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- Padalsalai's NEWS - Group
https://t.me/joinchat/NIfCqVRBNj9hhV4wu6_NqA
- Padalsalai's Channel - Group
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- Lesson Plan - Group
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- 9th Standard - Group
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CLASS: XI

UNIT - VII

①

EXERCISE - 7.1MATRICES AND
DETERMINANTS

$$(i) a_{ij} = \frac{(i-2j)^2}{2} \text{ when } m=2, n=3$$

solution:General form of 2×3 matrix

$$\text{is given by } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

$$a_{11} = \frac{(1-2)^2}{2} = \frac{(-1)^2}{2} = \frac{1}{2} ; a_{21} = \frac{(2-2(1))^2}{2} = \frac{(2-2)^2}{2} = 0$$

$$a_{12} = \frac{(1-2(2))^2}{2} = \frac{(1-4)^2}{2} = \frac{(-3)^2}{2} = \frac{9}{2} ; a_{22} = \frac{(2-2(2))^2}{2} = \frac{(2-4)^2}{2} = \frac{(-2)^2}{2} = \frac{4}{2}$$

$$a_{13} = \frac{(1-2(3))^2}{2} = \frac{(1-6)^2}{2} = \frac{(-5)^2}{2} = \frac{25}{2} ; a_{23} = \frac{(2-2(3))^2}{2} = \frac{(2-6)^2}{2} = \frac{(-4)^2}{2} = \frac{16}{2}$$

$$\therefore A = \begin{bmatrix} \frac{1}{2} & \frac{9}{2} & \frac{25}{2} \\ 0 & \frac{4}{2} & \frac{16}{2} \end{bmatrix}$$

$$A = \frac{1}{2} \begin{bmatrix} 1 & 9 & 25 \\ 0 & 4 & 16 \end{bmatrix}$$

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$$(ii) a_{ij} = \frac{|3i-4j|}{4} \text{ , } m=3, n=4$$

General form of 3×4 matrix is given
by

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$$

$$a_{11} = \frac{|3(1)-4(1)|}{4} = \frac{|-1|}{4} = \frac{1}{4}$$

$$a_{21} = \frac{|3(2)-4(1)|}{4} = \frac{|6-4|}{4} = \frac{2}{4}$$

$$a_{12} = \frac{|3(1)-4(2)|}{4} = \frac{|-5|}{4} = \frac{5}{4}$$

$$a_{22} = \frac{|3(2)-4(2)|}{4} = \frac{2}{4}$$

$$a_{13} = \frac{|3(1)-4(3)|}{4} = \frac{|-9|}{4} = \frac{9}{4}$$

$$a_{23} = \frac{|3(2)-4(3)|}{4} = \frac{6}{4}$$

$$a_{31} = \frac{|3(3)-4(1)|}{4} = \frac{5}{4}$$

$$a_{32} = \frac{|3(3)-4(2)|}{4} = \frac{1}{4}$$

$$a_{33} = \frac{|3(3)-4(3)|}{4} = \frac{|-3|}{4} = \frac{3}{4}$$

$$a_{34} = \frac{|3(3)-4(4)|}{4} = \frac{|-7|}{4} = \frac{7}{4}$$

$$\therefore A = \begin{pmatrix} \frac{1}{4} & \frac{5}{4} & \frac{9}{4} & \frac{13}{4} \\ \frac{2}{4} & \frac{2}{4} & \frac{6}{4} & \frac{10}{4} \\ \frac{5}{4} & \frac{1}{4} & \frac{3}{4} & \frac{7}{4} \end{pmatrix} = \frac{1}{4} \begin{pmatrix} 1 & 5 & 9 & 13 \\ 2 & 2 & 6 & 10 \\ 5 & 1 & 3 & 7 \end{pmatrix}$$

2) Given

$$\begin{bmatrix} p^2-1 & 0 & -31-q^3 \\ 7 & 2+1 & 9 \\ -2 & 8 & 5-1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -4 \\ 7 & 3/2 & 9 \\ -2 & 8 & -\pi \end{bmatrix}$$

As the orders of the two matrices are same, they are equal iff the corresponding entries are equal. Thus comparing the corresponding elements,

$$\begin{array}{l|l|l|l} \text{we get,} & -31-q^3 = -4 & 2+1 = 3/2 & 5-1 = -\pi \\ p^2-1 = 1 & -31+1 = q^3 & 2 = 3/2 - 1 & 5 = -\pi + 1 \\ p^2 = 1+1 & q^3 = -27 & \boxed{2 = 1/2} & \boxed{5 = 1-\pi} \\ p^2 = 2 & \boxed{q = -3} & & \\ \boxed{p = \pm\sqrt{2}} & & & \end{array}$$

3)

(2)

Given

$$\begin{bmatrix} 2x+y & 4x \\ 5x-7 & 4x \end{bmatrix} = \begin{bmatrix} 7 & 7y-13 \\ y & x+6 \end{bmatrix}$$

Soln

The orders of the two matrices are same, Thus by comparing the corresponding elements

$$\begin{array}{l|l} \text{we get,} & 2x+y = 7 \\ 4x = x+6 & 2(2)+y = 7 \\ 4x-x = 6 & 4+y = 7 \\ 3x = 6 & \boxed{y = 3} \\ \boxed{x = 2} & \end{array}$$

$$x+y = 2+3$$

$$\boxed{x+y = 5}$$

4)

$$2A - B + \begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix} = 0$$

$$A - 2B = \begin{bmatrix} 3 & 2 & 8 \\ -2 & 1 & -7 \end{bmatrix}$$

Soln

$$2A - B = \begin{bmatrix} -6 & 6 & 0 \\ 4 & -2 & -1 \end{bmatrix} \quad \text{--- (1)}$$

$$A - 2B = \begin{bmatrix} 3 & 2 & 8 \\ -2 & 1 & -7 \end{bmatrix} \quad \text{--- (2)}$$

$$\textcircled{1} \Rightarrow 2A - B = \begin{bmatrix} -6 & 6 & 0 \\ 4 & -2 & -1 \end{bmatrix}$$

$$\textcircled{2} \times 2 \Rightarrow 2A - 4B = \begin{bmatrix} +6 & 4 & 16 \\ 4 & 2 & 14 \end{bmatrix}$$

$$\text{--- (1) --- (2)}$$

$$3B = \begin{bmatrix} -12 & 2 & -16 \\ 8 & -4 & 13 \end{bmatrix}$$

$$B = \frac{1}{3} \begin{bmatrix} -12 & 2 & -16 \\ 8 & -4 & 13 \end{bmatrix} \text{ in } \textcircled{1}$$

We get

$$A = \frac{1}{3} \begin{bmatrix} -15 & 10 & -8 \\ 10 & -5 & 5 \end{bmatrix}$$

5.

$$A = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$$

$$\begin{aligned} A^2 &= \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1+0 & a+a \\ 0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 1 & 2a \\ 0 & 1 \end{bmatrix} \end{aligned}$$

$$A^3 = A^2 \times A$$

$$\begin{aligned} &= \begin{bmatrix} 1 & 2a \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1+0 & a+2a \\ 0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 1 & 3a \\ 0 & 1 \end{bmatrix} \end{aligned}$$

$$A^4 = A^3 \times A$$

$$\begin{aligned} &= \begin{bmatrix} 1 & 3a \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1+0 & a+3a \\ 0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 1 & 4a \\ 0 & 1 \end{bmatrix} \end{aligned}$$

$$\boxed{A^4 = \begin{bmatrix} 1 & 4a \\ 0 & 1 \end{bmatrix}}$$

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(b) Given $A_\alpha = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$

(3)

Similarly $A_\beta = \begin{bmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{bmatrix}$

$$A_\alpha \cdot A_\beta = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{bmatrix}$$

$$= \begin{bmatrix} \cos \alpha \cos \beta - \sin \alpha \sin \beta & -\cos \alpha \sin \beta - \sin \alpha \cos \beta \\ \sin \alpha \cos \beta + \cos \alpha \sin \beta & -\sin \alpha \sin \beta + \cos \alpha \cos \beta \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\alpha + \beta) & -\sin(\alpha + \beta) \\ \sin(\alpha + \beta) & \cos(\alpha + \beta) \end{bmatrix} = A_{\alpha + \beta}$$

$$\therefore \boxed{A_\alpha \cdot A_\beta = A_{(\alpha + \beta)}}$$

(ii) $A_\alpha^T = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$

$$A_\alpha + A_\alpha^T = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} + \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$$

$$A_\alpha + A_\alpha^T = \begin{bmatrix} 2\cos \alpha & 0 \\ 0 & 2\cos \alpha \end{bmatrix}$$

Given $A_\alpha + A_\alpha^T = I$

$$\begin{bmatrix} 2\cos \alpha & 0 \\ 0 & 2\cos \alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$2\cos \alpha = 1$$

$$\cos \alpha = \frac{1}{2} = \cos \frac{\pi}{3}$$

$$\begin{bmatrix} \cos 0 = \cos \alpha \\ \theta = 2n\pi + \alpha, \forall n \in \mathbb{Z} \end{bmatrix}$$

$$\theta = 2n\pi + \alpha, \forall n \in \mathbb{Z}$$

$$\boxed{\alpha = 2n\pi \pm \frac{\pi}{3} \quad \forall n \in \mathbb{Z}}$$

(7) $A = \begin{pmatrix} 4 & 2 \\ -1 & x \end{pmatrix} \quad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$$A - 2I = \begin{pmatrix} 4 & 2 \\ -1 & x \end{pmatrix} - 2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 4-2 & 2+0 \\ -1+0 & x-2 \end{pmatrix} = \begin{pmatrix} 2 & 2 \\ -1 & x-2 \end{pmatrix}$$

$$A - 3I = \begin{pmatrix} 4 & 2 \\ -1 & x \end{pmatrix} - 3 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 4-3 & 2+0 \\ -1+0 & x-3 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ -1 & x-3 \end{pmatrix}$$

Given $(A - 2I)(A - 3I) = \begin{pmatrix} 2 & 2 \\ -1 & x-2 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ -1 & x-3 \end{pmatrix} = \begin{pmatrix} 2-2 & 4+2x-6 \\ -1-x+2 & -2+(x-2) \end{pmatrix}$

(4)

$$-1 - x + 2 = 0$$

$$-1 + 2 - x = 0$$

$$\boxed{x = 1}$$

$$(8) A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}$$

$$A^2 = A \times A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0+0 & 0+0+0 & 0+0+0 \\ 0+0+0 & 0+1+0 & 0+0+0 \\ a+0-a & 0+b-b & 0+0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

$\therefore A^2$ is a unit matrix

$$(9) A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

$$A^2 = A \times A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0+4 & 0+2+0 & 2+0+6 \\ 0+0+2 & 0+4+0 & 2+2+3 \\ 2+0+6 & 0+0+0 & 4+0+9 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 5 & 2 & 8 \\ 2 & 4 & 7 \\ 8 & 0 & 13 \end{bmatrix}$$

$$A^3 = A^2 \times A = \begin{bmatrix} 5 & 2 & 8 \\ 2 & 4 & 7 \\ 8 & 0 & 13 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5+0+16 & 0+4+0 & 10+2+24 \\ 2+0+14 & 0+8+0 & 4+2+21 \\ 8+0+26 & 0+0+0 & 16+0+39 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 4 & 48 \\ 16 & 8 & 30 \\ 34 & 0 & 55 \end{bmatrix}$$

$$\begin{bmatrix} 21 & 4 & 48 \\ 16 & 8 & 30 \\ 34 & 0 & 55 \end{bmatrix} - 6 \begin{bmatrix} 5 & 2 & 8 \\ 2 & 4 & 7 \\ 8 & 0 & 13 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} + k \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} 21 & 4 & 48 \\ 16 & 8 & 30 \\ 34 & 0 & 55 \end{bmatrix} + \begin{bmatrix} -30 & -12 & -48 \\ -12 & -24 & -42 \\ 48 & 0 & -78 \end{bmatrix}$$

$$+ \begin{bmatrix} 7 & 0 & 14 \\ 0 & 14 & 7 \\ 14 & 0 & 21 \end{bmatrix} + \begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix} = 0$$

$$21 - 30 + 7 + k = 0$$

$$-2 + k = 0$$

$$\boxed{k = 2}$$

(10)

(i) for our convenience

$$\text{Let } A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0 & -1+4 \\ 3+0 & -3+8 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 3 & 5 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1-3 & 2-4 \\ 0+6 & 0+8 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -2 \\ 6 & 8 \end{bmatrix}$$

$$AB \neq BA$$

(ii)

$$\text{Let } A = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \neq 0$$

$$B = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \neq 0$$

$$AB = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \\ = \begin{pmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{pmatrix} = 0$$

$$BA = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \\ = \begin{pmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{pmatrix} \\ = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = 0$$

Hence $AB=0=BA$

$$\text{(iii)} \quad A = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$AB = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \\ = \begin{pmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{pmatrix} = 0$$

$$BA = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \\ = \begin{pmatrix} 0+0 & 0+1 \\ 0+0 & 0+0 \end{pmatrix} \\ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \neq 0$$

Hence $AB=0$ $BA \neq 0$

$$\text{(11)} \quad \text{Given } f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{Similarly } f(y) = \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$f(x) \cdot f(y) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos x \cos y - \sin x \sin y + 0 & \cos x \sin y - \sin x \cos y + 0 & 0+0+0 \\ \sin x \cos y + \cos x \sin y + 0 & -\sin x \sin y + \cos x \cos y + 0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0+1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= f(x+y)$$

$$\therefore \boxed{f(x) + f(y) = f(x+y)}$$

(12) Given A is singular matrix

$$A^2 = A \Rightarrow A^3 = A^2 \times A = A \times A \\ = A^2 = A$$

$$\boxed{A^3 = A}$$

$$7A - (I+A)^3$$

$$= 7A - [I^3 + A^3 + 3I^2A + 3IA^2]$$

$$= 7A - [I - A^3 - 3A^2 - 3A]$$

$$= 7A - I - A - 3A - 3A = -I$$

(13)

$$\text{Given } A = \begin{bmatrix} 2 & 0 & -3 \\ 1 & 4 & 5 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 1 \\ -1 & 0 \\ 1 & -1 \end{bmatrix}$$

$$C = \begin{bmatrix} 4 & 7 \\ 2 & 1 \\ 1 & -1 \end{bmatrix}$$

$$\underline{\text{LHS}} \quad [A(B+C)]$$

$$B+C = \begin{bmatrix} 3 & 1 \\ -1 & 0 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} 4 & 7 \\ 2 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 7 & 8 \\ 1 & 1 \\ 5 & 1 \end{bmatrix}$$

$$A(B+C) = \begin{bmatrix} -1 & 13 \\ 36 & 17 \end{bmatrix} \quad \text{--- (1)}$$

RHS

$$AB = \begin{pmatrix} 2 & 0 & -3 \\ 1 & 4 & 5 \end{pmatrix} \begin{pmatrix} 3 & 1 \\ -1 & 0 \\ 1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 6+0-12 & 2+0-6 \\ 3-4+20 & 1+0+10 \end{pmatrix}$$

(6)

$$= \begin{pmatrix} -6 & -4 \\ 19 & 11 \end{pmatrix}$$

$$AC = \begin{pmatrix} 2 & 0 & -3 \\ 1 & 4 & 5 \end{pmatrix} \begin{pmatrix} 4 & 7 \\ 2 & 1 \\ 1 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 8+0-3 & 14+0-3 \\ 4+8+5 & 7+4-5 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & 17 \\ 17 & 6 \end{pmatrix}$$

$$AB+AC = \begin{pmatrix} -6 & -4 \\ 19 & 11 \end{pmatrix} + \begin{pmatrix} 5 & 17 \\ 17 & 6 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 13 \\ 36 & 17 \end{pmatrix} \text{ --- (2)}$$

From (1) & (2) $A(B+C) = AB+AC$.

$$(14) \quad A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2 \times 3} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}_{2 \times 3}$$

$$\text{Take } \begin{matrix} 2 \times 2 & 2 \times 3 \\ 2 \times 3 & \end{matrix} \quad \begin{matrix} n=2 \\ m=2 \end{matrix}$$

A is order of 2×2

$$\text{Let us take } A = \begin{pmatrix} p & q \\ r & s \end{pmatrix}$$

$$\begin{pmatrix} p & q \\ r & s \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} = \begin{pmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{pmatrix}$$

$$\begin{bmatrix} p+4q & 2p+5q & 3p+6q \\ r+4s & 2r+5s & 3r+6s \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

$$p+4q = -7 \text{ --- (1)} \quad r+4s = 2 \text{ --- (3)}$$

$$2p+5q = -8 \text{ --- (2)} \quad 2r+5s = 4 \text{ --- (4)}$$

Solve (1) & (2)

$$\text{we get } \boxed{p=1}$$

$$\boxed{q=-2}$$

Solve (3) & (4)

$$\text{we get } \boxed{r=2}$$

$$\boxed{s=0}$$

$$A = \begin{pmatrix} p & q \\ r & s \end{pmatrix} = \begin{pmatrix} 1 & -2 \\ 2 & 0 \end{pmatrix}$$

(15)

Given

$$A^T = \begin{bmatrix} 4 & 5 \\ -1 & 0 \\ 2 & 3 \end{bmatrix} \text{ and}$$

$$B = \begin{bmatrix} 2 & -1 & 1 \\ 7 & 5 & -2 \end{bmatrix}$$

$$B^T = \begin{bmatrix} 2 & 7 \\ -1 & 5 \\ 1 & -2 \end{bmatrix}$$

$$(i) (A+B)^T = A^T + B^T = B^T + A^T$$

$$(A+B) = \begin{pmatrix} 4 & -1 & 2 \\ 5 & 0 & 3 \end{pmatrix} + \begin{pmatrix} 2 & -1 & 1 \\ 7 & 5 & -2 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & -2 & 3 \\ 12 & 5 & 1 \end{pmatrix}$$

$$(A+B)^T = \begin{pmatrix} 6 & 12 \\ -2 & 5 \\ 3 & 1 \end{pmatrix} \text{ --- (1)}$$

$$A^T + B^T = \begin{pmatrix} 4 & 5 \\ -1 & 0 \\ 2 & 3 \end{pmatrix} + \begin{pmatrix} 2 & 7 \\ -1 & 5 \\ 1 & -2 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & 12 \\ -2 & 5 \\ 3 & 1 \end{pmatrix} \text{ --- (2)}$$

$$B^T + A^T = \begin{pmatrix} 2 & 7 \\ -1 & 5 \\ 1 & -2 \end{pmatrix} + \begin{pmatrix} 4 & 5 \\ -1 & 0 \\ 2 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & 12 \\ -2 & 5 \\ 3 & 1 \end{pmatrix} \text{ --- (3)}$$

From (1), (2), (3)

$$(A+B)^T = B^T + A^T = A^T + B^T$$

$$(ii) (A-B)^T = A^T - B^T$$

$$A-B = \begin{pmatrix} 4 & -1 & 2 \\ 5 & 0 & 3 \end{pmatrix} - \begin{pmatrix} 2 & -1 & 1 \\ 7 & 5 & -2 \end{pmatrix}$$

$$A-B = \begin{pmatrix} 2 & 0 & 1 \\ -2 & 5 & 5 \end{pmatrix}$$

$$(A-B)^T = \begin{pmatrix} 2 & -2 \\ 0 & -5 \\ 1 & 5 \end{pmatrix} \text{ --- (1)}$$

$$A^T - B^T = \begin{pmatrix} 4 & 5 \\ -1 & 0 \\ 2 & 3 \end{pmatrix} - \begin{pmatrix} 2 & 7 \\ -1 & 5 \\ 1 & -2 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & -2 \\ 0 & -5 \\ 1 & 5 \end{pmatrix} \text{ --- (2)}$$

16) Given A is a 3×4 matrix

A^T is a 4×3 matrix

Let order of B be $m \times n$

Given $A^T B$ and $B A^T$ are defined

(i) $A^T_{4 \times 3} \cdot B_{m \times n} \Rightarrow m=3$

(ii) $B A^T$

$m \times n \quad A_{4 \times 3} \Rightarrow n=4$

order of the matrix B can be 3×4 matrix.

