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chapter - 3 Theory of Equation.

∴ Quadratic equation. $ax^2 + bx + c = 0$.

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0 \quad \therefore \alpha + \beta = -\frac{b}{a}$$

$$x^2 - (SOR)x + (POR) = 0 \quad \alpha\beta = \frac{c}{a}$$

cubic equation. $ax^3 + bx^2 + cx + d = 0$.

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$$

$$\alpha\beta\gamma = -\frac{d}{a}$$

Equation: $x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma = 0$

Example 3.1

Let α and β are roots of $17x^2 + 43x - 73 = 0$.

$$\alpha + \beta = -\frac{43}{17}, \quad \alpha\beta = -\frac{73}{17}$$

The roots are $\alpha+2$ and $\beta+2$.

$$\alpha+2 + \beta+2 = \alpha + \beta + 4$$

$$= -\frac{43}{17} + 4$$

$$= \frac{25}{17}$$

$$(\alpha+2)(\beta+2) = \alpha\beta + 2\alpha + 2\beta + 4$$

$$= \alpha\beta + 2(\alpha + \beta) + 4$$

$$= -\frac{73}{17} + 2\left(-\frac{43}{17}\right) + 4$$

$$= -\frac{91}{17}$$

Equation $x^2 - (\text{SOR})x + (\text{POR}) = 0$

$$x^2 - \frac{25}{17}x - \frac{91}{17} = 0$$

$$17x^2 - 25x - 91 = 0.$$

Example 3.2

Let α and β are roots of $2x^2 - 7x + 13 = 0$.

$$\alpha + \beta = \frac{7}{2}, \quad \alpha\beta = \frac{13}{2}.$$

The roots are α^2 and β^2 .

$$\begin{aligned} \text{SOR} \Rightarrow \alpha^2 + \beta^2 &= (\alpha + \beta)^2 - 2\alpha\beta \\ &= \left(\frac{7}{2}\right)^2 - 2\left(\frac{13}{2}\right) \\ &= \frac{49}{4} - 13 \end{aligned}$$

$$\alpha^2 + \beta^2 = -\frac{3}{4}$$

$$\begin{aligned} \text{POR} \Rightarrow \alpha^2\beta^2 &= (\alpha\beta)^2 \\ &= \left(\frac{13}{2}\right)^2 \\ \alpha^2\beta^2 &= \frac{169}{4} \end{aligned}$$

Equation :-

$$x^2 - (\text{SOR})x + \text{POR} = 0$$

$$x^2 - \left(-\frac{3}{4}\right)x + \frac{169}{4} = 0$$

$$x^2 + \frac{3x}{4} + \frac{169}{4} = 0$$

$$4x^2 + 3x + 169 = 0$$

cubic equation.

$$x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma = 0.$$

EXE 8.1

2. (i) $\alpha = 1, \beta = 2$ and $\gamma = 3$.

$$x^3 - (1+2+3)x^2 + (2+6+3)x - (1)(2)(3) = 0$$

$$x^3 - 6x^2 + 11x - 6 = 0.$$

(ii) let $\alpha = 1, \beta = 1$ and $\gamma = -2$

$$x^3 - (1+1-2)x^2 + (1-2-2)x - (1)(1)(-2) = 0$$

$$x^3 - 3x + 2 = 0.$$

(iii) let $\alpha = 2, \beta = \frac{1}{2}$ and $\gamma = 1$

$$x^3 - (2 + \frac{1}{2} + 1)x^2 + (1 + \frac{1}{2} + 2)x - (2)(\frac{1}{2})(1) = 0$$

$$x^3 - \frac{7}{2}x^2 + \frac{7}{2}x - 1 = 0.$$

$$2x^3 - 7x^2 + 7x - 1 = 0.$$

3. let α, β, γ are roots of $x^3 + 2x^2 + 3x + 4 = 0$.

$$\alpha + \beta + \gamma = -2$$

$$ax^3 + bx^2 + cx + d = 0$$

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = 3$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$$

$$\alpha\beta\gamma = -4$$

$$\alpha\beta\gamma = -\frac{d}{a}$$

$$(i) \quad 2\alpha, 2\beta, 2\gamma$$

$$2\alpha + 2\beta + 2\gamma = 2(\alpha + \beta + \gamma)$$

$$= 2(-2) = -4$$

$$(2\alpha)(2\beta) + (2\beta)(2\gamma) + (2\gamma)(2\alpha) = 4(\alpha\beta + \beta\gamma + \gamma\alpha)$$

$$= 4(3)$$

$$= 12.$$

$$(2\alpha)(2\beta)(2\gamma) = 8(\alpha\beta\gamma)$$

$$= 8(-4)$$

$$= -32.$$

Equation:

$$x^3 - (-4)x^2 + 12x - (-32) = 0$$

$$x^3 + 4x^2 + 12x + 32 = 0.$$

$$(ii) \quad \frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$$

$$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma} = \frac{3}{-4} = -\frac{3}{4}$$

$$\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha} = \frac{\alpha\beta\gamma(\alpha + \beta + \gamma)}{(\alpha\beta\gamma)^2} = \frac{\alpha + \beta + \gamma}{\alpha\beta\gamma} = \frac{-2}{-4} = \frac{2}{4}$$

$$\frac{1}{\alpha} \cdot \frac{1}{\beta} \cdot \frac{1}{\gamma} = \frac{1}{\alpha\beta\gamma} = \frac{1}{-4}$$

Equation:

$$x^3 - \left(-\frac{3}{4}\right)x^2 + \frac{2}{4}x - \left(-\frac{1}{4}\right) = 0.$$

$$x^3 + \frac{3x^2}{4} + \frac{2x}{4} + \frac{1}{4} = 0.$$

$$4x^3 + 3x^2 + 2x + 1 = 0.$$

4. Let α, β and γ are roots of $3x^3 - 16x^2 + 23x - 6 = 0$

$$\alpha + \beta + \gamma = \frac{16}{3} \rightarrow (1)$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{23}{3}$$

$$\alpha\beta\gamma = \frac{6}{3} = 2. \quad \therefore$$

The product of two roots is 1

$$\alpha\beta = 1$$

$$\boxed{\alpha = \frac{1}{\beta}}$$

$$\alpha\beta\gamma = 2$$

$$(\alpha\beta)\gamma = 2$$

$$\boxed{\gamma = 2}$$

$\Rightarrow$

$$\alpha + \beta + \gamma = \frac{16}{3}$$

$$\frac{1}{\beta} + \beta + 2 = \frac{16}{3}$$

$$1 + \beta^2 + 2\beta = \frac{16}{3}\beta$$

$$3\beta^2 - 10\beta + 3 = 0$$

$$(3\beta - 1)(\beta - 3) = 0$$

$$\beta = \frac{1}{3} \quad \beta = 3$$

put $\beta = \frac{1}{3}$,

$$\alpha = 3, \beta = \frac{1}{3}, \gamma = 2$$

put $\beta = 3$,

$$\alpha = \frac{1}{3}, \beta = 3, \gamma = 2$$

$$(iii) -\alpha, -\beta, -\gamma$$

$$(-\alpha) + (-\beta) + (-\gamma) = -(\alpha + \beta + \gamma) = -(-2) = 2$$

$$(-\alpha)(-\beta) + (-\beta)(-\gamma) + (-\gamma)(-\alpha) = \alpha\beta + \beta\gamma + \gamma\alpha = 3$$

$$(-\alpha)(-\beta)(-\gamma) = -(\alpha\beta\gamma) = -(-4) = 4$$

Equation

$$x^3 - 2x^2 + 3x - 4 = 0$$

①

The side of the cubic box increased 1, 2 and 3.

$$\text{Volume of cuboid} = x^3 + 52$$

$$(x+1)(x+2)(x+3) = x^3 + 52$$

$$x^3 + (1+2+3)x^2 + (2+6+3)x + 6 = x^3 + 52$$

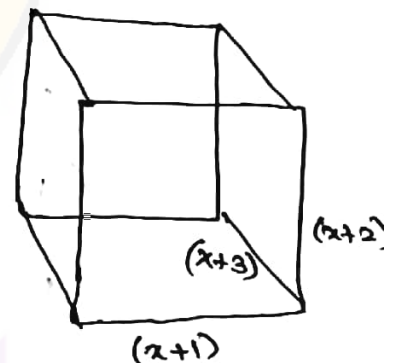
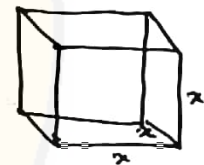
$$x^3 + 6x^2 + 11x + 6 = x^3 + 52$$

$$+ 6x^2 + 11x + 6 = 52$$

$$6x^2 + 11x - 46 = 0$$

$$(x + 23/6)(x - 2) = 0$$

$$x = -\frac{23}{6}, \quad \boxed{x = 2}$$



$$\text{Volume of cuboid} = x^3 + 52$$

$$= (2)^3 + 52$$

$$= 8 + 52$$

$$= 60$$

5. Let α, β, γ and δ are roots of $2x^4 - 8x^3 + 6x^2 - 3 = 0$,

then, $\alpha^2 + \beta^2 + \gamma^2 + \delta^2$

$$\alpha + \beta + \gamma + \delta = \frac{8}{2} = 4$$

$$\alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta = \frac{6}{2} = 3$$

$$(\alpha + \beta + \gamma + \delta)^2 = \alpha^2 + \beta^2 + \gamma^2 + \delta^2 + 2(\alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta)$$

$$(4)^2 = \alpha^2 + \beta^2 + \gamma^2 + \delta^2 + 2(3)$$

$$16 = \alpha^2 + \beta^2 + \gamma^2 + \delta^2 + 6$$

$$10 = \alpha^2 + \beta^2 + \gamma^2 + \delta^2$$

6. Let $\alpha, 3\beta$ and 2β are roots of $x^3 - 9x^2 + 14x + 24 = 0$

$$\alpha + 3\beta + 2\beta = 9$$

$$\alpha + 5\beta = 9$$

$$\boxed{\alpha = 9 - 5\beta}$$

$$(9 - 5\beta)(3\beta) + (3\beta)(2\beta) + (2\beta)(9 - 5\beta) = 14$$

$$45\beta - 15\beta^2 - 14 = 0$$

$$15\beta^2 - 45\beta + 14 = 0$$

$$(\beta - 2)\left(\beta - \frac{7}{3}\right) = 0$$

$$\boxed{\beta = 2}$$

$$\alpha, \beta, \gamma \Rightarrow (9 - 5\beta, 3\beta, 2\beta) = (-1, 6, 4)$$

7. Let α, β and γ are roots of $ax^3 + bx^2 + cx + d = 0$.

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$$

$$\alpha\beta\gamma = -\frac{d}{a}$$

$$\therefore \sum \frac{\alpha}{\beta\gamma} = \frac{\alpha}{\beta\gamma} + \frac{\beta}{\alpha\gamma} + \frac{\gamma}{\alpha\beta}$$

$$= \frac{\alpha^2\beta^2\gamma + \alpha\beta^2\gamma^2 + \alpha^2\beta\gamma^2}{(\alpha\beta\gamma)^2}$$

$$= \frac{\alpha\beta\gamma(\alpha^2 + \beta^2 + \gamma^2)}{(\alpha\beta\gamma)^2}$$

$$= \frac{(\alpha^2 + \beta^2 + \gamma^2)}{\alpha\beta\gamma}$$

$$= \frac{(\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)}{\alpha\beta\gamma}$$

$$= \frac{(-b/a)^2 - 2(c/a)}{-d/a}$$

$$= \frac{b^2/a^2 - \frac{2c}{a}}{-d/a}$$

$$= \frac{b^2 - 2ac}{-da}$$

$$\sum \frac{\alpha}{\beta\gamma} = \frac{2ac - b^2}{ad}$$

8. Let $\alpha, \beta, \gamma, \delta$ are roots of $2x^4 + 5x^3 - 7x^2 + 8 = 0$,

$$\alpha + \beta + \gamma + \delta = -\frac{5}{2},$$

$$\alpha\beta\gamma\delta = -\frac{8}{2} = -4$$

$$\text{SOR} \Rightarrow (\alpha + \beta + \gamma + \delta) + (\alpha\beta\gamma\delta) = -\frac{5}{2} - 4 = -\frac{13}{2},$$

$$\text{POR} \Rightarrow (\alpha + \beta + \gamma + \delta)(\alpha\beta\gamma\delta) = \left(-\frac{5}{2}\right)(-4) = 10$$

Required equation,

$$x^2 - (\text{SOR})x + (\text{POR}) = 0,$$

$$x^2 - \left(-\frac{13}{2}\right)x + 10 = 0$$

$$x^2 + \frac{13}{2}x + 10 = 0$$

$$2x^2 + 13x + 20 = 0,$$

Example 3.3

Let α, β and γ are roots of $x^3 + px^2 + qx + r = 0$,

$$\alpha + \beta + \gamma = -p,$$

$$\alpha\beta\gamma = -r$$

$$\sum \frac{1}{\alpha\beta} = \frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha}$$

$$= \frac{\alpha + \beta + \gamma}{\alpha\beta\gamma} = \frac{-p}{-r}$$

$$\sum \frac{1}{\alpha\beta} = \frac{p}{r}$$

Example 3.4

$$\alpha + \beta + \gamma + \delta = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta = \frac{c}{a}$$

$$(a+b+c+d)^2 = (a^2+b^2+c^2+d^2) + 2(ab+ac+ad+bc+bd+cd)$$

$$(\alpha+\beta+\gamma+\delta)^2 = (\alpha^2+\beta^2+\gamma^2+\delta^2) + 2(\alpha\beta+\alpha\gamma+\alpha\delta+\beta\gamma+\beta\delta+\gamma\delta)$$

$$\left(-\frac{b}{a}\right)^2 = \alpha^2+\beta^2+\gamma^2+\delta^2 + 2\left(\frac{c}{a}\right)$$

$$\begin{aligned}\alpha^2+\beta^2+\gamma^2+\delta^2 &= \frac{b^2}{a^2} - \frac{2c}{a} \\ &= \frac{b^2 - 2ac}{a^2}\end{aligned}$$

Example 3.5

The roots of cubic equation $x^3+ax^2+bx+c=0$ are in the ratio $p:q:r$.

The roots are $p\alpha, q\alpha, r\alpha$.

$$p\alpha + q\alpha + r\alpha = \alpha(p+q+r) = -a$$

$$\alpha = \left(\frac{-a}{p+q+r}\right)$$

$$(pq+qr+rp)\alpha^2 = b$$

$$pqr\alpha^3 = -c$$

$$\alpha^3 = \frac{-c}{pqr}$$

$$\left(\frac{-a}{p+q+r}\right)^3 = \frac{-c}{pqr}$$

$$\boxed{pqr a^3 = c(p+q+r)^3}$$

9. Let p and q are root of equation $lx^2 + nx + n = 0$.

$$p+q = -\frac{n}{l} \quad , \quad pq = \frac{n}{l}$$

$$\begin{aligned} \sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} &= \frac{\sqrt{p}}{\sqrt{q}} + \frac{\sqrt{q}}{\sqrt{p}} \\ &= \frac{\sqrt{p}\sqrt{p} + \sqrt{q}\sqrt{q}}{\sqrt{q}\sqrt{p}} \end{aligned}$$

$$= \frac{p+q}{\sqrt{pq}}$$

$$= \frac{-n/l}{\sqrt{n/l}}$$

$$\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} = -\sqrt{\frac{n}{l}}$$

$$\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = 0$$

10. Let α be the common root of equation
 $x^2 + px + q = 0$ and $x^2 + p'\alpha + q' = 0$.

$$\alpha^2 + p\alpha + q = 0$$

$$\alpha^2 + p'\alpha + q' = 0$$

By using cross multiplication method

	α^2	α	constant
p	q	1	p
p'	q'	1	p'

$$\frac{\alpha^2}{pq' - p'q} = \frac{\alpha}{q - q'} = \frac{\text{constant}}{p' - p}$$

$$\frac{\alpha^2}{pq' - p'q} = \frac{\alpha}{q - q'}$$

$$\frac{\alpha}{q - q'} = \frac{1}{p' - p}$$

$$\frac{\alpha}{pq' - p'q} = \frac{1}{q - q'}$$

$$\alpha = \frac{q - q'}{p' - p}$$

$$\alpha = \frac{pq' - p'q}{q - q'}$$

⑦.

Let x be the number.

$$3\sqrt{x} + x = b$$

$$3\sqrt{x} = b - x$$

$$x = (b - x)^3$$

$$x = b^3 - 3b^2x + 3bx^2 - x^3$$

$$x = 216 - 108x + 18x^2 - x^3$$

$$x^3 - 18x^2 + 109x - 216 = 0$$

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

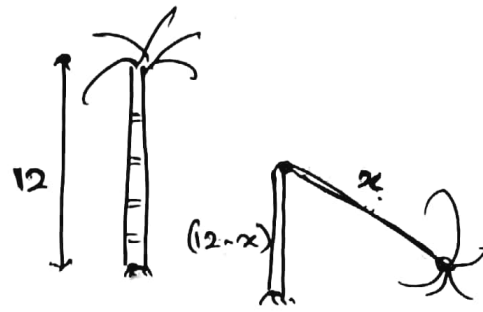
12. Height of the tree = 12m

Let x be the cut part

$$\sqrt[3]{12-x} = x$$

$$12-x = x^3$$

$x^3 + x - 12 = 0$ is the Required Equation.



Example 3.b.

Let α, β and γ are roots of $x^3 + ax^2 + bx + c = 0$.

$$\alpha + \beta + \gamma = -a$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = b$$

$$\alpha\beta\gamma = -c$$

The roots are α^2, β^2 and γ^2 .

$$\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)$$

$$= (-a)^2 - 2(b)$$

$$\alpha^2 + \beta^2 + \gamma^2 = a^2 - 2b$$

$$\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2 = (\alpha\beta)^2 + (\beta\gamma)^2 + (\gamma\alpha)^2$$

$$= (\alpha\beta + \beta\gamma + \gamma\alpha)^2 - 2(\alpha\beta^2\gamma + \beta\gamma^2\alpha + \gamma\alpha^2\beta)$$

$$= (\alpha\beta + \beta\gamma + \gamma\alpha)^2 - 2(\alpha\beta\gamma)(\beta + \gamma + \alpha)$$

$$= (b)^2 - 2(-c)(-a)$$

$$\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2 = b^2 - 2ac$$

$$\alpha^2 \beta^2 \gamma^2 = (\alpha \beta \gamma)^2 = (-c)^2 = c^2.$$

Equation:

$$x^2 - (\alpha^2 + \beta^2 + \gamma^2)x^2 + (\alpha^2 \beta^2 + \beta^2 \gamma^2 + \gamma^2 \alpha^2)x - \alpha^2 \beta^2 \gamma^2 = 0$$

$$x^2 - (a^2 - 2b)x^2 + (b^2 - 2ac)x - c^2 = 0.$$

Example: 3.7

nature of root, $4x^2 + 4px + (p+2) = 0.$

$$a^2 = 4, \quad b = 4p, \quad c = (p+2)$$

$$\Delta = b^2 - 4ac$$

$$= (4p)^2 - 4(4)(p+2)$$

$$= 16p^2 - 16(p+2)$$

$$= 16(p^2 - p - 2)$$

$$\Delta = 16(p+1)(p-2)$$

$\therefore \Delta < 0, \quad -1 < p < 2 \Rightarrow$ Imaginary roots.

$\Delta = 0, \quad p = -1, p = 2 \Rightarrow$ Equal real root.

$\Delta > 0, \quad -\infty < p < -1 \text{ (or)} \quad 2 < p < \infty$ distinct real roots.

EXE 3.2.

1.

 $2x^2 + kx + k = 0$, nature of the roots.

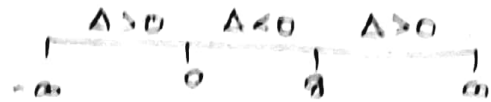
$$\therefore a=2, b=k, c=k.$$

$$\Delta = b^2 - 4ac.$$

$$= (k)^2 - 4(2)(k)$$

$$= k^2 - 8k.$$

$$\Delta = k(k-8)$$

 $\therefore$ (i) k is real.(ii) $\Delta < 0$, $0 < k < 8$ (Imaginary roots)(iii) $\Delta = 0$, $k=0$ (or) $k=8$ real and equal(iv) $\Delta > 0$, $(-\infty < k < 0)$ (or) $(8 < k < \infty)$ real and distinct.

