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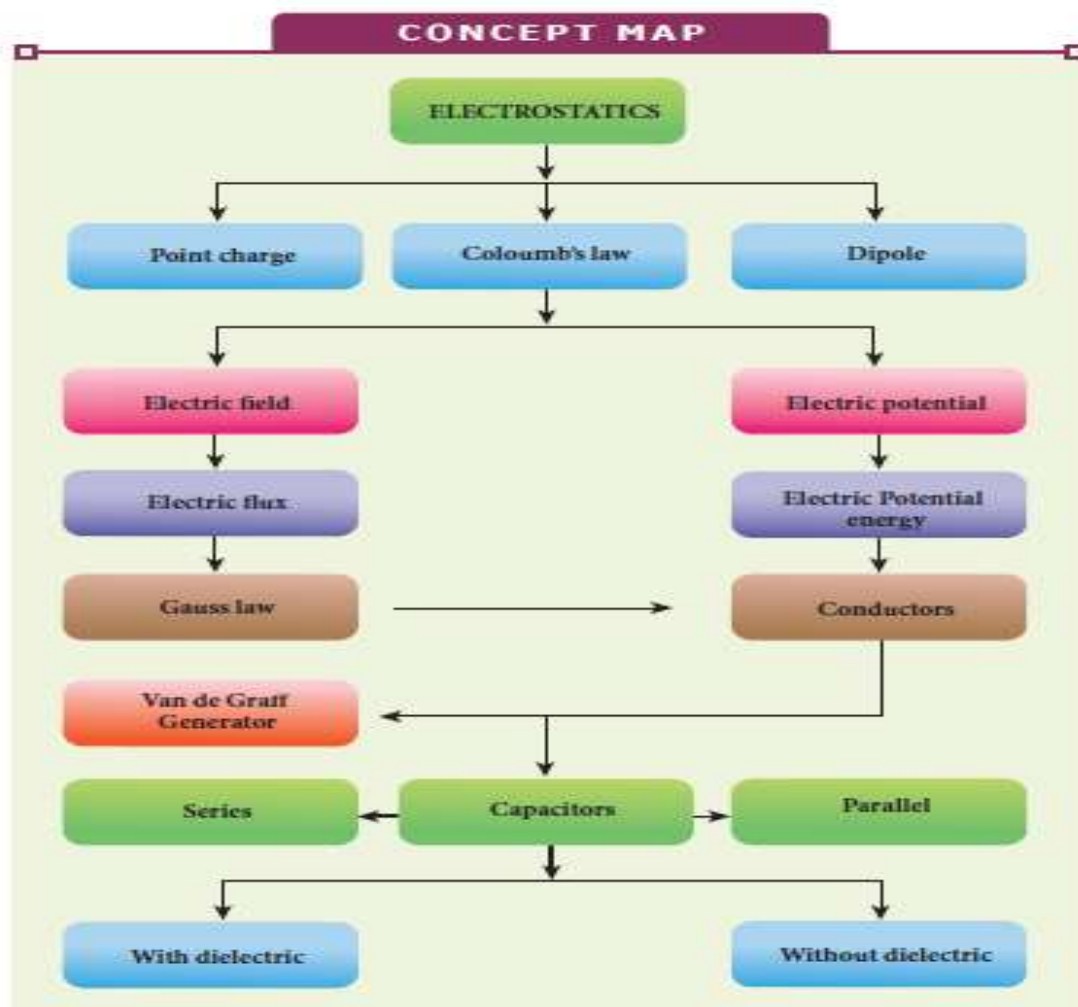
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# 12<sup>th</sup> PHYSICS

## UNIT -1

# ELECTROSTATICS

MR.THIVİYARAJ V.,M.Sc.,M.Phill.,B.Ed



**1. CHARGING BY RUBBING [ 2 MARK]**

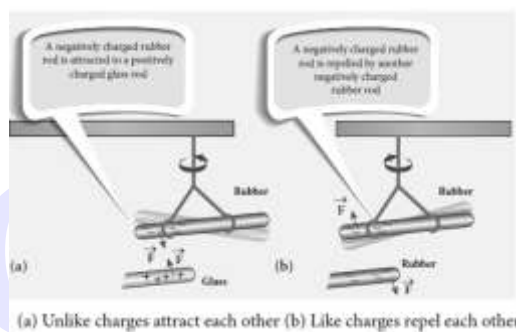
Some materials are found to be charged by rubbing with suitable materials.

**EX**

1. Amber rod is charged by rubbing with animal fur.
2. Glass rod is charged by rubbing with silk cloth.

**2. EXPERIMENT [ 3 MARK]**

Consider a charged rubbed rod hanging from a thread. When another charged rubber rod is brought near it will be repelled instead if a charged glass rod is brought near rubber rod, it will be attracted.

**3. CONCLUSION [ 2 MARK]**

- Like charges repel force.
- Unlike charges attract force.

**Two kinds of charges**

1. Positive charge (+ve)
2. Negative charge (-ve)

**4. NEUTRAL CHARGES [ 2 MARK]**

If the net charge is zero in the object it is said to be electricity neutral.

**CHARGE OF AN ATOM**

All material are made up of atoms which are electrically neutral.

But its constituent particles posses charges.

## ELECTROSTATICS

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- i. Protons → +ve
- ii. Electrons → -ve
- iii. Neutrons → zero

**5. TRIBOELECTRIC CHARGING[ 2 MARK]**

When an object is rubbed with another object [EX: Rubber with silk cloth], some amount of charges is transferred from one object to the other due to friction between them. The object is said to be '**Electrically charged**'. This method of charging the objects through rubbing is called '**Triboelectric charging**'.

**6. BASIC PROPERTIES OF CHARGES****1. ELECTRIC CHARGE [ 2 MARK]**

Electric charge is inherent and fundamental property of particles.

Charge is the physical property of particles is matter that experiences is directly

Proportional to the force when placed in an electromagnetic field.

S.I unit → Coulomb [C]

**2. CONSERVATION OF CHARGES[ 2 MARK]**

The total electric charge in the universe is constant and charge can be neither created nor destroyed. In any physical process, the net charge is always zero.

**3. QUANTISATION OF CHARGES[ 2 MARK]**

The charge of any object is equal to an integral multiple of the fundamental unit of charge  $e$ ,

$$q = ne$$

$n$  → integer  $[0, \pm 1, \pm 2, \dots]$

$e$  → charge of an electron.

The charge of electron is found to be ' $-e$ ' →  $-1.6 \times 10^{-19} \text{ C}$

## ELECTROSTATICS

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The charge of proton is found to be '+e'  $\rightarrow 1.6 \times 10^{-19} \text{ C}$

Charge Quantization is applicable for microscopic level and not for macroscopic.

When a glass rod is rubbed with silk cloth, approximately  $10^{10}$  charges are transferred and the charges are treated to be continuous.

The smallest charge in nature :-

Charge of electron (-e) and charge of proton (+e)

## 7. COULOMB'S LAW [ 5 MARK]

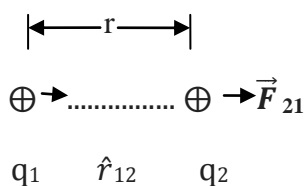
### STATEMENT [2 MARK]

The force of attraction or repulsion between two point charges [ $q_1$  and  $q_2$ ] separated by a distance 'r' is directly proportional to the product of the charges and inversely proportional to the square of the distance between them. The force acts along the line joining the two charges.

### PROOF

Consider two charges  $\rightarrow q_1$  and  $q_2$

distance  $r \rightarrow$



The force on the point charge ' $q_1$ ' exerted by charge ' $q_2$ ' is

$$\vec{F}_{21} \propto \frac{q_1 q_2}{r^2} \hat{r}_{12}$$

$$\vec{F}_{21} = K \frac{q_1 q_2}{r^2} \hat{r}_{12}$$

In the similar manner the force on the charge ' $q_1$ ' exerted by ' $q_2$ ' is,

$$\vec{F}_{12} = K \frac{q_1 q_2}{r^2} \hat{r}_{21}$$

$$K = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ Nm}^2\text{C}^{-2}$$

$\epsilon_0 \rightarrow$  Permittivity of free space  $\rightarrow 8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$

### 8. RELATIVE PERMITTIVITY [ Dielectric constant, $\epsilon_r$ ] [ 2 MARK]

Force between two point charges in medium is always less than that in vacuum.

In Vacuum:

$$[\vec{F}_{21}]_{\text{vac}} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{r}_{12}$$

In Medium:

$$[\vec{F}_{21}]_{\text{Med}} = \frac{1}{4\pi\epsilon} \frac{q_1 q_2}{r^2} \hat{r}_{12}$$

$$\frac{F_{21 \text{ vac}}}{F_{21 \text{ Med}}} = \frac{\epsilon}{\epsilon_0} > 1$$

Since  $\epsilon > \epsilon_0$

$$\vec{F}_{21 \text{ vac}} > \vec{F}_{21 \text{ Med}}$$

The ratio  $\frac{\epsilon}{\epsilon_0}$  is called relative permittivity of the Medium.

$$\epsilon_r = \frac{\epsilon}{\epsilon_0}$$

In air (or) vacuum  $\epsilon_r = 1$

For all other medium  $\epsilon_r > 1$

### 9. SIMILARITIES BETWEEN COULOMB'S LAW AND NEWTON'S LAW OF GRAVITATION [ 3 MARK]

S.NO	COULOMB'S LAW	NEWTON'S LAW
1	Electrostatic force between charges $q_1$ and $q_2$ .	Gravitational force between masses $m_1$ and $m_2$ .
2	$F \propto q_1 q_2$	$F \propto m_1 m_2$
3	$F \propto \frac{1}{r^2}$	$F \propto \frac{1}{r^2}$
4	Attractive or Repulsive depending on the nature of the charge.	Always attractive.

5	Depends on the nature of the Medium in which the charges are kept.	Independent of the medium.
6	Varies when they are in motion.	Remains the same when the masses are at rest as in Motion.
7	$K = 9 \times 10^9 \text{ Nm}^2\text{C}^{-2}$	$G = 6.67 \times 10^{-11} \text{ Nm}^2\text{kg}^{-2}$

K is much higher than G.

Electrostatic force is always greater [Magnitude] than gravitational force for smaller size object .

### 10. FORCE BETWEEN CHARGES [NEWTON'S III LAW ] [ 3 MARK]

Prove that electrostatic force obeys Newton's third law:

Two point charges  $q_1$  and  $q_2$  ; distance ' r'.

Force on the point charge ' $q_2$ ' exerted by charging ' $q_1$ '

$$\vec{F}_{21} = K \frac{q_1 q_2}{r^2} \hat{r}_{12}$$

Force on the point charge ' $q_1$ ' exerted by charging ' $q_2$ '

$$\vec{F}_{12} = K \frac{q_1 q_2}{r^2} \hat{r}_{21}$$

$$\text{But } \hat{r}_{21} = -\hat{r}_{12}$$

Substituting,

$$\vec{F}_{12} = K \frac{q_1 q_2}{r^2} [-\hat{r}_{12}] = -K \frac{q_1 q_2}{r^2} [\hat{r}_{12}]$$

$$\boxed{\vec{F}_{12} = -\vec{F}_{21}}$$

Equal in Magnitude but opposite in direction.

**11. LIMITATION OF COULOMB'S LAW [ 2 MARK]**

The coulomb force is the only for point charges and is applicable for charges whose size is smaller compared to the distance between them.

**12. MAGNITUDES OF ELECTROSTATIC AND GRAVITATIONAL FORCES****[3 MARK]**

Why a charged comb attracts an uncharged piece of paper with greater force even through the paper is attracted by the gravitational force of the earth?

The electrostatic force between a proton and electron is found to be  $10^{39}$  times than that of the gravitational force between them.

$$F_e = 10^{39} F_G$$

Thus, the gravitational force is negligible compared to the electrostatic force for small size objects in the atomic domain. This is the reason that a charged comb attracts an uncharged piece of paper with greater force even through the paper is attracted downwards by the gravitational force.

**13. SUPERPOSITION PRINCIPLE [ 3 MARK]**

“ The total force acting on a given charge is equal to the vector sum of forces Exerted on it by all the other charges “

Consider the system of 'n' charges, namely  $q_1, q_2, q_3, \dots, q_n$ .

The force on ' $q_1$ ' exerted by charging ' $q_2$ '

$$\vec{F}_{12} = K \frac{q_1 q_2}{r_{21}^2} \hat{r}_{21}$$

The force on ' $q_1$ ' exerted by charging ' $q_3$ '

$$\vec{F}_{13} = K \frac{q_1 q_2}{r_{13}^2} \hat{r}_{13}$$

$$\vec{F}_1^{\text{tot}} = \vec{F}_{12} + \vec{F}_{13} + \dots + \vec{F}_{1n}$$

$$\vec{F}_1^{\text{tot}} = K \left[ \frac{q_1 q_2}{r_{21}^2} \hat{r}_{21} + \frac{q_1 q_2}{r_{13}^2} \hat{r}_{13} + \dots + \frac{q_1 q_2}{r_{n1}^2} \hat{r}_{n1} \right]$$

### ELECTRIC FIELD AND ELECTRIC FIELD LINES

#### 14. ACTION AT A DISTANCE [ 2 MARK]

Consider a point charge kept at a point in space. If another charge is placed at some distance from the first charge, it experiences an attractive or repulsive force. This is called “action at a distance” [non-contact force]

#### 15. ELECTRIC FIELD [ 2 MARK]

The region of space around a charged particle which exerts a force on another charged particle brought near it.

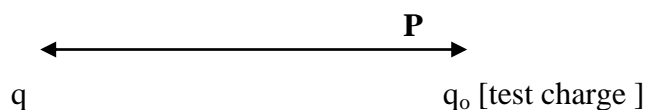
$\vec{F}$  is the force experienced by a charge  $q_0$ .

Electric field,  $\vec{E} = \frac{\vec{F}}{q_0}$

S.I unit  $\rightarrow \text{NC}^{-1}$

Direction of  $\vec{E}$  is along the direction of force.

#### 16. ELECTRIC FIELD DUE TO POINT CHARGE [ 3 MARK]



Consider a source point charge  $q$  placed at some point in space. Let another point charge  $q_0$  be placed at a point ‘p’ which is at a distance ‘r’ from  $q$ .

The force experienced by 'q<sub>0</sub>' is  $\vec{F} = K \frac{q q_0}{r^2} \hat{r}$

Electric field  $\vec{E} = \frac{\vec{F}}{q_0}$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$$

UNIT → NC<sup>-1</sup> ; Vector quantity.

### 17. PROPERTIES OF ELECTRIC FIELD [ 3 MARK ]

1. The electric field of a positive charge is directed away from the source charge and

the electric field of a negative charge is directed towards from the source charge.

2. **Coulomb's law in terms of electric field:-**

If the electric field at a point P is E, then the force experienced by q<sub>0</sub> at p is

$$\vec{F} = q_0 \vec{E}$$

3.  $\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$  .  $\vec{E}$  is independent of the test charge q<sub>0</sub> and depends only

on the source charge 'q'.

4.  $\vec{E}$  is Vector quantity which possesses both magnitude and direction .  $E \propto \frac{1}{r^2}$

$\vec{E}$  decreases in magnitude as the distance increases and vice versa.

5. It is assumed that the test charge is sufficiently smaller such that brightly it near the

source charge will not modify the electric field of the source charge.

6.  $E = \frac{F}{q}$  is valid only for point charges, for continuous and finite charge distributions

integration technique are used.

7. These are two kinds of electric fields, uniform and non uniform electric fields.

### 18. UNIFORM ELECTRIC FIELD[ 2 MARK]

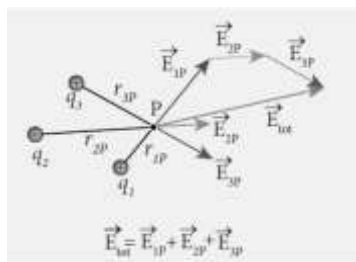
If the electric field has the same direction and constant magnitude at all points in space, then the electric field is said to be uniform.

### 19. NON- UNIFORM ELECTRIC FIELD[ 2 MARK]

If the electric field has different direction or different magnitude or both at different points in space, then the electric field is said to be non-uniform.

### 20. ELECTRIC FIELD DUE TO A SYSTEM OF POINT CHARGES[ 3 MARK]

Electric field obeys the superposition principle. "The electric field due to a collection of charges at any point is equal to the vector sum of the individual charges"



Consider a collection of point charges  $q_1, q_2, q_3, \dots, q_n$  located at various points in space.

$$\vec{E}_{\text{tot}} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \dots + \vec{E}_n$$

$$\vec{E}_1 = \frac{1}{4\pi\epsilon_0} \frac{q_1}{r_{1P}^2} \hat{r}_{1P}; \vec{E}_2 = \frac{1}{4\pi\epsilon_0} \frac{q_2}{r_{2P}^2} \hat{r}_{2P}; \vec{E}_n = \frac{1}{4\pi\epsilon_0} \frac{q_n}{r_{nP}^2} \hat{r}_{nP}.$$

$$\vec{E}_{\text{tot}} = \frac{1}{4\pi\epsilon_0} \left[ \frac{q_1}{r_{1P}^2} \hat{r}_{1P} + \frac{q_2}{r_{2P}^2} \hat{r}_{2P} + \dots + \frac{q_n}{r_{nP}^2} \hat{r}_{nP} \right]$$

$$\vec{E}_{\text{tot}} = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^n \frac{q_i}{r_{iP}^2} \hat{r}_{iP}$$

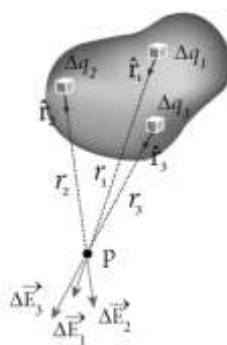
**21. ELECTRIC FIELD DUE TO A CONTINUOUS CHARGE DISTRIBUTION****[ 3 MARK ]**

In order to find the electric due to charged wire, charged sphere etc.,  
The charges are assumed to be distributed continuously, this is because the charges are closely spaced with lesser inter-particles distance. Hence, they are considered as continuously distributed and not discrete.

**22. EXPRESSION FOR  $\vec{E}$  OF A CONTINUOUS CHARGE DISTRIBUTION****[ 3 MARK ]**

Consider a charged irregular object. Divide the entire object into a large number of charge elements  $\Delta q_1, \Delta q_2, \Delta q_3, \dots, \Delta q_n$ . Each charge element  $\Delta q$  is taken as a point charge.

The electric field at a point P, due to the charged object is given by the sum of the field at 'P' due to all charge elements  $\Delta q_1, \Delta q_2, \Delta q_3, \dots, \Delta q_n$



$$\vec{E} \approx \frac{1}{4\pi\epsilon_0} \left[ \frac{q_1}{r_{1P}^2} \hat{r}_{1P} + \frac{q_2}{r_{2P}^2} \hat{r}_{2P} + \dots + \frac{q_n}{r_{nP}^2} \hat{r}_{nP} \right]$$

$$\vec{E} \approx \frac{1}{4\pi\epsilon_0} \sum_{i=1}^n \frac{q_i}{r_{iP}^2} \hat{r}_{iP} \longrightarrow \textcircled{1}$$

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$\Delta q_i$  is the  $i^{\text{th}}$  charge element,  $r_{ip}$  is the distance of the point P from the  $i^{\text{th}}$  charge element and  $\hat{r}_{ip}$  unit vector.

To incorporate the continuous charge distribution take the limit as  $\Delta q \rightarrow 0$

In this limit the summation in the equation (1) becomes an integral.

$$\vec{E} \approx \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r^2} \hat{r}$$

$r \rightarrow$  distance of the point P from the infinite similar charge  $dq$

**23. Show that  $F=qE$  is applicable for continuous charge distribution also:**

**[ 3 MARK ]**

**PROOF**

The electric field for a continuous charge distribution is,

$$E = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r^2}$$

$$E = K \int \frac{dq}{r^2}$$

$$E = \frac{K}{r^2} \int dq$$

$$E = \frac{Kq}{r^2} \rightarrow (1)$$

Multiplying both sides of equation (1) by q,

$$qE = \frac{Kq^2}{r^2}$$

$$F = \frac{Kq^2}{r^2}$$

$F = qE$  is applicable for continuous charge distribution.

**24. LINEAR CHARGE DENSITY ( $\lambda$ ) [ 2 MARK ]**

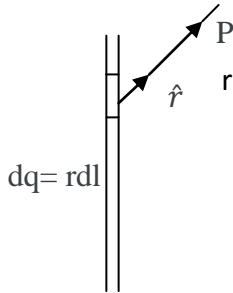
The charge (Q) per unit length (L).

S.I unit  $\rightarrow \text{cm}^{-1}$

$$\lambda = \frac{Q}{L}$$

**25. ELECTRIC FIELD DUE TO A LINEAR CHARGE DISTRIBUTION****[ 3 MARK]**

Consider a linear distribution of charges over a length  $L$  of a wire. Let  $Q$  be the total charge distributed over the length  $L$ . Consider an elementary length  $dl$  is  $dq$ .



The electric field ,

$$d\vec{E} = \frac{\lambda}{4\pi\epsilon_0} \frac{dq}{r^2} \hat{r}$$

$$\lambda = \frac{dq}{dl} \quad (\text{or}) \quad dq = \lambda dl$$

$$d\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{\lambda dl}{r^2} \hat{r}$$

$$\int dE = \int \frac{1}{4\pi\epsilon_0} \frac{\lambda dl}{r^2} \hat{r}$$

$$\vec{E} = \frac{\lambda}{4\pi\epsilon_0} \int \frac{dl}{r^2} \hat{r}$$

**26. SURFACE CHARGE DENSITY ( $\sigma$ ) [ 2 MARK]**

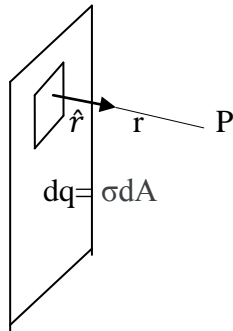
The charge ( $Q$ ) per unit surface area( $A$ ).

$$\sigma = \frac{Q}{A}$$

Unit  $\rightarrow \text{cm}^{-2}$

**27. ELECTRIC FIELD DUE TO A SURFACE CHARGE DISTRIBUTION****[ 3 MARK]**

Consider a charged surface .Let  $Q$  be the charges distributed over the surface of area  $A$ . Let an amount of charge  $dq$  is over  $dA$ .



$$\sigma = \frac{dq}{dA} \quad (\text{or}) \quad dq = \sigma dA$$

$$d\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2} \hat{r}$$

$$\int d\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{\sigma dA}{r^2} \hat{r}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{\sigma dA}{r^2} \hat{r}$$

$$\vec{E} = \frac{\sigma}{4\pi\epsilon_0} \int \frac{dA}{r^2} \hat{r}$$

## 28. VOLUME CHARGE DENSITY( $\rho$ ) [ 2 MARK]

Charge (Q) per unit volume (V)

$$\rho = \frac{Q}{V}$$

unit  $\rightarrow \text{cm}^{-3}$

## 29. ELECTRIC FIELD DUE TO A VOLUME CHARGE DISTRIBUTION

[ 3 MARK]

Consider a charged volume. Let Q charges are distributed over the volume V.

Consider an elementary dv over dq.



$$d\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2} \hat{r}$$

$$\rho = \frac{dq}{dv} \quad (\text{or}) \quad dq = \rho dv$$

$$d\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{\rho dv}{r^2} \hat{r}$$

$$\int d\vec{E} = \int \frac{1}{4\pi\epsilon_0} \frac{\rho dv}{r^2} \hat{r}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{\rho dv}{r^2} \hat{r}$$

$$\boxed{\vec{E} = \frac{\rho}{4\pi\epsilon_0} \int \frac{dv}{r^2} \hat{r}}$$

### 30. ELECTRIC FIELD LINES [ 2 MARK]

Electric field lines are the imaginary lines drawn to visualize the electric field in same region of space .

### 31. RULES FOR DRAWING ELECTRIC FIELD LINES FOR CHARGES

[ 3 MARK]

Electric field vectors are visualized by the concept of electric field lines . They form a set of continuous lines which represent electric field in some region of space visually.

1. The electric field lines start from a positive charge and end at negative Charge or at infinity . For a positive point charge the electric field lines point radially outward and negative point charge electric charge electric field lines point radially inward.

2. The electric field vectors at a point in space is tangential to the electric field lines at that point.

3. The electric field lines are denser [more close] in a region where the electric field has large magnitude and less dense in a region where the electric field is of smaller magnitude. In other words, the number of lines passing through a given surface area perpendicular to the lines is proportional to the magnitude of the electric field in that region.

4. No two electric field lines intersect each other. If two lines cross at a point, then there will be two different electric field vectors at the same point.

5. The number of electric field lines that emanate from the positive charge or end at a negative charge is directly proportional to the magnitude of the charges.

### ELECTRIC DIPOLE AND ITS PROPERTIES

#### 32. ELECTRIC DIPOLE [ 2 MARK]

Two equal and opposite charges separated by a small distance is called an Electric dipole.

#### 33. ELECTRIC DIPOLE MOMENT ( P ) [ 2 MARK]

The product of charge and the distance between them.

For a dipole of '+q' and '-q' separated by a distance 'r'

$$\vec{P} = q2\vec{d} = 2q\vec{d}$$

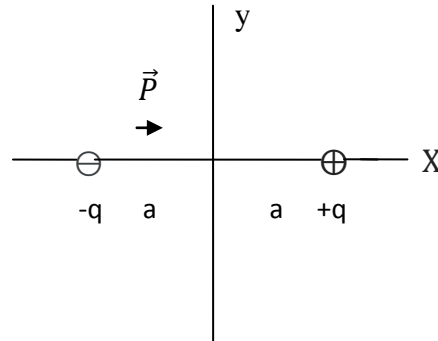
$$\vec{P} = 2q\vec{d}$$

It is vector quantity .

S.I unit  $\rightarrow$  cm.

**34. EXPRESSION FOR ELECTRIC DIPOLE MOMENT  $\vec{P}$  [ 3 MARK]**

Consider two equal and opposite charges (+q, -q) separated by a distance '2a' along the x- axis.



$$\vec{P} = (+q) \vec{r}_+ + (-q) \vec{r}_-$$

$\vec{r}_+ \rightarrow$  Position vector of +q from the origin .

$\vec{r}_- \rightarrow$  Position vector of -q from the origin .

Distance between the charges +q and -q is 2a.

$$\vec{r}_+ = a(\hat{i}) \quad \text{and} \quad \vec{r}_- = a(-\hat{i})$$

$$\vec{P} = qa(\hat{i}) - qa(-\hat{i})$$

$$\boxed{\vec{P} = 2qa\hat{i}}$$

The electric dipole moment vector  $\vec{P}$  lies along the line joining two charges and directed from -q and +q.

**35. PROPERTIES OF ELECTRIC DIPOLE MOMENT [ 3 MARK]**

1.  $\vec{P}$  will point from -q to +q whatever be the direction of its placement.
2. The magnitude of electric dipole moment is equal to the product of magnitude of the charges and the distance between them.

$$|\vec{P}| = 2qa$$

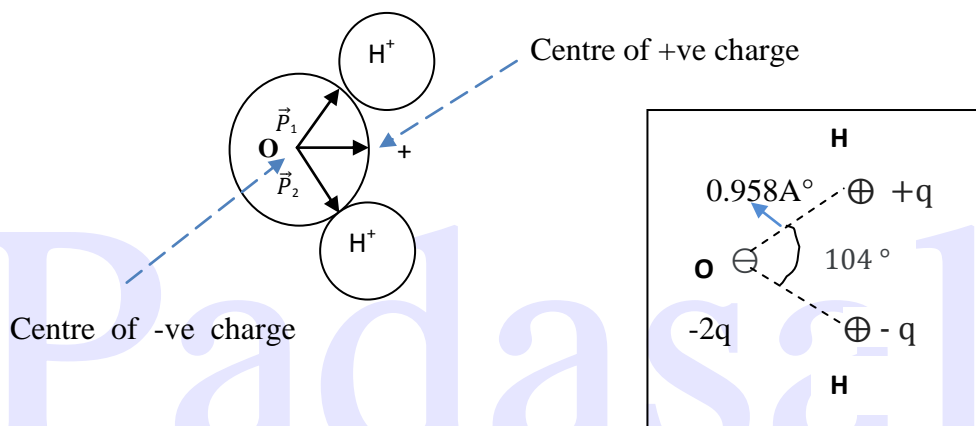
3. For a collection of point charges, the electric dipole moment is

$$\vec{P} = \sum_{i=1}^n q_i \vec{r}_i$$

where  $\vec{r}_i$  is the position vector of  $q_i$ , from the origin

### 36. DIPOLE MOMENT OF H<sub>2</sub>O [ 3 MARK ]

The water molecules (H<sub>2</sub>O) has this charge configuration. The water molecules has three atoms [ two H atom and one O atom ].  $1 \text{ \AA} = 10^{-10} \text{ m}$



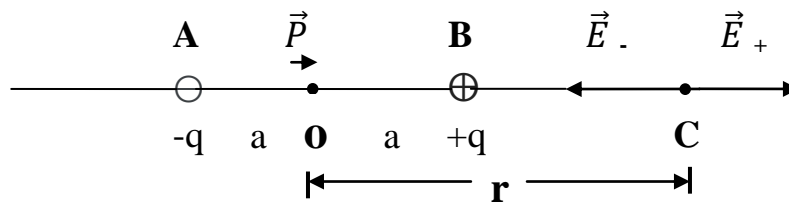
The centers of positive (H<sup>+</sup>) and negative (O) charges of a water molecules lie at different points, hence it possesses permanent dipole moment. The O-H bond length is  $0.958 \text{ \AA}$  due to which the electric dipole moment of water molecule has the magnitude

$$P = 6.1 \times 10^{-30} \text{ cm}$$

The electric dipole moment  $\vec{P}$  is directed from center of negative charge to the centre of positive charge.

**ELECTRIC FIELD DUE TO DIPOLE****37. ELECTRIC FIELD DUE TO AN ELECTRIC DIPOLE AT POINTS ON THE LINE [ 5 MARK]**

Consider an electric dipole separated by a distance  $2a$  placed on x-axis.



A point **C** is located at a distance **r** from the midpoint **O** along the axial line.

The electric field at the point **C** is obtained by summing the field due to **+q** and **-q** at **C**.

The charge **+q** is at a distance **r - a** from **C** and **-q** is at a distance **r + a** from **C**.

$\vec{E}_+$  due to **+q** at the point **C**

$$\vec{E}_+ = \frac{1}{4\pi\epsilon_0} \frac{q}{(r-a)^2} \text{ along BC}$$

$\vec{P}$  is -q to +q and is along BC.

$$\vec{E}_+ = \frac{1}{4\pi\epsilon_0} \frac{q}{(r-a)^2} \hat{P}$$

$\vec{E}_-$  due to **-q** at the point **C**

$$\vec{E}_- = - \frac{1}{4\pi\epsilon_0} \frac{q}{(r+a)^2} \hat{P} \text{ along BC}$$

Since **+q** is closer to the point **C** than **-q**,  $\vec{E}_+$  is stronger than  $\vec{E}_-$ . The length of the vector is a measure of its magnitude. Therefore, the length of  $\vec{E}_+$  vector is drawn larger than that of the point **C** is calculated from the superposition principle of the electric field as,

$$\vec{E}_{\text{tot}} = \vec{E}_+ + \vec{E}_-$$

$$\begin{aligned}
 &= \frac{1}{4\pi\epsilon_0} \frac{q}{(r-a)^2} \hat{P} - \frac{1}{4\pi\epsilon_0} \frac{q}{(r+a)^2} \hat{P} \\
 &= \frac{q}{4\pi\epsilon_0} \left[ \frac{1}{(r-a)^2} - \frac{1}{(r+a)^2} \right] \hat{P} \\
 &= \frac{q}{4\pi\epsilon_0} \left[ \frac{(r+a)^2 - (r-a)^2}{(r-a)^2(r+a)^2} \right] \hat{P} \\
 &= \frac{q}{4\pi\epsilon_0} \left[ \frac{r^2 + 2ra + a^2 - r^2 + 2ra + a^2}{(r-a)^2(r+a)^2} \right] \hat{P} \\
 &= \frac{q}{4\pi\epsilon_0} \left[ \frac{4ra}{(r-a)^2(r+a)^2} \right] \hat{P} \\
 \vec{E}_{\text{tot}} &= \frac{q}{4\pi\epsilon_0} \left[ \frac{4ra}{(r^2 - a^2)^2} \right] \hat{P}
 \end{aligned}$$

1. 'C' is very far away from dipole,  $r \gg a$

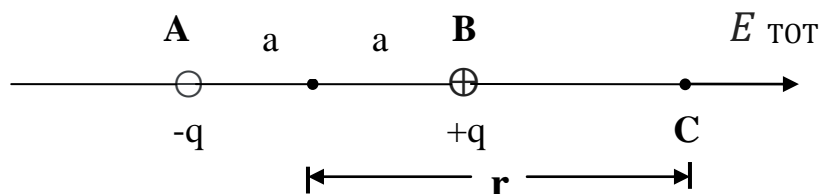
$$(r^2 - a^2)^2 \approx r^4$$

$$\vec{E}_{\text{tot}} = \frac{q}{4\pi\epsilon_0} \left[ \frac{4ra}{(r^2 - a^2)^2} \right] \hat{P}$$

$$\vec{P} = 2aq \cdot \hat{P}$$

$$\vec{E}_{\text{TOT}} = \frac{1}{4\pi\epsilon_0} \cdot \frac{\vec{P}}{r^3}$$

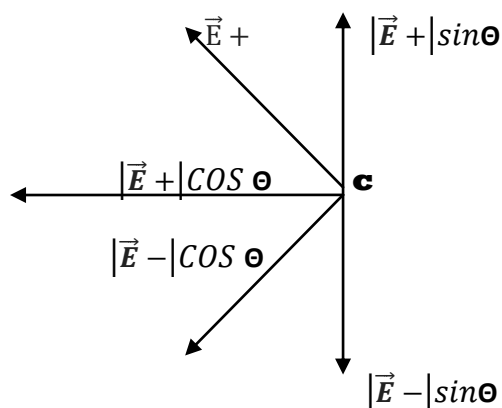
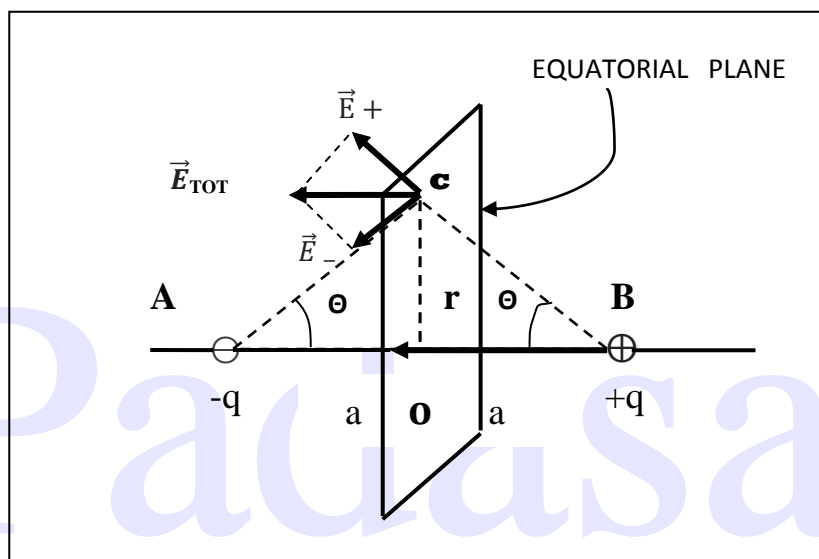
→ (a)



2. Points 'C' is left side of the dipole.  $\vec{E}_{\text{TOT}}$  still in the direction of  $\vec{P}$

**38. ELECTRIC FIELD DUE TO AN ELECTRIC DIPOLE AT A POINT ON THE EQUATORIAL PLANE [ 5 MARK ]**

Consider a dipole separated by a distance '2a'. A point 'C' at a distance 'r' from the midpoint 'O' of the dipole on the equatorial plane. Since C is at equal distance from the charges +q and -q, the magnitude of the electric field  $\vec{E}_+$  and  $\vec{E}_-$  will be the same.  $\vec{E}_+$  is along BC,  $\vec{E}_-$  is along CA.



$\vec{E}_+$  and  $\vec{E}_-$  are resolved into two components, one component parallel to

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the dipole axis and the other perpendicular to it. The perpendicular components are oppositely dissected and cancel each other. Hence the magnitude of the total electric field

at C is the sum of parallel components of  $\vec{E}_+$  and  $\vec{E}_-$ . Total electric field  $\vec{E}_{\text{tot}}$  are along  $-\vec{P}$

$$\vec{E}_{\text{tot}} = -|\vec{E}_+| \cos \theta \hat{P} - |\vec{E}_-| \cos \theta \hat{P}$$

Electric field  $\vec{E}_+$  at C due to  $+q$  is

$$\vec{E}_+ = \frac{1}{4\pi\epsilon_0} \frac{q}{r_+^2} \text{ along BC}$$

$r_+$  is the distance between  $+q$  and C.

From the triangle OCB  $r_+^2 = r^2 + a^2$

$$|\vec{E}_+| = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2 + a^2}$$

Electric field  $\vec{E}_-$  at C due to  $-q$  is

$$|\vec{E}_-| = \frac{1}{4\pi\epsilon_0} \frac{q}{r_-^2} \text{ along CA}$$

From the triangle OCB  $r_-^2 = r^2 + a^2$

$$|\vec{E}_-| = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2 + a^2}$$

$$|\vec{E}_+| = |\vec{E}_-| = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2 + a^2}$$

$$\vec{E}_{\text{tot}} = -\frac{1}{4\pi\epsilon_0} \frac{q}{(r^2 + a^2)} \cos \theta \hat{P} - \frac{1}{4\pi\epsilon_0} \frac{q}{(r^2 + a^2)} \cos \theta \hat{P}$$

$$\vec{E}_{\text{tot}} = -\frac{1}{4\pi\epsilon_0} \frac{2q}{(r^2 + a^2)} \cos \theta \hat{P}$$

In  $\triangle OCB$ ,  $OB=a$ ;  $BC^2 = r^2 + a^2$

$$\cos \theta = \frac{OB}{BC} = \frac{a}{\sqrt{r^2 + a^2}}$$

$$\vec{E}_{\text{tot}} = -\frac{1}{4\pi\epsilon_0} \frac{2qa}{\sqrt{r^2+a^2}(r^2+a^2)} \hat{P}$$

$$\vec{E}_{\text{tot}} = -\frac{1}{4\pi\epsilon_0} \frac{2qa}{(r^2+a^2)^{\frac{3}{2}}} \hat{P}$$

$$\vec{E}_{\text{tot}} = -\frac{1}{4\pi\epsilon_0} \frac{\vec{P}}{(r^2+a^2)^{\frac{3}{2}}}$$

$$\vec{P} = 2qa\hat{P}$$

Point 'C' is very far away from the dipole

$$r \gg a \rightarrow (r^2 + a^2)^{\frac{3}{2}} \approx (r^2)^{\frac{3}{2}} \approx r^3$$

$$\boxed{\vec{E}_{\text{tot}} = -\frac{1}{4\pi\epsilon_0} \frac{\vec{P}}{r^3}} \longrightarrow \text{(b)}$$

### 39. IMPORTANT INFERENCES [ 3 MARK ]

#### 1. RELATION BETWEEN THE MAGNITUDE OF ELECTRIC FIELD DUE TO A DIPOLE AT LARGE DISTANCE ON THE DIPOLE AXIS AND EQUATORIAL PLANE

Comparing (a) and (b) magnitude of the electric field on the dipole axis is twice the magnitude of electric field on the dipole axis is twice the magnitude of electric field at points on the equatorial plane.

Direction  $\vec{E}_{\text{tot}}$  and  $\vec{P}$  are the same at a point at large distance on the axis, but they are oppositely directed at a point at large distance on the equatorial plane.

#### 2. COMPARISON BETWEEN ELECTRIC FIELD DUE TO A DIPOLE AND POINT CHARGE AT LARGE DISTANCE :

At large distance, the electric field due to a dipole  $[E_{\text{tot}}]$  varies as  $\frac{1}{r^3}$ .