17) $A = \begin{pmatrix} 4 & -2 \\ 3 & -5 \end{pmatrix}$

We can write

$A = \frac{1}{2}(A+A^T) + \frac{1}{2}(A-A^T) \quad \text{--- (1)}$

$A^T = \begin{pmatrix} 4 & 3 \\ -2 & -5 \end{pmatrix}$

$A+A^T = \begin{pmatrix} 4 & -2 \\ 3 & -5 \end{pmatrix} + \begin{pmatrix} 4 & 3 \\ -2 & -5 \end{pmatrix}$
 $= \begin{pmatrix} 8 & 1 \\ 1 & -10 \end{pmatrix}$

$A-A^T = \begin{pmatrix} 4 & -2 \\ 3 & -5 \end{pmatrix} - \begin{pmatrix} 4 & 3 \\ -2 & -5 \end{pmatrix}$
 $= \begin{pmatrix} 0 & -5 \\ 5 & 0 \end{pmatrix}$

$A = \frac{1}{2} \begin{pmatrix} 8 & 1 \\ 1 & -10 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 0 & -5 \\ 5 & 0 \end{pmatrix}$
 $= \begin{pmatrix} 4 & -2 \\ 3 & -5 \end{pmatrix} = A$

(ii) $A = \begin{pmatrix} 3 & 8 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{pmatrix}$

$A^T = \begin{pmatrix} 3 & -2 & -4 \\ 8 & -2 & -5 \\ -1 & 1 & 2 \end{pmatrix}$

$A+A^T = \begin{pmatrix} 3 & 8 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{pmatrix} + \begin{pmatrix} 3 & -2 & -4 \\ 8 & -2 & -5 \\ -1 & 1 & 2 \end{pmatrix}$

$= \begin{pmatrix} 6 & 6 & -5 \\ 6 & -4 & -4 \\ 5 & -4 & 4 \end{pmatrix}$

$A-A^T = \begin{pmatrix} 3 & 8 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{pmatrix} - \begin{pmatrix} 3 & -2 & -4 \\ 8 & -2 & -5 \\ -1 & 1 & 2 \end{pmatrix}$
 $= \begin{pmatrix} 0 & 10 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{pmatrix}$

(1) $\Rightarrow$

$A = \frac{1}{2} \begin{pmatrix} 6 & 6 & -5 \\ 6 & -4 & -4 \\ -5 & -4 & 4 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 0 & 10 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{pmatrix}$

18) Given $\begin{pmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{pmatrix} 3 \times 2$

$A^T = \begin{pmatrix} -1 & -8 & -10 \\ 1 & 2 & -5 \\ 9 & 22 & 15 \end{pmatrix} 3 \times 3$

$3 \times 2 \quad m \times n = 3 \times 3$

$m=2 \quad n=3$

Let us take $A^T = \begin{pmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \end{pmatrix}$

$\begin{pmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{pmatrix} \begin{pmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \end{pmatrix}$
 $= \begin{pmatrix} -1 & -8 & -10 \\ 1 & 2 & -5 \\ 9 & 22 & 15 \end{pmatrix}$

$\begin{pmatrix} 2a_1 - a_4 & 2a_2 - a_5 & 2a_3 - a_6 \\ a_1 + 0 & a_2 + 0 & a_3 + 0 \\ -3a_1 + 4a_4 & -3a_2 + 4a_5 & -3a_3 + 4a_6 \end{pmatrix}$
 $= \begin{pmatrix} -1 & -8 & -10 \\ 1 & 2 & -5 \\ 9 & 22 & 15 \end{pmatrix}$

$a_1 = 1 \quad a_2 = 2 \quad a_3 = -5$

$2a_1 - a_4 = -1 \quad 2a_2 - a_5 = -8 \quad 2a_3 - a_6 = -10$
 $2(1) - a_4 = -1 \quad 2(2) - a_5 = -8 \quad 2(-5) - a_6 = -10$
 $a_4 = 3 \quad a_5 = 2 \quad a_6 = 0$

$A^T = \begin{pmatrix} 1 & 2 & -5 \\ 3 & 2 & 0 \end{pmatrix} \Rightarrow A = \begin{pmatrix} 1 & 3 \\ 2 & 12 \end{pmatrix}$

8

19.

$$\text{Given } A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ x & 2 & y \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 2 & x \\ 2 & 1 & 2 \\ 2 & -2 & y \end{bmatrix}$$

$$AA^T = 9I$$

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ x & 2 & y \end{bmatrix} \begin{bmatrix} 1 & 2 & x \\ 2 & 1 & 2 \\ 2 & -2 & y \end{bmatrix} = 9 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1+4+4 & 2+2-4 & x+4+2y \\ 2+2-4 & 4+1+4 & 2x+2-2y \\ x+4+2y & 2x+2-2y & x^2+4+y^2 \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$\begin{bmatrix} 9 & 0 & x+2y+4 \\ 0 & 9 & 2x-2y+2 \\ x+4+2y & 2x-2y & x^2+4+y^2 \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$x+2y+4=0$$

$$x+2y=-4 \quad \text{--- (1)}$$

$$2x-2y+2=0$$

$$2x-2y=-2 \quad \text{--- (2)}$$

$$\text{Add (1) \& (2)} \quad 3x = -6$$

$$x = -2$$

$$\text{Put } x = -2 \text{ in (1)}$$

$$-2+2y=-4$$

$$-2+4=-2y$$

$$y = -1$$

$$\therefore x = -2 \text{ and } y = -1$$

$$(ii) A = \begin{pmatrix} 0 & p & 3 \\ 2 & q^2 & -1 \\ 3 & 1 & 0 \end{pmatrix}$$

$$A^T = \begin{pmatrix} 0 & 2 & 3 \\ p & q^2 & 1 \\ 3 & -1 & 0 \end{pmatrix}$$

$$A^T = -A$$

$$\begin{pmatrix} 0 & 2 & 3 \\ p & q^2 & 1 \\ 3 & -1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -p & -3 \\ -2 & -q^2 & 1 \\ -3 & -1 & 0 \end{pmatrix}$$

$$-p = 2$$

$$p = -2$$

$$q^2 = -9$$

$$q^2 + q^2 = 0$$

$$2q^2 = 0$$

$$q^2 = 0$$

The value of

$$p = -2$$

$$q = 0$$

$$r = -3$$

21)

$$A = [a_{ij}]_{3 \times 3}$$

In General form 3×3

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$a_{ij} = i - j$$

$$\begin{array}{l} a_{11} = 1-1=0 \quad a_{21} = 2-1=1 \quad a_{31} = 3-1=2 \\ a_{12} = 1-2=-1 \quad a_{22} = 2-2=0 \quad a_{32} = 3-2=1 \\ a_{13} = 1-3=-2 \quad a_{23} = 2-3=-1 \quad a_{33} = 3-3=0 \end{array}$$

$$A = \begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & -1 \\ 2 & -1 & 0 \end{bmatrix}$$

$$-A = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & -1 \\ -2 & -1 & 0 \end{bmatrix}$$

$$A^T = -A$$

A is a skew matrix

$$20) \text{ Given } A = \begin{pmatrix} 0 & 1 & -2 \\ -1 & 0 & x^3 \\ 2 & -3 & 0 \end{pmatrix} \text{ is skew matrix}$$

$$A^T = -A$$

$$A^T = \begin{pmatrix} 0 & -1 & 2 \\ 1 & 0 & -x^3 \\ -2 & x^3 & 0 \end{pmatrix}$$

$$A^T = -A \Rightarrow \begin{pmatrix} 0 & -1 & 2 \\ 1 & 0 & -x^3 \\ -2 & x^3 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 & 2 \\ 1 & 0 & -x^3 \\ -2 & 3 & 0 \end{pmatrix}$$

$$x^3 = 3$$

$$x = 3^{\frac{1}{3}} = \sqrt[3]{3}$$

23)

Given A and B are symmetric

(i) $(i.e.) A^T = A \quad B^T = B$

(ii) $AB + BA$ is a symmetric matrix

$$(AB + BA)^T = (AB)^T + (BA)^T$$

$$= B^T A^T + A^T B^T$$

$$= BA + AB$$

 $(AB + BA)$ is a symmetric matrix(ii) $AB - BA$ is a symmetric matrix

$$(AB - BA)^T = (AB)^T - (BA)^T$$

$$= B^T A^T - A^T B^T$$

$$= BA - AB$$

$$= -(AB - BA)$$

 $AB - BA$ is a skew matrix.

22) Given A and B be two symmetric matrices.

$$\Rightarrow A^T = A \quad B^T = B$$

Given $AB = BA$

$$\Leftrightarrow (AB)^T = B^T A^T$$

$$\Leftrightarrow (AB)^T = BA$$

$$\Leftrightarrow (AB)^T = AB$$

 AB is a symmetric matrix

24)

	PI	PII	PIII	
Gift Pack matrix A =	100	200	250	Cashew
	100	100	250	raisins
	50	100	150	almonds

Cost of 50 gm of Cashew is ₹ 50

" " " " " " ₹ 1

Cost of 50 gm of raisins is ₹ 10

" " " " " " ₹ 1/5

Cost of 50 gm of almonds

is ₹ 60

1 gm of raisins is 1/5

Cost matrix $B = \begin{pmatrix} 1 & 1/5 & 1/5 \end{pmatrix}$

Cost of Package in AB

$$AB = \begin{pmatrix} 1 & 1/5 & 1/5 \end{pmatrix} \begin{bmatrix} 100 & 200 & 250 \\ 100 & 100 & 250 \\ 50 & 100 & 150 \end{bmatrix}$$

$$= \begin{bmatrix} 100 + 20 + 60 \\ 200 + 20 + 120 \\ 250 + 50 + 150 \end{bmatrix} = \begin{bmatrix} 180 \\ 340 \\ 480 \end{bmatrix}$$

Pack I Cost ₹ 180

Pack II Cost ₹ 340

Pack III Cost ₹ 480

Qstn No: 24, Any doubt for this matrix multiplication please refer the text book page 5x-7.11.

EXERCISE - 7.2

1.

LHS

$$\begin{bmatrix} s & a^2 & b^2+c^2 \\ s & b^2 & c^2+a^2 \\ s & c^2 & a^2+b^2 \end{bmatrix}$$

Apply $C_2 \rightarrow C_2 + C_3$

$$= \begin{bmatrix} s & a^2+b^2+c^2 & b^2+c^2 \\ s & a^2+b^2+c^2 & c^2+a^2 \\ s & a^2+b^2+c^2 & a^2+b^2 \end{bmatrix}$$

 $= 0$ Since C_1 and C_2 are proportional.

(10)

2) LHS:

$$\begin{vmatrix} b+c & bc & b^2c^2 \\ ca & ca & c^2a^2 \\ a+b & ab & a^2b^2 \end{vmatrix}$$

multiply R_1, R_2, R_3 by a, b, c

$$= \frac{1}{abc} \begin{vmatrix} ab+ac & abc & ab^2c^2 \\ bc+ab & abc & c^2a^2b \\ ac+bc & abc & a^2b^2c \end{vmatrix}$$

Take abc from C_2 and C_3

$$= \frac{(abc)abc}{abc} \begin{vmatrix} ab+ac & 1 & bc \\ bc+ab & 1 & ca \\ ac+bc & 1 & ab \end{vmatrix}$$

Apply $C_1 \rightarrow C_1 + C_2$

$$= abc \begin{vmatrix} ab+ac+bc & 1 & bc \\ ab+bc+ca & 1 & ca \\ ab+bc+ca & 1 & ab \end{vmatrix}$$

$$= abc(0) = 0 = RHS$$

Since C_1, C_2 are Proportional.

$$3) \underline{\text{LHS}} \begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix}$$

Take a, b, c from C_1, C_2, C_3

$$= abc \begin{vmatrix} a & c & a+c \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$$

Apply $C_1 \rightarrow C_1 + C_2 - C_3$

$$= abc \begin{vmatrix} 0 & c & a+c \\ ab & b & a \\ ab & b+c & c \end{vmatrix}$$

$$= abc \begin{vmatrix} 0 & c & a+c \\ 0 & -c & a-c \\ ab & b+c & c \end{vmatrix} \quad R_1 \rightarrow R_2 - R_3$$

$$= abc [2b[c(a-c)+c(a+c)]]$$

$$= 2ab^2c [ac - c^2 + ac + c^2]$$

$$= 2ab^2c(2ac)$$

$$= 4a^2b^2c^2 = RHS$$

4)

$$\underline{\text{LHS}} \begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$$

$$= \begin{vmatrix} a & 0 & 1 \\ -b & b & 1 \\ 0 & -c & 1+c \end{vmatrix} \quad \begin{matrix} C_1 \rightarrow C_1 - C_2 \\ C_2 \rightarrow C_2 - C_3 \end{matrix}$$

$$= a(b+bc+c)+0+1(bc-0)$$

$$= ab+abc+ac+bc$$

$$= \frac{1}{abc} \left[\frac{1}{c} + 1 + \frac{1}{b} + \frac{1}{a} \right]$$

$$= \frac{1}{abc} \left[1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right]$$

$$= RHS$$

Hence Proved

5)

$$\underline{\text{LHS}} = \begin{vmatrix} \sec^2\theta & \tan^2\theta & 1 \\ \tan^2\theta & \sec^2\theta & -1 \\ 38 & 36 & 2 \end{vmatrix}$$

$$= \begin{vmatrix} \sec^2\theta & \tan^2\theta+1 & 1 \\ \tan^2\theta & \sec^2\theta-1 & -1 \\ 38 & 36+2 & 2 \end{vmatrix} \quad C_2 \rightarrow C_2 + C_3$$

$$= \begin{vmatrix} \sec^2\theta & \sec^2\theta & 1 \\ \tan^2\theta & \tan^2\theta & -1 \\ 38 & 38 & 2 \end{vmatrix}$$

$$= 0$$

Since C_1 and C_2 are Proportional

b)

(11)

Ans:

$$\begin{vmatrix} x+2a & y+2b & z+2c \\ x & y & z \\ a & b & c \end{vmatrix}$$

Applying $R_1 \rightarrow R_1 - R_2$

$$= \begin{vmatrix} 2a & 2b & 2c \\ x & y & z \\ a & b & c \end{vmatrix}$$

$$= 0 \quad \left[\begin{array}{l} \text{Since } R_1 \text{ and } R_3 \text{ are} \\ \text{Proportional} \end{array} \right]$$

7)

Soln

Suppose that $n=3$ is an odd integer.

Let A be a 3×3 skew matrix.

