Padasalai's Telegram Groups!

(தலைப்பிற்கு கீழே உள்ள லிங்கை கிளிக் செய்து குழுவில் இணையவும்!)

- Padasalai's NEWS - Group
https://t.me/joinchat/NIfCqVRBNj9hhV4wu6_NqA

- Padasalai's Channel - Group
https://t.me/padasalaichannel

- Lesson Plan - Group
https://t.me/joinchat/NIfCqVWwo5iL-21gpzrXLw

- 12th Standard - Group
https://t.me/Padasalai_12th

- 11th Standard - Group
https://t.me/Padasalai_11th

- 10th Standard - Group
https://t.me/Padasalai_10th

- 9th Standard - Group
https://t.me/Padasalai_9th

- 6th to 8th Standard - Group
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- 1st to 5th Standard - Group
https://t.me/Padasalai_1to5

- TET - Group
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- PGTRB - Group
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- TNPSC - Group
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X MATHS

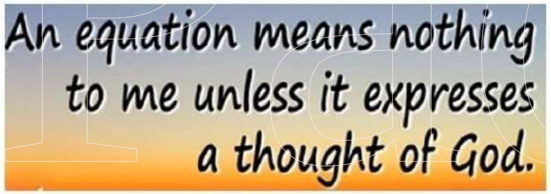
CONFIDENT

TWO MARKS

STUDY MATERIAL

ENGLISH MEDIUM

9 QUESTION – 18 MARKS



WITH

YOUR HAPPY.......

A ZAYAAN ABDUL M.SC.,B.ED.,
FATIMA MATRIC.HR.SEC.SCHOOL
JAYANKONDAM, ARIYALUR DT.
9524103797

1. RELATIONS AND FUNCTION

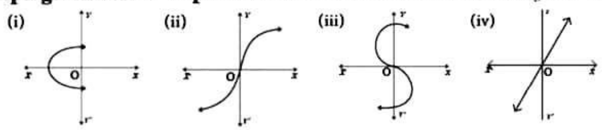
Let A = {1, 2, 3, 4} and B = N. Let $f : A \to B$ be defined by $f(x) = x^3$ then, (i) find the range of f (ii) identify the type of function

Solution : A = {1, 2, 3, 4}, B = N $f(x) = x^3$ $x = 1 \Rightarrow f(1) = 1$ $x = 3 \Rightarrow f(3) = 27$

$x = 2 \Rightarrow f(2) = 8$ $x = 4 \Rightarrow f(4) = 64$

(i) Range of f = {1, 8, 27, 64} (ii) f is one-one

Determine whether the graph given below represent functions. Give reason for your answers concerning each graph.



Solution :

(i) The curve meets y-axis at 2 points. it is not function

(ii) The curve meets x-axis or y-axis at only one point. it is function

(iii) The curve meets y-axis at 2 points. it is not function

(iv) The line meets axes at origin. it is function

Show that the function $f : N \to N$ defined by $f(m) = m^2 + m + 3$ is one-one function.

Solution : Given $f : N \to N$ defined by $f(m) = m^2 + m + 3$

$m = 1 \Rightarrow f(1) = 1 + 1 + 3 = 5$ $m = 3 \Rightarrow f(3) = 9 + 3 + 3 = 15$

$m = 2 \Rightarrow f(2) = 4 + 2 + 3 = 9$ $m = 4 \Rightarrow f(4) = 16 + 4 + 3 = 23$ ....

different elements in N are different images in N $\therefore f$ is one-one function.

Show that the function $f : N \to N$ defined $f(x) = 2x - 1$ is one-one but not onto.

Solution : Given $f : N \to N$ defined by $f(x) = 2x - 1$.

$x = 1 \Rightarrow f(1) = 2 - 1 = 1$ $x = 3 \Rightarrow f(3) = 6 - 1 = 5$

$x = 2 \Rightarrow f(2) = 4 - 1 = 3$ $x = 4 \Rightarrow f(4) = 8 - 1 = 7$ ....

different elements in N are different images in N $\therefore f$ is one-one function.

$\therefore$ Range $\neq$ Co-domain. $\therefore f$ is not on-to.

Define - one - one function.

A function $f : A \to B$ is called one - one function if distinct elements of A have distinct images in B.

Define - **many-one function**

A function $f : A \to B$ is called **many-one function** if two or more elements of A have same image in B.

Define - **onto function**

A function $f : A \to B$ is said to be **onto function** if the range of f is equal to the co-domain of f.

Define - **into function**

A function $f : A \to B$ is called an **into function** if there exists atleast one element in B which is not the image of any element of A.

Define - **constant function**

A function $f : A \to B$ is called a **constant function** if the range of f contains only one element. That is $f(x) = c$, for all $x \in A$ and for some fixed $c \in B$.

Define - **identity function**

Let A be a non-empty set. Then the function $f : A \to A$ defined by $f(x) = x$ for all $x \in A$ is called an **identity function** on A and is denoted by I_A.

Define - **identity function**

A function $f : A \to B$ is called a **real valued function** if the range of f is a subset of the set of all real numbers R. That is, $f(A) \subseteq R$.

Let $f = \{(-1, 3), (0, -1), (2, -9)\}$ be a linear function from Z into Z. Find $f(x)$.

Solution : $f = \{(-1, 3), (0, -1), (2, -9)\}$ is a linear function from Z into Z.

linear function $\Rightarrow y = ax + b$

When $x = 0$, $y = -1 \Rightarrow -1 = 0 + b \quad \therefore b = -1$

When $x = -1, y = 3 \Rightarrow 3 = -a + b$ —(1) $\quad \therefore (1) \Rightarrow 3 = -a - 1 \Rightarrow a = -4$

$\therefore a = -4, b = -1 \quad \therefore y = -4x - 1$ is the required linear function.

A function f is defined by $f(x) = 3 - 2x$. Find x such that $f(x^2) = (f(x))^2$.

Solution : $f(x) = 3 - 2x$ and $f(x^2) = (f(x))^2$

$\Rightarrow 3 - 2x^2 = (3 - 2x)^2 \Rightarrow 3 - 2x^2 = 9 + 4x^2 - 12x \Rightarrow 6x^2 - 12x + 6 = 0$

$\Rightarrow x^2 - 2x + 1 = 0$

$\Rightarrow (x - 1)^2 = 0$

$\Rightarrow x = 1$ (twice)

Let X = {3, 4, 6, 8}. Determine whether the relation $R = \{(x, f(x)) \mid x \in X, f(x) = x^2 + 1\}$ is a function from X to N ?

Solution : X = {3, 4, 6, 8} $\quad R = \{(x, f(x)) \mid x \in X, f(x) = x^2 + 1\}$

$x = 3 \Rightarrow f(3) = 9 + 1 = 10 \quad x = 6 \Rightarrow f(6) = 36 + 1 = 37$

$x = 4 \Rightarrow f(4) = 16 + 1 = 17 \quad x = 8 \Rightarrow f(8) = 64 + 1 = 65$

R = {(3, 10), (4, 17), (6, 37), (8, 65)} $\quad \therefore$ The relation $R : X \to N$ is a function.

A relation 'f' is defined by $f(x) = x^2 - 2$ where, $x \in \{-2, -1, 0, 3\}$ (i) List the elements of f (ii) If f a function ?

Solution : $f(x) = x^2 - 2$

$x \in \{-2, -1, 0, 3\}$

(i) List the elements of f $\quad f(-2) = (-2)^2 - 2 = 2$; $f(-1) = (-1)^2 - 2 = -1$

$f(0) = (0)^2 - 2 = -2$; $f(3) = (3)^2 - 2 = 7$

$f = \{(-2, 2), (-1, -1), (0, -2), (3, 7)\}$

(ii) Since all the elements has unique image. f is a function.

Let $f(x) = 2x + 5$. If $x \neq 0$ then find $\dfrac{f(x+2) - f(2)}{x}$.

Solution : $f(x) = 2x + 5$

$f(x + 2) = 2(x + 2) + 5 = 2x + 9$

$f(2) = 2(2) + 5 = 9$

$\therefore \dfrac{f(x+2) - f(x)}{x} = \dfrac{2x + 9 - 9}{x} = 2$

Let A = {9, 10, 11, 12, 13, 14, 15, 16, 17} and let $f : A \to N$ be defined by $f(n)$ = the highest prime factor of $n \in A$. Write f as a set of ordered pairs and find the range of f.

Solution :

f = {(9, 3), (10, 5), (11, 11), (12, 3), (13, 13), (14, 7), (15, 5), (16, 2), (17, 17)}

Range of f = {2, 3, 5, 7, 11, 13, 17}

Let A = {1, 2} and B = {1, 2, 3, 4}, C = {5, 6} and D = {5, 6, 7, 8}. Verify whether A × C is a subset of B × D =?

Solution : A = {1, 2}, B = {1, 2, 3, 4}, C = {5, 6}, D = {5, 6, 7, 8}

A × C = {(1, 5), (1, 6), (2, 5), (2, 6)}

B × D = {**(1, 5)**, **(1, 6)**, (1, 7), (1, 8), **(2, 5)**, **(2, 6)**, (2, 7), (2, 8), (3, 5), (3, 6), (3, 7), (3, 8), (4, 5), (4, 6), (4, 7), (4, 8).}

$\therefore$ A × C is a subset of B × D.

If $f(x) = 4x^2 - 1, g(x) = 1 + x$, find $f \circ g$ and $g \circ f$. Check whether $f \circ g = g \circ f$.

Solution : $f(x) = \dfrac{2}{x}, g(x) = 2x^2 - 1$

$(f \circ g) = \left(\dfrac{2}{x}\right)(2x^2 - 1) = \dfrac{2}{2x^2 - 1}$

$(g \circ f) = \left(\dfrac{2}{x}\right)(2x^2 - 1) = 2\left(\dfrac{2}{x}\right)^2 - 1 = \dfrac{8}{x^2} - 1 \quad \therefore f \circ g \neq g \circ f$

If $f(x) = \dfrac{x+6}{3}, g(x) = 3 - x$, find $f \circ g$ and $g \circ f$. Check whether $f \circ g = g \circ f$.

Solution : $f(x) = \dfrac{x+6}{3}, g(x) = 3 - x$

$(f \circ g) = \left(\dfrac{x+6}{3}\right)(3 - x) = \dfrac{(3-x)+6}{3} = \dfrac{9-x}{3}$

$(g \circ f)(x) = \left(\dfrac{x+6}{3}\right)(3 - x) = 3 - \dfrac{x+6}{3} = \dfrac{9-x-3}{3} = \dfrac{6-x}{3} \quad \therefore f \circ g \neq g \circ f$

If $f(x) = x^2 - 1$, $g(x) = x - 2$ find a, if $g \circ f(a) = 1$.

Solution : $g \circ f(a) = 1$

$(x - 2)(x^2 - 1) = 1$

$(x - 2)(a^2 - 1) = 1$

$a^2 - 1 - 2 = 1 \Rightarrow a^2 - 3 = 1 \Rightarrow a^2 = 4 \quad \therefore a = \pm 2$

Find k, if $f(k) = 2k - 1$ and $f \circ f(k) = 5$.

Solution : $f(k) = 2k - 1 \Rightarrow f \circ f(k) = 5$

$(2k - 1)(2k - 1) = 5$

$2(2k - 1) - 1 = 5 \Rightarrow 4k - 2 = 6 \Rightarrow 4k = 8 \quad \therefore k = 2$

If $f(x) = 2x - k$, $g(x) = 4x + 5$ Find k, $f \circ g = g \circ f$

$\Rightarrow (f \circ g) = (g \circ f) \Rightarrow (2x - k)(4x + 5) = (4x + 5)(2x - k)$

$\Rightarrow 2(4x + 5) - k = 4(2x - k) + 5 \Rightarrow 8x + 10 - k = 8x - 4k + 5$

$\Rightarrow 10 - k = -4k + 5 \Rightarrow -k + 4k = 5 - 10 \Rightarrow 3k = -5$

Let A, B, C ⊆ N and a function $f : A \to B$ be defined by $f(x) = 2x + 1$ and $g : B \to C$ be defined by $g(x) = x^2$. Find the range of $f \circ g$ and $g \circ f$.

Solution : **Range of $f \circ g$** $= (2x + 1)(x^2) = 2x^2 + 1 \quad \therefore$ Range of $f \circ g = \{y / y = 2x^2 + 1, x \in N\}$.

Range of $g \circ f$ $= (x^2)(2x + 1) = (2x + 1)^2 \quad \therefore$ Range of $g \circ f = \{y / y = (2x+1)^2, x \in N\}$.

Let $f(x) = x^2 - 1$. Find $f \circ f$

Solution : $(f \circ f) = (x^2 - 1)(x^2 - 1) = (x^2 - 1)^2 - 1 = x^4 - 2x^2 + 1 - 1 = x^4 - 2x^2$

If $f : R \to R$ and $g : R \to R$ are defined by $f(x) = x^5$ and $g(x) = x^4$ then check if f, g are one-one and $f \circ g$ is one-one ?

Solution : Let A be the domain. B be the co-domain.

For every element A, there is a unique image in B. Since f is an odd function $\therefore f$ is 1 – 1.

$g(x)$ is an even function. $\therefore$ Two elements of domain will have image in co-domain. $\therefore g$ is not 1 – 1.

$R_3 = \{(2, -1), (7, 7), (1, 3)\}$ $R_3 \subseteq A \times B$ $\therefore R_3$ is a relation.

Let $A = \{3, 4, 7, 8\}$ and $B = \{1, 7, 10\}$. $R_3 = \{(3, 7), (4, 10), (7, 7), (7, 8), (8, 11), (8, 7), (8, 10)\}$ are relations from A to B ?

Solution : $A \times B = \{(3, 1), (3, 7), (3, 10), (4, 1), (4, 7), (4,10), (7,1), (7,7), (7,10), (8, 1), (8, 7), (8, 10)\}$

$R_3 = \{(3, 7), (4, 10), (7, 7), (7, 8), (8, 11), (8, 7), (8, 10)\}$

$(7, 8) \in R_3$, but $(7, 8) \notin A \times B$. So R_3 is not a relation from A to B.

A Relation R is given by the set $\{(x, y) / y = x + 3, x \in \{0, 1, 2, 3, 4, 5\}\}$. Determine its domain and range.

Solution : $R = \{(x, y) / y = x + 3, \quad x \in \{0, 1, 2, 3, 4, 5\}\}$

$x = 0 \Rightarrow y = 3$ $x = 2 \Rightarrow y = 5$ $x = 4 \Rightarrow y = 7$

$x = 1 \Rightarrow y = 4$ $x = 3 \Rightarrow y = 6$ $x = 5 \Rightarrow y = 8$

$\therefore R = \{(0, 3), (1, 4), (2, 5), (3, 6), (4, 7), (5, 8)\}$ $\therefore$ Domain : $\{0, 1, 2, 3,4, 5\}$ Range : $\{3, 4, 5, 6, 7, 8\}$

Let $A = \{-1, 1\}$ and $B = \{0, 2\}$. If the function $f : A \to B$ defined by $f(x) = ax + b$ is an onto function ? Find a and b.

Solution : $A = \{-1, 1\}, B = \{2, 2\}$ $f(x) = ax + b$ is on to function.

$\therefore f(-1) = 0 \Rightarrow -a + b = 0$ — (1) $f(1) = 2 \Rightarrow a + b = 2$ — (2)

$-a + b = 0$

$a + b = 2$

$2b = 2 \Rightarrow b = 1 \Rightarrow a = 1$ $\therefore a = 1, b = 1$

If $f(x) = x^2 - 1$, **find** $f \circ f \circ f$

Solution :

$(f \circ f \circ f)(x) = (x^2 - 1)(x^2 - 1)(x^2 - 1) = (x^2 - 1)((x^2 - 1)^2 - 1) = (x^2 - 1)(x^4 - 2x^2) = (x^4 - 2x^2)^2 - 1$

$f : R \to R$ defined by $f(x) = 2x + 1$ whether the function is bijective or not. Justify your answer.

Solution : $f : R \to R$ defined by $f(x) = 2x + 1$ Let $f(x_1) = f(x_2)$

$\Rightarrow 2x_1 + 1 = 2x_2 + 1 \Rightarrow 2x_1 = 2x_2 \Rightarrow x_1 = x_2$ $\therefore f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

$\therefore f$ is $1 - 1$ function.

$y = 2x + 1 \Rightarrow \therefore 2x = y - 1 \Rightarrow x = \frac{y-1}{2}$ $\therefore f(x) = 2\left(\frac{y-1}{2}\right) + 1 = y$ $\therefore f$ is onto.

$\therefore f$ is one-one and onto $\Rightarrow f$ is bijective.

Given that $f(x) = \begin{cases} \sqrt{x-1} & x \geq 1 \\ 4 & x < 1 \end{cases}$. Find (i) $f(0)$ (ii) $f(3)$ (iii) $f(a + 1)$ in terms of a. (Given that $a \geq 0$)

Solution :

Given $f(x) = \begin{cases} \sqrt{x-1} & x \geq 1 \\ 4 & x < 1 \end{cases}$ *i)* $f(0) = 4$ *ii)* $f(3) = \sqrt{3-1} = \sqrt{2}$ **(iii)** $f(a + 1) = \sqrt{a+1-1} = \sqrt{a}$

Find k if $f \circ f(k) = 5$ where $f(k) = 2k - 1$.

Solution : $f \circ f(k) = (2k - 1)(2k - 1) = 2(2k - 1) - 1 = 4k - 3$.

$f \circ f(k) = 5 \Rightarrow 4k - 3 = 5 \Rightarrow 4k = 5 + 3 \Rightarrow 4k = 8 \Rightarrow k = 2$.

Represent the function $f(x) = \sqrt{2x^2 - 5x + 3}$ as a composition of two functions.

Solution : $f_2(x) = 2x^2 - 5x + 3$ and $f_1(x) = \sqrt{x}$

$f(x) = \sqrt{2x^2 - 5x + 3} = \sqrt{f_2(x)} = f_1 f_2(x)$

If $f(x) = 4x^2 - 1$ and $g(x) = 1 + x$, find $f \circ g$ and $g \circ f$. Check whether $f \circ g = g \circ f$.

Solution : $(f \circ g) = (4x^2 - 1)(1 + x) = 4(1 + x)^2 - 1 = 4(1 + x^2 + 2x) - 1 = 4x^2 + 8x + 3$

$(g \circ f) = (1 + x)(4x^2 - 1) = 1 + 4x^2 - 1 = 4x^2$ $\therefore f \circ g \neq g \circ f$.

Represent each of the given relation by (a) an arrow diagram, (b) a graph and (c) a set in roster form, wherever possible. $\{(x, y) \mid y = x + 3, x, y$ are natural numbers $< 10\}$

Solution : x, y **are natural numbers < 10**

$y = x + 3$

$x = 1 \Rightarrow y = 4$

$x = 2 \Rightarrow y = 5$

$x = 3 \Rightarrow y = 6$

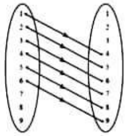
$x = 4 \Rightarrow y = 7$

$x = 5 \Rightarrow y = 8$

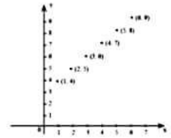
$x = 6 \Rightarrow y = 9$

c) a set in roster : $\{(1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6, 9)\}$

a) Arrow Diagram :



b) Graph :



Represent each of the given relation by (a) an arrow diagram, (b) a graph and (c) a set in roster form, wherever possible. $\{(x, y) \mid x = 2y, x \in \{2, 3, 4, 5\}, y \in \{1, 2, 3, 4\}$

Solution :

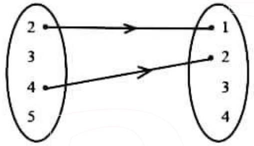
$y = 1 \Rightarrow x = 2$

$y = 2 \Rightarrow x = 4$

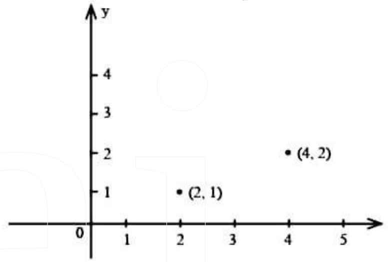
$y = 3 \Rightarrow x = 6$

$y = 4 \Rightarrow x = 8$

a) Arrow diagram :



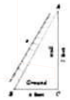
b) Graph :



c) a set in roster : $\{(2, 1), (4, 2)\}$

What length of ladder is needed to reach a height of 7 ft along the wall when the base of the ladder is 4 ft from the wall? Round off your answer to the next tenth place.

Solution : By Pythagoras theorem, $x^2 = 7^2 + 4^2 \Rightarrow x^2 = 49 + 16 \Rightarrow x = \sqrt{65}$



Therefore, length of the ladder is approximately 8.1 ft.

If ΔABC is similar to ΔDEF such that $BC = 3$ cm, $EF = 4$ cm and area of $\Delta ABC = 54$ cm². Find the area of ΔDEF.