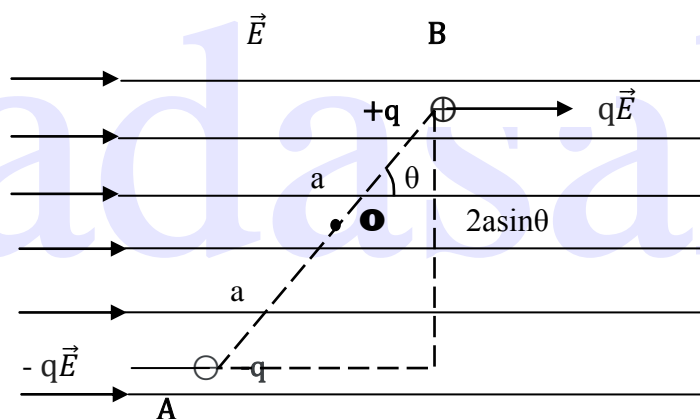
But for a point charge Electric field varies as  $\frac{1}{r^2}$ . Thus at large distance, the electric field due to a dipole goes to zero faster than that of a point charge. Hence at very large distance the two charges of the dipole appear close and neutralize each other.

#### 3. POINT DIPOLE :

If the distance  $2a$  approaches zero and  $q$  approaches infinity such that the product  $\vec{P} = 2qa\hat{P}$  is finite. Then the dipoles is called a point dipole equations (a) and (b) hold true. Thus if the distance between the charge in a dipole is zero, then it is called a point dipole.

#### 40. TORQUE EXPERIENCE BY AN ELECTRIC DIPOLE IN UNIFORM ELECTRIC FIELD: [ 3 MARK]

Consider an electric dipole of dipole moment  $\vec{P}$  placed in a uniform Electric field  $\vec{E}$ .



\* Torque is in to the paper

The charge  $+q$  in the dipole experience a force  $q\vec{E}$  in the direction of the field and the charge  $-q$  experience force  $-q\vec{E}$  in a direction opposite to the direction of  $\vec{E}$ . The forces acting at the two ends of the dipole are equal and opposite. Hence the total force acting on the dipole is zero because these two forces cancel each other. But the torque is not zero.

This torque makes the dipole to rotate through an angle  $\theta$ . The total torque acting at the two ends of the dipole. OA and OB are the perpendicular distance and the force acting at the ends are  $+q\vec{E}$  and  $-q\vec{E}$ .

The total torque on the dipole about the point 'O'

$$\vec{\tau} = \vec{OA} \times (-q\vec{E}) + \vec{OB} \times (q\vec{E})$$

The total torque [from right – hand corkscrew rule] is perpendicular to the plane of paper and direction into it. Since  $\theta$  is angle between  $\vec{OA}$  and  $-q\vec{E}$

$$\vec{OA} \times (-q\vec{E}) = |\vec{OA}| |-q\vec{E}| \sin \theta \hat{n}$$

Similarly,

$$\vec{OB} \times (q\vec{E}) = |\vec{OB}| |q\vec{E}| \sin \theta \hat{n}$$

$$\vec{\tau} = |\vec{OA}| |-q\vec{E}| \sin \theta \hat{n} + |\vec{OB}| |q\vec{E}| \sin \theta \hat{n}$$

$$|\vec{OA}| = |\vec{OB}| = a$$

$$\vec{\tau} = q a E \sin \theta \hat{n} + q a E \sin \theta \hat{n}$$

$$\vec{\tau} = 2 q a E \sin \theta \hat{n}$$

$$|\vec{\tau}| = |2 q a E \sin \theta \hat{n}|$$

$$\vec{\tau} = 2 q a E \sin \theta \hat{n}$$

$$\vec{\tau} = 2 q a E \sin \theta$$

$$\text{Since } |\vec{P}| = 2aq$$

$$\vec{\tau} = \vec{P} \times \vec{E}$$

$$\vec{\tau} = |\vec{P}| |\vec{E}| \sin \theta$$

$$\vec{\tau} = |\vec{P}| |\vec{E}| \sin \theta \hat{n}$$

Case (1) :

$$\tau \text{ is maximum, } \theta = 90^\circ$$

$$\tau = PE \sin 90^\circ$$

$$\tau = PE$$

This torque rotates the dipole and when it is align with the electric field  $\vec{E}$ .

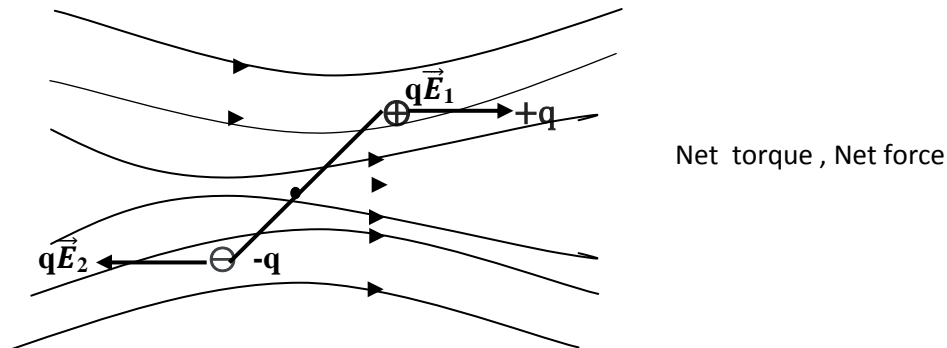
$$\theta = 0 ; \tau = 0$$

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Case (2) : [ 2 MARK]

If the electric field is non – uniform.



The force experienced by  $+q$  will be the same as experienced by  $-q$ . The net force is not zero. Hence in addition to the torque there will be a net force acting on the dipole.

## ELECTROSTATIC POTENTIAL AND ELECTROSTATIC POTENTIAL ENERGY

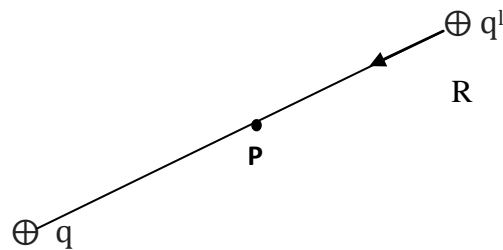
### 41. ELECTICSTATIC POTENTIAL( V ) [ 2 MARK]

The workdone to bring a unit positive charge from infinity to a point in the region of external electric field.

### 42. POTENTIAL DIFFERENCE ( $\Delta V$ ) [ 2 MARK]

The workdone by an external force to bring a unit positive charge from one point to another point in space.

Unit  $\rightarrow \text{Jc}^{-1}$  (or) Volt

**43. EXPRESSION FOR POTENTIAL DIFFERENCE [ 3 MARK]**

Consider a positive charge  $q$  in space. It produces an electric field  $\vec{E}$  which is pointed outwards around it. Let a +ve test charge  $q^l$  is brought from a point R to the point P. The charge  $q^l$  experience a electro static repulsive force due to  $+q$ . Work must be done by the charge  $q^l$  to overcome this repulsive force. This workdone is stored as potential energy. The test charge  $q^l$  is brought from R to P with a constant velocity so that the external force needed to bring the charge  $q^l$  from R to P must equal and opposite to the coulomb force of repulsion.

$$\vec{F}_{ext} = \vec{F}_{cou}$$

The workdone to move the charge from R to P

$$W = \text{Force} \times \text{distance}$$

Let  $dr$  be the small distance moved by  $q^l$  and  $dw$  is the workdone.

$$dw = \vec{F} \cdot d\vec{r}$$

Let the potential energy of  $q^l$  be  $U_P$  at P and  $U_R$  at R.

$$W = U_P - U_R$$

$$U_P - U_R = \Delta U$$

$$W = \Delta U$$

$$\Delta U = \int_R^P \vec{F}_{ext} \cdot d\vec{l}$$

$$\vec{F}_{ext} = -q^l \vec{E}$$

Negative sign  $\rightarrow \vec{F}_{ext}$  is opposite to  $\vec{E}$ .

$$\Delta U = \int_R^P (-q^l \vec{E}) \cdot d\vec{r}$$

$$\Delta U = q^l \int_R^P (-\vec{E}) \cdot d\vec{r}$$

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The potential energy difference per unit charge

$$\frac{\Delta U}{q^l} = \frac{q^l \int_R^P (-\vec{E}) \cdot d\vec{r}}{q^l} = - \int_R^P \vec{E} \cdot d\vec{r}$$

$$\boxed{\frac{\Delta U}{q^l} = - \int_R^P \vec{E} \cdot d\vec{r}}$$

The above equation is independent of  $q^l$ .

The quantity  $\frac{\Delta U}{q^l} = - \int_R^P \vec{E} \cdot d\vec{r}$  is called electro potential difference R and P.

$$V_P - V_R = \Delta V$$

$$V_P - V_R = \Delta V = - \int_R^P \vec{E} \cdot d\vec{r}$$

$$\frac{\Delta U}{q^l} = - \int_R^P \vec{E} \cdot d\vec{r}$$

$$\Delta U = -q^l \int_R^P \vec{E} \cdot d\vec{r}$$

$$\Delta U = \Delta V q^l \quad \left[ \text{Since, } \Delta V = - \int_R^P \vec{E} \cdot d\vec{r} \right]$$

If the point R is taken at infinity the potential is zero.  $[V_\infty = 0 \text{ because } V = \frac{Kq}{r} = \frac{Kq}{\infty} = 0]$

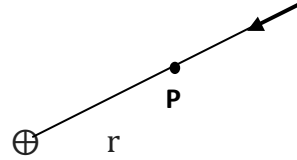
The electric potential at a point P is equal to the workdone by an external force to bring a unit positive charge with constant velocity from infinity to a point P in the velocity from infinity to a point P in the region external field which can be expressed mathematically as,

$$\boxed{\Delta V = - \int_R^P \vec{E} \cdot d\vec{r}}$$

The electric potential at a point P depends only on the electric field due to the source charge q and not on the test charge  $q^l$ .

**44. ELECTRIC POTENTIAL DUE TO A POINT CHARGE : [ 3 MARK ]**

Consider a positive charge  $q$  at the origin. Let P be a point at a distance 'r' from the charge  $q$ .



Electric potential at the point 'P' is,

$$V = - \int_{\infty}^r \vec{E} \cdot d\vec{r}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$$

$$V = - \int_{\infty}^r \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r} \cdot d\vec{r}$$

Small displacement  $d\vec{r}$  is along the direction of  $\vec{r}$ . Let  $\hat{r}$  be the unit vector along  $d\vec{r}$ ,

$$d\vec{r} = dr \cdot \hat{r}$$

$$V = - \frac{1}{4\pi\epsilon_0} \int_{\infty}^r \frac{q}{r^2} \hat{r} \cdot dr \cdot \hat{r}$$

$$= - \frac{1}{4\pi\epsilon_0} \int_{\infty}^r \frac{q}{r^2} dr (\hat{r} \cdot \hat{r})$$

$$\hat{r} \cdot \hat{r} = 1$$

$$= - \frac{1}{4\pi\epsilon_0} \int_{\infty}^r \frac{q}{r^2} dr$$

$$= - \frac{q}{4\pi\epsilon_0} \left[ \frac{r^{-2+1}}{-2+1} \right]_{\infty}^r$$

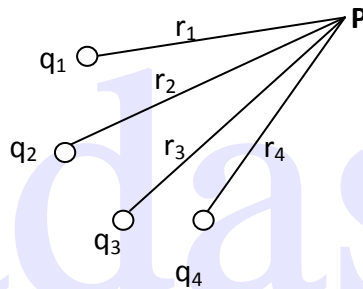
$$= - \frac{q}{4\pi\epsilon_0} \left[ -\frac{1}{r} \right]_{\infty}^r$$

$$= \frac{q}{4\pi\epsilon_0} \left[ \frac{1}{r} - \frac{1}{\infty} \right]$$

$$V = \frac{q}{4\pi\epsilon_0 r} \quad (\text{or}) \quad V = \frac{Kq}{r}$$

**45. PROPERTIES OF POTENTIAL : [ 3 MARK]**

1. The potential  $V$  is positive for a positive source charge ( $V > 0$ ) and  $V$  is negative if the source charge is negative ( $V < 0$ ).
2. The motion of charges can be described easily in terms of potential that that of field.
3.  $V \propto \frac{q}{r}$  Hence as the distance  $r$  increases the potential decreases. If the potential charge  $q$  is negative  $V \propto -\frac{q}{r}$  the potential increases as the distance is increased. The electric potential is zero. [ $V = 0$ ]
4. Super position principle :



Electric potential  $V$  obeys superposition principle. Consider a collection of charges  $q_1, q_2, q_3, \dots, q_n$ . The electric potential at a point 'P' due to these charges are the sum of electric potential due to individual charges.

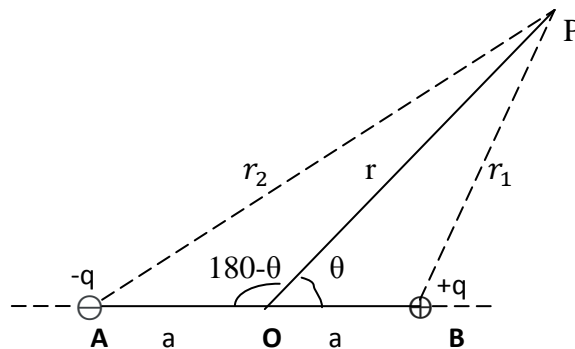
$$V_{\text{tot}} = \frac{Kq_1}{r_1} + \frac{Kq_2}{r_2} + \frac{Kq_3}{r_3} + \dots + \frac{Kq_n}{r_n}$$

$$V_{\text{tot}} = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^n \frac{q_i}{r_i}$$

Potential is always a scalar quantity .

**46. ELECTROSTATIC POTENTIAL AT THE POINT DUE TO AN ELECTRIC DIPOLE [ 5 MARK ]**

Consider two equal and opposite charges separated by a small distance  $2a$ . The point P is located at a distance  $r$  from the midpoint of the dipole. Let  $\theta$  be the angle between the line OP and dipole axis AB.



$$BP = r_1; AP = r_2$$

$$\text{Potential at P due to } +q = \frac{1}{4\pi\epsilon_0} \frac{q}{r_1}$$

$$\text{Potential at P due to } -q = -\frac{1}{4\pi\epsilon_0} \frac{q}{r_2}$$

Total potential at the point P,

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r_1} - \frac{1}{4\pi\epsilon_0} \frac{q}{r_2}$$

$$V = \frac{q}{4\pi\epsilon_0} \left[ \frac{1}{r_1} - \frac{1}{r_2} \right]$$

$$a \ll r$$

By the cosine law for triangle BOP,

$$r_1^2 = r^2 + a^2 - 2ra \cos\theta$$

$$r_1^2 = r^2 \left[ 1 + \frac{a^2}{r^2} - \frac{2a}{r} \cos\theta \right]$$

$a \ll r$ , the term  $\frac{a^2}{r^2}$  is very small and can be neglected

$$r_1^2 = r^2 \left[ 1 - \frac{2a}{r} \cos\theta \right]$$

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$$r_1 = r \left[ 1 - \frac{2a}{r} \cos\theta \right]^{\frac{1}{2}}$$

$$\frac{1}{r_1} = \frac{1}{r \left[ 1 - \frac{2a}{r} \cos\theta \right]^{\frac{1}{2}}}$$

$$\frac{1}{r_1} = \frac{1}{r} \left[ 1 - \frac{2a}{r} \cos\theta \right]^{\frac{-1}{2}}$$

$\frac{a}{r} \ll 1$ , we can use binomial theorem.

$$\boxed{\frac{1}{r_1} = \frac{1}{r} \left[ 1 + \frac{a}{r} \cos\theta \right]}$$

Similarly applying the cosine law for triangle AOP,

$$r_2^2 = r^2 + a^2 - 2ra \cos(180 - \theta)$$

$$r_2^2 = r^2 + a^2 + 2ra \cos\theta \quad [\cos(180 - \theta) = -\cos\theta]$$

$$r_2^2 = r^2 \left[ 1 + \frac{a^2}{r^2} + \frac{2a}{r} \cos\theta \right]$$

$$r_2^2 = r^2 \left[ 1 + \frac{2a}{r} \cos\theta \right] \quad \text{Neglecting } \frac{a^2}{r^2} [a \ll r]$$

$$r_2 = r \left[ 1 + \frac{2a}{r} \cos\theta \right]^{\frac{1}{2}}$$

$$\frac{1}{r_2} = \frac{1}{r} \left[ 1 + \frac{2a}{r} \cos\theta \right]^{\frac{-1}{2}}$$

Use Binomial theorem,

$$\boxed{\frac{1}{r_2} = \frac{1}{r} \left[ 1 - \frac{a}{r} \cos\theta \right]}$$

$$V = \frac{q}{4\pi\epsilon_0} \left[ \frac{1}{r} \left( 1 + \frac{a}{r} \cos\theta \right) - \frac{1}{r} \left( 1 - \frac{a}{r} \cos\theta \right) \right]$$

$$V = \frac{q}{4\pi\epsilon_0} \left[ \frac{1}{r} \left( 1 + \frac{a}{r} \cos\theta - 1 + \frac{a}{r} \cos\theta \right) \right]$$

$$V = \frac{1}{4\pi\epsilon_0} \frac{2aq}{r_2} \cos\theta$$

$$P = 2aq$$

$$\boxed{V = \frac{1}{4\pi\epsilon_0} \frac{P}{r_2} \cos\theta}$$

$$P \cos\theta = \vec{P} \cdot \vec{r}$$

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$$V = \frac{1}{4\pi\epsilon_0} \frac{\vec{P} \cdot \vec{r}}{r^2}$$

**Special cases :****Case (1)**

If the point P lies on the axial line of the dipole on the side of +q, then  $\theta=0^\circ$ .

$$V = \frac{1}{4\pi\epsilon_0} \frac{P}{r^2}$$

$$\cos 0^\circ = 1$$

**Case (2)**

The point P lies on the axial line of the dipole on the side of -q, then  $\theta=180^\circ$ .

$$V = - \frac{1}{4\pi\epsilon_0} \frac{P}{r^2}$$

$$\cos 180^\circ = -1$$

**Case (3)**

The point P lies on the equatorial line of the dipole, then  $\theta=90^\circ$ .

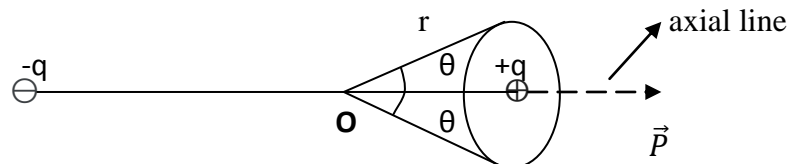
$$V = 0$$

$$\cos 90^\circ = 0$$

**IMPORTANT POINTS**

1. The potential due to an electric dipole falls as  $\frac{1}{r^2}$  and the potential due to single point charge falls as  $\frac{1}{r}$ . Thus, the potential point charge falls faster than that due to a monopole. As the distance increase from electric dipole, the effects of positive and negative charges nullify each other.

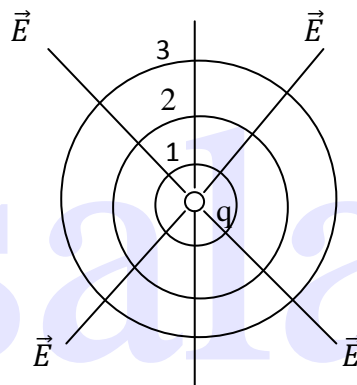
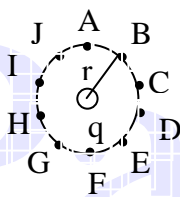
2. The potential due to a point charge is spherically symmetric since it depends only on the distance r. But the potential due to a dipole is not spherically symmetric because the potential depends on the angle between  $\vec{P}$  and position vector  $\vec{r}$  of the point.



However, the dipole potential is axially symmetric, If the position vector  $\vec{r}$  is rotated about  $\vec{P}$  by keeping  $\theta$  fixed, then all points, on the cone potential as shown in figure.

#### 47. EQUI-POTENTIAL SURFACE[ 3 MARK]

An equipotential surface is a surface on which all the points are at the same Potential .



Consider a point charged 'q' in space. Assume an imaginary spherical surface of radius 'r' with the charge of its centre .

The potential of the point charge is  $V = \frac{Kq}{r}$  . As the radius r is the same at each and every point along the spherical surface . The potential V at all points on the surface of the sphere is also the same . Such a surface is called an equipotential surface.

All spherical surface drawn concentric to the equipotential surface will also be equipotential surface . But the value of the potential will differ for different spherical surfaces [ as r varies ] . For a uniform field , the equipotential surfaces form a set of planes normal to the electric field  $\vec{E}$

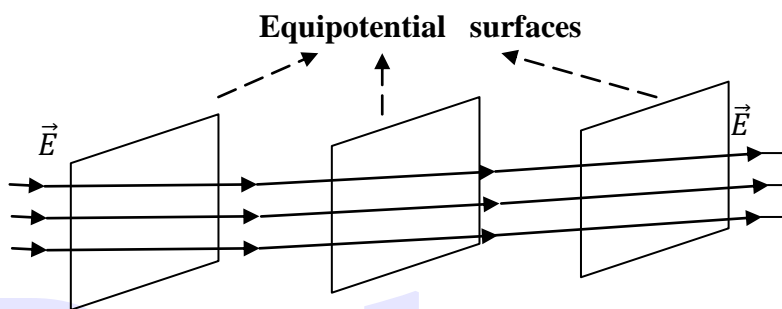
**48. PROPERTIES OF EQUIPOTENTIAL SURFACES[ 3 MARK]**

1. The workdone to move a charge  $q$  from one point to another on the same equipotential surface is zero.

Example :

Consider two points A and B. The workdone to move a charge from A to B is  $W = q(V_B - V_A)$ . Since A and B have the same potential  $V_B - V_A = 0$  (or)  $W = 0$

2. The electric field  $\vec{E}$  is always normal to an equipotential surfaces.

**RELATION BETWEEN ELECTRIC FIELD AND POTENTIAL**

Consider a positive charge  $q$  kept fixed at the origin. To move a unit positive charge by a small distance  $dx$  in the electric field  $E$ , the work done is given by,

workdone = Force  $\times$  distance

$$W = F \cdot x$$

$$F = qE$$

$$W = qE \cdot x$$

$$dW = -qE \cdot dx$$

Given  $q$  is a unit point charge

$$q = 1C$$

$$dW = -E \cdot dx$$

$$E = -\frac{dv}{dx}$$

The electric field is the negative gradient of the electric potential.

$$\vec{E} = - \left[ \frac{\partial v}{\partial x} \vec{i} + \frac{\partial v}{\partial y} \vec{j} + \frac{\partial v}{\partial z} \vec{k} \right]$$

#### 49. ELECTROSTATIC POTENTIAL ENERGY FOR COLLECTION OF POINT CHARGE WORKDONE TO MOVE A CHARGE [ 3 MARK]

The electric potential at a point at a distance  $r$  from a point charge  $q_1$  is

$$V = \frac{1}{4\pi\epsilon_0} \frac{q_1}{r}$$

In order to bring another charge  $q_2$  from infinity to a point in the field of  $q_1$  at a distance  $r$  from  $q_1$  work has to be done.

$$W = q_2 V$$

Workdone is the product of charge and potential. This workdone is stored as the electrostatic potential energy ( $U$ ) in a system of charges  $q_1$  and  $q_2$  is

$$U = q_2 V = q_2 \frac{1}{4\pi\epsilon_0} \frac{q_1}{r}$$

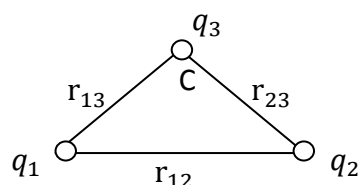
The electrostatic potential energy ( $U$ ) in a system of charges  $q_1$  and  $q_2$  is

$$U = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$$

Electrostatic potential energy depends only on the distance between the two charges  $q_1$  and  $q_2$ . This equation holds good for any collection of point charges.

#### 50. EXPRESSION FOR ELECTROSTATIC POTENTIAL ENERGY OF A COLLECTION OF POINT CHARGES [ 3 MARK]

Consider three point charges  $q_1$ ,  $q_2$  and  $q_3$ . In order to assemble these three charges in a region of space work has to be done. Workdone is stored in the form of potential energy. Bring the three charges one by one to a configuration as shown in the figure.



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Let the charge  $q_1$  is brought from infinity to that point A. The charge  $q_1$  does not experience any field no other charges was present in the present in the vicinity of  $q_1$ .

Hence, the workdone is zero. The potential due to the charge  $q_1$  is zero. But the charge  $q_1$  creates an electric field. Let another charge  $q_2$  is brought to a point B which is at a distance of  $r_{12}$  from  $q_1$ . Hence work must be done against the field. The workdone is bringing the charge  $q_1$  to B is

$$W = q_1 V_{1B}$$

Since, Workdone = charge X potential.

$V_{1B} \rightarrow$  Electrostatic potential energy due to the charge  $q_1$  at B.

The electrostatic potential energy due to  $q_2$  is,

$$U = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}}$$

Now the electric field is produced by the two charges  $q_1$  and  $q_2$ . Let another charge  $q_3$  be brought to the point C which is at a distance of  $r_{13}$  from  $q_1$  and  $r_{23}$  from  $q_2$ .

The workdone to bring the charge  $q_2$  to C is,

$$W = q_3 [V_{1C} + V_{2C}]$$

$V_{1C} \rightarrow$  Electrostatic potential energy due to the charge  $q_1$  at C

$V_{2C} \rightarrow$  Electrostatic potential energy due to the charge  $q_2$  at C

The electrostatics potential energy due to the charge  $q_3$  is,

$$U = \frac{1}{4\pi\epsilon_0} \left[ \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right]$$

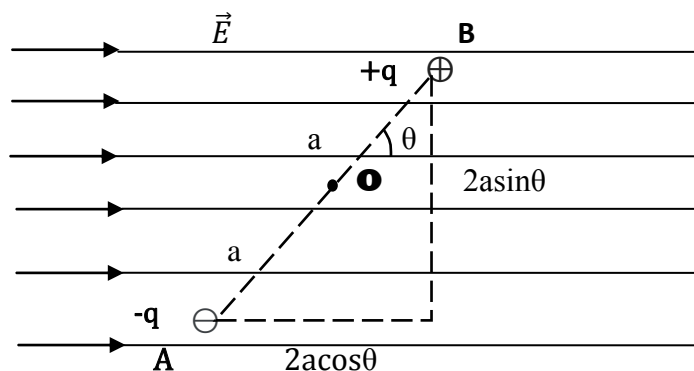
The total electrostatic potential energy for assembly  $q_1$ ,  $q_2$  and  $q_3$  is,

$$U = \frac{1}{4\pi\epsilon_0} \left[ \frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right]$$

The above expression is the same if the charges are brought to these positions in any other order. (i.e.) The electrostatic potential energy is independent of the manner in which the charges arrived.

**51. ELECTROSTATIC POTENTIAL ENERGY OF A DIPOLE IN A UNIFORM ELECTRIC FIELD [ 3 MARK ]**

Consider a dipole placed in a uniform electric field  $\vec{E}$  as shown in the figure



The electric field exerts a force on the dipole. As a result, the dipole experiences a torque and aligns the dipole in the direction of electric field. To rotate the dipole at constant angular velocity from an initial angle ' $\theta^1$ ' to another angle  $\theta$  against the torque exerted by the electric field, an equal and opposite external torque must be applied on the dipole.

The work done by the external torque to rotate the dipole from angle  $\theta^1$  to  $\theta$  at constant angular velocity is,

$$W = \int_{\theta^1}^{\theta} \tau_{ext} \cdot d\theta$$

Torque due to the field

$$\vec{\tau}_E = \vec{P} \times \vec{E}$$

$$|\tau_{ext}| = |\vec{\tau}_E| = |\vec{P} \times \vec{E}|$$

$$W = \int_{\theta^1}^{\theta} |\vec{P} \times \vec{E}| \cdot d\theta$$

$$= \int_{\theta^1}^{\theta} |\vec{P}| |\vec{E}| \sin\theta \cdot d\theta$$

$$= PE [-\cos\theta]_{\theta^1}^{\theta}$$

$$= PE [-\cos\theta + \cos\theta^1]$$

$$W = PE [\cos\theta^1 - \cos\theta]$$

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The above equation for workdone is equal to the potential energy difference between the angular position  $\theta$  and  $\theta^1$ .

$$U(\theta) - U(\theta^1) = \Delta U = -PE\cos\theta + PE\cos\theta^1$$

If  $\theta^1 = 90^\circ$  and is taken as reference point.

$$U(\theta^1) = PE\cos 90^\circ = 0$$

$$U = -PE\cos\theta = -\vec{P} \cdot \vec{E}$$

$$U = -\vec{P} \cdot \vec{E}$$

The potential energy depends on  $\vec{P}$  and  $\vec{E}$ , but also on the oriented  $\theta$  of the dipole with respect to the electric field.

**Special cases:**

1. If  $\theta = \pi$ , the dipole is aligned antiparallel to the electric field and potential energy is maximum.
2. If  $\theta = 0$ , the dipole is aligned parallel to the electric field and potential energy is Minimum.

**GAUSS LAW AND ITS APPLICATIONS****52. ELECTRIC FLUX ( $\phi_E$ ) [ 2 MARK]**

The number of electric field lines crossing a given area kept normal to the electric field lines is called electric flux.

$$\text{Unit} \rightarrow \text{Nm}^2\text{c}^{-1}$$

$\phi_E \rightarrow$  Scalar and it may +ve or -ve.

**53. ELECTRIC FLUX FOR UNIFORM ELECTRIC FIELD [ 2 MARK]**

Consider a uniform electric field in a region of space. Let us choose an area  $A$  normal to the electric field lines as shown in Figure. The electric flux for this case is

$$\phi_E = EA$$

Suppose the same area  $A$  is kept parallel to the uniform electric field, then no

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electric field lines pierce through the area A, The electric flux for this case is zero.

$$\phi_E = 0$$

If the area is inclined at an angle  $\theta$  with the field, then the component of the electric field perpendicular to the area alone contributes to the electric flux. The electric field component parallel to the surface area will not contribute to the electric flux. The electric flux,

$$\phi_E = (E \cos\theta) A$$

Further,  $\theta$  is also the angle between the electric field and the direction normal to the area. Hence in general, for uniform electric field, the electric flux is

$$\phi_E = \vec{E} \cdot \vec{A} = E A \cos\theta$$

Here, note that  $\vec{A}$  is the area vector  $\vec{A} = A\hat{n}$ . Its magnitude is simply the area A and the direction is along the unit vector  $\hat{n}$  perpendicular to the area

$$\phi_E = \vec{E} \cdot \vec{A}$$

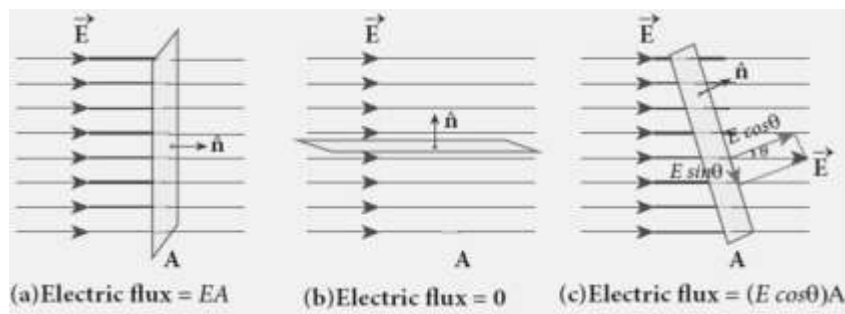
$$\theta = 0^\circ ; \quad \phi_E = E A \cos 0^\circ = EA$$

$$\phi_E = EA$$

$$\theta = 90^\circ ; \quad \phi_E = E A \cos 90^\circ$$

$$\phi_E = 0$$

Here,  $\vec{A} = A\hat{n}$



**54. ELECTRIC FLUX IN A NON – UNIFORM ELECTRIC FIELD AND AN ARBITRARILY SHAPED AREA: [ 3 MARK]**

Suppose the electric field is not uniform and the area A is not flat then the entire area is divided into 'n' small area segments  $\Delta\vec{A}_1, \Delta\vec{A}_2, \Delta\vec{A}_3, \dots, \Delta\vec{A}_n$  such that each area element is almost flat and the electric field over each area element is considered to be uniform. The electric flux for the entire area A is approximately written as

$$\phi_E = \vec{E}_1 \cdot \Delta\vec{A}_1 + \vec{E}_2 \cdot \Delta\vec{A}_2 + \vec{E}_3 \cdot \Delta\vec{A}_3 + \dots + \vec{E}_n \cdot \Delta\vec{A}_n$$

$$\phi_E = \sum_{i=1}^n \vec{E}_i \cdot \Delta\vec{A}_i \longrightarrow (a)$$

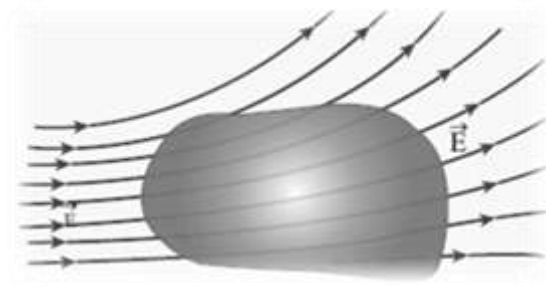
By taking  $\Delta\vec{A}_i \rightarrow 0$ , the summation in equation (a) becomes an integral,

$$\phi_E = \int \vec{E} \cdot d\vec{A}$$

Thus electric flux depends on the electric field on the surface and the orientation of the surface with the electric field.

**55. ELECTRIC FLUX FOR CLOSED SURFACE [ 3 MARK]**

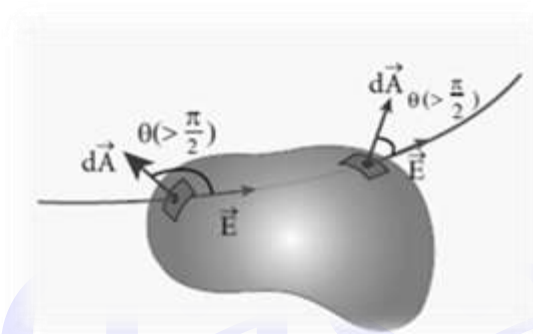
Consider a closed surface in a region of non – uniform electric field as shown in figure.



(a)

The total electric flux over the closed surface is

$$\phi_E = \oint \vec{E} \cdot d\vec{A}$$



(b)

Let the flux lines enter and leave the surface through an elementary area  $d\vec{A}$ , as shown in figure (b). For each elemental area, the outward normal is the direction of  $d\vec{A}$ . In the figure (b) the angle between  $d\vec{A}$  and  $\vec{E}$  is greater than  $90^\circ$  for one area element less than  $90^\circ$  for the other elementary area. For  $\theta < 90^\circ$ , the electric flux is positive and for  $\theta > 90^\circ$ , the electric field is negative.

[  $\phi = EA \cos \theta$ ,  $\cos \theta$  has negative values if  $\theta > 90^\circ$ , and the value is positive for  $\theta < 90^\circ$  ]

Thus, the electric flux  $\phi_E$  is negative if the electric field lines enter the closed surface and positive if the electric field lines leave the closed surface.

$$\phi_E = \oint \vec{E} \cdot d\vec{A}$$

$$\phi_E = \oint E \cdot d\vec{A} \cos\theta$$

$\theta$  is the angle between  $\vec{E}$  and  $d\vec{A}$ .

### 56. GAUSS LAW [ 2 MARK ]

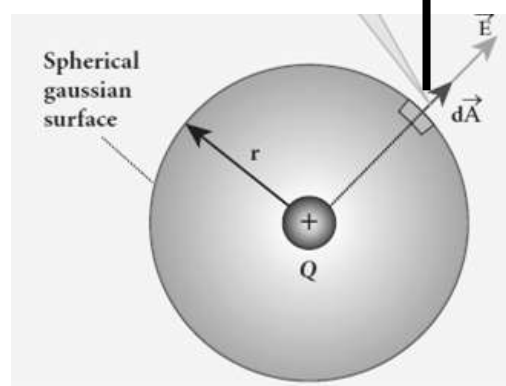
If a charge  $Q$  is enclosed by an arbitrary closed surface, then the total electric flux  $\phi_E$  over the closed surface is  $\frac{1}{\epsilon_0}$  times the total charges  $Q$  enclosed by the surfaces.

$$\phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

' $Q'_{encl}$  represents the charges inside the closed surfaces.

### PROOF [ 3 MARK ]

When the charge is at the centre of the sphere, the electric field is everywhere normal to the surface and constant in magnitude.



Consider a point  $+q$  placed at the centre of a sphere, as shown in figure. The electric field  $\vec{E}$  is directed radially outward at all points on the surface of the sphere. Let  $d\vec{A}$  be the elementary area on the surface of the sphere. The direction of  $d\vec{A}$

is along the same direction as the electric field  $\vec{E}$ . Hence the angle between  $\vec{E}$  and  $d\vec{A}$  is zero. (i.e)  $\theta=0^\circ$ .

$$\phi_E = \oint \vec{E} \cdot d\vec{A} = \oint E \cdot dA \cos\theta$$

$$\theta=0^\circ.$$

$$\phi_E = \oint E \cdot dA \cos\theta$$

$$\phi_E = \oint E \cdot dA$$

As the electric field has the same magnitude along the surface of the sphere .

$$\phi_E = E \oint dA = EA$$

A  $\rightarrow$  Area of the sphere ,

r  $\rightarrow$  radius of the sphere , then  $A = 4\pi r^2$

$$\phi_E = EA = E \cdot 4\pi r^2$$

E is the electric field due to point charge Q

$$\phi_E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} 4\pi r^2$$

$$\boxed{\phi_E = \frac{Q}{\epsilon_0}}$$

This law is applicable for any arbitrary shaped closed surfaces which encloses a charge Q.

### 57. IMPORTANT ASPECTS OF GAUSS LAW : [ 3 MARK]

1. From the equation  $\phi_E = \frac{Q}{\epsilon_0}$  , it is observed that the total electric flux depends only on the charges enclosed by the closed surface and independent of the charges present outside the surface , and the surface of the closed surface .
2. The total electric flux is independent of the location of the charges inside the closed surface

**3. Gaussian Surface :**

The imaginary surfaces chosen to find the electric field using Gauss law is called a **Gaussian surface** depends on the type of charge configuration and the type of symmetry existing in that charge distribution.

- a) For a point charge and spherical charge distribution the Gaussian surface is in the form of a sphere concentric to the charge distribution
- b) For a cylindrical charge distribution the Gaussian surface is taken in the form of a cylinder
- c) For a plane charge distribution, the Gaussian surface is taken in the form of a Pill box.

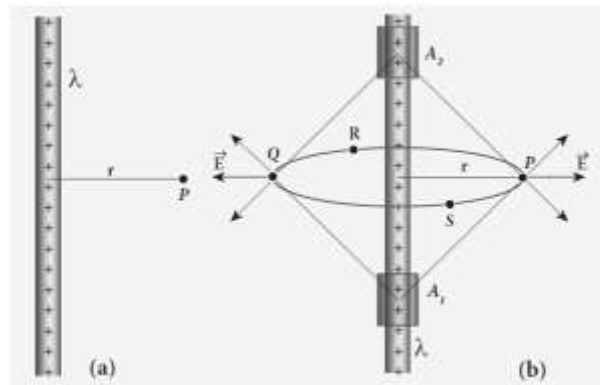
4. The electric field  $\vec{E}$  depends only on the charges enclosed by the closed surfaces

5. Gaussian surface is not applicable for discrete charges, because the electric field is not well defined. Gaussian surface is applicable for continuous charge distribution.

6. Gauss law is another form of Coulomb's law and it is also applicable to the charges in motion. Thus, Gauss law is treated as a more general law than Coulomb's law.

**APPLICATION OF GAUSS LAW:****58. ELECTRIC FIELD DUE TO AN INFINITY LONG WIRE (or)****CYLINDRICAL CHARGE DISTRIBUTION [ 5 MARK]**

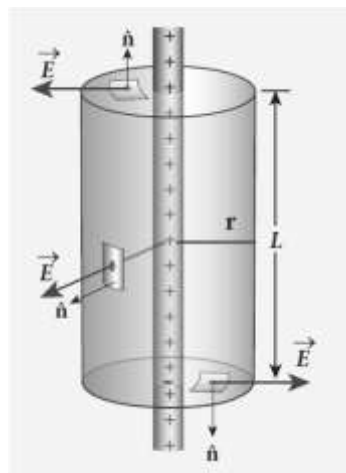
Consider an infinity long straight wire with linear charge density  $\lambda$



The electric field due to the charged wire at a point P which is at a distance  $r$  from the wire is determined using Gauss law. Consider two charge elements  $A_1$  and  $A_2$  on the wire which are at equal distance from P. The resultant field due to these charge elements points radially outwards from the wire. Moreover, the magnitude of the electric field at a distance  $r$  is the same around the wire. Hence, the wire possesses cylindrical symmetry.

