Padasalai's Telegram Groups!

(தலைப்பிற்கு கீழே உள்ள லிங்கை கிளிக் செய்து குழுவில் இணையவும்!)

- Padasalai's NEWS - Group

https://t.me/joinchat/NIfCqVRBNj9hhV4wu6_NqA

- Padasalai's Channel - Group

https://t.me/padasalaichannel

- Lesson Plan - Group

https://t.me/joinchat/NIfCqVWwo5iL-21gpzrXLw

- 12th Standard - Group

https://t.me/Padasalai_12th

- 11th Standard - Group

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- 10th Standard - Group

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- 9th Standard - Group

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- 6th to 8th Standard - Group

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- 1st to 5th Standard - Group

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- TET - Group

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- PGTRB - Group

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- TNPSC - Group

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EXE 2.8

$\omega^3 = 1$

$\omega^{3n} = 1$

1. $$\frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2} + \frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2} = -1.$$

$$\frac{a\omega^3+b\omega+c\omega^2}{b+c\omega+a\omega^2} + \frac{a+b\omega+c\omega^2}{c\omega^3+a\omega+b\omega^2} = -1$$

$$\frac{\omega(a\omega^2+b+c\omega)}{b+c\omega+a\omega^2} + \frac{a+b\omega+c\omega^2}{\omega(\omega^2c+a+b\omega)} = -1$$

$$\omega + \frac{1}{\omega} = -1$$

$$\omega + \frac{\omega^3}{\omega} = -1$$

$$\omega + \omega^2 = -1$$

$$1+\omega+\omega^2 = 0.$$

2. $$\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^5 = -\sqrt{3}.$$

Let $x+iy = \frac{\sqrt{3}}{2}+\frac{i}{2}.$

modulus $r = \sqrt{x^2+y^2}$

$= \sqrt{\frac{3}{4}+\frac{1}{4}}$

$= \sqrt{\frac{4}{4}}$

$\boxed{r=1}$

principal argument.

$\therefore \theta$ lies on I quadrant.

$\theta = \tan^{-1}|y/x|$

$= \tan^{-1}\left|\frac{1/2}{\sqrt{3}/2}\right|$

$= \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$\boxed{\theta = \pi/6}$

polar form.

$x+iy = r(\cos\theta + i\sin\theta)$

$\frac{\sqrt{3}}{2}+\frac{i}{2} = 1\left[\cos\pi/6 + i\sin\pi/6\right]$

$\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5 = \left[\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right]^5$ $\qquad (\cos\theta+i\sin\theta)^n = \cos n\theta + i\sin n\theta$

$\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5 = \cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}.$

i replace $-i$.

$\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^5 = \cos\frac{5\pi}{6} - i\sin\frac{5\pi}{6}$

$$\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5+\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^5 = 2\cos\frac{5\pi}{6}$$

$$= 2\left[\cos\left(\pi-\frac{\pi}{6}\right)\right]$$

$$= -2\cos\pi/6$$

$$= -2\left(\frac{\sqrt{3}}{2}\right)$$

$$= -\sqrt{3}.$$

(3)

Let $z = \sin\frac{\pi}{10}+i\cos\frac{\pi}{10}$

$\frac{1}{z} = \sin\frac{\pi}{10} - i\cos\frac{\pi}{10}$

$$\left[\frac{1+\sin\frac{\pi}{10}+i\cos\frac{\pi}{10}}{1+\sin\frac{\pi}{10}-i\cos\frac{\pi}{10}}\right]^{10} = \left[\frac{1+z}{1+\frac{1}{z}}\right]^{10} = z^{10}$$

$$= \left[\sin\frac{\pi}{10}+i\cos\frac{\pi}{10}\right]^{10}$$

$\sin n\pi = 0$

$\cos n\pi = (-1)^n$

$$= \left[\cos\left(\pi/2-\frac{\pi}{10}\right)+i\sin\left(\pi/2-\frac{\pi}{10}\right)\right]^{10}$$

$$= \left[\cos\left[\frac{4\pi}{10}\right]+i\sin\left(\frac{4\pi}{10}\right)\right]^{10}$$

$$= \cos 4\pi + i\sin 4\pi$$

$$= (-1)^4 + 0$$

$$= 1$$

4. $x + \frac{1}{x} = 2\cos\alpha$

$x^2 + 1 = 2x\cos\alpha.$

$x^2 - 2x\cos\alpha + 1 = 0$

$x^2 - 2x\cos\alpha + \cos^2\alpha + \sin^2\alpha = 0$

$(x - \cos\alpha)^2 + \sin^2\alpha = 0$

$(x - \cos\alpha)^2 = -\sin^2\alpha$

$x - \cos\alpha = \pm\sqrt{-\sin^2\alpha}$

$x - \cos\alpha = \pm i\sin\alpha$

$x = \cos\alpha \pm i\sin\alpha.$ $\therefore x = \cos\alpha + i\sin\alpha$

lly. $y + \frac{1}{y} = 2\cos\beta$

$y = \cos\beta \pm i\sin\beta.$ $\therefore y = \cos\beta + i\sin\beta.$

i) $\frac{x}{y} = \frac{\cos\alpha + i\sin\alpha}{\cos\beta + i\sin\beta} = \cos(\alpha-\beta) + i\sin(\alpha-\beta)$

