq 0$ ,

$$F(x) = P(X \leq x) = \int_{-\infty}^x 0 du = 0.$$

when  $x > 0$

$$F(x) = P(X \leq x) = \int_{-\infty}^0 f(u) du + \int_0^x f(u) du.$$

$$\begin{aligned}
 &= 0 + \int_0^{\infty} 2e^{-2x} dx \\
 &= 2 \left[ \frac{e^{-2x}}{-2} \right]_0^{\infty} \\
 &= -[e^{-2x} - e^0] \\
 &= 1 - e^{-2x}
 \end{aligned}$$

$$(iii) F(x) = \begin{cases} 0 & , x \leq 0 \\ 1 - e^{-2x} & , x > 0 \end{cases}$$

$$\begin{aligned}
 (ii) P(x < 2) &= P(x \leq 2) = F(2) \\
 &= 1 - e^{-2(2)} \\
 &= 1 - e^{-4}
 \end{aligned}$$

The probability  $x$  is at least four unit of time.

$$\begin{aligned}
 (iv) P(x \geq 4) &= 1 - P(x < 4) \\
 &= 1 - (1 - e^{-2(4)}) \\
 &= 1 - 1 + e^{-8} \\
 &= e^{-8}
 \end{aligned}$$

$$v) P(x=3) = 0, \text{ for example } P(x=a) = 0 \text{ condition.}$$

$$P(a \leq x \leq b) = \int_a^b f(x) dx = F(b) - F(a)$$

$$P(x=a) = \int_a^a f(x) dx = F(a) - F(a) = 0.$$

## MATHEMATICAL EXPECTATION.

Average :  $\sum (\text{value of } x) \times \text{probability of } x$ .

Mean.

Suppose  $x$  is a random variable with probability mass (or) density function  $f(x)$ . The expected value or mean or mathematical expectation of  $x$ , denoted by  $E(x)$  or  $M$ .

$$E(x) = \begin{cases} \sum x f(x) & , x \text{ is discrete} \\ \int_{-\infty}^{\infty} x f(x) dx & , x \text{ is continuous,} \end{cases}$$

Theorem:- 11.3

Suppose  $x$  is a random variable with probability mass (or) density function  $f(x)$ . The expected value of the function  $g(x)$ , a new random variable is,

$$E(g(x)) = \begin{cases} \sum g(x) f(x) & , g(x) \text{ is discrete} \\ \int_{-\infty}^{\infty} g(x) f(x) dx & , g(x) \text{ is continuous,} \end{cases}$$

$\therefore g(x) = x^k$ ,  $k$ -th moment

The origin of the random variable  $x$ ,

$$E(x^k) = \begin{cases} \sum x^k f(x) & , x \text{ is discrete.} \\ \int_{-\infty}^{\infty} x^k f(x) dx & , x \text{ is continuous.} \end{cases}$$

when  $k = 0$ .

$$E(1) = E(x^0) = \begin{cases} \sum f(x) = 1 & , x \text{ is discrete.} \\ \int_{-\infty}^{\infty} f(x) dx = 1 & , x \text{ is continuous.} \end{cases}$$

VARIANCE :

The variance of a random variable  $x$  denoted by  $\text{Var}(x)$  or  $V(x)$  (or)  $\sigma^2$  (or)  $\sigma_x^2$  is.

$$V(x) = E(x - E(x))^2 = E(x - M)^2.$$

properties of mathematical expectation and variance.

(i)  $E(ax+b) = a E(x) + b$ , where  $a$  and  $b$  are constant.

proof let  $x$  be a discrete random variable.

$$\begin{aligned} E(ax+b) &= \sum_{i=1}^n (ax_i + b) f(x_i) \\ &= \sum_{i=1}^n [ax_i f(x_i) + b f(x_i)] \\ &= a \sum_{i=1}^n x_i f(x_i) + b \sum_{i=1}^n f(x_i) \\ &= a E(x) + b(1) \quad \text{① } \sum f(x_i) = 1. \\ E(ax+b) &= a E(x) + b. \end{aligned}$$

Corollary 1 : when  $b=0$ ,

$$E(ax+0) = a E(x) + 0 \Rightarrow E(ax) = a E(x).$$

Corollary 2 :  $E(b) = b$ , when  $a=0$ .

$$(ii) \text{Var}(x) = E(x^2) - E(x)^2$$

proof :  $\therefore E(x) = M.$

$$\text{Var}(x) = E(x - M)^2$$

$$= E(x^2 - 2xM + M^2)$$

$$= E(x^2) - 2ME(x) + E(M^2)$$

$\therefore M$  is constant.

$$= E(x^2) - 2M(M) + M^2$$

$$= E(x^2) - 2M^2 + M^2$$

$$= E(x^2) - M^2.$$

$$\text{Var}(x) = E(x^2) - [E(x)]^2.$$

(iii)  $\text{Var}(ax + b) = a^2 \text{Var}(x)$ , where  $a$  and  $b$  are constant.

$$\text{Var}(x) = E(x - M)^2 \quad \therefore E(x) = M.$$

$$\text{Var}(ax + b) = E[(ax + b) - E(ax + b)]^2$$

$$= E[(ax + b) - aE(x) - b]^2$$

$$= E[ax + b - aE(x) - b]^2$$

$$= E[ax - aE(x)]^2$$

$$= a^2 E[x - E(x)]^2$$

$$\text{Var}(ax + b) = a^2 \text{Var}(x)$$

Corollary : when  $b=0 \Rightarrow \text{Var}(x) = a^2 \text{Var}(x)$

when  $a=0 \Rightarrow \text{Var}(b) = 0$

Example 11.16.

|        |       |        |        |        |     |      |
|--------|-------|--------|--------|--------|-----|------|
| $x$    | 1     | 2      | 3      | 4      | 5   | 6    |
| $f(x)$ | $c^2$ | $2c^2$ | $3c^2$ | $4c^2$ | $c$ | $2c$ |

(i) the value of  $c$ .The probability mass function,  $\sum f(x) = 1$ .

$$f(1) + f(2) + f(3) + f(4) + f(5) + f(6) = 1$$

$$c^2 + 2c^2 + 3c^2 + 4c^2 + c + 2c = 1$$

$$10c^2 + 3c - 1 = 0$$

$$10c^2 + 5c - 2c - 1 = 0$$

$$10c^2 + 5c - 2c - 1 = 0$$

$$5c(2c+1) - 1(2c+1) = 0$$

$$(5c-1)(2c+1) = 0$$

$$5c-1=0 \quad 2c+1=0$$

$$c = \frac{1}{5} \quad c = -\frac{1}{2} \text{ (not possible)}$$

The probability mass function.

|        |                |                |                |                |               |               |
|--------|----------------|----------------|----------------|----------------|---------------|---------------|
| $x$    | 1              | 2              | 3              | 4              | 5             | 6             |
| $f(x)$ | $\frac{1}{25}$ | $\frac{2}{25}$ | $\frac{3}{25}$ | $\frac{4}{25}$ | $\frac{1}{5}$ | $\frac{2}{5}$ |

(ii) mean and variance.

$$\text{Mean} \Rightarrow E(x) = \sum x f(x)$$

$$E(x^2) = \sum x^2 f(x)$$

$$\text{variance} \Rightarrow V(x) = E(x^2) - [E(x)]^2$$



| $x$ | $f(x)$            | $x f(x)$                                   | $x^2 f(x)$                                     |
|-----|-------------------|--------------------------------------------|------------------------------------------------|
| 1   | $\frac{1}{25}$    | $\frac{1}{25}$                             | $\frac{1}{25}$                                 |
| 2   | $\frac{2}{25}$    | $\frac{4}{25}$                             | $\frac{8}{25}$                                 |
| 3   | $\frac{3}{25}$    | $\frac{9}{25}$                             | $\frac{27}{25}$                                |
| 4   | $\frac{4}{25}$    | $\frac{16}{25}$                            | $\frac{64}{25}$                                |
| 5   | $\frac{1}{5}$     | $\frac{5}{5}$                              | $\frac{25}{5}$                                 |
| 6   | $\frac{2}{5}$     | $\frac{12}{5}$                             | $\frac{72}{5}$                                 |
| -   | $\Sigma f(x) = 1$ | $\Sigma x f(x) = \frac{115}{25}$<br>$E(x)$ | $\Sigma x^2 f(x) = \frac{585}{25}$<br>$E(x^2)$ |

Mean :  $E(x) = \Sigma x f(x) = \frac{115}{25} = 4.6$

Variance :  $V(x) = E(x^2) - [E(x)]^2$

$$= \frac{585}{25} - \left(\frac{115}{25}\right)^2$$

$$= \frac{585}{25} - \frac{13225}{625}$$

$$= \frac{14625 - 13225}{625}$$

$$= \frac{1400}{625}$$

$$= 2.24$$

$\therefore$  The mean and variance are 4.6 and 2.24 respectively.

Example 11.17

| White | Black | Total |
|-------|-------|-------|
| 8     | 4     | 12    |

$$x(BB) = 2(20) = 40$$

$$x(WB) = 20 - 10 = 10$$

$$x(WW) = -10 - 10 = -20$$

Total no. of balls is 12.

$$n(S) = {}^{12}C_2 = \frac{12 \times 11}{1 \times 2} = 66$$

$$x(40) = {}^8C_0 \times {}^4C_2 = 1 \times \frac{4 \times 3}{1 \times 2} = 6$$

$$x(10) = {}^8C_1 \times {}^4C_1 = 8 \times 4 = 32$$

$$x(-20) = {}^8C_2 \times {}^4C_0 = \frac{8 \times 7}{1 \times 2} \times 1 = 28$$

| Random variable x               | 40 | 10 | -20 | Total |
|---------------------------------|----|----|-----|-------|
| No. of element in Inverse image | 6  | 32 | 28  | 66    |

probability mass function

| x    | 40             | 10              | -20             |
|------|----------------|-----------------|-----------------|
| f(x) | $\frac{6}{66}$ | $\frac{32}{66}$ | $\frac{28}{66}$ |

$$E(x) = \sum x f(x) = \left(40 \times \frac{6}{66}\right) + \left(10 \times \frac{32}{66}\right) + \left(-20 \times \frac{28}{66}\right)$$

$$E(x) = 0$$

$$E(x)^2 = \sum x^2 f(x) = \left(40^2 \times \frac{6}{66}\right) + \left(10^2 \times \frac{32}{66}\right) + \left((-20)^2 \times \frac{28}{66}\right)$$

$$= \frac{4000}{11}$$

$$\text{Variance} = E(x^2) - [E(x)]^2 = \frac{4000}{11} - 0 = \frac{4000}{11}$$

$$E(x) = 0, \quad \text{Var}(x) = \frac{4000}{11}$$

## EXERCISE 11.4.

1.

|    |        |        |        |        |        |        |        |
|----|--------|--------|--------|--------|--------|--------|--------|
| i) | $x$    | 0      | 1      | 2      | 3      | 4      | 5      |
|    | $f(x)$ | $2/10$ | $2/10$ | $1/10$ | $2/10$ | $2/10$ | $1/10$ |

| $x$             | $f(x)$ | $E(x) = \sum x f(x)$         | $E(x^2) = \sum x^2 f(x)$       |
|-----------------|--------|------------------------------|--------------------------------|
| 0               | $2/10$ | 0                            | 0                              |
| 1               | $2/10$ | $2/10$                       | $2/10$                         |
| 2               | $1/10$ | $2/10$                       | $4/10$                         |
| 3               | $2/10$ | $6/10$                       | $18/10$                        |
| 4               | $2/10$ | $8/10$                       | $32/10$                        |
| 5               | $1/10$ | $5/10$                       | $25/10$                        |
| $\sum f(x) = 1$ |        | $E(x) = \frac{23}{10} = 2.3$ | $E(x^2) = \frac{81}{10} = 8.1$ |

$$\text{Mean} = E(x) = M = 2.3$$

$$\text{Variance} = E(x^2) - [E(x)]^2$$

$$= 8.1 - (2.3)^2$$

$$= 8.1 - 5.29$$

$$= 2.81$$

$$ii) f(x) = \frac{4-x}{6}, \quad x = 1, 2, 3$$

|        |       |       |       |
|--------|-------|-------|-------|
| $x$    | 1     | 2     | 3     |
| $f(x)$ | $3/6$ | $2/6$ | $1/6$ |

| $x$ | $f(x)$          | $E(x) = \sum x f(x)$         | $E(x^2) = \sum x^2 f(x)$       |
|-----|-----------------|------------------------------|--------------------------------|
| 1   | $3/6$           | $3/6$                        | $\frac{3}{6}$                  |
| 2   | $2/6$           | $4/6$                        | $\frac{8}{6}$                  |
| 3   | $1/6$           | $3/6$                        | $\frac{9}{6}$                  |
|     | $\sum f(x) = 1$ | $E(x) = \frac{10}{6} = 1.67$ | $E(x^2) = \frac{20}{6} = 3.33$ |

$$\text{Mean} = E(x) = \mu = 1.67$$

$$\text{Variance} = E(x^2) - [E(x)]^2$$

$$= \frac{10}{3} - \left(\frac{5}{3}\right)^2$$

$$= \frac{10}{3} - \frac{25}{9}$$

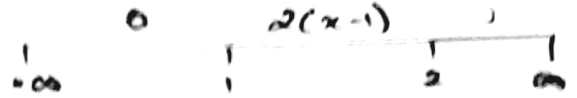
$$= \frac{30-25}{9}$$

$$= \frac{5}{9}$$

$$= 0.56$$

11)

$$E(x) = \int_{-\infty}^{\infty} x f(x) dx$$



$$E(x) = \int_{-\infty}^1 x f(x) dx + \int_1^2 x f(x) dx + \int_2^{\infty} x f(x) dx$$

$$= 0 + \int_1^2 x [2(x-1)] dx + 0$$

$$= 2 \int_1^2 (x^2 - x) dx$$

$$= 2 \left[ \frac{x^3}{3} - \frac{x^2}{2} \right]_1^2$$

$$= 2 \left[ \left( \frac{8}{3} - \frac{4}{2} \right) - \left( \frac{1}{3} - \frac{1}{2} \right) \right]$$

$$= 2 \left[ \frac{7}{3} - \frac{3}{2} \right]$$

$$E(x) = \frac{5}{3}$$

$$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x) dx$$

$$= \int_{-\infty}^1 x^2 f(x) dx + \int_1^2 x^2 f(x) dx + \int_2^{\infty} x^2 f(x) dx$$

$$= 0 + \int_1^2 x^2 [2(x-1)] dx + 0$$

$$= 2 \int_1^2 (x^3 - x^2) dx$$

$$= 2 \left[ \frac{x^4}{4} - \frac{x^3}{3} \right]_1^2$$

$$= 2 \left[ \left( \frac{16}{4} - \frac{8}{3} \right) - \left( \frac{1}{4} - \frac{1}{3} \right) \right]$$

$$= 2 \left[ \frac{15}{4} - \frac{1}{3} \right]$$

$$= 2 \left[ \frac{17}{12} \right]$$

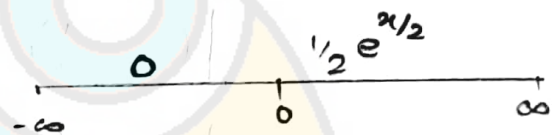
$$E(x^2) = \frac{17}{6}$$

$$\begin{aligned}
 \text{variance} &= E(x^2) - [E(x)]^2 \\
 &= \frac{17}{6} - \left(\frac{5}{3}\right)^2 \\
 &= \frac{17}{6} - \frac{25}{9} \\
 &= \frac{51-50}{18}
 \end{aligned}$$

$$\text{variance} = \frac{1}{18}$$

(iv)

$$\begin{aligned}
 E(x) &= \int_{-\infty}^{\infty} x f(x) dx \\
 &= \int_{-\infty}^0 x f(x) dx + \int_0^{\infty} x f(x) dx \\
 &= 0 + \int_0^{\infty} x \left[ \frac{1}{2} e^{-x/2} \right] dx \\
 &= \frac{1}{2} \int_0^{\infty} x e^{-x/2} dx \\
 &= \frac{1}{2} \left[ \frac{1!}{(1/2)^2} \right]
 \end{aligned}$$



Gamma integration.

$$\int_0^{\infty} x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$$

$$E(x) = 2$$

$$\begin{aligned}
 E(x^2) &= \int_{-\infty}^{\infty} x^2 f(x) dx \\
 &= \int_{-\infty}^0 x^2 f(x) dx + \int_0^{\infty} x^2 f(x) dx \\
 &= 0 + \int_0^{\infty} x^2 \left[ \frac{1}{2} e^{-x/2} \right] dx \\
 &= \frac{1}{2} \int_0^{\infty} x^2 e^{-x/2} dx \\
 &= \frac{1}{2} \left[ \frac{2!}{(1/2)^3} \right]
 \end{aligned}$$

$$E(x^2) = 8$$

$$\text{Mean} = E(x) = \mu = 2.$$

$$\text{Variance} = E(x^2) - [E(x)]^2$$

$$= 8 - (2)^2$$

$$= 8 - 4$$

$$\text{Variance} = 4$$

2.

$$P(x=0) = P(BB) = \frac{{}^3C_2}{{}^7C_2} = \frac{1}{7}$$

$$P(x=1) = P(RB) = \frac{{}^4C_1 \times {}^3C_1}{{}^7C_2} = \frac{4}{7}$$

$$P(x=2) = P(RR) = \frac{{}^4C_2}{{}^7C_2} = \frac{2}{7}$$

| Red | Black | Total |
|-----|-------|-------|
| 4   | 3     | 7     |

| x    | 0             | 1             | 2             |
|------|---------------|---------------|---------------|
| f(x) | $\frac{1}{7}$ | $\frac{4}{7}$ | $\frac{2}{7}$ |

| x                 | f(x)          | x f(x)               | x <sup>2</sup> f(x)     |
|-------------------|---------------|----------------------|-------------------------|
| 0                 | $\frac{1}{7}$ | 0                    | 0                       |
| 1                 | $\frac{4}{7}$ | $\frac{4}{7}$        | $\frac{4}{7}$           |
| 2                 | $\frac{2}{7}$ | $\frac{4}{7}$        | $\frac{8}{7}$           |
| $\Sigma f(x) = 1$ |               | $E(x) = \frac{8}{7}$ | $E(x^2) = \frac{12}{7}$ |

$$\text{Mean} : E(x) = \mu = \frac{8}{7}.$$



3.

$$E(x+3) = 10$$

$$E(x) + E(3) = 10$$

$$E(x) + 3 = 10$$

$$E(x) = 10 - 3$$

$$E(x) = 7.$$

$$\text{Mean} = E(x) = \mu = 7.$$

$$E(x+3)^2 = E(x^2 + 2(3)x + 9)$$

$$= E(x^2) + 6E(x) + E(9)$$

$$116 = E(x^2) + 6E(x) + 9$$

$$116 - 9 = E(x^2) + 6(7)$$

$$107 = E(x^2) + 42$$

$$107 - 42 = E(x^2)$$

$$65 = E(x^2)$$

$$\text{variance} = E(x^2) - (E(x))^2$$

$$= 65 - (7)^2$$

$$= 65 - 49$$

$$\text{variance} = \sigma^2 = 16$$

4.