**To find the electric field  $\vec{E}$**

Consider a cylindrical Gaussian surface of radius  $r$  and length  $L$ .



Because of cylindrical symmetry, the Gaussian surface should be taken in the form of a cylinder.

From Gauss law,

$$\phi_E = \oint \vec{E} \cdot d\vec{A}$$

The cylindrical charge distribution contains three surfaces, top, bottom and curved surface. All these surfaces contribute to the total electric flux

$$\phi_E = \int_{\text{Curved surface}} \vec{E} \cdot d\vec{A} + \int_{\text{top surface}} \vec{E} \cdot d\vec{A} + \int_{\text{Bottom surface}} \vec{E} \cdot d\vec{A}$$

For the top and bottom surface  $\vec{E}$  and  $\hat{n}$  vectors are perpendicular.

Therefore,  $\vec{E}$  and  $d\vec{A}$  are perpendicular.

$$\vec{E} \cdot d\vec{A} = |\vec{E}| |d\vec{A}| \cos 90^\circ$$

$$\vec{E} \cdot d\vec{A} = E \times dA \times 0$$

$$\vec{E} \cdot d\vec{A} = 0$$

For the top and bottom surfaces,

$$\int_{\text{top surface}} \vec{E} \cdot d\vec{A} = 0 \quad \text{and} \quad \int_{\text{Bottom surface}} \vec{E} \cdot d\vec{A} = 0$$

$$\phi_E = \int_{\text{Curved surface}} \vec{E} \cdot d\vec{A}$$

For the curved surface, the  $\hat{n}$  parallel to  $\vec{E}$ . Hence,  $\vec{E}$  is parallel to  $d\vec{A}$

$$\vec{E} \cdot d\vec{A} = |\vec{E}| |d\vec{A}| \cos 0^\circ$$

$$\vec{E} \cdot d\vec{A} = E \cdot dA \times 1$$

$$\vec{E} \cdot d\vec{A} = E \cdot dA$$

$$\phi_E = \int E \cdot dA$$

$\vec{E}$  has constant magnitude around the closed surface.

$$\phi_E = E \int dA$$

$$\phi_E = EA$$

Charge density  $\lambda = \frac{dQ}{dL}$ ;  $dQ = \lambda dL$

$$\int dQ = \int \lambda \cdot dL$$

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$$Q = \int \lambda \cdot dL$$

Using Gauss law,

$$\phi_E = \int \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

Curved  
surface

$$\phi_E = EA = \frac{\int \lambda \cdot dL}{\epsilon_0}$$

$$EA = \frac{1}{\epsilon_0} \lambda \int dL$$

$$EA = \frac{1}{\epsilon_0} \lambda L.$$

A → Area of the curved surface.

$$A = 2\pi rL \quad [\text{circumference} \times \text{Length}]$$

$$E \cdot 2\pi rL = \frac{\lambda L}{\epsilon_0}$$

$$E = \frac{\lambda}{2\pi r\epsilon_0}$$

Vector form,

$$\vec{E} = \frac{1}{2\pi r\epsilon_0} \frac{\lambda}{r} \hat{r} \quad \longrightarrow \quad (a)$$

The electric field due to an infinitely long charged wire is  $\vec{E} = \frac{1}{2\pi r\epsilon_0} \frac{\lambda}{r} \hat{r}$

$\vec{E}$  is depends on  $\frac{1}{r}$ .

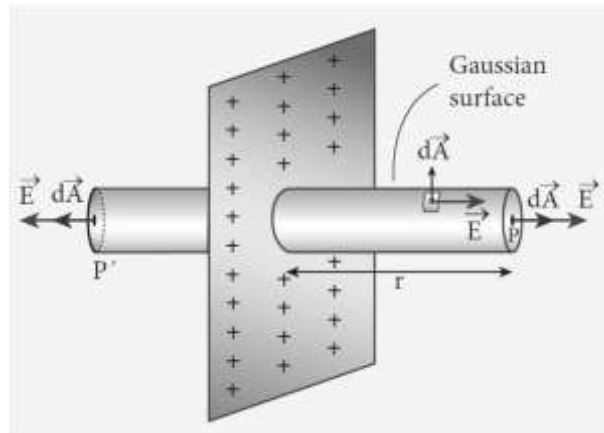
This equation shows that ,

1.  $\vec{E}$  is always perpendicular to the length of the wire .
2. If  $\lambda > 0$  ,  $\vec{E}$  points perpendicular in outward direction ( $\hat{r}$ )
3. If  $\lambda < 0$  ,  $\vec{E}$  points perpendicular in inward direction ( $-\hat{r}$ )

4. Eq (a) is only for an infinitely long charged wire. If the length of the wire is true around the midpoint of the wire and far away from the both ends of the wire.

**59. ELECTRIC FIELD DUE TO A CHARGED INFINITE SHEET [ 2 MARK ]**

Consider an infinite plane sheet of charge as shown in the fig. Let  $\sigma$  be the surface charge density.



Consider a point P at a distance r from the sheet. The electric field points radially outward on both side of the sheet.  $\vec{E}$  should be same at equal distance on both sides. The Gaussian surface is taken in the form of cylinder of length 2r extending at equal distance of r on both sides of the sheet. Let A be the area of the flat surface of the cylinder. The plane sheet passes perpendicularly through the middle of Gaussian surface. Use Gauss law for the cylindrical surface.

$$\phi_E = \oint \vec{E} \cdot d\vec{A}$$

$$\phi_E = \int_{\text{Curved surface}} \vec{E} \cdot d\vec{A} + \int_P \vec{E} \cdot d\vec{A} + \int_{P'} \vec{E} \cdot d\vec{A}$$

$$\phi_E = \frac{Q_{enc}}{\epsilon_0}$$

$\vec{E}$  is perpendicular to the element area  $d\vec{A}$  at all points on the curved surface.

$$\vec{E} \cdot d\vec{A} = 0$$

$$\phi_E = \int_P \vec{E} \cdot d\vec{A} + \int_{P'} \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

P and P'  $\rightarrow$   $\vec{E}$  and  $d\vec{A}$  are parallel.

$$\vec{E} \cdot d\vec{A} = |\vec{E}| |d\vec{A}| \cos 0^\circ \quad \cos 0^\circ = 1$$

$$\vec{E} \cdot d\vec{A} = E \cdot dA$$

$$\phi_E = \int_P E \cdot dA + \int_{P'} E \cdot dA = \frac{Q_{enc}}{\epsilon_0}$$

Since the magnitude of the electric field at these two equal surfaces [P and P'] is uniform E can be taken out of the integration .

$$\phi_E = E \int_P dA + E \int_{P'} dA = \frac{Q_{enc}}{\epsilon_0}$$

$$\phi_E = 2E \int_P dA = \frac{Q_{enc}}{\epsilon_0}$$

Charge density,  $\sigma = \frac{dQ}{dA}$

$$dQ = \sigma \cdot dA$$

$$\int dQ = \int \sigma \cdot dA$$

$$Q = \int \sigma \cdot dA$$

$$\phi_E = 2E \int_P dA = \frac{\int \sigma \cdot dA}{\epsilon_0}$$

$$2EA = \frac{\sigma}{\epsilon_0} \int dA$$

$$2EA = \frac{\sigma A}{\epsilon_0}$$

$\hat{n}$  → unit vector normal to the plane

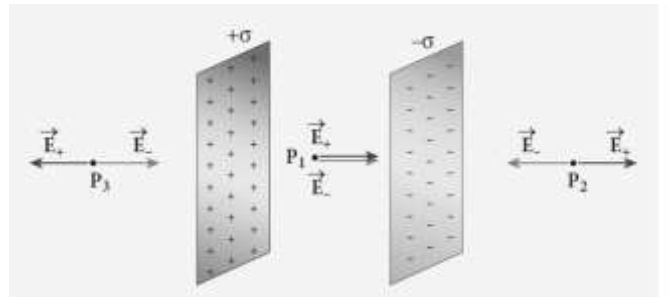
$$\boxed{\vec{E} = \frac{\sigma}{2\epsilon_0} \hat{n}} \longrightarrow (a)$$

$\vec{E}$  due to an infinite plane sheet of charge depends on the surface charge density ' $\sigma$ ' and independent of ' $r$ ' .

1. If  $\sigma > 0$ ,  $\vec{E}$  at any point P points perpendicularly outwards to the point ( $\hat{n}$ ) and if  $\sigma < 0$ ,  $\vec{E}$  at any point P points perpendicularly inwards to the point ( $-\hat{n}$ )
2. Eq (a) is true only in the middle region of the plane and at points far away from both ends.

### 60. ELECTRIC FIELD DUE TO TWO PARALLEL CHARGED INFINITE SHEETS [ 3 MARKS]

Consider two infinitely charged plane sheets with equal and opposite charge density  $+\sigma$  and  $-\sigma$  placed parallel to each other as shown in figure .



The electric field between the plates and outside the plates is found using Gauss law

The magnitude of the electric field due to an infinite charged plane sheet is  $\frac{\sigma}{2\epsilon_0}$  and it points perpendicular outward if  $\sigma > 0$  and points inwards if  $\sigma < 0$ .

Consider a point  $P_1$  in between the plates. The electric field  $\vec{E}_+$  due to  $+\sigma$  points outwards and the electric field  $\vec{E}_-$  due to  $-\sigma$  points radially inwards.

The electric field at the point  $P_1$  due to  $+\sigma$  and  $-\sigma$  are in the same direction. That towards the right.

The electric field  $\vec{E}_+$  at  $P_1$  due to  $+\sigma$  is,

$$\vec{E}_+ = \frac{\sigma}{2\epsilon_0}$$

The electric field  $\vec{E}_-$  at  $P_1$  due to  $-\sigma$  is,

$$\vec{E}_- = \frac{\sigma}{2\epsilon_0}$$

Total electric field at the point  $P_1$  is

$$E_{\text{inside}} = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0}$$

$$E_{\text{inside}} = \frac{\sigma + \sigma}{2\epsilon_0}$$

$$E_{\text{inside}} = \frac{2\sigma}{2\epsilon_0}$$

$$E_{\text{inside}} = \frac{\sigma}{\epsilon_0}$$

The direction of the electric field between the plates is directed from positively charged plate to the negatively charged plate and is uniform everywhere inside the plate.

Consider the two points  $P_2$  and  $P_3$  on the right and left sides of the plates .

The electric field at  $P_2$  due to  $+\sigma$  and  $-\sigma$  are  $\vec{E}_+$  and  $\vec{E}_-$

$\vec{E}_+$  and  $\vec{E}_-$  are equal in magnitude and opposite in direction . Hence the net electric field  $\vec{E}$  at  $P_2$  is zero .

Similarly, the electric field at ' $P_3$ '

$\vec{E}_+$  and  $\vec{E}_-$  are equal in magnitude and opposite in direction . Hence the net electric field  $\vec{E}$  at  $P_3$  is zero .

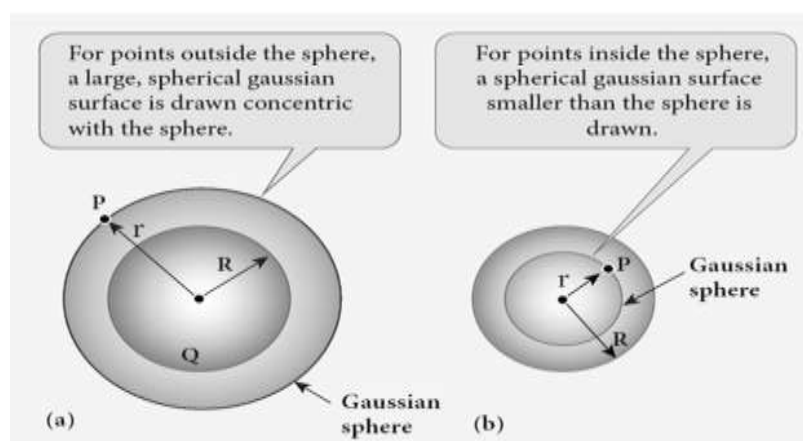
Hence, the net field  $\vec{E}$  is zero at the points  $P_2$  and  $P_3$  outside the plates .

Hence , the electric field due to two parallel charged infinite sheets is,

$$\vec{E} = \frac{\sigma}{\epsilon_0} \hat{n}$$

### 61. ELECTRIC FIELD DUE TO A UNIFORMLY CHARGED SPHERICAL SHELL[ 5 MARK]

Consider a uniformly charged spherical shell as shown in figure. The electric field  $\vec{E}$  points radially outwards in all directions around the shell . For the spherical charge distribution , the Gaussian surface is taken in the form of a sphere concentric to the charge distribution .



#### a) ELECTRIC FIELD AT A POINT OUTSIDE THE SHELL : ( $r > R$ )

Consider a point P at a distance  $r$  from the centre of the shell , as

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shown in figure (a). The charges are uniformly distributed on the spherical surface.

If  $Q$  is positive ( $Q > 0$ ) the electric field points radially outwards and if  $Q$  is negative ( $Q < 0$ ) the electric field points radially inwards. Applying Gauss law to the Gaussian surface of radius  $r$ .

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

$\vec{E}$  and  $d\vec{A}$  points in the same direction radially outwards at all points on to Gaussian surface.

Angle between  $\vec{E}$  and  $d\vec{A}$  is  $\theta = 0^\circ$

Hence ,  $\vec{E} \cdot d\vec{A} = |\vec{E}| |d\vec{A}| \cos 0^\circ$   $\cos 0^\circ = 1$

$$\vec{E} \cdot d\vec{A} = E \cdot dA$$

$$\oint E \cdot dA = \frac{Q_{enc}}{\epsilon_0}$$

$$E \oint dA = \frac{Q_{enc}}{\epsilon_0}$$

$$E \cdot A = \frac{Q_{enc}}{\epsilon_0}$$

$$A = 4\pi r^2$$

$A \rightarrow$  Surface area of the sphere.

$$E \cdot 4\pi r^2 = \frac{Q_{enc}}{\epsilon_0}$$

The electric field at a point outside the shell is ,

$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$$

$$\boxed{\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \hat{r}}$$

If  $Q > 0$ ,  $\vec{E}$  points outwards and if  $Q < 0$ , then  $\vec{E}$  points inwards.

The electric field  $\vec{E}$  at a point outside a charged shell looks as though the entire charges are concentrated at the centre of the spherical sphere.

**b) ELECTRIC FIELD AT A POINT ON THE SURFACE OF THE  
SPHERICAL SHELL (r = R)**

On the surface of the shell  $r = R$ .

The electric field at points on the spherical shell is,

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{R^2} \hat{r}$$

**c) ELECTRIC FIELD AT A POINT ON THE SURFACE OF THE  
SPHERICAL SHELL (r < R)**

Consider a point P inside the shell at a distance  $r$  from the centre.

Imagine a Gaussian surface of radius ' $r$ ' passing through the point P, Applying Gauss law to the Gaussian surface .

$$\phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0}$$

$\vec{E}$  and  $d\vec{A}$  points in the same direction radially outwards at all points on to Gaussian surface.

$$\theta = 0.$$

$$\vec{E} \cdot d\vec{A} = |\vec{E}| |d\vec{A}| \cos 0^\circ = E \cdot dA$$

$$\phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0}$$

$$E \oint dA = \frac{Q}{\epsilon_0}$$

$$EA = \frac{Q}{\epsilon_0}$$

$$A = 4\pi r^2$$

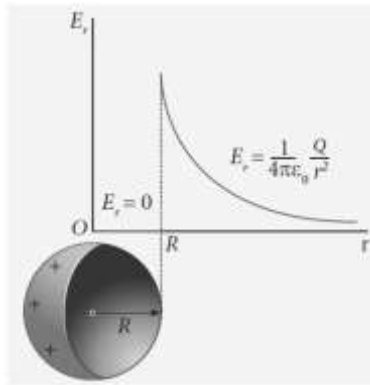
$$E \cdot 4\pi r^2 = \frac{Q}{\epsilon_0}$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$$

Since Gaussian surface encloses no charges ,

$$Q = 0$$

$$E = 0$$



The electric field due to the charged spherical sphere is zero at all the points inside the shell.

As  $\vec{E} \propto \frac{1}{r^2}$  the electric field decreases as  $r$  increases and vice versa. The plot of 'E' versus 'r' of a spherical shell. From the graph,  $\vec{E}$  inside the sphere is zero, and it maximum at the surface ( $r = R$ ) and decreases exponentially with distance  $r$ .

## ELECTROSTATICS OF CONDUCTORS AND DIELECTRICS :

### 62. CONDUCTORS [ 2 MARK ]

Electrical are materials which has a large number of mobile charges [ free electrons ] which are free to move in the material .

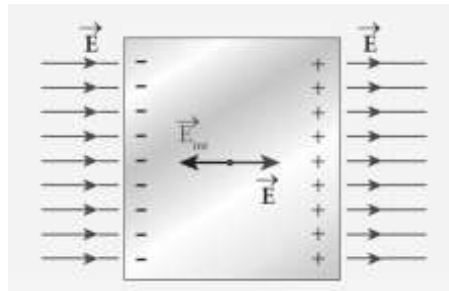
Ex : copper

### 63. ELECTROSTATIC EQUILIBRIUM: [ 2 MARK ]

The free charges in a conductor are not bound and they are free to move in all directions , Hence there is no net motion of charges [ namely electrons ] along a particular direction and there is no net current in the conductor .This state is called **Electrostatic equilibrium.**

**64.PROPERTIES OF CONDUCTOR AT ELECTROSTATIC EQUILIBRIUM:****[ 3 MARK]**

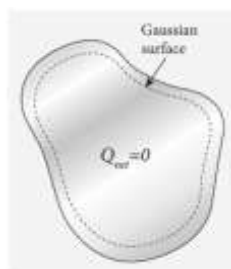
i. The electric field  $\vec{E}$  is zero everywhere inside a conductor. This is true for all conductors, whether it is solid or hollow.

**EXPERIMENTAL PROOF:**

Consider a conductor placed in an external electric field as shown in the figure. Before the application of electric field  $\vec{E}$  the free electrons are uniformly distributed in all the possible directions in the conductor. After the application of electric field  $\vec{E}$ , the negative electrons are accelerated to the left side of the plate and the positive charges are accelerated to the right side of the plate. [ This is because the external field  $\vec{E}$  has its positive potential in the left and negative potential in the right side ].

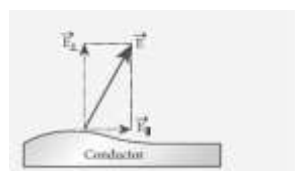
Due to this alignment of free electrons an internal field  $\vec{E}_{\text{int}}$  is created inside the conductor in a direction opposite to the applied field. This internal field  $\vec{E}_{\text{int}}$  increases, until it nullifies the external field. Hence, the net field inside the conductor is zero. At this stage the conductor is said to be in electrostatic equilibrium. The time taken by the conductor to reach electrostatic equilibrium state is instantaneous of the order of  $10^{-16}$  sec.

ii. There is no net charge inside the conductors. The charges reside only on the surface of the conductors.

**PROOF :**

Consider an arbitrarily shaped conductor as shown in figure. To find the effect of charges inside the conductor. Imagine, a Gaussian surface inside the conductor such that it is very close to the surface of the conductor. The electric field  $\vec{E}$  is zero everywhere inside a conductor, which shows that there is no net charge inside the conductor. No charge is enclosed by the Gaussian surface. Even if any charge is introduced inside the conductor, it immediately searches the surface of the conductor.

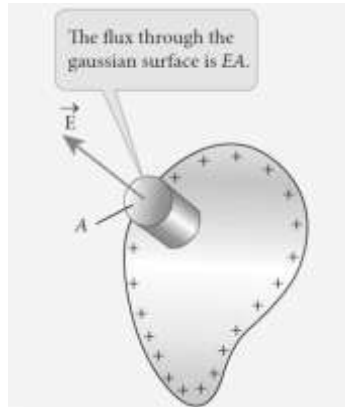
iii. The electric field outside the conductor is perpendicular to the surface of the conductor. The electric field is perpendicular to the surface of the conductor and has a magnitude of  $\frac{\sigma}{\epsilon_0}$ , where  $\sigma$  is the surface charge density at that point.

**PROOF :**

To show that the electric field is perpendicular to the surface of the conductor at electrostatic equilibrium. The electric field on the surface of the conductor may have parallel and perpendicular components as shown in figure. If the electric field has components parallel to the surface, this field would exert a force on the neighbouring charges. This creates an acceleration of the free electrons on the surface. This means that the conductor is not in electrostatic equilibrium. Hence the electric field must be

perpendicular to the surface as shown in figure .When the conductor is at electrostatic equilibrium.

**65. TO SHOW THAT  $E = \frac{\sigma}{\epsilon_0}$  AT ANY POINT OUTSIDE THE SURFACE OF THE CONDUCTOR: [ 3 MARK]**



Consider a conductor as shown in figure. To find the electric field at a point outside the conductor imagine a Gaussian surface in the form of a cylinder, such that one half of the cylinder is embedded inside the conductor since the electric field is normal to the surface of the conductor, the curved part of the cylindrical surface has zero electric flux. The bottom part of the Gaussian surface is inside the conductor where the electric field is zero. The top flat surface of the cylinder alone contributes to the total electric flux. let A be the area of cross section of the top surface.

Applying Gauss law,

$$\phi_E = \oint E \cdot dA = \frac{Q}{\epsilon_0}$$

Q is the charge over the area A. If  $\sigma$  is the surface density of charges.

$$\sigma = \frac{Q}{A} \quad (\text{or}) \quad Q = \sigma A.$$

$$E \oint dA = \frac{Q}{\epsilon_0}$$

$$E \cdot A = \frac{Q}{\epsilon_0}$$

$$E.A = \frac{\sigma A}{\epsilon_0}$$

$$E = \frac{\sigma}{\epsilon_0}$$

$$\vec{E} = \frac{\sigma}{\epsilon_0} \hat{n}$$

$\hat{n}$  is the unit vector pointing outwards normal to the surface of the conductor.

If  $\sigma > 0$ , then the electric field points inwards and  $\hat{n} = -\hat{n}$

iv. The electrostatic potential has the same value on the surface and inside of the conductor

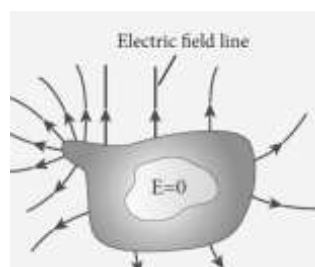
### EXPLANATION

The electric field outside the conductor is perpendicular to the surface of the conductor. Therefore, electric field has no parallel component on the surface. The charges on the surface does not experience any field in the parallel direction. Since there is no force due to the field in the parallel direction. The work done along the direction parallel to the surface of the conductor will be zero. Hence charges can be moved on the surface without doing any work. As  $w=qv$ , this is possible only if the Potential  $V$  is constant at all points on the surface, and there is no potential difference between any two points on the surface.

The electric field inside the conductor is zero. The potential is the same on the surface of the conductor. Thus at electrostatic equilibrium, the conductor is always at equipotential.

### 66. ELECTROSTATIC SHIELDING[ 2 MARK]

Using Gauss law, we proved that the electric field inside a charged spherical shell is zero. Further, we showed that the electric field inside both hollow and solid conductors is zero. It is a very interesting property which has an important consequence.



Consider a cavity inside the conductor as shown in figure. whatever the charges at the surfaces and whatever the electrical disturbances outside, the electric field inside a cavity is zero. A sensitive electrical instrument which is to be productive from external electrical disturbance is kept inside the cavity this is called electrostatic shielding.

Faraday cage is an instrument used to demonstrate this effect .It is made up of metal bars. If an artificial lighting jolt is created outside, the person inside is not affected.

### 67. ELECTROSTATIC INDUCTION[ 2 MARK]

Charging of conductors without actual contact is called **electrostatic induction**.

### 68. EXPERIMENT TO DEMONSTRATE ELECTROSTATIC INDUCTION

[ 3 MARK]



Consider an uncharged equal distribution of charges cancel each other conducting sphere at rest on an insulating stand as shown in figure. Let a negatively charged conducting rod is brought near the sphere without touching it . The negative charges in the rod repels the negative charges of the sphere. As a result, the negative charges are moved further side, But the total charge is zero.

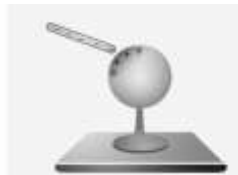


Now connect the conducting sphere to the ground, through a wire. This is called grounding. [Ground can receive any amount of electrons] The electrons in the sphere are removed

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from the sphere as shown in the figure. The positive charges remain near the region of the rod, as they are attracted by the negative charges of the rod.



The grounding wire is now removed as shown in the figure. The sphere has positive charges alone near the region of the rod.



Now that charged rod is taken away from the conductor. The positive charge are now uniformly distributed on the surface of the conductor as shown in figure.

The neutral conducting sphere is now positively charged without direct contact between the rod and the sphere for an arbitrary shaped sphere conductor distribution of positive charges is not be uniform.

### 69. DIELECTRICS OR INSULATORS[ 2 MARK]

A dielectric is a non conducting material with no free electrons.

**Ex:** Ebonite, glass, mica. A dielectric is made up of either polar molecules or non polar molecules.

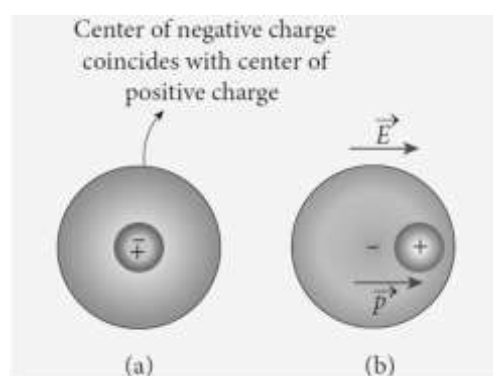
### 70. NON – POLAR MOLECULES[ 2 MARK]

Non-polar molecules are molecules which has no permanent dipole moments.

**Ex :** Hydrogen ( $H_2$ ), oxygen ( $O_2$ ), carbon dioxide( $CO_2$ )

**71. EFFECT OF ELECTRIC FIELD ON NON-POLAR MOLECULES****[ 3 MARK]**

The atoms constituting the molecule has positively charged nucleus and orbiting electrons . Each atom is considered as a dipole. The nucleus is at the centre and the e moving electrons create an electron cloud. The centre of the positive nucleus and the centre of the negatively electron clouds coincides in a non-polar molecule without any external electric field as shown in the figure.



When an external electric field is applied the positive charge moves a bit to the right and the negative charge moves a bit to the left .The positive and negative charges are separated by a small distance . This indicates a dipole moment  $\vec{P}$  in the direction of the external field as shown in the figure . The dielectric is now said to be the polarized. Thus ,a non-polar molecule is polarized by an external external field

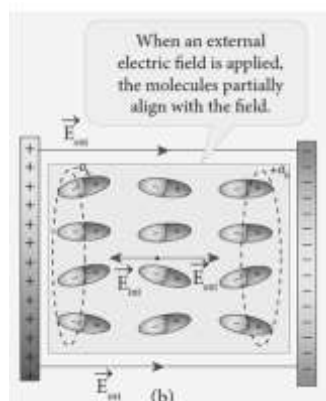
**72. POLAR MOLECULES[ 2 MARK]**

Polar molecules are molecules which have **permanent dipole moments**.

**Ex:**  $\text{H}_2\text{O}$  ,  $\text{N}_2\text{O}$  ,  $\text{NH}$

**73. EFFECT OF ELECTRIC FIELD ON POLAR MOLECULES [ 2 MARK]**

In polar molecules ,the centre of positive and negative charges are separated even in the absence of an external electric field. This, they have a permanent dipole moment one to thermal motion ,the direction of each dipole moment is oriented randomly . Hence the dipole moment gets cancelled due to this random orientation as shown in figure . The net dipole moment is zero in the absence of an external electric field.



When an external electric field is applied, the dipoles align in the direction of the external electric field. This indicates a dipole moment and the dipole are polarized by the external electric field as shown in the figure. The external electric field indicates polarization in both the polar and non-polar molecule.

#### 74. POLARISATION $\vec{P}$ [ 2 MARK]

The total dipole moment per unit volume in a dielectric is called polarization ( $\vec{P}$ )

For a linear dielectric, the polarization is directly proportional to the applied electric field.

$$\vec{P} \propto \vec{E}_{\text{ext}}$$

$$\vec{P} = X_e \vec{E}_{\text{ext}}$$

Where,  $X_e$  is constant called the electric susceptibility of the dielectric.

#### 75. SUSCEPTIBILITY ( $X_e$ ) [ 2 MARK]

Susceptibility is defined as the case with which a dielectric is electric field.

$$X_e = \frac{\vec{P}}{E}$$

It is the polarisation induced in a dielectric per unit electric field.

### 76. INDUCED ELECTRIC FIELD INSIDE A DIELECTRIC : [ 3 MARK]

Consider a rectangular dielectric slab placed between two oppositely charged capacitor plates as shown in figure . The opposite charges on the plates provide external electric field for the dielectric. The dipole try to align in the direction of the external electric field  $\vec{E}_{\text{ext}}$  as shown in figure. Consider a linear string of dipoles in the field as shown in the figure .Each positive charge cancels with neighbouring negative charge at the centre .But at the ends there are no neighbouring charges to get cancelled .Hence a negative charge and positive charge are left at both the ends. These charges cannot be removed and they are cancelled bound charges. So, the dielectric in the external field is equivalent to two oppositely charged sheets with surface charge densities  $+\sigma_b$  and  $-\sigma_a$  . These bound charges are not free to move like free electrons in conductors. These Bound charges also produce electric field within the dielectric.

#### EXAMPLE

A charged balloon after rubbing sticks on a wall ,because the negatively charged balloon when brought near the wall indicates opposite charges on the surface of the wall, which attracts the balloon.

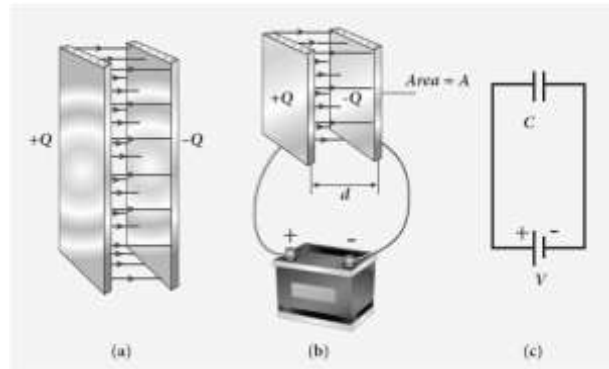
#### DIELECTRIC STRENGTH:

When the external electric field applied to a dielectric is very large ,it tears the atoms apart so that they bound charges become free charges. Then the dielectric starts to conduct electricity. This is called a dielectric breakdown. The maximum electric field the dielectric can withstand before it breakdowns is called dielectric strength .For example, that dielectric of air is  $3 \times 10^6 \text{ Vm}^{-1}$ . If the applied electric field increases beyond this, a spark is produced in the air. The dielectric strengths of some dielectrics are given in table below.

Substance	Dielectric strength ( $\text{Vm}^{-1}$ )
Mica	$100 \times 10^6$
Teflon	$60 \times 10^6$
Paper	$16 \times 10^6$
Air	$3 \times 10^6$
Pyrex glass	$^{64} 14 \times 10^6$

**CAPACITORS AND CAPACITANCE****77. CAPACITOR [ 2 MARK ]**

Capacitor is a device used to store electric charge and electric energy.

**CAPACITANCE OF A CAPACITOR**

A capacitor consists of two parallel metal plates separated by a small distance as shown in figure (a). When the capacitor is connected to a battery of potential difference  $V$  the plate connected to the positive terminal of the battery acquires a charge of  $+Q$  and the plate connected to the negative terminal acquires charge of  $-Q$ . The potential difference between the plates is equal to the battery's terminal voltage as shown in figure (b).

As the battery voltage  $V$  is increased. The amount of charge stored in the plates also increase. Thus, the charge stored in the capacitor is proportional to the potential difference  $V$ . (i.e)  $Q \propto V$

$$Q = cV$$

$c$  is proportionality constant called the capacitance.

**78. CAPACITANCE [ 2 MARK ]**

The capacitance  $C$  of a capacitor is defined as the ratio of the magnitude of charge on the conductor plates in the potential difference existing between the conductors .

$$C = \frac{Q}{V}$$

Unit of capacitance is COULOMB per volt or farad (F).

Farad is the larger unit of capacitance and capacitors are available in the range of

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Microfarad [ $1\mu\text{F} = 10^{-6}\text{ F}$ ] to picofarad [ $1\text{PF} = 10^{-12}\text{ F}$ ]. The capacitor is represented by the symbol  $\text{||} \text{--} \text{||}$  or  $\text{||} \text{--} \text{||}$

**79. TOTAL CHARGE STORED IN A CAPACITOR [ 2 MARK]**

The total charge stored in the capacitor is the sum of charges deposited on the two plates. The total charges stored in a capacitor is  $Q - Q = 0$  [because equal number of positive charge and negative charges are deposited on the plates which gets cancelled]

When a capacitor is said to store charges ,it actually represents the amount of charges stored in any one of the plates.

**80. TYPES OF CAPACITOR[ 2 MARK]**

Available shapes of capacitors:

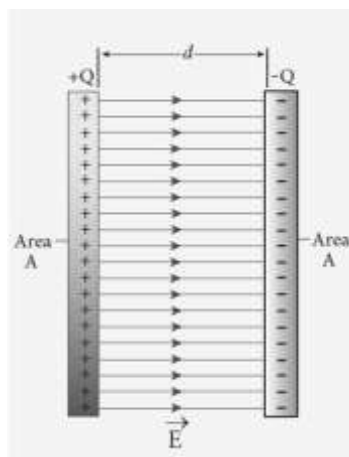
Cylindrical, disc.

Available types of capacitors:

Tantalum, ceramic ,electrolytic

**81. CAPACITANCE OF A PARALLEL PLATE CAPACITOR [ 3 MARK]**

Consider a capacitor with two parallel plates as shown in the figure.



Let 'A' be the cross sectional area of the plates and 'd' is the distance between them . The electric field between these two infinite parallel plates which is uniform and is given by  $E = \frac{\sigma}{\epsilon_0}$ .

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$\sigma = \frac{Q}{A}$ ; If  $d$  is much smaller than  $A$  then the above result is used even for finite sized parallel plate capacitor. Using Gauss law, the electric field  $E$  between the plates is,

$$\int \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0}$$

$$E \cdot A = \frac{Q}{\epsilon_0}$$

$$E = \frac{Q}{\epsilon_0 A}$$

In the above equation the R.H.S is a constant.

$\vec{E}$  is thus uniform,

$$V = E \cdot d$$

$$V = \frac{Q \cdot d}{\epsilon_0 A}$$

Capacitance  $C$  of a capacitor is  $C = \frac{Q}{V}$

$$C = \frac{Q}{\left[ \frac{Q \cdot d}{\epsilon_0 A} \right]} = \frac{\epsilon_0 A}{d}$$

$$C = \frac{\epsilon_0 A}{d}$$

From above equation, the capacitance  $C$  is directly proportional to area of cross section and inversely proportional to the distance between the plates.

From above equation, as the area of cross section of the capacitor plates is increased, more charges are distributed for same potential difference  $V$ . As a result, the capacitance is increased.

From above equation,  $V = E \cdot d$ , if the distance between the plates ' $d$ ' is reduced, the potential difference decreases with ' $E$ ' constant voltage difference  $V$  increases to keep  $E$  constant. This leads to an additional form of charges to the battery, till the voltage on the capacitor is equal to the battery voltage with the distance ' $d$ ' is increased [ $V \propto d$ ], the capacitor voltage increases and when it is greater than the battery voltage, then the charges flow from capacitor plates to battery till both the voltages are equal.

**82. ENERGY STORED IN A CAPACITOR : [ 3 MARK]**

Capacitor not only store charges, but it also stores energy. Consider a capacitor connected to a battery. The electrons from the plate connected to the negative terminal of the battery will be transferred to the plate connected to the positive terminal. Let the total charge transferred is  $-Q$ . To transfer the charge work is done by the battery and this work done is stored as potential energy in the capacitor.

Let the potential difference  $V$  transfers a small infinitesimal charge  $dQ$  and let  $dw$  the work done to transfer this charge.

$$W = VQ$$

$$dw = V \cdot dQ$$

$$V = \frac{Q}{C}$$

$$dw = \frac{Q}{C} \cdot dQ$$

$$\int dw = \int \frac{Q}{C} \cdot dQ$$

$$W = \int \frac{Q}{C} \cdot dQ = \frac{1}{C} \frac{Q^2}{2}$$

This workdone is stored as electrostatic potential energy ( $U_E$ ) in the capacitor.

$$U_E = \frac{Q^2}{2C}$$

$$Q = VC$$

$$U_E = \frac{V^2 C^2}{2C}$$

$$U_E = \frac{1}{2} CV^2$$

The energy stored (i.e) potential energy is proportional to the capacitance of the capacitor and square of the voltage between the plates.

**83. ENERGY DENSITY ( $U_E$ ) [ 2 MARK]**

The energy stored per unit volume of space is defined as the energy density .

$$U_E = \frac{U}{\text{Volume}}$$

**84. EXPRESSION FOR ENERGY DENSITY ( $U_E$ ) [ 3 MARK]**

For a parallel plate capacitor of capacitance is,

$$C = \frac{\epsilon_0 A}{d}$$

A → Area of the plates

d → separation between the plates

The potential V in between the plates is

$$V = E \cdot d$$

The electrostatic potential energy  $U_E$  of the capacitor is

$$U_E = \frac{1}{2} CV^2$$

$$U_E = \frac{1}{2} \left[ \frac{\epsilon_0 A}{d} \right] [E \cdot d]^2$$

Where  $A_d$  is the volume of the space between the capacitor plates. From the definition of energy density.

$$U_E = \frac{U}{\text{Volume}}$$

$$\text{Volume} = A_d ; \quad U_E = \frac{\frac{1}{2} \left[ \frac{\epsilon_0 A}{d} \right] [E \cdot d]^2}{A_d}$$

$$U_E = \frac{1}{2} \epsilon_0 E^2$$

Above equation shows that the energy is stored in the electric field existing in between the plates of the capacitor. Thus, the energy density depends only on the electric field and not on the size of the capacitor plates .Equation holds for any types of charge configurations.

**85. APPLICATIONS OF CAPACITORS[ 2 MARK]**

Capacitors are used in various electronic circuits. A few of the applications are ,

a) Most people are now familiar with the digital camera .The flash which comes from the camera when we take photographs is due to the energy released from the capacitor called the flash capacitor .

b) During cardiac arrest, device called heart defibrillator is used to give a sudden surge of a large amount of electrical energy to the patient's chest to retrieve the normal heart function. This defibrillator uses a capacitor of  $175\text{ }\mu\text{F}$  charged to a high voltage of around  $2000\text{ V}$ .

c) Capacitors are used in the ignition system of automobile engine to eliminate sparking.

d) Capacitors are used to reduce power fluctuations in power supplies and to increase efficiency of power transmission.

However, Capacitors have disadvantages as well. Even after the battery or power supply is removed the capacitor stores charges and energy for some time .For example, if the TV is switched off it is always advisable to not touch the back side of the TV panel .

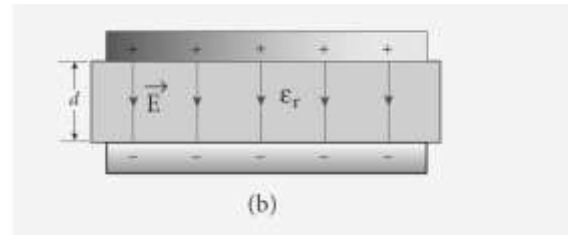
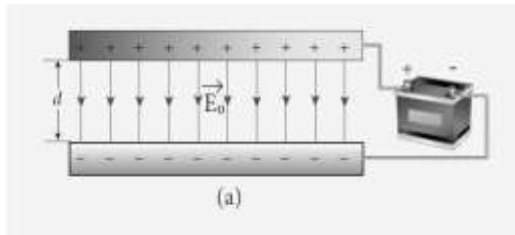
**86. EFFECT OF DIELECTRIC IN CAPACITORS[ 5 MARK]**

The capacitance of a capacitor is altered by the insertion of dielectric materials like mica ,glass or paper in between the plates. The dielectric can be inserted in two ways,

- i. When the capacitor is disconnected from the battery.
- ii. When the capacitor is connected to the battery.