2.

 $2+i\sqrt{3}$ is one of the roots. $2-i\sqrt{3}$ is another root.

$$\text{Sum of the roots: } (2+i\sqrt{3}) + (2-i\sqrt{3}) = 4.$$

$$\text{product of the roots: } (2+i\sqrt{3}) \times (2-i\sqrt{3}) = 4+3=7$$

Equation:

$$x^2 - (\text{SOR})x + (\text{POR}) = 0$$

$$x^2 - 4x + 7 = 0.$$

3.

$3+2i$ is one of the roots

$3-2i$ is another root.

Sum of the roots: $(3+2i) + (3-2i) = 6$.

Product of the roots: $(3+2i)(3-2i) = 9+4 = 13$.

Equation:

$$x^2 - (\text{SOR})x + \text{POR} = 0$$

$$x^2 - 6x + 13 = 0.$$

4. Theore 3.4

$\sqrt{p}$ and $\sqrt{q}$ are irrational number.

polynomial equation $\pm\sqrt{p} \pm\sqrt{q}$, there must have four roots.

$\sqrt{5}-\sqrt{3}$ is irrational number.

$\sqrt{5}+\sqrt{3}$ is another irrational number.

Sum of the roots: $(\sqrt{5}-\sqrt{3}) + (\sqrt{5}+\sqrt{3}) = 2\sqrt{5}$

product of the roots: $(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3}) = 5-3 = 2$

Equation $x^2 - (\text{SOR})x + \text{POR} = 0$

$$x^2 - 2\sqrt{5}x + 2 = 0 \longrightarrow \textcircled{1}$$

another equation $(x^2 + 2\sqrt{5}x + 2) = 0 \longrightarrow \textcircled{2}$

$$(x^2 - 2\sqrt{5}x + 2)(x^2 + 2\sqrt{5}x + 2) = 0.$$

$$x^4 - 16x^2 + 4 = 0.$$

5. Let the equation of st. line $y = mx + c$,

Let the equation of parabola $y^2 = 4ax$,

$$y^2 = 4ax.$$

$$(mx + c)^2 = 4ax$$

$$m^2x^2 + c^2 + 2mxc = 4ax$$

$$mx^2 + 2x(mc - 2a) + c^2 = 0.$$

Second degree equation in x the st. line and parabola intersect at two point.

Example 3.14

Let the equation of the circle $x^2 + y^2 = a^2$

Let the equation of the st. line $y = mx + c$.

$$x^2 + y^2 = a^2.$$

$$x^2 + (m^2x^2 + c^2 + 2mxc) = a^2.$$

$$x^2(1 + m^2) + 2mxc + (c^2 - a^2) = 0.$$

Second degree equation in x the st. line and circle intersect at two point.

$\therefore$ nature of root.

$\Delta < 0$, imaginary root

$\Delta = 0$, Real and equal

$\Delta > 0$, Real and distinct.

Example 3.8

$2 - \sqrt{3}i$ one of the root

$2 + \sqrt{3}i$ another root

$$\text{SOR} \Rightarrow (2 + \sqrt{3}i) + (2 - \sqrt{3}i) = 4$$

$$\text{POR} \Rightarrow (2 + \sqrt{3}i)(2 - \sqrt{3}i) = 7.$$

$$\text{Equation } x^2 - (\text{SOR})x + (\text{POR}) = 0$$

$$x^2 - 4x + 7 = 0.$$

Example 3.9

The coefficients are rational number.

$2 - \sqrt{3}$ one of the root

$2 + \sqrt{3}$ another root.

$$\text{SOR} \Rightarrow (2 - \sqrt{3}) + (2 + \sqrt{3}) = 4$$

$$\text{POR} \Rightarrow (2 - \sqrt{3})(2 + \sqrt{3}) = 4 - 3 = 1.$$

$$\text{Equation } x^2 - (\text{SOR})x + \text{POR} = 0$$

$$x^2 - 4x + 1 = 0.$$

Example 3.10

$\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ is a root, $x - \sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ is a factor.

$x + \sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ is another factor.

$$(x - \sqrt{\frac{\sqrt{2}}{\sqrt{3}}})(x + \sqrt{\frac{\sqrt{2}}{\sqrt{3}}}) = x^2 - \left(\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}\right)^2$$

$= x^2 - \frac{\sqrt{2}}{\sqrt{3}}$ and included another factor $x^2 + \frac{\sqrt{2}}{\sqrt{3}}$.

$$(x^2 - \frac{\sqrt{2}}{\sqrt{3}})(x^2 + \frac{\sqrt{2}}{\sqrt{3}}) = x^4 - \left(\frac{\sqrt{2}}{\sqrt{3}}\right)^2 = x^4 - \frac{2}{3}.$$

Example 3.11

$$a = 2, b = -6, c = 7$$

$$\Delta = b^2 - 4ac$$

$$= 36 - 4(2)(7)$$

$$= 36 - 56$$

$$\Delta = -20, \text{ The roots are imaginary, } \Delta < 0,$$

Example 3.12.

$$a = 1, b = 2(k+2), c = 9k$$

$$\Delta = b^2 - 4ac$$

$$= (2(k+2))^2 - 4(1)(9k)$$

$$= 4(k^2 + 4k + 4) - 36k$$

$$= 4k^2 + 16k + 16 - 36k$$

$$= 4k^2 - 20k + 16$$

$$= 4(k^2 - 5k + 4)$$

$$\Delta = 4(k-4)(k-1)$$

Example 3.13.

$$a = 1, b = -2p, c = p^2 - q^2 + 2qr - r^2 = 0.$$

$$\Delta = b^2 - 4ac$$

$$= (-2p)^2 - 4(1)(p^2 - q^2 + 2qr - r^2)$$

$$= 4(p^2 - 2qr + r^2)$$

$$= 4(p-r)^2, \text{ the roots are rational,}$$

which is a perfect square.

Theorem 3.2. Complex conjugate root theorem,

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0 = 0,$$

Let z_0 is a root of $p(z_0) = 0$,

$$p(z_0) = a_n z_0^n + a_{n-1} z_0^{n-1} + \dots + a_1 z_0 + a_0 = 0.$$

$$p(\bar{z}_0) = a_n \bar{z}_0^n + a_{n-1} \bar{z}_0^{n-1} + \dots + a_1 \bar{z}_0 + a_0 = 0$$

$$= \overline{a_n z_0^n + a_{n-1} z_0^{n-1} + \dots + a_1 z_0 + a_0} = 0$$

$$p(\bar{z}_0) = \overline{a_n z_0^n + a_{n-1} z_0^{n-1} + \dots + a_1 z_0 + a_0} = 0$$

z_0 is a root of $p(z_0) = 0$ and also $\bar{z}_0$ is root of $p(\bar{z}_0) = 0$.

Theorem 3.3.

Let p and q are rational number

$\sqrt{q}$ is an irrational number

$p + \sqrt{q}$ is root of $0x^3 + x^2 + bx + c = 0$.

$$\text{SoR} \Rightarrow \alpha + \beta + \sqrt{q} = -b.$$

$$\alpha + \sqrt{q} = -b - \beta.$$

$$\alpha + \sqrt{q} = s, \quad s = -b - \beta. \text{ (say)}$$

$$\boxed{\alpha = s - \sqrt{q}}$$

$$\text{POR} \Rightarrow (p + \sqrt{q})(s - \sqrt{q}) = (ps - q) + \sqrt{q}(s - p) = c$$

$$\Rightarrow s - p = 0$$

$$\boxed{s = p}$$

$\alpha = p - \sqrt{q}$ is another root.

Ex 3.3.

1. The sum of the two roots are zero.

So, $\alpha, -\alpha$ and β are the roots.

$$2x^3 - x^2 - 18x + 9 = 0.$$

$$\text{SOR} \Rightarrow \alpha - \alpha + \beta = 1/2$$

$$\boxed{\beta = 1/2}$$

$$\text{POR} \Rightarrow (\alpha)(-\alpha)\beta = -9/2$$

$$-\alpha^2\beta = -9/2$$

$$\alpha^2(1/2) = 9/2$$

$$\alpha^2 = 9$$

$$\alpha = \pm 3.$$

hence, 3, -3, 2 are the roots.

2. The roots are in Arithmetic progression (AP).

So, $\alpha+d, \alpha, \alpha-d$ are the roots. $9x^3 - 36x^2 + 44x - 16 = 0$

$$\text{SOR} \Rightarrow (\alpha+d) + (\alpha-d) + \alpha = \frac{36}{9}$$

$$3\alpha = 4$$

$$\boxed{\alpha = \frac{4}{3}}$$

$$9x^2 - 24x + 12 = 0.$$

$$\div 3 \quad 3x^2 - 8x + 4 = 0.$$

$$(x-2)(x-2/3) = 0$$

$$x=2, x=2/3.$$

The roots are, 2, 2/3 and 4/3.

$$\begin{array}{r|rrrr} 4/3 & 9 & -36 & 44 & -16 \\ & 0 & 12 & -32 & 16 \\ \hline & 9 & -24 & 12 & 0 \end{array}$$

3. The roots are in geometric progression (GP).

So, $\frac{\alpha}{r}$, α , αr are the roots of $3x^3 - 26x^2 + 52x - 24 = 0$

$$\text{FOR } \Rightarrow \left(\frac{\alpha}{r}\right)(\alpha)(\alpha r) = \frac{24}{3}$$

$$\alpha^3 = 8$$

$$\boxed{\alpha = 2}$$

$$3x^2 - 20x + 12 = 0.$$

$$(x-6)(x-2/3) = 0$$

$$x = 6, x = 2/3.$$

The roots are 2, 6, $2/3$.

2	3	-26	52	-24
	0	6	-40	24
	3	-20	12	0

4. Given, $2x^3 - 6x^2 + 3x + 2 = 0$, let α, β, γ are roots.

one of its root is twice the sum of the other two root.

$$\alpha = 2(\beta + \gamma) \rightarrow \textcircled{1}$$

$$\text{SOR } \Rightarrow \alpha + \beta + \gamma = 6/2.$$

$$2(\beta + \gamma) + (\beta + \gamma) = 3.$$

$$3(\beta + \gamma) = 3.$$

$$\boxed{\beta + \gamma = 1}$$

Sub $\beta + \gamma = 1$ in eq $\textcircled{1}$.

$$\alpha = 2(\beta + \gamma)$$

$$= 2(1)$$

$$\boxed{\alpha = 2}$$

$$\therefore k - 2 = 0$$

$$\boxed{k = 2}$$

$$\begin{array}{r|rrrr} 2 & 2 & -6 & 3 & k \\ & 0 & 4 & -4 & -2 \\ \hline & 2 & -2 & -1 & 0 \end{array}$$

$$\textcircled{1} \quad 2x^2 - 2x - 1 = 0$$

The roots are $k=2, 2, \frac{1 \pm \sqrt{3}}{2}$

Rough Column:

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = 2, b = -2, c = -1$$

$$x = \frac{2 \pm \sqrt{4 + 8}}{4}$$

$$= \frac{2 \pm 2\sqrt{3}}{4}$$

$$x = \frac{1 \pm \sqrt{3}}{2}$$

5.

$$\text{Given: } x^6 - 3x^5 - 5x^4 + 22x^3 - 39x^2 - 39x + 135 = 0$$

$1+2i$ one of the root

$1-2i$ another root

$$\text{SOR} \Rightarrow (1+2i) + (1-2i) = 2$$

$$\text{POR} \Rightarrow (1+2i)(1-2i) = 5$$

Equation,

$$x^2 - (\text{SOR})x + (\text{POR}) = 0$$

$$x^2 - 2x + 5 = 0$$

$\sqrt{3}$ one of the root

$-\sqrt{3}$ another root.

$$\text{SOR} \Rightarrow \sqrt{3} + (-\sqrt{3}) = 0$$

$$\text{POR} \Rightarrow (\sqrt{3})(-\sqrt{3}) = -3$$

Equation

$$x^2 - (\text{SOR})x + (\text{POR}) = 0$$

$$x^2 - 3 = 0$$

Compare the equation,

$$x^6 - 3x^5 - 5x^4 + 22x^3 - 39x^2 - 39x + 135 = (x^2 - 2x + 5)(x^2 - 3)(x^2 + px + q)$$

Equating x term.

$$-39 = (-2)(-3)(q) + p(-3)(5)$$

$$-39 = -54 - 15p$$

$$\boxed{p = -1}$$

another factor is $x^2 - x - 9$

$$x^2 - x - 9 = 0.$$

$$x = \frac{1 \pm \sqrt{37}}{2}$$

$$a = 1, b = -1, c = -9$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{+1 \pm \sqrt{1 + 36}}{2}$$

$$x = \frac{1 \pm \sqrt{37}}{2}$$

The roots are $1 \pm 2i$, $\pm \sqrt{3}$ and $\frac{1 \pm \sqrt{37}}{2}$

NOTE:
(i) zero sum of all co-efficient.

$$P(x) = 0.$$

let $x = 1$, $P(1) = 0$ 1 is a root of $P(x)$.

$(x-1)$ is one of the factors.

(ii) Equal sum of co-efficient of odd and even power.

$$\therefore P(x) \neq 0, \quad P(-x) = 0 \text{ means,}$$

$+1$ is a root of $P(-x) = 0$

which says that

-1 is a root of $P(x) = 0$.

$(x+1)$ is one of the factors.

$$b. (i) 2x^3 - 9x^2 + 10x - 3 = 0.$$

$$2x^3 - 9x^2 + 10x - 3 = 0.$$

Sum of the co-efficient of the equation.

$$2 - 9 + 10 - 3 = 0.$$

$x - 1$ is a factor.

$$P(1) = 0.$$

1 is a root of $P(x) = 0$.

$(x - 1)$ is a factor.

$$2x^2 - 7x + 3 = 0.$$

$$(x - 3)(x - 1/2) = 0$$

$$x = 3, x = 1/2.$$

The roots are 1, 3, $1/2$.

1	2	-9	10	-3
	0	2	-7	3
	2	-7	3	0

$$(ii) 8x^3 - 2x^2 - 7x + 3 = 0.$$

Sum of co-efficient of (odd power = even power)

$$8 - 7 = -2 + 3$$

$$1 = 1.$$

$x + 1$ is a factor.

$$P(x) \neq 0$$

$$P(-x) = 0.$$

1 is root of $P(-x) = 0$

$\therefore -1$ is root of $P(x) = 0$.

$x + 1$ is a factor.

-1	8	-2	-7	3
	0	-8	10	-3
	8	-10	3	0

$$8x^2 - 10x + 3 = 0$$

$$(x - 3/4)(x - 1/2) = 0$$

$$(x - 3/4)(x - 1/2) = 0.$$

$$x = 3/4, x = 1/2.$$

The roots are $-1, 3/4, 1/2$.

7.

$$x^4 - 14x^2 + 45 = 0.$$

$$\text{let } y = x^2.$$

$$(x^2)^2 - 14x^2 + 45 = 0$$

$$y^2 - 14y + 45 = 0.$$

$$(y-9)(y-5) = 0$$

$$y-9 = 0$$

$$y-5 = 0$$

$$y = 9$$

$$y = 5$$

$$\text{sub } y = x^2,$$

$$x^2 = 9$$

$$x^2 = 5$$

$$x = \pm 3$$

$$x = \pm \sqrt{5}$$

The roots are ± 3 and $\pm \sqrt{5}$.

Example 3.15

$$x^6 - 13x^5 + 62x^4 - 126x^3 + 65x^2 + 127x - 140 = 0.$$

2+2 one of the root

$3-\sqrt{2}$ one of the root

2-2 another root

$3+\sqrt{2}$ another root

$$\text{SOR} \Rightarrow (2+2)(2-2) = 4$$

$$\text{SOR} \Rightarrow (3-\sqrt{2})(3+\sqrt{2}) = 6$$

$$\text{POR} \Rightarrow (2+2)(2-2) = 4+1 = 5$$

$$\text{POR} \Rightarrow (3-\sqrt{2})(3+\sqrt{2}) = 9-2 = 7.$$

$$\text{factor } x^2 - (\text{SOR})x + (\text{POR})$$

$$\text{factor } x^2 - (\text{SOR})x + \text{POR}.$$

$$x^2 - 4x + 5 = 0$$

$$x^2 - 6x + 7 = 0$$

Compare the equation,

$$x^6 - 13x^5 + 62x^4 - 126x^3 + 65x^2 + 127x - 140 =$$

$$(x^2 - 4x + 5)(x^2 - 6x + 7)(x^2 + px - 4)$$

Equating the x term,

$$127 = (-4)(7)(-4) + (-6)(5)(-4) + (5)(7)(p)$$

$$127 = 112 + 120 + 35p$$

$$127 - 232 = 35p$$

$$35p = -105$$

$$\boxed{p = -3}$$

another factor $x^2 - 3x - 4 = 0$

$$(x - 4)(x + 1) = 0$$

$$x = 4, x = -1.$$

The roots of the equation are $2 \pm i$, $3 \pm \sqrt{2}$, 4 and -1.

Example 3.16,

$$x^4 - 9x^2 + 20 = 0$$

$$(x^2)^2 - 9x^2 + 20 = 0$$

Let $x^2 = y$ $y^2 - 9y + 20 = 0$

$$(y - 5)(y - 4) = 0$$

$$y = 5$$

$$y = 4$$

$$x^2 = 5$$

$$x^2 = 4$$

$$x = \pm\sqrt{5}$$

$$x = \pm 2.$$

The roots are $\pm\sqrt{5}$ and ± 2 .

Example 3.17

$$x^3 - 3x^2 - 33x + 35 = 0.$$

Sum of all co. efficient is zero

$$1 - 3 - 33 + 35 = 0$$

$\therefore (x-1)$ is a factor.

$$\begin{array}{r|rrrr} 1 & 1 & -3 & -33 & 35 \\ & 0 & 1 & -2 & -35 \\ \hline & 1 & -2 & -35 & 0 \end{array}$$

$$x^2 - 2x - 35 = 0$$

$$x^2 - 7x + 5x - 35 = 0$$

$$x(x-7) + 5(x-7) = 0$$

$$(x+5)(x-7) = 0$$

$$x = -5, x = 7.$$

The roots are 1, -5 and 7.

Example 3.18

$$2x^3 + 11x^2 - 9x - 18 = 0.$$

Sum of the co. efficient (odd power = even power)

$$2 - 9 = 11 - 18$$

$$-7 = -7$$

$x+1$ is a factor

$$\begin{array}{r|rrrr} -1 & 2 & 11 & -9 & -18 \\ & 0 & -2 & -9 & 18 \\ \hline & 2 & 9 & -18 & 0 \end{array}$$

$$2x^2 + 9x - 18 = 0.$$

$$(x+6)(x-3/2) = 0$$

$$x = -6, x = 3/2.$$

$$\begin{array}{r} 9 \\ 2 \frac{1}{2} \wedge -\frac{3}{2} \\ \times \\ -36 \end{array}$$

The roots are -1, -6 and $3/2$.

Example. 3.22.

$$x^3 - 6x^2 - 4x + 24 = 0.$$

The roots are $a-d$, a , $a+d$.

Sum of the roots $a-d + a + a+d = +6$

$$3a = +6$$

$$\boxed{a = 2}$$

$$x^2 - 4x - 12 = 0$$

$$(x-6)(x+2) = 0$$

$$x = 6, x = -2$$

2	1	-6	-4	24
	0	2	-8	-24
	1	-4	-12	0

The roots are 2, -2 and 6.

Example 3.19.

$$x^3 + px^2 + qx + r = 0.$$

Let the roots are A, P , $a-d$, a , $a+d$.

