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1. Applications of Matrices and determinants

EXERCISE 1.1

1) Find the adjoint of the following: [Each 2M]

(i) $\begin{bmatrix} -3 & 4 \\ 6 & 2 \end{bmatrix}$

Soln:

Let $A = \begin{bmatrix} -3 & 4 \\ 6 & 2 \end{bmatrix}$

$\text{adj}A = \begin{bmatrix} 2 & -4 \\ -6 & -3 \end{bmatrix}$

(ii) $\begin{bmatrix} 2 & 3 & 1 \\ 3 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$

Soln:

Let $A = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$

$[A_{ij}] = \begin{bmatrix} 4 & 1 & 3 & 4 \\ 7 & 2 & 3 & 7 \\ 3 & 1 & 2 & 3 \\ 4 & 1 & 3 & 4 \end{bmatrix}$

$= \begin{bmatrix} 8-7 & 3-6 & 21-12 \\ 7-6 & 4-3 & 9-14 \\ 3-4 & 3-2 & 8-9 \end{bmatrix}$

$= \begin{bmatrix} 1 & -3 & 9 \\ 1 & 1 & -5 \\ -1 & 1 & -1 \end{bmatrix}$

$\text{adj}A = [A_{ij}]^T = \begin{bmatrix} 1 & 1 & -1 \\ -3 & 1 & 1 \\ 9 & -5 & -1 \end{bmatrix}$

$\therefore \text{adj}A = \begin{bmatrix} 1 & 1 & -1 \\ -3 & 1 & 1 \\ 9 & -5 & -1 \end{bmatrix}$

(iii) $\frac{1}{3}\begin{bmatrix} 2 & 2 & 1 \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix}$

Soln:

Let $A = \frac{1}{3}\begin{bmatrix} 2 & 2 & 1 \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix}$

$[A_{ij}] = \begin{bmatrix} 1 & 2 & -2 & 1 \\ -2 & 2 & 1 & -2 \\ 2 & 1 & 2 & 2 \\ 1 & 2 & -2 & 1 \end{bmatrix}$

$= \begin{bmatrix} 2+4 & 2+4 & 4-1 \\ -2-4 & 4-1 & 2+4 \\ 4-1 & -2-4 & 2+4 \end{bmatrix}$

$[A_{ij}] = \begin{bmatrix} 6 & 6 & 3 \\ -6 & 3 & 6 \\ 3 & -6 & 6 \end{bmatrix}$

$\text{adj}A = [A_{ij}]^T = \begin{bmatrix} 6 & -6 & 3 \\ 6 & 3 & -6 \\ 3 & 6 & 6 \end{bmatrix}$

$\therefore \text{adj}A = \begin{bmatrix} 6 & -6 & 3 \\ 6 & 3 & -6 \\ 3 & 6 & 6 \end{bmatrix}$

$\boxed{\text{adj}(kA) = k^{n-1}\,\text{adj}A}$

$\text{adj}A = \left(\frac{1}{3}\right)^{3-1}\begin{bmatrix} 6 & -6 & 3 \\ 6 & 3 & -6 \\ 3 & 6 & 6 \end{bmatrix}$

$= \left(\frac{1}{3}\right)^{2}\begin{bmatrix} 6 & -6 & 3 \\ 6 & 3 & -6 \\ 3 & 6 & 6 \end{bmatrix}$

$= \frac{1}{\cancel{9}\,3} \times \cancel{3}\begin{bmatrix} 2 & -2 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 2 \end{bmatrix}$

$\text{adj}A = \frac{1}{3}\begin{bmatrix} 2 & -2 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 2 \end{bmatrix}$

2) Find the inverse (if it exists) of the following:

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(i) $\begin{bmatrix} -2 & 4 \\ 1 & -3 \end{bmatrix}$

Soln:

Let $A = \begin{bmatrix} -2 & 4 \\ 1 & -3 \end{bmatrix}$

$|A| = \begin{vmatrix} -2 & 4 \\ 1 & -3 \end{vmatrix} = 6-4 = 2$

$|A| = 2 \neq 0$

A is non-singular and hence A^{-1} exists

$\text{adj}A = \begin{bmatrix} -3 & -4 \\ -1 & -2 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|}(\text{adj}A)$

$A^{-1} = \frac{1}{2}\begin{bmatrix} -3 & -4 \\ -1 & -2 \end{bmatrix}$

(ii) $\begin{bmatrix} 5 & 1 & 1 \\ 1 & 5 & 1 \\ 1 & 1 & 5 \end{bmatrix}$

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Soln:

Let $A = \begin{bmatrix} 5 & 1 & 1 \\ 1 & 5 & 1 \\ 1 & 1 & 5 \end{bmatrix}$

$|A| = \begin{vmatrix} 5 & 1 & 1 \\ 1 & 5 & 1 \\ 1 & 1 & 5 \end{vmatrix}$

$= 5(25-1) - 1(5-1) + 1(1-5)$

$= 120 - 4 - 4 = 112$

$|A| = 112 \neq 0$

A is non-Singular and A^{-1} exists

$[A_{ij}] = \begin{bmatrix} 5 & 1 & 1 & 5 \\ 1 & 5 & 1 & 1 \\ 1 & 1 & 5 & 1 \\ 5 & 1 & 1 & 5 \end{bmatrix}$

$= \begin{bmatrix} 25-1 & 1-5 & 1-5 \\ 1-5 & 25-1 & 1-5 \\ 1-5 & 1-5 & 25-1 \end{bmatrix}$

$[A_{ij}] = \begin{bmatrix} 24 & -4 & -4 \\ -4 & 24 & -4 \\ -4 & -4 & 24 \end{bmatrix}$

$\text{adj} A = [A_{ij}]^T = \begin{bmatrix} 24 & -4 & -4 \\ -4 & 24 & -4 \\ -4 & -4 & 24 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|}(\text{adj} A)$

$= \frac{1}{112}\begin{bmatrix} 24 & -4 & -4 \\ -4 & 24 & -4 \\ -4 & -4 & 24 \end{bmatrix}$

$= \frac{\cancel{4}\,1}{\cancel{112}\,28}\begin{bmatrix} 6 & -1 & -1 \\ -1 & 6 & -1 \\ -1 & -1 & 6 \end{bmatrix}$

$A^{-1} = \frac{1}{28}\begin{bmatrix} 6 & -1 & -1 \\ -1 & 6 & -1 \\ -1 & -1 & 6 \end{bmatrix}$

(iii) $\begin{bmatrix} 2 & 3 & 1 \\ 3 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$

Soln:

Let $A = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$

$|A| = \begin{vmatrix} 2 & 3 & 1 \\ 3 & 4 & 1 \\ 3 & 7 & 2 \end{vmatrix}$

$= 2(8-7) - 3(6-3) + 1(21-12)$

$= 2(1) - 3(3) + 1(9)$

$= 2 - \cancel{9} + \cancel{9} = 2$

$|A| = 2 \neq 0$

A is non-singular and hence A^{-1} exists

$[A_{ij}] = \begin{bmatrix} 4 & 1 & 3 & 4 \\ 7 & 2 & 3 & 7 \\ 3 & 1 & 2 & 3 \\ 4 & 1 & 3 & 4 \end{bmatrix}$

$= \begin{bmatrix} 8-7 & 3-6 & 21-12 \\ 7-6 & 4-3 & 9-14 \\ 3-4 & 3-2 & 8-9 \end{bmatrix}$

$[A_{ij}] = \begin{bmatrix} 1 & -3 & 9 \\ 1 & 1 & -5 \\ -1 & 1 & -1 \end{bmatrix}$

$\text{adj} A = [A_{ij}]^T = \begin{bmatrix} 1 & 1 & -1 \\ -3 & 1 & 1 \\ 9 & -5 & -1 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|}(\text{adj} A)$

$A^{-1} = \frac{1}{2}\begin{bmatrix} 1 & 1 & -1 \\ -3 & 1 & 1 \\ 9 & -5 & -1 \end{bmatrix}$

3) If $F(\alpha) = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}$ Show that $[F(\alpha)]^{-1} = F(-\alpha)$

Soln:

$F(\alpha) = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}$

$|F(\alpha)| = \begin{vmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{vmatrix}$

$= \cos\alpha(\cos\alpha - 0) - \cancel{0(0+0)}$
$+ \sin\alpha(0 + \sin\alpha)$

$= \cos^2\alpha + \sin^2\alpha = 1$

$|F(\alpha)| = 1 \neq 0$

$F(\alpha)$ is non-singular and hence $[F(\alpha)]^{-1}$ exists

$$[F_{ij}(\alpha)] = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & \cos\alpha & -\sin\alpha & 0 \\ 0 & \sin\alpha & \cos\alpha & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & \cos^2\alpha+\sin^2\alpha & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}$$

$$[F_{ij}(\alpha)] = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}$$

$$adj(F(\alpha)) = [F_{ij}(\alpha)]^T = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix}$$

$$[F(\alpha)]^{-1} = \frac{1}{|F(\alpha)|}(adj(F(\alpha)))$$

$$= \frac{1}{1}\begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix}$$

$$[F(\alpha)]^{-1} = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix}$$

$$F(-\alpha) = \begin{bmatrix} \cos(-\alpha) & 0 & \sin(-\alpha) \\ 0 & 1 & 0 \\ -\sin(-\alpha) & 0 & \cos(-\alpha) \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix}$$

$$F(-\alpha) = [F(\alpha)]^{-1}$$

4) If $A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$ Show that $A^2-3A-7I_2=0$ Hence find A^{-1}

Soln:

$$A^2 = AA = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 25-3 & 15-6 \\ -5+2 & -3+4 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 22 & 9 \\ -3 & 1 \end{bmatrix}$$

$$A^2 - 3A - 7I_2$$

$$= \begin{bmatrix} 22 & 9 \\ -3 & 1 \end{bmatrix} - \begin{bmatrix} 15 & 9 \\ -3 & -6 \end{bmatrix} - \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

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$$A^2 - 3A - 7I_2 = 0$$

Also, $|A| = \begin{vmatrix} 5 & 3 \\ -1 & -2 \end{vmatrix}$

$= -10+3 = -7$

$|A| = -7 \neq 0$

A is non-singular and hence A^{-1} exists

$$adj A = \begin{bmatrix} -2 & -3 \\ 1 & 5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}(adj A)$$

$$= \frac{1}{-7}\begin{bmatrix} -2 & -3 \\ 1 & 5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & 3 \\ -1 & -5 \end{bmatrix}$$

