Padasalai's

Telegram Groups!

(தலைப்பிற்கு கீழே உள்ள லிங்கை கிளிக் செய்து குழுவில் இணையவும்!)

- Padasalai's NEWS - Group

https://t.me/joinchat/NIfCqVRBNj9hhV4wu6_NqA

- Padasalai's Channel - Group

https://t.me/padasalaichannel

- Lesson Plan - Group

https://t.me/joinchat/NIfCqVWwo5iL-21gpzrXLw

- 12th Standard - Group

https://t.me/Padasalai_12th

- 11th Standard - Group

https://t.me/Padasalai_11th

- 10th Standard - Group

https://t.me/Padasalai_10th

- 9th Standard - Group

https://t.me/Padasalai_9th

- 6th to 8th Standard - Group

https://t.me/Padasalai_6to8

- 1st to 5th Standard - Group

https://t.me/Padasalai_1to5

- TET - Group

https://t.me/Padasalai_TET

- PGTRB - Group

https://t.me/Padasalai_PGTRB

- TNPSC - Group

https://t.me/Padasalai_TNPSC

CHAPTER - 2 complex number.

Introduction of complex number.

Equation : $x^2-1=0$

$x^2=1$

$x=\pm\sqrt{1}$

$x=\pm 1$

Equation : $x^2+1=0$

$x^2=-1$

$x=\pm\sqrt{-1}$

$\boxed{\sqrt{-1} \text{ denoted has } i}$

$x=\pm i$

Powers of imaginary units.

$\boxed{i \rightarrow \text{iotta}}$

$i^0=1$	$i^{-0}=1$	$i^{4n-0}=1$
$i^1=i$	$i^{-1}=-i$	$i^{4n-3}=i$
$i^2=-1$	$i^{-2}=-1$	$i^{4n-2}=-1$
$i^3=-i$	$i^{-3}=i$	$i^{4n-1}=-i$
$i^4=1$	$i^{-4}=1$	$i^{4n-4}=1$

Example 2.1

i) $i^7=i^{4+3}=i^4\cdot i^3=i^3=-i$

ii) $i^{1729}=i^{1728+1}=i^1=i$

iii) $i^{-1924}+i^{2018}=(i)^{-1924+0}+i^{2016+2}$

$=i^0+i^2$

$=1+(-1)$

$i^{-1924}+i^{2018}=0.$

EXE 2.1

1. $i^{1947} + i^{1950} = i^{1944+3} + i^{1948+2}$

$= i^3 + i^2$

$= -i - 1$

(ii) $i^{1948} - i^{-1869} = i^{1948+0} - i^{-1868-1}$

$= i^0 - i^{-1}$

$= 1 - (-i)$

$i^{1948} - i^{-1869} = 1 + i.$

(iii) $\sum_{n=1}^{12} i = (i + i^2 + i^3 + i^4) + (i^5 + i^6 + i^7 + i^8) + (i^9 + i^{10} + i^{11} + i^{12})$

$= 0 + 0 + 0$

$= 0.$

Example 2.1

(iv) $\sum_{n=1}^{102} = (i + i^2 + i^3 + i^4) + \dots + i^{98} + i^{100}) + i^{101} + i^{102}$

$= i^{101} + i^{102}$

$= i + i^2$

$= i - 1$

multiple of complex number.

$$i\, i^2\, i^3\, i^4 \ldots i^n = i^{\frac{(n)\times(n+1)}{2}}$$

(v) $$i\, i^2\, i^3\, i^4 \ldots i^{40} = i^{\frac{40\times 41}{2}} = i^{820} = i^0 = 1$$

EXE 2.1

4) $\Rightarrow$ $$i^{59} + \frac{1}{i^{59}} = i^{58+1} + \frac{1}{i^{58+1}}$$

$$= i + \frac{1}{i}$$

$$= i + i^{-1}$$

$$= i - i$$

$$= 0.$$

5) $$i\, i^2\, i^3 \ldots i^{2000} = i^{\left(\frac{2000\times 2001}{2}\right)}$$

$$= i^{2001000}$$

$$= 1$$

6. $$\sum_{i=1}^{n} i^{n+50} = i^{51} + i^{52} + i^{53} + i^{54} + i^{55} + i^{56} + i^{57} + i^{58} + i^{59} + i^{60}$$

$$= i^4 + i^3 + 0 + 0$$

$$= -i + 1$$

$$= 1 - i$$

Rectangular form of a complex number

A complex number is of the form $x+iy$, where x and y are real number, x is called real part and y is called the imaginary part of the complex number.

Definition : 2.

Two complex number $z_1 = x_1+iy_1$ and $z_2 = x_2+iy_2$ are said to be equal if and only if $Re(z_1) = Re(z_2)$ and $Im(z_1) = Im(z_2)$, that is $x_1 = x_2$ and $y_1 = y_2$.

1) Scalar multiple of complex number.

let $z = x+iy$.

$zk = k(x+iy) = kx+iky$.

2) Addition of complex number.

let $z_1 = x_1+iy_1$ & $z_2 = x_2+iy_2$.

$z_1+z_2 = (x_1+iy_1)+(x_2+iy_2)$

$z_1+z_2 = (x_1+x_2)+i(y_1+y_2)$

3. Subtraction of complex number.

let $z_1 = x_1+iy_1$ & $z_2 = x_2+iy_2$.

$z_1-z_2 = (x_1+iy_1)-(x_2+iy_2)$

$= (x_1-x_2)+i(y_1-y_2)$

4. multiple of complex number

let $z_1 = x_1 + iy_1$ & $z_2 = (x_2 + iy_2)$

$$z_1 z_2 = (x_1 + iy_1)(x_2 + iy_2)$$

$$z_1 z_2 = (x_1 x_2 - y_1 y_2) + i(x_1 y_2 + y_1 x_2)$$

multiple of complex number z by i

let $z = x + iy$.

$$iz = i(x + iy)$$

$$= ix + i^2 y$$

$$= ix - y$$

$$iz = -y + ix$$

$\therefore cis\,0 = 1$

$cis\,\pi = -1$

$cis\,\pi/2 = i$

$cis(-\pi/2) = -i$

EXE 2.2

1. $z = 5 - 2i$ $\quad w = -1 + 3i$

(i)