## I. WHEN THE CAPACITOR IS DISCONNECTED FROM THE BATTERY

Consider a capacitor of cross-sectional area  $A$  and separated by a distance  $d$ . Let the capacitor is charged by a voltage  $V_0$ .  $Q_0$  is that charge stored in the capacitor and.  $E_0$  is the electric field between the plates. The space between the plate is empty. The capacitance of a capacitor is taken as  $C_0$ .



The battery is now disconnected from the capacitor and let a dielectric is inserted between the plates are shown in the figure. The dielectric constant of the dielectric is  $\epsilon_r$ . The introduction of dielectric modifies the electric field. Let the new field be  $E$ .

$$E = \frac{E_0}{\epsilon_r}$$

$E_0$  is the electric field inside the capacitor plates without the dielectric. For any dielectric material,  $\epsilon_r > 1$ . Hence,  $E < E_0$ . The new electric field  $E$  decreases when the

dielectric is inserted. The new potential difference  $V$  is,

$$V = Ed.$$

$$V = \frac{E_0}{\epsilon_r} d$$

$$V = \frac{V_0}{\epsilon_r}$$

$V < V_0$ , The potential decreases by the introduction of dielectric.

Capacitance increases when the dielectric is introduced between the plates of the capacitor.

### PROFF :

The new capacitance in the presence of dielectric is

$$C = \frac{Q_0}{V} = \frac{Q_0}{\frac{V_0}{\epsilon_r}} = \frac{\epsilon_r Q_0}{V}$$

$$C = \epsilon_r C_0$$

Since  $\epsilon_r > 1$ ,  $C > C_0$ . Thus, the capacitance of the capacitor is increased in the presence of dielectric.

$$C_0 = \frac{\epsilon_0 A}{d}$$

$$C = \frac{\epsilon_r \epsilon_0 A}{d}$$

$$\epsilon_r = \frac{\epsilon}{\epsilon_0} ; \quad \epsilon = \epsilon_0 \epsilon_r$$

$$C = \frac{\epsilon A}{d}$$

$\epsilon \rightarrow$  Permittivity of the dielectric medium.

### ENERGY STORED IN THE CAPACITOR BEFORE AND AFTER THE INSERTION

Before the insertion of dielectric, the energy stored in the capacitor  $U_0$  is,

$$U_0 = \frac{1}{2} \frac{Q_0^2}{C_0}$$

After the insertion of dielectric the charge  $Q_0$  remains constant and the capacitance is increased by an amount  $C = \epsilon_r \epsilon_0$ . The energy stored after the insertion of dielectric is

$$U = \frac{1}{2} \frac{Q_0^2}{C}$$

$$U = \frac{1}{2} \frac{Q_0^2}{\epsilon_r C_0} = \frac{U_0}{\epsilon_r} \quad C = \epsilon_r C_0$$

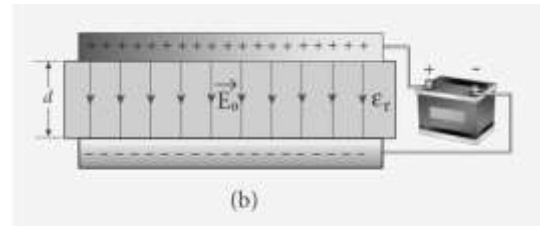
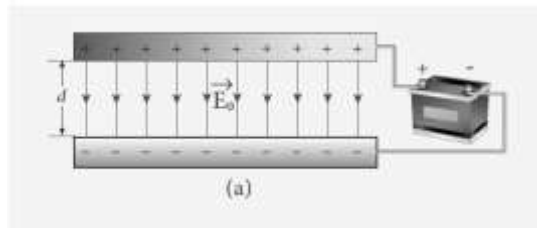
$$\boxed{U = \frac{U_0}{\epsilon_r}}$$

As  $\epsilon_r > 1$ ,  $U < U_0$ . Thus, the energy stored in the capacitor decreases as the dielectric introduced. This decrease is due to the fact that the capacitor spends some energy in pulling the dielectric inside.

## II. WHEN THE BATTERY REMAINS CONNECTED TO THE CAPACITOR

Consider a dielectric inserted between the plates of a capacitor, when a battery of voltage  $V_0$  is connected across it as shown in the figure. The charge stored in a capacitor increases by a factor  $\epsilon_r$  when a dielectric is introduced between the plates.

$$Q = \epsilon_r Q_0$$



As  $Q$  is increased, the capacitance  $C$  is also increased. The new capacitance  $C$  is,

$$C = \frac{Q}{V_0} = \frac{\epsilon_r Q_0}{V_0}$$

$$C_0 = \frac{Q_0}{V_0}$$

$$C = \epsilon_r C_0$$

As the battery is connected, the potential remains the same. The capacitance is increased by the introduction of dielectric.

## ENERGY STORED IN A CAPACITOR BEFORE AND AFTER THE INSERTION OF DIELECTRIC

Before inserting the dielectric, the energy stored in the capacitor is,

$$U_0 = \frac{1}{2} C_0 V_0^2$$

After the insertion of dielectric, the capacitance increases as  $C = \epsilon_r C_0$ . Hence, the energy stored is

$$U = \frac{1}{2} C V_0^2 = \frac{1}{2} \epsilon_r C_0 V_0^2$$

$$U = \epsilon_r U_0$$

Since  $\epsilon_r > 1$ ,  $U > U_0$ . The energy (U) of the capacitor increases after the introduction of dielectric. Since the voltage between the capacitor plates  $V_0$  is constant, the electric field ( $E_0$ ) also remains constant.

The energy density U is given by

$$U = \frac{1}{2} \epsilon E_0^2$$

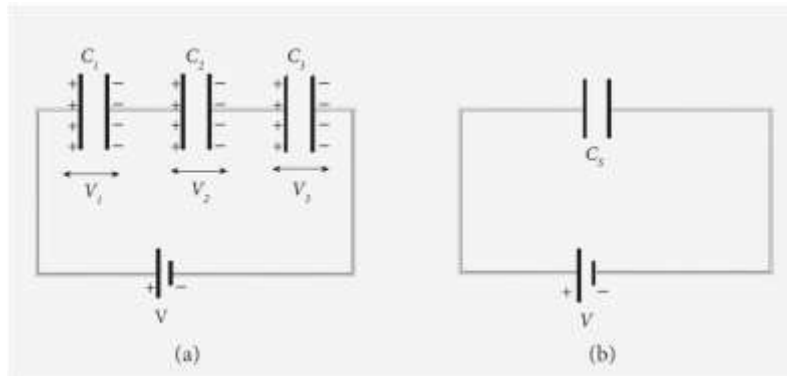
Where,  $\epsilon$  is permittivity of the dielectric material.

S.No	Dielectric is inserted	Charge Q	Voltage V	Electric field E	Capacitance C	Energy U
1	When the battery is disconnected	Constant	decreases	Decreases	Increases	Decreases
2	When the battery is connected	Increases	Constant	Constant	Increases	Increases

### 87. CAPACITOR IN SERIES : [ 3 MARK]

Consider three capacitors  $C_1$ ,  $C_2$  and  $C_3$  are connected in series with a battery of voltage V as shown in the figure. Let electrons of charge - Q are transferred from the negative terminal of the battery to the right plate of  $C_3$ . [ This is because the negative potential repels the negative electrons away from the battery ]. This induces an equal amount of positive charge on the left plate of  $C_3$ . The transferred - Q electrons from the right plate of the  $C_3$  pushes the same amount of - Q to the right plate of  $C_2$  due to electrostatic induction. Similarly, the positive charges induced on the left to plate of  $C_2$  pushes the equal amount of charge - Q on the right plate of  $C_1$ . At the same time the electron of charge - Q are transferred from left plate of  $C_1$  to the positive terminal of battery. Hence the same amount of charge Q is store in all the capacitors  $C_1$ ,  $C_2$  and  $C_3$

But the voltage across the capacitor varies .Let  $V_1$ ,  $V_2$  and  $V_3$  be the voltage across  $C_1$ ,  $C_2$  and  $C_3$ .

(a) Capacitors connected in series (b) Equivalence capacitors  $C_s$ 

The battery voltage across each capacitor is equal to the battery voltage  $v$

$$V = V_1 + V_2 + V_3$$

As  $Q = cV \rightarrow V = \frac{Q}{c}$

$$V_1 = \frac{Q}{C_1}, V_2 = \frac{Q}{C_2}, V_3 = \frac{Q}{C_3} \text{ as the charge } Q \text{ remains the same in } C_1, C_2 \text{ and } C_3.$$

$$V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

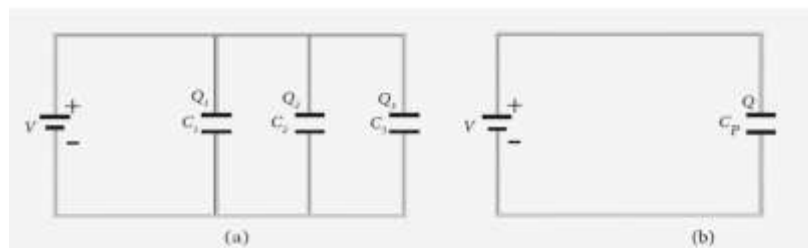
$$V = \frac{Q}{C_s}, C_s \rightarrow \text{Equivalent capacitor when the capacitors are connected in series.}$$

$$\frac{Q}{C_s} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$\boxed{\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

The inverse of equivalent capacitance ' $C_s$ ' of the three capacitors connected in series is equal to the sum of the inverse of each capacitance. The equivalent capacitance  $C_s$  is always less than the smallest individual capacitance in series.

### 88. CAPACITORS IN PARALLEL [ 3 MARK ]



(a) capacitors in parallel (b) equivalent capacitance with the same total charge

Consider three capacitors  $C_1$ ,  $C_2$  and  $C_3$  connected in parallel with the battery of voltage  $V$  are shown in the figure. Since  $C_1$ ,  $C_2$  and  $C_3$  connected across the battery, the potential across  $C_1$ ,  $C_2$  and  $C_3$  will be same equal to the battery voltage  $V$ . Since the capacitance of the capacitor are different, the charges flowing through the capacitors varies. According to the conservation of charge, the sum of charges through  $C_1$ ,  $C_2$  and  $C_3$  is equal to the total charge transferred by the battery.

$$\text{i.e) } Q = Q_1 + Q_2 + Q_3$$

$$\text{Since, } Q = CV \longrightarrow Q_1 = C_1V, Q_2 = C_2V \text{ and } Q_3 = C_3V$$

Let the combined capacitance of the parallel combination be  $C_p$ .

$$Q = C_p V$$

$$C_p V = C_1V + C_2V + C_3V$$

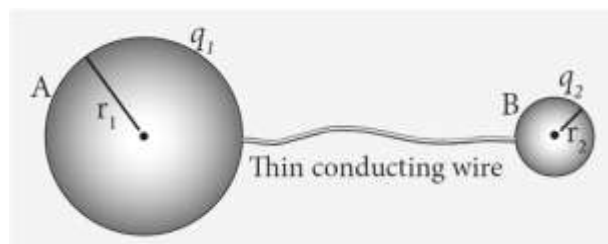
$$C_p V = V [C_1 + C_2 + C_3]$$

$$C_p = C_1 + C_2 + C_3$$

Thus, the equivalent capacitance of capacitors connected in parallel is equal to the sum of individual capacitors  $C_p$  is always larger than the largest individual capacitance.

### 89. DISTRIBUTION OF CHARGES IN A CONDUCTOR AND ACTION AT POINTS [ 3 MARK]

Considering two conducting spheres A and B of radii  $r_1$  and  $r_2$  respectively connected to each other by a thin conducting wire as shown in figure.



The distance between the sphere is much greater than the radii of the spheres.

If a charge  $Q$  is introduced into any one of the spheres, this charge  $Q$  is redistributed

## ELECTROSTATICS

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into both the sphere such that the electrostatic potential is same in both the space. They are now uniformly charged and attain electrostatic equilibrium. Let  $q_1$  be the charge residing on the surface of sphere A and  $q_2$  is the charge residing on the surface of sphere B such that the  $Q = q_1 + q_2$ . The charges are distributed only on the surface and there is no net charge inside the conductor.

The electrostatic potential at the surface of the sphere A is given by,

$$V_A = \frac{1}{4\pi\epsilon_0} \frac{q_1}{r_1}$$

The Electrostatic potential at the surface of the sphere B is given by,

$$V_B = \frac{1}{4\pi\epsilon_0} \frac{q_2}{r_2}$$

The surface of the conductor is an equipotential. Since the spheres are connected by the conducting wire, the surfaces of both the spheres together form an equipotential surface. This implies that,

$$V_A = V_B$$

$$\frac{1}{4\pi\epsilon_0} \frac{q_1}{r_1} = \frac{1}{4\pi\epsilon_0} \frac{q_2}{r_2}$$

$$\frac{q_1}{r_1} = \frac{q_2}{r_2}$$

Let us take the charge density on the surface of sphere A and  $\sigma_1$  charge density on the surface of sphere B and  $\sigma_2$  implies that  $q_1 = 4\pi r_1^2 \sigma_1$  and  $q_2 = 4\pi r_2^2 \sigma_2$ .

Substituting these values,

$$\frac{4\pi r_1^2 \sigma_1}{r_1} = \frac{4\pi r_2^2 \sigma_2}{r_2}$$

$$\sigma_1 r_1 = \sigma_2 r_2$$

From which we concluded that,

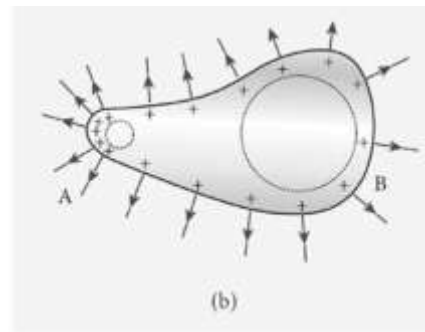
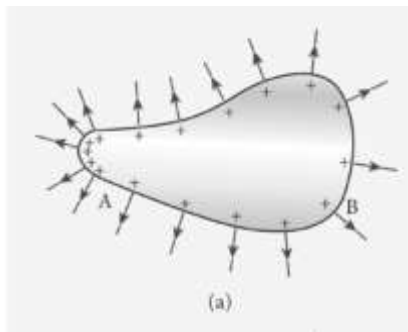
$$\sigma r = \text{constant}$$

Thus, the surface charge density ' $\sigma$ ' is inversely proportional to the radius of the sphere.

For a smaller radius, the charge density will be larger and vice versa.

### 90. ACTION OF POINTS (OR) CORONA DISCHARGE [ 2 MARK]

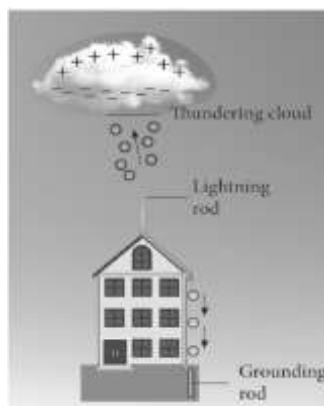
Consider a charged conductor of irregular shaped as shown in the figure. In conductors, if the radius curvature is smaller, the charge density around it will be larger, because of the large accumulator of charges over a small area.



The electric field near the edge of smaller curvature is very high and it ionizes the surrounding air. (i.e) The positive ions are repelled at the sharp edge and negative ions are attracted towards the sharper edges. This causes neutralisation of charges and reduces the charge of the conductor near the sharp edge. This is called **action at points (or) corona discharge**.

### 91. LIGHTNING ARRESTER [OR] LIGHTNING CONDUCTOR [ 3 MARK]

This is a device used to protect tall buildings from lightning strikes. It works on the principle of action at points or corona discharge. This device consists of a long thick copper rod passing from the top of the building to the ground. The upper end of the rod has a sharp spike or a sharp needle as shown in the figure.

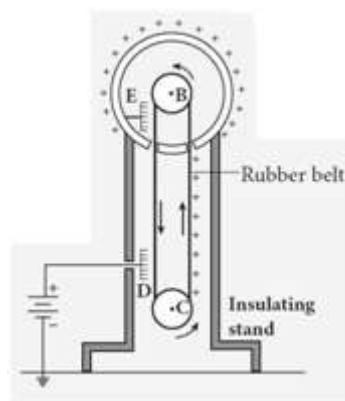


The lower end of the rod is connected to the copper plate which is buried deep into the ground. When a negatively charged cloud is passing above the building, it induces a positive charge on the spike. Since the induced charge density on the sharp spike is large, it results in corona discharge. This positive charge ionizes the surrounding air which in turn neutralizes the negative charge in the cloud. The negative charge pushes to the spikes passes through the copper rod and is safely diverted to the earth. The Lightning arrester does not stop lightning; rather it diverts the lightning to the ground safely.

## 92. VAN DE GRAFF GENERATOR [5 MARK]

In the year 1929, Robert van de Graaff designed a machine which produces large amount of electrostatic potential difference, up to several million volts [ $10^7$ ]. This van de Graaff generator works on the principle of electrostatic induction and action at points.

A large hollow spherical conductor is fixed on the insulating stand as shown in the figure. A pulley B is mounted at the centre of a hollow sphere and another pulley C is fixed at the bottom. A belt made up of insulating materials like silk or rubber runs over both pulleys. The pulley C is driven continuously by the electric motor. Two comb shaped metallic conductors E and D are fixed near the pulleys.



The comb D is maintained at a positive potential of  $10^4$  V by a power supply. The upper comb E is connected to the inner side of the hollow metallic sphere.

Because of the highest field near comb D air between the belt and comb D gets ionized. The positive charges are pushed towards the belt and Negative charges are attracted towards the comb D. The positive charges stick to the belt and move up. When the positive charges reach the comb E, a large amount of negative and positive charges are induced on either side of comb E, due to electrostatic induction. As a result, the positive charges are pushed away from the comb E and they reach the outer surface of the sphere. Since the sphere is a conductor, the positive charges are distributed uniformly on the outer surface of the hollow sphere. At the same time, the negative charges nullify the positive charge in the belt due to Corona discharge before it passes over the pulley.

When the belt descends, it has almost no net charge. At the bottom, it again gains a large positive charge. The belt goes up and delivers the positive charge to the outer surface of the sphere. This process continues until the outer surface produces the potential difference of the order of  $10^7$  V. Which is the limiting value we can't store charges beyond this limit. Since the extra charges start leaking to the surroundings due to ionization of air. The leakage of charges can be reduced by enclosing the machine in a gas filled steel chamber at very high pressure.

The high voltage produced in this Van De graaff generator is used to accelerator positive ions [ protons and neutrons ] for nuclear disintegration and other applications.

**IMPORTANT FORMULAE:**

1. Coulomb's law,  $\vec{F}_{21} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{r}_{12}$

2. Electric field due to a point charges ,  

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$$

3. Electric dipole moment ,  $\vec{P} = 2q\vec{d}$

4. Electric field due to a dipole along its axial line ,

$$\vec{E}_{\text{tot}} = \frac{1}{4\pi\epsilon_0} \cdot \frac{2\vec{P}}{r^3}$$

5. Electric field due to a dipole along its equatorial line ,

$$\vec{E}_{\text{tot}} = \frac{1}{4\pi\epsilon_0} \frac{\vec{P}}{r^3}$$

6. Torque experience by an electric dipole in a uniform electric field .

$$\vec{\tau} = \vec{P} \times \vec{E}; \quad \tau = qE2a \sin \theta$$

7. Electric potential at a point due to a point charge,

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

8. Electric potential at a point due to electric field ,

$$V = \frac{1}{4\pi\epsilon_0} \frac{P}{r^2} \cos \theta = \frac{1}{4\pi\epsilon_0} \frac{\vec{P} \cdot \vec{r}}{r^2}$$

9. Electric field  $E = -\frac{dv}{dx} = -\left[\frac{\partial v}{\partial x} \hat{i} + \frac{\partial v}{\partial y} \hat{j} + \frac{\partial v}{\partial z} \hat{k}\right]$

10. Electrostatic potential energy between two charges is ,

$$U = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$$

11. Electrostatic potential energy between three charges is ,

$$U = \frac{1}{4\pi\epsilon_0} \left[ \frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right]$$

12. Electrostatic potential energy stored in a dipole in an uniform electric field ,

$$U = -PE\cos\theta = -\vec{P} \cdot \vec{E}$$

13. Electrostatic potential energy difference between two angular positions  $\theta$  and  $\theta^l$  of a dipole kept in an uniform electric field ,

$$\Delta U = -PE\cos\theta + PE\cos\theta^l$$

14. Electric flux ,  $\phi_E = \vec{E} \cdot \vec{A} = EA\cos\theta$

15. Total electric flux through a closed surface ,

$$\phi_E = \frac{Q_{net}}{\epsilon_0}$$

16. Electric field due to an infinitely long charged wire is,

$$E = \frac{\lambda}{2\pi r\epsilon_0}$$

17. Electric field due to charged infinite plane sheet is,

$$E = \frac{\sigma}{2\epsilon_0}$$

18. Electric field due to two parallel charged infinite sheet at a point between the sheet

$$E = \frac{\sigma}{\epsilon_0}$$

19. Electric field due to uniformly charged spherical shell of radius 'R' at a point outside the shell ,

$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$$

20. Electric field due to uniformly charged spherical shell of radius 'R' at a point on the shell ,

$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{R^2}$$

21. For a parallel plate capacitor,

$$\text{Electric field , } E = \frac{Q}{\epsilon_0 A}$$

$$\text{Potential difference , } V = \frac{Q.d}{\epsilon_0 A}$$

$$\text{Capacitance , } C = \frac{\epsilon_0 A}{d}$$

$$\text{Stored electrostatic energy , } U = \frac{1}{2} E_0 A d E^2$$

$$\text{Energy density , } U_E = \frac{1}{2} \epsilon_0 E^2$$

22. (a) Battery is disconnected from the capacitor and a dielectric is inserted between the plates ,

$$C_0 = \frac{Q_0}{V_0} , \quad E = \frac{E_0}{\epsilon_r} , \quad V = \frac{V_0}{\epsilon_r} , \quad C = \frac{\epsilon_r \epsilon_0 A}{d} , \quad U_0 = \frac{1}{2} C_0 V_0^2 , \quad U = \frac{U_0}{\epsilon_r}$$

- (b) Battery remains connected and a dielectric inserted between the plates of the capacitor ,

$$Q = \epsilon_r Q_0 , \quad C = \epsilon_r C_0 , \quad C = \frac{\epsilon_r \epsilon_0 A}{d} , \quad U_0 = \frac{1}{2} C_0 V_0^2 , \quad U = \epsilon_r U_0$$

23. Capacitor in series :  $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

$$\text{Capacitor in parallel : } C_p = C_1 + C_2 + C_3$$

24. Two charged spheres connected through a wire ,

$$\frac{q_1}{r_1} = \frac{q_2}{r_2} , \quad Q = q_1 + q_2 , \quad \sigma_1 r_1 = \sigma_2 r_2 .$$

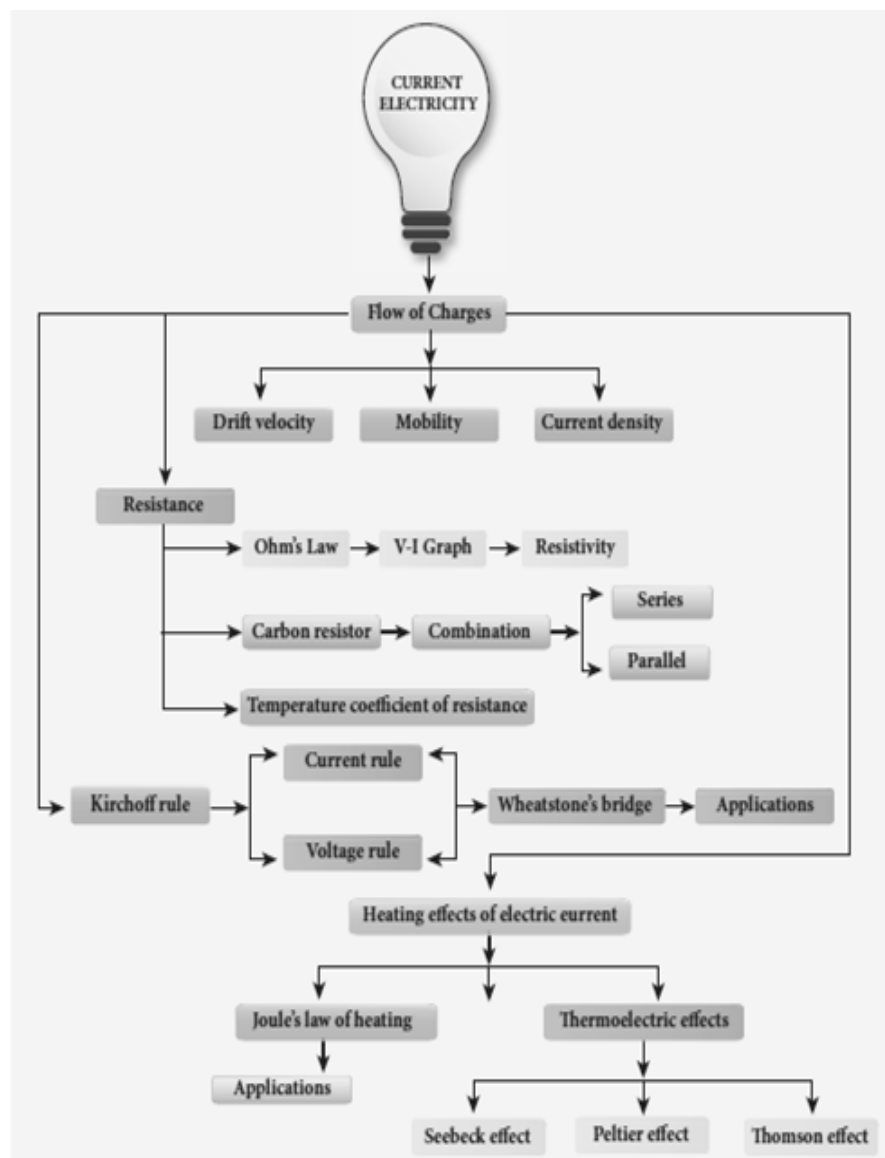
$$q_2 = Q \left[ \frac{r_2}{r_1 + r_2} \right]$$

25. In van de graaff generator , maximum potential difference created ,

$$V_{max} = 10^7 \text{ V.}$$

**12<sup>th</sup> PHYSICS****UNIT 2- CURRENT ELECTRICITY**

▸ MR. THIVYARAJ V, M.Sc.,M.PHILL.,B.ED

**LECTURE VIDEOS**

**1. CURRENT [ 2 MARK ]**

If a net charge Q passes through any cross section of a conductor in time t.

$$I = \frac{Q}{t}$$

It is a scalar quantity.

S.I unit ampere ( A)

**2. ONE AMPERE [ 2 MARK ]**

1 coulomb of charge passing through a perpendicular cross section in 1 second.

$$1A = \frac{1c}{1s}$$

**3. DRIFT VELOCITY [ 2 MARK ]**

The average velocity acquired by the electrons inside the conductor when it is subjected to an electric field.

$$\vec{V}_d = \vec{a}\tau \quad (\text{or}) \quad \vec{V}_d = -\mu\vec{E}$$

$$\text{Unit} \rightarrow \frac{m}{s} \quad (\text{or}) \quad \text{ms}^{-1}$$

**4. MOBILITY [ $\mu$ ] [ 2 MARK]**

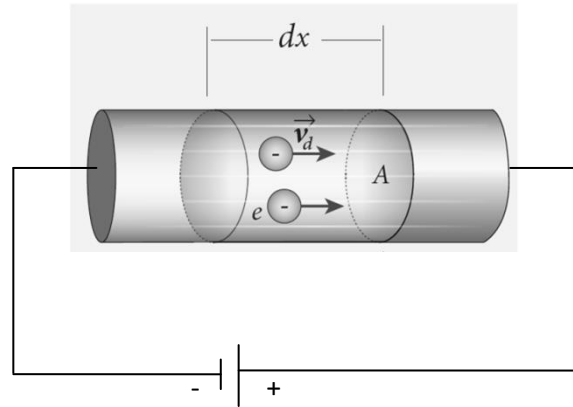
The magnitude of the drift velocity per unit electric field.

$$\mu = \frac{|\vec{V}_d|}{|\vec{E}|}$$

$$\text{Unit} \rightarrow \frac{m^2}{Vs} \quad (\text{or}) \quad \text{ms}^{-1}$$

**5. MICROSCOPIC MODEL OF CURRENT [ 5 MARK ]**

Consider a conductor with area of cross section 'A' and an electric field  $\vec{E}$  applied from right to left.



$V_d \rightarrow$  drift velocity of the electron

$n \rightarrow$  Electrons per unit volume in the conductor

Electrons move a distance 'dx' within a small interval of 'dt'.

$$V_d = \frac{dx}{dt}, \quad dx = V_d dt.$$

The electrons available in the volume of length  $dx$  is,

= volume  $\times$  number of electrons per unit volume.

$$= A dx \times n$$

$$= [A V_d dt] n$$

Total charge in volume element  $[dQ]$

= (charge)  $\times$  (number of electrons in the volume element)

$$dQ = e \times [A V_d dt] n$$

$$I = \frac{dQ}{dt} = \frac{neAV_d dt}{dt}$$

$$I = neAV_d$$

**6. CURRENT DENSITY [ 2 MARK ]**

The current per unit area of cross section of the conductor.

$$J = \frac{I}{A}$$

$$\vec{J} = ne\vec{V}_d$$

It is the vector quantity.

$$\text{SI unit} \rightarrow \frac{A}{m^2} \quad (\text{or}) \quad Am^{-2}$$

Substituting  $\vec{V}_d$

$$\vec{J} = -\frac{ne^2\tau}{m} \vec{E}$$

$$\vec{J} = -\sigma \vec{E}$$

$$\sigma = \frac{ne^2\tau}{m} \text{ is called conductivity.}$$

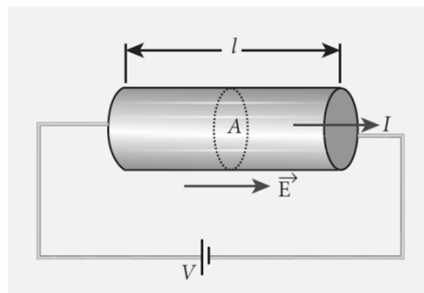
**Resistivity ( $\rho$ )**

The inverse of conductivity

$$\rho = \frac{1}{\sigma} = \frac{m}{ne^2\tau}$$

**7. OHM 'S LAW [ 5 MARK ]**

Consider a segment of wire of length 'l' and cross section area 'A'.



A potential difference 'V' is applied across the wire, a net electric field is created in the wire which constitutes the current.

$$V = E.l$$

$$J = \sigma E = \sigma \frac{V}{l}$$

$$\frac{I}{A} = \sigma \frac{V}{l}$$

$$J = \frac{I}{A}$$

$$V = I \left[ \frac{l}{\sigma A} \right]$$

The quantity  $\frac{l}{\sigma A}$  is called resistance of the conductor. [ R ]

$$R = \frac{l}{\sigma A}$$

$$R \propto \frac{l}{A}$$

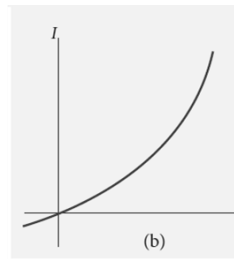
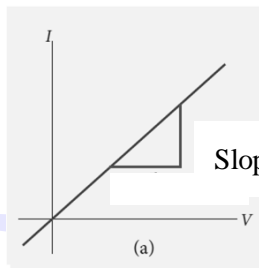
Macroscopic form of ohm's law.

$$V = IR$$

The resistance is the ratio of potential difference across the given conductor to the current passing through the conductor.

$$R = \frac{V}{I}$$

S.I Unit  $\rightarrow$  ohm ( $\Omega$ )



The graph between current versus voltage is straight line with a slope equal to the inverse of resistance R of the conductor. It is shown in the graph (a). Material for which the current against voltage graph is a straight line through the origin are said to obey Ohm's law and their behaviour is said to be ohmic materials or device that do not follow Ohm's law are said to be non-ohmic. They do not have a constant resistance.

### 8. RESISTIVITY ( $\rho$ ) [ 2 MARKS ]

The resistance offered to current flow by a conductor of unit length having unit area of cross section.

$$\rho = \frac{RA}{l}$$

SI Unit  $\rightarrow$  ohm meter ( $\Omega m$ )

**9. RESISTORS IN SERIES AND PARALLEL [ 5 MARK ]****RESISTORS IN SERIES :**

Consider three resistors  $R_1$ ,  $R_2$  and  $R_3$  connected in series. The amount of charge passing through resistors  $R_1$  must be pass through resistors  $R_2$  and  $R_3$ .

Since ,the charges cannot accumulate anywhere in the circuit . Due to this reason,this current  $I$  passing through all the three resistors is the same. According to Ohm's law ,if same current pass through different resistors of different values ,then the potential difference across each resistor must be different. Let  $V_1$ ,  $V_2$  and  $V_3$  be the potential difference across each of resistor  $R_1$ ,  $R_2$  and  $R_3$  respectively.  $V_1 = IR_1$ ,  $V_2 = IR_2$  and  $V_3 = IR_3$  .But the total voltage  $V$  is equal to the sum of voltage across each resistor .

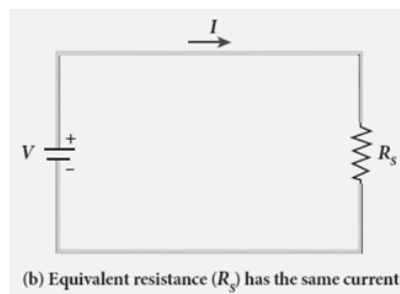
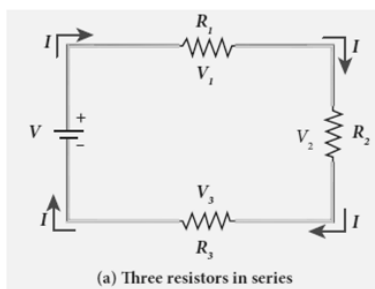
$$V = V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3$$

$$V = I ( R_1 + R_2 + R_3 )$$

$$V = IR_s$$

$$R_s = R_1 + R_2 + R_3$$

Where  $R_s$  is Equivalent resistance.



The total or equivalent resistance is the sum of the individual resistance.

The value of equivalent resistance in series connection will be greater than each individual resistance.

**RESISTORS IN PARALLEL :**

Consider three resistors  $R_1$ ,  $R_2$  and  $R_3$  are in parallel when they are connected across the same potential difference.

The total current  $I$  that leave the battery is split into three separate paths.

Let  $I_1$ ,  $I_2$  and  $I_3$  be the current through the resistors  $R_1$ ,  $R_2$  and  $R_3$  respectively. Due to the conservation of charges, total current in the  $I$  is equal to sum of the current through each of the three resistors.

$$I = I_1 + I_2 + I_3$$

Since, the voltage across each resistor is the same, applying ohm 's law to each resistors.

$$I_1 = \frac{V}{R_1}, I_2 = \frac{V}{R_2}, I_3 = \frac{V}{R_3}$$

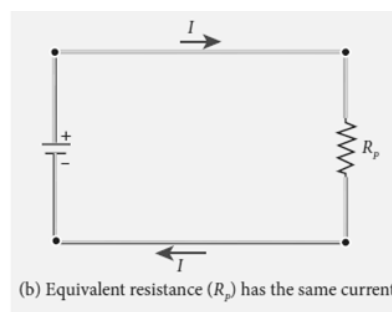
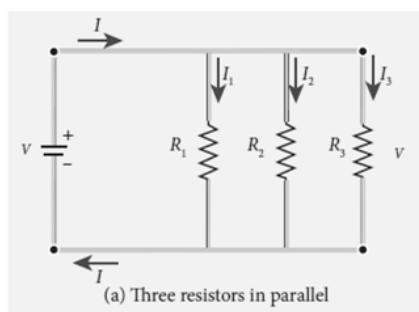
$$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$I = V \left[ \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right]$$

$$I = \frac{V}{R_p}$$

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$R_p \rightarrow$  Equivalent resistance of the parallel combination of resistors.



The reciprocal of the effective resistance is equal to the sum of the reciprocal of the values of resistance of the individual resistor. The value of equivalent resistance will be lesser than each individual resistance

**10. COLOUR CODE FOR THE CARBON RESISTORS [ 3 MARKS ]**

Carbon resistor consists of a ceramic core ,on which a thin layer of Crystalline carbon is deposited

**USES**

1. Inexpensive.
2. Stable.
3. Compact in size.

**VALUE OF THE RESISTANCE :**

Using colour rings.

Three coloured rings are used.

First two rings are significant figures of resistances, the third ring indicates the decimal multiplier after them. The fourth colour, silver or Gold shows the tolerance of resistor at 10 % or 5% . If there is no forth ring, the tolerance is 20 %.

**EXAMPLE**

First ring → green

Second ring → blue

Third ring → orange

Tolerance → gold

The value of resistance

$$5 \quad 6 \quad 10^3 \quad \pm 5 \% \rightarrow 56 \times 10^3 \, \Omega \text{ (or) } 56 \, \text{k}\Omega \pm 5\%$$

Color	Number	Multiplier	Tolerance
Black	0	1	
Brown	1	$10^1$	
Red	2	$10^2$	
Orange	3	$10^3$	
Yellow	4	$10^4$	
Green	5	$10^5$	
Blue	6	$10^6$	
Violet	7	$10^7$	
Gray	8	$10^8$	
White	9	$10^9$	
Gold		$10^{-1}$	5%
Sliver		$10^{-2}$	10%
Colorless			20%

**11. TEMPERATURE COEFFICIENT OF RESISTIVITY [ $\alpha$ ] [ 2 MARKS]**

The ratio of increase in resistivity per degree rise in temperature to its resistivity at  $T_0$ .

$$\alpha = \frac{\rho_T - \rho_0}{\rho_0 [T - T_0]} = \frac{\Delta \rho}{\rho_0 \Delta T}$$

Unit  $\rightarrow$   $^{\circ}\text{C}$

 **$\alpha$  of conductors**

$\alpha$  is positive, if the temperature of a conductor increases, the resistivity increases.

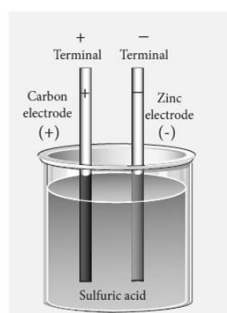
 **$\alpha$  of semiconductors**

$\alpha$  is negative, the resistivity decreases with increase in temperature.

**12. ENERGY AND POWER IN ELECTRICAL CIRCUITS:**

**DISTINGUISH** [ 3 MARK (or) 2 MARK ]

S.NO	ELECTRIC POWER	ELECTRIC ENERGY
1	The rate at which the electrical potential energy is delivered.	Multiplying the power and duration of the time.
2	$P = VI$	Energy = VIT
3	Watt (or) $\frac{J}{s}$	Joules (or) Watt hour In practice , 1 Kwh = 1000 wh = $3.6 \times 10^6$ J

**13. ELECTRIC CELLS AND BATTERIES:** [ 3 MARK ]

An electric cell converts chemical energy into electrical energy to produce electricity. It contains two electrode immersed in an electrolyte. Several electric cells connected together

form a battery. When it's connected to a circuit, electrons flow from the positive to negative terminal through the circuit. By using chemical reactions, a battery produces potential difference across its terminals. It provides the energy to move the electrons through the circuit.

#### 14. ELECTROMOTIVE FORCE [ $\xi$ ] [ 2 MARK]

The amount of work a battery or cell does to move a certain amount of charge around the circuit.

UNIT  $\rightarrow$  Volt ( V )

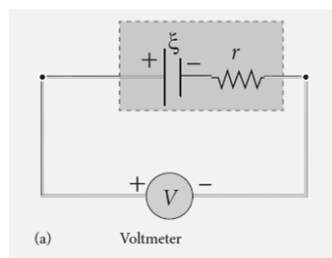
#### 15. INTERNAL RESISTANCE [r] [ 2 MARK]

There is resistance to the flow of charges within the battery.