5) If $A = \frac{1}{9}\begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}$ Prove that $A^{-1} = A^T$

Soln:

Given: To Prove $A^{-1} = A^T$

Pre multiple by A on both sides we get,

$$AA^{-1} = AA^T$$

$$I = AA^T$$

It is enough to prove $AA^T = I$

$$AA^T = \frac{1}{9}\begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}\frac{1}{9}\begin{bmatrix} -8 & 4 & 1 \\ 1 & 4 & -8 \\ 4 & 7 & 4 \end{bmatrix}$$

$$= \frac{1}{81}\begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}\begin{bmatrix} -8 & 4 & 1 \\ 1 & 4 & -8 \\ 4 & 7 & 4 \end{bmatrix}$$

$$= \frac{1}{81}\begin{bmatrix} 81 & 0 & 0 \\ 0 & 81 & 0 \\ 0 & 0 & 81 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

$\therefore AA^T = I$

6) If $A = \begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$ verify that $A(\text{adj}A) = (\text{adj}A)A = |A| I_2$

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Soln:

$|A| = \begin{vmatrix} 8 & -4 \\ -5 & 3 \end{vmatrix} = 24 - 20 = 4$

$|A| I_2 = 4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$ — (1)

$\text{adj}A = \begin{bmatrix} 3 & 4 \\ 5 & 8 \end{bmatrix}$

$A(\text{adj}A) = \begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}\begin{bmatrix} 3 & 4 \\ 5 & 8 \end{bmatrix}$

$A(\text{adj}A) = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$ — (2)

$(\text{adj}A)A = \begin{bmatrix} 3 & 4 \\ 5 & 8 \end{bmatrix}\begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$

$(\text{adj}A)A = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$ — (3)

from (1), (2) and (3) we get,

$A(\text{adj}A) = (\text{adj}A)A = |A| I_2$

7) If $A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & -3 \\ 5 & 2 \end{bmatrix}$ verify that $(AB)^{-1} = B^{-1}A^{-1}$

Soln:

$AB = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}\begin{bmatrix} -1 & -3 \\ 5 & 2 \end{bmatrix}$

$= \begin{bmatrix} -3+10 & -9+4 \\ -7+25 & -21+10 \end{bmatrix}$

$AB = \begin{bmatrix} 7 & -5 \\ 18 & -11 \end{bmatrix}$

$|AB| = \begin{vmatrix} 7 & -5 \\ 18 & -11 \end{vmatrix}$

$= -77 + 90 = 13$

$|AB| = 13 \neq 0$

AB is non-singular and hence $(AB)^{-1}$ exists

$\text{adj}AB = \begin{bmatrix} -11 & 5 \\ -18 & 7 \end{bmatrix}$

$(AB)^{-1} = \frac{1}{|AB|}(\text{adj}AB)$

$(AB)^{-1} = \frac{1}{13}\begin{bmatrix} -11 & 5 \\ -18 & 7 \end{bmatrix}$ — (1)

$|B| = \begin{vmatrix} -1 & -3 \\ 5 & 2 \end{vmatrix} = -2 + 15 = 13$

$|B| = 13 \neq 0$

B is non-singular and hence B^{-1} exists

$\text{adj}B = \begin{bmatrix} 2 & 3 \\ -5 & -1 \end{bmatrix}$

$B^{-1} = \frac{1}{|B|}(\text{adj}B)$

$B^{-1} = \frac{1}{13}\begin{bmatrix} 2 & 3 \\ -5 & -1 \end{bmatrix}$

$|A| = \begin{vmatrix} 3 & 2 \\ 7 & 5 \end{vmatrix} = 15 - 14 = 1$

$|A| = 1 \neq 0$

A is non-singular and hence A^{-1} exists

$\text{adj}A = \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|}(\text{adj}A)$

$= \frac{1}{1}\begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$

$B^{-1}A^{-1} = \frac{1}{13}\begin{bmatrix} 2 & 3 \\ -5 & -1 \end{bmatrix}\begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$

$B^{-1}A^{-1} = \frac{1}{13}\begin{bmatrix} -11 & 5 \\ -18 & 7 \end{bmatrix}$ — (2)

from (1) & (2) we get,

$(AB)^{-1} = B^{-1}A^{-1}$

8) If $adj(A) = \begin{bmatrix} 2 & -4 & 2 \\ -3 & 12 & -7 \\ -2 & 0 & 2 \end{bmatrix}$ find A

Soln:

$$|adj(A)| = \begin{vmatrix} 2 & -4 & 2 \\ -3 & 12 & -7 \\ -2 & 0 & 2 \end{vmatrix}$$

$$= 2(24+0) + 4(-6-14) + 2(0+24)$$

$$= 48 - 80 + 48 = 16$$

$$\sqrt{|adj(A)|} = \sqrt{16} = 4$$

$$adj(adj(A)) = \begin{bmatrix} 12 & -7 & -3 & 12 \\ 0 & 2 & -2 & 0 \\ -4 & 2 & 2 & -4 \\ 12 & -7 & -3 & 12 \end{bmatrix}^T$$

$$= \begin{bmatrix} 24+0 & 14+6 & 0+24 \\ 0+8 & 4+4 & 8-0 \\ 28-24 & -6+14 & 24-12 \end{bmatrix}^T$$

$$= \begin{bmatrix} 24 & 20 & 24 \\ 8 & 8 & 8 \\ 4 & 8 & 12 \end{bmatrix}^T$$

$$adj(adj(A)) = \begin{bmatrix} 24 & 8 & 4 \\ 20 & 8 & 8 \\ 24 & 8 & 12 \end{bmatrix}$$

$$A = \pm \frac{1}{\sqrt{|adj(A)|}} (adj(adj A))$$

$$= \pm \frac{1}{4} \begin{bmatrix} 24 & 8 & 4 \\ 20 & 8 & 8 \\ 24 & 8 & 12 \end{bmatrix}$$

$$= \pm \frac{\cancel{4}}{\cancel{4}} \begin{bmatrix} 6 & 2 & 1 \\ 5 & 2 & 2 \\ 6 & 2 & 3 \end{bmatrix}$$

$$A = \pm \begin{bmatrix} 6 & 2 & 1 \\ 5 & 2 & 2 \\ 6 & 2 & 3 \end{bmatrix}$$

9) If $adj(A) = \begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$ find A^{-1}

Soln:

$$|adj(A)| = \begin{vmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{vmatrix}$$

$$= \cancel{0(12+0)} + 2(36-18) + \cancel{0(0+6)}$$

$$= 2(18) = 36$$

$$\sqrt{|adj(A)|} = \sqrt{36} = 6$$

$$A^{-1} = \pm \frac{1}{\sqrt{|adj(A)|}} (adj(A))$$

$$A^{-1} = \pm \frac{1}{6} \begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$$

10) Find adj(adj(A)) if $adj\,A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

Soln:

$$adj(adj(A)) = \begin{bmatrix} 2 & 0 & 0 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 2 & 0 & 0 & 2 \end{bmatrix}^T$$

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$$= \begin{bmatrix} 2 & 0 & 2 \\ 0 & 2 & 0 \\ -2 & 0 & 2 \end{bmatrix}^T$$

$$adj(adj(A)) = \begin{bmatrix} 2 & 0 & -2 \\ 0 & 2 & 0 \\ 2 & 0 & 2 \end{bmatrix}$$

11) $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$ Show that $A^T A^{-1} = \begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$

Soln:

$$A^T = \begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & \tan x \\ -\tan x & 1 \end{vmatrix}$$

$$= 1 + \tan^2 x = \sec^2 x$$

$$|A| = \sec^2 x$$

$$adj\,A = \begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} (adj\,A)$$

$$= \frac{1}{\sec^2 x} \begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}$$

$$A^{-1} = \cos^2 x \begin{bmatrix} 1 & -\frac{\sin x}{\cos x} \\ \frac{\sin x}{\cos x} & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \cos^2 x & -\sin x \cos x \\ \sin x \cos x & \cos^2 x \end{bmatrix}$$

$$A^T A^{-1} = \begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix} \begin{bmatrix} \cos^2 x & -\sin x \cos x \\ \sin x \cos x & \cos^2 x \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -\frac{\sin x}{\cos x} \\ \frac{\sin x}{\cos x} & 1 \end{bmatrix} \begin{bmatrix} \cos^2 x & -\sin x\cos x \\ \sin x\cos x & \cos^2 x \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2 x - \sin^2 x & -\sin x\cos x - \sin x\cos x \\ \sin x\cos x + \sin x\cos x & \cos^2 x - \sin^2 x \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2x & -2\sin x\cos x \\ 2\sin x\cos x & \cos 2x \end{bmatrix}$$

$$A^T A^{-1} = \begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$$

12) Find the matrix A for which $A\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 14 & 7 \\ 7 & 7 \end{bmatrix}$

Soln:

Let $B = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$ and

$C = \begin{bmatrix} 14 & 7 \\ 7 & 7 \end{bmatrix}$

$AB = C$

$A = CB^{-1}$ —— (1)

$|B| = \begin{vmatrix} 5 & 3 \\ -1 & -2 \end{vmatrix} = -10+3$

$|B| = -7$

$\text{adj}\,B = \begin{bmatrix} -2 & -3 \\ 1 & 5 \end{bmatrix}$

$B^{-1} = \frac{1}{|B|}(\text{adj}\,B)$

$= \frac{1}{-7}\begin{bmatrix} -2 & -3 \\ 1 & 5 \end{bmatrix}$

$B^{-1} = \frac{1}{7}\begin{bmatrix} 2 & 3 \\ -1 & -5 \end{bmatrix}$

(1) ⇒

$A = \begin{bmatrix} 14 & 7 \\ 7 & 7 \end{bmatrix} \frac{1}{7}\begin{bmatrix} 2 & 3 \\ -1 & -5 \end{bmatrix}$

$= \cancel{7}\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \frac{1}{\cancel{7}}\begin{bmatrix} 2 & 3 \\ -1 & -5 \end{bmatrix}$