$$z + w = (5 - 2i) + (-1 + 3i)$$

$$= 4 + i.$$

(ii) $$z - iw = (5 - 2i) - i(-1 + 3i)$$

$$= (5 - 2i) + (i + 3)$$

$$= 8 - i$$

3. $2z+3w = 2(5-2i)+3(-1+3i)$

$= (10-4i)+(-3+9i)$

$= 7+5i$

(iv) $zw = (5-2i)(-1+3i)$

$= (-5+6)+i(15+2)$

$zw = 1+i17$ (or) $1+17i$

(v) $z^2+2zw+w^2 = (5-2i)^2+2(5-2i)(-1+3i)+(-1+3i)^2$

$= [25-20i+(-4)]+2(1+17i)+[1-6i-9]$

$= 21-20i+2+34i-8-6i$

$= 15+8i$

(vi) $(z+w)^2 = z^2+2wz+w^2$

$= 15+8i$

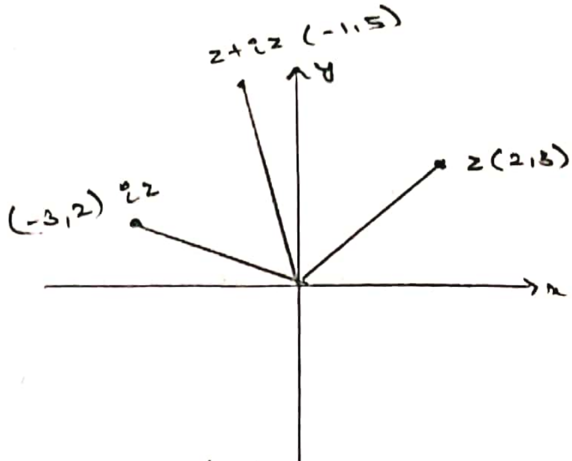
(2) (i) Let $z = 2+3i$

$iz = i(2+3i)$

$= -3+2i$

$z + iz = (2+3i) + (-3+2i)$

$= -1+5i$



Argand diagram

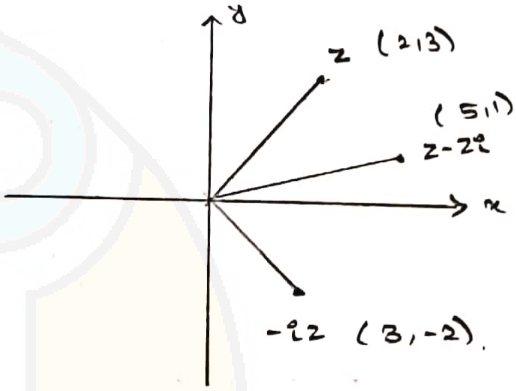
(ii) Let $z = 2+3i$

$-iz = -i(2+3i)$

$= 3-2i$

$z - iz = (2+3i) + (3-2i)$

$= 5+i$



Argand diagram.

Example 2.2.

$(2+i)x + (1-i)y + 2i - 3 = x + (-1+2i)y + 1 + i.$

$(2x+y-3) + i(x-y+2) = (x-y+1) + i(2y+1)$

Equating real and imaginary part.

$2x+y-3 = x-y+1$, $x-y+2 = 2y+1$.

$x+2y = 4 \rightarrow (1)$ $x - 3y = -1. \rightarrow (2)$.

Solve the equation (1) & (2)

$5y = 5$ sub $y=1$ in eq (1)

$\boxed{y=1}$

$x+2=4$

$\boxed{x=2}$

Scanned with CamScanner

8. $(3-i)x-(2-i)y+2i+5 = 2x+(-1+2i)y+3+2i.$

$(3x-2y+5)+i(-x+y+2) = (2x-y+3)+i(2y+2)$

Equating Real and Imaginary part.

$3x-2y+5 = 2x-y+3$, $-x+y+2 = 2y+2.$

$x-y+2=0 \rightarrow (1)$ $\quad -x-y=0 \rightarrow (2)$

solve the equation (1) & (2).

$-2y+2=0$

$2y=2$

$\boxed{y=1}$

sub $y=1$ in eq (1)

$x-1+2=0$

$\boxed{x=-1}$

properties of complex number.

property	Addition	multiple.
closure	z_1+z_2	$z_1z_2.$
Associative	$(z_1+z_2)+z_3=z_1+(z_2+z_3)$	$(z_1z_2)z_3=z_1(z_2z_3)$
Commutative	$z_1+z_2=z_2+z_1$	$z_1z_2=z_2z_1$
Additive Identity	$z+0=0+z=z$	Multiple Identity $z\cdot1=1\cdot z=z.$
Inverse.	Additive inverse $z+(-z)=-z+z=0$	multiple inverse. $zw=wz=1$

Multiple distributive over addition

$z_1(z_2+z_3) = z_1z_2+z_1z_3$

commutative property under addition.

let z_1 and z_2 two complex number.

$z_1 = x_1 + iy_1$, $z_2 = x_2 + iy_2$.

$$z_1 + z_2 = (x_1 + iy_1) + (x_2 + iy_2)$$

$$= (x_1 + x_2) + i(y_1 + y_2)$$

$$= (x_2 + x_1) + i(y_2 + y_1)$$

$$= (x_2 + iy_2) + (x_1 + iy_1)$$

$$z_1 + z_2 = z_2 + z_1$$

Inverse property under multiplication.

$z = x + iy$, z is non zero complex number

$$\frac{x}{x^2+y^2} + \frac{-iy}{x^2+y^2} = \frac{x}{x^2+y^2} + i\frac{-y}{x^2+y^2}.$$

Inverse property under addition.

$z = x + iy$, z is non zero complex number

$$-z = -(x+iy) = -x - iy.$$

EXE 2.3.

1.(i) $z_1 = 1-3i$, $z_2 = -4i$, $z_3 = 5$

$$LHS = (z_1+z_2)+z_3$$

$$= [(1-3i)+(-4i)]+5$$

$$= (1-7i)+5$$

$$LHS = 6-7i \longrightarrow (1)$$

$$RHS = z_1+(z_2+z_3)$$

$$= (1-3i)+[-4i+5]$$

$$RHS. = 6-7i \longrightarrow (2)$$

from (1) & (2)

$$(z_1+z_2)+z_3 = z_1+(z_2+z_3)$$

(ii) $(z_1z_2)z_3 = z_1(z_2z_3)$

$$LHS: (z_1z_2)z_3 = [(1-3i)(4i)]5$$

$$= [4i+12]5$$

$$= 60+20i. \longrightarrow (1)$$

$$RHS \quad z_1(z_2z_3) = (1-3i)[(4i)(5)]$$

$$= (1-3i)(20i)$$

$$= 20i+60 \longrightarrow (2).$$

from 1 & 2, we get

$$(z_1z_2)z_3 = z_1(z_2z_3).$$

2. $z_1 = 3$, $z_2 = -7i$ $z_3 = 5+4i$

(i) LHS $\Rightarrow$ $z_1(z_2+z_3) = 3[(-7i)+(5+4i)]$

$= 3[5-3i]$

$= 15-9i \longrightarrow (1)$

RHS $\Rightarrow$ $z_1z_2+z_1z_3 = 3(-7i)+3(5+4i)$

$= -21i+15+12i$,

$= 15-9i \longrightarrow (2)$

from (1) & (2), we get.

$z_1(z_2+z_3) = z_1z_2+z_1z_3$.

(ii)

LHS $\Rightarrow$ $(z_1+z_2)z_3 = [3+(-7i)](5+4i)$

$= [3-7i](5+4i)$

$= (15+28)+i(12-35)$

$= 43-23i. \longrightarrow (1)$

RHS $\Rightarrow$ $z_1z_3+z_2z_3 = 3(5+4i)+(-7i)(5+4i)$

$= (15+12i)+(-35i+28)$

$= (15+28)+i(12-35)$

$= 43-23i. \longrightarrow (2)$

from (1) & (2), we get.