Then $A^T = -A$

Using Property (1)

$$\det(A) = \det(A^T)$$

$$= \det(-A) \quad [A \text{ is a skew matrix}]$$

$$= (-1)^3 \det A \quad [\text{Property 2}]$$

$$\det A = -\det A$$

$$(1) \det A = -\det A$$

$$\det A + \det A = 0$$

$$2 \det A = 0$$

$$\det A = 0$$

Hence Proved

8. Given

$$\begin{vmatrix} a & b & a+b \\ b & c & a+c \\ a+b & b+c & 0 \end{vmatrix} = 0$$

Using determinant of order 3 by using Sarrus Rule [Refer textbook]

$$\begin{array}{ccccc} a & b & a+b & a & b \\ & b & c & a+c & \\ a+b & b+c & 0 & a+b & b+c \end{array}$$

$$\Rightarrow [0 + 2b(b+c)(a+b) +$$

$$c(a+b)b(b+c)] - [c(a+b)^2 + a(b+c)^2 + 0] = 0$$

$$\Rightarrow 2b(ab^2 + b^2c + abc + bc^2) - c(a^2 + b^2 + 2ab) - a(b^2 + c^2 + 2bc) = 0$$

$$2ab^2c + 2b^3c + 2ab^2c + 2b^2c^2$$

$$- a^2c^2 - b^2c^2 - 2ab^2c$$

$$- ab^2c^2 - ac^2 - 2ab^2c = 0$$

$$\Rightarrow ab^2c^2 + b^2c^2 - a^2c^2 - ac^2 + 2b^3c - 2ab^2c = 0$$

$$\Rightarrow b^2c - ac^2 + ab^2c - a^2c^2 + 2b^3c - 2ab^2c = 0$$

$$c(b^2 - ac) + a^2(b^2 - ac) + 2b^2c(b^2 - ac) = 0$$

$$(b^2 - ac)(c + a^2 + 2b^2) = 0$$

$$b^2 - ac = 0 \quad (\because) \quad a^2 + 2b^2 + c = 0$$

$$\Rightarrow a, b, c \text{ are in G.P.} \quad \Rightarrow \alpha \text{ is the root of } a^2 + 2bx + c = 0$$

Hence Proved

9)

$$\text{LHS} \begin{vmatrix} 1 & a & a^2 - bc \\ 1 & b & b^2 - ca \\ 1 & c & c^2 - ab \end{vmatrix}$$

Using Property (7)

$$= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$$

$$= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \frac{1}{abc} \begin{vmatrix} a & a^2 & abc \\ b & b^2 & abc \\ c & c^2 & abc \end{vmatrix}$$

Multiply R_1, R_2, R_3 by a, b, c

$$= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \frac{abc}{abc} \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix}$$

Take abc from (3)

$$= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \begin{matrix} C_1 \leftrightarrow C_3 \\ C_3 \leftrightarrow C_2 \end{matrix}$$

$$= 0 = \text{RHS.}$$

Hence Proved.

10)

Given a, b, c are $P^{\text{th}}, Q^{\text{th}}, R^{\text{th}}$ terms of an AP.

$$P^{\text{th}} \text{ term} \Rightarrow A + (P-1)R = a \Rightarrow A + PR - R = a$$

$$Q^{\text{th}} \text{ term} \Rightarrow A + (Q-1)R = b \Rightarrow A + QR - R = b$$

$$R^{\text{th}} \text{ term} \Rightarrow A + (R-1)R = c \Rightarrow A + RR - R = c$$

Here $A \rightarrow$ First term, $R \rightarrow$ Common difference

$$\text{LHS} \begin{vmatrix} a & b & c \\ P & Q & R \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} A+PR-R & A+QR-R & A+RR-R \\ P & Q & R \\ 1 & 1 & 1 \end{vmatrix}$$

multiply R_2 and R_3 by R respectively

$$= \frac{1}{R^2} \begin{vmatrix} A+PR-R & A+QR-R & A+RR-R \\ PR & QR & RR \\ R & R & R \end{vmatrix}$$

Applying $R_1 \rightarrow R_1 - R_2 + R_3$

$$= \frac{1}{R^2} \begin{vmatrix} A & A & A \\ PR & QR & RR \\ R & R & R \end{vmatrix}$$

$$= \frac{1}{R^2} (0) = 0$$

Since R_1 and R_2 are proportional

11)

$$\begin{vmatrix} a^2+x^2 & ab & ac \\ ab & b^2+x^2 & bc \\ ac & bc & c^2+x^2 \end{vmatrix}$$

$$= (a^2+x^2)(b^2x^2+c^2x^2+x^4) - ab(abx^2) + ac(-a^2x^2)$$

$$= a^2b^2x^2 + a^2c^2x^2 + a^2x^4 - a^2b^2x^2 - a^2c^2x^2 - a^2b^2x^2$$

$$= x^4(a^2+b^2+c^2+x^2)$$

which is divisible by x^4 .

Hence Proved.

12)

Given a, b, c are $p^{\text{th}}, q^{\text{th}}$ and r^{th} terms of a G.P.

$$p^{\text{th}} \text{ term} \Rightarrow AR^{p-1} = a$$

$$q^{\text{th}} \text{ term} \Rightarrow AR^{q-1} = b$$

$$r^{\text{th}} \text{ term} \Rightarrow AR^{r-1} = c$$

$$AR^{p-1} = a$$

Taking $\log$ on both side

$$\log A + (p-1) \log R = \log a$$

Similarly

$$\log A + (q-1) \log R = \log b$$

$$\log A + (r-1) \log R = \log c$$

LHS

$$\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix}$$

$$= \begin{vmatrix} \log A + p \log R - R & p & 1 \\ \log A + q \log R - R & q & 1 \\ \log A + r \log R - R & r & 1 \end{vmatrix}$$

$$= \begin{vmatrix} \log A & p & 1 \\ \log A & q & 1 \\ \log A & r & 1 \end{vmatrix} + \begin{vmatrix} p \log R & p & 1 \\ q \log R & q & 1 \\ r \log R & r & 1 \end{vmatrix}$$

$$= \begin{vmatrix} R & p & 1 \\ R & q & 1 \\ R & r & 1 \end{vmatrix} \quad \left[\begin{array}{l} \text{Property} \\ \text{No 7} \end{array} \right]$$

Since in all determinants C_1 and C_3 are proportional.

$$= 0 + 0 - 0 = 0 = \text{RHS}$$

Hence Proved.

13)

$$\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$$

$$= 1(1 - \log_x y \cdot \log_y z)$$

$$- \log_x y (\log_y x - \log_y z \log_z y) + \log_x z (\log_z x \log_z y - \log_z z)$$

$$= 1(1 - \log 1) - \log_x y (\log_y x - \log_y z)$$

$$+ \log_x z (\log_z x - \log_z z)$$

$$= 1(1-1) - \log_x y (0) + \log_x z (0)$$

$$= 0 = \text{RHS}$$

14)

$$A = \begin{pmatrix} \frac{1}{2} & \alpha \\ 0 & \frac{1}{2} \end{pmatrix}$$

$$|A| = \frac{1}{4} - 0 = \frac{1}{4}$$

$$A^2 = A \times A = \begin{pmatrix} \frac{1}{2} & \alpha \\ 0 & \frac{1}{2} \end{pmatrix} \begin{pmatrix} \frac{1}{2} & \alpha \\ 0 & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} \frac{1}{4} + 0 & \frac{1}{2}\alpha + \frac{1}{2}\alpha \\ 0+0 & 0 + \frac{1}{4} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{4} & \alpha \\ 0 & \frac{1}{4} \end{pmatrix}$$

$$|A^2| = \frac{1}{4^2} - 0 = \frac{1}{4^2}$$

$$A^3 = A^2 \times A$$

$$= \begin{pmatrix} \frac{1}{4} & \alpha \\ 0 & \frac{1}{4} \end{pmatrix} \begin{pmatrix} \frac{1}{2} & \alpha \\ 0 & \frac{1}{2} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{8} + 0 & \frac{1}{4}\alpha + \frac{1}{2}\alpha \\ 0+0 & 0 + \frac{1}{8} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{8} & \frac{3}{4}\alpha \\ 0 & \frac{1}{8} \end{pmatrix}$$

$$|A^3| = \frac{1}{64} - 0 = \frac{1}{64} = \frac{1}{4^3}$$

$$\boxed{|A^n| = \frac{1}{4^n}} \text{ and so on.}$$

(14)

$$\sum_{k=1}^n \det A^k = A + A^2 + A^3 + \dots + A^n$$

$$= \frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \dots + \frac{1}{4^n}$$

$$= \frac{1}{4} \left[1 + \frac{1}{4} + \frac{1}{4^2} + \dots + \frac{1}{4^{n-1}} \right]$$

$$= \frac{1}{4} \left[\frac{1(1 - \frac{1}{4^n})}{1 - \frac{1}{4}} \right] \quad \left[S_n = \frac{a(1-r^n)}{1-r} \right]$$

$$= \frac{1}{4} \left[\frac{1 - \frac{1}{4^n}}{\frac{3}{4}} \right]$$

$$= \frac{1}{3} \left(1 - \frac{1}{4^n} \right)$$

= RHS. Hence Proved.

(15)

$$(i) \begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 62 & 92 & 122 \end{vmatrix}$$

Take 32 from R_3

$$= 32 \begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 2 & 3 & 4 \end{vmatrix}$$

$$= 32(0) = 0 \quad \left[\text{Since } R_1 \text{ and } R_3 \text{ are proportional} \right]$$

$$(ii) \begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$$

$$R_1 \rightarrow R_1 + R_2$$

$$= \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$$

$$= (x+y+z) \begin{vmatrix} 1 & 1 & 1 \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$$

$$= (x+y+z)(0) = 0$$

Since R_1 and R_3 are proportional

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(16)

$$\text{Given } |A| = 2$$

$$|A| = |A^T| \quad (\text{by property 1})$$

$$|A^T| = 2$$

$$|AA^T| = |A||A^T|$$

$$= 2 \times 2 = 4$$

$$(17) \text{ Given } |A| = -1 \quad |B| = 3$$

We know that

$$|kA| = k^n |A| \quad \text{Here } n = 3$$

$$|3AB| = 3^3 |A||B|$$

$$= 27(-1)(3)$$

$$= -81$$

$$(18) \text{ Given } \lambda = -2$$

LHS

$$\begin{vmatrix} 0 & 2\lambda & 1 \\ \lambda^2 & 0 & 3\lambda^2 + 1 \\ -1 & \lambda - 1 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & -4 & 1 \\ 4 & 0 & 13 \\ -1 & -3 & 0 \end{vmatrix}$$

$$= 0(0+12) + 4(0+13) + 1(-52) = 0$$

$$= 4(13) - 52$$

$$= 52 - 52 = 0$$

$$(19) \text{ Given } \begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 22 & 52 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 4 & 20 & 1 & 4 \\ 1 & -2 & 5 & 1 & -2 \\ 1 & 22 & 52 & 1 & 22 \end{vmatrix}$$

$$(-10x^2 + 20 + 40x) - (-40 + 10x + 20x^2) = 0$$

$$-10x^2 + 20 + 40x + 40 - 10x - 20x^2 = 0$$

$$-30x^2 + 30x + 60 = 0$$

$$-x^2 + x + 2 = 0$$

$$(x-2)(x+1) = 0$$

$$\begin{matrix} x = 2 \text{ or } \\ x = -1 \end{matrix}$$

(15)

20)

$$\text{Given } A = \begin{bmatrix} 4 & 3 & -2 \\ 1 & 0 & 7 \\ 2 & 3 & -5 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 3 & 3 \\ -2 & 4 & 0 \\ 9 & 7 & 5 \end{bmatrix}$$

Ans

$$\det A = 4(0 \cdot 21) - 3(-5 \cdot 14) - 2(3 \cdot 0)$$

$$= 4(-21) - 3(-14) - 2(3)$$

$$= -84 + 57 - 6$$

$$\boxed{\det A = -33}$$

$$\det B = 1(20 \cdot 0) - 3(-10 \cdot 0) + 3(-14 \cdot 36)$$

$$= 20 + 30 + 3(-50)$$

$$= -100$$

$$\boxed{\det B = -100}$$

$$\det A \cdot \det B = (-33)(-100) = 3300 \text{ --- (1)}$$

LHS

$$AB = \begin{bmatrix} 4 & 3 & -2 \\ 1 & 0 & 7 \\ 2 & 3 & -5 \end{bmatrix} \begin{bmatrix} 1 & 3 & 3 \\ -2 & 4 & 0 \\ 9 & 7 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 4-6-18 & 12+12-14 & 12+0-10 \\ 1+0+63 & 3+0+49 & 3+0+35 \\ 2-6-45 & 6+12-35 & 6+0-25 \end{bmatrix}$$

$$AB = \begin{bmatrix} -20 & 10 & 2 \\ 64 & 52 & 38 \\ -49 & -17 & -19 \end{bmatrix}$$

$$\det(AB) = \begin{vmatrix} -20 & 10 & 2 \\ 64 & 52 & 38 \\ -49 & -17 & -19 \end{vmatrix}$$

$$= -20(-988 + 646) - 10(1216 + 1862) + 2(-1088 + 2518)$$

$$= -20(342) - 10(2478) + 2(1430)$$

$$= 6840 - 24780 + 2860$$

$$\boxed{\det AB = -3300}$$

From (1) & (2)

$$\det(AB) = \det A \cdot \det B$$

Hence Proved.

21)

$$\text{Given } A = \begin{bmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{bmatrix}$$

Co-factors of elements of

second row.

$$|A| = -2 \begin{vmatrix} 3 & 8 \\ 2 & 3 \end{vmatrix} + 0 - 1 \begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix}$$

$$= -2(9-16) - 1(10-3)$$

$$= -2(9-16) - 1(10-3)$$

$$= -2(-7) - 1(7)$$

$$= 14 - 7 = 7$$

$$\boxed{|A| = 7}$$

EXERCISE 57.3

$$1) \quad |A| = \begin{vmatrix} x & a & a \\ a & x & a \\ a & a & x \end{vmatrix}$$

Put $x=a$

$$|A| = \begin{vmatrix} a & a & a \\ a & a & a \\ a & a & a \end{vmatrix} = 0$$

Since all the three rows are identical

 $\therefore (x-a)^2$ is a factor of $|A|$ Put $x = -2a$ in $|A|$

$$|A| = \begin{vmatrix} -2a & a & a \\ a & -2a & a \\ a & a & -2a \end{vmatrix}$$

$$= \begin{vmatrix} 0 & a & a \\ 0 & -2a & a \\ 0 & a & -2a \end{vmatrix} \quad C_1 \rightarrow C_1 + C_2 + C_3$$

$$= 0$$

 $(x+2a)$ is a factor of $|A|$

(16)

The degree of the Product of the factor $(x-a)^2(x+2a)$ is 3

The determinant is a cubic Polynomial in x .

$$m = 3 - 3 = 0$$

the other factor must be a Constant k .

$$\begin{vmatrix} x & a & a \\ a & x & a \\ a & a & x \end{vmatrix} = k(x-a)^2(x+2a) \quad \text{--- (1)}$$

Equating x^3 term on both sides we get $\boxed{1=k}$

Put in (1)

$$|A| = 1(x-a)^2(x+2a)$$

Hence Proved.

2).

$$|A| = \begin{vmatrix} b+c & a-c & a-b \\ b-c & c+a & b-a \\ c-b & c-a & a+b \end{vmatrix}$$

Put $a=0$

$$|A| = \begin{vmatrix} b+c & -c & -b \\ b+c & c & b \\ c-b & c & b \end{vmatrix} \quad (c_2 \cong c_3)$$

$= 0$

$(a=0)$ is a factor. i.e. a is a factor.

Since $|A|$ is in cyclic symmetric form in a, b, c and hence b, c also factors.

The degree of the Product of the factor a, b, c is 3.

The determinant is a cubic Polynomial. The other factor must be a Constant k .

$$\begin{vmatrix} b+c & a-c & a-b \\ b-c & c+a & b-a \\ c-b & c-a & a+b \end{vmatrix} = k(abc)$$

Put $a=1 \quad b=1 \quad c=1$

$$\begin{vmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{vmatrix} = k(1)(1)(1)$$

$$\boxed{8=k}$$

$$|A| = 8abc$$

Hence Proved.

3)

$$|A| = \begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix}$$

Put $x=0$

$$|A| = \begin{vmatrix} a & b & c \\ a & b & c \\ a & b & c \end{vmatrix} \quad (c_1 \cong c_2 \cong c_3)$$

$= 0$

Since all the three rows are identical

$\therefore (x=0)^2 = x^2$ is a factor of $|A|$

Put $x = -a-b-c$

$$|A| = \begin{vmatrix} -a-b-c+a & b & c \\ a & -a-b-c+b & c \\ a & b & -a-b-c+c \end{vmatrix}$$

$$= \begin{vmatrix} 0 & b & c \\ 0 & -a-b-c+b & c \\ 0 & b & -a-b-c+c \end{vmatrix} \quad (c_1 \rightarrow c_1 + c_2 - c_3)$$

$= 0$

$[x+(a+b+c)]$ is a factor of $|A|$.

The number of roots must be 3.

$$x = 0, 0, -(a+b+c).$$

(17)

(A)

$$|A| = \begin{vmatrix} b+c & a & a^2 \\ c+a & b & b^2 \\ a+b & c & c^2 \end{vmatrix}$$

Take $a=b$

$$|A| = \begin{vmatrix} a+c & a & a^2 \\ c+a & a & a^2 \\ a+b & c & c^2 \end{vmatrix}$$

$$= 0 \quad [R_1 \cong R_2]$$

 $(a-b)$ is a factor of $|A|$ Since $|A|$ is in cyclic symmetric form in a, b, c . $(b-c)$ and $(c-a)$ are also factors.The degree of the product of the factors $(a-b)(b-c)(c-a)$ is 3 and

The degree of the product of the leading diagonal elements

$$(b+c) + b + c^2 \text{ is } 4.$$

$$m = 4 - 3 = 1$$

The other factor is $k(a+b+c)$.

$$\therefore \begin{vmatrix} b+c & a & a^2 \\ c+a & a & b^2 \\ a+b & c & c^2 \end{vmatrix} = k(a+b+c)(a-b)(b-c)(c-a)$$

$$\text{Put } a=0 \quad b=1 \quad c=2 \quad \rightarrow \textcircled{1}$$

$$\begin{vmatrix} 3 & 0 & 0 \\ 2 & 1 & 1 \\ 1 & 2 & 4 \end{vmatrix} = k(3)(-1)(-1)(2)$$

$$3(4-2) = 6k$$

$$6 = 6k$$

$$\boxed{k=1} \text{ in } \textcircled{1}$$

$$|A| = (a+b+c)(a-b)(b-c)(c-a)$$

5) Soln

$$|A| = \begin{vmatrix} 4-x & 4+x & 4+x \\ 4+x & 4-x & 4+x \\ 4+x & 4+x & 4-x \end{vmatrix}$$

Put $x=0$

$$|A| = \begin{vmatrix} 4 & 4 & 4 \\ 4 & 4 & 4 \\ 4 & 4 & 4 \end{vmatrix} = 0$$

Since all the three rows are identical

$$\therefore (x-0)^2 = x^2 \text{ is a factor of } |A|$$

Applying $C_1 \rightarrow C_1 + C_2 + C_3$

$$|A| = \begin{vmatrix} 12+x & 4+x & 4+x \\ 12+x & 4-x & 4+x \\ 12+x & 4+x & 4-x \end{vmatrix}$$

Put $x=-12$

$$= \begin{vmatrix} 0 & 4+x & 4+x \\ 0 & 4-x & 4+x \\ 0 & 4-x & 4-x \end{vmatrix} = 0$$

 $\therefore (x+12)$ is another factor of $|A|$.