$= \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -1 & -5 \end{bmatrix}$

$= \begin{bmatrix} 4-1 & 6-5 \\ 2-1 & 3-5 \end{bmatrix}$

$A = \begin{bmatrix} 3 & 1 \\ 1 & -2 \end{bmatrix}$

13) Given $A = \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}$

$B = \begin{bmatrix} 3 & -2 \\ 1 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$

Find a matrix X such that $AXB = C$

Soln:

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$AXB = C$

$AX = CB^{-1}$

$X = A^{-1}CB^{-1}$ —— (1)

$|A| = \begin{vmatrix} 1 & -1 \\ 2 & 0 \end{vmatrix} = 0+2 = 2$

$\text{adj}\,A = \begin{bmatrix} 0 & 1 \\ -2 & 1 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|}(\text{adj}\,A)$

$A^{-1} = \frac{1}{2}\begin{bmatrix} 0 & 1 \\ -2 & 1 \end{bmatrix}$

$|B| = \begin{vmatrix} 3 & -2 \\ 1 & 1 \end{vmatrix} = 3+2 = 5$

$\text{adj}\,B = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$

$B^{-1} = \frac{1}{|B|}(\text{adj}\,B)$

$B^{-1} = \frac{1}{5}\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$

(1) ⇒

$X = \frac{1}{2}\begin{bmatrix} 0 & 1 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}\frac{1}{5}\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$

$= \frac{1}{10}\begin{bmatrix} 2 & 2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$

$= \frac{1}{10}\begin{bmatrix} 0 & 10 \\ 0 & 0 \end{bmatrix}$

$X = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

14) If $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

Show that

$A^{-1} = \frac{1}{2}(A^2 - 3I)$

Soln: $A^{-1} = \frac{1}{2}(A^2 - 3I)$

$2A^{-1} = A^2 - 3I$

Pre multiple by A on both sides,

$A^3 - 3A = 2I$

It is enough to prove

$A^3 - 3A = 2I$

$A^2 = AA = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

$A^2 = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$

$A^3 = A^2A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

$A^3 = \begin{bmatrix} 2 & 3 & 3 \\ 3 & 2 & 3 \\ 3 & 3 & 2 \end{bmatrix}$

$A^3 - 3A = \begin{bmatrix} 2 & 3 & 3 \\ 3 & 2 & 3 \\ 3 & 3 & 2 \end{bmatrix} - \begin{bmatrix} 0 & 3 & 3 \\ 3 & 0 & 3 \\ 3 & 3 & 0 \end{bmatrix}$

$A^3 - 3A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

$= 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$A^3 - 3A = 2I$

15) Decrypt the received encoded message $[2\ -3][20\ 4]$ with the encryption matrix $\begin{bmatrix} -1 & -1 \\ 2 & 1 \end{bmatrix}$ and the decryption matrix as its inverse, where the system of codes are described by the numbers 1 – 26 to the letters A – Z respectively and the number 0 to a blank space

Soln: Let $A = \begin{bmatrix} -1 & -1 \\ 2 & 1 \end{bmatrix}$

$|A| = \begin{vmatrix} -1 & -1 \\ 2 & 1 \end{vmatrix} = -1+2 = 1$

$\text{adj} A = \begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|}(\text{adj} A)$

$= \frac{1}{1}\begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix}$

	Coded row matrix	decoding matrix		decoded row matrix
I	$[2\ \ -3]$	$\begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix}$	=	$[8\ \ 5]$
II	$[20\ \ 4]$	$\begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix}$	=	$[12\ \ 16]$

∴ Decoded row matrix is $[8\ \ 5]$, $[12\ \ 16]$

Receiver Reads the Message as "HELP"

EX 1.1 If $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ Verify that $A(\text{adj}A) = (\text{adj}A)A = |A|I_3$

Soln: $|A| = \begin{vmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{vmatrix}$

$= 8(21-16)+6(-18+8)+2(24-14)$

$= 8(5)+6(-10)+2(10)$

$= 40-60+20 = 0$

$|A|I_3 = 0\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 0$ ——(1)

$[A_{ij}] = \begin{bmatrix} 7 & -4 & -6 & 7 \\ -4 & 3 & 2 & -4 \\ -6 & 2 & 8 & -6 \\ 7 & -4 & -6 & 7 \end{bmatrix}$

$= \begin{bmatrix} 21-16 & -8+18 & 24-14 \\ -8+18 & 24-4 & -12+32 \\ 24-14 & -12+32 & 56-36 \end{bmatrix}$

$= \begin{bmatrix} 5 & 10 & 10 \\ 10 & 20 & 20 \\ 10 & 20 & 20 \end{bmatrix}$

$\text{adj} A = [A_{ij}]^T = \begin{bmatrix} 5 & 10 & 10 \\ 10 & 20 & 20 \\ 10 & 20 & 20 \end{bmatrix}$

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$A(\text{adj}A) = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}\begin{bmatrix} 5 & 10 & 10 \\ 10 & 20 & 20 \\ 10 & 20 & 20 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$A(\text{adj}A) = |A| I_3$ —— (2)

$(\text{adj}A)A = \begin{bmatrix} 5 & 10 & 10 \\ 10 & 20 & 20 \\ 10 & 20 & 20 \end{bmatrix}\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$= 0\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$(\text{adj}A)A = |A| I_3$ —— (3)

from (1), (2) and (3) we get,

$A(\text{adj}A) = (\text{adj}A)A = |A| I_3$

EX 1.2 If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is non-singular, find A^{-1}

Soln: $|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$

$|A| = ad - bc \neq 0$

A is non-singular and hence A^{-1} exists.

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$\text{adj}A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$A^{-1} = \frac{1}{|A|}(\text{adj}A)$

$A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

EX 1.3 Find the inverse of the Matrix $\begin{bmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{bmatrix}$

Soln:

Let $A = \begin{bmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{bmatrix}$

$|A| = \begin{vmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{vmatrix}$

$= 2(9-2)+1(-15+3)+3(-10+9)$

$= 2(7)+1(-12)+3(-1)$

$= 14-12-3 = -1$

$|A| = -1 \neq 0$

A is non-singular and hence A^{-1} exists

$[A_{ij}] = \begin{bmatrix} 3 & 1 & -5 & 3 \\ 2 & 3 & -3 & 2 \\ -1 & 3 & 2 & -1 \\ 3 & 1 & -5 & 3 \end{bmatrix}$

$= \begin{bmatrix} 9-2 & -3+15 & -10+9 \\ 6+3 & 6+9 & 3-4 \\ -1-9 & -15-2 & 6-5 \end{bmatrix}$

$= \begin{bmatrix} 7 & 12 & -1 \\ 9 & 15 & -1 \\ -10 & -17 & 1 \end{bmatrix}$

$\text{adj}A = [A_{ij}]^T = \begin{bmatrix} 7 & 9 & -10 \\ 12 & 15 & -17 \\ -1 & -1 & 1 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|}(\text{adj}A)$

$= \frac{1}{-1}\begin{bmatrix} 7 & 9 & -10 \\ 12 & 15 & -17 \\ -1 & -1 & 1 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} -7 & -9 & 10 \\ -12 & -15 & 17 \\ 1 & 1 & -1 \end{bmatrix}$

EX 1.4 If A is a non-singular matrix of odd order, Prove that $|\text{adj}A|$ is Positive

Soln: Let A be a non-singular matrix of order $2m+1$. where $m = 0, 1, 2 \ldots\ldots$

Given: $|A| \neq 0$,

By Property,

$|\text{adj}A| = |A|^{n-1}$

Put $n = 2m+1$

$|\text{adj}A| = |A|^{(2m+1)-1}$

$|\text{adj}A| = |A|^{2m}$ is always positive

$\therefore |\text{adj}A|$ is positive

EX 1.5 Find a matrix A if $\text{adj}(A) = \begin{bmatrix} 7 & 7 & -7 \\ -1 & 11 & 7 \\ 11 & 5 & 7 \end{bmatrix}$

Soln:

$$|adj(A)| = \begin{vmatrix} 7 & 7 & -7 \\ -1 & 11 & 7 \\ 11 & 5 & 7 \end{vmatrix}$$

$$= 7(77-35) - 7(-7-77) - 7(-5-121)$$

$$= 7(42) - 7(-84) - 7(-126)$$

$$= 294 + 588 + 882 = 1764$$

$$|adj(A)| = 1764$$

$$\sqrt{|adj(A)|} = \sqrt{1764} = 42$$

$$adj(adj(A)) = \begin{bmatrix} 11 & 7 & -1 & 11 \\ 5 & 7 & 11 & 5 \\ 7 & -7 & 7 & 7 \\ 11 & 7 & -1 & 11 \end{bmatrix}^T$$

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$$= \begin{bmatrix} 77-35 & 77+7 & -5-121 \\ -35-49 & 49+77 & 77-35 \\ 49+77 & 7-49 & 77+7 \end{bmatrix}^T$$

$$= \begin{bmatrix} 42 & 84 & -126 \\ -84 & 126 & 42 \\ 126 & -42 & 84 \end{bmatrix}^T$$

$$adj(adj(A)) = \begin{bmatrix} 42 & -84 & 126 \\ 84 & 126 & -42 \\ -126 & 42 & 84 \end{bmatrix}$$

$$A = \pm \frac{1}{\sqrt{|adj(A)|}} (adj(adj A))$$

$$= \pm \frac{1}{42} \begin{bmatrix} 42 & -84 & 126 \\ 84 & 126 & -42 \\ -126 & 42 & 84 \end{bmatrix}$$

$$= \pm \frac{\cancel{42}}{\cancel{42}} \begin{bmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ -3 & 1 & 2 \end{bmatrix}$$

$$A = \pm \begin{bmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ -3 & 1 & 2 \end{bmatrix}$$

Ex 1.6 If $adj A = \begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ find A^{-1}.

