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WAY TO SUCCESS

Leads to Success

# 12th

# Maths - Study material

## Chapter 1. Applications of Matrices and Determinants

- We publish this study material for request of many teachers and students.
- This study material contains only solution for exercise questions.
- Way to success – $12^{th}$ Maths guide will be published very shortly.
- Way to success is preparing $12^{th}$ maths guide with help of expert-experienced teachers to give an assurance for you to score more marks in your public examination.

# 1. Applications of Matrices and Determinants

**EXERCISE 1.1**

**1. Find the adjoint of the following: (i)** $\begin{bmatrix} -3 & 4 \\ 6 & 2 \end{bmatrix}$ **(ii)** $\begin{bmatrix} 2 & 3 & 1 \\ 3 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$ **(iii)** $\frac{1}{3}\begin{bmatrix} 2 & 2 & 1 \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix}$

(i) Let $A = \begin{bmatrix} -3 & 4 \\ 6 & 2 \end{bmatrix}$

$A_{11} =$ co-factor of $-3 = 2$

$A_{12} =$ co-factor of $4 = -6$

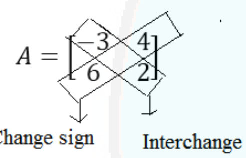
$A_{21} =$ co-factor of $6 = -4$

$A_{22} =$ co-factor of $2 = -3$

$$adjA = \begin{bmatrix} 2 & -6 \\ -4 & -3 \end{bmatrix}^T$$

$$= \begin{bmatrix} 2 & -4 \\ -6 & -3 \end{bmatrix}$$

**Note:**




$$adjA = \begin{bmatrix} 2 & -4 \\ -6 & -3 \end{bmatrix}$$

(ii) Let $A = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$

$$adjA = \begin{bmatrix} \begin{vmatrix} 4 & 1 \\ 7 & 2 \end{vmatrix} & -\begin{vmatrix} 3 & 1 \\ 3 & 2 \end{vmatrix} & \begin{vmatrix} 3 & 4 \\ 3 & 7 \end{vmatrix} \\ -\begin{vmatrix} 3 & 1 \\ 7 & 2 \end{vmatrix} & \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} & -\begin{vmatrix} 2 & 3 \\ 3 & 7 \end{vmatrix} \\ \begin{vmatrix} 3 & 1 \\ 4 & 1 \end{vmatrix} & -\begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix} & \begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix} \end{bmatrix}^T$$

$$= \begin{bmatrix} (8-7) & -(6-3) & (21-12) \\ -(6-7) & (4-3) & -(14-9) \\ (3-4) & -(2-3) & (8-9) \end{bmatrix}^T$$

$$= \begin{bmatrix} 1 & -3 & 9 \\ 1 & 1 & -5 \\ -1 & 1 & -1 \end{bmatrix}^T$$

$$\therefore adj\ A = \begin{bmatrix} 1 & 1 & -1 \\ -3 & 1 & 1 \\ 9 & -5 & -1 \end{bmatrix}^T$$

(iii) Let $B = \frac{1}{3}\begin{bmatrix} 2 & 2 & 1 \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix}$

Let us consider $A = \begin{bmatrix} 2 & 2 & 1 \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix}$

$$adjA = \begin{bmatrix} \begin{vmatrix} 1 & 2 \\ -2 & 2 \end{vmatrix} & -\begin{vmatrix} -2 & 2 \\ 1 & 2 \end{vmatrix} & \begin{vmatrix} -2 & 1 \\ 1 & -2 \end{vmatrix} \\ -\begin{vmatrix} 2 & 1 \\ -2 & 2 \end{vmatrix} & \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} & -\begin{vmatrix} 2 & 2 \\ 1 & -2 \end{vmatrix} \\ \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} & -\begin{vmatrix} 2 & 1 \\ -2 & 2 \end{vmatrix} & \begin{vmatrix} 2 & 2 \\ -2 & 1 \end{vmatrix} \end{bmatrix}^T$$

$$= \begin{bmatrix} (2+4) & -(-4-2) & (4-1) \\ -(4+2) & (4-1) & -(-4-2) \\ (4-1) & -(4+2) & (2+4) \end{bmatrix}^T$$

$$= \begin{bmatrix} 6 & 6 & 3 \\ -6 & 3 & 6 \\ 3 & -6 & 6 \end{bmatrix}^T$$

$$adj\ A = \begin{bmatrix} 6 & -6 & 3 \\ 6 & 3 & -6 \\ 3 & 6 & 6 \end{bmatrix}$$

$$adj\ B = adj\ \left(\frac{1}{3}A\right)$$

$$= \left(\frac{1}{3}\right)^{3-1} adjA$$

$$= \frac{1}{3^2}\begin{bmatrix} 6 & -6 & 3 \\ 6 & 3 & -6 \\ 3 & 6 & 6 \end{bmatrix}$$

$$= \frac{1}{9} \times 3\begin{bmatrix} 2 & -2 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 2 \end{bmatrix}$$

$$\therefore adj\ B = \frac{1}{3}\begin{bmatrix} 2 & -2 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 2 \end{bmatrix}$$

[If $A$ is a matrix of order $n$, then $adj(\lambda A) = \lambda^{n-1} adjA$]

**2. Find the inverse (if it exists) of the following (i) $\begin{bmatrix} -2 & 4 \\ 1 & -3 \end{bmatrix}$ (ii) $\begin{bmatrix} 5 & 1 & 1 \\ 1 & 5 & 1 \\ 1 & 1 & 5 \end{bmatrix}$ (iii) $\begin{bmatrix} 2 & 3 & 1 \\ 3 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$**

(i) Let $A = \begin{bmatrix} -2 & 4 \\ 1 & -3 \end{bmatrix}$

$|A| = 6 - 4 = 2 \neq 0.$ $\quad \therefore A^{-1}$ exists

$adj\ A = \begin{bmatrix} -3 & -4 \\ -1 & -2 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} adj\ A$

$\therefore A^{-1} = \frac{1}{2}\begin{bmatrix} -3 & -4 \\ -1 & -2 \end{bmatrix}$

(ii) Let $A = \begin{bmatrix} 5 & 1 & 1 \\ 1 & 5 & 1 \\ 1 & 1 & 5 \end{bmatrix}$

$|A| = 5[25-1] - 1[5-1] + 1[1-5]$

$= 5(24) - 1(4) + 1(-4)$

$= 120 - 4 - 4 = 112 \neq 0$

$\therefore A^{-1}$ exists.

$$adjA = \begin{bmatrix} \begin{vmatrix} 5 & 1 \\ 1 & 5 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 1 & 5 \end{vmatrix} & \begin{vmatrix} 1 & 5 \\ 1 & 1 \end{vmatrix} \\ -\begin{vmatrix} 1 & 1 \\ 1 & 5 \end{vmatrix} & \begin{vmatrix} 5 & 1 \\ 1 & 5 \end{vmatrix} & -\begin{vmatrix} 5 & 1 \\ 1 & 1 \end{vmatrix} \\ \begin{vmatrix} 1 & 1 \\ 5 & 1 \end{vmatrix} & -\begin{vmatrix} 5 & 1 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} 5 & 1 \\ 1 & 5 \end{vmatrix} \end{bmatrix}^T$$

$$= \begin{bmatrix} (25-1) & -(5-1) & (1-5) \\ -(5-1) & (25-1) & -(5-1) \\ (1-5) & -(5-1) & (25-1) \end{bmatrix}^T$$

$$= \begin{bmatrix} 24 & -4 & -4 \\ -4 & 24 & -4 \\ -4 & -4 & 24 \end{bmatrix}^T$$

$$adj\ A = \begin{bmatrix} 24 & -4 & -4 \\ -4 & 24 & -4 \\ -4 & -4 & 24 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} adj\ A = \frac{1}{112}\begin{bmatrix} 24 & -4 & -4 \\ -4 & 24 & -4 \\ -4 & -4 & 24 \end{bmatrix}$$

$$A^{-1} = \frac{1}{112} \times 4 \begin{bmatrix} 6 & -1 & -1 \\ -1 & 6 & -1 \\ -1 & -1 & 6 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{28}\begin{bmatrix} 6 & -1 & -1 \\ -1 & 6 & -1 \\ -1 & -1 & 6 \end{bmatrix}$$

(iii) Let $A = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$

$|A| = 2(8-7) - 3(6-3) + 1(21-12)$

$= 2(1) - 3(3) + 1(9)$

$= 2 - 9 + 9$

$= 2 \neq 0$

$\therefore A^{-1}$ exists

$$adjA = \begin{bmatrix} \begin{vmatrix} 4 & 1 \\ 7 & 2 \end{vmatrix} & -\begin{vmatrix} 3 & 1 \\ 3 & 2 \end{vmatrix} & \begin{vmatrix} 3 & 4 \\ 3 & 7 \end{vmatrix} \\ -\begin{vmatrix} 3 & 1 \\ 7 & 2 \end{vmatrix} & \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} & -\begin{vmatrix} 2 & 3 \\ 3 & 7 \end{vmatrix} \\ \begin{vmatrix} 3 & 1 \\ 4 & 1 \end{vmatrix} & -\begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix} & \begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix} \end{bmatrix}^T$$

$$= \begin{bmatrix} (8-7) & -(6-3) & (21-12) \\ -(6-7) & (4-3) & -(14-9) \\ (3-4) & -(2-3) & (8-9) \end{bmatrix}^T$$

$$= \begin{bmatrix} 1 & -3 & 9 \\ 1 & 1 & -5 \\ -1 & 1 & -1 \end{bmatrix}^T$$

$$\therefore adj\ A = \begin{bmatrix} 1 & 1 & -1 \\ -3 & 1 & 1 \\ 9 & -5 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} adj\ A$$

$$A^{-1} = \frac{1}{2}\begin{bmatrix} 1 & 1 & -1 \\ -3 & 1 & 1 \\ 9 & -5 & -1 \end{bmatrix}$$

**3. If $F(\alpha)=\begin{bmatrix} cos\alpha & 0 & sin\alpha \\ 0 & 1 & 0 \\ -sin\alpha & 0 & cos\alpha \end{bmatrix}$, show that $[F(\alpha)]^{-1}=F(-\alpha)$.**

$$F(\alpha)=\begin{bmatrix} cos\alpha & 0 & sin\alpha \\ 0 & 1 & 0 \\ -sin\alpha & 0 & cos\alpha \end{bmatrix}$$

$$F(-\alpha)=\begin{bmatrix} \cos(-\alpha) & 0 & \sin(-\alpha) \\ 0 & 1 & 0 \\ -\sin(-\alpha) & 0 & \cos(-\alpha) \end{bmatrix}$$

$$[\because \sin(-\alpha)=-\sin\alpha, \quad \cos(-\alpha)=cos\alpha]$$

$$F(-\alpha)=\begin{bmatrix} cos\alpha & 0 & -sin\alpha \\ 0 & 1 & 0 \\ sin\alpha & 0 & cos\alpha \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots(1)$$

**To find $[F(\alpha)]^{-1}$:**

$$|F(\alpha)| = cos\alpha[cos\alpha-0]-0+\sin\alpha[0+\sin\alpha]$$
$$=\cos^2\alpha+\sin^2\alpha$$
$$=1\neq 0$$

$\therefore [F(\alpha)]^{-1}$ exists.

$$adj[F(\alpha)]=\begin{bmatrix} \begin{vmatrix} 1 & 0 \\ 0 & cos\alpha \end{vmatrix} & -\begin{vmatrix} 0 & 0 \\ -\sin\alpha & \cos\alpha \end{vmatrix} & \begin{vmatrix} 0 & 1 \\ -\sin\alpha & 0 \end{vmatrix} \\ -\begin{vmatrix} 0 & \sin\alpha \\ 0 & \cos\alpha \end{vmatrix} & \begin{vmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{vmatrix} & -\begin{vmatrix} \cos\alpha & 0 \\ -\sin\alpha & 0 \end{vmatrix} \\ \begin{vmatrix} 0 & +\sin\alpha \\ 1 & 0 \end{vmatrix} & -\begin{vmatrix} \cos\alpha & +\sin\alpha \\ 0 & 0 \end{vmatrix} & \begin{vmatrix} \cos\alpha & 0 \\ 0 & 1 \end{vmatrix} \end{bmatrix}^T$$

$$=\begin{bmatrix} (cos\alpha-0) & -(0-0) & (0+sin\alpha) \\ -(0-0) & (\cos^2\alpha+\sin^2\alpha) & -(0-0) \\ (0-sin\alpha) & -(0-0) & (cos\alpha-0) \end{bmatrix}^T$$

$$=\begin{bmatrix} cos\alpha & 0 & sin\alpha \\ 0 & 1 & 0 \\ -sin\alpha & 0 & cos\alpha \end{bmatrix}^T$$

$$[\because \sin^2 x+\cos^2 x=1]$$

$$adj[F(\alpha)]=\begin{bmatrix} cos\alpha & 0 & -sin\alpha \\ 0 & 1 & 0 \\ sin\alpha & 0 & cos\alpha \end{bmatrix}$$

$$[F(\alpha)]^{-1}=\frac{1}{|F(\alpha)|}adj\,[F(\alpha)]$$

$$=\frac{1}{1}\begin{bmatrix} cos\alpha & 0 & -sin\alpha \\ 0 & 1 & 0 \\ sin\alpha & 0 & cos\alpha \end{bmatrix}$$

$$\therefore [F(\alpha)]^{-1}=\begin{bmatrix} cos\alpha & 0 & -sin\alpha \\ 0 & 1 & 0 \\ sin\alpha & 0 & cos\alpha \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots(2)$$

From (1) and (2), we get

$$[F(\alpha)]^{-1}=F(-\alpha)$$

**4. If $A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$, show that $A^2 - 3A - 7I_2 = O_2$. Hence find $A^{-1}$.**

$$A^2 = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 25-3 & 15-6 \\ -5+2 & -3+4 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 22 & 9 \\ -3 & 1 \end{bmatrix}$$

$$3A = 3\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 15 & 9 \\ -3 & -6 \end{bmatrix}$$

$$7I_2 = 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$A^2 - 3A - 7I_2 = \begin{bmatrix} 22 & 9 \\ -3 & 1 \end{bmatrix} - \begin{bmatrix} 15 & 9 \\ -3 & -6 \end{bmatrix} - \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 22-15-7 & 9-9-0 \\ -3+3-0 & 1+6-7 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\therefore A^2 - 3A - 7I_2 = 0$

**To find $A^{-1}$:**

$A^2 - 3A - 7I_2 = 0$

Post – multiplying by $A^{-1}$, we get

$A - 3I - 7A^{-1} = 0$

$$A^{-1}A^{-2} = (A^{-1}A)A = IA = A$$
$$A^{-1}3A = 3(A^{-1}A) = 3I$$
$$A^{-1}I_2 = A^{-1}$$

$$7A^{-1} = A - 3I$$

$$7A^{-1} = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$7A^{-1} = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} + \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}$$

$$7A^{-1} = \begin{bmatrix} 2 & 3 \\ -1 & -5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & 3 \\ -1 & -5 \end{bmatrix}$$

