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MATHS QUESTION & ANSWER BOOKLET
12th Std (E/M)

Prepared under the guidance of our respectable CEO of Tiruvallur District

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1. APPLICATION OF MATRICES AND DETERMINANTS

I. 2 MARK

1.If adj A = $\begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, find A^{-1}

Solution:

Adj(A) = $\begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

$A^{-1} = \pm\frac{1}{\sqrt{|adj(A)|}}$ adj(A)

Therefore |adj A| = $\begin{vmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix}$

= -1[1-4] -2[1-4]+2[2-2]

= -1[-3] -2[-3] +2[0]

= 3+6 = 9

$A^{-1} = \pm\frac{1}{\sqrt{9}}\begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

$= \pm\frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

2. If A=$\begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$, verify that

A(adjA) = (adj A)A =|A| I_2

Solution:

A= $\begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$; adj A = $\begin{bmatrix} 3 & 4 \\ 5 & 8 \end{bmatrix}$

=|A| = 24-20 =4 ≠ 0

A(adj A) = $\begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}\begin{bmatrix} 3 & 4 \\ 5 & 8 \end{bmatrix}$

$=\begin{bmatrix} 24-20 & 32-32 \\ -15+15 & -20+24 \end{bmatrix}$

$= \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$

$|A|I_2 = 4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Therefore A(adj A) = (adj A) A = |A| I_2 is verified.

3. Find the rank of the following matrices by the row reduction method $\begin{bmatrix} 3 & -8 & 5 & 2 \\ 2 & -5 & 1 & 4 \\ -1 & 2 & 3 & -2 \end{bmatrix}$

Solution:

$A = \begin{bmatrix} 3 & -8 & 5 & 2 \\ 2 & -5 & 1 & 4 \\ -1 & 2 & 3 & -2 \end{bmatrix}$

$\sim \begin{bmatrix} -1 & 2 & 3 & -2 \\ 2 & -5 & 1 & 4 \\ 3 & -8 & 5 & 2 \end{bmatrix} R_1 <-> R_3$

$\sim \begin{bmatrix} -1 & 2 & 3 & -2 \\ 2 & 5 & 1 & 4 \\ 3 & -8 & 5 & 2 \end{bmatrix} R_1 -> (\_R_1)$

$\sim \begin{bmatrix} 1 & -2 & -3 & 2 \\ 0 & -1 & 7 & 0 \\ 0 & -2 & -14 & -4 \end{bmatrix}$ $R_2 \to R_2$-2R_1 $R_3 \to R_3$-3R_1

$\sim \begin{bmatrix} 1 & -2 & -3 & 2 \\ 0 & -1 & 7 & 0 \\ 0 & 0 & 0 & -4 \end{bmatrix} R_3 \to R_3-2R_2$

:: P(A) = 3

4. Solve the following system of linear equations by matrix inversion method 2x+5y = -2, x+2y = -3.

Solution:

2x+5y = -2, x+2y = -3

$\begin{bmatrix} 2 & 5 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ -3 \end{bmatrix}$

$A \times = B$

$\times = A^{-1}$ B

$A = \begin{bmatrix} 2 & 5 \\ 1 & 2 \end{bmatrix}$

Adj A $= \begin{bmatrix} 2 & -5 \\ -1 & 2 \end{bmatrix}$

$|A| = 4 - 5 = -1$

$A^{-1} = \frac{1}{|A|}$ adj(A)

$= \frac{1}{-1} \begin{bmatrix} 2 & -5 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 5 \\ 1 & -2 \end{bmatrix}$

$X = A^{-1}B$

$= \begin{bmatrix} -2 & 5 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} -2 \\ -3 \end{bmatrix}$

$= \begin{bmatrix} 4 & -15 \\ -2 & 6 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -11 \\ 4 \end{bmatrix}$

5. Solve the following systems of linear equation by Cramer's rule $\frac{3}{x} + 2y = 12$, $\frac{2}{x} + 3y = 13$.

Solution:

$\frac{3}{x} + 2y = 12, \frac{2}{x} + 3y = 13.$

Let $a = \frac{1}{x}$, $3a + 2y = 12$

$2a + 3y = 13$

$\Delta = \begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix} = 9 - 4 = 5 \neq 0$

$\Delta a = \begin{vmatrix} 12 & 2 \\ 13 & 3 \end{vmatrix} = 36 - 26 = 10$

$\Delta y = \begin{vmatrix} 3 & 12 \\ 2 & 13 \end{vmatrix} = 39 - 24 = 15$

$a = \frac{\Delta a}{\Delta} = \frac{10}{5} = 2$

$x = \frac{1}{a} = \frac{1}{2}$

$y = \frac{\Delta y}{\Delta} = \frac{15}{5} = 3$

:: The solution is $(x,y) = (\frac{1}{2}, 3)$

II. 3 MARKS.

1. Find a matrix A if adj (A) = $\begin{bmatrix} 7 & 7 & -7 \\ -1 & 11 & 7 \\ 11 & 5 & 7 \end{bmatrix}$

Sol.

adj (A) $= \begin{bmatrix} 7 & 7 & -7 \\ -1 & 11 & 7 \\ 11 & 5 & 7 \end{bmatrix}$

|adj (A)| $= \begin{bmatrix} 7 & 7 & -7 \\ -1 & 11 & 7 \\ 11 & 5 & 7 \end{bmatrix}$

$= 7(77 - 35) - 7(-7 - 77) - 7(-5 - 121)$

$= 7(42) - 7(-84) - 7(-126)$

$= 294 + 588 + 882 = 1764$

$A = \pm \frac{1}{\sqrt{|adj(A)|}}$ adj (adj (A))

$= \pm \frac{1}{\sqrt{1764}}$

$\begin{vmatrix} +(77-35) & -7(-7-77) & (-5-121) \\ -(49+35) & +(49+77) & -(35-77) \\ +(49+77) & -(49-7) & +(77+7) \end{vmatrix}^{\wedge T}$

$= \pm \frac{1}{42} \begin{bmatrix} 42 & 84 & -126 \\ -84 & 126 & 42 \\ 126 & -42 & 84 \end{bmatrix}^{\wedge T}$

$A = \pm \begin{bmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ 3 & 1 & 2 \end{bmatrix}$

2. If A = $\frac{1}{9}\begin{bmatrix} -8 & 1 & 4 \\ -4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}$ prove that $A^{-1} = A^T$

Sol.

$A = \frac{1}{9} \begin{bmatrix} -8 & 1 & 4 \\ -4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}$

$A^T = \frac{1}{9} \begin{bmatrix} -8 & 1 & 4 \\ 1 & 4 & -8 \\ 4 & 7 & 4 \end{bmatrix}$

$AA^{-1} = \frac{1}{9}\begin{bmatrix}-8 & 1 & 4\\ -4 & 4 & 7\\ 1 & -8 & 4\end{bmatrix}\frac{1}{9}\begin{bmatrix}-8 & 4 & 1\\ 1 & 4 & -8\\ 4 & 7 & 4\end{bmatrix}$

$= \frac{1}{81}\begin{bmatrix}64+1+16 & -32+4+28 & -8-8+16\\ -32+4+28 & 16+16+49 & 4-32+28\\ -8-8+16 & 4-32+28 & 1+64+16\end{bmatrix}$

$= \frac{1}{81}\begin{bmatrix}81 & 0 & 0\\ 0 & 81 & 0\\ 0 & 0 & 81\end{bmatrix}$

$= \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} = \text{I}$

AA^T=I We get

:: $A^{-1}=A^T$

3). Find the inverse of the non singular matrix A = $\begin{bmatrix}0 & 5\\ -1 & 6\end{bmatrix}$ by the gauss Jordan method.

Sol:

A= $\begin{bmatrix}0 & 5\\ -1 & 6\end{bmatrix}$

$[\frac{A}{I_2}] = \left[\begin{array}{cc|cc}0 & 5 & 1 & 0\\ -1 & 6 & 0 & 1\end{array}\right]$

$R_1 <-> R_2 \sim \left[\begin{array}{cc|cc}-1 & 6 & 1 & 0\\ 0 & 5 & 0 & 1\end{array}\right]$

$R_1 -> -R_1 \sim \left[\begin{array}{cc|cc}1 & -6 & 0 & 0-1\\ 0 & 5 & 1 & 0\end{array}\right]$

$R_2 -> \frac{R_2}{5} \sim \left[\begin{array}{cc|cc}1 & -6 & 0 & -1\\ 0 & 1 & 1/5 & 0\end{array}\right]$

$R_1 -> R_1 + 6R_2 \sim \left[\begin{array}{cc|cc}1 & 0 & 6/5 & -1\\ 0 & 1 & 1/5 & 0\end{array}\right]$

$A^{-1} = \begin{bmatrix}\frac{6}{5} & -1\\ \frac{1}{5} & 0\end{bmatrix}$

$= \frac{1}{5}\begin{bmatrix}6 & -1\\ 1 & 0\end{bmatrix}$

III. 5 MARKS.

1. $\frac{3}{x} - \frac{4}{y} - \frac{2}{z} - 1 = 0$, $\frac{1}{x} + \frac{2}{y} + \frac{1}{z} - 2 = 0$, $\frac{2}{x} - \frac{5}{y} - \frac{4}{z} + 1 = 0$, Solve using Cramer's rule.

Solution:

Let $\frac{1}{x} = a, \frac{1}{y} = b, \frac{1}{z} = c$

3a-4b-2c = 1

a+2b+c = 2

2a-5b-4c = -1

$\Delta = \begin{vmatrix}3 & -4 & -2\\ 1 & 2 & 1\\ 2 & -5 & -4\end{vmatrix}$

= 3[-8+5] +4[-4-2] -2[-5-4]

= 3[-3] +4[-6] -2[-9]

= -9-24+18

= -15 ≠ 0

$\Delta a = \begin{vmatrix}1 & -4 & -2\\ 2 & 2 & 1\\ -1 & -5 & -4\end{vmatrix}$

= 1[-8+5] +[-8+1] -2[-10+2]

= -3-28+16

= -15

$\Delta b = \begin{vmatrix}3 & 1 & -2\\ 1 & 2 & 1\\ 2 & -1 & -4\end{vmatrix}$

= 3[-8+1] -1[-4-2] -2[-1-4]

= -21+6+10

= -5

$\Delta c = \begin{vmatrix}3 & -4 & 1\\ 1 & 2 & 2\\ 2 & -5 & -1\end{vmatrix}$

= 3[-2+10] +4[-1-4] +1[-5-4]

= 24-20-9

= -5

a = $\frac{\Delta a}{\Delta} = \frac{-15}{-15} = 1$

b = $\frac{\Delta b}{\Delta} = \frac{-5}{-15} = \frac{1}{3}$

c = $\frac{\Delta c}{\Delta} = \frac{-5}{-15} = \frac{1}{3}$

=> x =1, y = 3, z = 3

2. Solve the following equations by using Gauss Elimination method, 4x+3y+6z=25, x+y+7z=13, 2x+9y+z=1.

Solution:

$$A = \begin{bmatrix} 4 & 3 & 6 \\ 1 & 5 & 7 \\ 2 & 9 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 25 \\ 13 \\ 1 \end{bmatrix}$$

$$[A/B] = \begin{bmatrix} 4 & 3 & 6 \\ 1 & 5 & 7 \\ 2 & 9 & 1 \end{bmatrix} \begin{bmatrix} 25 \\ 13 \\ 1 \end{bmatrix}$$

$$R_1 \rightarrow R_2 \quad \left[\begin{array}{ccc|c} 1 & 5 & 7 & 13 \\ 4 & 3 & 6 & 25 \\ 2 & 9 & 1 & 1 \end{array}\right]$$

$R_2 \rightarrow R_2 - 4R_1$

$R_3 \rightarrow R_3 - 2R_1$

$$\left[\begin{array}{ccc|c} 1 & 5 & 7 & 13 \\ 0 & -17 & -22 & -27 \\ 0 & -1 & -13 & -25 \end{array}\right]$$

$R_2 \rightarrow R_2 - (-1)$

$R_3 \rightarrow R_3 - (-1)$

$$\left[\begin{array}{ccc|c} 1 & 5 & 7 & 13 \\ 0 & 17 & 22 & 27 \\ 0 & 1 & 13 & 25 \end{array}\right]$$

$R_3 \rightarrow 17R_3 - R_2$

$$\left[\begin{array}{ccc|c} 1 & 5 & 7 & 13 \\ 0 & 17 & 22 & 27 \\ 0 & 0 & 199 & 398 \end{array}\right]$$

X+5y+7z = 13 → ①

17y+22z = 27 → ②

199z = 398 → ③

Z = 398/199 = 2

② => 17y+22(2) = 27

⇨ 17y = 27 -44

⇨ = -17

⇨ y = -1

①=> x=5(-1)+7(2) =13

⇨ x-5+14 =13

⇨ x+9 =13

⇨ x =13-9

⇨ =4

Therefore the solution is x=4, =-1, z=2.

3) Investigate for what values of λ and μ the system of linear equations, x+2y+z = 7,x+y+ λz= μ ,x+3y-5z = 5 has i) no solution ii) a unique solution iii) an infinite number of solutions.

Soln:

Number of unknowns = 3

x+2y+z = 7,x+y+ λz= μ ,x+3y-5z = 5

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & \lambda \\ 1 & 3 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ \mu \\ 5 \end{bmatrix}$$

$$[A/\ B] = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 7 \\ 1 & 1 & \lambda & \mu \\ 1 & 3 & -5 & 5 \end{array}\right]$$

$R_2 \rightarrow R_2 - R_1$

$R_3 \rightarrow R_3 - R_1$

$$\begin{bmatrix} 1 & 2 & 1 & 7 \\ 1 & 1 & \lambda - 1 & \mu - 7 \\ 1 & 3 & -5 & 5 \end{bmatrix}.$$

$R_3 \to R_2$-R_3

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 7 \\ 1 & 1 & \lambda-1 & \mu-7 \\ 1 & 3 & \lambda-7 & \mu-9 \end{array}\right]$$

Case i) If -7 , $\mu - 9$ P(A) =P(A/B) =2<3.

Hence the given system is consistent and has an infinite number of solutions.

Case ii) If $\lambda \neq 7$, $\mu \neq 9$ P(A) =P(A/B) =3

The given system is consistent and has only one solutions.

Case iii) If $\lambda = 7$, $\mu \neq 9$ P(A) =2 P(A/B) =3

:. P(A) ≠P(A/B)

The given system is inconsistent and has no solution.

4) Find the value of k for which the equations kx-2y+z = 1,x-2ky+z = -2,

x-2y+kz=1 have i) no solution ii) unique solution iii) infinitely many solutions.

Soln:

No of unknown = 3

$$\begin{bmatrix} k & -2 & 1 \\ 1 & -2k & 1 \\ 1 & -2 & k \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

$$[A/ \quad B] = \left[\begin{array}{ccc|c} k & 2 & 1 & 1 \\ 1 & -2k & 1 & -2 \\ 1 & 3 & k & 1 \end{array}\right]$$

$R_1 \leftrightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 1 & -2k & 1 & -2 \\ k & 2 & 1 & 1 \end{array}\right]$$

$R_2 \to R_2$-R_1

$R_3 \to R_3 k R_1$

$$\begin{bmatrix} 1 & 2 & k & 1 \\ 0 & -2k+2 & 1-k & -3 \\ 0 & -2k+2 & 1-k^2 & 1-k \end{bmatrix}$$

$R_3 \to R_3 + R_2$

$$\begin{bmatrix} 1 & 2 & k & 1 \\ 0 & -2(1-k) & 1-k & -3 \\ 0 & 0 & 2-k-k^2 & -k-2 \end{bmatrix}$$

$2\text{-}k\text{-}k^2$

$-(k^2+k-2)$

$-(k-1)(k+2)$

$$\begin{bmatrix} 1 & -2 & k & 1 \\ 0 & 2(1-k) & 1-k & -3 \\ 0 & 0 & (k-1)(k+2) & k+2 \end{bmatrix}$$

Case (i)

If $K = -2$, p(A) =2, p(A/B) = 2< 3 ,

The given system is consistent and has infinitely many solution.

Case (ii)

If $K = 1$, $K \neq -2$, p(A) = 2, p(A/B) =3

p(A) ≠ p(A/B)

The given system is inconsistent and has no solution

Case (iii)

If $K \neq -1$, $K \neq -2$,

p(A) = p(A/B) =3 no.of unknowns

The given system is consistent and has only on solutions.

5. Inverstigate the values of λ and μ the system of linear equations 2x+3y+5z = 9, 7x+3y-5z = 8, 2x+3y3+ $\lambda z = \mu$ have (i) no

solution (ii) a unique solution (iii) an infinite number of solutions.

Solution:-

Number of unknowns = 3 $\begin{bmatrix} 2 & 3 & 5 \\ 7 & 3 & -5 \\ 2 & 3 & \lambda \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$= \begin{bmatrix} 9 \\ 8 \\ \mu \end{bmatrix}$

$[A/B] = \left[\begin{array}{ccc|c} 2 & 3 & 5 & 9 \\ 7 & 3 & -5 & 8 \\ 2 & 3 & \lambda & \mu \end{array}\right]$

$R_2 \to 2R_2 - 7R_1$
$R_3 \to R_3 - R_1$

$\left[\begin{array}{ccc|c} 2 & 3 & 5 & 9 \\ 0 & -15 & -45 & -47 \\ 0 & 0 & \lambda - 5 & \mu - 9 \end{array}\right]$

Case (i)

If $\lambda = 5, \mu \neq 9$, p(A) =2, p(A/B) =3, p(A)≠p(A,B)

The given system is inconsistent and has no solution.

Case (ii)

If $\lambda \neq 5, \mu \neq 9$, p(A) = p(A/B) =3

3≤ (no of unknowns)

The given system is consistent and has unique solution

Case (iii)

If $\lambda = 5, \mu = 9$, p(A) = p(A/B) =2

2< 3

The given system is consistent and has infinitely many solutions.

6. By using Gaussian elimination method balance the chemical reaction equation:

$C_5H_8 + O_2 \rightarrow CO_2 + H_2O$

Solution:-

$x_1C_5H_8 + X_2O_2 = x_3CO_3 + x_4H_2O$

Carbon:

$5x_1 = x_3$
$5x_1 - x_3 = 0$

Hydrogen:

$8x_1 = 2x_4$
$4x_1 = x_4$
$4x_1 - x_4 = 0$

Oxygen:

$2x_2 = 2x_3 + x_4$
$2x_2 - 2x_3 - x_4 = 0$

The equations are

$5x_1 - x_3 = 0$
$4x_1 - x_4 = 0$
$2x_2 - 2x_3 - x_4 = 0$

The Augmented matrix

$[A/0] = \begin{bmatrix} 5 & 0 & -1 & 0 & 0 \\ 4 & 0 & 0 & -1 & 0 \\ 0 & 2 & -2 & -1 & 0 \end{bmatrix}$

$R_1 \leftrightarrow R_2 \begin{bmatrix} 4 & 0 & 0 & -1 & 0 \\ 5 & 0 & -1 & 0 & 0 \\ 0 & 2 & -2 & -1 & 0 \end{bmatrix}$

$R_2 \leftrightarrow R_3 \begin{bmatrix} 4 & 0 & 0 & -1 & 0 \\ 0 & 2 & -2 & -1 & 0 \\ 5 & 0 & -1 & 0 & 0 \end{bmatrix}$

$R_3 \rightarrow 4R_3 - 5R_1 \begin{bmatrix} 4 & 0 & 0 & -1 & 0 \\ 0 & 2 & -2 & -1 & 0 \\ 0 & 0 & -4 & 5 & 0 \end{bmatrix}$

Number of unknowns = 4
P(A) =P(A/0) = 3 < 4
The given system is consistent and has infinitely many solution
Therefore

$4x_1 - x_4 = 0 \qquad \rightarrow ①$

$2x_2 - 2x_3 - x_4 = 0 \quad \rightarrow (2)$

$-4x_3 + 5x_4 = 0 \quad \rightarrow (3)$

Let $x_2 = t$

(3) => $-4x_3 + 5t = 0$

$x_3 = -5t / -4$

$= 5t / 4$

⇨ $2x_2 - 2(\frac{5t}{4}) - t = 0$

$2x_2 - \frac{5t}{2} - t = 0$

$2x_2 - \frac{5t-2t}{2} = 0$

$2x_2 - \frac{7t}{2} = 0$

$2x_2 = \frac{7t}{2}$

$X_2 = \frac{7t}{4}$

(1)=>

$4x_1 - t = 0$

$4x_1 = t$

$x_1 = t/4$

Therefore t =4, since x_1, x_2, x_3, x_4 are whole numbers.