SoR $\Rightarrow a-d + a + a+d = -p.$

$$3a = -p$$

$$\boxed{a = -p/3}$$

$$\left(-\frac{p}{3}\right)^3 + p\left(-\frac{p}{3}\right)^2 + q\left(-\frac{p}{3}\right) + r = 0.$$

$$-\frac{p^3}{27} + \frac{(+p^3)}{9} + \frac{(-pq)}{3} + r = 0.$$

$$\times 27 \Rightarrow -p^3 + 3p^3 - 9pq + 27r = 0.$$

$$2p^3 - 9pq + 27r = 0.$$

$$9pq = 2p^3 + 27r.$$

Example 3.20

$$ax^3 + bx^2 + cx + d = 0.$$

The roots are in HP $\frac{a}{r}, a, ar$.

$$\frac{a}{r} + a + ar = -\frac{b}{a}.$$

$$a\left(\frac{1}{r} + 1 + r\right) = -\frac{b}{a} \rightarrow (1).$$

$$\left(\frac{a}{r}\right)(a) + (a)(ar) + (ar)\left(\frac{a}{r}\right) = \frac{c}{a}$$

$$a^2\left(\frac{1}{r} + r + 1\right) = \frac{c}{a} \rightarrow (2).$$

$$\left(\frac{a}{r}\right)(a)(ar) = -\frac{d}{a}$$

$$a^3 = -\frac{d}{a} \rightarrow (3).$$

from (1) & (3)

$$\frac{(2)}{(1)} \Rightarrow \frac{a^2\left(\frac{1}{r} + 1 + r\right)}{a\left(\frac{1}{r} + 1 + r\right)} = \frac{c/a}{-b/a}.$$

$$\boxed{a = -\frac{c}{b}}$$

sub $a = -\frac{c}{b}$ in eq (3)

$$\left(-\frac{c}{b}\right)^3 = -\frac{d}{a}$$

$$\frac{c^3}{b^3} = \frac{d}{a}$$

$$\boxed{ac^3 = b^3d}$$

Example 3.21.

$x^3 + px^2 + qx + r = 0$, are in HP.

$(\frac{1}{x})^3 + p(\frac{1}{x})^2 + q(\frac{1}{x}) + r = 0$ are in AP.

$$\frac{1}{x^3} + \frac{p}{x^2} + \frac{q}{x} + r = 0.$$

$$1 + px + qx^2 + rx^3 = 0 \rightarrow \textcircled{1}.$$

The roots are in AP, $a-d, a, a+d$.

$$\text{SOR} \Rightarrow a-d + a + a+d = -\frac{q}{r},$$

$$3a = -\frac{q}{r}.$$

$$\boxed{a = -\frac{q}{3r}}$$

$$1 + p\left(-\frac{q}{3r}\right) + q\left(-\frac{q}{3r}\right)^2 + r\left(-\frac{q}{3r}\right)^3 = 0.$$

$$1 + \frac{(-pq)}{3r} + \frac{q^3}{9r^2} + \frac{-q^3r}{27r^3} = 0.$$

$$\times 27r^2 \Rightarrow 27r^2 - 9pqr + 3q^3 - q^3 = 0.$$

$$27r^2 - 9pqr + 2q^3 = 0.$$

$$\boxed{9pqr = 2q^3 + 27r^2}$$

Ex 3.4

$$2. (2x-1)(x+3)(x-2)(2x+3) + 20 = 0.$$

$$(2x-1)(2x+3)(x+3)(x-2) = -20$$

$$(4x^2 + 4x - 3)(x^2 + x - 6) = -20$$

$$4(x^2 + x - 3/4)(x^2 + x - 6) = -20$$

$$\text{Let } x^2 + x = y$$

$$4(y - 3/4)(x^2 + x - 6) + 20 = 0.$$

$$\div 4 \quad (y - 3/4)(y - 6) + \frac{20}{4} = 0.$$

$$(y - 3/4)(y - 6) + 5 = 0$$

$$y^2 = \frac{3y}{4} - 6y + \frac{9}{2} + 5 = 0.$$

$$4y^2 - 3y - 24y + 18 + 20 = 0,$$

$$4y^2 - 27y + 38 = 0,$$

$$(y-2)(y-19/4) = 0$$

$$y - 2 = 0$$

$$y - 19/4 = 0$$

$$x^2 + x - 2 = 0$$

$$(x-1)(x+2) = 0$$

$$x = 1, -2,$$

$$x^2 + x - 19/4 = 0$$

$$4x^2 + 4x - 19 = 0.$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{16 + 4(4)(19)}}{8}$$

$$x = \frac{-4 \pm 4\sqrt{20}}{8}$$

$$x = \frac{-1 \pm \sqrt{20}}{2} = \frac{-1 \pm 2\sqrt{5}}{2}.$$

The roots are 1, -2 and

$$\frac{-1 \pm 2\sqrt{5}}{2}.$$

partly factored polynomial.

Example 3.23

$$(x-2)(x-7)(x-3)(x+2) + 19 = 0$$

$$(x-2)(x-3)(x-7)(x+2) + 19 = 0.$$

$$(\underline{x^2 - 5x + 6})(\underline{x^2 - 5x - 14}) + 19 = 0.$$

∴ let $y = x^2 - 5x$.

$$(y+6)(y-14) + 19 = 0.$$

$$(y^2 - 8y - 84) + 19 = 0$$

$$y^2 - 8y - 65 = 0$$

$$(y-13)(y+5) = 0.$$

$$y - 13 = 0$$

&

$$y + 5 = 0.$$

sub $y = x^2 - 5x$

$$x^2 - 5x - 13 = 0$$

$$x^2 - 5x + 5 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{5 \pm \sqrt{25 + 52}}{2}$$

$$= \frac{5 \pm \sqrt{25 - 20}}{2}$$

$$x = \frac{5 \pm \sqrt{77}}{2}$$

$$x_1 = \frac{5 \pm \sqrt{5}}{2}$$

The solution of the equation. $\frac{5 \pm \sqrt{77}}{2}$ and $\frac{5 \pm \sqrt{5}}{2}$.

EXE 3.A.

①.

$$(x-5)(x-7)(x+6)(x+4) = 504$$

$$(x-5)(x+4)(x-7)(x+6) = 504$$

$$(x^2 - x - 20)(x^2 - x - 42) = 504$$

$$\text{let } y = x^2 - x$$

$$(y-20)(y-42) = 504$$

$$y^2 - 62y + 840 = 504$$

$$y^2 - 62y + 336 = 0$$

$$(y-6)(y-56) = 0$$

$$y-6=0 \quad \& \quad y-56=0$$

$$\text{sub } y = x^2 - x$$

$$x^2 - x - 6 = 0$$

$$x^2 - x - 56 = 0$$

$$(x-3)(x+2) = 0$$

$$(x-8)(x+7) = 0$$

$$x-3=0, x+2=0$$

$$x=8, -7$$

$$x=3, x=-2$$

The root of the equation is, 3, -2, 8 and -7

$$(ii) \quad (x-4)(x-7)(x-2)(x+1) = 16$$

$$(x-4)(x-2)(x-7)(x+1) = 16$$

$$(x^2 - 6x + 8)(x^2 - 6x - 7) = 16$$

$$\text{let } y = x^2 - 6x$$

$$(y+8)(y-7) = 16$$

$$y^2 + y - 56 = 16$$

$$y^2 + y - 72 = 0$$

$$(y+9)(y-8) = 0$$

$$y+9=0 \quad \& \quad y-8=0$$

$$\text{Sub } y = x^2 - 6x$$

$$x^2 - 6x + 9 = 0$$

$$(x-3)(x-3) = 0$$

$$x = 3, 3$$

$$x^2 - 6x - 8 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{6 \pm \sqrt{36 + 32}}{2}$$

$$= \frac{6 \pm \sqrt{68}}{2}$$

$$= \frac{6 \pm 2\sqrt{17}}{2}$$

$$x = 3 \pm \sqrt{17}$$

The roots of the equation, 3, 3 and $3 \pm \sqrt{17}$.

Example 8.24

$$(2x-3)(6x-1)(3x-2)(x-12) - 7 = 0$$

$$(2x-3)(3x-2)(6x-1)(x-12) - 7 = 0$$

$$(6x^2 - 13x + 6)(6x^2 - 13x + 12) - 7 = 0.$$

$$\text{Let } y = 6x^2 - 13x$$

$$(y+6)(y+12) - 7 = 0.$$

$$y^2 + 18y + 72 - 7 = 0$$

$$y^2 + 18y + 65 = 0.$$

$$y^2 + 13y + 5y + 65 = 0$$

$$(y+13)(y+5) = 0.$$

$$y+5 = 0.$$

$$y+13 = 0$$

$$\text{Sub } y = 6x^2 - 13x.$$

$$6x^2 - 13x + 13 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{13 \pm \sqrt{169 - 312}}{2(6)}$$

$$= \frac{13 \pm \sqrt{-143}}{12}$$

$$x = \frac{13 \pm \sqrt{-143}}{12}.$$

$$6x^2 - 13x + 5 = 0.$$

$$(x - \frac{10}{6})(x - \frac{3}{6}) = 0$$

$$(x - \frac{5}{3})(x - \frac{1}{2}) = 0$$

$$x = \frac{5}{3}, x = \frac{1}{2}.$$

The roots of the equation $\frac{1}{2}, \frac{5}{3}$ and $\frac{13 \pm \sqrt{-143}}{12}.$

Exe 8.5

1. (i) $\sin^2 x - 5 \sin x + 4 = 0.$

Let $\sin x = y.$

$$y^2 - 5y + 4 = 0$$

$$(y-4)(y-1) = 0$$

$$y-4=0 \quad y-1=0.$$

$$y=4, \quad y=1$$

$\therefore y=4 \Rightarrow \sin x = 4$ is not possible of the solution.

$\therefore$ The range $[-1, 1]$

$y=1 \Rightarrow \sin x = 1$

$$\sin x = \sin \pi/2.$$

$$\boxed{x = \pi/2}$$

General solution $x = (2n\pi \pm \pi/2), n \in \mathbb{Z}$

(ii) $12x^3 + 8x = 29x^2 - 4.$

$$12x^3 + 8x - 29x^2 + 4 = 0.$$

$$P(x) = 12x^3 + 8x - 29x^2 + 4$$

$$P(2) = 12(2)^3 + 8(2) - 29(2)^2 + 4 = 0.$$

$x-2$ is factor of equation.

$$\begin{array}{r|rrrr} 2 & 12 & -29 & 8 & 4 \\ & 0 & 24 & -10 & -4 \\ \hline & 12 & -5 & -2 & 0 \end{array}$$

$$12x^2 - 5x - 2 = 0$$

$$12x^2 - 8x + 3x - 2 = 0.$$

$$4x(3x-2) + 1(3x-2) = 0$$

$$(4x+1)(3x-2) = 0$$

$$x = -1/4 \quad x = 2/3$$

The roots of the equation, $2, 2/3, -1/4.$

$$2. (i) \quad 2x^3 - x^2 - 1 = 0$$

sum of the all co-efficient is zero.

$(x-1)$ is a factor.

Compare the equation.

$$2x^3 - x^2 - 1 = (x-1)(2x^2 + px + 1)$$

Equating the x term.

$$0 = 1 - p$$

$$\boxed{p = 1}$$

another factor is $2x^2 + x + 1 = 0$.

$$2x^2 + x + 1 = 0.$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{1 - 8}}{4}$$

$$= \frac{-1 \pm \sqrt{-7}}{4}$$

$$x = \frac{-1 \pm \sqrt{-7}}{4}$$

$x = 1$ is a rational root.

$$(ii) \quad x^3 - 3x + 1 = 0$$

First term $b_n = a_n = 1$

$$(p/q) = 1$$

Last term $b_0 = a_0 = 1$

p is a factor of $a_0 = 1$

The root of $\frac{p}{q} = \frac{1}{1} = 1$,

q is a factor of $a_n = 1$

Since 1 has no factor.

There is no rational root.

$$3. \quad 8x^{\frac{3}{2n}} - 8x^{-\frac{3}{2n}} = 63.$$

$$\text{Let } x^{\frac{3}{2n}} = y.$$

$$8y - 8y^{-1} = 63.$$

$$\times y \Rightarrow 8y^2 - 8 = 63y.$$

$$8y^2 - 63y - 8 = 0.$$

$$8y^2 - 64y + y - 8 = 0.$$

$$8y(y-8) + 1(y-8) = 0$$

$$(8y+1)(y-8) = 0.$$

$$y = -1/8$$

$$\boxed{y = 8}$$

$$x^{\frac{3}{2n}} = 8$$

$$x = (8)^{\frac{2n}{3}}$$

$$x = [2^3]^{\frac{2n}{3}}$$

$$x = (2)^{2n}$$

$$x = [2^2]^n$$

$$x = 4^n.$$

$$1. \quad 2\sqrt{\frac{x}{a}} + 3\sqrt{\frac{a}{x}} = \frac{b}{a} + \frac{6a}{b}$$

Taking square on both side.

$$\left(2\sqrt{\frac{x}{a}} + 3\sqrt{\frac{a}{x}}\right)^2 = \left(\frac{b}{a} + \frac{6a}{b}\right)^2$$

$$\frac{4x}{a} + \frac{9a}{x} + 12 = \frac{b^2}{a^2} + \frac{36a^2}{b^2} + 12$$

$$\times \frac{ax}{4} \Rightarrow x^2 + \frac{9a^2}{4} = \frac{b^2x}{4a} + \frac{9a^3}{b^2}x$$

$$x^2 - \frac{b^2x}{4a} + \frac{9a^2}{4} - \frac{9a^3x}{b^2} = 0.$$

$$x\left(x - \frac{b^2}{4a}\right) - \frac{9a^3}{b^2}\left[x - \frac{b^2}{4a}\right] = 0$$

$$\left(x - \frac{9a^3}{b^2}\right)\left(x - \frac{b^2}{4a}\right) = 0$$

$$x = \frac{9a^3}{b^2} \quad \& \quad x = \frac{b^2}{4a}.$$

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$$5. (\therefore) 6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$$

$$\div x^2 \quad 6x^2 - 35x + 62 - \frac{35}{x} + \frac{6}{x^2} = 0$$

$$6\left(x^2 + \frac{1}{x^2}\right) - 35\left(x + \frac{1}{x}\right) + 62 = 0$$

$$\therefore \text{ let } \boxed{y = x + \frac{1}{x}} \quad \text{taking square on both side}$$

$$y^2 = \left(x + \frac{1}{x}\right)^2$$

$$y^2 = x^2 + \frac{1}{x^2} + 2$$

$$\boxed{y^2 - 2 = x^2 + \frac{1}{x^2}}$$

$$6(y^2 - 2) - 35(y) + 62 = 0$$

$$6y^2 - 12 - 35y + 62 = 0.$$

$$6y^2 - 35y + 50 = 0$$

$$6y^2 - 15y - 20y + 50 = 0$$

$$3y(2y - 5) - 10(2y - 5) = 0.$$

$$(3y - 10)(2y - 5) = 0.$$

$$3y - 10 = 0$$

$$2y - 5 = 0$$

$$3\left(x + \frac{1}{x}\right) - 10 = 0$$

$$2\left(x + \frac{1}{x}\right) + 5 = 0$$

$$3x^2 + 3 - 10x = 0$$

$$2x^2 + 2 - 5x = 0$$

$$3x^2 - 10x + 3 = 0$$

$$2x^2 - 5x + 2 = 0$$

$$3x^2 - 9x - x + 3 = 0$$

$$2x^2 - 4x - x + 2 = 0$$

$$3x(x - 3) - 1(x - 3) = 0$$

$$2x(x - 2) - 1(x - 2) = 0$$

$$(x - 3)(3x - 1) = 0$$

$$(2x - 1)(x - 2) = 0$$

$$x = 3, x = \frac{1}{3}$$

$$x = \frac{1}{2}, x = 2.$$

(ii)

$$x^4 + 3x^3 - 3x - 1 = 0$$

Sum of the all co-efficient zero, $P(x) = 0$

$(x-1)$ is factor

$$P(1) = 0.$$

Equal sum of co-efficient; odd power = even power.

$(x+1)$ is factor.

$$P(-1) = 0.$$

compare the equation.

$$x^4 + 3x^3 - 3x - 1 = (x-1)(x+1)(x^2 + px + 1)$$

$$x^4 + 3x^3 - 3x - 1 = (x^2 - 1)(x^2 + px + 1)$$

Equating the x term.

$$-3 = -p$$

$$\boxed{p = 3}$$

another factor $x^2 + 3x + 1$

$$x^2 + 3x + 1 = 0.$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-3 \pm \sqrt{9 - 4}}{2}$$

$$x = \frac{-3 \pm \sqrt{5}}{2}$$

The roots are $-1, 1, \frac{-3 + \sqrt{5}}{2}$ and $\frac{-3 - \sqrt{5}}{2}$.

$$6. \quad 4^x - 3(2^{x+2}) + 2^5 = 0.$$

$$(2^2)^x - 3 \cdot 2^x (2^2) + 2^5 = 0.$$

$$(2^x)^2 - 12(2^x) + 2^5 = 0$$

$$\text{Let } 2^x = y$$

$$y^2 - 12y + 32 = 0$$

$$(y-8)(y-4) = 0$$

$$y = 8 \quad y = 4$$

$$2^x = 8 \quad 2^x = 2^2$$

$$2^x = 2^3 \quad \boxed{x=2}$$

$$\therefore \boxed{x=3}$$

Example 3.25

$$x^3 - 5x^2 - 4x + 20 = 0.$$

Sum of all coefficients

Sum of coefficients [odd = even]

not statistics,

2 is one of the root

$$\therefore p(2) = 0.$$

$$x^2 - 3x - 10 = 0.$$

$$(x+5)(x-2) = 0$$

$$x = 5, -2.$$

$$\begin{array}{r|rrrr} 2 & 1 & -5 & -4 & 20 \\ & 0 & 2 & -6 & -20 \\ \hline & 1 & -3 & -10 & 0 \end{array}$$

The roots are -2, 2 and 5.

Example 3-26.

$$2x^3 + 3x^2 + 2x + 3 = 0.$$

$$a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x^1 + a_0 = 0.$$

$$\therefore a_n = 2, \quad a_0 = 3.$$

$\therefore p$ must divides a_0

q must divides a_n .

The possible roots $\pm 1, \pm 2, \pm 3$.

The roots are $\pm 1/2, \pm 1, \pm 3, \pm 3/2$.

$-3/2$ is only one possible root.

$$\begin{array}{r|rrrr} -3/2 & 2 & 3 & 2 & 3 \\ & 0 & -3 & 0 & -3 \\ \hline & 2 & 0 & 2 & 0 \end{array}$$

$$2x^2 + 2 = 0$$

$$x^2 = -1$$

$$x = \pm i.$$

It is imaginary root.

The roots are $-3/2, \pm i$

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$$7. \quad 6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0.$$

$$\div x^2 \quad 6x^2 - 5x - 38 - \frac{5}{x} + \frac{6}{x^2} = 0$$

$$6\left(x^2 + \frac{1}{x^2}\right) - 5\left(x + \frac{1}{x}\right) - 38 = 0.$$

$$\therefore \text{let } y = x + \frac{1}{x}.$$

$$(y)^2 = \left(x + \frac{1}{x}\right)^2$$

$$y^2 = x^2 + \frac{1}{x^2} + 2.$$

$$y^2 - 2 = x^2 + \frac{1}{x^2}.$$

$$6(y^2 - 2) - 5y - 38 = 0.$$

$$6y^2 - 12 - 5y - 38 = 0$$

$$6y^2 - 5y - 50 = 0$$

$$6y^2 - 20y + 15y - 50 = 0$$

$$2y(3y - 10) + 5(3y - 10) = 0$$

$$(2y + 5)(3y - 10) = 0.$$

$$2\left(x + \frac{1}{x}\right) + 5 = 0 \quad 3\left(x + \frac{1}{x}\right) - 10 = 0$$

$$2x^2 + 5x + 2 = 0$$

$$3x^2 - 10x + 3 = 0.$$

$$2x^2 + 4x + x + 2 = 0$$

$$3x^2 - 9x - x + 3 = 0$$

$$2x(x+2) + 1(x+2) = 0$$

$$3x(x-3) - 1(x-3) = 0$$

$$(2x+1)(x+2) = 0$$

$$(3x-1)(x-3) = 0$$

$$x = -1/2, x = -2.$$

$$x = 1/3, x = 3.$$

The roots are $1/3, -1/2, -2$ and 3 .