$$S = \{ \text{HHHH, HTHH, THHH, TTHH,} \\ \text{HHTH, HTTH, THTH, TTTH} \\ \text{HHHT, HTHT, THHT, TTHT} \\ \text{HHTT, HTTT, THTT, TTTT} \}$$

$$n(S) = 16.$$

$$P(X=0) = \frac{1}{16}$$

$$P(X=1) = \frac{4}{16}$$

$$P(X=2) = \frac{6}{16}$$

$$P(X=3) = \frac{4}{16}$$

$$P(X=4) = \frac{1}{16}$$

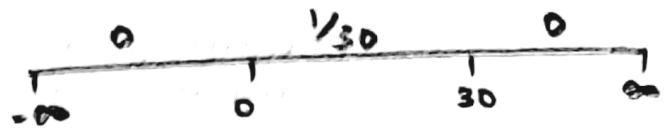
| $x$    | 0              | 1              | 2              | 3              | 4              |
|--------|----------------|----------------|----------------|----------------|----------------|
| $f(x)$ | $\frac{1}{16}$ | $\frac{4}{16}$ | $\frac{6}{16}$ | $\frac{4}{16}$ | $\frac{1}{16}$ |

| $x$ | $f(x)$         | $xf(x)$                    | $x^2f(x)$                    |
|-----|----------------|----------------------------|------------------------------|
| 0   | $\frac{1}{16}$ | 0                          | 0                            |
| 1   | $\frac{4}{16}$ | $\frac{4}{16}$             | $\frac{4}{16}$               |
| 2   | $\frac{6}{16}$ | $\frac{12}{16}$            | $\frac{24}{16}$              |
| 3   | $\frac{4}{16}$ | $\frac{12}{16}$            | $\frac{36}{16}$              |
| 4   | $\frac{1}{16}$ | $\frac{4}{16}$             | $\frac{16}{16}$              |
|     | 1              | $E(X) = \frac{32}{16} = 2$ | $E(X^2) = \frac{80}{16} = 5$ |

$$\begin{aligned} \text{Mean} = E(X) = \mu &= 2 & \text{variance} &= E(X^2) - (E(X))^2 \\ & & &= 5 - (2)^2 \\ & & &= 5 - 4 \\ & & &= 1 \end{aligned}$$

5.

$$E(x) = \int_{-\infty}^{\infty} x f(x) dx$$



$$= \int_{-\infty}^0 x f(x) dx + \int_0^{30} x f(x) dx + \int_{30}^{\infty} x f(x) dx$$

$$= 0 + \int_0^{30} x \left(\frac{1}{30}\right) dx + 0$$

$$= \frac{1}{30} \int_0^{30} x dx$$

$$= \frac{1}{30} \left[ \frac{x^2}{2} \right]_0^{30}$$

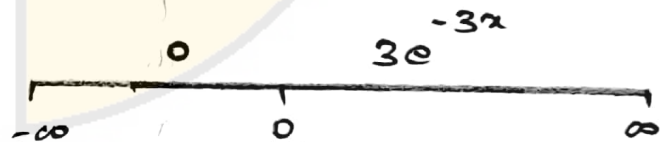
$$= \frac{1}{30} \left[ \frac{(30)^2}{2} - 0 \right]$$

$$= \frac{1}{30} \left[ \frac{30 \times 30}{2} \right]$$

$$E(x) = 15 \text{ min.}$$

6.

$$E(x) = \int_{-\infty}^{\infty} x f(x) dx$$



$$= \int_{-\infty}^0 x f(x) dx + \int_0^{\infty} x f(x) dx$$

$$= 0 + \int_0^{\infty} x [3e^{-3x}] dx$$

$$= 3 \int_0^{\infty} x e^{-3x} dx$$

$$= 3 \left[ \frac{1!}{3^2} \right]$$

$$E(x) = 1/3$$

Gamma integration

$$\int_0^{\infty} x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$$

$$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x) dx$$

$$= \int_{-\infty}^0 x^2 f(x) dx + \int_0^{\infty} x^2 f(x) dx$$

$$= 0 + \int_0^{\infty} x^2 [3e^{-3x}] dx$$

$$= 3 \int_0^{\infty} x^2 e^{-3x} dx$$

$$= 3 \left[ \frac{2!}{(3)^3} \right]$$

$$E(x^2) = \frac{2}{9}$$

$$\text{variance} = E(x^2) - [E(x)]^2$$

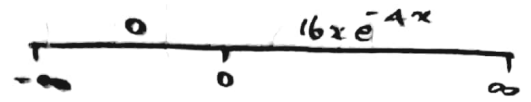
$$= \frac{2}{9} - \left(\frac{1}{3}\right)^2$$

$$= \frac{2}{9} - \frac{1}{9}$$

$$\text{variance} = \frac{1}{9}$$

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$$\begin{aligned}
 7. \quad E(x) &= \int_{-\infty}^{\infty} x f(x) dx \\
 &= \int_{-\infty}^0 x f(x) dx + \int_0^{\infty} x f(x) dx \\
 &= 0 + \int_0^{\infty} x [16x e^{-4x}] dx \\
 &= 16 \int_0^{\infty} x^2 e^{-4x} dx \\
 &= 16 \left[ \frac{2!}{4^3} \right] \\
 &= 16 \left[ \frac{2}{64} \right]
 \end{aligned}$$



Gamma integration,

$$\int_0^{\infty} x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$$

$$E(x) = \frac{1}{2}$$

$$\begin{aligned}
 E(x^2) &= \int_{-\infty}^{\infty} x^2 f(x) dx \\
 &= \int_{-\infty}^0 x^2 f(x) dx + \int_0^{\infty} x^2 f(x) dx \\
 &= 0 + \int_0^{\infty} x^2 [16x e^{-4x}] dx \\
 &= 16 \int_0^{\infty} x^3 e^{-4x} dx \\
 &= 16 \left[ \frac{3!}{(4)^4} \right]
 \end{aligned}$$

$$= 16 \left[ \frac{6}{16 \times 16} \right]$$

$$E(x^2) = \frac{3}{8}$$

$$\text{Mean} = E(x) = \mu = \frac{1}{2}$$

$$\begin{aligned}
 \text{variance} &= E(x^2) - [E(x)]^2 \\
 &= \frac{3}{8} - \left(\frac{1}{2}\right)^2
 \end{aligned}$$

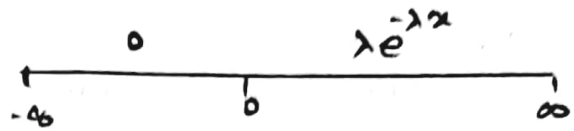
$$= \frac{3}{8} - \frac{1}{4}$$

$$= \frac{3-2}{8}$$

$$\text{variance} = \frac{1}{8}$$

Example 11.18.

$$\begin{aligned}
 E(x) &= \int_{-\infty}^{\infty} x f(x) dx \\
 &= \int_{-\infty}^0 x f(x) dx + \int_0^{\infty} x f(x) dx \\
 &= 0 + \int_0^{\infty} x (\lambda e^{-\lambda x}) dx
 \end{aligned}$$



Gamma integration,

$$\int_0^{\infty} x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$$

$$\begin{aligned}
 &= \lambda \int_0^{\infty} x e^{-\lambda x} dx \\
 &= \lambda \left[ \frac{1!}{\lambda^2} \right]
 \end{aligned}$$

$$E(x) = \frac{1}{\lambda}$$

$$\begin{aligned}
 E(x^2) &= \int_{-\infty}^{\infty} x^2 f(x) dx \\
 &= \int_{-\infty}^0 x^2 f(x) dx + \int_0^{\infty} x^2 (\lambda e^{-\lambda x}) dx \\
 &= \lambda \int_0^{\infty} x^2 e^{-\lambda x} dx \\
 &= \lambda \left[ \frac{2!}{\lambda^3} \right]
 \end{aligned}$$

$$E(x^2) = \frac{2}{\lambda^2}$$

$$\text{Mean} = E(x) = \mu = \frac{1}{\lambda}$$

$$\text{Variance} = E(x^2) - [E(x)]^2$$

$$= \frac{2}{\lambda^2} - \left(\frac{1}{\lambda}\right)^2$$

$$= \frac{2}{\lambda^2} - \frac{1}{\lambda^2}$$

$$\text{Variance} = \frac{1}{\lambda^2}$$

|              |                 |                 |                 |                   |
|--------------|-----------------|-----------------|-----------------|-------------------|
| Prize        | 200             | 100             | 50              | 0                 |
| Ticket cost  | 2               | 2               | 2               | 2                 |
| Winning cost | 198             | 98              | 48              | -2                |
| $f(x)$       | $\frac{1}{600}$ | $\frac{4}{600}$ | $\frac{6}{600}$ | $\frac{589}{600}$ |

$$E(x) = \sum x f(x).$$

$$= \left(198 \times \frac{1}{600}\right) + \left(98 \times \frac{4}{600}\right) + \left(48 \times \frac{6}{600}\right) + \left(-2 \times \frac{589}{600}\right)$$

$$= \frac{198 + 392 + 288 - 1178}{600}$$

$$= \frac{-1178 + 878}{600}$$

$$= \frac{-300}{600}$$

$$= -\frac{1}{2}$$

$$E(x) = -0.5$$

The loss is 0.5

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## Theoretical distribution.

## one point distribution

$$f(x) = P(x=x_0) = 1$$

$$F(x) = \begin{cases} 0 & , -\infty < x < x_0 \\ 1 & , x_0 \leq x < \infty \end{cases}$$

mean  $E(x) = x_0$

variance  $V(x) = 0$

## Two point distribution.

## A) Unsymmetrical case:

$$f(x) = \begin{cases} p & , \text{for } x=x_1 \\ 1-p & , \text{for } x=x_2 \end{cases}$$

where  $0 < p < 1$

$$F(x) = \begin{cases} 0 & , \text{if } x < x_1 \\ p & , \text{if } x_1 \leq x < x_2 \\ 1 & , \text{if } x \geq x_2 \end{cases}$$

$$E(x) = px_1 + qx_2$$

$$V(x) = pq(x_2 - x_1)^2$$

## B) Symmetrical case:

$\therefore p = q = 1/2$  = two point distribution.

$$f(x) = \begin{cases} 1/2 & , \text{for } x=x_1 \\ 1/2 & , \text{for } x=x_2 \end{cases}$$

$$F(x) = \begin{cases} 0 & , \text{if } x < x_1 \\ 1/2 & , \text{if } x_1 \leq x < x_2 \\ 1 & , \text{if } x \geq x_2 \end{cases}$$

Mean:  $E(x) = \frac{x_1 + x_2}{2}$

variance:  $V(x) = \frac{(x_2 - x_1)^2}{4}$

Bernoulli's distribution:-

Let  $x$  be a random variable associated with a Bernoulli trial by defining it as  $x(\text{success}) = 1$  and  $x(\text{failure}) = 0$ ,

$$f(x) = \begin{cases} p & , x = 1 \\ q = 1 - p & , x = 0 \end{cases} \quad \text{where } 0 < p < 1$$

then  $x$  is called a Bernoulli random variable and  $f(x)$  is called Bernoulli distribution.

$$f(x) = p^x (1-p)^{1-x}, \quad x = 0, 1$$

$$F(x) = \begin{cases} 0 & , \text{if } x < 0 \\ q = 1 - p & , \text{if } 0 \leq x < 1 \\ 1 & , \text{if } x \geq 1 \end{cases}$$

Mean :  $E(x) = p$

Variable :  $V(x) = pq$ .

$$\therefore P(x=x) = {}^nC_x p^x (1-p)^{n-x}, \quad x = 0, 1, 2, \dots, n.$$

Binomial random variable:

A discrete random variable  $x$  is called binomial random variable, if  $x$  is the number of successes in  $n$ -repeated trials.

- i) The  $n$ -repeated trials are independent and  $n$  is finite.
- ii) Each trial result only two possible outcomes, labelled as success and failure.
- iii) The probability of a success in each trial, denoted as  $p$ , remains constant.

Binomial distribution,

The binomial random variable  $x$  equals the number of success with probability  $p$  for a success and  $q = 1 - p$  for a failure in independent trials, has binomial distribution denoted by

$$x \sim B(n, p)$$

$$f(x) = {}^nC_x p^x (1-p)^{n-x}, \quad x = 0, 1, 2, \dots, n.$$

Binomial Expansion,  $(a+b)^n = \sum_{x=0}^n {}^nC_x a^x b^{n-x}.$

If  $x$  is a binomial random variable which follows binomial distribution with parameters  $p$  and  $n$ , the mean  $\mu$  and variance  $\sigma^2$  are,

$$\mu = np \quad \text{and} \quad \sigma^2 = np(1-p)$$

EXE 11.5

1.

(i)  $n=6, p=1/3, x=3.$

$$P(X=x) = f(x) = {}^nC_x p^x (1-p)^{n-x}, \quad x = 0, 1, 2, \dots, n.$$

$$P(X=3) = f(3) = {}^6C_3 \left(\frac{1}{3}\right)^3 \left(1 - \frac{1}{3}\right)^{6-3}$$

$$= {}^6C_3 \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^3$$

$$= \frac{20 \times 2^3}{3^6}$$

$$= \frac{160}{729}$$

=

$$ii) \quad n=10, \quad p=\frac{1}{5}, \quad x=4$$

$$P(X=x) = f(x) = {}^n C_x p^x (1-p)^{n-x}, \quad x=0, 1, 2, \dots, n$$

$$P(X=4) = f(4) = {}^{10} C_4 \left(\frac{1}{5}\right)^4 \left(1-\frac{1}{5}\right)^{10-4}$$

$$= {}^{10} C_4 \left(\frac{1}{5}\right)^4 \left(\frac{4}{5}\right)^6$$

$$= \frac{{}^{10} C_4 \times 4^6}{5^{10}}$$

$$= 210 \times \frac{4^6}{5^{10}}$$

$$iii) \quad n=9, \quad p=\frac{1}{2}, \quad x=7$$

$$P(X=x) = f(x) = {}^n C_x p^x (1-p)^{n-x}, \quad x=0, 1, 2, \dots, n$$

$$P(X=7) = f(7) = {}^9 C_7 \left(\frac{1}{2}\right)^7 \left(1-\frac{1}{2}\right)^{9-7}$$

$$= {}^9 C_7 \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right)^2$$

$$= {}^9 C_7 \left(\frac{1}{2}\right)^9$$

$$P(X=7) = 36 \left(\frac{1}{2}\right)^9$$

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## CHAPTER - II PROBABILITY DISTRIBUTION.

Two - Binomial function:

Success failure.  $\therefore$  sum of success and failure = 1.

$p$   $q$

$$p + q = 1.$$

$$q = 1 - p \text{ (or) } p = 1 - q.$$

$x$  - Binomial random variable.

Binomial distribution

$$x \sim B(n, p)$$

$$\binom{n}{x} = nC_x.$$

$$f(x) = \binom{n}{x} p^x q^{n-x}, \quad x = 0, 1, 2, \dots, n.$$

(or)

$$(*) f(x) = nC_x p^x q^{n-x}, \quad x = 0, 1, 2, \dots, n.$$

binomial expansion.

$$(a+b)^n = \sum_{x=0}^n \binom{n}{x} a^x b^{n-x}.$$

$$(a+b)^n = nC_0 a^n b^0 + nC_1 a^{n-1} b^1 + nC_2 a^{n-2} b^2 +$$

Binomial distribution - - - + nC\_n a^0 b^n.

Parameters:

$$* \text{ mean} = \mu = np$$

$$* \text{ variance} = \sigma^2 = npq.$$

$$P(X=k), B(n, p)$$

Binomial distribution

$$P(X=x) = f(x) = {}^nC_x p^x q^{n-x}, \quad x=0, 1, 2, \dots, n.$$

$$P(X=x) = f(x) = {}^nC_x p^x (1-p)^{n-x}, \quad p+q=1,$$

$$q=1-p.$$

$$(i) \quad n=6, p=1/3, k=3.$$

$$\begin{aligned} P(X=3) &= f(3) = {}^6C_3 \left(\frac{1}{3}\right)^3 \left(1-\frac{1}{3}\right)^{6-3} \\ &= {}^6C_3 \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^3 \\ &= 20 \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^3 \\ &= \frac{20 \times 2^3}{3^6} \\ &= \frac{20 \times 8}{729} \end{aligned}$$

$$P(X=3) = f(3) = \frac{160}{729}.$$

$$(ii) \quad n=10, p=1/5, k=4$$

$$P(X=x) = f(x) = {}^nC_x p^x (1-p)^{n-x}, \quad x=0, 1, 2, \dots, n.$$

$$P(X=4) = f(4) = {}^{10}C_4 \left(\frac{1}{5}\right)^4 \left(1-\frac{1}{5}\right)^{10-4}$$

$$\begin{aligned} {}^{10}C_4 &= \frac{10 \times 9 \times 8 \times 7}{1 \times 2 \times 3 \times 4} \\ &= 210 \times \frac{4^6}{5^{10}} \end{aligned}$$

G1)  $n=9, p=1/2, k=7.$

$$P(X=x) = f(x) = {}^nC_x p^x (1-p)^{n-x}, \quad x=0,1,2,\dots,n.$$

$$P(X=7) = f(7) = {}^9C_7 \left(\frac{1}{2}\right)^7 \left(1-\frac{1}{2}\right)^{9-7}$$

$$= {}^9C_7 \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right)^2$$

$${}^nC_x = {}^nC_{n-x}$$

$${}^9C_7 = {}^9C_{9-7}$$

$${}^9C_7 = {}^9C_2 = \frac{9 \times 8}{1 \times 2}$$

$$= {}^9C_7 \left(\frac{1}{2}\right)^9$$

$$= 36 \left(\frac{1}{2}\right)^9$$

$$= \frac{36}{2^9}$$

② Mr. A hits a target  $p = \frac{1}{4}$

$$p+q=1$$

$$q=1-p$$

$$q=1-\frac{1}{4}$$

$$q = \frac{3}{4}$$

(i) exactly 4 time :  $P(X=4)$

$$P(X=x) = f(x) = {}^nC_x p^x (1-p)^{n-x}, \quad x=0,1,\dots,10.$$

$$P(X=4) = f(4) = {}^{10}C_4 \left(\frac{1}{4}\right)^4 \left(1-\frac{1}{4}\right)^{10-4}$$

$$= {}^{10}C_4 \left(\frac{1}{4}\right)^4 \left(\frac{3}{4}\right)^6$$

$$= {}^{10}C_4 \times \frac{3^6}{4^{10}}$$

(ii) atleast one time  $P(X \geq 1)$

$$P(X \geq 1) = 1 - P(X < 1)$$

$$= 1 - f(0)$$

$$= 1 - \left\{ {}^{10}C_0 \left(\frac{1}{4}\right)^0 \left(1-\frac{1}{4}\right)^{10-0} \right\}$$

$${}^nC_0 = 1$$

$${}^{10}C_0 = 1$$

$$= 1 - 1 \left(\frac{3}{4}\right)^{10}$$

+

$$= 1 - \left(\frac{3}{4}\right)^{10}$$





Example 11.19

$$(i) \quad n=5, \quad p=\frac{1}{2}, \quad q=\frac{1}{2}$$

$$p(x=x) = f(x) = {}^n C_x p^x (1-p)^{n-x}, \quad x=0,1,2,3,4,5$$

$$p(x=x) = f(x) = {}^5 C_x \left(\frac{1}{2}\right)^x \left(1-\frac{1}{2}\right)^{5-x}$$

$$= {}^5 C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{5-x}$$

$$= {}^5 C_x \left(\frac{1}{2}\right)^5$$

(ii)

$$n=10, \quad S = \{1, 2, 3, 4, 5, 6\} \quad p = \frac{1}{6}, \quad q = \frac{5}{6}$$

$$p(x=x) = f(x) = {}^n C_x p^x (1-p)^{n-x}$$

$$p(x=x) = f(x) = {}^{10} C_x \left(\frac{1}{6}\right)^x \left(\frac{5}{6}\right)^{10-x}$$