$x_1 = \frac{t}{4}$	$x_2 = \frac{7t}{4}$	$x_3 = \frac{5t}{4}$	$x_4 = t$
$x_1 = 1$	$x_2 = 7$	$x_3 = 5$	$x_4 = 4$

The required chemical reaction is

$C_5H_8 + 7O_2 \longrightarrow 5CO_2 + 4H_2O$

2. COMPLEX NUMBERS

I. 2 MARK

1) simplify the following

$i^{1948}-i^{1869}$

soln:

$1948 \div 4 = 487$ $1869 \div 4 = 467 \times 4 + 1$

$= (i)^{4x487}-(i)^{-(4x467+1)}$

$= (i^4)^{487}-(i)^{-4x467-1}$

$= 1^{487} - i^{-4x467}.(i)^{-1}$

$= 1-(i^4)^{-467}.\frac{1}{i}$

$= 1 - \frac{1*i}{i*i}$

$= 1 - \frac{i}{-1}$

Ans = 1+i

2) Given the complex no Z = 2+3i represent the complex no in argand diagram

I) z , iz and z + iz

iz = i(2 +3i)

$= 2i + 3i^2$

= 2 i + 3(-1)

=2 i-3

iz = - 3 +2 i

z+iz = 2+3i-3+2i

z+iz = -1+5i

3)if $z_1 = 1 - 3i$, $z_2 = -4i$, $z_3 = 5$ S.T

I)$(z_1+z_2)+z_3 = z_1+(z_2+z_3)$

ii) $(z_1z_2)\ z_3 = z_1(z_2z_3)$

soln:

L.H.S

(z_1+z_2) = 1- 3i - 4i

= 1- 7i

$(z_1+z_2)+z_3$ = 1- 7i+ 5

= 6 – 7i

R.H.S

$z_1+(z_2+z_3)$

z_2+z_3= - 4i +5

=5-4 i

$z_1+(z_2+z_3)$ =1- 3i+5- 4i

=**6 – 7i**

L.H.S = R.H.S

Hence verified

ii) $(z_1z_2)\ z_3 = z_1(z_2z_3)$

L.H.S

$(z_1z_2)\ z_3$

z_1z_2 =(1-3i)(- 4i)

= -4i – 12

= **-12 -4i**

$(z_1 z_2)\ z_3 = (-12 - 4i)\ 5$

$= -60 - 20i$

$= \mathbf{-20(3 + i)}$

= L.H.S

R.H.S

$z_1\ (z_2\ z_3)$

$(z_2\ z_3) = -4i \times 5$

$= \mathbf{-20\ i}$

$z_1\ (z_2\ z_3) = (1-3i)(-20\ i)$

$= -20\ i + 60\ i^2$

$= -20\ i - 60$

$= -60 - 20\ i$

$= \mathbf{-20\ (3 + i)}$

= R.H.S

L.H.S = R.H.S

Hence verified

4) If $z_1 = 3+4i$, $z_2 = 5-12i$,$z_3 = 6+8i$ find $|z_1|$, $|z_2|$, $|z_3|$ $|z_1 + z_2|$ $|z_2 - z_3|$ $|z_1 + z_3|$

Soln:

i) $\mathbf{z_1 = 3+4i}$

$|z_1| = \sqrt{(3)^2 + (4)^2}$

$= \sqrt{9 + 16}$

$= \sqrt{25}$

$\mathbf{|z_1| = 5}$

ii) $\mathbf{Z_2 = 5 - 12\ i}$

$|Z_2| = \sqrt{(5)^2 + (-12)^2}$

$= \sqrt{25 + 144}$

$= \sqrt{169}$

$\mathbf{|Z_2| = 13}$

iii) $\mathbf{z_3 = 6+8i}$

$|z_3| = \sqrt{(6)^2 + (8)^2}$

$= \sqrt{36 + 64}$

$= \sqrt{100}$

$|z_3| = 10$

iv) $|z_1 + z_2| = |(3+4i) + (5 - 12i)|$

$= |3+4i + 5 - 12i|$

$= |8 - 8i|$

$|z_1 + z_2| = \sqrt{(8)^2 + (-8)^2}$

$= \sqrt{64 + 64}$

$= \sqrt{128}$

$|z_1 + z_2| = 8\sqrt{2}$

v) $|z_2 - z_3| = |(5 - 12i) - (6+8i)|$

$= |5 - 12i - 6 + 8i|$

$= \sqrt{-1 + 20i}$

$= \sqrt{(-1)^2 + (-20)^2}$

$= \sqrt{1 + 400}$

$= \sqrt{401}$

vi) $|z_1 + z_3| = |3+4i + 6 + 8i|$

$= |(9 + 12i)|$

$= \sqrt{9^2 + 12^2}$

$= \sqrt{81 + 144}$

$= \sqrt{225}$

$|z_1 + z_3| = 15$

5) write in polar form of the following complex numbers

i) $2 + i2\sqrt{3}$

<u>soln:</u>

$z = x + iy$

modulus : $|z| = \sqrt{x^2 + y^2}$

arg $= \tan^{-1}\left(\frac{y}{x}\right)$

$x = 2$, $y = 2\sqrt{3}$

$|z| = \sqrt{4 + 4(3)}$

$= \sqrt{4 + 12}$

$= \sqrt{16}$

modulus : $|z| = 4$

$\alpha = \tan^{-1}\left(\frac{y}{x}\right)$

$= \tan^{-1}\left|\left(\frac{2\sqrt{3}}{2}\right)\right|$

$\alpha = \tan^{-1}(\sqrt{3})$

arg: $\alpha = \pi/3$

$\alpha = \pi/3$

$\Theta = \pi/3$

$\therefore \Theta = \alpha$

Polar form:

$Z = r(\cos\Theta + i\sin\Theta)$

$= 4\,[\cos \pi/3 + i\sin \pi/3]$

$2 + i2\sqrt{3} = 4\,[\cos(\pi/3 + 2k\pi) + i\sin(\pi/3 + 2k\pi)]$ $k \in z$

II. 3 MARK.

1) Find the values of the real numbers of x and y if the complex nos

(3- i) x – (2 – i) y + 2i + 5 and 2x + (-1 + 2 i) y + 3 + 2i are equal

<u>Soln:</u>

$(3 - i)x - (2 - i)y + 2i + 5 =$

$2x + (-1 + 2i)y + 3 + 2i$

$(3x - 2y + 5) + i(-x + y + 2) =$

$(2x - y + 3) + i(2y + 2)$

Equating real numbers on both sides :

$3x - 2y + 5 = 2x - y + 3$	Equating Imaginary
$3x - 2x - 2y + y + 5 - 3 = 0$	$-x + y + 2 = 2y + 2$
$x - y + 2 = 0$	$-x + y - 2y + 2 - 2 = 0$
$x - y = -2$ →①	$-x - y = 0$ ---②

solve:

$x - y = -2$ →①

$-x - y = 0$ →②

$-2y = -2$

$y = 1$

subt $y = 1$ in equation →①

$x-1 = -2$

$x = -2 + 1$

$x = -1$

∴ (x,y) = (-1 , 1)

2) Find the least value of the positive integer n for which $(\sqrt{3}+i)^n$ i) real ii)purely imaginary

Soln:

$(\sqrt{3}+i)^n$

Take n =1 $= (\sqrt{3}+i)$

Take n = 2

$(\sqrt{3}+i)^2 = [\,[\sqrt{3}]^2 + 2\sqrt{3}i + i2\,]$

[∵ $[a+b]^2=a^2 + 2ab+b^2$]

$= 3 + 2\sqrt{3}$ i - 1

$= 2+ 2\sqrt{3}$ i

$= 2(1 + i\sqrt{3})$

Take n = 3

$(\sqrt{3}+i)^3 = (\sqrt{3}+i)^2 (\sqrt{3}+i)$

$= 2(1 +i\sqrt{3}) [\sqrt{3} +i]$

{Subt ; $(\sqrt{3}+i)^2 = 2[1 + i\sqrt{3}]$

$= 2 [\sqrt{3} + i + i3 + ((\sqrt{3})^2 i)$

$= 2 [\sqrt{3} + 4i - \sqrt{3}]$

$=2[4i]$

$= 8i$

$(\sqrt{3}+i)^3 = 8i$

Purely imaginary

n=3

$[\,(\sqrt{3}+i)^3\,]^2 = [8i]^2$

$(\sqrt{3}+i)^6 = 64 i^2$

$= -64$ (real)

n=6 **i) n=6 for real**

ii) n= 3 for purely imaginary

3) If $|z_1| = 3$ state that $7 \le |z + 6 - 8i| \le 13$

Soln:

$||z_1| - |z_2|| \le |z_1+z_2| \le |z_1| + |z_2|$

$z_1 = 6 - 8i$

$|z_1| = \sqrt{36+64} = \sqrt{100} = 10$

$||z| - |z_1|| \le |z+z_1| \le |z| + |z_1|$

$|3-10| \le |z + 6- 8i| \le 3+ 10$

$7 \le |z + 6- 8i| \le 13$

Hence proved

4) Obtain the Cartesian equation for the locus of z = x+iy in each of the following cases

i) $|z-4| = 16$

soln:

put : z = x+ iy

$|z-4| = 16$

$|x+ iy| = 16$

$|x+ iy - 4| = 16$

$|x - 4+ iy| = 16$

$[\sqrt{(x-1)^2+y^2}\ ]^2$ - $[\sqrt{(x-1)^2+y^2}$ $]^2 = 16$

Squaring on both sides;

$(x-4)^2 + y^2 = 16^2$

$(x-4)^2 + y^2 = 256$

$x^2-8x+16+y^2-256 = 0$

$x^2+y^2-8x-240 = 0$

i) $|z-4|^2 - |z-1|^2 = 16$

put : z = x + iy

$|z-4|^2 - |z-1|^2 = 16$

$|x+iy-4|^2 - |x+iy-1|^2 = 16$

$|x-4+iy|^2 - |x-1+iy|^2 = 16$

$[\sqrt{(x-4)^2}+\sqrt{y^2}\ ]^2 -$

$[\sqrt{(x-1)^2}+\sqrt{y^2}\ ]^2 = 16$

$[(x-4)^2+y^2] - [(x-1)^2+y^2] = 16$

$x^2-8x+16+y^2 - [x^2-2x+1+y^2] = 16$

$-8x+16+2x-1 = 16$

$-6x-1+16 = 16$

$-6x-1 = 0$

$6x+1 = 0$

5) find the value of $\left[\frac{1+\sin\frac{\pi}{10}+i\cos\frac{\pi}{10}}{1+\sin\frac{\pi}{10}-i\cos\frac{\pi}{10}}\right]^{10}$

<u>Soln:</u>

$Z = \sin\frac{\pi}{10} + i\cos\frac{\pi}{10}$

$\frac{1}{z} = \frac{1}{\sin\frac{\pi}{10}+i\cos\frac{\pi}{10}} \times \frac{\sin\frac{\pi}{10}-i\cos\frac{\pi}{10}}{\sin\frac{\pi}{10}-i\cos\frac{\pi}{10}}$

$= \frac{\sin\frac{\pi}{10}-i\cos\frac{\pi}{10}}{\sin\frac{\pi}{10}+i\cos\frac{\pi}{10}}$

$[\because \sin^2\theta + \cos^2\theta]$

$\frac{1}{z} = \sin\frac{\pi}{10} - i\cos\frac{\pi}{10}$

$\left(\frac{1+z}{1+\frac{1}{z}}\right)^{10} = \left(\frac{1+z}{\frac{z+1}{z}}\right)^{10}$

$= \left(\frac{1}{\frac{1}{z}}\right)^{10} = z^{10}$

$= z^{10} = \left[\sin\frac{\pi}{10} + i\cos\frac{\pi}{10}\right]^{10}$

[Demorire's theorem]

$[\because (\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin\theta]$

$= 2^{10} = [\cos(\frac{\pi}{2} - \frac{\pi}{10}) + i\sin(\frac{\pi}{2} - \frac{\pi}{10})]^{10}$

$= [\cos\frac{4\pi}{10} + i\sin\frac{4\pi}{10}]^{10}$

$= [\cos\frac{4\pi}{10} \times 10 + i\sin \times 10\frac{4\pi}{10}]^{10}$

$= [\cos 4\pi + i\sin 4\pi]$

$= [\cos\pi + i\sin\pi]^4$

$= [-1 + i(0)]^4$

$= [-1]^4 = 1$

$\left[\frac{1+z}{1+\frac{1}{z}}\right]^{10} = 1$

III. 5 MARKS

1) If z_1, z_2 and z_3 are three complex numbers such that $|z_1|=1$, $|z_2|=2$, $|z_3|=3$ and $|z_1+z_2+z_3|=1$ state that $|9z_1z_2+4z_1z_3+z_2z_3|=6$

Soln:

$|z1| = 1,\ |z_2| = 2,\ |z_3| = 3$

$|z_1|^2 = 1,\ |z_2|^2 = 4,\ |z_3|^2 = 9$

$z_1\bar{z}_1 = 1 \quad z_2\bar{z}_2 = 4 \quad z_3\bar{z}_3 = 9$

$z_1 = \frac{1}{\bar{z}1},\ z_2 = \frac{4}{\bar{z}2},\ z_3 = \frac{9}{\bar{z}3}$

$|z_1+z_2+z_3| = 1$

$\left|\frac{1}{\bar{z}1} + \frac{4}{\bar{z}2} + \frac{9}{\bar{z}3}\right| = 1$

$\left|\frac{\bar{z}2\bar{z}3+4\bar{z}1\bar{z}3+9\bar{z}1\bar{z}2}{\bar{z}1\,\bar{z}2\,\bar{z}3}\right| = 1$

$\left\{ \text{By property } \left|\frac{z1}{z2}\right| = \frac{|z1|}{|z2|} \right\}$

$\frac{|\bar{z}1\bar{z}3+4\bar{z}1\bar{z}3+9\bar{z}1\bar{z}2|}{|\bar{z}1||\bar{z}2||\bar{z}3|} = 1$

$[\ \therefore\ |\bar{z}1| = |z1|\ ]$

$\frac{|\overline{z1z3+4z1z3+9z1z2}|}{|z1||z2||z3|} = 1$

$|z1z3+4z1z3+9z1z2| = |z1||z2||z3|$

$= (1)(2)(3)$

$|9z1z2+4z1z3+z1z3| = 6$

2) If $z = x+iy$ is a complex numbers such that $\text{Im}\left(\frac{2z+1}{iz+1}\right)$ state that the locus of z is $2x^2+2y^2+x-2y$.

Soln:

$\text{Im}\left[\frac{2(x+iy)+1}{i(x=iy)+1}\right] = 0$

$\text{Im}\left[\frac{2x+2iy+1}{ix-y+1}\right] = 0$

$\text{Im}\left[\frac{(2x+1)+i(2y)}{i-y+ix}\right] = 0$

$\text{Im}\left[\frac{(2x+1)+i(2y)}{i-y+ix}\right] \times \left[\frac{(i-y)-ix}{(i-y)-ix}\right] = 0$

$\text{Im}\left[\frac{-x(2x+1)+(2y)(1-y)}{(i-y)^2+x^2}\right] = 0$

[Take only imaginary parts]

⇨ $-x(2x+1)+(2y)(1-y)=0$

⇨ $-2x^2+x+2y-2y^2=0$

⇨ $\mathbf{2x^2+2y^2+x-2y=0}$

3) Find all cube root of $\sqrt{3}+i$

Soln:

$z^3 = \sqrt{3}+i = r(\cos\theta + i\sin\theta)$

[By polar form]

$r = \sqrt{(\sqrt{3})^2+1^2} = \sqrt{3+1} = \sqrt{4} = 2$

$r = 2$

$\alpha = \tan^{-1}\left(\frac{y}{x}\right)$

$\alpha = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$\alpha = \frac{\pi}{6}$

$\alpha = \theta = \frac{\pi}{6}$

$z^3 = 2[\cos\pi/6 + i\sin\pi/6]$ [:. r =2

$\theta = \pi/6$]

$z = 2^{1/3}[\cos\pi/6 + i\sin\pi/6]^{1/3}$

Adding $2k\pi$

$z = 2^{1/3}[\cos(2k\pi+\pi/6)+ i\sin(2k\pi+\pi/6)]^{1/3}$

Apply Demovire's theorem

$z = 2^{1/3}\left[\cos\left(\frac{(2k\pi+\pi/6)}{3}\right) + \sin\left(\frac{(2k\pi+\pi/6)}{3}\right)\right]$ k = 0,1,2

If K=0

$z = 2^{1/3}[\cos\pi/18 + i\sin\pi/18]$

If K = 1

$z = 2^{1/3}[\cos 13\pi/18 + i\sin 13\pi/18]$

If K = 2

$z = 2^{1/3}[\cos 25\pi/18 + i\sin 25\pi/18]$

$z = 2^{1/3}[\cos(\pi + \frac{7\pi}{18}) + i\sin(\pi + \frac{7\pi}{18})]$

$z = 2^{1/3}[-\cos\frac{7\pi}{18} - i\sin\frac{7\pi}{18}]$

4) If $2\cos\alpha = x + \frac{1}{x}$, $2\cos\beta = y + \frac{1}{y}$ state that

i) $\frac{x}{y} + \frac{y}{x} = 2\cos(\alpha - \beta)$

ii) $xy + \frac{1}{2y} = 2i\sin(\alpha + \beta)$

iii) $\frac{x^m}{y^n} - \frac{y^n}{x^m} = 2i\sin(m\alpha - n\beta)$

iv) $x^m y^n + \frac{1}{x^m y^n} = 2\cos(m\alpha + n\beta)$

soln:

$2\cos\alpha = x + \frac{1}{x}$

$2\cos\alpha = \frac{x^2+1}{x}$

$(2\cos\alpha)x = x^2 + 1$

$x^2 - (2\cos\alpha)x + 1 = 0$

[:. $ax^2+bx+c = 0$]

a = 1 , b = -2 cosα , c = 1

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

[quadratic equation]

$$x = \frac{2\cos\alpha \pm \sqrt{4\cos^2\alpha - 4(1)(1)}}{2(1)}$$

$$= \frac{2\cos\alpha \pm 2\sqrt{-(1-\cos^2\alpha)}}{2}$$

$$= \frac{2\cos\alpha \pm 2\sqrt{1(1-\cos^2\alpha)}}{2}$$

$$= \frac{2\cos\alpha \pm 2i\sin\alpha}{2}$$

$$= \frac{2[\cos\alpha \pm i\sin\alpha]}{2}$$

$x = \cos\alpha + i\sin\alpha$ (or)

$x = \cos\alpha - i\sin\alpha$

Let us consider $x = \cos\alpha + i\sin\alpha$

$$\frac{1}{x} = \frac{1}{\cos\alpha + i\sin\alpha} \times \frac{\cos\alpha - i\sin\alpha}{\cos\alpha - i\sin\alpha}$$

$$\frac{1}{x} = \frac{\cos\alpha - i\sin\alpha}{\cos^2\alpha + \sin^2\alpha}$$

$$\frac{1}{x} = \frac{\cos\alpha - i\sin\alpha}{\cos\alpha + \sin^2\alpha}$$ **[:. i^2= 1]**

$$\mathbf{\frac{1}{x} = \cos\alpha - i\sin\alpha}$$

⇨ $x = \cos\alpha + i\sin\alpha$

⇨ $\mathbf{\frac{1}{x} = \cos\alpha - i\sin\alpha}$

⇨ $\mathbf{y} = \cos\beta + i\sin\beta$

$\Rightarrow \frac{1}{y}$ = cosβ - isinβ

i) $\frac{x}{y} = \frac{\cos\alpha + i\sin\alpha}{\cos\beta + i\sin\beta}$

= cos(α -β)+ isin(α − β)

$\frac{y}{x}$ = cos(α -β) - isin(α − β)

L.H.S

$\frac{x}{y} + \frac{y}{x}$ = cos(α -β) + isin(α − β) + cos(α - β) - isin(α − β)

$\frac{x}{y} + \frac{y}{x}$ = 2 cos(α -β)

ii) xy = (cosα + isinα)(cosβ + isinβ)

= cos (α+β) + i sin(α+β)

$\frac{1}{xy}$ = cos (α+β) + i sin(α+β)

L.H.S

xy = $\frac{1}{xy}$ = 2 isin (α+β).

iii) $\frac{x^m}{y^n} = \frac{(\cos\alpha + i\sin\alpha)^m}{(\cos n\beta + i\sin n\beta)}$

$\frac{x^m}{y^n}$ = cos (mα –nβ) + i sin(mα –nβ)

$\frac{y^n}{x^m}$ = - cos (mα –nβ) + i sin(mα –nβ)

$\frac{x^m}{y^n} + \frac{y^n}{x^m}$ = 2 i sin(mα –nβ)

iv) $x^m y^n$ = [cosα +isinα]^m [cosβ+isinβ]^n

=[cosmα +isinmα] [cosnβ+isinnβ]

$x^m y^n$ =

cos(mα+nβ) + isin(mα+nβ)→①

$\frac{1}{x^m y^n}$ = cos(mα+nβ) - isin(mα+nβ)→②

① +②→

L.H.S

$x^m y^n + \frac{1}{x^m y^n}$ =2cos(mα+nβ)

5) If ω≠ 1 is a cube root of unity state that

(i) $(1-\omega+\omega^2)^6 + (1+\omega-\omega^2)^6 = 128$

(ii) $(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)+\ldots+(1+\omega^{2n}) = 1$

Sol:

L.H.S.

$= (1-\omega+\omega^2)^6 + (1+\omega-\omega^2)^6$

$= (1+\omega^2-\omega)^6 + (1+\omega-\omega^2)^6$

$= (-\omega-\omega)^6 + (-\omega^2-\omega^2)^6$

$= (-2\omega)^6 + (-2\omega^2)^6$

$= (-2)^6\omega^6 + (-2)^6(\omega^2)^6$

$= (-2)^6[\omega^6 + \omega^{12}]$

$= 2^6[1+1] = 2^6(2)$

$=2^7 = 128$ [:. $2^7 = 2x2x2x2x2x2x2$]

ii)**$(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)$**

$(1+\omega^{16})(1+\omega^{32})(1+\omega^{64})(1+\omega^{128})$

$(1+\omega^{256})(1+\omega^{512})(1+\omega^{1024})(1+\omega^{2048})$

$(1+\omega)(1+\omega^2)$ $(1+\omega)(1+\omega^2)$ $(1+\omega)(1+\omega^2)\ldots$

6 terms.

:.$[(1+\omega)(1+\omega^2)]^6$

$[- \omega^2 \cdot (- \omega)]^6 = [- \omega^2 \cdot (- \omega)]^6$

$=[\omega^3]^6 = (1)^6 = 1$

=1

= R.H.S.

3. THEORY OF EQUATIONS

2 MARKS & 3 MARKS

I. Construct a cubic equation with roots 1) 1,2,3 2) 1,1,-2 3) 2, ½,1

Roots 1, 2, 3
α β γ

$\Sigma_1 = \alpha + \beta + \gamma = 1+2+3=6$

$\Sigma_2 = \alpha\beta + \beta\gamma + \gamma\alpha$
$= 1(2) + 2(3) + 3(1)$
$= 2+6+3=11$

$\Sigma_3 = \alpha\beta\gamma = 1(2)(3) = 6$

The required equation
$x^3 - \Sigma_1 x^2 + \Sigma_2 x - \Sigma_3 = 0$
$x^3 - 6x^2 + 11x - 6 = 0$

II. Find the polynomial equation of minimum degree with integer co-efficient having the following roots 1) 2 - √3 i 2) $\overline{2 - \sqrt{3}}$ 3) 2+ √3i 4)2i + 3 5) √ √2 / √3 6) √5-√3

1 roots : 2-√3i , 2+√3i
α β

$\Sigma_1 = \alpha + \beta = 2-\sqrt{3}i + 2+\sqrt{3}i$

$\Sigma_1 = 4$

$\Sigma_2 = \alpha\beta = (2-\sqrt{3}i)(2+\sqrt{3}i)$
$= 2^2 - (\sqrt{3}i)^2$
$= 4+3 = 7$

The required equation is
$x^2 - \Sigma_1 x + \Sigma_2 = 0$
$x^2 - 4x + 7 = 0$

III. Prove that the following cannot intersect more than two points.

1. Parabola and St.line
2. Circle and St.line
   1. Parabola $y^2=4ax$ →(1)
   2. St.line $y = mx+c$ →(2)

Sub.equation 2 in 1
$(mx+c)^2 = 4ax$
$m^2x^2+2mcx+c^2-4ax=0$
$m^2x^2+(2mc-4a)x+c^2=0$

It is a quadratic equation. Which cannot have more than two solutions and hence cannot intersect at more than two points.