**5. If $A = \frac{1}{9}\begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}$, prove that $A^{-1} = A^T$.**

$$A = \frac{1}{9}\begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}$$

$$A^T = \frac{1}{9}\begin{bmatrix} -8 & 4 & 1 \\ 1 & 4 & -8 \\ 4 & 7 & 4 \end{bmatrix} \text{.....................(1)}$$

$$|A| = \left(\frac{1}{9}\right)^3 [-8(16+56) - 1(16-7) + 4(-32-4)]$$

$[\because |KA| = K^n|A|]$

$= \frac{1}{729}[-8(72) - 1(9) + 4(-36)]$

$= \frac{1}{729}[-576 - 9 - 144] = \frac{1}{729}(-729)$

$|A| = -1 \neq 0$

$\therefore A^{-1}$ exists.

$$adj\ A = \left(\frac{1}{9}\right)^{3-1}\begin{bmatrix} \begin{vmatrix} 4 & 7 \\ -8 & 4 \end{vmatrix} & -\begin{vmatrix} 4 & 7 \\ 1 & 4 \end{vmatrix} & \begin{vmatrix} 4 & 4 \\ 1 & -8 \end{vmatrix} \\ -\begin{vmatrix} 1 & 4 \\ -8 & 4 \end{vmatrix} & \begin{vmatrix} -8 & 4 \\ 1 & 4 \end{vmatrix} & -\begin{vmatrix} -8 & 1 \\ 1 & -8 \end{vmatrix} \\ \begin{vmatrix} 1 & 4 \\ 4 & 7 \end{vmatrix} & -\begin{vmatrix} -8 & 4 \\ 4 & 7 \end{vmatrix} & \begin{vmatrix} -8 & 1 \\ 4 & 4 \end{vmatrix} \end{bmatrix}^T$$

$[\because adj(\lambda A) = \lambda^{n-1}(adj\ A)]$

$$= \frac{1}{81}\begin{bmatrix} (16+56) & -(16-7) & (-32-4) \\ -(4+32) & (-32-4) & -(64-1) \\ (7-16) & -(-56-16) & (-32-4) \end{bmatrix}$$

$$= \frac{1}{81}\begin{bmatrix} 72 & -9 & -36 \\ -36 & -36 & -63 \\ -9 & 72 & -36 \end{bmatrix}^T$$

$$adj\ A = \frac{1}{81}\begin{bmatrix} 72 & -36 & -9 \\ -9 & -36 & +72 \\ -36 & -63 & -36 \end{bmatrix} = \frac{1}{81} \times 9\begin{bmatrix} 8 & -4 & -1 \\ -1 & 4 & 8 \\ -4 & -7 & -4 \end{bmatrix} = \frac{1}{9} \times \begin{bmatrix} 8 & -4 & -1 \\ -1 & 4 & 8 \\ -4 & -7 & -4 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}adj\ A$$

$$= \frac{1}{-1} \cdot \frac{1}{9}\begin{bmatrix} 8 & -4 & -1 \\ -1 & 4 & 8 \\ -4 & -7 & -4 \end{bmatrix}$$

$$A^{-1} = \frac{1}{9}\begin{bmatrix} -8 & 4 & 1 \\ 1 & -4 & -8 \\ 4 & 7 & 4 \end{bmatrix} \text{.................}(2)$$

From (1) and (2), we get $A^{-1} = A^T$

**6. If $A = \begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$, verify that $A(adjA) = (adjA)A = |A|I_2$.**

$A = \begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$

$adj\ A = \begin{bmatrix} 3 & 4 \\ 5 & 8 \end{bmatrix}$

$A(adj\ A) = \begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}\begin{bmatrix} 3 & 4 \\ 5 & 8 \end{bmatrix}$

$= \begin{bmatrix} 24-20 & 32-32 \\ -15+15 & -20+24 \end{bmatrix}$

$A(adj\ A) = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$......................(1)

$(adj\ A)A = \begin{bmatrix} 3 & 4 \\ 5 & 8 \end{bmatrix}\begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$

$= \begin{bmatrix} 24-20 & -12+12 \\ 40-40 & -20+24 \end{bmatrix}$

$(adj\ A) = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$........................(2)

$|A| = 24 - 20 = 4$

$|A|I_2 = 4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$ ......................(3)

From (1), (2) and (3), we get $A(adj\ A) = (adjA)A = |A|I_2$.

**7. If $A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & -3 \\ 5 & 2 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1}A^{-1}$**

$AB = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}\begin{bmatrix} -1 & -3 \\ 5 & 2 \end{bmatrix}$

$= \begin{bmatrix} -3+10 & -9+4 \\ -7+25 & -21+10 \end{bmatrix}$

$AB = \begin{bmatrix} 7 & -5 \\ +18 & -11 \end{bmatrix}$

$|AB| = -77 + 90$

$= 13 \neq 0$

$(AB)^{-1}$ exists.

$adj\ (AB) = \begin{bmatrix} -11 & 5 \\ -18 & 7 \end{bmatrix}$

$(AB)^{-1} = \frac{1}{|AB|} adj\ (AB)$

$= \frac{1}{13}\begin{bmatrix} -11 & 5 \\ -18 & 7 \end{bmatrix}$.............(1)

$A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$

$|A| = 15 - 14 = 1 \neq 0$

$A^{-1}$ exists

$adj\ A = \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} adj\ A$

$A^{-1} = \frac{1}{1}\begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$

$B = \begin{bmatrix} -1 & -3 \\ 5 & 2 \end{bmatrix}$

$|B| = -2 + 15$

$= 13 \neq 0$

$\therefore B^{-1}$ exists

$adj\ B = \begin{bmatrix} 2 & 3 \\ -5 & -1 \end{bmatrix}$

$B^{-1} = \frac{1}{|B|} adj\ B$

$B^{-1} = \frac{1}{13}\begin{bmatrix} 2 & 3 \\ -5 & -1 \end{bmatrix}$

$B^{-1}A^{-1} = \frac{1}{13}\begin{bmatrix} 2 & 3 \\ -5 & -1 \end{bmatrix}\begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$

$= \frac{1}{13}\begin{bmatrix} 10-21 & -4+9 \\ -25+7 & 10-3 \end{bmatrix}$

$= \frac{1}{13}\begin{bmatrix} -11 & 5 \\ -18 & 7 \end{bmatrix}$................(2)

From (1) and (2), we get $(AB)^{-1} = B^{-1}A^{-1}$

**8. If $adj(A) = \begin{bmatrix} 2 & -4 & 2 \\ -3 & 12 & -7 \\ -2 & 0 & 2 \end{bmatrix}$, find $A$.**

$|adj\ A| = 2[24-0] + 4[-6-14] + 2[0+24] = 48 - 80 + 48 = 16$

$$adj(adjA) = \begin{bmatrix} \begin{vmatrix} 12 & -7 \\ 0 & 2 \end{vmatrix} & -\begin{vmatrix} -3 & -7 \\ -2 & 2 \end{vmatrix} & \begin{vmatrix} -3 & 12 \\ -2 & 0 \end{vmatrix} \\ -\begin{vmatrix} -4 & 2 \\ 0 & 2 \end{vmatrix} & \begin{vmatrix} 2 & 2 \\ -2 & 2 \end{vmatrix} & -\begin{vmatrix} 2 & -4 \\ -2 & 0 \end{vmatrix} \\ \begin{vmatrix} -4 & 2 \\ 12 & -7 \end{vmatrix} & -\begin{vmatrix} 2 & 2 \\ -3 & -7 \end{vmatrix} & \begin{vmatrix} 2 & -4 \\ -3 & 12 \end{vmatrix} \end{bmatrix}^T$$

$$= \begin{bmatrix} (24-0) & -(-6-14) & (0+24) \\ -(-8-0) & (4+4) & -(0-8) \\ (28-24) & -(-14+6) & (24-12) \end{bmatrix}^T$$

$$= \begin{bmatrix} 24 & 20 & 24 \\ 8 & 8 & 8 \\ 4 & 8 & 12 \end{bmatrix}^T$$

$$= \begin{bmatrix} 24 & 8 & 4 \\ 20 & 8 & 8 \\ 24 & 8 & 12 \end{bmatrix}$$

$$A = \pm \frac{1}{\sqrt{|adj\ A|}} adj\ (adj\ A)$$

$$= \pm \frac{1}{\sqrt{16}} \begin{bmatrix} 24 & 8 & 4 \\ 20 & 8 & 8 \\ 24 & 8 & 12 \end{bmatrix}$$

$$= \pm \frac{1}{4} \times 4 \begin{bmatrix} 6 & 2 & 1 \\ 5 & 2 & 2 \\ 6 & 2 & 3 \end{bmatrix}$$

$$\therefore A = \pm \begin{bmatrix} 6 & 2 & 1 \\ 5 & 2 & 2 \\ 6 & 2 & 3 \end{bmatrix}$$

**9. If $adj(A) = \begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$, find $A^{-1}$.**

$$adj(A) = \begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$$

$$|adjA| = 0 + 2[36-18] + 0 = 36$$

$$A^{-1} = \pm \frac{1}{\sqrt{|adj\ A|}} adj\ (adj\ A)$$

$$= \pm \frac{1}{\sqrt{36}} \begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$$

$$A^{-1} = \pm \frac{1}{6} \begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$$

**10. Find $adj(adj(A)$ if adj $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}$**

$$\text{adj } A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$$adj(adjA) = \begin{bmatrix} \begin{vmatrix} 2 & 0 \\ 0 & 1 \end{vmatrix} & -\begin{vmatrix} 0 & 0 \\ -1 & 1 \end{vmatrix} & \begin{vmatrix} 0 & 2 \\ -1 & 0 \end{vmatrix} \\ -\begin{vmatrix} 0 & 1 \\ 0 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 0 \\ -1 & 0 \end{vmatrix} \\ \begin{vmatrix} 0 & 1 \\ 2 & 0 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix} & \begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix} \end{bmatrix}^T$$

$$= \begin{bmatrix} (2-0) & -(0+0) & (0+2) \\ -(0-0) & (1+1) & -(0+0) \\ (0-2) & -(0-0) & (2-0) \end{bmatrix}^T$$

$$= \begin{bmatrix} 2 & 0 & 2 \\ 0 & 2 & 0 \\ -2 & 0 & 2 \end{bmatrix}^T$$

$$\therefore\ adj\ (adjA) = \begin{bmatrix} 2 & 0 & -2 \\ 0 & 2 & 0 \\ 2 & 0 & 2 \end{bmatrix}$$

**11.** $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$**, show that** $A^T A^{-1} = \begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$

$$A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}$$

$$|A| = 1 + \tan^2 x = \sec^2 x$$

$$adj\ A = \begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} adj\ A$$

$$A^{-1} = \frac{1}{\sec^2 x}\begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}$$

$$A^T A^{-1} = \begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix} \frac{1}{\sec^2 x} \begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}$$

$$= \frac{1}{\sec^2 x}\begin{bmatrix} 1-\tan^2 x & -2\tan x \\ 2\tan x & 1-tan^2 x \end{bmatrix}$$

$$= \cos^2 x \begin{bmatrix} 1-\dfrac{\sin^2 x}{\cos^2 x} & -\dfrac{2\sin x}{\cos x} \\ 2\dfrac{\sin x}{\cos x} & 1-\dfrac{\sin^2 x}{\cos^2 x} \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{\cos^2 x - \sin^2 x}{\cos^2 x}\cos^2 x & \dfrac{-2\sin x \cos^2 x}{\cos x} \\ \dfrac{2\sin x}{\cos x}\cos^2 x & \dfrac{\cos^2 x - \sin^2 x}{\cos^2 x}\cos^2 x \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2 x - \sin^2 x & -2\sin x\cos x \\ 2\sin x\cos x & \cos^2 x - \sin^2 x \end{bmatrix} \quad \begin{bmatrix} \because \cos^2 x - \sin^2 x = cos2x \\ 2sinx\cos x = \sin 2x \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2x & -sin2x \\ \sin 2x & \cos 2x \end{bmatrix}$$

Hence proved

**12. Find the matrix $A$ for which $A\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 14 & 7 \\ 7 & 7 \end{bmatrix}$.**

Let $B = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$ and $C = \begin{bmatrix} 14 & 7 \\ 7 & 7 \end{bmatrix}$

Then $AB = C$

Post multiplying by $B^{-1}$, we get $A(BB^{-1}) = CB^{-1}$

$AI = CB^{-1}$

$A = CB^{-1}$..................(1)

$|B| = -10 + 3 = -7 \neq 0$

$\therefore B^{-1}$ exists

$adjB = \begin{bmatrix} -2 & -3 \\ 1 & 5 \end{bmatrix}$

$B^{-1} = \frac{1}{|B|} adj\ B$

$= \frac{1}{-7}\begin{bmatrix} -2 & -3 \\ 1 & 5 \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 2 & 3 \\ -1 & -5 \end{bmatrix}$

$(1) \Rightarrow A = \begin{bmatrix} 14 & 7 \\ 7 & 7 \end{bmatrix}\frac{1}{7}\begin{bmatrix} 2 & 3 \\ -1 & -5 \end{bmatrix}$

$= \frac{1}{7} \times 7\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -1 & -5 \end{bmatrix}$

$A = \begin{bmatrix} 4-1 & 6-5 \\ 2-1 & 3-5 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 1 & -2 \end{bmatrix}$

**13. Given $A = \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -2 \\ 1 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$, find a matrix $X$ such that $A \times B = C$**

$A = \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}$

$|A| = 0 + 2 = 2 \neq 0$

$\therefore A^{-1}$ exists

$adjA = \begin{bmatrix} 0 & -1 \\ -2 & 1 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} adj\ A$

$A^{-1} = \frac{1}{2}\begin{bmatrix} 0 & 1 \\ -2 & 1 \end{bmatrix}$

$B = \begin{bmatrix} 3 & -2 \\ 1 & 1 \end{bmatrix}$

$|B| = 3 + 2 = 5 \neq 0$

$\therefore B^{-1}$ exists

$adj\ B = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$

$B^{-1} = \frac{1}{|B|} adj\ B$

$B^{-1} = \frac{1}{5}\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$

Given: $AXB = C$

Pre – multiplying by $A^{-1}$. On both sides, we get $(A^{-1}A) \times B = A^{-1}C$

$(IX)B = A^{-1}C$

$XB = A^{-1}C$

Post – multiplying by $B^{-1}$ on both sides, we get $X(BB^{-1}) = A^{-1}CB^{-1}$

$XI = A^{-1}CB^{-1}$

$X = A^{-1}(CB^{-1})$

$X = \frac{1}{2}\begin{bmatrix} 0 & 1 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}\frac{1}{5}\begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix}$

$X = \frac{1}{10}\begin{bmatrix} 0+2 & 0+2 \\ -2+2 & -2+2 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$

$X = \frac{1}{10}\begin{bmatrix} 2 & 2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$

$X = \frac{1}{10}\begin{bmatrix} 2-2 & 4+6 \\ 0+0 & 0+0 \end{bmatrix}$

$= \frac{1}{10}\begin{bmatrix} 0 & 10 \\ 0 & 0 \end{bmatrix}$

$\therefore X = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

**14. If $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, show that $A^{-1} = \frac{1}{2}(A^2 - 3I)$.**

$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

$|A| = 0 - 1[0 - 1] + 1[1 - 0]$

$= 1 + 1 = 2 \neq 0$

$\therefore A^{-1}$ exists

$$adj\ A = \begin{bmatrix} \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} & \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} \\ -\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} & \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} & -\begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} \\ \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} & -\begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} \end{bmatrix}^T$$

$$= \begin{bmatrix} (0-1) & -(0-1) & (1-0) \\ -(0-1) & (0-1) & -(0-1) \\ (1-0) & -(0-1) & (0-1) \end{bmatrix}^T$$

$$= \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}^T$$

$$adj\ A = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} adj\ A$$

$$A^{-1} = \frac{1}{2}\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} \text{.................}(1)$$

$$A^2 = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0+1+1 & 0+0+1 & 0+1+0 \\ 0+0+1 & 1+0+1 & 1+0+0 \\ 0+1+0 & 1+0+0 & 1+1+0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

$$3I = 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$A^2 - 3I = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

$$\frac{1}{2}(A^2 - 3I) = \frac{1}{2}\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} \text{...............}(2)$$

From (1) & (2), we get

$$A^{-1} = \frac{1}{2}(A^2 - 3I)$$

**15. Decrypt the received encoded message $[2\ \ -3][20\ \ 4]$ with the encryption matrix $\begin{bmatrix} -1 & -1 \\ 2 & 1 \end{bmatrix}$ and the decryption matrix as its inverse, where the system of codes are described by the numbers 1-26 to the letters $A - Z$ respectively, and the number 0 to a blank space.**

Let the encoding matrix be $A = \begin{bmatrix} -1 & -1 \\ 2 & 1 \end{bmatrix}$

$|A| = -1 + 2 = 1 \neq 0$

$A^{-1}$ exists

$adj\ A = \begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} adj\ A = \frac{1}{1}\begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix}$

$\therefore$ Decoding matrix $= A^{-1} = \begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix}$

| Coded row matrix | decoding matrix | decoded row matrix |
|---|---|---|
| $[2 - 3]$ | $\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$ | $= [2+6 \ \ 2+3] = [8 \ \ 5]$ |
| $[20 \ \ 4]$ | $\begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix}$ | $= [20-8 \ \ 20-4] = [12 \ 16]$ |

So, the sequence of decoded row matrix $= [8 \ \ 5][12 \ 16]$

The message received = HELP

Thus, the message received is "HELP."