Example 11.20

$$n=10, \quad p = \frac{1}{4}, \quad q = \frac{3}{4}$$

$$p(x=x) = f(x) = {}^n C_x p^x (1-p)^{n-x}, \quad x=1, 2, \dots, 10$$

$$(i) \quad p(x=x) = f(x) = {}^{10} C_x \left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{10-x}$$

(ii) probability for seven correct answer:

$$p(x=7) = f(7) = {}^{10} C_7 \left(\frac{1}{4}\right)^7 \left(\frac{3}{4}\right)^{10-7}$$

$${}^{10} C_7 = {}^{10} C_3 = \frac{10 \times 9 \times 8}{1 \times 2 \times 3} = 120 \times \frac{3^3}{4^{10}}$$

$$(iii) \quad p(x \geq 1) = 1 - p(x < 1)$$

$$= 1 - f(0)$$

$$= 1 - {}^{10} C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^{10-0} = 1 - \left(\frac{3}{4}\right)^{10}$$

② binomial distribution

$$\text{mean} = N = np$$

$$\text{variance} : \sigma^2 = npq$$

(i)

$$n = 100, \quad p = 1/2 \quad q = 1/2$$

$$\text{mean} = np = 100 \times 1/2 = 50$$

$$\text{variance} = npq = 100 \times 1/2 \times 1/2 = 25$$

(ii)

$$n = 240, \quad p = 1/6 \quad q = 5/6$$

$$\text{mean} = np = 240 \times 1/6 = 40$$

$$\text{variance} = npq = 240 \times \frac{1}{6} \times \frac{5}{6} = \frac{200}{6} = \frac{100}{3}$$

③

$$p = \frac{3}{4}, \quad q = 1 - p = 1 - \frac{3}{4}, \quad p(x=3), \quad n=5$$

$$q = \frac{1}{4}$$

$$P(x=x) = f(x) = {}^nC_x p^x q^{n-x}$$

$$P(x=3) = f(3) = {}^5C_3 \left(\frac{3}{4}\right)^3 \left(\frac{1}{4}\right)^{5-3}$$

$$= {}^5C_3 \left(\frac{3}{4}\right)^3 \left(\frac{1}{4}\right)^2$$

$$= {}^5C_3 \times \frac{3^3}{4^5}$$

$$4^5 = (2^2)^5 = 2^{10}$$

$$= \frac{{}^5C_3 \times 3^3}{2^{10}}$$

$$= \frac{10 \times 27}{1024}$$

$$P(x=3) = f(3) = \frac{270}{1024}$$

Example 11.21

$$\text{mean} = np = 2$$

$$\text{variance} = npq = 1.5$$

$$q = \frac{npq}{np} = \frac{1.5}{2} = \frac{3/2}{2} = \frac{3}{4}$$

$$p + q = 1$$

$$p = 1 - q = 1 - 3/4$$

$$\therefore np = 2$$

$$n(1/4) = 2$$

$$\boxed{n = 8}$$

$$, p = 1/4, q = 3/4$$

$$\boxed{p = 1/4}$$

Binomial distribution,

$$P(X=x) = f(x) = {}^n C_x p^x (1-p)^{n-x}, \quad x = 1, 2, \dots, 8$$

$$(i) P(X=0)$$

$$\begin{aligned} P(X=0) &= f(0) = {}^8 C_0 (1/4)^0 (1-1/4)^{8-0} \\ &= {}^8 C_0 (3/4)^8 \\ &= (3/4)^8 \end{aligned}$$

$$\begin{aligned} (ii) P(X=1) &= f(1) = {}^8 C_1 (1/4)^1 (1-1/4)^{8-1} \\ &= {}^8 C_1 (1/4)^1 (3/4)^7 \end{aligned}$$

$$= 8 \times \frac{3^7}{4^8} = 2 \times \left(\frac{3}{4}\right)^7$$

$$\begin{aligned} (iii) P(X \geq 1) &= 1 - P(X < 1) = 1 - f(0) \\ &= 1 - {}^8 C_0 (1/4)^0 (1-1/4)^{8-0} \\ &= 1 - (3/4)^8 \end{aligned}$$

EXE 11.5

$$5) \quad p = 5\% = \frac{5}{100} = 0.05$$

$$p + q = 1$$

$$n = 10$$

$$q = 1 - p$$

$$q = 1 - 0.05$$

$$q = 0.95$$

$$(i) \quad P(X \geq 1) = 1 - P(X = 0)$$

$$= 1 - f(0)$$

$$= 1 - {}^{10}C_0 (0.05)^0 (0.95)^{10-0}$$

Binomial distribution,  
 $P(X = x) = f(x) = {}^nC_x p^x q^{n-x}$

$$P(X \geq 1) = 1 - (0.95)^{10}$$

$$(ii) \quad P(X = 2) = f(2) = {}^{10}C_2 (0.05)^2 (0.95)^{10-2}$$

$$= 45 (0.05)^2 (0.95)^8$$

$$\frac{{}^{10}C_2}{1 \times 2} = \frac{5 \times 4}{1 \times 2} = 10$$

⑥.

$$p = 0.9$$

$$q = 1 - p = 1 - 0.9 = 0.1$$

$$q = 0.1$$

$$n = 12$$

$$(i) \quad P(X = 10) = f(10) = {}^{12}C_{10} (0.9)^{10} (0.1)^{12-10}$$

$$\therefore {}^{12}C_{10} = {}^{12}C_2 = 66$$

$$= 66 \times (0.9)^{10} (0.1)^2$$

$$(ii) \quad P(X \geq 11) = P(X = 11) + P(X = 12)$$

$$= {}^{12}C_{11} (0.9)^{11} (0.1)^{12-11} + {}^{12}C_{12} (0.9)^{12} (0.1)^{12-12}$$

$$= 12 (0.9)^{11} (0.1) + 1 (0.9)^{12}$$

$$= (0.9)^{11} (1.2 + 0.9)$$

$$= (0.9)^{11} (2.1)$$

$$(iii) P(X \geq 2)$$

$$\boxed{P = 0.1} \quad \boxed{q = 0.9}$$

$$P(X \geq 2) = 1 - \{P(X=0) + P(X=1)\}$$

$$= 1 - \{ {}^{12}C_0 (0.1)^0 (0.9)^{12-0} + {}^{12}C_1 (0.1)^1 (0.9)^{12-1} \}$$

$$= 1 - \{ (0.9)^{12} + 12(0.1)(0.9)^{11} \}$$

$$= 1 - (0.9)^{11} \{ 0.9 + 1.2 \}$$

$$P(X \geq 2) = 1 - (0.9)^{11} (2.1)$$

$$\therefore \text{mean} = M = np$$

$$\text{Variance} = \sigma^2 = npq$$

④

$$\text{mean} \Rightarrow np = 6$$

$$\text{S.D} \Rightarrow \sqrt{npq} = 2$$

$$\boxed{npq = 4}$$

$$\text{Standard deviation} = \sigma = \sqrt{npq}$$

$$\therefore q = \frac{npq}{np} = \frac{4}{6} = \frac{2}{3}$$

$$\therefore p = 1 - q = 1 - \frac{2}{3}$$

$$\boxed{q = \frac{2}{3}}$$

$$\boxed{p = \frac{1}{3}}$$

$$\therefore \text{mean} \Rightarrow np = 6$$

$$n(1/3) = 6$$

$$\boxed{n = 18}$$

(i) probability mass function.

$$P(X=x) = f(x) = {}^nC_x p^x q^{n-x}, x=0, \dots, n$$

$$P(X=x) = f(x) = {}^{18}C_x \left(\frac{1}{3}\right)^x \left(\frac{2}{3}\right)^{18-x}, x=0, \dots, 18$$

$$(ii) P(X=3) = f(3) = {}^{18}C_3 \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^{18-3}$$

$$= {}^{18}C_3 \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^{15}$$

$$(iii) P(X \geq 2) = 1 - \{P(X=0) + P(X=1)\}$$

$$= 1 - \left\{ {}^{18}C_0 \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^{18-0} + {}^{18}C_1 \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^{18-1} \right\}$$

$$(ii) P(X=3) = f(3) = {}^{18}C_3 \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^{18-3}$$

$$= (48 \times 17) \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^{15}$$

$$= (48 \times 17) \frac{2^{15}}{3^{18}}$$

$${}^{18}C_3 = \frac{{}^{18}P_3}{3!} = \frac{18 \times 17 \times 16}{1 \times 2 \times 3}$$

$$(iii) P(X \geq 2) = 1 - \{P(X=0) + P(X=1)\}$$

$$= 1 - \left\{ {}^{18}C_0 \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^{18-0} + {}^{18}C_1 \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^{17} \right\}$$

$$= 1 - \left(\frac{2}{3}\right)^{17} \left\{ \frac{2}{3} + 18 \left(\frac{1}{3}\right) \right\}$$

$$= 1 - \left(\frac{2}{3}\right)^{17} \left\{ \frac{2}{3} + 6 \right\}$$

$$= 1 - \left(\frac{2}{3}\right)^{17} \left[ \frac{18+2}{3} \right]$$

$$P(X \geq 2) = 1 - \left(\frac{2}{3}\right)^{17} \left(\frac{20}{3}\right)$$

$$(8). X \sim B(n, P) \text{ and } P(X=4) = P(X=2), \quad n=6$$

$$P(X=r) = f(r) = {}^nC_r P^r q^{n-r}, \quad r=0, 1, \dots, n$$

$$A \left\{ {}^6C_4 P^4 q^{6-4} \right\} = {}^6C_2 P^2 q^{6-2}$$

$$\therefore {}^6C_4 = {}^6C_2$$

$$A P^4 q^2 = P^2 q^4$$

$$4 P^2 = q^2$$

Taking square root on both side

$$\pm 2P = q$$

$$\pm 2P = (1-P)$$

$$\begin{array}{l} 2P = 1-P \\ 3P = 1 \\ P = \frac{1}{3} \end{array}$$

$$-2P = 1-P$$

$$H = P$$

not possible.



$$\therefore n=6, p=1/3 \quad q=1-p$$

$$q=1-1/3$$

$$q=2/3$$

$$\text{Mean} = np = 6(1/3) = 2.$$

$$S.D = \sqrt{npq} = \sqrt{6 \times 1/3 \times 2/3} = \sqrt{\frac{12}{9}} = \sqrt{4/3} = \frac{2}{\sqrt{3}}$$

⑨

$$n=5$$

$$P(X=1) = 0.4096$$

$$P(X=2) = 0.2048$$

$$P(X=1) = 0.4096$$

$$5C_1 P^1 q^{5-1} = 0.4096$$

$$5Pq^4 = 0.4096 \rightarrow \textcircled{1}$$

$$P(X=2) = 0.2048.$$

$$5C_2 P^2 q^{5-2} = 0.2048$$

$$10P^2 q^3 = 0.2048 \rightarrow \textcircled{2}$$

from ① P ②

$$\frac{5Pq^4}{10P^2q^3} = \frac{0.4096}{0.2048}$$

$$\frac{q}{2P} = 2.$$

$$q = 4P$$

$$(1-p) = 4P$$

$$1 = 5P$$

$$P = 1/5$$

$$q = 1-p = 1 - 1/5 = 4/5$$

$$2. \text{Mean} = np = 5 \times (1/5) = 1$$

$$\text{Variance} = npq = 5 \times 1/5 \times 4/5 = 4/5$$

$$S.D = \sqrt{npq} = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}}$$



# CHAPTER -10 ORDINARY DIFFERENTIAL EQUATION.

## DIFFERENTIAL EQUATION:

A differential equation is any equation which contains atleast one derivative of an unknown function, either ordinary derivative or partial derivative.

## ORDER:

The order of a differential equation is the highest order derivative present in the differential equation.

## DEGREE:

If a differential equation is expressible in a polynomial form, then the integral power of the highest order derivative appears is called the degree of the differential equation.

∴ Equation:-

$$y = x^2 + 3x + 4 \quad \text{O. Equation}$$

$$y = mx' + c \quad \text{Linear Equation}$$

$$y = x^3 + 5x^2 + 7x + 3 \quad \text{O. Equation}$$

$$y = x^4 - 3x^3 + 7x^2 + 3x - 5$$

quadratic Equation

Differential Equation,

$$\frac{dy}{dx} = m.$$

First order derivative.

$$\frac{d^2y}{dx^2} + 3 \frac{dy}{dx} + 4 = 0.$$

second order derivative

for example:

$$(i) \frac{dy}{dx} = 0$$

$$(ii) \frac{dy}{dx} = \sin x$$

$$(iii) \frac{dy}{dx} = y + 7x + 5$$

$$(iv) \frac{d^2y}{dx^2} + \frac{dy}{dx} + y = \sin x$$

$$\frac{d^3y}{dx^3} + 5 \frac{d^2y}{dx^2} + 7 \frac{dy}{dx} = 0$$

third order derivative.

$$5) e^{\frac{dy}{dx}} = \log x, x > 0$$

Case (i)

$$6) \tan^{-1} \left[ \frac{d^2y}{dx^2} + y^2 + 2x \right] = \frac{dy}{dx}$$

1) power fraction,

2) inner function  
(No, degree)

3) Square root.

## EXERCISE 10-1

$$(1) \therefore \left( \frac{dy}{dx} \right)' + xy = \log x.$$

order, degree = 1, 1

$$(ii) \left( \frac{d^3y}{dx^3} \right)^{2/3} - 3 \frac{d^2y}{dx^2} + 5 \frac{dy}{dx} + 4 = 0$$

power multiple by 3.

$$\left( \frac{d^3y}{dx^3} \right)^2 - 27 \left( \frac{d^2y}{dx^2} \right)^3 + 125 \left( \frac{dy}{dx} \right)^3 + 64 = 0.$$

order, degree = 3, 2

$$(iii) \left( \frac{d^2y}{dx^2} \right)^2 + \left( \frac{dy}{dx} \right)^2 = x \sin \left( \frac{d^2y}{dx^2} \right)$$

inner function.

order, degree = 2, does not exist

$$(iv) \sqrt{\frac{dy}{dx}} - 4 \frac{dy}{dx} - 7x = 0$$

$$\left( \frac{dy}{dx} \right)^{1/2} - 4 \frac{dy}{dx} - 7x = 0$$

power multiple by 2.

$$\left( \frac{dy}{dx} \right) - (4)^2 \left( \frac{dy}{dx} \right)^2 - 49x^2 = 0$$

order, degree = 1, 2

$$v) \quad y \left( \frac{dy}{dx} \right) = \frac{x}{\left( \frac{dy}{dx} \right) + \left( \frac{dy}{dx} \right)^3}$$

$$y \left( \frac{dy}{dx} \right) \left[ \left( \frac{dy}{dx} \right) + \left( \frac{dy}{dx} \right)^3 \right] = x$$

$$y \left( \frac{dy}{dx} \right)^2 + y \left( \frac{dy}{dx} \right)^4 = x$$

order, degree = 1, 4

$$(vi) \quad x^2 \frac{d^2y}{dx^2} + \left[ 1 + \left( \frac{dy}{dx} \right)^2 \right]^{1/2} = 0$$

power multiple by 2.

$$x^4 \left( \frac{d^2y}{dx^2} \right)^2 + \left[ 1 + \left( \frac{dy}{dx} \right)^2 \right] = 0$$

order, degree = 2, 2

(vii)

$$\left( \frac{d^2y}{dx^2} \right)^3 = \sqrt{1 + \left( \frac{dy}{dx} \right)^2}$$

$$\left( \frac{d^2y}{dx^2} \right)^6 = \left( 1 + \frac{dy}{dx} \right)^2$$

power multiple by 2.

$$\left( \frac{d^2y}{dx^2} \right)^6 = 1 + \frac{dy}{dx}$$

order, degree = 2, 6

(viii)  $\frac{d^2y}{dx^2} = xy + \log\left(\frac{dy}{dx}\right)$

order, degree = 2, does not exist.

ix  $\frac{d^2y}{dx^2} + 5 \frac{dy}{dx} + \int y dx = x^3$

D. w. r to x.

$$\frac{d}{dx} \left[ \frac{d^2y}{dx^2} \right] + \frac{d}{dx} \left[ 5 \frac{dy}{dx} \right] + \frac{d}{dx} \left[ \int y dx \right] = \frac{d}{dx} [x^3]$$

$$\frac{d^3y}{dx^3} + 5 \frac{d^2y}{dx^2} + yx = 3x^2$$

order, degree = 3, 1

(x)  $x = e^{xy \left( \frac{dy}{dx} \right)}$

Taking log on both side.

$$\log x = \log e^{xy \left( \frac{dy}{dx} \right)}$$

$$\log A^m = m \log A$$

$$\log x = xy \left( \frac{dy}{dx} \right) \log e$$

$$\log e = 1,$$

$$\log x = xy \left( \frac{dy}{dx} \right)$$

order, degree = 1, 1

Example 10-1

$$i) \frac{dy}{dx} = x + y + 5$$

order, degree = 1, 1

$$ii) \left(\frac{d^4y}{dx^4}\right)^3 + 4\left(\frac{dy}{dx}\right)^7 + by = 5 \cos 3x \rightarrow \text{not inner function}$$

order, degree = 4, 3

(iii)

$$\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 = x^2 \log\left(\frac{d^2y}{dx^2}\right)$$

inner function.

order, degree = 2, does not exist.

iv)

$$3\left(\frac{d^2y}{dx^2}\right) = \left[4 + \left(\frac{dy}{dx}\right)\right]^{3/2}$$

power multiply by 2

$$9\left(\frac{d^2y}{dx^2}\right)^2 = \left[4 + \left(\frac{dy}{dx}\right)\right]^3$$

order, degree = 2, 2

$$v) dy + (xy - \cos x) dx = 0$$

divided by dx

$$\frac{dy}{dx} + (xy - \cos x) = 0$$

order, degree = 1, 1



ordinary differential equation :-

It a differential equation contains only derivatives of one or more functions with respect to a single independent variable, it is said to be an ODE.

partial differential equation :-

An equation involving only partial derivatives of one or more function of two or more independent variable is called partial differential equation (PDE)

\* A general linear ordinary differential equation of order  $n$  is any differential equation that can be written in the following form.

$$a_n(x)y^n + a_{n-1}(x)y^{n-1} + a_{n-2}(x)y^{n-2} + \dots + a_1(x)y' + a_0y^0 = g(x)$$

$a_n \neq 0$ ,  $a_0(x)$ ,  $a_1(x)$ , ...,  $a_{n-1}(x)$  and  $g(x)$  are

any function of independent variable  $x$ .

\* A nonlinear ordinary differential equation is simply one that is not linear.

\* If  $g(x) = 0$  in equation, then the above equation is said to be homogeneous, otherwise it is called as non-homogeneous.