IV. Find the Least possible number of imaginary roots and maximum number of positive and negative roots.

1. $9x^9+2x^5-x^4-7x^2+2=0$
2. $x^5-19x^4+2x^3+5x^2+11=0$
3. $x^9+9x^7+7x^5+5x^3+3x=0$
4. $x^{2018}+1947x^{1950}+15x^8+26x^6+1=0$

Ans: 1. $P(x)= 9x^9+2x^5-x^4-7x^2+2=0$

	Sign of co-eff.	No.of.sing changes
P(x)	+ + - - +	2
P(-x)	- - - - +	1

(Change the sign of odd power only)

Max.+ve roots = 2
Max.-ve roots = 1
Total real roots 3
But total power is 9
Minimum of imaginary roots = 9-3=6

V. find the solution if any

1. $2\cos^2x - 9\cos x+4=0$
2. $\sin^2x-5\sin x+4=0$
3. $2\cos^2x-9\cos x+20=0$

1. $2\cos^2x - 9\cos x + 4 = 0$

Put Cos x= t $2t^2-9t + 4 = 0$
$(t-½)(t-4) = 0$
$t = ½ \quad t=4$
Cos x=4
(Impossible)

$t = ½$
$\cos x = ½$
$\cos x = \cos \pi/3$

$x = 2n\pi \pm \pi/3$

State and prove Complex conjucate root theorem

(Theorem 3.2 in book)

I. Find the sum of squares of the roots of the following:

1. $ax^4 + bx^3 + cx^2 + dx + e = 0$
2. $2x^4 - 8x^3 + 6x^2 - 3 = 0$

(1) $ax^4 + bx^3 + cx^2 + dx + e = 0$

Let the roots : α, β, γ , δ

Sum of the squares of roots

$\alpha^2+\beta^2+\gamma^2+\delta^2 = (\alpha+\beta+\gamma+\delta)^2 - 2(\alpha\beta+\alpha\gamma+\alpha\delta+\beta\alpha+\beta\gamma+\beta\gamma+\beta\delta+\gamma\delta)$

$\alpha^2+\beta^2+\gamma^2+\delta^2 = (\Sigma_1)^2 - 2(\Sigma_2)$

$= (-b/a)^2 - 2(c/a)$

$= \frac{b^2}{a^2} - \frac{2c}{a}$

$= \frac{b^2 - 2ac}{a^2}$

II. Solve :

1. $x^3 - 3x^2 - 33x + 35 = 0$
2. $2x^3 - 9x^2 + 10x = 3$
3. $8x^3 - 2x^2 - 7x + 3 = 0$
4. $2x^3 + 11x^2 - 9x - 18 = 0$
5. $x^3 - 5x^2 - 4x + 20 = 0$
6. $2x^3 + 3x^2 + 2x + 3 = 0$
7. $x^4 - 9x^2 + 20 = 0$
8. $x^4 - 14x^2 + 45 = 0$

Note:

* If sum of the co-eff.is Zero 1 is the root

*Co-eff. odd power=co-eff. Even power -1 is a root

*If x^4, x^2 are the only co-eff. sub. $x^2 = t$

1) $x^3 - 3x^2 - 33x + 35 = 0$

Sum of co-eff: 1-3-33+35=0

1	1	-3	-33	35
	0	1	-2	-35
	1	-2	-35	0

Remaining factor: $x^2 - 2x - 35 = 0$

$(x-7)(x+5) = 0$

35 → -7, +5 $x = 7, -5$

.: roots are: 1,7,-5

III. If α, β (Or) α ,β, γ are the root of the equation whose roots are given against them. Equation

Equation	New roots
1. $x^3+2x^2+3x+4=0$	
2. $x^3+2x^2+3x+4=0$	
3. $x^3+2x^2+3x+4=0$	-α, -β, -γ
4. $17x^2+43x-73=0$	2α, 2β, 2γ
5. $2x^2-7x+13=0$	1/α, 1/β, 1/γ
	α+2, β+2
	α^2, β^2

1) $x^3+2x^2+3x+4=0$

Given roots	New roots
$\alpha+\beta+\gamma = -2$	$\Sigma_1 = -\alpha - \beta - \gamma = -(\alpha+\beta+\gamma) = -(-2) = 2$
$\alpha\beta + \beta\gamma + \gamma\delta = 3$	$\Sigma_2 = \alpha\beta + \beta\gamma + \gamma\delta = 3$
$\alpha\beta\gamma = -4$	$\Sigma_3 = (-\alpha)(-\beta)(-\gamma) = 4$

The required equation

$x^3 - \Sigma_1 x^2 + \Sigma_2 x - \Sigma_3 = 0$

$x^3 - 2x^2 + 3x - 4 = 0$

IV. Obtain the condition:

1. The root of $x^3+px^2+qx+r = 0$ are in A.P
2. The roots of $ax^3+bx^2+cx+d=0$ are in G.P
3. The roots of $x^3+px^2+qx+r=0$ are in H.P
4. Solve: $9x^3-36x^2+44x-16=0$ roots are in A.P
5. Solve $3x^3-26x^2+52x-24=0$ roots are in G.P

Note: In A.P, take the roots as α –d, α, α+d

In G.P take the roots as α/t , α, α t

In H.P reverse the co-efficient of given equation and use A.P. roots

3. $x^3+px^2+qx+r=0$ here the roots are in H.P Reverse the co-efficient $rx^3+qx^2+px+1=0$ its roots are in A.P
   roots are α-d, α , α+d sum of roots
   α-d + α + α+d = -q / r
   3 α = -q/r

$\alpha = \frac{-q}{3r}$

Sub in $rx^3 + qx^2 + px + 1 = 0$

$r\left(\frac{-q^3}{27r^2}\right) + q\left(\frac{q^2}{qr^2}\right) + p\left(\frac{-q}{3r}\right) + 1 = 0$

$\frac{-q^3}{27r^2} + \frac{q^3}{9r^2} - \frac{pq}{3r} + 1 = 0$

Multiply by $27r^2$

$-q^3 + 3q^3 - 9pqr + 27r^2 = 0$

$2q^3 + 27r^2 = 9pqr$

1). S.T $2x^2 - 6x + 7 = 0$

Cannot have real roots

2). $X^2 + 2(k+2)x + 9k = 0$

Has real and equal roots find k

3). Discuss the nature of the root of

$2x^2 + kx + k = 0$ in terms of K

4). Discuss the nature of the roots of

$4x^2 + 4px + p + 2 = 0$ in terms of P

5). S.T if P,Q,R are rational the roots of

$X^2 - 2px + p^2 - q^2 + 2qr - r^2 = 0$ are

Rationals

<u>Note</u>

- ➔ $b^2 - 4ac = 0$ roots are real and equal
- ➔ If $b^2 - 4ac > 0$ roots are real and different
- ➔ If $b^2 - 4ac < 0$ roots are imaginary.

1). $2x^2 - 6x + 7 = 0$
(a = 2, b = −6, c = 7)

$b^2 - 4ac = 36 - 4(2)(7)$

$= 36 - 56 = -20 < 0$

:: The roots are imaginary

From the equation to find a number such that when its cube root is added to it, the result is 6

$x + x^{\frac{1}{3}} = 6$

$x^{\frac{1}{3}} = 6 - x \Rightarrow (x^{1/3})^3 = (6 - x)^3$

:: $x = 216 - 3(6^2)x + 3(6)(x^2) - x^3$

$X = 216 – 108x + 18x^2 – x^3$

$X^3 – 18x^2 + 109x – 216 = 0$

5 Marks

1). If 2 + i, 3-$\sqrt{2}$ are roots of

$x^6 – 13x^5 + 62x^4 – 126x^3 + 65x^2 + 127x - 140 = 0$

$x^6 - 13x^5 + 62x^4 – 126x^3 + 65x^2 + 127x – 140 = 0$

roots are:

2 + i, 2 – i, 3 -$\sqrt{2}$, 3+ $\sqrt{2}$, α, β

Sum of roots

2 + 2 + 3 + 3 + α + β = 13

$\alpha + \beta = 3$

Product of roots

$(2 + i)(2 – i)(3- \sqrt{2})(3 + \sqrt{2})\alpha\beta = -140$

(4+1) (9 – 2) αβ = -140

35 αβ = -140

$\alpha\beta = \frac{-140}{35}$

αβ = -4

Two numbers whose product is – 4 and sum is 3 are -1, 4

:: The roots are

(2 + i) (2 –i) (3 -$\sqrt{2}$) (3 +$\sqrt{2}$) -1 , 4

Solve :

1). (x-2) (x-7) (x-3) (x+2) + 19 = 0

2). (2x -3) (6x -1) (3x – 2) (x-2) -7=0

3). (x-5) (x-7) (x+6) (x+4) = 504

4). (2x-1) (x+3) (x-2) (2x+3) +20 =0

5). (x-4) (x-7) (x-2) (x+1) = 16

1). (x-2) (x-7) (x-3) (x+2) + 19 = 0

Re arrange them

[(x-2) (x-3)] [(x-7) (x+2)] +19 =0

$[x^2 – 5x + 6] [x^2 – 5x – 14] + 19 = 0$

$X^2 – 5x = t$

(t+6) (t-14) + 19 = 0

$t^2 - 8t – 84 + 19 = 0$

$t^2 – 8t – 65 = 0$

(t- 13) (t + 5) = 0

t = 13, -5

$x^2 - 5x = 13$

$x^2 - 5x - 13 = 0$

$x = \frac{5 \pm \sqrt{25-4(-13)}}{2(1)}$

$= \frac{5\pm\sqrt{25+52}}{2}$

$= \frac{5\pm\sqrt{77}}{2}$

:: The root are

$\frac{5\pm\sqrt{77}}{2}$,

$x^2 - 5x = -5$

$x^2 - 5x + 5 = 0$

$x = \frac{5\pm\sqrt{25-4(1)(5)}}{2(1)}$

$= \frac{5\pm\sqrt{25-20}}{2}$

$= \frac{5\pm\sqrt{5}}{2}$

The root are

$\frac{5\pm\sqrt{5}}{2}$

3). Solve $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$

If one root is $\frac{1}{3}$

$6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$ is a reciprocal equation

:: if $\frac{1}{3}$ is a root 3 also root

$\frac{1}{3}$	6	-5	-38	-5	6
	0	2	-1	-13	-6
3	6	-3	-39	-18	0
	0	18	45	18	
	6	15	6	0	

:: remaining factor :

$6x^2 + 15x + 6 = 0$

36 → $\frac{12}{6}$, $\frac{3}{6}$

$(x+2)\left(x + \frac{1}{2}\right)$

$x = -2, -\frac{1}{2}$

:: Roots are $\frac{1}{3}$, 3, -2, $\frac{-1}{2}$

4).solve $6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$

Last constant term is 6

:: by verification we can find out 2 is a root

It is reciprocal equation

:: $\frac{1}{2}$ is also a root

	6	-35	62	-35	6
2	0	12	-46	32	-6
	6	-23	16	-3	0

$\frac{1}{2}$	0	3	-10	3
		6	-20	6
	6	-20	6	0

Remaining factor $6x^2 - 20x + 6 = 0$

36 → $\frac{-2}{6}$, $\frac{-18}{6}$

$(3 - \frac{1}{3})\ (x - 3) = 0$

$x = \frac{1}{3}, 3$

$\therefore$ roots $2, \frac{1}{2}, \frac{1}{3}, 3$

5). $X^4 - 10x^3 + 26x^2 - 10x + 1 = 0$ solve

$\div x^2 : x^2 - 10x + 26 - \frac{10}{x} + \frac{1}{x^2} = 0$

$(x^2 + \frac{1}{x^2}) - 10\ (x + \frac{1}{x}) + 26 = 0$

$X + \frac{1}{x} = t \Rightarrow x^2 + \frac{1}{x^2} = t - 2$

$(t^2 - 2) - 10t + 26 = 0$

$\therefore t^2 - 10t + 24 = 0$

$(t-6)(t-4) = 0$

$\therefore t = 6$ or $t = 4$

$X + \frac{1}{x} = 6$	$x + \frac{1}{x} = 4$
$X^2 + 1 = 6x$	$x^2 + 1 = 4x$

$X^2 - 6x + 1 = 0$ $\quad x^2 - 4x + 1 = 0$

$X = \frac{6 \pm \sqrt{36-4}}{2}$ $\quad x = \frac{4 \pm \sqrt{16-4}}{2}$

$= \frac{6 \pm \sqrt{32}}{2}$ $\quad = \frac{4 \pm \sqrt{12}}{2}$

$= \frac{6 \pm 4\sqrt{2}}{2}$ $\quad = \frac{4 \pm 2\sqrt{3}}{2}$

$= 3 \pm 2\sqrt{2}$ $\quad = 2 \pm \sqrt{3}$

6). $(1 + 2i), \sqrt{3}$ are the roots of

$X^6 - 3x^5 - 5x^4 + 22x^3 - 39x^2 - 39x + 135 = 0$

Find the others roots.

Roots : $(1 + 2i), (1 - 2i), \sqrt{3}, -\sqrt{3}, \alpha, \beta$

Sum of roots : $1 + 1 + \alpha + \beta = 3$

$\alpha + \beta = 1$

product of roots:

$(1+2i)\ (1-2i)\ (\sqrt{3}), (-\sqrt{3})\ (\alpha\beta) = 135$

$(1+4) \quad (-3) \quad \alpha\beta = 135$

$\alpha\beta = -9$

remaining factor :

$x^2 - (\alpha + \beta)x + \alpha\beta + 0$

$x^2 - 1x - 9 = 0$

$x = \frac{1 \pm \sqrt{1-4(-9)}}{2}$

$x = \frac{1 \pm \sqrt{37}}{2}$ :: The roots are

$(1+2i), (1-2i), \sqrt{3}, -\sqrt{3}, \frac{1 \pm \sqrt{37}}{2}$

7). If the one root of is twice the sum of the other two roots, find K and solve

$2x^3 - 6x^2 + 3x + k = 0$

Let the roots α, β, γ

$\alpha + \beta + \gamma = \frac{6}{2}$

$\alpha + \frac{\alpha}{2} = 3$

$\alpha = \frac{3*2}{3}$

$\alpha = 2$

$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{3}{2}$

$\alpha(\beta+\gamma) + \beta\gamma = \frac{3}{2}$

$2(1) - \frac{k}{4} = \frac{3}{2}$

$2 - \frac{3}{2} = \frac{k}{4}$

$\frac{1}{2} = \frac{k}{4}$

K = 2

one root is twice the sum of other two

$\alpha = 2(\beta + \gamma)$

$\frac{\alpha}{2} = \beta + \gamma$

$1 = \beta + \gamma$

$\alpha\beta\gamma = \frac{-k}{2}$

$2(\beta\gamma) = \frac{-k}{2}$

$\beta\gamma = \frac{-k}{4}$

:: $\beta\gamma = \frac{-2}{4} = \frac{-1}{2}$

:: remaining factor:

$x^2 - (\beta + \gamma)x + \beta\gamma = 0$

$x^2 - 1x - \frac{1}{2} = 0$

$2x^2 - 2x - 1 = 0$

$x = 2 \pm \sqrt{4 - 4(2)(-1)} / 2(2)$

$= \frac{+2 \pm \sqrt{12}}{4}$ => $x = \frac{+2 \pm 2\sqrt{3}}{4}$

$X = \frac{1 \pm \sqrt{3}}{2}$

Roots : 2, $\frac{1 \pm \sqrt{3}}{2}$

Solve : $3x^3 - 16x^2 + 23x - 6 = 0$

If the product of two roots is 1

4. INVERSE TRIGNOMETRIC FUNCTIONS

I.TWO MARK

1.find the value of $\sin^{-1}(\sin(5\pi/6))$

$\sin^{-1}(\sin(5\pi/6)) = \sin^{-1}(\sin(\pi - \pi/6))$

$= \sin^{-1}(\sin \pi/6)$

$= \pi/6 \;\ldots\; \pi/6 \in [-\pi/2, \pi/2]$

2. find the value of

$\sin^{-1}(\sin(5\pi/9 \cos \pi/9 + \cos 5\pi/9 \sin \pi/9$

sol : $\sin^{-1} \sin((5\pi/9 + \pi/9))$

$= \sin^{-1}(\sin 6\pi/9)$

$= \sin^{-1}(\sin 2\pi/3)$

$= \sin^{-1}(\sin(\pi - \pi/3))$

$= \sin^{-1}(\sin \pi/3)$

$= \pi/3$

3. Find the value of $\cos^{-1}(1/2) + \sin^{-1}(-1)$

Sol: $\cos^{-1}(1/2) + \sin^{-1}(-1) = \pi/3 - \pi/2$

$= \frac{2\pi - 3\pi}{6}$

$= -\pi/6$

4. Find the value of $\sec^{-1}\left(-\frac{2\sqrt{3}}{3}\right)$

Sol:let, $\sec^{-1}\left(-\frac{2\sqrt{3}}{3}\right) = \theta$

$\sec\theta = \frac{-2}{\sqrt{3}}$

$\theta \in [0, \pi] \setminus \{\pi/2\}$

we have,

$\cos\theta = -\sqrt{3}/2$ or $\sec\theta = -2/\sqrt{3}$

then, $\cos 5\pi/6 = -\sqrt{3}/2$

now, $\theta = 5\pi/6$

$\sec^{-1}(-2\sqrt{3}/3) = 5\pi/6$

5.Prove that $\tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24} = \tan^{-1}\frac{1}{2}$

$= \tan^{-1}\frac{\frac{2}{11}+\frac{7}{24}}{1-\left(\frac{2}{11}\right)\left(\frac{7}{24}\right)}$

$= \tan^{-1}\frac{\frac{48+77}{264}}{\frac{264-14}{264}}$

$= \tan^{-1}(125/250)$

$= \tan^{-1}(1/2)$

$\tan^{-1}(2/11) + \tan^{-1}(7/24) = \tan^{-1}(1/2)$

Hence the proved.

II.THREE MARK

Find the domain of $\sin^{-1}(2 - 3x^2)$

1. Sol : Range of $\sin^{-1}(x)$ is $[-1,1]$

$-1 \le 2-3x^2 \le 1$

Add $-2 \rightarrow -3 \le 3x^2 \le -1$

$-3 \le -3x^2$ then $x^2 \le 1$ _____(1)

$-3x^2 \le -1$ then $x^2 \ge 1/3$_____(2)

From equations (1) and (2)we have get,

$1/3 \le x^2 \le 1$

Then $1/\sqrt{3} \le |x| \le 1$

Since $a \le |x| \le$,

implies $x \in [-b,-a] \cup [a,b]$

combining the equations (1) and (2)

$X \in [-1,-1/\sqrt{3}] \cup [1/\sqrt{3},1]$

2. Find the domain of $f(x) = \sin^{-1}\frac{x^2+1}{2x}$

Sol : range of $\sin^{-1} x$ is $[-1,1]$

$-1 \le \frac{x^2+1}{2x} \le 1$

Multiply by 2x

$-2x \leq x^2 + 1 \leq 2x$	$x^2 - 2x + 1 \leq 0$
$0 \leq x^2 + 1 + 2x$	$(x-1)^2 \leq 0$
$0 \leq (x+1)^2$	$x - 1 \leq 0$
$0 \leq x + 1$	$x = 1$

$x = -1$

solution is {-1,1}

1. Find the domain of $f(x) = \sin^{-1} x + \cos^{-1} x$

   Sol : range of $\sin^{-1}(x)$ is [-1,1]

   Range of $\cos^{-1}(x)$ is [-1,1]

   Then $-1 \leq x \leq 1$

   X €[-1,1]

2. Find the domain $\tan^{-1}(\sqrt{9 - x^2})$

   Soln:

   $9 - x^2 \geq 0$

   $9 \geq x^2$

   $x^2 \leq 9$

   $x \leq \pm 3$

   domain[-3,3]

3. $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z$ show that $x+y+z = xyz$

   proof: $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$

   $\tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right] = \pi$

   $\therefore \tan\pi = 0$

   $\frac{x+y+z-xyz}{1-xy-yz-zx} = \tan\pi$

   $\frac{x+y+z-xyz}{1-xy-yz-zx} = 0$

   $x+y+z-xyz = 0$

   $x+y+z = xyz$

   Hence the proved.

III. FIVE MARK

1) If $a_1, a_2 \dots a_n$ is an arithmetic progression with common difference d prove that,

$\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{d}{1+a_2a_3}\right) + \dots\dots + \tan^{-1}\left(\frac{d}{1+a_na_{n-1}}\right)\right] = \left(\frac{a_n - a1}{1+a_1a_2}\right)$

Proof:-

$\tan^{-1}\left(\frac{d}{1+a_1a_2}\right) = \tan^{-1}\left(\frac{a_2 - a1}{1+a_1a_2}\right)$

$\tan^{-1}a_2 - a_n^{-1}a_1 \dashrightarrow 1$

III ly,

$\tan^{-1}\left(\frac{d}{1+a_2a_3}\right) = \tan^{-1}\left(\frac{a_3 - a_2}{1+a_1a_2}\right)$

$\tan^{-1}a_3 - \tan^{-1}a_2 \dashrightarrow 2$

Continuing:-

$\tan^{-1}\left(\frac{d}{1+a_na_{n-1}}\right)) =$

$\tan^{-1}\left(\frac{a_na_{n-1}}{1+a_{n-1}a_n}\right)$

$\tan^{-1}a_n - \tan^{-1}a_{n-1} \dashrightarrow 3$

Adding equations 1, 2 and 3 We get

$[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{d}{1+a_2a_3}\right) + \dots\dots + \tan^{-1}\left(\frac{d}{1+a_na_{n-1}}\right)$

$= \cancel{\tan^{-1}a_2} - \tan^{-1}a_1 + \cancel{\tan^{-1}a_3} - \cancel{\tan^{-1}a_2} + \dots\dots + \tan^{-1}a_n - \cancel{\tan^{-1}a_{n-1}}$

$= \tan^{-1}a_n - \tan^{-1}a_1$

Then,

tan [tan^{-1} (($\frac{d}{1+a_1a_2}$))+ tan^{-1}($\frac{d}{1+a_2a_3}$)+…….+ ($\frac{d}{1+a_na_{n-1}}$))

= tan [tan^{-1} a_n -tan^{-1} a_1]

= tan [tan^{-1}($\frac{a_na_1}{1+a_{n-a1}}$)

= ($\frac{a_n-a1}{1+a_1a_2}$)

Hence the proved.

2.slove $\tan^{-1}(\frac{x-1}{x-2})$ + $\tan^{-1}(\frac{x+1}{x-2})$ = Π /4

tan^{-1} [($\frac{x-1}{x-2}$) + ($\frac{x+1}{x+2}$) / 1– ($\frac{x-1}{x-2}$)($\frac{x+1}{x+2}$)=Π/4

$$\frac{\frac{x2+x-2x-2+x2-x+2x-2}{x2-4}}{1-\frac{x2-x+x-1}{x2-4}} = \tan(\Pi/4)$$

$$\frac{2x^2-4}{-3} = 1$$

$2x^2 -4 = - 3$

$2x^2 =1$

$X^2 =1/2$

X= ± 1/ √2

3. slove : cos (sin^{-1}($\frac{x}{\sqrt{1+x^2}}$) = Sin{$\cot^{-1}$ 3/4}

Sol: we know $\sin^{-1}\frac{x}{\sqrt{1+x^2}}$ = $\cos^{-1}\frac{1}{\sqrt{1+x^2}}$

cos($\sin^{-1}\frac{x}{\sqrt{1+x^2}}$ =cos ($\cos^{-1}\frac{1}{\sqrt{1+x^2}}$

=$\frac{1}{\sqrt{1+x^2}}$-------1

Let us $\cot^{-1}$ 3/4 =θ

cot θ=3/4

θ is active angle then

sin {$\cot^{-1}(3/4)$} = 4/5 ------2

From equation 1and 2 equal

$\frac{1}{\sqrt{1+x^2}}$) =4/5

$\sqrt{1+x^2}$= 5/4

$1+x^2$ =25/16

x^2 =25/16 - 1

x^2 =25-16 /16

=9/16

x=± ¾

4. Prove that , tan^{-1}x+ tan^{-1}y+ tan^{-1}z = tan^{-1}[$\frac{x+y+z-xyz}{1-xy-yz-zx}$] Proof:-

tan^{-1}x+ tan^{-1}y+ tan^{-1}z = tan$^{-1}\frac{x+y}{1-xy}$+ tan^{-1}z

= tan^{-1}[$\frac{\frac{x+y}{1-xy}+z}{1-(\frac{x+y}{1-xy})z}$]

= tan^{-1} . $\left[\frac{\frac{x+y+z-xyz}{1-xy}}{\left(\frac{1-xy-yz-zx}{1-xy}\right)}\right]$

= tan^{-1} . $\left[\frac{X+Y+Z-XYZ}{(1-XY-YZ-Zx)}\right]$

Hence the proved

5 . slove :2 $\tan^{-1} x$ = cos^{-1} $\frac{1-a}{1+a2}$ – cos$^{-1}\frac{1-b^2}{1+b^2}$,a>0,b>0

Sol:

2 tan^{-1} x =$\cos^{-1}$.$\frac{1-x^2}{1+x^2}$

2 tan^{-1} x = 2 tan^{-1}a - 2 tan^{-1} b

=2[tan^{-1} a - tan^{-1} b]

=2 tan^{-1}($\frac{a-b}{1+ab}$)

X=$\frac{a-b}{1+ab}$

5. TWO DIMENSIONAL ANALYTICAL GEOMENTRY-II

2 MARKS :

1) Find the equation of the circle with centre (2,-1) and passing through the point (3,6) in standard form.

Soln:

Centre : $(h,k) = (2,-1)$

Equation of circle $(x-h)^2 + (y-k)^2 = r^2$

$\rightarrow (x-2)^2 + (y+1)^2 = r^2$

It passes through (3,6):

$\rightarrow (3-2)^2 + (6+1)^2 = r^2$

$1^2 + 7^2 = r^2$

$1 + 49 = r^2$

$r^2 = 50$

① $\rightarrow (x-2)^2 + (y+1)^2 = 50$

2)Find the general equation of the circle whose diameter is the line segment joining the points

(-4,-2) and (1,1).