$(z_1+z_2)z_3 = z_1z_3+z_2z_3$

③ $z_1 = 2+5i$, $z_2 = -3-4i$, $z_3 = 1+i$

	Question	additive inverse	multiple inverse.
(i)	$2+5i$	$-z_1 = -2-5i$	$\frac{1}{z_1} = \frac{1}{29}(2-5i)$
(ii)	$-3-4i$	$-z_2 = 3+4i$	$\frac{1}{z_2} = \frac{1}{25}(-3+4i)$
(iii)	$1+i$	$-z_3 = -1-i$	$\frac{1}{z_3} = \frac{1}{2}(1-i)$.

Conjugate of a complex number:

The conjugate of the complex number $x+iy$ is defined as the complex number $x-iy$.

properties of complex conjugate:-

1. $\overline{z_1+z_2} = \overline{z_1} + \overline{z_2}$
2. $\overline{z_1-z_2} = \overline{z_1} - \overline{z_2}$
3. $\overline{z_1 z_2} = \overline{z_1} \cdot \overline{z_2}$
4. $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$, $z_2 \neq 0$.
5. $Re(z) = \frac{z+\overline{z}}{2}$
6. $Im(z) = \frac{z-\overline{z}}{2i}$
7. $(\overline{z})^n = \overline{(z^n)}$, where n is integer.

8. z is real if only if $z = \bar{z}$

9. z is purely imaginary if and only if $z = -\bar{z}$

10. $\bar{\bar{z}} = z$.

EXE : 2.4.

1.

i) $\overline{(5+9i)+(2-4i)} = \overline{7+5i} = 7-5i$.

(ii) $\dfrac{10-5i}{6+2i} = \dfrac{10-5i}{6+2i} \times \dfrac{6-2i}{6-2i}$

$= \dfrac{60-20i-30i+(-10)}{(6)^2-(2i)^2}$

$= \dfrac{50-50i}{36+4}$

$= \dfrac{50(1-i)}{40}$

$\dfrac{10-5i}{6+2i} = \frac{5}{4}(1-i)$

(iii) $\overline{3i} + \dfrac{1}{2+i} = -3i + \dfrac{1}{2+i} \times \dfrac{2+i}{2+i}$

$= -3i + \dfrac{2+i}{5}$

$= \dfrac{-15i+2+i}{5}$

$= \dfrac{2-14i}{5}$

$\overline{3i} + \dfrac{1}{2+i} = \frac{2}{5}(1-7i)$

2. $z = x+iy$.

(i) $Re\left(\frac{1}{z}\right) = Re\left(\frac{1}{x+iy}\right)$

$= Re\left[\frac{1}{x+iy} \times \frac{x-iy}{x-iy}\right]$

$= Re\left[\frac{x-iy}{x^2+y^2}\right]$

$= Re\left[\frac{x}{x^2+y^2} - i\frac{y}{x^2+y^2}\right]$

$Re\left(\frac{1}{z}\right) = \frac{x}{x^2+y^2}$

(ii) $Re(i\bar{z}) = Re[i(x-iy)]$

$= Re[y+ix]$

$Re(i\bar{z}) = y$.

(iii) $Im(3z+4\bar{z}-4i) = Im[3(x+iy)+4(x-iy)-4i]$

$= Im[(3x+4x)+i(3y-4y-4)]$

$= Im[7x+i(-y-4)]$

$Im(3z+4\bar{z}-4i) = -y-4$.

3. $z_1 = 2 - i$, $z_2 = -4 + 3i$.

(i) $z_1 z_2 = (2 - i)(-4 + 3i)$

$= (-8 + 3) + i(6 + 4)$

$z_1 z_2 = -5 + 10i = -5(1 + (-2i)]$

$\frac{1}{z_1 z_2} = \frac{1}{-5(1-2i)}$

$= \frac{1}{-5(1-2i)} \times \frac{(1+2i)}{(1+2i)}$

$\frac{1}{z_1 z_2} = \frac{(1+2i)}{-25}$

(ii) $\frac{z_1}{z_2} = \frac{2 - i}{-4 + 3i} \times \frac{-4 - 3i}{-4 - 3i}$

$= \frac{(-8 - 3) + i(4 - 6)}{(-4)^2 - (3i)^2}$

$\frac{z_1}{z_2} = \frac{-11 - 2i}{25}$

$\left(\frac{z_1}{z_2}\right)^{-1} = \frac{-25}{11+2i} \times \frac{11-2i}{11-2i}$

$= \frac{-25(11-2i)}{(11)^2 - (2i)^2}$

$= \frac{-25(11-2i)}{121 + 4}$

$\left(\frac{z_1}{z_2}\right)^{-1} = \left(\frac{z_2}{z_1}\right) = \frac{-25(11-2i)}{125} = -\frac{(11-2i)}{5}$

4. $v = 3-4i$, $w = 4+3i$.

$$\frac{1}{u} = \frac{1}{v} + \frac{1}{w}$$

$$\frac{1}{u} = \frac{w+v}{vw}$$

$$u = \frac{vw}{v+w}$$

$$= \frac{(3-4i)(4+3i)}{(3-4i)+(4+3i)}$$

$$= \frac{(12+12) + i(-16+9)}{7-i}$$

$$u = \frac{24-7i}{7-i} \times \frac{7+i}{7+i}$$

$$= \frac{(168+7) + i(24-49)}{(7)^2-(i)^2}$$

$$= \frac{175-25i}{50}$$

$$= \frac{25(7-i)}{50}$$

$u = \frac{1}{2}(7-i)$.

5. (i) z is real if and only if $z = \bar{z}$

let $z = x+iy$, $\bar{z} = x-iy$.

$x+iy = x-iy$.

$2iy = 0$

$y = 0$.

z is a purely real.

(ii) z is real and purey imaginary if and only if $z = -\bar{z}$

let $z = x+iy$ $\bar{z} = x-iy$.

$z = -\bar{z}$

$x+iy = -(x-iy)$

$2x = 0$

$x = 0$.

z is a purely imaginary.

(iii) $Re(z) = \frac{z+\bar{z}}{2}$,

let $z = x+iy$, $\bar{z} = x-iy$.

$z+\bar{z} = (x+iy)+(x-iy)$

$z+\bar{z} = 2x$

$\frac{z+\bar{z}}{2} = x$

$\frac{z+\bar{z}}{2} = Re(z)$.

$$Im(z) = \frac{z-\bar{z}}{2i}$$

$$z - \bar{z} = (x+iy) - (x-iy)$$

$$z - \bar{z} = 2iy.$$

$$\frac{z-\bar{z}}{2i} = y.$$

$$\frac{z-\bar{z}}{2i} = Im(z).$$

6.

$$(\sqrt{3}+i)^2 = (\sqrt{3})^2 + (i)^2 + 2i\sqrt{3}$$

$$= 3 - 1 + 2i\sqrt{3}$$

$$= 2 + 2i\sqrt{3}$$

$$(\sqrt{3}+i)^2 = 2(1+i\sqrt{3})$$

$$(\sqrt{3}+i)^3 = (\sqrt{3}+i)^2(\sqrt{3}+i)$$

$$= 2(1+i\sqrt{3})(\sqrt{3}+i)$$

$$= 2[\sqrt{3} + i + i(3) + i^2\sqrt{3}]$$

$$= 2[\sqrt{3} + 4i - \sqrt{3}]$$

$$= 2[4i]$$

$$(\sqrt{3}+i)^3 = 8i$$

$n=3$ is purely imaginary.

$$(\sqrt{3}+i)^6 = [(\sqrt{3}+i)^3]^2 = [8i]^2 = -64$$

$n=6$ is purely real.