## EXE 10.2.

$$1) \quad \therefore \quad \frac{d\theta}{dt} \propto \theta$$

$$\frac{d\theta}{dt} = k\theta$$

$$(ii) \quad \frac{dp}{dt} \propto p(5,00,000 - p)$$

$$\frac{dp}{dt} = kp(5,00,000 - p)$$

$$(iii) \quad \frac{dv}{dT} \propto p, \quad \frac{dv}{dT} \propto \frac{1}{T^2}$$

$$\frac{dv}{dT} \propto \frac{p}{T^2}$$

$$\frac{dv}{dT} = \frac{kp}{T^2}$$

$$(iv) \quad \frac{dA}{dt} = \frac{8x}{100} + 400$$

$$\frac{dA}{dt} = \frac{2x}{25} + 400$$



2)

$$\frac{dv}{dt} \propto s$$

$$\text{volume : } \frac{4}{3}\pi r^3.$$

$$\text{surface area : } 4\pi r^2.$$

$$\frac{dv}{dt} = -k s.$$

$$\frac{d}{dt} \left[ \frac{4}{3}\pi r^3 \right] = -k [4\pi r^2]$$

$$\frac{4}{3}\pi (3r^2) \frac{dr}{dt} = -k (4\pi r^2)$$

$$\frac{dr}{dt} = -k$$

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EXE 10.3.

1. The equation of non-vertical line is  $ax + by + c = 0$ .

$$\therefore a \in \mathbb{R}, b \neq 0$$

$a, b$  are arbitrary constant.

D.W.r to  $x$

$$a + by' = 0$$

$$by'' = 0$$

$$\boxed{y'' = 0}$$

The equation of non-horizontal line is  $ax + by + c = 0$

D.W.r to  $y$

$$ax' + b = 0$$

$$ax'' = 0$$

$$\boxed{x'' = 0}$$

$$b \in \mathbb{R}, a \neq 0$$

$a, b$  are arbitrary constant

2. The straight line touching the circle  $x^2 + y^2 = r^2$ .

$$y = mx \pm r\sqrt{1+m^2} \rightarrow \text{①}$$

$m$  is arbitrary constant.

D.W.r to  $x$

$$y' = m$$

sub  $m = y'$  in equation.

$$y = y'x \pm r\sqrt{1+y'^2}$$

$$y - y'x = \pm r\sqrt{1+y'^2}$$

Taking square on both side.

$$(y - y'x)^2 = r^2(1+y'^2)$$

$$\boxed{(y'x - y)^2 = r^2(1+y'^2)}$$

3. The equation of the family of circle passing through the origin and having their centres on the  $x$ -axis

$$(x-a)^2 + (y-0)^2 = a^2$$

\*  $a$  is arbitrary constant

$$(x-a)^2 + y^2 = a^2$$

$$(x-h)^2 + (y-k)^2 = a^2$$

General equation of circle

$$(h, k) = (a, 0)$$

D. w. r to " $x$ "

$$2(x-a) + 2y'y' = 0$$

$$(x-a) = -yy'$$

$$a = x + yy'$$

Equation

$$[x - (x + yy')]^2 + y^2 = [x + yy']^2$$

$$(-yy')^2 + y^2 = x^2 + y^2y'^2 + 2xyy'$$

$$+ y^2y'^2 + y^2 = x^2 + y^2y'^2 + 2xyy'$$

$$y^2 = x^2 + 2xyy'$$

$$x^2 + 2xyy' - y^2 = 0$$

4. The equation of the family of all the parabola with Latus rectum  $4a$  and parallel to  $x$  axis is.

$$(y-k)^2 = 4a(x-h) \rightarrow (1)$$

where  $h$  and  $k$  are arbitrary constant

D. w. r to " $x$ "

$$2(y-k)y' = 4a \rightarrow (2)$$

$$(y-k) = \frac{4a}{2y'} = \frac{2a}{y'}$$

$$2(y-k)y' = 4a$$

Again D. w. r to  $x$ .

$$(y-k)y'' + y'^2 = 0.$$

$$y-k = \frac{-y_1^2}{y''}$$

②  $\Rightarrow$

$$2\left(\frac{-y_1^2}{y''}\right)y' = 4a.$$

$$\frac{-y_1^3}{y''} = 2a$$

$$-y_1^3 = 2ay''$$

$$2ay'' + y_1^3 = 0$$

5). Equation of parabola,  $y$ -axis,  $(h, k) = (0, -1)$

$$(x-h)^2 = 4a(y-k) \quad \text{where } a \text{ is arbitrary constant}$$

$$(x-0)^2 = 4a(y+1)$$

$$x^2 = 4a(y+1) \rightarrow (1)$$

D. w. r to  $x$ .

$$2x = 4ay'$$

$$\frac{2x}{y'} = 4a$$

①  $\Rightarrow$

$$x^2 = \frac{2x}{y'} (y+1)$$

$$xy' = 2y + 2.$$

$$xy' - 2y - 2 = 0$$

b. The equation of ellipses,  $x$ -axis,

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1.$$

where  $a^2$  and  $b^2$  are arbitrary constant

$$\times a^2 b^2 \Rightarrow a^2 x^2 + b^2 y^2 = a^2 b^2$$

D. w. r to  $x$

$$2a^2 x + 2b^2 y y' = 0 \rightarrow (1)$$

Again D. w. r to  $x$

$$2a^2 + 2b^2 (y y'' + y'^2) = 0 \rightarrow (2)$$

Eliminating  $2a^2$  and  $2b^2$  from eq (1) and (2)

$$\begin{vmatrix} x & y y' \\ 1 & y y'' + y'^2 \end{vmatrix} = 0$$

$$x (y y'' + y'^2) - y y' = 0.$$

$$\boxed{x y y'' + x y'^2 - y y' = 0}$$

8.

$$xy = a e^x + b e^{-x} + x^2.$$

$$xy - x^2 = a e^x + b e^{-x}$$

D. w. r to  $x$

$$x y' + y - 2x = a e^x - b e^{-x}.$$

Again D. w. r to  $x$ .

$$x y'' + y' + y' - 2 = a e^x + b e^{-x}$$

$$x y'' + 2y' - 2 = xy - x^2.$$

$$\boxed{x y'' + 2y' - 2 - xy + x^2 = 0}$$

7.

$$y = Ae^{8x} + Be^{-8x}$$

where A & B are arbitrary constant.

D. w. r to x

$$y' = 8Ae^{8x} - 8Be^{-8x} \rightarrow \textcircled{1}$$

Again D. w. r to x.

$$y'' = 64Ae^{8x} + 64Be^{-8x} \rightarrow \textcircled{2}$$

$$y'' = 64 [Ae^{8x} + Be^{-8x}]$$

$$y'' = 64y$$

$$y'' - 64y = 0$$

Example 10.2.

The family of st. line passing through origin

$$y = mx \rightarrow \textcircled{1} \quad \text{where } m \text{ is arbitrary constant}$$

$$y' = m$$

$$\textcircled{1} \Rightarrow y = y'x$$

$$y - y'x = 0$$

Example 10.3.

$$y = A \cos x + B \sin x$$

D. w. r to x

$$y' = -A \sin x + B \cos x$$

Again D. w. r to x.

$$y'' = -A \cos x - B \sin x$$

$$y'' = -[A \cos x + B \sin x]$$

$$y'' = -y$$

$$\boxed{y'' + y = 0}$$

Example 10.4

A circle passing through the point  $(a, 0)$  and  $(-a, 0)$  has its centre on  $y$ -axis. Let  $(0, b)$  be the centre of the circle.

$$(x-0)^2 + (y-b)^2 = a^2 + b^2, \text{ where } b \text{ are arbitrary constant}$$

$$x^2 + (y-b)^2 = a^2 + b^2 \rightarrow \textcircled{1}$$

D. w. r to  $x$ .

$$2x + 2(y-b)y' = 0$$

$$x + (y-b)y' = 0$$

$$(y-b)y' = -x$$

$$y-b = -\frac{x}{y'}$$

$$b = y + \frac{x}{y'}$$

$$x^2 + \left(-\frac{x}{y'}\right)^2 = a^2 + \left(y + \frac{x}{y'}\right)^2$$

$$x^2 + \frac{x^2}{y'^2} = a^2 + \frac{(yy' + x)^2}{y'^2}$$

$$x^2 y'^2 + x^2 = a^2 y'^2 + (yy' + x)^2 \quad \therefore (x^2 - y'^2 - a^2)y' - 2xy = 0$$



## Example 10.5

The equation of parabola

$$y^2 = 4ax \rightarrow \textcircled{1} \quad a \text{ is arbitrary constant.}$$

D. w. r to  $x$

$$2yy' = 4a$$

Equation  $\textcircled{1} \Rightarrow y^2 = 2yy'x$

$$y = 2y'x$$

$$y' = \frac{y}{2x}$$

## Example 10.6

The equation of ellipses,  $x - axb$ .

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

where  $a^2$  and  $b^2$  are arbitrary constant.

$$\times a^2 b^2 \Rightarrow b^2 x^2 + a^2 y^2 = a^2 b^2$$

D. w. r to  $x$

$$2b^2x + 2a^2yy' = 0 \rightarrow \textcircled{1}$$

Again D. w. r to  $x$

$$2b^2 + 2a^2[yy'' + y'^2] = 0 \rightarrow \textcircled{2}$$

Eliminating  $2a^2$  and  $2b^2$  from the equation  $\textcircled{1}$  &  $\textcircled{2}$

$$\begin{vmatrix} x & yy' \\ 1 & yy'' + y'^2 \end{vmatrix} = 0$$

$$x(yy'' + y'^2) - yy' = 0$$

$$x yy'' + x y'^2 - yy' = 0$$

Solution of DE :-

A solution of a differential equation is an expression for the dependent variable in terms of the independent variables, which satisfies the differential equation.

General solution :-

The solution which contains as many arbitrary constants as the order of the differential equation is called as general solution.

Particular solution :-

If we give particular values to the arbitrary constants in the general solution of differential equation, the resulting solution is called a Particular solution.

## EXERCISE: 10.4

$$\textcircled{1} \quad \therefore y = 2x^2 \rightarrow \textcircled{1}, \quad xy' = 2y$$

$$y' = 4x.$$

$$\boxed{\frac{y'}{4} = x}$$

$$\textcircled{1} \Rightarrow y = 2x^2$$

$$y = 2x(x)$$

$$y = 2x\left(\frac{y'}{4}\right)$$

$$y = \frac{xy'}{2}$$

$$2y = xy'$$

The solution of the differential equation is  $xy' = 2y$ .

$$2) \quad \text{The value of } m, \quad y = e^{mx}$$

$$y' = me^{mx}$$

$$y'' = m^2 e^{mx}$$

$$(i) \quad y' + 2y = 0.$$

$$me^{mx} + 2e^{mx} = 0$$

$$e^{mx}(m+2) = 0$$

$$e^{mx} \neq 0 \quad (m+2) = 0$$

$$\boxed{m = -2}$$

$$\textcircled{1}$$

$$(ii) \quad y = ae^x + be^{-x}$$

$$y' = ae^x - be^{-x}$$

$$y'' = ae^x + be^{-x}$$

$$y'' = y.$$

$$y'' - y = 0$$

The solution of differential equation is  $y'' - y = 0$

$$\textcircled{2}$$

$$(iii) \quad y'' - 5y' + 6y = 0$$

$$m^2 e^{mx} - 5me^{mx} + 6e^{mx} = 0.$$

$$e^{mx}(m^2 - 5m + 6) = 0$$

$$e^{mx} \neq 0 \quad m^2 - 5m + 6 = 0$$

$$(m-3)(m-2) = 0$$

$$m-3=0, \quad m-2=0$$

$$m=3, \quad m=2.$$

$$\boxed{m = 2, 3}$$

③ The slope of the tangent  $m = \frac{dy}{dx} = \frac{1}{4y}$

$$\frac{dy}{dx} = \frac{1}{4y}$$

$$4y dy = dx$$

$$4 \int y dy = \int dx$$

$$4 \frac{y^2}{2} = x + C$$

$$2y^2 = x + C$$

At (2, 5)

$$2(5)^2 = 2 + C$$

$$50 = 2 + C$$

$$\boxed{C = 48}$$

The equation of the curve is  $2y^2 = x + 48$

④

$$y = e^{-x} + mx + n, \quad e^x y'' - 1 = 0$$

$$y' = -e^{-x} + m$$

$$y'' = e^{-x}$$

$$y'' = \frac{1}{e^x}$$

$$e^x y'' = 1$$

$$\boxed{e^x y'' - 1 = 0}$$

$$\textcircled{6} \quad y = ae^{-3x} + b$$

$$y' = -3ae^{-3x}$$

$$y'' = -3[-3ae^{-3x}]$$

$$y'' = -3y'$$

$$\boxed{y'' + 3y' = 0}$$

$$\textcircled{8} \quad y = a \cos bx$$

$$y' = -ba \sin bx$$

$$y'' = -b^2 a \cos bx$$

$$y'' = -b^2 [a \cos bx]$$

$$y'' = -b^2 (y)$$

$$y'' = -b^2 y$$

$$\boxed{y'' + b^2 y = 0}$$

$$\textcircled{9} \quad y^2 = 2a(x + a^{2/3}) \rightarrow \textcircled{1}$$

$$2yy' = 2a$$

$$yy' = a$$

$$y^2 = 2yy'(x + (yy')^{2/3})$$

$$y^2 = 2yy'x + 2yy'(yy')^{2/3}$$

$$y^2 = 2yy'x + 2(yy')^{5/3}$$

$$y^2 - 2yy'x = 2(yy')^{5/3}$$

$$(y^2 - 2yy'x)^3 = (2)^3 [yy']^5$$

$$(y^2 - 2yy'x)^3 = 8(yy')^5$$

$$\textcircled{1} \quad y = ax + \frac{b}{x}, \quad x \neq 0 \quad \rightarrow \textcircled{1}$$

$$y' = a + \left( \frac{-b}{x^2} \right)$$

$$d\left(\frac{1}{x^n}\right) = \frac{-b}{x^{n+1}} \checkmark$$

$$y' = a - \frac{b}{x^2} \quad \rightarrow \textcircled{2}$$

$$y'' = \frac{2b}{x^3}$$

$$\boxed{\frac{x^3 y''}{2} = b.}$$

$$\textcircled{2} \quad y' = a - \frac{(x^3 y'')/2}{x^2}$$

$$y' = a - \frac{xy''}{2}$$

$$y' + \frac{xy''}{2} = a$$

$$\boxed{\frac{2y' + xy''}{2} = a}$$

$$y = \left( \frac{2y' + xy''}{2} \right) x + \frac{(x^3 y'')/2}{x}$$

$$y = \left( y' + \frac{xy''}{2} \right) x + \frac{x^2 y''}{2}$$

$$y = xy' + \frac{x^2 y''}{2} + \frac{x^2 y''}{2}$$

$$\boxed{y = xy' + x^2 y''}$$

Variable separable method.

$$y = f(x) = x^2 + 3x + 2$$

EXE 10.5

$$m \frac{dv}{dt} = F - kv$$

$$\frac{dv}{F - kv} = \frac{dt}{m}$$

$$\int \frac{f'(x)}{f(x)} dx = \log |f(x)|.$$

$$\int \frac{dv}{F - kv} = \int \frac{dt}{m}$$

-k multiple on both side

$$\int \frac{-k dv}{F - kv} = \frac{-k}{m} \int dt$$

$$\log |F - kv| = \frac{-kt}{m} + c \rightarrow \textcircled{1}$$

when  $t=0$ ,  $v=0$ .

$$\log |F - k(0)| = \frac{-k}{m}(0) + c$$

$$\boxed{\log |F| = c}$$

$$\textcircled{1} \Rightarrow \log |F - kv| = \frac{-kt}{m} + \log |F|$$

$$\frac{kt}{m} = \log |F| - \log |F - kv|$$

$$\frac{kt}{m} = \log \left| \frac{F}{F - kv} \right|$$

$$e^{\frac{kt}{m}} = \frac{F}{F - kv}$$

$$\boxed{F = (F - kv) e^{\frac{kt}{m}}}$$



(2)

$$v \frac{dv}{dx} = g \left( 1 - \frac{v^2}{k^2} \right)$$

$$\frac{v dv}{\left( 1 - \frac{v^2}{k^2} \right)} = g dx$$

$$\int \frac{v dv}{\left( 1 - \frac{v^2}{k^2} \right)} = g \int dx$$

$-2/k^2$  multiple on both side.

$$\int \frac{-\frac{2v}{k^2} dv}{\left( 1 - \frac{v^2}{k^2} \right)} = -\frac{2g}{k^2} \int dx$$

$$\log \left( 1 - \frac{v^2}{k^2} \right) = -\frac{2gx}{k^2} + C$$

when  $v=0$ ,  $x=0$

$$\log(1-0) = -\frac{2g}{k^2}(0) + C$$

$$\log 1 = 0$$

$$\log 0 = 1$$

$$\log 1 = C$$

$$\boxed{C=0}$$

$$\log \left( 1 - \frac{v^2}{k^2} \right) = -\frac{2gx}{k^2}$$

$$1 - \frac{v^2}{k^2} = e^{-\frac{2gx}{k^2}}$$

$$\frac{v^2}{k^2} = 1 - e^{-\frac{2gx}{k^2}}$$

$$\boxed{v^2 = k^2 \left( 1 - e^{-\frac{2gx}{k^2}} \right)}$$

③

$$\frac{dy}{dx} = \frac{y-1}{x^2+x}, \text{ At } (1,0)$$

$$\frac{dy}{y-1} = \frac{dx}{x^2+x}$$

$$\frac{1}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}$$

$$\boxed{A=1} \quad \boxed{B=-1}$$

$$\frac{dy}{y-1} = \frac{dx}{x(x+1)}$$

$$\frac{dy}{y-1} = \left( \frac{1}{x} + \frac{(-1)}{x+1} \right) dx$$

$$\int \frac{dy}{y-1} = \int \left( \frac{1}{x} - \frac{1}{x+1} \right) dx$$

$$\log |y-1| = \log |x| - \log (x+1) + \log c$$

$$\log |y-1| = \log \left| \frac{xc}{x+1} \right|$$

$$y-1 = \frac{xc}{x+1} \rightarrow \textcircled{1}$$

At (1,0)

$$0-1 = \frac{1c}{1+1}$$

$$-1 = \frac{c}{2}$$

$$\boxed{c=-2}$$

$$y-1 = \frac{-2x}{x+1}$$

$$y = 1 - \frac{2x}{x+1}$$

$$y = \frac{(x+1) - 2x}{x+1}$$

$$\boxed{y = \frac{1-x}{x+1}}$$

EXE 10.5

$$1. (i) \frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x$$

$$\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$$

$$\frac{dy}{\sqrt{1-y^2}} = \frac{dx}{\sqrt{1-x^2}}$$

$$\int \frac{dy}{\sqrt{1-y^2}} = \int \frac{dx}{\sqrt{1-x^2}}$$

$$\sin^{-1} y = \sin^{-1} x + \sin^{-1} c$$

$$\sin^{-1} y = \sin^{-1} x + c$$

$$(ii) y dx + (1+x^2) \tan^{-1} x dy = 0$$

$$(1+x^2) \tan^{-1} x dy = -y dx$$

$$\frac{dy}{y} = \frac{-dx}{(1+x^2) \tan^{-1} x}$$

$$d(\tan^{-1} x) = \frac{1}{1+x^2} dx$$

$$\frac{dy}{y} = - \frac{\left(\frac{1}{1+x^2}\right) dx}{\tan^{-1} x}$$

$$\int \frac{dy}{y} = - \int \frac{\left(\frac{1}{1+x^2}\right) dx}{\tan^{-1} x}$$