Soln:

Equation of the circle with end points of the diameter as (x_1,y_1) and (x_2,y_2) is,

$(x-x_1)(x-x_2) + (y-y_1)(y-y_2) = 0$ [Theorem 5.2]

$(x+4)(x-1) + (y+2)(y-1) = 0$

$X^2 + 4x - x - 4 + y^2 + 2y - y - 2 = 0$

$X^2 + y^2 + 3x + y - 6 = 0$

$\rightarrow$This is the required equation of circle.

3) Find the equation of the parabola with vertex (-1,-2), axis parallel to y – axis and passing through (3 ,6).

Soln:

Axis parallel to y-axis ,the equation of parabola is

$(x+1)^2 = 4a\,(x+2) \rightarrow$ ①

It passes through (3,6)

① $\rightarrow (3+1)^2 = 4a\,(6+2)$

$4^2 = 4a\,(8)$

$16 = 32\,a$

$a = \frac{16}{32}$

$\rightarrow a = \frac{1}{2}$

① $\rightarrow (x+1)^2 = 4(\frac{1}{2})(y+2)$

$X^2 + 2x + 1 = 2y + 4$

$X^2 + 2x - 2y - 3 = 0$

4) Find the equation of the ellipse with foci($\pm$3,0), e = $\frac{1}{2}$.

Soln:

As per given condition the major axis is along x-axis

$\rightarrow \frac{x2}{a2} + \frac{y2}{b2} = 1$ -------①

C.F = ae = 3 $\rightarrow a(\frac{1}{2}) = 3$

$a = 6$

$a^2 = 36$

$b^2 = a^2(1 - e^2)$

$= 36\,(1 - \frac{1}{4})$

$= 36\,(\frac{3}{4}) = 27 \Rightarrow b^2 = 27$

① $\rightarrow \frac{x2}{36} + \frac{y2}{27} = 1$

5) Find the vertices ,foci for the hyperbola $9x^2 - 16y^2 = 144$

Soln :

Reducing $9x2 - 16\,y2 = 144$ to the standard form, we have

$\frac{x2}{16} - \frac{y2}{9} = 1$
[÷ by 144]

$a^2=16$, $b^2=16$ => a=4, b= 3

with the transverse axis is along x-axis, the vertices are (-4,0) and (4 ,0) and

$c^2 = a^2 + b^2 = 16 + 9 = 25$

⇨ c=25

Hence, the faci are (-5,0) and (5, 0).

3 MARKS

1) Find the equation of the circle described on the chords 3x + y + 5 = 0 of the circle $x^2 + y^2 = 16$ as diameter.

Soln:

Equation of the circle passing through the points of intersection of the chords and circle is

$X^2+y^2-16+\lambda(3x + y + 5) = 0$
[by Theorem 5.1]

The chord 3x + y + 5 = 0 is a diameter of this circle if the centre ($\frac{-3\lambda}{2}$, $\frac{-\lambda}{2}$) lies on the chord.

So ,we have $3\left(\frac{-3\lambda}{2}\right) - \frac{\lambda}{2} + 5 = 0$

$\frac{-9\lambda}{2} - \frac{\lambda}{2} + 5 = 0$

$\frac{-10\lambda}{2} + 5 = 0$

$-5\lambda + 5 = 0$

=> $\lambda = 1$

The required equations are ,

$X^2+y^2-16+\lambda(3x + y + 5) = 0$

$X^2 + y^2 + 3x + y - 11 = 0$

2) Find the equation of the circle passing through the points (1,1),(2,-1) and (3,2) .

Soln:

General equation : $x^2 + y^2 + 2gx + 2fy + c = 0$ →①

It passes through (1,1), (2,1) and (3,2)

(1,1) => $(1)^2 + (1)^2 + 2g(1) + 2f(1) + c = 0$

2g + 2f + c = -2 →②

(2,-1) => $(2)^2 + (-1)^2 + 2g(2) + 2f(-1) + c = 0$

4g - 2f + c = -5 →③

(3,2) => $(3)^2 + (2)^2 + 2g(3) + 2f(2) + c = 0$

6g +4f + c = -13 →④

②-③ => -2g +4f = 3 →⑤

④-③ => 2g +6f = -8 →⑥

⑤ +⑥ => 0+10f = -5

$f = \frac{-1}{2}$

subt:

$f = \frac{-1}{2}$ in ⑥ , we get

2g – 3 = -8

$2g = -8+3$

$\rightarrow \quad g = \frac{-5}{2}$

Subt the value of f and g in ②

$2\left(\frac{-5}{2}\right) + 2\left(\frac{-1}{2}\right) + c = -2$

$-5 - 1 + c = -2$

$C = 4$

Thus required equation of the circle is $x^2 + y^2 + 2\left(\frac{-5}{2}\right)x + 2\left(\frac{-1}{2}\right)y + 4 = 0$

$x^2 + y^2 + 5x - y + 4 = 0$

3) Find centre,foci,vertices and directrices of ellipse $\left(\frac{x2}{25}\right) + \left(\frac{y2}{9}\right) = 1$

Soln:

$\left(\frac{x2}{25}\right) + \left(\frac{y2}{9}\right) = 1$

Here:

$a^2 = 25 \quad , \quad b^2 = 9$

$a = 5 \qquad b = 3$

$e = \sqrt{1 - \frac{b2}{a2}} = \sqrt{1 - \frac{9}{25}} = \sqrt{1 - \frac{16}{25}} = \frac{4}{5}$

$ae = 5 * \frac{4}{5} = 4 \quad \Rightarrow ae = 4$

i)centre : (0,0)

ii)Foci : $(\pm ae, 0) = (\pm 4, 0)$

iii)vertices : $(\pm a, 0) = (\pm 5, 0)$

iv)Directrices : $x = \pm\frac{a}{e} = \pm\frac{5}{4/5} = \pm\frac{25}{4}$

4) Find the equations of tangent to the hyperbola $\left(\frac{x2}{16}\right) + \left(\frac{y2}{64}\right) = 1$ which are parallel to $10x - 3y + 9 = 0$.

Soln:

$\left(\frac{x2}{16}\right) + \left(\frac{y2}{64}\right) = 1 \qquad \Rightarrow a^2 = 16$, $b^2 = 64$

Slop of the line $10x - 3y + 9 = 0$. Is

$-3y = -10x - 9$

$Y = \frac{-10x}{-3} - \left(\frac{9}{3}\right)$

slop $m = \frac{10}{3}$

Equation of tangents are

$y = mx \pm \sqrt{a2m2 - b2}$

$= \frac{10}{3}x \pm \sqrt{16\left(\frac{100}{9}\right) - 64}$

$= \frac{10}{3}x \pm \sqrt{\frac{1600-576}{9}}$

$= \frac{10}{3}x \pm \frac{32}{3}$

$3y = 10x \pm 32$

$10x - 3y \pm 32 = 0$

5) If the normal point 't_1' on the parabola $y^2 = 4ax$ meets the parabola again at the point 't_2',then prove that $t_2 = -\left(t_1 + \frac{2}{t1}\right)$.

Soln:

$(a t_1^2, 2a t_1)$ equations of the normal

$y + x t_1 = a t_1^3, 2a t_1$

$\rightarrow y - 2at_1 = -xt_1 + a t_1^3$

$Y - 2at_1 = - t_1 (x - at_1^2)$

Parabola passes through $(a t_2^2, 2a t_2)$

$2a t_2 - 2a t_1 = -t_1(a t_2^2, a t_1^2)$

$2a(t_2 - t_1) = -at_1 (t_2^2 - t_1^2)$

$2a(t_2 - t_1) = -at_1 (t_2 + t_1)(t_2 - t_1)$

$2 = -t_1 (t_2 + t_1)$

$t_2 + t_1 = \frac{-2}{t1}$

$t_2 = -t_1 - \frac{2}{t1}$

$t_2 = -(t_1 + \frac{2}{t2})$.

Hence proved.

5 MARKS

1) Find the foci,vertices and length of the major and minor axis of the conic $4x^2+36y^2+40x-288y+532=0$.

Soln:

Completing the square on x & y

$4x^2+36y^2+40x-288y+532 = 0.$

$4(x^2+10x) + 36(y^2-8y) = -532$

$4(x^2+10x+5^2-5^2) + 36(y^2-8y+4^2-4^2) = -532$

$4[(x+5)^2 - 25] + 36[(y-4)^2 - 16] = -532$

$4(x+5)^2 + 36(y-4)^2 = -532 + 100 + 576$

$\Rightarrow \frac{(x+5)2}{36} + \frac{(y-4)2}{4} = 1$

$a^2 = 36, b^2 = 4$

This is an ellipse major axis is parallel to x-axis.

Centre : $x+5 = 0$ $y-4 = 0$

$x = -5$ $y = 4$

$C(-5, 4) = (h,k)$

Vertices : $(h \pm a, k)$

$= (-5+6, 4), (-5-6, 4)$

$= (1,4), (-11,4)$

Foci : $(h \pm c, k)$

$=(-5-4\sqrt{2}, 4), (-5+4\sqrt{2}, 4)$

Where ;

$C^2 = a^2+b^2$

$= 36 - 4$

$=32$

$C = \sqrt{32} = \pm 4\sqrt{2}$

Length of major axis = 2a = 2(6) = 12 units

Length of minor axis = 2b = 2(2) = 4 units

2) A semielliptical arch way over a one-way road road has a height of 3m and a width of 12m. Thetruck has a width of 3m and a height of 2.7m will the truck clear the opening of the archway?

Soln:

From the diagram

a=6 ,b = 3

- the equation of the ellipse is $(\frac{x2}{62}) + (\frac{y2}{32}) = 1$ → ①
- The edge of th 3m wide truck corresponding to x =1.5m from centre.

① → $(\frac{(1.5)2}{36}) + (\frac{y2}{9}) = 1$

$(\frac{y2}{9}) = 1 - (\frac{3/2)2}{36})$

$Y^2 = 9(1 - (\frac{9}{144}))$

$= 9 (\frac{9(135)}{144}) = (\frac{135}{16})$

$Y = \sqrt{(\frac{135}{16})}$

$= (\frac{11.62}{4})$

= 2.90

Thus the height of archway 1.5m from the centre is 2.90 mapprox. So,the truck will clear the archway.

3) parabolic of a 60 m portion of the roadbed of a suspension bridge are positioned as shown in figure .vertical cables are to be spaced every 6m along this position of the roadbed .calculate the length of first two of these vertical cables from the vertex.

Soln :

Vertex = (h , k)

= (0, 3)

The equation of the parabola

$(x - h)^2 = 4a (y-k)$

$X^2 = 4a (y-3)$ →① [subt. Point (0,3)]

It passes through (30,16)

$30^2 = 4a (16 - 3)$

$= 4a (13)$

$a = (\frac{30*30}{4*13})$ →②

① → $x^2 = 4 (\frac{30*30}{4*13}) (y - 3)$

$X^2 = (\frac{30*30}{13}) (y - 3)$ →③

i) If x = 6 ,then

③ → $36 = (\frac{30*30}{13}) (y - 3)$

$(y - 3) = (\frac{36*13}{30*30}) = (\frac{52}{100}) = 0.52$

Y = 3+0.52

Y= 3.52 m

ii)If x=12 ,then

③ → $144 = (\frac{30*30}{13}) (y - 3)$

$(y - 3) = (\frac{144*13}{30*30}) = (\frac{208}{100}) = 2.08$

Y =3 + 2.08

Y = 5.08m

The length of he cables are 5.08m and 3.52m.

4) Cross section of a nuclear cooling tower is in the shape of a hyperbola with the equation$(\frac{x2}{(30)2}) + (\frac{y2}{(44)2}) = 1$. This tower is 150m tall and the distance from the top of the tower to the centre of the hyperbola is half the distance from the base of the tower to the center of the hyperbola.Find the diameter of the top and base of the tower.

Soln:

Given :

p+2p = 150

3p = 150

P=50

- Distance from the top of the tower to the centre = 50 m.
- Distance from the base of the tower to the centre = 100m.

$(\frac{x2}{(30)2}) + (\frac{y2}{(44)2}) = 1 \rightarrow ①$

i) if y =50 , then

$① \rightarrow (\frac{x2}{(30)2}) + (\frac{y2}{(44)2}) = 1 \Rightarrow (\frac{x2}{(30)2}) = 1 + (\frac{(50)2}{(44)2})$

$(\frac{x2}{(30)2}) = 1 + (\frac{2500}{1936}) = (\frac{4436}{1936}) = 2.291$

- $x^2 = 30^2 * 2.291$
- $x = 30\sqrt{2.291} = 30 * (2.4839)$
- x = 74.51m

ii) if y = 100, then

$① \rightarrow (\frac{x2}{(30)2}) + (\frac{(100)2}{(44)2}) = 1 \Rightarrow (\frac{x2}{(30)2}) = 1 + (\frac{10000}{1936})$

$(\frac{x2}{(30)2}) = (\frac{11936}{1936}) = 6.17$

$x^2 = 30^2 * 6.17$

$x = 30\sqrt{6.17} = 30 * (2.4839)$

x = 74.51m

The diameter of the top = 45.41m

The diameter of the base = 74.51m.

5) point A and point B are 10 km apart and it is determined from the sound of the explosion heard at those points at different times that the location of the explosion is 6km closer to A than B shows that the location if the explosion is restricted to a particular curve and find an equation of it.

Soln:

sp – s'p = 6 =>2a = 6

a = 3

$a^2 = 9$

Midpoint of ss' is c (0,0)

Then equation

$\frac{x2}{a2} - \frac{y2}{b2} = 1 \rightarrow ①$

cs = 5 => ae = 5

3e = 5

$e = \frac{5}{3}$

$b^2 = a^2(e^2-1)$

$= 9\left(\frac{25}{9} - 1\right)$

$= 9\left(\frac{16}{9}\right)$

$b^2 = 16$

① → $\frac{x^2}{9} - \frac{y^2}{16} = 1$

This is a hyperbola.

6. VECTOR ALGEBRA

2 MARKS

1). Find the Cartesian equation if a line passing through the points A(2, -1, 3) and B(4, 2, 1)

Sol.

$x_1 = 2, y_1 = -1, z_1 = 3$

$x_2 = 4, y_2 = 2, z_2 = 1$

$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$

$\frac{x-2}{4-2} = \frac{y-(-1)}{2-(-1)} = \frac{z-3}{1-3}$

$\frac{x-2}{2} = \frac{y+1}{3} = \frac{z-3}{-2}$

2). If the plane $\vec{r}.\left(\wedge + \vec{2j} + \vec{3k}\right) = 7$ and

$\vec{r}.\left(\vec{\lambda i} + \vec{2j} - \vec{7k}\right) = 26$ are perpendicular. Find the value of λ.

Sol.

$\vec{r}.\left(\vec{i} + \vec{2j} + \vec{3k}\right) = 7,$

$\vec{n_1}$

$\vec{r}.\left(\vec{\lambda i} + \vec{2j} - \vec{7k}\right) = 26$

$\vec{n_2}$

Perpendicular $\vec{n_1}.\vec{n_2} = 0$

$\left(\vec{i} + \vec{2j} + \vec{3k}\right).\left(\vec{\lambda i} + \vec{2j} - \vec{7k}\right) = 0$

$\lambda + 4 - 21 = 0$

$\lambda - 17 = 0, \ \lambda = 17$

3). Find the acute angle between the following line $\frac{x+4}{3} = \frac{y-7}{4} = \frac{z+5}{5}$.

$\vec{r} = 4k + t\left(\overrightarrow{2i} + \vec{j} + \vec{k}\right)$

$\vec{b} = \vec{3i} + \vec{4j} + \vec{5k}, \quad \vec{d} = \vec{2i} + \vec{j} + \vec{k}$

Sol.

$\cos\theta = \frac{\vec{b}.\vec{d}}{|\vec{b}||\vec{d}|} = \frac{(3i+4j+5k)}{\sqrt{3^2+4^2+5^2}}\frac{(2i+j+k)}{\sqrt{2^2+1^2+1^2}}$

$= \frac{6+4+5}{\sqrt{9+16+25}\sqrt{4+1+1}}$

$= \frac{15}{\sqrt{50}\sqrt{6}} = \frac{15}{5\sqrt{2}\sqrt{6}}$

$= \frac{3}{\sqrt{4*3}} = \frac{\sqrt{3}*\sqrt{3}}{2\sqrt{3}}$

$\Theta = \frac{\sqrt{3}}{2}$

$\Theta = cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$\Theta = \frac{\pi}{6}$

4). For any vector $\vec{a}$ prove that

$\vec{i} * \left(\vec{a} * \vec{i}\right) + \vec{j} * \left(\vec{a} * \vec{j}\right) + \vec{k}\left(\vec{a} * \vec{k}\right)$

$= 2\vec{a}$

$\vec{a} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}$

Sol.

$\vec{a} * \left(\vec{b} * \vec{c}\right) = \left(\vec{a}.\vec{c}\right)\vec{b} - \left(\vec{a}.\vec{b}\right)\vec{c}$

$:: \vec{i} * \left(\vec{a} * \vec{i}\right) = \left(\vec{i}.\vec{i}\right)\vec{a} - \left(\vec{i}.\vec{a}\right)\vec{i} =$

$\vec{a} - a_1\vec{i}$

$:: \vec{j} * \left(\vec{a} * \vec{j}\right) = \left(\vec{j}.\vec{j}\right)\vec{a} - \left(\vec{j}.\vec{a}\right)\vec{j} =$

$\vec{a} - a_2\vec{j}$

$:: \vec{k} * (\vec{a} * \vec{k}) = (\vec{k}.\vec{k})\vec{a} - (\vec{k}.\vec{a})\vec{k} = \vec{a} - a_3\vec{k}$

$= 3\vec{a} - (a_1\vec{i} + a_2\vec{j} + a_3\vec{k})$

$= 3\vec{a} - \vec{a}$

$= 2\vec{a}$

5). Show that the points (2,3,4), (-1,4,5) and (8,1,2) are collinear, can it passing through 2 points.

$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$

$\frac{x-2}{-1-2} = \frac{y-3}{4-3} = \frac{z-4}{5-4}$

$\frac{x-2}{-3} = \frac{y-3}{1} = \frac{z-4}{1}$ sub (x,y,z) = (8,1,2)

$\Rightarrow \frac{8-2}{-3} = \frac{1-3}{1} = \frac{2-4}{1}$

$\frac{6}{-3} = -2 = -2$

$-2 = -2 = -2 \Rightarrow$ it is collinear

Part c

1). Prove by vector method that sin (α+β) = sinα cosβ + cosα sinβ

Sol.

Let $\vec{a} = \vec{OA}$

$\vec{b} = \vec{OB}$

$\vec{a} = \cos\alpha\,\vec{i} + \sin\alpha\,\vec{j}$

$\vec{b} = \cos\beta\,\vec{i} + \sin\beta\,\vec{j}$

$\vec{b} * \vec{a} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \cos\beta & -\sin\beta & 0 \\ \cos\alpha & \sin\alpha & 0 \end{vmatrix}$

$\vec{b} * \vec{a} = \vec{k}$ (sinα cos β + cos α sinβ) ---(1)

$\vec{b} * \vec{a} = |\vec{b}||\vec{a}|$ sin (α +β)$\vec{k}$ ----- (2)

sum (1) and (2)

sin (α +β) = sinα cosβ + cosα sinβ

2). Find the shortest distance between the 2 given lines

$\vec{r} = (\vec{2i} + \vec{3j} + \vec{4k}) + t(-\vec{2i} + \vec{j} - \vec{2k})$

and $\frac{x-3}{2} = \frac{y}{-1} = \frac{z+1}{2}$

$\vec{a} = \vec{2i} + \vec{3j} + \vec{4k}$, $\vec{b} = -\vec{2i} + \vec{j} - \vec{2k}$

$\vec{c} = \vec{3i} - \vec{2k}$, $\vec{d} = \vec{2i} - \vec{j} + \vec{2k}$

$\vec{b}$ is parallel to $\vec{d}$

$\vec{c} - \vec{a} = \vec{3i} - \vec{2k} - \vec{2i} - \vec{3j} - \vec{4k}$

$\vec{c} - \vec{a} = \vec{i} - \vec{3j} - \vec{6k}$

$:: d = \frac{|(\vec{c} - \vec{a}) * \vec{b}|}{|\vec{b}|}$

$(\vec{c} - \vec{a}) * \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -3 & -6 \\ -2 & 1 & -2 \end{vmatrix}$

$= \vec{12i} + \vec{14j} - \vec{5k}$

$|\vec{b}| = \sqrt{(2)^2 + (1)^2 + (-2)^2}$

$= \sqrt{4+1+4} = \sqrt{9} = 3$

$d = \frac{|\vec{12i} + \vec{14j} - \vec{5k}|}{3}$

$= \frac{\sqrt{144+196+25}}{3} = \frac{\sqrt{365}}{3}$

3). Prove by vector method that the Area of quadrilateral ABCD having diagonal AC and BD is ½ $|\vec{AC} * \vec{BD}|$

Sol.

Area of Quadrilateral ABCD

= Area of [Δ ABC + Δ ACD]

$= \frac{1}{2} (\vec{AB} * \vec{AC}) + \frac{1}{2} (\vec{AC} * \vec{AD})$

$= \frac{1}{2} (-\vec{AC} * \vec{AB}) + \frac{1}{2} (\vec{AC} * \vec{AD})$

$= \frac{1}{2} \vec{AC} * (-\vec{AB} + \vec{AD})$

$= \frac{1}{2} \vec{AC} * (\vec{BA} + \vec{AD})$

$= \frac{1}{2} (\vec{AC} * \vec{BD})$

Area of quadrilateral ABCD $= \frac{1}{2} \left|\vec{AC} * \vec{BD}\right|$

4). Find the image of the point $\vec{i} + \vec{2j} + \vec{3k}$ in the plane $\vec{r}.(\vec{i} + \vec{2j} + \vec{4k}) = 38$

Sol.

$\vec{u} = \vec{i} + \vec{2j} + \vec{3k}$, $\vec{n} = \vec{i} + \vec{2j} + \vec{4k}$, p=38

The vector of the image

$$\vec{v} = \vec{u} + 2\frac{[p - (\vec{u}.\vec{n})]}{\left|\vec{n}\right|^2}\vec{n}$$

$\vec{v}.\vec{u} = (\vec{i} + \vec{2j} + \vec{3k}) . (\vec{i} + \vec{2j} + \vec{4k})$

= 1 + 4 + 12 = 17

$\left|\vec{n}\right|^2 = (1)^2 + (2)^2 + (4)^2$

= 1 + 4 + 16 = 21

$$\vec{v} = \frac{(\vec{i} + \vec{2j} + \vec{3k}) + 2\left((38-17)(\vec{i} + \vec{2j} + \vec{4k})\right)}{21}$$

$$= (\vec{i} + \vec{2j} + \vec{3k}) + 2\,(21)\frac{(\vec{i} + \vec{2j} + \vec{4k})}{21}$$

$= \vec{i} + \vec{2j} + \vec{3k} + \vec{2i} + \vec{4j} + \vec{8k}$

$\vec{v} = \vec{3i} + \vec{6j} + \vec{11k}$

Part c 3 marks

1). Find the Area of the triangle whose vertices are A(3, -1, 2) B(1, -1, -3) and c(4, -3, 1)

Sol.