Solution : $\frac{Area\ (\Delta ABC)}{Area\ (\Delta DEF)} = \frac{BC^2}{EF^2} \Rightarrow \frac{54}{Area\ (\Delta DEF)} = \frac{3^2}{4^2} \Rightarrow \frac{54}{Area\ (\Delta DEF)} = \frac{9}{16} \Rightarrow Area\ (\Delta DEF) = \frac{16 \times 54}{9} = 96\ cm^2$

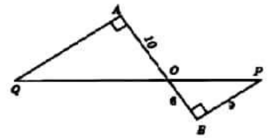
In Fig. QA and PB are perpendiculars to AB. If $AO = 10$ cm, $BO = 6$ cm and $PB = 9$ cm. Find AQ.

Solution : In ΔAOQ and ΔBOP, $\angle OAQ = \angle OBP = 90^0$

$\angle AOQ = \angle BOP$ (Vertically opposite angles)

by AA Criterion $\Delta AOQ \sim \Delta BOP$

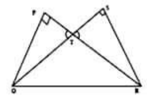
$\frac{AO}{BO} = \frac{AQ}{BP} = \frac{10}{6} = \frac{AQ}{9} \Rightarrow AQ = \frac{10 \times 9}{6} = 15cm$



Two triangles QPR and QSR, right angled at P and S respectively are drawn on the same base QR and on the same side of QR. If PR and SQ intersect at T, prove that $PT \times TR = ST \times TQ$.

Solution : ΔPQT and ΔSRT $\angle P = \angle S = 90^0$ $\angle PTQ = \angle STR$ (Vertically Opp.angle)

$\therefore$ By AA similarity, $\Delta PQT \sim \Delta SRT$ $\therefore \frac{QT}{TR} = \frac{PT}{ST} \Rightarrow PT \times TR = ST \times TQ$



If $A \times B = \{(3, 2), (3, 4), (5, 2), (5, 4)\}$ then find A and B.

Solution : $A \times B = \{(3, 2), (3, 4), (5, 2), (5, 4)\}$

$A = \{3, 5\}$ and $B = \{2, 4\}$.

If $A = \{2, -2, 3\}$ and $B = \{1, -4\}$ then find **$A \times B$, $A \times A$ and $B \times A$**

Solution: Given $A = \{2, -2, 3\}$, $B = \{1, -4\}$.

$R_3 = \{(2, -1), (7, 7), (1, 3)\}$

$A \times B = \{2, -2, 3\} \times \{1, -4\} = \{(2, 1), (2, -4), (-2, 1), (-2, -4), (3, 1), (3, -4)\}$

$A \times A = \{2, -2, 3\} \times \{2, -2, 3\} = \{(2, 2), (2, -2), (2, 3), (-2, 2), (-2, -2), (-2, 3), (3, 2), (3, -2), (3, 3)\}$

$B \times A = \{1, -4\} \times \{2, -2, 3\} = \{(1, 2), (1, -2), (1, 3), (-4, 2), (-4, -2), (-4, 3)\}$

If **$A = B = \{p, q\}$** find **$A \times B$, $A \times A$ and $B \times A$**

Solution: $A \times B = A \times A = B \times A = \{p, q\} \times \{p, q\} = \{(p, p), (p, q), (q, p), (q, q)\}$

If **$A = \{m, n\}$ and $B = \phi$** then find **$A \times B$, $A \times A$ and $B \times A$**

Solution: $A \times B = B \times A = \phi$ **and** $A \times A = \{(m, m), (m, n), (n, m), (n, n)\}$

Let $A = \{1, 2, 3\}$ and $B = \{x \mid x$ is a prime number less than 10$\}$. Find $A \times B$ and $B \times A$.

Solution : $A = \{1, 2, 3\}$, $B = \{x \mid x$ is a prime number less than $10\} = \{2, 3, 5, 7\}$

$A \times B = \{(1, 2), (1, 3), (1, 5), (1, 7), (2, 2), (2, 3), (2, 5), (2, 7), (3, 2), (3, 3), (3, 5), (3, 7)\}$

$B \times A = \{(2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (5, 1), (5, 2), (5, 3), (7, 1), (7, 2), (7, 3)\}$

If $B \times A = \{(-2, 3), (-2, 4), (0, 3), (0, 4), (3, 3), (3, 4)\}$ find A and B.

Solution : $B \times A = \{(-2, 3), (-2, 4), (0, 3), (0, 4), (3, 3), (3, 4)\}$ $\therefore B = \{-2, 0, 3\}$, $A = \{3, 4\}$

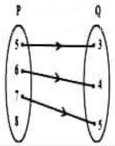
Define - Cartesian Product.

If A and B are two non-empty sets, then the set of all ordered pairs (a, b) such that $a \in A$, $b \in B$ is called the Cartesian Product of A and B, and is denoted by $A \times B$. Thus, $A \times B = \{(a, b) \mid a \in A, b \in B\}$

Define - **Relation**

A relation f between two non-empty sets X and Y is called a function from X to Y if, for each $x \in X$ there exists only one $y \in Y$ such that $(x, y) \in f$. That is, $f = \{(x, y) \mid$ for all $x \in X$, $y \in Y\}$.

The arrow diagram shows a relationship between the sets P and Q. Write the relation in (i) Set builder form (ii) Roster form (iii) What is the domain and range of R.



Solution : (i) Set builder form $R = \{(x, y) \mid y = x - 2, x \in P, y \in Q\}$

(ii) Roster form $R = \{(5, 3), (6, 4), (7, 5)\}$

(iii) Domain $= \{5, 6, 7\}$ and Range $= \{3, 4, 5\}$

Let $A = \{1, 2, 3, 4$ ....., $45\}$ and R be the relation defined as "is square of" on A. Write R as a subset of $A \times A$. Also, find the domain and range of R.

Solution : $A = \{1, 2, 3, 4, ... 45\}$ **and** R **"is square of" on A.** $\Rightarrow R = \{1, 4, 9, 16, 25, 36\}$

R is a subset of A.

$\therefore$ Domain $= \{1, 2, 3, 4, 5, 6\}$ $\therefore$ Range $= \{1, 4, 9, 16, 25, 36\}$

Let $A = \{1, 2, 3, 7\}$ and $B = \{3, 0, -1.7)$, $R_4 = \{(7, -1), (0, 3), (3, 3), (0, 7)\}$ are relation from A to B ?

Solution : $\therefore A \times B = \{(1, 3), (1, 0), (1, -1), (1, 7), (2, 3), (2, 0), (2, -1), (2, 7), (3, 3), (3, 0), (3, -1), (3, 7), (7, 3), (7, 0), (7, -1), (7, 7)\}$

$R_4 = \{(7, -1), (0, 3), (3, 3), (0, 7)\}$ **and** $(0, 3), (0, 7) \in R_4$ but not in $A \times B$.

$\therefore R_4$ is not a relation.

Let $A = \{1, 2, 3, 7\}$ and $B = \{3, 0, -1.7)$, $R_4 = \{(7, -1), (0, 3), (3, 3), (0, 7)\}$ are relation from A to B ?

Solution : $\therefore A \times B = \{(1, 3), (1, 0), (1, -1), (1, 7), (2, 3), (2, 0), (2, -1), (2, 7), (3, 3), (3, 0), (3, -1), (3, 7), (7, 3), (7, 0), (7, -1), (7, 7)\}$

A plane is flying at a speed of 500km per hour. Express the distance d travelled by the plane as function of time t in hours.

Solution : Speed of the plane $= 500$ km / h

$\therefore$ Distance $=$ Time $\times$ Speed $= 500t$

Let f be a function from R to R defined by $f(x) = 3x - 5$. Find the values of a and b given that $(a, 4)$ and $(1, b)$ belong to f.

Solution : $3a - 5 = 4 \Rightarrow a = 3$

$3(1) - 5 = b \Rightarrow b = -2$

If $A = \{-2, -1, 0, 1, 2\}$ and $f : A \to B$ is an onto function defined by $f(x) = x^2 + x + 1$ then find B.

Solution : $A = \{-2, -1, 0, 1, 2\}$ and $f(x) = x^2 + x + 1$

$f(-2) = (-2)^2 + (-2) + 1 = 3$; $f(0) = 0^2 + 0 + 1 = 1$;

$f(-1) = (-1)^2 + (-1) + 1 = 1$; $f(1) = 1^2 + 1 + 1 = 3$; $f(2) = 2^2 + 2 + 1 = 7$

$B = \{1, 3, 7\}$.

The Cartesian product $A \times A$ has 9 elements among which $(-1, 0)$ and $(0, 1)$ are found. Find the set A and the remaining elements of $A \times A$.

Solution : $n(A \times A) = 9$ and $(-1, 0), (0, 1) \in A \times A$

$\therefore A = \{-1, 0, 1\}$

The **remaining elements of $A \times A$** $= \{(-1, -1), (-1, 1), (0, -1), (0, 0), (1, -1), (1, 0), (1, 1)\}$

Find the domain of the function $f(x) = \sqrt{1 + \sqrt{1 - \sqrt{1 - x^2}}}$.

Solution : If $x > 1$ and $x < -1$, $f(x)$ leads to unreal

$\therefore$ The domain of $f(x) = \{-1, 0, 1\}$

Write the domain $f(x) = \dfrac{2x + 1}{x - 9}$

Solution : If $x = 9 \Rightarrow f(x) \to \infty$

The domain is $R - \{9\}$

Write the domain $g(x) = \sqrt{x - 2}$

Solution : The function exists only if $x \geq 2$ $\therefore$ The domain is $[2, \infty)$.

Let $A = \{-1, 1\}$ and $B = \{0, 2\}$. If the function $f : A \to B$ defined by $f(x) = ax + b$ is an onto function ? Find a and b.

Solution : $\therefore f(-1) = 0 \Rightarrow -a + b = 0$ — (1)

$f(1) = 2 \Rightarrow$ $a + b = 2$ — (2)

$2b = 2 \Rightarrow b = 1$

$\therefore a = 1, b = 1$

If the ordered pairs $(x^2 - 3x, y^2 + 4y)$ and $(-2, 5)$ are equal, then find x and y.

Solution : Given $(x^2 - 3x, y^2 + 4y) = (-2, 5)$

$\therefore x^2 - 3x = -2$	$y^2 + 4y = 5$
$\Rightarrow x^2 - 3x + 2 = 0$	$y^2 + 4y - 5 = 0$
$\Rightarrow (x - 2)(x - 1) = 0$	$(y + 5)(y - 1) = 0$
$\therefore x = 2, 1$	$y = -5, 1$

2 .NUMBERS AND SEQUENCES

Write the Euclid's Division Lemma.

Let a and b $(a > b)$ be any two positive integers. Then, there exist unqiue integers q and r such that $a = bq + r, 0 \le r < b$.

Write the Fundamental theorem of arithmetic

Every natural number except 1 can be factorized as a product of primes and this factorization is unique except for the order in which the prime factors are written.

Find the sum $3 + 1 + \frac{1}{3} + \dots \infty$

Solution : $a = 3,\quad r = \frac{t_2}{t_1} = \frac{1}{3}$ Sum of infinite terms $= \frac{a}{1-r} = \frac{3}{1-\frac{1}{3}} = \frac{9}{2}$

Find the sum to infinity of $21 + 14 + \frac{28}{3} + \dots$

Solution :

$$a = 21,\ r = 14/21 = 2/3 < 1$$

$$\therefore S_\infty = \frac{a}{1-r} = \frac{21}{1-2/3} = \frac{21}{1-2/3} = \frac{21}{1/3} = 63$$

If the first term of an infinite G.P. is 8 and its sum to infinity is $\frac{32}{3}$ then find the common ratio.

Solution : $a = 8,\ S_\infty = \frac{32}{3},\ r = ?$

$$\Rightarrow \frac{a}{1-r} = \frac{32}{3} \Rightarrow \frac{8}{1-r} = \frac{32}{3} \Rightarrow 3 = 4 - 4r \Rightarrow 4r = 1 \quad \therefore\ r = 1/4$$

Find the rational form of the number $0.\overline{123}$.

Solution : Let $x = 0.\overline{123}$ $\quad x = 0.123123123$

$\Rightarrow 1000x = 123.123123$

$\Rightarrow 1000x = 123 + 0.123123123$

$\Rightarrow 1000x = 123 + x$

$1000x - x = 123 \Rightarrow 999x = 123 \Rightarrow x = \frac{123}{999} \quad \therefore x = \frac{41}{333}$

Find the 8[th] term of the G.P. 9, 3, 1, ...

Solution : First term $a = 9$, common ratio $r = \frac{t_2}{t_1} = \frac{3}{9} = \frac{1}{3}$

$$t_8 = 9 \times \left(\frac{1}{3}\right)^{8-1} = 9 \times \left(\frac{1}{3}\right)^7 = \frac{1}{243}$$

If $1^3 + 2^3 + 3^3 + \dots + k^3 = 44100$, then find $1 + 2 + 3 + \dots + k$.

Solution : Given $1^3 + 2^3 + 3^3 + \dots + k^3 = 44100$

$$\Rightarrow \left(\frac{k(k+1)}{2}\right)^2 = 44100 \quad \Rightarrow \frac{k(k+1)}{2} = 210 \quad \Rightarrow 1 + 2 + 3 + \dots + k = 210$$

Find the sum of the following series $3 + 6 + 9 + \dots + 96$

Solution : $3 + 6 + 9 + \dots + 96$

$$= 3(1 + 2 + 3 + \dots + 32) = 3\left(\frac{32 \times 33}{2}\right)$$

$$= 3 \times 16 \times 33 = 1584$$

Find the first five terms of the following sequence. $a_1 = 1,\ a_2 = 1,\ a_n = \frac{a_{n-1}}{a_{n-2} + 3};\ n \ge 3,\ n \in N$

Solution : $a_3 = \frac{a_{3-1}}{a_{3-2} + 3} = \frac{a_2}{a_1 + 3} = \frac{1}{1+3} = \frac{1}{4}$

$$a_4 = \frac{a_{4-1}}{a_{4-2} + 3} = \frac{a_3}{a_2 + 3} = \frac{\frac{1}{4}}{1+3} = \frac{\frac{1}{4}}{4} = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16} \qquad a_5 = \frac{a_{5-1}}{a_{5-2} + 3} = \frac{a_4}{a_3 + 3} = \frac{\frac{1}{16}}{\frac{1}{4} + 3} = \frac{1}{16} \times \frac{4}{13} = \frac{1}{52}$$

The first five terms of the sequence are $1, 1, \frac{1}{4}, \frac{1}{16}, \frac{1}{25}$

Find the 19[th] term of an A.P. −11, −15, −19,...

Solution : A.P is −11, −15, −19, ... $a = -11,\quad d = -15 - (-11) = -15 + 11 = -4$

$$t_n = a + (n-1)d$$

$$t_{19} = a + 18d = (-11) + 18(-4) = -11 - 72 = -83$$

Which term of an A.P. 16, 11, 6, 1,... is −54?

Solution : A.P. is 16, 11, 6, 1, ... − 54

$a = 16,\ d = -5,\ t_n = -54$

$\Rightarrow\ a + (n-1)d = -54 \quad \Rightarrow 16 + (n-1)(-5) = -54 \quad \Rightarrow\ 16 - 5n + 5 = -54$

$\Rightarrow\ -5n + 21 = -54$

$\Rightarrow\ -5n + 21 = -54$

$\Rightarrow\ -5n = -54 - 21$

$\Rightarrow\ -5n = -75$

$\therefore n = 15$

$\therefore$ 15th term of A.P. is − 54

Find the middle term(s) of an A.P. 9, 15, 21, 27,…,183.

Solution : Given A.P is 9, 15, 21, 27, ... 183 $\quad a = 9,\ d = 6,\ l = 183$

$$n = \frac{l-a}{d} + 1 = \frac{183-9}{6} + 1 = \frac{174}{6} + 1 = 29 + 1 = 30$$

$\therefore$ Middle terms are $30/2,\ 30/2 + 1 = 15^{th},\ 16^{th}$

$t_{15} = a + 14d$	$t_{16} = a + 15d$
$= 9 + 14(6)$	$= 9 + 15(6)$
$= 9 + 84$	$= 9 + 90$
$= 93$	$= 99$

A milk man has 175 litres of cow's milk and 105 litres of buffalow's milk. He wishes to sell milk by filling the two types of milk in cans of equal capacity. Claculate the following (i) Capacity of a can (ii) Number of cans of cow's milk (iii) Number of cans of buffalow's milk

Solution : Cow's milk = 175 lrs. Buffalow's milk = 105 lrs.

i) Capacity of a can = HCF of 175 and 105 = 35 litres

ii) Number of cans of Cow's milk $= \frac{175}{35} = 5$ iii) Number of cans of buffalow's milk $= \frac{105}{35} = 3$

If $3 + k,\ 18 - k,\ 5k + 1$ are in A.P. then find k.

Solution : a, b, c are in A.P. $\Rightarrow 2b = a + c$

$\Rightarrow 2(18 - k) = (3 + k) + (5k + 1)$

$36 - 2k = 6k + 4$

$8k = 32 \Rightarrow k = 4$

Find x, y and z, given that the numbers x, 10, y, 24, z are in A.P.

Solution : Given that x, 10, y, 24, z are in A.P. $\quad \therefore\ y$ is the arithmetic mean of 10 & 24

$2y = 10 + 24 \Rightarrow y = \frac{10+24}{2} = \frac{34}{2} = 17$ Clearly $d = 7$

$\therefore x = 10 - 7 = 3$ & $z = 24 + 7 = 31$

$\therefore x = 3, y = 17, z = 31$

In a theatre, there are 20 seats in the front row and 30 rows were allotted. Each successive row contains two additional seats than its front row. How many seats are there in the last row?

Solution : $a = 20,\ d = 2,\ n = 30$ $\quad t_{30} = a + 29d = 20 + 29(2) = 20 + 58 = 78$

$t_n = a + (n-1)d$ $\quad \therefore$ The no. of seats in 30[th] row = 78

Find the sum of all odd positive integers less than 450.

Solution : $1 + 3 + 5 + 7 + \dots + 449 = \left[\frac{(l+1)}{2}\right]^2 = \left[\frac{449+1}{2}\right]^2 = \left[\frac{450}{2}\right]^2 = [225]^2 = 50,625$

In a G.P. 729, 243, 81,... find t_7.

Solution : 729, 243, 21, ... $a = 729$, $r = \frac{81}{243} = \frac{1}{3}$

$\therefore t_n = a \cdot r^{n-1}$

$\Rightarrow t_7 = a \cdot r^6 = 729 \times \left(\frac{1}{3}\right)^6 = 729 \times \left(\frac{1}{729}\right) = 1$

Find x so that $x + 6, x + 12$ and $x + 15$ are consecutive terms of a Geometric Progression.

Solution : Given $x + 6, x + 12, x + 15$ are consecutive terms of a G.P.

a, b, c are in G.P. $\Rightarrow$ $b^2 = ac$

$\Rightarrow (x+12)^2 = (x+15)(x+6) \Rightarrow x^2 + 24x + 144 = x^2 + 21x + 90 \Rightarrow 3x = -54 \Rightarrow x = -18$

Find the 10th term of a G.P. whose 8th term is 768 and the common ratio is 2.

Solution : $t_8 = 768$, $r = 2$

$\Rightarrow a \cdot r^7 = 768 \Rightarrow a \times 2^7 = 768 \Rightarrow a \times 128 = 768 \Rightarrow a = 6$

$\therefore t_{10} = a \cdot r^9 = 6 \times 2^9 = 6 \times 512 = 3072$

If a, b, c are in A.P. then show that $3^a, 3^b, 3^c$ are in G.P.

Solution : Given a, b, c are in A.P. $\Rightarrow 2b = a + c$ .....(1)

$(3^b)^2 = 3^{2b} = 3^{a+c} = 3^a \cdot 3^c$ (from (1))

$\therefore 3^a, 3^b, 3^c$ are in G.P.

Find the sum of $2 + 4 + 6 + ... + 80$

Solution : $2 + 4 + 6 + ... + 80 = 2(1 + 2 + 3 + ... + 40) = 2 \times \frac{40 \times (40+1)}{2} = 1640$

Find the sum of $1 + 3 + 5 + ... + 55$

Solution : $1 + 3 + 5 + ... + 55 = \left[\frac{(l+1)}{2}\right]^2 = \left[\frac{(55+1)}{2}\right]^2 = \left[\frac{56}{2}\right]^2 = (28)^2 = 784.$

Find the sum of $0.40 + 0.43 + 0.46 + ... + 1$.

Solution : $a = 0.40$ and $l = 1$, $d = 0.43 - 0.40 = 0.03$. $n = \left(\frac{l-a}{d}\right) + 1 = \left(\frac{1 - 0.40}{0.03}\right) + 1 = 21$

$S_n = \frac{n}{2}[a + l]$ $S_{21} = \frac{21}{2}[0.40 + 1] = 14.7$

When the positive integers a, b and c are divided by 13, the respective remainders are 9,7 and 10. Show that $a + b + c$ is divisible by 13.