**EXCERSICE 1.2**

**1. Find the rank of the following matrices by minor method.**

**(i)** $\begin{bmatrix} 2 & -4 \\ -1 & 2 \end{bmatrix}$ **(ii)** $\begin{bmatrix} -1 & 3 \\ 4 & -7 \\ 3 & -4 \end{bmatrix}$ **(iii)** $\begin{bmatrix} 1 & -2 & -1 & 0 \\ 3 & -6 & -3 & 1 \end{bmatrix}$ **(iv)** $\begin{bmatrix} 1 & -2 & 3 \\ 2 & 4 & -6 \\ 5 & 1 & -1 \end{bmatrix}$ **(v)** $\begin{bmatrix} 0 & 1 & 2 & 1 \\ 0 & 2 & 4 & 3 \\ 8 & 1 & 0 & 2 \end{bmatrix}$

(i) Let $A = \begin{bmatrix} 2 & -4 \\ -1 & 2 \end{bmatrix}$

Order of the matrix $A$ is $2 \times 2$

$\therefore \rho(A) \leq \min\{2,2\} = 2$

$|A| = 4 - 4 = 0$

$\therefore \rho(A) \neq 2$

Thus $\rho(A) = 1$

(ii) Let $A = \begin{bmatrix} -1 & 3 \\ 4 & -7 \\ 3 & -4 \end{bmatrix}$

order of the matrix $A$ is $3 \times 2$

$\therefore \rho(A) \leq \min\{3,2\} = 2$

We find that the second order minor

$\begin{vmatrix} -1 & 3 \\ 4 & -7 \end{vmatrix} = 7 - 12 = -5 \neq 0.$

$\therefore \rho(A) = 2.$

(iii) Let $A = \begin{bmatrix} 1 & -2 & -1 & 0 \\ 3 & -6 & -3 & 1 \end{bmatrix}$

order of the matrix $A$ is $2 \times 4$

$\therefore \rho(A) \leq \min\{2,4\} = 2$

We find that the second order minor

$\begin{vmatrix} -1 & 0 \\ -3 & 1 \end{vmatrix} = -1 + 0 = -1 \neq 0.$

$\therefore \rho(A) = 2$

(iv) Let $A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 4 & -6 \\ 5 & 1 & -1 \end{bmatrix}$

order of the matrix $A$ is $3 \times 3$

$\therefore \rho(A) \leq \min\{3,3\} = 3$

There is only one third order minor of $A$

$|A| = 1(-4+6) + 2(-2+30) + 3(2-20)$

$= 1(2) + 2(28) + 3(-18)$

$= 2 + 56 - 54 = 4 \neq 0.$

$\therefore \rho(A) = 3$

(v) Let $A = \begin{bmatrix} 0 & 1 & 2 & 1 \\ 0 & 2 & 4 & 3 \\ 8 & 1 & 0 & 2 \end{bmatrix}$

order of the matrix $A$ is $3 \times 4$

$\therefore \rho(A) \leq \min\{3,4\} = 3$

We find that the third order minor

$\begin{vmatrix} 0 & 1 & 1 \\ 0 & 2 & 2 \\ 8 & 1 & 2 \end{vmatrix} = 0 - 0 + 8(3-2)$

$= 8(1) = 8 \neq 0$

$\therefore \rho(A) = 3$

**2. Find the rank of the following matrices by row reduction method:**

(i) $\begin{bmatrix} 1 & 1 & 1 & 3 \\ 2 & -1 & 3 & 4 \\ 5 & -1 & 7 & 11 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 1 & -2 & 3 \\ 1 & -1 & 1 \end{bmatrix}$ (iii) $\begin{bmatrix} 3 & -8 & 5 & 2 \\ 2 & -5 & 1 & 4 \\ -1 & 2 & 3 & -2 \end{bmatrix}$

(i) Let $A = \begin{bmatrix} 1 & 1 & 1 & 3 \\ 2 & -1 & 3 & 4 \\ 5 & -1 & 7 & 11 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & -3 & 1 & -2 \\ 0 & -6 & 2 & -4 \end{bmatrix} \begin{matrix} \\ R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 5R_1 \end{matrix}$

$\sim \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & -3 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix} R_3 \to R_3 - 2R_2$

The last equivalent matrix is in row-echelon form. It has two non-zero rows.

$\therefore \rho(A) = 2$

(iii) Let $A = \begin{bmatrix} 3 & -8 & 5 & 2 \\ 2 & -5 & 1 & 4 \\ -1 & 2 & 3 & -2 \end{bmatrix}$

$\sim \begin{bmatrix} 3 & -8 & 5 & 2 \\ 0 & 1 & -7 & 8 \\ 0 & -2 & 14 & -4 \end{bmatrix} \begin{matrix} \\ R_2 \to 3R_2 - 2R_1 \\ R_3 \to 3R_3 - R_1 \end{matrix}$

$\sim \begin{bmatrix} 3 & -8 & 5 & 2 \\ 0 & 1 & -7 & 8 \\ 0 & 0 & 0 & 12 \end{bmatrix} R_3 \to R_3 - 2R_2$

The last equivalent matrix is in row – echelon form. It has three non-zero rows.

$\therefore \rho(A) = 3$

(ii) Let $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 1 & -2 & 3 \\ 1 & -1 & 1 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 2 & -1 \\ 0 & -7 & 5 \\ 0 & -4 & 4 \\ 0 & -3 & 2 \end{bmatrix} \begin{matrix} \\ R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 - R_1 \\ R_4 \to R_4 - R_1 \end{matrix}$

$\sim \begin{bmatrix} 1 & 2 & -1 \\ 0 & -7 & 5 \\ 0 & 0 & 8 \\ 0 & 0 & -4 \end{bmatrix} \begin{matrix} \\ \\ R_3 \to 7R_3 - 4R_2 \\ R_4 \to 4R_4 - 3R_3 \end{matrix}$

$\sim \begin{bmatrix} 1 & 2 & -1 \\ 0 & -7 & 5 \\ 0 & 0 & 8 \\ 0 & 0 & 0 \end{bmatrix} R_4 \to 2R_4 + R_3$

The last equivalent matrix is in row-echelon form. It has three non-zero row.

$\therefore \rho(A) = 3$

**3. Find the inverse of each of the following by Gauss-Jordan method:**

(i) $\begin{bmatrix} 2 & -1 \\ 5 & -2 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 6 & -2 & -3 \end{bmatrix}$ (iii) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$

(i) Let $A = \begin{bmatrix} 2 & -1 \\ 5 & -2 \end{bmatrix}$

Applying Gauss – Jordan method, we get

$[A|I] = \left[\begin{array}{cc|cc} 2 & -1 & 1 & 0 \\ 5 & -2 & 0 & 1 \end{array}\right]$

$\to \left[\begin{array}{cc|cc} 1 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 5 & -2 & 0 & 1 \end{array}\right] R_1 \to \frac{1}{2}R_1$

$\to \left[\begin{array}{cc|cc} 1 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & -\frac{5}{2} & 1 \end{array}\right] R_2 \to R_2 - 5R_1$

$$\rightarrow \left[\begin{array}{cc|cc} 1 & 0 & -2 & 1 \\ 0 & \frac{1}{2} & -\frac{5}{2} & 1 \end{array}\right] R_1 \rightarrow R_1 + R_2$$

$$\rightarrow \left[\begin{array}{cc|cc} 1 & 0 & -2 & 1 \\ 0 & 1 & -5 & 2 \end{array}\right] R_2 \rightarrow 2R_2$$

$$\therefore A^{-1} = \begin{bmatrix} -2 & 1 \\ -5 & 2 \end{bmatrix}$$

(ii) Let $A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 6 & -2 & -3 \end{bmatrix}$ applying Gauss – Jordan method, we get

$$[A|I] = \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 1 & 0 \\ 6 & -2 & -3 & 0 & 0 & 1 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 4 & -3 & -6 & 0 & 1 \end{array}\right] \begin{array}{l} R_2 \rightarrow R_2 - R_1 \\ R_3 \rightarrow R_3 - 6R_1 \end{array}$$

$$\rightarrow \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & -2 & -4 & 1 \end{array}\right] R_3 \rightarrow R_3 - 4R_2$$

$$\rightarrow \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -3 & -3 & 1 \\ 0 & 0 & 1 & -2 & -4 & 1 \end{array}\right] R_2 \rightarrow R_2 + R_3$$

$$\rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & -3 & 1 \\ 0 & 1 & 0 & -3 & -3 & 1 \\ 0 & 0 & 1 & -2 & -4 & 1 \end{array}\right] R_1 \rightarrow R_1 + R_2$$

$$\therefore A^{-1} = \begin{bmatrix} -2 & -3 & 1 \\ -3 & -3 & 1 \\ -2 & -4 & 1 \end{bmatrix}$$

(iii) Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$ applying Gauss – Jordan method, we get

$$[A|I] = \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 5 & 3 & 0 & 1 & 0 \\ 1 & 0 & 8 & 0 & 0 & 1 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & -2 & 5 & -1 & 0 & 1 \end{array}\right] \begin{array}{l} R_2 \rightarrow R_2 - 2R_1 \\ R_3 \rightarrow R_3 - R_1 \end{array}$$

$$\rightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 0 & -1 & -5 & 2 & 1 \end{array}\right] R_3 \rightarrow R_3 + 2R_2$$

$$\rightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right] R_3 \rightarrow (-1)R_3$$

$$\rightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & -14 & 6 & 3 \\ 0 & 1 & 0 & 13 & -5 & -3 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right] \begin{array}{l} R_1 \rightarrow R_1 - 3R_3 \\ R_2 \rightarrow R_2 + 3R_3 \end{array}$$

$$\rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -40 & 16 & 9 \\ 0 & 1 & 0 & 13 & -5 & -3 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right] R_1 \rightarrow R_1 - 2R_2$$

$$\therefore A^{-1} = \begin{bmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{bmatrix}$$

## EXERCISE 1.3

**1. Solve the following system of linear equations by matrix inversion method.**

**(i) $2x + 5y = -2, x + 2y = -3$ (ii) $2x - y = 8, 3x + 2y = -2$**

**(iii) $2x + 3y - y = 9, x + y + z = 9, 3x - y - z = -1$**

**(iv) $x + y + z - 2 = 0, 6x - 4y + 5z - 31 = 0, 5x + 2y + 2z = 13$**

**(i) $2x + 5y = -2, x + 2y = -3$**

The matrix form of the system is $AX = B$, where

$A = \begin{bmatrix} 2 & 5 \\ 1 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} -2 \\ -3 \end{bmatrix}$

$|A| = 4 - 5 = -1 \neq 0$

$\therefore A^{-1}$ exists

$adjA = \begin{bmatrix} 2 & -5 \\ -1 & 2 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} adjA$

$A^{-1} = \frac{1}{-1}\begin{bmatrix} 2 & -5 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 5 \\ 1 & -2 \end{bmatrix}$

$AX = B$

$\Rightarrow X = A^{-1}B$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 & 5 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} -2 \\ -3 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -4 + 15 \\ -2 + 6 \end{bmatrix} = \begin{bmatrix} -11 \\ 4 \end{bmatrix}$

So the solution is $(x = -11, y = 4)$

**(ii) $2x - y = 8, 3x + 2y = -2$**

The matrix form of the system is

$AX = B$ where

$A = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 8 \\ -2 \end{bmatrix}$

$|A| = 4 + 3 = 7 \neq 0 \therefore A^{-1}$ exists

$adjA = \begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} adjA = \frac{1}{7}\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$

$AX = B \Rightarrow X = A^{-1}B$

$= \frac{1}{7}\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 8 \\ -2 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 16 - 2 \\ -24 - 4 \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 14 \\ -28 \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \end{bmatrix}$

So the solution is $(x = 2, y = -4)$

**(iii) $2x + 3y - z = 9, x + y + z = 9, 3x - y - z = -1$**

The matrix form of the system is $AX = B$ where $A = \begin{bmatrix} 2 & 3 & -1 \\ 1 & 1 & 1 \\ 3 & -1 & -1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 9 \\ 9 \\ -1 \end{bmatrix}$

$|A| = 2[-1 + 1] - 3[-1 - 3] - 1[-1 - 3]$

$= 0 + 12 + 4 = 16 \neq 0.$

$\therefore A^{-1}$ exists

$$adjA = \begin{bmatrix} \begin{vmatrix} 1 & 1 \\ -1 & -1 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 3 & -1 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 3 & -1 \end{vmatrix} \\ -\begin{vmatrix} 3 & -1 \\ -1 & -1 \end{vmatrix} & \begin{vmatrix} 2 & -1 \\ 3 & -1 \end{vmatrix} & -\begin{vmatrix} 2 & 3 \\ 3 & -1 \end{vmatrix} \\ \begin{vmatrix} 3 & -1 \\ 1 & 1 \end{vmatrix} & -\begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} \end{bmatrix}^T$$

$$= \begin{bmatrix} (-1+1) & -(-1-3) & (-1-3) \\ -(-3-1) & (-2+3) & -(-2-9) \\ (3+1) & -(2+1) & (2-3) \end{bmatrix}^T$$

$$= \begin{bmatrix} 0 & 4 & -4 \\ 4 & 1 & 11 \\ 4 & -3 & -1 \end{bmatrix}^T$$

$$adjA = \begin{bmatrix} 0 & 4 & 4 \\ 4 & 1 & -3 \\ -4 & 11 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} adjA = \frac{1}{16} \begin{bmatrix} 0 & 4 & 4 \\ 4 & 1 & -3 \\ -4 & 11 & -1 \end{bmatrix}$$