UNIT  $\rightarrow$  Ohm (  $\Omega$  )

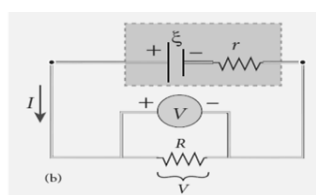
#### 16. DETERMINATION OF INTERNAL RESISTANCE [ 5 MARK ]

The EMF of cell  $\xi$  is measured by connecting a high resistance voltmeter across it without connecting the external resistance R. [ Fig (A) ]



Hence, the voltmeter reading gives the EMF of the cell. Then, external resistance R is included in the circuit and current I is established in the circuit.

The potential difference across R is equal to the potential difference across the cell (V). [ Fig (b) ]



$$V = IR$$

Due to internal resistance 'r' of the cell, the voltmeter reads a value V, which is less than the EMF of cells certain amount of voltage( Ir) has dropped.

$$V = \xi - Ir$$

$$Ir = \xi - V$$

$$\frac{Ir}{IR} = \frac{\xi - V}{V}$$

$$r = \left[ \frac{\xi - V}{V} \right] R$$

Since  $\xi$ , V and R an internal resistance r can be determined.

Due to this internal resistance, the power delivered to the circuit is not equal to power rating mentioned in the battery.

The power,

$$P = I \xi = I [V + Ir]$$

$$P = I [IR + Ir]$$

$$P = [I^2R + I^2r]$$

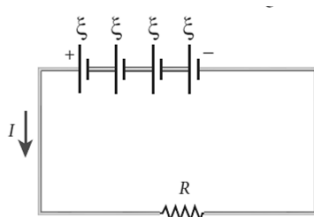
Hence,  $I^2r$  is the power delivered to the internal resistance.  $I^2R$  is the power delivered to the electrical circuit. For a good battery, the internal resistance 'r' is very small, then  $I^2r \ll I^2R$  and almost entire power is delivered to the resistance.

## 17. CELL IN SERIES [ 3 MARK ]

In series connection, the negative terminal of one cell is connected to the positive terminal of second cell, the negative terminal of second cell is connected to the positive

terminal of the third cell and so on. The free positive terminal of the first cell and the free negative terminal of the last cell become the terminals of the battery.

Suppose,  $n$  cells, each of EMF  $\xi$  volts and internal resistance ' $r$ ' ohms are connected in series with an external resistance ' $R$ '.



Cells in series (circuit diagram)

The total emf of the battery =  $n \xi$

The total resistance in the circuit =  $nr + R$

By ohm's law,

$$I = \frac{\text{total emf}}{\text{total resistance}} = \frac{n \xi}{nr + R}$$

**Case (a) : [  $r \ll R$  ]**

$$I = \frac{n \xi}{R} \approx n I_1$$

$I_1$  is the current due to a single cell =  $\frac{\xi}{R}$

The current supplied by the battery is ' $n$ ' times that supplied by a single cell.

**Case (b) : [  $r \gg R$  ]**

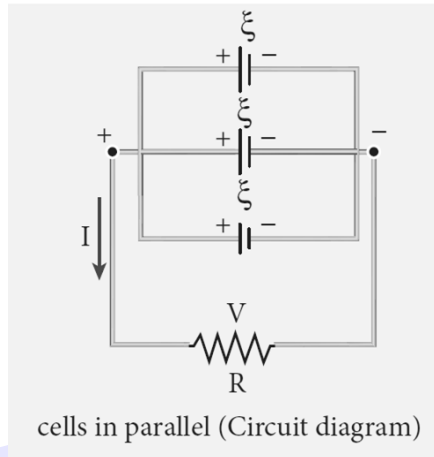
$$I = \frac{n \xi}{nr} \approx \frac{\xi}{r}$$

It is the current due to single cell.

There is no advantage in connecting several cells.

Thus series connection of cells is advantageous only when the effective internal resistance of the cell is negligible small compared with 'R' .

### 18. CELLS IN PARALLEL : [ 3 MARK]



In parallel connection all the positive terminals of the cells are connected to one point and all the negative terminals to a second point. These two points form the positive and the negative terminal of the battery.

Let 'n' cells can be connected in parallel between the points A and B and a resistance R is connected between that points A and B.

$\xi \rightarrow$  EMF of the cell

$r \rightarrow$  Internal resistance .

The equivalent resistance,

$$\frac{1}{r_{eq}} = \frac{1}{r} + \frac{1}{r} + \dots \dots \dots \frac{1}{r} = [n \text{ terms}] = \frac{n}{r}$$

$$r_{eq} = \frac{r}{n}$$

The total resistance =  $R + \frac{r}{n}$

The total emf is the potential difference between the points A and B which is equal to  $\xi$

$$I = \frac{\xi}{R + \frac{r}{n}} = \frac{n \xi}{r + nR}$$

**Case (a) : [  $r \gg R$  ]**

$$I = \frac{n \xi}{r} = nI_1$$

The current through the external resistance due to the whole battery is n times the current due to a single cell.

**Case (a) : [  $r \gg R$  ]**

$$I = \frac{\xi}{R}$$

The current due to the whole battery is the same as that due to a single cell.

Hence it is advantages to connect cells in parallel when the external resistance is very small compared to the internal resistance of the cells.

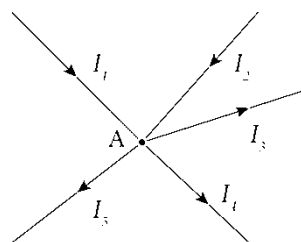
### 19. KIRCHHOFF'S RULES : [ 3 MARK]

Ohm's law is useful only for simple circuits .For more Complex circuits, kirchoff's law can be used to find current and voltage.

- i. Current rule
- ii. Voltage rule

### 20. CURRENT RULE OR JUCTION RULE : [ 3 MARK]

The algebraic sum of the currents at any junction of a circuit is zero. current entering the junction is taken as positive and current leaving the junction is taken as negative.



$$I_1 + I_2 - I_3 - I_4 - I_5 = 0$$

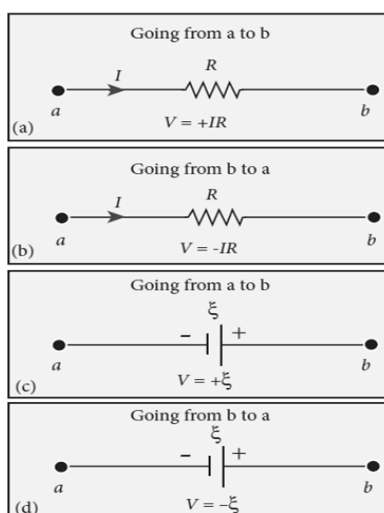
$$I_1 + I_2 = I_3 + I_4 + I_5$$

The sum of the entering current in a Junction is equal to the sum of the leaving current from the junction. This is follow from the law of conservation of charges.

## 21. VOLTAGE RULE OR LOOP RULE : [ 2 MARK ]

In a closed circuit the algebraic sum of the products of the current and resistance of each part of the circuit is equal to the total emf included in the circuit. This rule follows from the law of conservation of energy for an isolated system.

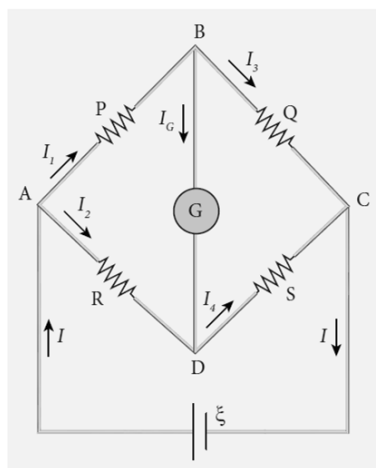
The product of current and resistance is taken as positive when the direction of current followed. Suppose, if the direction of current is opposite to the direction of the loop, then product of current and voltage across the resistance is negative.



The emf is considered positive when proceeding from negative to the positive terminal of the cell. This rule applied only when all currents in the circuit reach a steady state condition.

**22. WHEATSTONE'S BRIDGE [ 5 MARK]**

An important application of Kirchhoff's rules is the Wheatstone's Bridge. The Bridge consists of four resistances P, Q, R and S connected. A galvanometer G is connected between the points B and D. The battery is connected between the points A and C. The current through the Galvanometer is ' $I_G$ ' and its resistance is ' $G$ '.



Applying Kirchhoff's current rule to junction B,

$$I_1 - I_G - I_3 = 0$$

①

Applying Kirchhoff's current rule to junction D,

$$I_2 + I_G - I_4 = 0$$

②

Applying voltage rule to loop ABDA,

$$I_1 P + I_G G - I_3 R = 0$$

③

Applying voltage rule to loop ABCDA,

$$I_1 P + I_3 Q - I_4 S - I_2 R = 0$$

④

The points B and D are at same potential the bridge is said to be balanced. No current flows through galvanometer ( $I_G = 0$ ). Substituting  $I_G = 0$  in above equation ①, ② and ③

$$I_1 = I_3$$

$$I_2 = I_4$$

$$I_1P = I_2R \longrightarrow \textcircled{5}$$

Above equation substituting eq  $\textcircled{4}$

$$I_1P + I_3Q - I_4S - I_2R = 0$$

$$I_1(P + Q) = I_2(S + R)$$

Dividing equation  $\textcircled{6}$  by eq  $\textcircled{5}$

$$\frac{P + Q}{P} = \frac{R + S}{R}$$

$$1 + \frac{Q}{P} = 1 + \frac{S}{R}$$

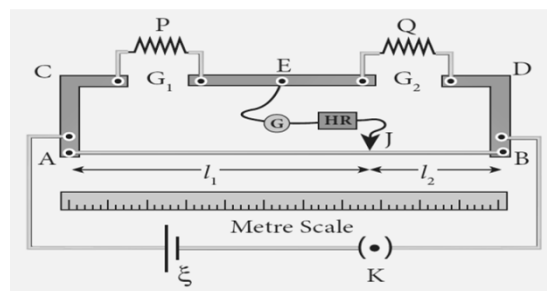
$$\frac{Q}{P} = \frac{S}{R}$$

$$\frac{P}{Q} = \frac{R}{S}$$

This is the bridge balance condition .Only under this condition, Galvanometer shows a deflection. If three of the resistance are known ,the value of unknown resistance can be determined.

### 23. METER BRIDGE [ 5 MARK ]

The metre bridge is another form of wheatstone's bridge. It consists of a uniform maganin wire AB of one meter length. This wire is stretched along a metre scale on a wooden board between two copper strips C and D. Between these two copper strips another copper strip E is mounted to enclose two gaps  $G_1$  and  $G_2$ . An unknown resistance 'P' is connected in  $G_1$  and a standard resistance Q is connected in  $G_2$ . A Jockey is connected to the terminal E on the central copper strip through a galvonometer (G) and high resistance(HR) . The exact position of Jockey on the wire can be read on the scale . A lechlanche cell and a key [K] are connected across the ends of bridge wire.



A position of the jockey on the wire is adjusted so that the Galvanometer shows zero deflection. Let the point be J. The lengths AJ and JB of the bridge wire now replaced a resistance R and S of the wheatstone's bridge.

$$\frac{P}{Q} = \frac{R}{S} = \frac{R^l \cdot AJ}{R^l \cdot JB}$$

$R^l \rightarrow$  Resistance per unit length of wire.

$$\frac{P}{Q} = \frac{AJ}{JB} = \frac{l_1}{l_2}$$

$$P = Q \frac{l_1}{l_2}$$

The bridge wire is soldered at the ends of the copper strips. Due to imperfect contact, some resistance might be introduced at the contact. These are called end resistance. This error can be eliminated, if another set of readings are taken with P and Q interchange and the average value of P is found.

To find the specific resistance of the material of the wire in the coil P.

$r \rightarrow$  Radius of the wire.

$l \rightarrow$  Length of the wire

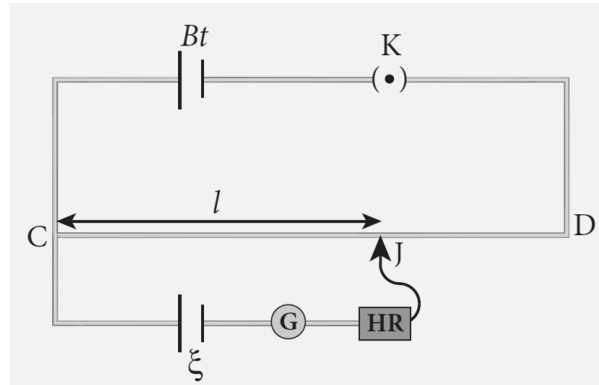
$$\text{Resistance} = \rho \frac{l}{A}$$

$$\rho = P \frac{A}{l}$$

$$\rho = P \frac{\pi r^2}{l}$$

**24. POTENTIOMETER [ 3 MARK]**

Potential is used for the accurate measurement of potential difference, current and resistance. It consists of ten metre long uniform wire of Maganin or constantan stretched in parallel rows each of one metre length, on a wooden board. The two free ends A and B are brought to the same side and fixed to Copper stripes with binding screws. A metre scale is fixed parallel to the wire. A Jockey is provided for making contact.

**PRINCIPLE OF THE POTENTIOMETER.**

A steady current is maintained across the wire CD by a battery Bt. The battery, key and potentiometer wire are connected in series forms the primary circuit. The positive terminal of the cell of emf  $\xi$  is connected to the point C and negative terminal is connected to jockey through a galvanometer G and a High resistance HR. This forms the secondary circuit.

Let Contact be make at any point J on the wire by jockey. If the potential difference across CJ is equal to the emf of the cell  $\xi$  then no current will flow through the Galvanometer and it will show zero deflection. CJ is the balancing length l. The potential difference across CJ is equal to  $Irl$ . Where 'I' is the current flowing through the wire and 'r' is the resistance per unit length of the wire.

$$\xi = Irl$$

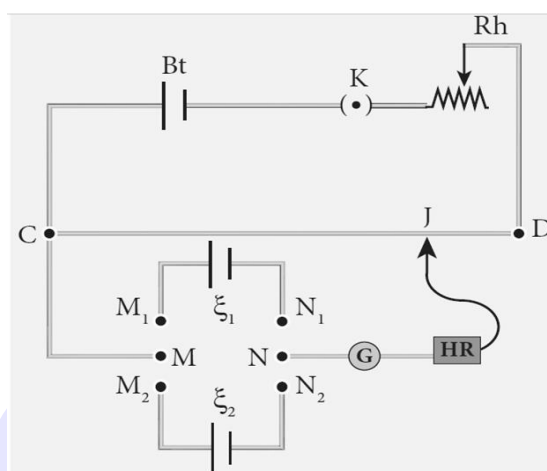
Since I and r constant,

$$\xi \propto l$$

The emf of the cell is directly proportional to the balancing length.

## 25. COMPARISON OF EMF OF TWO CELLS WITH A POTENTIOMETER [ 5 MARK]

To compare the EMF of two cells the circuit connections are made as shown in figure. Potentiometer wire CD is connected to a battery Bt and a key K in series. This is the primary circuit. The end C of the wire is connected to the terminal M of DPDT [double pole double throw] switch and the other terminal N is connected to a jockey through a galvanometer G and high resistance HR. The cells whose emf  $\xi_1$  and  $\xi_2$  to be compared are connected to the terminals  $M_1 N_1$  and  $M_2 N_2$  of the DPDT switch .



The DPDT switch is pressed towards  $M_1, N_1$  so that cell  $\xi_1$  is included in the secondary circuit and balancing length  $l_1$  is found by adjusting then jockey for zero deflection. Then the second cell  $\xi_2$  is included in the circuit and the balancing length  $l_2$  is determined. Let  $r$  be the resistance per unit length of the Potentiometer wire and  $I$  be the current flowing through the wire.

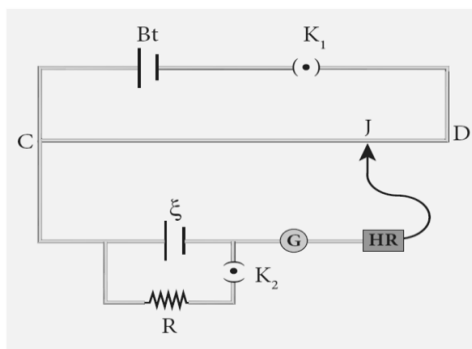
$$\xi_1 = Irl_1 \quad \xi_2 = Irl_2$$

$$\boxed{\frac{\xi_1}{\xi_2} = \frac{l_1}{l_2}}$$

By including a Rheostat [Rh] in the primary circuit, the experiment can be repeated several times by changing the current flowing through it

## 26. MEASUREMENT OF INTERNAL RESISTANCE OF A CELL BY POTENTIOMETER [ 3 MARK]

The end C of the potentiometer wire is connected to the positive terminal of the battery Bt and negative terminal of the battery is connected to the end D through a key  $K_1$ . This forms the primary circuit .



The positive terminal of the cell  $\xi$  whose internal resistance is to be determined is also connected to the end C of the wire. The negative terminal of the cell  $\xi$  is connected to a jockey through a galvanometre and a high resistance. A resistance box R and key  $K_2$  are connected across the cell . With key  $K_2$  open, the balancing point J is obtained and the balancing length CJ =  $l_1$  is measured. Since the cell is in open circuit ,its emf is

$$\xi \propto l_1$$

A suitable resistance [  $10 \Omega$  ] is included in the resistance box and  $K_2$  is closed.

The current is,

$$I = \frac{\xi}{R + r}$$

The potential difference across R is,

$$V = \frac{\xi}{R + r} R$$

This potential difference is balanced on the Potentiometer wire [  $l_2$  ]

Then

$$\frac{\xi}{R+r} R \propto l_2$$

$$\frac{R+r}{R} = \frac{l_1}{l_2}$$

$$1 + \frac{r}{R} = \frac{l_1}{l_2}$$

$$r = R \left[ \frac{l_1}{l_2} - 1 \right] = R \left[ \frac{l_1 - l_2}{l_2} \right]$$

Substituting the values of R, L<sub>1</sub> and L<sub>2</sub> the value r determined .The experiment can be repeated for different values of R. The external resistance increases and also increases the internal resistance.

### 27. WHY IS COPPER WIRE NOT SUITABLE FOR A POTENTIOMETER? [ 2 MARK ]

1. The value of temperature coefficient of resistance for copper is **high**.
2. Further its resistivity is **low**.

### 28. HEATING EFFECT OF ELECTRIC CURRENT [ 2 MARK]

When current flows through a resistor, some of the electrical energy delivered on to the system is converted into heat energy and is dissipated .This heating effect of current is known as Joule's heating effect.

### 29. JOULE'S LAW [ 2 MARK]

The heat developed in an electrical circuit due to the flow of current varies directly as,

1. The square of the current.
2. The resistance of the circuit and

3. The time of flow.

$$H = I^2RT$$

This relation was experimentally verified by Joule and is known as Joule's law of heating.

### 30. APPLICATION OF JOULE'S HEATING EFFECT [ 3 MARK]

#### 1. ELECTRIC HEATERS:

Electric iron ,electric heater , electric toaster are some of the home appliances that utilise the heating effect of current. The heating elements are made of nichrome and alloy of Nickel and chromium.

#### WHY NICHROME IS USED AS A HEATING ELEMENT ? [ 2 MARK]

1. High specific resistance.
2. very high temperature. ( high melting point )
3. Without oxidation.

#### 2. ELECTRICAL FUSES

Fuses are connected in series in a circuit to protect the electrical devices from the heat development by the passage of excessive current. It is short length of a wire made of a low melting point material. It melts and breaks the circuit if current exceeds a certain Value lead and copper wire melts And burns out when the current increases above 5A and 35 A respectively.

#### 3. ELECTRICAL FURNACE

Furnace are used to manufacture such as Steel ,silicon carbide, quartz , gallium arsenide etc... To produce temperature of 1500°C molybdenum - nichrome Wire wound on a silica tube is used carbon arc furnaces produce temperature upto 3000 °C.

#### 4. ELECTRICAL LAMP

It consists of tungsten filament [melting point 3380°C] kept inside a glass bulb and heated to incandescent by current . Incandescent electric lamps only about 5 % of

electrical energy is converted into light and the rest is wasted as heat. And also electric discharge lamps, electric welding and electric arc .

### 31. THERMOELECTRIC EFFECT [ 2 MARK]

Conversion of temperature difference into electrical voltage and vice versa is known as **thermoelectric effect**

### 32. SEEBECK EFFECT [ 2 MARK]

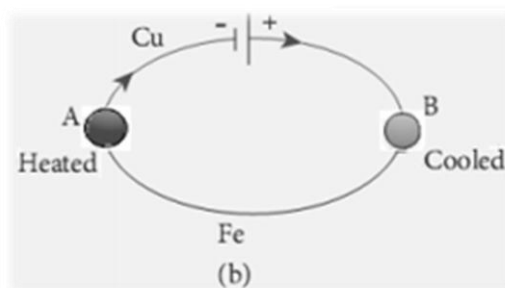
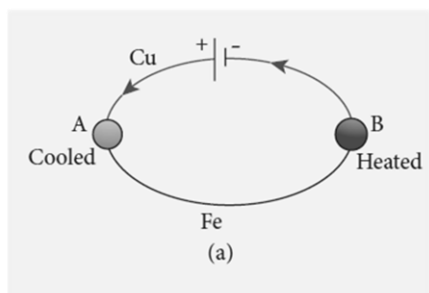
A closed circuit consisting of two dissimilar metals ,when the junctions are maintained at different temperatures an emf is developed .That current is called thermoelectric current .The two dissimilar metals connected to form two junctions is known as Thermocouple.It is reversible.

### 33. APPLICATION OF SEEBECK EFFECT [ 2 or 3 MARK ]

1. Seebeck effect is used in electric generators.
2. This effect is utilised in automobiles as automotive thermoelectric generators for increasing fuel efficiency.
3. Seebeck effect is used in thermocouples and thermophiles to measure the temperature difference between two objects.

### 34. PELTIER EFFECT [ 2 MARK]

An electric current is passed through a circuit of a thermocouple, heat is evolved at one junction and observed at the other Junction.



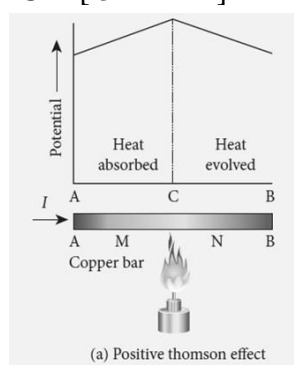
In the cu-Fe Thermocouple junctions A and B are maintained at the same

temperature. Let a current from your battery flow through the thermocouple. At the junction A, where the current flows from Cu to Fe, heat is observed and the junction A becomes cold. At the junction B, where the current flows from Fe to Cu, heat is liberated and it becomes hot. When the direction of current is reversed, junction A gets heated and junction B gets cooled. Hence Peltier effect is reversible.

### 35. THOMSON EFFECT [ 2 MARK ]

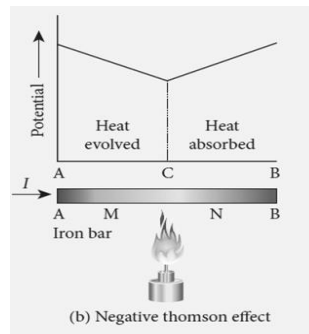
If two points in a conductor are at different temperatures, the density of electrons at these points will differ and as a result the potential difference is created between these points. Thomson effect is reversible.

### 36. POSITIVE THOMSON EFFECT [ 3 MARK ]



If current is passed through a copper bar AB which is heated at the middle point C, the point C will be at higher potential. This indicates that the heat is observed along AC and evolved along CB of the conductor. Thus heat is transferred due to the current flow in the direction of the current. It is called positive effect.

**EX :** Silver, zinc and cadmium.

**37. NEGATIVE THOMSON EFFECT [ 3 MARK]**

When the copper bar is replaced by an iron bar , heat is evolved along CA and absorbed along BC. Thus heat is transferred due to the current flow in the direction opposite to the direction of current. It is called negative Thomson effect.

**EX:** Platinum ,Nickel , Cobalt and mercury .

**38. WHAT ARE THE FACTORS DEPENDS ON THE MAGNITUDE OF THE THERMOELECTRIC EMF? [ 2 MARKS]**

- i. The nature of the metal forming the couple.
- ii. The temperature difference between the junctions.

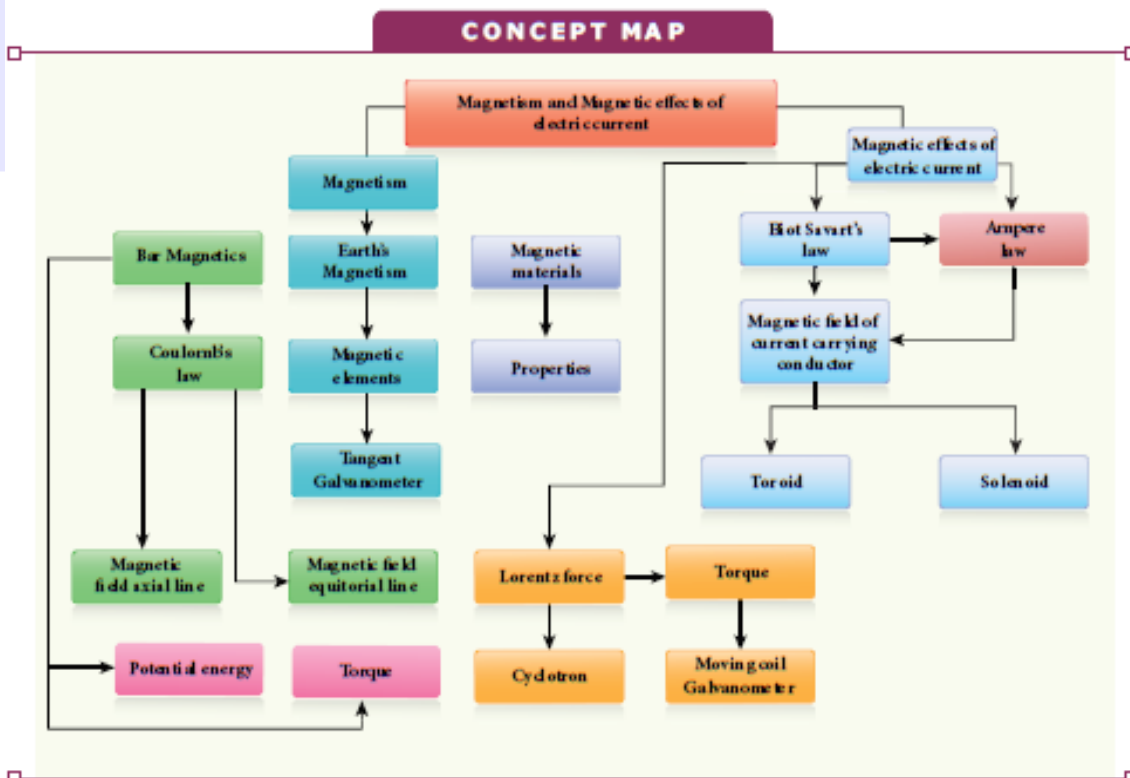
# 12<sup>th</sup> PHYSICS

## UNIT-3

### MAGNETISM AND MAGNETIC EFFECTS OF ELECTRIC CURRENT

MR. THIVYARAJ V., M.Sc., M.Phil., B.Ed

LECTURE VIDEOS 



**1. GEOMAGNETISM OR TERRESTRIAL MAGNETISM**

[2 MARK]

The branch of Physics which deals with the Earth's magnetic field is called **Geomagnetism or Terrestrial magnetism**.

**2. DECLINATION OR MAGNETIC DECLINATION [ D ]**

[ 2 MARK]

The angle between magnetic meridian at a point and geographical Meridian is called the **declination**.

**3. DIP OR MAGNETIC INCLINATION [ I ]**

[ 2 MARK]

The angle subtended by the Earth's total magnetic field with the horizontal direction in the magnetic meridian is called **dip or magnetic inclination**.

**4. HORIZONTAL COMPONENT OF EARTH'S MAGNETIC FIELD [B<sub>H</sub> ]**

[2 MARK]

The component of Earth's magnetic field along the horizontal direction in the magnetic meridian.

$$B_H = B_E \cos I$$

**5. MAGNETIC DIPOLE MOMENT [  $\vec{P}_m$  ]**

[ 2 mark]

The product of its pole strength and magnetic length.

$$\vec{P}_m = q_m \vec{d}$$

$$P_m = 2q_m l$$

$$\text{SI unit} \rightarrow \text{Am}^2$$

**6. MAGNETIC FIELD [  $\vec{B}$  ]**

[ 2 MARK]

A force experienced by the bar magnet of unit pole strength.

$$\vec{B} = \frac{1}{q_m} \vec{F}$$

$$\text{Unit} \rightarrow \text{NA}^{-1} \text{m}^{-1}$$

**7. PROPERTIES OF MAGNET**

[ 3 MARK]

1. A freely suspended bar magnet will always point along the north-south direction.
2. A magnet attracts another magnet or magnetic substances towards itself. The attractive force is maximum near the end of the bar magnet.
3. When a magnet is broken into pieces, each piece behaves like a magnet with poles at its ends.
4. Two poles of a magnet have pole strength equal to one another.
5. The length of the bar magnet is called geometrical length and the length between two magnetic poles in a bar magnet is called a magnetic length.

Magnetic length < geometrical length.

The ratio of magnetic length and geometrical length is = 0.833.

**8. MAGNETIC FIELD LINES**

[ 3 MARK ]

1. Magnetic field lines are continuous closed curves. The direction of magnetic field lines is from north pole to south pole of the magnet and South Pole to north pole inside the magnet.
2. The direction of magnetic field at any point on the curve is known by drawing tangent to the magnetic line of force at that point.
3. Magnetic field lines never intersect each other.
4. The degree of closeness of the field line determines the relative strength of the magnetic field. The magnetic field is strong where magnetic field lines crowd and weak magnetic field lines thin out.

**9. MAGNETIC FLUX [  $\phi_B$  ]**

[ 2 MARK]

The number of magnetic field lines crossing per unit area is called magnetic flux  $\phi_B$ .

$$\phi_B = \vec{B} \cdot \vec{A} = BA \cos \theta$$

$\theta$  is angle between  $\vec{B}$  and  $\vec{A}$ .

a)  $\vec{B}$  is normal to the surface

$$\theta = 0^\circ$$

$$\phi_B = BA$$

b)  $\vec{B}$  is parallel to the surface

$$\theta = 90^\circ$$

$$\phi_B = 0$$

Suppose the magnetic field is not uniform over the surface,

$$\phi_B = \int \vec{B} \cdot d\vec{A}$$

SI unit  $\rightarrow$  Weber [wb]

It is Scalar quantity.

$$1 \text{ wb} = 10^8 \text{ Maxwell [ CGS unit]}$$

## 10. MAGNETIC FLUX DENSITY

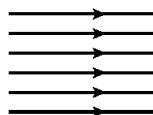
[ 2 MARK]

The number of magnetic field lines crossing unit area kept normal to the direction of line of force.

$$\text{unit} \rightarrow \text{wb m}^{-2} [ \text{ or } ] \text{ tesla}$$

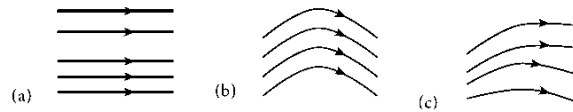
## 11. UNIFORM MAGNETIC FIELD

[ 3 MARK ]



Magnetic field is said to be uniform if it has same magnitude and direction at all the points in a given region.

**Ex:** local Earth's magnetic field is uniform.

**NON-UNIFORM MAGNETIC FIELD**

Magnetic field is said to be non-uniform if the magnitude and direction or both where is at all its points.

**Ex:** magnetic field of a bar magnet.

**12. COULOMB'S INVERSE SQUARE LAW OF MAGNETISM [ 2 MARK]**

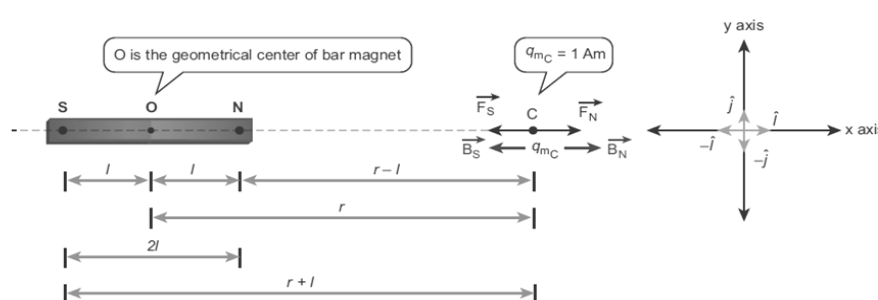
The force of attraction or repulsion between two magnetic force is directly proportional to the product of the of their pole strengths and inversely proportional to the square of the distance between them.

$$\vec{F} \propto \frac{q_{mA} q_{mB}}{r^2} \hat{r}$$

$$\vec{F} = k \frac{q_{mA} q_{mB}}{r^2} \hat{r}$$

$$k \approx \frac{\mu_0}{4\pi} \approx 10^{-7} \text{ Hm}^{-1}$$

[ $\mu_0 \rightarrow$  absolute permeability of free space ]

**13. MAGNETIC FIELD AT A POINT ALONG THE AXIAL LINE OF THE MAGNETIC DIPOLE [ 5 MARK]**

Consider a bar magnet NS shown in the figure. let N be the north pole and S be the South Pole of the bar magnet, which of pole strength  $q_m$  and separated by a distance of  $2\ell$ . The magnetic field at a point C at a distance from the geometrical centre O of the bar magnet can be computed by keeping unit north pole [ $q_{mc} = 1 \text{ Am}$ ] at C.

The force of repulsion between north pole of the bar magnet and unit north pole at point C. [Due to coulomb's law]

$$\vec{F}_N = \frac{\mu_0}{4\pi} \frac{q_m}{[r-l]^2} \hat{i}$$

The force of attraction between south pole of the bar magnet and unit north pole at point C.

$$\vec{F}_S = -\frac{\mu_0}{4\pi} \frac{q_m}{[r+l]^2} \hat{i}$$

Net force at point C ,

$$\vec{F} = \vec{F}_N + \vec{F}_S$$

$$\vec{F} = \vec{B}$$

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q_m}{[r-l]^2} \hat{i} + \left[ -\frac{\mu_0}{4\pi} \frac{q_m}{[r+l]^2} \hat{i} \right]$$

$$\vec{B} = \frac{\mu_0 q_m}{4\pi} \left[ \frac{1}{[r-l]^2} - \frac{1}{[r+l]^2} \right] \hat{i}$$

$$\vec{B} = \frac{\mu_0 2r}{4\pi} \left[ \frac{q_m \cdot [2l]}{[r^2 - l^2]^2} \right] \hat{i}$$

$$\vec{B} = \frac{\mu_0}{4\pi} \left[ \frac{2rP_m}{[r^2 - l^2]^2} \right] \hat{i}$$

$$|\vec{P}_m| = P_m = q_m \cdot 2\ell$$

$$[r^2 - l^2]^2 \approx r^4$$

$$r \gg \ell$$

$$\vec{B}_{axial} = \frac{\mu_0}{4\pi} \left[ \frac{2P_m}{r^3} \right] \hat{i}$$

$$\hat{P}_m = P_m \hat{i}$$

$$\vec{B}_{axial} = \frac{\mu_0}{4\pi} \frac{2}{r^3} \vec{P}_m$$

#### 14. MAGNETIC FIELD AT A POINT ALONG THE EQUATORIAL LINE DUE TO A MAGNETIC DIPOLE

Consider a bar magnet NS. Let N be the north pole and S be South Pole of the bar magnet, each with pole strength  $q_m$  and separated by a distance  $2\ell$ . The magnetic field at a point c at distance  $r$  from the geometrical centre O of the bar magnet can be computed by keeping unit north pole [ $q_{mc} = 1 \text{ Am}$ ] at c. The force of repulsion between North pole of the bar magnet and unit north pole at a point c.

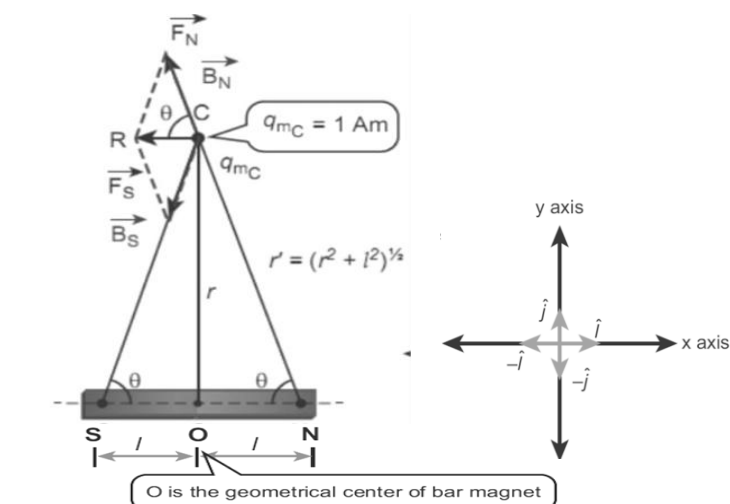
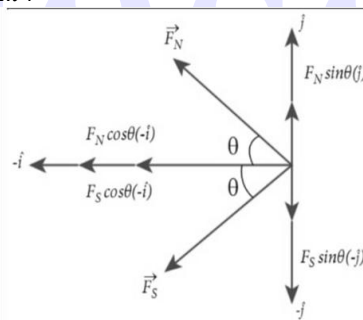
$$\vec{F}_N = -F_N \cos \theta \hat{i} + F_N \sin \theta \hat{j}$$

$$\text{where } F_N = \frac{\mu_0}{4\pi} \frac{q_m}{r^2}$$

The force of attraction between South Pole of the bar magnet and unit north pole at a point c.

$$\vec{F}_S = -F_S \cos \theta \hat{i} - F_S \sin \theta \hat{j}$$

$$\text{where } F_S = \frac{\mu_0}{4\pi} \frac{q_m}{r^2}$$



The net force at a point c is,

$$\vec{F} = \vec{F}_N + \vec{F}_S$$

This net force is equal to the magnetic field at the point c

$$\vec{B} = -[F_N + F_S] \cos \theta \hat{i}$$

$$F_N = F_S$$

$$\begin{aligned}\vec{B} &= -\frac{2\mu_0}{4\pi} \frac{q_m}{r^2} \cos \theta \hat{i} \\ &= -\frac{2\mu_0}{4\pi} \frac{q_m}{[r^2 + l^2]^{3/2}} \cos \theta \hat{i}\end{aligned}$$

In a right angle triangle NOC,

$$\cos \theta \hat{i} = \frac{l}{r} = \frac{l}{[r^2 + l^2]^{1/2}}$$

$$\vec{B} = -\frac{\mu_0}{4\pi} \frac{q_m \times [2l]}{[r^2 + l^2]^{3/2}} \hat{i}$$

$$|\vec{P}_m| = P_m = q_m 2l$$

$$\vec{B}_{\text{equa}} = -\frac{\mu_0}{4\pi} \frac{P_m}{[r^2 + l^2]^{3/2}} \hat{i}$$

$$[r^2 + l^2]^{3/2} \approx r^3$$

$$r \gg l$$

$$\vec{B}_{\text{equa}} = -\frac{\mu_0}{4\pi} \frac{P_m}{r^3} \hat{i}$$

$$P_m \hat{i} = \vec{P}_m$$

$$\vec{B}_{\text{equa}} = -\frac{\mu_0}{4\pi} \frac{\vec{P}_m}{r^3}$$

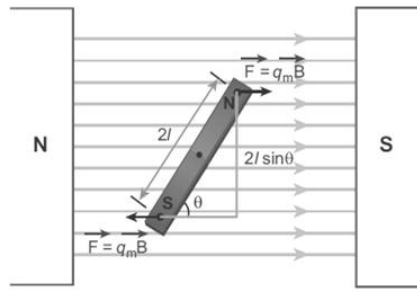
Magnitude of the  $B_{\text{axial}}$  is twice that of magnitude of  $B_{\text{equa}}$  and the direction of

$B_{\text{axial}}$  and  $B_{\text{equa}}$  are opposite.

**15. TORQUE ACTING ON A BAR MAGNET IN UNIFORM MAGNETIC FIELD**

[ 3 MARK]

Consider a magnet of length  $2l$  of pole strength  $q_m$  kept in a uniform magnetic field  $\vec{B}$ . Each pole experiences a force of magnitude  $q_m B$  but acts in opposite direction. Therefore, the net force exerted on the magnet is zero, so that there is no translatory motion. These two forces constitute a couple which will rotate and try to align in the direction of the magnetic field  $\vec{B}$ .