Solution : When a is divided by 13, remainder is 9 i.e., $a = 13q + 9$ .... (1)

When b is divided by 13, remainder is 7 i.e., $b = 13q + 7$ .... (2)

When c is divided by 13, remainder is 11 i.e., $c = 13q + 11$ .... (3)

Adding (1), (2) & (3) $a + b + c = 39q + 26 = 13(2q + 2)$

$a + b + c$ is divisible by 13

Find the least number that is divisible by the first ten natural numbers.

Solution : The required number is the LCM of (1, 2, 3, ... 10)

$2 = \underline{2} \times 1$ $4 = \underline{2} \times 2$ $6 = 3 \times \underline{2}$ $8 = 2 \times 2 \times \underline{2}$ $9 = 3 \times 3$

$10 = 5 \times \underline{2}$ and 1, 3, 5, 7 L.C.M $= 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7 = 2520$

Find the sum of the following series $1 + 4 + 9 + 16 + ... + 225$

Solution : $1 + 4 + 9 + 16 + ... + 225 = 1^2 + 2^2 + 3^2 + ... + 15^2 = \frac{15 \times 16 \times 31}{6} = 1240$ $\sum n^2 = \frac{n(n+1)(2n+1)}{6}$

Find the 12th term from the last term of the A.P – 2, – 4, – 6, ... –100.

Solution : Given A.P is $-2, -4, -6, ... -100$

12th term from the last term $a = -100$, $d = 2$

$t_{12} = a + 11d = -100 + 11(2) = -100 + 22 = -78$

If $1 + 2 + 3 + ... + k = 325$, then find $1^3 + 2^3 + 3^3 + ... + k^3$

Solution : $1 + 2 + 3 + ... + k = 325 \Rightarrow \frac{k(k+1)}{2} = 325$

$\therefore 1^3 + 2^3 + 3^3 + ... + k^3 = \left(\frac{k(k+1)}{2}\right)^2 = (325)^2 = 105625$

Show that the square of an odd integer is of the form $4q + 1$, for some integer q.

Solution : Let $x = 2k + 1$ be any odd integer.

The square of an odd integer $x^2 = (2k+1)^2 = 4k^2 + 4k + 1 = 4k(k+1) + 1 = 4q + 1$

A man has 532 flower pots. He wants to arrange them in rows such that each row contains 21 flower pots. Find the number of completed rows and how many flower pots are left over

Solution : No. of flower pots = 532 each row to contain 21 flower pots.

$\Rightarrow 532 = 21 \times 25 + 7$

$\therefore$ Number of completed rows = 25

Number of flower pots left out = 7

```
      25
21 ) 532
     42
     ---
     112
     105
     ---
       7
```

'a' and 'b' are two positive integers such that $a^b \times b^a = 800$. Find 'a' and 'b'.

Solution : $800 = 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 = 2^5 \times 5^2$

$a^b \times b^a = 2^5 \times 5^2$

$a = 2$ and $b = 5$ (or) $a = 5$ and $b = 2$.

Prove that two consecutive positive integers are always coprime.

Solution : Let x, $x + 1$ be two consecutive integers.

G.C.D. of $(x, x + 1) = 1$

x & $x + 1$ are Co-prime.

Prove that $2^n + 6 \times 9^n$ is always divisible by 7 for any positive integer n.

Solution : When n = 1, 2n + 6 × 9n = 2 + (6 × 9) = 56, divisible by 7.

Kala and Vani are friends, Kala says, "Today is my birthday" and she asks Vani, "When will you celebrate your birthday?" Vani replies, "Today is Monday and I celebrated my birthday 75 days ago". Find the day when Vani celebrated her birthday.

Solution : Let 0, 1, 2, 3, 4, 5, 6 to represent the weekdays from Sunday to Saturday respecitvely.

$-74 \pmod 7 \equiv -4 \pmod 7 \equiv 7 - 4 \pmod 7 \equiv 3 \pmod 7$

The day for the number 3 is Wednesday. Vani's birthday must be on Wednesday.

Today is Tuesday. My uncle will come after 45 days. In which day my uncle will be coming?

Solution : Today is Tuesday Day after 45 days = ?

When we divide 45 by 7, remainder is 3.

$\therefore$ The 3rd day from Tuesday is <u>Friday</u>

3 . ALGEBRA

Brief explaination of types of matrices.

A matrix is said to be a **column matrix** if it has only one column and any number of rows. It is also called as a column vector.

A matrix in which the number of rows is equal to the number of columns is called a **square matrix**.

A square matrix, all of whose elements, except those in the leading diagonal are zero is called a **diagonal matrix.**

A diagonal matrix in which all the leading diagonal elements are equal is called a **scalar matrix.**

A square matrix in which elements in the leading diagonal are all "1" and rest are all zero is called an **identity matrix** (or) **unit matrix**.

A square matrix in which all the entries above the leading diagonal are zero is called a lower **triangular matrix.**

If all the entries below the leading diagonal are zero, then it is called an **upper triangular matrix.**

A matrix is said to be a **row matrix** if it has only one row and any number of columns. A row matrix is also called as a row vector.

Define – Matrix.

A matrix is a rectangular array of elements. The horizontal arrangements are called rows and vertical arrangements are called columns. **Example:** $A = \begin{pmatrix} 23 & 18 \\ 47 & 36 \\ 15 & 16 \end{pmatrix}$

Write the three conditions of nature of roots.

Solution :

Values of Discriminant $\Delta = b^2 - 4ac$

Real and Unequal roots	$\Delta > 0$
Real and Equal roots	$\Delta = 0$
No Real root	$\Delta < 0$

Write relation between GCD and LCM.

Solution : The product of two polynomials is the product of their LCM and GCD.

That is, $f(x) \times g(x) = \text{LCM}[f(x), g(x)] \times \text{GCD}[f(x), g(x)]$.

If $\begin{pmatrix} 5 & 4 & 3 \\ 1 & -7 & 9 \\ 3 & 8 & 2 \end{pmatrix}$ **then find the transpose of A.** ***Solution :*** $A = \begin{pmatrix} 5 & 4 & 3 \\ 1 & -7 & 9 \\ 3 & 8 & 2 \end{pmatrix}$ $\therefore A^T = \begin{pmatrix} 5 & 1 & 3 \\ 4 & -7 & 8 \\ 3 & 9 & 2 \end{pmatrix}$

If a matrix has 18 elements, what are the possible orders it can have? What if it has 6 elements?

Solution : Given, a matrix has 18 elements . The possible orders $18 \times 1, 1 \times 18, 9 \times 2, 2 \times 9, 6 \times 3, 3 \times 6$

The matrix has 6 elements . The order are $1 \times 6, 6 \times 1, 3 \times 2, 2 \times 3$

If $A = \begin{pmatrix} \sqrt{7} & -3 \\ -\sqrt{5} & 2 \\ \sqrt{3} & -5 \end{pmatrix}$ **then find the transpose of -A.**

Solution : $A = \begin{pmatrix} \sqrt{7} & -3 \\ -\sqrt{5} & 2 \\ \sqrt{3} & -5 \end{pmatrix}$ $\quad -A = \begin{pmatrix} -\sqrt{7} & 3 \\ \sqrt{5} & -2 \\ -\sqrt{3} & 5 \end{pmatrix}$ $\quad \therefore$ Transpose of $-A = \begin{pmatrix} -\sqrt{7} & \sqrt{5} & -\sqrt{3} \\ 3 & -2 & 5 \end{pmatrix}$

Subtract $\frac{1}{x^2+2}$ **from** $\frac{2x^3+x^2+3}{(x^2+2)^2}$

Solution :

$$\frac{2x^3+x^2+3}{(x^2+2)^2} - \frac{1}{x^2+2} = \frac{(2x^3+x^2+3)-(x^2+2)}{(x^2+2)^2} = \frac{2x^3+x^2+3-x^2-2}{(x^2+2)^2} = \frac{2x^3+1}{(x^2+2)^2}$$

Find the square root of $256(x-a)^8(x-b)^4(x-c)(x-d)^{20}$

Solution : $\sqrt{256(x-a)^8(x-b)^4(x-c)^{16}(x-d)^{20}} = 16\left|(x-a)^4(x-b)^2(x-c)^8(x-d)^{10}\right|$

Find the square root of $\frac{144a^8b^{12}c^{16}}{81f^{12}g^4h^{14}}$ ***Solution :*** $\sqrt{\frac{144a^8b^{12}c^{16}}{81f^{12}g^4h^{14}}} = \frac{4}{3}\left|\frac{a^4b^6c^8}{f^6g^2h^7}\right|$

Find the square root of $16x^2+9y^2-24xy+24x-18y+9$

Solution : $\sqrt{16x^2+9y^2-24xy+24x-18y+9} = \sqrt{(4x-3y+3)^2} = |4x-3y+3|$

Find the sum and product of the roots for the quadratic equation $3+\frac{1}{a}=\frac{10}{a^2}$

Solution : Given $3+\frac{1}{a}=\frac{10}{a^2} \Rightarrow \frac{3a+1}{a}=\frac{10}{a^2} \Rightarrow 3a+1=\frac{10}{a} \Rightarrow 3a^2+a-10=0$

$A=3, B=1, C=-10 \quad \therefore \alpha+\beta=\frac{-B}{A}=\frac{-1}{3} \quad \therefore \alpha\beta=\frac{C}{A}=\frac{-10}{3}$

Solve $x^4-13x^2+42=0$

Solution : $(x^2)^2-13x^2+42=0 \Rightarrow (x^2-7)(x^2-6)=0 \Rightarrow x=\pm\sqrt{7}$ or $x=\pm\sqrt{6}$

If the difference between a number and its reciprocal is $\frac{24}{5}$**, find the number.**

Solution :

Let x be the required number $\frac{1}{x}$ be its reciprocal Given $x-\frac{1}{x}=\frac{24}{5}$

$\Rightarrow \frac{x^2-1}{x}=\frac{24}{5} \Rightarrow 5x^2-24x-5=0 \Rightarrow \therefore$ The required numbers are $5, -1/5$

The number of volleyball games that must be scheduled in a league with n teams is given by $G(n)=\frac{n^2-n}{2}$ **where each team plays with every other team exactly once. A league schedules 15 games. How many teams are in the league?**

Solution : $G(n)=\frac{n^2-n}{2}=15 \Rightarrow n^2-n=30 \Rightarrow n^2-n-30=0 \Rightarrow (n-6)(n+5)=0 \Rightarrow n=6, -5$

$\therefore$ Number of terms in the league $= 6$

Find the excluded values of $\frac{x}{x^2+1}$ expressions

Solution : $x^2+1 \neq 0$ for any x. Therefore, no real excluded values

Reduce to lowest form. $\frac{x^2-1}{x^2+x}$ ***Solution :*** $\frac{x^2-1}{x^2+x} = \frac{(x+1)(x-1)}{x(x+1)} = \frac{x-1}{x}$

Find the LCM and GCD for $(x^2y+xy^2), (x^2+xy)$ **and verify that** $f(x) \times g(x) = \text{LCM} \times \text{GCD}$

Solution : $f(x) = x^2y+xy^2 = xy(x+y)$ $\quad g(x) = x^2+xy = x(x+y)$

$\therefore \text{GCD} = x(x+y) \quad \therefore \text{LCM} = xy(x+y)$

$\because f(x) \times g(x) = xy(x+y) \times x(x+y) = x^2y(x+y)^2 = \text{LCM} \times \text{GCD}$

Find the LCM of $x^3-27, (x-3)^2, x^2-9$

Solution : $x^3-27 = (x-3)(x^2+3x+9)$ | $(x-3)^2 = (x-3)^2$ | $(x^2-9) = (x+3)(x-3)$

$\text{LCM} = (x-3)^2(x+3)(x^2+3x+9)$

Find the excluded values of $\frac{x^2+6x+8}{x^2+x-2}$ **Solution :** $\frac{x^2+6x+8}{x^2+x-2}=\frac{(x+4)(x+2)}{(x+2)(x-1)}=\frac{x+4}{x-1}$

∴ The excluded values is 1

Find the excluded values of $\frac{x^3-27}{x^3+x^2-6x}$

Solution :

$\frac{x^3-27}{x^3+x^2-6x}=\frac{x^3-3^3}{x(x^2+x-6)}=\frac{(x-3)(x^2+3x+9)}{x(x+3)(x-2)}$ ∴ The excluded values are 0, – 3, 2

Simplify $\frac{x+2}{4y}\div\frac{x^2-x-6}{12y^2}$

Solution : $\frac{x+2}{4y}\div\frac{x^2-x-6}{12y^2}=\frac{x+2}{4y}\times\frac{12y^2}{x^2-x-6}=\frac{x+2}{4y}\times\frac{12y^2}{(x-3)(x+2)}=\frac{3y}{x-3}$

Multiply $\frac{x^4b^2}{x-1}$ by $\frac{x^2-1}{a^4b^3}$ $\frac{x^4b^2}{x-1}\times\frac{x^2-1}{a^4b^3}=\frac{x^4\times b^2}{x-1}\times\frac{(x+1)(x-1)}{a^4\times b^3}=\frac{x^4(x+1)}{a^4b}$

Simplify $\frac{x^2-16}{x+4}\div\frac{x-4}{x+4}$ **Solution :** $\frac{x^2-16}{x+4}\div\frac{x-4}{x+4}=\frac{(x+4)(x-4)}{(x+4)}\times\left(\frac{x+4}{x-4}\right)=x+4$

Simplify $\frac{x+4}{3x+4y}\times\frac{9x^2-16y^2}{2x^2+3x-20}$ **Solution :** $\frac{x+4}{3x+4y}\times\frac{9x^2-16y^2}{2x^2+3x-20}=\frac{x+4}{3x+4y}\times\frac{(3x+4y)(3x-4y)}{(x+4)(2x-5)}=\frac{3x-4y}{2x-5}$

Simplify $\frac{x^3-y^3}{3x^2+9xy+6y^2}\times\frac{x^2+2xy+y^2}{x^2-y^2}$ **Solution :** $\frac{x^3-y^3}{3x^2+9xy+6y^2}\times\frac{x^2+2xy+y^2}{x^2-y^2}$

$=\frac{(x-y)(x^2+xy+y^2)}{3(x^2+3xy+2y^2)}\times\frac{(x+y)(x+y)}{(x+y)(x-y)}=\frac{(x^2+xy+y^2)(x+y)}{3(x+2y)(x+y)}=\frac{x^2+xy+y^2}{3(x+2y)}$

Subtract $\frac{1}{x^2+2}$ **from** $\frac{2x^3+x^2+3}{(x^2+2)^2}$

Solution :

$\frac{2x^3+x^2+3}{(x^2+2)^2}-\frac{1}{x^2+2}=\frac{(2x^3+x^2+3)-(x^2+2)}{(x^2+2)^2}=\frac{2x^3+x^2+3-x^2-2}{(x^2+2)^2}=\frac{2x^3+1}{(x^2+2)^2}$

Find the square root of $256(x-a)^8(x-b)^4(x-c)(x-d)^{20}$

Solution : $\sqrt{256(x-a)^8(x-b)^4(x-c)^{16}(x-d)^{20}}=16\left|(x-a)^4(x-b)^2(x-c)^8(x-d)^{10}\right|$

Find the square root of $\frac{144a^8b^{12}c^{16}}{81f^{12}g^4h^{14}}$ **Solution :** $\sqrt{\frac{144a^8b^{12}c^{16}}{81f^{12}g^4h^{14}}}=\frac{4}{3}\left|\frac{a^4b^6c^8}{f^6g^2h^7}\right|$

Find the square root of $16x^2+9y^2-24xy+24x-18y+9$

Solution : $\sqrt{16x^2+9y^2-24xy+24x-18y+9}=\sqrt{(4x-3y+3)^2}=|4x-3y+3|$

Simplify $\frac{x^3}{x-y}+\frac{y^3}{y-x}$

Solution : $\frac{x^3}{x-y}+\frac{y^3}{y-x}=\frac{x^3}{x-y}-\frac{y^3}{x-y}=\frac{x^3-y^3}{x-y}=\frac{(x-y)(x^2+xy+y^2)}{x-y}=x^2+xy+y^2$

If a polynomial $p(x)=x^2-5x-14$ is divided by another polynomial $q(x)$ we get $\frac{x-7}{x+2}$ find $q(x)$.

Solution : $\frac{p(x)}{q(x)}=\frac{x-7}{x+2}\Rightarrow\frac{x^2-5x-14}{q(x)}=\frac{x-7}{x+2}\Rightarrow\frac{(x-7)(x+2)}{q(x)}=\frac{x-7}{x+2}\Rightarrow q(x)=(x+2)^2$

$\Rightarrow q(x)=x^2+4x+4$ is another polynomial

Simplify $\frac{x(x+1)}{x-2}+\frac{x(1-x)}{x-2}$ **Solution :** $\frac{x(x+1)}{x-2}+\frac{x(1-x)}{x-2}=\frac{x^2+x+x-x^2}{x-2}=\frac{2x}{x-2}$

Find the excluded values of $\frac{t}{t^2-5t+6}$

Solution : $\frac{t}{t^2-5t+6}=\frac{t}{(t-3)(t-2)}$ ∴ The excluded values are 3, 2

If A is of order p × q and B is of order q × r what is the order of AB and BA?

Solution : A is of order p × q B is of order q × r

∴ Order of AB = p × r Order of BA is not defined

Find the square root $\frac{7x^2+2\sqrt{14}x+2}{x^2-\frac{1}{2}x+\frac{1}{16}}$ **Solution :** $\sqrt{\frac{7x^2+2\sqrt{14}x+2}{x^2-\frac{1}{2}x+\frac{1}{16}}}=\sqrt{\frac{(\sqrt{7}x+\sqrt{2})^2}{\left(x-\frac{1}{4}\right)^2}}=4\left|\frac{\sqrt{7}x+\sqrt{2}}{(4x-1)}\right|$

Find the square root of $4x^2+20x+25$ **Solution :** $\sqrt{4x^2+20x+25}=\sqrt{(2x+5)^2}=|2x+5|$

Find the square root of $9x^2-24xy+30xz-40yz+25z^2+16y^2$

Solution : $\sqrt{9x^2-24xy+30xz-40yz+25z^2+16y^2}=\sqrt{(3x-4y+5z)^2}=|3x-4y+5z|$

Find the square root of $1+\frac{1}{x^6}+\frac{2}{x^3}$ **Solution :** $\sqrt{1+\frac{1}{x^6}+\frac{2}{x^3}}=\sqrt{\left(1+\frac{1}{x^3}\right)^2}=\left|1+\frac{1}{x^3}\right|$

Which rational expression should be subtracted from $\frac{x^2+6x+8}{x^3+8}$ **to get** $\frac{3}{x^2-2x+4}$

Solution : $\frac{x^2+6x+8}{x^3+8}-\frac{3}{x^2-2x+4}=\frac{(x+4)(x+2)}{(x+2)(x^2-2x+4)}-\frac{3}{x^2-2x+4}=\frac{x+4-3}{x^2-2x+4}=\frac{x+1}{x^2-2x+4}$

Solve $\sqrt{a(a-7)}=3\sqrt{2}$ **by factorization method**

Solution : Given $\sqrt{a(a-7)}=3\sqrt{2}$ Squaring on both sides $a^2-7a=18\Rightarrow a^2-7a-18=0$

$\Rightarrow(a-9)(a+2)=0\Rightarrow$ Roots are 9, – 2

Solve $\sqrt{2}x^2+7x+5\sqrt{2}=0$ **by factorization method**

Solution : $\Rightarrow\sqrt{2}x^2+2x+5x+5\sqrt{2}=0\Rightarrow(x+\sqrt{2})(\sqrt{2}x+5)=0$ ∴ Roots are $-5/\sqrt{2},-\sqrt{2}$

Solve $2x^2-x+\frac{1}{8}=0$ **by factorization method**

Solution : $\Rightarrow16x^2-8x+1=0\Rightarrow(4x-1)(4x-1)=0$

∴ Roots are $1/4$, $1/4$

Find the square root of $16x^4+8x^2+1$ by division method

Solution : $\sqrt{16x^4+8x^2+1}=\sqrt{(4x^2+1)^2}=|4x^2+1|$

Verify that $A^2 = I$ when $A=\begin{pmatrix}5&-4\\6&-5\end{pmatrix}$

Solution : $A=\begin{pmatrix}5&-4\\6&-5\end{pmatrix}$ $\quad A^2 = A.A=\begin{pmatrix}5&-4\\6&-5\end{pmatrix}\begin{pmatrix}5&-4\\6&-5\end{pmatrix}=\begin{pmatrix}25-24&-20+20\\30-30&-24+25\end{pmatrix}=\begin{pmatrix}1&0\\0&1\end{pmatrix}=I$

If $\cos\theta\begin{pmatrix}\cos\theta&\sin\theta\\-\sin\theta&\cos\theta\end{pmatrix}+\sin\theta\begin{pmatrix}x&-\cos\theta\\\cos\theta&x\end{pmatrix}=I_2$ find x.