$\vec{OA} = \vec{3i} - \vec{j} + \vec{2k}$, $\vec{OB} = \vec{i} - \vec{j} - \vec{3k}$,

$\vec{OC} = \vec{4i} - \vec{3j} + \vec{k}$

$\vec{AB} = \vec{OB} - \vec{OA} = \vec{i} - \vec{j} - \vec{3k} - \vec{3i} + \vec{j} - \vec{2k} = -\vec{2i} - \vec{5K}$

$\vec{AC} = \vec{OC} - \vec{OA} = \vec{4i} - \vec{3j} + \vec{k} - \vec{3i} + \vec{j} - \vec{2k} = \vec{i} - \vec{5j} - \vec{K}$

Area of $\Delta = \frac{1}{2} \left|\vec{AB} * \vec{AC}\right|$

$$\left|\vec{AB} * \vec{AC}\right| = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & 0 & -5 \\ 1 & -2 & -1 \end{vmatrix}$$

$= \vec{i}\,(0 - 10) - \vec{j}\,(2 + 5) + \vec{k}\,(4 - 0)$

$= \vec{10i} - \vec{7j} + \vec{4k}$

$\left|\vec{AB} * \vec{AC}\right| = \sqrt{(-10)^2 + (-7)^2 + (4)^2}$

$= \sqrt{100 + 49 + 16} = \sqrt{165}$

Area of triangle = $\frac{1}{2}\sqrt{165}$ sq.units

2). Let $\vec{a}, \vec{b}, \vec{c}$ be unit vector

$\vec{a} - \vec{b} = \vec{a} - \vec{c} = 0$ and the angle between $\vec{b}$ *and* $\vec{c}$ is $\frac{\pi}{6}$. Prove that $\vec{a} = \pm 2(\vec{b} * \vec{c})$

Sol.

Given $\vec{a} . \vec{b} = \vec{a} . \vec{c} = 0 \quad => \vec{a} \perp \vec{b}$ *and* $\vec{a} \perp \vec{c}$

$=> \vec{a} \perp r\ to$

$(\vec{b}\ and\ \vec{c}) => \vec{b}\ and\ \vec{c}\ are\ parallel$

$\vec{b} * \vec{c} = |\vec{b}||\vec{c}|\ since\ \vec{n}$

$\vec{b} * \vec{c} = 1 * 1 .\sin\frac{\pi}{6} \vec{n}$

$\vec{b} * \vec{c} = \frac{1}{2} \vec{a}$

$\vec{a} = 2(\vec{b} * \vec{c})$

5 Marks

1). Show that x + 1 = 2y = -12z and

x = y+2 = 6z-6 are show lines

Sol.

$\frac{x+1}{1} = 2y = -12z => \frac{x+1}{1} = \frac{y}{\frac{1}{2}} = \frac{z}{\frac{-1}{12}}$ ---(1)

$\frac{x}{1} = \frac{y+2}{1} = \frac{6z-6}{1} = \frac{x}{1} = \frac{y+2}{1} = \frac{z-1}{\frac{1}{6}}$ ----(2)

$\vec{a} = -\vec{i}$

$\vec{b} = \vec{i} + \frac{1}{2}\vec{j} - \frac{1}{12}\vec{k})$

$\vec{c} = -2\vec{j} + \vec{k}$

$\vec{d} = \vec{i} + \vec{j} + \frac{1}{6}\vec{k}$

$\vec{c} - \vec{d} = -2\vec{j} + \vec{k} + \vec{i} = \vec{i} - 2\vec{j} + \vec{k}$

$$\vec{b} * \vec{d} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & \frac{1}{2} & -\frac{1}{12} \\ 1 & 1 & \frac{1}{6} \end{vmatrix}$$

$= \vec{i}\left(\frac{1}{12} + \frac{1}{12}\right) - \vec{j}\left(\frac{1}{6} + \frac{1}{12}\right) + \vec{k}\left(1 - \frac{1}{2}\right)$

$\vec{b} * \vec{c} = \frac{2}{12}\vec{i} - \frac{3}{12}\vec{j} + \frac{1}{2}\vec{k}$

$\vec{b} * \vec{c} = \frac{1}{6}\vec{i} - \frac{1}{4}\vec{j} + \frac{1}{2}\vec{k}$

$(\vec{c} * \vec{d}).(\vec{b} * \vec{a})$

$= (\vec{i} - 2\vec{j} + \vec{k}).(\frac{1}{6}\vec{i} - 4\vec{j} + \frac{1}{2}\vec{k})$

$= \frac{1}{6} + \frac{2}{4} + \frac{1}{2} = \frac{1}{6} + 1$

$= \frac{7}{6} \neq 0$ hence it is shown

2). $\mathbf{2\vec{i} + 3\vec{j} + \vec{k}\ ,\ \vec{i} - 2\vec{j} + 2\vec{k})\ ,\ 3\vec{i} + \vec{j} + 3\vec{k})}$ **are coplanar**

Sol.

$\vec{a} = (2\vec{i} + 3\vec{j} + \vec{k})$

$\vec{b} = (\vec{i} - 2\vec{j} + 2\vec{k})$

$\vec{c} = (3\vec{i} + \vec{j} + 3\vec{k})$

Coplanar $\vec{a}(\vec{b} * \vec{c}) = 0$

$$\vec{a}(\vec{b} * \vec{c}) = \begin{vmatrix} 2 & 3 & 1 \\ 1 & -2 & 2 \\ 3 & 1 & 3 \end{vmatrix}$$

= 2(-6-2) -3(3-6) + 1(1+6)

= 2(-8) -3(-3) + 1(7)

= -16 +9 + 7

= 0 :: It is coplanar

3). Find the two parameter sum of vector can if the plane follow through (2,3,6) and parallel to $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-3}{1}$ *and*

$\frac{x+3}{2} = \frac{y-3}{-5} = \frac{z+1}{-3}$

Non parameter can$(\vec{r} - \vec{a}) * \left(\vec{b} * \vec{c}\right) = 0$

$\vec{a} = (\vec{2i} + \vec{3j} + \vec{6k})$

$\vec{b} = (\vec{2i} + \vec{3j} + \vec{k})$

$\vec{c} = (\vec{2i} - \vec{5j} - \vec{3k})$

$(\vec{b} * \vec{c}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 3 & 1 \\ 2 & -5 & -3 \end{vmatrix}$

$= \vec{i}(-9+5) - \vec{j}(-6-2) + \vec{k}(-10-6)$

$= -\vec{4i} + \vec{8j} - \vec{16k})$

$(\vec{r} - \vec{a}) * \left(\vec{b} * \vec{c}\right) = (\vec{r} - (\vec{2i} + \vec{3j} + \vec{6k})) \cdot (-\vec{4i} + \vec{8j} - \vec{16k})$ 50

$\vec{r} \cdot (-\vec{4i} + \vec{8j} - \vec{16k})$ - $(-8 + 24 - 96)$ 50

-4x +8y – 16z = -80

÷ by – 4 => x +2y + 4z = 20

X – 2y + 4z -20 = 0

4). Find the parameter forum of vector equation and Cartesian equation the plane following through the points (2, 2, 1), (9,3,6) and perpendicular to the plane 2x + 6y + 6z = 9

Sol.

$\vec{a} = (\vec{2i} + \vec{2j} + \vec{k})$

$\vec{b} = (\vec{9i} + \vec{3j} + \vec{6k})$

$\vec{c} = (\vec{2i} + \vec{6j} + \vec{6k})$

Vector equation :

$\vec{r} = (1-s)\vec{a} + s\vec{b} + t\vec{c}$

$\vec{r} = (1-s)(\vec{2i} + \vec{2j} + \vec{k}) + s(\vec{9i} + \vec{3j} + \vec{6k})$ + t $(\vec{2i} + \vec{6j} + \vec{6k})$

Cartesian equation = $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0$

$\begin{vmatrix} x-2 & y-2 & z-1 \\ 9-2 & 3-2 & 6-1 \\ 2 & 6 & 6 \end{vmatrix} = 0$

$\begin{vmatrix} x-2 & y-2 & z-1 \\ 7 & 1 & 5 \\ 2 & 6 & 6 \end{vmatrix} = 0$

(x-2) (6-30) – (y-2) (42-10) + (z-1) (42-2) = 0

(x-2) (-24) – (y-2) (32) + (z-1) (40) = 0

-24x + 48 – 32y + 64 + 40z -40 = 0

÷ by -8 => 3x + 4y -5z -9 = 0

5). Prove by vector method that the perpendicular form of vector to the opposite scale of a triangle area concurrent

Sol.

Δ ABC

$\vec{OA} = \vec{a}$

$\vec{OB} = \vec{b}$

$\vec{OC} = \vec{c}$

$\vec{AD}$ ⊥r to $\vec{BC}$ => $\vec{OA}$ ⊥r $\vec{BC}$

=> $\vec{a} * \left(\vec{c} - \vec{b}\right) = 0$

$\left(\vec{a}\ \vec{c}\right) . \left(\vec{a} \times \vec{b}\right) = 0$ --- (1)

$\vec{BE}$ ⊥r to $\vec{CA}$ => $\vec{OB}$ ⊥r $\vec{CA}$

=> $\vec{b} * \left(\vec{a} - \vec{c}\right) = 0$

$\left(\vec{a} * \vec{b}\right) - \left(\vec{b} * \vec{c}\right) = 0$ -- (2)

(1) + (2) => $\left(\vec{a} * \vec{c}\right) - \left(\vec{b} * \vec{c}\right) = 0$

$\left(\vec{c}\right) . \left(\vec{a} - \vec{b}\right) = 0$

$\vec{OC}$ ⊥r to $\vec{BA}$

$\vec{CF}$ ⊥r to $\vec{BA}$

:: All the altitude are concurrent

7. APPLICATIONS OF DIFFERENTIALCALCULUS

TWO MARKS

1). A particle moves so that the distance moved is according to the law

$s(t) = \frac{t3}{3} - t^2 + 3$. At what time the velocity and acceleration are zero respectively ?

sol.

$S(t) = \frac{3t3}{3} - t^2 + 3$

V: $S'(t) = \frac{3t2}{3} - 2t$

A: $S''(t) = 2t - 2$

$S'(t) = 0$ =) $t^2 - 2t = 0$

$t(t-2) = 0$

$t=0, t = 2$

: Velocity is zero at t = 0, 2

$S''(t) = 0$ =) $2t - 2 = 0$

$2t = 2, t = 1$

Acceleration is zero at t = 1

2) Find the tangent and normal to the curve

$Y = x^4 + 2e^x$ at (0,2)

$\frac{dy}{dx} = 4x^3 + 2e^2$

$M = \left(\frac{dy}{dx}\right)$ (0,2), $=4(0) + 2e^2$

$M = 0 + 2(1)$

$M = 2$

Equation of tangent at (0,2)

$y-y_1 = m(x-x_1)$

$y-2 = 2(x-0)$

$y-2 = 2x$

=) $2x - y + 2 = 0$

Equation of normal at (0,2)

$y-y_1 = -\frac{1}{m}(x-x_1)$

$y-2 = -\frac{1}{2}(x-0)$

$2(y-2) = -1(x)$

$2y - 4 = -x$

$X+2y-4 = 0$

3). Show that the value in the conclusion of the mean value theorem for $f(x) = \frac{1}{x}$ on a closed interval of positive numbers a,b is $\sqrt{ab}$

F(x) is continuous in (a,b) and differentiable in (a,b) by mean value theorem, there exist c (a,b) such that

$F'(c) = \frac{f(b)-f(a)}{b-a}$ - (1)

$F(x) = \frac{1}{x}$

=) $f'(x) = \frac{-1}{x2}$ =) $f'(c) = \frac{-1}{c2}$

(1)=) $-\frac{1}{c2} = \frac{\left(\frac{1}{b}\right)-\left(\frac{1}{a}\right)}{b-a}$

$-\frac{1}{c2} = \frac{1}{b-a}(1/b)-(1/a)$

$= \frac{1}{b-a}(a-b/ba)$

$-\frac{1}{c2} = ab$, $c=\sqrt{ab}$

4). Find the absolute extrema of the function $f(x) = 3x^4 - 4x^3$ on (-1,2)

$F'(x) = 12x^3 - 12x^2$

$= 12x^2 (x-1)$

$F'(x) = 0 \Rightarrow 12x^3 (x-1) = 0$

X=0, x=1

Critical numbers: x = 0,1

$F(x) = 3x^4 - 4x^3$

$F(0) = 3(0) - 4(0) = 0$

$F(1) = 3(1) - 4(1) = 3-4 = -1$

$F(-1) = 3(-1)^4 - 4(-1)^3 = 3(1)-4(-1)$

$= 3+4 = 7$

$F(2) = 3(2)^4 - 4(2)^3$

$=3(16)-4(8)$

$=48 - 32 \quad =16$

Absolute maximum is 16

Absolute minimum is -1

5). Find the asymptote of the curve

$f(x) = \frac{x2}{x2-1}$

$x^2-1 = 0 \Rightarrow x^2-1$

$x = \pm 1$

$\lim_{x\to1} f(x) = \lim_{x\to1} \frac{x2}{x2-1}$

$= \frac{1}{1-1} = \frac{1}{0} = \infty$

$\lim_{x\to-1} f(x) = \lim_{x\to-1} \frac{x2}{x2-1}$

$= \frac{(-1)^2}{(-1)^2-1} = \frac{1}{1-1} = \frac{1}{0} = \infty$

X = 1 and x =-1 are vertical asymptotes

$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{x^2}{x^2\left(1-\frac{1}{x^2}\right)}$

$= \lim_{x\to\infty} \frac{1}{1-x^2}$

$= \frac{1}{1-0}, = 1$

≈ y =1 is the horizontal asymptote

3 marks

1). Show that the two curves

$X^2-y^2 = r^2$ and $xy = c^2$

Where c,r are constant orthogonally

$X^2 - y^2 = r^2$

Diff w.r.to x

$2x - 2y\frac{dy}{dx} = 0$

$-2y\frac{dy}{dx} = -2x$

$\frac{dy}{dx} = \frac{-2x}{-2y}$

$m_1 = \left(\frac{dy}{dx}\right)(x_1,y_1) = \frac{x_1}{y_1}$

Let(x1,y1)be the point of intersection

$Xy = c^2$

$Y = \frac{c^2}{x}$

$\frac{dy}{dx} = \frac{-c^2}{x^2}$

$m_2 = \left(\frac{dy}{dx}\right)(x_1,y_1) = \frac{-c^2}{x_1^2}$

$m_1 x m_2 = \left(\frac{x_1}{y_1}\right)\left(\frac{-c^2}{x_1^2}\right)$

$= \frac{-c^2}{x_1 y_1}$

$= \frac{-c^2}{c^2} \Rightarrow x_1 y_1 = c^2$

$m_1 m_2 = -1$

The given curves cut orthogonally

2). Expand log (1+x) as a maclaurins series upto 4 non zero terms for -1<x≤1

	Log(1+x) And its derivatives	Values at X=0
F(x)	Log(1+x)	Log 1 = 0
F'(x)	$\frac{1}{1+x}$	$\frac{1}{1+0}$=1
F''(x)	$\frac{-1}{(1+x)2}$	$\frac{-1}{(1+0)^2}$ =-1
F'''(x)	$\frac{(-1)(-2)}{(1+x)3}=\frac{2}{(1+x)^3}$	$\frac{2}{(1+0)^3}$=2
F^{1v}(x)	$\frac{2(-3)}{(1+x)^4}=\frac{-6}{(1+4)^4}$	$\frac{-6}{(1+0)^4}$=-6

Maclaurins series

f(x) = f(0) + $\frac{x}{1!}$f'(0)+$\frac{x^2}{2!}$f''(0)+…

log (1+x) = 0 + $\frac{x}{1!}$(1)+$\frac{x^2}{2!}$(-1)+$\frac{x^3}{3!}$(2)+$\frac{x^4}{4!}$(-6)+..

=$\frac{x}{1}$ - $\frac{x^2}{1*2}$ +$\frac{2x^3}{1*2*3}$ - $\frac{6x^4}{1*2*3*4}$ +…..

Log (1+x) = x - $\frac{x^2}{2}$ + $\frac{x^3}{3}$ - $\frac{x^4}{4}$ +….

3) Evaluation :$\lim_{x\to\frac{\pi}{2}}(sinx)^{tanx}$

Let g(x) = $(sinx)^{tanx}$

Log g(x) = log $(sinx)^{tanx}$

= tan x. log (sin x)

Log g(x) = $\frac{\log(sinx)}{\cot x}$

$\lim_{x\to\frac{\pi}{2}}\log g(x)$=$\lim_{x\to\frac{\pi}{2}}\frac{\log(sinx)}{\cot x}$ $\left(\frac{0}{0}\right)$form

Applying L Hospital rule

=$\lim_{x\to\frac{\pi}{2}}\frac{\frac{1}{sinx}.cosx}{-cosec_x^2}$

=$\lim_{x\to\frac{\pi}{2}}(-cosx\ sinx)$

= - cos $\frac{\pi}{2}$ sin $\frac{\pi}{2}$

$\lim_{x\to\frac{\pi}{2}}\log g(x)$ = -(0)(1)

Log $\left[\lim_{x\to\frac{\pi}{2}} g(x)\right]$ = 0

$\lim_{x\to\frac{\pi}{2}} g(x)$ = e0 = 1

4). Find two positive numbers whose sum is 12 and their product is maximum

Let the numbers be x,y

Sum = 12

X+y =12

Y = 12-x

Product A =XY

A = x(12-x)

A(x) = 12x – x^2

A'(x) = 12 -2x

A''(x) = -2

For maximum, A'(x) =0

⇨ 12 – 2x = 0
-2x = 12
x= 6
When x=6, A''(X) = -2 < 0
A (x) is maximum at n=6
X=6 => y = 12 – 6, y = 6
Required numbers = 6, 6
Maximum value = xy
= (6)(6)
= 36

5) Show that there lies point on the curves

F(x) = x(x+3) $e^{-\frac{\pi}{2}}$, -3≤ $x \le 0$

i.e $f(x) = (x^2+3x)\, e^{-\frac{\pi}{2}}$

$f'(x) = (2x+3)\, e^{-\frac{\pi}{2}}$

f(x) is continuous in (-3,0)

and differentiable in (-3,0)

and f(-3) = f (0) =0 By Rolle's theorem there exist c € (-3,0) such that f'(c) = 0

:: $(2c+3)\, e^{-\frac{\pi}{2}} = 0$

$2c + 3 = 0, \; e^{-\frac{\pi}{2}} \neq 0$

2c = -3

$C = -\frac{3}{2} = -1.5$ €(-3,0)

:: At x = -1.5 the tangent is parallel to the X –axis

5 MARKS

1. If we blow air into a ballon of spherical shape at a rate of 1000 cm^3per second,at what rate the radius of the balloon changes when the radious is 7 cm ? also compute the rate at which the surface area changes.

Let r be the radious of spherical ballon

Volume : $v = \frac{4}{3}\pi r^3$

$\frac{dv}{dt} = \frac{4}{3}\pi\,(3r^2\frac{dr}{dt})$

$\frac{dv}{dt} = 4\pi r^2\frac{dr}{dt}$ -(1)

Given : r = 7cm, $\frac{dv}{dt} = 1000$cm

(1) => $1000 = 4\pi\,(7)^2\,\frac{dr}{dt}$

$\frac{dr}{dt} = \frac{1000}{4\pi(7)2} = \frac{250}{49\pi}$

:: Rate of change of radius is $\frac{250}{49\pi}$cm/sec

Surface area :: $S = 4\pi r^2$

$\frac{ds}{dt} = 4\pi\,(2r\,\frac{dr}{dt})$

$= 8\pi\,(7)\left(\frac{250}{49\pi}\right)$

$\frac{ds}{dt} = \frac{2000}{7}$

Rate of change of surface area is $\frac{2000}{7}$cm²/sec

2)Find the angle between $y=x^2$ and

$Y= (x-3)^2$

Sol

$Y = x^2$ -(1)

$Y = (x-3)^2$ –(2)

From (1) And (2)

$X^2 = (x-3)^2$

$X^2 = x^2 - 6x +9$

6x = 9

$X = \frac{3}{2}$

$Y= x^2 \Rightarrow y = (\frac{3}{2})^2 = 9/4$

Intersection point is (3/2), (9/4)

$Y = x^2$

$\frac{dy}{dx} = 2x$

$m_1 = \left(\frac{dy}{dx}\right), \left(\frac{3}{2},\frac{9}{4}\right) = 2\left(\frac{3}{2}\right)$

$m_1=3$

$Y = (x-3)^2$

$\frac{dy}{dx}=2(x-3)$

$m_2=\left(\frac{dy}{dx}\right),\left(\frac{3}{2}\frac{9}{4}\right) = 2\left(\frac{3-3}{2}\right)$

$= 2\left(\frac{3-6}{2}\right)$

$m_2 = -3$

Let
θ *be the angle between the curves*

$\tan\theta = \left|\frac{m1-m2}{1+m1m2}\right|$

$= \left|\frac{3-(-3)}{1+(3)(3)}\right|$

$= \left|\frac{3+3}{1-9}\right|$

$= \left|\frac{6}{-8}\right|$

$\tan\Theta = \frac{3}{4}$

$\Theta = \tan^{-1}\left(\frac{3}{4}\right)$

3)For the function f(x) =$4x^3 + 3x^2 - 6x + 1$ find the intervals of monotonicity and local extreme

$F(x) = 4x^3 + 3x^2 - 6x + 1$

$F'(x) = 12x^2 + 6x - 6$

$F'(x) = 6(2x^2 + x - 1)$

$F'(x) = 6(x+1)(2x-1)$

$F'(x) = 0$

⇨ $6(x+1)(2x-1) = 0$
$X = -1$,or $2x - 1 = 0$
$2x = 1$
$X = ½$

Critical numbers are x = -1, ½

Intervals : (-∞, -1), (-1,1/2),(-1/2,∞)

Intervals	(-∞,-1) X = -2	(-1,1/2) X = 0	(1/2, ∞)
Sign of f'(x)= 6(x+1)(2x-1)	(-)(-)=+	(-)(+)= -	(+)(+) = +
monotonicity	Strictly increasing	Strictly decreasing	Strictly increasing

F(x) strictly increasing on (-∞,-1), (1/2,∞), strictly decreasing on (-1,1/2)

F'(x) changes from positive to negative when possing through x = -1

F (x) has local maximum at x = -1

Maximum value = f (-1)

$= 4(-3)^3 + 3(-1)^2 - 6(-1)$
+1

$= -4 + 3 + 6 + 1$

$= 6$

F'(x) changes from negative to positive thoughts x = ½

:: f (x) has local minimum at x = ½

Local minimum value

= f(1/2)

$= 4\left(\frac{1}{2}\right)^3 + 3\left(\frac{1}{2}\right)^2 - 6\left(\frac{1}{2}\right) + 1$

$= 4\left(\frac{1}{8}\right) + 3\left(\frac{1}{4}\right) - 3 + 1$

$= \frac{1}{2} + \frac{3}{4} - 2$

$= \frac{2+3-8}{4}, = -\frac{3}{4}$

5) A rectangular page is to contain 24cm² of print, The margins at the top and bottom of the page are 1.5 cm and the margins at other sides of the page is 1cm. What should be the dimensions of the page so that the area of the paper used is minimum

Sol

Let x and y be the length and breadth of the printed page

Length of pole = x +2

Breadth of paper = y + 3

Area : A = (x+ 2) (y + 3)

A = XY + 3X + 2Y + 6

By data, xy = 24

Y = 24/x

:: A = 24+ 3x + $2\left(\frac{24}{x}\right)$ + 6

A (X) = 3X + $\frac{48}{X}$ + 30

A' (X) = 3 - $\frac{48}{X^2}$

A'' (X) = $\frac{(-48)(-2)}{X^3} = \frac{96}{X^3}$

A'(X) =0=>3-$\frac{48}{X^2}$ = 0

3 = $\frac{48}{X^2}$

X^2 = 16 => X = 4

When x = 4, A'' (x) = $\frac{96}{4^3}$>0

:: A (X) is minimum at x = 4

X = 4 => y =$\frac{24}{x}$ = $\frac{24}{4}$ =6

For minimum area

Length = x + 2 = 4+2 = 6cm

Breadth = y + 3 = 6 + 3 = 9cm

6)Sketch the curve

y = f(x) = $x^2 - x - 6$

f(x) = $x^2 - x - 6$

=>y =(x – 3) (x + 2)

1. Domain : (-∞,∞)

2.Intercepts :

Y = 0 => x =3, x = -2

X = 0 = > y = -6

X intercepts : (3,0) , (-2,0)

Y intercepts : = -6

3. f' (x) = 2x -1

F' (x) =0 => 2x -1 = 0

2x = 1

X = ½

:: Critical point occurs at x = ½

4. f''(x) = 2> 0 ≠ x

:: At x = ½, f(x) has a local minimum

Minumum value = f($\frac{1}{2}$)

$= \frac{(1)^2}{(2)} - \left(\frac{1}{2}\right) - 6$

$= \frac{1}{4} - \frac{1}{2} - 6$

$= \frac{1-2-24}{4} = \frac{-25}{4}$

5. Range of f(x) is

Y ≥ -$\frac{25}{4}$

6. f'' (x) = 2, ≠ x

:: f(x) is concave upward in the entire real line

7. f(x) has no points of inflection

8. The curve has no asymptotes

8. DIFFERENTIALS AND PARTIAL DERIVATIVES

2 MARKS

1) **Find df for $f(x) = x^2 +3x$ and evaluate it for x=3 and dx = 0.02.**

$f(x) =x^2 +3x \to f'(x) = 2x +3$

$\to df = (2*3+3)\ 0.02$

$= 9(0.02)$ $\quad [\because f'(x) = \frac{df}{dx}]$

$df =0.18$

2)Find a linear approximation for the following function at the indicated points

$f(x) = x^3-5x+12$, $x_0 = 2$.