7. (i) $z = (2+i\sqrt{3})^{10} - (2-i\sqrt{3})^{10}$

$\bar{z} = \overline{(2+i\sqrt{3})^{10} - (2-i\sqrt{3})^{10}}$

$= \overline{(2+i\sqrt{3})^{10}} - \overline{(2-i\sqrt{3})^{10}}$

$= (2-i\sqrt{3})^{10} - (2+i\sqrt{3})^{10}$

$= -\left[(2+i\sqrt{3})^{10} - (2-i\sqrt{3})^{10}\right]$

$\bar{z} = -z$

z is purely imaginary.

(ii) $z = \left(\frac{19-7i}{9+i}\right)^{12} + \left(\frac{20-5i}{7-6i}\right)^{12}$

$\frac{19-7i}{9+i} = \frac{19-7i}{9+i} \times \frac{9-i}{9-i} = \frac{164-82i}{82} = \frac{82(2-i)}{82} = 2-i.$

$\frac{20-5i}{7-6i} = \frac{20-5i}{7-6i} \times \frac{7+6i}{7+6i} = \frac{170+85i}{85} = \frac{85(2+i)}{85} = 2+i.$

$z = \left(\frac{19-7i}{9+i}\right)^{12} + \left(\frac{20-5i}{7-6i}\right)^{12} = (2-i)^{12} + (2+i)^{12}$

$\bar{z} = \overline{(2-i)^{12} + (2+i)^{12}}$

$= \overline{(2-i)^{12}} + \overline{(2+i)^{12}}$

$= (2+i)^{12} + (2-i)^{12}$

$\bar{z} = z$

z is purely real.

Example 2.8

(i) $z = (2+i\sqrt{3})^{10} + (2-i\sqrt{3})^{10}$

$$\bar{z} = \overline{(2+i\sqrt{3})^{10} + (2-i\sqrt{3})^{10}}$$

$$= \overline{(2+i\sqrt{3})}^{10} + \overline{(2-i\sqrt{3})}^{10}$$

$$= (2-i\sqrt{3})^{10} + (2+i\sqrt{3})^{10}$$

$\bar{z} = z$

z is purely real.

(ii) $z = \left(\frac{19+9i}{5-3i}\right)^{15} - \left(\frac{8+i}{1+2i}\right)^{15}$

$$\frac{19+9i}{5-3i} = \frac{19+9i}{5-3i} \times \frac{5+3i}{5+3i} = \frac{68+102i}{34} = \frac{34(2+3i)}{34} = 2+3i$$

$$\frac{8+i}{1+2i} = \frac{8+i}{1+2i} \times \frac{1-2i}{1-2i} = \frac{10-15i}{5} = \frac{5(2-3i)}{5} = 2-3i$$

$$z = \left(\frac{19+9i}{5-3i}\right)^{15} - \left(\frac{8+i}{1+2i}\right)^{15} = (2+3i)^{15} - (2-3i)^{15}$$

$$\bar{z} = \overline{(2+3i)^{15} - (2-3i)^{15}}$$

$$= \overline{(2+3i)}^{15} - \overline{(2-3i)}^{15}$$

$$= (2-3i)^{15} - (2+3i)^{15}$$

$$= -\left[(2+3i)^{15} - (2-3i)^{15}\right]$$

$\bar{z} = -z.$

z is purely imaginary.

Example 2.7

$$z = (2+3i)(1-i) = 5+i$$

$$\frac{1}{z} = \frac{1}{5+i} \times \frac{5-i}{5-i}$$

$$= \frac{5-i}{25+1}$$

$$z^{-1} = \frac{5-i}{26}$$

Example 2.6

$$z_1 = 3-2i \ , \quad z_2 = 6+4i$$

$$\frac{z_1}{z_2} = \frac{3-2i}{6+4i} \times \frac{6-4i}{6-4i}$$

$$= \frac{(18-8)+i(-12-12)}{36+16}$$

$$= \frac{10-24i}{52}$$

$$\frac{z_1}{z_2} = \frac{10-24i}{52}.$$

Example 2.5

$$\frac{z+3}{z-5i} = \frac{1+4i}{2}$$

$$2(z+3) = (1+4i)(z-5i)$$

$$2z+6 = z-5i+4iz+20$$

$$z = \frac{14-5i}{1-4i} \times \frac{1+4i}{1+4i} = \frac{34+51i}{17} = 2+3i.$$

Example 2.4

$$\frac{1+i}{1-i} = \frac{1+i}{1-i} \times \frac{1+i}{1+i} = \frac{(1+i)^2}{2} = \frac{2i}{2} = i$$

$$\frac{1-i}{1+i} = \frac{1-i}{1+i} \times \frac{1-i}{1-i} = \frac{(1-i)^2}{2} = \frac{-2i}{2} = -i$$

$$\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3 = (i)^3 - (-i)^3 = i^3 + i^3 = -i - i = -2i$$

Example 2.3.

$$\frac{3+4i}{5-12i} = \frac{3+4i}{5-12i} \times \frac{5+12i}{5+12i} = \frac{(15-48) + i(20+36)}{25+144}$$

$$\frac{3+4i}{5-12i} = \frac{-33+56i}{169}.$$

modulus of a complex number.

If $z = x+iy$, then the modulus of z, denoted by $|z|$, is defined by $|z| = \sqrt{x^2+y^2}$. , $|z|^2 = x^2+y^2$.

$$|z|^2 = z\bar{z}.$$

properties of modulus of a complex number.

1. $|z| = |\bar{z}|$
2. $|z_1+z_2| \leq |z_1| + |z_2|$ (triangle inequality)
3. $|z_1 z_2| = |z_1||z_2|$
4. $|z_1 - z_2| \geq |z_1| - |z_2|$

5. $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$, $z_2 \neq 0$.

6. $|z^n| = |z|^n$, where n is an integer.

7. $Re(z) \leq |z|$

8. $Im(z) \leq |z|$.

Triangle inequality.