$AX = B \Rightarrow X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{16} \begin{bmatrix} 0 & 4 & 4 \\ 4 & 1 & -3 \\ -4 & 11 & -1 \end{bmatrix} \begin{bmatrix} 9 \\ 9 \\ -1 \end{bmatrix}$$

$$= \frac{1}{16} \begin{bmatrix} 0+36-4 \\ 36+9+3 \\ -36+99+1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{16} \begin{bmatrix} 32 \\ 48 \\ 64 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

$\therefore$ The solution is $(x = 2, y = 3, z = 4)$

**(iv)** $\boldsymbol{x + y + z = 2, 6x - 4y + 5z - 31 = 0, 5x + 2y + 2z = 13}$

The matrix form of the system is $AX = B$, where $A = \begin{bmatrix} 1 & 1 & 1 \\ 6 & -4 & 5 \\ 5 & 2 & 2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 2 \\ 31 \\ 13 \end{bmatrix}$

$|A| = 1(-8-10) - 1(12-25) + 1(12+20)$

$= -18 + 13 + 32 = 27 \neq 0$

$\therefore A^{-1}$ exists.

$$adjA = \begin{bmatrix} (-8-10) & -(12-25) & (12+20) \\ -(2-2) & (2-5) & -(2-5) \\ (5+4) & -(5-6) & (-4-6) \end{bmatrix}^T$$

$$= \begin{bmatrix} -18 & 13 & 32 \\ 0 & -3 & 3 \\ 9 & 1 & -10 \end{bmatrix}^T = \begin{bmatrix} -18 & 0 & 9 \\ 13 & -3 & 1 \\ 32 & 3 & -10 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} adjA = \frac{1}{27} \begin{bmatrix} -18 & 0 & 9 \\ 13 & -3 & 1 \\ 32 & 3 & -10 \end{bmatrix}$$

$$AX = B \Rightarrow X = A^{-1}B = \frac{1}{27}\begin{bmatrix} -18 & 0 & 9 \\ 13 & -3 & 1 \\ 32 & 3 & -10 \end{bmatrix}\begin{bmatrix} 2 \\ 31 \\ 13 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{27}\begin{bmatrix} -36+0+117 \\ 26-93+13 \\ 64+93-130 \end{bmatrix} = \frac{1}{27}\begin{bmatrix} 81 \\ -54 \\ 27 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$$

$\therefore$ The solution is $(x = 3, y = -2, z = 1)$

**2. If $A = \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$, find the products $AB$ and $BA$ and hence solve the system of equations $x + y + 2z = 1, 3x + 2y + z = 7, 2x + y + 3z = 2$.**

$$AB = \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -5+3+6 & -5+2+3 & -10+1+9 \\ 7+3-10 & 7+2-5 & 14+1-15 \\ 1-3+2 & 1-2+1 & 2-1+3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} = 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore AB = 4I_3$

$$BA = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -5+7+2 & 1+1-2 & 3-5+2 \\ -15+14+1 & 3+2-1 & 9-10+1 \\ -10+7+3 & 2+1-3 & 6-5+3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} = 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore BA = 4I_3$

$[\because AB = BA = I \Rightarrow B^{-1} = A (or) A^{-1} = B]$

So, we get $AB = BA = 4I_3$

$$\Rightarrow \left(\frac{1}{4}A\right)B = B\left(\frac{1}{4}A\right) = I_3$$

Hence $B^{-1} = \frac{1}{4}A$

Matrix form of the given system of equations:

$$\begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$$

$$\Rightarrow B\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = B^{-1}\begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix} = \frac{1}{4}A\begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} -5+7+6 \\ 7+7-10 \\ 1-7+2 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 8 \\ 4 \\ -4 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$$

$\therefore$ The solution is $(x = 2, y = 1, z = -1)$

3. **A man is appointed in a job with a monthly salary of certain amount and a fixed amount of annual increment. If his salary was ₹19,800 per month at the end of the first month after 3 years of service and ₹23,400 per month at the end of the first month after 9 years of service, find his starting salary and his annual increment. (Use matrix inversion method to solve the problem.)**

Let the monthly salary =₹ x

Annual increment =₹y

From the given information, we have

$x + 3y = 19800$

$x + 9y = 23400$

The matrix form is $AX = B$ where $A = \begin{bmatrix} 1 & 3 \\ 1 & 9 \end{bmatrix}, x = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} 19800 \\ 23400 \end{bmatrix}$

$|A| = 9 - 3 = 6 \neq 0 \therefore A^{-1} exists$

$adjA = \begin{bmatrix} 9 & -3 \\ -1 & 1 \end{bmatrix}$

$$A^{-1} = \frac{1}{|A|}adjA = \frac{1}{6}\begin{bmatrix} 9 & -3 \\ -1 & 1 \end{bmatrix}$$

$$AX = B \Rightarrow X = A^{-1}B = \frac{1}{6}\begin{bmatrix} 9 & -3 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 19800 \\ 23400 \end{bmatrix}$$

$$X = \frac{1}{6}\begin{bmatrix} 178200 & -70200 \\ -19800 & +23400 \end{bmatrix}$$

$$X = \frac{1}{6}\begin{bmatrix} 108000 \\ 3600 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 18000 \\ 600 \end{bmatrix}$$

The solution is $x = 18000, y = 600$

$\therefore$ Monthly salary=₹18,000

Annual increment = ₹600

**4. 4 men and 4 women can finish a piece of work jointly in 3 days while 2 men and 5 women can finish the same work jointly in 4 days. Find the time taken by one man alone and that of one woman alone to finish the same work by using matrix inversion method.**

Let

The number of days taken by a man to complete the work $= x$

The number of days taken by a woman to complete the work $= y$

From the given information, we have

$\frac{4}{x} + \frac{4}{y} = \frac{1}{3}$

$\frac{2}{x} + \frac{5}{y} = \frac{1}{4}$

The matrix form is $AX = B$ where

$$A = \begin{bmatrix} 4 & 4 \\ 2 & 5 \end{bmatrix}, X = \begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \end{bmatrix}, B = \begin{bmatrix} \frac{1}{3} \\ \frac{1}{4} \end{bmatrix}$$

$|A| = 20 - 8 = 12 \neq 0 \therefore A^{-1}$ exists

$$A^{-1} = \frac{1}{|A|} adjA = \frac{1}{12} \begin{bmatrix} 5 & -4 \\ -2 & 4 \end{bmatrix}$$

$AX = B \Rightarrow X = A^{-1}B$

$$X = \frac{1}{12} \begin{bmatrix} 5 & -4 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} \frac{1}{3} \\ \frac{1}{4} \end{bmatrix}$$

$$X = \frac{1}{12} \begin{bmatrix} \frac{5}{3} & -1 \\ -\frac{2}{3} & +1 \end{bmatrix} = \frac{1}{12} \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \end{bmatrix} = \begin{bmatrix} \frac{1}{12} \times \frac{2}{3} \\ \frac{1}{12} \times \frac{1}{3} \end{bmatrix} = \begin{bmatrix} \frac{1}{18} \\ \frac{1}{36} \end{bmatrix}$$

$$\frac{1}{x} = \frac{1}{18} \Rightarrow x = 18$$

$$\frac{1}{y} = \frac{1}{36} \Rightarrow y = 36$$

$\therefore$ Number of days taken by a man to complete the work $= 18$ days

Number of days taken by a woman to complete the work $= 36$ days

5. **The prices of three commodities $A, B$ and $C$ are ₹ $x, y$ and $z$ per units respectively. A person $P$ purchases 4 units of $B$ and sells two units of $A$ and 5 units of $C$. Person Q purchases 2 units of $C$ and sells 3 units of $A$ and one unit of $B$. Person R purchases one unit of $A$ and sells 3 units of B and one unit of $C$. In the process, P,Q and R earn ₹ 15,000, ₹ 1,000 and ₹ 4,000 respectively. Find the prices per unit of $A, B$ and $C$. (Use matrix inversion method to solve the problem.)**

Let the price per unit of

Commodity $A =$₹$x$, Commodity $B =$₹$y$, Commodity $C =$₹$z$

| |
|---|
| **Note:** 1. The amount separate on purchasing the commodity is negative<br>2. The amount earned by selling the commodity is positive. |

From the given information, we get

$2x - 4y + 5z = 15000$

$3x + y - 2z = 1000$

$-x + 3y + z = 4000$

The matrix from is $AX = B$ where

$$A = \begin{bmatrix} 2 & -4 & 5 \\ 3 & 1 & -2 \\ -1 & 3 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 15000 \\ 1000 \\ 4000 \end{bmatrix}$$

$|A| = 2(1+6) + 4(3-2) + 5(9+1)$

$= 14 + 4 + 50 = 68 \neq 0$

$\therefore A^{-1}$ exists

$$adjA = \begin{bmatrix} (1+6) & -(3-2) & (9+1) \\ -(-4-15) & (2+5) & -(6-4) \\ (8-5) & -(-4-15) & (2+12) \end{bmatrix}^T$$

$$adjA = \begin{bmatrix} 7 & -1 & 10 \\ 19 & 7 & -2 \\ 3 & 19 & 14 \end{bmatrix}^T = \begin{bmatrix} 7 & 19 & 3 \\ -1 & 7 & 19 \\ 10 & -2 & 14 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} adjA = \frac{1}{68}\begin{bmatrix} 7 & 19 & 3 \\ -1 & 7 & 19 \\ 10 & -2 & 14 \end{bmatrix}$$

$AX = B \Rightarrow X = A^{-1}B$

$$X = \frac{1}{68}\begin{bmatrix} 7 & 19 & 3 \\ -1 & 7 & 19 \\ 10 & -2 & 14 \end{bmatrix}\begin{bmatrix} 15000 \\ 1000 \\ 4000 \end{bmatrix}$$

$$X = \frac{1}{68}\begin{bmatrix} 105000 + 19000 + 12000 \\ -15000 + 7000 + 76000 \\ 150000 - 2000 + 56000 \end{bmatrix}$$

$$X = \frac{1}{68}\begin{bmatrix} 136000 \\ 68000 \\ 204000 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2000 \\ 1000 \\ 3000 \end{bmatrix}$$

$\therefore$ The solution is $x = 2000, y = 1000, z = 3000$

Price per unit of $A =$ ₹2000

Price per unit of $B =$ ₹1000

Price per unit of $C =$ ₹3000

**EXERCISE 1.4**

**1. Solve the following systems of linear equations by Cramer's rule:**

**(i)** $\boldsymbol{5x - 2y + 16 = 0, x + 3y - 7 = 0}$ **(ii)** $\boldsymbol{\frac{3}{x} + 2y = 12, \frac{2}{x} + 3y = 13}$

**(iii)** $\boldsymbol{3x + 3y - z = 11, 2x - y + 2z = 9, 4x + 3y + 2z = 25}$

**(iv)** $\boldsymbol{\frac{3}{x} - \frac{4}{y} - \frac{2}{z} - 1 = 0, \frac{1}{x} + \frac{2}{y} + \frac{1}{z} - 2 = 0, \frac{2}{x} - \frac{5}{y} - \frac{4}{z} + 1 = 0}$

**(i)** $\boldsymbol{5x - 2y = -16, x + 3y = 7}$

$\Delta = \begin{vmatrix} 5 & -2 \\ 1 & 3 \end{vmatrix}$

$= 15 + 2$

$= 17$

$\Delta_1 = \begin{vmatrix} -16 & -2 \\ 7 & 3 \end{vmatrix}$

$= -48 + 14$

$= -34$

$\Delta_2 = \begin{vmatrix} 5 & -16 \\ 1 & 7 \end{vmatrix}$

$= 35 + 16$

$= 51$

By Cramer's rule, we get

$$x = \frac{\Delta_1}{\Delta} = -\frac{34}{17} = -2$$

$$y = \frac{\Delta_2}{\Delta} = \frac{51}{17} = 3$$

$\therefore$ The solution is $(x = -2, y = 3)$.

**(ii)** $\boldsymbol{\frac{3}{x} + 2y = 12, \frac{2}{x} + 3y = 13}$

$\Delta = \begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix}$

$= 9 - 4$

$= 5$

$\Delta_1 = \begin{vmatrix} 12 & 2 \\ 13 & 3 \end{vmatrix}$

$= 36 - 26$

$= 10$

$\Delta_2 = \begin{vmatrix} 3 & 12 \\ 2 & 13 \end{vmatrix}$

$= 39 - 24$

$= 15$

$$\frac{1}{x} = \frac{\Delta_1}{\Delta} = \frac{10}{5} = 2 \Rightarrow x = \frac{1}{2}$$

$$y = \frac{\Delta_2}{\Delta} = \frac{15}{5} = 3$$

$\therefore$ The solution is $\left(x = \frac{1}{2}, y = 3\right)$

**(iii)** $\boldsymbol{3x + 3y - z = 11, 2x - y + 2z = 9, 4x + 3y + 2z = 25}$

$\Delta = \begin{vmatrix} 3 & 3 & -1 \\ 2 & -1 & 2 \\ 4 & 3 & 2 \end{vmatrix}$

$= 3[-2 - 6] - 3[4 - 8] - 1[6 + 4]$

$= -24 + 12 - 10 = -22$

$\Delta = -22$

$\Delta_1 = \begin{vmatrix} 11 & 3 & -1 \\ 9 & -1 & 2 \\ 25 & 3 & 2 \end{vmatrix}$

$= 11(-2-6) - 3(18-50) - 1(27+25)$

$= 11(-8) - 3(-32) - 1(52)$

$= -88 + 96 - 52 = -44$

$\Delta_1 = -44$

$\Delta_2 = \begin{vmatrix} 3 & 11 & -1 \\ 2 & 9 & 2 \\ 4 & 25 & 2 \end{vmatrix}$

$= 3(18-50) - 11(4-8) - 1(50-36)$

$= 3(-32) - 11(-4) - 1(14)$

$= -96 + 44 - 14 = -66$

$\Delta_2 = -66$

$\Delta_3 = \begin{vmatrix} 3 & 3 & 11 \\ 2 & -1 & 9 \\ 4 & 3 & 25 \end{vmatrix}$

$= 3(-25-27) - 3(50-36) + 11(6+4)$

$= 3(-52) - 3(14) + 11(10)$

$= -156 - 42 + 110 = -88$

$\Delta_3 = -88$

By Cramer's Rule, we get

$x = \frac{\Delta_1}{\Delta} = \frac{-44}{-22} = 2$

$y = \frac{\Delta_2}{\Delta} = \frac{-66}{-22} = 3$

$z = \frac{\Delta_3}{\Delta} = \frac{-88}{-22} = 4$

$\therefore$ The solution is $(x = 2, y = 3, z = 4)$

**(iv)** $\mathbf{\frac{3}{x} - \frac{4}{y} - \frac{2}{z} = 1}$

$\frac{1}{x} + \frac{2}{y} + \frac{1}{z} = 2$

$\frac{2}{x} - \frac{5}{y} - \frac{4}{z} = -1$

$\Delta = \begin{vmatrix} 3 & -4 & -2 \\ 1 & 2 & 1 \\ 2 & -5 & -4 \end{vmatrix}$

$= 3(-8+5)+4(-4-2)-2(-5-4)$

$= 3(-3)+4(-6)-2(-9)$

$= -9-24+18 = -15$

$\Delta = -15$

$$\Delta_1 = \begin{vmatrix} 1 & -4 & -2 \\ 2 & 2 & 1 \\ -1 & -5 & -4 \end{vmatrix}$$