Soln;

$f(x_0) = 2^3-5(2)+12$

$= 8 -10 +12$

$f(x_0) = 10$

$f'(x) = 3x^2-5$

$f'(0) = f'(2) = 3(2)^2-5$

$= 3(4) -5 = 7$

$L(x) = f(x_0) +f'(x_0) (x-x_0)$

$L(x) = 10 +7(x-2) \Rightarrow 10+7x-14 \Rightarrow$ **7x-4**

3)Let $v(x,y,z) = xy +yz+zx$, $x,y,z \in R$. Find the differential dv.

Soln:

$v(x,y,z) = xy +yz + zx$

$\frac{\partial v}{\partial x} = y+0+z = y+z$

$\frac{\partial v}{\partial y} = x+z+0 = x+z$

$\frac{\partial v}{\partial z} = 0+y+x = y+x$

$dv = \frac{\partial v}{\partial x} dx+ \frac{\partial v}{\partial y} dy + \frac{\partial v}{\partial z} dz$

$dv= (y+z)\ dx + (x+z)\ dy + (x+y)\ dz$

4) An egg of a particular bird is very nearly spherical .If the radius to the inside of the shell is 5mm and radius to the ouside of the shell is 5.3mm , find the volume of the shell approximately.

Soln:

Volume of the sphere = $\frac{4}{3}\pi r^3$

Given r =5mm ,Δr = dr =5.3-5 = 0.3mm

$\frac{dv}{dr} = \frac{4}{3}\pi\, 3 r^2$

$dv= 4\pi r^2 dr \Rightarrow 4\pi(5*5)\ (0.3)$

$= 30\pi mm^3$

5) Evaluate $\lim_{(x,y)\to(1,2)} g(x,y)$, if the limit exist , where $g(x,y) = \frac{3x^2-xy}{x^2+y^2+3}$.

Given : $g(x,y) = \frac{3x^2-xy}{x^2+y^2+3}$

$\lim_{(x,y)\to(1,2)} g(x,y)=$

$\lim_{(x,y)\to(1,2)} \frac{3x^2-xy}{x^2+y^2+3}$

$= \frac{3(1)^2-1(2)}{x^2+y^2+3} = \frac{3-2}{8} = \frac{1}{8}$

3 MARKS

1 . Find Δf and df for the function f for the indicated values of x ,Δx and compare

$f(x) = x^3 -2x^2$; x=2 , Δx =dx = 0.5

$df = f'(x)\ \Delta x$

$= (3x^3 - 4x)\,\Delta x$

$= [3(2)^2 - 4(2)]\,0.5$

$= [3(4) - 8]\,0.5$

$= (12 - 8)\,0.5$

$= 4(0.5)$

$= 2.0$

$\Delta f = f(x + \Delta x) - f(x)$

$= f(2+0.5) - f(2)$

$= f(2.5) - f(2)$

$= [(2.5)^3 - 2(2.5)^2] - [2^3 - 2(2)^2]$

$= [15.625 - 12.5] - [8 - 8]$

= 3.125

2 . show that $f(x,y) = \frac{x^2+y^2}{y^2+1}$ is continuous at every $(x,y) \in R^2$.

Let $(a,b) \in R^2$ be an arbitrary point

i) $f(a,b) = \frac{a^2+b^2}{b^2+1}$ is defined for $\forall (a,b) \in R^2$

ii) $\lim_{(x,y)\to(a,b)} f(x,y) = \lim_{(x,y)\to(a,b)} \frac{x^2+y^2}{y^2+1}$

$= \frac{a^2+b^2}{b^2+1} = L$

Limit exist at $(a,b) \in R^2$

iii) $\lim_{(x,y)\to(a,b)} f(x,y) = L = f(a,b)$

$= \frac{a^2+b^2}{b^2+1}$

:. f is continuous at every point on R^2.

3) If $U(x,y,z) = \log(x^3+y^3+z^3)$

$\frac{\partial U}{\partial x} = \frac{3x^3}{x3+y3+z3}$, $\frac{\partial U}{\partial y} = \frac{3y^3}{x3+y3+z3}$,

$\frac{\partial U}{\partial z} = \frac{3z^3}{x3+y3+z3}$

$\frac{\partial U}{\partial x}+\frac{\partial U}{\partial y}+\frac{\partial U}{\partial z} = = \frac{3x^3}{x^3+y^3+z^3} + \frac{3y^3}{x^3+y^3+z^3} + \frac{3z^3}{x^3+y^3+z^3}$

$=> \frac{3(x2+y+z3)}{x^3+y^3+z^3}$

4) If $u(x,y) = \frac{x^2+y^2}{\sqrt{x+y}}$,prove that $x\frac{\partial U}{\partial x}+y\frac{\partial U}{\partial y} = \frac{3}{2}u$

Soln:

$u(x,y) = \frac{x^2+y^2}{\sqrt{x+y}}$

$u(\lambda x, \lambda y) = \frac{(\lambda x)^2+(\lambda y)^2}{\sqrt{\lambda x+\lambda y}}$

$\lambda^{2^{\frac{-1}{2}}}\,u(x,y) = \lambda^{\frac{3}{2}}\,u(x,y)$

Thus U is homogenous with degree $\frac{3}{2}$, and so by Euler's Theorem .

$x\frac{\partial U}{\partial x} + y\frac{\partial U}{\partial y} = n\,u$

$\mathbf{x\frac{\partial U}{\partial x} + y\frac{\partial U}{\partial y} = \frac{3}{2}\,u}$ **.**

5 . If $v(x,y,z) = x^3+y^3+z^3+3xyz$,shows that $\frac{\partial^2 v}{\partial y\partial z} = \frac{\partial^2 v}{\partial y\partial z}$.

Given : $v(x,y,z) = x^3+y^3+z^3+3xyz$

$\frac{\partial v}{\partial z} = 0 + 0 + 3z^2 + 3xy = 3z^2 + 3xz$

$\frac{\partial v}{\partial y} = 0 + 3y^2 + 0 + 3xz = 3y^2 + 3xz$

$\frac{\partial^2 v}{\partial y\partial z} = \frac{\partial}{\partial z}\left(\frac{\partial v}{\partial z}\right) = 0 + 3x = 3x \to ①$

$\frac{\partial^2 v}{\partial y \partial z} = \frac{\partial}{\partial z}\left(\frac{\partial v}{\partial y}\right) = 0 + 3x = 3x \rightarrow$ ②

From ① & ②

$$\frac{\partial^2 v}{\partial y \partial z} = \frac{\partial^2 v}{\partial y \partial z}.$$

5 MARKS

1) The radius of a circular plate is measured as 12.65 cm instead of actual length 12.5cm .find the following in calculating the area of the circular plate.

r= 12.65 , Δ r = $\pm$ 0.15

Area of circle A = π r^2

$\frac{dA}{dr} = 2\pi r$

= 2 π * **12.65 * (+0.15)**

Approximate error = 3.795π cm^2

Actual error = A (12.5) – A (12.65)

= $\pi(12.5)^2 - \pi(12.65)^2$

= π(156.25 – 160.0225)

= 3.7725 π cm^2

i) Absolute Error = Actual error – Appropriate error.

= 3.7725 π - (3.795 π)

= 0.0225 π cm^2

ii) Relative error = $\frac{Absolute\ error}{Actual\ error}$

$= \frac{0.0225\,\pi}{3.7725\,\pi}$

= 0.00596

= 0.006

iv) Percentage error = Relative error * 100

= **0.6 %**

2) w(x,y,z) = xy +yz + zx ; x=u-v ; y =uv; z = u+v .

Given :

w(x,y,z) = xy +yz + zx ; x=u-v ;

y =uv; z = u+v .

$\frac{\partial w}{\partial x} = y + z$; $\frac{\partial w}{\partial y} = x + z$; $\frac{\partial w}{\partial z} + y+x.$

x= u-v y = uv z = u+v .

$\frac{\partial x}{\partial u} = 1$, $\frac{\partial x}{\partial v} = -1$; $\frac{\partial y}{\partial u} = v$, $\frac{\partial y}{\partial v} = u$;

$\frac{\partial z}{\partial u} = 1$, $\frac{\partial z}{\partial v} = 1.$

(uv + u + v) (1) + 2u (v) + (uv +u – v)(1).

= uv + u+ v+ 2uv + uv+ u-v.

$\frac{\partial w}{\partial u}$ = 4uv +2u = 2u (2v+1)

$\left(\frac{\partial w}{\partial u}\right)_{\left(\frac{1}{2}\,-1\right)} = 2 * \frac{1}{2}$ (2+1) = 1 (2+1) =3

$\frac{\partial w}{\partial v} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial v} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial v}$

= (uv+ v+u) (-1) + (2u) (u) + (uv+u-v)(1).

= -u –u –v +2u^2 +uv +u –v

= **2u^2 – 2v = 2(u^2 –v)**

$\therefore \left(\frac{\partial w}{\partial v}\right)_{\left(\frac{1}{2}\ -1\right)} = 2\left(\frac{1}{4} - 1\right) = 2\left(\frac{-3}{4}\right)$

$= \left(\frac{-3}{2}\right).$

3)If u = sin $^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$,shows that x $\frac{\partial u}{\partial x}$ + y $\frac{\partial u}{\partial y} = \frac{1}{2}$ tan u .

Soln:

f(x,y) = $\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ = sin u is homogeneous.

f(tx ,ty) = $\left(\frac{tx+ty}{\sqrt{tx}+\sqrt{ty}}\right) = \left(\frac{t}{\sqrt{t}}\right)$

$= t^{\frac{1}{2}}$ f(x,y) ,∀ x,y,t ≥ 0.

Thus f is homogeneous with degree $\frac{1}{2}$,by Euler's theorem.

$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = \frac{1}{2}\ f(x,y)$

put , f = sin u

$x\frac{\partial \sin u}{\partial x} + y\frac{\partial \sin u}{\partial y} = \frac{1}{2}\ \sin u.$

$x \cos u\frac{\partial u}{\partial x} + y\cos u\frac{\partial u}{\partial y} = \frac{1}{2}\ \sin u.$

Dividing bothsides by cos u

$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}$ **tan u.**

4) Let z (x,y) = $x^2y + 3xy^4$, x, y ∈ R . Find the linear approximation for z at (2, -1)

Soln:

Let z (x,y) = $x^2y + 3xy^4$, x, y ∈ R.

z(2, -1) = $2^2(-1) + 3(2)(-1)^4$

= - 4 + 6 = 2

$\frac{\partial z}{\partial x} = 2xy + 3xy^4$,

$\frac{\partial z}{\partial x}_{(2,-1)} = 2(2)(-1) + 3(-1)^4$

= -4 + 3 = **-1.**

$\frac{\partial z}{\partial y} = x^2 + 12xy^3$

$\frac{\partial z}{\partial y}_{(2,-1)} = 2^2 + 12(2)(-1)^3$

= 4- 24 = **-20**

Linear appropriation

L (x, y) = z (2, -1) + $\frac{\partial z}{\partial x}_{(2,-1)}$ (x -2) + $\frac{\partial z}{\partial y}_{(2,-1)}$ (y + 1).

= 2 + (-1)(x- 2) + (-20) (y + 1)

= 2 – x + 2 – 20 y -20

= -x - 20y + 16

L (x, y) = -(x + 20y – 16).

5) Show that the percentage error in the n^th root of a number is approximately $\frac{1}{2}$ times the percentage error in the number.

Soln:

Let the number be x its n^th root $x^{\frac{1}{n}} = y$

$y = x^{\frac{1}{n}}$

taking log ,

log y = log $x^{\frac{1}{n}}$

log y = $\frac{1}{n}$ log x.

Differntiate with respect to x

$$\frac{1}{y}\,\frac{dy}{dx} = \frac{1}{n}\,\frac{1}{x}$$

$$\frac{dy}{y} = \frac{1}{n}\left(\frac{dx}{x}\right)$$

$$\frac{dy}{y} * 100 = \frac{1}{n}\left(\frac{dx}{x} * 100\right)$$

$$\frac{\Delta y}{y} * 100 \approx \frac{dy}{y} * 100 = \frac{1}{n}\left(\frac{dx}{x} * 100\right)$$

% error of y $\approx \frac{1}{n}$ **(% error on x).**

9. APPLICATION ON INTEGRATION

TWO MARKS

1). Evaluate $\int_0^1 x\,dx$ as the limit of sum

Sol.

F(x) = x a = 0 and b = 1

$\int_a^b f(x)dx = \lim_{n\to\infty} \frac{1}{n}\sum_{r=1}^{n} f(\frac{r}{n})$

$\int_0^1 x\,dx = \lim_{n\to\infty} \frac{1}{n}\sum_{r=1}^{n}(\frac{r}{n})$

$= \lim_{n\to\infty} \frac{1}{n^2}[1+2+3+\ldots n]$

$= \lim_{n\to\infty} \frac{1}{n^2}\left(\frac{n(n+1)}{2}\right)$

$= \lim_{n\to\infty} \frac{1}{n^2} n^2\left(\frac{1+\frac{1}{n}}{2}\right)$

$= \lim_{n\to\infty} \frac{1}{2}(n+\frac{1}{n})$

$= \frac{1}{2}(1+0) = \frac{1}{2}$

2). Evaluate $\int_{-1}^{1} \frac{dx}{x^2+2x+5}$

Sol.

$X^2 + 2x + 5 = x^2 + 2x + 1 + 4 = (x+1)^2 + 2^2$

$I = \int_{-1}^{1} \frac{dx}{2^2+(x+1)^2}$

$I = \left(\frac{1}{2}tan^{-1}(\frac{x+1}{2})\right)_{-1}^{1}$

$I = \frac{1}{2}\{\tan^{-1}\frac{1+1}{2} - \tan^{-1}\frac{-1+1}{2}\}$

$= \frac{1}{2}\tan^{-1}(\frac{2}{2})$

$= \frac{1}{2}\tan^{-1}(1)$

$= \frac{1}{2}\frac{\pi}{4} = \frac{\pi}{8}$

3). Evaluate $\int_0^{\pi} x^2 \cos nx dx$ where n is a positive integer

Sol.

Take $u = x^2$ $dv = \cos nx$ :: integration

$u' = 2x$ $v = \frac{\sin nx}{n}$ $\int dr = \int cosnxdx$

$u'' = 2$ $v_1 = \frac{-cosnx}{n^2}$

$v_2 = \frac{-sinnx}{n^3}$

$I = \int_0^{\pi} cosnxdx$

$= \{x^2(\frac{sinnx}{n}) - 2x(\frac{-cosnx}{n^2}) + 2(\frac{-sinnx}{n^3})\}_0^{\pi}$

$I = \frac{2\pi(-1)^n}{n^2}$ $\sin n\pi = 0, \cos n\pi = (-1)^n$

4). Evaluate $\int_0^{\frac{\pi}{2}} sin^{10}xdx$

Sol.

$I = \int_0^{\frac{\pi}{2}} sin^{10}xdx$

$= \left(\frac{10-1}{10}\right)\left(\frac{10-3}{10-2}\right)\left(\frac{10-5}{10-4}\right)\left(\frac{10-7}{10-6}\right)\left(\frac{10-9}{10-8}\right)\frac{\pi}{2}$

$= \frac{9}{10}.\frac{7}{8}.\frac{5}{6}.\frac{3}{4}.\frac{1}{2}.\frac{\pi}{2}$

$= \frac{63\pi}{512}$

5). Evaluate $\int_0^{\infty} x^5 e^{-3x}\,dx$

Sol.

$\int_0^{\infty} x^n e^{-ax}dx = \frac{n!}{a^{n+1}}$

n=5, a=3

$\int_0^{\infty} x^5 e^{-3x}dx = \frac{5!}{3^{5+1}} = \frac{5!}{3^6}$

6). Find the area of two redion bounded by the line 6x+5y = 30, x-axis and the line x=-1 and x=3

Sol.

Area bounded by two line 6x + 5y = 30,

x-axis

A = $\int_a^b y dx$

$= \int_{-1}^{3} \frac{30-6x}{5}$ dx

$= \frac{1}{5}(30x - 3x^2)_{-1}^{3}$

$= \frac{1}{5}(90 - 27) - \frac{1}{5}(-30 - 3)$

$= \frac{96}{5}$ sq.units

7). Find the volume of the solid generated by revolving about two x – axis, the region enclosed by y = 2x², y = 0 and

x = 1

Sol.

$Y = 2x^2$

Y=0, => x= 0, x = 1

Volume $v = \pi\int_a^b y^2 \, dx$

$= \pi \int_0^1 4x^4$ dx

$= 4\pi \left(\frac{x^5}{5}\right)_0^1$

$= 4\pi \left(\frac{1}{5}\right)$

$= \frac{4\pi}{5}$

3 MARKS

1). Evaluate $\int_0^1 x^3 dx$ as two limit of sum

Sol.

$F(x) = x^3$ a = 0 and b = 1

$\int_a^b f(x)dx = \lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{n} f\left(\frac{r}{n}\right)$

$\int_0^1 x^3 dx = \lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{n} f\left(\frac{r^3}{n^3}\right)$

$= \lim_{n\to\infty} \frac{1}{n^4} \{1^3 + 2^3 + \ldots + n^3\}$

$= \lim_{n\to\infty} \frac{1}{n^4} \frac{n^2(n+1)^2}{4}$

$= \lim_{n\to\infty} \frac{n^4}{n^4} \left(\frac{1+\frac{1}{n}}{4}\right)^2$

$= \frac{(1+0)^2}{4} = \frac{1}{4}$

2). Find the approximate value of $\int_1^{1.5} x\,dx$ by applying two left end rule with the partition{1.1, 1.2, 1.3, 1.4, 1.5}

Sol.

$\Delta x = 1.1 - 1 = 0.1$

n = 5

$x_0 = 1, x_1 = 1.1, x_2 = 1.2, x_3 = 1.3,$

$x_4 = 1.4, x_5 = 1.5$

$\int_a^b f(x)dx = \{f(x) + f(x_1) + f(x_2) + f(x_3) + f(x_4) + \}\Delta_x$

$\int_1^{1.5} x\,dx = \{f(1) + f(1.1) + f(1.2) + f(1.3) + f(1.4) + \}\, 0.1$

$= \{1 + 1.1 + 1.2 + 1.3 + 1.4\}0.1$

$= (6)(0.1)$

$\int_{1}^{1.5} x\,dx$ = 0.6

3). Evaluate $\int_0^1 \sqrt{\frac{1-x}{1+x}}$ dx

Sol.

$\int_0^1 \frac{\sqrt{1-x}}{\sqrt{1-x}}$ dx = $\int_0^1 \frac{\sqrt{1-x}}{\sqrt{1+x}} * \frac{\sqrt{1-x}}{\sqrt{1-x}}$ dx

= $\int_0^1 \frac{1-x}{\sqrt{1-x^2}}$ dx   x= sint

= $\int_0^{\frac{\pi}{2}} \frac{1-sint}{\sqrt{1-sin^2t}}$ cos tdt ,   dx = cos tdt

= $\int_0^{\frac{\pi}{2}} 1 - sint\ dt$

= (t+ cos t)$^{\pi/2}$

= ($\frac{\pi}{2}$ + cos $\frac{\pi}{2}$) – (0 + cos 0)

= $\frac{\pi}{2}$ - 1

4). Evaluate $\int_0^1 x^5(1\text{-}x^2)^5$dx

Sol.

I = $\int_0^1 x^5$ (1-x^2)^{5}dx ,   [x =sin Θ]

= $\int_0^{\frac{\pi}{2}}$ sin^5 ⍰$(1 - sin^2\theta)^5$ cos Θ dΘ

= $\int_0^{\frac{\pi}{2}} sin^5\theta\ cos^{11}$Θ dΘ :: [dx = cosΘdΘ]

= $\frac{10}{16} * \frac{8}{14} * \frac{6}{12} * \frac{4}{10} * \frac{2}{8} * \frac{1}{6}$ [x=0,sinΘ=0]

= $\frac{1}{336}$ [x=1, sin Θ=1, Θ =$\frac{\pi}{2}$]

5). Find the area of the region bounded between the parabolic y^2 = 4ax and its latus rectum.

Sol.

Equation of the parabola y^2 = 4ax

Y = 2 $\sqrt{a}\sqrt{x}$

Equation of the latus rectum x =a

Parabola symmetrical about x –axis

Required area A = 2 {Above x – axis }

X=0, x=a

= 2 $\int_0^a y\,dx$

= 2 $\int_0^a$ 2 $\sqrt{a}\sqrt{x}$ dx

= 4$\sqrt{a}$ $\{\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\}_0^a$

= $\frac{8}{3}$ $\sqrt{a}$(a$\sqrt{a}$)

= $\frac{8}{3} a^2$

5 MARKS

1). Evaluate : $\int_1^4 (2x^2 + 3)$ dx as the limit of a sum.

Sol.

$\int_a^b f(x)dx$

= $\lim_{n\to\infty} \frac{b-a}{n} \sum_{r=1}^{n} f(a + (b-a)\frac{r}{n})$

F(x) = 2x^2 + 3

a = 1, b =4

f(a+(b-a) $\frac{r}{n}$) = 2 ($\frac{1+3r}{n}$)^2 + 3

= 5 + $\frac{18r^2}{n^2}$ +$\frac{12r}{n}$

$\int_1^4 (2x^2 + 3)dx$ = $\lim_{n\to\infty} \frac{3}{n} \sum_{r=1}^{n} (5 + \frac{18r^2}{n^2} + \frac{12r}{n})$

$= \lim_{n\to\infty}\{\frac{15}{n}\sum_{r=1}^{n} 1 + \frac{54}{n^3}\sum_{r=1}^{n} r^2 + \frac{36}{n^2}$
$\sum_{r=1}^{n} r\}$

$= \lim_{n\to\infty}[\frac{15}{n} n + \frac{54}{n^3}\frac{n(n+1)(2n+n)}{6} + \frac{36}{n^2}\frac{n(n+1)}{2}]$

$= \lim_{n\to\infty}\{15 + 9(1+\frac{1}{n})(2+\frac{1}{n}) + 18\,(1+\frac{1}{n})\}$

= 15 + 9 (1) (2) + 18 (1)

= 15 + 18 + 18 = 51

2). Evaluate $\int_0^1 \frac{\log(1+x)}{1+x^2}$ dx

Sol.