$$\log y = - \log \tan^{-1} x + \log c$$

$$\log y + \log \tan^{-1} x = \log c$$

$$\log (y \tan^{-1} x) = \log c$$

$$\boxed{y \tan^{-1} x = c}$$

$$(iii) \sin \frac{dy}{dx} = a, \quad y(0) = 1.$$

$$\frac{dy}{dx} = \sin^{-1} a.$$

$$dy = (\sin^{-1} a) dx$$

$$\int dy = (\sin^{-1} a) \int dx$$

$$y = (\sin^{-1} a)x + c \rightarrow \text{①}$$

$$\text{when } x=0, y=1.$$

$$1 = (\sin^{-1} a) \cdot 0 + c$$

$$\boxed{1 = c}$$

$$\text{①} \Rightarrow y = (\sin^{-1} a)x + 1$$

$$y-1 = (\sin^{-1} a)x$$

$$\frac{y-1}{x} = \sin^{-1} a$$

$$\boxed{\sin\left(\frac{y-1}{x}\right) = a}$$

$$(iv) \frac{dy}{dx} = e^{x+y} + x^3 e^y$$

$$\frac{dy}{dx} = e^x e^y + x^3 e^y$$

$$\frac{dy}{dx} = e^y (e^x + x^3)$$

$$\frac{dy}{e^y} = (e^x + x^3) dx$$

$$e^{-y} dy = (e^x + x^3) dx$$

$$\int e^{-y} dy = \int (e^x + x^3) dx$$

$$-e^{-y} = e^x + \frac{x^4}{4} + c.$$

$$-c = e^x + e^{-y} + \frac{x^4}{4}$$

$$c = e^x + e^{-y} + \frac{x^4}{4}$$

$$\text{⑤} \quad (e^y + 1) \cos x \, dx + e^y \sin x \, dy = 0$$

$$e^y \sin x \, dy = -(e^y + 1) \cos x \, dx$$

$$\int \frac{e^y dy}{e^y + 1} = - \int \frac{\cos x}{\sin x} dx$$

$$\log(e^y + 1) = -\log(\sin x) + \log c$$

$$\log(e^y + 1) + \log(\sin x) = \log c$$

$$\log[(e^y + 1) \sin x] = \log c$$

$$(e^y + 1) \sin x = c$$

$$(vi) (y dx - x dy) \cot(x/y) = ny^2 dx$$

$$d\left(\frac{x}{y}\right) = \frac{y dx - x dy}{y^2}$$

$$\frac{(y dx - x dy)}{y^2} \cot(x/y) = n dx$$

$$d\left(\frac{y}{x}\right) = \frac{y dx - x dy}{x^2}$$

$$\cot(x/y) d(x/y) = n dx$$

$$\int \frac{\cot(x/y)}{\sin(x/y)} d(x/y) = n \int dx$$

$$\log |\sin(x/y)| = nx + c$$

$$\sin(x/y) = e^{(nx+c)}$$

(v)

$$\frac{dy}{dx} - x \sqrt{25-x^2} = 0$$

$$\frac{dy}{dx} = x \sqrt{25-x^2}$$

$$dy = x \sqrt{25-x^2} dx$$

$$\int dy = \int x \sqrt{25-x^2} dx$$

$$\text{let } t = 25-x^2$$

$$dt = -2x dx$$

$$\int dy = \int \sqrt{t} \left(\frac{dt}{-2}\right)$$

$$\frac{dt}{-2} = x dx$$

$$\int dy = -\frac{1}{2} \int \sqrt{t} dt$$

$$y = -\frac{1}{2} \left( \frac{t^{3/2}}{3/2} \right) + c$$

$$y = -\frac{t^{3/2}}{3} + c$$

$$y = -\frac{(25-x^2)^{3/2}}{3} + c$$

$$3y + (25-x^2)^{3/2} = 3c$$

$$3y + (25-x^2)^{3/2} = c$$

$$(viii) \quad x \cos y \, dy = e^x (x \log x + 1) \, dx.$$

$$\cos y \, dy = \frac{e^x (x \log x + 1)}{x} \, dx$$

$$\cos y \, dy = e^x \left[ \log x + \frac{1}{x} \right] dx$$

$$\int e^x [f(x) + f'(x)] \, dx =$$

$$e^x f(x) + C.$$

$$\int \cos y \, dy = \int e^x \left[ \log x + \frac{1}{x} \right] dx$$

$$\sin y = e^x \log x + C.$$

(ix)

$$\tan y \frac{dy}{dx} = \cos(x+y) + \cos(x-y)$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\tan y \frac{dy}{dx} = 2 \cos x \cos y.$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B.$$

$$\frac{\tan y}{\cos y} \, dy = 2 \cos x \, dx.$$

$$\sec y \tan y \, dy = 2 \cos x \, dx.$$

$$\int \sec y \tan y \, dy = 2 \int \cos x \, dx$$

$$\sec y = 2 \sin x + C$$

$$x. \quad \frac{dy}{dx} = \tan^2(x+y) \rightarrow (1)$$

$$\text{let } t = x+y.$$

$$\text{D. w. r to } x.$$

$$\frac{dt}{dx} = 1 + \frac{dy}{dx}$$

$$\frac{dt}{dx} - 1 = \frac{dy}{dx} \rightarrow (2)$$

$$\frac{dt}{dx} - 1 = \tan^2 t$$

$$\frac{dt}{dx} = \tan^2 t + 1$$

$$\frac{dt}{dx} = \sec^2 t$$

$$\frac{dt}{\sec^2 t} = dx$$

$$\cos^2 t \, dt = dx$$

$$\left( \frac{1 + \cos 2t}{2} \right) dt = dx$$

$$\Rightarrow \frac{1}{2} \left[ (x+y) + \frac{\sin(x+y) \cos(x+y)}{2} \right] = x + c$$

$$\frac{1}{2} \int (1 + \cos 2t) dt = \int dx$$

$$\frac{1}{2} [(x+y) + \sin(x+y) \cos(x+y)] = x + c.$$

$$\frac{1}{2} \left[ t + \frac{\sin 2t}{2} \right] = x + c.$$

$$\frac{1}{2} \left[ (x+y) + \frac{\sin 2(x+y)}{2} \right] = x + c$$



Example 10.11

$$(1+x^2) \frac{dy}{dx} = (1+y^2)$$

$$\frac{dy}{1+y^2} = \frac{dx}{1+x^2}$$

$$\int \frac{dy}{1+y^2} = \int \frac{dx}{1+x^2}$$

$$\tan^{-1}y = \tan^{-1}x + \tan^{-1}c$$

$$\tan^{-1}y - \tan^{-1}x = \tan^{-1}c$$

$$\tan^{-1} \left[ \frac{y-x}{1+xy} \right] = \tan^{-1}c$$

$$\frac{y-x}{1+xy} = c$$

$$y-x = (1+xy)c$$

Example 10.12.

$$(1+x^3) dy - x^2 y dx = 0$$

$$(1+x^3) dy = x^2 y dx$$

$$\frac{dy}{y} = \frac{x^2 dx}{1+x^3}$$

$$\int \frac{3dy}{y} = \int \frac{3x^2 dx}{1+x^3}$$

$$3 \log y = \log(1+x^3) + \log c$$

$$\log y^3 = \log(1+x^3) + \log c$$

$$\log y^3 - \log(1+x^3) = \log c$$

$$\log\left(\frac{y^3}{1+x^3}\right) = \log c$$

$$\frac{y^3}{1+x^3} = c \rightarrow \textcircled{1}$$

when,  $x=1$ ,  $y=2$ ,  $[y(1)=2]$

$$\frac{8}{1+1} = c$$

$$c = 4$$

$$\textcircled{1} \Rightarrow \frac{y^3}{1+x^3} = 4$$

$$y^3 = 4(1+x^3)$$

Example 10-13

$$y' = \sin^2(x-y+1)$$

$$\frac{dy}{dx} = \sin^2(x-y+1)$$

$$y' = \frac{dy}{dx}$$

$$\text{let } z = x - y + 1$$

$$\text{D.W. } x \rightarrow 0 \rightarrow x$$

$$1 - \frac{dz}{dx} = \sin^2 z$$

$$\frac{dz}{dx} = 1 - \sin^2 z$$

$$\frac{dz}{dx} = \cos^2 z$$

$$\frac{dz}{\cos^2 z} = dx$$

$$\frac{dz}{dx} = 1 - \frac{dy}{dx}$$

$$\frac{dy}{dx} = 1 - \frac{dz}{dx}$$

$$\therefore \cos^2 z + \sin^2 z = 1$$

$$\int \sec^2 z \, dz = \int dx$$

$$\tan z = x + c$$

$$\tan(x - y + 1) = x + c.$$

Example 10.14.

$$\frac{dy}{dx} = \sqrt{4x+2y-1}$$

$$z \frac{dz}{dx} - 2 = z$$

$$z \frac{dz}{dx} = z + 2$$

$$\frac{z \, dz}{z^2 + 2} = dx$$

$$\int \frac{z \, dz}{z^2 + 2} = \int dx$$

$$\int \left( \frac{z+2}{z+2} - \frac{2}{z+2} \right) dz = \int dx$$

$$\int \left( 1 - \frac{2}{z+2} \right) dz = \int dx$$

$$z - 2 \log(z+2) = x + c$$

$$\sqrt{4x+2y-1} - 2 \log [\sqrt{4x+2y-1} + 2] = x + c$$

$$\text{let } z^2 = 4x + 2y - 1$$

$$\text{D.W. } x \rightarrow 2x$$

$$2z \frac{dz}{dx} = 4 + 2 \frac{dy}{dx}$$

$$z \frac{dz}{dx} = 2 + \frac{dy}{dx}$$

$$z \frac{dz}{dx} - 2 = \frac{dy}{dx}$$

Example 10.15

$$\frac{dy}{dx} = \frac{x-y+5}{2(x-y)+7}$$

$$1 - \frac{dz}{dx} = \frac{z+5}{2z+7}$$

$$\frac{dz}{dx} = 1 - \frac{z+5}{2z+7}$$

$$\frac{dz}{dx} = \frac{z+2}{2z+7}$$

$$\frac{2z+7}{z+2} dz = dx$$

$$\frac{2z+4+3}{z+2} dz = dx$$

$$\left( \frac{2(z+2)+3}{z+2} \right) dz = dx$$

$$\int \left( 2 + \frac{3}{z+2} \right) dz = \int dx$$

$$2z + 3 \log |z+2| = x + C.$$

$$2(x-y) + 3 \log |(x-y)+2| = x + C.$$

$$\text{let } z = x-y$$

$$D.W. r to x$$

$$\frac{dz}{dx} = 1 - \frac{dy}{dx}$$

$$\frac{dy}{dx} = 1 - \frac{dz}{dx}$$

Example 10.15

$$\frac{dy}{dx} = \frac{x-y+5}{2(x-y)+7}$$

$$1 - \frac{dz}{dx} = \frac{z+5}{2z+7}$$

$$\frac{dz}{dx} = 1 - \frac{z+5}{2z+7}$$

$$\frac{dz}{dx} = \frac{z+2}{2z+7}$$

$$\frac{2z+7}{z+2} dz = dx$$

$$\frac{2z+4+3}{z+2} dz = dx$$

$$\left( \frac{2(z+2)+3}{z+2} \right) dz = dx$$

$$\int \left( 2 + \frac{3}{z+2} \right) dz = \int dx$$

$$2z + 3 \log |z+2| = x + C.$$

$$2(x-y) + 3 \log |(x-y)+2| = x + C.$$

$$\text{let } z = x-y$$

$$\text{D.W. r to } x$$

$$\frac{dz}{dx} = 1 - \frac{dy}{dx}$$

$$\frac{dy}{dx} = 1 - \frac{dz}{dx}$$

Example 10.16

$$\frac{dy}{dx} = (3x + y + 4)^2$$

$$\text{let } z = 3x + y + 4$$

$$\text{D. w. r. to } x$$

$$\frac{dz}{dx} - 3 = z^2$$

$$\frac{dz}{dx} = 3 + \frac{dy}{dx}$$

$$\frac{dz}{dx} = z^2 + 3$$

$$\frac{dy}{dx} = \frac{dz}{dx} - 3$$

$$\frac{dz}{dx} = z^2 + (\sqrt{3})^2$$

$$\frac{dz}{z^2 + (\sqrt{3})^2} = dx$$

$$\int \frac{dz}{z^2 + (\sqrt{3})^2} = \int dx$$

$$\int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + c$$

$$\frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{z}{\sqrt{3}}\right) = x + c$$

$$\frac{1}{\sqrt{3}} \tan^{-1}\left[\frac{3x + y + 4}{\sqrt{3}}\right] = x + c$$

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homogeneous differential equation.

case (i)

$$\frac{dy}{dx} = f(y/x) \rightarrow (1)$$

Put  $v = y/x$

$$y = vx \quad \text{shortcut}$$

D.W. r to  $x$   $y=v, x=1$

$$\frac{dy}{dx} = v + x \frac{dv}{dx} \rightarrow (2)$$

from (1) & (2)

$$v + x \frac{dv}{dx} = f(y/x)$$

EXE 10.6.

$$1. [x + y \cos(y/x)] dx = x \cos(y/x) dy.$$

$$\frac{dy}{dx} = \frac{x + y \cos(y/x)}{x \cos(y/x)}$$

Shortcut:  $y = vx$

$$\frac{dy}{dx} = \frac{1 + v \cos v}{\cos v} \rightarrow (1)$$

from (1) & (2)

$$v + x \frac{dv}{dx} = \frac{1 + v \cos v}{\cos v}$$

$$x \frac{dv}{dx} = \frac{1 + v \cos v}{\cos v} - v$$

$$x \frac{dv}{dx} = \frac{1 + v \cos v - v \cos v}{\cos v}$$

case (ii)

$$\frac{dx}{dy} = f(x/y) \rightarrow (1)$$

Put  $u = x/y$

$$x = uy$$

D.W. r to  $y$   $x=u, y=1$

$$\frac{dx}{dy} = u + y \frac{du}{dy} \rightarrow (2)$$

from (1) & (2)

$$u + y \frac{du}{dy} = f(x/y)$$

Put  $v = y/x$

$$y = vx$$

D.W. r to  $x$

$$\frac{dy}{dx} = v + x \frac{dv}{dx} \rightarrow (2)$$



$$x \frac{dv}{dx} = \frac{1}{\cos v}$$

$$\therefore v = y/x$$

$$\cos v \, dv = \frac{dx}{x}$$

$$\int \cos v \, dv = \int \frac{dx}{x}$$

$$\sin v = \log x + \log c.$$

$$\sin v = \log (xc).$$

$$\sin(y/x) = \log (xc).$$

$$(2) \quad (x^3 + y^3) dy = x^2 y \, dx = 0$$

$$(x^3 + y^3) dy = x^2 y \, dx.$$

$$\text{put } v = y/x$$

$$y = vx.$$

$$\text{D.W.r to } x$$

$$\frac{dy}{dx} = \frac{x^2 y}{x^3 + y^3} \rightarrow (1)$$

$$\text{Shortcut } y = vx, x = 1 \quad \frac{dy}{dx} = \frac{v}{1+v^3} \rightarrow (1) - A$$

$$\frac{dy}{dx} = v + x \frac{dv}{dx} \rightarrow (2)$$

from (1) & (2)

$$v + x \frac{dv}{dx} = \frac{v}{1+v^3}$$

$$x \frac{dv}{dx} = \frac{v}{1+v^3} - v$$

$$x \frac{dv}{dx} = \frac{v - v(1+v^3)}{1+v^3}$$

$$= \frac{\cancel{v} - \cancel{v} - v^4}{1+v^3}$$

$$x \frac{dv}{dx} = - \frac{v^4}{1+v^3}$$

$$\frac{1+v^3}{v^4} dv = - \frac{dx}{x}$$

$$\left( \frac{1}{v^4} + \frac{1}{v} \right) dv = - \frac{dx}{x}$$

$$\int \left( \frac{1}{x^n} \right) dx = \frac{-1}{(n-1)x^{n-1}} + c$$

$$\int \frac{1}{x} dx = \log|x| + c$$

$$\int \left( \frac{1}{v^4} + \frac{1}{v} \right) dv = - \int \frac{dx}{x}$$

$$\frac{-1}{3v^3} + \log v = -\log x + \log c$$

$$\log v + \log x - \log c = \frac{1}{3v^3}$$

$$\log \left( \frac{vx}{c} \right) = \frac{1}{3v^3}$$

$$\frac{vx}{c} = e^{\left( \frac{1}{3v^3} \right)}$$

$$vx = c e^{\left( \frac{1}{3v^3} \right)}$$

$$\left( \frac{y}{x} \right) x = c e^{\left( \frac{1}{3v^3} \right)}$$

$$y = c e^{\left( \frac{1}{3v^3} \right)}$$

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$$(9) \quad y e^{(x/y)} dx = (x e^{(x/y)} + y) dy.$$

$$\frac{dx}{dy} = \frac{x e^{(x/y)} + y}{y e^{(x/y)}}$$

put  $v = x/y$ .

$$x = vy.$$

D. w. r to  $y$ .

$$\frac{dx}{dy} = v + y \frac{dv}{dy} \rightarrow (2)$$

Shortcut  
[ $x = v, y = 1$ ]

$$\frac{dx}{dy} = \frac{v e^v + 1}{e^v} \rightarrow (1)$$

from (1) (2)

$$v + y \frac{dv}{dy} = \frac{v e^v + 1}{e^v}$$

$$y \frac{dv}{dy} = \frac{v e^v + 1}{e^v} - v$$

$$y \frac{dv}{dy} = \frac{v e^v + 1 - v e^v}{e^v}$$

$$y \frac{dv}{dy} = \frac{1}{e^v}$$

$$e^v dv = \frac{dy}{y}$$

$$\int e^v dv = \int \frac{dy}{y}$$

$$e^v = \log y + \log c$$

$$e^v = \log (yc)$$

$$e^{(x/y)} = \log (yc).$$

4.  $2xy \, dx + (x^2 + 2y^2) \, dy = 0$ .