$= 1(-8+5)+4(-8+1)-2(-10+2)$

$= 1(-3)+4(-7)-2(-8)$

$= -3-28+16 = -15$

$\Delta_1 = -15$

$$\Delta_2 = \begin{vmatrix} 3 & 1 & -2 \\ 1 & 2 & 1 \\ 2 & -1 & -4 \end{vmatrix}$$

$= 3(-8+1)-1(-4-2)-2(-1-4)$

$= 3(-7)-1(-6)-2(-5)$

$= -21+6+10 = -5$

$\Delta_2 = -5$

$$\Delta_3 = \begin{vmatrix} 3 & -4 & 1 \\ 1 & 2 & 2 \\ 2 & -5 & -1 \end{vmatrix}$$

$= 3(-2+10)+4(-1-4)+1(-5-4)$

$= 3(8)+4(-5)+1(-9)$

$= 24-20-9 = -5$

$\Delta_3 = -5$

$\frac{1}{x} = \frac{\Delta_1}{\Delta} = \frac{-15}{-15} = 1 \Rightarrow x = 1$

$\frac{1}{y} = \frac{\Delta_2}{\Delta} = -\frac{5}{-15} = \frac{1}{3} \Rightarrow y = 3$

$\frac{1}{z} = \frac{\Delta_3}{\Delta} = -\frac{5}{-15} = \frac{1}{3} \Rightarrow z = 3$

$\therefore$ The solution is $(x = 1, y = 3, z = 3)$

**2. In a competitive examination, one mark is awarded for every correct answer while $\frac{1}{4}$ marks is deducted for every wrong answer. A student answered 100 questions and got 80 marks. How many questions did he answer correctly? (Use Cramer's rule to solve the problem).**

Let the number of question Answered correctly $= x$

The number of question Answered wrongly $= y$

$\therefore x + y = 100$.....................(1)

Marks awarded for one

Correct answer $= 1$

Wrong answer $= -\frac{1}{4}$

$\therefore (1 \times x) + \left(-\frac{1}{4} \times y\right) = 80$

Multiplying by 4, we get $4x - y = 320$...........(2)

$x + y = 100$

$4x - y = 320$

$\Delta = \begin{vmatrix} 1 & 1 \\ 4 & -1 \end{vmatrix} = -1 - 4 = -5$

$\Delta = -5$

$\Delta_1 = \begin{vmatrix} 100 & 1 \\ 320 & -1 \end{vmatrix} = -100 - 320 = -420$

$\Delta_1 = -420$

$\Delta_2 = \begin{vmatrix} 1 & 100 \\ 4 & 320 \end{vmatrix} = 320 - 400 = -80$

$\Delta_2 = -80$

$x = \frac{\Delta_1}{\Delta} = \frac{-420}{-5} = 84$

$y = \frac{\Delta_2}{\Delta} = \frac{-80}{-5} = 16$

$\therefore$ The student answered 84 questions correctly.

3. **A chemist has one solution which is 50% acid and another which is 25% acid. How much each should be mixed to make 10 litres of a 40% acid solution? (Use Cramer's rule to solve the problem).**

Let $x$ and $y$ be the amount of solution containing 50% and 25% acid respectively.

From the given data,

$x + y = 10$ .............(1)

50% of $x$ +25% of $y$ = 40% of 10

$\frac{50}{100}x + \frac{25}{100}y = \frac{40}{100}(10)$

$50x + 25y = 400$ ............(2)

$\Delta = \begin{vmatrix} 1 & 1 \\ 50 & 25 \end{vmatrix}$

$= 25 - 50 = -25$

$\Delta_x = \begin{vmatrix} 10 & 1 \\ 400 & 25 \end{vmatrix}$

$= 250 - 400 = -150$

$\Delta_y = \begin{vmatrix} 1 & 100 \\ 50 & 400 \end{vmatrix}$

$= 400 - 500 = -100$

$x = \frac{\Delta_x}{\Delta} = \frac{-150}{-25} = 6$

$y = \frac{\Delta_y}{\Delta} = \frac{-100}{-5} = 4$

6 litres of solution containing 50% acid and 4 litres of solution containing 25% acid must be mixed to make 40% acid solution

**4. A fish tank can be filled in 10 minutes using both pumps A and B simultaneously. However, pump B can pump water in or out at the same rate. If pump B is inadvertently run in reverse, then the tank will be filled in 30 minutes. How long would it take each pump to fill the tank by itself? (use Cramer's rule to solve the problem).**

Let

The time taken by pump A to fill the tank by itself $= x$ minutes

The time taken by pump B to fill the tank by itself $= y$ minutes

So, the part of the tank filled by pump A in 1 minute $= \frac{1}{x}$

The part of the tank filled by pump B in 1 minute $= \frac{1}{y}$

The part of the tank filled by both pumps A & B in 1 minute $= \frac{1}{10}$

$\therefore \frac{1}{x} + \frac{1}{y} = \frac{1}{10}$

If pump B runs in reverse, then the tank will be filled by both pumps in 30 minutes.

In this case, the part of the tank filled by both pumps A & B in 1 minute $= \frac{1}{30}$

$\therefore \frac{1}{x} - \frac{1}{y} = \frac{1}{30}$

Let $a = \frac{1}{x}$ and $b = \frac{1}{y}$

$a + b = \frac{1}{10} \Rightarrow 10a + 10b = 1$

$a - b = \frac{1}{30} \Rightarrow 30a - 30b = 1$

$\Delta = \begin{vmatrix} 10 & 10 \\ 30 & -30 \end{vmatrix}$

$= -300 - 300 = -600$

$\Delta_1 = \begin{vmatrix} 1 & 10 \\ 1 & -30 \end{vmatrix}$

$= -30 - 10 = -40$

$\Delta_2 = \begin{vmatrix} 10 & 1 \\ 30 & 1 \end{vmatrix}$

$= 10 - 30 = -20$

$a = \dfrac{\Delta_1}{\Delta} = \dfrac{-40}{-600} = \dfrac{1}{15}$

$b = \dfrac{\Delta_2}{\Delta} = \dfrac{-20}{-600} = +\dfrac{1}{30}$

$a = \dfrac{1}{x} = \dfrac{1}{15}$

$\Rightarrow x = 15$

$b = \dfrac{1}{y} = \dfrac{1}{30}$

$\Rightarrow y = 30.$

Pump A will take 15 minutes to fill the tank by itself.

Pump B will take 30 minutes to fill the tank by itself.

5. **A family of 3 people went out for dinner in a restaurant. The cost of two dosai, three idlies and two vadais is ₹150. The cost of the two dosai, two idlies and four vadais is ₹200. The cost of five dosai, four idlies and two vadais is ₹250. The family has ₹350 in hand and they ate 3 dosai and six idlies and six vadais. Will they be able to manage to pay the bill within the amount they had?**

Let

The cost of one dosai $=₹x$

The cost of one idly $=₹y$

The cost of one vadai $=₹z$

According to the given information, we get

$2x + 3y + 2z = 150$

$2x + 2y + 4z = 200$

$5x + 4y + 2z = 250$

$\Delta = \begin{vmatrix} 2 & 3 & 2 \\ 2 & 2 & 4 \\ 5 & 4 & 2 \end{vmatrix}$

$= 2(4 - 16) - 3(4 - 20) + 2(8 - 10)$

$= 2(-12) - 3(-16) + 2(-2)$

$= -24 + 48 - 4 = 20$

$\Delta = 20$

$$\Delta_1 = \begin{vmatrix} 150 & 3 & 2 \\ 200 & 2 & 4 \\ 250 & 4 & 2 \end{vmatrix}$$

$= 150(4-16) - 3(400-1000) + 2(800-500)$

$= 150(-12) - 3(-600) + 2(300)$

$= -1800 + 1800 + 600 = 600$

$\therefore \Delta_1 = 600$

$$\Delta_2 = \begin{vmatrix} 2 & 150 & 2 \\ 2 & 200 & 4 \\ 5 & 250 & 2 \end{vmatrix}$$

$= 2(400-1000) - 150(4-20) + 2(500-1000)$

$= 2(-600) - 150(-16) + 2(-500)$

$= -1200 + 2400 - 1000 = 200$

$\therefore \Delta_2 = 200$

$$\Delta_3 = \begin{vmatrix} 2 & 3 & 150 \\ 2 & 2 & 200 \\ 5 & 4 & 250 \end{vmatrix}$$

$= 2(500-800) - 3(500-1000) + 150(8-10)$

$= 2(-300) - 3(-500) + 150(-2)$

$= -600 + 1500 - 300 = 600$

$\Delta_3 = 600$

By Cramer's Rule, we get

$$x = \frac{\Delta_1}{\Delta} = \frac{600}{20} = 30$$

$$y = \frac{\Delta_2}{\Delta} = \frac{200}{20} = 10$$

$$z = \frac{\Delta_3}{\Delta} = \frac{600}{20} = 30$$

$\therefore$ The cost of one dosai =₹30

The cost of one idly = ₹10

The cost of one vadai =₹30

The cost of 3 dosai and six idly and six vadai $= 3(30) + 6(10) + 6(30)$

$= 90 + 60 + 180 =$ ₹330

Since the family has ₹350 in hand, they able to manage to pay the bill.

**EXERCISE 1.5**

**1. Solve the following systems of linear equations by Gaussian elimination method.**

**(i) $2x-2y+3z=2, x+2y-z=3,$ $3x-y+2z=1$**

The augmented matrix is

$$[A|B]=\left[\begin{array}{ccc|c}2&-2&3&2\\1&2&-1&3\\3&-1&2&1\end{array}\right]$$

$$\rightarrow\left[\begin{array}{ccc|c}1&2&-1&3\\2&-2&3&2\\3&-1&2&1\end{array}\right] R_1\leftrightarrow R_2$$

$$\rightarrow\left[\begin{array}{ccc|c}1&2&-1&3\\0&-6&5&-4\\0&-7&5&-8\end{array}\right] \begin{array}{l}R_2\rightarrow R_2-2R_1\\R_3\rightarrow R_3-3R_1\end{array}$$

$$\rightarrow\left[\begin{array}{ccc|c}1&2&-1&3\\0&-6&5&-4\\0&0&-5&-20\end{array}\right] R_3\rightarrow 6R_3-7R_2$$

Writing the equivalent system of equation from the row-echelon matrix, we get

$x+2y-z=3$……………(1)

$-6y+5z=-4$……………(2)

$-5z=-20$……………………(3)

(3)$\Rightarrow z=\frac{20}{5}=4\Rightarrow z=4$

Substituting $z=4$ in 2, we get

$-6y+5(4)=-4$

$-6y=-4-20=-24$

$y=\frac{-24}{-6}=4$

$y=4$

Substituting $y=4$ and $z=4$ in (1), we get

$x+2(4)-4=3$

$x+4=3$

$x=3-4=-1$

$x=-1$

$\therefore$ The solution is $(x=-1, y=4, z=4)$

**(ii) $2x-4y+6z=22,$ $3x+8y-5z=27, -x+y+2z=2$**

The augmented matrix is

$$[A|B]=\left[\begin{array}{ccc|c}2&4&6&22\\3&8&5&27\\-1&1&2&2\end{array}\right]$$

$$\rightarrow\left[\begin{array}{ccc|c}1&2&3&11\\3&8&5&27\\-1&1&2&2\end{array}\right] R_1\rightarrow\frac{1}{2}R_1$$

$$\rightarrow\left[\begin{array}{ccc|c}1&2&3&11\\0&2&-4&-6\\0&3&5&13\end{array}\right] \begin{array}{l}R_2\rightarrow R_2-3R_1\\R_3\rightarrow R_3+R_1\end{array}$$

$$\rightarrow\left[\begin{array}{ccc|c}1&2&3&11\\0&1&-2&-3\\0&3&5&13\end{array}\right] R_2\rightarrow\tfrac{1}{2}R_2$$

$$\rightarrow\left[\begin{array}{ccc|c}1&2&3&11\\0&1&-2&-3\\0&0&11&22\end{array}\right] R_3\rightarrow R_3-3R_2$$

Writing the equivalent system of equations from the row – echelon matrix, we get

$x+2y+3z=11$…………(1)

$y-2x=-3$………………(2)

$11z=22$………………………(3)

(3) $\Rightarrow z=\frac{22}{11}=2$

$z=2$

Substituting $z=2$ in (2), we get

$y-2(2)=-3$

$y-4=-3$

$y=-3+4=1$

$y=1$

Substituting $y=1$ & $z=2$ in 1, we get

$x+2(1)+3(2)=11$

$x+2+6=11\Rightarrow x=11-8=3$

$x=3$

$\therefore$ The solution is $(x=3, y=1, z=2)$

**2. If $ax^2+bx+c$ is divided by $x+3, x-5,$ and $x-1$, the remainders are 21, 61 and 9 respectively. Find $a, b$ and $c$. (Use Gaussian elimination method.)**

Let $P(x)=ax^2+bx+c$

<u>Given:</u> $P(x)\div(x+3)$ and leaves the remainder 21

$\therefore P(-3)=a(-3)^2+b(-3)+c=21$

$$9a - 3b + c = 21$$

Given: $P(x) \div (x-5)$ and leaves the remainder 61

$\therefore P(5) = a(5)^2 + b(5) + c = 61$

$$25a + 5b + c = 61$$

Given: $P(x) \div (x-1)$ and leave the remainder 9

$\therefore P(+1) = a(+1)^2 + b(+1) + c = 9$

$a + b + c = 9$

$\therefore$ The system of linear equations:

$9a - 3b + c = 21$

$25a + 5b + c = 61$

$a + b + c = 9$

The augmented matrix is $[A|B] = \left[\begin{array}{ccc|c} 9 & -3 & 1 & 21 \\ 25 & 5 & 1 & 61 \\ 1 & 1 & 1 & 9 \end{array}\right]$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 25 & 5 & 1 & 61 \\ 9 & -3 & 1 & 21 \end{array}\right] R_1 \leftrightarrow R_3$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & -20 & -24 & -164 \\ 0 & -12 & -8 & -60 \end{array}\right] \begin{array}{l} R_2 \rightarrow R_2 - 25R_1 \\ R_3 \rightarrow R_3 - 9R_1 \end{array}$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & -5 & -6 & -41 \\ 0 & -3 & -2 & -15 \end{array}\right] \begin{array}{l} R_2 \rightarrow \frac{R_2}{4} \\ R_3 \rightarrow \frac{R_3}{4} \end{array}$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & -5 & -6 & -41 \\ 0 & 0 & 8 & 48 \end{array}\right] R_3 \rightarrow 5R_3 - 3R_2$$

Writing the equivalent equation from the row – echelon matrix, we get

$a + b + c = 9$……………..(1)

$-5b - 6c = -41$…………..(2)

$8c = 48$…………………………(3)