I = $\int_0^1 \frac{\log(1+x)}{1+x^2}$ dx

Put x= tan t dx = $\sec^2$t dt

X = 0 t = 0

X =1 t = $\frac{\pi}{4}$

I = $\int_0^{\frac{\pi}{4}} \frac{\log(1+\tan t)}{1+\tan^2 t}$ $\sec^2$t dt

= $\int_0^{\frac{\pi}{4}} \frac{\log(1+\tan t)}{\sec^2 t}\sec^2 t$ dt

I = $\int_0^{\frac{\pi}{4}} \log(1+\tan t)\, dt$ –(1)

F(t) = log (1 + tan t)

F($\frac{\pi}{4} - t$) = log (1 + tan($\frac{\pi}{4}$ - t))

= log (1 + $\frac{1-\tan t}{1+\tan t}$)

= log ($\frac{1+\tan t+1-\tan t}{1+\tan t}$)

F($\frac{\pi}{4}$ - t) = log ($\frac{2}{1+\tan t}$)

$\int_0^a f(x)dx = \int_0^a f(a-x)dx$

I = $\int_0^{\frac{\pi}{4}} \log(\frac{2}{1+\tan t})$ dt - (1)

(1) + (2)

I + I = $\int_0^{\frac{\pi}{4}} \log(1+\tan t)$ + log ($\frac{2}{1+\tan t}$) dt

2I = $\int_0^{\frac{\pi}{4}} \log(1+\tan t)\frac{2}{(1+\tan t)}$ dt

= log 2$\int_0^{\frac{\pi}{4}} dt$

= log 2 $(t)_0^{\frac{\pi}{4}}$

= log 2 ($\frac{\pi}{4}$)

I = $\frac{\pi}{8}$ log 2

$\int_0^1 \frac{\log(1+x)}{1+x^2}$ dx = $\frac{\pi}{8}$ log 2

3). Evaluate $\int_0^{\frac{1}{2}} \frac{e^{\sin^{-1}x}\sin^{-1}x}{\sqrt{1-x^2}}$ dx

Sol.

t= $\sin^{-1}x$ dt = $\frac{1}{\sqrt{1-x^2}}$ dx

x = 0 t = 0

x = $\frac{1}{\sqrt{2}}$ t = $\frac{\pi}{4}$

I = $\int_0^{\frac{\pi}{4}} e^t$ t dt

$\int u\,dv$ = uv – $\int v\,du$

$\int_0^{\frac{\pi}{4}} te^t$ dt = $\{te^t - e^t\}_0^{\frac{\pi}{4}}$

= $(e^{\frac{\pi}{4}}\frac{\pi}{4} - e^{\frac{\pi}{4}}) - (0 - e^0)$

= $e^{\frac{\pi}{4}}\{\frac{\pi}{4} - 1\} + 1$

4). Find the area of the region common to the circle $x^2 + y^2 = 16$ and the parabola

$y^2 = 6x$

Sol.

Equation of the circle $x^2 + y^2 = 16$ --- (1)

Equation of the parabola $y^2 = 6x$ -----(2)

Solve (1) and (2)

$X^2 + 6x - 16 = 0$

(x+8) (x-2) = 0

X = -8 , x =2

X = -8 $y^2 = 6(-8) = -48$ not valid

X = 2 $y^2 = 6(2) = 12$

$Y = \pm 2\sqrt{3}$

Area boundary by the region = 2 {Area lie on the first quadrant}

$= 2\{\int_0^2 \sqrt{6}\, x^{\frac{1}{2}}\, dx + \int_2^4 \sqrt{4^2 - x^2}\, dx\}$

$= 2\{\{\sqrt{6}\, \frac{x^{\frac{3}{2}}}{\frac{3}{2}}\}_0^2 + \{\frac{x}{2}\sqrt{16 - x^2} + \frac{16}{2} sin^{-1}(\frac{x}{4})\}_2^4\}$

$= \frac{4\sqrt{6\,(2\sqrt{2})}}{3} + 16\,\frac{\pi}{2} - 2\sqrt{12} - 16(\frac{\pi}{6})$

$= \frac{4^3}{3}\{4\pi + \sqrt{3}\}$

5). Find the volume of the solid formed by revolving the region bounded by the parabola y = x² + 4x + 5, x- axis ordinates x= 0, and x = 1 about the x- axis.

Sol.

Equation of the parabola

$Y = x^2 + 4x + 5$

The region revolved about x- axis

Limit x =0, x =1

Volume $V = \pi\int_0^1 (X^4 + 4x + 5)^2\, dx$

$= \pi\int_0^1 (x^4 + 16x^2 + 25 + 8x^3 + 40x + 10x^2)\, dx$

$= \pi\{\frac{x^5}{5} + \frac{8x^4}{4} + \frac{26x^3}{3} + \frac{40x^2}{2} + 25x\}_0^1$

$= \pi\{\frac{1}{5} + 2 + \frac{26}{3} + 20 + 25\}$

$= \frac{838}{15}\pi$

10. ORDINARY DIFFERENTIAL EQUATION

2 MARK

1. **Determine the order and degree of the differential equation?**

$\left(\frac{d3y}{dx3}\right)^{2/3}$ - $3\frac{d2y}{dx2}$ + $5\frac{dy}{dx}$ + 4 = 0

Sol.

$\left(\frac{d3y}{dx3}\right)^{2/3}$ = $3\frac{d2y}{dx2}$ - $5\frac{dy}{dx}$ - 4

$\left(\frac{d3y}{dx3}\right)^{2} = \left(3\frac{d2y}{dx2} - 5\frac{dy}{dx} - 4\right)^{3}$

Order = 3

Order = 2

2. **Determine the order and degree if exists?**

$\frac{d2y}{dx2} + 3\left(\frac{dy}{dx}\right)^2 = x^2 \log\left(\frac{d2y}{dx2}\right)$

Sol.

Order = 2

It is not a polynomial equation in its derivatives

Degree is not defined.

3. **For the certain substance, the rate of change of vapor pressure P with respect to temperature T is proportional to the pressure and inversely proportional to the square of the temperature express this physical statement in the form of differential equation?**

Sol.

$\frac{dP}{dT}$ α $\frac{P}{T2}$

$\frac{dP}{dT} = \frac{KP}{T2}$

4. **Assume that a spherical rain drop evaporates at a rate proportional to its surface area. From a differential equation involving the rate of change of the radius of the rain drop.**

Sol.

Radius – r, volume – v, S.A = S

Rate of change of volume α S.A evaporates

V=$\frac{4}{3}\pi r3$, S.A = $4\pi r2$

$\frac{dv}{dt}$ α – A , $\frac{dv}{dt}$ = - KA

$\frac{4}{3}\pi(3r^2)\frac{dr}{dt}$ = - K $4\pi r^2$

$\frac{dr}{dt}$ = - K

5. **Show that xy' = 2y is the solution of the differential equation y = $2x^2$?**

Sol.

Y = $2x^2$

Y' = 4x

Multiply by x on both sides

Xy' = $4x^2$

Xy' = $2(2x^2)$

Xy' = 2y (using 1)

3 MARK

1. **Find the differential equation of the family of circles passing through the points (a,0) and (-a,0)**

Sol.

From the given information the centre is on y axis

Centre : (0, b)

- Radius (r) = $\sqrt{a2 + b2}$
- Euations of circle

$x^2 + (y – b)^2 = a^2 + b^2$

d.w.r. to x

$2x + 2(y – b)\frac{dy}{dx} = 0$

$y – b = -\frac{x}{\frac{dy}{dx}}$

$b = \frac{x}{\frac{dy}{dx}} + y$

substituting – (1)

$x^2\left(\frac{x2}{\left(\frac{dy}{dx}\right)2}\right) = a^2 + \left[\frac{x}{\frac{dy}{dx}} + y\right]^2$

$x^2\left(\frac{dy}{dx}\right)^2 + x^2 = a^2\left(\frac{dy}{dx}\right)^2 + \left[x + y\left(\frac{dy}{dx}\right)\right]^2$

$(x^2 - y^2 - a^2)\frac{dy}{dx} - 2xy = 0$

2) Find the differential equations of the family of all the ellipses having foci on the y – axis and centre at the origin?

Sol.

Equation of ellipse

$\frac{x2}{b2} + \frac{y2}{a2} = 1$

Arbitrary constants are a,b

Differentiating – (1)

$\frac{2x}{b2} + \frac{2yy'}{a2} = 0$ - (2)

Differentiating – (2)

$\frac{1}{b2} + \frac{yy''+yy''}{a2} = 0$ – (3)

From (1), (2), & (3)

$\begin{vmatrix} x2 & y2 & 1 \\ x & yy' & 0 \\ 1 & yy'' + y'2 & 0 \end{vmatrix} = 0$

Expanding along – (3)

$1[x(yy'' + y'2) - yy'] = 0$

$Xyy'' + x(y')^2 - yy' = 0$

3) Solve $\frac{dy}{dx}$ = (3x +y+4)^2

Sol.

Z= 3x + y + 4 say

$\frac{dz}{dx} = 3 + \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{dz}{dx} - 3$

Substituting in $\frac{dy}{dx} = (3x + y + 4)^2$

$\frac{dz}{dx} - 3 = z^2$

$\frac{dz}{dx} = z^2 + 3$

$\int \frac{1}{Z2+\sqrt{3}^2} dz = \int dx$

$\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{z}{\sqrt{3}}\right) = x+c$

$\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{3x+y+4}{\sqrt{3}}\right) = x+c$

4) Find the differential equation corresponding to the family of curves represented by the equation y = A e^{8x}+Be^{-8x}, Where A and B are arbitrary constants

Sol.

$Y = Ae^{8x}+Be^{-8x}$

$\frac{dy}{dx} = 8Ae^{8x} - 8Be^{-8x}$

$\frac{d2y}{dx2} = 64\,Ae^{8x} + 64\,Be^{-8x}$

$= 64\,(Ae^{8x} + Be^{-8x})$

$\frac{d2y}{dx2} = 64y$

4) **Solve $y^2+x^2\frac{dy}{dx} = xy\frac{dy}{dx}$**

Sol

$\frac{dy}{dx} = \frac{y2}{xy-x2}$

Put y = vx and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$V + x\frac{dv}{dx} = \frac{v2x2}{x(vx)-x2} = \left(\frac{v2}{v-1}\right)\frac{x2}{x2}$

$x\frac{d}{dx} = \frac{v2}{v-1} - v = \frac{v2-v2+v}{v-1}$

$\int \frac{v-1}{v} dv = \int \frac{dx}{x}$

$\int \left(1-\frac{1}{v}\right)dv = \int \frac{dx}{x}$

V – log $|v|$ = log $|x|$ + log $|c|$

V = log $|vxc|$

From y = vx , sub in v = y/x

y/x = log $\left|\frac{y}{x}xc\right|$

$c^{y/x}$ = cy

y = ke $^{y/x}$

5 MARK

1). A tank initially contains 50l of pure water starting at time t=0 a brine containing with 2 grams of dissolved salt per litre flows into the tank at the rate of 3l per minute the mixture is kept uniform by strings and the well stirred mixture simultaneously flows out of the tank at the same rate. Find the amount of salt present in the tank at any time t>0?

Sol.

$\frac{dx}{dt}$ = in flow rate – out flow rate

= (2*3L) – (3/50x)

= 6- 3/50x

$\frac{dx}{dt} = \frac{-3}{50}(x - \frac{6*50}{3}) = \frac{-3}{50}$(x-100)

$\frac{dx}{x-100} = \frac{-3}{50}$dt

$\int \frac{dx}{x-100} = \frac{-3}{50}\int dt$

Log (x-100) = $\frac{-3}{50}$ + log c

Log $\left(\frac{x-100}{c}\right) = \frac{-3t}{50}$

$\frac{x-100}{c} = e^{-3t/50}$

x-100 = $ce^{-3t/50}$ - (1)

t = 0, x = 0

(1)» 0-100 = ce^0 » c = - 100

(1) » x - 100 = - 100 $e^{-3t/50}$

X = 100 – 100 $e^{-3t/50}$

X = 100 (1-$e^{-3t/50}$)

2). Solve $\frac{dy}{dx}$+ 2y cot x = $3x^2$ $cosec^2$ x

$\frac{dy}{dx}$ = py = Q , where P = 2cot x,

Q = $3x^2$ $cosec^2x$

IF = $e^{\int pdx}$ = $e^{\int 2\cot x\, dx}$

= $e^{2\log|sinx|} = e^{\log|sinx|^2}$ = $\sin^2 x$

Sol.

y $e^{\int pdx} = \int qe^{\int pdx}$dx + c

y $\sin^2x = \int 3x^2 cosec^2x \sin^2x$ dx +c

y $\sin^2 x = \int 3x^2$dx + c

y $\sin^2x = 3\frac{x^3}{3}$ + c

y $\sin^2 x = x^3$ + c

3). Solve $\frac{dy}{dx} = \frac{x-y+5}{2(x-y)+7}$

Let Z = x – y

$\frac{dz}{dx} = 1 - \frac{dy}{dx} = \frac{dy}{dx} = 1 - \frac{dz}{dx}$

$\frac{dy}{dx} = \frac{x-y+5}{2(x-y)+7}$

$1 - \frac{dz}{dx} = \frac{Z+5}{2Z+7}$

$\frac{dz}{dx} = 1 - \frac{Z+5}{2Z+7}$

$\frac{dz}{dx} = \frac{2Z+7-Z-5}{2Z+7} = \frac{Z+2}{2Z+7}$

$\frac{2Z+7}{Z+2}dz = dx$

$\frac{2(Z+2)+3}{Z+2}dz = dx$

$\left(2 + \frac{3}{Z+2}\right)dz = dx$

Integrating

$2Z + 3\log|Z+2| = x+c$

$2(x+y) + 3\log|x-y+2| = x+c$

Example 10.30

4). A tank contains 1000 liters of water in which 100 grams of salt is dissolved. Brine (Brine is a high concentration solution of salt (usually sodium chloride)) in water runs in a rate of 10 liters per minute, and each litre contains 5 gramsof dissolved salt, The mixture of the tank is kept uniform by strring. Brine runs out at 10 litres per minute. Find the amount of salt at anytime t.

Sol.

Let x(t) denote the amount of salt in the tank at time 't'

$\frac{dx}{dt}$ = in flow rate – out flowrate

$\frac{dx}{dt} = 50 - \frac{10}{100}x$

$= 50 - 0.01x = -0.01(x-5000)$

$\frac{dx}{dt} = -0.01(x-5000)$

$\frac{dx}{x-5000} = -0.01\,dt$

$\log|x-5000| = -0.01t + \log c$

$x-5000 = ce^{-0.01t}$

$x = 5000 + ce^{-0.01t}$ --(1)

t=0, x=100, 100 = 5000+ c

-c = 5000 – 100

-c = 4900

C = -4900

(1) =) $x = 5000 - 4900\,e^{-0.01t}$

5) A pot of boiling water at 100^{0c} is removed from a stove at time t = 0 and left to cool in the kitchen. After 5 minutes, the water temperature has decreased to 80^{0c} and another 5 minutes later it has dropped to 65^{0c}. Determine the temperature of the kitchen.

Sol.

At time 't'

T – Temperature of water

S – Room temperature

$\frac{dT}{dt} \alpha$ T-S

$\frac{dT}{dt} = K(T-S)$

$\frac{dT}{T-S} = kdt$

Log (T-S) = Kt – C

$T - S = e^{Kt + C}$

$T - S = ce^{kt}$ - (1)

t=0, T=100 (1)= $100 - S = Ce^0$

c = 100 – S

(1)= $T - S = (100 - S)e^{kt}$ --(2)

t = 5, T = 80

$80 - S = (100 - S)e^{5k}$

$e^{5k} = \frac{80-s}{100-s}$

$65 - s = \frac{(100-s)(80-s)}{(100-s)} * \frac{(80-s)}{100-s}$

$(100-s)(65-s) = (80 - s)^2$

$6500 - 165s + s^2 = 6400 - 160s + s^2$

$6500 - 6400 = 165\ s - 160\ s$

$5s = 100$

$S = 20^0c$

Room temperature $S = 20^0C$

11. PROBABLITY DISTRIBUTIONS

2 MARK

1) Suppose X is the number of tails occurred when three fair coins are tossed once simultaneously, find the values of the random variables X and number of points in the inverse images?

Sol.

S = {HHH, HHT, HTH, THH, TTH, THT, HTT, TTT}

:: Be the no of tails

:: X = 0, 1, 2, 3

X^{-1} ({0}) = { HHH }

X^{-1} ({1}) = { HHT, HTH, THH }

X^{-1} ({2}) = { TTH, THT, HTT }

X^{-1} ({3}) = { TTT }

Values of random variable	0	1	2	3	Total
No of points inverse image	1	3	3	1	8

2) An jar contains 2 white and 3 red balls. A sample of 3 balls chosen. If X denotes the no of red balls ,find the value of random variables X and its no of inverse images?

Sol.

n(s) = $5c_3 = \frac{5*4*3}{1*2*3}$ = 10

X be the no of red balls in 3 drawn

W	R	T
2	3	5

:: X = 1, 2, 3 [0 is not possible here]

X = 1 => X (One red ball) = $3c_1 * 2c_2$

= 3*1 = 3

X = 2 => X (2 red balls) = $3c_2 * 2c_1$

= 3*2 = 6

X = 3 => X (3 red balls) = $3c_3$ = 1

Value of random variable	1	2	3	Total
No of points in inverse images	3	6	1	10

3) Three fair coins are tossed simultaneously, find the probability mass function for numbers of heads occurred?

Sol.

S = { HHH, HHT, HTH, THH, TTH, THT, HTT, TTT}

n(s) = 8

X be the r.v denotes no of heads

:: X = 0, 1, 2, 3

f(0) = P (x=0) = 1/8

f(1) = P (x=1) = 3/8

f(2) =P (X=2) = 3/8

f(3) = P (X=3) = 1/8

:: Probability mass function is

$$f(x) = \begin{cases} \frac{1}{8} & x=0,3 \\ \frac{3}{8} & x=1,2 \end{cases}$$

4) The probability density function of X is given by

$$F(x) \begin{cases} Kxe^{-2x} & x>0 \\ 0 & x\leq 0 \end{cases}$$

Find the value of K

Sol.

F(x) is a pdf

⇨ $\int_{-\infty}^{\infty} f(x)dx = 1$

⇨ $\int_0^{\infty} kxe^{-2x}dx = 1$

$\int_0^{\infty} x^n e^{-ax}dx = \frac{n!}{a^{n+1}}$

⇨ $k\int_0^{\infty} xe^{-2x}dx = 1$

$k\left[\frac{1!}{2^{1+1}}\right] = 1$

$k\left(\frac{1}{4}\right) = 1$ => k =4

5) A fair die is rolled 10 times and X denotes the no of times 4 appeared. Find the binomial distribution?

Sol.

n= 10

X = no of 4'S APPEARING

P = Probability of getting 4 in one throw

= 1/6

Q = 1-P = 1- 1/6 = 5/6

F(X) = $nc_x p^x q^{n-x}$

:: f(x) = $10\, c_x \left(\frac{1}{6}\right)^x \left(\frac{5}{6}\right)^{10-x}$

X = 0, 1, 2, ……, 10

3 MARK

1) A random variable X has the following probability mass function?

X	1	2	3	4	5
F(X)	k^2	$2k^2$	$3k^2$	2k	3k

(i)Find K (ii) P (2≤ X < 5) (iii) P(3<X)

Sol.

(i) F(x) is a pmf

⇨ $\sum \square(\square) = 1$

$K^2 + 2k^2 + 3k^2 + 2k + 3k = 1$

$6k^2 + 5K - 1 = 0$

$6k^2+6k-k-1=0$

$6k^2 (k+1) - 1(k+1)=0$

(6k-1)(k-1) =0

K=1/6 , k= -1(not possible)

K = 1/6

(ii) P(2≤ □ < 5)

= P(X=2) + P(X=3)+ P(X=4)

= $2k^2 + 3k^2 + 2k$

= $5k^2 + 2k$

= 5(1/6) + 2(1/6)

= $\frac{5+12}{36} = \frac{17}{36}$

(iii) P(3<x) = P(x>3)

= P (X=4) + P(x=5)

= 2k + 3k = 5k

= 5(1/6) = 5/6

2) If X is the random variable with distribution function f(x) given by

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{2}(x^2 + x) & 0 \leq x < 1 \\ 1 & x \geq 1 \end{cases}$$

Then find (i) pdf f(x)

(ii) P(0.3 ≤ X ≤ 0.6)

Sol.

F(x) = f' (x)

$$F(x) = f'(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{2}(2x+1) & 0 \le x < 1 \\ 0 & x > 1 \end{cases}$$

⇨ F(x) =

$$\begin{cases} \frac{1}{2}(2x+1) & 0 \le x < 1 \\ 0 & \text{□□□□□□h□□□} \end{cases}$$

(ii) $P(0.3 \le x \le 0.6)$

$= F(0.6) - F(0.3)$

$= \frac{1}{2}[(0.6)^2 + 0.6] - \frac{1}{2}[(0.3)^2 + 0.3]$

$= \frac{1}{2}[0.36 + 0.6] - \frac{1}{2}[0.09 + 0.3]$

$= \frac{1}{2}[0.96] - \frac{1}{2}[0.39]$

$= 0.48 - 0.195 = 0.285$

3) For the random variable X with the probability mass function

$F(x) = \{ \frac{4-x}{6}$, x=1, 2, 3

Find the mean and variance

Sol.

$X = 1 \Rightarrow F(x) = \frac{4-1}{6} = 3/6$

$X = 2 \Rightarrow F(x) = \frac{4-2}{6} = 2/6$

$X = 3 \Rightarrow F(x) = \frac{4-3}{6} = 1/6$

x	1	2	3	Total
F(x)	3/6	2/6	1/6	1
x. f(x)	3/6	4/6	3/6	10/6

$:: E(x) = \sum x f(x) = \frac{10}{6}$

$= 5/3 = 1.667$

Mean = 1.667

X^2	1	4	9	Total
F(X)	3/6	2/6	1/6	1
$X^2F(X)$	3/6	8/6	9/6	20/6

$E(X^2) = \sum x^2 f(x)$

$= 20/6 = 10/3$

Variable(x) $= E(x^2) - [E(X)]^2$

$= 10/3 - (5/3)^2$

$= 10/3 - 25/9$

$= \frac{30-25}{9} = 5/9 = 0.56$

Variance = 0.56

4) A lottery with 600 tickets gives one prize of Rs.200, four prizes of Rs. 100, and six prizes of Rs. 50,, If the ticket costs is Rs. 2 find the expected winning amont of tickets?

Sol.

n(s) = 600, X be the amount of winning

:: X = 200, 100, 50, 0

Probability mass function is

X	200	100	50	0
F(X)	$\frac{1}{600}$	$\frac{4}{600}$	$\frac{6}{600}$	$\frac{589}{600}$
X.F(X)	$\frac{200}{600}$	$\frac{400}{600}$	$\frac{300}{600}$	0

$:: E(X) = \sum x f(x) = \frac{900}{600}$

$= 3/2 = 1.5$

Rate of ticket = Rs. 2

:: Amount of winning = 1.5 – 2

= Rs. -0.50

5) If X~ B (n,p) such that 4P (x=4) = P(X=2) and n=6 find the distribution mean and SD?

Sol.

n(6) P (X=x) = $n\square_{\square}\square^{\square}\square^{\square-\square}$

4(p(x=4)) = p(x=2)

$4[6\square_4\square^4\square^{6-4}] = 6\,\square_2\square^2\square^{\square-\square}$

$4[6\square_2\square^4\square^2] = 6\square_2\square^2\square^4$

$4\frac{\square^4}{\square^2} = \frac{\square^4}{\square^2}$

$4p^2 = q^2 = (1-p)^2$

$= 1 + p^2 - 2p$

$\Rightarrow 4p^2 - 1 - p^2 + 2p = 0$

$3p^2 + 2p - 1 = 0$

$(3p-1)(p+1) = 0$

$P = \frac{1}{3}$, p = -1 (not possible)

$P = \frac{1}{3} \Rightarrow q = 1 - \frac{1}{3} = \frac{2}{3}$

$P(X = x) = 6\square_3(\frac{1}{3})^{\square}(\frac{2}{3})^{6-\square}$

Mean = np = $6 * \frac{1}{3} = 2$

$D = \sqrt{\square\square\square} = \sqrt{6} * 1/3*2/3$

$= \sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}}$

5 MARK

1) Suppose a pair of unbiased dice is rolled once, If X denotes the total score of two dice, write down (i) sample space (ii) Values taken by the random variable (iii) inverse image of 10, (iv) the no of elements in inverse image of X

Sol.