The number of roots must be 3.

$$\boxed{x=0, 0, -12}$$

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$$6) |A| = \begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix}$$

Put $x=y$

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ y & y & z \\ y^2 & y^2 & z^2 \end{vmatrix} = 0$$

 $(x-y)$ is a factor of $|A|$ The given determinant is in cyclic symmetric form in x, y, z . $\therefore (y-z)$ and $(z-x)$ are also factors.The degree of the product of the factors $(x-y)(y-z)(z-x)$ is 3.

The degree of product of the leading diagonal elements

$$1 \cdot y \cdot z^2 = 3$$

$$m = 3 - 3 = 0$$

The other factor is k .

$$\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix} = k(x-y)(y-z)(z-x) \quad (1)$$

Put $x=1, y=-1$ and $z=0$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 1 & 0 \end{vmatrix} = k(2)(-1)(-1)$$

$$1(1+1) = 2k$$

$$2 = 2k$$

$$\boxed{k=1} \text{ in (1)}$$

$$|A| = (x-y)(y-z)(z-x)$$

Hence Proved

EXERCISE - 7-4(1) Given vertices are $(0,0)$, $(1,2)$ and $(4,3)$ x_1, y_1
 x_2, y_2
 x_3, y_3

$$\text{Area of The triangle} = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2} \begin{vmatrix} 0 & 0 & 1 \\ 1 & 2 & 1 \\ 4 & 3 & 1 \end{vmatrix}$$

$$= \frac{1}{2} [1(3-8)]$$

$$= \frac{1}{2} (-5) = -\frac{5}{2} = -2.5$$

Area of the triangle = 2.5 square units

2)

Given vertices are $(k,2)$, $(2,4)$ and $(3,2)$

$$\text{Area of a triangle} = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= 4$$

$$\frac{1}{2} \begin{vmatrix} k & 2 & 1 \\ 2 & 4 & 1 \\ 3 & 2 & 1 \end{vmatrix} = 4$$

$$\frac{1}{2} (k(4-2) - 2(2-3) + 1(4-12))$$

$$\frac{1}{2} (2k+2-8) = 4$$

$$\frac{1}{2} (2k+2-8) = \pm 4$$

$$2k-6 = \pm 8$$

$$2k-6 = -8$$

$$2k-6 = -8$$

$$2k = -8+6$$

$$2k = -8+6$$

$$\boxed{k=-1}$$

$$\boxed{k=7}$$

 $\therefore k = -1 \text{ and } 7$

3)

$$(i) |A| = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$$

$$= 1(45 - 48) - 2(36 - 42) + 3(32 - 35)$$

$$= 1(-3) - 2(-6) + 3(-3)$$

$$= -3 + 12 - 9$$

$$|A| = 0$$

A is Singular

$$(ii) |A| = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$$

$$= 2(0 - 20) + 3(-42 - 4) + 5(30 - 0)$$

$$= 2(-20) + 3(-46) + 5(30)$$

$$= -40 - 138 + 150$$

$$= -28 \neq 0$$

$$|A| \neq 0$$

A is non Singular

$$(iii) |A| = \begin{vmatrix} 0 & a-b & k \\ b-a & 0 & 5 \\ -k & -5 & 0 \end{vmatrix}$$

$$= 0 - (a-b)[0 + 5k] + k(-5(b-a) - 0)$$

$$= -(-a+b)(5k) + k(-5b+5a)$$

$$= -5ka + 5kb - 5kb$$

$$|A| = 0$$

A is Singular

MATRICES AND DETERMINANTS

4)

$$(i) A = \begin{bmatrix} 7 & 3 \\ -2 & 9 \end{bmatrix}$$

Soln

Given A is Singular

Thus $|A| = 0$

$$\begin{vmatrix} 7 & 3 \\ -2 & 9 \end{vmatrix} = 0$$

$$7a + 6 = 0$$

$$7a = -6$$

$$a = -\frac{6}{7}$$

(ii) Given B is Singular

$$|B| = 0$$

$$\begin{vmatrix} b-1 & 2 & 3 \\ 3 & 1 & 2 \\ 1 & -2 & 4 \end{vmatrix} = 0$$

$$(b-1)(4+4) - 2(12-2) + 3(-6-1) = 0$$

$$(b-1)(8) - 2(10) + 3(-7) = 0$$

$$8b - 8 - 20 - 21 = 0$$

$$8b - 49 = 0$$

$$8b = 49$$

$$b = \frac{49}{8}$$

MATRICES AND DETERMINANTS

5) Given $\cos 2\theta = 0$

$$\cos 2\theta = \cos \pi/2$$

$$2\theta = \pi/2$$

$$\theta = \pi/4$$

Let $\begin{vmatrix} 0 & \cos \theta & \sin \theta \\ \cos \theta & \sin \theta & 0 \\ \sin \theta & 0 & \cos \theta \end{vmatrix} = 2$

$$\begin{vmatrix} 0 & \cos \theta & \sin \theta \\ \cos \theta & \sin \theta & 0 \\ \sin \theta & 0 & \cos \theta \end{vmatrix} = \begin{vmatrix} 0 & \cos \theta & \sin \theta \\ \cos \theta & \sin \theta & 0 \\ \sin \theta & 0 & \cos \theta \end{vmatrix}$$

$$= \begin{vmatrix} 0 + \cos^2 \theta + \sin^2 \theta & \cos \theta \sin \theta + 0 + 0 & 0 + 0 + \sin \theta \cos \theta \\ 0 + \sin \theta \cos \theta + 0 & \cos^2 \theta + \sin^2 \theta + 0 & \cos \theta \sin \theta + 0 + 0 \\ 0 + 0 + \sin \theta \cos \theta & \cos \theta \sin \theta + 0 + 0 & \sin^2 \theta + 0 + \cos^2 \theta \end{vmatrix}$$

$$= \begin{vmatrix} 1 & \cos \pi/4 \sin \pi/4 & \sin \pi/4 \cos \pi/4 \\ \sin \pi/4 \cos \pi/4 & 1 & \cos^2 \pi/4 \sin \pi/4 \\ \sin \pi/4 \cos \pi/4 & \cos^2 \pi/4 \sin \pi/4 & 1 \end{vmatrix} \quad (\theta = \pi/4)$$

$$= \begin{vmatrix} 1 & 1/2 & 1/2 \\ 1/2 & 1 & 1/2 \\ 1/2 & 1/2 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 1/2 & 1/2 \\ 1/2 & 1 & 1/2 \\ 1/2 & 1/2 & 1 \end{vmatrix}$$

$$= 1 \left(1 - \frac{1}{4} \right) - \frac{1}{2} \left(\frac{1}{2} - \frac{1}{4} \right) + \frac{1}{2} \left(\frac{1}{4} - \frac{1}{2} \right)$$

$$= 1 - \frac{1}{4} - \frac{1}{4} + \frac{1}{8} + \frac{1}{8} - \frac{1}{4}$$

$$= \frac{8 - 2 - 2 + 1 + 1 - 2}{8} = \frac{4}{8} = \frac{1}{2}$$

$$= \frac{1}{2}$$

MATRICES AND DETERMINANTS
R.V

6).

(21)

Soln

$$\begin{vmatrix} \log_3 64 & \log_4 3 \\ \log_3 8 & \log_4 9 \end{vmatrix} \times \begin{vmatrix} \log_2 3 & \log_6 3 \\ \log_3 4 & \log_2 4 \end{vmatrix}$$

$$= \begin{vmatrix} \log_3 64 \cdot \log_2 3 + \log_4 3 \cdot \log_3 4 & \log_3 64 \log_6 3 + \log_4 3 \log_3 4 \\ \log_3 8 \cdot \log_2 3 + \log_4 9 \cdot \log_3 4 & \log_3 8 \cdot \log_6 3 + \log_4 9 \log_3 4 \end{vmatrix}$$

$$[\log_a b \log_b c = \log_a c]$$

$$= \begin{vmatrix} \log_3 64 + \log 1 & \log_3 64 + \log 1 \\ \log_2 3 + \log_3 9 & \log 1 + \log_3 9 \end{vmatrix}$$

$$= \begin{vmatrix} 6 \log_2 2 & 2 \log_3 8 \\ 3 \log_2 2 + 2 \log_3 3 & 2 \log_2 3 \end{vmatrix}$$

$$[\log_a a = 1]$$

$$= \begin{vmatrix} 6 & 2 \\ 3 & 2 \end{vmatrix}$$

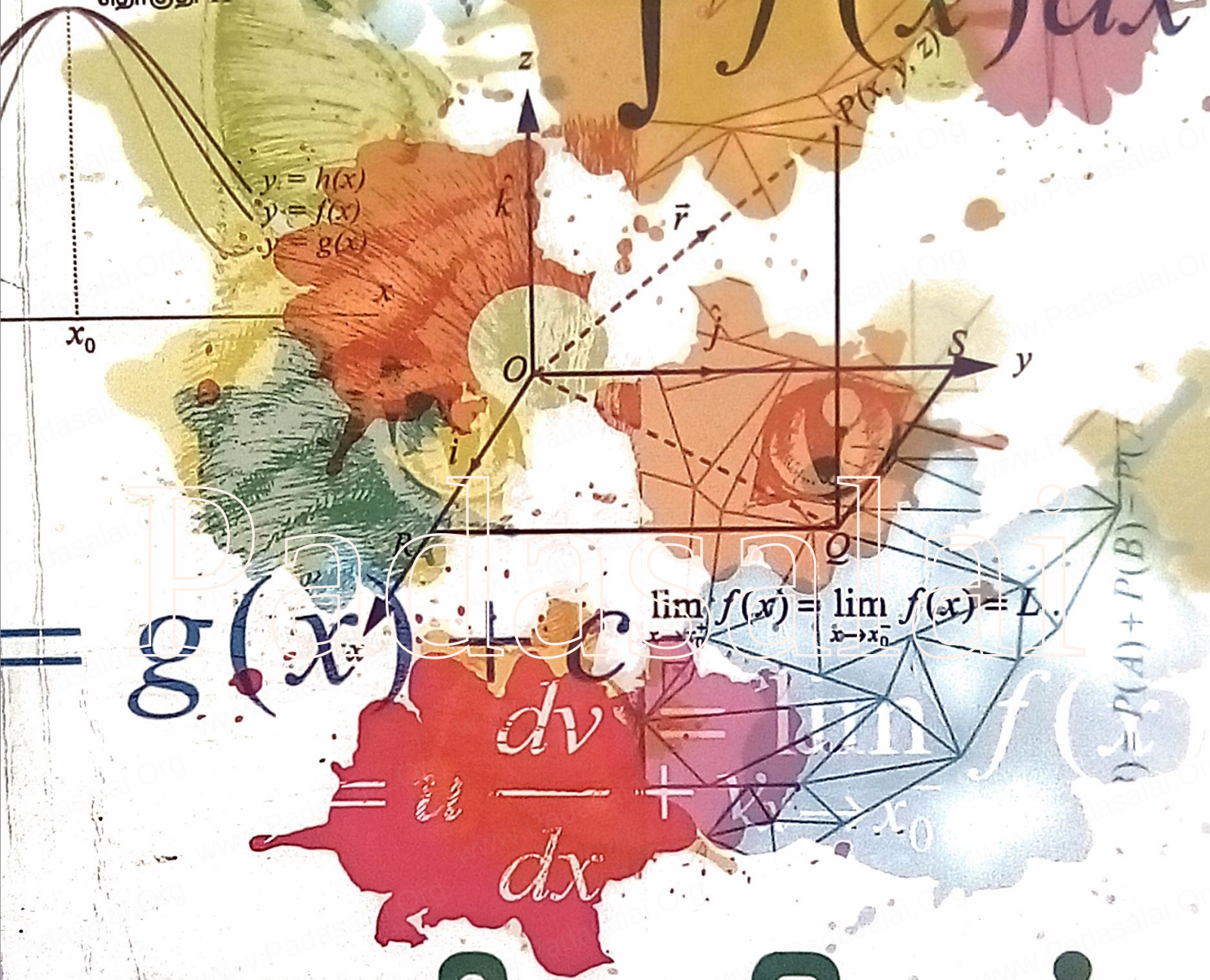
$$= 12 - 6$$

$$= 6$$

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தொகுதி II



கணிதவியல்



பள்ளிக் கல்வித்துறை
தமிழ்நாடு அரசு

CLASS: XI

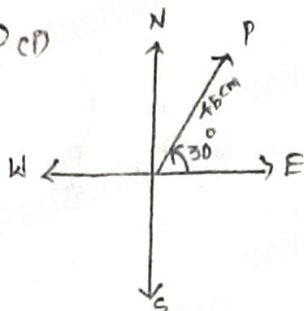
UNIT - VIII

VECTOR ALGEBRA

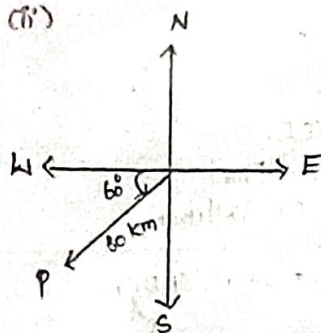
EXERCISE - 8.1

①

① (i)



(ii)



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PG ASST IN MATHEMATICS,

ARUNODHAYA HIGHER SEC SCHOOL,
SANDANGADU.② Given The Relation R is defined $\vec{a} R \vec{b}$ if $\vec{a} = \vec{b}$.

To Prove, R is Equivalence relation.

(i) Reflexive:-

We know that $\vec{a}$ is equal to $\vec{a}$

$$\Rightarrow \vec{a} R \vec{a}.$$

R is Reflexive.

(ii) Symmetric:-

Let $\vec{a} R \vec{b} \Rightarrow \vec{a}$ is equal to $\vec{b}$.Then $\vec{b}$ is equal to $\vec{a}$

$$\Rightarrow \vec{b} R \vec{a}.$$

R is Symmetric.

(iii) Transitive:-

Let $\vec{a} R \vec{b} \Rightarrow \vec{a}$ is equal to $\vec{b}$ and $\vec{b} R \vec{c} \Rightarrow \vec{b}$ is equal to $\vec{c}$.Then $\vec{a}$ is equal to $\vec{c}$.

$$\Rightarrow \vec{a} R \vec{c}.$$

R is Transitive.

From (i), (ii) & (iii) Hence

The Relation R is equivalence relation.

(3) Let P and Q be the Point of trisection of AB.

Let $AP = PQ = QB = \lambda$ (say)

P divides AB in the ratio 1:2

$$\text{Position Vector of P} = \vec{OP} = \frac{1\vec{OB} + 2\vec{OA}}{1+2}$$

$$P = \frac{1\vec{b} + 2\vec{a}}{1+2} = \frac{\vec{b} + 2\vec{a}}{3}$$

$$P = \frac{\vec{b} + 2\vec{a}}{3}$$

Q is the Point of PB.

$$\vec{OQ} = \frac{1\vec{OP} + 1\vec{OB}}{1+1} = \frac{\frac{\vec{b} + 2\vec{a}}{3} + \vec{b}}{2}$$

$$= \frac{\vec{b} + 2\vec{a} + 3\vec{b}}{6}$$

$$= \frac{2\vec{a} + 4\vec{b}}{6}$$

$$= \frac{2(\vec{a} + 2\vec{b})}{6}$$

$$= \frac{\vec{a} + 2\vec{b}}{3}$$

$$\vec{OQ} = \frac{\vec{a} + 2\vec{b}}{3}$$

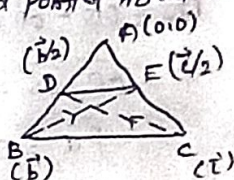
④ For our Convenience, We choose

A as the origin.

Let $\vec{b}$ and $\vec{c}$ are the Position Vectors of B and C.

Since D and E are the mid point of AB and AC.

$$\therefore D = \vec{b}/2, E = \vec{c}/2.$$



To Prove,

$$\vec{BE} + \vec{DC} = \frac{3}{2}(\vec{BC}).$$

$$\vec{BE} = \vec{OE} - \vec{OB} = \vec{c}/2 - \vec{b} = \vec{c}/2 - \vec{b}$$

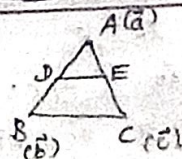
$$\vec{DC} = \vec{OC} - \vec{OD} = \vec{c} - \vec{b}/2 = \vec{c} - \vec{b}/2$$

$$\vec{BE} + \vec{DC} = \vec{c}/2 - \vec{b} + \vec{c} - \vec{b}/2 = \frac{3}{2}\vec{c} - \frac{3}{2}\vec{b}$$

$$= \frac{3}{2}(\vec{c} - \vec{b})$$

$$\vec{BE} + \vec{DC} = \frac{3}{2}(\vec{BC}).$$

⑤

Let ABC be a triangle.
and let O be the origin of reference.