$(3) \Rightarrow c = \frac{48}{8} = 6$

$c = 6$

Substituting $c = 6$ in (2), we get

$-5b - 6(6) = -41$

$-5b - 36 = -41$

$-5b = -41 + 36 = -5$

$b = \frac{-5}{-5} = 1$

$b = 1$

Substituting $b = 1, c = 6$ in 1, we get

$a + 1 + 6 = 9$

$a = 9 - 7 = 2$

$a = 2$

$\therefore$ The solution is $(a = 2, b = 1, c = 6)$

**3. An amount of ₹65,000 is invested in three bonds at the rates of 6%, 8% and 10% per respectively. The total annual income is ₹5000. The income from the third bond is ₹800 more than that from the second bond. Determine the price of each bond. (Use Gaussian elimination method.)**

Let

The price of the first bond =₹$x$

The price of the second bond =₹$y$

The price of the third bond =₹$z$

According to the given information, we get

$x + y + z = 65000$

(6% of $x$) + (8% of $y$)+ (10% of $z$) = 5000

$\Rightarrow \dfrac{6x + 8y + 10z}{100} = 5000$

$\Rightarrow 6x + 8y + 10z = 500000$

$\div$ by 2 $\Rightarrow 3x + 4y + 5z = 250000$

(10% of $z$) − (8% of $y$) = 800

$\Rightarrow \frac{10z-8y}{100} = 800$

$\Rightarrow -8y + 10z = 8000$

$\div$ by 2 $\Rightarrow -4y + 5z = 40000$

The augmented matrix is $[A|B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 65000 \\ 3 & 4 & 5 & 250000 \\ 0 & -4 & 5 & 40000 \end{array}\right]$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 65000 \\ 0 & 1 & 2 & 55000 \\ 0 & -4 & 5 & 40000 \end{array}\right] R_2 \rightarrow R_2 - 3R_1$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 65000 \\ 0 & 1 & 2 & 55000 \\ 0 & 0 & 13 & 260000 \end{array}\right] R_3 \rightarrow R_3 + 4R_2$$

Writing the equivalent equations from the row-echelon matrix, we get

$x + y + z = 65000$................(1)

$y + 2z = 55000$ ................(2)

$13z = 260000$ ................(3)

$(3) \Rightarrow z = \frac{260000}{13} = 20000$

$z = 20000$

Substituting $z = 20000$ in (2), we get

$y + 2(20000) = 55000$

$y = 55000 - 40000 = 15000$

$y = 15000$

Substituting $y = 15000, z = 20000$ in (1), we get

$x + 15000 + 20000 = 65000$

$x = 65000 - 35000 = 30000$

$x = 30000$

∴ The price of the first bond =₹ 30,000

The price of the second bond =₹ 15,000

The price of the third bond =₹20,000

**4. A boy is walking along the path $y = ax^2 + bx + c$ through the points $(-6, 8), (-2, 12)$, and $(3, 8)$. He wants to meet his friend at $P(7, 60)$. Will he meet his friend? (Use Gaussian elimination method.)**

The path $y = ax^2 + bx + c$ passes through the points $(-6,8), (-2,12)$ and $(3,8)$

So, we get the system of equations

$8 = a(-6)^2 + b(-6) + c \Rightarrow 36a - 6b + c = 8$

$-12 = a(-2)^2 + b(-2) + c \Rightarrow 4a - 2b + c = -12$

$8 = a(3)^2 + b(3) + c \Rightarrow 9a + 3b + c = 8$

The augmented matrix is

$$[A|B] = \left[\begin{array}{ccc|c} 36 & -6 & 1 & 8 \\ 4 & -2 & 1 & -12 \\ 9 & 3 & 1 & 8 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{ccc|c} 36 & -6 & 1 & 8 \\ 0 & -12 & 8 & 116 \\ 0 & 18 & 3 & 24 \end{array}\right] \begin{array}{l} \\ R_2 \rightarrow 9R_2 - R_1 \\ R_3 \rightarrow 4R_3 - R_1 \end{array}$$

$$\rightarrow \left[\begin{array}{ccc|c} 36 & -6 & 1 & 8 \\ 0 & -3 & 2 & -29 \\ 0 & 6 & 1 & 8 \end{array}\right] \begin{array}{l} R_2 \rightarrow \frac{1}{4}R_2 \\ R_3 \rightarrow \frac{1}{3}R_3 \end{array}$$

$$\rightarrow \left[\begin{array}{ccc|c} 36 & -6 & 1 & 8 \\ 0 & -3 & 2 & -29 \\ 0 & 0 & 5 & -50 \end{array}\right] R_3 \rightarrow R_3 + 2R_2$$

Writing the equivalent equations from the row-echelon matrix, we get

$36a - 6b + c = 8$……………..(1)

$-3b + 2c = -29$………………(2)

$5c = -50$……………………… (3)

$(3) \Rightarrow c = -\frac{50}{5} = -10$

$(2) \Rightarrow -3b + 2(-10) = -29$

$-3b = -29 + 20 = -9$

$b = \frac{-9}{-3} = 3$

$b = 3$

$(1) \Rightarrow 36a - 6(3) - 10 = 8$

$36a - 28 = 8$

$36a = 8 + 28 = 36$

$a = \frac{36}{36} = 1$

$a = 1$

$\therefore$ Equation of the path is $y = x^2 + 3x - 10$

Substituting $x = 7$, we get

$y = 7^2 + 3(7) - 10$

$= 49 + 21 - 10 = 60$

Thus, the path passes through the point $P(7,60)$.

Hence, the boy will meet his friend.

**EXERCISE 1.6**

**1. Test for consistency and if possible, solve the following systems of equations by rank method.**

**(i) $x - y + 2z = 2, 2x + y + 4z = 7, 4x - y + z = 4$**

The matrix form of the system is $AX = B$, where

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & 4 \\ 4 & -1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 2 \\ 7 \\ 4 \end{bmatrix}$$

The augmented matrix is

$$[A|B] = \left[\begin{array}{ccc|c} 1 & -1 & 2 & 2 \\ 2 & 1 & 4 & 7 \\ 4 & -1 & 1 & 4 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 2 & 2 \\ 0 & 3 & 0 & 3 \\ 0 & 3 & -7 & -4 \end{array}\right] \begin{array}{l} \\ R_2 \rightarrow R_2 - 2R_1 \\ R_3 \rightarrow R_3 - 4R_1 \end{array}$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 2 & 2 \\ 0 & 3 & 0 & 3 \\ 0 & 0 & -7 & -7 \end{array}\right] R_3 \rightarrow R_3 - R_2$$

The last equivalent matrix is in row-echelon form and has three non-zero rows.

$\therefore \rho[A|B] = 3$

Also $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 0 \\ 0 & 0 & -7 \end{bmatrix}$

It is also in the echelon form and it has also three non-zero rows.

$\therefore \rho(A) = 3$

Since $\rho(A) = \rho[A|B] = 3$ =no.of unknowns, the given system is consistent and has a unique solution.

The equivalent system of equations:

$x - y + 2z = 2$....................(1)

$3y = 3$ ................................(2)

$-7z = -7$.............................(3)

(3) $\Rightarrow -7z = -7$

$z = \frac{-7}{-7} = 1$

$z = 1$

(2) $\Rightarrow 3y = 3$

$y = \frac{3}{3} = 1$

$y = 1$

Substituting, $y = 1, z = 1$ in (1), we get

$x - 1 + 2 = 2$

$x + 1 = 2 \Rightarrow x = 2 - 1 = 1 \Rightarrow x = 1$

$\therefore$ The solution is $(x = 1, y = 1, z = 1)$

**(ii) $3x + y + z = 2, x - 3y + 2z = 1, 7x - y + 4z = 5$**

The matrix form of the system is $AX = B$, where

$A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & -3 & 2 \\ 7 & -1 & 4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix}$

The augmented matrix is

$$[A,B]=\left[\begin{array}{ccc|c}3&1&1&2\\1&-3&2&1\\7&-1&4&5\end{array}\right]$$

$$\to\left[\begin{array}{ccc|c}3&-3&2&1\\3&1&1&2\\7&-1&4&5\end{array}\right] R_1\leftrightarrow R_2$$

$$\to\left[\begin{array}{ccc|c}1&-3&2&1\\0&10&-5&-1\\0&20&-10&-2\end{array}\right] \begin{array}{l}R_2\to R_2-3R_1\\R_3\to R_3-7R_1\end{array}$$

$$\to\left[\begin{array}{ccc|c}1&-3&2&1\\0&10&-5&-1\\0&0&0&0\end{array}\right] R_3\to R_3-2R_1$$

The last equivalent matrix is in row-echelon form and has three non-zero rows.

$\therefore \rho[A|B]=2$

Also $A=\begin{bmatrix}1&-3&2\\0&10&-5\\0&0&0\end{bmatrix}$

It has two non-zero rows

$\therefore \rho(A)=2$

Since $\rho(A)=\rho[A|B]=2<$ Number of unknowns, the given system is consistent and has infinitely many solutions.

The equivalent system of equations:

$x-3y+2z=1$……………………..(1)

$10y-5z=-1$……………………….(2)

Let $z=t, t\in R$

$(2)\Rightarrow 10y-5t=-1\Rightarrow 10y=5t-1\Rightarrow y=\dfrac{5t-1}{10}$

$(1)\Rightarrow x-3\left(5t-\dfrac{1}{10}\right)+2t=1$

$x+\frac{-15t+3+20t}{10}=1$

$x+\frac{5t+3}{10}=1$

$x=\frac{10-5t-3}{10}=\frac{7-5t}{10}$

$\therefore$ The solution is $\left(x=\frac{1}{10}(7-5t),\ y=\frac{1}{10}(5t-1), z=t\right), t\in R$

**(iii) $2x+2y+z=5, x-y+z=1, 3x+y+2z=4$**

The matrix form of the system is $AX=B$, where

$$A=\begin{bmatrix}2&2&1\\1&-1&1\\3&1&2\end{bmatrix}, X=\begin{bmatrix}x\\y\\z\end{bmatrix}, B=\begin{bmatrix}5\\1\\4\end{bmatrix}$$

The augmented matrix is

$$[A,B]=\left[\begin{array}{ccc|c}2&2&1&5\\1&-1&1&1\\3&1&2&4\end{array}\right]$$

$$\rightarrow\left[\begin{array}{ccc|c}1&-1&1&1\\2&2&1&5\\3&1&2&4\end{array}\right] R_1\leftrightarrow R_2$$

$$\rightarrow\left[\begin{array}{ccc|c}1&-1&1&1\\0&4&-1&3\\0&4&-1&1\end{array}\right] \begin{array}{l}R_2\rightarrow R_2-2R_1\\R_3\rightarrow R_3-3R_1\end{array}$$

$$\rightarrow\left[\begin{array}{ccc|c}1&-1&1&1\\0&4&-1&3\\0&0&0&-2\end{array}\right] R_3\rightarrow R_3-R_1$$

The last equivalent matrix is in row-echelon form and it has three non-zero rows.

$\therefore \rho[A|B]=3$

Also $A=\begin{bmatrix}1&-1&1\\0&4&-1\\0&0&0\end{bmatrix}$

It is in row – echelon form and it has two non-zero rows

$\therefore \rho(A)=2$

Since $\rho(A)\neq\rho[A|B]$ the given system is inconsistent and has no solutions.

**(iv) $2x-y+z=2, 6x-3y+3z=6, 4x-2y+2z=4$**

The matrix form of the system is $AX=B$, where

$$A=\begin{bmatrix}2&-1&1\\6&-3&3\\4&-2&2\end{bmatrix}, X=\begin{bmatrix}x\\y\\z\end{bmatrix}, B=\begin{bmatrix}2\\6\\4\end{bmatrix}$$

The augmented matrix is

$$[A|B]=\left[\begin{array}{ccc|c}2&-1&1&2\\6&-3&3&6\\4&-2&2&4\end{array}\right]$$

$$\rightarrow\left[\begin{array}{ccc|c}2&-1&1&2\\0&0&0&0\\0&0&0&0\end{array}\right] \begin{array}{l}R_2\rightarrow R_2-3R_1\\R_3\rightarrow R_3-2R_1\end{array}$$

The last equivalent matrix is in row-echelon form and has one non-zero rows.

$\therefore \rho[A|B]=1$

Also $A=\begin{bmatrix}2&-1&1\\0&0&0\\0&0&0\end{bmatrix}$

A is in row – echelon form and it has two non-zero row.

$\therefore \rho(A)=1$

Since $\rho(A)=\rho[A|B]=1$ <No.of unknowns, the given system is consistent and it has infinitely many solutions.

The equivalent system of equations:

$2x - y + z = 2$

Let $y = s$ and $z = t$ where $s, t \in R$

$2x - s + t = 2$

$2x = 2 + s - t$

$x = \frac{1}{2}(2 + s - t)$

$\therefore$ The solution is $\left(x = \frac{1}{2}(2 + s - t)y = s, z = t\right) s, t \in R$

**2. Find the value of $k$ for which the equations $kx - 2y + z = 1, x - 2ky + z = -2$, $x - 2y + kz = 1$ have**

**(i) no solution (ii) unique solution (iii) infinitely many solution**

$kx - 2y + z = 1, x - 2ky + z = -2, x - 2y + kz = 1$

The matrix form of the system is $AX = B$, where

$A = \begin{bmatrix} k & -2 & 1 \\ 1 & -2k & 1 \\ 1 & -2 & k \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$

The augmented matrix is

$$[A|B] = \left[\begin{array}{ccc|c} k & -2 & 1 & 1 \\ 1 & -2k & 1 & -2 \\ 1 & -2 & k & 1 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & -2 & k & 1 \\ 1 & -2k & 1 & -2 \\ k & -2 & 1 & 1 \end{array}\right] R_1 \leftrightarrow R_3$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & -2 & k & 1 \\ 0 & 2-2k & 1-k & -3 \\ 0 & 2k-2 & 1-k^2 & 1-k \end{array}\right] \begin{array}{l} R_2 \rightarrow R_2 - R_1 \\ R_3 \rightarrow R_3 - kR_1 \end{array}$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & -2 & k & 1 \\ 0 & 2-2k & 1-k & -3 \\ 0 & 0 & 2-k-k^2 & -2-k \end{array}\right] R_3 \rightarrow R_3 + R_1$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & -2 & k & 1 \\ 0 & 2(1-k) & 1-k & -3 \\ 0 & 0 & (2+k)(1-k) & -(2+k) \end{array}\right]$$

**Case (i)**

If $k = 1$, then $\rho[A|B] = 3$ and$\rho(A) = 1$

Since $\rho(A) \neq \rho[A|B]$, the given system of equations is inconsistent and has no solution.

**Case (ii)**

If $k \neq 1, k \neq 2$, then $\rho[A|B] = 3$ and $\rho[A] = 3$

Since $\rho[A] = \rho[A|B] = 3 = no. of\ unknowns$, the given system is consistent and has a unique solution

**Case (iii)**

If $k = -2$, then $\rho[A|B] = 2$ and $\rho[A] = 2$

Since $\rho[A|B] = \rho(A) = 2 < number\ of\ unknowns$, the given system of equations is consistent and has infinitely many solution.