Soln:

$$|adj A| = \begin{vmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix}$$

$$= -1(1-4) - 2(1-4) + 2(2-2)$$

$$= -1(-3) - 2(-3) + 2(0)$$

$$= 3 + 6 = 9$$

$$|adj A| = 9$$

$$A^{-1} = \pm \frac{1}{\sqrt{|adj A|}} (adj A)$$

$$= \pm \frac{1}{\sqrt{9}} \begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

$$A^{-1} = \pm \frac{1}{3} \begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

Ex 1.7 If A is Symmetric Prove that then adjA is also Symmetric

Soln: A is Symmetric

Then $A^T = A$

By Property,

$$adj(A^T) = (adj A)^T$$

Put $A^T = A$

$$adj A = (adj A)^T$$

$\Rightarrow adj A$ is Symmetric

Ex 1.8 Verify the Property $(A^T)^{-1} = (A^{-1})^T$ with $A = \begin{bmatrix} 2 & 9 \\ 1 & 7 \end{bmatrix}$

Soln: $|A| = \begin{vmatrix} 2 & 9 \\ 1 & 7 \end{vmatrix} = 14 - 9$

$$|A| = 5 \neq 0$$

A is non-singular and hence A^{-1} exists

$$adj A = \begin{bmatrix} 7 & -9 \\ -1 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}(adj A)$$

$$A^{-1} = \frac{1}{5} \begin{bmatrix} 7 & -9 \\ -1 & 2 \end{bmatrix}$$

$$(A^{-1})^T = \frac{1}{5} \begin{bmatrix} 7 & -1 \\ -9 & 2 \end{bmatrix} \quad -(1)$$

$$A^T = \begin{bmatrix} 2 & 1 \\ 9 & 7 \end{bmatrix}$$

$$|A^T| = \begin{vmatrix} 2 & 1 \\ 9 & 7 \end{vmatrix} = 14 - 9 = 5$$

$$adj A^T = \begin{bmatrix} 7 & -1 \\ -9 & 2 \end{bmatrix}$$

$(A^T)^{-1} = \frac{1}{|A^T|}(\text{adj } A^T)$

$(A^T)^{-1} = \frac{1}{5}\begin{bmatrix} 7 & -1 \\ -9 & 2 \end{bmatrix}$ — ②

from ①&② we get,

$(A^{-1})^T = (A^T)^{-1}$

EX 1.9 verify $(AB)^{-1} = B^{-1}A^{-1}$ with $A = \begin{bmatrix} 0 & -3 \\ 1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} -2 & -3 \\ 0 & -1 \end{bmatrix}$

Soln:

$AB = \begin{bmatrix} 0 & -3 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} -2 & -3 \\ 0 & -1 \end{bmatrix}$

$AB = \begin{bmatrix} 0 & 3 \\ -2 & -7 \end{bmatrix}$

$|AB| = \begin{vmatrix} 0 & 3 \\ -2 & -7 \end{vmatrix} = 0+6 = 6$

$|AB| = 6 \neq 0$

AB is non-singular and hence $(AB)^{-1}$ exists

$\text{adj } AB = \begin{bmatrix} -7 & -3 \\ 2 & 0 \end{bmatrix}$

$(AB)^{-1} = \frac{1}{|AB|}(\text{adj } AB)$

$(AB)^{-1} = \frac{1}{6}\begin{bmatrix} -7 & -3 \\ 2 & 0 \end{bmatrix}$ — ①

$|B| = \begin{vmatrix} -2 & -3 \\ 0 & -1 \end{vmatrix} = 2+0 = 2$

$\text{adj } B = \begin{bmatrix} -1 & 3 \\ 0 & -2 \end{bmatrix}$

$B^{-1} = \frac{1}{|B|}(\text{adj } B)$

$B^{-1} = \frac{1}{2}\begin{bmatrix} -1 & 3 \\ 0 & -2 \end{bmatrix}$

Also, $|A| = \begin{vmatrix} 0 & -3 \\ 1 & 4 \end{vmatrix} = 0+3 = 3$

$|A| = 3$

$\text{adj } A = \begin{bmatrix} 4 & 3 \\ -1 & 0 \end{bmatrix}$

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$A^{-1} = \frac{1}{|A|}(\text{adj } A)$

$A^{-1} = \frac{1}{3}\begin{bmatrix} 4 & 3 \\ -1 & 0 \end{bmatrix}$

$B^{-1}A^{-1} = \frac{1}{2}\begin{bmatrix} -1 & 3 \\ 0 & -2 \end{bmatrix}\frac{1}{3}\begin{bmatrix} 4 & 3 \\ -1 & 0 \end{bmatrix}$

$B^{-1}A^{-1} = \frac{1}{6}\begin{bmatrix} -1 & 3 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 4 & 3 \\ -1 & 0 \end{bmatrix}$

$B^{-1}A^{-1} = \frac{1}{6}\begin{bmatrix} -7 & -3 \\ 2 & 0 \end{bmatrix}$ — ②

from ①&② we get,

$(AB)^{-1} = B^{-1}A^{-1}$

EX 1.10

If $A = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$ find x and y such that $A^2 + xA + yI_2 = 0$ Hence, find A^{-1}

Soln: $A^2 = AA = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$

$A^2 = \begin{bmatrix} 22 & 27 \\ 18 & 31 \end{bmatrix}$

$A^2 + xA + yI_2 = 0$

$\begin{bmatrix} 22 & 27 \\ 18 & 31 \end{bmatrix} + x\begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix} + y\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\begin{bmatrix} 22+4x+y & 27+3x \\ 18+2x & 31+5x+y \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$18+2x = 0 \Rightarrow 2x = -18$

$\boxed{x = -9}$

$31+5x+y = 0$

$31+5(-9)+y = 0$

$31-45+y = 0$

$-14+y = 0$

$\boxed{y = 14}$

$A^2 + xA + yI_2 = 0$

$A^2 - 9A + 14I_2 = 0$

Multiple by A^{-1},

$A - 9I + 14A^{-1} = 0$

$14A^{-1} = 9I - A$

$A^{-1} = \frac{1}{14}(9I - A)$

$$A^{-1} = \frac{1}{14}\left[9\begin{bmatrix}1&0\\0&1\end{bmatrix} - \begin{bmatrix}4&3\\2&5\end{bmatrix}\right]$$

$$= \frac{1}{14}\left[\begin{bmatrix}9&0\\0&9\end{bmatrix} - \begin{bmatrix}4&3\\2&5\end{bmatrix}\right]$$

$$A^{-1} = \frac{1}{14}\begin{bmatrix}5&-3\\-2&4\end{bmatrix}$$

EX 1.11 Prove that $\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$ is orthogonal

Soln: Let $A = \begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$

$$A^T = \begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$$

$$AA^T = \begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$$

$$= \begin{bmatrix}\cos^2\theta+\sin^2\theta & \sin\theta\cos\theta-\sin\theta\cos\theta\\ \sin\theta\cos\theta-\sin\theta\cos\theta & \sin^2\theta+\cos^2\theta\end{bmatrix}$$

$$= \begin{bmatrix}1&0\\0&1\end{bmatrix}$$

$AA^T = I_2$

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$|||^{ly}$ $A^TA = I_2$

$\therefore AA^T = A^TA = I_2$

$\therefore$ A is orthogonal.

EX 1.12

If $A = \frac{1}{7}\begin{bmatrix}6&-3&a\\b&-2&6\\2&c&3\end{bmatrix}$ is orthogonal, find a, b and c and hence A^{-1}

Soln: If A is orthogonal then $AA^T = A^TA = I_3$

Now, $AA^T = I_3$

$$\frac{1}{7}\begin{bmatrix}6&-3&a\\b&-2&6\\2&c&3\end{bmatrix}\frac{1}{7}\begin{bmatrix}6&b&2\\-3&-2&c\\a&6&3\end{bmatrix} = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$\frac{1}{49}\begin{bmatrix}36+9+a^2 & 6b+6+6a & 12-3c+3a\\ 6b+6+6a & b^2+4+36 & 2b-2c+18\\ 12-3c+3a & 2b-2c+18 & 4+c^2+9\end{bmatrix} = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$\begin{bmatrix}45+a^2 & 6b+6+6a & 12-3c+3a\\ 6b+6+6a & 40+b^2 & 2b-2c+18\\ 12-3c+3a & 2b-2c+18 & 13+c^2\end{bmatrix} = 49\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$= \begin{bmatrix}49&0&0\\0&49&0\\0&0&49\end{bmatrix}$$

$45+a^2 = 49$, $40+b^2 = 49$ and $13+c^2 = 49$

$a^2 = 49-45 = 4 \Rightarrow \boxed{a=\pm 2}$

$b^2 = 49-40 = 9 \Rightarrow \boxed{b=\pm 3}$

$c^2 = 49-13 = 36 \Rightarrow \boxed{c=\pm 6}$

$6b+6+6a = 0$

$6b+6a = -6$

$\div 6$, $\boxed{b+a=-1}$ —— (1)

$12-3c+3a = 0$

$3c-3a = 12$

$\div 3$, $\boxed{c-a=4}$ —— (2)

$2b-2c+18 = 0$

$2c-2b = 18$

$\div 2$, $\boxed{c-b=9}$ —— (3)

(1)+(2)+(3) $\Rightarrow 2c = 12$

$\boxed{c=6}$

(2) $\Rightarrow 6-a = 4$

$a = 6-4 = 2$

$\boxed{a=2}$

(1) $\Rightarrow b+2 = -1$

$b = -1-2 = -3$

$\boxed{b=-3}$

$\therefore a=2, b=-3, c=6$

$$A = \frac{1}{7}\begin{bmatrix}6&-3&2\\-3&-2&6\\2&6&3\end{bmatrix}$$

WKT, $A^{-1} = A^T$

$$A^{-1} = \frac{1}{7}\begin{bmatrix}6 & -3 & 2\\ -3 & -2 & 6\\ 2 & 6 & 3\end{bmatrix}$$

1.2.5 Application of Matrices to Cryptography

Let the encoding Matrix be $A = \begin{bmatrix}1 & -1 & 1\\ 2 & -1 & 0\\ 1 & 0 & 0\end{bmatrix}$

Let the message to be send by the sender be "WELCOME". But Matrix order is 3

∴ (WEL), (COM), (E _ _)

[23 5 12], [3 15 13], [5 0 0]