For any two complex number z_1 and z_2,

$$|z_1+z_2| \leq |z_1|+|z_2|.$$

Proof.

$$|z_1+z_2|^2 = (z_1+z_2)\overline{(z_1+z_2)}$$ $\quad |z|^2 = z\bar{z}$

$$= (z_1+z_2)(\bar{z}_1+\bar{z}_2)$$ $\quad \overline{z_1+z_2} = \bar{z}_1+\bar{z}_2$

$$= z_1\bar{z}_1 + z_1\bar{z}_2 + z_2\bar{z}_1 + z_2\bar{z}_2$$

$$= z_1\bar{z}_1 + (z_1\bar{z}_2 + \overline{z_2\bar{z}_1}) + z_2\bar{z}_2$$

$$= |z_1|^2 + 2\,Re(z_1\bar{z}_2) + |z_2|^2$$

$$|z_1+z_2|^2 \leq |z_1|^2 + 2|z_1||z_2| + |z_2|^2$$

$$|z_1+z_2|^2 \leq (|z_1|+|z_2|)^2.$$

Taking square root on both side.

$$|z_1+z_2| \leq |z_1|+|z_2|.$$

EXE 2.5

① i) $\left|\frac{2i}{3+4i}\right| = \frac{|2i|}{|3+4i|} = \frac{\sqrt{4}}{\sqrt{9+16}} = \frac{\sqrt{4}}{\sqrt{25}} = \frac{2}{5}$

ii) $\left|\frac{2-i}{1+i} + \frac{1-2i}{1-i}\right| = \left|\frac{(2-i)(1-i)+(1-2i)(1+i)}{(1+i)\ (1-i)}\right|$

$= \left|\frac{(1-3i) + (3-i)}{\sqrt{2}}\right|$

$= \left|\frac{4-4i}{\sqrt{2}}\right|$

$= \frac{\sqrt{16+16}}{\sqrt{2}}$

$= \frac{\sqrt{32}}{\sqrt{2}}.$

(ii) $|1-i|^{10} = \left(\sqrt{1+1}\right)^{10} = (\sqrt{2})^{10} = \left((\sqrt{2})^2\right)^5 = 2^5 = 32.$

(iv) $|2i(3-4i)(4-3i)| = |2i|\,|3-4i|\,|4-3i|$

$= \sqrt{4} \cdot \sqrt{9+16} \cdot \sqrt{16+9}$

$= \sqrt{4} \cdot \sqrt{25} \cdot \sqrt{25}$

$= 2\times5\times5$

$= 50.$

EXE 2.5

(2) Let z_1 and z_2 be two complex number.

$|z_1| = |z_2| = 1$, $z_1 z_2 \neq 0$. $\boxed{\bar{z} = z}$ purely real

$|z_1| = 1$ $|z_2| = 1$. $|z|^2 = z\bar{z}$

$|z_1|^2 = 1$ $|z_2|^2 = 1$

$z_1\bar{z}_1 = 1$ $z_2\bar{z}_2 = 1$.

$z_1 = \frac{1}{\bar{z}_1}$ $z_2 = \frac{1}{\bar{z}_2}$

$$\therefore \frac{z_1+z_2}{1+z_1z_2} = \frac{\frac{1}{\bar{z}_1} + \frac{1}{\bar{z}_2}}{1 + \frac{1}{\bar{z}_1}\frac{1}{\bar{z}_2}}$$

$$= \frac{\frac{\bar{z}_2 + \bar{z}_1}{\bar{z}_1\bar{z}_2}}{\frac{\bar{z}_1\bar{z}_2 + 1}{\bar{z}_1\bar{z}_2}}$$

$$= \frac{\bar{z}_2 + \bar{z}_1}{\bar{z}_1\bar{z}_2 + 1}$$

properties.

$$= \frac{\overline{z_1+z_2}}{1+\overline{z_1z_2}}$$

$\overline{z_1+z_2} = \bar{z}_1 + \bar{z}_2$

$\overline{z_1z_2} = \bar{z}_1\bar{z}_2$

$$\frac{z_1+z_2}{1+z_1z_2} = \overline{\left[\frac{z_1+z_2}{1+z_1z_2}\right]}$$

$\frac{z_1+z_2}{1+z_1z_2}$ is real number.

7. Let z_1, z_2 and z_3 are three complex number.

$|z_1| = 1$ $\quad |z_2| = 2$ $\quad |z_3| = 3$ $\quad |z_1+z_2+z_3| = 1$

$|z_1|^2 = 1$ $\quad |z_2|^2 = 4$ $\quad |z_3| = 9.$ $\quad \therefore |z|^2 = z\bar{z}$

$z_1\bar{z}_1 = 1$ $\quad z_2\bar{z}_2 = 4$ $\quad z_3\bar{z}_3 = 9$

$z_1 = \frac{1}{\bar{z}_1}$ $\quad z_2 = \frac{4}{\bar{z}_2}$ $\quad z_3 = \frac{9}{\bar{z}_3}$

$$|z_1+z_2+z_3| = \left|\frac{1}{\bar{z}_1} + \frac{4}{\bar{z}_2} + \frac{9}{\bar{z}_3}\right|$$

$$= \left|\frac{\bar{z}_2\bar{z}_3 + 4\bar{z}_1\bar{z}_3 + 9\bar{z}_1\bar{z}_2}{\bar{z}_1\bar{z}_2\bar{z}_3}\right|$$

$$= \left|\frac{\overline{z_2z_3 + 4z_1z_3 + 9z_1z_2}}{\overline{z_1z_2z_3}}\right| \quad \therefore |z| = |\bar{z}|$$

$$= \left|\frac{z_2z_3 + 4z_1z_3 + 9z_1z_2}{z_1z_2z_3}\right|$$

$$= \frac{|z_2z_3 + 4z_1z_3 + 9z_1z_2|}{|z_1||z_2||z_3|}$$

$$1 = \frac{|z_2z_3 + 4z_1z_3 + 9z_1z_2|}{(1)(2)(3)}$$

$$6 = |z_2z_3 + 4z_1z_3 + 9z_1z_2|.$$

4. $|z|=3$, $7 \le |z+6-8i| \le 13$.

$$|x+iy| - |z| \le |z+(x+iy)| \le |x+iy| + |z|$$

$$|6-8i| - |z| \le |z+(6-8i)| \le |6-8i| + |z|$$

$$\sqrt{36+64} - |z| \le |z+(6-8i)| \le \sqrt{36+64} + |z|$$

$$\sqrt{100} - |z| \le |z+(6-8i)| \le \sqrt{100} + |z|$$

$$10-3 \le |z+(6-8i)| \le 10+3$$

$$7 \le |z+(6-8i)| < 13.$$

5. $|z|=1$, $2 \le |z^2-3| \le 4$

$$|x+iy| - |z| \le |z+(x+iy)| \le |x+iy| + |z|$$

$$|-3| - |z| \le |z^2-3| \le |-3| + |z|$$

$$3-1 \le |z^2-3| \le 3+1$$

$$2 \le |z^2-3| \le 4.$$

3. Let $z = 1+i$ and $z_1 = 10-8i$, $z_2 = 11+6i$.

$|z-z_1| = |9-9i|$
$= \sqrt{81+81}$
$= \sqrt{2\times81}$
$= 9\sqrt{2}$.

$|z-z_2| = |10-5i|$
$= \sqrt{100+25}$
$= \sqrt{125}$
$= \sqrt{25\times5}$
$= 5\sqrt{5}$

$11+6i$ is closed to $1+i$.