**The given system has**

(i) no solution when $k = 1$

(ii) unique solution when $k \neq 1, k \neq -2$

(iii) infinitely many solution when $k = -2$

**3. Investigate the values of $\lambda$ and $\mu$ the system of linear equations $2x + 3y + 5z = 9$, $7x + 3y - 5z = 8, 2x + 3y + \lambda z = \mu$, have**

**(i) no solution (ii) a unique solution (iii) an infinite number of solutions.**

$2x + 3y + 5z = 9, 7x + 3y - 5z = 8, 2x + 3y + \lambda z = \mu$

The matrix form of the system is $AX = B$, where

$$A = \begin{bmatrix} 2 & 3 & 5 \\ 7 & 3 & -5 \\ 2 & 3 & \lambda \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 9 \\ 8 \\ \mu \end{bmatrix}$$

The augmented matrix is

$$[A|B] = \left[\begin{array}{ccc|c} 2 & 3 & 5 & 9 \\ 7 & 3 & -5 & 8 \\ 2 & 3 & \lambda & \mu \end{array}\right]$$

$$\rightarrow \left[\begin{array}{ccc|c} 2 & 3 & 5 & 9 \\ 0 & -15 & -45 & -47 \\ 0 & 0 & \lambda - 5 & \mu - 9 \end{array}\right] \begin{array}{l} R_2 \rightarrow 2R_2 - 7R_1 \\ R_3 \rightarrow R_3 - R_1 \end{array}$$

**Case (i)**

If $\lambda = 5$ and $\mu \neq 9$, then $\rho(A) = 2$ and $\rho[A|B] = 3$

Since $\rho(A) \neq \rho[A|B]$, the given system of equations is inconsistent and has no solution.

**Case (ii)**

If $\lambda \neq 5$ and $\mu \in R$, then

Since $\rho(A) = \rho[A|B] = 3 =$ no. of. unknowns, the given system of equations is consistent and has a unique solution.

**Case (iii)**

If $\lambda = 5$ and $\mu = 9$ then $\rho(A) = 2$ and $\rho[A|B] = 2 <$ No. of unknowns the given system of equations is consistent and has infinitely many solutions.

**The given system has**

(i) no solution when $\lambda = 5$ and $\mu \neq 9$

(ii) unique solution when $\lambda \neq 5$ and $\mu \in R$

(iii) infinitely many solution when $\lambda = 5$ and $\mu = 9$.

**EXERCISE 1.7**

**1. Solve the following system of homogenous equations.**

**(i)** $\mathbf{3x+2y+7z=0, 4x-3y-2z=0, 5x+9y+23z=0}$

The matrix form of the system is $AX=O$, where

$$A=\begin{bmatrix}3&2&7\\4&-3&-2\\5&9&23\end{bmatrix}, X=\begin{bmatrix}x\\y\\z\end{bmatrix}, O=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

The augmented matrix is

$$[A|O]=\left[\begin{array}{ccc|c}3&2&7&0\\4&-3&-2&0\\5&9&23&0\end{array}\right]$$

$$\rightarrow\left[\begin{array}{ccc|c}3&2&7&0\\0&-17&-34&0\\0&17&34&0\end{array}\right]\begin{matrix}\\R_3\rightarrow 3R_2-4R_1\\R_3\rightarrow 3R_3-5R_1\end{matrix}$$

$$\rightarrow\left[\begin{array}{ccc|c}3&2&7&0\\0&-17&-34&0\\0&0&0&0\end{array}\right]\begin{matrix}\\\\R_3\rightarrow R_3+R_1\end{matrix}$$

$$\rightarrow\left[\begin{array}{ccc|c}3&2&7&0\\0&1&2&0\\0&0&0&0\end{array}\right]R_2\rightarrow -\frac{1}{17}R_2$$

$\therefore \rho(A)=\rho([A|O])=2<$number of unknowns.

$\therefore$ The system is consistent and has infinite number of non-trivial solution.

The equivalent system of equation

$3x+2y+7z=0$..................................(1)

$y+2z=0$........................................(2)

Let $z=t, t\in R$

$(2)\Rightarrow y+2t=0\Rightarrow y=-2t$

$(1)\Rightarrow 3x+2(-2)t+7(t)=0$

$$3x-4t+7t=0$$
$$3x+3t=0$$
$$x+t=0$$
$$x=-t$$

$\therefore$ The solution is $(x=-t, y=-2t, z=t)$ where $t\in R$

**(ii)** $\mathbf{2x+3y-z=0, x-y-2z=0, 3x+y+3z=0}$

The matrix form of the system is $AX=O$, where

$$A=\begin{bmatrix}2&3&-1\\1&-1&-2\\3&1&3\end{bmatrix}, X=\begin{bmatrix}x\\y\\z\end{bmatrix}, O=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

The augmented matrix is

$$[A|O] = \left[\begin{array}{ccc|c} 2 & 3 & -1 & 0 \\ 1 & -1 & -2 & 0 \\ 3 & 1 & 3 & 0 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & -1 & -2 & 0 \\ 2 & 3 & -1 & 0 \\ 3 & 1 & 3 & 0 \end{array}\right] R_1 \leftrightarrow R_2$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & -1 & -2 & 0 \\ 0 & 5 & 3 & 0 \\ 0 & 4 & 9 & 0 \end{array}\right] \begin{array}{l} R_2 \rightarrow R_2 - 2R_1 \\ R_3 \rightarrow R_3 - 3R_1 \end{array}$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & -1 & -2 & 0 \\ 0 & 5 & 3 & 0 \\ 0 & 0 & 33 & 0 \end{array}\right] R_3 \rightarrow 5R_3 - 4R_2$$

$\rho(A) = \rho([A|O]) = 3$ =number of unknowns.

∴ The given system is consistent and has a trivial solution.

∴ The trivial solution is $(x = 0, y = 0, z = 0)$

**2. Determine the values of $\lambda$ for which the following system of equations $x + y + 3z = 0$, $4x + 3y + \lambda z = 0, 2x + y + 2z = 0$ has**
**(i) a unique solution (ii) a non-trivial solution.**

$x + y + 3z = 0, 4x + 3y + \lambda z = 0, 2x + y + 2z = 0$

The matrix form of the system is $AX = O$, where

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 4 & 3 & \lambda \\ 2 & 1 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, O = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

The augmented matrix is

$$[A|O] = \left[\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 4 & 3 & \lambda & 0 \\ 2 & 1 & 2 & 0 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 0 & -1 & \lambda - 12 & 0 \\ 0 & -1 & -4 & 0 \end{array}\right] \begin{array}{l} R_2 \rightarrow R_2 - 4R_1 \\ R_3 \rightarrow R_3 - 2R_1 \end{array}$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 0 & -1 & \lambda - 12 & 0 \\ 0 & 0 & 8 - \lambda & 0 \end{array}\right] R_3 \rightarrow R_3 - R_2$$

**Case (i)**

If $\lambda \neq 8$, then $\rho(A) = \rho([A|O]) = 3$ = number of unknowns

∴ The given system of equations is consistent and has a unique solution or trivial solution.

**Case (ii)**

If $\lambda = 8$, then $\rho(A) = \rho([A|0]) = 2 <$ number of unknowns

∴ The given system is consistent and has a non-trival solution.

**The given system has**

(i) a unique solution when $\lambda \neq 8$

(ii) a non-trivial solution when $\lambda = 8$

3. **By using Gaussian elimination method, balance the chemical reaction equation:**
$\boldsymbol{C_2H_6 + O_2 \rightarrow H_2O + CO_2}$

We are searching for positive integers $x_1, x_2, x_3$ and $x_4$ such that

$x_1C_2H_6 + x_2O_2 \rightarrow x_3H_2O + x_4CO_2$....................(1)

Equating carbon, Hydrogen and Oxygen atoms on the left-hand side of (1) to the respective carbon, Hydrogen and Oxygen atoms on the right-hand side of (1), we get the system of linear equations.

$2x_1 = x_4 \Rightarrow 2x_1 - x_4 = 0$

$6x_1 = 2x_3 \Rightarrow 3x_1 - x_3 = 0$

$2x_2 = x_3 + 2x_4 \Rightarrow 2x_2 - x_3 - 2x_4 = 0$

The matrix form is $AX = 0$, where $A = \begin{bmatrix} 2 & 0 & 0 & -1 \\ 3 & 0 & -1 & 0 \\ 0 & 2 & -1 & -2 \end{bmatrix}, X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, O = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

The augmented matrix is

$$[A|O] = \begin{bmatrix} 2 & 0 & 0 & -1 \\ 3 & 0 & -1 & 0 \\ 0 & 2 & -1 & -2 \end{bmatrix}$$

$$\rightarrow \left[\begin{array}{cccc|c} 2 & 0 & 0 & -1 & 0 \\ 0 & 0 & -2 & 3 & 0 \\ 0 & 2 & -1 & -2 & 0 \end{array}\right] R_2 \rightarrow 2R_2 - 3R_1$$

$$\rightarrow \left[\begin{array}{cccc|c} 2 & 0 & 0 & -1 & 0 \\ 0 & 2 & -1 & -2 & 0 \\ 0 & 0 & -2 & 3 & 0 \end{array}\right] R_2 \leftrightarrow R_3$$

$\rho(A) = \rho([A|O]) = 3 <$ no. of unknowns

$\therefore$ The system is consistent and has infinite number of solutions. The equivalent system of equations:

$2x_1 - x_4 = 0$..................................(1)

$2x_2 - x_3 - 2x_4 = 0$........................(2)

$-2x_3 + 3x_4 = 0$.............................(3)

Let $x_4 = t, t \in R - \{0\}$

$(1) \Rightarrow 2x_1 - t = 0 \Rightarrow x_1 = \frac{t}{2}$

$(3) \Rightarrow -2x_3 + 3t = 0 \Rightarrow x_3 = \frac{3t}{2}$

$(2) \Rightarrow -2x_2 - \frac{3t}{2} - 2t = 0$

$-2x_2 - \frac{7t}{2} = 0$

$-2x_2 = \frac{7t}{2} \Rightarrow x_2 = \frac{7t}{4}$

Let us choose $t = 4$

$x_1 = \frac{4}{2} = 2$, $x_2 = \frac{7\times4}{4} = 7$, $x_3 = \frac{3\times4}{2} = 6$, $x_4 = 4$

So, the balanced equation is

$2C_2H_6 + 7O_2 \rightarrow 6H_2O + 4CO_2$

**EXERCISE 1.8**

**Choose the correct answer:**

1. If $|adj(adjA)| = |A|^9$, then the order of the square matrix $A$ is

(1) 3 **(2) 4** (3) 2 (4) 5

We know that $|adj(adjA)| = |A|^{(n-1)^2}$ where A is a non-singular matrix of order n.
So, $(n-1)^2 = 9 \Rightarrow n - 1 = 3$
$\therefore n = 4$
$\therefore$ order of the square matrix $A$ is 4

2. If $A$ is a $3 \times 3$ non-singular matrix such that $AA^T = A^TA$ and $B = A^{-1}A^T$, then $BB^T =$

(1) $A$ (2) $B$ **(3) $I$** (4) $B^T$

$BB^T = [A^{-1}A^T][A^{-1}A^T]^T$
$= [A^{-1}A^T][(A^T)^T(A^{-1})^T]$ $[\because (AB)^T = B^TA^T]$
$= (A^{-1}A^T)(A(A^T)^{-1})$ $[\because (A^T)^T = A, (A^{-1})^T = (A^T)^{-1}]$
$= A^{-1}(A^TA)(A^T)^{-1}$
$= A^{-1}(AA^T)(A^T)^{-1}$ $[Given\ that\ AA^T = A^TA]$
$= (A^{-1}A)[A^T(A^T)^{-1}]$
$= (I)(I)$
$\therefore BB^T = I$

3. If $A = \begin{bmatrix}3 & 5\\1 & 2\end{bmatrix}$, $B = adjA$ and $C = 3A$, then $\frac{|adj\,B|}{|C|} =$

(1) $\frac{1}{3}$ **(2) $\frac{1}{9}$** (3) $\frac{1}{4}$ (4) 1

$\frac{|adj\,B|}{|C|} = \frac{|adj(adjA)|}{|3A|}$
$= \frac{|A|^{(2-1)^2}}{3^2|A|}$ $[\because |adj(adjA)| = |A|^{(n-1)^2}$, here $n = 2]$
$= \frac{|A|}{9|A|} = \frac{1}{9}$ $[|KA| = K^n|A| Here\ n = 2]$

4. If $A\begin{bmatrix}1 & -2\\1 & 4\end{bmatrix} = \begin{bmatrix}6 & 0\\0 & 6\end{bmatrix}$, then $A =$

(1) $\begin{bmatrix}1 & -2\\1 & 4\end{bmatrix}$ (2) $\begin{bmatrix}1 & 2\\-1 & 4\end{bmatrix}$ **(3) $\begin{bmatrix}4 & 2\\-1 & 1\end{bmatrix}$** (4) $\begin{bmatrix}4 & -1\\2 & 1\end{bmatrix}$

$A\begin{bmatrix}1 & -2\\1 & 4\end{bmatrix} = \begin{bmatrix}6 & 0\\0 & 6\end{bmatrix} = 6\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$
$A\frac{1}{6}\begin{bmatrix}1 & -2\\1 & 4\end{bmatrix} = I_2$

Comparing with $AA^{-1} = I$, we get

$A^{-1} = \frac{1}{|A|} adjA = \frac{1}{6}\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}$

$\Rightarrow adjA = \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}$

$\therefore A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$

5. If $A = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$, then $9I - A =$

(1) $A^{-1}$ (2) $\frac{A^{-1}}{2}$ (3) $3A^{-1}$ **(4) $2A^{-1}$**

$A = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$

$|A| = 14 - 12 = 2$

$adj\ A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} adj\ A = \frac{1}{2}\begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$

$2A^{-1} = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$................(1)

$9I - A = 9\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$

$= \begin{bmatrix} 9-7 & 0-3 \\ 0-4 & 9-2 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$.............(2)

From (1) & (2), $9I - A = 2A^{-1}$

6. If $A = \begin{bmatrix} 2 & 0 \\ 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 2 & 0 \end{bmatrix}$ then $|adj(AB)| =$

(1) $-40$ **(2) $-80$** (3) $-60$ (4) $-20$

$|A| = 10 - 0 = 10$

$|B| = 0 - 8 = -8$

$|adj(AB| = |(adj\ B)(adjA)|$ $[\because adj(AB) = (adjB)(adjA)]$

$= |adj\ B||adj\ A|$ $[\because |AB| = |A||B|]$

$= |B|^{2-1}|A|^{2-1}$ $[|adj\ A| = |A|^{n-1}]$

$= |B||A|$

$= (-8)(10)$

$= -80$

7. If $P = \begin{bmatrix} 1 & x & 0 \\ 1 & 3 & 0 \\ 2 & 4 & -2 \end{bmatrix}$ is the adjoint of $3 \times 3$ matrix $A$ and $|A| = 4$, then $x$ is

(1) 15 (2) 12 (3) 14 **(4) 11**

$adjA = P = \begin{bmatrix} 1 & x & 0 \\ 1 & 3 & 0 \\ 2 & 4 & -2 \end{bmatrix}$

We know that $|adj\ A| = |A|^{n-1}$ (Here $n = 3$)

$\Rightarrow 1(-6-0) - x(-2-0) + 0 = (4)^{3-1}$

$$-6 + 2x = 16$$
$$2x = 16 + 6 = 22$$
$$x = \frac{22}{2} = 11$$

8. If $A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & -2 & 0 \\ 1 & 2 & -1 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ then the value of $a_{23}$ is

(1) 0 (2) – 2 (3) $-3$ **(4) $-1$**

$|A| = 3(2-0) - 1(-2-0) - 1(4+2)$
$= 6 + 2 - 6 = 2$

Co-factor of 2 in $A$ is $= -\begin{vmatrix} 3 & -1 \\ 2 & 0 \end{vmatrix} = -(0+2) = -2$

The value of $a_{23}$ in $A^{-1} = \frac{1}{|A|}$(co-facor of 2 in $a_{32}$ in $A$)
$= \frac{1}{2}(-2) = -1$

9. If $A, B$ and $C$ are invertible matrices of some order, then which one of the following is not true?

(1) $adjA = |A|A^{-1}$ **(2) $adj(AB) = (adjA)(adjB)$**

(3) $detA^{-1} = (detA)^{-1}$ (4) $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

We know that $adj(AB) = (adjB)(adjA)$
Thus option (2) is not true.