Uncoded row matrix	Encoding Matrix	Coded row Matrix

$$\begin{bmatrix}23 & 5 & 12\end{bmatrix}\begin{bmatrix}1 & -1 & 1\\ 2 & -1 & 0\\ 1 & 0 & 0\end{bmatrix} = \begin{bmatrix}45 & -28 & 23\end{bmatrix}$$

$$\begin{bmatrix}3 & 15 & 13\end{bmatrix}\begin{bmatrix}1 & -1 & 1\\ 2 & -1 & 0\\ 1 & 0 & 0\end{bmatrix} = \begin{bmatrix}46 & -28 & 3\end{bmatrix}$$

$$\begin{bmatrix}5 & 0 & 0\end{bmatrix}\begin{bmatrix}1 & -1 & 1\\ 2 & -1 & 0\\ 1 & 0 & 0\end{bmatrix} = \begin{bmatrix}5 & -5 & 5\end{bmatrix}$$

∴ The encoded Message is [45 −28 23], [46 −28 3] and [5 −5 5]

Let the decoding Matrix be A^{-1}

$$|A| = \begin{vmatrix}1 & -1 & 1\\ 2 & -1 & 0\\ 1 & 0 & 0\end{vmatrix}$$

$$= 1(\cancel{0-0}) + 1(\cancel{0-0}) + 1(0+1)$$

$|A| = 1 \neq 0$

A is non-singular and hence A^{-1} exists

$$[A_{ij}] = \begin{bmatrix}0 & 0 & 1\\ 0 & -1 & -1\\ 1 & 2 & 1\end{bmatrix}$$

$$\text{adj}A = [A_{ij}]^T = \begin{bmatrix}0 & 0 & 1\\ 0 & -1 & 2\\ 1 & -1 & 1\end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}(\text{adj}A) = \frac{1}{1}\begin{bmatrix}0 & 0 & 1\\ 0 & -1 & 2\\ 1 & -1 & 1\end{bmatrix}$$

$$A^{-1} = \begin{bmatrix}0 & 0 & 1\\ 0 & -1 & 2\\ 1 & -1 & 1\end{bmatrix}$$

Coded row matrix	Decoding Matrix	Decoded row matrix

$$\begin{bmatrix}45 & -28 & 23\end{bmatrix}\begin{bmatrix}0 & 0 & 1\\ 0 & -1 & 2\\ 1 & -1 & 1\end{bmatrix} = \begin{bmatrix}23 & 5 & 12\end{bmatrix}$$

$$\begin{bmatrix}46 & -18 & 3\end{bmatrix}\begin{bmatrix}0 & 0 & 1\\ 0 & -1 & 2\\ 1 & -1 & 1\end{bmatrix} = \begin{bmatrix}3 & 15 & 13\end{bmatrix}$$

$$\begin{bmatrix}5 & -5 & 5\end{bmatrix}\begin{bmatrix}0 & 0 & 1\\ 0 & -1 & 2\\ 1 & -1 & 1\end{bmatrix} = \begin{bmatrix}5 & 0 & 0\end{bmatrix}$$

The Sequence of decoded row matrices are [23 5 12], [3 15 13] and [5 0 0]

The receiver reads the Message as "WELCOME"

1.3 Elementary transformations of a Matrix

EXERCISE 1.2

1) Find the rank of the following matrices by minor method

(i) $\begin{bmatrix}2 & -4\\ -1 & 2\end{bmatrix}$

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Soln:

Let $A = \begin{bmatrix}2 & -4\\ -1 & 2\end{bmatrix}$

$\rho(A) \leq \min\{2,2\} = 2$

$\therefore \rho(A) \leq 2$

$\begin{vmatrix}2 & -4\\ -1 & 2\end{vmatrix} = 4-4 = 0$

$\therefore \rho(A) = 1$

(ii) $\begin{bmatrix} -1 & 3 \\ 4 & -7 \\ 3 & -4 \end{bmatrix}$

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Soln:

Let $A = \begin{bmatrix} -1 & 3 \\ 4 & -7 \\ 3 & -4 \end{bmatrix}$

$\rho(A) \leq \min\{3,2\} = 2$

$\rho(A) \leq 2$

$\begin{vmatrix} -1 & 3 \\ 4 & -7 \end{vmatrix} = 7-12 = -5 \neq 0$

$\therefore \rho(A) = 2$

(iii) $\begin{bmatrix} 1 & -2 & -1 & 0 \\ 3 & -6 & -3 & 1 \end{bmatrix}$

Soln:

Let $A = \begin{bmatrix} 1 & -2 & -1 & 0 \\ 3 & -6 & -3 & 1 \end{bmatrix}$

$\rho(A) \leq \min\{2,4\} = 2$

$\rho(A) \leq 2$

$\begin{vmatrix} 1 & -2 \\ 3 & -6 \end{vmatrix} = -6+6 = 0$

But $\begin{vmatrix} -1 & 0 \\ -3 & 1 \end{vmatrix} = -1 \neq 0$

$\therefore \rho(A) = 2$

(iv) $\begin{bmatrix} 1 & -2 & 3 \\ 2 & 4 & -6 \\ 5 & 1 & -1 \end{bmatrix}$

Soln:

Let $A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 4 & -6 \\ 5 & 1 & -1 \end{bmatrix}$

$\rho(A) \leq \min\{3,3\} = 3$

$\rho(A) \leq 3$

$\begin{vmatrix} 1 & -2 & 3 \\ 2 & 4 & -6 \\ 5 & 1 & -1 \end{vmatrix} = 1(-4+6)+2(-2+30)+3(2-20)$

$= 2+56-54 = 4 \neq 0$

$\therefore \rho(A) = 3$

(v) $\begin{bmatrix} 0 & 1 & 2 & 1 \\ 0 & 2 & 4 & 3 \\ 8 & 1 & 0 & 2 \end{bmatrix}$

Soln:

Let $A = \begin{bmatrix} 0 & 1 & 2 & 1 \\ 0 & 2 & 4 & 3 \\ 8 & 1 & 0 & 2 \end{bmatrix}$

$\rho(A) \leq \min\{3,4\} = 3$

$\rho(A) \leq 3$

$\begin{vmatrix} 0 & 1 & 2 \\ 0 & 2 & 4 \\ 8 & 1 & 0 \end{vmatrix} = 0\cancel{(4-4)} - 1(0-32) + 2(0-16)$

$= 32-32 = 0$

But $\begin{vmatrix} 1 & 2 & 1 \\ 2 & 4 & 3 \\ 1 & 0 & 2 \end{vmatrix} = 1(8-0) - 2(4-3) + 1(0-4)$

$= 8-2-4 = 2 \neq 0$

$\therefore \rho(A) = 3$

2) Find the rank of the following matrices by row reduction method:

(i) $\begin{bmatrix} 1 & 1 & 1 & 3 \\ 2 & -1 & 3 & 4 \\ 5 & -1 & 7 & 11 \end{bmatrix}$

Soln:

Let $A = \begin{bmatrix} 1 & 1 & 1 & 3 \\ 2 & -1 & 3 & 4 \\ 5 & -1 & 7 & 11 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & -3 & 1 & -2 \\ 0 & -6 & 2 & -4 \end{bmatrix}$ $R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 5R_1$

$\sim \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & -3 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $R_3 \to R_3 - 2R_2$

$\therefore \rho(A) = 2$

(ii) $\begin{bmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 1 & -2 & 3 \\ 1 & -1 & 1 \end{bmatrix}$

Soln:

Let $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 1 & -2 & 3 \\ 1 & -1 & 1 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 2 & -1 \\ 0 & -7 & 5 \\ 0 & -4 & 4 \\ 0 & -3 & 2 \end{bmatrix}$ $R_2 \to R_2 - 3R_1$, $R_3 \to R_3 - R_1$, $R_4 \to R_4 - R_1$

$\sim \begin{bmatrix} 1 & 2 & -1 \\ 0 & -7 & 5 \\ 0 & 0 & 8 \\ 0 & 0 & -4 \end{bmatrix}$ $R_3 \to 7R_3 - 4R_2$, $R_4 \to 4R_4 - 3R_3$

$\sim \begin{bmatrix} 1 & 2 & -1 \\ 0 & -7 & 5 \\ 0 & 0 & 8 \\ 0 & 0 & 0 \end{bmatrix}$ $R_4 \to 2R_4 + R_3$

$\therefore \rho(A) = 3$

(iii) $\begin{bmatrix} 3 & -8 & 5 & 2 \\ 2 & -5 & 1 & 4 \\ -1 & 2 & 3 & -2 \end{bmatrix}$

Soln:

Let $A = \begin{bmatrix} 3 & -8 & 5 & 2 \\ 2 & -5 & 1 & 4 \\ -1 & 2 & 3 & -2 \end{bmatrix}$

$\sim \begin{bmatrix} -1 & 2 & 3 & -2 \\ 2 & -5 & 1 & 4 \\ 3 & -8 & 5 & 2 \end{bmatrix} R_1 \leftrightarrow R_3$

$\sim \begin{bmatrix} 1 & -2 & -3 & 2 \\ 2 & -5 & 1 & 4 \\ 3 & -8 & 5 & 2 \end{bmatrix} R_1 \to (-1)R_1$

$\sim \begin{bmatrix} 1 & -2 & -3 & 2 \\ 0 & -1 & 7 & 0 \\ 0 & -2 & 14 & -4 \end{bmatrix} \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{matrix}$

$\sim \begin{bmatrix} 1 & -2 & -3 & 2 \\ 0 & -1 & 7 & 0 \\ 0 & 0 & 0 & -4 \end{bmatrix} R_3 \to R_3 - 2R_2$

$\therefore \rho(A) = 3$

3) Find the inverse of each of the following by Gauss-Jordan method:

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(i) $\begin{bmatrix} 2 & -1 \\ 5 & -2 \end{bmatrix}$

Soln: Let $A = \begin{bmatrix} 2 & -1 \\ 5 & -2 \end{bmatrix}$

$[A \mid I_2] = \left[\begin{array}{cc|cc} 2 & -1 & 1 & 0 \\ 5 & -2 & 0 & 1 \end{array}\right]$

$\sim \left[\begin{array}{cc|cc} 1 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 5 & -2 & 0 & 1 \end{array}\right] R_1 \to \frac{R_1}{2}$

$\sim \left[\begin{array}{cc|cc} 1 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & -\frac{5}{2} & 1 \end{array}\right] R_2 \to R_2 - 5R_1$