9. $z^3 + 2\bar{z} = 0$.

$z^3 = -2\bar{z} \longrightarrow (1)$

$|z^3| = |-2\bar{z}|$

$|z^3| = 2|\bar{z}|$ $\quad \therefore |\bar{z}| = |z|$

$|z|^3 - 2|z| = 0$ $\quad |\overline{z^n}| = |\bar{z}|^n$.

$|z|\,[|z|^2 - 2] = 0$. $\quad |z| =$.

$|z| = 0$, $|z|^2 - 2 = 0$.

$|z|^2 = 2$.

$|z| = \pm\sqrt{2}$.

$z\bar{z} = \sqrt{2}$.

$\boxed{\bar{z} = \frac{\sqrt{2}}{z}}$

from the equation (1)

(1) $\Rightarrow$. $z^3 = -2\bar{z}$

$= -2\left(\frac{\sqrt{2}}{z}\right)$

$z^3 = \frac{-2\sqrt{2}}{z}$

$z^3 z = -2\sqrt{2}$.

$z^4 = -2\sqrt{2}$.

Square root of the complex number.

$$\sqrt{a+ib} = x+iy \longrightarrow (1)$$

$$a+ib = (x+iy)^2$$

$$a+ib = x^2-y^2+2ixy.$$

∴ Equating real and imaginary.

$$a = x^2-y^2, \quad b = 2xy.$$

$$(x^2+y^2)^2 = (x^2-y^2)^2 + 4x^2y^2.$$

$$= (x^2-y^2)^2 + (2xy)^2.$$

$$(x^2+y^2)^2 = a^2+b^2.$$

$$(x^2+y^2) = \sqrt{a^2+b^2}.$$

$$\therefore \quad x^2-y^2 = a \quad \text{and} \quad x^2+y^2 = \sqrt{a^2+b^2}.$$

from the equation.

$$x = \pm\sqrt{\frac{\sqrt{a^2+b^2}+a}{2}}, \quad y = \pm\sqrt{\frac{\sqrt{a^2+b^2}-a}{2}}.$$

$$\therefore (1) \Rightarrow$$

$$\sqrt{a+ib} = \pm\left[\sqrt{\frac{|z|+a}{2}} + \frac{b}{|b|} i \sqrt{\frac{|z|-a}{2}}\right]$$

∴ $\frac{b}{|b|} = -1$, x and y are different sign

$\frac{b}{|b|} = 1$, x and y are same sign.

10.

i) $|4+3i| = \sqrt{16+9} = \sqrt{25} = 5$

$$\sqrt{4+3i} = \pm\left[\sqrt{\frac{5+4}{2}} + i\sqrt{\frac{5-4}{2}}\right]$$

$$= \pm\left[\sqrt{\frac{9}{2}} + i\sqrt{\frac{1}{2}}\right]$$

$$= \pm \frac{1}{\sqrt{2}}\left[3+i\right]$$

(ii) $|-6+8i| = \sqrt{36+64} = \sqrt{100} = 10.$

$$\sqrt{-6+8i} = \pm\left[\sqrt{\frac{10-6}{2}} - i\sqrt{\frac{10+6}{2}}\right]$$

$$= \pm\left[\sqrt{\frac{4}{2}} - i\sqrt{\frac{16}{2}}\right]$$

$$= \pm\left[\sqrt{2} - i\sqrt{8}\right]$$

$$= \pm\sqrt{2}\left[1-i2\right]$$

(iii) $\sqrt{-5-12i}$

$|-5-12i| = \sqrt{25+144} = \sqrt{169} = 13.$

$$\sqrt{-5-12i} = \pm\left[\sqrt{\frac{13-5}{2}} - i\sqrt{\frac{13+5}{2}}\right]$$

$$= \pm\left[\sqrt{\frac{8}{2}} - i\sqrt{\frac{18}{2}}\right]$$

$$= \pm\left[\sqrt{4} - i\sqrt{9}\right]$$

$$= \pm\left[2-3i\right]$$

Example 2.17

$|6-8i| = \sqrt{36+64} = \sqrt{100} = 10$

$\sqrt{6-8i} = \pm\left(\sqrt{\frac{10+6}{2}} - i\sqrt{\frac{10-6}{2}}\right)$

$= \pm\left(\sqrt{\frac{16}{2}} - i\sqrt{\frac{4}{2}}\right)$

$= \pm\left(\sqrt{8} - i\sqrt{2}\right)$

$= \pm\left(2\sqrt{2} - i\sqrt{2}\right)$

$\sqrt{6-8i} = \pm\sqrt{2}(2-i)$.

8. z, iz and $z+iz$,

Let $z = x+iy \longrightarrow (1)$

$iz = i(x+iy) = -y+ix \longrightarrow (2)$

$z+iz = (x+iy)+(-y+ix)$

$= (x-y)+i(x+y) \longrightarrow (3)$

$|z-zi| = |(x+iy)-(-y+ix)|$

$= |(x+y)+i(y-x)|$

$= \sqrt{(x+y)^2+(y-x)^2}$

$|z-zi| = \sqrt{2(x^2+y^2)}$.

$|zi-(z+iz)| = |-z| = |z|$

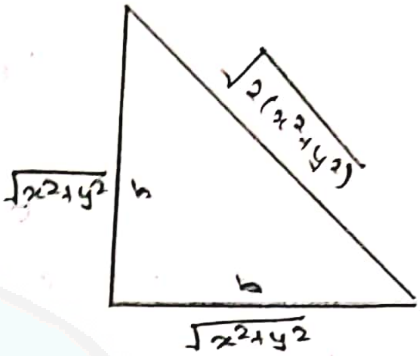
$= |x+iy|$

$= \sqrt{x^2+y^2}$

$$|(z+iz)-z| = |iz| = |y-ix|$$

$$= \sqrt{x^2+y^2}$$

Area of triangle $= 50$



$$\tfrac{1}{2} bh = 50$$

$$bh = 100$$

$$\sqrt{x^2+y^2}\cdot\sqrt{x^2+y^2} = 100$$

$$x^2+y^2 = 100$$

$$|z|^2 = 100$$

$$|z| = 10$$

Example 2.9

$$|z_1| = |3+4i| = \sqrt{9+16} = \sqrt{25} = 5$$

$$|z_2| = |5-12i| = \sqrt{25+144} = \sqrt{169} = 13$$

$$|z_3| = |6+8i| = \sqrt{36+64} = \sqrt{100} = 10$$

$$|z_1+z_2| = |8-8i| = \sqrt{64+64} = 8\sqrt{2}$$

$$|z_1-z_3) = |-1-20i| = \sqrt{1+400} = \sqrt{401}$$

$$|z_1+z_3| = |9+12i| = \sqrt{81+144} = \sqrt{225} = 15$$

Example 2.10

(i) $$\left|\frac{2+i}{-1+2i}\right| = \frac{|2+i|}{|-1+2i|} = \frac{\sqrt{5}}{\sqrt{5}} = 1.$$

(ii) $$|\overline{(1+i)}(2+3i)(4i-3)| = |1-i||2+3i||4i-3|$$

$$= \sqrt{2}\cdot\sqrt{13}\cdot\sqrt{25}$$

$$= 5\sqrt{26}.$$

$$\therefore \left|\frac{i(2+i)^3}{(1+i)^2}\right| = \frac{|i(2+i)|^3}{|1+i|^2} = \frac{|-1+2i|^3}{|1+i|^2} = \frac{(\sqrt{5})^3}{\sqrt{2}} = \frac{5\sqrt{5}}{\sqrt{2}}.$$

Example 2.11

The distance between origin i, $-2+i$ and 3,

$|i| = 1$

$|-2+i| = \sqrt{5}$

$|3| = 3.$

$\therefore$ $1 < \sqrt{5} < 3$, the farthest point from the origin 3.