Solution : $\cos\theta\begin{pmatrix}\cos\theta&\sin\theta\\-\sin\theta&\cos\theta\end{pmatrix}+\sin\theta\begin{pmatrix}x&-\cos\theta\\\cos\theta&x\end{pmatrix}=I_2 \Rightarrow \begin{pmatrix}\cos^2\theta+x\sin\theta&0\\0&\cos^2\theta+x\sin\theta\end{pmatrix}=\begin{pmatrix}1&0\\0&1\end{pmatrix}$

$\Rightarrow \cos^2\theta + x\sin\theta = 1 \Rightarrow x\sin\theta = 1-\cos^2\theta \Rightarrow x\sin\theta=\sin^2\theta \Rightarrow x=\sin\theta$

Let $A=\begin{pmatrix}1&2\\1&3\end{pmatrix}$, $B=\begin{pmatrix}4&0\\1&5\end{pmatrix}$, $C=\begin{pmatrix}2&0\\1&2\end{pmatrix}$ Show that $(A-B)^T = A^T - B^T$

Solution : $A=\begin{pmatrix}1&2\\1&3\end{pmatrix}$, $B=\begin{pmatrix}4&0\\1&5\end{pmatrix}$, $C=\begin{pmatrix}2&0\\1&2\end{pmatrix}$ $\quad (A-B)^T=\begin{pmatrix}-3&2\\0&-2\end{pmatrix}^T=\begin{pmatrix}-3&0\\2&-2\end{pmatrix}$ ....(1)

$A^T-B^T=\begin{pmatrix}1&1\\2&3\end{pmatrix}-\begin{pmatrix}4&1\\0&5\end{pmatrix}=\begin{pmatrix}-3&0\\2&-2\end{pmatrix}$.......(2)

From (1) & (2) $\quad (A-B)^T = A^T - B^T$

Show that the matrices $A=\begin{pmatrix}1&2\\3&1\end{pmatrix}$, $B=\begin{pmatrix}1&-2\\-3&1\end{pmatrix}$ satisfy commutative property $AB = BA$

Solution :

$AB=\begin{pmatrix}1&2\\3&1\end{pmatrix}\begin{pmatrix}1&-2\\-3&1\end{pmatrix}=\begin{pmatrix}1-6&-2+2\\3-3&-6+1\end{pmatrix}=\begin{pmatrix}-5&0\\0&-5\end{pmatrix}$

$BA=\begin{pmatrix}1&-2\\-3&1\end{pmatrix}\begin{pmatrix}1&2\\3&1\end{pmatrix}=\begin{pmatrix}1-6&2-2\\-3+3&-6+1\end{pmatrix}=\begin{pmatrix}-5&0\\0&-5\end{pmatrix}$

$\therefore AB = BA$

$\therefore$ Commutative property is true.

If $A=\begin{pmatrix}\cos\theta&0\\0&\cos\theta\end{pmatrix}$, $B=\begin{pmatrix}\sin\theta&0\\0&\sin\theta\end{pmatrix}$ then show that $A^2+B^2=I$.

Solution : $A=\begin{pmatrix}\cos\theta&0\\0&\cos\theta\end{pmatrix}$, $B=\begin{pmatrix}\sin\theta&0\\0&\sin\theta\end{pmatrix}$

$A^2=\begin{pmatrix}\cos\theta&0\\0&\cos\theta\end{pmatrix}\begin{pmatrix}\cos\theta&0\\0&\cos\theta\end{pmatrix}=\begin{pmatrix}\cos^2\theta&0\\0&\cos^2\theta\end{pmatrix}$

$B^2=\begin{pmatrix}\sin\theta&0\\0&\sin\theta\end{pmatrix}\begin{pmatrix}\sin\theta&0\\0&\sin\theta\end{pmatrix}=\begin{pmatrix}\sin^2\theta&0\\0&\sin^2\theta\end{pmatrix}$

$\therefore A^2+B^2=\begin{pmatrix}\cos^2\theta+\sin^2\theta&0\\0&\cos^2\theta+\sin^2\theta\end{pmatrix}=\begin{pmatrix}1&0\\0&1\end{pmatrix}$ $\quad A^2+B^2=I.$

Reduce to lowest form $\dfrac{x^{3a}-8}{x^{2}a+2xa+4}$ ***Solution :*** $\dfrac{x^{3a}-8}{x^2a+2xa+4}=\dfrac{(x^a-2)(x^{2a}+2x^a+4)}{x^{2a}+2x^a+4}=x^a-2$

Solve $\sqrt{2}f^2-6f+3\sqrt{2}=0$ by formula method

Solution : $\sqrt{2}f^2-6f+3\sqrt{2}=0 \qquad a=\sqrt{2},\ b=-6,\ c=3\sqrt{2}$

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}=\dfrac{6\pm\sqrt{36-4(\sqrt{2})(3\sqrt{2})}}{2\sqrt{2}}=\dfrac{3+\sqrt{3}}{\sqrt{2}},\ \dfrac{3-\sqrt{3}}{\sqrt{2}}$

Determine the nature of the roots $9a^2b^2x^2-24abcdx+16c^2d^2=0,\ a\neq 0\ b\neq 0$

Solution : $A=9a^2b^2, B=-24abcd, C=16c^2d^2$

$\therefore \Delta = B^2-4AC = 576a^2b^2c^2d^2-576a^2b^2c^2d^2=0$

$\therefore$ The roots are real & equal.

Discuss the nature of solutions of the equations $\dfrac{y+z}{4}=\dfrac{z+x}{3}=\dfrac{x+y}{2};\ x+y+z=27$

Solution :

$\Rightarrow \dfrac{y+z}{4}=\dfrac{z+x}{3}$ & $\dfrac{z+x}{3}=\dfrac{x+y}{2} \Rightarrow 3y+3z=4z+4x$ & $2z+2x=3x+3y$

$\Rightarrow 4x-3y+z=0$ & $x+3y-2z=0$

$\therefore 4x-3y+z=0$ ...... (1) $\quad x+3y-2z=0$ ..... (2) $\quad x+y+z=27$ .... (3)

From (1) & (2) $\dfrac{a_1}{a_2}\neq\dfrac{b_1}{b_2}\neq\dfrac{c_1}{c_2}$ ie $\dfrac{4}{1}\neq\dfrac{-3}{3}\neq\dfrac{-1}{2}$ $\quad$ From (2) & (3), $\dfrac{a_1}{a_2}\neq\dfrac{b_1}{b_2}\neq\dfrac{c_1}{c_2}$ ie $1\neq 3\neq -2$

$\therefore$ The system of equations has a unique solutions.

If $A=\begin{pmatrix}1&3&-2\\5&-4&6\\-3&2&9\end{pmatrix}$ $B=\begin{pmatrix}1&8\\3&4\\9&6\end{pmatrix}$ find A+B. ***Solution :*** A is of order 3×3 B is of order 3×2

It is not possible to add A and B because different orders.

Given $A=\begin{pmatrix}p&0\\0&2\end{pmatrix}$, $B=\begin{pmatrix}0&-q\\1&0\end{pmatrix}$, $C=\begin{pmatrix}2&-2\\2&2\end{pmatrix}$ and if $BA=C^2$, find p and q.

Solution : Given $BA=C^2 \Rightarrow \begin{pmatrix}0&-q\\1&0\end{pmatrix}\begin{pmatrix}p&0\\0&2\end{pmatrix}=\begin{pmatrix}2&-2\\2&2\end{pmatrix}\begin{pmatrix}2&-2\\2&2\end{pmatrix}\Rightarrow\begin{pmatrix}0&-2q\\p&0\end{pmatrix}=\begin{pmatrix}0&-8\\8&0\end{pmatrix}$

$\therefore p=8,\quad -2q=-8,\quad q=4$

If $A=\begin{pmatrix}\cos\theta&\sin\theta\\-\sin\theta&\cos\theta\end{pmatrix}$ prove that $AA^T=I$

Solution : $AA^T=\begin{pmatrix}\cos\theta&\sin\theta\\-\sin\theta&\cos\theta\end{pmatrix}\begin{pmatrix}\cos\theta&-\sin\theta\\\sin\theta&\cos\theta\end{pmatrix}=\begin{pmatrix}\cos^2\theta+\sin^2\theta&-\cos\theta\sin\theta+\sin\theta\cos\theta\\-\sin\theta\cos\theta+\cos\theta\sin\theta&\sin^2\theta+\cos^2\theta\end{pmatrix}=\begin{pmatrix}1&0\\0&1\end{pmatrix}=I$

Hence proved.

Solve $2x-3y=6$, $x+y=1$

Solution : $2x-3y=6$ —(1)

$x+y=1$ —(2)

(1) × 1 gives, $2x-3y=6$

(2) × 2 gives, $2x+2y=2$

(−) (−) (−)

$-5y=4$ gives, $y=\dfrac{-4}{5}$

$y=\dfrac{-4}{5}$ in (2),

$x-\dfrac{4}{5}=1=1+\dfrac{4}{5}=\dfrac{9}{5}$

Find the LCM of $x^4-1,\ x^2-2x+1$

Solution : $x^4-1=(x^2+1)(x+1)(x-1)\ 1)$

$x^2-2x+1=(x-1)(x-1)=(x-1)^2$

LCM $=(x^2+1)(x+1)(x-1)^2$

(1) ⇒ $2x - y = 2$
(2) ⇒ $-x + y = 2$
Adding, $x = 4$

Sub x = 4 in (2)
$-4 + y = 2 \Rightarrow y = 6$

$\therefore x = 4, y = 6$

Find the values of x, y, z if $\begin{pmatrix} x-3 & 3x-z \\ x+y+7 & x+y+z \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 6 \end{pmatrix}$

Solution : $\Rightarrow x - 3 = 1$, $\therefore x = 4$ | $3x - z = 0$, $12 - z = 0 \Rightarrow z = 12$ | $x + y + 7 = 1$, $\Rightarrow x + y = -6 \Rightarrow 4 + y = -6 \Rightarrow y = -10$

At t minutes past 2 pm, the time needed to 3 pm is 3 minutes less than $\frac{t^2}{4}$ **Find t.**

Solution :
$\therefore$ As per the data given, $\frac{t^2}{4} - 3 = 60 - t \Rightarrow t^2 - 12 = 240 - 4t \Rightarrow t^2 + 4t - 252 = 0$
$\Rightarrow (t+18)(t-14) = 0 \quad \therefore t = 14$ min.

If a matrix has 16 elements, what are the possible orders it can have?
Solution : The matrix has 16 elements . Hence possible orders are 1 × 16, 16 × 1, 4 × 4, 2 × 8, 8 × 2.

If $A = \begin{pmatrix} 1 & 2 & 0 \\ 3 & 1 & 5 \end{pmatrix}, B = \begin{pmatrix} 8 & 3 & 1 \\ 2 & 4 & 1 \\ 5 & 3 & 1 \end{pmatrix}$ find AB.

Solution :

$$AB = \begin{pmatrix} 1 & 2 & 0 \\ 3 & 1 & 5 \end{pmatrix} \times \begin{pmatrix} 8 & 3 & 1 \\ 2 & 4 & 1 \\ 5 & 3 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 8+4+0 & 3+8+0 & 1+2+0 \\ 24+2+25 & 9+4+15 & 3+1+5 \end{pmatrix} = \begin{pmatrix} 12 & 11 & 3 \\ 51 & 28 & 9 \end{pmatrix}$$

If $A = \begin{pmatrix} 2 & -2\sqrt{2} \\ \sqrt{2} & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 2\sqrt{2} \\ -\sqrt{2} & 2 \end{pmatrix}$ Show that A and B satisfy commutative property with respect to matrix multiplication.

Solution :

$$AB = \begin{pmatrix} 2 & -2\sqrt{2} \\ \sqrt{2} & 2 \end{pmatrix} \times \begin{pmatrix} 2 & 2\sqrt{2} \\ -\sqrt{2} & 2 \end{pmatrix} = \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix} \quad BA = \begin{pmatrix} 2 & 2\sqrt{2} \\ -\sqrt{2} & 2 \end{pmatrix} \times \begin{pmatrix} 2 & -2\sqrt{2} \\ \sqrt{2} & 2 \end{pmatrix} = \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix}$$

A and B satisfy commutative property

Solve $\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$

Solution :

By matrix multiplication $\begin{pmatrix} 2x+y \\ x+2y \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$ $2x + y = 4$ .... (1)
$x + 2y = 5$ .... (2)

(1) − 2 × (2) gives $2x + y = 4$
$2x + 4y = 10$ (−)
$-3y = -6$ gives y = 2

Substituting y = 2 in (1), 2x + 2 = 4 gives x = 1 Therefore, x = 1, y = 2.

If $x = \frac{a^2 + 3a - 4}{3a^2 - 3}$ **and** $y = \frac{a^2 + 2a - 8}{2a^2 - 2a - 4}$ **find the value of** $x^2 y^2$**.**

Solution : $x = \frac{a+4}{3(a+1)}$ | $y = \frac{a+4}{2(a+1)}$ $\quad \therefore x^2 y^{-2} = \frac{x^2}{y^2} = \frac{(a+4)^2}{9(a+1)^2} \times \frac{4(a+1)^2}{(a+4)^2} = \frac{4}{9}$

If one root of the equation $2y^2 - ay + 64 = 0$ **is twice the other then find the values of** a**.**

Solution : Let α, β be the roots of $2y^2 - ay + 64 = 0$ $\quad \alpha + \beta = a/2 \quad \alpha\beta = 32$

Given α = 2β $\quad \alpha\beta = 32 \Rightarrow 2\beta . \beta = 32 \quad \therefore \alpha + \beta = a/2 \Rightarrow 3\beta = a/2 \Rightarrow \beta = a/6$
$\Rightarrow \beta^2 = 16 \Rightarrow \beta = \pm 4$
$\therefore a/6 = \pm 4 \Rightarrow a = \pm 24$

The product of Kumaran's age (in years) two years ago and his age four years from now is one more than twice his present age. What is his present age?

Solution : Let the present age of Kumaran ⇒x years.
Two years ago, (x − 2) years. Four years from now, (x + 4) years.
Given, $(x - 2)(x + 4) = 1 + 2x \Rightarrow x^2 + 2x - 8 = 1 + 2x$ gives $(x - 3)(x + 3) = 0$
Kumaran's present age is 3 years.

If the difference between the roots of the equation $x^2 - 13x + k = 0$ is 17 find k.

Solution : $x^2 - 13x + k = 0$ here, a = 1, b = −13, c = k

$\alpha + \beta = \frac{-b}{a} = \frac{-(-13)}{1} = 13$ ...... (1) $\quad \alpha - \beta = 17$ .... (2)

(1) + (2) we get, 2α = 30 gives α = 15 Therefore, 15 + β = 13 (from (1)) gives β = −2

But $\alpha\beta = \frac{c}{a} = \frac{k}{1}$ gives $15 \times (-2) = k$ we get, $k = -30$

A has 'a' rows and 'a + 3' columns. B has 'b' rows and '17–b' columns, and if both products AB and BA exist, find a, b?

Solution : Given Order of A is a × (a + 3) Order of B is b × (17 − b)
⇒ a + 3 = b ⇒ a − b = − 3........ (1) Solving (1) & (2) 2a = 14 a = 7
⇒ 17 − b = a ⇒ a + b = 17 .... (2) Sub a = 7 in (1) 7 − b = − 3 b = 10

In the matrix $A = \begin{pmatrix} 8 & 9 & 4 & 3 \\ -1 & \sqrt{7} & \frac{\sqrt{3}}{2} & 5 \\ 1 & 4 & 3 & 0 \\ 6 & 8 & -11 & 1 \end{pmatrix}$ **write**

(i) The number of elements
(ii) The order of the matrix
(iii) Write the elements $a_{22}, a_{23}, a_{24}, a_{34}, a_{43}, a_{44}$

Solution : i) A has 4 rows and 4 columns Number of elements = 16
ii) Order of the matrix = 4 × 4
iii) $a_{22} = \sqrt{7}$, $a_{23} = \sqrt{3}/2$ $\quad a_{24} = 5$ $a_{34} = 0$, $a_{43} = -11$, $a_{44} = 1$

Find the values of x, y, z if $(x \quad y-z \quad z+3) + (y \quad 4 \quad 3) = (4 \quad 8 \quad 16)$

Solution : $(x \quad y-z \quad z+3) + (y \quad 4 \quad 3) = (4 \quad 8 \quad 16)$

⇒ $x + y = 4$
⇒ $x + 14 = 4$
⇒ $x = -10$

$y - z + 4 = 8$
$y - z = 4$
$y - 10 = 4$
∴ $y = 14$

$z + 6 = 16$
$z = 10$

∴ x = − 10, y = 14, z = 10

Find x and y if $x\begin{pmatrix} 4 \\ -3 \end{pmatrix} + y\begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$

Solution :

Given $x\begin{pmatrix} 4 \\ -3 \end{pmatrix} + y\begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$ $\quad 4x - 2y = 4$ .... (1)
$-3x + 3y = 6$ .... (2)

Find the value of a, b, c, d, x, y from the following matrix equation. $\begin{pmatrix} d & 8 \\ 3b & a \end{pmatrix}+\begin{pmatrix} 3 & a \\ -2 & -4 \end{pmatrix}=\begin{pmatrix} 2 & 2a \\ b & 4c \end{pmatrix}+\begin{pmatrix} 0 & 1 \\ -5 & 0 \end{pmatrix}$

Solution : $\begin{pmatrix} d+3 & 8+a \\ 3b-2 & a-4 \end{pmatrix}=\begin{pmatrix} 2 & 2a+1 \\ b-5 & 4c \end{pmatrix}$ $d+3=2 \Rightarrow d=2-3 \Rightarrow d=-1$

$8+a=2a+1 \Rightarrow 8-1=2a-a \Rightarrow a=7$

$3b-2=b-5 \Rightarrow 3b-b=-5+2 \Rightarrow 2b=-3 \Rightarrow b=\frac{-3}{2}$

Substituting a = 7 in $a-4=4c \Rightarrow 7-4=4c \Rightarrow 3=4c \Rightarrow c=\frac{3}{4}$

If $A=\begin{pmatrix} 1 & 9 \\ 3 & 4 \\ 8 & -3 \end{pmatrix}, B=\begin{pmatrix} 5 & 7 \\ 3 & 3 \\ 1 & 0 \end{pmatrix}$ **then verify that** $A+(-A)=(-A)+A=O$

Solution :

$$A+(-A)=\begin{pmatrix} 1 & 9 \\ 3 & 4 \\ 8 & -3 \end{pmatrix}+\begin{pmatrix} -1 & -9 \\ -3 & -4 \\ -8 & 3 \end{pmatrix}=\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}=O$$

$$(-A)+A=\begin{pmatrix} -1 & -9 \\ -3 & -4 \\ -8 & 3 \end{pmatrix}+\begin{pmatrix} 1 & 9 \\ 3 & 4 \\ 8 & -3 \end{pmatrix}=\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}=O$$

If $A=\begin{pmatrix} 1 & 8 & 3 \\ 3 & 5 & 0 \\ 8 & 7 & 6 \end{pmatrix}, B=\begin{pmatrix} 8 & -6 & -4 \\ 2 & 11 & -3 \\ 0 & 1 & 5 \end{pmatrix}, C=\begin{pmatrix} 5 & 3 & 0 \\ -1 & -7 & 2 \\ 1 & 4 & 3 \end{pmatrix}$ compute $\frac{1}{2}A-\frac{3}{2}B$

Solution :

$$\frac{1}{2}A-\frac{3}{2}B=\frac{1}{2}(A-3B)=\frac{1}{2}\left(\begin{pmatrix} 1 & 8 & 3 \\ 3 & 5 & 0 \\ 8 & 7 & 6 \end{pmatrix}-3\begin{pmatrix} 8 & -6 & -4 \\ 2 & 11 & -3 \\ 0 & 1 & 5 \end{pmatrix}\right)=\begin{pmatrix} -\frac{23}{2} & 13 & \frac{15}{2} \\ -\frac{3}{2} & -14 & \frac{9}{2} \\ 4 & 2 & -\frac{9}{2} \end{pmatrix}$$

If $A=\begin{pmatrix} 0 & 4 & 9 \\ 8 & 3 & 7 \end{pmatrix}, B=\begin{pmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{pmatrix}$ **find the value of B − 5A**

Solution :

$$B-5A=\begin{pmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{pmatrix}-\begin{pmatrix} 0 & 20 & 45 \\ 40 & 15 & 35 \end{pmatrix}=\begin{pmatrix} 7 & -17 & -37 \\ -39 & -11 & -26 \end{pmatrix}$$