Let D and E be the midpoints of AB and AC.

$$\vec{OA} = \vec{a}, \vec{OB} = \vec{b}, \vec{OC} = \vec{c} \quad \vec{OD} = \frac{\vec{a} + \vec{b}}{2}, \vec{OE} = \frac{\vec{a} + \vec{c}}{2}$$

$$\begin{aligned}\vec{DE} &= \vec{OE} - \vec{OD} \\ &= \left(\frac{\vec{a} + \vec{c}}{2}\right) - \left(\frac{\vec{a} + \vec{b}}{2}\right) \\ &= \frac{1}{2}(\vec{c} - \vec{b}) = \frac{1}{2}(\vec{OC} - \vec{OB}) \\ \vec{DE} &= \frac{1}{2}(\vec{BC})\end{aligned}$$

$$\vec{DE} = \frac{1}{2} \vec{BC} \Rightarrow DE \parallel BC$$

$$\text{Also, } \vec{DE} = \frac{1}{2} \vec{BC}$$

$$\Rightarrow |\vec{DE}| = \frac{1}{2} |\vec{BC}|$$

$$DE = \frac{1}{2} BC$$

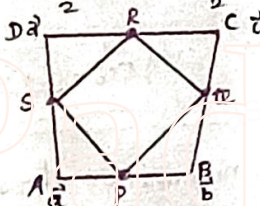
Hence $DE \parallel BC$ and $DE = \frac{1}{2} BC$

(6) Let ABCD be a quadrilateral and let P, Q, R, S be the midpoints of the sides AB, BC, CD and DA.

The position vectors of

$$P = \frac{\vec{a} + \vec{b}}{2}, Q = \frac{\vec{b} + \vec{c}}{2}$$

$$R = \frac{\vec{c} + \vec{d}}{2}, S = \frac{\vec{d} + \vec{a}}{2}$$



To Prove PQRS is a Parallelogram it is enough to prove that $\vec{PQ} = \vec{SR}$ and $\vec{PS} = \vec{QR}$.

$$\vec{PQ} = \vec{OQ} - \vec{OP} = \left(\frac{\vec{b} + \vec{c}}{2}\right) - \left(\frac{\vec{a} + \vec{b}}{2}\right) = \frac{\vec{c} - \vec{a}}{2}$$

$$\vec{SR} = \vec{OR} - \vec{OS} = \left(\frac{\vec{c} + \vec{d}}{2}\right) - \left(\frac{\vec{d} + \vec{a}}{2}\right) = \frac{\vec{c} - \vec{a}}{2}$$

$$\therefore \vec{PQ} = \vec{SR} \Rightarrow PQ \parallel SR \text{ and } PQ = SR$$

$$PQ = SR$$

Similarly we can prove that

$$PS = QR \text{ and } PS \parallel QR$$

Hence PQRS is a Parallelogram.

(7) ABCD is a Parallelogram.

$$\vec{AB} = \vec{DC} \text{ and } \vec{AD} = \vec{BC}$$

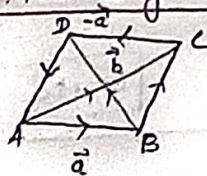
$$\vec{AB} = \vec{a} \Rightarrow \vec{CD} = -\vec{a}$$

$$\text{Since, } \vec{AC} = \vec{AB} + \vec{BC}$$

$$\vec{b} = \vec{a} + \vec{BC}$$

$$\vec{BC} = \vec{b} - \vec{a}$$

$$\vec{BC} = +\vec{DA} = -\vec{CB} = -(\vec{b} - \vec{a}) = \vec{a} - \vec{b}$$



$$\vec{DA} = \vec{a} - \vec{b}$$

2

$$\vec{BD} = \vec{BC} + \vec{CD} = \vec{b} - \vec{a} - \vec{a}$$

$$\vec{BD} = \vec{b} - 2\vec{a}$$

$\therefore$ The Required sides are

$$\vec{CD} = -\vec{a}, \vec{BC} = \vec{b} - \vec{a}$$

$$\vec{DA} = \vec{a} - \vec{b}$$

Other diagonal $\vec{BD} = \vec{b} - 2\vec{a}$.

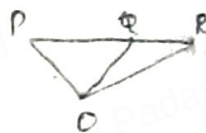
(8) Given

$$\vec{PO} + \vec{OR} = \vec{PR}$$

$$\vec{QO} + \vec{OR} = \vec{QR}$$

$$\text{Now } \vec{PQ} = \vec{QR}$$

$$\Rightarrow \vec{PQ} \parallel \vec{QR}$$



But O is a Common Point

$\Rightarrow P, Q, R$ are Collinear

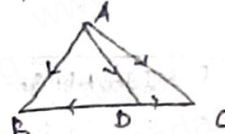
(9) D is the mid point of ABC,

$$\Rightarrow \vec{BD} = \vec{DC}$$

Join AD.

$$\vec{AB} = \vec{AD} + \vec{DB}$$

$$\vec{AC} = \vec{AD} + \vec{DC}$$



$$\text{LHS } \vec{AB} + \vec{AC} = \vec{AD} + \vec{DB} + \vec{AD} + \vec{DC}$$

$$= 2\vec{AD} + (\vec{DB} + \vec{DC})$$

$$= 2\vec{AD} + 0 \quad [\vec{DB} \text{ and } \vec{DC} \text{ are equal and opposite}]$$

$$= 2\vec{AD}$$

$$= \text{RHS}$$

(10)

Let O be the origin.

We know that

$$\vec{OG} = \frac{\vec{OA} + \vec{OB} + \vec{OC}}{3}$$

To Prove $\vec{GA} + \vec{GB} + \vec{GC} = \vec{0}$

LHS:

$$\vec{GA} + \vec{GB} + \vec{GC} = (\vec{OA} - \vec{OG}) +$$

$$(\vec{OB} - \vec{OG}) +$$

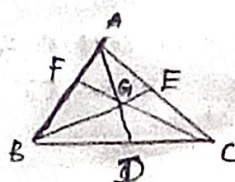
$$(\vec{OC} - \vec{OG})$$

$$= \vec{OA} + \vec{OB} + \vec{OC} - 3\vec{OG}$$

$$= 3\left(\frac{\vec{OA} + \vec{OB} + \vec{OC}}{3}\right) - 3\vec{OG}$$

$$= 3\vec{OG} - 3\vec{OG}$$

$$= \vec{0}$$



(12) ABCD is a quadrilateral in which E and F are midpoints of AC and BD.

$$\vec{AB} = \vec{AE} + \vec{EF} + \vec{FB}$$

$$\vec{AD} = \vec{AE} + \vec{EF} + \vec{FD}$$

$$\vec{CB} = \vec{CE} + \vec{EF} + \vec{FB}$$

$$\vec{CD} = \vec{CE} + \vec{EF} + \vec{FD}$$

$$\text{So, } \vec{AB} + \vec{AD} + \vec{CB} + \vec{CD} = 4\vec{EF} +$$

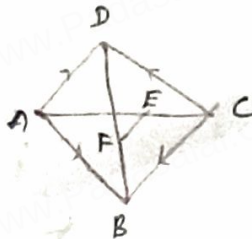
$$2(\vec{AE} + \vec{CE}) +$$

$$2(\vec{FB} + \vec{FD})$$

$$= 4\vec{EF} + \vec{0} + \vec{0}$$

$$= 4\vec{EF}$$

$\therefore \vec{AE}$ and $\vec{CE}$ are equal and opposite, $\vec{FB}$ and $\vec{FD}$ are equal and opposite vectors



$$(ii) \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2$$

$$= \frac{1}{2} + \frac{1}{4} + \frac{1}{4} = \frac{2+1+1}{4} = \frac{4}{4} = 1$$

Thus, direction cosines

$$(iii) \left(\frac{4}{3}\right)^2 + (0)^2 + \left(\frac{3}{4}\right)^2$$

$$= \frac{16}{9} + 0 + \frac{9}{16} = \frac{337}{144} \neq 1, \text{ Thus}$$

not direction cosines

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2.

(i) 1, 2, 3.

$$\text{Given } (x, y, z) = (1, 2, 3)$$

$$r = \sqrt{x^2 + y^2 + z^2} = \sqrt{1+4+9} = \sqrt{14}$$

Direction cosines are $\left(\frac{x}{r}, \frac{y}{r}, \frac{z}{r}\right)$

$$= \left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)$$

(ii) 3, -1, 3.

$$\text{Given } (x, y, z) = (3, -1, 3)$$

$$r = \sqrt{x^2 + y^2 + z^2} = \sqrt{9+1+9} = \sqrt{19}$$

$$= \sqrt{9+1+9} = \sqrt{19}$$

Direction cosines are $\left(\frac{3}{\sqrt{19}}, \frac{-1}{\sqrt{19}}, \frac{3}{\sqrt{19}}\right)$

(iii) $(x, y, z) = (0, 0, 7)$

$$r = \sqrt{x^2 + y^2 + z^2} = \sqrt{0+0+49} = \sqrt{49} = 7$$

Direction cosines are $\left(\frac{0}{7}, \frac{0}{7}, \frac{7}{7}\right)$

$$= (0, 0, 1)$$

(3)

$$(i) 3\hat{i} - 4\hat{j} + 8\hat{k}$$

The direction ratios of

$$3\hat{i} - 4\hat{j} + 8\hat{k} \text{ are } (3, -4, 8)$$

Direction cosines are $\left(\frac{x}{r}, \frac{y}{r}, \frac{z}{r}\right)$

$$r = \sqrt{x^2 + y^2 + z^2} = \sqrt{3^2 + (-4)^2 + 8^2} = \sqrt{9+16+64} = \sqrt{89}$$

Direction cosines are $\left(\frac{3}{\sqrt{89}}, \frac{-4}{\sqrt{89}}, \frac{8}{\sqrt{89}}\right)$

EXERCISE - 8.2

1)

(i) Sum of the Squares of the

(ii) Direction cosine is 1.

Given $\frac{1}{5}, \frac{3}{5}, \frac{4}{5}$.

$$\left(\frac{1}{5}\right)^2 + \left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2 = \frac{1}{25} + \frac{9}{25} + \frac{16}{25}$$

$$= \frac{26}{25} \neq 1. \text{ Thus}$$

Not direction cosine.

(ii) $3\hat{i} + \hat{j} + \hat{k}$

The Direction Ratio of $3\hat{i} + \hat{j} + \hat{k}$ are $(3, 1, 1)$

$$r = \sqrt{x^2 + y^2 + z^2} = \sqrt{3^2 + 1^2 + 1^2}$$

$$= \sqrt{9 + 1 + 1} = \sqrt{11}$$

Direction Cosines are

$$\left(\frac{3}{\sqrt{11}}, \frac{1}{\sqrt{11}}, \frac{1}{\sqrt{11}} \right)$$

(iii) $\hat{j}$. It can be written as $0\hat{i} + \hat{j} + 0\hat{k}$. $\therefore$ Direction Ratio of $\hat{j}$ are $(0, 1, 0)$.

$$r = \sqrt{0^2 + 1^2 + 0^2} = \sqrt{1} = 1$$

 $\therefore$ Direction Cosines are $(0, 1, 0)$.

(iv) $5\hat{i} - 3\hat{j} - 4\hat{k}$

The Direction Ratio of $5\hat{i} - 3\hat{j} - 4\hat{k}$ are $(5, -3, -4)$.

$$r = \sqrt{5^2 + (-3)^2 + (-4)^2} = \sqrt{25 + 9 + 16}$$

$$= \sqrt{2338}$$

Direction Cosines are

$$\left(\frac{5}{\sqrt{2338}}, \frac{-3}{\sqrt{2338}}, \frac{-4}{\sqrt{2338}} \right)$$

(v) $3\hat{i} + 4\hat{j} - 3\hat{k}$

The Direction Ratio of $3\hat{i} + 4\hat{j} - 3\hat{k}$ are $(3, 4, -3)$.

$$r = \sqrt{(3)^2 + (4)^2 + (-3)^2} = \sqrt{9 + 16 + 9}$$

$$= \sqrt{34}$$

Direction Cosines are $\left(\frac{3}{\sqrt{34}}, \frac{4}{\sqrt{34}}, \frac{-3}{\sqrt{34}} \right)$

(vi) $\hat{i} - \hat{k}$

It can be written $\hat{i} + 0\hat{j} - \hat{k}$. $\therefore$ Direction Ratio of $\hat{i} - \hat{k}$ are $(1, 0, -1)$.

$$r = \sqrt{1^2 + (0)^2 + (-1)^2} = \sqrt{2}$$

The Direction Cosines are $\left(\frac{1}{\sqrt{2}}, 0, \frac{-1}{\sqrt{2}} \right)$

VECTOR ALGEBRA

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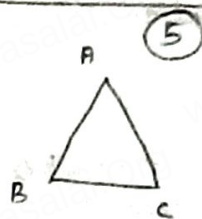
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The S

4) Let A, B, C be the vertices of the triangle ABC.

D, E, F be the midpoints of the sides BC, CA and AB respectively.

Take A(1, 0, 0) B(0, 1, 0) and C(0, 0, 1).



Midpoint of BC

$$D = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Midpoint of CA:

$$E = \left(\frac{1}{2}, 0, \frac{1}{2} \right)$$

$$D = \left(0, \frac{1}{2}, \frac{1}{2} \right)$$

Midpoint of AB:

$$F = \left(\frac{1}{2}, \frac{1}{2}, 0 \right)$$

$\therefore$ Median $\vec{AD} = \vec{OD} - \vec{OA} = (0\vec{i} + \frac{1}{2}\vec{j} + \frac{1}{2}\vec{k}) - (\vec{i} - 0\vec{j} + 0\vec{k})$

$$\vec{AD} = -\vec{i} + \frac{1}{2}\vec{j} + \frac{1}{2}\vec{k}$$

$$r = \sqrt{x^2 + y^2 + z^2} = \sqrt{(-1)^2 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \sqrt{1 + \frac{1}{4} + \frac{1}{4}} = \sqrt{\frac{3}{2}} = \frac{\sqrt{6}}{2}$$

$$r = \frac{\sqrt{6}}{2}$$

Direction Cosines of $\vec{AD}$ are $-\frac{1}{\frac{\sqrt{6}}{2}}, \frac{\frac{1}{2}}{\frac{\sqrt{6}}{2}}, \frac{\frac{1}{2}}{\frac{\sqrt{6}}{2}}$

$$\text{D.C. of AD are} = \left(-\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}} \right)$$

Median $\vec{BE} = \vec{OE} - \vec{OB}$

$$= \left(\frac{1}{2}\vec{i} - 0\vec{j} + \frac{1}{2}\vec{k} \right) - (0\vec{i} + \vec{j} + 0\vec{k})$$

$$= \frac{1}{2}\vec{i} - \vec{j} + \frac{1}{2}\vec{k}$$

$$r = \sqrt{\left(\frac{1}{2}\right)^2 + (-1)^2 + \left(\frac{1}{2}\right)^2} = \sqrt{\frac{1}{4} + 1 + \frac{1}{4}} = \sqrt{\frac{3}{2}} = \frac{\sqrt{6}}{2}$$

$$r = \frac{\sqrt{6}}{2}$$

Direction Cosines of $\vec{BE}$ are

$$\frac{1}{2 \times \frac{\sqrt{6}}{2}}, \frac{-1}{\frac{\sqrt{6}}{2}}, \frac{\frac{1}{2}}{2 \times \frac{\sqrt{6}}{2}}$$

$$\text{D.C. of BE are} \left(\frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}} \right)$$

Median $\vec{CF} = \vec{OF} - \vec{OC}$

$$= \left(\frac{1}{2}\vec{i} + \frac{1}{2}\vec{j} + 0\vec{k}\right) - (0\vec{i} + 0\vec{j} + \vec{k})$$

$$= \frac{1}{2}\vec{i} + \frac{1}{2}\vec{j} - \vec{k}$$

$$r = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + (-1)^2} = \sqrt{\frac{1}{4} + \frac{1}{4} + 1} = \frac{\sqrt{6}}{2}$$

$$r = \frac{\sqrt{6}}{2}$$

Direction cosines of CF are $\frac{1}{2 \times \frac{\sqrt{6}}{2}}, \frac{1}{2 \times \frac{\sqrt{6}}{2}}, \frac{-1}{\frac{\sqrt{6}}{2}}$

$$\text{D.C of CF are } = \left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}\right)$$

$\therefore$ Direction cosines of medians are $\left(\frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$

$$\left(\frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right) \text{ and}$$

$$\left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}\right)$$

5) Given the Direction cosines are $\frac{1}{2}, \frac{1}{\sqrt{2}}, a$.

We know That
Sum of the square of the Direction cosines is equal to 1.

$$\therefore \left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + a^2 = 1$$

$$\frac{1}{4} + \frac{1}{2} + a^2 = 1$$

$$\frac{3}{4} + a^2 = 1$$

$$3 + 4a^2 = 4$$

$$4a^2 = 4 - 3$$

$$a^2 = \frac{1}{4}$$

$$a = \pm \frac{1}{2}$$

6) Given Points are A(1,0,0) and B(0,1,0)
one set of Direction ratio a, a+b, a+b+c.

(i) $\vec{AB} = \vec{OB} - \vec{OA} = (0\vec{i} + \vec{j} + 0\vec{k}) - (\vec{i} + 0\vec{j} + 0\vec{k})$
 $= -\vec{i} + \vec{j}$

D.R of the line AB are (-1, 1, 0).