$$2xy \, dx = -(x^2 + 2y^2) \, dy$$

$$\frac{dx}{dy} = -\frac{(x^2 + 2y^2)}{2xy}$$

Shortcut  
 $x = u, y = 1$

$$\frac{dx}{dy} = -\frac{(u^2 + 2)}{2u} \rightarrow$$

from ① & ②

$$u + y \frac{du}{dy} = -\frac{(u^2 + 2)}{2u}$$

$$y \frac{du}{dy} = -\frac{(u^2 + 2)}{2u} - u$$

$$y \frac{du}{dy} = \frac{-u^2 - 2 - u(2u)}{2u}$$

$$y \frac{du}{dy} = \frac{-u^2 - 2 - 2u^2}{2u}$$

$$y \frac{du}{dy} = -\frac{3u^2 + 2}{2u}$$

$$y \frac{du}{dy} = -\frac{(3u^2 + 2)}{2u}$$

$$\frac{2u}{3u^2 + 2} \, du = -\frac{dy}{y}$$

$$\int \frac{6u}{3u^2 + 2} \, du = -3 \int \frac{dy}{y}$$

$$\log(3u^2 + 2) = -3 \log y + \log c$$

Put  $u = xy$

$$y^x = x$$

$$D. w. r. to y$$

$$\frac{dx}{dy} = u + y \frac{du}{dy}$$

②

$$m \log A = \log A^m$$

$$\log |3v^2 + 2| = -\log y^3 + \log c$$

$$\log |3v^2 + 2| + \log y^3 = \log c$$

$$\log |(3v^2 + 2)y^3| = \log c.$$

$$(3v^2 + 2)y^3 = c.$$

$$\therefore v = xy.$$

$$[3(xy)^2 + 2] y^3 = c$$

$$\left[ \frac{3x^2 + y^2}{y^2} \right] y^3 = c.$$

$$[3x^2 + y^2] y = c.$$

$$3x^2 y + y^3 = c.$$

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ExE 10.6

$$5. (y^2 - 2xy) dx = (x^2 - 2xy) dy$$

$$\frac{dy}{dx} = \frac{y^2 - 2xy}{x^2 - 2xy}$$

$$\text{put } \Rightarrow v = y/x$$

$$y = vx$$

$$D, w, r \rightarrow x$$

$$\text{Shortcut} \\ y = vx, x = 1 \quad \frac{dy}{dx} = \frac{v^2 - 2v}{1 - 2v} \rightarrow \textcircled{1}$$

$$\frac{dy}{dx} = v + x \frac{dv}{dx} \rightarrow \textcircled{2}$$

from  $\textcircled{1}$  &  $\textcircled{2}$ 

$$v + x \frac{dv}{dx} = \frac{v^2 - 2v}{1 - 2v}$$

$$x \frac{dv}{dx} = \frac{v^2 - 2v}{1 - 2v} - v$$

$$x \frac{dv}{dx} = \frac{v^2 - 2v - v(1 - 2v)}{1 - 2v}$$

$$x \frac{dv}{dx} = \frac{v^2 - 2v - v + 2v^2}{1 - 2v}$$

$$x \frac{dv}{dx} = \frac{3v^2 - 3v}{1 - 2v}$$

$$x \frac{dv}{dx} = \frac{-3(v - v^2)}{1 - 2v}$$

$$\frac{1 - 2v}{v - v^2} dv = -3 \frac{dx}{x}$$

$$\int \frac{1 - 2v}{v - v^2} dv = -3 \int \frac{dx}{x}$$

$$\log(v - v^2) = -3 \log x + \log c$$

$$\log (v-v^2) + \log x^3 = \log c$$

$$\log | (v-v^2) x^3 | = \log c$$

$$(v-v^2) x^3 = c$$

$$\therefore v = y/x$$

$$\left[ \frac{y}{x} - \left( \frac{y}{x} \right)^2 \right] x^3 = c$$

$$\left[ \frac{y}{x} - \frac{y^2}{x^2} \right] x^3 = c$$

$$x^2 y - x y^2 = c$$

$$b. \quad x \frac{dy}{dx} = y - x \cos^2(y/x)$$

$$\frac{dy}{dx} = \frac{y - x \cos^2(y/x)}{x}$$

$$\text{Put } v = y/x$$

$$y = vx$$

$$D.w.r to x$$

$$\frac{dy}{dx} = v + x \frac{dv}{dx} \rightarrow (1)$$

Short cut  
 $y=v, x=1$

$$\frac{dy}{dx} = \frac{v - \cos^2 v}{1} \rightarrow (2)$$

from (1) & (2)

$$v + x \frac{dv}{dx} = v - \cos^2 v$$

$$x \frac{dv}{dx} = -\cos^2 v$$

$$\frac{dv}{\cos^2 v} = -\frac{dx}{x}$$

$$\sec^2 v \, dv = -\frac{dx}{x}$$



$$\int \sec^2 v \, dv = - \int \frac{dx}{x}$$

$$\tan v = -\log x + \log c$$

$$\tan v = \log \left| \frac{c}{x} \right|$$

$$e^{\tan v} = \frac{c}{x}$$

$$x e^{\tan v} = c$$

$$\therefore v = y/x$$

$$x e^{\tan(y/x)} = c$$

⑦

$$(1 + 3e^{y/x}) dy = -3e^{y/x} (1 - y/x) dx$$

$$\frac{dy}{dx} = \frac{-3e^{y/x} (1 - y/x)}{1 + 3e^{y/x}}$$

$$\text{put } v = y/x$$

$$y = vx$$

$$D.W. \text{ r t o m}$$

$$\frac{dy}{dx} = v + x \frac{dv}{dx} \rightarrow \textcircled{1}$$

$$\frac{dy}{dx} = \frac{-3e^v (1-v)}{1+3e^v} \rightarrow \textcircled{2}$$

from ① & ②.

$$v + x \frac{dv}{dx} = \frac{-3e^v (1-v)}{1+3e^v}$$

$$x \frac{dv}{dx} = \frac{-3e^v (1-v)}{1+3e^v} - v$$

$$x \frac{dv}{dx} = \frac{-3e^v + 3e^v v - v(1+3e^v)}{1+3e^v}$$

$$= \frac{-3e^v + 3e^v v - v - 3ve^v}{1+3e^v}$$

short cut  
y=v, x=1

$$x \frac{dv}{dx} = \frac{-3e^u - u}{1+3e^u} = \frac{-(3e^u + u)}{1+3e^u}$$

$$\frac{1+3e^u}{3e^u + u} dv = - \frac{dx}{x}$$

$$\int \frac{1+3e^u}{3e^u + u} dv = - \int \frac{dx}{x}$$

$$\log |3e^u + u| = -\log x + \log c.$$

$$\log |3e^u + u| + \log x = \log c.$$

$$\log |(3e^u + u)x| = \log c$$

$$(3e^u + u)x = c$$

$$\therefore u = y/x$$

$$[3e^{y/x} + y/x]x = c$$

$$3xe^{y/x} + y = c.$$

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$$(8) \quad (x^2 + y^2) dy = xy dx$$

$$\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$$

short cut  
 $y = vx, x = 1$

$$\frac{dy}{dx} = \frac{v}{1+v^2} \rightarrow (2)$$

$$\text{put } v = y/x$$

$$y = vx$$

$$\text{D.w.r to } x$$

$$\frac{dy}{dx} = v + x \frac{dv}{dx} \rightarrow (1)$$

from (1) & (2)

$$v + x \frac{dv}{dx} = \frac{v}{1+v^2}$$

$$x \frac{dv}{dx} = \frac{v}{1+v^2} - v$$

$$x \frac{dv}{dx} = \frac{v - v(1+v^2)}{1+v^2}$$

$$x \frac{dv}{dx} = \frac{\cancel{v} - v - v^3}{1+v^2}$$

$$\frac{1+v^2}{v^3} dv = - \frac{dx}{x}$$

$$\int \frac{1}{x^n} dx = \frac{-1}{(n-1)x^{n-1}} + c$$

$$\int \left( \frac{1}{v^3} + \frac{1}{v} \right) dv = - \int \frac{dx}{x}$$

$$\int \frac{1}{x} dx = \log x + c$$

$$\frac{-1}{2v^2} + \log v = -\log x + \log c$$

$$\log v + \log x - \log c = \frac{1}{2v^2}$$

$$\log \left| \frac{vx}{c} \right| = \frac{1}{2v^2}$$

$$\frac{vx}{c} = e^{\left(\frac{1}{2vx}\right)}$$

$$\therefore \frac{y}{x} = v$$

$$\frac{(y/x)x}{c} = e^{\left(\frac{1}{2(y/x)^2}\right)}$$

$$y = ce^{\left(\frac{x^2}{2y^2}\right)}$$

→ ①

when  $x=1$ ,  $y=1$ .

$$1 = ce^{\left(\frac{1}{2}\right)}$$

$$\frac{1}{e^{1/2}} = c$$

$$e^{-1/2} = c$$

$$y = e^{-1/2} e^{\frac{x^2}{2y^2}}$$

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Example 10.17

$$(x^2 - 3y^2) dx + 2xy dy = 0.$$

$$2xy dy = -(x^2 - 3y^2) dx$$

$$\frac{dy}{dx} = \frac{-(x^2 - 3y^2)}{2xy}$$

$$\frac{dy}{dx} = \frac{-(1 - 3v^2)}{2v} \rightarrow \textcircled{2}$$

$$\text{Put } v = y/x$$

$$y = vx$$

$$\frac{dy}{dx} = v + x \frac{dv}{dx} \rightarrow \textcircled{1}$$

from ① &amp; ②

$$v + x \frac{dv}{dx} = \frac{-(1 - 3v^2)}{2v}$$

$$x \frac{dv}{dx} = \frac{-(1 - 3v^2)}{2v} - v$$

$$x \frac{dv}{dx} = \frac{-1 + 3v^2 - 2v^2}{2v}$$

$$x \frac{dv}{dx} = \frac{-1 + v^2}{2v}$$

$$\frac{2v}{-1 + v^2} dv = \frac{dx}{x}$$

$$\int \frac{2v}{v^2 - 1} dv = \int \frac{dx}{x}$$

$$\log(v^2 - 1) = \log x + \log c.$$

$$\log(v^2 - 1) = \log xc$$

$$v^2 - 1 = xc$$

$$\frac{y^2 - x^2}{x^2} = xc$$

$$\boxed{y^2 - x^2 = x^3 c}$$

Example 10.18

$$(y + \sqrt{x^2 + y^2}) dx - x dy = 0.$$

$$x dy = (y + \sqrt{x^2 + y^2}) dx$$

$$\frac{dy}{dx} = \frac{y + \sqrt{x^2 + y^2}}{x}$$

$$\text{put } v = y/x$$

$$y = vx$$

$$\frac{dy}{dx} = v + x \frac{dv}{dx} \rightarrow \textcircled{1}$$

$$\frac{dy}{dx} = v + \sqrt{1+v^2} \rightarrow \textcircled{2}$$

from  $\textcircled{1}$  &  $\textcircled{2}$ ,

$$v + \sqrt{1+v^2} = v + x \frac{dv}{dx}$$

$$x \frac{dv}{dx} = \sqrt{1+v^2}$$

$$\frac{dv}{\sqrt{1+v^2}} = \frac{dx}{x}$$

$$\int \frac{dv}{\sqrt{1+v^2}} = \int \frac{dx}{x}$$

$$\log |v + \sqrt{1+v^2}| = \log x + \log e$$

$$\log |v + \sqrt{1+v^2}| = \log x e.$$

$$v + \sqrt{1+v^2} = x e$$

$$\frac{y}{x} + \frac{\sqrt{x^2 + y^2}}{x} = x e$$

$$y + \sqrt{x^2 + y^2} = x^2 e.$$

$$\text{when } x = 1, y = 0 \quad \boxed{e = 1}$$

$$\text{Required equation } y + \sqrt{x^2 + y^2} = x^2.$$

Example 10.19

$$(2x+3y) dx + (y-x) dy = 0$$

$$(y-x) dy = -(2x+3y) dx$$

$$\frac{dy}{dx} = \frac{2x+3y}{x-y}$$

$$\text{put } v = y/x$$

$$y = vx$$

$$\frac{dy}{dx} = v + x \frac{dv}{dx} \quad \text{--- (1)}$$

$$\frac{dy}{dx} = \frac{2+3v}{1-v} \quad \text{--- (2)}$$

from (1) &amp; (2)

$$v + x \frac{dv}{dx} = \frac{2+3v}{1-v}$$

$$x \frac{dv}{dx} = \frac{2+3v}{1-v} - v$$

$$x \frac{dv}{dx} = \frac{2+3v - v + v^2}{1-v}$$

$$x \frac{dv}{dx} = \frac{v^2 + 2v + 2}{1-v}$$

$$\frac{1-v}{v^2+2v+2} dv = \frac{dx}{x}$$

$$- \frac{1}{2} \int \frac{2v+2+4}{v^2+2v+2} dv = \int \frac{dx}{x}$$

$$\int \left( \frac{2v+2}{v^2+2v+2} - \frac{4}{v^2+2v+2} \right) dv = -2 \int \frac{dx}{x}$$

$$\log |v^2+2v+2| - 4 \tan^{-1}(v+1) = -2 \log x + \log c.$$



$$\log |v^2 + 2v + 2| + \log x^2 - 4 \tan^{-1}(v+1) = \log c.$$

$$\log |(v^2 + 2v + 2)x^2| - 4 \tan^{-1}(v+1) = \log c.$$

$$\log \left| \left( \frac{y^2}{x^2} + 2 \frac{y}{x} + 2 \right) x^2 \right| - 4 \tan^{-1} \left( \frac{y}{x} + 1 \right) = c$$

$$\log |y^2 + 2xy + x^2| - 4 \tan^{-1} \left( \frac{x+y}{x} \right) = c$$

Example 10.20.

$$y^2 + x^2 \frac{dy}{dx} = xy \frac{dy}{dx}$$

$$x^2 \frac{dy}{dx} - xy \frac{dy}{dx} = -y^2.$$

$$(x^2 - xy) \frac{dy}{dx} = -y^2$$

$$\frac{dy}{dx} = \frac{-y^2}{x^2 - xy}$$

$$\frac{dy}{dx} = \frac{y^2}{xy - x^2}$$

$$\frac{dy}{dx} = \frac{v^2}{v-1} \rightarrow \textcircled{2}.$$

put  $v = y/x$

$y = vx$

D. w. r to  $x$

$$\frac{dy}{dx} = v + x \frac{dv}{dx} \rightarrow \textcircled{1}$$

from  $\textcircled{1}$  &  $\textcircled{2}$ .

$$v + x \frac{dv}{dx} = \frac{v^2}{v-1}$$

$$x \frac{dv}{dx} = \frac{v^2 - v^2 + v}{v-1}$$

$$x \frac{dv}{dm} = \frac{v}{v-1}$$

$$\frac{v-1}{v} dv = \frac{dm}{m}$$

$$\int (1 - 1/v) dv = \int \frac{dm}{m}$$

$$v - \log v = \log m + \log e$$

$$v = \log v \times c.$$

$$e^v = v \times c.$$

$$e^{y/x} = y/x \times c$$

$$e^{y/x} = y \times c$$

Example 10-21.

$$(1 + 2e^{x/y}) dx + 2e^{x/y} (1 - x/y) dy = 0.$$

$$\frac{dx}{dy} = \frac{-2e^{x/y} (1 - x/y)}{(1 + 2e^{x/y})}$$

$$v = x/y$$

$$x = vy$$

$$dx = v dy + y dv.$$

Shortcut  
x=v, y=1

$$\frac{dx}{dy} = \frac{-2e^v (1-v)}{1+2e^v} \rightarrow (2)$$

$$\frac{dx}{dy} = v + y \frac{dv}{dy} \rightarrow (1)$$

from (1) & (2)

$$v + y \frac{dv}{dy} = \frac{-2e^v (1-v)}{1+2e^v}$$

$$y \frac{dv}{dy} = \frac{-2e^v + 2e^v \cdot v}{1 + 2e^v} - v$$

$$= \frac{-2e^v + 2ve^v - 2e^v - v}{1 + 2e^v}$$

$$y \frac{dv}{dy} = - \frac{(v + 2e^v)}{1 + 2e^v}$$

$$\frac{1 + 2e^v}{v + 2e^v} dv = - \frac{dy}{y}$$

$$\int \frac{1 + 2e^v}{v + 2e^v} dv = - \int \frac{dy}{y}$$

$$\log(v + 2e^v) = -\log y + \log c$$

$$\log[y(v + 2e^v)] = \log c$$

$$y(v + 2e^v) = c$$

$$y \left[ \frac{x}{y} + 2e^{x/y} \right] = c$$

$$\boxed{x + 2ye^{x/y} = c}$$

First Order Linear Differential equation.

case (i)

The function linear in  $y$ .

$$\frac{dy}{dx} + py = Q.$$

Integrating factor.

$$IF = e^{\int p dx}$$

General solution:

$$y(IF) = \int Q(IF) dx$$

Example 10.22.

$$\frac{dy}{dx} + 2y = e^{-x}.$$

The function linear in  $y$ ,  $\frac{dy}{dx} + py = Q.$

$$\therefore p = 2, \quad Q = e^{-x}.$$

$$IF = e^{\int p dx} = e^{\int 2 dx} = e^{2 \int dx} = e^{2x}$$

General solution

$$y(IF) = \int Q(IF) dx.$$

$$y(e^{2x}) = \int e^{-x} e^{2x} dx$$

$$y e^{2x} = \int e^x dx$$

$$y e^{2x} = e^x + C.$$

case (ii)

The function linear in  $x$ .

$$\frac{dx}{dy} + px = Q.$$

Integrating factor.

$$IF = e^{\int p dy}$$

General solution

$$x(IF) = \int Q(IF) dy.$$

ExE 10-7

$$1. \quad \cos x \frac{dy}{dx} + y \sin x = 1.$$

$$\div \cos x \Rightarrow \frac{dy}{dx} + \frac{\sin x}{\cos x} \cdot y = \frac{1}{\cos x}$$

$$\frac{dy}{dx} + \tan x \cdot y = \sec x$$

The function linear in  $y$ ,  $\frac{dy}{dx} + py = Q$ .

$$p = \tan x, \quad Q = \sec x.$$

Integrating factor.

$$I.F. = e^{\int p dx} = e^{\int \tan x dx} = e^{\log \sec x} = \sec x$$

$$\therefore \int \tan x dx = \log \sec x$$

General solution.

$$y(I.F.) = \int Q(I.F.) dx.$$

$$y(\sec x) = \int \sec x \sec x dx.$$

$$= \int \sec^2 x dx$$

$$\therefore \int \sec^2 x dx = \tan x + c$$

$$y \sec x = \tan x + c$$

$$\frac{y}{\cos x} = \frac{\sin x}{\cos x} + c$$

$$\times \cos x \Rightarrow y = \sin x + c \cos x$$

$$y = \sin x + c$$

$$(5) (1-x^2) \frac{dy}{dx} - xy = 1$$

$$\div (1-x^2) \Rightarrow \frac{dy}{dx} - \frac{x}{(1-x^2)} \cdot y = \frac{1}{1-x^2}$$

the function linear in  $y$ ,  $\frac{dy}{dx} + py = Q$ .

$$p = \frac{-x}{1-x^2} \quad , \quad Q = \frac{1}{1-x^2}$$

Integrating factor.

$$\begin{aligned} IF = e^{\int p dx} &= e^{\int \frac{-x}{1-x^2} dx} = e^{\frac{1}{2} \int \frac{-2x}{1-x^2} dx} = e^{\frac{1}{2} \log(1-x^2)} \\ &= e^{\log(1-x^2)^{1/2}} \\ IF &= (1-x^2)^{1/2} = \sqrt{1-x^2}. \end{aligned} \quad m \log A = \log A^m.$$

General solution.

$$y(IF) = \int Q(IF) dx.$$

$$y(\sqrt{1-x^2}) = \int \frac{1}{1-x^2} \cdot \sqrt{1-x^2} dx$$

$$y\sqrt{1-x^2} = \int \frac{1}{\sqrt{1-x^2}} dx \quad \therefore \int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1}x + c$$

$$y\sqrt{1-x^2} = \sin^{-1}x + c.$$

$$y = \frac{\sin^{-1}x}{\sqrt{1-x^2}} + c.$$

$$3. \quad \frac{dy}{dx} + \frac{y}{x} = \sin x.$$

The function linear in  $y$ ,  $\frac{dy}{dx} + py = Q$ ,

$$p = 1/x \quad Q = \sin x.$$

Integrating factor.