$\sim \left[\begin{array}{cc|cc} 1 & 0 & -2 & 1 \\ 0 & \frac{1}{2} & -\frac{5}{2} & 1 \end{array}\right] R_1 \to R_1 + R_2$

$\sim \left[\begin{array}{cc|cc} 1 & 0 & -2 & 1 \\ 0 & 1 & -5 & 2 \end{array}\right] R_2 \to 2R_2$

$\therefore A^{-1} = \begin{bmatrix} -2 & 1 \\ -5 & 2 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 6 & -2 & -3 \end{bmatrix}$

Soln:

Let $A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 6 & -2 & -3 \end{bmatrix}$

$[A \mid I_3] = \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 1 & 0 \\ 6 & -2 & -3 & 0 & 0 & 1 \end{array}\right]$

$\sim \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 4 & -3 & -6 & 0 & 1 \end{array}\right] \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - 6R_1 \end{matrix}$

$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 4 & -3 & -6 & 0 & 1 \end{array}\right] R_1 \to R_1 + R_2$

$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & -2 & -4 & 1 \end{array}\right] R_3 \to R_3 - 4R_2$

$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & -3 & 1 \\ 0 & 1 & 0 & -3 & -3 & 1 \\ 0 & 0 & 1 & -2 & -4 & 1 \end{array}\right] \begin{matrix} R_1 \to R_1 + R_3 \\ R_2 \to R_2 + R_3 \end{matrix}$

$\therefore A^{-1} = \begin{bmatrix} -2 & -3 & 1 \\ -3 & -3 & 1 \\ -2 & -4 & 1 \end{bmatrix}$

(iii) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$

Soln:

Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$

$[A \mid I_3] = \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 5 & 3 & 0 & 1 & 0 \\ 1 & 0 & 8 & 0 & 0 & 1 \end{array}\right]$

$\sim \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & -2 & 5 & -1 & 0 & 1 \end{array}\right] \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$

$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 8 & 0 & 0 & 1 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & -2 & 5 & -1 & 0 & 1 \end{array}\right] R_1 \to R_1 + R_3$

$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 8 & 0 & 0 & 1 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 0 & -1 & -5 & 2 & 1 \end{array}\right] R_3 \to R_3 + 2R_2$

$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -40 & 16 & 9 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 0 & -1 & -5 & 2 & 1 \end{array}\right] R_1 \to R_1 + 8R_3$

$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -40 & 16 & 9 \\ 0 & 1 & 0 & 13 & -5 & -3 \\ 0 & 0 & -1 & -5 & 2 & 1 \end{array}\right] R_2 \to R_2 - 3R_3$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -40 & 16 & 9 \\ 0 & 1 & 0 & 13 & -5 & -3 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right] \quad R_3 \to (-1)R_3$$

$$\therefore A^{-1} = \begin{bmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{bmatrix}$$

EX 1.13

Reduce the Matrix $\begin{bmatrix} 3 & -1 & 2 \\ -6 & 2 & 4 \\ -3 & 1 & 2 \end{bmatrix}$ to a row-echelon form

Soln:

Let $A = \begin{bmatrix} 3 & -1 & 2 \\ -6 & 2 & 4 \\ -3 & 1 & 2 \end{bmatrix}$

$$\sim \begin{bmatrix} 1 & -\frac{1}{3} & \frac{2}{3} \\ -6 & 2 & 4 \\ -3 & 1 & 2 \end{bmatrix} \quad R_1 \to \frac{R_1}{3}$$

$$\sim \begin{bmatrix} 1 & -\frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 8 \\ 0 & 0 & 4 \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 + 6R_1 \\ R_3 \to R_3 + 3R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & -\frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 8 \\ 0 & 0 & 0 \end{bmatrix} \quad R_3 \to 2R_3 - R_2$$

This is also a row-echelon form of the given matrix

EX 1.14

Reduce the Matrix $\begin{bmatrix} 0 & 3 & 1 & 6 \\ -1 & 0 & 2 & 5 \\ 4 & 2 & 0 & 0 \end{bmatrix}$ to row-echelon form

Soln: Let $A = \begin{bmatrix} 0 & 3 & 1 & 6 \\ -1 & 0 & 2 & 5 \\ 4 & 2 & 0 & 0 \end{bmatrix}$

$$\sim \begin{bmatrix} 4 & 2 & 0 & 0 \\ -1 & 0 & 2 & 5 \\ 0 & 3 & 1 & 6 \end{bmatrix} \quad R_1 \leftrightarrow R_3$$

$$\sim \begin{bmatrix} 1 & \frac{1}{2} & 0 & 0 \\ -1 & 0 & 2 & 5 \\ 0 & 3 & 1 & 6 \end{bmatrix} \quad R_1 \to \frac{R_1}{4}$$

$$\sim \begin{bmatrix} 1 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 2 & 5 \\ 0 & 3 & 1 & 6 \end{bmatrix} \quad R_2 \to R_2 + R_1$$

$$\sim \begin{bmatrix} 1 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 4 & 10 \\ 0 & 3 & 1 & 6 \end{bmatrix} \quad R_2 \to 2R_2$$

$$\sim \begin{bmatrix} 1 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 4 & 10 \\ 0 & 0 & -11 & -24 \end{bmatrix} \quad R_3 \to R_3 - 3R_2$$

This is row-echelon form of given matrix

EX 1.15

Find the rank of each of the following Matrices (i) $\begin{bmatrix} 3 & 2 & 5 \\ 1 & 1 & 2 \\ 3 & 3 & 6 \end{bmatrix}$ (ii) $\begin{bmatrix} 4 & 3 & 1 & -2 \\ -3 & -1 & -2 & 4 \\ 6 & 7 & -1 & 2 \end{bmatrix}$

Soln:

(i) Let $A = \begin{bmatrix} 3 & 2 & 5 \\ 1 & 1 & 2 \\ 3 & 3 & 6 \end{bmatrix}$

$\rho(A) \leq \min\{3,3\} = 3$

$\rho(A) \leq 3$

$$\begin{vmatrix} 3 & 2 & 5 \\ 1 & 1 & 2 \\ 3 & 3 & 6 \end{vmatrix} = 3(6-6) - 2(6-6) + 5(3-3) = 0$$

But $\begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} = 3 - 2 = 1 \neq 0$

$\therefore \rho(A) = 2$

(ii) Let $A = \begin{bmatrix} 4 & 3 & 1 & -2 \\ -3 & -1 & -2 & 4 \\ 6 & 7 & -1 & 2 \end{bmatrix}$

$\rho(A) \leq \min\{3,4\} = 3$

$\rho(A) \leq 3$

$$\begin{vmatrix} 4 & 3 & 1 \\ -3 & -1 & -2 \\ 6 & 7 & -1 \end{vmatrix} = 4(1+14) - 3(3+12) + 1(-21+6)$$

$= 4(15) - 3(15) + 1(-15)$

$= 60 - 45 - 15 = 0$

III^{ly} $\begin{vmatrix} 4 & 3 & -2 \\ -3 & -1 & 4 \\ 6 & 7 & 2 \end{vmatrix} = 0$

$\begin{vmatrix} 4 & 1 & -2 \\ -3 & -2 & 4 \\ 6 & -1 & 2 \end{vmatrix} = 0$

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$\begin{vmatrix} 3 & 1 & -2 \\ -1 & -2 & 4 \\ 7 & -1 & 2 \end{vmatrix} = 0$

But $\begin{vmatrix} 4 & 3 \\ -3 & -1 \end{vmatrix} = -4 + 9 = 5 \neq 0$

$\therefore \rho(A) = 2$

Ex 1.16

Find the rank of the following matrices which are in row-echelon form

(i) $\begin{bmatrix} 2 & 0 & -7 \\ 0 & 3 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ (ii) $\begin{bmatrix} -2 & 2 & -1 \\ 0 & 5 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

(iii) $\begin{bmatrix} 6 & 0 & -9 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Soln:

(i) Let $A = \begin{bmatrix} 2 & 0 & -7 \\ 0 & 3 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

$\rho(A) \le \min\{3,3\} = 3$

$\rho(A) \le 3$

$\begin{vmatrix} 2 & 0 & -7 \\ 0 & 3 & 1 \\ 0 & 0 & 1 \end{vmatrix} = 2(3-0) - 0(\cancel{0}-0) - 7(\cancel{0}-0)$

$= 6 \ne 0$

$\therefore \rho(A) = 3$

(ii) Let $A = \begin{bmatrix} -2 & 2 & -1 \\ 0 & 5 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

$\rho(A) \le \min\{3,3\} = 3$

$\rho(A) \le 3$

$\begin{vmatrix} -2 & 2 & -1 \\ 0 & 5 & 1 \\ 0 & 0 & 0 \end{vmatrix} = -2(0-0) - 2(0-0) - 1(0-0)$

$= 0$

But $\begin{vmatrix} -2 & 2 \\ 0 & 5 \end{vmatrix} = -10 \ne 0$

$\therefore \rho(A) = 2$

(iii) Let $A = \begin{bmatrix} 6 & 0 & -9 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\rho(A) \le \min\{4,3\} = 3$

$\rho(A) \le 3$

$\begin{vmatrix} 6 & 0 & -9 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{vmatrix} = (6)(2)(0) = 0$

But $\begin{vmatrix} 6 & 0 \\ 0 & 2 \end{vmatrix} = 12 \ne 0$

$\therefore \rho(A) = 2$

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Ex 1.17

Find the rank of the Matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 0 & 5 \end{bmatrix}$ by reducing it to a row-echelon form

Soln:

Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 0 & 5 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -2 \\ 0 & -6 & -4 \end{bmatrix} \begin{matrix} \\ R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{matrix}$

$\sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -2 \\ 0 & 0 & 0 \end{bmatrix} R_3 \to R_3 - 2R_2$

$\therefore \rho(A) = 2$

Ex 1.18 Find the rank of the Matrix $\begin{bmatrix} 2 & -2 & 4 & 3 \\ -3 & 4 & -2 & -1 \\ 6 & 2 & -1 & 7 \end{bmatrix}$

Soln:

Let $A = \begin{bmatrix} 2 & -2 & 4 & 3 \\ -3 & 4 & -2 & -1 \\ 6 & 2 & -1 & 7 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & -1 & 2 & \frac{3}{2} \\ -3 & 4 & -2 & -1 \\ 6 & 2 & -1 & 7 \end{bmatrix} R_1 \to \frac{R_1}{2}$