Given $(a, a+b, a+b+c) = (-1, 1, 0)$

$$\boxed{a=-1} \left| \begin{array}{l} a+b=1 \\ -1+b=1 \\ \boxed{b=2} \end{array} \right. \left| \begin{array}{l} a+b+c=0 \\ -1+2+c=0 \\ 1+c=0 \end{array} \right.$$

$$1+c=0$$

$$\boxed{c=-1}$$

(ii) $\vec{BA} = \vec{OA} - \vec{OB}$
 $= \vec{i} + 0\vec{j} + 0\vec{k} - (0\vec{i} + \vec{j} + 0\vec{k})$
 $= \vec{i} - \vec{j}$

D.R of the line BA are (1, -1, 0)

Given $(a, a+b, a+b+c) = (1, -1, 0)$

$$\boxed{a=1} \left| \begin{array}{l} a+b=-1 \\ 1+b=-1 \\ \boxed{b=-2} \end{array} \right. \left| \begin{array}{l} a+b+c=0 \\ 1-2+c=0 \\ -1+c=0 \end{array} \right. \Rightarrow \boxed{c=1}$$

7)

Let the given vectors be

$$\vec{a} = 2\hat{i} - \hat{j} + \hat{k}, \vec{b} = 3\hat{i} - 4\hat{j} - 4\hat{k}$$

$$\text{and } \vec{c} = \hat{i} - 3\hat{j} - 5\hat{k}$$

$$\vec{a} + \vec{c} = (2\hat{i} - \hat{j} + \hat{k}) + (\hat{i} - 3\hat{j} - 5\hat{k})$$

$$= 3\hat{i} - 4\hat{j} - 4\hat{k}$$

$$= \vec{b}$$

$$\boxed{\vec{a} + \vec{c} = \vec{b}}$$

$$\text{Now } |\vec{a}| = |2\hat{i} - \hat{j} + \hat{k}|$$

$$= \sqrt{(2)^2 + (-1)^2 + (1)^2}$$

$$= \sqrt{4+1+1}$$

$$|\vec{a}| = \sqrt{6} \Rightarrow \boxed{|\vec{a}|^2 = 6}$$

$$|\vec{b}| = \sqrt{(3)^2 + (-4)^2 + (-4)^2}$$

$$= \sqrt{9+16+16}$$

$$|\vec{b}| = \sqrt{41} \Rightarrow \boxed{|\vec{b}|^2 = 41}$$

$$|\vec{c}| = \sqrt{(1)^2 + (-3)^2 + (-5)^2} = \sqrt{1+9+25}$$

$$|\vec{c}| = \sqrt{35}$$

$$|\vec{c}| = \sqrt{35} \Rightarrow \boxed{|\vec{c}|^2 = 35}$$

$$|\vec{b}|^2 = |\vec{a}|^2 + |\vec{c}|^2 = 6 + 35$$

$$= 41$$

$$\therefore \vec{a} + \vec{c} = \vec{b} \text{ and}$$

$$|\vec{b}|^2 = |\vec{a}|^2 + |\vec{c}|^2, \text{ Thus}$$

Given vectors form the sides of a right angled triangle.

8) Given The vectors.

$$\vec{a} = 3\hat{i} + 2\hat{j} + 9\hat{k} \text{ and } \vec{b} = \hat{i} + \lambda\hat{j} + 3\hat{k}$$

We know That if $\vec{a}$ and $\vec{b}$ are Parallel,

$$a_1 = b_1, a_2 = b_2, a_3 = b_3.$$

$$\vec{a} = 3\hat{i} + 2\hat{j} + 9\hat{k}$$

$$\vec{b} = \hat{i} + \lambda\hat{j} + 3\hat{k}$$

7)

$$\text{Now, } a_1 = b_1, a_2 = b_2, a_3 = b_3$$

Equating components of $\hat{j}$

from $\vec{a}$ and $\vec{b}$, we get

$$\frac{2}{3} = \lambda$$

$$\boxed{\lambda = 2/3}$$

9)

(i) Let

$$\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$$

$$\vec{b} = -2\hat{i} + 3\hat{j} - 4\hat{k}$$

$$\vec{c} = 0\hat{i} - \hat{j} + 2\hat{k}$$

We know That, when we are able to write one vector as a linear combination of the other two vectors, then the given vectors are called coplanar vectors.

Let

$$\vec{a} = s\vec{b} + t\vec{c}$$

$$\hat{i} - 2\hat{j} + 3\hat{k} = s(-2\hat{i} + 3\hat{j} - 4\hat{k}) + t(0\hat{i} - \hat{j} + 2\hat{k})$$

Equating $\hat{i}, \hat{j}$ and $\hat{k}$ components.

$$1 = -2s \quad \text{--- (1)}$$

$$-2 = 3s - t \quad \text{--- (2)}$$

$$3 = -4s + 2t \quad \text{--- (3)}$$

① $\Rightarrow$

$$-2s = 1$$

$$\boxed{s = -1/2} \text{ in (2)}$$

$$3(-1/2) - t = -2$$

$$-3/2 - t = -2$$

$$-t = -2 + 3/2$$

$$-t = \frac{-4+3}{2} = -1/2$$

$$\boxed{t = 1/2}$$

Substituting the values of s and t in (3)

From (3)

$$LHS = 3$$

$$RHS = -4s + 2t$$

$$= -4(-\frac{1}{2}) + 2(\frac{1}{2})$$

$$= 2 + 1$$

$$= 3$$

$$LHS = RHS$$

one vector is a linear combination of other two vectors,

Thus $\vec{a}, \vec{b}, \vec{c}$ are coplanar.

(ii)

$$\text{Let } \vec{a} = 5\hat{i} + 6\hat{j} + 7\hat{k}$$

$$\vec{b} = 7\hat{i} - 8\hat{j} + 9\hat{k}$$

$$\vec{c} = 3\hat{i} + 20\hat{j} + 5\hat{k}$$

We are able to write

$$\vec{a} = s\vec{b} + t\vec{c} \quad (s, t \text{ are scalars})$$

$$5\hat{i} + 6\hat{j} + 7\hat{k} = s(7\hat{i} - 8\hat{j} + 9\hat{k}) + t(3\hat{i} + 20\hat{j} + 5\hat{k})$$

Equating $\hat{i}, \hat{j}, \hat{k}$ components

$$5 = 7s + 3t \quad \text{--- (1)}$$

$$6 = -8s + 20t \quad \text{--- (2)}$$

$$7 = 9s + 5t \quad \text{--- (3)}$$

$$\frac{(2)}{2} \Rightarrow 3 = -4s + 10t \quad \text{--- (4)}$$

Solving (1) & (4)

$$(1) \times 4 \Rightarrow 28s + 12t = 20$$

$$(4) \times 7 \Rightarrow -28s + 70t = 21$$

$$82t = 41$$

$$t = \frac{1}{2}$$

$t = \frac{1}{2}$ in (1) we get

$$s = \frac{1}{2}$$

$$s = \frac{1}{2}, t = \frac{1}{2} \text{ in (3)}$$

From (3) $\Rightarrow$

$$LHS = 7$$

$$RHS = 9(\frac{1}{2}) + 5(\frac{1}{2})$$

$$= \frac{9}{2} + \frac{5}{2} = \frac{14}{2} = 7$$

$$LHS = RHS$$

$\Rightarrow \therefore$ one vector is a linear combination of other two vectors.
 $\Rightarrow \vec{a}, \vec{b}, \vec{c}$ are coplanar.

(10)

Let the given points A, B, C and D. To Prove that the points A, B, C and D are coplanar.

We have to prove that $\vec{AB}, \vec{AC}$ and $\vec{AD}$ are coplanar.

$$\begin{aligned} \vec{OA} &= 4\hat{i} + 5\hat{j} + \hat{k} & \vec{OB} &= -\hat{j} - \hat{k} \\ \vec{OC} &= 3\hat{i} + 9\hat{j} + 4\hat{k} & \vec{OD} &= -4\hat{j} + \hat{i} + 4\hat{k} \end{aligned}$$

$$\vec{AB} = \vec{OB} - \vec{OA} = (-\hat{j} - \hat{k}) - (4\hat{i} + 5\hat{j} + \hat{k})$$

$$\vec{AB} = -4\hat{i} - 6\hat{j} - 2\hat{k} = \vec{a}$$

$$\vec{AC} = \vec{OC} - \vec{OA} = (3\hat{i} + 9\hat{j} + 4\hat{k}) - (4\hat{i} + 5\hat{j} + \hat{k})$$

$$\vec{AC} = -\hat{i} + 4\hat{j} + 3\hat{k} = \vec{b}$$

$$\vec{AD} = \vec{OD} - \vec{OA} = (-4\hat{j} + \hat{i} + 4\hat{k}) - (4\hat{i} + 5\hat{j} + \hat{k})$$

$$\vec{AD} = -3\hat{i} - 9\hat{j} + 3\hat{k} = \vec{c}$$

To Prove $\vec{a}, \vec{b}, \vec{c}$ are coplanar. we have to prove that

$$\vec{a} = s\vec{b} + t\vec{c}$$

Let

$$-4\hat{i} - 6\hat{j} - 2\hat{k} = s(-\hat{i} + 4\hat{j} + 3\hat{k}) + t(-3\hat{i} - 9\hat{j} + 3\hat{k})$$

Equating $\hat{i}, \hat{j}, \hat{k}$ components

$$-4 = -s - 3t$$

$$s + 3t = 4 \quad \text{--- (1)}$$

$$-6 = 4s - t$$

$$4s - t = -6 \quad \text{--- (2)}$$

$$3s + 3t = -2 \quad \text{--- (3)}$$

solving (1) & (2)

$$s + 8t = 4$$

$$\textcircled{2} \Rightarrow 32s - 8t = -48$$

$$33s = -44$$

$$s = \frac{-44}{33}$$

$$\boxed{s = -4/3}$$

$$s = -4/3 \text{ in (1)}$$

$$-4/3 + 8t = 4$$

$$8t = 4 + 4/3$$

$$8t = \frac{12+4}{3}$$

$$8t = 16/3$$

$$t = \frac{16/3}{8} = 2/3$$

$$\boxed{t = 2/3}$$

$$s = -4/3 \text{ and } t = 2/3 \text{ in (3)}$$

$$\underline{\text{LHS}} \quad -2$$

RHS

$$3(-4/3) + 3(2/3)$$

$$= -4 + 2$$

$$= -2$$

$$\text{LHS} = \text{RHS}$$

We are able to express vector $\vec{c}$ as a linear combination of the other two vectors.

$\Rightarrow$ The given vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar.

$\Rightarrow$ The given points A, B, C, D are coplanar.

(9)

11 Given

$$\vec{a} = 2\hat{i} + 3\hat{j} - 4\hat{k}$$

$$\vec{b} = 3\hat{i} - \hat{j} - 5\hat{k}$$

$$\vec{c} = -3\hat{i} + 2\hat{j} + 3\hat{k}$$

$$(i) \vec{a} + \vec{b} + \vec{c} =$$

$$(2\hat{i} + 3\hat{j} - 4\hat{k}) + (3\hat{i} - \hat{j} - 5\hat{k}) + (-3\hat{i} + 2\hat{j} + 3\hat{k})$$

$$= 2\hat{i} + \hat{j} - 6\hat{k}$$

$$\boxed{\vec{a} + \vec{b} + \vec{c} = 2\hat{i} + \hat{j} - 6\hat{k}}$$

magnitude :-

$$|\vec{a} + \vec{b} + \vec{c}| = |2\hat{i} + \hat{j} - 6\hat{k}|$$

$$= \sqrt{(2)^2 + (1)^2 + (-6)^2}$$

$$= \sqrt{4+1+36}$$

$$\boxed{|\vec{a} + \vec{b} + \vec{c}| = \sqrt{41}}$$

Direction cosines are $\left(\frac{2}{\sqrt{41}}, \frac{1}{\sqrt{41}}, \frac{-6}{\sqrt{41}}\right)$

(ii)

$$\vec{a} = 2\hat{i} + 3\hat{j} - 4\hat{k}$$

$$\boxed{3\vec{a} = 6\hat{i} + 9\hat{j} - 12\hat{k}}$$

$$\vec{b} = 3\hat{i} - \hat{j} - 5\hat{k}$$

$$\boxed{-2\vec{b} = -6\hat{i} + 2\hat{j} + 10\hat{k}}$$

$$\vec{c} = -3\hat{i} + 2\hat{j} + 3\hat{k}$$

$$\boxed{5\vec{c} = -15\hat{i} + 10\hat{j} + 15\hat{k}}$$

$$3\vec{a} - 2\vec{b} + 5\vec{c} =$$

$$(6\hat{i} + 9\hat{j} - 12\hat{k}) + (-6\hat{i} + 2\hat{j} + 10\hat{k})$$

$$+ (-15\hat{i} + 10\hat{j} + 15\hat{k})$$

$$= 6\hat{i} + 9\hat{j} - 12\hat{k} - 6\hat{i} + 2\hat{j} + 10\hat{k} - 15\hat{i} + 10\hat{j} + 15\hat{k}$$

$$= -15\hat{i} + 21\hat{j} + 13\hat{k}$$

$$\boxed{3\vec{a} - 2\vec{b} + 5\vec{c} = -15\hat{i} + 21\hat{j} + 13\hat{k}}$$

Magnitude :-

$$|3\vec{a} - 2\vec{b} + 5\vec{c}| = \sqrt{(-15)^2 + (21)^2 + (13)^2}$$

$$|3\vec{a} - 2\vec{b} + 5\vec{c}| = \sqrt{1123}$$

Direction cosine are

$$\left(\frac{-15}{\sqrt{1123}}, \frac{27}{\sqrt{1123}}, \frac{13}{\sqrt{1123}} \right)$$

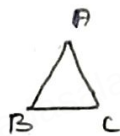
12)

Let A, B, C be the Position Vectors. (i.e)

$$\vec{OA} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\vec{OB} = 3\hat{i} - 4\hat{j} + 5\hat{k}$$

$$\vec{OC} = -2\hat{i} + 3\hat{j} - 7\hat{k}$$



$$\vec{AB} = \vec{OB} - \vec{OA}$$

$$= (3\hat{i} - 4\hat{j} + 5\hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k})$$

$$\vec{AB} = 2\hat{i} - 6\hat{j} + 2\hat{k} \Rightarrow |\vec{AB}| = \sqrt{14}$$

$$\vec{BC} = \vec{OC} - \vec{OB}$$

$$= -2\hat{i} + 3\hat{j} - 7\hat{k} - (3\hat{i} - 4\hat{j} + 5\hat{k})$$

$$= -2\hat{i} + 3\hat{j} - 7\hat{k} - 3\hat{i} + 4\hat{j} - 5\hat{k}$$

$$\vec{BC} = -5\hat{i} + 7\hat{j} - 12\hat{k}$$

$$|\vec{BC}| = \sqrt{(-5)^2 + (7)^2 + (-12)^2}$$

$$= \sqrt{25 + 49 + 144}$$

$$|\vec{BC}| = \sqrt{218}$$

$$\vec{CA} = \vec{OA} - \vec{OC}$$

$$= \hat{i} + 2\hat{j} + 3\hat{k} - (-2\hat{i} + 3\hat{j} - 7\hat{k})$$

$$= \hat{i} + 2\hat{j} + 3\hat{k} + 2\hat{i} - 3\hat{j} + 7\hat{k}$$

$$\vec{CA} = 3\hat{i} - \hat{j} + 10\hat{k}$$

$$|\vec{CA}| = \sqrt{(3)^2 + (-1)^2 + (10)^2}$$

$$= \sqrt{9 + 1 + 100} = \sqrt{110}$$

$$|\vec{CA}| = \sqrt{110}$$

Perimeter of the triangle =

$$|\vec{AB}| + |\vec{BC}| + |\vec{CA}|$$

$$= \sqrt{14} + \sqrt{218} + \sqrt{110}$$

10

13) $\vec{a} = 3\hat{i} - \hat{j} - 4\hat{k}$

$$\vec{b} = -2\hat{i} + 4\hat{j} - 3\hat{k}$$

$$\vec{c} = \hat{i} + 2\hat{j} - \hat{k}$$

$$3\vec{a} - 2\vec{b} + 4\vec{c} = 3(3\hat{i} - \hat{j} - 4\hat{k})$$

$$- 2(-2\hat{i} + 4\hat{j} - 3\hat{k}) + 4(\hat{i} + 2\hat{j} - \hat{k})$$

$$= 9\hat{i} - 3\hat{j} - 12\hat{k} + 4\hat{i} - 8\hat{j} + 6\hat{k}$$

$$+ 4\hat{i} + 8\hat{j} - 4\hat{k}$$

$$= 17\hat{i} - 3\hat{j} - 10\hat{k}$$

Let $\vec{r} = 3\vec{a} - 2\vec{b} + 4\vec{c}$

Then $\vec{r} = 17\hat{i} - 3\hat{j} - 10\hat{k}$

Unit vectors Parallel to

$$\vec{r} \text{ is } \frac{\vec{r}}{|\vec{r}|}$$

$$|\vec{r}| = \sqrt{17^2 + (-3)^2 + 10^2} = \sqrt{289 + 9 + 100}$$

$$|\vec{r}| = \sqrt{398}$$

Unit vectors Parallel

$$\text{to } \vec{r} = \pm \frac{17\hat{i} - 3\hat{j} - 10\hat{k}}{\sqrt{398}}$$

14)

Given

$$2\vec{a} - 7\vec{b} + 5\vec{c} = 0$$

$$2\vec{a} + 5\vec{c} = 7\vec{b}$$

$$\div 7 \Rightarrow \frac{2}{7}\vec{a} + \frac{5}{7}\vec{c} = \vec{b}$$

$$s\vec{a} + t\vec{c} = \vec{b}$$

$\therefore \vec{b}$ is a linear combination of $\vec{a}$ and $\vec{c}$.

$\therefore \vec{a}, \vec{b}, \vec{c}$ are collinear.