Find the non-zero values of x satisfying the matrix equation $x\begin{pmatrix} 2x & 2 \\ 3 & x \end{pmatrix}+2\begin{pmatrix} 8 & 5x \\ 4 & 4x \end{pmatrix}=2\begin{pmatrix} x^2+8 & 24 \\ 10 & 6x \end{pmatrix}$

Solution :

$$\Rightarrow \begin{pmatrix} 2x^2 & 2x \\ 3x & x^2 \end{pmatrix}+\begin{pmatrix} 16 & 10x \\ 8 & 8x \end{pmatrix}=\begin{pmatrix} 2x^2+16 & 48 \\ 20 & 12x \end{pmatrix} \Rightarrow \begin{pmatrix} 2x^2+16 & 12x \\ 3x+8 & x^2+8x \end{pmatrix}=\begin{pmatrix} 2x^2+16 & 48 \\ 20 & 12x \end{pmatrix}$$

$\therefore 12x=48 \Rightarrow x=4$

Construct a 3 × 3 matrix whose elements are $a_{ij}=i^2j^2$

Solution : The general 3 × 3 matrix is given by $A=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ $a_{ij}=i^2j^2$

$a_{11}=1^2\times1^2=1\times1=1$; $a_{12}=1^2\times2^2=1\times4=4$; $a_{13}=1^2\times3^2=1\times9=9$

$a_{21}=2^2\times1^2=4\times1=4$; $a_{22}=2^2\times2^2=4\times4=16$; $a_{23}=2^2\times3^2=4\times9=36$

$a_{31}=3^2\times1^2=9\times1=9$; $a_{32}=3^2\times2^2=9\times4=36$; $a_{33}=3^2\times3^2=9\times9=81$

$$A=\begin{pmatrix} 1 & 4 & 9 \\ 4 & 16 & 36 \\ 9 & 36 & 81 \end{pmatrix}$$

Find the values of x, y and z from the fol lowing equations $\begin{pmatrix} x+y+z \\ x+z \\ y+z \end{pmatrix}=\begin{pmatrix} 9 \\ 5 \\ 7 \end{pmatrix}$

Solution :

$$\begin{pmatrix} x+y+z \\ x+z \\ y+z \end{pmatrix}=\begin{pmatrix} 9 \\ 5 \\ 7 \end{pmatrix}$$

$\Rightarrow x+y+z=0$ | $x+z=5$ | $y+z=7$

$\Rightarrow 5+y=9$ | $\Rightarrow x+3=5$ | $\Rightarrow 4+z=7$

$\Rightarrow y=4$ | $\Rightarrow x=2$ | $\Rightarrow z=3$

If $A=\begin{pmatrix} 5 & 4 & -2 \\ \frac{1}{2} & \frac{3}{4} & \sqrt{2} \\ 1 & 9 & 4 \end{pmatrix}$ $B=\begin{pmatrix} -7 & 4 & -3 \\ \frac{1}{4} & \frac{7}{2} & 3 \\ 5 & -6 & 9 \end{pmatrix}$ then Find $4A-3B$

Solution :

$$4A-3B=4\begin{pmatrix} 5 & 4 & -2 \\ \frac{1}{2} & \frac{3}{4} & \sqrt{2} \\ 1 & 9 & 4 \end{pmatrix}-3\begin{pmatrix} -7 & 4 & -3 \\ \frac{1}{4} & \frac{7}{2} & 3 \\ 5 & -6 & 9 \end{pmatrix}=\begin{pmatrix} 20 & 6 & -8 \\ 2 & 3 & 4\sqrt{2} \\ 4 & 36 & 16 \end{pmatrix}+\begin{pmatrix} 21 & -12 & 9 \\ -\frac{3}{4} & -\frac{21}{2} & -9 \\ -15 & 18 & -27 \end{pmatrix}=\begin{pmatrix} 41 & 4 & 1 \\ \frac{5}{4} & -\frac{15}{2} & 4\sqrt{2}-9 \\ -11 & 54 & -11 \end{pmatrix}$$

If $A=\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}, B=\begin{pmatrix} 1 & 7 & 0 \\ 1 & 3 & 1 \\ 2 & 4 & 0 \end{pmatrix}$, find A + B

Solution :

$$A+B=\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}+\begin{pmatrix} 1 & 7 & 0 \\ 1 & 3 & 1 \\ 2 & 4 & 0 \end{pmatrix}=\begin{pmatrix} 1+1 & 2+7 & 3+0 \\ 4+1 & 5+3 & 6+1 \\ 7+2 & 8+4 & 9+0 \end{pmatrix}=\begin{pmatrix} 2 & 9 & 3 \\ 5 & 8 & 7 \\ 9 & 12 & 9 \end{pmatrix}$$

If $A=\begin{pmatrix} 1 & 8 & 3 \\ 3 & 5 & 0 \\ 8 & 7 & 6 \end{pmatrix}, B=\begin{pmatrix} 8 & -6 & -4 \\ 2 & 11 & -3 \\ 0 & 1 & 5 \end{pmatrix}, C=\begin{pmatrix} 5 & 3 & 0 \\ -1 & -7 & 2 \\ 1 & 4 & 3 \end{pmatrix}$ compute $3A+2B-C$

Solution :

$$3A+2B-C=3\begin{pmatrix} 1 & 8 & 3 \\ 3 & 5 & 0 \\ 8 & 7 & 6 \end{pmatrix}+2\begin{pmatrix} 8 & -6 & -4 \\ 2 & 11 & -3 \\ 0 & 1 & 5 \end{pmatrix}-\begin{pmatrix} 5 & 3 & 0 \\ -1 & -7 & 2 \\ 1 & 4 & 3 \end{pmatrix}$$

$$=\begin{pmatrix} 3 & 24 & 9 \\ 9 & 15 & 0 \\ 24 & 21 & 18 \end{pmatrix}+\begin{pmatrix} 16 & -12 & -8 \\ 4 & 22 & -6 \\ 0 & 2 & 10 \end{pmatrix}+\begin{pmatrix} -5 & -3 & 0 \\ 1 & 7 & -2 \\ -1 & -4 & -3 \end{pmatrix}=\begin{pmatrix} 14 & 9 & 1 \\ 14 & 44 & -8 \\ 23 & 19 & 25 \end{pmatrix}$$

If $A=\begin{pmatrix} 1 & 9 \\ 3 & 4 \\ 8 & -3 \end{pmatrix}, B=\begin{pmatrix} 5 & 7 \\ 3 & 3 \\ 1 & 0 \end{pmatrix}$ **then verify that A + B = B + A**

Solution :

$$\therefore A+B=\begin{pmatrix} 1 & 9 \\ 3 & 4 \\ 8 & -3 \end{pmatrix}+\begin{pmatrix} 5 & 7 \\ 3 & 3 \\ 1 & 0 \end{pmatrix}=\begin{pmatrix} 6 & 16 \\ 6 & 7 \\ 9 & -3 \end{pmatrix} \quad B+A=\begin{pmatrix} 5 & 7 \\ 3 & 3 \\ 1 & 0 \end{pmatrix}+\begin{pmatrix} 1 & 9 \\ 3 & 4 \\ 8 & -3 \end{pmatrix}=\begin{pmatrix} 6 & 16 \\ 6 & 7 \\ 9 & -3 \end{pmatrix}$$

$\therefore A+B=B+A$

If $A=\begin{pmatrix} 0 & 4 & 9 \\ 8 & 3 & 7 \end{pmatrix}, B=\begin{pmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{pmatrix}$ **find the value of 3A − 9B**

Solution :

$$3A-9B=\begin{pmatrix} 0 & 12 & 27 \\ 24 & 9 & 21 \end{pmatrix}-\begin{pmatrix} 63 & 27 & 72 \\ 9 & 36 & 81 \end{pmatrix}=\begin{pmatrix} -63 & -65 & -45 \\ 15 & -27 & -60 \end{pmatrix}$$

Construct a 3 × 3 matrix whose elements are given by $a_{ij} = |i-2j|$

Solution : Given $a_{ij} = |i-2j|$, 3 × 3

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$a_{11} = |1-2| = |-1| = 1$ $a_{12} = |1-4| = |-3| = 3$

$a_{13} = |1-6| = |-5| = 5$ $a_{21} = |2-2| = 0$

$a_{22} = |2-4| = |-2| = 2$ $a_{23} = |2-6| = |-4| = 4$ $\therefore A = \begin{pmatrix} 1 & 3 & 5 \\ 0 & 2 & 4 \\ 1 & 1 & 3 \end{pmatrix}$

$a_{31} = |3-2| = |1| = 1$ $a_{32} = |3-4| = |-1| = 1$

$a_{33} = |3-6| = |-3| = 3$

If $A = \begin{pmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{pmatrix}$ **then verify** $(A^T)^T = A$

Solution :

$$A^T = \begin{pmatrix} 5 & -\sqrt{17} & 8 \\ 2 & 0.7 & 3 \\ 2 & 5/2 & 1 \end{pmatrix} \quad (A^T)^T = \begin{pmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & 5/2 \\ 8 & 3 & 1 \end{pmatrix} = A$$

Find the values of x, y and z from the following equations $\begin{pmatrix} 12 & 3 \\ x & \frac{3}{2} \end{pmatrix} = \begin{pmatrix} y & z \\ 3 & 5 \end{pmatrix}$

Solution :

$$\begin{pmatrix} 12 & 3 \\ x & \frac{3}{2} \end{pmatrix} = \begin{pmatrix} y & z \\ 3 & 5 \end{pmatrix} \Rightarrow x = 3,\ y = 12,\ z = 3$$

Construct a 3 × 3 matrix whose elements are given by $a_{ij} = \dfrac{(i+j)^3}{3}$

Solution :

$a_{11} = 8/3$, $a_{12} = 27/3 = 9$, $a_{13} = 64/3$ $a_{21} = 27/3 = 9$, $a_{22} = 64/3$,

$a_{23} = 125/3$ $a_{31} = 64/3$, $a_{32} = 125/3$, $a_{33} = 216/3 = 72$

$$\therefore A = \begin{pmatrix} 8/3 & 9 & 64/3 \\ 9 & 64/3 & 125/3 \\ 64/3 & 125/3 & 72 \end{pmatrix}$$

If $A = \begin{pmatrix} 7 & 8 & 6 \\ 1 & 3 & 9 \\ -4 & 3 & -1 \end{pmatrix}$ $B = \begin{pmatrix} 4 & 11 & -3 \\ -1 & 2 & 4 \\ 7 & 5 & 0 \end{pmatrix}$ then Find 2A + B

Solution :

$$2A+B = 2\begin{pmatrix} 7 & 8 & 6 \\ 1 & 3 & 9 \\ -4 & 3 & -1 \end{pmatrix} + \begin{pmatrix} 4 & 11 & -3 \\ -1 & 2 & 4 \\ 7 & 5 & 0 \end{pmatrix} = \begin{pmatrix} 14 & 16 & 12 \\ 2 & 6 & 18 \\ -8 & 6 & -2 \end{pmatrix} + \begin{pmatrix} 4 & 11 & -3 \\ -1 & 2 & 4 \\ 7 & 5 & 0 \end{pmatrix} = \begin{pmatrix} 18 & 27 & 9 \\ 1 & 8 & 22 \\ -1 & 11 & -2 \end{pmatrix}$$

Find the values of x, y and z from the following equations $\begin{pmatrix} x+y & 2 \\ 5+z & xy \end{pmatrix} = \begin{pmatrix} 6 & 2 \\ 5 & 8 \end{pmatrix}$

Solution :

$$\begin{pmatrix} x+y & 2 \\ 5+z & xy \end{pmatrix} = \begin{pmatrix} 6 & 2 \\ 5 & 8 \end{pmatrix} \Rightarrow x+y = 6,\ xy = 8,$$

x = 2 (or) 4, y = 4 (or) 2 5 + z = 5 ⇒ z = 0

4 . GEOMETRY

Define – Cevian.

A cevian is a line segment that extends from one vertex of a triangle to the opposite side.

Write the Ceva's Theorem

Let ABC be a triangle and let D,E,F be points on lines BC, CA, AB respectively. Then the cevians AD, BE, CF are concurrent if and only if $\frac{BD}{DC} \times \frac{CE}{EA} \times \frac{AF}{FB} = 1$ where the lengths are directed.

Write the Menelaus Theorem

A necessary and sufficient condition for points P, Q, R on the respective sides BC, CA, AB of a triangle ABC to be collinear is that $\frac{BP}{PC} \times \frac{CQ}{QA} \times \frac{AR}{RB} = 1$ where all segments in the formula are directed segments.

Write the **Alternate Segment theorem**

The angles between the tangent and the chord are equal to the angles in the corresponding alternate segments.

Define – Tangent.

If a line touches the given circle at only one point, then it is called tangent to the circle.

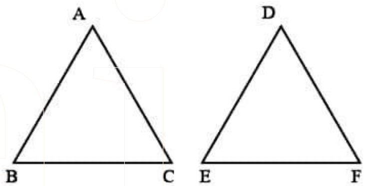
If ΔABC ~ ΔDEF such that area of ΔABC is 9cm² and the area of ΔDEF is 16cm² and BC = 2.1 cm. Find the length of EF.

Solution : Given ΔABC ~ ΔDEF

$$\therefore \frac{\text{Area of } \Delta\, ABC}{\text{Area of } \Delta\, DEF} = \frac{BC^2}{EF^2} \Rightarrow \frac{9}{16} = \frac{(2.1)^2}{EF^2}$$

$$\Rightarrow EF^2 = \frac{16 \times (2.1)^2}{9}$$

$$\therefore EF = \frac{4 \times 2.1}{3} = 2.8\ cm$$

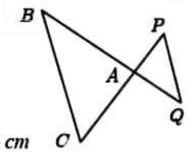


In the adjacent figure, ΔACB ~ ΔAPQ. If BC = 8 cm, PQ = 4 cm, BA = 6.5 cm and AP = 2.8 cm, find CA and AQ.

Solution : Given Δ ACB ~ Δ APQ $\therefore \frac{BC}{PQ} = \frac{AC}{AP} = \frac{AB}{AQ} \Rightarrow \frac{8}{4} = \frac{AC}{2.8} = \frac{6.5}{AQ}$

$\therefore \frac{AC}{2.8} = 2$ | $\frac{6.5}{AQ} = 2$

$\Rightarrow AC = 5.6\ cm$ | $\Rightarrow AQ = \frac{6.5}{2} = 3.25\ cm$

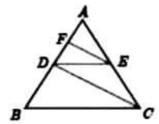


In figure DE || BC and CD || EF. Prove that AD² = AB × AF

Solution : In figure DE || BC and CD || EF

In Δ ACD, by BPT, $\frac{AF}{AD} = \frac{AE}{AC}$ ..... (1) In Δ ABC, by BPT, $\frac{AD}{AB} = \frac{AE}{AC}$ ..... (2)

∴ From (1) & (2) $\frac{AF}{AD} = \frac{AD}{AB} \Rightarrow$ **AD² = AB × AF**

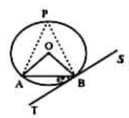


A tangent ST to a circle touches it at B. AB is a chord such that ∠ABT = 65⁰. Find ∠AOB, where "O" is the centre of the circle.

Solution:

(angles in alternate segment). ∠TBA = 65⁰ ⇒ ∠APB = 65⁰

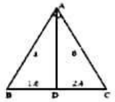
∴ ∠AOB=2∠APB=2(65⁰)=130⁰



Check whether AD is bisector of $\angle A$ of ΔABC, AB = 4 cm, AC = 6 cm, BD = 1.6. cm and CD = 2.4 cm.

Solution : $\frac{AB}{AC}=\frac{4}{6}=\frac{2}{3}$, $\frac{BD}{DC}=\frac{1.6}{2.4}=\frac{2}{3}$

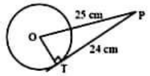
$\therefore$ By Converse of ABT, $\therefore \frac{AB}{AC}=\frac{BD}{DC}$ AD is the bisector of $\angle A$.



The length of the tangent to a circle from a point P, which is 25 cm away from the centre is 24 cm. What is the radius of the circle?

Solution: $\therefore OT=\sqrt{25^2-24^2}=\sqrt{625-576}=\sqrt{49}=7$ cm

$\therefore$ Radius = 7 cm



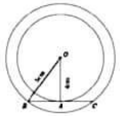
If radii of two concentric circles are 4 cm and 5 cm then find the length of the chord of one circle which is a tangent to the other circle.

Solution : $OB^2=OA^2+AB^2$

$5^2=4^2+AB^2$

$AB^2=9 \Rightarrow AB=3$ cm

$BC=2AB \Rightarrow BC=2\times 3=6$ cm

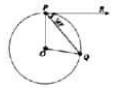


In Figure O is the centre of a circle. PQ is a chord and the tangent PR at P makes an angle of 50° with PQ. Find $\angle POQ$

Solution : $\angle OPQ=90^0-50^0=40^0$ OP = OQ (Radii of a circle are equal)

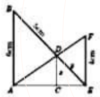
$\angle OPQ=\angle OQP=40^0$ (ΔOPQ is isosceles) $\angle POQ=180^0-\angle OPQ-\angle OQP$

$\angle POQ=180^0-40^0-40^0=100^0$



In the given figure AB || CD || EF. If AB = 6 cm, CD = x cm, EF = 4 cm, BD = 5 cm and DE = y cm. Find x and y.

Solution : $x=\frac{ab}{a+b}=\frac{6\times 4}{6+4}=\frac{24}{10}=\frac{12}{5}$



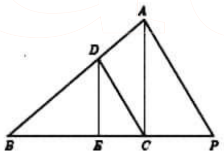
In the Fig. DE || AC and DC || AP. Prove that $\frac{BE}{EC}=\frac{BC}{CP}$

Solution : In ΔBPA, DC || AP. In ΔBCA, DE || AC.

$\frac{BC}{CP}=\frac{BD}{DA}$ ......(1) $\frac{BE}{EC}=\frac{BD}{DA}$ ......(2)

By Basic Proportionality Theorem,

From (1) and (2) we get, $\frac{BE}{EC}=\frac{BC}{CP}$ Hence proved.

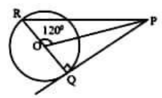


PQ is a tangent drawn from a point P to a circle with centre O and QOR is a diameter of the circle such that $\angle POR=120°$. Find $\angle OPQ$.

Solution: Given $\angle POR=120^0 \Rightarrow \angle POQ=60^0$ (linear pair)

$\angle OQP=90^0$ (Radius $\perp$ tangent)

$\therefore \angle OPQ=90^0-60^0=30^0$

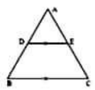


O is any point inside a triangle ABC. The bisector of $\angle AOB$, $\angle BOC$ and $\angle COA$ meet the sides AB, BC and CA in point D, E and F respectively. Show that AD × BE × CF = DB × EC × FC

Solution : By using Ceva's Theorem, $\frac{AD}{DB}\times\frac{BE}{EC}\times\frac{AF}{FC}=1$

$\Rightarrow AD\times BE\times AF=DB\times EC\times FC$

Hence proved.



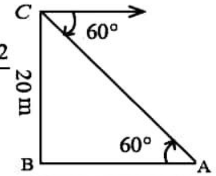
6. TRIGONOMETRY

A player sitting on the top of a tower of height 20 m observes the angle of depression of a ball lying on the ground as 60°. Find the distance between the foot of the tower and the ball. ($\sqrt{3}=1.732$)

Solution : $\tan 60^0=\frac{BC}{AB} \Rightarrow \sqrt{3}=\frac{20}{AB} \Rightarrow AB=\frac{20}{\sqrt{3}} \Rightarrow AB=\frac{20\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}}=\frac{20\times 1.732}{3}$

$=11.54$m

distance between the foot of the tower and the ball is 11.54 m.

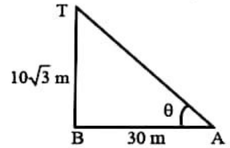


Find the angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of a tower of height $10\sqrt{3}$ m.

Solution :

$\tan\theta=\frac{10\sqrt{3}}{30} \Rightarrow \tan\theta=\frac{\sqrt{3}}{3} \Rightarrow \tan\theta=\frac{1}{\sqrt{3}}$

$\therefore \theta=30^0$

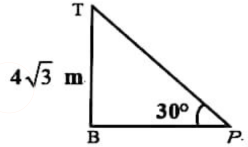


A road is flanked on either side by continuous rows of houses of height $4\sqrt{3}$ m with no space in between them. A pedestrian is standing on the median of the road facing a row house. The angle of elevation from the pedestrian to the top of the house is 30°. Find the width of the road.