The force experienced by the North and South Pole,

$$\vec{F}_N = q_m \vec{B} \quad ; \quad \vec{F}_S = -q_m \vec{B}$$

The net force acting on a dipole,

$$\vec{F} = \vec{F}_N + \vec{F}_S = \vec{0}$$

The moment of force or torque experienced by North and South Pole about O point is,

$$\begin{aligned} \vec{\tau} &= \vec{ON} \times \vec{F}_N + \vec{OS} \times \vec{F}_S \\ &= \vec{ON} \times q_m \vec{B} + \vec{OS} \times [-q_m \vec{B}] \end{aligned}$$

By using right hand corkscrew rule ,

$$|\vec{ON}| = |\vec{OS}| = l \quad \text{and} \quad |q_m \vec{B}| = |-q_m \vec{B}|$$

The magnitude of total point about O is,

$$\begin{aligned}\tau &= \ell \times q_m B \sin \theta + \ell \times q_m B \sin \theta & [q_m \times 2\ell = P_m] \\ &= 2 \ell \times q_m B \sin \theta\end{aligned}$$

$$\tau = P_m B \sin \theta$$

In vector notation,

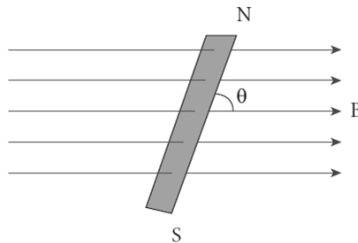
$$\vec{\tau} = \vec{P}_m \times \vec{B}$$

## 16. POTENTIAL ENERGY OF A MAGNET IN A UNIFORM MAGNETIC FIELD [ 3 MARK]

When a bar magnet of dipole moment  $\vec{P}_m$  is held at an angle  $\theta$  with the direction of

a uniform magnetic field  $\vec{B}$ . The magnitude of the torque acting on the dipole is,

$$|\vec{\tau}_B| = |\vec{P}_m| |\vec{B}| \sin \theta$$



If the dipole is rotated through a very small angular displacement ' $d\theta$ ' against the torque  $\tau_B$  at constant angular velocity, then the work done by external torque,

$$dw = |\vec{\tau}_{ext}| d\theta$$

$$|\vec{\tau}_B| = |\vec{\tau}_{ext}|$$

$$dw = P_m B \sin \theta d\theta$$

Total work done in rotating the dipole from  $\theta^I$  to  $\theta$  is,

$$\begin{aligned}W &= \int_{\theta^I}^{\theta} \tau d\theta = \int_{\theta^I}^{\theta} P_m B \sin \theta d\theta \\ &= P_m B [-\cos \theta]_{\theta^I}^{\theta}\end{aligned}$$

$$W = - P_m B [ \cos \theta - \cos \theta^l ]$$

This work done is stored as potential energy in bar magnet at an angle  $\theta$ .

$$U = - P_m B [ \cos \theta - \cos \theta^l ]$$

The reference point  $\theta^l = 90^\circ$

$$U = - P_m B \cos \theta$$

The potential energy stored in a bar magnet in a uniform magnetic field is given by,

$$U = - \vec{P}_m \cdot \vec{B}$$

### Case 1:

$$(i) \quad \theta = 0^\circ$$

$$U = - P_m B [ \cos 0^\circ ]$$

$$U = - P_m B$$

The bar magnet is aligned along the  $\vec{B}$

The potential energy is maximum.

### Case 2:

$$(ii) \quad \theta = 180^\circ$$

$$U = - P_m B [ \cos 180^\circ ]$$

$$U = P_m B$$

The bar magnet is aligned anti- parallel to external magnetic field.

The potential energy is maximum

## 17. TANGENT LAW

[ 2 MARK ]

When a magnetic needle or magnet is freely suspended in two mutually perpendicular uniform magnetic fields, it will come to rest in the direction of the resultant of

the two fields .

$$B = B_H \tan\theta$$

### 18. TANGENT GALVANOMETER [ TG]

[ 5 MARK]

Tangent Galvanometer is a device used to measure very small current . It is a moving type galvanometer. Its working is based on tangent law.

#### CONSTRUCTION

It consists of copper coil wound on non-magnetic circular frame. The same is made up of brass or wood which is mounted vertically on a horizontal base table with three leveling screws. The tangent galvanometer is provided with two or more coils of different number of turns consists of 2 turns ,5 turns and 50 turns which are of different thickness and or used for measuring currents for different strengths.

At the centre of turntable, small upright projection is seen on which a compass box is placed. Compass box consists of a small magnetic needle which is pivoted at the centre, such that arrangement shows the centre of both magnetic needle and circular coil

exactly coincide . A thin aluminium pointer is attached to the magnetic needle normally and moves over circular scale. The surface scale is divided into four quadrant and graduated in degrees which are used to measure the deflection of aluminium pointer on a circular degree scale. In order to avoid parallel error, in measure mirror is placed below the aluminium pointer.

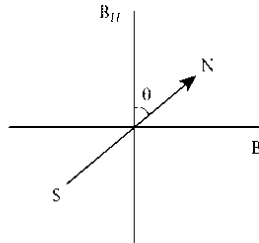
#### THEORY

When no current is passed through the coil , the small magnetic needle lie along horizontal component of Earth's magnetic field. When the circuit is switched ON, the electric current will pass through the circular coil and produce magnetic field. Now there are two field which are active mutually perpendicular to each other.

1. The magnetic field ( **B** ) due to the electric current in the coil acting#

normal to the plane of the coil.

## 2. The component of Earth's magnetic field ( $B_H$ )



Because of these crossed fields, the pivoted magnetic needle deflects and angle  $\theta$ ,

From tangent law,

$$B = B_H \tan \theta$$

When an electric current is passed through a circular coil of radius  $R$  having  $N$  turns, the magnitude of magnetic field at the centre is,

$$B = \frac{\mu_0 NI}{2R}$$

$$\frac{\mu_0 NI}{2R} = B_H \tan \theta$$

The horizontal component of Earth's magnetic field

$$B_H = \frac{\mu_0 NI}{2R} \frac{1}{\tan \theta}$$

in tesla

### PRECAUTIONS

1. All the nearby magnets and magnetic materials are kept away from the instrument.
2. Using spirit level, the levelling screws at the base are adjusted so that the small magnetic needle is exactly horizontal and also coil is exactly vertical.
3. The coil remains in magnetic meridian.
4. The compass box is rotated such that the point reads  $0^\circ - 0^\circ$

**19 . RIGHT HAND RULE**

[ 2 MARK]

If we hold the current carrying conductor in our right hand such that the thumb points in the direction of current flows ,then the fingers encircling the wire points in the direction of magnetic field lines produced.

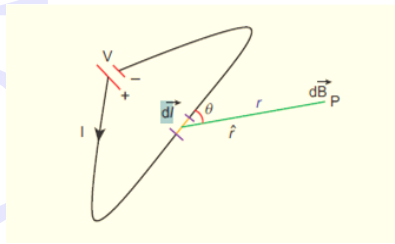
**20 .MAXWELL'S RIGHT HAND CORK SCREW RULE**

[ 2 MARK ]

If we rotate a right-handed screw then the direction of current is same as the direction in which screw advances and the direction of rotation of the screw gives the direction of the magnetic field.

**21 .BIOT-SAVART LAW**

[ 3MARK]



The magnitude of magnetic field  $\vec{dB}$  at a point P at a distance r from the small elemental length taken on a conductor carrying current varies ,

- (i) Directly as the strength of the current I
- (ii) Directly as the magnitude of the length element  $\vec{dl}$
- (iii) Directly as the sine of the angle between  $\vec{dl}$  and  $\vec{r}$
- (iv) Inversely as the square of the distance between the point P and length element  $\vec{dl}$

$$dB \propto \frac{Idl}{r^2} \sin\theta$$

$$dB = K \frac{Idl}{r^2} \sin\theta$$

$$K = \frac{\mu_0}{4\pi} \text{ in SI units \& } K = 1 \text{ in CGS units.}$$

$\vec{dB}$  is perpendicular to both  $I \vec{dl}$  and the unit vector  $\hat{r}$  director from  $\vec{dl}$  towards a point P.

The net magnetic field at P due to the conduct is obtained from principle of superposition by considering the contribution from all current element  $I \vec{dl}$

$$\vec{B} = \int \vec{dB} = \frac{\mu_0 I}{4\pi} \int \frac{\vec{dl} \times \hat{r}}{r^2}$$

**Cases :**

1. If the point P lies on the conductor

$$\theta = 0^\circ$$

$$\vec{dB} = 0$$

2. If the point P lies in perpendicular conductor

$$\theta = 90^\circ$$

$$\vec{dB} = \frac{\mu_0}{4\pi} \frac{Idl}{r^2} \hat{n}$$

$\hat{n} \rightarrow$  Unit vector perpendicular to both  $\vec{dl}$  and  $\hat{r}$

## 21.SIMILARITIES BETWEEN COULOMB'S LAW AND BIOT SAVORT LAW

[ 2 MARK]

Electric and magnetic field

$\Rightarrow$  Obey inverse Square Law, so they are long-range fields.

$\Rightarrow$  Obey the principle of superposition and are linear with respect to source.

$$E \propto q$$

$$B \propto Idl$$

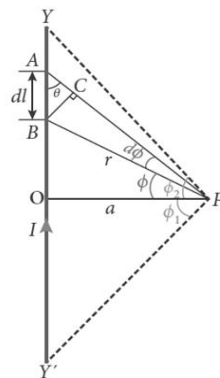
**22. DIFFERENCE BETWEEN COULOMB'S LAW AND BIOT SAVORT LAW**

[ 3 MARK]

S.NO	ELECTRIC FIELD	MAGNETIC FIELD
1	Produced by a scalar source i.e., an electric charge $q$	Produced by a vector source i.e., an current element $I \vec{dl}$
2	It is directed along the position vector joining the source and the point at which the field is calculated	It is directed perpendicular to the position vector $\vec{r}$ and the current element $I \vec{dl}$
3	Does not depend on angle	Depends on the angle between the position vector $\vec{r}$ and the current element $I \vec{dl}$

**23. MAGNETIC FIELD DUE TO LONG STRAIGHT CONDUCTOR CARRYING CURRENT**

[ 5 MARK ]



Let  $YY^l$  be an infinitely long straight conductor and  $I$  be the steady current through the conductor. A point P which is at a distance  $a$  from the wire, let us consider a small line element  $dl$ .

According to Biot- savart law, The magnetic field at point 'P' due to the element is ,

$$\vec{dB} = \frac{\mu_0}{4\pi} \frac{I \vec{dl} \sin \theta}{r^2} \hat{n}$$

$\hat{n} \rightarrow$  unit vector ,  $\theta$  is the angle between  $I dl$  and line joining  $dl$  and the point P. Let ' $r$ ' be the distance between line element at A to the point P.

To apply trigonometry, draw a perpendicular AC to the line BP

$$\Delta ABC, \sin\theta = \frac{AC}{AB}$$

$$AC = AB \sin\theta$$

$$AB = dl \Rightarrow AC = dl \sin\theta$$

Let  $d\theta$  be the angle subtended between AP and BP

$$\angle APB = \angle BPC = d\theta$$

$$\Delta APC; \sin[d\theta] = \frac{AC}{AP}$$

$$d\theta \text{ is very small, } \sin[d\theta] \approx d\theta$$

$$AP = r \Rightarrow AC = rd\theta$$

$$AC = dl \sin\theta = rd\theta$$

$$\vec{dB} = \frac{\mu_0 I}{4\pi r^2} [rd\theta] \hat{n}$$

$$\vec{dB} = \frac{\mu_0}{4\pi} \frac{Id\theta}{r} \hat{n}$$

Let  $\theta$  be the angle between BP and OP

$$\text{In a } \Delta OPA, \cos\theta = \frac{OP}{BP} = \frac{a}{r}$$

$$r = \frac{a}{\cos\theta}$$

$$\vec{dB} = \frac{\mu_0}{4\pi} \frac{I}{\frac{a}{\cos\theta}} d\theta \hat{n}$$

$$\vec{dB} = \frac{\mu_0 I}{4\pi a} \cos\theta d\theta \hat{n}$$

The total magnetic field at P due to the conductor  $YY'$

$$\vec{B} = \int_{\theta_1}^{\theta_2} \vec{dB} = \int_{\theta_1}^{\theta_2} \frac{\mu_0 I}{4\pi a} \cos\theta d\theta \hat{n}$$

$$= \frac{\mu_0 I}{4\pi a} [\sin\theta]_{-\theta_1}^{\theta_2} \hat{n}$$

$$\vec{B} = \frac{\mu_0 I}{4\pi a} [\sin\phi_1 + \sin\phi_2] \hat{n}$$

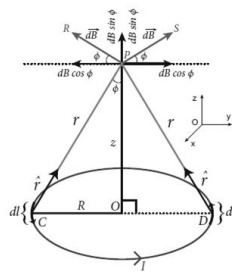
For an Infinitely long conductor  $\phi_1 = \phi_2 = 90^\circ$

$$\vec{B} = \frac{\mu_0 I}{4\pi a} \times 2\hat{n}$$

$$\vec{B} = \frac{\mu_0 I}{2\pi a} \hat{n}$$

## 24. MAGNETIC FIELD PRODUCED ALONG THE AXIS OF THE CURRENT CARRYING CIRCULAR COIL

[ 5 MARK ]



Consider a current carrying circular loop of radius R and let I be the current flowing through the wire in the direction .

The magnetic field at a point 'P' on the axis of the circular coil at a distance z from its centre of the coil O is computed by taking two diametrically opposite line element of the coil each of length  $d\vec{l}$  at c and D . let  $\vec{r}$  be the vector joining the current element[  $d\vec{l}$ ] at C to the point P.

According to Biot- Savart's law,

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \hat{r}}{r^2}$$

The magnetic field of  $d\vec{B}$  is,

$$dB = \frac{\mu_0}{4\pi} \frac{Idl \sin \theta}{r^2} = \frac{\mu_0}{4\pi} \frac{Idl}{r^2}$$

$\theta$  is angle  $d\vec{l}$  and  $\vec{r}$  ;  $\theta = 90^\circ$

The direction of  $d\vec{B}$  is perpendicular to the current element  $d\vec{l}$  and CP. It is therefore along PR perpendicular to CP.

The magnitude of magnetic field at P due to current element at D is same as that for the element at C because of equal distances from the coil. But its direction is along PS.

$\vec{dB}$  resolved into two components ;

$dB \cos\theta$  along Y – direction

$dB \sin\theta$  along Z – direction

The Horizontal components cancels out.

The Vertical components along contribute to net magnetic field  $\vec{B}$  at the point P.

$$\vec{B} = \int \vec{dB} = \int dB \sin\theta \hat{k}$$

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{dl}{r^2} \sin\theta \hat{k}$$

From  $\Delta OCP$ ,

$$\sin\theta = \frac{R}{[R^2 + Z^2]^{1/2}} \text{ and } r^2 = R^2 + Z^2$$

Substituting above equation,

$$\vec{B} = \frac{\mu_0 I}{4\pi} \frac{R}{[R^2 + Z^2]^{3/2}} \int_0^{2\pi R} dl \hat{k}$$

We integrate the line element from 0 to  $2\pi R$

$$\vec{B} = \frac{\mu_0 NI}{2} \frac{R^2}{[R^2 + Z^2]^{3/2}} \hat{k} \quad N \rightarrow \text{Turns in coil}$$

The magnetic field points at the centre of the coil is ,

$$\boxed{\vec{B} = \frac{\mu_0 NI}{2R} \hat{k}} \quad [Z = 0]$$

## 25. CURRENT LOOP AS A MAGNETIC DIPOLE

[ 3 MARK ]

The magnetic field at a point on the axis of the current-carrying circular loop,

$$\vec{B} = \frac{\mu_0 I}{2} \frac{R^2}{[R^2 + Z^2]^{3/2}} \hat{k}$$

At larger distance  $z \gg R$ ,  $R^2 + Z^2 \approx Z^2$

$$\vec{B} = \frac{\mu_0 I R^2}{2 Z^3} \hat{k} \text{ [ or ] } \vec{B} = \frac{\mu_0 I \pi R^2}{2\pi Z^3} \hat{k}$$

Let A be the area of the circular loop ,  $A = \pi R^2$

$$\vec{B} = \frac{\mu_0 I A}{2\pi Z^3} \hat{k} \text{ [ or ] } \vec{B} = \frac{\mu_0 I 2\pi A}{4\pi Z^3} \hat{k}$$

$$P_m = IA$$

$P_m \Rightarrow$  magnetic dipole moment

Vector notation,

$$\vec{P}_m = I\vec{A}$$

A current carrying circular loop behaves as a magnetic dipole of magnetic moment  $\vec{P}_m$

The magnetic dipole moment of any current loop is equal to the product of the current and area of the loop.

DIRECTION : Using right hand thumb rule

## 26. RIGHT HAND THUMB RULE

[ 2 MARK ]

### STATEMENT :-

If we curl the fingers of right hand in the direction of current in the loop, then the stretched thumb gives the direction of the magnetic moment associated with the loop.

## 27. END RULE

[ 2 MARK ]

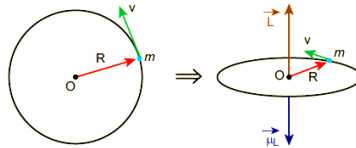
A current in circular loop in anticlockwise direction ,the polarity is North Pole.

A current in circular loop in clockwise direction, the polarity is South Pole .

**28. MAGNETIC DIPOLE MOMENT OF REVOLVING ELECTRON [ 5 MARK ]**

An electron undergoes circular motion around the nucleus . The circulating electron in a loop is like current in a circular loop . The magnetic dipole moment due to current carrying circular loop is ,

$$\vec{\mu}_L = I\vec{A}$$



In Magnitude,  $\mu_L = IA$

T → time period of revolution of an electron,

The current,  $I = -\frac{e}{T}$

e → Charge of an electron

R → radius of the circular Orbit

V → Velocity of the electron in the circular Orbit

$$T = \frac{2\pi R}{V}$$

$$\mu_L = -\frac{2\pi R}{V} \pi R^2 = -\frac{eVR}{2}$$

$A = \pi R^2$  → Area of the circular loop

Angular momentum of electron about O.

$$\vec{L} = \vec{R} \times \vec{P}$$

In magnitude ,  $L = RP = mVR$

$$\frac{\mu_L}{L} = -\frac{eVR/2}{mVR} = -\frac{e}{2m}$$

$$\vec{\mu}_L = -\frac{e}{2m} \vec{L}$$

The negative sign indicates that the magnetic moment and angular momentum are in

opposite direction.

$$\text{In magnitude, } \frac{\mu_L}{L} = \frac{e}{2m} = \frac{1.6 \times 10^{-19}}{2 \times 9.11 \times 10^{-31}}$$

$$\frac{\mu_L}{L} = 0.0878 \times 10^{12} \text{ ckg}^{-1} = 8.78 \times 10^{10} \text{ ckg}^{-1} \text{ constant.}$$

The ratio  $\frac{\mu_L}{L}$  is a constant known as gyro -magnetic ratio

According to Neil 's Bohr quantisation rule,

$$L = n\hbar = \frac{nh}{2\pi}$$

$h \rightarrow$  Planck's Constant  $\rightarrow 6.63 \times 10^{-34} \text{ Js}$

$n \rightarrow$  orbit number [  $n = 1, 2, 3, \dots$  ]

$$\mu_L = \frac{e}{2m} L = n \frac{eh}{4\pi m}$$

$$\mu_L = n \times \frac{[1.6 \times 10^{-19}]h}{4\pi m} \text{ Am}^2$$

$$\mu_L = n \times \frac{[1.6 \times 10^{-19}][6.63 \times 10^{-34}]}{4 \times 3.14 \times [9.11 \times 10^{-31}]}$$

$$\mu_L = n \times 9.27 \times 10^{-24} \text{ Am}^2$$

The minimum value of magnetic moment,  $n = 1$

$$\mu_L = 9.27 \times 10^{-24} \text{ Am}^2 = 9.27 \times 10^{-24} \text{ JT}^{-1}$$

$$[\mu_L]_{\min} = \mu_B$$

$$\mu_B = \frac{eh}{4\pi m} = 9.27 \times 10^{-24} \text{ Am}^2 \text{ is called } \underline{\text{Bohr magnetron}} \text{ which is used to}$$

measure atomic magnetic moments.

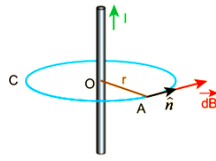
## 29. AMPERE'S CIRCUITAL LAW

The line integral of magnetic field over a closed loop is times net current enclosed by the loop.

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I \text{ enclosed}$$

### 30. MAGNETIC FIELD DUE TO THE CURENET CARRYING WIRE OF INFINITE LENGHT USING AMPERE'S LAW

[ 3 MARK ]



Consider a straight conduct of infinite length carrying current  $I$  and the direction of magnetic field lines is shown in figure. Since the wire is geometrically cylindrical in the shape and symmetrical about its axis, we construct an amperian loop in the form of a circular shape of at a distance  $r$  from the centre of the conductor .

From the Ampere 's law,

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I$$

$d\vec{l}$  is the line element along Amperian loop. Hence the angle between magnetic field vector and the line element is zero

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I$$

$$B \oint_C dl = \mu_0 I$$

$B \rightarrow$  Uniform over the Amperian loop

For a circular loop,

$$B \int_0^{2\pi R} dl = \mu_0 I$$

$$B [ 2\pi R ] = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi R}$$

In vector form,

$$\vec{B} = \frac{\mu_0 I}{2\pi R} \hat{n}$$

$\hat{n} \rightarrow$  Unit vector along that tangent on the Amperian loop

**31. SOLENOID**

[ 2 MARK]

A solenoid is a long coil of wire closely wound in the form of helix .

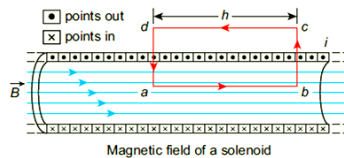
The magnetic field of the solenoid is due to the superposition of magnetic fields of each turn of the solenoid .

**Direction :** By right hand palm - rule .

**32. MAGNETIC FIELD DUE TO A LONG CURRENT CARRYING SOLENOID**

[ 5 MARK ]

Consider a solenoid of length L having N turns. The diameter of the solenoid is assumed to be much smaller when compared to its length and that is wound very closely.



To calculate the magnetic field at any point inside the solenoid ,using ampere's circuital law , consider a rectangle look abcd

From Ampere's circuital law ,

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enclosed}}$$

LHS

$$\oint_C \vec{B} \cdot d\vec{l} = \int_a^b \vec{B} \cdot d\vec{l} + \int_b^c \vec{B} \cdot d\vec{l} + \int_c^d \vec{B} \cdot d\vec{l} + \int_d^a \vec{B} \cdot d\vec{l}$$

Since the elemental length along bc and da are perpendicular to the magnetic field which is along the axis of the solenoid.

$$\int_b^c \vec{B} \cdot d\vec{l} = \int_b^c B \cdot dl \cos 90^\circ = 0;$$

$$\text{Similarly } \int_d^a \vec{B} \cdot d\vec{l} = 0;$$

The magnetic field outside of the solenoid is zero,

$$\int_c^d \vec{B} \cdot d\vec{l} = 0$$

For the path along ab ,

$$\int_a^b \vec{B} \cdot d\vec{l} = B \int_a^b dl \cos 0^\circ = B \int_a^b dl$$

$$B \int_a^b dl = BL$$

Let I be the current passing through the solenoid of N turns ,

$$\int_a^b \vec{B} \cdot d\vec{l} = BL = \mu_0 NI$$

$$B = \mu_0 \frac{NI}{L}$$

The number of turns per unit length is,

$$\frac{N}{L} = n$$

$$B = \mu_0 \frac{nLI}{L} = \mu_0 nI$$

$n$  &  $\mu_0 \rightarrow$  constant

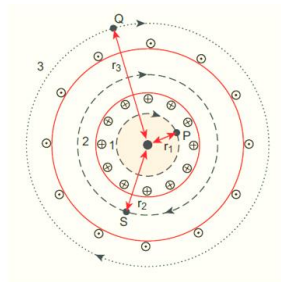
For a fixed current I , the magnetic field inside the solenoid is also a constant .

### 33. TOROID

[ 5 MARKS ]

A Solenoid is bent in such a way its ends are joined together to form a closed ring shape , is called a toroid.

The magnetic field has constant magnitude inside the toroid whereas in the interior region [ At Point P ] and exterior region [ At Point Q ] the magnetic field is zero .



**a ) OPEN SPACE INTERIOR TO THE TOROID**

Let us calculate the magnetic field  $B_P$  at point P . We construct an amperian loop1 of radius  $r_1$  around the point P we take a circular loop so that the length of the loop is its circumference .

$$L_1 = 2\pi r_1$$

Ampere's circuital law for the loop 1 is ,

$$\oint_{\text{Loop 1}} \vec{B}_P \cdot d\vec{l} = \mu_0 I_{\text{enclosed}}$$

Loop 1 encloses no current,  $I_{\text{enclosed}} = 0$

$$\oint_{\text{Loop 1}} \vec{B}_P \cdot d\vec{l} = 0$$

This is possible only if the magnetic field at a point P vanishes ,

$$\vec{B}_P = 0$$

**b) OPEN SPACE EXTERIOR TO THE TOROID**

Let us calculate the magnetic field  $B_Q$  at a point Q. We construct an amperian loop 3 of radius of  $r_3$  around the point Q the length of the loop is

$$L_3 = 2\pi r_3$$

Ampere's circuital law for the loop 3 is ,

$$\oint_{\text{Loop 3}} \vec{B}_Q \cdot d\vec{l} = \mu_0 I_{\text{enclosed}}$$

Each turn of the loop , current coming out of the plane of paper is cancelled by the current going into the plane of the paper

$$I_{\text{enclosed}} = 0$$

$$\oint_{\text{Loop 3}} \vec{B}_Q \cdot d\vec{l} = 0$$

The magnetic field at a point Q vanishes ,

$$\vec{B}_Q = 0$$

**c) INSIDE THE TOROID**

Let us calculate the magnetic field  $B_s$  at a point  $s$  by constructing an amperian loop 2 of radius  $r_2$  around the point  $S$ . The length of the loop is

$$L_2 = 2\pi r_2$$

Ampere's circuital law for the loop 2 is ,

$$\oint_{\text{Loop 2}} \vec{B}_s \cdot d\vec{l} = \mu_0 I_{\text{enclosed}}$$

Let  $I$  be the current passing through the toroid and  $N$  is no of turns

$$I_{\text{enclosed}} = NI$$

$$\oint_{\text{Loop 2}} \vec{B}_s \cdot d\vec{l} = \oint_{\text{Loop 2}} B_s \cdot dl \cos\theta = B_s 2\pi r_2$$

$$B_s 2\pi r_2 = \mu_0 NI$$

$$B_s = \mu_0 \frac{NI}{2\pi r_2}$$

$$\text{No of turns per unit length } n = \frac{N}{2\pi r_2}$$

$$B_s = \mu_0 nI$$

**34. LORENTZ FORCE**

[ 2 MARK ]

If the charges moves into the magnetic field, It experiences a force. This force is known as Magnetic Lorentz Force

$$\vec{F} = q [ \vec{V} \times \vec{B} ]$$

If the charge is moving in both the magnetic and electric fields , the total force experienced by the charge is,

$$\vec{F} = q [ \vec{E} + \vec{V} \times \vec{B} ]$$

**35. FORCE ON A MOVING CHARGE IN A MAGNETIC FIELD**

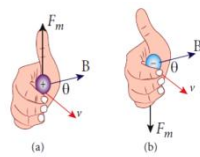
[ 5 MARK ]

When an electric charge  $q$  is moving with velocity  $\vec{V}$  in the magnetic field  $\vec{B}$  it experiences a force called magnetic Lorentz Force  $\vec{F}_m$

$$\vec{F}_m = q [ \vec{V} \times \vec{B} ]$$

In magnetic,

- i.  $\vec{F}_m$  is directly proportional to the magnetic field  $\vec{B}$
- ii.  $\vec{F}_m$  is directly proportional to the velocity  $\vec{V}$  of the moving charge
- iii.  $\vec{F}_m$  is directly proportional to the sine of the angle between the velocity and magnetic field
- iv.  $\vec{F}_m$  is directly proportional to the magnitude of the charge  $q$
- v. The direction of  $\vec{F}_m$  is always perpendicular to  $\vec{V}$  and  $\vec{B}$  as  $\vec{F}_m$  is a cross product of  $\vec{V}$  and  $\vec{B}$



- vi. The direction of  $\vec{F}_m$  on negative charge is opposite to the direction of  $\vec{F}_m$  on positive charge
- vii. If the velocity  $\vec{V}$  of charge  $q$  is along magnetic field  $\vec{B}$  then  $\vec{F}_m$  is zero

**36. TESLA**

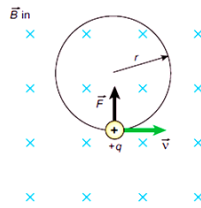
[ 2 MARK]

If a unit charge moving in it with unit velocity experiences unit force then the magnetic field strength is 1 tesla

$$1 \text{ T} = \frac{1 \text{ NS}}{\text{Cm}} = 1 \frac{\text{N}}{\text{Am}} = 1 \text{ N A}^{-1} \text{ m}^{-1}$$

**37. MOTION OF A CHARGED PARTICLE IN A UNIFORM MAGNETIC FIELD**

[ 5 MARK]



Consider a charged particles of charge  $q$  having mass  $m$  entering into a region of uniform magnetic field  $\vec{B}$  with velocity  $\vec{V}$  such that velocity is perpendicular to the magnetic field. As soon as the particle enters into the field. Lorentz Force acts on it in a direction perpendicular to both magnetic field and velocity  $\vec{V}$

As a result, the charged particle moves into a circular orbit. The Lorentz Force on the charged particle is given by ,

$$\vec{F} = q [ \vec{V} \times \vec{B} ]$$

Since , Lorentz Force alone acts on the particle, the magnitude of the net force on the particle is

$$\sum_i F_i = F_m = qvB$$

This Lorentz Force acts as centripetal force for the particle causing it to execute circular motion. Therefore ,

$$qvB = \frac{mv^2}{r}$$

The radius of the circular path is

$$r = \frac{mv}{qB} = \frac{P}{qB}$$

Linear momentum of a particle ,  $P = mv$

Let  $T$  be the time taken by the particle to finish one complete circular motion, then

$$T = \frac{2\pi r}{v}$$

$$T = \frac{2\pi r}{qB}$$

T is called the cyclotron period. The reciprocal of time period is the frequency f,

$$f = \frac{1}{T}$$

$$f = \frac{qB}{2\pi r}$$

In terms of angular frequency,

$$\omega = 2\pi f = \frac{q}{m}B$$

These are called as cyclotron frequency or Gyro frequency.

The component of velocity of the particle perpendicular to the field keeps charging due to Lorentz Force, it is a helical around the field lines.

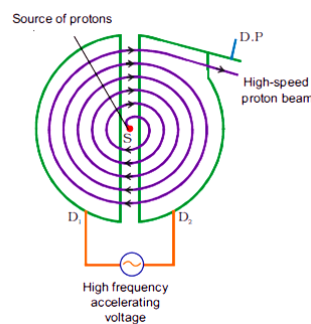
### 38. CYCLOTRON

[ 5 MARK]

To accelerate the charged particles to gain large kinetic energy. It is called as high energy accelerator .

#### PRINCIPLE

When a charged particle moves perpendicular to the magnetic field, it experiences magnetic Lorentz Force .



## CONSTRUCTION

The particles are allowed to move in between two semicircular metal containers called Dees. Dees are present in an evacuated chamber and it is kept in a region with uniform magnetic field controlled by an Electromagnet. The direction of magnetic field is normal to the plane of the dees. The two dees are kept separated with a gap and the source  $S$  is placed at the centre in the gap between the dees. Dees are connected to high frequency alternating potential difference.

## WORKING

The ion ejected from source  $S$  is positively charged. It is directed towards a Dee  $-1$  which has negative potential at that time. Since the magnetic field is normal to the plane of the Dees, the ion moves in a circular path. After one semicircular path inside Dee  $-1$ , the ion searches that gap between Dees. At this time, the polarities of the Dees are reversed so that the ion is now accelerated towards Dee  $-2$  with greater velocity. For this circular motion, the centripetal force on charged particle  $q$  is provided by Lorentz Force.

$$\frac{mv^2}{r} = Bqv$$

$$r = \frac{m}{qB} v$$

$$r \propto v$$

The increase in velocity increases the radius of the circular path. This process continues and hence the particle moves in a spiral path of increasing radius. Increase it reaches near the edge, it is taken out with the help of deflector plate and allowed to hit the target  $T$ .

The important condition the frequency  $f$  at which the positive ion circulates in the magnetic field must be equal to the constant frequency of the electrical oscillator.

This is called resonance condition.

$$f_{osc} = \frac{qB}{2\pi m}$$

The time period of oscillation is,

$$T = \frac{2\pi m}{qB}$$

The kinetic energy of a charged particle is,

$$\begin{aligned} KE &= \frac{1}{2}mv^2 \\ &= \frac{q^2 m^2 r^2}{2m} \end{aligned}$$

### LIMITATIONS OF CYCLOTRON

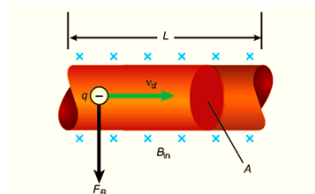
[ 2 MARK ]

- The speed of the ion is limited.
- Electron cannot be accelerated.
- Uncharged particles cannot be accelerated.

### 39. FORCE ON A CURRENT CARRYING CONDUCTOR PLACED IN THE MAGNETIC FIELD

[ 5 MARK ]

When a current carrying conductor is placed in a magnetic field, the force experienced by the conductor is equal to the sum of Lorentz forces on the individual charge carriers in the conductor. Consider a small segment of conductor of length  $dl$  with cross sectional area  $A$  and current  $I$ .



The free electrons drift opposite to the direction of current. So the relation between current and drift velocity.

$$I = nAeVd$$

If the conductor is kept in a magnetic field  $\vec{B}$ , then average force experienced by the electron in the conductor is

$$\vec{f} = -e[ \vec{V}_d \times \vec{B} ]$$

$n \rightarrow$  Number of free electrons present in unit volume

$$n = \frac{N}{V}$$

$N \rightarrow$  Number of free electrons in the small element of volume

$$V = A \cdot dl$$

Hence Lorentz Force on the elementary section of length  $dl$  is the product of the number of the electrons and the force acting on each electron .

$$\vec{F} = -enAdl[ \vec{V}_d \times \vec{B} ]$$

The current element in the conductor is  $I\vec{dl} = -enA\vec{V}_d dl$ . Therefore, the force on the smaller elemental section of the current carrying conductor is,

$$d\vec{F} = [ I\vec{dl} \times \vec{B} ]$$

The force on a conductor carrying current of length ' $\ell$ ' placed in a uniform magnetic field is

$$\vec{F}_{total} = [ I\vec{\ell} \times \vec{B} ]$$

In magnitude,  $F_{total} = BI\ell \sin\theta$

- a) The conductor is placed along the direction of the magnetic field,  $\theta=0^\circ$

$$F = 0$$

- b) the conductor is placed perpendicular to the magnetic field,  $\theta = 90^\circ$

$$F_{total} = BI\ell \quad [ \text{Maximum} ]$$

**40. FLEMING'S LEFT HAND RULE**

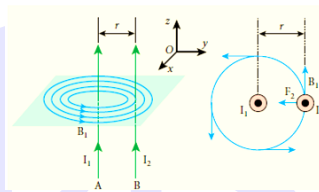
[ 2 MARK ]

Stretch out forefinger , the middle finger and that thumb of the left-hand such that they are in three mutually perpendicular directions. If the fore finger points in the direction of magnetic field the middle finger ,in the direction of electric current. Then thumb will point in the direction of the force experienced by the conductor.

**41. FORCE BETWEEN TWO LONG PARALLEL CURRENT CARRYING CONDUCTORS**

[ 3 MARK ] or [ 5 MARK ]

Let two long straight parallel current carrying conductor separated by a distance 'r' are kept in air medium.



Let  $I_1$  and  $I_2$  be the electric current passing through the conductors A and B in same direction respectively.

The net magnetic field is , [ conductor A]

$$\vec{B}_1 = \frac{\mu_0 I_1}{2\pi r} [-\hat{t}] = -\frac{\mu_0 I_1}{2\pi r} \hat{t}$$

From thumb rule , the direction of magnetic field is perpendicular to plane of the paper and inwards .i.e., along negative  $\hat{t}$  direction.

Let us consider a small elemental length  $dl$  in conductor B at which the magnetic field  $\vec{B}_1$  is present. Lorentz Force on the element  $dl$  of conductor B is,

$$d\vec{F} = [I_2 d\vec{l} \times \vec{B}_1] = -I_2 dl \frac{\mu_0 I_1}{2\pi r} [\hat{k} \times \hat{t}]$$

$$= - \frac{\mu_0 I_1 I_2 dl}{2\pi r} \hat{j}$$

∴ The force on dl of the wire B is directed towards the conductor A. Hence the force per unit length of the conductor B due to current in the conductor A is ,

$$\frac{\vec{F}}{l} = - \frac{\mu_0 I_1 I_2}{2\pi r} \hat{j}$$

III<sup>ly</sup> ,The net magnetic induction ,[ conductor B ]

$$\vec{B}_2 = - \frac{\mu_0 I_2}{2\pi r} \hat{i}$$

From the Thumb Rule ,direction of magnetic field is perpendicular to the plane of the paper and outwards, along  $\hat{i}$  direction. Hence, the magnetic force acting on element dl of the conducted A is,

$$\begin{aligned} d\vec{F} &= [I_1 d\vec{l} \times \vec{B}_2] = I_1 dl \frac{\mu_0 I_2}{2\pi r} [\hat{k} \times \hat{i}] \\ &= - \frac{\mu_0 I_1 I_2 dl}{2\pi r} \hat{j} \end{aligned}$$

∴ The force on dl of conductor A is directed towards the conductor B. The force acting per unit length of the conductor A due to current in conductor B is ,

$$\frac{\vec{F}}{l} = - \frac{\mu_0 I_1 I_2}{2\pi r} \hat{j}$$

Thus the force between two parallel current carrying conductor is attractive if they carry current in the same direction. The force between two parallel current carrying conductors is repulsive if they carry current in opposite directions.

#### 42. AMPERE

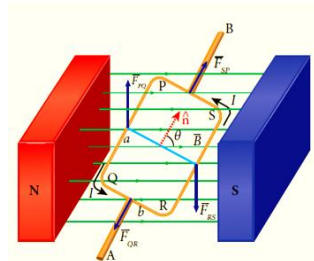
[ 2 MARK ]

A constant current which when passed through each of the two infinitely long parallel straight conductors kept side-by-side parallelly at a distance of one metre apart

in air or vacuum causes each conductor to experience a force of  $2 \times 10^{-7}$  Newton per metre length of conductor .

#### 43. TORQUE ON A CURRENT LOOP PLACED IN A MAGNETIC FIELD [5MARK]

Consider a rectangular loop PQRS carrying current  $I$  is placed in a uniform magnetic field  $\vec{B}$  . Let  $a$  and  $b$  be the length and breadth of a rectangular loop respectively. The unit vector normal  $\hat{n}$  to the plane of the loop makes an angle  $\theta$  with the magnetic field.



The magnitude of the magnetic force acting on the arm PQ and RS is

$$F_{PQ} = I_a B \sin \left[ \frac{\pi}{2} \right] = I_a B$$

The direction of the force is upwards in arm PQ and downwards in [using right hand corkscrew rule ] in the arm RS.

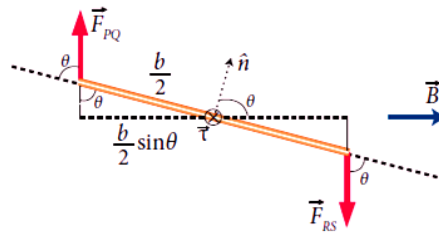
The magnitude of the magnetic force acting on the arm QR is,

$$F_{QR} = I_b B \sin \left[ \frac{\pi}{2} - \theta \right] = I_b B \cos \theta$$

The force acting on the arm SP is,

$$F_{SP} = I_b B \sin \left[ \frac{\pi}{2} + \theta \right] = I_b B \cos \theta$$

Since the forces  $F_{QR}$  and  $F_{SP}$  are equal ,opposite and collinear, they cancel each other. But the forces  $F_{PQ}$  and  $F_{RS}$  which are equal in magnitude and opposite in direction of not acting along same straight line. The four constitute a couple which exerts a torque on the loop .