Exe 3.6.

$$\textcircled{1} \quad p(x) = \underbrace{9x^9}_{\textcircled{1}} - \underbrace{4x^8}_{\textcircled{2}} + \underbrace{4x^7}_{\textcircled{3}} - \underbrace{3x^6}_{\textcircled{4}} + 2x^5 + x^3 + 7x^2 + 7x + 2,$$

The no. of sign change in $p(x)$ is 4

no. of positive root of $p(x)$

$$p(-x) = -9x^9 - 4x^8 - 4x^7 - 3x^6 - 2x^5 - x^3 + 7x^2 - 7x + 2,$$

$\underbrace{-x^3}_{\textcircled{1}} \quad \underbrace{7x^2}_{\textcircled{2}} \quad \underbrace{-7x}_{\textcircled{3}} \quad \underbrace{+2}_{\textcircled{4}}$

The no. of sign change in $p(-x)$ is 3,

no. of negative root of $p(-x)$.

It has atmost four positive root and atmost 3 negative root

The no. of imaginary root is 2.

2.

$$p(x) = \underbrace{x^2}_{\textcircled{1}} - \underbrace{5x}_{\textcircled{2}} + 6$$

$$q(x) = \underbrace{x^2}_{\textcircled{1}} - \underbrace{5x}_{\textcircled{2}} + 16$$

The no. of sign change in $p(x)$ and $q(x)$ are 2.

no. of positive root of $p(x)$ and $q(x)$

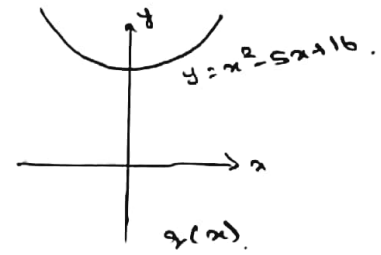
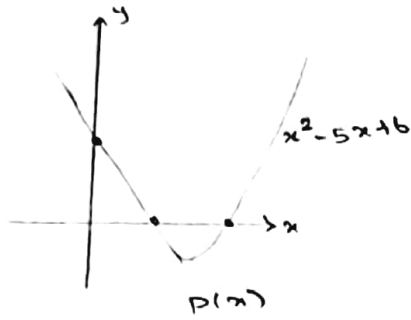
$$p(-x) = x^2 + 5x + 6$$

$$q(-x) = x^2 + 5x + 16$$

The no. of sign change in $p(-x)$ and $q(-x)$ is zero

No. of Negative root of $p(-x)$ and $q(-x)$.

It has atmost two positive root and No negative root.



$$y = (x^2 - 5x + 6)$$

$$y = (x-2)(x-3)$$

when $y=0$ $x=2, 3$

$x=0$ $y=6$.

$$y = x^2 - 5x + 16$$

when $y=0$, $x^2 - 5x + 16 = 0$.

$x=0$, $y=16$.

3.

$$p(x) = x^9 - 5x^5 + 4x^4 + 2x^2 + 1$$

The no. of sign change in $p(x) = 2$

no. of positive root in $p(x)$.

$$p(-x) = -x^9 + 5x^5 + 4x^4 + 2x^2 + 1$$

The no. of sign change in $p(-x) = 1$

no. of negative root in $p(-x)$

It has at most 2 positive root and one negative root

The no. of imaginary root is 6.

4. $p(x) = x^9 - 5x^8 - 14x^7$

①

The no. of sign change in $p(x)$ is 1

no. of positive root in $p(x)$

$p(-x) = -x^9 - 5x^8 + 14x^7$

①

The no. of sign change in $p(-x)$ is 1

no. of negative root in $p(-x)$

It has one positive root and one negative root.

The no. of imaginary root is 7.

5. $p(x) = x^9 + 9x^7 + 7x^5 + 5x^3 + 3x$

The no. of sign change in $p(x)$ is zero.

no. of positive root in $p(x)$

$p(-x) = -x^9 - 9x^7 - 7x^5 - 5x^3 - 3x$

The no. of sign change in $p(-x)$ is zero

no. of negative root in $p(-x)$.

It has no positive root and no negative root.

All the roots are the polynomial only imaginary roots

Example 3.30.

$$p(x) = 9x^9 + 2x^5 - x^4 - 7x^2 + 2.$$

The no. of sign change in $p(x)$ is 2.

no. of positive root in $p(x)$

$$p(-x) = -9x^9 - 2x^5 - x^4 - 7x^2 + 2.$$

The no. of sign change in $p(-x)$ is 1

no. of negative root in $p(-x)$

It has at most 2 positive root and one negative root.

The no. of imaginary roots are 6.

Example 3.31.

$$(i) \quad p(x) = x^{2018} + 1947x^{1950} + 15x^8 + 26x^6 + 2019.$$

The no. of sign change in $p(x)$ and $p(-x)$ is Zero.

It has no positive root and negative roots.

The polynomial has no real roots.

$\therefore$ All the roots of polynomial are imaginary root.

$$(i) \quad p(x) = x^5 - 19x^4 + 2x^3 + 5x^2 + 11.$$

$\swarrow \quad \searrow$
 ① ②

The no. of sign change in $p(x)$ is 2.

The no. of positive root in $p(x)$

$$p(-x) = -x^5 - 19x^4 - 2x^3 + 5x^2 + 11$$

$\swarrow$
 ①

The no. of sign change in $p(-x)$ is 1

no. of negative root in $p(-x)$

It has atmost 2 positive root and one negative root

The no. of imaginary root is 2.

Example. 3, 27.

$$7x^3 - 43x^2 = 43x - 7,$$

$$7x^3 - 43x^2 - 43x + 7 = 0.$$

Sum of the Co-efficient Odd power = even power,

$$7 - 43 = -43 + 7$$

$$-36 = -36,$$

$x+1$ is one of factor

$$7x^2 - 50x + 7 = 0.$$

$$(7x-1)(x-7) = 0$$

$$x = 1/7, x = 7.$$

$$\begin{array}{r|rrrr}
 -1 & 7 & -43 & -43 & +7 \\
 & 0 & -7 & 50 & -7 \\
 \hline
 & 7 & -50 & 7 & 0
 \end{array}$$

The roots are -1, 7, and $1/7$.

Example 3.28

$$x^4 - 10x^3 + 26x^2 - 10x + 1 = 0,$$

$$\div x^2 \quad x^2 - 10x + 26 - \frac{10}{x} + \frac{1}{x^2} = 0$$

$$\left(x^2 + \frac{1}{x^2}\right) - 10\left(x + \frac{1}{x}\right) + 26 = 0$$

$$\text{Let } y = x + \frac{1}{x}$$

$$y^2 = \left(x + \frac{1}{x}\right)^2$$

$$y^2 = x^2 + \frac{1}{x^2} + 2,$$

$$y^2 - 2 = x^2 + \frac{1}{x^2},$$

$$(y^2 - 2) - 10y + 26 = 0$$

$$y^2 - 10y + 24 = 0$$

$$(y - 6)(y - 4) = 0$$

$$y - 6 = 0 \quad y - 4 = 0$$

$$x + \frac{1}{x} - 6 = 0$$

$$x^2 - 6x + 1 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{6 \pm \sqrt{36 - 4}}{2}$$

$$x = 3 \pm 2\sqrt{2}$$

$$x + \frac{1}{x} - 4 = 0$$

$$x^2 - 4x + 1 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{4 \pm \sqrt{16 - 4}}{2}$$

$$x = 2 \pm \sqrt{3}$$

The roots are $3 \pm 2\sqrt{2}$ and $2 \pm \sqrt{3}$.

Example 3.29.

$$2 \cos^2 x - 9 \cos x + 4 = 0$$

$$\text{Let } \cos x = y$$

$$2y^2 - 9y + 4 = 0,$$

$$2y^2 - 8y - y + 4 = 0$$

$$2y(y-4) - 1(y-4) = 0$$

$$(2y-1)(y-4) = 0$$

$$y = 1/2 \quad y = 4,$$

$$\cos x = 1/2$$

$$x = \cos^{-1}(1/2)$$

$$x = \pi/3$$

$\forall x = 2n\pi \pm \pi/3$, are solution of the equation.

$\cos x = 4$ is never possible.

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CHAPTER-5 TWO DIMENSIONAL ANALYTICAL GEOMETRY - II

Example 5.1

Equation of the circle is

$$(x-h)^2 + (y-k)^2 = r^2$$

$(h, k) = (-3, -4)$ and radius $r = 3$.

$$(x+3)^2 + (y+4)^2 = 3^2$$

$$x^2 + 6x + 9 + y^2 + 8y + 16 = 9$$

$$x^2 + y^2 + 6x + 8y + 16 = 0$$

Example 5.2.

Chord : $3x + y + 5 = 0$

Circle : $x^2 + y^2 = 16 \Rightarrow x^2 + y^2 - 16 = 0$

The required Equation of the circle

$$(x^2 + y^2 - 16) + \lambda(3x + y + 5) = 0$$

$$x^2 + y^2 - 16 + 3\lambda x + \lambda y + 5\lambda = 0$$

$$x^2 + 2hx + y^2 + 2ky + c = 0$$

Here $2g = 3\lambda$, $2f = \lambda$

$$g = \frac{3\lambda}{2}, f = \frac{\lambda}{2}$$

Centre $\Rightarrow (-g, -f) = \left(-\frac{3\lambda}{2}, -\frac{\lambda}{2}\right)$

$\therefore$ Centre lies on the chord.

$$3x + y + 5 = 0$$

$$3\left(-\frac{3\lambda}{2}\right) + \left(-\frac{\lambda}{2}\right) + 5 = 0$$

$$\frac{-10\lambda}{2} = -5$$

$$-10\lambda = -10$$

$$\boxed{\lambda = 1}$$

required Equation of the circle

$$(x^2 + y^2 - 16) + 1(3x + y + 5) = 0$$

$$x^2 + y^2 + 3x + y - 11 = 0$$

Example : 5.3.

Circle : $x^2 + y^2 - 6x + 4y + c = 0$

Here $2g = -6$ $2f = 4$

$$g = -3 \quad f = 2$$

Centre $(-g, -f) = (3, -2)$, which lies on $x + y - 1 = 0$.

$x + y - 1 = 0$ passes through the centre

$x + y - 1 = 0$ is a diameter of the

circle for all possible value of c .

Example 5.4

Equation of the circle (diameter form)

$$(x-x_1)(x-x_2) + (y-y_1)(y-y_2) = 0$$

$$(x_1, y_1) = (-4, -2), (x_2, y_2) = (1, 1)$$

$$(x+4)(x-1) + (y+2)(y-1) = 0$$

$$x^2 - x + 4x - 4 + y^2 - y + 2y - 2 = 0$$

$$x^2 + y^2 + 3x + y - 6 = 0$$

Example 5.5

Point (2, 3)

$$x^2 + y^2 - 6x - 8y + 12 = (2)^2 + (3)^2 - 6(2) - 8(3) + 12$$

$$= 4 + 9 - 12 - 24 + 12$$

$$= -11 < 0$$

$\therefore (2, 3)$ lies inside the circle.

Example 5.6

$$3x + 4y = 12$$

Intercept $\Rightarrow \frac{x}{4} + \frac{y}{3} = 1$

At (4, 0) A & B (0, 3).

Equation of the circle (diameter form)

$$(x-x_1)(x-x_2) + (y-y_1)(y-y_2) = 0$$

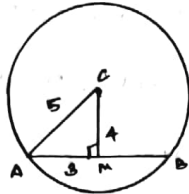
$$(x-4)(x-0) + (y-0)(y-3) = 0$$

$$x^2 + y^2 - 4x - 3y = 0$$

Example 5.7

Centre = (2, 1)

$3x + 4y + 10 = 0$ cuts a chord AB on the circle.



Let M be the midpoint of AB.

$\therefore AM = MB = 3$, [$\because AB = 6$]

$$CM = \frac{|2x + my + n|}{\sqrt{l^2 + m^2}}$$

$$= \frac{|3(2) + 4(1) + 10|}{\sqrt{3^2 + 4^2}}$$

$$= \frac{120}{25}$$

$$= \frac{20}{5}$$

$$CM = 4$$

$$\triangle BMC \quad AC^2 = AM^2 + MC^2$$

$$= 3^2 + 4^2$$

$$AC^2 = 25$$

$$AC = 5 \text{ (radius)}$$

Equation of the circle

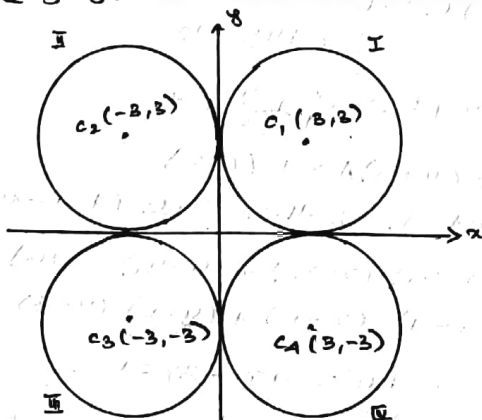
$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x-2)^2 + (y-1)^2 = 5^2$$

$$x^2 - 4x + 4 + y^2 - 2y + 1 = 25$$

$$x^2 + y^2 - 4x - 2y - 20 = 0$$

Example 5.8.



Equation of the circle,

$$(x-h)^2 + (y-k)^2 = r^2$$

i) Centre: (h, k) = (3, 3) C₁

$$(x-3)^2 + (y-3)^2 = 3^2$$

$$x^2 - 6x + 9 + y^2 - 6y + 9 = 9$$

$$x^2 + y^2 - 6x - 6y + 9 = 0$$

ii) Centre: (h, k) = (-3, 3) C₂

$$(x+3)^2 + (y-3)^2 = 3^2$$

$$x^2 + 6x + 9 + y^2 - 6y + 9 = 9$$

$$x^2 + y^2 + 6x - 6y + 9 = 0$$

iii) Centre: (h, k) = (-3, -3) C₃

$$(x+3)^2 + (y+3)^2 = 3^2$$

$$x^2 + 6x + 9 + y^2 + 6y + 9 = 9$$

$$x^2 + y^2 + 6x + 6y + 9 = 0$$

iv) Centre: (h, k) = (3, -3) C₄

$$(x-3)^2 + (y+3)^2 = 3^2$$

$$x^2 - 6x + 9 + y^2 + 6y + 9 = 9$$

$$x^2 + y^2 - 6x + 6y + 9 = 0$$

The required equation of the circle

$$x^2 + y^2 \pm 6x \pm 6y + 9 = 0$$

Example: 5.9

$$\text{circle: } 3x^2 + (a+1)y^2 + 6x - 9y + a + 4 = 0$$

Co-efficient of x^2 = Co-efficient of y^2

$$3 = a + 1$$

$$\boxed{a = 2}$$

Equation of the circle

$$3x^2 + 3y^2 + 6x - 9y + 6 = 0$$

$$x^2 + y^2 + 2x - 3y + 2 = 0$$

Here $2g = 2$, $2f = -3$

$$g = 1, \quad f = -3/2$$

$$\text{Centre} = (-g, -f) = (-1, 3/2)$$

Radius(r)

$$= \sqrt{g^2 + f^2 - c}$$

$$= \sqrt{1 + \frac{9}{4} - 2}$$

$$= \sqrt{5/4} \text{ units}$$

Example 5.10

General Equation of the circle.

$$x^2 + y^2 + 2gx + 2fy + c = 0.$$

It passes through the point (1,1),
(2,-1) and (3,2).

At (1,1) $\Rightarrow 1+1+2g+2f+c=0.$

$$2g+2f+c=-2 \rightarrow \textcircled{1}$$

At (2,-1) $\Rightarrow 4+1+4g-2f+c=0$

$$4g-2f+c=-5 \rightarrow \textcircled{2}$$

At (3,2) $\Rightarrow 9+4+6g+4f+c=0$

$$6g+4f+c=-13 \rightarrow \textcircled{3}$$

$$\textcircled{1}-\textcircled{2} \Rightarrow -2g+4f=3 \rightarrow \textcircled{4}$$

$$\textcircled{2}-\textcircled{3} \Rightarrow 2g+6f=-8 \rightarrow \textcircled{5}$$

$$\textcircled{4}+\textcircled{5} \Rightarrow 10f=-5$$

$$\boxed{f = -1/2}$$

Sub $f = -1/2$ in eq 4.

$$-2g-2=3$$

$$-2g=5$$

$$\boxed{g = -5/2}$$

Sub $f = -1/2$ in eq 1. $\boxed{g = -5/2}$

$$-5-1+c=-2$$

$$-6+c=-2$$

$$\boxed{c=4}$$

Equation of the circle.

$$x^2 + y^2 + 2(-5/2)x + 2(-1/2)y + 4 = 0$$

$$x^2 + y^2 - 5x - y + 4 = 0.$$

Example 5.11

$$x^2 + y^2 = 25 \text{ at } P(-3, 4) = (x_1, y_1)$$

Equation of the tangent

$$xx_1 + yy_1 = 25$$

$$-3x + 4y = 25$$

Equation of the normal.

$$4x + 3y = k.$$

At (-3, 4)

$$4(-3) + 3(4) = k$$

$$-12 + 12 = k$$

$$\boxed{k=0}$$

Equation of the normal is.

$$4x + 3y = 0.$$

Example 5.12.

$$y = mx + c$$

$$y = 4x + c$$

$$\therefore m=4$$

$$x^2 + y^2 = a^2$$

$$x^2 + y^2 = 9$$

$$a^2 = 9,$$

The condition is $c^2 = a^2(1+m^2)$

$$c^2 = 9(1+16)$$

$$c^2 = 9(17)$$

$$c = \pm 3\sqrt{17}.$$

Example 5.13

Let $O_1(12,0)$ and $O_2(34,0)$ be the centres of the semicircle and radius $r=10$.

Equation of the semi-circle.

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x-h)^2 + (y-k)^2 = 100$$

Case (i) centre = $(12,0) = (h,k)$

$$(x-12)^2 + (y-0)^2 = 100$$

$$x^2 + y^2 - 24x + 44 = 0, \quad y > 0$$

Case (ii) centre $(34,0) = (h,k)$

$$(x-34)^2 + (y-0)^2 = 100$$

$$x^2 + y^2 - 68x + 1056 = 0, \quad y > 0$$

EXERCISE 5.1

1. $r=5$, Centre $= (0, \pm 5)$

Equation of the circle

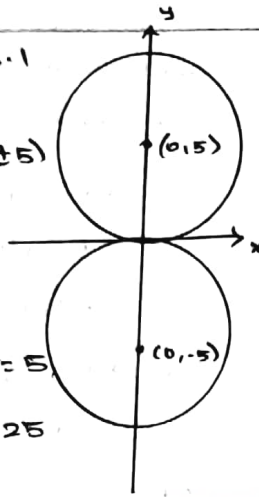
$$(x-h)^2 + (y-k)^2 = r^2$$

Here $(h,k) = (0, \pm 5)$, $r=5$

$$(x-0)^2 + (y \pm 5)^2 = 25$$

$$x^2 + y^2 + 25 \pm 10y = 25$$

$$x^2 + y^2 \pm 10y = 0$$



2. Equation of the circle

$$(x-h)^2 + (y-k)^2 = r^2$$

Here $(h,k) = (2, -1)$

$$(x-2)^2 + (y+1)^2 = r^2$$

At $(3, 6)$

$$(3-2)^2 + (6+1)^2 = r^2$$

$$1 + 49 = r^2$$

$$50 = r^2$$

Equation of the circle

$$(x-2)^2 + (y+1)^2 = 50$$

$$x^2 + y^2 - 4x + 2y - 45 = 0$$

3. Equation of the circle

$$(x-h)^2 + (y-k)^2 = r^2$$

Centre $(h,k) = (-r, -r)$

$$(x+r)^2 + (y+r)^2 = r^2$$

At $(-4, -2)$

$$(-4+r)^2 + (-2+r)^2 = r^2$$

$$16 - 8r + r^2 + 4 - 4r + r^2 = r^2$$

$$r^2 - 12r + 20 = 0$$

$$(r-10)(r-2) = 0$$

$$\therefore r = 10, 2$$

$$\begin{array}{r} 20 \\ -2 \\ \hline -10 \end{array}$$

case (i) $r=10$

$$(x+10)^2 + (y+10)^2 = 10^2$$

$$x^2 + y^2 + 20x + 20y + 100 = 0$$

case (ii) $r=2$

$$(x+2)^2 + (y+2)^2 = 2^2$$

$$x^2 + y^2 + 4x + 4y + 4 = 0$$

4. Centre $(h,k) = (2, 3)$

Equation of the circle

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x-2)^2 + (y-3)^2 = r^2 \rightarrow \textcircled{1}$$

Given lines

$$3x - 2y - 1 = 0 \rightarrow \textcircled{2}$$

$$4x + y - 27 = 0 \rightarrow \textcircled{3}$$

from 2 & 3, $x=5$ & $y=7$

Point of intersection $(5, 7)$

$$\textcircled{1} \Rightarrow (5-2)^2 + (7-3)^2 = r^2$$

$$3^2 + 4^2 = r^2$$

$$25 = r^2$$

$$\textcircled{1} \Rightarrow (x-2)^2 + (y-3)^2 = 25$$

$$x^2 + y^2 - 4x - 6y - 12 = 0$$

5. Equation of the circle (diameter form)

$$(x-x_1)(x-x_2) + (y-y_1)(y-y_2) = 0$$

$$(x-x_1)(x-x_2) + (y-y_1)(y-y_2) = 0$$

Here $(x_1, y_1) = (3, 4)$

$(x_2, y_2) = (2, -7)$

$$(x-3)(x-2) + (y-4)(y+7) = 0$$

$$x^2 - 2x - 3x + 6 + y^2 + 7y - 4y - 28 = 0$$

$$x^2 + y^2 - 5x + 3y - 22 = 0$$

6.