Solution :

$\tan 30^0=\frac{4\sqrt{3}}{PB} \Rightarrow \frac{1}{\sqrt{3}}=\frac{4\sqrt{3}}{PB} \Rightarrow PB=12$ m

$\therefore$ Width of the road $=2\ PB=2\ (12)=24$ m

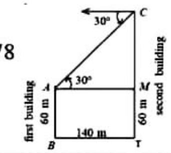


The horizontal distance between two buildings is 140 m. The angle of depression of the top of the first building when seen from the top of the second building is 30°. If the height of the first building is 60 m, find the height of the second building. ($\sqrt{3}=1.732$)

Solution :

$\tan 30^0=\frac{CM}{140}$ $\frac{1}{\sqrt{3}}=\frac{CM}{140} \Rightarrow CM=\frac{140}{\sqrt{3}}=\frac{140\sqrt{3}}{3}=\frac{140\times 1.732}{3}=80.78$

height of the second building $=60+80.78=140.78$ m



Prove $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}=\sec\theta+\tan\theta$

Solution : $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}=\sqrt{\frac{1+\sin\theta}{1-\sin\theta}\times\frac{1+\sin\theta}{1+\sin\theta}}=\sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}}=\frac{1+\sin\theta}{\cos\theta}=\frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}=\sec\theta+\tan\theta$

Prove $\frac{\cos\theta}{1+\sin\theta}=\sec\theta-\tan\theta$

Solution : $\frac{\cos\theta}{1+\sin\theta}=\frac{\cos\theta}{1+\sin\theta}\times\frac{1-\sin\theta}{1-\sin\theta}=\frac{\cos\theta(1-\sin\theta)}{\cos^2\theta}=\frac{\cos\theta(1-\sin\theta)}{\cos\theta\ \cos\theta}$

$=\frac{1-\sin\theta}{\cos\theta}=\frac{1}{\cos\theta}-\frac{\sin\theta}{\cos\theta}=\sec\theta-\tan\theta$

Prove that $\tan^2 A-\tan^2 B=\frac{\sin^2 A-\sin^2 B}{\cos^2 A\cos^2 B}$

Solution :

$\tan^2 A-\tan^2 B=\frac{\sin^2 A}{\cos^2 A}-\frac{\sin^2 B}{\cos^2 B}=\frac{\sin^2 A\cos^2 B-\sin^2 B\cos^2 A}{\cos^2 A\cos^2 B}=\frac{\sin^2 A\left(1-\sin^2 B\right)-\sin^2 B\left(1-\sin^2 A\right)}{\cos^2 A\cos^2 B}$

$=\frac{\sin^2 A-\sin^2 A\sin^2 B-\sin^2 B+\sin^2 A\sin^2 B}{\cos^2 A\cos^2 B}=\frac{\sin^2 A-\sin^2 B}{\cos^2 A\cos^2 B}$

Prove that $\frac{\sin A}{1+\cos A}+\frac{\sin A}{1-\cos A}=2\text{cosec}\, A$

Solution : $\frac{\sin A}{1+\cos A}+\frac{\sin A}{1-\cos A}=\frac{2\sin A}{(1-\cos A)\times(1+\cos A)}=\frac{2\sin A}{1-\cos^2 A}=\frac{2\sin A}{\sin^2 A}=\frac{2\sin A}{\sin A\,\sin A}=2\text{cosec}\, A$

Prove that $\sin^2 A\cos^2 B+\cos^2 A\sin^2 B+\cos^2 A\cos^2 B+\sin^2 A\sin^2 B=1$

Solution : $\sin^2 A\cos^2 B+\cos^2 A\sin^2 B+\cos^2 A\cos^2 B+\sin^2 A\sin^2 B$

$=\sin^2 A\cos^2 B+\sin^2 A\sin^2 B+\cos^2 A\sin^2 B+\cos^2 A\cos^2 B$

$=\sin^2 A(\cos^2 B+\sin^2 B)+\cos^2 A(\sin^2 B+\cos^2 B)$

$=\sin^2 A(1)+\cos^2 A(1)=\sin^2 A+\cos^2 A=1$

Prove $\sec^6\theta=\tan^6\theta+3\tan^2\theta\sec^2\theta+1$

Solution Take $a=1$ $b=\tan^2\theta$

$(1+\tan^2\theta)^3=1+\tan^6\theta+3(1)\tan^2\theta)(1+\tan^2\theta)$ $(\because a+b)^3=a^3+b^3+3ab(a+b))$

$\sec^6\theta=1+\tan^6\theta+3\tan^2\theta\,.\sec^2\theta$

Prove $(\sin\theta+\sec\theta)^2+(\cos\theta+\text{cosec}\,\theta)^2=1+(\sec\theta+\text{cosec}\,\theta)^2$

Solution : $(\sin\theta+\sec\theta)^2+(\cos\theta+\text{cosec}\,\theta)^2=\sin^2\theta+\sec^2\theta+2\sin\theta\,.\sec\theta+\cos^2\theta+\text{cosec}^2\theta+2\cos\theta\,.\text{cosec}\,\theta$

$=1+(\sec\theta+\text{cosec}\,\theta)^2$

Prove that $\tan^2\theta-\sin^2\theta=\tan^2\theta\sin^2\theta$

Solution : $\tan^2\theta-\sin^2\theta=\frac{\sin^2\theta}{\cos^2\theta}-\sin^2\theta$

$=\sin^2\theta\left(\frac{1}{\cos^2\theta}-1\right)$

$=\sin^2\theta(\sec^2\theta-1)=\tan^2\theta\sin^2\theta$

Prove $\frac{1-\tan^2\theta}{\cot^2\theta-1}=\tan^2\theta$

Solution : $1-\tan^2\theta=\tan^2\theta(\cot^2\theta-1)$

$1-\tan^2\theta=\tan^2\theta\cot^2\theta-\tan^2\theta$

$1-\tan^2\theta=1-\tan^2\theta$

Prove $\tan^4\theta+\tan^2\theta=\sec^4\theta-\sec^2\theta$

Solution : $\tan^2\theta(\tan^2\theta+1)=\sec^2\theta\,.(\sec^2\theta-1)$

$\tan^2\theta\sec^2\theta=\tan^2\theta\sec^2\theta$

Prove $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}+\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}=2\sec\theta$

Solution : $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}+\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}=(\sec\theta+\tan\theta)+(\sec\theta-\tan\theta)=\sec\theta+\sec\theta=2\sec\theta$

Prove that $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}=\text{cosec}\,\theta+\cot\theta$

Solution : $\sqrt{\frac{1+\cos\theta}{1\quad\cos\theta}}=\sqrt{\frac{(1+\cos\theta)\times(1+\cos\theta)}{(1-\cos\theta)\times(1+\cos\theta)}}=\sqrt{\frac{(1+\cos\theta)^2}{\sin^2\theta}}$

$=\frac{1+\cos\theta}{\sin\theta}=\text{cosec}\,\theta+\cot\theta$

The horizontal distance between two buildings is 70 m. The angle of depression of the top of the first building when seen from the top of the second building is 45°. If the height of the second building is 120 m, find the height of the first building.

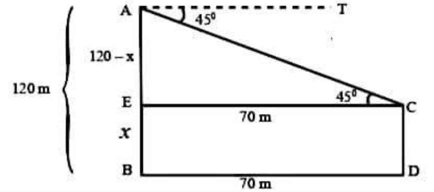
Solution: $\tan 45^0=\frac{AE}{EC}\Rightarrow 1=\frac{120-x}{70}$

$\Rightarrow 70=120-x$

$\Rightarrow x=120-70$

$\Rightarrow x=50$

$\therefore$ Height of 1st building = 50 m

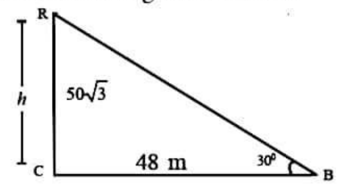


A tower stands vertically on the ground. From a point on the ground, which is 48 m away from the foot of the tower, the angle of elevation of the top of the tower is 30°. Find the height of the tower.

Solution : $\tan 30^0=\frac{h}{48}$

$\Rightarrow\frac{1}{\sqrt{3}}=\frac{h}{48}\Rightarrow h=16\sqrt{3}$

The height of the tower is $16\sqrt{3}$ m

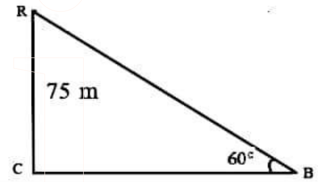


A kite is flying at a height of 75 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string, assuming that there is no slack in the string.

Solution : $\sin 60^0=\frac{75}{RB}\Rightarrow\frac{\sqrt{3}}{2}=\frac{75}{RB}$

$\Rightarrow RB=\frac{150}{\sqrt{3}}=50\sqrt{3}$

length of the string is $50\sqrt{3}$ m

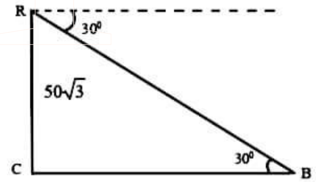


From the top of a rock $50\sqrt{3}$ m high, the angle of depression of a car on the ground is observed to be 30°. Find the distance of the car from the rock.

Solution: $\tan 30^0=\frac{RC}{CB}\Rightarrow\frac{1}{\sqrt{3}}=\frac{50\sqrt{3}}{CB}$

$\Rightarrow CB=50\sqrt{3}\sqrt{3}\Rightarrow CB=150\text{m}$

$\therefore$ Dist. of the car from the rock = 150m



Prove that $\frac{\sin A}{1+\cos A}=\frac{1-\cos A}{\sin A}$

Solution : $\sin A\times\sin A=(1+\cos A)\times(1-\cos A)\Rightarrow\sin^2 A=1-\cos^2 A\Rightarrow\sin^2 A=\sin^2 A$

Show that $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right)=\left(\frac{1-\tan A}{1-\cot A}\right)^2$

Solution : $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right)=\left(\frac{1-\tan A}{1-\cot A}\right)^2\Rightarrow\left(\frac{1+\tan^2 A}{\frac{\tan^2 A+1}{\tan^2 A}}\right)=\left(\frac{1-\tan A}{\frac{\tan A-1}{\tan A}}\right)^2\Rightarrow\tan^2 A=(-\tan A)^2\Rightarrow\tan^2 A=\tan^2 A$

Prove that $(\text{cosec}\theta-\sin\theta)(\sec\theta-\cos\theta)(\tan\theta+\cot\theta)=1$

Solution : $(\text{cosec}\theta-\sin\theta)(\sec\theta-\cos\theta)(\tan\theta+\cot\theta)=\left(\frac{1}{\sin\theta}-\sin\theta\right)\left(\frac{1}{\cos\theta}-\cos\theta\right)\left(\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}\right)$

$=\frac{1-\sin^2\theta}{\sin\theta}\times\frac{1-\cos^2\theta}{\cos\theta}\times\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}=\frac{\cos^2\theta\sin^2\theta\times 1}{\sin^2\theta\cos^2\theta}=1$

Prove that $\left(\frac{\cos^3 A - \sin^3 A}{\cos A - \sin A}\right) - \left(\frac{\cos^3 A + \sin^3 A}{\cos A + \sin A}\right) = 2\sin A\cos A$

Solution : $\text{LHS} = \left(\frac{(\cos A - \sin A)(\cos^2 A + \sin^2 A + \cos A\sin A)}{\cos A - \sin A}\right) - \left(\frac{(\cos A + \sin A)(\cos^2 A + \sin^2 A - \cos A\sin A)}{\cos A + \sin A}\right)$

$= (1 + \cos A\sin A) - (1 - \cos A\sin A) = 2\cos A\sin A$

5. COORDINATE GEOMETRY

The line through the points (–2, 6) and (4, 8) is perpendicular to the line through the points (8, 12) and (x, 24). FInd the value of x.

Solution : Slope of line joining (–2, 6), (4, 8) $m_1 = \frac{8-6}{4+2} = \frac{2}{6} = \frac{1}{3}$

Slope of line joining (8, 12), (x, 24) $m_2 = \frac{24-12}{x-8} = \frac{12}{x-8}$

Since two lines are perpendicular, $\Rightarrow \frac{1}{3} \times \frac{12}{x-8} = -1 \Rightarrow \frac{4}{x-8} = -1 \Rightarrow -x+8=4 \Rightarrow x=4$

Find the intercepts made by the line $4x - 9y + 36 = 0$ on the coordinate axes.

Solution : put $x=0 \Rightarrow 4x=-36$ x intercept $a=-9$

put $y=0 \Rightarrow -9y+36=0.\ -9y=-36 \Rightarrow y$ intercept $b=4$

Show that the straight lines $2x + 3y - 8 = 0$ and $4x + 6y + 18 = 0$ are parallel.

Solution :
Slope of the straight line $2x + 3y - 8 = 0$ is $m_1 = \frac{-\text{coefficient of } x}{\text{coefficient of } y} = \frac{-2}{3}$

Slope of the straight line $4x + 6y + 18 = 0$ is $m_2 = \frac{-4}{6} = \frac{-2}{3}$ Here, $m_1 = m_2$

That is, slopes are equal. Hence, the two straight lines are parallel.

Show that the straight lines $x - 2y + 3 = 0$ and $6x + 3y + 8 = 0$ are perpendicular.

Solution : Slope of the straight line $x - 2y + 3 = 0$ is $m_1 = \frac{-1}{-2} = \frac{1}{2}$

Slope of the straight line $6x + 3y + 8 = 0$ is $m_2 = \frac{-6}{3} = -2$

Now, $m_1 \times m_2 = \frac{1}{2} \times (-2) = -1$ Hence, the two straight lines are perpendicular.

Find the equation of a straight line which is parallel to the line $3x - 7y = 12$ and passing through the point (6, 4).

Solution : Equation of the straight line, parallel to $3x - 7y - 12 = 0$ is $3x - 7y + k = 0$

$3(6) - 7(4) + k = 0 \Rightarrow k = 28 - 18 = 10$

The required straight line is $3x - 7y + 10 = 0$.

Find the equation of a straight line perpendicular to the line $y = \frac{4}{3}x - 7$ and passing through the point (7, – 1).

Solution : The equation $y = \frac{4}{3}x - 7 \Rightarrow 4x - 3y - 21 = 0$.

Equation of a straight line perpendicular to $4x - 3y - 21 = 0$ is $3x + 4y + k = 0$

it is passes through the point (7, –1), $21 - 4 + k = 0 \Rightarrow k = -17$

The required straight line is $3x + 4y - 17 = 0$.

Find the slope and y intercept of $\sqrt{3}x + (1-\sqrt{3})y = 3$.

Solution : $a = \sqrt{3}$ $b = (1-\sqrt{3})$ $c = -3$

Slope of the line $= \frac{-a}{b} = \frac{-\sqrt{3}}{(1-\sqrt{3})} = \frac{3+\sqrt{3}}{2}$

Interecept on y-axis $= \frac{-c}{b} = \frac{-(-3)}{1-\sqrt{3}} = \frac{3+3\sqrt{3}}{-2}$

Find the equation of a line whose inclination is 30° and making an intercept -3 on the Y axis.

Solution : Given $\theta = 30° \Rightarrow m = \tan 30° = \frac{1}{\sqrt{3}}$ and y - intercept $= -3$

The required equation of the line is $y = mx + c$ $\Rightarrow y = \frac{1}{\sqrt{3}}x - 3 \Rightarrow \sqrt{3}y = x - 3\sqrt{3}$

$\Rightarrow x - \sqrt{3}y - 3\sqrt{3} = 0$

Find the equation of a line through the given pair of points $\left(2, \frac{2}{3}\right)$ and $\left(\frac{-1}{2}, -2\right)$

Solution : Given points are $\left(2, \frac{2}{3}\right), \left(\frac{-1}{2}, -2\right)$ two-point form $\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$

$\frac{y-\frac{2}{3}}{-2-\frac{2}{3}} = \frac{x-2}{\frac{-1}{2}-2} \Rightarrow \frac{3y-2}{-8} = \frac{x-2}{\frac{-5}{2}} \Rightarrow \frac{3y-2}{-8} = \frac{2x-4}{-5} \Rightarrow$

$15y - 10 = 16x - 32$

$16x - 15y - 22 = 0$

The equation of a straight line is $2(x-y)+5=0$. Find its slope, inclination and intercept on the Y axis.

Solution : $2(x-y)+5=0 \Rightarrow 2x-2y+5=0$

i) Slope of the line $= \frac{-a}{b} = \frac{-2}{-2} = 1$

ii) The slope of the straight line is $m = \tan\theta$

Slope of the line $= 1$ $\therefore \tan\theta = 1$ $\therefore \theta = 45°$.

iii) Intercept on y-axis $= \frac{-c}{b} = \frac{-5}{-2}$ $\therefore y$ - intercept $= 5/2$

Find the value of 'a', if the line through (–2, 3) and (8, 5) is perpendicular to $y = ax + 2$.

Solution :

Slope of the line joining (– 2, 3), (8, 5). $\Rightarrow \frac{y_2-y_1}{x_2-x_1} = \frac{5-3}{8+2} = \frac{2}{10} \Rightarrow m_1 = \frac{1}{5}$

Slope of the line $y = ax + 2 \Rightarrow ax - y + 2 = 0$ Slope of the line $= \frac{-a}{b} = \frac{-a}{-1} \Rightarrow m_2 = a$.

$m_1 m_2 = -1 \Rightarrow \frac{1}{5} \times a = -1 \Rightarrow a = -5$

Find the equation of a straight line passing through (5, –3) and (7, – 4).

Solution : $\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1} \Rightarrow \frac{y+3}{-4+3} = \frac{x-5}{7-5}$

$2y + 6 = -x + 5$

$x + 2y + 1 = 0.$

The required equation of the line is $x + 2y + 1 = 0$.

Find the equation of a straight line passing through the mid-point of a line segment joining the points (1,-5), (4,2) and parallel to (i) X axis (ii) Y axis

Solution : Equation of a Straight line parallel to the Y axis is $x = c$.
Equation of a straight line parallel to X axis is $y = b$

Mid point of the line joining the points $(1,-5), (4,2)$ is $= \left[\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right] = \left(\frac{1+4}{2}, \frac{-5+2}{2}\right) = \left(\frac{5}{2}, \frac{-3}{2}\right)$

(i) Parallel to x-axis is $y = -3/2$ (ii) Parallel to y-axis is $x = 5/2$

Determine the sets of points are collinear ? $(a, b+c), (b, c+a)$ and $(c, a+b)$

Solution : Area of triangle $= \frac{1}{2}\begin{bmatrix} a & b & c & a \\ b+c & c+a & a+b & b+c \end{bmatrix}$

$= \frac{1}{2}[(a^2+b^2+c^2+ab+bc+ca) - (a^2+b^2+c^2+ab+bc+ca)] = \frac{1}{2}[0] = 0$

$\therefore$ The 3 points are collinear.

If the straight lines $12y = -(p+3)x + 12$, $12x - 7y = 16$ are perpendicular then find 'p'.

Solution : $12y = -(p+3)x + 12, \Rightarrow (p+3)x + 12y = 12$ **and** $12x - 7y = 16$ are perpendicular

$m_1 = \frac{-(p+3)}{12}, \; m_2 = \frac{12}{7}$

$m_1 \times m_2 = -1 \Rightarrow \frac{-(p+3)}{12} \times \frac{12}{7} = -1 \Rightarrow p = 4$

Determine the sets of points are collinear ? $\left(-\frac{1}{2}, 3\right)$, (–5, 6) and (–8, 8)

Solution

Area of triangle $= \frac{1}{2}\begin{bmatrix} -1/2 & -5 & -8 & -1/2 \\ 3 & 6 & 8 & 3 \end{bmatrix} = \frac{1}{2}[(-3-40-24)-(-15-48-4)]$

$= \frac{1}{2}[-67-(67)] = \frac{1}{2}(0) = 0$

$\therefore$ The 3 points are collinear.

If the points **$(-a+1, 2a)$ and $(-4-a, 6-2a)$ are collinear**, then find **the value of 'a'**

Solution : $\frac{1}{2}\begin{bmatrix} a & -a+1 & -4-a & a \\ 2-2a & 2a & 6-2a & 2-2a \end{bmatrix}$

$\Rightarrow 8a^2 + 4a - 4 = 0 \quad \Rightarrow 2a^2 + a - 1 = 0 \quad \Rightarrow a = -1, \frac{1}{2}$

Find the slope of a line joining the given points $\left(-\frac{1}{3}, \frac{1}{2}\right)$ and $\left(\frac{2}{7}, \frac{3}{7}\right)$

Solution : The slope $= \frac{y_2-y_1}{x_2-x_1} = \frac{\left(\frac{3}{7}\right)-\left(\frac{1}{2}\right)}{\left(\frac{2}{7}\right)-\left(-\frac{1}{3}\right)} = \frac{\frac{3}{7}-\frac{1}{2}}{\frac{2}{7}+\frac{1}{3}} = \frac{\frac{6-7}{14}}{\frac{6+7}{21}} = -\frac{1}{14} \times \frac{21}{13} = -\frac{3}{26}$.