The magnitude of torque acting on the PQ about AB is  $\tau_{PQ} = \left[ \frac{b}{2} \sin \theta \right] I_{aB}$

and it points in the direction of AB.

The magnitude of torque acting on the arm RS about AB is  $\tau_{RS} = \left[ \frac{b}{2} \sin \theta \right] I_{aB}$

and it points also in the same direction AB.

The total torque acting on the entire loop about an axis AB is

$$\begin{aligned} \tau &= \left[ \frac{b}{2} \sin \theta \right] F_{PQ} + \left[ \frac{b}{2} \sin \theta \right] F_{RS} \\ &= I_a [b \sin \theta] B \\ &= I AB \sin \theta \text{ along the direction AB.} \end{aligned}$$

$$\vec{\tau} = [I \vec{A}] \times \vec{B}$$

$$\vec{\tau} = [\vec{P}_m] \times \vec{B} \quad [\vec{P}_m = I\vec{A}]$$

The torque is to rotate the loop so as to align its normal vector with direction of the magnetic field.

$$\tau = NIAB \sin \theta \quad [N \text{—turns is the rectangular loop}]$$

#### Special cases

a)  $\theta = 90^\circ$ , The plane of the loop is parallel to the  $\vec{B}$ , torque is maximum

$$\tau_{\max} = IAB$$

b)  $\theta = 90^\circ / 180^\circ$ , The plane of the loop is perpendicular to the  $\vec{B}$ , torque is zero

$$\tau = 0$$

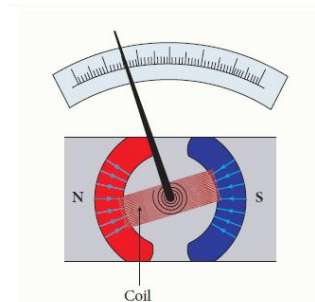
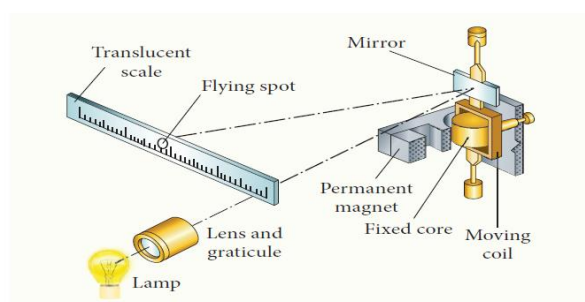
**44. MOVING COIL GALVANOMETER**

[ 5 MARK ]

Used to detect the flow of current in an electric circuit .

**PRINCIPLE**

When a current carrying loop is placed in a uniform magnetic field, it experiences a torque.

**CONSTRUCTION**

A moving coil galvanometer consists of a rectangular coil PQRS of insulated thin copper wire. The coil contains a large number of turns wound over a light metallic frame. The cylindrical soft-iron core placed is symmetrical inside the coil. The rectangular coil is suspended freely between two pole pieces of a horseshoe magnet.

The upper end of the rectangular coil is attracted to one end of the points of phosphor bronze and the lower end of the coil is connected to hair spring which also made up of phosphor bronze. In a fine suspension strip, a small plane mirror is attached in order to measure the deflection of the coil with the help of the lamp and scale arrangement. The other end of the mirror is connected to the Torsion head. In order to pass electric current through the Galvanometer the suspension strip and the spring 's' are connected to terminals.

**WORKING**

Consider a single turn of the rectangular coil PQRS when length is  $\ell$  and breadth  $b$ .

$PQ=RS = \ell$  and  $QR =SP= b$ .

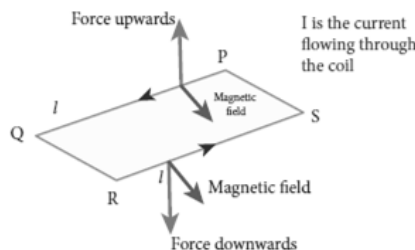
Let  $I$  be the electric current flowing through the rectangular coil PQRS. The horse shoe magnet have a hemispherical magnetic poles which produces a radial magnetic field ,the sides QR and SP are parallel to the magnetic field  $B$  and experiences no force. The sides PQ and RS are always perpendicular to the magnetic field and experience equal forces in opposite direction .Due to this ,torque is produced.

For single turn, the deflecting torque is

$$\tau = bF = bBI\ell = [\ell b ] BI = ABI \quad [ A = \ell b ]$$

For coil with  $N$  turns ,

$$\tau = NABI$$



Due to this deflecting torque, the coil gets twisted under restoring torque is

developed.Hence the moment of the restoring torque is proportional to the amount of Twist  $\theta$ ,

$$\tau = k\theta$$

$k \rightarrow$  Restoring couple per unit twist

At equilibrium ,

The deflection couple= the storing couple.

$$NABI = k\theta$$

$$I = \frac{k}{NAB} \theta$$

$$I = G \theta$$

$G = \frac{k}{NAB} \rightarrow$  Galvanometer constant [or] current reducing factor of the galvanometer.

#### 45. FIGURE OF MERIT OF A GALVANOMETER

[ 2 MARK ]

The current required to produce a deflection of one scale division in the galvanometer

#### 46. CURRENT SENSITIVITY

[ 2 MARK ]

The deflection produced per unit current flowing through the galvanometer.

$$I_s = \frac{\theta}{I} = \frac{NAB}{k} \Rightarrow I_s = \frac{1}{G}$$

#### 47. HOW CAN BE INCREASE THE CURRENT SENSITIVITY OF A GALVANOMETER?

[ 2 MARK ]

$$\text{CURRENT SENSITIVITY } I_s = \frac{\theta}{I} = \frac{NAB}{K}$$

The current sensitivity of the Galvanometer can be increased by

- (i) Increasing the number of turns ,N
- (ii) increasing their magnetic induction ,B
- (iii) increasing the area of the coil ,A
- (iv) decreasing the couple per unit Twist of the suspension wire ,k

#### 48. WHY IS PHOSPHOR - BRONZE WIRE USED AS THE SUSPENSION WIRE ?

[ 2 MARK ]

Because the couple per unit test is very small, so we use the moving coil galvanometer.

#### VOLTAGE SENSITIVITY

The deflection produced per unit voltage applied across the galvanometer.

$$V_s = \frac{\theta}{V} = \frac{\theta}{IR_g} = \frac{NAB}{KR_g}$$

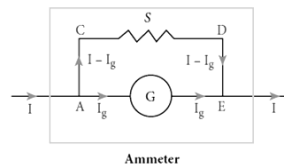
$$V_s = \frac{1}{GR_g} = \frac{I_s}{R_g}$$

**49. CONVERSION OF GALVANOMETER TO AN AMMETER**

[ 3 MARK ]

Ammeter is a instrument used to measure current flowing in the electrical circuit .

A galvanometer is converted into an ammeter by connecting yellow resistance in parallel with the galvanometer. This low resistance is called shunt resistance 'S'. The scale is now calibrated in ampere and the range of ammeter depends on the values of the shunt resistance.



When current  $I$  reaches the junction A it divides into two components. Let  $I_g$  be the current passing through the Galvanometer of resistance  $R_g$  through a path AGE and the remaining current  $[ I - I_g ]$  passes along the path ACDE through shunt resistance  $S$ . The value of shunt resistance is so adjusted that current  $I_g$  produces full scale deflection in the galvanometer.

$$V_{\text{galvanometer}} = V_{\text{shunt}}$$

$$I_g R_g = [ I - I_g ] S$$

$$S = \frac{I_g}{[I - I_g]} R_g$$

$$I_g = \frac{S}{[S + R_g]} I$$

Since,

$$\theta = \frac{1}{G} I_g \Rightarrow \theta \propto I_g \Rightarrow \theta \propto I$$

The deflection produced in the galvanometer is a measure of the current  $I$  passing through the circuit.

Shunt resistance is connected in parallel to Galvanometer, the resistance of ammeter  $[ R_a ]$  is

$$\frac{1}{R_{eff}} = \frac{1}{R_g} + \frac{1}{s} \Rightarrow R_{eff} = \frac{R_g s}{R_g + s}$$

$$R_a = \frac{R_g s}{R_g + s}$$

$R_a$  is small, the resistance offered by the ammeter is small. So when we connect ammeter in series with ammeter will not change appreciably the current in the circuit.

For an ideal ammeter, the resistance must be equal to zero. But in reality, the reading in ammeter is always less than the actual current in the circuit.

$I_{ideal} \Rightarrow$  Current in ideal ammeter

$I_{actual} \Rightarrow$  Actual current in the circuit

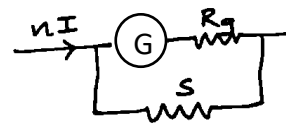
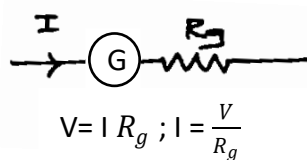
The percentage error,

$$\frac{\Delta I}{I} \times 100 \% = \frac{I_{ideal} - I_{actual}}{I_{ideal}} \times 100 \%$$

In order to increase the range of an ammeter  $n$  times, the value of shunt resistance to be connected in parallel is,

$$S = \frac{R_g}{n-1}$$

### NOT FOR EXAM



$$\frac{V}{\frac{R_g s}{R_g + s}} = nI$$

$$\frac{V}{\frac{R_g s}{R_g + s}} = n \frac{V}{R_g}$$

$$\frac{R_g + s}{R_g s} = \frac{n}{R_g}$$

$$R_g + s = nS = nS - S$$

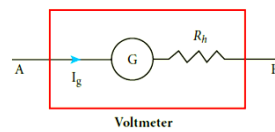
$$R_g = S [n - 1]$$

**50. CONVERSION OF GALVANOMETER TO AN VOLTMETER****[ 3 MARK]**

A voltmeter is an instrument used to measure potential difference across any two points in the electrical circuits.

A Galvanometer is converted into a voltmeter by connecting high resistance  $R_h$  in series with galvanometer.

The value of resistance is so adjusted so that current ' $I_g$ ' produces full scale deflection in the galvanometer .



$R_g \Rightarrow$  Resistance of galvanometer.

$$I = I_g = \frac{\text{potential difference}}{\text{total resistance}}$$

Effective resistance in the circuit gives the resistance of voltmeter.

$$R_v = R_g + R_h$$

$$I_g = \frac{V}{R_g + R_h}$$

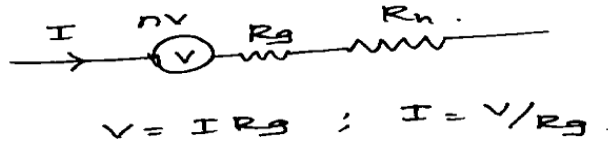
$$R_g = \frac{V}{I_g} - R_g$$

$$I_g \propto V$$

The resistance of voltmeter is very large, a voltmeter connected in parallel in an electrical circuit means least current in the circuit. An ideal voltmeter is one which has infinite resistance.

In order to increase the range of voltmeter 'n' times the value of resistance to be connected in series with the Galvanometer is

$$R_h = [n - 1] R_g$$

**NOT FOR EXAM**

$$nV = I [R_g + R_h]$$

$$\frac{nV}{R_g + R_h} = \frac{V}{R_g}$$

$$nR_g = R_g + R_h$$

$$R_h = nR_g - R_g$$

$$R_h = [n - 1] R_g$$

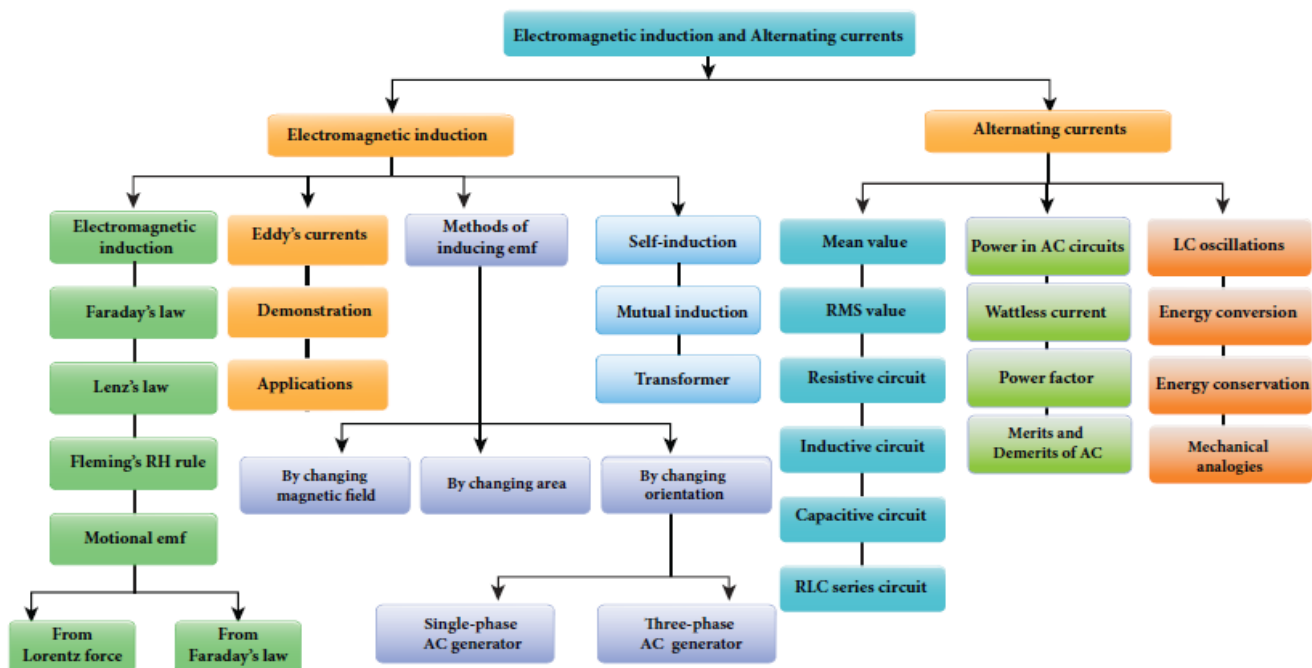
# Padasalai

# 12<sup>th</sup> PHYSICS

## UNIT 4 - ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENT

-MR.THIVYARAJ V., M.Sc., M.Phill. B.Ed

**LECTURE VIDEOS**





## ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENT

**1. MAGNETIC FLUX [ $\phi_B$ ]****[ 2 MARK]**

The number of magnetic field lines passing through that area normally.

$$\phi_B = \int_A \vec{B} \cdot d\vec{A} = BA \cos\theta$$

SI unit  $\Rightarrow \text{Tm}^2$  [ or ] weber

**2. ELECTROMAGNETIC INDUCTION****[ 2 MARK]**

Whenever the magnetic flux linked with a closed coil changes, an emf is induced and hence an electric current flows in the circuit. This current is called an induced current and the emf giving rise to such current is called an induced emf. This phenomenon is known as electromagnetic induction.

**3. FARADAY'S LAW OF ELECTROMAGNETIC INDUCTION****[ 2 MARK]****FIRST LAW**

Whenever magnetic flux linked with a closed circuit changes, an emf is induced in the circuit which lasts in the circuit as long as the magnetic flux is changing.

**SECOND LAW**

The magnetic of induced emf in a closed circuit is equal to the time rate of change of magnetic flux linked with the circuit

$$\xi = \frac{d\phi_B}{dt}$$

$$\xi = N \frac{d\phi_B}{dt} \quad N \Rightarrow \text{No of turns in a coil}$$

**4. FLUX LINKAGE [ $N\phi_B$ ]****[ 2 MARK]**

The product of number of turns 'N' of the coil and the magnetic flux linking each turn of the coil  $\phi_B$



## ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENT

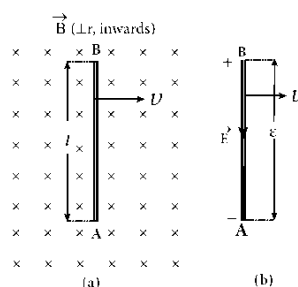
**5. LENZ'S LAW****[ 2 MARK]**

The direction of the induced current is such that it always opposes the cause responsible for its production.

$$\xi = - \frac{d[N\Phi_B]}{dt}$$

**6. FLEMING'S RIGHT HAND RULE****[ 2 MARK]**

The thumb, index finger and the middle finger of right hand are stretched out in mutually perpendicular direction. If the index finger points the direction of the magnetic field and the thumb indicates the direction of motion of their conductors, then the middle finger will indicate the direction of the induced current.

**7. MOTIONAL EMF FROM LORENTZ FORCE****[ 3 MARK]**

Consider a straight conductor rod AB of length  $\ell$  in a uniform magnetic field  $\vec{B}$  which is directed perpendicular into the plane of the paper. The length of the rod is normal to the magnetic field. Let the rod move with a constant velocity  $\vec{v}$  towards right side. The Lorentz force acts on free electrons in the direction from B to A and is,

$$\vec{F}_B = -e [\vec{v} \times \vec{B}]$$

The action of this Lorentz force is to accumulate the free electrons at the end A and produces a potential difference across the rod which in turn establishes an electric field  $\vec{E}$  directed along BA the coulomb force starts along AB.

$$\vec{F}_E = -e \vec{E}$$



## ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENT

At equilibrium, the magnetic Lorentz Force  $\vec{F}_B$  and the coulomb force  $\vec{F}_E$  balance each other and no further accumulation of free electrons at the end A takes place,

$$|\vec{F}_B| = |\vec{F}_E|$$

$$|-e [\vec{v} \times \vec{B}]| = |-e \vec{E}|$$

$$vB \sin\theta = E$$

$$vB = E \quad [\theta = 90^\circ]$$

The potential difference between two ends of the rod is ,

$$V = E\ell$$

$$V = vB\ell$$

The Lorentz force on the free electron is responsible to maintain this potential difference and hence produces an emf.

$$\xi = B\ell v$$

As this emf is produced due to the movement of the rod, it is often called as motional emf. If the ends A and B are connected by an external circuit of total resistance R,

then current  $i = \frac{\xi}{R}$

$$i = \frac{Blv}{R} \text{ flows in it.}$$

The direction of the current is found from right -hand thumb rule.

## 8. EDDY CURRENT

[ 3 MARK]

An emf induced in a conductor when the magnetic flux passing through it changes.

## 9. APPLICATION OF EDDY CURRENT

Though the production of eddy current is undesirable in some cases, it is useful in some other cases. A few of them are

- i. Induction stove
- ii. Eddy current brake
- iii. Eddy current testing
- iv. Electromagnetic damping

**i. Induction stove**

Induction stove is used to cook the food quickly and safely with less energy consumption. Below the cooking zone, there is a tightly wound coil of insulated wire. The cooking pan made of suitable material, is placed over the cooking zone. When the stove is switched on, an alternating current flowing in the coil produces high frequency alternating magnetic field which induces very strong eddy currents in the cooking pan. The eddy currents in the pan produce so much of heat due to Joule heating which is used to cook the food

**ii. Eddy current brake**

This eddy current braking system is generally used in high speed trains and roller coasters. Strong electromagnets are fixed just above the rails. To stop the train, electromagnets are switched on. The magnetic field of these magnets induces eddy currents in the rails which oppose or resist the movement of the train. This is Eddy current linear brake. In some cases, the circular disc, connected to the wheel of the train through a common shaft, is made to rotate in between the poles of an electromagnet. When there is a relative motion between the disc and the magnet, eddy currents are induced in the disc which stop the train. This is Eddy current circular brake

**iii. Eddy current testing**

It is one of the simple non-destructive testing methods to find defects like surface cracks, air bubbles present in a specimen. A coil of insulated wire is given an alternating electric current so that it produces an alternating magnetic field. When this coil is brought near the test surface, eddy current is induced in the test surface. The presence of defects causes the change in phase and amplitude of the eddy current that can be detected by some other means. In this way, the defects present in the specimen are identified

**iv. Electro magnetic damping**

The armature of the galvanometer coil is wound on a soft iron cylinder. Once the armature is deflected, the relative motion between the soft iron cylinder and the radial magnetic field induces eddy current in the cylinder. The



## ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENT

damping force due to the flow of eddy current brings the armature to rest immediately and then galvanometer shows a steady deflection. This is called electromagnetic damping.

**10. SELF – INDUCTION****[ 3 MARK]**

An electric current flowing through a coil will set up a magnetic field around it. Therefore, the magnetic flux of the magnetic field is linked with that coil itself. If this flux is changed by changing the current, an emf is induced in that same coil. This phenomenon is known as **self-induction**.

**11. CO-EFFICIENT OF SELF-INDUCTION OF THE COIL****(or) SELF INDUCTANCE****[ 3 MARK]**

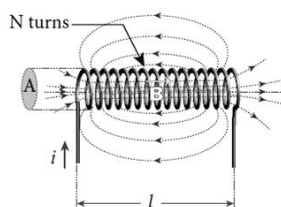
The opposing emf induced in the coil when the rate of change of current through the coil is  $1 \text{ A s}^{-1}$ .

$$L = - \frac{\xi}{\frac{di}{dt}}$$

**12. UNIT OF INDUCTANCE (or) ONE HENRY OF SELF-INDUCTANCE****[ 2 MARK]**

One henry if a current changing at the rate of  $1 \text{ A s}^{-1}$  induces an opposing emf of 1 V in it.

$$1 \text{ H} = 1 \text{ wb A}^{-1} = 1 \text{ VsA}^{-1}$$

**13. SELF INDUCTANCE OF A LONG SOLENOID****[ 3 MARK]**

Consider a long solenoid of length  $l$  and cross-sectional area  $A$ . Let  $n$  be the number of turns per unit length (or turn density) of the solenoid. When an electric current  $i$  is passed through the solenoid, a magnetic field produced inside is almost uniform and is directed along the axis of the solenoid

$$B = \mu_0 n i$$



## ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENT

The magnetic flux passing through each turn is

$$\phi_B = \int_A \vec{B} d\vec{A} = BA \cos\theta$$

$$= BA$$

$$[\theta = 0^\circ]$$

$$\phi_B = [\mu_0 n i] A$$

The total magnetic flux linked or flux linkage of the solenoid with  $N$  turns

$$N\phi_B = [n\ell][\mu_0 n i] A$$

$$N\phi_B = [\mu_0 n^2 A \ell] i$$

$$N\phi_B = L i$$

$$L = \mu_0 n^2 A \ell$$

If the solenoid is filled with a dielectric medium of relative permeability  $\mu_r$ ,

$$L = \mu n^2 A \ell \quad [\text{or}]$$

$$L = \mu_0 \mu_r n^2 A \ell$$

#### 14. ENERGY STORED IN AN INDUCTOR

[ 3 MARK ]

Whenever a current is established in the circuit, the inductance opposes the growth of the current. In order to establish a current in the circuit, work is done against this opposition by some external agency. This work done is stored as magnetic potential energy.

Let us assume that electrical resistance of the inductor is negligible and inductor effect alone is considered. The induced emf  $\xi$  at any instant ' $t$ ' is

$$\xi = -L \frac{di}{dt}$$

Let  $dW$  be work done in moving a charge  $dq$  in a time  $dt$  against the opposition, then

$$dW = -\xi dq$$

$$= -\xi i dt$$

$$dW = - \left[ -L \frac{di}{dt} \right] i dt$$

$$dW = L i dt$$

The Total work done ,

$$W = \int dW = \int_0^i L i dt = L \left[ \frac{i^2}{2} \right]_0^i$$



## ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENT

$$W = \frac{1}{2} Li^2$$

This work done is stored as magnetic potential energy

$$\therefore U_B = \frac{1}{2} Li^2$$

The energy density is the energy stored per unit volume of the space,

$$u_B = \frac{U_B}{Al} \quad [\text{volume of the solenoid} = Al]$$

$$u_B = \frac{Li^2}{2Al}$$

$$= \frac{[\mu_0 n^2 Al] i^2}{2Al}$$

$$L = \mu_0 n^2 Al$$

$$= \frac{\mu_0 n^2 i^2}{2}$$

$$B = \mu_0 ni$$

$$u_B = \frac{B^2}{2\mu_0}$$

**15. MUTUAL INDUCTION****[ 2 MARK ]**

When an electric current passing through a coil changes with time, an emf is induced in the neighbouring coil. This phenomenon is known as **mutual induction** and the emf induced is called mutually induced emf.

**16. CO-EFFICIENT OF MUTUAL INDUCTION OF THE COIL****(or) MUTUAL INDUCTANCE****[ 3 MARK ]**

The opposing emf induced in the coil 2 when the rate of change of current through the coil 1 is  $1\text{As}^{-1}$ .

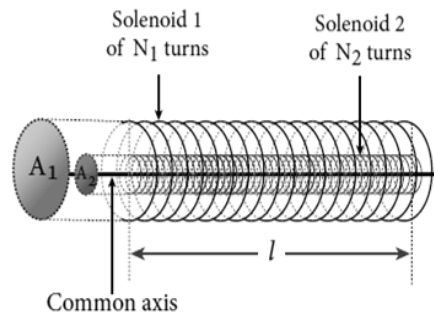
**17. UNIT OF MUTUAL - INDUCTANCE (or) ONE HENRY OF MUTUAL – INDUCTANCE****[ 2 MARK ]**

The mutual inductance between two neighbouring coils is one henry if a current changing at the rate of  $1\text{A s}^{-1}$  in one coil induces an opposing emf of 1V in neighbouring coil.



### 18. MUTUAL INDUCTANCE BETWEEN THE LONG CO-AXIAL SOLENOIDS.

[5 MARK]



Consider two long co-axial solenoids of same length  $l$ . The length of these solenoids is large when compared to their radii so that the magnetic field produced inside the solenoids is uniform and the fringing effect at the ends may be ignored. Let  $A_1$  and  $A_2$  be the area of cross section of the solenoids with  $A_1$  being greater than  $A_2$ . The turn density of these solenoids are  $n_1$  and  $n_2$  respectively.

Let  $i_1$  be the current flowing through solenoid 1, then the magnetic field produced inside it is

$$B_1 = \mu_0 n_1 i_1$$

$\vec{B}_1$  are passing through the area bounded by solenoid 2, the magnetic flux is linked with each turn of solenoid 2 due to current  $i_1$ ,

$$\phi_{21} = \int_{A_2} \vec{B}_1 \cdot d\vec{A} = B_1 A_1 \quad [\theta = 0^\circ]$$

$$= [\mu_0 n_1 i_1] A_2$$

The flux linkage with solenoid 2 ,

$$N_2 \phi_{21} = [n_2 \ell] [\mu_0 n_1 i_1] A_2 \quad [N_2 = n_2 \ell]$$

$$N_2 \phi_{21} = [\mu_0 n_1 n_2 A_2 \ell] i_1$$

$$N_2 \phi_{21} = M_{21} i_1$$

$$M_{21} = \mu_0 n_1 n_2 A_2 \ell$$

Mutual inductance  $M_{21}$  of the solenoid 2 with respect to solenoid 1



## ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENT

The magnetic field produced by the solenoid 2 when carrying a current  $i_2$

$$B_2 = \mu_0 n_2 i_2$$

This magnetic field  $B_2$  is uniform inside the solenoid 2 but outside the solenoid 2, it is almost zero. Therefore for solenoid 1, the area  $A_2$  is the effective area over which the magnetic field  $B_2$  is present, not area  $A_1$ . Then the magnetic flux  $\phi_{12}$

$$\phi_{12} = \int_{A_2} \vec{B}_1 \cdot d\vec{A} = B_2 A_2 \quad [ \theta = 0^\circ ]$$

$$= [\mu_0 n_1 i_1] A_2$$

The flux linkage of solenoid 1 with total turns  $N_1$  is

$$N_1 \phi_{12} = [n_1 \ell] [\mu_0 n_2 i_2] A_2 \quad [N_2 = n_2 \ell]$$

$$N_1 \phi_{12} = [\mu_0 n_1 n_2 A_2 \ell] i_1$$

$$N_1 \phi_{12} = M_{12} i_1$$

$$M_{12} = \mu_0 n_1 n_2 A_2 \ell$$

$$M_{12} = M_{21} = M$$

The mutual inductance between two long co-axial solenoids

$$M = \mu_0 n_1 n_2 A_2 \ell$$

If a dielectric medium of relative permeability  $\mu_r$  is present inside the solenoids,

$$M = \mu n_1 n_2 A_2 \ell$$

$$M = \mu_0 \mu_r n_1 n_2 A_2 \ell$$

### 19. METHODS OF PRODUCING INDUCED EMF

[ 2 MARK ]

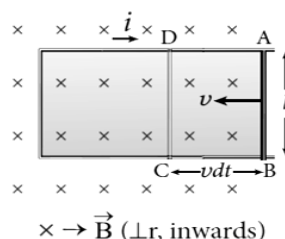
$$\xi = \frac{d}{dt} [ BA \cos \theta ]$$

1. By changing the magnetic field  $B$
2. By changing the area  $A$  of the coil and
3. By changing the relative orientation  $\theta$  of the coil with magnetic field

**20. PRODUCTION OF INDUCED EMF BY CHANGING THE MAGNETIC****FIELD****[ 2 MARK]**

The change in flux is brought about by

- (i) relative motion between the circuit and the magnet
- (ii) variation in current flowing through the nearby coil

**21. PRODUCTION OF INDUCED EMF BY CHANGING THE AREA OF THE COIL****[ 3 MARK]**

Consider a conducting rod of length ' $l$ ' moving with a velocity ' $V$ ' towards left on a rectangular fixed metallic framework. The whole arrangement is placed in a uniform magnetic field  $\vec{B}$  whose magnetic lines are perpendicularly directed into the plane of the paper.

As the rod moves from  $AB$  to  $DC$  in a time  $dt$ , the area enclosed by the loop and hence the magnetic flux through the loop decreases.

The change in magnetic flux in time  $dt$  is

$$\begin{aligned} d\phi_B &= B \times [\text{change in area}] \\ &= B \times \text{Area } ABCD \quad [\text{Area } ABCD = l(Vdt)] \end{aligned}$$

$$d\phi_B = B l V dt$$

The induced emf is ,

$$\begin{aligned} \xi &= \frac{d\phi_B}{dt} \\ \xi &= B l V \end{aligned}$$

This emf is known as Motional emf.

The direction of induced current is found to be clockwise from Fleming's right hand rule.

Induced current in the loop is,

$$\begin{aligned} i &= \frac{\xi}{R} \\ i &= \frac{B l V}{R} \end{aligned}$$



## ENERGY CONSERVATION

The current-carrying movable rod AB kept in the perpendicular  $\vec{B}$  experiences a force  $\vec{F}_B$  in the outward direction,

$$\vec{F}_B = i \ell B$$

A constant force that is equal and opposite to the magnetic force, must be applied.

$$|\vec{F}_{app}| = |\vec{F}_B| = i \ell B$$

∴ Mechanical work is done by the applied force to move the rod. The rate of doing work or power is

$$\begin{aligned} P &= \vec{F}_{app} \cdot \vec{V} = F_{app} \cdot V \cos\theta \\ &= i \ell B V \quad [\theta = 0^\circ] \end{aligned}$$

$$= \left[ \frac{BlV}{R} \right] \ell B V$$

$$P = \frac{B^2 l^2 V^2}{R}$$

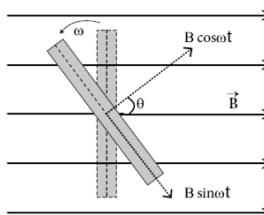
Joule heating takes place in the loop ,

$$\begin{aligned} P &= I^2 R \\ &= \left[ \frac{BlV}{R} \right]^2 R \end{aligned}$$

$$P = \frac{B^2 l^2 V^2}{R}$$

The mechanical energy needed to move the rod is converted into electrical energy which then appears as thermal energy in the loop.

## 22. PRODUCTION OF INDUCED EMF BY CHANGING RELATIVE ORIENTATION OF THE COIL WITH THE MAGNETIC FIELD [ 5 MARK ]



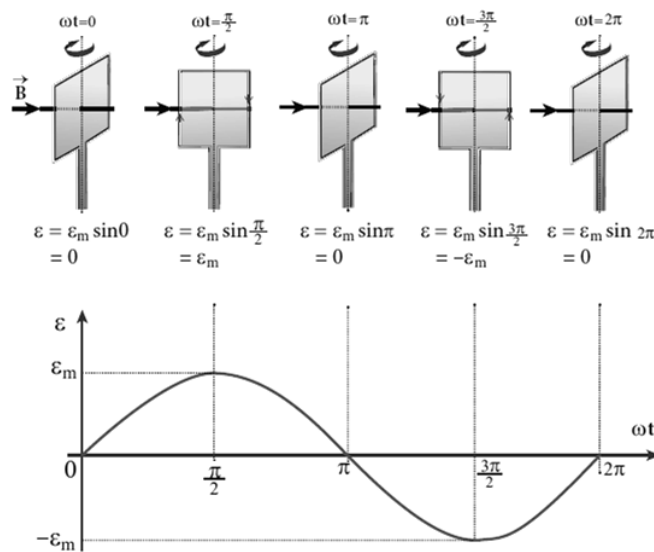


## ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENT

Consider a rectangular coil of  $N$  turns kept in a uniform magnetic field  $\vec{B}$ . The coil rotates in anti-clockwise direction with an angular velocity  $\omega$  about an axis, perpendicular to the field and to the plane of the paper.

At time  $t = 0$ , the plane of the coil is perpendicular to the field and the flux linked with the coil has its maximum value

$$\Phi_B = NBA$$



In a time  $t$  seconds, the coil is rotated through an angle  $\theta (= \omega t)$  in anti-clockwise direction.

The flux linked  $NBA \cos \omega t$  is due to the component of  $\vec{B}$  normal to the plane of the coil. The component  $(B \sin \omega t)$  parallel to the plane has no role in electromagnetic induction.

$\therefore$  The flux linkage with the coil

$$N\Phi_B = NBA \cos \theta = NBA \cos \omega t$$

According to Faraday's law,

$$\begin{aligned} \xi &= - \frac{d[N\Phi_B]}{dt} = - \frac{d[NBA \cos \omega t]}{dt} \\ &= - NBA [-\sin \omega t] \cdot \omega \\ &= NBA \omega \sin \omega t \end{aligned}$$



## ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENT

When the coil is rotated through  $90^\circ$  from initial position,  $\sin \omega t = 1$ . Then the maximum value of induced emf is

$$\xi_m = NBA\omega$$

The value of induced emf is

$$\xi = \xi_m \sin \omega t$$

The induced emf varies as sine function of the time angle  $\omega t$ . The graph between induced emf and time angle for one rotation of the coil will be a sine curve and the emf varying in this manner is called **sinusoidal emf** or **alternating emf**.

This current is called **alternating current**

$$i = I_m \sin \omega t$$

$I_m \Rightarrow$  maximum value of induced current.

### 23. AC GENERATOR

[ 3 MARK ]

1. It is an energy conversion device.
2. It converts mechanical energy used to rotate the coil or field magnet into electrical energy.
3. Alternator produces a large scale electrical power for use in homes and industries.

### PRINCIPLE

Electromagnetic induction.

### CONSTRUCTION

Two major parts ,

1. Stator
2. Rotor

#### • STATOR

The stationary part which has armature windings mounted in it is called stator.



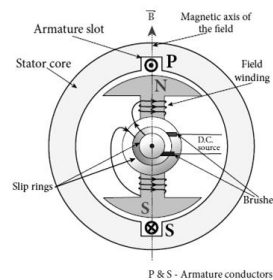
## ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENT

**(i) Stator core**

It is made up of iron or steel alloy. It is a hollow cylinder and is laminated to minimize eddy current loss. The slots are cut on inner surface of the core to accommodate armature windings.

**(ii) Armature winding**

It is the coil, wound on slots provided in the armature core

**• ROTOR**

It contains magnetic field windings. The magnetic poles are magnetized by DC source. The ends of field windings are connected to a pair of slip rings, attached to a common shaft about which rotor rotates. Slip rings rotate along with rotor. To maintain connection between the DC source and field windings, two brushes are used which continuously slide over the slip rings. This is 2-pole rotor

**24. ADVANTAGES OF STATIONARY ARMATURE – ROTATING FIELD****ALTERNATING****[ 5 MARK**

- 1) The current is drawn directly from fixed terminals on the stator without the use of brush contacts.
- 2) The insulation of stationary armature winding is easier.
- 3) The number of sliding contacts (slip rings) is reduced. Moreover, the sliding contacts are used for low-voltage DC Source.
- 4) Armature windings can be constructed more rigidly to prevent deformation due to any mechanical stress.



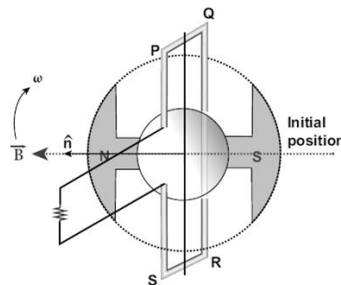
## 25. SINGLE PHASE AC GENERATOR

[ 5 MARK ]

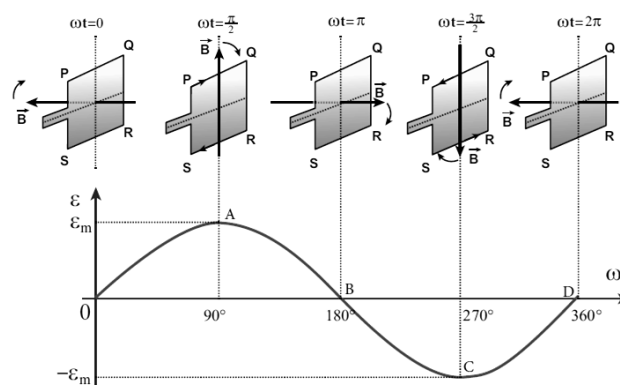
The armature conductors are connected in series so as to form a single circuit which generates a single-phase alternating emf and hence it is called single-phase alternator.

A single-turn rectangular loop PQRS is mounted on the stator. The field winding is fixed inside the stator and it can be rotated about an axis, perpendicular to the plane of the paper.

The loop PQRS is stationary and is also perpendicular to the plane of the paper. When field windings are excited, magnetic field is produced around it. Let the field magnet be rotated in clockwise direction by some external means



Assume that initial position of the field magnet is horizontal. At that instant, the direction of magnetic field is perpendicular to the plane of the loop PQRS. The induced emf is zero. This is represented by origin O in the graph drawn between induced emf and time angle.



When field magnet rotates through  $90^\circ$ , magnetic field becomes parallel to PQRS. The induced emfs across PQ and RS would become maximum. Since they are connected in series, emfs are added up and the direction of total induced emf is given by Fleming's right hand rule. The direction of the induced emf is at right angles to the plane of



## ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENT

the paper. For PQ, it is inwards and for RS it is outwards. Therefore, the current flows along PQRS. The point A in the graph represents this maximum emf.

For the rotation of  $180^\circ$  from the initial position, the field is again perpendicular to PQRS and the induced emf becomes zero. This is represented by point B.

The field magnet becomes again parallel to PQRS for  $270^\circ$  rotation of field magnet. The induced emf is maximum but the direction is reversed. Thus the current flows along SRQP. This is represented by point C.

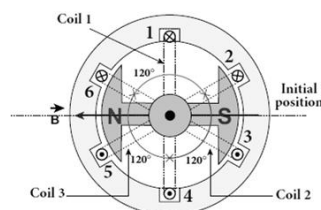
On completion of  $360^\circ$ , the induced emf becomes zero and is represented by the point D. From the graph, it is clear that emf induced in PQRS is alternating in nature. Therefore, when field magnet completes one rotation, induced emf in PQRS finishes one cycle.

**26. POLY-PHASE AC GENERATOR****[ 2 MARK ]**

Some AC generators may have more than one coil in the armature core and each coil produces an alternating emf. In these generators, more than one emf is produced. Thus they are called **poly-phase generators**.