$\frac{y}{x} = \cos(\alpha-\beta) - i\sin(\alpha-\beta)$

$\therefore \frac{x}{y} + \frac{y}{x} = 2\cos(\alpha-\beta)$

(ii) $xy = (\cos\alpha + i\sin\alpha)(\cos\beta + i\sin\beta) = \cos(\alpha+\beta) + i\sin(\alpha+\beta)$

$\frac{1}{xy} = \cos(\alpha+\beta) - i\sin(\alpha+\beta)$

$xy - \frac{1}{xy} = 2i\sin(\alpha+\beta)$

(iii) $$\frac{x^m}{y^n} = \frac{(\cos\alpha + i\sin\alpha)^m}{(\cos\beta + i\sin\beta)^n} = \frac{\cos m\alpha + i\sin m\alpha}{\cos n\beta + i\sin n\beta}$$

$$\frac{x^m}{y^n} = \cos(m\alpha - n\beta) + i\sin(m\alpha - n\beta)$$

$$\frac{y^n}{x^m} = \cos(m\alpha - n\beta) - i\sin(m\alpha - n\beta)$$

$$\therefore \frac{x^m}{y^n} - \frac{y^n}{x^m} = 2i\sin(m\alpha - n\beta)$$

(iv) $$x^m y^n = (\cos\alpha + i\sin\alpha)^m (\cos\beta + i\sin\beta)^n$$

$$= (\cos m\alpha + i\sin m\alpha)(\cos n\beta + i\sin n\beta)$$

$$x^m y^n = \cos(m\alpha + n\beta) + i\sin(m\alpha + n\beta)$$

$$\frac{1}{x^m y^n} = \cos(m\alpha + n\beta) - i\sin(m\alpha + n\beta)$$

$$x^m y^n + \frac{1}{x^m y^n} = 2\cos(m\alpha + n\beta).$$

5) $z^3 + 27 = 0$

$$z^3 = -27$$

$$z = (-27)^{1/3}$$

$$= (-1 \times 27)^{1/3}$$

$$= (-1)^{1/3} (27)^{1/3}$$

$$= 3(-1)^{1/3}$$

$$= 3\left[\cos\pi + i\sin\pi\right]^{1/3}$$

$$= 3\left[\cos(2\pi k + \pi) + i\sin(2\pi k + \pi)\right]^{1/3}$$

$$= 3\left[\cos(2k+1)\pi + i\sin(2k+1)\pi\right]^{1/3}$$

$$z = 3\left[\cos(2k+1)\pi/3 + i\sin(2k+1)\pi/3\right]$$

$1 = \cos 0 + i\sin 0$

$-1 = \cos\pi + i\sin\pi$

$i = \cos\pi/2 + i\sin\pi/2$

$-i = \cos\pi/2 - i\sin\pi/2$.

where $k = 0, 1, 2.$

when $k=0$, $z = 3(\cos\pi/3 + i\sin\pi/3)$

$k=1$, $z = 3\left[\cos\frac{3\pi}{3} + i\sin\frac{3\pi}{3}\right]$

$k=2$, $z = 3\left[\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right]$

6.

$$(z-1)^3 = -8$$

$$z-1 = (-8)^{1/3}$$

$$z-1 = (-1)^{1/3}(8)^{1/3}$$

$$z-1 = -2(1)^{1/3}.$$

$$z = 1-2(1)^{1/3}$$

where $1, \omega, \omega^2$.

when : 1

$z = 1-2(1)^{1/3}$

$= 1-2$

$z = -1$

when : ω

$z = 1-2(\omega)^{1/3}$

$z = 1-2\omega$

when : ω^2

$z = 1-2(\omega^2)^{1/3}$

$z = 1-2\omega^2$

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$$\sum_{k=1}^{8}\left(\cos\frac{2k\pi}{9}+i\sin\frac{2k\pi}{9}\right) = \omega+\omega^2+\omega^3+\omega^4+\omega^5+\omega^6+\omega^7+\omega^8$$