Find the slope of a line joining the given points (14, 10) and (14, – 6)

Solution : The slope $= \frac{y_2-y_1}{x_2-x_1} = \frac{(-6)-(10)}{(14)-(14)}$

$= \frac{-6-10}{14-14} = \frac{-16}{0}$. The slope is undefined.

If the three points (3, –1), (a, 3) and (1, –3) are collinear, find the value of a.

Solution : $\therefore$ Slope of AB = Slope of BC

$\frac{4}{a-3} = \frac{-6}{1-a} \Rightarrow 4 - 4a = -6a + 18 \Rightarrow 2a = 14 \Rightarrow a = 7$

$\therefore$ Slope $= \frac{y_2-y_1}{x_2-x_1}$

What is the slope of a line perpendicular to the line joining A (5, 1) and P where P is the mid-point of the segment joining (4, 2) and (–6, 4).

Solution : P is the midpoint of $(4, 2), (-6, 4) \Rightarrow P = \left(\frac{4-6}{2}, \frac{2+4}{2}\right) = (-1, 3)$

$\therefore$ Slope of the line joining A (5, 1), P (–1, 3) $\Rightarrow m = \frac{y_2-y_1}{x_2-x_1} = \frac{3-1}{-1-5} = \frac{2}{-6} = \frac{-1}{3}$

$\therefore$ Slope of the line perpendicular = 3

Find the equation of a straight line passing through (5,7) and is (i) parallel to X axis (ii) parallel to Y axis.

Solution : (i) The equation of any straight line parallel to X axis is y=b $\Rightarrow$ y=7.
(ii) The equation of any straight line parallel toY axis is x=c $\Rightarrow$ x = 5.

Find the equation of a straight line whose Slope is 5 and x intercept is – 9

Solution : Given, Slope = 5, x intercept, d = –9
The equation of a straight line is $y = m(x-d)$
$y = 5(x+9)$
$y = 5x + 45$

Find the equation of a line passing through the point (3, – 4) and having slope $\frac{-5}{7}$.

Solution : Given slope of the line is $-\frac{5}{7}$ and (3, – 4) is a point on the line.

$y - y_1 = m(x - x_1)$

$y + 4 = \frac{-5}{7}(x-3)$

$5x + 7y + 13 = 0$.

Find the equation of a straight line which has slope $\frac{-5}{4}$ and passing through the point (–1,2).

Solution : slope of the line is $\frac{-5}{4}$ and (–1, 2) is a point on the line.

$y - y_1 = m(x - x_1)$

$y - 2 = \frac{-5}{4}(x+1)$

$4y - 8 = -5x - 5 \Rightarrow 5x + 4y - 3 = 0$

Without using Pythagoras theorem, show that the points (1,-4) , (2, –3) and (4, –7) form a right angled triangle.

Solution : Let the given points be A(1, – 4) , B(2, –3) and C(4, –7) .

The slope of AB $= \frac{-3+4}{2-1} = \frac{1}{1} = 1$ The slope of AC $= \frac{-7+4}{4-1} = \frac{-3}{3} = -1$

Slope of AB $\times$ slope of AC = (1)(–1) = –1

AB is perpendicular to AC. $\angle A = 90°$

Therefore, ΔABC is a right angled triangle.

7.MENSURATION

Define – Right circular cylinder.

A right circular cylinder is a solid generated by the revolution of a rectangle about one of its sides as axis.

Define – Right circular cone.

A right circular cone is a solid generated by the revolution of a right angled triangle about one of the sides containing the right angle as axis.

Define –Frustum.

When a cone is cut through by a plane parallel to its base, the portion of the cone between the cutting plane and the base is called a frustum of the cone.

The curved surface area of a right circular cylinder of height 14 cm is 88 cm². Find the diameter of the cylinder.

Solution : C.S.A. of the cylinder =88 sq. cm $\Rightarrow$ $2\pi rh = 88$ cm².

$$2\times\frac{22}{7}\times r\times 14 = 88 \text{ (given } h = 14 \text{ cm)}$$

$$2r = \frac{88\times 7}{22\times 14} = 2 \Rightarrow \text{Diameter} = 2 \text{ cm}$$

A cylindrical drum has a height of 20 cm and base radius of 14 cm. Find its curved surface area and the total surface area.

Solution : h = 20 cm ; r =14 cm

C.S.A. of the cylinder $= 2\pi rh$ sq. units $= 2\times\frac{22}{7}\times 14\times 20 = 2\times 22\times 2\times 20 = 1760$ cm²

T.S.A. of the cylinder $= 2\pi r(h+r)$ sq. units $= 2\times\frac{22}{7}\times 14\times(20+14) = 2\times\frac{22}{7}\times 14\times 34 = 2992$ cm²

The slant height of a frustum of a cone is 5 cm and the radii of its ends are 4 cm and 1 cm. Find its curved surface area.

Solution : Given that, l = 5 cm, R = 4 cm, r = 1 cm

C.S.A. of the frustum $= \pi(R+r)\,l$ sq. units $= \frac{22}{7}\times(4+1)\times 5 = \frac{550}{7} = 78.57$ cm²

The radius of a spherical balloon increases from 12 cm to 16 cm as air being pumped into it. Find the ratio of the surface area of the balloons in the two cases.

Solution : Let r_1 and r_2 be the radii of the balloons. $\boxed{\frac{r_1}{r_2} = \frac{12}{16} = \frac{3}{4}}$

$$\text{Ratio of C.S.A. of balloons} = \frac{4\pi r_1^2}{4\pi r_2^2} = \frac{r_1^2}{r_2^2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$$

Ratio of C.S.A. of balloons is 9:16.

If the base area of a hemispherical solid is 1386 sq. metres, then find its total surface area?

Solution : Let r be the radius of the hemisphere. $\pi r^2 = 1386$ sq. m

T.S.A. $= 3\pi r^2$ sq.m $= 3\times 1386 = 4158$ m².

A sphere, a cylinder and a cone are of the same radius, where as cone and cylinder are of same height. Find the ratio of their curved surface areas.

Solution : Required Ratio = C.S.A. of the sphere: C.S.A. of the cylinder : C.S.A. of the cone

$$= 4\pi r^2 : 2\pi rh : \pi rl = 4:2:\sqrt{2} = 2\sqrt{2}:\sqrt{2}:1$$

Find the diameter of a sphere whose surface area is 154 m².

Solution : Let r be the radius of the sphere.

Given that, $4\pi r^2 = 154 \Rightarrow 4\times\frac{22}{7}\times r^2 = 154 \Rightarrow r^2 = 154\times\frac{1}{4}\times\frac{7}{22} \Rightarrow r^2 = \frac{49}{4} \Rightarrow r = \frac{7}{2}$

$\Rightarrow$ Diameter = 7 m

If the total surface area of a cone of radius 7cm is 704 cm², then find its slant height.

Solution : Given that, radius r = 7 cm

$$\text{T.S.A.} = \pi r(l+r) \text{ sq. units} \Rightarrow 704 = \frac{22}{7}\times 7(l+7) \Rightarrow 32 = l+7 \Rightarrow l = 25 \text{ cm}$$

The slant height of the cone is 25 cm.

A garden roller whose length is 3 m long and whose diameter is 2.8 m is rolled to level a garden. How much area will it cover in 8 revolutions?

Solution : Given that, diameter d = 2.8 m and height = 3 m

radius r = 1.4 m

Area covered in one revolution = curved surface area of the cylinder $= 2\pi rh$ sq. units $= 2\times\frac{22}{7}\times 1.4\times 3 = 26.4$

Area covered in 8 revolutions = 8 × 26.4 = 211.2 m²

4 persons live in a conical tent whose slant height is 19 cm. If each person require 22 cm² of the floor area, then find the height of the tent.

Solution : Given slant height of the cone $l = 19$ cm

Total floor area of 4 persons = 88 cm² $\Rightarrow \pi r^2 = 88 \Rightarrow \frac{22}{7}\times r^2 = 88 \Rightarrow r^2 = 28$

$\therefore h = \sqrt{l^2 - r^2} = \sqrt{19^2 - 28} = \sqrt{333} \Rightarrow$ height of cone $\simeq$ 18.25 cm.

Find the volume of a cylinder whose height is 2 m and whose base area is 250 m².

Solution : Let r and h be the radius and height of the cylinder respectively.

Given that, height h = 2 m, base area = 250 m²

volume of a cylinder $= \pi r^2 h$ cu. units = base area × h = 250 × 2 = 500 m³

The volumes of two cones of same base radius are 3600 cm³ and 5040 cm³. Find the ratio of heights.

Solution : Given volumes of 2 cones = 3600 cm³ & 5040 cm³ & base radius are equal

$$\therefore \text{Ratio of volumes} = \frac{V_1}{V_2} = \frac{3600}{5040} \Rightarrow \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2} = \frac{3600}{5040} \Rightarrow \frac{h_1}{h_2} = \frac{40}{56} = \frac{5}{7} \quad \therefore h_1 : h_2 = 5:7$$

An aluminium sphere of radius 12 cm is melted to make a cylinder of radius 8 cm. Find the height of the cylinder.

Solution: Given radius of sphere = 12 cm = R & radius of cylinder = 8 cm = r

Volume of sphere = Volume of Cylinder

$$\frac{4}{3}\pi R^3 = \pi r^2 h \Rightarrow \frac{4}{3}\times 12\times 12\times 12 = 8\times 8\times h \Rightarrow h = 36 \text{ cm}$$

A hemi-spherical hollow bowl has material of volume $\frac{436\pi}{3}$ cubic cm. Its external diameter is 14 cm. Find its thickness.

Solution : D = 14 cm, R = 7 cm, t = ? Volume $= \frac{436\pi}{3}$ cm³ $\Rightarrow \frac{2}{3}\pi(R^3 - r^3) = \frac{436\pi}{3}$

$$\Rightarrow 7^3 - r^3 = 218 \Rightarrow 343 - r^3 = 218 \therefore r^3 = 125 \therefore r = 5 \text{ cm}$$

$\therefore$ thickness, $t = R - r = 7 - 5 = 2$ cm

Find the maximum volume of a cone that can be carved out of a solid hemisphere of radius r units

Solution : Volume of a cone that can be carved Out of hemisphere $= \frac{2}{3}\pi r^3 - \frac{1}{3}\pi r^2 h$

But volume is maximum (given) $\therefore$ h = r $\therefore$ Required volume $= \frac{2}{3}\pi r^3 - \frac{1}{3}\pi r^3 = \frac{1}{3}\pi r^3$

8. STATISTICS AND PROBABILITY

Write the Different Measures of Dispersion.

1. Range 2. Mean deviation 3. Quartile deviation 4. Standard deviation 5. Variance 6. Coefficient of Variation

Find the range and coefficient of range of the following data: 25, 67, 48, 53, 18, 39, 44.

Solution : L = 67; S = 18

$R = L - S = 67 - 18 = 49$ **and** Coefficient of range $= \frac{L-S}{L+S} = \frac{67-18}{67+18} = \frac{49}{85}$ (or) 0.576

The range of a set of data is 13.67 and the largest value is 70.08. Find the smallest value.

Solution : R = 13.67; L = 70.08

$R = L - S \Rightarrow 13.67 = 70.08 - S \Rightarrow S = 70.08 - 13.67 = 56.41 \Rightarrow$ The smallest value is 56.41.

Find the range of the following distribution.

Age (in years)	16-18	18-20	20-22	22-24	24-26	26-28
Number of students	0	4	6	8	2	2

Solution : L = 28; S = 18

$R = L - S \Rightarrow R = 28 - 18 = 10$ Years.

Write the mean and variance of the first n natural numbers.

Solution : Mean $\bar{x} = \frac{n+1}{2}$, Variance $\sigma^2 = \frac{n^2-1}{12}$

Find the range and coefficient of range of 63, 89, 98, 125, 79, 108, 117, 68

Solution:

Range $= L - S = 125 - 63 = 62$ **and** Coefficient of range $= \frac{L-S}{L+S} = \frac{125-63}{125+63} = \frac{62}{185}$ (or) 0.33

Find the range and coefficient of range of 43.5, 13.6, 18.9, 38.4, 61.4, 29.8

Solution:

Range $= L - S = 61.4 - 13.6 = 47.8$ Coefficient of range $= \frac{L-S}{L+S} = \frac{61.4-13.6}{61.4+13.6} = \frac{47.8}{75}$ (or) 0.64

If the range and the smallest value of a set of data are 36.8 and 13.4 respectively, then find the largest value.

Solution : R = 36.8 ; S = 13.4 $\therefore R = L - S \Rightarrow 36.8 = L - 13.4$ $\therefore L = 36.8 + 13.4 = 50.2$

Calculate the range of the following data.

Income	400-450	450-500	500-550	550-600	600-650
Number of workers	8	12	30	21	6

Solution : L = 650 ; S = 450

$\therefore$ Range $= L - S = 650 - 400 = 250$

A wall clock strikes the bell once at 1 o' clock, 2 times at 2 o' clock, 3 times at 3 o' clock and so on. How many times will it strike in a particular day. Find the standard deviation of the number of strikes the bell make a day.

Solution : $\therefore$ Number of times it strikes in a particular day $= 2(1+2+3+\ldots\ldots 12) = 2\left(\frac{12\times13}{2}\right) = 156$ times

S.D of 2 (1, 2, 3, ...12) $= 2\left[\sqrt{\frac{n^2-1}{12}}\right] = 2\left[\sqrt{\frac{144-1}{12}}\right] = 2\sqrt{\frac{143}{12}} = 6.9$

Find the standard deviation of first 21 natural numbers.

Solution :

SD of first 21 natural numbers $= \sqrt{\frac{n^2-1}{12}} = \sqrt{\frac{441-1}{12}} = \sqrt{\frac{440}{12}} = 6.05$

Find the standard deviation of first 13 natural numbers.

Solution :

SD of first 21 natural numbers $= \sqrt{\frac{n^2-1}{12}} = \sqrt{\frac{169-1}{12}} = \sqrt{\frac{168}{12}} = 3.74$

Define – sample space and give example.

Solution : The set of all possible outcomes in a random experiment is called a **sample space.**

Example : When two coins are tossed together, the sample space is S = {HH,HT,TH,TT}; n(S) = 4

The probability that a student will pass the final examination in both English and Tamil is 0.5 and the probability of passing neither is 0.1. If the probability of passing the English examination is 0.75, what is the probability of passing the Tamil examination?

Solution : $P(E\cup T) = 1 - 0.1 = 0.9$

$P(E\cup T) = P(E) + P(T) - P(E\cap T) \Rightarrow 0.9 = 0.75 + P(T) - 0.5$

$P(T) = 0.9 - 0.25 = 0.65 = \frac{65}{100} = \frac{13}{20}$

A bag contains 5 white and some black balls. If the probability of drawing a black ball from the bag is twice the probability of drawing a white ball then find the number of black balls.

Solution : Given $n(S) = 5 + x$, 5 white balls **and** x black balls

By daa given, $P(B) = 2.\ P(W) \Rightarrow \frac{x}{5+x} = 2.\left(\frac{5}{5+x}\right) \Rightarrow x = 10$ $\therefore$ No. of black balls = 10

If A and B are two mutually exclusive events of a random experiment and P(not A) = 0.45, P(A∪B)=0.65, then find P(B).

Solution : P (not A) = 0.45 $\Rightarrow P(\bar{A}) = 0.45$ $\therefore 1 - P(A) = 0.45 \Rightarrow P(A) = 0.55$

$P(A\cup B) = P(A) + P(B) \Rightarrow P(B) = P(A\cup B) - P(A) = 0.65 - 0.55 = 0.10$

If $P(A) = \frac{2}{3}$, $P(B) = \frac{2}{5}$, $P(A\cup B) = \frac{1}{3}$ then find $P(A\cap B)$.

Solution : $P(A\cap B) = P(A) + P(B) - P(A\cup B) = \frac{2}{3} + \frac{2}{5} - \frac{1}{3} = \frac{10+6-5}{15} = \frac{11}{15}$

The probability that atleast one of A and B occur is 0.6. If A and B occur simultaneously with probability 0.2, then find $P(\bar{A}) + P(\bar{B})$.

Solution : Given $P(A\cup B) = 0.6$, $P(A\cap B) = 0.2$ $\therefore P(A) + P(B) = P(A\cup B) + P(A\cap B) = 0.6 + 0.2 = 0.8$

$\therefore P(\bar{A}) + P(\bar{B}) = 1 - P(A) + 1 - P(B) = 2 - (P(A) + P(B)) = 2 - 0.8$

The probability of happening of an event A is 0.5 and that of B is 0.3. If A and B are mutually exclusive events, then find the probability that neither A nor B happen.

Solution : Given $P(A) = 0.5$, $P(B) = 0.3$, $P(A\cap B) = 0$

P (neither A nor B) $= 1 - P(A\cup B) = 1 - [P(A) + P(B) - P(A\cap B)] = 1 - (0.8) = 0.2$

If A is an event of a random experiment such that $P(A) : P(\bar{A}) = 17:15$ and n(S) = 640 then find (i) $P(\bar{A})$ (ii) n(A).

Solution : Given $P(A) : P(\bar{A}) = 17 : 15 \Rightarrow n(S) = 32$ (i) $P(\bar{A}) = \frac{15}{32}$ (ii) $n(A) = \frac{17}{32} \times 640 = 340$

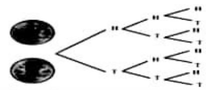
A coin is tossed thrice. What is the probability of getting two consecutive tails?

Solution : S = {(HHH), (HHT), (HTH), (HTT), (THH), (THT), (TTH), (TTT)} $\Rightarrow n(S) = 8$

Let A bet the event of getting 2 tails A = {(HTT), (TTH), (TTT)} $\Rightarrow n(A) = 3 \Rightarrow P(A) = \frac{3}{8}$

Write the sample space for tossing three coins using tree diagram.

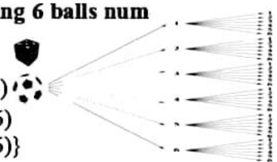
Solution :



S = {(HHH), (HHT), (HTH), (HTT), (THH), (THT), (TTH), (TTT)}

Write the sample space for selecting two balls from a bag containing 6 balls numbered 1 to 6 (using tree diagram). Example 8.18 (same answer also)

Solution :



S = {(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)
(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)
(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)}

Find the standard deviation of first 10 natural numbers.

Solution :

SD of first 21 natural numbers $= \sqrt{\frac{n^2-1}{12}} = \sqrt{\frac{100-1}{12}} = \sqrt{\frac{99}{12}} = 2.84$

If the standard deviation of a data is 4.5 and if each value of the data is decreased by 5, then find the new standard deviation.

Solution : S.D of a data = **4.5** $\Rightarrow$ new S.D of a data = **4.5**

($\because$ S.D will not be changed when we add (or) subtract fixed constant to all the values of the data).

If the standard deviation of a data is 3.6 and each value of the data is divided by 3, then find the new variance and new standard deviation.

Solution : S.D of a data = 3.6 $\Rightarrow$ new S.D $= \frac{3.6}{3} = 1.2$

New Variance $= (1.2)^2 = 1.44$

The mean of a data is 25.6 and its coefficient of variation is 18.75. Find the standard deviation.

Solution : Mean $\bar{x} = 25.6$, Coefficient of variation, C.V. = 18.75

Coefficient of variation, C.V. $= \frac{\sigma}{\bar{x}} \times 100\%$ $\Rightarrow$ $18.75 = \frac{\sigma}{25.6} \times 100 \Rightarrow \sigma = 4.8$

The standard deviation and mean of a data are 6.5 and 12.5 respectively. Find the coefficient of variation.

Solution : $\sigma = 6.5, \bar{x} = 12.5$ $\therefore$ C.V $= \frac{\sigma}{\bar{x}} \times 100 = \frac{6.5}{12.5} \times 100 = 52\%$

The standard deviation and coefficient of variation of a data are 1.2 and 25.6 respectively. Find the value of mean.

Solution : Given $\sigma = 1.2$, C.V = 25.6 $\therefore$ C.V $= \frac{\sigma}{\bar{x}} \times 100 \Rightarrow 25.6 = \frac{1.2}{\bar{x}} \times 100 \Rightarrow \bar{x} = \frac{120}{25.6} = 4.69$

If the mean and coefficient of variation of a data are 15 and 48 respectively, then find the value of standard deviation.

Solution : Given $\bar{x} = 15$, CV = 48, σ = ? $\therefore$ C.V $= \frac{\sigma}{\bar{x}} \times 100 \Rightarrow 48 = \frac{\sigma}{15} \times 100 \Rightarrow \sigma = \frac{15 \times 48}{100} = 7.2$

If $n = 5$, $\bar{x} = 6$, $\Sigma x^2 = 765$, then calculate the coefficient of variation.