From the diagram.

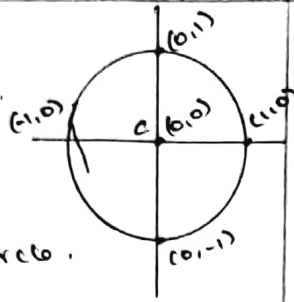
Centre: $(0,0)$ radius: $r=1$

Equation of the circle.

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x-0)^2 + (y-0)^2 = 1^2$$

$$x^2 + y^2 = 1.$$



$$\text{At } (x_1, y_1) = (2, 2)$$

$$2x + 2y - 3(x+2) + 3(y+2) - 8 = 0$$

$$2x + 2y - 3x - 6 + 3y + 6 - 8 = 0$$

$$-x + 5y - 8 = 0$$

$$x - 5y + 8 = 0$$

Equation of normal,

$$5x + y + k = 0$$

$$\text{At } (2, 2)$$

$$10 + 2 + k = 0$$

$$\boxed{k = -12}$$

Equation of normal is $5x + y - 12 = 0$.7. Area of the circle = 9π

$$\pi r^2 = 9\pi$$

$$\boxed{r^2 = 9}$$

Two diameters are $x+y=5$ and

$$x-y=1.$$

From the equation $x=3$ & $y=2$.Centre: $(h, k) = (3, 2)$.

Equation of the circle.

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x-3)^2 + (y-2)^2 = 9$$

$$x^2 + y^2 - 6x - 4y + 4 = 0.$$

$$8. \quad y = mx + c. \quad x^2 + y^2 = a^2$$

$$y = 2\sqrt{3}x + c \quad x^2 + y^2 = 16$$

$$\therefore m = 2\sqrt{3} \quad \therefore a^2 = 16$$

$$\text{Condition: } c^2 = a^2(1+m^2)$$

$$= 16(1+8)$$

$$= 16(9)$$

$$c^2 = 144$$

$$\boxed{c = \pm 12}$$

$$9. \quad x^2 + y^2 - 6x + 4y - 8 = 0.$$

Equation of the tangent.

$$xx_1 + yy_1 - 6\left(\frac{x+x_1}{2}\right) + 4\left(\frac{y+y_1}{2}\right) - 8 = 0$$

$$xx_1 + yy_1 - 3(x+x_1) + 2(y+y_1) - 8 = 0$$

10.

$$x^2 + y^2 - 5x + 2y - 5 = 0 \rightarrow \text{①}$$

$$\text{At } (-2, 1)$$

$$\text{①} \Rightarrow 4 + 1 + 10 + 2 - 5 = 12 > 0.$$

 $(-2, 1)$ lies outside the circle.

$$\text{At } (0, 0)$$

$$\text{①} \Rightarrow 0 + 0 - 0 + 0 - 5 = -5 < 0.$$

 $(0, 0)$ lies inside the circle.

$$\text{At } (-4, -3)$$

$$\text{①} \Rightarrow 16 + 9 + 20 - 6 - 5 = 34 > 0.$$

 $(-4, -3)$ lies outside the circle.

11.

Equation of the circle

$$(x-h)^2 + (y-k)^2 = r^2.$$

$$(i) \quad x^2 + (y+2)^2 = 0.$$

$$\text{Centre } (-g, -f) = (0, -2)$$

$$\text{radius } r = 0.$$

$$(ii) \quad x^2 + y^2 + 2gx + 2fy + c = 0$$

$$x^2 + y^2 + 6x - 4y + 4 = 0$$

$$2g = 6$$

$$g = 3$$

$$2f = -4$$

$$f = -2.$$

$$\text{Centre } (-g, -f) = (-3, 2)$$

$$\text{radius } r = \sqrt{g^2 + f^2 - c}$$

$$= \sqrt{9 + 4 - 4}$$

$$= \sqrt{9}$$

$$\boxed{r=3}$$

$$\text{iii) } x^2 + y^2 - x + 2y - 3 = 0.$$

$$\begin{array}{c|c|c} 2g = -1 & 2f = 2 & c = -3 \\ \hline g = -1/2 & f = 1 & \end{array}$$

$$\text{Centre } (-g, -f) = (1/2, -1)$$

$$\text{radius } r = \sqrt{g^2 + f^2 - c}$$

$$= \sqrt{\frac{1}{4} + 1 + 3}$$

$$= \sqrt{\frac{1+4+12}{4}}$$

$$= \sqrt{\frac{17}{4}}$$

$$\boxed{r = \sqrt{17}/2}$$

$$\text{iv) } 2x^2 + 2y^2 - 6x + 4y + 2 = 0.$$

$$x^2 + y^2 - 3x + 2y + 1 = 0$$

$$\begin{array}{c|c|c} 2g = -3 & 2f = 2 & c = 1 \\ \hline g = -3/2 & f = 1 & \end{array}$$

$$\text{Centre } (-g, -f) = (3/2, -1)$$

$$\text{radius } r = \sqrt{g^2 + f^2 - c}$$

$$= \sqrt{\frac{9}{4} + 1 - 1}$$

$$= \sqrt{9/4}$$

$$\boxed{r = \frac{3}{2}}$$

12.

$$3x^2 + (3-p)xy + 9y^2 - 2px = 8pq$$

$$\text{Co-efficient } x^2 = \text{Co-efficient } y^2$$

$$3 = 9$$

$$\therefore \boxed{p=3}$$

Again, Co-efficient $xy = 0$,

$$3-p=0$$

$$\boxed{p=3}$$

$$\textcircled{1} \quad 3x^2 + (3-3)xy + 3y^2 - 2(3)x = 8(3)(3)$$

$$3x^2 + 3y^2 - 6x - 72 = 0$$

$$\div 3$$

$$x^2 + y^2 - 2x - 24 = 0$$

$$2g = -2 \quad 2f = 0 \quad c = -24$$

$$g = -1 \quad f = 0$$

$$\text{Centre } (-g, -f) = (1, 0)$$

$$\text{radius } r = \sqrt{g^2 + f^2 - c}$$

$$= \sqrt{1 + 0 + 24}$$

$$= \sqrt{25}$$

$$r = 5 \text{ units.}$$

CONICS:

A conic is the locus of a point which moves in a plane, so that its distance from a fixed point bears a constant ratio to its distance from a fixed line not containing the fixed point.

fixed point $\rightarrow$ focus

fixed line $\rightarrow$ directrix

constant ratio $\rightarrow$ eccentricity.

$e = 1$, parabola $\Rightarrow B^2 - 4AC = 0$

$e < 1$, ellipse $\Rightarrow B^2 - 4AC < 0$

$e > 1$, hyperbola, $B^2 - 4AC > 0$

General second grade equation,

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

ELLIPSE

$a^2 > b^2$

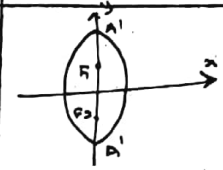
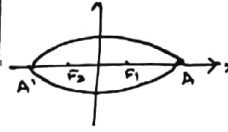
$b^2 > a^2$

Equation

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

Diagram



Centre

$(0,0)$

$(0,0)$

Focus

$(\pm ae, 0)$

$(0, \pm ae)$

vertices

$(\pm a, 0)$

$(0, \pm a)$

eccentricity

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

Directrix

$x = \pm \frac{a}{e}$

$y = \pm \frac{a}{e}$

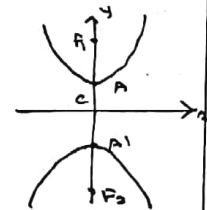
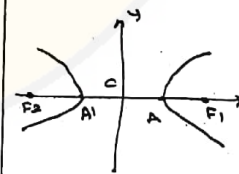
Hyperbola

Equation

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

Diagram



Focus

$(\pm ae, 0)$

$(0, \pm ae)$

vertices

$(\pm a, 0)$

$(0, \pm a)$

eccentricity

$$e = \sqrt{1 + \frac{b^2}{a^2}}$$

$$e = \sqrt{1 + \frac{b^2}{a^2}}$$

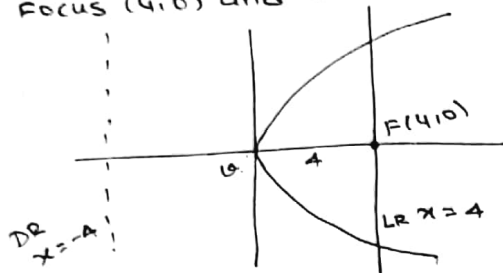
Directrix

$x = \pm \frac{a}{e}$

$y = \pm \frac{a}{e}$

Equation	Graph	vertex	focus	Axis	Equation Directrix	Length of the Latus rectum
$y^2 = 4ax$		(0,0)	(a,0)	y=0	x=-a	4a
$y^2 = -4ax$		(0,0)	(-a,0)	y=0	x=a	4a
$x^2 = 4ay$		(0,0)	(0,a)	x=0	y=-a	4a
$x^2 = -4ay$		(0,0)	(0,-a)	x=0	y=a	4a

EXE 5.2 (i) (i)

Focus $(4, 0)$ and directrix $x = -4$ 

The parabola open rightward.

$$(y-k)^2 = 4a(x-h)$$

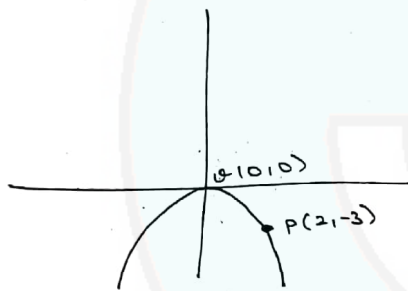
$$(h, k) = (0, 0)$$

$$(a, 0) = (4, 0)$$

$$(y-0)^2 = 4(4)(x-0)$$

$$y^2 = 16x$$

EXE 5.2 (i) (ii)



The parabola open downward.

$$(x-h)^2 = -4a(y-k)$$

$$V(h, k) = (0, 0)$$

$$(x, y) = (2, -3)$$

$$(2-0)^2 = -4a(-3-0)$$

$$4 = 4a(3)$$

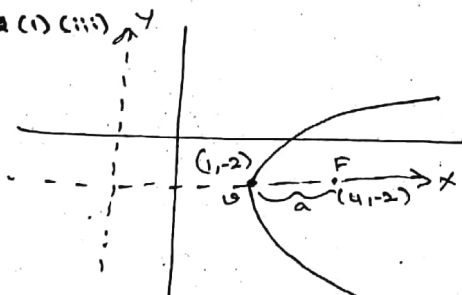
$$a = \frac{1}{3}$$

$$\text{Equation. } (x-0)^2 = -4\left(\frac{1}{3}\right)(y-0)$$

$$x^2 = -\frac{4}{3}y$$

$$3x^2 = -4y$$

EXE 5.2 (i) (iii)



The parabola open rightward.

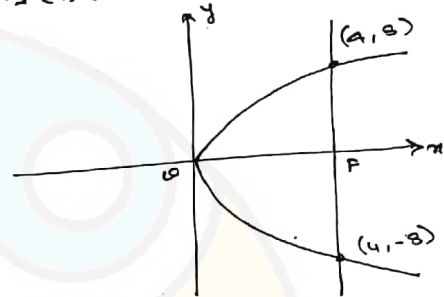
$$(y-k)^2 = 4a(x-h)$$

$$\therefore a = 3, V(h, k) = (1, -2)$$

$$(y+2)^2 = 4(3)(x-1)$$

$$(y+2)^2 = 12(x-1)$$

EXE 5.2 (i) (iv)



The parabola open rightward.

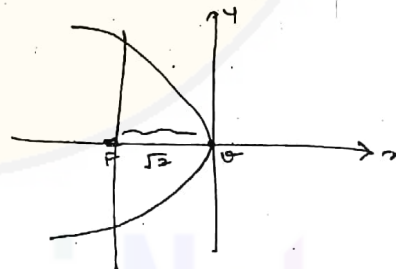
$$(y-k)^2 = 4a(x-h)$$

$$V(h, k) = (0, 0), \boxed{a = 4} \therefore 4a = 8$$

$$(y-0)^2 = 4(4)(x-0)$$

$$y^2 = 16x$$

EXAMPLE 5.14.



The parabola open leftward.

Symmetric as x -axis.

$$F(a, 0) = (-\sqrt{2}, 0), \therefore a = \sqrt{2}$$

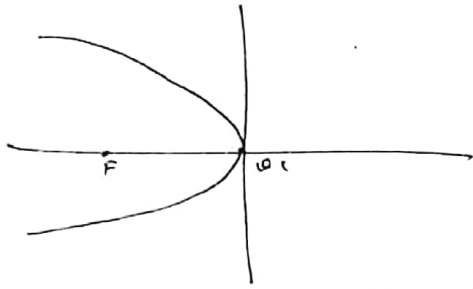
$$V(h, k) = (0, 0)$$

$$(y-k)^2 = -4a(x-h)$$

$$(y-0)^2 = -4(\sqrt{2})(x-0)$$

$$y^2 = -4\sqrt{2}x$$

EXAMPLE : 5.15



The parabola open leftward

$$(y-k)^2 = -4a(x-h)$$

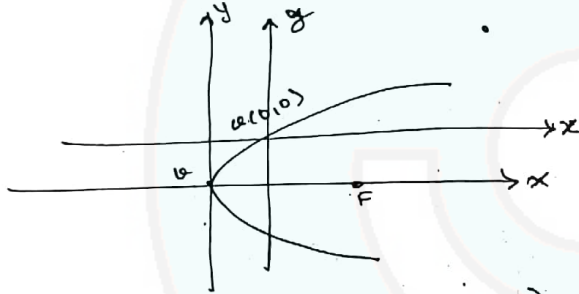
$$V(h,k) = (5,-2), F(-a,0) = (2,-2)$$

$$a=3$$

$$(y+2)^2 = -4(3)(x-5)$$

$$(y+2)^2 = -12(x-5)$$

EXAMPLE : 5.16



The parabola open ward (up).

$$(x-h)^2 = 4a(y-k)$$

$$V(h,k) = (-1,-2), \text{ passes } (3,6)$$

$$(3+1)^2 = 4a(6+2)$$

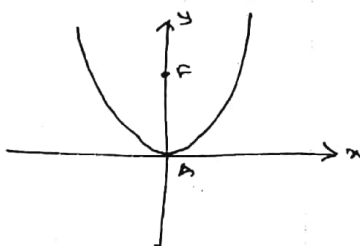
$$16 = 4a(8)$$

$$a = 1/2$$

$$(x+1)^2 = 4(1/2)(y+2)$$

$$(x+1)^2 = 2(y+2)$$

EXAMPLE : 5.17



The parabola $x^2 - 4x - 5y - 1 = 0$

$$x^2 - 4x - 1 + 5 = 5y + 5$$

$$x^2 - 4x + 4 = 5y + 5$$

$$(x-2)^2 = 5(y+1)$$

$4a=5$, length of latus rectum is 5

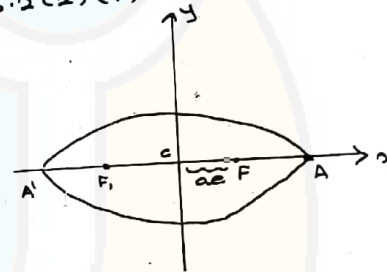
$$a = 5/4$$

$$\text{Focus } (0,a), x-2=0 \quad \left| \begin{array}{l} y+1 = 5/4 \\ y = 1/4 \end{array} \right.$$

$$\text{Focus } (0,a) = (2, 1/4)$$

$$\text{vertices } (h,k) = (2, -1)$$

EXE 5.2 (2) (i)



The Equation of the ellipses.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$ae = 3, e = 1/2$$

$$a(1/2) = 3$$

$$a = 6$$

$$\therefore b^2 = a^2 - (ae)^2$$

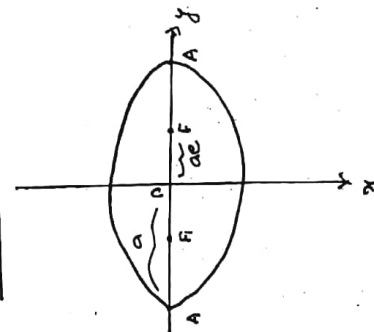
$$= 36 - 9$$

$$b^2 = 27$$

$$\text{Equation } \frac{x^2}{36} + \frac{y^2}{27} = 1$$

EXE 5.2 (2) (ii)

$$\begin{aligned} C &= (0,0) \\ F &= (0,3) \\ F' &= (0,-3) \\ A &= (0,4) \\ A' &= (0,-4) \end{aligned}$$



$$ae = 4, a = 5$$

$$5e = 4$$

$$e = 4/5$$

$$\therefore b^2 = a^2 - (ae)^2$$

$$= 25 - 16$$

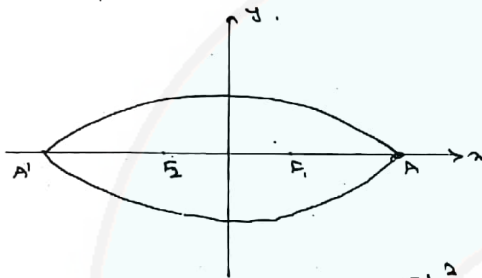
$$b^2 = 9$$

Equation of the ellipses.

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1.$$

$$\frac{x^2}{9} + \frac{y^2}{25} = 1.$$

ExE (5.2) (2) (iii).



Length of the Latus rectum $\frac{2b^2}{a} = 8$

$$2b^2 = 8a$$

$$b^2 = 4a$$

$$e = 3/5$$

$$\therefore b^2 = a^2 - (ae)^2$$

$$4a = a^2 [1 - e^2]$$

$$4 = a [1 - 9/25]$$

$$4 = a \left[\frac{16}{25} \right]$$

$$\boxed{a = \frac{25}{4}}$$

$$\therefore b^2 = 4a$$

$$b^2 = 4 \left(\frac{25}{4} \right) = 25$$

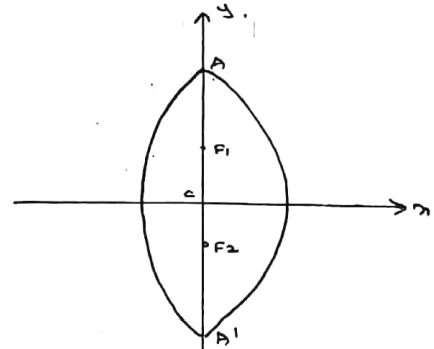
Equation of the ellipses

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

$$\frac{x^2}{\left(\frac{625}{16} \right)} + \frac{y^2}{25} = 1.$$

$$\frac{16x^2}{625} + \frac{y^2}{25} = 1.$$

ExE 5.2 (2) (iv).