Example 2.12.

$|z_1| = |z_2| = |z_3| = 1$,

$|z_1|^2 = 1$, $|z_2|^2 = 1$, $|z_3|^2 = 1$.

$z_1\bar{z}_1 = 1$ $\quad z_2\bar{z}_2 = 1$ $\quad z_3\bar{z}_3 = 1$

$z_1 = \frac{1}{\bar{z}_1}$ $\quad z_2 = \frac{1}{\bar{z}_2}$ $\quad z_3 = \frac{1}{\bar{z}_3}$

(or) (or) (or)

$\bar{z}_1 = \frac{1}{z_1}$ $\quad \bar{z}_2 = \frac{1}{z_2}$ $\quad \bar{z}_3 = \frac{1}{z_3}$

$$\left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_2}\right| = |\bar{z}_1 + \bar{z}_2 + \bar{z}_3|$$

$$= |\overline{z_1 + z_2 + z_3}|$$

$$= |z_1 + z_2 + z_3|$$

$$\left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right| = 1.$$

Example 2.13. $|z| = 2$,

$$|x+iy| - |z| \le |z+(x+iy)| \le |x+iy| + |z|$$

$$|3+4i| - |z| \le |z+3+4i| \le |3+4i| + |z|$$

$$\sqrt{9+16} - |z| \le |z+3+4i| \le \sqrt{9+16} + |z|$$

$$5 - |z| \le |z+3+4i| \le 5 + |z|$$

$$5 - 2 \le |z+3+4i| \le 5+2.$$

$$3 \le |z+3+4i| \le 7.$$

Example 2.14.

$z_1 = 1$, $z_2 = \frac{-1}{2} + i\frac{\sqrt{3}}{2}$ & $z_3 = \frac{-1}{2} - \frac{i\sqrt{3}}{2}$.

$$|z_1 - z_2| = \left|1 - \left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)\right| = \left|\frac{3}{2} - \frac{i\sqrt{3}}{2}\right| = \sqrt{\frac{9}{4} + \frac{3}{4}} = \sqrt{\frac{12}{4}} = \sqrt{3}.$$

$$|z_2 - z_3| = \left|\left(-\tfrac{1}{2} + \frac{i\sqrt{3}}{2}\right) - \left(-\tfrac{1}{2} - \frac{i\sqrt{3}}{2}\right)\right| = \sqrt{(\sqrt{3})^2} = \sqrt{3}.$$

$$|z_3 - z_1| = \left|\left(-\tfrac{1}{2} + \frac{i\sqrt{3}}{2}\right) - 1\right| = \sqrt{\frac{9}{4} + \frac{3}{4}} = \sqrt{\frac{12}{4}} = \sqrt{3}.$$

The sides are equal, The points form an equilateral triangle.

Example 2.15

$$|z_1| = |z_2| = |z_3| = r > 0,$$

$$|z_1| = r \qquad |z_2| = r \qquad |z_3| = r.$$

$$|z_1|^2 = r^2 \qquad |z_2|^2 = r^2 \qquad |z_3|^2 = r^2.$$

$$z_1\overline{z_1} = r^2 \qquad z_2\overline{z_2} = r^2 \qquad z_3\overline{z_3} = r^2.$$

$$z_1 = \frac{r^2}{\overline{z_1}} \qquad z_2 = \frac{r^2}{\overline{z_2}} \qquad z_3 = \frac{r^2}{\overline{z_3}}$$

$$z_1 + z_2 + z_3 = \frac{r^2}{\overline{z_1}} + \frac{r^2}{\overline{z_2}} + \frac{r^2}{\overline{z_3}}$$

$$z_1 + z_2 + z_3 = r^2\left(\frac{1}{\overline{z_1}} + \frac{1}{\overline{z_2}} + \frac{1}{\overline{z_3}}\right)$$

$$|z_1 + z_2 + z_3| = |r^2| \left|\frac{1}{\overline{z_1}} + \frac{1}{\overline{z_2}} + \frac{1}{\overline{z_3}}\right|$$

$$= r^2 \left| \frac{\overline{z_2}\,\overline{z_3} + \overline{z_1}\,\overline{z_3} + \overline{z_1}\,\overline{z_2}}{\overline{z_1}\,\overline{z_2}\,\overline{z_3}} \right|$$

$$= r^2 \left| \overline{\frac{z_2z_3 + z_1z_3 + z_1z_3}{z_1z_2z_3}} \right|$$

$$= r^2 \left| \frac{z_2z_3 + z_1z_3 + z_1z_3}{z_1z_2z_3} \right|$$

$$= \frac{r^2\,|z_1z_2 + z_2z_3 + z_1z_3|}{|z_1||z_2||z_3|}$$

$$= \frac{r^2\,|z_1z_2 + z_2z_3 + z_1z_3|}{r^3}.$$

$$|z_1 + z_2 + z_3| = \frac{|z_1z_2 + z_2z_3 + z_1z_3|}{r}$$

$$r = \frac{|z_1z_2 + z_2z_3 + z_1z_3|}{|z_1 + z_2 + z_3|}$$

Example 2.16.

$$z^2 = \bar{z} \longrightarrow (1)$$

$$|z|^2 = |\bar{z}|$$

$$|z|^2 = |z|$$

$$|z|^2 - |z| = 0.$$

$$|z|(|z| - 1) = 0$$

$$|z| = 0, \quad |z| - 1 = 0.$$

$$|z| = 1$$

$$|z|^2 = 1$$

$$z\bar{z} = 1$$

$$\boxed{\bar{z} = \frac{1}{z}}$$

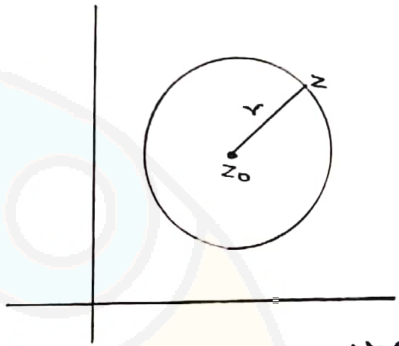
$$(1) \Rightarrow z^2 = \frac{1}{z}$$

$$z^2 \cdot z = 1$$

$$\boxed{z^3 = 1}$$

Circle:

A circle is defined as the locus of a point which moves in a plane such that its distance from a fixed point in that plane is always a constant. The fixed point is the centre and the constant distance is the radius of the circle.