10. If $(AB)^{-1} = \begin{bmatrix} 12 & -17 \\ -19 & 27 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$, then $B^{-1} =$

**(1) $\begin{bmatrix} 2 & -5 \\ -3 & 8 \end{bmatrix}$** (2) $\begin{bmatrix} 8 & 5 \\ 3 & 2 \end{bmatrix}$ (3) $\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$ (4) $\begin{bmatrix} 8 & -5 \\ -3 & 2 \end{bmatrix}$

$(AB)^{-1} = B^{-1}A^{-1} = \begin{bmatrix} 12 & -17 \\ -19 & 27 \end{bmatrix}$

$B^{-1}(A^{-1}A) = \begin{bmatrix} 12 & -17 \\ -19 & 27 \end{bmatrix} A$

$B^{-1} = \begin{bmatrix} 12 & -17 \\ -19 & 27 \end{bmatrix} (A^{-1})^{-1}$ $\left[(A^{-1})^{-1} = \frac{1}{|A^{-1}|} adj(A^{-1})\right]$

$B^{-1} = \begin{bmatrix} 12 & -17 \\ -19 & 27 \end{bmatrix} \frac{1}{1} \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$

$= \begin{bmatrix} 36 - 34 & 12 - 17 \\ -57 + 54 & -19 + 27 \end{bmatrix}$

$= \begin{bmatrix} 2 & -5 \\ -3 & 8 \end{bmatrix}$

11. If $A^T A^{-1}$ is symmetric, then $A^2 =$

(1) $A^{-1}$ **(2) $(A^T)^2$** (3) $A^T$ (4) $(A^{-1})^2$

Given that $A^T A^{-1}$ is symmetric

$\therefore (A^T A^{-1})^T = A^T A^{-1}$

$(A^{-1})^T (A^T)^T = A^T A^{-1}$

$(A^{-1})^T (A) = A^T A^{-1}$

$(A^T)^{-1}(A)(A) = A^T(A^{-1}A)$

$$(A^T)^{-1}A^2 = A^T(I) = A^T$$
$$(A^T)(A^T)^{-1}A^2 = (A^T)(A^T)$$
$$IA^2 = (A^T)^2$$
$$\therefore A^2 = (A^T)^2$$

12. If $A$ is a non-singular matrix such that $A^{-1} = \begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$, then $(A^T)^{-1} =$

(1) $\begin{bmatrix} -5 & 3 \\ 2 & 1 \end{bmatrix}$ (2) $\begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$ (3) $\begin{bmatrix} -1 & -3 \\ 2 & 5 \end{bmatrix}$ **(4) $\begin{bmatrix} 5 & -2 \\ 3 & -1 \end{bmatrix}$**

$$A^{-1} = \begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$$
$$(A^T)^{-1} = (A^{-1})^T = \begin{bmatrix} 5 & -2 \\ 3 & -1 \end{bmatrix}$$

13. If $A = \begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ x & \frac{3}{5} \end{bmatrix}$ and $A^T = A^{-1}$, then the value of $x$ is

**(1) $-\frac{4}{5}$** (2) $-\frac{3}{5}$ (3) $\frac{3}{5}$ (4) $\frac{4}{5}$

$$A = \begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ x & \frac{3}{5} \end{bmatrix}$$
$$|A| = \frac{9}{25} - \frac{4x}{5} = \frac{9-20x}{25}$$
$$adjA = \begin{bmatrix} \frac{3}{5} & -\frac{4}{5} \\ -x & \frac{3}{5} \end{bmatrix}$$
$$A^{-1} = \frac{1}{\frac{9-20x}{25}} \begin{bmatrix} \frac{3}{5} & -\frac{4}{5} \\ -x & \frac{3}{5} \end{bmatrix}$$
$$= \frac{25}{90-20x} \times \frac{1}{5} \begin{bmatrix} 3 & -4 \\ -5x & 3 \end{bmatrix}$$
$$A^{-1} = \frac{5}{90-20x} \begin{bmatrix} 3 & -4 \\ -5x & 3 \end{bmatrix}$$
$$A^T = A^{-1} \Rightarrow \begin{bmatrix} \frac{3}{5} & x \\ \frac{4}{5} & \frac{3}{5} \end{bmatrix} = \frac{5}{9-20x} \begin{bmatrix} 3 & -4 \\ -5x & 3 \end{bmatrix}$$
$$\frac{3}{5} = \frac{15}{9-20x}$$
$$27 - 60x = 75$$
$$27 - 75 = 60x$$

$$-\frac{48}{60} = x$$

$$x = -\frac{12\times4}{12\times5} = -\frac{4}{5}$$

14. If $A = \begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}$ and $AB = I$, then $B =$

(1) $\left(\cos^2\frac{\theta}{2}\right)A$ **(2) $\left(\cos^2\frac{\theta}{2}\right)A^T$** (3) $(\cos^2\theta)I$ (4) $\left(\sin^2\frac{\theta}{2}\right)A$

$$AB = I \Rightarrow B = A^{-1} = \frac{1}{|A|} adj\ A$$

$$B = \frac{1}{1+tan^2\frac{\theta}{2}}\begin{bmatrix} 1 & -tan\frac{\theta}{2} \\ tan\frac{\theta}{2} & 1 \end{bmatrix}$$

$$= \frac{1}{sec^2\frac{\theta}{2}}A^T$$

$$= \left(cos^2\frac{\theta}{2}\right)A^T$$

15. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ and $A(adjA) = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$, then $k =$

(1) 0 (2) $\sin\theta$ (3) $\cos\theta$ **(4)1**

$$A(adjA) = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$$

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$$

$$\begin{bmatrix} \cos^2\theta + sin^2\theta & -\sin\theta cos\,\theta + sin\,\theta cos\,\theta \\ -\sin\theta\cos\theta + sin\theta cos\,\theta & sin^2\theta + cos^2\theta \end{bmatrix} = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$$

$$\therefore k = 1$$

16. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $\lambda A^{-1} = A$, then $\lambda$ is

(1) 17 (2) 14 **(3) 19** (4)21

$$\lambda A^{-1} = A$$

$$\lambda(AA^{-1}) = (A)A$$

$$\lambda I = A^2$$

$$\lambda\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$$

$$\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} 4+15 & 6-6 \\ 10-10 & 15+4 \end{bmatrix}$$

$$\lambda = 4 + 15 = 19$$

17. If $adj\ A = \begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$ and $adj\ B = \begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix}$ then $adj(AB)$ is

(1) $\begin{bmatrix} -7 & -1 \\ 7 & -9 \end{bmatrix}$ **(2)** $\begin{bmatrix} -6 & 5 \\ -2 & -10 \end{bmatrix}$ (3) $\begin{bmatrix} -7 & 7 \\ -1 & -9 \end{bmatrix}$ (4) $\begin{bmatrix} -6 & -2 \\ 5 & -10 \end{bmatrix}$

$adj\ (AB) = (adjB)(adjA)$

$= \begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$

$= \begin{bmatrix} 2-8 & 3+2 \\ -6+4 & -9-1 \end{bmatrix} = \begin{bmatrix} -6 & 5 \\ -2 & -10 \end{bmatrix}$

18. The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ -1 & -2 & -3 & -4 \end{bmatrix}$ is

**(1) 1** (2) 2 (3) 4 (4)3

$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ -1 & -2 & -3 & -4 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} \\ R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 + R_1 \end{matrix}$

$\therefore \rho(A) = 1$

19. If $x^a y^b = e^m, x^c y^d = e^n, \Delta_1 = \begin{vmatrix} m & b \\ n & d \end{vmatrix}, \Delta_2 = \begin{vmatrix} a & m \\ c & n \end{vmatrix}, \Delta_3 = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$, then the values of $x$ and $y$ are respectively.

(1) $e^{\left(\frac{\Delta_2}{\Delta_1}\right)}, e^{\left(\frac{\Delta_3}{\Delta_1}\right)}$ (2) $\log(\Delta_1/\Delta_3), \log(\Delta_2/\Delta_3)$

(3) $\log(\Delta_2/\Delta_1), \log(\Delta_3/\Delta_1)$ **(4)** $e^{\left(\frac{\Delta_1}{\Delta_3}\right)}, e^{\left(\frac{\Delta_2}{\Delta_3}\right)}$

$x^a y^b = e^m$

$log(x^a y^b) = loge^m$

$a\ logx + b\ logy = m\ loge$

$a\ logx + b\ logy = m$ ............................(1)

$x^c y^d = e^n$

$log(x^c y^d) = loge^n$

$c\ logx + d\ logy = n$ ..................................(2)

By Crammer's rule, we get

$\Delta = \begin{vmatrix} a & b \\ c & d \end{vmatrix}, \Delta_1 = \begin{vmatrix} m & b \\ n & d \end{vmatrix}, \Delta_2 = \begin{vmatrix} a & m \\ c & n \end{vmatrix}$

But, it is given that $\Delta_3 = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$

$\therefore \Delta = \Delta_3$

So, $log_e^x = \frac{\Delta_1}{\Delta} = \frac{\Delta_1}{\Delta_3} \Rightarrow x = e^{\left(\frac{\Delta_1}{\Delta_3}\right)}$

$log_e^y = \frac{\Delta_2}{\Delta} = \frac{\Delta_2}{\Delta_3} \Rightarrow y = e^{\left(\frac{\Delta_2}{\Delta_3}\right)}$

20. Which of the following is / are correct?
(i) Adjoint of a symmetric matrix is also a symmetric matrix.
(ii) Adjoint of a diagonal matrix is also a diagonal matrix.
(iii) If $A$ is a square matrix of order $n$ and $\lambda$ is a scalar, then $adj(\lambda A) = \lambda^n adj(A)$.
(iv) $A(adjA) = (adjA)A = |A|I$
(1) Only (i) (2) (ii) and (iii) (3) (iii) and (iv) **(4) (i), (ii)and (iv)**

$adj\ (\lambda A) = \lambda^{n-1}\ adj\ (A)$ But given that $adj(\lambda A) = \lambda^n adj\ (A)$
So, (iii) only a wrong statement.

21. If $\rho(A) = \rho([A|B])$, then the system $AX = B$ of linear equations is
(1) consistent and has a unique solution **(2) consistent**
(3) consistent and has infinitely many solution (4) inconsistent

22. If $0 \le \theta \le \pi$ and the system of equations $x + (\sin\theta)y - (\cos\theta)z = 0$, $(\cos\theta)x - y + z = 0$, $(\sin\theta)\,x + y - z = 0$ has a non-trivial solution then $\theta$ is
(1) $\frac{2\pi}{3}$ (2) $\frac{3\pi}{4}$ (3) $\frac{5\pi}{6}$ **(4) $\frac{\pi}{4}$**

The system has a non-trivial solution.

$$\therefore \Delta = \begin{vmatrix} 1 & sin\ \theta & -cos\ \theta \\ cos\ \theta & -1 & 1 \\ sin\ \theta & 1 & -1 \end{vmatrix} = 0$$

$$\Rightarrow 1(1-1) - sin\theta(-cos\ \theta - sin\theta) - cos\theta(cos\ \theta + sin\ \theta) = 0$$

$$\Rightarrow sin\theta cos\theta + sin^2\theta - cos^2\theta - sin\ \theta\ cos\ \theta = 0$$

$$sin^2\theta - cos^2\theta = 0$$

$$\frac{sin^2\theta}{cos^2\theta} - \frac{cos^2\theta}{cos^2\theta} = 0$$

$$\frac{sin^2\theta}{cos^2\theta} - 1 = 0$$

$$tan^2\theta = 1$$

$$\tan\theta = 1$$

$$\Rightarrow \theta = \frac{\pi}{4} [\because 0 \le \theta \le \pi]$$

23. The augmented matrix of a system of linear equations is $\begin{bmatrix} 1 & 2 & 7 & 3 \\ 0 & 1 & 4 & 6 \\ 0 & 0 & \lambda - 7 & \mu + 5 \end{bmatrix}$. The system has infinitely many solutions if
(1) $\lambda = 7, \mu \ne -5$ (2) $\lambda = -7, \mu = 5$ (3) $\lambda \ne 7, \mu \ne -5$ **(4)$\boldsymbol{\lambda = 7, \mu = -5}$**

Let $A = \begin{bmatrix} 1 & 2 & 7 & 3 \\ 0 & 1 & 4 & 6 \\ 0 & 0 & \lambda - 7 & \mu + 5 \end{bmatrix}$

If $\lambda = 7, \mu = -5$ then $\rho(A) = 2, \rho[(A|B)] = 2$

$\therefore \rho(A) = \rho([A|B]) = 2 < no. of\ unknowns$

Thus, the given system is consistent and has infinitely many solutions if $\lambda = 7, \mu = -5$

24. Let $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and $4B = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & x \\ -1 & 1 & 3 \end{bmatrix}$. If $B$ is the inverse of $A$, then the value of $x$ is

(1) 2 (2) 4 (3) 3 **(4) 1**

$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

$|A| = 2(4-1) + 1(-2+1) + 1(1-2)$

$= 6 - 1 - 1 = 4 \neq 0$

$adj\ A = \begin{bmatrix} (4+1) & -(-2+1) & (1-2) \\ -(-2+1) & (4-1) & -(-2+1) \\ (1-2) & -(-2+1) & (4-1) \end{bmatrix}^T$

$= \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}^T = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} adj\ A = \frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$

Given: $4B = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & x \\ -1 & 1 & 3 \end{bmatrix} \Rightarrow B = \frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & x \\ -1 & 1 & 3 \end{bmatrix} \Rightarrow B = A^{-1} \Rightarrow x = 1$

25. If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, then $adj(adjA)$ is

**(1)** $\begin{bmatrix} \mathbf{3} & \mathbf{-3} & \mathbf{4} \\ \mathbf{2} & \mathbf{-3} & \mathbf{4} \\ \mathbf{0} & \mathbf{-1} & \mathbf{1} \end{bmatrix}$ (2) $\begin{bmatrix} 6 & -6 & 8 \\ 4 & -6 & 8 \\ 0 & -2 & 2 \end{bmatrix}$ (3) $\begin{bmatrix} -3 & 3 & -4 \\ -2 & 3 & -4 \\ 0 & 1 & -1 \end{bmatrix}$ (4) $\begin{bmatrix} 3 & -3 & 4 \\ 0 & -1 & 1 \\ 2 & -3 & 4 \end{bmatrix}$

$A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$

$|A| = 3[-3+4] + 3[2-0] + 4[-2-0] = 3(1) + 3(2) + 4(-2) = 3 + 6 - 8 = 1$

$adj(adjA) = |A|^{n-2}A = (1)^{3-2}\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$

$\therefore adj(adjA) = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$