Equation of the ellipse $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

Length of the latus rectum $\frac{2b^2}{a} = 4$

$$2b^2 = 4a$$

$$\boxed{b^2 = 2a}$$

distance between foci

$$2ae = 4\sqrt{2}$$

$$ae = 2\sqrt{2}$$

$$\therefore b^2 = a^2 - (ae)^2$$

$$2a = a^2 [1 - e^2]$$

$$2a = a^2 - (2\sqrt{2})^2$$

$$2a = a^2 - 8$$

$$a^2 - 2a - 8 = 0.$$

$$(a-4)(a+2) = 0$$

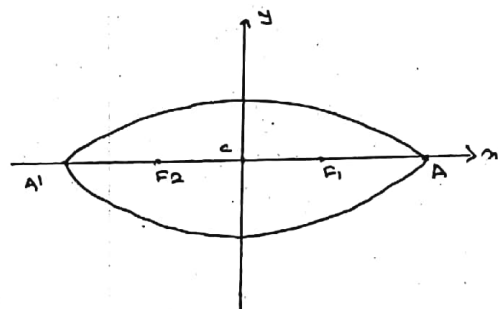
$$\therefore a = 4 \text{ and } a = -2 \text{ (not poss. b/c)}$$

$$b^2 = 2(4) = 8.$$

Equation of the ellipses.

$$\frac{x^2}{8} + \frac{y^2}{16} = 1.$$

EXAMPLE 5.18.



FOCI: $ae = 2$, vertices $a = 3$.

$$b^2 = a^2 - (ae)^2$$

$$= 9 - 4$$

$$b^2 = 5$$

Equation of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{9} + \frac{y^2}{5} = 1$$

EXAMPLE: 5.19.

$$\frac{FM}{PM} = e$$

$$FM^2 = e^2 PM^2$$

$$(x - ae)^2 + (y - 0)^2 = e^2 \left[\left(x - \frac{ae}{e} \right)^2 + 0 \right]$$

FOCI: $(ae, 0) = (2, 3)$, $e = \frac{1}{2}$.

directrix $x = \frac{a}{e} = 7$.

$$(x - 2)^2 + (y - 3)^2 = \left(\frac{1}{2} \right)^2 [(x - 7)^2 - 0]$$

$$(x - 2)^2 + (y - 3)^2 = \frac{1}{4} (x - 7)^2$$

$$x^2 - 4x + 4 + y^2 - 6y + 9 = \frac{1}{4} (x^2 - 14x + 49)$$

$$4x^2 + 4y^2 - 16x - 24y + 52 = x^2 - 14x + 49$$

$$3x^2 + 4y^2 - 2x - 24y + 3 = 0$$

$$3\left(x - \frac{1}{3}\right)^2 + 4(y - 3)^2 = \frac{1}{3} + 36 - 3$$

$$3\left(x - \frac{1}{3}\right)^2 + 4(y - 3)^2 = \frac{100}{3}$$

$$\frac{\left(x - \frac{1}{3}\right)^2}{\frac{100}{9}} + \frac{4(y - 3)^2}{\frac{100}{3}} = 1$$

$$\frac{\left(x - \frac{1}{3}\right)^2}{100/9} + \frac{(y - 3)^2}{100/12} = 1$$

length of the major axis $= 2a \Rightarrow 2\sqrt{\frac{100}{9}}$

length of the minor axis $= 2b \Rightarrow 2\sqrt{\frac{100}{12}}$

EXAMPLE 8.20.

$$4x^2 + y^2 + 24x - 2y + 21 = 0$$

$$4[x^2 + 6x] + 1[y^2 - 2y] = -21$$

$$4[(x+3)^2 - 9] + 1[(y-1)^2 - 1] = -21$$

$$4(x+3)^2 + 1(y-1)^2 = -21 + 36 + 1$$

$$4(x+3)^2 + (y-1)^2 = 16$$

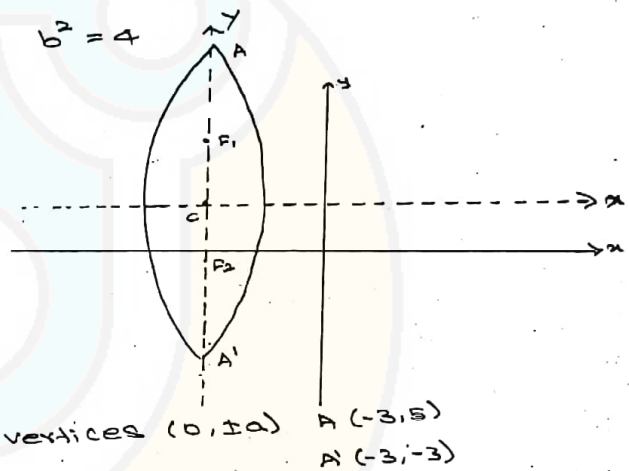
$$4 \frac{(x+3)^2}{16} + \frac{(y-1)^2}{16} = 1$$

$$\frac{(x+3)^2}{4} + \frac{(y-1)^2}{16} = 1$$

$(h, k) = (-3, 1)$ centre

$$a^2 = 16$$

$$b^2 = 4$$



vertices $(0, \pm a)$ $A(0, 5)$
 $A'(-3, -3)$

$$b^2 = a^2 - (ae)^2$$

$$4 = 16 - 16e^2$$

$$-12 = -16e^2$$

$$\frac{3}{4} = e^2$$

$$e = \frac{\sqrt{3}}{2}$$

FOCI: $(0, \pm ae)$, $ae = 4\left(\frac{\sqrt{3}}{2}\right) = 2\sqrt{3}$

$$x + 3 = 0, \quad y - 1 = \pm 2\sqrt{3}$$

$$x = -3, \quad y = \pm 2\sqrt{3} + 1$$

FOCI: $(0, \pm ae) = (-3, \pm 2\sqrt{3} + 1)$

EXAMPLE 5.20

$$4x^2 + 36y^2 + 40x - 288y + 532 = 0$$

$$4[x^2 + 10x] + 36[y^2 - 8y] = -532$$

$$4[(x+5)^2 - 25] + 36[(y-4)^2 - 16] = -532$$

$$4(x+5)^2 + 36(y-4)^2 = -532 + 100 + 576$$

$$4(x+5)^2 + 36(y-4)^2 = 144$$

$$\frac{(x+5)^2}{36} + \frac{(y-4)^2}{4} = 1$$

$$\text{Centre } (-5, 4) = (h, k)$$

$$a^2 = 36, \quad b^2 = 4$$

$$a = 6, \quad b = 2$$

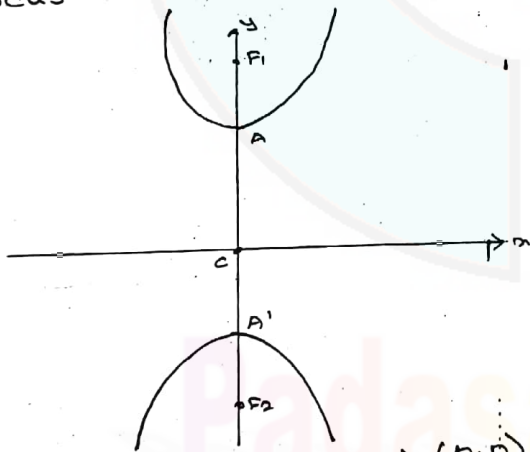
Length of the major axis $2a = 12$

Length of the minor axis $2b = 4$

EXAMPLE 5.22.

$$\text{vertices } (0, \pm a) = (0, \pm 4)$$

$$\text{Focus } (0, \pm ae) = (0, \pm 6)$$



The midpoint of the foci $(0, 0)$

$$2a = 8 \quad 2ae = 12$$

$$a = 4 \quad ae = 6$$

$$b^2 = (ae)^2 - a^2$$

$$= 36 - (4)^2 = 36 - 16$$

$$b^2 = 20$$

Equation of the hyperbola

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

$$\frac{y^2}{16} - \frac{x^2}{20} = 1$$

EXAMPLE 5.23.

$$9x^2 - 16y^2 = 144$$

$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

$$a^2 = 16 \Rightarrow a = \pm 4$$

$$b^2 = 9 \Rightarrow b = \pm 3$$

$$\text{vertices } (\pm a, 0) = (\pm 4, 0)$$

$$b^2 = (ae)^2 - a^2$$

$$(ae)^2 = a^2 + b^2$$

$$= 16 + 9$$

$$(ae)^2 = 25$$

$$ae = \pm 5$$

$$\text{Focus } (\pm ae, 0) = (\pm 5, 0)$$

EXAMPLE 5.24.

$$11x^2 - 25y^2 - 44x + 50y - 256 = 0$$

$$11[x^2 - 4x] - 25[y^2 - 2y] = 256$$

$$11[(x-2)^2 - 4] - 25[(y-1)^2 - 1] = 256$$

$$11(x-2)^2 - 25(y-1)^2 = 256 + 44 - 25$$

$$11(x-2)^2 - 25(y-1)^2 = 275$$

$$\frac{(x-2)^2}{25} - \frac{(y-1)^2}{11} = 1$$

$$a^2 = 25, \quad a = \pm 5$$

$$b^2 = 11, \quad b = \pm \sqrt{11}$$

$$\therefore b^2 = (ae)^2 - a^2$$

$$(ae)^2 = a^2 + b^2 = 25 + 11 = 36$$

$$ae = \pm 6$$

$$e = \frac{6}{5}$$

$$\text{Focus } (\pm ae, 0) = (\pm 6, 0)$$

$$x-2 = \pm 6 \quad y-1 = 0$$

$$x = \pm 6 + 2 \quad y = 1$$

$$\text{Focus } (\pm 6+2, 1)$$

$$(8, 1) \quad (-4, 1)$$

EXAMPLE : 5.25.

Astronomical unit long $2a = 36.18$
 wide (or) short $2b = 9.12$

$$b^2 = (ae)^2 - a^2$$

$$a^2 + b^2 = a^2 e^2$$

$$e^2 = \frac{a^2 + b^2}{a^2}$$

$$e = \frac{\sqrt{a^2 + b^2}}{a}$$

$$= \frac{\sqrt{(18.09)^2 - (4.56)^2}}{18.09}$$

$$e \approx 0.97$$

EXE 5.2 (3) (i)

$$\text{Foci } (\pm ae, 0) = (\pm 2, 0)$$

$$ae = 2$$

$$e = 3/2$$

$$a(3/2) = 2$$

$$a = \frac{4}{3}$$

$$\therefore b^2 = (ae)^2 - a^2$$

$$= 4 - \frac{16}{9}$$

$$= \frac{36 - 16}{9}$$

$$b^2 = \frac{20}{9}$$

Equation of the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{16/9} - \frac{y^2}{20/9} = 1$$

$$\frac{9x^2}{16} - \frac{9y^2}{20} = 1$$

EXE 5.2 (3) (ii)

Centre $(h, k) = (2, 1)$, Foci $(8, 1)$

The distance between centre & focus

$$ae = \sqrt{(8-2)^2 + (1-1)^2}$$

$$= \sqrt{6^2}$$

$$ae = 6$$

Corresponding directrix $x = 4$

$$(4, 1)$$

The distance between centre & directrix

$$\frac{a}{e} = \sqrt{(4-2)^2 + (1-1)^2}$$

$$= \sqrt{2^2}$$

$$\frac{a}{e} = 2$$

$$\rightarrow (ae) \left(\frac{a}{e} \right) = 6 \times 2$$

$$a^2 = 12$$

$$(ae)^2 = 36$$

$$b^2 = (ae)^2 - a^2$$

$$= 36 - 12$$

$$b^2 = 24$$

Equation of the hyperbola

$$\frac{(x-2)^2}{a^2} - \frac{(y-1)^2}{b^2} = 1$$

$$\frac{(x-2)^2}{12} - \frac{(y-1)^2}{24} = 1$$

EXE 5.2 (3) (iii)

passing through $(5, 2) = (x, y)$ length of transverse axis, long x-axis
length 8 units.

$$2a = 8$$

$$a = 4$$

Centre $(h, k) = (0, 0)$

Equation of the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

At (5, -2) $a = 4$

$$\frac{25}{16} - \frac{4}{b^2} = 1$$

$$\frac{25}{16} - 1 = \frac{4}{b^2}$$

$$\frac{25-16}{16} = \frac{4}{b^2}$$

$$\frac{9}{16} = \frac{4}{b^2}$$

$$\frac{b^2}{4} = \frac{16}{9}$$

$$b^2 = \frac{64}{9}$$

Equation of the hyperbola.

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{16} - \frac{y^2}{64/9} = 1$$

$$\frac{x^2}{16} - \frac{9y^2}{64} = 1$$

EXE 5.2 (4).

$$(i) y^2 = 16x \Rightarrow y^2 = 4ax$$

$$y^2 = 4(4)x$$

$$a = 4$$

The parabola open rightward

vertices (0, 0)

Focus (a, 0) = (4, 0)

Latus rectum $x = a \Rightarrow x = 4$

Directrix $x = -a \Rightarrow x = -4$

length of L.R. $4a = 4(4) = 16$

$$(ii) x^2 = 24y \quad x^2 = 4ay$$

$$x^2 = 4(6)y \quad a = 6$$

The parabola open upward
vertices (0, 0) = (h, k)

focus (0, a) = (0, 6)

Latus rectum $y = a \Rightarrow y = 6$

Directrix $y = -a \Rightarrow y = -6$

length of L.R. $4a = 4(6) = 24$

$$(iii) y^2 = -8x \quad y^2 = -4ax$$

$$y^2 = -4(2)x \quad a = 2$$

The parabola open leftward

vertices (0, 0) = (h, k)

focus (-a, 0) = (-2, 0)

Latus rectum $x = -a \Rightarrow x = -2$

Directrix $x = a \Rightarrow x = 2$

length of the Latus rectum
 $4a = 4(2) = 8$

$$iv) x^2 - 2x + 8y + 17 = 0$$

$$x^2 - 2x + 17 = -8y$$

$$x^2 - 2x + 1 = -8y - 16$$

$$(x-1)^2 = -8(y+2)$$

The parabola open downward

Centre (h, k) = (1, -2)

Vertices (0, h, -a+k) = (1, -4)

Latus rectum $y = -a$
 $y+2 = -2$
 $y = -4$

Directrix $y = a$
 $y+2 = +2$
 $y = 0$

length of the latus rectum
 $4a = 4(2) = 8$

$$(iv) y^2 - 4y - 8x + 12 = 0$$

$$y^2 - 4y + 4 = +8x - 8$$

$$(y-2)^2 = +8(x-1)$$

$$y^2 = +4ax \therefore a=2$$

The parabola open ^{Right} leftward

$$\text{Vertices } (h, k) = (1, 2)$$

$$\text{Focus } (h+a, 0+k) = (3, 2)$$

$$\text{Latus rectum } x = +a$$

$$x-1 = +2$$

$$x = +1+2$$

$$\boxed{x=3}$$

$$\text{Directrix } x = -a$$

$$x-1 = -2$$

$$\boxed{x=-1}$$

Length of the latus rectum.

$$4a = 4(2) = 8.$$

EXERCISE 2 (5).

$$(i) \frac{x^2}{25} + \frac{y^2}{9} = 1$$

$$a^2 = 25 \Rightarrow a = \pm 5$$

$$b^2 = 9 \Rightarrow b = \pm 3.$$

$$(ae)^2 = a^2 - b^2 = 25 - 9 = 16$$

$$\boxed{ae=4}$$

$$\text{Centre } (h, k) = (0, 0)$$

$$\text{Focus } (h \pm ae, k) = (\pm 4, 0).$$

$$\text{Vertices } (h \pm a, k) = (\pm 5, 0).$$

$$\text{Directrix } x = \pm \frac{a}{e}$$

$$e = \sqrt{\frac{a^2 - b^2}{a^2}} = \sqrt{\frac{25 - 9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

$$x = \pm \frac{5}{(4/5)} = \pm \frac{25}{4}$$

$$(ii) \frac{x^2}{3} + \frac{y^2}{10} = 1.$$

$$a^2 = 10, b^2 = 3.$$

$$(ae)^2 = a^2 - b^2 = 10 - 3 = 7$$

$$ae = \sqrt{7}$$

$$\text{Centre } (h, k) = (0, 0).$$

$$\text{Vertices } (h, \pm a) = (0, \pm \sqrt{10}).$$

$$\text{Focus } (h, \pm ae) = (0, \pm \sqrt{7})$$

$$e = \sqrt{\frac{a^2 - b^2}{a^2}} = \sqrt{\frac{10 - 3}{10}} = \frac{\sqrt{7}}{\sqrt{10}}$$

$$\text{Directrix } x = \pm \frac{a}{e} = \pm \frac{\sqrt{10}}{(\sqrt{7}/\sqrt{10})}$$

$$y = \pm \frac{10}{\sqrt{7}}$$

$$(iii) \frac{x^2}{25} - \frac{y^2}{144} = 1.$$

$$a^2 = 25, b^2 = 144.$$

$$(ae)^2 = a^2 + b^2 = 25 + 144 = 169$$

$$ae = 13.$$

$$\text{Centre } (h, k) = (0, 0)$$

$$\text{Focus } (h \pm ae, k) = (\pm 13, 0).$$

$$\text{Vertices } (h \pm a, k) = (\pm 5, 0).$$

$$e = \sqrt{\frac{a^2 + b^2}{a^2}} = \frac{13}{5}$$

$$\text{Directrix } x = \pm \frac{a}{e} = \pm \frac{5}{(13/5)}$$

$$x = \pm \frac{25}{13}.$$

$$(iv) \frac{y^2}{16} - \frac{x^2}{9} = 1$$

$$a^2 = 16, b^2 = 9.$$

$$(ae)^2 = a^2 + b^2 = 16 + 9 = 25$$

$$\boxed{ae=5}$$

Centre $(h, k) = (0, 0)$
 Focus $(h, k \pm ae) = (0, \pm 5)$
 Vertices $(h, k \pm a) = (0, \pm 4)$

$$e = \sqrt{\frac{a^2 + b^2}{a^2}} = \frac{5}{4}$$

$$\text{Directrix } y = \pm \frac{a}{e} = \frac{(4)}{(5/4)}$$

$$y = \pm \frac{16}{5}$$

EXE 5.2 (8)