$\sim \begin{bmatrix} 1 & -1 & 2 & \frac{3}{2} \\ 0 & 1 & 4 & \frac{7}{2} \\ 0 & 8 & -13 & -2 \end{bmatrix} \begin{matrix} \\ R_2 \to R_2 + 3R_1 \\ R_3 \to R_3 - 6R_1 \end{matrix}$

$\sim \begin{bmatrix} 1 & -1 & 2 & \frac{3}{2} \\ 0 & 2 & 8 & 7 \\ 0 & 8 & -13 & -2 \end{bmatrix} R_2 \to 2R_2$

$\sim \begin{bmatrix} 1 & -1 & 2 & \frac{3}{2} \\ 0 & 2 & 8 & 7 \\ 0 & 0 & -45 & -30 \end{bmatrix} R_3 \to R_3 - 4R_2$

$\sim \begin{bmatrix} 1 & -1 & 2 & \frac{3}{2} \\ 0 & 2 & 8 & 7 \\ 0 & 0 & 3 & 2 \end{bmatrix} R_3 \to \frac{R_3}{-15}$

$\therefore \rho(A) = 3$

Ex 1.19 Show that the

Matrix $\begin{bmatrix} 3 & 1 & 4 \\ 2 & 0 & -1 \\ 5 & 2 & 1 \end{bmatrix}$ is non-singular and reduces it to the identity matrix by elementary row transformations

Soln:

Let $A = \begin{bmatrix} 3 & 1 & 4 \\ 2 & 0 & -1 \\ 5 & 2 & 1 \end{bmatrix}$

$$|A| = \begin{vmatrix} 3 & 1 & 4 \\ 2 & 0 & -1 \\ 5 & 2 & 1 \end{vmatrix}$$

$= 3(0+2) - 1(2+5) + 4(4-0)$

$= 6 - 7 + 16 = 15 \neq 0$

$\therefore A$ is non-singular

$$A = \begin{bmatrix} 3 & 1 & 4 \\ 2 & 0 & -1 \\ 5 & 2 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & \frac{1}{3} & \frac{4}{3} \\ 2 & 0 & -1 \\ 5 & 2 & 1 \end{bmatrix} R_1 \to \frac{R_1}{3}$$

$$\sim \begin{bmatrix} 1 & \frac{1}{3} & \frac{4}{3} \\ 0 & -2/3 & -11/3 \\ 0 & 1/3 & -17/3 \end{bmatrix} \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 5R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1/3 & 4/3 \\ 0 & 1 & 11/2 \\ 0 & 1/3 & -17/3 \end{bmatrix} R_2 \to R_2\left(\frac{-3}{2}\right)$$

$$\sim \begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & 11/2 \\ 0 & 1/3 & -17/3 \end{bmatrix} R_1 \to R_1 - R_3$$

$$\sim \begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & 11/2 \\ 0 & 1 & -17 \end{bmatrix} R_3 \to 3R_3$$

$$\sim \begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & 11/2 \\ 0 & 0 & -\frac{45}{2} \end{bmatrix} R_3 \to R_3 - R_2$$

$$\sim \begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & 11/2 \\ 0 & 0 & -45 \end{bmatrix} R_3 \to 2R_3$$

$$\sim \begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & 11/2 \\ 0 & 0 & 1 \end{bmatrix} R_3 \to \frac{R_3}{-45}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{matrix} R_1 \to R_1 - 7R_3 \\ R_2 \to R_2 - \frac{11}{2}R_3 \end{matrix}$$

EX 1.20 Find the inverse of the non-singular matrix $A = \begin{bmatrix} 0 & 5 \\ -1 & 6 \end{bmatrix}$ by Gauss-Jordan Method

Soln:

$$[A|I_2] = \left[\begin{array}{cc|cc} 0 & 5 & 1 & 0 \\ -1 & 6 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{cc|cc} -1 & 6 & 0 & 1 \\ 0 & 5 & 1 & 0 \end{array}\right] R_1 \leftrightarrow R_2$$

$$\sim \left[\begin{array}{cc|cc} 1 & -6 & 0 & -1 \\ 0 & 5 & 1 & 0 \end{array}\right] R_1 \to (-1)R_1$$

$$\sim \left[\begin{array}{cc|cc} 1 & -6 & 0 & -1 \\ 0 & 1 & \frac{1}{5} & 0 \end{array}\right] R_2 \to \frac{R_2}{5}$$

$$\sim \left[\begin{array}{cc|cc} 1 & 0 & \frac{6}{5} & -1 \\ 0 & 1 & \frac{1}{5} & 0 \end{array}\right] R_1 \to R_1 + 6R_2$$

$$A^{-1} = \begin{bmatrix} \frac{6}{5} & -1 \\ \frac{1}{5} & 0 \end{bmatrix}$$

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$$\therefore A^{-1} = \frac{1}{5}\begin{bmatrix} 6 & -5 \\ 1 & 0 \end{bmatrix}$$

EX 1.21 Find the inverse of $A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$ by Gauss-Jordan method

Soln:

$$[A|I_3] = \left[\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 3 & 2 & 1 & 0 & 1 & 0 \\ 2 & 1 & 2 & 0 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 3 & 2 & 1 & 0 & 1 & 0 \\ 2 & 1 & 2 & 0 & 0 & 1 \end{array}\right] R_1 \to \frac{R_1}{2}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{1}{2} & -\frac{3}{2} & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right] \begin{matrix} R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 - 2R_1 \end{matrix}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & -1 & -3 & 2 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right] R_2 \to 2R_2$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 2 & -1 & 0 \\ 0 & 1 & -1 & -3 & 2 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right] R_1 \to R_1 - \frac{1}{2}R_2$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -1 & -1 \\ 0 & 1 & 0 & -4 & 2 & 1 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right] \begin{array}{l} R_1 \to R_1 - R_3 \\ R_2 \to R_2 + R_3 \end{array}$$

$$A^{-1} = \begin{bmatrix} 3 & -1 & -1 \\ -4 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

1.4 Applications of Matrices Solving System of linear Equations

EXERCISE 1.3

1) Solve the following system of linear equations by matrix Inversion Method

(i) $2x+5y=-2$, $x+2y=-3$

Soln:

$$\begin{bmatrix} 2 & 5 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ -3 \end{bmatrix}$$

$$AX = B$$

$$X = A^{-1}B \quad \text{—— (1)}$$

$$|A| = \begin{vmatrix} 2 & 5 \\ 1 & 2 \end{vmatrix} = 4-5 = -1 \neq 0$$

A is non-singular and hence A^{-1} exists

$$\text{adj}A = \begin{bmatrix} 2 & -5 \\ -1 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}(\text{adj}A)$$

$$= \frac{1}{-1}\begin{bmatrix} 2 & -5 \\ -1 & 2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -2 & 5 \\ 1 & -2 \end{bmatrix}$$

$$(1) \Rightarrow \quad X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 & 5 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} -2 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} 4-15 \\ -2+6 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -11 \\ 4 \end{bmatrix}$$

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$$\therefore \ x = -11, \ y = 4$$

(ii) $2x-y=8$, $3x+2y=-2$

Soln:

$$\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ -2 \end{bmatrix}$$

$$AX = B$$

$$X = A^{-1}B \quad \text{—— (1)}$$

$$|A| = \begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix} = 4+3 = 7 \neq 0$$

A is non-Singular and hence A^{-1} exists

$$\text{adj}A = \begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}(\text{adj}A)$$

$$A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$$

$$(1) \Rightarrow \quad X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 8 \\ -2 \end{bmatrix}$$

$$= \frac{1}{7}\begin{bmatrix} 16-2 \\ -24-4 \end{bmatrix}$$

$$= \frac{1}{7}\begin{bmatrix} 14 \\ -28 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \end{bmatrix}$$

$$\therefore \ x = 2, \ y = -4$$

(iii) $2x+3y-z=9$, $x+y+z=9$, $3x-y-z=-1$

Soln:

$$\begin{bmatrix} 2 & 3 & -1 \\ 1 & 1 & 1 \\ 3 & -1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 9 \\ -1 \end{bmatrix}$$

$$AX = B$$

$$X = A^{-1}B \quad \text{—— (1)}$$

$$|A| = \begin{vmatrix} 2 & 3 & -1 \\ 1 & 1 & 1 \\ 3 & -1 & -1 \end{vmatrix}$$

$$= 2(\cancel{-1+1}) - 3(-1-3) - 1(-1-3)$$

$$= 12 + 4 = 16 \neq 0$$

A is non-singular and hence A^{-1} exists

$$[A_{ij}] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & -1 & 3 & -1 \\ 3 & -1 & 2 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$[A_{ij}] = \begin{bmatrix} -1+1 & 3+1 & -1-3 \\ 1+3 & -2+3 & 9+2 \\ 3+1 & -1-2 & 2-3 \end{bmatrix}$$

$$[A_{ij}] = \begin{bmatrix} 0 & 4 & -4 \\ 4 & 1 & 11 \\ 4 & -3 & -1 \end{bmatrix}$$

$$\text{adj}A = [A_{ij}]^T = \begin{bmatrix} 0 & 4 & 4 \\ 4 & 1 & -3 \\ -4 & 11 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}(\text{adj}A)$$

$$A^{-1} = \frac{1}{16}\begin{bmatrix} 0 & 4 & 4 \\ 4 & 1 & -3 \\ -4 & 11 & -1 \end{bmatrix}$$

$$① \Rightarrow X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{16}\begin{bmatrix} 0 & 4 & 4 \\ 4 & 1 & -3 \\ -4 & 11 & -1 \end{bmatrix}\begin{bmatrix} 9 \\ 9 \\ -1 \end{bmatrix}$$

$$= \frac{1}{16}\begin{bmatrix} 0+36-4 \\ 36+9+3 \\ -36+99+1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{16}\begin{bmatrix} 32 \\ 48 \\ 64 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