(i) $|z-z_0| < r$, represented the point interior of the circle.

(ii) $|z-z_0| > r$, represented the point exterior of the circle.

EXE 2.6.

4. $|z-z_0| = r$. centre $= (x+iy)$

$|z-(x+iy)| = r$. radius $= r$.

i) $|z-2-i| = 3$.

$|z-(2+i)| = 3$.

centre : $(2+i)$ radius $= 3$.

ii) $|2z+2-4i| = 2$.

$2|z-(-1+2i)| = 2$.

$|z-(-1+2i)| = 1$

centre $= -1+2i$, radius $= 1$.

(iii) $|3z-6+12i| = 8$

$3|z-(2-4i)| = 8$

$|z-(2-4i)| = 8/3$

centre $= 2-4i$, radius $= 8/3$

Example 2.19

$|3z-5+i| = 4$

$3\left|z-\left(-5/3+\frac{i}{3}\right)\right| = 4$

$\left|z-\left(-5/3+\frac{i}{3}\right)\right| = 4/3.$

centre $= -\frac{5}{3}+\frac{i}{3}$, radius $= 4/3.$

Example 2.20

$|z+2-i| = 2.$

$|z-(-2+i)| = 2.$

centre $= -2+i$ radius $= 2.$

$|z+2-i| < 2$, represented all points inside the circle.

Example 2.21

(i) $|z| = |z-i|$ let $z = x+iy$

$|x+iy| = |x+iy-i|$

$\sqrt{x^2+y^2} = \sqrt{x^2+(y-1)^2}$

$x^2+y^2 = x^2+y^2-2y+1$

$2y-1=0$.

(ii) $|2z-3-i| = 3$.

$|2(x+iy)-3-i| = 3$

$|(2x-3)+i(2y-1)| = 3$

$(2x-3)^2+(2y-1)^2 = 9$.

$4x^2+4y^2-12x-4y+1=0$.

Exe 2.6

5.

i) $|z-4| = 16$

$|(x+iy)-4| = 16$

$(x-4)^2+y^2 = (16)^2$

$x^2-8x+16+y^2 = 256$

$x^2-8x+y^2-240 = 0$.

(ii) $|z-4|^2 - |z-1|^2 = 16$

$|x+iy-4|^2 - |x+iy-1|^2 = 16$

$\{(x-4)^2+y^2\} - \{(x-1)^2+y^2\} = 16$

$(x^2-8x+16+y^2) - (x^2-2x+1+y^2) = 16$

$-8x+2x-1 = 0$

$-6x-1 = 0$

$6x+1 = 0.$

①. $z = x+iy.$

$\left|\frac{z-4i}{z+4i}\right| = 1.$

$\frac{|z-4i|}{|z+4i|} = 1$

$|z-4i| = |z+4i|$

$|(x+iy)-4i| = |(x+iy)+4i|$

$|x+i(y-4)| = |x+i(y+4)|$

$x^2+(y-4)^2 = x^2+(y+4)^2$

$x^2+y^2-8y+16 = x^2+y^2+8y+16$

$16y = 0$

$\boxed{y=0}$

$z = x+iy$

$z = x+i(0)$

$z = x$ is a real axis.

2. $z = x+iy$.

$$\operatorname{Im}\left[\frac{2z+1}{iz+1}\right] = 0.$$

$$\operatorname{Im}\left[\frac{2(x+iy)+1}{i(x+iy)+1}\right] = 0$$

$$\operatorname{Im}\left[\frac{(2x+1)+2iy}{(1-y)+ix}\right] = 0$$

Long

s

$$\operatorname{Im}\left[\frac{(2x+1)+2iy}{(1-y)+ix} \times \frac{(1-y)-ix}{(1-y)-ix}\right] = 0.$$

$$\frac{2y(1-y) - x(2x+1)}{(1-y)^2 + x^2} = 0.$$

$$2y(1-y) - x(2x+1) = 0.$$

$$2y - 2y^2 - 2x^2 - x = 0.$$

$$2x^2 + 2y^2 + x - 2y = 0.$$

Imaginary part

$$\frac{\text{short} - \text{long}}{(Re)^2 + (Im)^2} \begin{array}{l} \to \text{Num} \\ \to \text{Denmonitator.} \end{array}$$

Real part

$$\frac{ReRe + ImIm}{(Re)^2 + (Im)^2.} \begin{array}{l} \to \text{Num} \\ \to \text{Denmominator.} \end{array}$$

argument

only
Numerator $\quad \dfrac{\text{short} - \text{long}}{ReRe + ImIm.}$

3. (i) $[Re(iz)]^2 = 3$. Let $z = x+iy$.

$[Re\{i(x+iy)\}]^2 = 3$.

$[Re(ix-y)]^2 = 3$.

$(-y)^2 = 3$

$y^2 = 3$.

(ii)

$[Im[(1-i)z+1]] = 0$ Let $z = x+iy$.

$[Im[(1-i)(x+iy)+1]] = 0$

$Im[(x+y)+i(y-x)+1] = 0$

$Im[(x+y+1)+i(y-x)] = 0$

$y-x = 0$

$-x \Rightarrow$. $x-y = 0$.

(iii)

$|z+i| = |z-1|$ Let $z = x+iy$.

$|(x+iy)+i| = |(x+iy)-1|$

$|x+i(y+1)| = |(x-1)+iy|$.

$x^2+(y+1)^2 = (x-1)^2+y^2$.

$\cancel{x^2}+\cancel{y^2}+2y+\cancel{1} = \cancel{x^2}-2x+\cancel{1}+\cancel{y^2}$

$2x+2y = 0$

$\div 2 \Rightarrow$ $x+y = 0$.

(iv) $\bar{z} = z^{-1}$

let $z = x+iy$ $\quad \bar{z} = x-iy$.

$\bar{z} = \frac{1}{z}$

$(x-iy) = \frac{1}{(x+iy)}$

$(x-iy)(x+iy) = 1.$

$x^2-(iy)^2 = 1.$

$x^2+y^2 = 1.$

polar form of a complex number.

Any non zero complex number $z = x+iy$ can be expressed as $z = r\cos\theta + i r\sin\theta$ (or)

$z = r(\cos\theta + i\sin\theta).$

let r and θ be polar co-ordinates of the point $P(x,y)$ that corresponds to the non zero complex number $z = x+iy$. The polar form or trigonometric form of complex number P is.

$z = r(\cos\theta + i\sin\theta).$

properties of argument.

i) $\arg(z_1 z_2) = \arg z_1 + \arg z_2.$

(ii) $\arg\left(\frac{z_1}{z_2}\right) = \arg z_1 - \arg z_2.$

(iii) $\arg(z^n) = n\arg z.$

(iv) alternative form $\cos\theta + i\sin\theta$ are $\cos(2k\pi+\theta) + i\sin(2k\pi+\theta)$