15)

Given

$$\vec{OP} = \hat{i} + \hat{j} + \hat{k}$$

$$\vec{OA} = 2\hat{i} + 5\hat{k}$$

$$\vec{OR} = 3\hat{i} + 2\hat{j} - 3\hat{k}$$

$$\vec{OS} = \hat{i} - 6\hat{j} - \hat{k}$$

$$\vec{PA} = \vec{OA} - \vec{OP}$$

$$= (2\hat{i} + 5\hat{j}) - (\hat{i} + \hat{j} + \hat{k})$$

$$= 2\hat{i} + 5\hat{j} - \hat{i} - \hat{j} - \hat{k}$$

$$\boxed{\vec{PA} = \hat{i} + 4\hat{j} - \hat{k}}$$

$$\vec{RS} = \vec{OS} - \vec{OR}$$

$$= \hat{i} - 6\hat{j} - \hat{k} - (3\hat{i} + 2\hat{j} - 3\hat{k})$$

$$= \hat{i} - 6\hat{j} - \hat{k} - 3\hat{i} - 2\hat{j} + 3\hat{k}$$

$$= -2\hat{i} - 8\hat{j} + 2\hat{k}$$

$$= -2(\hat{i} + 4\hat{j} - \hat{k})$$

$$= -2(\vec{PA})$$

$$\vec{RS} = -2(\vec{PA})$$

Hence $\vec{PA} \parallel \vec{RS}$.

16)

$$\text{Let } \vec{a} = m(\hat{i} + \hat{j} + \hat{k})$$

$$|\vec{a}| = m\sqrt{1^2 + 1^2 + 1^2}$$

$$|\vec{a}| = m\sqrt{3}$$

To make $\vec{a}$ as a unit vector

$$\text{Thus } |\vec{a}| = 1$$

$$\therefore m\sqrt{3} = \pm 1$$

$$\boxed{m = \pm \frac{1}{\sqrt{3}}}$$

17)

$$\text{Let } \vec{OA} = \hat{i} + \hat{j} + \hat{k}$$

$$\vec{OB} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\vec{OC} = 2\hat{i} - \hat{j} + \hat{k}$$

$$\vec{AB} = \vec{OB} - \vec{OA}$$

$$= (\hat{i} + 2\hat{j} + 3\hat{k}) - (\hat{i} + \hat{j} + \hat{k})$$

$$\vec{AB} = \hat{j} + 2\hat{k}$$

$$|\vec{AB}| = \sqrt{1^2 + 2^2} = \sqrt{5}$$

$$\boxed{|\vec{AB}| = \sqrt{5}}$$

$$\vec{BC} = \vec{OC} - \vec{OB}$$

$$= 2\hat{i} - \hat{j} + \hat{k} - (\hat{i} + 2\hat{j} + 3\hat{k})$$

$$\vec{BC} = \hat{i} - 3\hat{j} - 2\hat{k}$$

$$|\vec{BC}| = \sqrt{1^2 + (-3)^2 + (-2)^2}$$

$$= \sqrt{1+9+4}$$

$$\boxed{|\vec{BC}| = \sqrt{14}}$$

$$\vec{CA} = \vec{OA} - \vec{OC}$$

$$= (\hat{i} + \hat{j} + \hat{k}) - (2\hat{i} - \hat{j} + \hat{k})$$

$$\vec{CA} = -\hat{i} + 2\hat{j}$$

$$|\vec{CA}| = \sqrt{(-1)^2 + (2)^2} = \sqrt{1+4} = \sqrt{5}$$

$$\boxed{|\vec{CA}| = \sqrt{5}}$$

$$|\vec{AB}| = |\vec{CA}| \text{ Therefore}$$

The given points form an isosceles triangle.

VECTOR ALGEBRA -
[8.1 & 8.2]
R. VIJAYARAGHAVAN MSc, MEDICAL
PG ASST IN MATHEMATICS
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A black and white portrait of a man with a mustache, wearing a light-colored button-down shirt and dark trousers. He is sitting and looking directly at the camera. A smartphone is visible in his left shirt pocket. He is wearing a watch on his left wrist and rings on both hands. The background is slightly blurred, showing an indoor setting.

R. Vijayaragavan
M.sc., M.ed., D.ted.,
Pg asst in maths.

Class: XI

UNIT XII

PROBABILITY THEORYEXERCISE XII-I [12.1]

1)

(i) $P(A) = 0.15$ $P(B) = 0.30$

$P(C) = 0.43$ $P(D) = 0.12$

$P(A) + P(B) + P(C) + P(D) > 1$

Also,

$P(S) = P(A) + P(B) + P(C) + P(D)$

$= 0.15 + 0.30 + 0.43 + 0.12$

$P(S) = 1.0$

 $\therefore$ The Assignment of Probability is Permissible

(ii) Given $P(A) = 0.22$ $P(B) = 0.38$

$P(C) = 0.16$ $P(D) = 0.34$

$P(A) + P(B) + P(C) + P(D) > 1$

Also,

$P(S) = P(A) + P(B) + P(C) + P(D)$

$= 0.22 + 0.38 + 0.16 + 0.34$

$= 1.1$

$P(S) \neq 1$

 $\therefore$ The Assignment of Probability is not Permissible.

(iii) Given $P(A) = \frac{2}{5}$ $P(B) = \frac{3}{5}$

$P(C) = -\frac{1}{5}$ $P(D) = \frac{1}{5}$

$P(A) \geq 0$ $P(B) \geq 0$

$P(C) \geq 0$ But $P(C) = -\frac{1}{5} < 0$

 $P(C)$ is Negative, Thus

The Assignment of Probability is not Permissible.

(2)

$S = \{HH, HT, TH, TT\}$

$n(S) = 4$

(i) Let A be the event of getting one head and one tail.

$\therefore A = \{HT, TH\}$

$n(A) = 2$

$P(A) = \frac{n(A)}{n(S)} = \frac{2}{4} = \frac{1}{2}$

$P(A) = \frac{1}{2}$

(ii) Let B be the event of getting at most two tails

$B = \{HH, HT, TH, TT\}$

$n(B) = 4$

$P(B) = \frac{n(B)}{n(S)} = \frac{4}{4} = 1$

$P(B) = 1$

3)

$S = \{5 \text{ mangoes, 4 apples}\}$

$n(S) = (5+4)C_2$

$n(S) = 9C_2$

[Since two Fruits taken from 9 Fruits] $(9)C_2$

$n(S) = 9C_2 = \frac{9 \times 8}{2 \times 1} = 36$

$n(S) = 36$

(i) Let A be the event getting one mango and one apple.

$$n(A) = {}^5C_1 \times {}^4C_1 \\ = 5 \times 4 = 20$$

$$n(A) = 20$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{20}{36} = \frac{5}{9}$$

$$P(A) = 5/9$$

(ii) Let B be the event of getting 2 fruits from the same variety.

$$n(B) = {}^5C_2 + {}^4C_2 \\ = \frac{5 \times 4}{2 \times 1} + \frac{4 \times 3}{2 \times 1} = \frac{10}{2} + \frac{12}{2}$$

$$n(B) = 16$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{16}{36} = \frac{4}{9}$$

$$P(B) = 4/9$$

4) Non leap year consists of 365 days.

$$365 \text{ days} = 52 \text{ weeks} + 1 \text{ day.}$$

Remaining one Sunday can be selected from following days.

$$S = \{\text{Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}\}$$

$$n(S) = 7$$

$$\text{Here } n(A) = 1 \text{ [one Sunday]}$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{7}$$

$$P(A) = 1/7$$

(ii) Leap year consists of 366 days.

$$366 \text{ days} = 52 \text{ weeks} + 2 \text{ days.}$$

These two days can be selected from following days.

$$S = \{(S-M), (M-T), (T-W), (W-TH), (TH-F), (F-S), (SAT-Sunday)\}$$

$$n(S) = 7$$

$$B = \{(\text{Sunday-Monday}), (\text{Saturday-Sunday})\}$$

$$n(B) = 2$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{2}{7}$$

$$P(B) = 2/7$$

5.

$$n(S) = 2^8 = 256. \text{ Here } n = 8$$

(i) Let A be the event of getting exactly two tails.

$$n(A) = {}^8C_2 = \frac{8 \times 7}{1 \times 2} = 28$$

$$n(A) = 28$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{28}{256} = \frac{7}{64}$$

(ii) Let B be the event of getting at least two tails.

$$n(B) = {}^8C_2 + {}^8C_3 + {}^8C_4 + {}^8C_5 + {}^8C_6 + {}^8C_7 + {}^8C_8$$

$$n(B) = n(S) - [{}^8C_0 + {}^8C_1]$$

$$= 256 - [1 + 8]$$

$$= 256 - 9$$

$$n(B) = 247$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{247}{256}$$

$$P(B) = \frac{247}{256}$$

(iii) Let C be the event of getting at most two tails.

$$n(C) = 8C_0 + 8C_1 + 8C_2$$

$$n(C) = 1 + 8 + \frac{8 \times 7}{2 \times 1}$$

$$= 1 + 8 + 28$$

$$n(C) = 37$$

$$P(C) = \frac{n(C)}{n(S)} = \frac{37}{256}$$

$$P(C) = 37/256$$

6)

Let $S = \{1, 2, 3, \dots, 100\}$.

$$n(S) = 100$$

Let A be the event of getting a Prime number

$$A = \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97\}$$

$$n(A) = 25$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{25}{100}$$

Let B be the event of getting multiple of 8

$$B = \{8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96\}$$

$$n(B) = 12$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{12}{100}$$

A and B are mutually exclusive events thus $P(A \cap B) = 0$

$$P(A \cup B) = P(A) + P(B)$$

$$= \frac{25}{100} + \frac{12}{100}$$

$$P(A \cup B) = \frac{37}{100}$$

$$P(A \cup B) = \frac{37}{100}$$

7) $S = \{7 \text{ Red, } 4 \text{ Black balls}\}$

$$n(S) = 11C_3 =$$

[3 balls drawn out of 11 balls]

(i) Let A be the events of getting 3 Red balls.

$$n(A) = 7C_3$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{7C_3}{11C_3}$$

$$= \frac{7 \times 6 \times 5}{1 \times 2 \times 3} \times \frac{1}{11 \times 10 \times 9}$$

$$= \frac{5 \times 2 \times 1}{5 \times 2 \times 1}$$

$$P(A) = \frac{7}{33}$$

(ii) Let B be the event of getting one red and 2 black balls

$$n(B) = 7C_1 \times 4C_2$$

$$= 7 \times \frac{4 \times 3}{1 \times 2} = 42$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{42}{11C_3}$$

$$P(B) = \frac{14}{55}$$

(8) $S = \{\text{Pack of 52 cards}\}$

$$n(S) = 52$$

$$\begin{aligned} \text{(i) } P(\text{ace card or a king card}) \\ &= P(\text{ace card}) + P(\text{king card}) \\ &= \frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \frac{2}{13} \end{aligned}$$

$$\begin{aligned} \text{(ii) } P(\text{card will be 6 or smaller}) \\ &= \frac{5+5+5+5}{52} \quad \left[\begin{array}{l} 5 \text{ cards with one} \\ 6 \text{ or smaller from} \\ \text{each variety} \end{array} \right] \\ &= \frac{20}{52} = \frac{5}{13} \end{aligned}$$

$$P(\text{card will be 6 or smaller}) = \frac{5}{13}$$

$$\begin{aligned} \text{(iii) } P(\text{queen card or 9}) \\ &= \frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \frac{2}{13} \end{aligned}$$

(9)

$S = \{\text{Members of Cricket Club}\}$

$$n(S) = 16 (11)$$

Let A be the event of selecting at least 3 bowlers.

At least 3 bowlers can be selected as follows.

	5 Bowlers	11 Non Bowlers		
(i)	3	8	${}^5C_3 \times {}^{11}C_8$	1650
(ii)	4	7	${}^5C_4 \times {}^{11}C_7$	1650
(iii)	5	6	${}^5C_5 \times {}^{11}C_6$	462
				3762

$$\begin{aligned} {}^{16}C_{11} &= {}^{16}C_{16-11} \\ &= {}^{16}C_5 \end{aligned}$$

$$= \frac{16 \times 15 \times 14 \times 13 \times 12}{5 \times 4 \times 3 \times 2 \times 1}$$

$$n(S) = 4368$$

No. of Favourable Cases = 3762

$$\begin{aligned} P(A) &= \frac{n(A)}{n(S)} \\ &= \frac{3762}{4368} \end{aligned}$$

$$P(A) = \frac{627}{728}$$

R. VIJAYARAGHAVAN MSC, MED, DTE

PG-ASST IN MATHEMATICS

ARUNODHAYA HIGHER SEC. SCHOOL,

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10) The odds of an event A are $a:b$ in favour of an event and

$$P(A) = \frac{a}{a+b}$$

(i) Given that odds of an event A occurs is $5:7$

$$P(A) = \frac{5}{5+7} = \frac{5}{12}$$

(ii) $P(B) = \frac{2}{5} = \frac{2}{2+3}$

The odds that the event B occurs is 2 to 3.

Probability Theory

R.V

EXERCISE - 12.2

D Given A and B are mutually exclusive events.
Thus $P(A \cap B) = 0$.

$$P(A) = \frac{3}{8} \quad P(B) = \frac{1}{8}$$

(i) $P(\bar{A})$:-

$$P(A) + P(\bar{A}) = 1$$

$$\frac{3}{8} + P(\bar{A}) = 1$$

$$P(\bar{A}) = 1 - \frac{3}{8}$$

$$P(\bar{A}) = \frac{5}{8}$$

(ii) $P(A \cup B)$:-

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{3}{8} + \frac{1}{8} - 0$$

$$= \frac{4}{8} = \frac{1}{2}$$

$$P(A \cup B) = \frac{1}{2}$$

(iii) $P(\bar{A} \cap B)$:-

$$P(\bar{A} \cap B) = P(B) - P(A \cap B)$$

$$= \frac{1}{8} - 0$$

$$P(\bar{A} \cap B) = \frac{1}{8}$$

(iv) $P(\bar{A} \cup \bar{B})$:-

$$P(\bar{A} \cup \bar{B}) = P(\overline{A \cap B})$$

$$= 1 - P(A \cap B)$$

$$= 1 - 0$$

$$P(\bar{A} \cup \bar{B}) = 1$$

2)

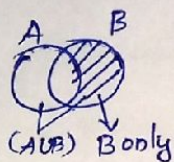
$$\text{Given } P(A) = 0.85$$

$$P(A \cup B) = 0.85$$

$$P(A \cup B) = 0.85$$

$$P(A \text{ and } B) = 0.15$$

$$P(A \cap B) = 0.15$$

(i) $P(B \text{ only})$ $P(B \text{ only})$ 

$$= P(A \cup B) - P(A)$$

$$= 0.85 - 0.35$$

$$P(B \text{ only}) = 0.50$$

(ii) $P(\bar{B})$:-

$$P(\bar{B}) = P(A \cup B) - P(B)$$

$$= 0.85 - 0.50$$

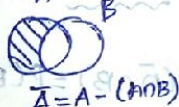
$$P(\bar{B}) = 0.35$$

(iii) $P(\text{only } A)$:-

$$P(A) - P(A \cap B)$$

$$= 0.85 - 0.15$$

$$P(\text{only } A) = 0.20$$



3)

$$S = \{(1,1), (1,2), \dots, (6,6)\}$$

$$n(S) = 36$$

Let A be the event of first die shows 5

$$A = \{(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)\}$$

$$n(A) = 6$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{36}$$

$$P(A) = \frac{6}{36}$$

Let B be the event of second die shows 5.

$$B = \{(1,5), (2,5), (3,5), (4,5), (5,5), (6,5)\}$$

$$n(B) = 6$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{6}{36}$$

$$P(B) = \frac{6}{36}$$

$$A \cap B = \{(5,5)\}$$

$$n(A \cap B) = 1 \Rightarrow P(A \cap B) = \frac{1}{36}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{6}{36} + \frac{6}{36} - \frac{1}{36}$$

$$P(A \cup B) = \frac{11}{36}$$

4)

$$P(A) = 0.5$$

$$P(B) = 0.3$$

A and B are mutually exclusive events. $P(A \cap B) = 0$.

(i) $P(A \cup B)$:-

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 0.5 + 0.3 - 0$$

$$P(A \cup B) = 0.8$$

(ii) $P(A \cap \bar{B})$:-

$$= 0.5 - 0$$

$$P(A \cap \bar{B}) = 0.5$$

(iii) $P(\bar{A} \cap B) = ?$

$$P(\bar{A} \cap B) = P(B) - P(A \cap B) \\ = 0.3 - 0$$

$$P(\bar{A} \cap B) = 0.3$$

5)

Given $P(A) = 0.96$

$$P(B) = 0.96$$

$$P(A \cap B) = P(A) \cdot P(B)$$

$$= 0.96 \times 0.96$$

$$P(A \cap B) = 0.9216$$

P (Fire engine is available when needed)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \\ = 0.96 + 0.96 - 0.9216$$

$$P(A \cup B) = 0.9984$$

(ii) P (Neither fire engine is available when needed)

$$P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B) \\ = 1 - 0.9984$$

$$P(\bar{A} \cap \bar{B}) = 0.0016$$

6)

Let A be the event of getting an award for its design.

Let B be the event of getting an award for efficient use of materials.

$$P(A) = 0.48$$

$$P(B) = 0.36$$

$$P(A \cap B) = 0.4$$

(i) P (getting at least one of the two awards) :-

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \\ = 0.48 + 0.36 - 0.2$$

$$P(A \cup B) = 0.64$$

(ii) P (getting only one of the awards) :-

$$P(A \cap \bar{B}) + P(\bar{A} \cap B)$$

$$= [P(A) - P(A \cap B)] + [P(B) - P(A \cap B)]$$

$$= P(A) + P(B) - 2P(A \cap B)$$

$$= 0.48 + 0.36 - 2(0.2)$$

$$= 0.84 - 0.40$$

$$= 0.44$$

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EXERCISE - 12.3

1) Two events cannot be mutually exclusive and independent simultaneously.