Solution : Given $n = 5$, $\bar{x} = 6$, $\Sigma x^2 = 765$, CV = ?

$$\sigma = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2} = \sqrt{\frac{765}{5} - (6)^2} = 10.82 \quad \therefore \text{C.V} = \frac{\sigma}{\bar{x}} \times 100 = \frac{10.82}{6} \times 100 = 180.33\%$$

The standard deviation of some temperature data in degree celsius (°C) is 5. If the data were converted into degree Farenheit (°F) then what is the variance?

Solution : Given $\sigma_c = 5$

$$F = \frac{9c}{5} + 32 \Rightarrow \sigma_F = \frac{9}{5}\sigma_c = \frac{9}{5} \times 5 = 9$$

If the range and coefficient of range of the data are 20 and 0.2 respectively, then find the largest and smallest values of the data.

Solution : Given range = 20, Co.eff. of range = 0.2 $\Rightarrow L - S = 20$ ...(1)

$$\frac{L-S}{L+S} = 0.2 \Rightarrow \frac{20}{L+S} = 0.2 \Rightarrow \frac{20}{L+S} = 0.2 \Rightarrow L + S = 100 \quad ...(2)$$

Solviong (1) and (2) L = 60, S = 40

In a two children family, find the probability that there is at least one girl in a family.

Solution : S = {(BB), (BG), (GB), (GG)} $\Rightarrow$ n(S) = 4

Let A be the event of getting atleast one girl. A = {(BG), (GB), (GG)} $\Rightarrow$ $\therefore$ n(A) = 3 $\Rightarrow$ P(A) $= \frac{3}{4}$

In a game, the entry fee is ₹150. The game consists of tossing a coin 3 times. Dhana bought a ticket for entry . If one or two heads show, she gets her entry fee back. If she throws 3 heads, she receives double the entry fees. Otherwise she will lose. Find the probability that she (i) gets double entry fee (ii) just gets her entry fee (iii) loses the entry fee.

Solution : S = {(HHH), (HHT), (HTH), (THH), (HTT), (THT), (HTT), (TTT)} $\Rightarrow$ n(S) = 8

i) P (gets double entry fee) $= \frac{1}{8}$ ($\because$ 3 heads) iii) P (loses the entry fee) $= \frac{1}{8}$ ($\because$ 3 no heads (TTT) only)

ii) P (just gets for her entry fee) $= \frac{6}{8} = \frac{3}{4}$ ($\because$ 1 (or) 2 heads)

Two customers Priya and Amuthan are visiting a particular shop in the same week (Monday to Saturday). Each is equally likely to visit the shop on any one day as on another day. What is the probability that both will visit the shop on (i) the same day (ii) different days (iii) consecutive days?

Solution : Given n(S) = 6. (Monday - Saturday)

i) Prob. that both of them will visit the shop on the same day $= \frac{1}{6}$

ii) Prob. that both of them will visit the shop in different days $= \frac{5}{6}$

iii) Prob. that both of them will visit the shop in consecutive days.

A = {(Mon, Tue), (Tue, Wed), (Wed, Thu), (Thu, Fri), (Fri, Sat)} $\Rightarrow$ n(A) = 5 $\Rightarrow P(A) = \frac{5}{6}$

What is the probability that a leap year selected at random will contain 53 saturdays. (Hint: 366 = 52×7 +2)

Solution : S = {(Sun-Mon, Mon-Tue, Tue-Wed, Wed-Thu, Thu-Fri, Fri-Sat, Sat-Sun)} $\Rightarrow$ n(S) = 7

Let A be the event of getting 53rd Saturday. A = {Fri-Sat, Sat-Sun}; n(A) = 2 $\Rightarrow P(A) = \frac{2}{7}$

A die is rolled and a coin is tossed simultaneously. Find the probability that the die shows an odd number and the coin shows a head.

Solution : S = {1H,1T,2H,2T,3H,3T,4H,4T,5H,5T,6H,6T}; n (S) = 12

Let A be the event of getting an odd number and a head. A = {1H, 3H, 5H}; n (A) = 3 $\Rightarrow P(A) = \frac{3}{12} = \frac{1}{4}$

A bag contains 6 green balls, some black and red balls. Number of black balls is as twice as the number of red balls. Probability of getting a green ball is thrice the probability of getting a red ball. Find (i) number of black balls (ii) total number of balls.

Solution : Number of green balls is n (G) = 6 ; Let number of red balls is n (R) = x

The number of black balls is n (B) = $2x$; Total number of balls n(S) = $6 + x + 2x = 6 + 3x$

It is given that, P(G) = 3 × P(R) $\Rightarrow \frac{6}{6+3x} = 3 \times \frac{x}{6+3x} \Rightarrow 3x = 6 \Rightarrow x = 2$.

(i) Number of black balls = 2×2 = 4 (ii) Total number of balls = 6 +(3×2) = 12

Two coins are tossed together. What is the probability of getting different faces on the coins?

Solution : S = {HH,HT,TH,TT}; n(S) = 4

Let A be the event of getting different faces on the coins. A = {HT,TH}; n(A) = 2 $\Rightarrow P(A) = \frac{2}{4} = \frac{1}{2}$

A bag contains 5 blue balls and 4 green balls. A ball is drawn at random from the bag. Find the probability that the ball drawn is (i) blue (ii) not blue.

Solution : Total number of possible outcomes n(S) = 5 + 4 = 9

(i) Let A be the event of getting a blue ball. P(A) $= \frac{5}{9}$ (ii) $\bar{A}$ will be the event of not getting a blue ball. P($\bar{A}$) $= \frac{4}{9}$

XII MATHS

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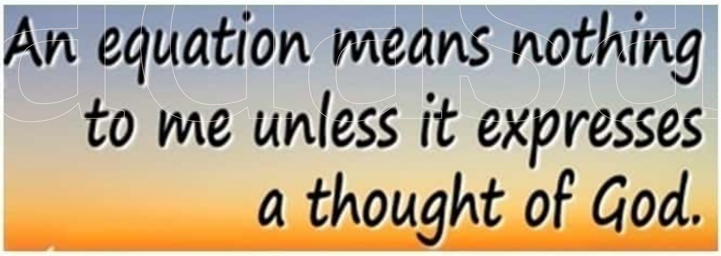


WITH

YOUR HAPPY.......

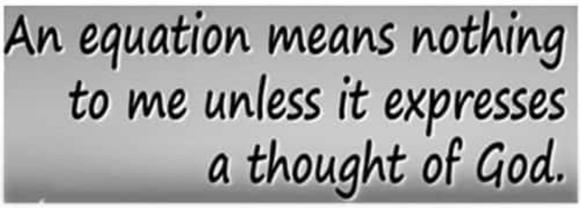
A ZAYAAN ABDUL M.SC.,B.ED.,
FATIMA MATRIC.HR.SEC.SCHOOL
JAYANKONDAM, ARIYALUR DT.
9524103797

A ABDUL MUNAB M.SC.,B.ED.,

XII MATHS CONFIDENT PART – B & C IMPORTANT QUESTIONS

FATIMA MATRIC. HR. SEC.SCHOOL, JAYANKONDAM,ARIYALUR DT.
CELL: 9524103797

21. 31.	Example **1.3 , 1.4 , 1.6 , 1.8** , 1.14, 1.15(ii) , **1.18 , 1.20 , 1.25 , 1.22** , Theorem 1.2 . **Theorem 1.3.** Theorem 1.4(ii) , Theorem 1.7 , Theorem 1.8 , **Theorem 1.9(v ,vi).** **Write Rouche- capelli theorem ,** EX 1.1 => 2(i),**3 ,4 ,6 , 7 , 8 , 9 , 10 ,11** , 12 , 13. EX 1.2 => **1(iii) , 3(i).** EX 1.3 => 4. EX 1.4 => **2 , 3 ,4.** Define – Orthogonal.
22. 32.	**Write the cube roots of unity, Write de- movers theorem , Property 3(81).** **Write equ/- of circle complex form, Property 9(62), Prove that inverse property under multiplication of complex number(59),** Example **2.1(iv) , 2.4 , 2.5 , 2.7 , 2.17 , 2.19 ,2.26. 2.29 ,2.30.** EX 2.1=> **5,6.** EX 2.2 =>**2(ii)** .EX 2.4 =>**1(iii), 2(ii ,iii),5(i ,ii) ,6.** EX 2.5 => **6 ,8 ,7,9.** EX 2.6 => **3(i ,iv), 4(ii ,iii) ,5(i).** EX 2.7 => **1(iv), 2(ii) .** EX 2.8 => **1,3,5 ,7,8(i, ii) ,9(iii).**
23. **(OR)** 33. **ANY ONE**	Example **3.1 ,3.2 ,3.7,3.10, 3.14 ,3.16 ,3.21 ,3.29, 3.31(i) , Theorem 3.7 , Theorem 3.5**. EX 3.1 => **7,8 ,9** ,11 ,**3(i , ii, iii).** EX 3.2 => **1**, 2, **4, 5.** EX 3.3 => **7**. EX 3.5 => 1(i) , 2(ii) , **6.** EX 3.6 => **1 , 4 , 5.** **(OR)** Example **4.2**,4.3(iii),**4.4** , 4.6(ii) ,**4.9(ii) ,4.10** , 4.12(ii) ,**4.13**,4.14,**4.16 ,4.17(i,iv)** 4.18(ii ,iii). **4.19 , 4.21(iii) ,4.24 , 4.25.** EX 4.1 => **1(ii), 2(ii ,iii) , 3 , 5** ,6(ii). EX 4.2 => 2 , **3 , 7 ,8(i), 5(i).** EX 4.3 => 1(ii) , 4(iii) . EX 4.4 => **2(iii**,ii). EX 4.5 => **1(iii**,ii),**3(ii). Property – V(i). Property – VIII(i).** **Write the inverse trigonometry functions domain and its range.** **Draw the graph of all inverse functions.**
24. 34.	Example 5.2 ,**5.6 ,5.8,** 5.11, 5.12, **5.15 , 5.16 ,5.18 ,5.22 , 5.25,5.31 , 5.32, 5.38.** **Theorem 5.4, Theorem 5.6.** EX 5.1 => **2, 3** , 5 , 7 ,**8** ,10 , **11(iv) ,12.** EX 5.2 => 1(i,**ii) ,2(iii,iv)** ,3(ii) ,5(iii),**8(i,iii).** EX 5.4 => **4 ,5.** **Define – focal chord and latus rectum.** **Write the condition and point of contact tangent to the circle.**
25. 35.	Example **6.11** ,6.13, **6.14 , 6.15, 6.17 ,6.18** , 6.21,6.19,6.29 ,**6.32 , 6.36 , 6.42 , 6.52 ,6.55** **Theorem 6.4. Prove that Jocobis identity, Prove that Lagranges identity.** EX 6.1 => 1 ,**14.** EX 6.2 => **2 , 3 ,9** ,8. EX 6.3 => **2 ,3**, 6 ,**8** . EX 6.5 => **4.** EX 6.6 => **5** , 6. EX 6.9 => **4.**
26. **(OR)** 36. **ANY ONE**	Example **10.6 ,10.4,10.3 ,10.2 ,**10.10 ,**10.13.** EX 10.1 => **1**(x ,**ix)** . EX 10.2=> **2.** EX 10.3 => 1(all) , **3 , 4 ,5 ,6.** EX 10.4 => **8.** EX 10.5 => **1** , 2 , **4 (iii ,vi ,x ,ix).** **(OR)** **Definition 8.1, 8.7, 8.9, 8.13 .** Example **8.21, 8.17 , 8.15** ,8.11 , **8.7 , 8.1, 8.2.** EX 8.1 => **1 ,2(i),** 3(ii). EX 8.2 => 1(iii) ,**4, 6 , 9.** EX 8.3 => **2, 4.** EX 8.4 => **3 ,4** ,9. EX 8.5 => **4 , 3.** EX 8.7 => **1(iv).**



A ABDUL MUNAB M.SC.,B.ED.,

XII MATHS CONFIDENT PART – B & C IMPORTANT QUESTIONS

FATIMA MATRIC. HR. SEC.SCHOOL, JAYANKONDAM,ARIYALUR DT.
CELL: 9524103797

27. 37.	Example **9.3 ,9.9 ,9.10 ,9.18 ,** 9.14 ,**9.7 ,9.25 ,9.31 ,** 9.34, **9.45 ,** 9.46 ,9.50, **9.52 ,9.53** ,9.58. **9.62 , 9.69 , 9.38 ,** 9.37 , **9.41, Theorem 9.1 , Theorem 9.2 , Property – 7, 12** . EX 9.1 => **3.** EX 9.3 => **1(ii, iv)** ,2(i, **vi)**. EX 9.6 => 1(vi). EX 9.7 => **2.** EX 9.8 => 1 ,**3**. EX 9.9 => **2.**
28. 38.	Example 7.37 , **7.49, 7.51** ,7.55 ,7.58 ,**7.67 ,7.29. Theorem 7.1,7.2,7.3,7.5,7.11,7.12 ,7.13. Definition 7.6 , 7.7, 7.7.** EX 7.3 => **6 ,** 3(ii) , **8 ,9.** EX 7.4 => **2 , 1(i ,ii ,iii).** EX 7.5 => **1 , 8 , 9.** EX 7.7 => 1(iii) , **2(ii).** EX 7.8 => **1 ,2 , 3.** EX 7.9 => **1(ii).**
29. **(OR)** 39. **ANY ONE**	Example **12.17 , 12.15 ,** 12.14, **12.8, 12.13(ii) ,12.1(i ,ii).Definition 12.2 ,12.3 . Theorem 12.1, 12.2. Write the Identity , De- morgans ,Complement laws. Write the three logical statement of consequences.** EX 12.2 =>1(iv ,**v)** , 3(i ,**ii** ,iv) ,5(i) ,**6(i ,ii) , 8(i,ii) ,9, 10,11.** EX 12.1 => 1(ii) , **2, 3 ,4 , 6.** **(OR)** **Definition 11.1 , 11.2 , 11.3 , 11.6, .** Example 11.1 , 11.2 , **11.3 , 11.4 , 11.5 , 11.6 , 11.15(i). Prove that mathematical expectation properties - i, ii, iii. Write mean and variance one point and two point distribution.** EX 11.1 => **2.** EX 11.2 => **1 , 4(i) , 3.** EX 11.3 => **1** ,3(i) ,4(i) , **5.** EX 11.4 => 4 , **8.** EX 11.5 => 1(ii), **4. Write the any three properties of cum. Distribution.**
30. 40.	**CREATIVE QUESTIONS ALL CHAPTER ANY ONE HERE**

NOTE : SLOW LEARNER

i) **Part – A => 1 MARKS [ALL CHAPTER] TOTAL = 10M**

ii) **Part – B => 2 MARKS [1 , 3 , 4 , 8 , 11 , 12] TOTAL = 8M**

iii) **Part – C => 3 MARKS [1 , 3 , 4 , 8 , 11,12] TOTAL = 12M**

iv) **Part – D => 5 MARKS [ANY ONE OPTION] TOTAL= 30M**

FINAL TOTAL =68/90

An equation means nothing to me unless it expresses a thought of God.

A ABDUL MUNAB M.SC.,B.ED.,

XII MATHS CONFIDENT FIVE MARKS PART – D IMPORTANT QUESTIONS

FATIMA MATRIC. HR. SEC.SCHOOL, JAYANKONDAM,ARIYALUR DT.
CELL: 9524103797

41.	a) Example 1.1 , 1.2 , 1.5 , 1.9 , 1.10, 1.12 , 1.21 , 1.23 1.24 1.26 , 1.27,1.28 1.29 , 1.32 , 1.34 1.35 , 1.36 1.37 , 1. 38 1.39 , 1.40 , Theorem 1.1. Welcome Pbm. EX 1.1 => 1(iii),5 ,8 ,14 , 15. EX 1.2 => 2 (ii , iii) ,3 (iii). EX 1.3 => 1(iv), 2 ,5. EX 1.4 => 1(iv) , 5. EX 1.5 => 1(ii),2 , 3 ,4.EX 1.6 => 2 ,3. EX 1. 7 => 1(i), 2 ,3.
	b) Example 2.2 ,2.14 ,2.16,2.18,2.36 ,2.35 , 2.34 , 2.27 , 2.8 (ii) , 2.31(ii) ,2.32 ,2.33. Triangle inequality , EX 2.2 => 3. EX 2.5 => 6 ,8 ,7 , 9. EX 2.6 => 1 ,2. EX 2.7 => 3 , 4 , 5 ,6. EX 2.8 => 2 , 4 , 6 ,10.
42.	a) Example 3.4 , 3.5 , 3.6 , 3.15 , 3.18 , 3.22 , 3.23 , 3.24, 3.28, 3.30. Complex Conjugate Therem. EX 3.1 => 1,4 ,6 ,10 ,12. EX 3.3 => 1, 2, 3, 4, 5. EX 3.4 => 1(ii), 2 . EX 3.5 => 3 , 4 , 5(i ,ii) , 7. EX 3.6 => 2 , 3.
	b) Example 4.11 , 4.15 , 4.7 , 4.20 , 4.22 , 4.23 , 4.26 , 4.27, 4.28 , 4.29. Property IV – (i). EX 4.1 => 6(i),7. EX 4.2 => 8(ii), 6(i) , 5(iii). EX 4.3 => 4(iii) . EX 4.5 => 5 ,6 ,10 ,9 (iii,iv).
43.	a) Example 5.10 , 5.19 , 5.20 ,5.21 , 5.24 ,5.30 , 5.39 ,Theorem 5,5. EX 5.1 => 6 EX 5.2 => 4(iv,v) ,8(v,vi) ,6 ,7. EX 5.4 => 7 , 8 . EX 5.5 =>1 ,2 ,3 ,4 , 5 ,6 ,7, 8 ,9,10.
	b) Example 7.7 , 7.9 , 7.15 , 7.18 , 7.31 , 7.44 , 7.45 , 7.61 , 7.62 , 7.64 , 7.63,7.65 7.69 7.70 7.71 , 7.72. EX 7.9 => 2(all). EX 7.1 => 6 , 7 , 8 , 10. EX 7.2 => 6 , 7. EX 7.5 => 10 , 11 , 12. EX 7.7 => 3. EX 7.8 => 4 , 5 , 6 , 7 , 8 , 9.
44.	a) Example 6.1 ,6.2 , 6.3 , 6.4, 6.5 , Prove that Apollonious Theorem, 6.7 , 6.23(i,ii). EX 6.1 => 3 , 4 ,5 , 7 , 8 , 9 , 10. EX 6.3 => 4(i , ii).
	b) Example 6.26 , 6.27 , 6.33 ,6.34 ,6.44. EX 6.4 => 3, 6 ,8 ,9. EX 6.5 => 7. EX 6.7 => 1 , 3 , 4 , 6. EX 6.8 => 2 , 3 , 4. EX 6.9 => 1, 2 , 5 , 8.
45.	a) Example 8.22 , 8.19 , 8.14 ,8.13 , 8.9 , 8.10, 8.8. EX 8.1 => 6 ,7. EX 8.3 => 5, 7. EX 8.4 => 2(ii) , 6 ,7 , 8 , 10. EX 8.6 => 4 , 6 , 7 , 8 , 9. EX 8.7 => 3 ,4, 5 ,6.
	b) Example 10.23 , 10.21,10.27 ,10.28 ,10.29 ,10.30 ,10.19 , 10.15. EX 10.6 => 3 ,6 ,7,8. EX 10.7 => 10 , 11 ,13 , 14. EX 10.8 => 1 , 2 , 3 , 4 ,5 ,6 , 7 8 , 9 , 10.
46.	a) Example 11.7 , 11.8 , 11.11 , 11.10 , 11.14 , 11.15 , 11. 16 , 11.17 , 11. 18 11.19 11.20 , 11.21 11.22 . EX 11.1 => 3 , 5. EX 11.2 => 4 , 6. EX 11.3 => 3 , 4. EX 11.4 => 1(iv) ,2 , 3 , 7. EX 11.5 => 2, 7 , 8 , 9.
	b)Example 12.19 , 12.18 , 12.16, 12.9, 12.10 ,12.7 , 12.6 , 12.2 , 12.3 , 12.4 ,12.5 Construct the truth table Associative,Distributive,De-morgans, Absorption laws EX 12.2 => 6(iv) , 7(I,ii,iii,iv) ,12 , 13 , 14 , 15. EX 12.1 => 5 , 8 ,9 , 10.
47.	a) Example 9.1 , 9.4 ,9.6 , 9.13 , 9.15 , 9.27 , 9.28 , 9.54 , 9.55 , 9.63 , 9.64 , 9.66 9.61 ,9.60 , 9.59 , 9.56 , 9.50 , 9.49 , 9.36 , 9.30 .
	b) EX 9.2 => 1(ii). EX 9.3 => 2(vi,vii, xi). EX 9.4 => 2 , 3. EX 9.5 => 1(i,ii). EX 9.8 => 5 , 6 , 8 , 9, 10. EX 9.9 => 4 , 5 , 6.