$$IF = e^{\int p dx} = e^{\int 1/x dx} = e^{\log x} = x.$$

General solution.

$$y(IF) = \int Q(IF) dx$$

$$yx = \int x \sin x dx$$

ILATE

$$u = x \quad dv = \sin x dx$$

$$u' = 1 \quad v = -\cos x$$

$$v' = -\sin x.$$

$$\int u dv = uv - u'v' + u''v_2$$

$$xy = x(-\cos x) - (1)(-\sin x) + C.$$

$$xy = -x \cos x + \sin x + C.$$

$$xy + x \cos x = \sin x + C.$$

$$x(y + \cos x) = \sin x + C.$$



4.  $(x^2+1) \frac{dy}{dx} + 2xy = \sqrt{x^2+4}$

$$\frac{dy}{dx} + \frac{2x}{x^2+1} \cdot y = \frac{\sqrt{x^2+4}}{x^2+1}$$

The function linear in  $y$ ,  $\frac{dy}{dx} + Py = Q$

$$P = \frac{2x}{x^2+1} \quad Q = \frac{\sqrt{x^2+4}}{x^2+1}$$

Integrating factor,

$$IF = e^{\int P dx} = e^{\int \frac{2x}{x^2+1} dx} = e^{\log|x^2+1|} = x^2+1$$

General solution,

$$y(IF) = \int Q(IF) dx$$

$$y(x^2+1) = \int \frac{\sqrt{x^2+4}}{(x^2+1)} \cdot (x^2+1) dx$$

$$y(x^2+1) = \int \sqrt{x^2+4} dx$$

$$\int \sqrt{x^2+a^2} dx = \frac{x}{2} \sqrt{x^2+a^2} + \frac{a^2}{2} \log|x+\sqrt{x^2+a^2}| + C$$

$$y(x^2+1) = \frac{x}{2} \sqrt{x^2+4} + \frac{4}{2} \log|x+\sqrt{x^2+4}| + C$$

$$y(x^2+1) = \frac{x}{2} \sqrt{x^2+4} + 2 \log|x+\sqrt{x^2+4}| + C$$

$$5. (2x - 10y^3) dy + y dx = 0$$

$$y dx = -(2x - 10y^3) dy$$

$$\div dy \quad y \frac{dx}{dy} = -2x + 10y^3.$$

$$\div y \quad \frac{dx}{dy} = -\frac{2x}{y} + 10y^2$$

$$\frac{dx}{dy} + \frac{2}{y} \cdot x = +10y^2.$$

The function linear in  $x$ ,  $\frac{dx}{dy} + Px = Q$ .

$$P = \frac{+2}{y} \quad Q = +10y^2.$$

Integrating factor.

$$IF = e^{\int P dy} = e^{\int \frac{2}{y} dy} = e^{2 \log y} = e^{\log y^2}$$

$$IF = y^2.$$

General solution.

$$x(IF) = \int Q(IF) dy.$$

$$xy^2 = \int 10y^2 \cdot y^2 dy.$$

$$xy^2 = 10 \int y^4 dy.$$

$$xy^2 = 10 \frac{y^5}{5} + C$$

$$xy^2 = 2y^5 + C.$$

$$b. \quad x \sin x \frac{dy}{dx} + (x \cos x + \sin x) y = \sin x.$$

$$\div x \sin x \Rightarrow \frac{dy}{dx} + \left[ \frac{x \cos x + \sin x}{x \sin x} \right] y = \frac{\sin x}{x \sin x}$$

$$\frac{dy}{dx} + \left[ \frac{x \cos x + \sin x}{x \sin x} \right] y = \frac{1}{x}.$$

The function linear in  $y$ ,  $\frac{dy}{dx} + Py = Q$ .

$$P = \frac{x \cos x + \sin x}{x \sin x}, \quad Q = \frac{1}{x}$$

Integrating factor.

$$IF = e^{\int P dx} = e^{\int \frac{x \cos x + \sin x}{x \sin x} dx} = e^{\log(x \sin x)}$$

$$IF = x \sin x.$$

General solution,

$$y(IF) = \int Q(IF) dx$$

$$y(x \sin x) = \int \left(\frac{1}{x}\right)(x \sin x) dx$$

$$xy \sin x = \int \sin x dx$$

$$xy \sin x = -\cos x + C.$$

$$xy \sin x + \cos x = C.$$

$$7. (y - e^{\sin^{-1}x}) \frac{dx}{dy} + \sqrt{1-x^2} = 0$$

$$(y - e^{\sin^{-1}x}) \frac{dx}{dy} = -\sqrt{1-x^2}$$

$$\frac{y - e^{\sin^{-1}x}}{\sqrt{1-x^2}} = -\frac{dy}{dx}$$

$$\frac{y}{\sqrt{1-x^2}} - \frac{e^{\sin^{-1}x}}{\sqrt{1-x^2}} = -\frac{dy}{dx}$$

$$\frac{dy}{dx} + \frac{y}{\sqrt{1-x^2}} = \frac{e^{\sin^{-1}x}}{\sqrt{1-x^2}}$$

The function linear in  $y$ ,  $\frac{dy}{dx} + Py = Q$ .

$$P = \frac{1}{\sqrt{1-x^2}} \quad Q = \frac{e^{\sin^{-1}x}}{\sqrt{1-x^2}}$$

Integrating factor,

$$IF = e^{\int P dx} = e^{\int \frac{1}{\sqrt{1-x^2}} dx} = e^{\sin^{-1}x}$$

$$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1}x$$

General solution,

$$y(IF) = \int Q(IF) dx$$

$$y e^{\sin^{-1}x} = \int \frac{e^{\sin^{-1}x}}{\sqrt{1-x^2}} \cdot e^{\sin^{-1}x} dx$$

$$\therefore \text{let } t = \sin^{-1}x$$

$$dt = \frac{1}{\sqrt{1-x^2}} dx$$

$$= \int e^{2t} dt$$

$$y e^{\sin^{-1}x} = \frac{e^{2t}}{2} + C$$

$$2y e^{\sin^{-1}x} = e^{2\sin^{-1}x} + C$$

$$8. \quad \frac{dy}{dx} + \frac{y}{(1-x)\sqrt{x}} = 1 - \sqrt{x}.$$

The function linear in  $y$ ,  $\frac{dy}{dx} + Py = Q$ .

$$P = \frac{1}{\sqrt{x}(1-x)} \quad Q = 1 - \sqrt{x}.$$

Integrating factor.

$$IF = e^{\int P dx} = e^{\int \frac{1}{\sqrt{x}(1-x)} dx}$$

$$= e^{\int \frac{2dt}{(1-t^2)}}$$

$$= e^{\frac{2 \log\left(\frac{1+t}{1-t}\right)}{2}}$$

$$= e^{\log\left(\frac{1+t}{1-t}\right)}$$

$$= \frac{1+t}{1-t}$$

$$IF = \frac{1+\sqrt{x}}{1-\sqrt{x}}$$

$$\text{let } t = \sqrt{x} \Rightarrow \boxed{t^2 = x}$$

$$dt = \frac{1}{2\sqrt{x}} dx$$

$$2dt = \frac{dx}{\sqrt{x}}$$

General solution.

$$y(IF) = \int Q(IF) dx.$$

$$y \left[ \frac{1+\sqrt{x}}{1-\sqrt{x}} \right] = \int (1-\sqrt{x}) \left[ \frac{1+\sqrt{x}}{1-\sqrt{x}} \right] dx$$

$$= \int (1+\sqrt{x}) dx$$

$$= x + \frac{x^{3/2}}{3/2} + C$$

$$\int x^{1/2} dx = \frac{x^{1/2+1}}{1/2+1} + C$$

$$y \left[ \frac{1+\sqrt{x}}{1-\sqrt{x}} \right] = x + \frac{2x^{3/2}}{3} + C$$

9.  $(1+x+xy^2) \frac{dy}{dx} + (y+y^3) = 0$

$$y(1+y^2) = - (1+x+xy^2) \frac{dy}{dx}$$

$$y(1+y^2) \frac{dx}{dy} = -1 - x(1+y^2)$$

$$\div y(1+y^2)$$

$$\frac{dx}{dy} = -\frac{1}{y(1+y^2)} - \frac{x}{y}$$

$$\frac{dx}{dy} + \frac{x}{y} = -\frac{1}{y(1+y^2)}$$

The function linear in  $x$ ,  $\frac{dx}{dy} + px = Q$ .

$$p = 1/y \quad Q = -\frac{1}{y(1+y^2)}$$

Integrating factor.

$$IF = e^{\int p dy} = e^{\int 1/y dy} = e^{\log y} = y.$$

General solution.

$$x(IF) = \int Q(IF) dy.$$

$$xy = \int -\frac{1}{y(1+y^2)} \cdot y dy$$

$$= - \int \frac{1}{1+y^2} dy$$

$$xy = -\tan^{-1}y + c$$

$$xy + \tan^{-1}y = c.$$

$$10. \quad \frac{dy}{dx} + \frac{y}{x \log x} = \frac{\sin 2x}{\log x}$$

The function linear in  $y$ ,  $\frac{dy}{dx} + py = Q$ .

$$P = \frac{1}{x \log x} \quad Q = \frac{\sin 2x}{\log x}$$

Integrating factor.

$$IF = e^{\int p dx} = e^{\int \frac{1}{x \log x} dx} = e^{\int \frac{(1/x)}{\log x} dx} = e^{\log(\log x)}$$

$$IF = \log x.$$

General solution.

$$y(IF) = \int Q(IF) dx$$

$$y(\log x) = \int \frac{\sin 2x}{\log x} \cdot \log x dx$$

$$= \int \sin 2x dx$$

$$y \log x = -\frac{\cos 2x}{2} + c$$

$$2y \log x = -\cos 2x + c$$

$$2y \log x + \cos 2x = c.$$



11.

$$(x+a) \frac{dy}{dx} - 2y = (x+a)^4$$

$$\div (x+a) \Rightarrow \frac{dy}{dx} - \frac{2y}{(x+a)} = (x+a)^3$$

The function linear in  $y$ ,  $\frac{dy}{dx} + py = Q$

$$P = \frac{-2}{(x+a)} \quad Q = (x+a)^3$$

Integrating factor

$$IF = e^{\int P dx} = e^{\int \frac{-2}{x+a} dx} = e^{-2 \log(x+a)}$$

$$IF = \frac{1}{(x+a)^2}$$

$$e^{-m \log A} = \frac{1}{A^m}$$

General solution.

$$y(IF) = \int Q(IF) dx$$

$$y \left( \frac{1}{(x+a)^2} \right) = \int (x+a)^3 \frac{1}{(x+a)^2} dx$$

$$\frac{y}{(x+a)^2} = \int (x+a) dx$$

$$\frac{y}{(x+a)^2} = \frac{(x+a)^2}{2} + C$$

$$\times 2(x+a)^2$$

$$2y = (x+a)^4 + 2C(x+a)^2$$

$$2y = (x+a)^4 + C$$

$$12. \frac{dy}{dx} = \frac{\sin^2 x}{1+x^3} - \frac{3x^2}{1+x^3} y.$$

$$\frac{dy}{dx} + \frac{3x^2}{1+x^3} y = \frac{\sin^2 x}{1+x^3}$$

The function linear in  $y$ ,  $\frac{dy}{dx} + py = Q$ .

$$P = \frac{3x^2}{1+x^3} \quad Q = \frac{\sin^2 x}{1+x^3}$$

Integrating factor.

$$IF = e^{\int P dx} = e^{\int \frac{3x^2}{1+x^3} dx} = e^{\log(1+x^3)} = 1+x^3.$$

$$IF = (1+x^3).$$

General solution.

$$y(IF) = \int Q(IF) dx$$

$$y(1+x^3) = \int \frac{\sin^2 x}{1+x^3} \cdot (1+x^3) dx$$

$$= \int \sin^2 x dx$$

$$= \int \left( \frac{1 - \cos 2x}{2} \right) dx$$

$$= \frac{1}{2} \int (1 - \cos 2x) dx$$

$$y(1+x^3) = \frac{1}{2} \left[ x - \frac{\sin 2x}{2} \right] + C$$

$$4y(1+x^3) = 2x - \sin 2x + C$$

13.  $x \frac{dy}{dx} + y = x \log x$

$$\frac{dy}{dx} + \frac{y}{x} = \log x.$$

The function linear in  $y$ ,  $\frac{dy}{dx} + py = Q$ .

$$p = 1/x, Q = \log x$$

Integrating factor.

$$IF = e^{\int p dx} = e^{\int \frac{dx}{x}} = e^{\log x} = x$$

$$IF = x$$

General solution.

$$y(IF) = \int Q(IF) dx$$

$$y x = \int \log x (x) dx$$

$$= \int x \log x dx$$

ILATE

$$u = \log x \quad du = \frac{1}{x} dx$$

$$u' = 1/x \quad v = \frac{x^2}{2}$$

$$= \frac{x^2}{2} \log x - \int \frac{x^2}{2} \frac{1}{x} dx \quad \int u dv = uv - \int v du$$

$$= \frac{x^2}{2} \log x - \int \frac{x}{2} dx$$

$$y(x) = \frac{x^2}{2} \log x - \frac{x^2}{4} + C.$$

$$4xy = 2x^2 \log x - x^2 + C.$$

$$14. \quad x \frac{dy}{dx} + 2y - x^2 \log x = 0.$$

$$x \frac{dy}{dx} + 2y = x^2 \log x$$

$$\frac{dy}{dx} + \frac{2y}{x} = x \log x$$

The function linear in  $y$ ,  $\frac{dy}{dx} + py = Q$ .

$$P = 2/x, \quad Q = x \log x$$

Integrating factor.

$$IF = e^{\int P dx} = e^{\int \frac{2}{x} dx} = e^{2 \log x} = x^2$$

$$IF = x^2.$$

$$\therefore e^{m \log A} = A^m.$$

General solution.

$$y(IF) = \int Q(IF) dx$$

$$y x^2 = \int x \log x (x^2) dx$$

$$= \int \frac{x^3 \log x}{u} dx$$

$$u = \log x \quad du = \frac{1}{x} dx$$

$$u' = 1/x \quad v = \frac{x^4}{4}$$

$$= \log x \left( \frac{x^4}{4} \right) - \int \frac{x^4}{4} \frac{1}{x} dx$$

$$= \frac{x^4 \log x}{4} - \int \frac{x^3}{4} dx.$$

$$x^2 y = \frac{x^4 \log x}{4} - \frac{x^4}{16} + C.$$

15.  $\frac{dy}{dx} + \frac{3}{x} y = \frac{1}{x^2}$ .

The function linear in  $y$ ,  $\frac{dy}{dx} + py = Q$ .

$$P = \frac{3}{x}, \quad Q = \frac{1}{x^2}$$

Integrating factor

$$IF = e^{\int p dx} = e^{\int \frac{3}{x} dx} = e^{3 \log x} = x^3$$

$$IF = x^3.$$

$$e^{m \log A} = A^m.$$

General solution.

$$y(IF) = \int Q(IF) dx$$

$$y(x^3) = \int \frac{1}{x^2} (x^3) dx$$

$$x^3 y = \int x dx$$

$$x^3 y = \frac{x^2}{2} + C.$$

when  $y = 2$ ,  $x = 1$

$$2 = \frac{1}{2} + C.$$

$$C = 2 - \frac{1}{2}$$

$$C = \frac{3}{2}.$$

The Required equation.

$$x^3 y = \frac{x^2}{2} + \frac{3}{2}$$

$$2x^3 y = x^2 + 3 //$$

Example 10.22

$$\frac{dy}{dx} + 2y = e^{-x}$$

The function linear in  $y$ ,  $\frac{dy}{dx} + py = Q$ .

$$P = 2, \quad Q = e^{-x}$$

Integrating factor

$$IF = e^{\int p dx} = e^{\int 2 dx} = e^{2x}$$

General solution,

$$y(IF) = \int Q(IF) dx$$

$$y e^{2x} = \int e^{-x} e^{2x} dx$$

$$= \int e^x dx$$

$$y e^{2x} = e^x + C$$

$$\div e^{2x} \Rightarrow$$

$$y = e^{-x} + C e^{-2x}$$

Example 10.23

$$\{ [y(1-x \tan x) + x^2 \cos x] dx - x dy = 0 \}$$

$$[y(1-x \tan x) + x^2 \cos x] dx = x dy$$

$$\frac{dy}{dx} = \frac{y(1-x \tan x) + x^2 \cos x}{x}$$

$$\frac{dy}{dx} - \frac{(1-x \tan x)}{x} y = x \cos x$$

$$\frac{dy}{dx} + \left( \frac{x \tan x - 1}{x} \right) y = x \cos x$$

The function linear in  $y$ ,  $\frac{dy}{dx} + py = Q$ .