$$(i) \frac{(x-3)^2}{225} + \frac{(y-4)^2}{289} = 1$$

$$b^2 = 225 \quad a^2 = 289$$

$$(ae)^2 = a^2 - b^2 = 289 - 225$$

$$(ae)^2 = 64$$

$$\boxed{ae = 8}$$

Centre $(h, k) = (3, 4)$

Focus $(h, k \pm ae) = (3, 4 \pm 8)$

$F_1(3, 12) \quad F_2(3, -4)$

Vertices $(h, k \pm a) = (3, 4 \pm 17)$

$A(3, 21) \quad A'(3, -13)$

$$e = \sqrt{\frac{a^2 - b^2}{a^2}} = \frac{8}{17}$$

$$\text{Directrix } y - 4 = \pm \frac{a}{e}$$

$$y - 4 = \pm \frac{17}{(8/17)}$$

$$y - 4 = \pm \frac{289}{8}$$

$$y = 4 \pm \frac{289}{8}$$

$$(ii) \frac{(x+1)^2}{100} + \frac{(y-2)^2}{64} = 1$$

$$a^2 = 100, \quad b^2 = 64$$

$$(ae)^2 = a^2 - b^2 = 100 - 64 = 36$$

$$\boxed{ae = 6}$$

Centre $(h, k) = (-1, 2)$

Focus $(h \pm ae, k) = (-1 \pm 6, 2)$

$F_1(5, 2) \quad F_2(-7, 2)$

Vertices $(h \pm a, k) = (-1 \pm 10, 2)$

$A(9, 2) \quad A'(-11, 2)$

$$e = \sqrt{\frac{a^2 - b^2}{a^2}} = \frac{6}{10} = \frac{3}{5}$$

Equation of Directrices $x = \pm \frac{a}{e}$

$$x + 1 = \pm \frac{a}{e}$$

$$x + 1 = \pm \frac{10}{(3/5)}$$

$$x + 1 = \pm \frac{50}{3}$$

$$x = -1 \pm \frac{50}{3}$$

$$(iii) \frac{(x+3)^2}{225} - \frac{(y-4)^2}{64} = 1$$

$$a^2 = 225, \quad b^2 = 64$$

$$(ae)^2 = a^2 + b^2 = 225 + 64 = 289$$

$$\boxed{ae = 17}$$

Centre $(h, k) = (-3, 4)$

Focus $(h \pm ae, k) = (-3 \pm 17, 4)$

$F_1(14, 4) \quad F_2(-20, 4)$

Vertices $(h \pm a, k) = (-3 \pm 15, 4)$

$A(12, 4) \quad A'(-18, 4)$

$$\text{Directrix } x = \pm \frac{a}{e} = \frac{15}{17/15} = \frac{225}{17}$$

$$x + 3 = \pm \frac{225}{17}$$

$$x = -3 \pm \frac{225}{17}$$

$$iv) \frac{(y-2)^2}{25} - \frac{(x+1)^2}{16} = 1$$

$$a^2 = 25, \quad b^2 = 16$$

$$(ae)^2 = a^2 + b^2 = 25 + 16 = 41$$

$$\boxed{ae = \sqrt{41}} \quad e = \frac{\sqrt{41}}{5}$$

$$\text{centre } (h, k) = (-1, 2)$$

$$\text{Focus } (h, k \pm ae) = (-1, 2 \pm \sqrt{41})$$

$$F_1(-1, 2 + \sqrt{41}) \text{ \& } F_2(-1, 2 - \sqrt{41})$$

$$\text{Vertices } (h, k \pm a) = (-1, 2 \pm 5)$$

$$V \Rightarrow A(-1, 7) \text{ \& } A'(-1, -3)$$

$$\text{Directrices } y = \pm \frac{a}{e}$$

$$y - 2 = \pm \frac{\sqrt{25}}{\sqrt{41}/5}$$

$$y - 2 = \pm \frac{25}{\sqrt{41}}$$

$$y = 2 \pm \frac{25}{\sqrt{41}}$$

$$v) 18x^2 + 12y^2 - 144x + 48y + 120 = 0$$

$$18[x^2 - 8x] + 12[y^2 + 4y] = -120$$

$$18[(x-4)^2 - 16] + 12[(y+2)^2 - 4] = -120$$

$$18(x-4)^2 + 12(y+2)^2 = -120 + 288 + 48$$

$$18(x-4)^2 + 12(y+2)^2 = 216$$

$$\frac{(x-4)^2}{12} + \frac{(y+2)^2}{18} = 1$$

$$a^2 = 18, \quad b^2 = 12$$

$$(ae)^2 = a^2 - b^2 = 18 - 12 = 6$$

$$ae = \sqrt{6}$$

$$\text{centre } (h, k) = (4, -2)$$

$$\text{Focus } (h, k \pm ae) = (4, -2 \pm \sqrt{6})$$

$$F_1(4, -2 + \sqrt{6}) \text{ \& } F_2(4, -2 - \sqrt{6})$$

$$\text{Vertices } (h, k \pm a) = (4, -2 \pm \sqrt{18})$$

$$A(4, -2 + \sqrt{18}) \text{ \& } A'(4, -2 - \sqrt{18})$$

$$e = \frac{ae}{a} = \frac{\sqrt{6}}{\sqrt{18}} = \frac{1}{\sqrt{3}}$$

$$\text{Directrix } y = \pm \frac{a}{e}$$

$$y + 2 = \pm \frac{\sqrt{18}}{1/\sqrt{3}}$$

$$y + 2 = \pm \sqrt{54}$$

$$y = -2 \pm \sqrt{54}$$

$$vi) 9x^2 - y^2 - 36x - 6y + 18 = 0$$

$$9[x^2 - 4x] - [y^2 + 6y] = -18$$

$$9[(x-2)^2 - 4] - [(y+3)^2 - 9] = -18$$

$$9(x-2)^2 - (y+3)^2 = -18 + 36 - 9$$

$$9(x-2)^2 - (y+3)^2 = 9$$

$$\frac{(x-2)^2}{1} - \frac{(y+3)^2}{9} = 1$$

$$a^2 = 1, \quad b^2 = 9$$

$$(ae)^2 = a^2 + b^2 = 1 + 9 = 10$$

$$\boxed{ae = \sqrt{10}} \quad e = \frac{ae}{a} = \frac{\sqrt{10}}{1} = \sqrt{10}$$

$$\text{centre } (h, k) = (2, 3)$$

$$\text{Focus } (h, k \pm ae) = (2, 3 \pm \sqrt{10})$$

$$F_1(2, 3 + \sqrt{10}) \text{ \& } F_2(2, 3 - \sqrt{10})$$

$$\text{Vertices } (h, k \pm a) = (2, 3 \pm 1)$$

$$A(2, 4), \quad A'(2, 2)$$

$$\text{Focus } (h \pm ae, k) = (2 \pm \sqrt{10}, 3)$$

$$F_1(2 + \sqrt{10}, 3) \text{ \& } F_2(2 - \sqrt{10}, 3)$$

$$\text{Vertices } (h \pm a, k) = (2 \pm 1, 3)$$

$$A(3, 3), \quad A'(1, 3)$$

$$\text{Directrix } x = \pm \frac{a}{e} = \frac{1}{\sqrt{10}}$$

EXERCISE 5-3

1. $2x^2 - y^2 = 7$

Compare with

$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

$A=2, B=0, C=-1$

$B^2 - 4AC = 0 - 4(2)(-1) = 8 > 0$

The given Equation is a Hyperbola

2. $3x^2 + 3y^2 - 4x + 3y + 10 = 0$

Compare $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

$A=3, B=0, C=3$

$A=C=3 \text{ \& } B=0$

The given Equation is a circle

3. $3x^2 + 2y^2 = 14$

Compare $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

$A=3, B=0, C=2$

$B^2 - 4AC = 0 - 4(3)(2) = -12 < 0$

The given Equation is an Ellipse

4. $x^2 + y^2 + x - y = 0$

Compare $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

$A=1, B=0, C=1$

$A=C=1, \text{ \& } B=0$

The given Equation is circle

5. $11x^2 - 25y^2 - 44x + 50y - 256 = 0$

Compare $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

$A=11, B=0, C=-25$

$B^2 - 4AC = 0 - 4(11)(25) = 1100 > 0$

The given Equation is Hyperbola

6. $y^2 + 4x + 3y + 4 = 0$

Compare $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

$A=0, B=0, C=1$

$B^2 - 4AC = 0 - 4(0)(1) = 0$

The Equation is a parabola

Example 5-26

(i) $16y^2 = -4x^2 + 64$

$4x^2 + 16y^2 - 64 = 0$

Compare $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

$A=4, B=0, C=16$

$B^2 - 4AC = 0 - 4(4)(16) = -256 < 0$

The Equation is an Ellipse

(ii) $x^2 + y^2 = -4x - y + 4$

$x^2 + y^2 + 4x + y + 4 = 0$

Compare $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

$A=1, B=0, C=1$

$B^2 - 4AC = 0 - 4(1)(1) = 0$

$\therefore A=C=1 \text{ \& } B=0$

The Equation is a circle

(iii) $x^2 - 2y = x + 3$

$x^2 - x - 2y - 3 = 0$

Compare $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

$A=1, B=0, C=0$

$B^2 - 4AC = 0 - 4(1)(0) = 0$

The Equation is a parabola

(iv)

$4x^2 - 9y^2 - 16x + 18y - 29 = 0$

Compare $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

$A=4, B=0, C=-9$

$B^2 - 4AC = 0 - 4(4)(-9) = 144 > 0$

The Equation is Hyperbola

EXERCISE 5.3.

1. $2x^2 - y^2 = 7$.

Compare with

$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$.

$A=2, B=0, C=-1$.

$B^2 - 4AC = 0 - 4(2)(-1) = 8 > 0$.

The given Equation is a Hyperbola.

2. $3x^2 + 3y^2 - 4x + 3y + 10 = 0$.

Compare $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

$A=3, B=0, C=3$.

$A=C=3 \text{ \& } B=0$.

The given Equation is a circle.

3. $3x^2 + 2y^2 = 14$

Compare $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

$A=3, B=0, C=2$.

$B^2 - 4AC = 0 - 4(3)(2) = -12 < 0$.

The given Equation is an Ellipse.

4. $x^2 + y^2 + x - y = 0$.

Compare $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

$A=1, B=0, C=1$.

$A=C=1, \text{ \& } B=0$.

The given Equation is circle.

5. $11x^2 - 25y^2 - 44x + 50y - 256 = 0$.

Compare $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

$A=11, B=0, C=-25$

$B^2 - 4AC = 0 - 4(11)(25) = 1100 > 0$.

The given Equation is Hyperbola

6. $y^2 + 4x + 3y + 4 = 0$

Compare $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$.

$A=0, B=0, C=1$.

$B^2 - 4AC = 0 - 4(0)(1) = 0$.

The Equation is a parabola.

Example 5.26

(i) $16y^2 = -4x^2 + 64$.

$4x^2 + 16y^2 - 64 = 0$.

Compare $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$.

$A=4, B=0, C=16$

$B^2 - 4AC = 0 - 4(4)(16) = -256 < 0$

The Equation is an Ellipse.

(ii) $x^2 + y^2 = -4x - y + 4$

$x^2 + y^2 + 4x + y + 4 = 0$

Compare $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$.

$A=1, B=0, C=1$.

$B^2 - 4AC = 0 - 4(1)(1) = 0$.

$\therefore A=C=1 \text{ \& } B=0$.

The Equation is a Circle.

(iii) $x^2 - 2y = x + 3$.

$x^2 - x - 2y - 3 = 0$

Compare $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$.

$A=1, B=0, C=0$.

$B^2 - 4AC = 0 - 4(1)(0) = 0$.

The Equation is a parabola.

(iv)

$4x^2 - 9y^2 - 16x + 18y - 29 = 0$.

Compare $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

$A=4, B=0, C=-9$.

$B^2 - 4AC = 0 - 4(4)(-9) = 144 > 0$.

The Equation is Hyperbola.

Note: The Tangent equation at (x_1, y_1) for all the curves.

$$x^2 \text{ by } x x_1,$$

$$y^2 \text{ by } y y_1$$

$$x \text{ by } \frac{x+x_1}{2}$$

$$y \text{ by } \frac{y+y_1}{2} \text{ and}$$

$$xy \text{ by } \frac{xy_1 + yx_1}{2}.$$

Example 5.27.

$$x^2 + 6x + 4y + 5 = 0.$$

Tangent equation at (x_1, y_1) is.

$$x x_1 + 6\left(\frac{x+x_1}{2}\right) + 4\left(\frac{y+y_1}{2}\right) + 5 = 0.$$

$$x x_1 + 3x + 3x_1 + 2y + 2y_1 + 5 = 0.$$

At $(1, -3) \Rightarrow$

$$x(1) + 3x + 3(1) + 2y + 2(-3) + 5 = 0.$$

$$4x + 2y + 2 = 0.$$

$$2x + y + 1 = 0.$$

Equation of Normal.

$$x - 2y + k = 0.$$

At $(1, -3)$

$$1 - 2(-3) + k = 0$$

$$\boxed{k = -7}$$

Equation of Normal is $x - 2y - 7 = 0$

Example 5.28

$$x^2 + 4y^2 = 32.$$

$$\frac{x^2}{32} + \frac{y^2}{8} = 1.$$

$$\therefore a^2 = 32 \Rightarrow a = 4\sqrt{2}$$

$$b^2 = 8 \Rightarrow b = 2\sqrt{2}$$

Equation of tangent at " θ "

$$\frac{x \cos \theta}{a} + \frac{y \sin \theta}{b} = 1.$$

At $\theta = \pi/4$

$$\frac{x \cos \pi/4}{4\sqrt{2}} + \frac{y \sin \pi/4}{2\sqrt{2}} = 1.$$

$$\frac{x}{8} + \frac{y}{4} = 1.$$

$$x + 2y = 8 \text{ (or) } x + 2y - 8 = 0$$

Equation of Normal at " θ ".

$$\frac{ax}{\cos \theta} - \frac{by}{\sin \theta} = a^2 - b^2.$$

At $\theta = \pi/4$

$$\frac{4\sqrt{2}x}{\cos \pi/4} - \frac{2\sqrt{2}y}{\sin \pi/4} = 32 - 8$$

$$8x - 4y = 24$$

$$2x - y = 6 \text{ (or) } 2x - y - 6 = 0$$

EXERCISE 5.4

$$1. \quad 2x^2 + 7y^2 = 14, \quad (5, 2).$$

$$\text{Intercept} \Rightarrow \frac{x^2}{7} + \frac{y^2}{2} = 1.$$

Any tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$y = mx \pm \sqrt{a^2 m^2 + b^2}$$

$$2 = m(5) \pm \sqrt{7m^2 + 2}$$

$$2 - 5m = \pm \sqrt{7m^2 + 2}.$$

Taking Squaring on both side.

$$(2 - 5m)^2 = 7m^2 + 2.$$

$$9m^2 - 10m + 1 = 0.$$

$$(9m - 1)(m - 1) = 0$$

$$m = 1/9 \text{ \& } m = 1.$$

Tangent Equation: $(5, 2)$

$$\text{At } m = 1, \quad y - y_1 = m(x - x_1)$$

$$y - 2 = 1(x - 5)$$

$$x - y - 3 = 0.$$

At $m = 1/9$

$$y - 2 = 1/9(x - 5)$$

$$x - 9y + 13 = 0.$$

2. $\frac{x^2}{16} - \frac{y^2}{64} = 1$, $y = mx + c$
 $10x - 3y + 9 = 0$
 $a^2 = 16$, $b^2 = 64$
 $m = -\frac{a}{b}$

Any tangent to the parabola $m = -\frac{10}{-3} = \frac{10}{3}$.

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is the form.

$$y = mx \pm \sqrt{a^2 m^2 - b^2}$$

$$y = \frac{10x}{3} \pm \sqrt{16\left(\frac{100}{9}\right) - 64}$$

$$y = \frac{10x}{3} \pm \sqrt{\frac{1024}{9}}$$

$$y = \frac{10x}{3} \pm \frac{32}{3}$$

$$3y = 10x \pm 32$$

The tangent equations are

$$10x - 3y + 32 = 0 \text{ \& } 10x - 3y - 32 = 0$$

3. $x - y + 4 = 0$, $x^2 + 3y^2 = 12$
 $y = x + 4$, $\frac{x^2}{12} + \frac{y^2}{4} = 1$
 $m = 1$, $c = 4$, $a^2 = 12$, $b^2 = 4$
 $c^2 = 16$

condition for tangency,

$$c^2 = a^2 m^2 + b^2$$

$$a^2 m^2 + b^2 = 12(1)^2 + 4 = 16 = c^2$$

The line is the tangent to the ellipse.

Point of contact,

$$\left(-\frac{a^2 m}{c}, \frac{b^2}{c}\right) = \left[\frac{-12(1)}{4}, \frac{4}{4}\right]$$

$$= (-3, 1)$$

1. $y^2 = 16x$, $y^2 = 4ax$
 $4a = 16$
 $a = 4$

Any line perpendicular to,

$$2x + 2y + 3 = 0 \Rightarrow 2x - 2y + k = 0$$

$$x - y + \frac{k}{2} = 0$$

$$y = x + \frac{k}{2}$$

$$\therefore m = 1, c = \frac{k}{2}$$

The condition for the line

$y = mx + c$ be a tangent to the parabola $y^2 = 4ax$ is $c = \frac{a}{m}$.

$$c = \frac{4}{1}$$

$$\frac{k}{2} = 4$$

$$k = 8$$

The required perpendicular line

$$\text{is } x - y + 4 = 0$$

5. $y^2 = 8x \Rightarrow y^2 = 4ax$
 $4a = 8$
 $a = 2$
 At $t = 2$, $a = 2$.

The Equation of tangent at "t" on the parabola

$$yt = x + at^2$$

$$y(2) = x + 2(2)^2$$

$$2y = x + 8$$

$$x - 2y + 8 = 0$$

6. $12x^2 - 9y^2 = 108$

$$\frac{x^2}{9} - \frac{y^2}{12} = 1.$$

$$a^2 = 9 \Rightarrow a = 3.$$

$$b^2 = 12 \Rightarrow b = 2\sqrt{3}.$$

Tangent Equation at " θ "

$$\frac{x \sec \theta}{a} - \frac{y \tan \theta}{b} = 1.$$

At $\theta = \pi/3$

$$\frac{x \sec \pi/3}{3} - \frac{y \tan \pi/3}{2\sqrt{3}} = 1$$

$$\frac{2x}{3} - \frac{\sqrt{3}y}{2\sqrt{3}} = 1.$$

$$4x - 3y - 6 = 0.$$

Normal Equation at " θ "

$$\frac{ax}{\sec \theta} + \frac{by}{\tan \theta} = a^2 + b^2$$

At $\theta = \pi/3$

$$\frac{3x}{\sec \pi/3} + \frac{2\sqrt{3}y}{\tan \pi/3} = 9 + 12.$$

$$\frac{3x}{2} + \frac{2\sqrt{3}y}{\sqrt{3}} = 21.$$

$$\frac{3x}{2} + 2y = 21$$

$$3x + 4y - 42 = 0.$$

7. Tangent equation at " t_1 " is

$$yt_1 = x + at_1^2 \rightarrow \textcircled{1}$$

at " t_2 " is $yt_2 = x + at_2^2 \rightarrow \textcircled{2}$.

$$\textcircled{1} - \textcircled{2} \Rightarrow y(t_1 - t_2) = a(t_1^2 - t_2^2)$$

$$y = \frac{a(t_1^2 - t_2^2)}{t_1 - t_2}.$$

$$y = \frac{a(t_1 - t_2)(t_1 + t_2)}{(t_1 - t_2)}$$

$$y = a(t_1 + t_2)$$

using $y = a(t_1 + t_2)$ in eq $\textcircled{1}$.

$$a(t_1 + t_2)t_1 = x + at_1^2$$

$$at_1^2 + at_1t_2 = x + at_1^2.$$

$$x = at_1t_2$$

The point of intersection is

$$[at_1t_2, a(t_1 + t_2)],$$

8.

Normal Equation of a parabola at

$P(at_1^2, 2at_1)$ is

$$y + xt_1 = at_1^3 + 2at_1$$

It meets the parabola again at $\theta (at_2^2, 2at_2)$ is.

$$2at_2 + (at_1^2)t_1 = at_1^3 + 2at_1$$

$$2at_2 + at_1t_1^2 = at_1^3 + 2at_1$$

$$2at_2 - 2at_1 = at_1^3 - at_1t_1^2$$

$$2a(t_2 - t_1) = at_1(t_1^2 - t_2^2)$$

$$2a(t_2 - t_1) = -at_1(t_2^2 - t_1^2)$$

$$2a(t_2 - t_1) = -at_1(t_2 - t_1)(t_2 + t_1)$$

$$2a = -at_1(t_2 + t_1)$$

$$-\frac{2}{t_1} = t_2 + t_1$$

$$-\frac{2}{t_1} - t_1 = t_2$$

$$t_2 = -\left(t_1 + \frac{2}{t_1}\right).$$

Example 5.30.Let $a = b$, $b = 3$.

Equation of the ellipse is

$$\frac{x^2}{36} + \frac{y^2}{9} = 1 \rightarrow (1)$$

When $x = 1.5 = 3/2$, Then y .

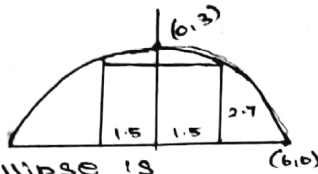
$$\frac{(3/2)^2}{36} + \frac{y^2}{9} = 1.$$

$$y^2 = 9 \left[1 - \frac{9}{144} \right]$$

$$y^2 = \frac{135}{16}$$

$$y = \frac{\sqrt{135}}{4} = 2.90.$$

The truck will clear the archway.