**27. THREE PHASE AC GENERATOR****[ 5 MARK ]**

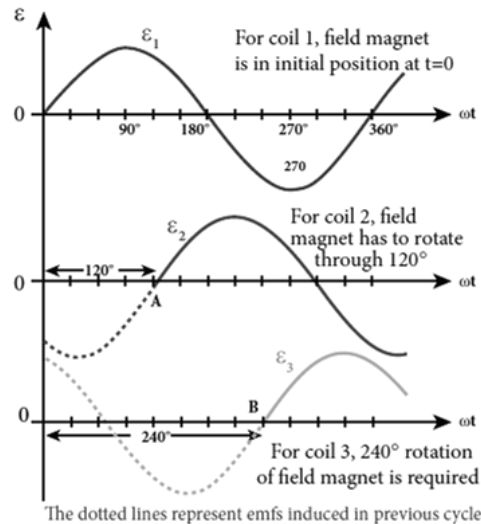
The armature core has 6 slots, cut on its inner rim. Each slot is  $60^\circ$  away from one another. Six armature conductors are mounted in these slots. The conductors 1 and 4 are joined in series to form coil 1. The conductors 3 and 6 form coil 2 while the conductors 5 and 2 form coil 3. So, these coils are rectangular in shape and are  $120^\circ$  apart from one another





## ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENT

The initial position of the field magnet is horizontal and field direction is perpendicular to the plane of the coil 1. As it is seen in single phase AC generator, when field magnet is rotated from that position in clockwise direction, alternating emf  $\xi_1$  in coil 1 begins a cycle from origin O.



An alternating emf  $\xi_2$  in coil 2 starts at point A after field magnet has rotated through  $120^\circ$ . Therefore, the phase difference between  $\xi_1$  and  $\xi_2$  is  $120^\circ$ . Similarly, emf  $\xi_3$  in coil 3 would begin its cycle at point B after  $240^\circ$  rotation of field magnet from initial position. Thus these emfs produced in the three phase AC generator have  $120^\circ$  phase difference between one another.

**28. ADVANTAGES OF THREE PHASE GENERATOR****[ 2 MARK]**

- 1) For a given dimension of the generator, it produces higher power output .
- 2) For the same capacity, three-phase alternator is smaller in size when compared to single-phase alternator.
- 3) Three-phase transmission system is cheaper. A relatively thinner wire is sufficient for transmission of three-phase power.

**29. TRANSFORMER****[ 5 MARK]**

Transformer is a stationary device used to transform electrical power from one circuit to another without changing its frequency.

If the transformer converts an alternating current with low voltage



## ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENT

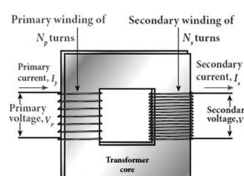
into an alternating current with high voltage, it is called **step-up transformer**. On the contrary, if the transformer converts alternating current with high voltage into an alternating current with low voltage, then it is called **step-down transformer**.

**PRINCIPLE**

The principle of transformer is the mutual induction between two coils.

**CONSTRUCTION**

There are two coils of high mutual inductance wound over the same transformer core. The core is generally laminated and is made up of a good magnetic material like silicon steel. Coils are electrically insulated but magnetically linked via transformer core



The assembled core and coils are kept in a container which is filled with suitable medium for better insulation and cooling purpose.

**WORKING**

The coil across which alternating voltage is applied is called primary coil P and the coil from which output power is drawn out is called secondary coil S. an alternating magnetic flux is set up in the laminated core. If there is no magnetic flux leakage, then whole of magnetic flux linked with primary coil is also linked with secondary coil. This means that rate at which magnetic flux changes through each turn is same for both primary and secondary coils. The emf induced in the primary coil or back emf  $\xi_P$  is

$$\xi_P = - N_P \frac{d\phi_B}{dt}$$

But the voltage applied  $V_p$  across the primary is equal to the back emf .

$$V_p = - N_P \frac{d\phi_B}{dt}$$

The frequency of alternating magnetic flux in the core is same as the frequency of the applied voltage. Therefore, induced emf in secondary will also have



## ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENT

same frequency as that of applied voltage. The emf induced in the secondary coil  $\xi_s$  is given by

$$\xi_s = -N_s \frac{d\phi_B}{dt}$$

The voltage across secondary coil,

$$V_s = -N_s \frac{d\phi_B}{dt}$$

[  $N_p$  and  $N_s$  are the numbers of turns in the primary and secondary coil ]

$$\frac{V_s}{V_p} = \frac{N_s}{N_p} = K$$

$K \rightarrow$  voltage transformer ratio

For an ideal transformer ,

$$\text{Input power } V_p i_p = \text{output power } V_s i_s$$

Where  $i_p$  and  $i_s$  are the currents in the primary and secondary coil respectively

$$\frac{V_s}{V_p} = \frac{N_s}{N_p} = \frac{i_p}{i_s} = K$$

- i.  $K > 1$  [ $N_s > N_p$ ,  $V_s > V_p$ , and  $I_s < I_p$ ]

This is the case of step-up transformer in which voltage is increased and the corresponding current is decreased

- ii.  $K < 1$  [ $N_s < N_p$ ,  $V_s < V_p$ , and  $I_s > I_p$ ]

This is step-down transformer where voltage is decreased and the current is increased.

### EFFICIENCY OF THE TRANSFORMER

The ratio of the useful output power to the input power

$$\eta = \frac{\text{Output power}}{\text{input power}}$$

Transformers are highly efficient devices having their efficiency in the range of 96 – 99%.

**30. ENERGY LOSSES IN THE TRANSFORMER****[ 3 MARK ]****i) Core loss or Iron loss**

This loss takes place in transformer core. Hysteresis loss and eddy current loss are known as core loss or Iron loss. When transformer core is magnetized and demagnetized repeatedly by the alternating voltage applied across primary coil, hysteresis takes place due to which some energy is lost in the form of heat. Hysteresis loss is minimized by using steel of high silicon content in making transformer core.

Alternating magnetic flux in the core induces eddy currents in it. Therefore there is energy loss due to the flow of eddy current, called eddy current loss which is minimized by using very thin laminations of transformer core.

**ii) Copper loss**

Transformer windings have electrical resistance. When an electric current flows through them, some amount of energy is dissipated due to Joule heating. This energy loss is called copper loss which is minimized by using wires of larger diameter.

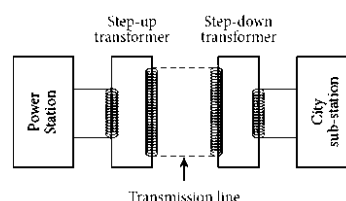
**iii) Flux leakage**

Flux leakage happens when the magnetic lines of primary coil are not completely linked with secondary coil. Energy loss due to this flux leakage is minimized by winding coils one over the other.

**31. ADVANTAGES OF AC IN LONG DISTANCE POWER TRANSMISSION****[ 3 MARK ]**

The electric power generated is transmitted over long distances through transmission lines to reach towns or cities where it is actually consumed. This process is called power transmission.

At the transmitting point, the voltage is increased and the corresponding current is decreased by using step-up transformer





Then it is transmitted through transmission lines. This reduced current at high voltage reaches the destination without any appreciable loss. At the receiving point, the voltage is decreased and the current is increased to appropriate values by using step-down transformer and then it is given to consumers. Thus power transmission is done efficiently and economically.

### ILLUSTRATION

An electric power of 2 MW is transmitted to a place through transmission lines of total resistance, say  $R = 40 \Omega$ , at two different voltages. One is lower voltage (10 kV) and the other is higher (100 kV). Let us now calculate and compare power losses in these two cases.

#### Case (i):

$$P = 2 \text{ MW}; R = 40 \Omega; V = 10 \text{ Kv}$$

$$\text{Power, } P = VI$$

$$\text{Current } I = \frac{P}{V} = \frac{2 \times 10^6}{10 \times 10^3} = 200 \text{ A}$$

$$\text{Power loss} = \text{Heat produced} = I^2 R$$

$$= [200]^2 \times 40 = 1.6 \times 10^6 \text{ W}$$

$$\begin{aligned} \% \text{ of power loss} &= \frac{1.6 \times 10^6}{2 \times 10^6} \times 100\% \\ &= 0.8 \times 100\% = 80\% \end{aligned}$$

#### Case (ii):

$$P = 2 \text{ MW}; R = 40 \Omega; V = 100 \text{ kV}$$

$$I = \frac{P}{V} = \frac{2 \times 10^6}{100 \times 10^3} = 20 \text{ A}$$

$$\text{Power loss} = I^2 R$$

$$= [20]^2 \times 40 = 0.016 \times 10^6 \text{ W}$$

$$\begin{aligned} \% \text{ of power loss} &= \frac{0.016 \times 10^6}{2 \times 10^6} \times 100\% \\ &= 0.008 \times 100\% = 0.8\% \end{aligned}$$

Thus it is clear that when an electric power is transmitted at higher voltage, the power loss is reduced to a large extent.



## ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENT

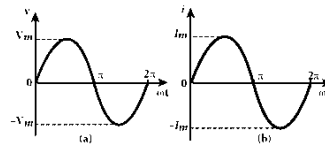
**32. ALTERNATING CURRENT****[ 2 MARK]**

An alternating voltage is the voltage which changes polarity at regular intervals of time and the direction of the resulting alternating current also changes accordingly.

**33. SINUSOIDAL ALTERNATING VOLTAGE****[ 2 MARK]**

If the wave form of alternating voltage is a sine wave , then it is known as in sinusoidal alternating voltage

$$V = V_m \sin \omega t$$



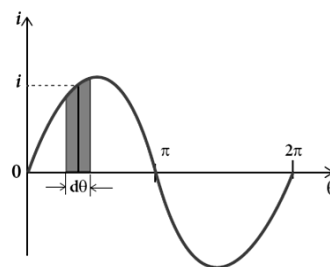
Alternating current  $i = I_m \sin \omega t$

**34. MEAN OR AVERAGE OF AC****[ 3 MARK]**

The average value of alternating current is defined as the average of all values of current over a positive half-cycle or negative half-cycle.

The instantaneous value of sinusoidal alternating current is

$$i = I_m \sin \omega t \text{ or } i = I_m \sin \theta$$



The sum of all current over a half-cycle is given by area of positive half-cycle

$$I_{av} = \frac{\text{Area of positive half-cycle} \text{ [or negative half-cycle]}}{\text{base length of half-cycle}}$$

Consider an elementary strip of thickness  $d\theta$  in the positive half-cycle of the current wave . Let  $i$  be the mid-ordinate of that strip.

$$\text{Area of the elementary strip} = i d\theta$$

$$\begin{aligned} \text{Area of positive half-cycle} &= \int_0^\pi i d\theta = \int_0^\pi I_m \sin \theta d\theta \\ &= I_m [ -\cos \theta ]_0^\pi \\ &= I_m [ \cos \pi - \cos 0 ] \end{aligned}$$



## ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENT

$$= 2 I_m$$

The base length of half-cycle is  $\pi$

$$\text{Average value of AC, } I_{av} = \frac{2 I_m}{\pi}$$

$$I_{av} = 0.637 I_m$$

Hence the average value of AC is 0.637 times the maximum value  $I_m$  of the alternating current. For negative half-cycle,  $I_{av} = -0.637 I_m$

'n' current in a half-cycle of AC,

$$I_{av} = \frac{\text{sum of all current over half cycle}}{\text{number of current}}$$

$$I_{av} = \frac{i_1 + i_2 + \dots + i_n}{n}$$

### 35. RMS VALUE OF AC [ $I_{RMS}$ ]

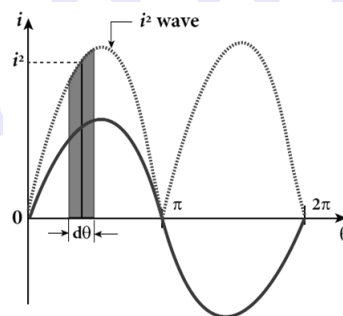
[ 5 MARK]

The square root of the mean of the squares of all currents over one cycle

For alternating voltages,  $V_{RMS}$

Alternating current  $i = I_m \sin \omega t$

$$i = I_m \sin \theta$$



The sum of the squares of all currents over one cycle is given by the area of one cycle of squared wave

$$I_{RMS} = \sqrt{\frac{\text{area of one cycle of squared wave}}{\text{Base length of one cycle}}}$$

An elementary area of thickness ' $d\theta$ '. Let  $i^2$  be the mid ordinate of the element.

$$\text{Area of the element} = i^2 d\theta$$

Area of one cycle of squared wave

$$= \int_0^{2\pi} i^2 d\theta$$

$$= \int_0^{2\pi} I_m^2 \sin^2 \theta \cdot d\theta$$

$$= I_m^2 \int_0^{2\pi} \sin^2 \theta \cdot d\theta$$



## ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENT

$$\begin{aligned}
 &= I_m^2 \left[ \int_0^{2\pi} d\theta - \int_0^{2\pi} \cos 2\theta d\theta \right] & [\sin^2 \theta = \frac{1 - \cos \theta}{2}] \\
 &= I_m^2 \left[ 0 - \frac{\sin 2\theta}{2} \right]_0^{2\pi} \\
 &= I_m^2 \left\{ \left[ 2\pi - \frac{\sin 2 \times 2\pi}{2} \right] - \left[ 0 - \frac{\sin \theta}{2} \right] \right\} \\
 &= \frac{I_m^2}{2} \times 2\pi = I_m^2 \pi & [\sin 0 = \sin 4\pi = 0]
 \end{aligned}$$

The base length of one cycle is  $2\pi$

$$\begin{aligned}
 I_{\text{RMS}} &= \sqrt{\frac{I_m^2 \pi}{2\pi}} \\
 &= \frac{I_m}{\sqrt{2}} \\
 I_{\text{RMS}} &= 0.707 I_m
 \end{aligned}$$

A symmetrical sinusoidal current rms value of current is 70.7 % of its peak value.

For alternating voltage,

$$I_{\text{RMS}} = 0.707 I_m$$

For ex ;

If we consider 'n' current, in one cycle of AC,

$$\begin{aligned}
 I_{\text{RMS}} &= \sqrt{\frac{\text{sum of squares of all current over one cycle}}{\text{number of current}}} \\
 I_{\text{RMS}} &= \sqrt{\frac{i_1^2 + i_2^2 + \dots + i_n^2}{n}}
 \end{aligned}$$

### 36. PHASOR

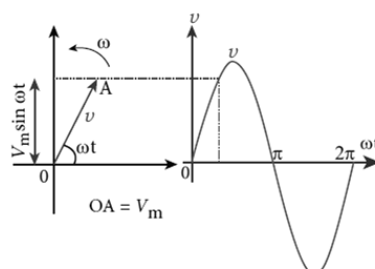
[2 MARK]

A sinusoidal alternating voltage (or current) can be represented by a vector which rotates about the origin in anti-clockwise direction at a constant angular velocity  $\omega$ . Such a rotating vector is called a phasor.

### 37. PHASOR DIAGRAM

[2 MARK]

The diagram which shows various phasors and their phase relations is called phasor diagram.

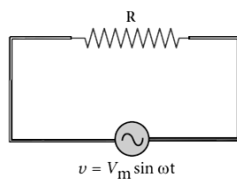




## ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENT

## 38. AC CIRCUIT CONTAINING PURE RESISTOR

[ 3 MARK]



Consider a circuit containing a pure resistor of resistance  $R$  connected across an alternating voltage source . The instantaneous value of the alternating voltage

$$V = V_m \sin \omega t$$

An alternating current  $i$  flowing in the circuit due to this voltage, develops a potential drop across  $R$  and is,

$$V_R = iR$$

Kirchoff's loop rule , For this resistive circuit,

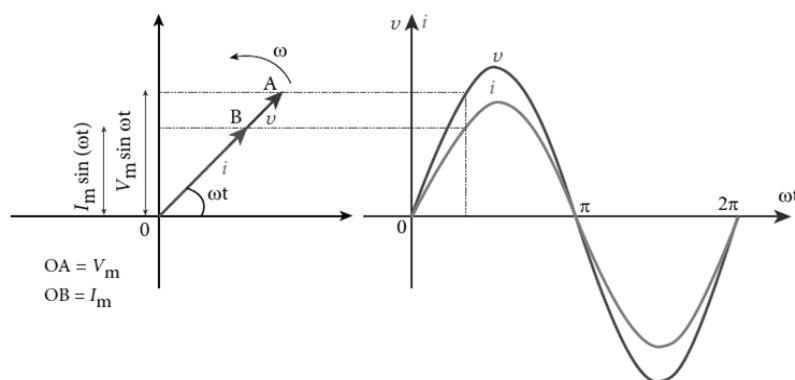
$$V - V_R = 0$$

$$V_m \sin \omega t = iR$$

$$i = \frac{V_m}{R} \sin \omega t$$

$$i = I_m \sin \omega t$$

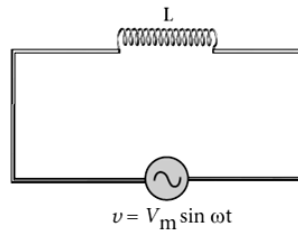
$$I_m = \frac{V_m}{R} \Rightarrow \text{the peak value of alternating current in the circuit.}$$



The applied voltage and the current are in phase with each other in a resistive circuit. It means that they reach their maxima and minima simultaneously. This is indicated in the phasor diagram . The wave diagram also depicts that current is in phase with the applied voltage.



## ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENT

**39. AC CIRCUIT CONTAINING ONLY AN INDUCTOR****[ 5 MARK]**

Consider a circuit containing a pure inductor of inductance  $L$  connected across an alternating voltage source. The instantaneous value of the alternating voltage is

$$V = V_m \sin \omega t$$

The alternating current flowing through the inductor induces a self-induced emf or back emf in the circuit.

$$\xi = -L \frac{di}{dt}$$

By applying Kirchoff's loop rule,

$$V + \xi = 0$$

$$V_m \sin \omega t = L \frac{di}{dt}$$

$$di = \frac{V_m}{L} \sin \omega t \cdot dt$$

Integrating both sides, we get

$$i = \frac{V_m}{L} \int \sin \omega t \cdot dt$$

$$i = \frac{V_m}{L\omega} [ -\cos \omega t ] + \text{constant}$$

The integration constant in the above equation is independent of time. So constant as zero.

$$i = \frac{V_m}{L\omega} \sin \left[ \omega t - \frac{\pi}{2} \right]$$

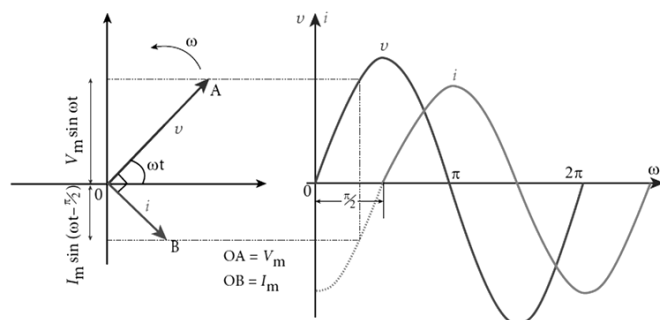
$$\left( -\cos \omega t = -\sin \left[ \frac{\pi}{2} - \omega t \right] = \sin \left[ \omega t - \frac{\pi}{2} \right] \right)$$

$$(\text{or}) \quad i = I_m \sin \left[ \omega t - \frac{\pi}{2} \right]$$

The peak value of the alternating current is  $I_m = \frac{V_m}{L\omega}$ . From the above equation that current lags behind the applied voltage by  $\frac{\pi}{2}$  in an inductive circuit.



## ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENT

**INDUCTIVE REACTANCE [ $X_L$ ]**

$I_m = \frac{V_m}{\omega L}$ ,  $\omega L$  plays the same role as the resistance in resistive circuit.

This is the resistance offered by the inductor, called inductive reactance [ $X_L$ ]

**UNIT:** ohm.

$$X_L = \omega L$$

The inductive reactance ( $X_L$ ) varies directly as the frequency.

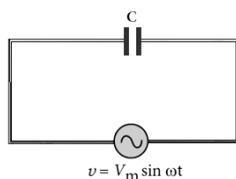
$$X_L = 2\pi fL$$

$f \Rightarrow$  frequency of the alternating current. For a steady current,  $f = 0$ ;  $X_L = 0$ . Thus an ideal inductor offers no resistance to steady DC current.

**40. AC CIRCUIT CONTAINING ONLY A CAPACITOR****[ 5 MARK]**

Consider a circuit containing a capacitor of capacitance  $C$  connected across an alternating voltage source. The instantaneous value of the alternating voltage is given by

$$V = V_m \sin \omega t$$



Let  $q$  be the instantaneous charge on the capacitor. The emf across the capacitor at that instant is  $q/C$ . According to Kirchhoff's loop rule,

$$V - \frac{q}{C} = 0$$

$$q = CV$$

$$q = CV_m \sin \omega t$$

$$i = \frac{dq}{dt} = \frac{d}{dt} [CV_m \sin \omega t]$$



## ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENT

$$= CV_m \frac{d}{dt} \sin \omega t$$

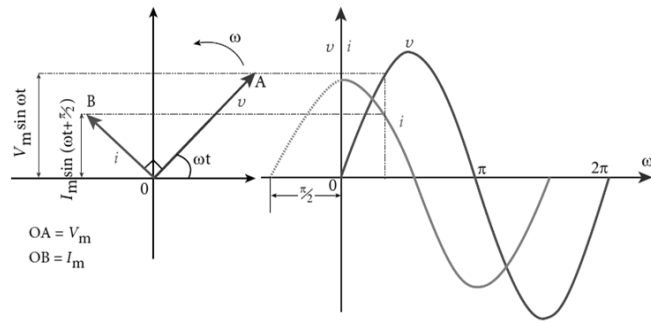
$$i = \frac{V_m}{1/\omega C} \sin\left[\omega t + \frac{\pi}{2}\right]$$

Instantaneous value of current,

$$i = I_m \sin\left[\omega t + \frac{\pi}{2}\right]$$

Peak value of the current  $I_m = \frac{V_m}{1/\omega C}$

Here, current leads the applied voltage by  $\frac{\pi}{2}$  in a capacitive circuit.



### CAPACITIVE REACTANCE

The peak value of current  $I_m$  is given by  $I_m = \frac{V_m}{1/\omega C}$ . Let us compare this equation with  $I_m = \frac{V_m}{R}$  for a resistive circuit. The quantity  $1/\omega C$  plays the same role as the resistance  $R$  in resistive circuit. This is the resistance offered by the capacitor, called capacitive reactance ( $X_C$ ).

**UNIT** ohm.

$$X_C = 1/\omega C$$

The capacitive reactance ( $X_C$ ) varies inversely as the frequency. For a steady current,  $f = 0$ .

$$\therefore X_C = 1/\omega C = 1/2\pi fC = 1/0 = \infty$$

Thus a capacitive circuit offers infinite resistance to the steady current. So that steady current cannot flow through the capacitor.

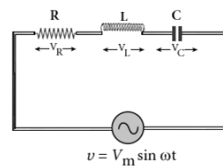
### 41. AC CIRCUIT CONTAINING A RESISTOR ,AN INDUCTOR AND A CAPACITOR IN SERIES –SERIES RLC CIRCUIT [ 5 MARK]

Consider a circuit containing a resistor of resistance  $R$ , an inductor of inductance  $L$



## ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENT

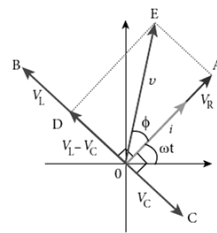
and a capacitor of capacitance  $C$  connected across an alternating voltage source



The instantaneous value of the alternating voltage is given by

$$V = V_m \sin \omega t$$

Let  $i$  be the resulting current in the circuit at that instant. As a result, the voltage is developed across  $R$ ,  $L$  and  $C$ .



From phasor diagram,

The voltage across  $R$   $[V_R] = OA = I_m R$

$[V_R]$  is in phase with  $i$

The voltage across  $L$   $[V_L] = OB = I_m L$

$[V_L]$  leads  $i$  by  $\frac{\pi}{2}$

The voltage across  $C$   $[V_C] = OC = I_m C$

$[V_C]$  lags behind  $i$  by  $\frac{\pi}{2}$

$$OI = I_m$$

The circuit is either effectively inductive or capacitive or resistive depending on the value of  $V_L$  or  $V_C$ .

Let us assume that  $V_L > V_C$  Therefore, net voltage drop across  $L$ - $C$  combination is  $V_L - V_C$  which is represented by a phasor  $\overline{OD}$

By parallelogram law, the diagonal  $\overline{OE}$  gives the resultant voltage  $v$  of  $V_R$  and  $(V_L - V_C)$  and its length  $OE$  is equal to  $V_m$ .



## ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENT

$$V_m^2 = V_R^2 + (V_L - V_C)^2$$

$$V_m = \sqrt{(I_m R)^2 + (I_m X_L - I_m X_C)^2}$$

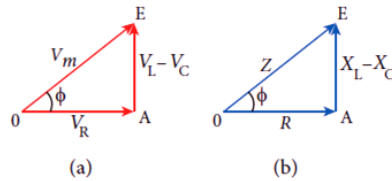
$$= I_m \sqrt{R^2 + (X_L - X_C)^2} \text{ or}$$

$$I_m = \frac{V_m}{\sqrt{R^2 + (X_L - X_C)^2}} \text{ or}$$

$$I_m = \frac{V_m}{Z}$$

$$\text{where } Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$Z \Rightarrow$  Impedance of the circuit which refers to the effective opposition to the current by the series  $RLC$  circuit.



From phasor diagram, the phase angle between  $v$  and  $i$  is

$$\tan \phi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R}$$

**SPECIAL CASES**

(i)  $X_L > X_C$

$(X_L - X_C)$  is positive and phase angle  $\phi$  is also positive. The applied voltage leads the current by  $\phi$  and The circuit is inductive.

$$\therefore V = V_m \sin \omega t ; i = I_m \sin[\omega t - \phi]$$

(ii)  $X_L < X_C$

$(X_L - X_C)$  is negative and  $\phi$  is also negative. The current leads voltage

by  $\phi$  and the circuit is capacitive.

$$\therefore V = V_m \sin \omega t ; i = I_m \sin[\omega t + \phi]$$

(iii)  $X_L = X_C$

$\phi$  is zero. Therefore current and voltage are in the same phase and the circuit is resistive.

$$\therefore V = V_m \sin \omega t ; i = I_m \sin \omega t$$

**42. RESONANCE IN SERIES RLC CIRCUIT :****RESONANCE FREQUENCY****[ 3 MARK]**



The frequency of the applied alternating source  $\omega_r$  is equal to the natural frequency  $1/\sqrt{LC}$  of the  $RLC$  circuit, the current in the circuit reaches its maximum value. Then the circuit is said to be in **electrical resonance**. The frequency at which resonance takes place is called **resonant frequency**.

$$\text{Resonant angular frequency, } \omega_r = 1/\sqrt{LC}$$

$$\text{Or } f_r = \frac{1}{2\pi\sqrt{LC}}$$

At series resonance,

$$\omega_r = 1/\sqrt{LC} \quad (\text{or}) \quad \omega_r^2 = 1/LC$$

$$\omega_r L = 1/\omega_r C \quad (\text{or}) \quad X_L = X_C$$

This is the condition for resonance in  $RLC$  circuit.

Since  $X_L$  and  $X_C$  are frequency dependent, the resonance condition ( $X_L = X_C$ ) can be achieved by varying the frequency of the applied voltage.

#### 43. EFFECTS OF SERIES RESONANCE

[ 3 MARK]

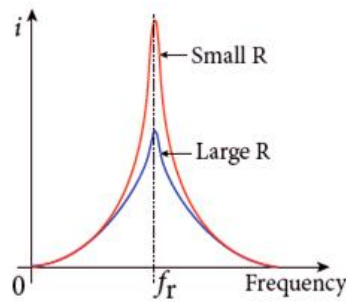
OR

#### RESONANCE CURVE

When series resonance occurs, the impedance of the circuit is minimum and is equal to the resistance of the circuit. As a result of this, the current in the circuit becomes maximum. This is shown in the resonance curve drawn between current and frequency



## ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENT



At resonance, the impedance is

$$Z = \sqrt{R^2 + [X_L - X_C]^2} = R \text{ since } X_L = X_C$$

The current in the circuit is,

$$I_m = \frac{V_m}{\sqrt{R^2 + [X_L - X_C]^2}}$$

$$I_m = \frac{V_m}{R}$$

The maximum current at series resonance is limited by the resistance of the circuit. For smaller resistance, larger current with sharper curve is obtained and vice versa.

#### 44. APPLICATION OF SERIES RLC RESONANT CIRCUIT [2 MARK]

1. RLC circuits have many applications like filter circuits, oscillators, voltage multipliers etc.
2. Tuning circuits of radio and TV systems

The tuning is commonly achieved by varying capacitance of a parallel plate variable capacitor.

#### 45. QUALITY FACTOR (or) Q-FACTOR [2 MARK]

The ratio of voltage across  $L$  or  $C$  at resonance to the applied voltage.

$$Q\text{-factor} = \frac{\text{Voltage across } L \text{ or } C \text{ at resonance}}{\text{Applied voltage}}$$

$$Q\text{-factor} = \frac{1}{R} \sqrt{\frac{L}{C}}$$

**46. POWER IN AC CIRCUITS****[ 3 MARK]**

The rate of consumption of electric energy in that circuit.

The alternating voltage and alternating current in the series inductive *RLC* circuit at an instant

$$V = V_m \sin \omega t \text{ and } i = I_m \sin[\omega t + \phi]$$

$\phi$  = Phase angle between  $V$  and  $i$

The instantaneous power is,

$$\begin{aligned} P &= Vi \\ &= V_m \sin \omega t \cdot I_m \sin[\omega t + \phi] \\ &= V_m I_m \sin \omega t [\sin \omega t \cos \phi + \cos \omega t \sin \phi] \\ P &= V_m I_m [\cos \phi \sin^2 \omega t + \sin \omega t \cos \omega t \sin \phi] \end{aligned}$$

Here the average of  $\sin^2 \omega$  over a cycle is  $\frac{1}{2}$  and that of  $\sin \omega t \cos \omega t \sin \phi$  is zero.

$$P_{av} = V_m I_m \cos \phi \times \frac{1}{2}$$

$$= \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos \phi$$

$$P_{av} = V_{RMS} I_{RMS} \cos \phi$$

$V_{RMS} I_{RMS} \Rightarrow$  Apparent power

$\cos \phi \Rightarrow$  Power factor

The average power of an AC circuit is also known as the true power of the circuit.

**SPECIAL CASES**

1. For a purely resistive circuit, the phase angle between voltage and current is zero and  $\cos \phi = 1$ .

$$\therefore P_{av} = V_{RMS} I_{RMS}$$

2. For a purely inductive or capacitive circuit,  $\phi$  is  $\pm \pi/2$  and  $\cos [\pm \pi/2] = 0$

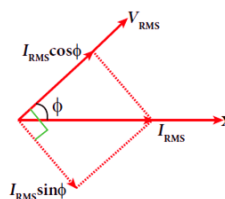
$$\therefore P_{av} = 0$$

3. For series RLC circuit,

$$\phi = \tan^{-1} \left[ \frac{X_L - X_C}{R} \right]$$

4. For series RLC circuit at resonance,  $\phi = 0$  ;  $\cos \phi = 1$ .

$$\therefore P_{av} = V_{RMS} I_{RMS}$$

**47. WATTFUL CURRENT AND WATTLESS CURRENT****[ 3 MARK]**

The component of current  $[I_{RMS} \cos \phi]$  which is in phase with the voltage is called active component. The power consumed by this current  $= V_{RMS} I_{RMS} \cos \phi$ . So that it is also known as **‘Wattful’ current**.

The other component  $[I_{RMS} \sin \phi]$  which has a phase angle of  $\pi/2$  with the voltage is called reactive component. The power consumed is zero. Hence it is also known as **‘Wattless’ current**.

The current in an AC circuit is said to be wattless current if the power consumed by it is zero. This wattless current occurs in a purely inductive or capacitive circuit

**48. POWER FACTOR****[ 2 MARK]**

- i. Power factor  $= \cos \phi = \text{cosine of the angle of lead or lag}$
- ii. Power factor  $= \frac{R}{Z} = \frac{\text{Resistance}}{\text{impedance}}$
- iii. Power factor  $= \frac{P_{av}}{V_{RMS} I_{RMS}} = \frac{\text{True power}}{\text{apparent power}}$

**EXAMPLES:**

- i. Power factor  $= \cos 0^\circ = 1$   
pure resistive circuit  
 $\phi$  between V and i is zero.
- ii. Power factor  $= \cos [\pm \pi/2] = 0$   
purely inductive or capacitive circuit  
 $\phi$  between V and i is  $\pm \pi/2$
- iii. Power factor lies between 0 and 1 for a circuit having R, L and C in varying proportions
- iv.

**49. ADVANTAGES AND DISADVANTAGES OF AC OVER DC****[3 MARK]****ADVANTAGES**

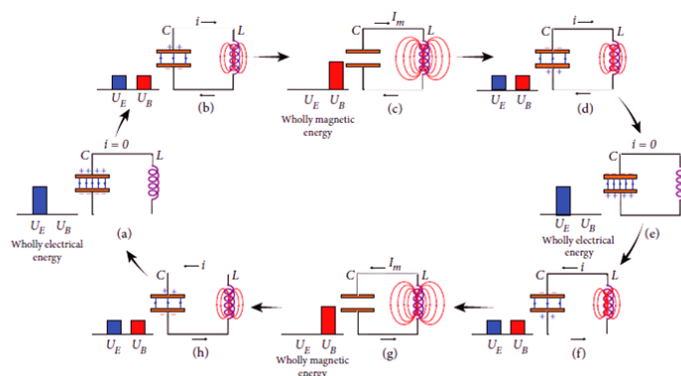
- (i) The generation of AC is cheaper than that of DC.
- (ii) When AC is supplied at higher voltages, the transmission losses are small compared to DC transmission.
- (iii) AC can easily be converted into DC with the help of rectifiers.

**DISADVANTAGES**

- (i) Alternating voltages cannot be used for certain applications such as charging of batteries, electroplating, electric traction etc.
- (ii) At high voltages, it is more dangerous to work with AC than DC.

**OSCILLATION IN LC CIRCUIT****50. ENERGY CONVERSION DURING LC OSCILLATION****[ 5 MARK]****LC OSCILLATION**

The energy oscillates back and forth between the magnetic field of the inductor and the electric field of the capacitor. Thus the electrical oscillations of definite frequency are generated. These oscillations are called **LC oscillations**.

**GENERATION OF LC OSCILLATION**

Let us assume that the capacitor is fully charged with maximum charge  $Q_m$  at the initial stage. So that the energy stored in the capacitor is maximum and is given by



## ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENT

$U_E = \frac{Q_m^2}{2C}$ . As there is no current in the inductor, the energy stored in it is zero i.e.,  $U_B = 0$ . Therefore, the total energy is wholly electrical.

The capacitor now begins to discharge through the inductor that establishes current  $i$  in clockwise direction. This current produces a magnetic field around the inductor and the energy stored in the inductor is given by  $U_B = \frac{Li^2}{2}$ . As the charge in the capacitor decreases, the energy stored in it also decreases and is given by  $U_E = \frac{q^2}{2C}$ . Thus there is a transfer of some part of energy from the capacitor to the inductor. At that instant, the total energy is the sum of electrical and magnetic energies.

When the charges in the capacitor are exhausted, its energy becomes zero i.e.,  $U_E = 0$ . The energy is fully transferred to the magnetic field of the inductor and its energy is maximum. This maximum energy is given by  $U_E = \frac{LI_m^2}{2}$  where  $I_m$  is the maximum current flowing in the circuit. The total energy is wholly magnetic. Even though the charge in the capacitor is zero, the current will continue to flow in the same direction because the inductor will not allow it to stop immediately. The current is made to flow with decreasing magnitude by the collapsing magnetic field of the inductor. As a result of this, the capacitor begins to charge in the opposite direction. A part of the energy is transferred from the inductor back to the capacitor. The total energy is the sum of the electrical and magnetic energies.

When the current in the circuit reduces to zero, the capacitor becomes fully charged in the opposite direction. The energy stored in the capacitor becomes maximum. Since the current is zero, the energy stored in the inductor is zero. The total energy is wholly electrical.

The state of the circuit is similar to the initial state but the difference is that the capacitor is charged in opposite direction. The capacitor then starts to discharge through the inductor with anti-clockwise current. The total energy is the sum of the electrical and magnetic energies.

As already explained, the processes are repeated in opposite direction. Finally, the circuit returns to the initial state. Thus, when the circuit goes through these stages, an alternating current flows in the circuit. As this process is repeated again and again, the electrical oscillations of definite frequency are generated. These are known as  $LC$  oscillations.

In the ideal  $LC$  circuit, there is no loss of energy. Therefore, the oscillations



will continue indefinitely. Such oscillations are called undamped oscillations.

### 51. CONSERVATION OF ENERGY IN LC OSCILLATIONS [3 MARK]

During *LC* oscillations in *LC* circuits, the energy of the system oscillates between the electric field of the capacitor and the magnetic field of the inductor. Although, these two forms of energy vary with time, the total energy remains constant.

$$\text{Total energy, } U = U_E + U_B = \frac{q^2}{2C} + \frac{1}{2} LI^2$$

#### Case (i)

When the charge in the capacitor,  $q = Q_m$  and the current through the inductor,  $i = 0$ , the total energy is

$$U = \frac{Q_m^2}{2C} + 0 = \frac{Q_m^2}{2C}$$

The total energy is wholly electrical.

#### Case (ii)

When  $q = 0$ ;  $i = I_m$ ,

The total energy is,  $U = 0 + \frac{1}{2} LI_m^2 = \frac{1}{2} LI_m^2$

$$\begin{aligned} &= \frac{L}{2} \times \left[ \frac{Q_m^2}{LC} \right] \quad \left\{ I_m = Q_m \omega = \frac{Q_m}{\sqrt{LC}} \right\} \\ &= \frac{Q_m^2}{2C} \end{aligned}$$

The total energy is wholly electrical.

#### Case (iii)

When charge =  $q$ ; current =  $i$ ,  
The total energy is

$$U = \frac{q^2}{2C} + \frac{1}{2} LI^2$$

Since  $q = Q_m \cos \omega t$

$$i = -\frac{dq}{dt} = Q_m \omega \sin \omega t$$

[−ve sign in current indicates that the charge in the capacitor decreases with time]

$$U = \frac{Q_m^2 \cos^2 \omega t}{2C} + \frac{L \omega^2 Q_m^2 \sin^2 \omega t}{2} \quad \left[ \omega^2 = \frac{1}{LC} \right]$$



## ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENT

$$= \frac{Q_m^2 \cos^2 \omega t}{2c} + \frac{LQ_m^2 \sin^2 \omega t}{2LC}$$

$$= Q_m^2 / 2c (\cos^2 \omega t + \sin^2 \omega t)$$

$$U = Q_m^2 / 2c$$

From above these cases, the total energy of the system remains constant.

## 52. ANALOGIES BETWEEN LC OSCILLATIONS AND SIMPLE HARMONIC OSCILLATIONS

[3 MARKS]

### QUALITATIVE TREATMENT

The electromagnetic oscillations of  $LC$  system can be compared with the mechanical oscillations of a spring-mass system.

There are two forms of energy involved in  $LC$  oscillations. One is electrical energy of the charged capacitor; the other magnetic energy of the inductor carrying current.

Likewise, the mechanical energy of the spring-mass system exists in two forms; the potential energy of the compressed or extended spring and the kinetic energy of the mass.

### Energy in two oscillatory systems

Element	LC oscillator Energy	Element	Spring-mass system Energy
Capacitor	Electrical Energy = $\frac{1}{2} \left( \frac{1}{C} \right) q^2$	Spring	Potential energy = $\frac{1}{2} k x^2$
Inductor	Magnetic energy = $\frac{1}{2} L i^2$ $i = \frac{dq}{dt}$	Mass	Kinetic energy = $\frac{1}{2} m v^2$ $v = \frac{dx}{dt}$



## ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENT

**Analogies between electrical and mechanical quantities**

Electrical system	Mechanical system
Charge $q$	Displacement $x$
Current $i = \frac{dq}{dt}$	Velocity $v = \frac{dx}{dt}$
Inductance $L$	Mass $m$
Reciprocal of capacitance $\frac{1}{C}$	Force constant $k$
Electrical energy $= \frac{1}{2} \left( \frac{1}{C} \right) q^2$	Potential energy $= \frac{1}{2} k x^2$
Magnetic energy $= \frac{1}{2} L i^2$	Kinetic energy $= \frac{1}{2} m v^2$
Electromagnetic energy $U = \frac{1}{2} \left( \frac{1}{C} \right) q^2 + \frac{1}{2} L i^2$	Mechanical energy $E = \frac{1}{2} k x^2 + \frac{1}{2} m v^2$

The angular frequency of oscillations of a spring-mass is

$$\omega = \sqrt{\frac{k}{m}}$$

$$k \Rightarrow \frac{1}{C} \text{ and } m \Rightarrow L$$

The angular frequency of LC oscillations is,

$$\omega = \frac{1}{\sqrt{LC}}$$