(i)S ={ (1,1) (1,2), (1,3), (1,4), (1,5), (1,6)

(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)

(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)

(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)

(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)

(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)}

n(s) = 36

(ii)X denotes the sum of two dice

:: X = 2, 3, 4, 5, 6, 7, 8, 9, 10,11, 12

(iii)Inverse image of 10

X^{-1} ({10}) ={(4,6) (5,5) (6,4)}

(iv)Number of elements in inverse image of X is

Value of random variable	2	3	4	5	6	7	8	9	10	11	12	Total
No of points in inverse image	1	2	3	4	5	6	5	4	3	2	1	36

2) Suppose a discrete random variable X can taken only the values 0,1 and 2. The pmf is defined by

$$F(x) = \begin{cases} \frac{x^2+1}{k} & x = 0,1,2 \\ 0 & otherwise \end{cases}$$

Find the (i) alue of K

(ii) Cumulative distribution function

(iii)P (X≥ 1)

Sol.

(i)F(X) is a pmf

X =0 = > f(0) = $\frac{0^2+1}{\square} = \frac{1}{\square}$

X = 1 => F(1) = $\frac{1^2+1}{\square} = \frac{2}{\square}$

X = 2 => F(2) = $\frac{2^2+1}{\square} = \frac{5}{\square}$

$\sum \square(\square) = 1$

⇨ $\frac{1}{\square} + \frac{2}{\square} + \frac{5}{\square} = 1$

⇨ $\frac{8}{\square} = 1$

⇨ $\square = 8$

(ii)F(X) = P(X≤ □)

X= 0 => f(0) = P (X≤ 0)

= P(X=0) = $\frac{1}{\square} = \frac{1}{8}$

X=1 => F(1) = P(X≤ 1)

= P (X=0) + P (X=1)

= $\frac{1}{\square} + \frac{2}{\square} = \frac{3}{\square} = \frac{3}{8}$

X=2 => F(2) = P(X≤ 2)

= P(X=0) + P(X=1) + P(X=2)

= $\frac{1}{\square} + \frac{2}{\square} + \frac{3}{\square} = \frac{5}{\square} = \frac{5}{8}$

$$:: F(X) = \begin{cases} 0 & \square<0 \\ \frac{1}{8} & 0\leq\square<1 \\ \frac{3}{8} & 1 \leq \square < 2 \\ 1 & 2 \leq \square < \infty \end{cases}$$

(iii)P(X≥ 1) = 1 – P(X<1)

= 1 – (P(X=0))

= 1 - $\frac{1}{8} = \frac{7}{8}$

3) A six sided die is marked '1' and '2' on two faces and '3' on its remaining three faces. The die is rolled twice. If X denotes the total on the two throws.

(i) Find probability mass function

(ii)find the cumulative distribution function

(iii)find P (3≤ X < 6)

(iv)find P(x≥ 4)

Sol.

Numbers on the dice are 1,2,2,3,3,3

X denotes the sum on two dice

Sample space

I/II	1	2	2	3	3	3
1	2	3	3	4	4	4
2	3	4	4	5	5	5
2	3	4	4	5	5	5
3	4	5	5	6	6	6
3	4	5	5	6	6	6
3	4	5	5	6	6	6

From the table

X = 2,3,4,5,6

X =2 => f(2) = p(x=2) = $\frac{1}{36}$

X = 3 => f(3) = p (x=3) = $\frac{4}{36}$

X=4 => f(4) = p(x=4) = $\frac{10}{36}$

X=5 => f(5) =p(x=5) = $\frac{12}{36}$

X=6 => f(6) = p(x=6) = $\frac{9}{36}$

(i)Probability mass function

x	2	3	4	5	6
F(x)	$\frac{1}{36}$	$\frac{4}{36}$	$\frac{10}{36}$	$\frac{12}{36}$	$\frac{9}{36}$

(ii)cumulative distribution function

F(2) = P(x≤ 2)

= P(x=2) = $\frac{1}{36}$

F(3) = P(x≤ 3)

$= P(x=2) + p(x=3)$

$= \frac{1}{36} + \frac{4}{36} = \frac{5}{36}$

$F(4) = P(x \leq 4)$

$= P(x=2) + P(x=3) + P(x=4)$

$= \frac{1}{36} + \frac{4}{36} + \frac{10}{36} = \frac{15}{36}$

$F(5) = P(x \leq 5)$

$= \frac{1}{36} + \frac{4}{36} + \frac{10}{36} + \frac{12}{36}$

$= \frac{27}{36}$

$F(6) = P(x \leq 6)$

$= \frac{1}{36} + \frac{4}{36} + \frac{10}{36} + \frac{12}{36} + \frac{9}{36}$

$= \frac{36}{36} = 1$

$$F(x) \begin{cases} 0 & -\infty < x < 2 \\ \frac{1}{36} & 2 \leq x < 3 \\ \frac{5}{36} & 3 \leq x < 4 \\ \frac{15}{36} & 4 \leq x < 5 \\ \frac{27}{36} & 5 \leq x < 6 \\ 1 & 6 \leq x < \infty \end{cases}$$

(iii)$P(3 \leq x < 6)$

$= P(x=3) + P(x=4) + P(x=5)$

$= \frac{4}{36} + \frac{10}{36} + \frac{12}{36} = \frac{26}{36}$

(Iv)$P(x \geq 4)$

$= P(x=4) + P(x=5) + P(x=5)$

$= \frac{10}{36} + \frac{12}{36} + \frac{9}{36} = \frac{31}{36}$

4) If f(x) is a pdf given by

F(x) $= \begin{cases} cx^2 & 1 < x < 4 \\ 0 & otherwise \end{cases}$

Find the value of C. Also find

(i) $P(1.5<x<3.5)$ (ii) $P(x \leq 2)$

(iii)$P(3<x)$

Sol.

F(x) is a pdf

⇨ $\int_{-\infty}^{\infty} f(x)dx = 1$

⇨ $\int_{1}^{4} cx^2 dx = 1$

$C\left[\frac{x^3}{3}\right]_1^4 = 1$

$C\left[\frac{4^3}{3} - \frac{1^3}{3}\right] = 1$

$C\left[\frac{64}{3} - \frac{1}{3}\right] = 1$

$C\left(\frac{63}{3}\right) = 1, C(21) = 1 \Rightarrow C = \frac{1}{21}$

(i)$P(1.5<X<3.5)$

$= \int_{1.5}^{3.5} F(X)dx$

$= \int_{1.5}^{3.5} cx^2\, dx$

$= C\left[\frac{x^3}{3}\right]_{1.5}^{3.5}$

$= \frac{1}{21}\left[\frac{(3.5)^3}{3} - \frac{(1.5)^3}{3}\right]$

$= \frac{1}{63}[42.875 - 3.375]$

$= \frac{1}{63}(39.5) = \frac{395}{630} = \frac{79}{126}$

(ii)$P(x \leq 2) = \int_{-\infty}^{2} f(x)dx$

$= \int_{1}^{2} f(x)dx = \int_{-\infty}^{2} f(x)dx$

$= C\left[\frac{x^3}{3}\right]_1^2 = C\left[\frac{2^3}{3} - \frac{1^3}{3}\right]$

$= \frac{1}{21}\left[\frac{8}{3} - \frac{1}{3}\right]$

$= \frac{1}{21} * \frac{7}{3} = \frac{1}{9}$

(iii)P(3<x) = P(x>3)

$= \int_3^\infty f(x)dx$

$= \int_3^4 cx^2\,dx = C\left[\frac{x^3}{3}\right]_3^4$

$= \frac{1}{21}\left[\frac{4^3}{3} - \frac{3^3}{3}\right]$

$= \frac{1}{63}[64 - 27] = \frac{37}{63}$

5) Find the mean and variance of a random variable x, which has pdf

$F(x) = \begin{cases} \lambda e^{-\lambda x} & x > 0 \\ 0 & otherwise \end{cases}$

Sol.

Mean = μ = E (X) = $\int_{-\infty}^{\infty} xf(x)dx$

$= \int_0^\infty x(\lambda e^{-\lambda x})dx$

$= \lambda \int_0^\infty xe^{-\lambda x}dx$

$= \left[\int_0^\infty x^n e^{-ax}dx = \frac{n!}{a^{n+1}}\right]$

$= \lambda\left[\frac{1!}{\lambda^{1+1}}\right] = \lambda\left(\frac{1}{\lambda^2}\right) = \frac{1}{\lambda}$

$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)dx$

$= \int_0^\infty (\lambda e^{-\lambda x})dx$

$= \lambda \int_0^\infty x^2 e^{-\lambda x}dx$

$= \lambda\left[\frac{2!}{\lambda^{2+1}}\right] = \lambda\left(\frac{2}{\lambda^3}\right) = \frac{2}{\lambda^2}$

Variance

$V(X) = E(X^2) - [E(X)]^2$

$= \frac{2}{\lambda^2} - (\frac{1}{\lambda})^2$

$= \frac{2}{\lambda^2} - \frac{1}{\lambda^2} = \frac{1}{\lambda^2}$

6) On the average 20% of the products manufactured by ABC company are formed to be defective. If we select 6 of these products at random and X denotes the numbers of defective products, find the probability that,

(i) Two products are defective

(ii) Atmost one product is defective
(iii)Atleast two products are defective

Sol.

n = 6

P =Probability of defective item

$= 20\% = \frac{20}{100} = \frac{1}{5}$

$Q = 1-P = 1- \frac{1}{5} = \frac{4}{5}$

$F(X) = nc_x p^x q^{n-x}$

$F(x) = 6c_x(\frac{1}{5})^x(\frac{4}{5})^{6-x}$

X = 0,1,2,…6

(i)Exactly two items are defective

$P(x=2) = 6c_2(\frac{1}{5})^2(\frac{4}{5})^{6-2}$

$= \frac{6*5}{1*2} * \frac{1}{5^2} * \frac{4^4}{5^4}$

$= 15\,(\frac{4^4}{5^6})$

(ii)Atmost one item is defective

$P(x=\le 1) = P(x = 0) + P(x = 1)$

$= 6c_0(\frac{1}{5})^0(\frac{4}{5})^{6-0} + 6c_1(\frac{1}{5})^1(\frac{4}{5})^{6-1}$

$= (1)\,(1)\,\frac{4^6}{5^6} + 6 * \frac{1}{5} * \frac{4^5}{5^5}$

$= \frac{4^6}{5^6} + 6\,(\frac{4^5}{5^6})$

$= \frac{4^5}{5^{5*5}} * 10^2$

$=2(\frac{4}{5})^5$

(iii)Atleast 2 items are defective

$P(x \geq 2) = 1 - P(x < 2)$

$= 1 - P(x \leq 1)$

$= 1 - (2(\frac{4}{5})^5)$

$= 1 - 2(\frac{4^5}{5^5})$

12. DISCRETE MATHEMATICS

2.MARKS

1. Prove that in an algebraic structure the identity (if exists) must be unique.

Proof:

Let (s,*) be an algebraic structure.

Let e_1 and e_2 be any two identity element of S. First treat e_1 as the identity element and e_2 as an arbitrary element of S.

By definition,

$e_2 * e_1 = e_1 * e_2 = e_2$ → ①

Interchanging the role of e_1 and e_2, we get

$e_1 * e_2 = e_2 * e_1 = e_1$ → ②

From 1 and 2,

$\boldsymbol{e_1 = e_2}$

Hence the proof.

2. Prove that in an algebraic structure the inverse of an element (if exists) must be unique.

Proof:

Let (S,*) be an algebraic structure and a ϵ s

Suppose that a has two inverses say a_1 & a_2.

Treating a_1 as an inverse of a, we get

a * a_1 = a_1 * a = e → ①

Next treating a_2 as the inverse of a, we get

a * a_2 = a_2 * a = e → ②

Now,

$a_1 = a_1 * e = a_1 * (a * a_2)$

$= (a_1 * a) * a_2$

$= e * a_2$

$= a_2$

By ① and ②

i.e, $\boldsymbol{a_1 = a_2}$. Hence the proof.

3. Let A = $\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$, B = $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, be any two Boolean matric of the same type. Find A∨B and A∧B.

Solution:

A∨B $= \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \vee \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0\vee1 & 1\vee1 \\ 1\vee0 & 1\vee1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

A∧B $= \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \wedge \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0\wedge1 & 1\wedge1 \\ 1\wedge0 & 1\wedge1 \end{bmatrix}$

$= \begin{bmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} \end{bmatrix}$

4. Let * be defined on R by a*b = a+b+ab-7. Is * binary on R? If so find 3* $[-7/15]$.

Solution:

Let a, b ϵ R. Clearly a, b, ab ϵ R.

:. a*b = a+b +ab – 7 ϵ R.

:. * is binary on R.

$3 * \frac{-7}{15} = 3 - \frac{7}{15} + 3 * \frac{-7}{15}$

$= \frac{45-7-21-105}{15}$

$= \frac{45-133}{15}$

$= \boldsymbol{\frac{-88}{15}}$

5 . Fill in the following table so that the binary operation * on A = {a,b,c}

Is communicative.

*	a	b	c
a	b		
b	c	b	a
c	a		c

Soln:

(i) From the table , b * a = c

(ii) From the table , a * b = c

(iii) c* a = a , => a*c = a

(iv) b*c = a , => c*b = a

*	a	b	c
a	b	c	a
b	c	b	a
c	a	a	c

6) Construct the truth table for $\overline{(p \vee q)}$ ∧ (p v ¬q)

Soln :

P	q	¬q	p v̄ q	p v̄ ¬q	(pv̄q) ∧(pv̄¬q)
T	T	F	F	T	F
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	F

3 MARKS QUESTION AND ANSWERS

1) Verify (i) closure (ii) commutative property (iii) Associative property of the following operation on the given set

a*b = a^b ; ∀ a,b ∈ N

Soln :

(i) a*b = a^b ∈ N ; ∀ a,b ∈ N

∴ * is a binary operation on N

(ii) a *b = a^b

b*a = b^a

put a = 2,b = 3

a*b = 2^3 =8

b*a = 3^2 = 9

a * b need not be equal to b*a

∴ * is not commutative

(iii) a * (b* c) = a * b^c = a^{bc} → ①

(a* b) * c = a^b * c = $(a^b)^c$ = a^{bc} →②

a * (b* c) ≠ (a* b) * c .

∴ * is not associate on N

2 . Check whether the statement (p↔ q) ∧

(p → ¬ q) is a tautology or contraction or contingency

Soln:

p	q	(p ↔ q)	¬ q	(p↔ ¬q)	¬ (p↔ q	(p ↔ q) ∧ ¬ (p → ¬ q)
T	T	T	F	F	T	T
T	F	F	T	T	F	F
F	T	F	F	T	F	F
F	F	T	T	T	F	F

The last column is a combination of T and R

∴ It is a contingency.

3 . Check whether the statement (p ⟶q) ∧ (q ⟶ r) ⟶ (p ⟶ r) is a tautology or a contradiction.

Soln:

p	q	r	p ⟶q	q ⟶ r	p ⟶r	(p ⟶q) ∧ (q ⟶ r)	(p ⟶q) ∧ (q ⟶ r) ⟶ (p ⟶ r)
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	T	F	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

Last column contains only T.

∴ **This is a tautology .**

4 .Prove p → (q → r) ≡ (p ∧ q) →r without using truth table.

Soln :

p → (q → r) ≡ p → (¬q v r)

≡ ¬ p v (¬ q v r)

≡ (¬ p v ¬ q) v r

[∴ Associative law]

≡ ¬ (p ∧ q) v r

[De – Morgan's Law]

≡ (p ∧ q) → r .

Hence proved.

5 . Write the converse , inverse and contrapositive of the following implication of the x and y are numbers such that x = y ,then x^2 =y^2

Soln:

i) converse :

p : x and y are numbers such that x = y

q : x^2 =y^2

given statement : p ⟶ q

converse :

q ⟶ p

ii) Inverse : ¬ p → ¬ q

If x and y are numbers such that x ≠y then $x^2 \neq y^2$.

iii) contrapositive : **¬ q → ¬ p.**

If x and y are numbers such that $x^2 \neq y^2$,then x ≠y

6 . prove the de morgan's law using truth table ¬(p ∧ q) ≡ ¬ p v ¬ q .

p	q	¬ p	¬ q	p ∧ q	¬(p ∧ q)	¬ p v ¬ q
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

From ① and ② , ¬(p ∧ q) ≡ ¬ p v ¬ q

5 MARK

1) Verify (i) closure property (ii) Commutative property (iii) Associate property (iv) existence of identity (v) existence of inverse for the operation x11

on a subset A = { 1,3,4,5,9 } of the set of remaining { 0,1,2,3,4,5,6,7,8,9,10 }

Soln:

A = { 1,3,4,5,9 }

x_{11}	1	3	4	5	9
1	1	3	4	5	9
3	3	9	1	4	5
4	4	1	5	9	3
5	5	4	9	3	1
9	9	5	3	1	4

i)Since each box has an unique element of x_{11} is a binary operation on A.

∴ **Closure property is true**

ii)From the table it is clear that x_{11} is commutative.

∴ **Commutative property is true**.

iii) x_{11} is always associative .

∴**Associative property is true**.

iv)1 is the identity element .

∴**Identity property is true.**

v) From ther table ,

inverse of 1 is 1 ,

inverse of 3 is 4,

inverse of 4 is 3,

inverse of 5 is 9,

and inverse of 9 is 5.

∴**Inverse property is true.**

2) Using the equivalence property ,show that $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$.

Soln:

$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

$\equiv (\neg p \vee q) \wedge (\neg q \vee p)$

$[\therefore p \rightarrow q \equiv \neg p \vee q]$

$\equiv (\neg p \vee q) \wedge (\neg p \vee q)$

[by com.law]

$\equiv (\neg p \wedge (p \vee \neg q)) \vee (q \wedge (p \vee \neg q))$

[by distributing law]

$\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q)) \vee (q \wedge p) \vee (q \wedge \neg q))$

[by distributing law]

$\equiv \mathbb{F} \vee (\neg p \wedge \neg q) \vee (q \wedge p) \vee \mathbb{F}$

[by complement law]

$\equiv (\neg p \wedge \neg q) \vee (q \wedge p)$

[by identity law]

$\equiv (q \wedge p) \vee (\neg p \wedge \neg q)$

[by com.law]

$\equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

[by com.law]

$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$.

3 . Let $M = \left\{ \begin{pmatrix} x & x \\ x & x \end{pmatrix} : x \in R - \{0\} \right\}$ and let * be the matrix multiplication . Determine whether M is closed under * .If so ,examine i) commutative proper

ty ii) associative property iii) existence of identity v) existence of inverse properties for the operation * on M.

Soln:

$$M = \left\{ \begin{pmatrix} x & x \\ x & x \end{pmatrix} : x \in R - \{0\} \right\}$$

i)Closure property :

Let A $= \begin{pmatrix} x & x \\ x & x \end{pmatrix}$, B $= \begin{pmatrix} y & y \\ y & y \end{pmatrix} \in$ M.
Where x,y ∈ R .

$$A*B = \begin{pmatrix} x & x \\ x & x \end{pmatrix}\begin{pmatrix} y & y \\ y & y \end{pmatrix}$$

$$= \begin{pmatrix} 2xy & 2xy \\ 2xy & 2xy \end{pmatrix} \in M .$$

[∴ 2 xy ∈ R-{0}]

∴ * is closed on M.

ii)Commutative property :

Let A , B ∈ M

$$A * B = \begin{pmatrix} x & x \\ x & x \end{pmatrix}\begin{pmatrix} y & y \\ y & y \end{pmatrix}$$

$$= \begin{pmatrix} 2xy & 2xy \\ 2xy & 2xy \end{pmatrix}$$

$$= \begin{pmatrix} 2yx & 2yx \\ 2yx & 2yx \end{pmatrix}$$

$$= \begin{pmatrix} y & y \\ y & y \end{pmatrix}\begin{pmatrix} x & x \\ x & x \end{pmatrix}$$

= B * A

A * B = B * A

∴ * is a commutative on M .

iii)Associative property :

Matrix multiplication is always associative is A * (B * C) = (A * B) * C ∀ A , B ,C ∈ M.

iv) Existence of identity :

Let A ∈ M. , E $= \begin{pmatrix} e & e \\ e & e \end{pmatrix}$ be the identity element .

∴AE = A

$$=> \begin{pmatrix} x & x \\ x & x \end{pmatrix}\begin{pmatrix} e & e \\ e & e \end{pmatrix} = \begin{pmatrix} x & x \\ x & x \end{pmatrix}$$

$$=> \begin{pmatrix} 2xe & 2xe \\ 2xe & 2xe \end{pmatrix} = \begin{pmatrix} x & x \\ x & x \end{pmatrix}$$

$2xe$ = x

2e = 1

e = $\frac{1}{2}$ **∈ R – {0}.**

$$\therefore \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \in M$$

llly EA = A ∀ A ∈ M

∴ * has identity element on M .

v) Existence of inverse:

Let A ∈ M , $A^{-1} = \begin{pmatrix} x^{-1} & x^{-1} \\ x^{-1} & x^{-1} \end{pmatrix}$ be the inverse of A .

$A A^{-1} = E$

$$\begin{pmatrix} x & x \\ x & x \end{pmatrix}\begin{pmatrix} x^{-1} & x^{-1} \\ x^{-1} & x^{-1} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$$\begin{pmatrix} 2xx^{-1} & 2xx^{-1} \\ 2xx^{-1} & 2xx^{-1} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$$2xx^{-1} = \frac{1}{2}$$

$x^{-1} = \frac{1}{2*2x} = \frac{1}{4x}$ **∈ R – {0}.**

∴ $A^{-1} = \begin{pmatrix} \frac{1}{4x} & \frac{1}{4x} \\ \frac{1}{4x} & \frac{1}{4x} \end{pmatrix} \in$ **M.**

llly $A^{-1}A = E \ \forall A \in M$

∴Inverse property is true.

4) Let A be a – {1} .Define * on A by x * y = x+ y –xy . Is * binary on A ?. If so examine the commutative ,associative, identity and inverse properties satisfied by * on A.

Soln:

i)Closure property :

Let x,y ∈ A , x ≠ 1 , y ≠ 1

x- 1 ≠ 0 , y -1 ≠ 0

(x – 1)(y – 1) ≠ 0

xy – x – y + 1 ≠ 0

1 ≠ x+y –xy

=>x*y ≠ 1

∴ x*y ∈ A

∴ *** is closed on A.**

ii) commutative property :

Let x,y ∈ A

x*y = x+y-xy

= y+x-yx

= y*x

∴ *** is Commutative on A.**

iii) Associative property :

(x*y)*z = (x+y-xy)*z

= (x+y-xy) + z – (x+y-xy)z

= x+y+z – xy –xz-yz +xyz→ ①

x*(y*z) = x* (y+z –yz)

= x+(y+z-yz) – x(y+z-yz)

= x+y+z –xy –xz –yz +xyz →②

From ①&②

(x*y) * z = x*(y*z)

4) Identity Property:

Let x ∈ A , e be the identity element

x*e = x

=>x+e –xe = x

=>e – xe = 0

=>e(1-x) = 0

=> $e = \frac{0}{1-x}$ = 0 ∈ A

:. Identity element e = 0 ∈ A

:. * has identity element on A.

5) Inverse property:

Let x ∈ A , x^{-1} be the inverse of x .

By definition:

$x*x^{-1} = e$

is $x + x^{-1} - xx^{-1} = 0$ [∴ e = 0]

=>x^{-1} (1-x) = -x

=>$x^{-1} = \frac{-x}{1-x}$ ∈ A

*** has inverse element**

∀ x ∈ A.