Integrating factor.

$$\begin{aligned} IF &= e^{\int p dx} = e^{\int (\tan x - \frac{1}{x}) dx} = e^{(\log \sec x - \log x)} \\ &= e^{\log \left( \frac{\sec x}{x} \right)} = \frac{\sec x}{x} = \frac{1}{x \cos x}. \end{aligned}$$

General solution.

$$y(IF) = \int Q dx (IF)$$

$$\frac{y}{x \cos x} = \int (x \cos x) \left( \frac{1}{x \cos x} \right) dx$$

$$= \int dx$$

$$\frac{y}{x \cos x} = x + C.$$

$$y = x^2 \cos x + x \cos x C.$$

$$y = x^2 \cos x + C$$

Example 10.24

$$\frac{dy}{dx} + 2y \cot x = 3x^2 \sec^2 x.$$

The function linear in  $y$ ,  $\frac{dy}{dx} + py = Q$ .

$$p = 2 \cot x$$

$$Q = 3x^2 \sec^2 x.$$

Integrating factor.

$$If = e^{\int p dx} = e^{\int 2 \cot x} = e^{2 \log \sin x} = e^{\log \sin^2 x}$$

$$If = \sin^2 x$$



General solution

$$y(IF) = \int Q(IF) dx.$$

$$y \sin^2 x = \int 3x^2 \cos^2 x \sin^2 x \, dx.$$

$$= \int 3x^2 \left( \frac{1}{\sin^2 x} \right) \sin^2 x \, dx.$$

$$= \int 3x^2 \, dx$$

$$= \frac{3x^3}{3} + C.$$

$$y \sin^2 x = x^3 + C.$$

Example 10.25

$$(1+x^3) \frac{dy}{dx} + 6x^2 y = 1+x^2.$$

$$\frac{dy}{dx} + \frac{6x^2}{(1+x^3)} y = \frac{1+x^2}{1+x^3}$$

The function linear in  $y$ ,  $\frac{dy}{dx} + py = Q$ .

$$P = \frac{6x^2}{1+x^3} \quad Q = \frac{1+x^2}{1+x^3}.$$

Integrating factor,

$$IF = e^{\int p \, dx} = e^{\int \frac{6x^2}{1+x^3} \, dx} = e^{2 \int \frac{3x^2}{1+x^3} \, dx}$$

$$IF = e^{2 \log(1+x^3)} = (1+x^3)^2.$$

General solution.

$$y(IF) = \int Q(IF) dx$$

$$y \cdot \frac{1}{(1+x^3)^2} = \int \left( \frac{1+x^2}{1+x^3} \right) (1+x^3)^2 dx$$

$$= \int (1+x^2)(1+x^3) dx$$

$$= \int (x^5 + x^3 + x^2 + 1) dx$$

$$y(1+x^3)^2 = \frac{x^6}{6} + \frac{x^4}{4} + \frac{x^3}{3} + x + C.$$

Example 10-26

$$ye^y dx = (y^3 + 2xe^y) dy.$$

$$\frac{dx}{dy} = \frac{y^3 + 2xe^y}{ye^y}$$

$$\frac{dx}{dy} = \frac{y^2}{e^y} + \frac{2x}{y}$$

$$\frac{dx}{dy} - \frac{2}{y} \cdot x = y^2 e^{-y}.$$

The function linear in  $x$ ,  $\frac{dx}{dy} + Px = Q$ .

$$P = -2/y \quad Q = y^2 e^{-y}.$$

General solution,  $IF = e^{\int P dy} = e^{\int -2/y dy} = e^{-2 \log y}.$

$$IF = 1/y^2.$$

$$x(IF) = \int Q(IF) dy.$$

$$x(1/y) = \int x(1/y) dy.$$

$$x(1/y^2) = \int y^2 e^{-y} (1/y^2) dy$$

$$= \int e^{-y} dy$$

$$x/y^2 = -e^{-y} + C.$$

$$x = -y^2 e^{-y} + C y^2.$$

$$x + y^2 e^{-y} = C$$

Application of First order differential equation

$$\frac{dA}{dt} \propto A$$

$$\frac{dA}{dt} = kA$$

$$\frac{dA}{A} = k dt$$

$$\int \frac{dA}{A} = k \int dt$$

$$\log A = kt + C.$$

$$A = e^{kt+C}$$

$$A = e^{kt} e^C$$

$$\therefore e^C = C$$

$$\boxed{A = Ce^{kt}}$$

EXE 10.8

1. let A be the number of bacteria at any time t.

$$\frac{dA}{dt} \propto A$$

$$\frac{dA}{dt} = kA$$

$$A = Ce^{kt} \rightarrow (1)$$

|   |                |                 |    |
|---|----------------|-----------------|----|
| t | 0              | 5               | 10 |
| A | A <sub>0</sub> | 3A <sub>0</sub> | -  |

when  $t=0$   $A=A_0$

$$A_0 = Ce^{0k}$$

$$A_0 = C$$

when  $t=5$ ,  $A=3A_0$

$$3A_0 = A_0 e^{5k}$$

$$3 = e^{5k}$$

when  $t=10$ , Then A

$$A = A_0 e^{10k}$$

$$= A_0 (e^{5k})^2$$

$$= A_0 (3)^2$$

$$A = 9A_0$$

After 10 hours  $A = 9$  (times of initial value)

2. Let  $A$  be the population of city at any time  $t$ .

$$\frac{dA}{dt} \propto A$$

$$\frac{dA}{dt} = kA$$

$$A = ce^{kt} \rightarrow (1)$$

|     |          |          |  |
|-----|----------|----------|--|
| $t$ | 0        | 40       |  |
| $A$ | 3,00,000 | 4,00,000 |  |

when  $t = 0$ ,  $A = 3,00,000$

$$3,00,000 = ce^{0k}$$

$$3,00,000 = c$$

when  $t = 40$ ,  $A = 4,00,000$

$$4,00,000 = 3,00,000 e^{40k}$$

$$4/3 = e^{40k}$$

$$4/3 = (e^k)^{40}$$

$$(4/3)^{1/40} = e^k$$

(1)  $\Rightarrow$

$$A = ce^{kt}$$

$$A = c(e^k)^t$$

$$A = 3,00,000 (4/3)^{t/40}$$

$$A = 3,00,000 (4/3)^{t/40}$$

5. let  $A$  be the amount at time  $t$ .

$$\frac{dA}{dt} \propto A$$

$$\frac{dA}{dt} = kA$$

$$k = 5\%$$

$$k = 0.05$$

$$A = ce^{kt}$$

$$A = ce^{0.05t} \rightarrow \textcircled{1}$$

when  $t = 0$ ,  $A = 10,000$

$$10,000 = ce^{0.05(0)}$$

$$10,000 = c$$

when  $t = 3/2$  (or) 18 month (or)  $1\frac{1}{2}$  yrs., then  $A$ .

$$A = 10,000 e^{0.05(3/2)}$$

$$A = 10,000 e^{0.075}$$

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b. let  $A$  be the radioactive nuclei at time  $t$ .

$$\frac{dA}{dt} \propto A$$

$$\frac{dA}{dt} = kA$$

$$\boxed{A = ce^{kt}} \rightarrow \textcircled{1}$$

when  $t=0$ ,  $A=A_0$ .

$$A_0 = ce^{0k}$$

$$\boxed{A_0 = c}$$

when  $t=100$ ,  $A = 90\% A_0$

$$A = \frac{9A_0}{10}$$

$$\frac{9A_0}{10} = A_0 e^{100k}$$

$$\boxed{\frac{9}{10} = e^{100k}}$$

when  $t = 1000$ , then  $A$

$$A = A_0 e^{1000k}$$

$$= A_0 (e^{100k})^{10}$$

$$= A_0 \left(\frac{9}{10}\right)^{10}$$

$$= A_0 \left(\frac{9}{10}\right)^{10} \times 100\%$$

$$A = A_0 \frac{9^{10}}{10^8} \%$$

$\frac{9^{10}}{10^8} \%$  of the radioactive element will remain after 1000 yrs.



Example 10.28

Let  $A$  be the mass of the isotope remaining after  $t$  time

$$\frac{dA}{dt} \propto A$$

$$\frac{dA}{dt} = -kA$$

$$A = ce^{-kt} \rightarrow (1)$$

when  $t=0$ ,  $A=200$

$$200 = ce^{-0k}$$

$$200 = c$$

when  $t=2$ ,  $A=50$

$$50 = 200e^{-2k}$$

$$\frac{1}{4} = e^{-2k}$$

$$\left(\frac{1}{2}\right)^2 = e^{-2k}$$

$$\frac{1}{2} = e^{-k}$$

$$A = ce^{-kt}$$

$$(1) \quad A = 200 \left(\frac{1}{2}\right)^t \rightarrow (2)$$

when  $t = t_n$  (half time),  $A = 100$ .

$$100 = 200 \left(\frac{1}{2}\right)^{t_n}$$

$$\left(\frac{1}{2}\right)^1 = \left(\frac{1}{2}\right)^{t_n}$$

$$\therefore t_n = 1 \text{ yrs}$$

Example 10.27

let  $A$  be the population at time  $t$

$$\frac{dA}{dt} \propto A$$

$$\frac{dA}{dt} = +kA$$

$$\boxed{A = Ce^{kt}} \rightarrow \textcircled{1}$$

when  $t=0$ ,  $A=A_0$

$$A_0 = Ce^{0k}$$

$$\boxed{A_0 = C}$$

when  $t=50$ ,  $A=2A_0$

$$2A_0 = A_0 e^{50k}$$

$$2 = e^{50k}$$

Taking log on both side

$$\log 2 = 50k$$

$$k = \frac{\log 2}{50}$$

$\textcircled{1} \Rightarrow$

$$A = A_0 e^{\left(\frac{\log 2}{50}\right)t}$$

$$= A_0 \left(e^{\log 2}\right)^{t/50}$$

$$A = A_0 (2)^{t/50} \rightarrow \textcircled{2}$$

when  $A = 3A_0$ ,  $t = t_1$

$$\textcircled{2} \Rightarrow 3A_0 = A_0 (2)^{t_1/50}$$

$$3 = (2)^{t_1/50}$$

Taking log on both side.

$$\log 3 = \log (2)^{t_1/50}$$

$$\log 3 = \frac{t_1}{50} (\log 2)$$

$$\frac{\log 3}{\log 2} = \frac{t_1}{50}$$

$$t_1 = 50 \left( \frac{\log 3}{\log 2} \right)$$

The population is tripled in  $50 \left( \frac{\log 3}{\log 2} \right)$  yrs.

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3.

$$E = Ri + L \frac{di}{dt}$$

$$L \frac{di}{dt} + Ri = E$$

$$\frac{di}{dt} + \frac{R}{L} i = \frac{E}{L}$$

The function linear in  $i$ ,  $\frac{di}{dt} + Pi = Q$ ,

$$P = \frac{R}{L} \quad Q = \frac{E}{L}$$

Integrating factor

$$IF = e^{\int P dt} = e^{\int \frac{R}{L} dt} = e^{\frac{Rt}{L}}$$

General solution,

$$i(IF) = \int Q(IF) dt$$

$$i e^{\frac{Rt}{L}} = \int \frac{E}{L} e^{\frac{Rt}{L}} dt$$

$$= \frac{E}{L} \frac{e^{\frac{Rt}{L}}}{\frac{R}{L}} + C$$

$$i e^{\frac{Rt}{L}} = \frac{E}{R} e^{\frac{Rt}{L}} + C$$

when  $E=0$ ,

$$i = \frac{C}{e^{\frac{Rt}{L}}}$$

$$i = C e^{-\frac{Rt}{L}}$$

4. let  $v$  be the velocity at time  $t$

Retardation = velocity.

$$-\frac{dv}{dt} = v$$

$$\frac{dv}{v} = -dt$$

$$\int \frac{dv}{v} = -\int dt$$

$$\log v = -t + \log c$$

$$\log v - \log c = -t$$

$$\log (v/c) = -t$$

$$v/c = e^{-t}$$

$$v = ce^{-t} \rightarrow \textcircled{1}$$

when  $t=0$ ,  $v=10$

$$10 = ce^{-0}$$

$$\boxed{10 = c}$$

$$\textcircled{1} \Rightarrow \therefore v = 10e^{-t}$$

when  $t=2$ , then  $v$ .

$$v = 10e^{-2}$$

$$\boxed{v = \frac{10}{e^2}}$$

7. let  $T$  be the temperature at time  $t$ .  
let  $S$  be the room temperature.

$$\frac{dT}{dt} \propto (T-S) \Rightarrow \frac{dT}{dt} = k(T-S)$$

$$T-S = Ce^{kt} \rightarrow (1)$$

|     |     |    |    |    |
|-----|-----|----|----|----|
| $t$ | 0   | 10 | 20 | -  |
| $T$ | 100 | 80 | -  | 40 |
| $S$ | 25  |    |    |    |

when  $t=0$ ,  $T=100$ ,  $S=25$

$$100-25 = Ce^{0k}$$

$$\boxed{75 = C}$$

when  $t=10$ ,  $T=80$ ,  $S=25$

$$80-25 = 75e^{10k}$$

$$55 = 75e^{10k}$$

$$\frac{55}{75} = e^{10k}$$

$$\boxed{e^{10k} = \frac{11}{15}}$$

Taking log on both side.

$$10k = \log \frac{11}{15}$$

$$10k = -0.3101$$

$$\boxed{k = -0.0310}$$

when  $T = 40$ ,

$$40 - 25 = 15 e^{kt}$$

$$15 = 15 e^{kt}$$

$$\frac{15}{15} = e^{kt}$$

$$1 = e^{kt}$$

Taking reciprocal

$$1 = e^{-kt}$$

Taking log on both side

$$\log 1 = -kt$$

$$0 = -(-0.0310)t$$

$$t = \frac{1.6094}{0.0310}$$

$$t = 51.89 \text{ min.}$$

- ⑤. Let  $T$  be the temperature of coffee at time  $t$   
Let  $S$  be the room temperature.

$$\frac{dT}{dt} \propto (T - S)$$

$$\frac{dT}{dt} = k(T - S)$$

$$T - S = Ce^{kt} \rightarrow (1)$$

when  $t = 0$ ,  $T = 180^\circ$ ,  $S = 70^\circ$

$$180^\circ - 70^\circ = Ce^{0k}$$

$$\boxed{110 = C}$$

|     |     |     |                |     |     |
|-----|-----|-----|----------------|-----|-----|
| $t$ | 0   | 10  | 15             | -   | -   |
| $T$ | 180 | 160 | -              | 130 | 140 |
|     |     |     | $S = 70^\circ$ |     |     |



when  $t = 10$ ,  $T = 160$ ,  $s = 70^\circ$

$$160 - 70 = 110 e^{10k}$$

$$90 = 110 e^{10k}$$

$$\boxed{\frac{9}{11} = e^{10k}}$$

$$(e^{5k})^2 = 9/11$$

Taking log on both side.  $e^{5k} = (9/11)^{1/2}$

$$10k = \log 9/11$$

$$10k = -0.2006$$

$$\boxed{k = -0.02006}$$

f) when  $t = 15$ ,  $T = ?$   $s = 70^\circ$

$$T = 70 + 110 e^{15k}$$

$$= 70 + 110 (e^{5k})^3$$

$$= 70 + 110 \left( (9/11)^{1/2} \right)^3$$

$$= 70 + 110 (9/11)^{3/2}$$

$$= 70 + 81.4$$

$$\boxed{T = 151.4^\circ \text{ F}}$$

when  $T = 130^\circ$ ,  $t =$  ,  $S = 70^\circ$

$$130^\circ - 70^\circ = 110^\circ e^{kt}$$

$$\frac{60}{110} = e^{kt}$$

$$\log \left( \frac{6}{11} \right) = kt$$

$$-0.606135 = -0.02006 t$$

$$t = \frac{-0.606135}{-0.02006}$$

$$t = 30.216 \text{ min.}$$

when  $T = 140^\circ$ ,  $t = ?$ ,  $S = 70^\circ$

$$140^\circ - 70^\circ = 110^\circ e^{kt}$$

$$\frac{70}{110} = e^{kt}$$

$$\log 7/11 = kt$$

$$-0.45198 = -0.02006 t$$

$$t = \frac{-0.45198}{-0.02006}$$

$$t = 22.53 \text{ min.}$$

Temperature between  $130^\circ$  to  $140^\circ$  between.

10.22 - 10.30 AM (app).

9. Let  $T$  be the temperature of boiling water at time  $t$ ,  
let  $S$  be the room temperature,

$$\frac{dT}{dt} \propto (T-S) \Rightarrow \frac{dT}{dt} = k(T-S)$$

$$T-S = Ce^{kt} \rightarrow \text{①}$$

|     |     |    |    |
|-----|-----|----|----|
| $t$ | 0   | 5  | 10 |
| $T$ | 100 | 80 | 65 |

Then  $S = ?$

when  $t = 0$ ,  $T = 100$

$$100 - S = Ce^{0k}$$

$$\boxed{100 - S = C}$$

when  $t = 5$ ,  $T = 80$

$$(80 - S) = (100 - S)e^{5k}$$

$$\boxed{e^{5k} = \frac{80 - S}{100 - S}}$$

when  $t = 10$ ,  $T = 65$

$$(65 - S) = (100 - S)e^{10k}$$

$$(65 - S) = (100 - S)(e^{5k})^2$$

$$(65 - S) = \frac{(100 - S)(80 - S)^2}{(100 - S)^2}$$

$$(65 - S)(100 - S) = (80 - S)^2$$

$$6500 - 165S + S^2 = 6400 + S^2 - 160S$$

$$100 - 5S = 0$$

$$100 = 5S$$

$$\boxed{S = 20^\circ}$$

Room temperature  $S = 20^\circ$

⑩ let  $x$  denote the amount of salt in the tank at time  $t$ .

$$\frac{dx}{dt} = \text{inflow} - \text{outflow},$$

$$\therefore \text{inflow} = 2 \times 3 = 6.$$

$$\frac{dx}{dt} = 6 - \frac{3x}{50}$$

$$\text{outflow} = \frac{3}{50}x = \frac{3x}{50}$$

$$\frac{dx}{dt} = -\frac{3}{50} \left[ x - \frac{6 \times 50}{3} \right]$$

$$\frac{dx}{dt} = -\frac{3}{50} [x - 100]$$

$$\frac{dx}{x-100} = -\frac{3}{50} dt$$

$$\int \frac{dx}{x-100} = -\frac{3}{50} \int dt$$

$$\log(x-100) = -\frac{3}{50}t + \log c.$$

$$\log(x-100) - \log c = -\frac{3t}{50}$$

$$\log\left(\frac{x-100}{c}\right) = -\frac{3t}{50}$$

$$(x-100) = c e^{-\frac{3t}{50}} \rightarrow \text{①}.$$

when  $t=0$ ,  $x=0$ .

$$\text{①} \Rightarrow \boxed{-100 = c}$$

$$\text{①} \rightarrow (x-100) = -100 e^{-\frac{3t}{50}}$$

$$x = 100 - 100 e^{-\frac{3t}{50}}$$

$$x = 100 \left( 1 - e^{-\frac{3t}{50}} \right).$$

Example 10.29

Let  $T$  be the temperature of the body at time  $t$ ,

$$\frac{dT}{dt} \propto (T-s) \Rightarrow \frac{dT}{dt} = k(T-s)$$

$$\boxed{T-s = ce^{kt}} \rightarrow \text{①}$$

|     |    |    |      |
|-----|----|----|------|
| $t$ | 0  | 2  | -    |
| $T$ | 70 | 60 | 98.6 |

When  $T = 70$ ,  $s = 50$ ,  $t = 0$ .

$$s = 50$$

$$70 - 50 = ce^{0k}$$

$$\boxed{20 = c}$$

When  $t = 2$ ,  $T = 60$ ,  $s = 50$

$$60 - 50 = 20 e^{2k}$$

$$\frac{10}{20} = e^{2k}$$

$$1/2 = e^{2k}$$

$$\boxed{e^k = (1/2)^{1/2}}$$

When  $T = 98.6$ ,  $s = 50$ ,  $t = t_{\text{ind}}$ .

$$98.6 - 50 = 20 e^{kt}$$

$$\frac{48.6}{20} = (1/2)^{t/2}$$

$$\log\left(\frac{48.6}{20}\right) = t/2 \log(1/2)$$

$$t = \frac{2 \times \log\left(\frac{48.6}{20}\right)}{\log(1/2)} = \frac{2(-0.88789)}{0.69315}$$

$$t = -2.56 \text{ hrs (app).}$$

$$\text{App time of death} = 8.00 - 2.56 = 5.30 \text{ PM.}$$

Example 10-30.

Let  $x$  denote the amount of salt in the tank at time  $t$ .

$$\frac{dx}{dt} = \text{inflow} - \text{outflow}$$

$$\text{inflow} = 5 \times 10$$

$$= 50$$

$$\text{outflow} = \frac{10}{1000} x$$

$$\frac{dx}{dt} = 50 - \frac{10x}{1000}$$

$$= 50 - \frac{x}{100}$$

$$\frac{dx}{dt} = -\frac{1}{100} [x - 5000]$$

$$\frac{dx}{(x - 5000)} = -\frac{1}{100} dt$$

$$\int \frac{dx}{x - 5000} = -\frac{1}{100} \int dt$$

$$\log (x - 5000) = -\frac{1}{100} t + \log c.$$

$$\log (x - 5000) - \log c = -\frac{t}{100}$$

$$\log \left( \frac{x - 5000}{c} \right) = -\frac{t}{100}$$

$$\frac{x - 5000}{c} = e^{-t/100}$$

$$x = 5000 + c e^{-t/100}$$

Initially, when  $t = 0$ ,  $x = 100$

$$100 = 5000 + ce^0$$

$$\boxed{-4900 = c}$$

$$\textcircled{1} \Rightarrow x = 5000 - 4900 e^{-t/100}$$

The amount of the salt in the tank at time  $t$  is.

$$x = 5000 - 4900 e^{-0.01t}$$

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