$= (\omega+\omega^2+1)+\omega^3(\omega+\omega^2+1)+\omega^6(\omega+\omega^2)$

$k=0$, $\text{cis}\frac{2\pi k}{\omega} = \omega^0$

$k=1$, $\text{cis}\frac{2\pi k}{\omega} = \omega^1$

$= 0+\omega^3(0)+\omega^6(-1)$

$= (1)(-1)$

$= -1$

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8. $\omega \neq 1$

(i) $(1-\omega+\omega^2)^6 + (1+\omega-\omega^2)^6 = (-\omega-\omega)^6 + (-\omega^2-\omega^2)^6$

$1+\omega+\omega^2=0$

$= (-2\omega)^6 + (-2\omega^2)^6$

$= 64\omega^6 + 64\omega^{12}$

$\omega^{3n}=1$

$\omega^3=1$

$= 64+64$

$= 128$

(ii) $(1+\omega)(1+\omega^2)(1+\omega^4)\dots(1+\omega^{2^{11}}) = (1+\omega^{2^0})(1+\omega^{2^1}) + (1+\omega^{2^2})$

$\dots(1+\omega^{2^{11}})$

$1+\omega+\omega^2=0.$

$= (-\omega^2)(-\omega)(-\omega^2)(-\omega)$

$(-\omega^2)(-\omega)(-\omega^2)(-\omega)$

$(-\omega^2)(-\omega)(-\omega^2)(\omega)$

$= (\omega^3)(\omega^3)(\omega^3)(\omega^3)(\omega^3)(\omega^3)$

$= 1$

9. $z = 2-2i.$

modulus $r=\sqrt{x^2+y^2}$

$r=\sqrt{4+4}$

$r=2\sqrt{2}$

principal argument.

θ lies on fourth quadrant.

$\theta = -\tan^{-1}\left|\frac{y}{x}\right|$

$= -\tan^{-1}\left|\frac{-2}{2}\right|$

$= -\tan^{-1}(1)$

$\theta = -\pi/4$

polar form.

$x+iy = r(\cos\theta + i\sin\theta)$

$2-2i = 2\sqrt{2}\left[\cos(-\pi/4) + i\sin(-\pi/4)\right]$

(i) $\theta = \pi/3$.

$$z = 2\sqrt{2}\left[\cos(-\pi/4) + i\sin(-\pi/4)\right]\left[\cos\pi/3 + i\sin\pi/3\right]$$

$$= 2\sqrt{2}\left[\cos(-\pi/4 + \pi/3) + i\sin(-\pi/4 + \pi/3)\right]$$

$$= 2\sqrt{2}\left[\cos\pi/12 + i\sin\pi/12\right]$$

(ii) $\theta = \frac{2\pi}{3}$

$$z = 2\sqrt{2}\left[\cos(-\pi/4) + i\sin(-\pi/4)\right]\left[\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right]$$

$$= 2\sqrt{2}\left[\cos\left(-\pi/4 + \frac{2\pi}{3}\right) + i\sin\left(-\pi/4 + \frac{2\pi}{3}\right)\right]$$

$$= 2\sqrt{2}\left[\cos\left(\frac{5\pi}{12}\right) + i\sin\left(\frac{5\pi}{12}\right)\right]$$

(iii) $\theta = \frac{3\pi}{2}$.

$$z = 2\sqrt{2}\left[\cos(-\pi/4) + i\sin(-\pi/4)\right]\left[\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right]$$

$$= 2\sqrt{2}\left[\cos\left(-\pi/4 + \frac{3\pi}{2}\right) + i\sin\left(-\pi/4 + \frac{3\pi}{2}\right)\right]$$

$$= 2\sqrt{2}\left[\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right]$$

10.

Let $x = \sqrt[4]{-1}$

$= (-1)^{1/4}$

$\cos 0 + i\sin 0 = 1$

$\cos\pi + i\sin\pi = -1$

$\cos\pi/2 + i\sin\pi/2 = i$

$\cos\pi/2 - i\sin\pi/2 = -i.$

$= [\cos\pi + i\sin\pi]^{1/4}$

$= [\cos(2\pi k + \pi) + i\sin(2\pi k + \pi)]^{1/4}$

$x. = [\cos(2k+1)\pi/4 + i\sin(2k+1)\pi/4]$

where $k = 0, 1, 2, 3.$

$x = \cos\pi/4 + i\sin\pi/4 = \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}.$

$x = \cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4} = \cos(\pi - \pi/4) + i\sin(\pi - \pi/4)$

$= -\cos\pi/4 + i\sin\pi/4$

$= -\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}.$

$x = \cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4} = \cos(\pi + \pi/4) + i\sin(\pi + \pi/4)$

$= -\cos\pi/4 - i\sin\pi/4$

$= -\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}.$

$x = \cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4} = \cos(2\pi - \pi/4) + i\sin(2\pi - \pi/4)$

$= \cos\pi/4 - i\sin\pi/4$

$= \frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}$

$x = \frac{1}{\sqrt{2}}(\pm 1 \pm i)$

$x = \pm 1/\sqrt{2}(1 \pm i)$