Since mutually exclusive events Thus $P(A \cap B) = 0$ and Independent events

$$P(A \cap B) = P(A) \cdot P(B)$$

2) Given

$$P(A \cup B) = 0.7$$

$$P(A \cap B) = 0.2$$

$$P(B) = 0.5$$

We have to prove that A and B are independent events.

$$\text{ie) } P(A \cap B) = P(A) \cdot P(B)$$

$$\text{Since, } P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$0.7 = P(A) + 0.5 - 0.2$$

$$0.7 = P(A) + 0.3$$

$$P(A) = 0.7 - 0.3$$

$$\boxed{P(A) = 0.4}$$

$$P(A \cap B) = 0.2 \quad \text{(i)}$$

$$P(A) \cdot P(B) = (0.4)(0.5)$$

$$= 0.2 \quad \text{(ii)}$$

From (i) & (ii)

$$P(A \cap B) = P(A) \cdot P(B) \text{ Thus}$$

A and B are Independent

events.

3) Given A and B are independent events.

$$P(A \cap B) = P(A) \cdot P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = P(A) + P(B) - P(A) \cdot P(B)$$

$$P(A \cup B) = P(A) + P(B) [1 - P(A)]$$

$$0.6 = 0.2 + P(B) [1 - 0.2]$$

$$0.6 - 0.2 = P(B) [0.8]$$

$$P(B) = \frac{0.4}{0.8} = \frac{1}{2}$$

$$\boxed{P(B) = 0.5}$$

4) $P(A) = 0.5$

$$P(B) = 0.8$$

$$P(B|A) = 0.8$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$0.8 = \frac{P(A \cap B)}{0.5}$$

$$\boxed{P(A \cap B) = 0.4}$$

$$\text{ii) } P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{0.4}{0.8} = \frac{1}{2}$$

$$\boxed{P(A|B) = 0.5}$$

$$\text{ii) } P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 0.5 + 0.8 - 0.4$$

$$= 1.3 - 0.4$$

$$\boxed{P(A \cup B) = 0.9}$$

5) Given $P(A) = \frac{3}{4}$
 $P(B) = \frac{2}{5}$

$$A \cup B = S$$

$$P(A \cup B) = P(S) \quad [P(S) = 1]$$

$$P(A) + P(B) - P(A \cap B) = 1$$

$$\frac{3}{4} + \frac{2}{5} - P(A \cap B) = 1$$

$$\frac{3}{4} + \frac{2}{5} - 1 = P(A \cap B)$$

$$\boxed{P(A \cap B) = \frac{3}{20}}$$

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{3/20}{2/5} = \frac{3}{20} \times \frac{5}{2} = \frac{3}{8}$$

$$\boxed{P(A/B) = \frac{3}{8}}$$

6)

Let $[A \text{ solves the Problem}]$

$$\boxed{P(A) = \frac{1}{3}}$$

$P[A \text{ will not solve the Problem}]$

$$P(\bar{A}) = 1 - \frac{1}{3} = \frac{2}{3}$$

$P[B \text{ solves the Problem}]$

$$\boxed{P(B) = \frac{1}{4}}$$

$P[B \text{ will not solve the Problem}]$

$$P(\bar{B}) = 1 - \frac{1}{4} = \frac{3}{4}$$

$P[C \text{ solve the Problem}]$

$$\boxed{P(C) = \frac{1}{5}}$$

$P[C \text{ will not solve the Problem}]$

$$P(\bar{C}) = 1 - \frac{1}{5} = \frac{4}{5}$$

$$\begin{aligned} P(\text{None of them will solve the Problem}) &= P(\bar{A} \cap \bar{B} \cap \bar{C}) \\ &= P(\bar{A}) \cdot P(\bar{B}) \cdot P(\bar{C}) \\ &= \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \\ &= \frac{2}{5} \end{aligned}$$

(i) $P(\text{Problem is solved})$

$$\begin{aligned} &= 1 - P(\text{None of them will solve the Problem}) \\ &= 1 - \frac{2}{5} \\ &= \frac{3}{5} \end{aligned}$$

(ii) $P(\text{Exactly one of them will solve it})$

$$\begin{aligned} &= P(A \cap \bar{B} \cap \bar{C}) + P(\bar{A} \cap B \cap \bar{C}) + P(\bar{A} \cap \bar{B} \cap C) \\ &= \frac{1}{3} \times \frac{3}{4} \times \frac{4}{5} + \frac{2}{3} \times \frac{1}{4} \times \frac{4}{5} + \frac{2}{3} \times \frac{3}{4} \times \frac{1}{5} \\ &= \frac{12}{60} + \frac{8}{60} + \frac{6}{60} = \frac{26}{60} \\ &= \frac{13}{30} \end{aligned}$$

Since $P(A \cap \bar{B} \cap \bar{C}) = P(A) \cdot P(\bar{B}) \cdot P(\bar{C})$.
 And A and B and C are independent events, $\therefore P(A \cap \bar{B} \cap \bar{C}) = P(A) \cdot P(\bar{B}) \cdot P(\bar{C})$

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7)

Let B be the event of a Car being filled with Petrol will also need an oil change.

$$P(B) = 0.30$$

Let A be the event of a Car needs a new oil filter

$$P(A) = 0.40$$

$$P(A \cap B) = 0.15$$

i) If the oil had to be changed The Probability That a new oil filter is needed

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.15}{0.30} = \frac{1}{2}$$

$$P(A \cup B) = 0.5$$

ii) If a new filter is needed The Probability that oil has to be changed

$$P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{0.15}{0.40}$$

$$P(B/A) = 0.375$$

8)

Bag I has 5 white, 3 black balls (Total 8 balls)

Bag II has 4 white, 6 black balls (Total 10 balls)

$$P(\text{white ball from Bag I}) = \frac{5}{8}$$

$$P(\text{white ball from Bag II}) = \frac{4}{10}$$

Required Probability:-

$$\frac{5}{8} \times \frac{4}{10} = \frac{20}{80} = \frac{1}{4}$$

$$P(\text{Black ball from Bag I}) = \frac{3}{8}$$

$$P(\text{Black ball from Bag II}) = \frac{6}{10}$$

Required Probability

$$= \frac{3}{8} \times \frac{6}{10} = \frac{18}{80} = \frac{9}{40}$$

iii) $P(\text{white ball from I Bag and Black ball from Bag II}) + P(\text{Black ball from Bag I and white ball from Bag II})$

$$= \frac{5}{8} \times \frac{6}{10} + \frac{3}{8} \times \frac{4}{10}$$

$$= \frac{30}{80} + \frac{12}{80} = \frac{42}{80} = \frac{21}{40}$$

9)

$$P(\text{selecting a boy in a class}) = \frac{2}{3}$$

$$P(\text{selecting a girl in a class}) = \frac{1}{3}$$

$$P(\text{boy getting I grade}) = 0.70$$

$$P(\text{girl getting I grade}) = 0.85$$

$$P(\text{student getting I Grade mark})$$

$$= [P(\text{selecting a boy in a class}) \times P(\text{boy getting I grade})] +$$

$$[P(\text{selecting a girl in a class}) \times P(\text{girl getting I grade})]$$

$$= \frac{2}{3} \times 0.70 + \frac{1}{3} \times 0.85$$

$$= \frac{1.4}{3} + \frac{0.85}{3}$$

$$= \frac{2.25}{3} = 0.75$$

$$= \frac{2.25}{3} = 0.75$$

10) Given $P(A) = 0.4$

$$P(A \cup B) = 0.7$$

$$P(B) = ?$$

(i) Given A and B are mutually exclusive events, Thus

$$P(A \cap B) = 0.$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$0.7 = 0.4 + P(B) - 0$$

$$P(B) = 0.7 - 0.4$$

$$P(B) = 0.3$$

(ii) A and B are Independent events, $P(A \cap B) = P(A) \cdot P(B)$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = P(A) + P(B) - P(A) \cdot P(B)$$

$$P(A \cup B) = P(A) + P(B) [1 - P(A)]$$

$$0.7 = 0.4 + P(B) [1 - 0.4]$$

$$0.7 - 0.4 = P(B) (0.6)$$

$$0.3 = P(B) (0.6)$$

$$P(B) = \frac{0.3}{0.6} = \frac{1}{2}$$

$$P(B) = 0.5$$

(iii) $P(A/B) = 0.4$

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A/B) P(B) = P(A) + P(B) - P(A \cup B)$$

$$0.4 P(B) = 0.4 + P(B) - 0.7$$

$$0.4 P(B) - P(B) = -0.3$$

$$P(B) (0.4 - 1) = -0.3$$

$$P(B) = \frac{-0.3}{-0.6} = \frac{1}{2} = 0.5$$

$$P(B) = 0.5$$

$$(iv) P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$P(B/A) = 0.5$$

$$P(B/A) \cdot P(A) = P(A) + P(B) - P(A \cup B)$$

$$(0.5) (0.4) = 0.4 + P(B) - 0.7$$

$$0.2 = P(B) - 0.3$$

$$P(B) = 0.2 + 0.3$$

$$P(B) = 0.5$$

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11) Non Leap years has 365 days = 52 weeks + 1 day

Remaining one day may be following days

[Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday]

$$n(S) = 7$$

$$n(A) = 1$$

$$P(A) = \frac{1}{7}$$

A Year is Selected Random.

It may be an ordinary year which the

Probability is $\frac{3}{4}$. (or)

A Leap year for which the Probability is $\frac{1}{4}$.

$$\text{Required Probability} = \frac{3}{4} \times \frac{1}{7} + \frac{1}{4} \times \frac{2}{7}$$

$$= \frac{3}{28} + \frac{2}{28} = \frac{5}{28}$$

(ii) Leap years which contain 53 Sundays.

A Leap year 366 days = 52 weeks + 2 days.

The two days may be { (Sun-Mon), (Mon-Tue), (Tue-Wed),

(Wed-Thurs), (Thurs-Fri), (Fri-Sat),

(Sat-Sunday) }

$$n(S) = 7$$

$$n(A) = 2$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{7}$$

$$A = \{ (\text{Sun-Mon}), (\text{Sat-Sun}) \}$$

Probability that the year selected is a leap and it contains 53 Sundays

$$= \frac{1}{4} \times \frac{2}{7} = \frac{1}{14}$$

12)

$$P(\text{target is hit by } x) = \boxed{P(x) = 3/4}$$

$$P(\bar{x}) = 1 - 3/4 = 1/4 \Rightarrow \boxed{P(\bar{x}) = 1/4}$$

$$P(\text{target is hit by } y) = \boxed{P(y) = 4/5}$$

$$P(\bar{y}) = 1 - 4/5 = 1/5 \Rightarrow \boxed{P(\bar{y}) = 1/5}$$

$$P(\text{target is hit by } z) = \boxed{P(z) = 2/3}$$

$$P(\bar{z}) = 1 - P(z) = 1 - 2/3 = 1/3 \Rightarrow \boxed{P(\bar{z}) = 1/3}$$

$P(\text{target is hit by exactly 2 hits}) =$

$$P(x \cap y \cap \bar{z}) + P(x \cap \bar{y} \cap z) + P(x \cap y \cap \bar{z})$$

$$= P(x) \cdot P(y) \cdot P(\bar{z}) + P(x) \cdot P(\bar{y}) \cdot P(z) + P(x) \cdot P(y) \cdot P(\bar{z})$$

$$= \frac{3}{4} \times \frac{4}{5} \times \frac{1}{3} + \frac{3}{4} \times \frac{1}{5} \times \frac{2}{3} + \frac{1}{4} \times \frac{4}{5} \times \frac{2}{3}$$

$$= \frac{12}{60} + \frac{6}{60} + \frac{8}{60} = \frac{26}{60} = \frac{13}{30}$$

EXERCISE - 12.4

1) Let A_1 be the event that the item one produced by Machine - I

Let A_2 be the event that the item one produced by Machine - II

Let B be the event drawn a defective item.

$$\text{Given } P(A_1) = 60/100 \quad P(B/A_1) = 2/100$$

$$P(A_2) = 40/100 \quad P(B/A_2) = 4/100$$

Since, A_1, A_2 are mutually exclusive events

$$P(B) = P(A_1) \cdot P(B/A_1) + P(A_2) \cdot P(B/A_2)$$

$$P(B) = \frac{60}{100} + \frac{2}{100} + \frac{40}{100} \times \frac{4}{100}$$

$$= \frac{120}{100} + \frac{160}{100} = 0.028$$

$$P(B) = 0.028$$

2)

Let A_1 be the event selecting urn - I

Let A_2 be the event selecting urn - II

Let B be the event of selecting the Black ball.

$$P(A_1) = \frac{1}{2} \quad P(B/A_1) = \frac{6}{10}$$

$$P(A_2) = \frac{1}{2} \quad P(B/A_2) = \frac{2}{4}$$

$$P(B) = P(A_1) \cdot P(B/A_1) + P(A_2) \cdot P(B/A_2)$$

$$= \frac{1}{2} \times \frac{6}{10} + \frac{1}{2} \times \frac{2}{4} = \frac{22}{40} = \frac{11}{20}$$

$$P(B) = \frac{11}{20}$$

(i)

$$P(A_1/B) = \frac{P(A_1) \cdot P(B/A_1)}{P(B)}$$

$$= \frac{\frac{1}{2} \times \frac{6}{10}}{\frac{11}{20}}$$

$$= \frac{3}{10} \times \frac{20}{11}$$

$$P(A_1/B) = \frac{6}{11}$$

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3) Let A_1 be the event of Production of the PVC Pipe from Plant X

Let A_2 be the event of Production of the PVC Pipe from Plant Y

Let A_3 be the event of Production of the PVC Pipe from Plant Z.

Let B be the selecting a defective Pipe

$$P(A_1) = \frac{2000}{2000+3000+5000} = \frac{2000}{10000} = \frac{2}{10} \quad \boxed{P(B/A_1) = \frac{2}{100}}$$

$$P(A_2) = \frac{3000}{2000+3000+5000} = \frac{3000}{10000} = \frac{3}{10} \quad \boxed{P(B/A_2) = \frac{4}{100}}$$

$$P(A_3) = \frac{5000}{2000+3000+5000} = \frac{5000}{10000} = \frac{5}{10} \quad \boxed{P(B/A_3) = \frac{2}{100}}$$

$$P(B) = P(A_1) \cdot P(B/A_1) + P(A_2) \cdot P(B/A_2) + P(A_3) \cdot P(B/A_3)$$

$$= \frac{2}{10} \cdot \frac{2}{100} + \frac{3}{10} \cdot \frac{4}{100} + \frac{5}{10} \cdot \frac{2}{100}$$

$$= \frac{6}{1000} + \frac{12}{1000} + \frac{10}{1000}$$

$$= \frac{28}{1000} = \frac{7}{250}$$

$$\boxed{P(B) = 0.028}$$

By Bayes' Theorem Probability of selecting defective Pipe

$$P(A_2/B) = \frac{P(A_2) \cdot P(B/A_2)}{P(B)} = \frac{\frac{3}{10} \times \frac{4}{100}}{\frac{7}{250}}$$

$$= \frac{\frac{12}{1000}}{\frac{7}{250}} = \frac{12}{1000} \times \frac{250}{7}$$

$$\boxed{P(A_2/B) = \frac{3}{7}}$$

4)

Let A_1 be the event of A becoming a Manager

Let A_2 be the event of B becoming a Manager

Let A_3 be the event of C becoming a Manager

Let B be the event of office Canteen will be improved.

$$\text{Given } P(A_1) = \frac{5}{5+3+2} = \frac{5}{10} \quad P(B/A_1) = 0.4$$

$$P(A_2) = \frac{3}{5+3+2} = \frac{3}{10} \quad P(B/A_2) = 0.5$$

$$P(A_3) = \frac{2}{5+3+2} = \frac{2}{10} \quad P(B/A_3) = 0.3$$

By Baye's Theorem,

$$P(A_2) \cdot P(B/A_2)$$

$$P(A_1) \cdot P(B/A_1) + P(A_2) \cdot P(B/A_2) + P(A_3) \cdot P(B/A_3)$$

$$\frac{5}{10} \times 0.4 + \frac{3}{10} \times 0.5 + \frac{2}{10} \times 0.3$$

$$= \frac{1.5}{10} \times \frac{10}{4.1} = \frac{1.5}{4.1} = \frac{15}{41}$$

$$P(A_2/B) = \frac{15}{41}$$

5) Let the events be defined as follows.

A_1 : Event of husband and watching the television

B : Husband is watching the television

Given $P(A_1) = 0.40$ $P(B/A_1) = 0.60$

$P(A_2) = 0.30$ $P(B/A_2) = 1 - 0.60 = 0.40$

i) $P(\text{Husband watching the television})$

$$P(B) = P(A_1) \cdot P(B/A_1) + P(A_2) \cdot P(B/A_2)$$

$$= (0.40) \cdot (0.60) + (0.30) \cdot (0.40)$$

$$= 0.24 + 0.12$$

$$= 0.36$$

$$= \frac{36}{100}$$

$$\boxed{P(B) = \frac{9}{25}}$$

ii) $P(\text{if the husband is watching, the wife is also watching the television})$

$$P(A_1/B) = \frac{P(A_1) \cdot P(B/A_1)}{P(A_1) \cdot P(B/A_1) + P(A_2) \cdot P(B/A_2)} = \frac{(0.40)(0.60)}{9/25}$$

$$= \frac{0.24}{1} \times \frac{25}{9} = \frac{24}{100} \times \frac{25}{9} = \frac{2}{3}$$

$$\boxed{P(A_1/B) = \frac{2}{3}}$$

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