Example 5.31.Let $SA = 94.5 \times 10^6$ km $SA' = 152 \times 10^6$ km.

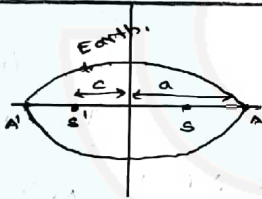
$$SA = a - c = 94.5 \times 10^6 \rightarrow (1)$$

$$SA' = a + c = 152 \times 10^6 \rightarrow (2)$$

$$(2) - (1) \Rightarrow 2c = 57.5 \times 10^6$$

$$2c = 575 \times 10^5$$

$$SS' = 575 \times 10^5 \text{ km.}$$

Example 5.32.The Equation of parabola is $x^2 = -4ay$ At $(20, -15)$

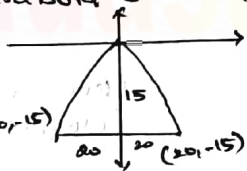
$$(20)^2 = -4a(-15)$$

$$400 = 4a(15)$$

$$4a = \frac{80}{3}$$

$$\text{Equation } x^2 = -\frac{80}{3}y$$

$$3x^2 = -80y.$$

Example 5.33.Equation of parabola is $y^2 = 4ax$ $\therefore a = 2.$

$$y^2 = 4(2)x$$

$$y^2 = 8x$$

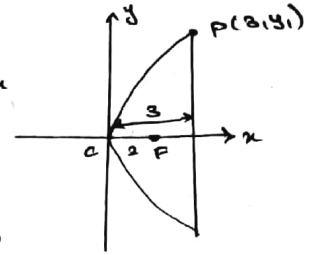
At $(3, y_1)$

$$y_1^2 = 8(3)$$

$$y_1^2 = 24$$

$$y_1 = 2\sqrt{6}$$

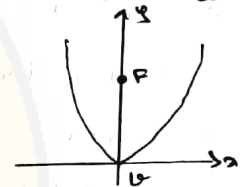
$$\text{Required width } 2y_1 = 2 \times 2\sqrt{6} = 4\sqrt{6} \text{ m.}$$

Example 5.34.Equation of the parabola is $y = \frac{1}{82}x^2$

$$x^2 = 82y.$$

$$x^2 = 4(8)y.$$

$$\therefore a = 8.$$



The Heating tube need to be placed 8 units above the vertex of the parabola.

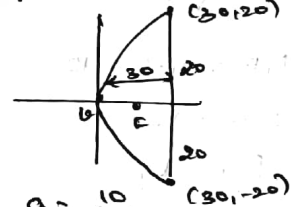
Example 5.35Equation of parabola is $y^2 = 4ax$ At $(30, 20)$

$$(20)^2 = 4a(30)$$

$$4a = \frac{40}{3} \Rightarrow a = \frac{10}{3}$$

Equation

$$y^2 = \frac{40}{3}x.$$

The bulb is at focus $(10/3, 0)$.Example 5.32.The Equation of parabola is $x^2 = -4ay$ At $(20, -15)$

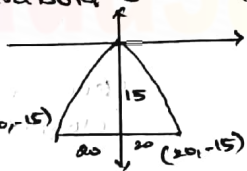
$$(20)^2 = -4a(-15)$$

$$400 = 4a(15)$$

$$4a = \frac{80}{3}$$

$$\text{Equation } x^2 = -\frac{80}{3}y$$

$$3x^2 = -80y.$$

Example 5.36Equation of ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1.$

$$a^2 = 16 \quad b^2 = 9.$$

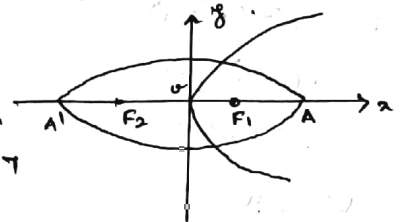
$$c^2 = a^2 - b^2.$$

$$c^2 = 16 - 9 = 7$$

$$c = \pm\sqrt{7}$$

Foci: $F_1(\sqrt{7}, 0)$ & $F_2(-\sqrt{7}, 0)$

$$\therefore a = \sqrt{7}$$

Equation of the parabola $y^2 = 4\sqrt{7}x$ 

Example 5.37.

$$a=17, b=8$$

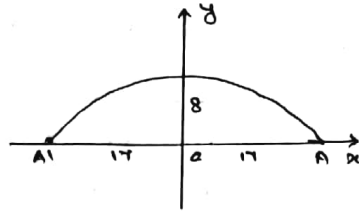
$$c^2 = a^2 - b^2$$

$$= 289 - 64$$

$$c^2 = 225$$

$$c = 15$$

$$\text{Foci} : F_1 (15, 0) \text{ \& } F_2 (-15, 0).$$

Example 5.38

$$\text{Equation of ellipse is } \frac{(x-1)^2}{484} + \frac{y^2}{64} = 1.$$

$$a^2 = 484, b^2 = 64$$

$$c^2 = a^2 - b^2$$

$$= 484 - 64$$

$$c^2 = 420$$

$$c = 20.5 \text{ (app).}$$

The patient's kidney stone should be placed 20.5cm from the centre of the ellipse.

Example 5.40.

$$F_1 F_2 = 14 - 2 = 12.$$

$$2ae = 12$$

$$\boxed{ae = 6}$$

$$V_2 F_1 = (CF_2 - CV_2)$$

$$1 = ae - a$$

$$1 = b - a$$

$$\boxed{a = 5}$$

$$b^2 = a^2 (e^2 - 1)$$

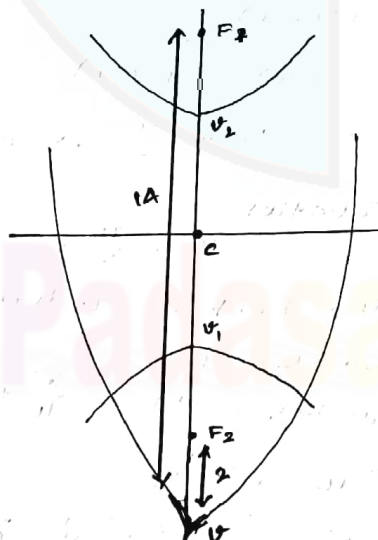
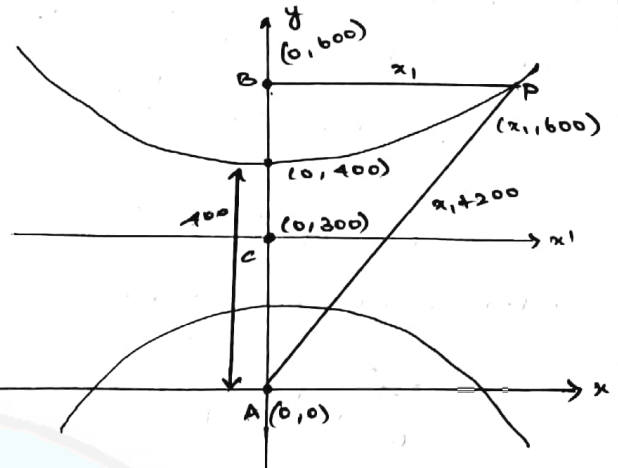
$$= (ae)^2 - a^2$$

$$= 36 - 25$$

$$b^2 = 11$$

Equation of the hyperbola is.

$$\frac{y^2}{25} - \frac{x^2}{11} = 1.$$

Example 5.39.

$$\text{Centre : } c (0, 300) = (h, k)$$

$$\text{Equation of hyperbola } \frac{y^2}{a^2} - \frac{x^2}{b^2} = 1.$$

$$\frac{(y-300)^2}{a^2} - \frac{(x-0)^2}{b^2} = 1.$$

$$\text{At } (0, 400).$$

$$\frac{(400-300)^2}{a^2} - 0 = 1.$$

$$a^2 = (100)^2$$

$$\boxed{a = 100}$$

$$\text{In } \triangle ABP \quad AP^2 = AB^2 + BP^2.$$

$$(x_1 + 200)^2 = 600^2 + x_1^2.$$

$$x_1^2 + 400x_1 + (200)^2 = (600)^2 + x_1^2$$

$$400x_1 = (600)^2 - (200)^2$$

$$400x_1 = (600-200)(600+200)$$

$$\boxed{x_1 = 800}$$

P (800, 600) lies on the hyperbola.

$$\frac{(y-300)^2}{a^2} - \frac{x^2}{b^2} = 1.$$

$$\frac{(600-300)^2}{(100)^2} - 1 = \frac{(800)^2}{b^2}$$

$$\boxed{b^2 = 80000}$$

The required equation of hyperbola

$$\frac{(y-300)^2}{10000} - \frac{x^2}{80000} = 1.$$

EXERCISE 5-5

1. Equation of parabola $x^2 = -4ay$.

At (15, -10)

$$(15)^2 = -4a(-10)$$

$$4a = \frac{225}{10}$$

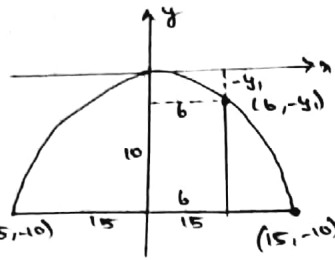
$$\text{Equation } x^2 = -4\left(\frac{225}{40}\right)y$$

At (6, -y₁)

$$36 = -4\left(\frac{225}{40}\right)y_1$$

$$y_1 = 1.6$$

$$\begin{aligned} \text{The required height} &= 10 - y_1 \\ &= 10 - 1.6 \\ &= 8.4 \text{ m.} \end{aligned}$$



2.

$$CA = a = 8$$

$$CB = b = 5$$

Equation of Ellipse

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

$$\frac{x^2}{25} + \frac{y^2}{64} = 1$$

At (4, y₁)

$$\frac{16}{25} + \frac{y_1^2}{64} = 1$$

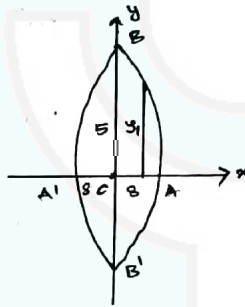
$$y_1^2 = 64\left(1 - \frac{16}{25}\right)$$

$$y_1^2 = 8\left(\frac{9}{25}\right)$$

$$y_1 = \frac{8 \times 3}{5}$$

$$y_1 = \frac{24}{5} \text{ (or) } 4.8 \text{ m.}$$

$$\begin{aligned} \text{The required width for the} \\ \text{opening is } 2y_1 &= 2(4.8) \\ &= 9.6 \text{ m.} \end{aligned}$$



3. Equation of the parabola is

$$x^2 = -4ay$$

At (0.5, -4)

$$(0.5)^2 = -4a(-4)$$

$$0.25 = 4a(4)$$

$$4a = \frac{0.25}{4}$$

Equation.

$$x^2 = -\left(\frac{0.25}{4}\right)y$$

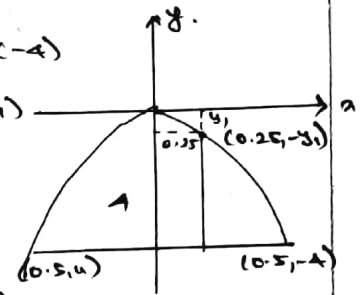
At (0.25, -y₁)

$$(0.25)^2 = -\left(\frac{0.25}{4}\right)(-y_1)$$

$$0.25 = \frac{y_1}{4}$$

$$y_1 = 1 \text{ m}$$

$$\begin{aligned} \text{Required height} &= 4 - y_1 \\ &= 4 - 1 \\ &= 3 \text{ m.} \end{aligned}$$



4.

Equation of parabola is

$$y^2 = 4ax$$

$$\therefore a = 1.2$$

$$y^2 = 4(1.2)x$$

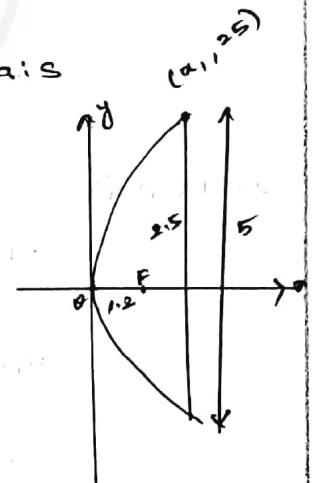
$$y^2 = 4.8x$$

At (x, 2.5)

$$(2.5)^2 = 4.8x$$

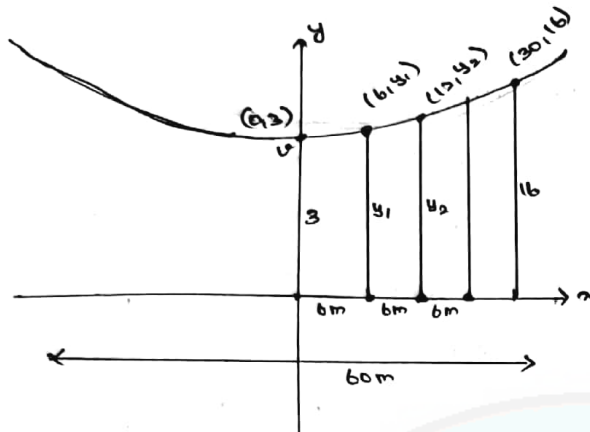
$$\frac{2.5 \times 2.5}{4.8} = x$$

$$x = 1.3 \text{ m}$$



The depth of the satellite dish
at the vertex is 1.3 m

5.



Equation of the parabola is.

$$(x-h)^2 = 4a(y-k)$$

At (0, 3)

$$(x-0)^2 = 4a(y-3)$$

$$x^2 = 4a(y-3)$$

At (30, 16)

$$(30)^2 = 4a(16-3)$$

$$a = \left(\frac{30 \times 30}{4} \right) \cdot \frac{1}{13}$$

Equation.

$$x^2 = 4 \left(\frac{30 \times 30}{4} \right) \left(\frac{1}{13} \right) (y-3)$$

$$x^2 = \frac{30 \times 30 (y-3)}{13}$$

At (6, 4)

$$36 = \frac{30 \times 30 (y_1-3)}{13}$$

$$y_1-3 = \frac{36 \times 13}{30 \times 30}$$

$$y_1 = 3 + 0.52$$

$$y_1 = 3.52 \text{ m.}$$

At (12, 5)

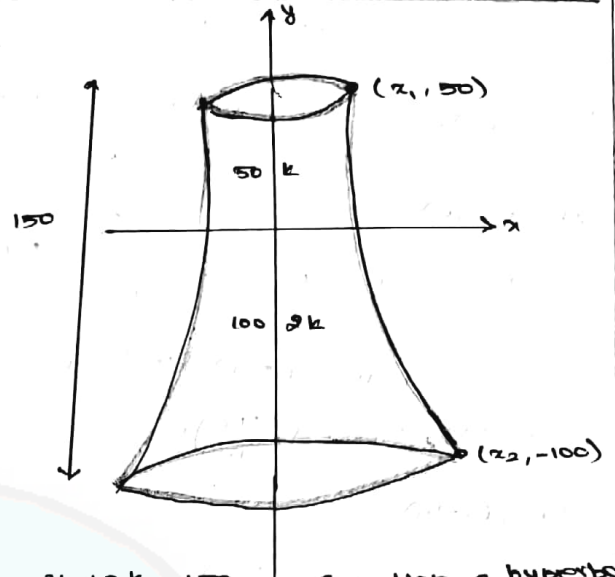
$$12^2 = \frac{900}{13} (y_2-3)$$

$$y_2-3 = \frac{144 \times 13}{900}$$

$$y_2 = 3 + 2.08$$

$$y_2 = 5.08 \text{ m.}$$

6.



$$2k + 2b = 150$$

$$3k = 150$$

$$k = 50$$

Equation of hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

(x1, 50)

$$\frac{x_1^2}{20^2} - \frac{50^2}{44^2} = 1$$

$$x_1^2 = 30^2 \left(1 + \frac{50^2}{44^2} \right)$$

$$x_1^2 = \left(\frac{30}{44} \right)^2 (1936 + 2500)$$

$$x_1 = \frac{30}{44} \sqrt{4436}$$

$$x_1 = 45.40$$

Radius of the top of the tower is

$$x_1 = 45.40 \text{ m.}$$

At (x2, -100) lies on hyperbola

$$\frac{x_2^2}{30^2} - \frac{100^2}{44^2} = 1$$

$$x_2^2 = \left(\frac{30}{44} \right)^2 (44^2 + 100^2)$$

$$x_2 = \frac{30}{44} \sqrt{11936}$$

$$x_2 = \frac{30}{44} \times 109.25$$

$$x_2 = 74.48 \text{ m.}$$

Radius of the base of the tower

$$x_2 = 74.48 \text{ m.}$$

7. $BAP \cong PCD$

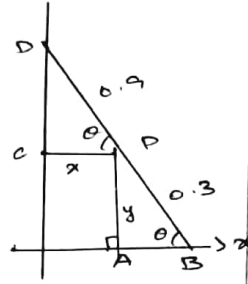
Let $\angle ABP = \angle CPD = \theta$.

In $\triangle PAB \Rightarrow \sin \theta = \frac{y}{0.3}$

In $\triangle PCD \Rightarrow \cos \theta = \frac{x}{0.9}$

$\cos^2 \theta + \sin^2 \theta = 1$

$\frac{x^2}{0.9^2} + \frac{y^2}{(0.3)^2} = 1$



eccentricity, $e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{(0.3)^2}{(0.9)^2}}$
 $= \sqrt{1 - \left(\frac{1}{9}\right)^2}$
 $= \sqrt{1 - \frac{1}{9}}$
 $= \sqrt{\frac{8}{9}}$
 $e = \frac{2\sqrt{2}}{3} \text{ m.}$

8.

The equation of the parabola is,

$x^2 = -4ay$

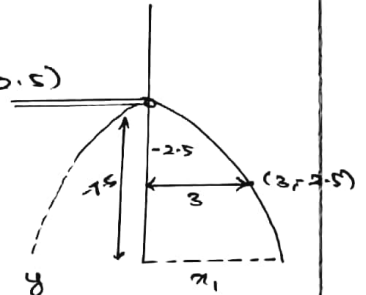
At $(3, -2.5)$

$9 = -4a(-2.5)$

$\frac{9}{2.5} = 4a$

Equation,

$x^2 = -\frac{9}{2.5}y$



At $(x_1, -1.5)$

$x_1^2 = -\frac{9}{2.5} \times (-1.5)$

$x_1^2 = 9 \times 3$

$x_1 = 3\sqrt{3}$

The water strikes the ground $3\sqrt{3} \text{ m}$ beyond the vertical line.

9.

The equation of parabola is

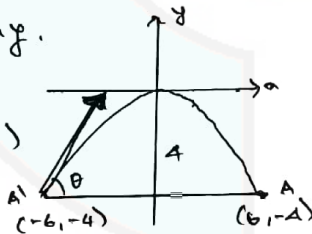
$x^2 = -4ay$

At $(6, -4)$

$36 = 4a(4)$

$4a = 9$

Equation $x^2 = 9y$



D. w. r to x ,

$2x = -9 \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{2x}{-9}$

At $(-6, -4)$,

$\tan \theta \Rightarrow \frac{dy}{dx} = \frac{2(-6)}{-9} = \frac{12}{9} = \frac{4}{3}$

$\tan \theta = \frac{4}{3}$

$\theta = \tan^{-1}\left(\frac{4}{3}\right)$

10.

$ae = 5$

$Ap \sim Bp \Rightarrow 2a = b$

$a = 3$

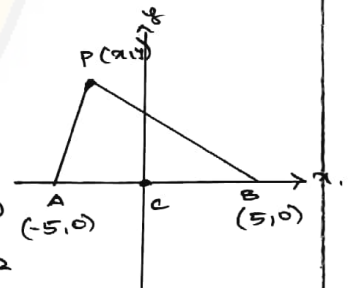
$b^2 = a^2(e^2 - 1)$

$= (ae)^2 - a^2$

$= (5)^2 - (3)^2$

$= 25 - 9$

$b^2 = 16$



The equation of the required hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\frac{x^2}{9} - \frac{y^2}{16} = 1$

The locus of P is hyperbola