$\therefore x = 2,\ y = 3,\ z = 4$

(iv) $x+y+z-2=0$, $6x-4y+5z-31=0$, $5x+2y+2z=13$

Soln:

$$\begin{bmatrix} 1 & 1 & 1 \\ 6 & -4 & 5 \\ 5 & 2 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 31 \\ 13 \end{bmatrix}$$

$$AX = B$$

$$X = A^{-1}B \text{ ——— } ①$$

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 6 & -4 & 5 \\ 5 & 2 & 2 \end{vmatrix}$$

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$$= 1(-8-10) - 1(12-25) + 1(12+20)$$

$$= 1(-18) - 1(-13) + 1(32)$$

$$= -18+13+32 = 27 \neq 0$$

A is non-singular and hence A^{-1} exists

$$[A_{ij}] = \begin{array}{cccc} -4 & 5 & 6 & -4 \\ 2 & 2 & 5 & 2 \\ 1 & 1 & 1 & 1 \\ -4 & 5 & 6 & -4 \end{array}$$

$$[A_{ij}] = \begin{bmatrix} -8-10 & 25-12 & 12+20 \\ 2-2 & 2-5 & 5-2 \\ 5+4 & 6-5 & -4-6 \end{bmatrix}$$

$$[A_{ij}] = \begin{bmatrix} -18 & 13 & 32 \\ 0 & -3 & 3 \\ 9 & 1 & -10 \end{bmatrix}$$

$$\text{adj}A = [A_{ij}]^T = \begin{bmatrix} -18 & 0 & 9 \\ 13 & -3 & 1 \\ 32 & 3 & -10 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}(\text{adj}A)$$

$$A^{-1} = \frac{1}{27}\begin{bmatrix} -18 & 0 & 9 \\ 13 & -3 & 1 \\ 32 & 3 & -10 \end{bmatrix}$$

$$① \Rightarrow X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{27}\begin{bmatrix} -18 & 0 & 9 \\ 13 & -3 & 1 \\ 32 & 3 & -10 \end{bmatrix}\begin{bmatrix} 2 \\ 31 \\ 13 \end{bmatrix}$$

$$= \frac{1}{27}\begin{bmatrix} -36+0+117 \\ 26-93+13 \\ 64+93-130 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{27}\begin{bmatrix} 81 \\ -54 \\ 27 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$$

$\therefore x = 3,\ y = -2,\ z = 1$

2) If $A = \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$ find the products AB and BA and hence solve the system of equations $x+y+2z=1$, $3x+2y+z=7$, $2x+y+3z=2$

Soln:

$$AB = \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$$

$$AB = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} = 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$AB = 4I_3$$

$$BA = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$$

$$BA = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} = 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$BA = 4I_3$$

$AB = BA = 4I_3$

$\left(\frac{1}{4}A\right)B = B\left(\frac{1}{4}A\right) = I_3$

$\therefore \boxed{B^{-1} = \frac{1}{4}A}$

The given system of eqns in matrix form.

$$\begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$$

$BX = C$

$X = B^{-1}C$

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$X = \left(\frac{1}{4}A\right)C$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} -5+7+6 \\ 7+7-10 \\ 1-7+2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 8 \\ 4 \\ -4 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$$

$\therefore x = 2, y = 1, z = -1$

3) A man is appointed in a job with a monthly salary of certain amount and a fixed amount of annual increment. If his salary was ₹19,800 per month at the end of the first month after 3 years of service and ₹23,400 per month at the end of the first month after 9 years of service, find his starting salary and his annual increment. (Use matrix inversion method to solve the problem)

Soln: Let x be the monthly salary and y be the annual increment

From the given information,

$x + 3y = 19800$

$x + 9y = 23400$

$$\begin{bmatrix} 1 & 3 \\ 1 & 9 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 19800 \\ 23400 \end{bmatrix}$$

$AX = B$

$X = A^{-1}B$ —— ①

$|A| = \begin{vmatrix} 1 & 3 \\ 1 & 9 \end{vmatrix} = 9 - 3 = 6 \neq 0$

A is non-singular and hence A^{-1} exists

$\text{adj}A = \begin{bmatrix} 9 & -3 \\ -1 & 1 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|}(\text{adj}A)$

$A^{-1} = \frac{1}{6}\begin{bmatrix} 9 & -3 \\ -1 & 1 \end{bmatrix}$

① $\Rightarrow X = A^{-1}B$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 9 & -3 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 19800 \\ 23400 \end{bmatrix}$$

$$= \frac{1}{6}\begin{bmatrix} 178200 - 70200 \\ -19800 + 23400 \end{bmatrix}$$

$$= \frac{1}{6}\begin{bmatrix} 108000 \\ 3600 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 18000 \\ 600 \end{bmatrix}$$

$\therefore x = 18000, y = 600$

$\therefore$ The starting salary is ₹18,000 and the annual increment is ₹600

4) 4 men and 4 women can finish a piece of work jointly in 3 days while 2 men and 5 women can finish the same work jointly in 4 days. Find the time taken by one man alone and

that of one women alone to finish the same work by using matrix inversion method.

Soln: Let x be the number of days taken by a men to complete a work and y be the number of days taken by a women to complete the same work.

From the given information,

$$\frac{4}{x}+\frac{4}{y}=\frac{1}{3}$$

$$\frac{2}{x}+\frac{5}{y}=\frac{1}{4}$$

Let $\frac{1}{x}=a$, $\frac{1}{y}=b$

$$4a+4b=\frac{1}{3}$$

$$12a+12b=1 \quad \text{—— (1)}$$

$$2a+5b=\frac{1}{4}$$

$$8a+20b=1 \quad \text{—— (2)}$$

$$\begin{bmatrix}12 & 12\\ 8 & 20\end{bmatrix}\begin{bmatrix}a\\ b\end{bmatrix}=\begin{bmatrix}1\\ 1\end{bmatrix}$$

$$AX=B$$

$$X=A^{-1}B \quad \text{—— (1)}$$

$$|A|=\begin{vmatrix}12 & 12\\ 8 & 20\end{vmatrix}=240-96=144$$

$$|A|=144\neq 0$$

A is non-singular and hence A^{-1} exists

$$\text{adj}A=\begin{bmatrix}20 & -12\\ -8 & 12\end{bmatrix}$$

$$A^{-1}=\frac{1}{|A|}(\text{adj}A)$$

$$A^{-1}=\frac{1}{144}\begin{bmatrix}20 & -12\\ -8 & 12\end{bmatrix}$$

(1) $\Rightarrow X=A^{-1}B$

$$\begin{bmatrix}a\\ b\end{bmatrix}=\frac{1}{144}\begin{bmatrix}20 & -12\\ -8 & 12\end{bmatrix}\begin{bmatrix}1\\ 1\end{bmatrix}$$

$$=\frac{1}{144}\begin{bmatrix}20-12\\ -8+12\end{bmatrix}$$

$$=\frac{1}{144}\begin{bmatrix}8\\ 4\end{bmatrix}$$

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$$\begin{bmatrix}a\\ b\end{bmatrix}=\begin{bmatrix}1/18\\ 1/36\end{bmatrix}$$

$$a=\frac{1}{18},\quad b=\frac{1}{36}$$

$$\frac{1}{x}=\frac{1}{18},\quad \frac{1}{y}=\frac{1}{36}$$

$$x=18,\quad y=36$$

$\therefore x=18$ days and $y=36$ days

$\therefore$ The time taken for men complete work is 18 days and the time taken for women complete same work is 36 days

5) The prices of three commodities A, B and C are ₹ x, y and z per units respectively. A person P purchases 4 units of B and sells two units of A and 5 units of C. Person Q purchases 2 units of C and sells 3 units of A and one unit of B. Person R purchases one unit of A and sells 3 unit of B and one unit of C. In the process, P, Q and R earn ₹15,000, ₹1,000 and ₹4,000 respectively. Find the prices per unit of A, B and C (use matrix inversion method to solve the problem)

Soln: From the given information,

$$2x-4y+5z=15000$$
$$3x+y-2z=1000$$
$$-x+3y+z=4000$$

$$\begin{bmatrix}2 & -4 & 5\\ 3 & 1 & -2\\ -1 & 3 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}15000\\ 1000\\ 4000\end{bmatrix}$$

$$AX=B$$

$$X=A^{-1}B \text{ ——— } (1)$$

$$|A|=\begin{vmatrix}2 & -4 & 5\\ 3 & 1 & -2\\ -1 & 3 & 1\end{vmatrix}$$

$$=2(1+6)+4(3-2)+5(9+1)$$
$$=14+4+50=68$$
$$|A|=68\neq 0$$

A is non-singular and hence A^{-1} exists

$$[A_{ij}]=\begin{bmatrix}1 & -2 & 3 & 1\\ 3 & 1 & -1 & 3\\ -4 & 5 & 2 & -4\\ 1 & -2 & 3 & 1\end{bmatrix}$$

$$[A_{ij}]=\begin{bmatrix}1+6 & 2-3 & 9+1\\ 15+4 & 2+5 & 4-6\\ 8-5 & 15+4 & 2+12\end{bmatrix}$$

$$[A_{ij}]=\begin{bmatrix}7 & -1 & 10\\ 19 & 7 & -2\\ 3 & 19 & 14\end{bmatrix}$$

$$\text{adj}A=[A_{ij}]^T=\begin{bmatrix}7 & 19 & 3\\ -1 & 7 & 19\\ 10 & -2 & 14\end{bmatrix}$$

$$A^{-1}=\frac{1}{|A|}(\text{adj}A)$$

$$A^{-1}=\frac{1}{68}\begin{bmatrix}7 & 19 & 3\\ -1 & 7 & 19\\ 10 & -2 & 14\end{bmatrix}$$

$$(1)\Rightarrow X=A^{-1}B$$

$$\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\frac{1}{68}\begin{bmatrix}7 & 19 & 3\\ -1 & 7 & 19\\ 10 & -2 & 14\end{bmatrix}\begin{bmatrix}15000\\ 1000\\ 4000\end{bmatrix}$$

$$=\frac{1}{68}\begin{bmatrix}105000+19000+12000\\ -15000+7000+76000\\ 150000-2000+56000\end{bmatrix}$$

$$=\frac{1}{68}\begin{bmatrix}136000\\ 68000\\ 204000\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}2000\\ 1000\\ 3000\end{bmatrix}$$

$\therefore x=2000,\ y=1000,\ z=3000$

The prices per unit of A, B and C are respectively ₹2,000, ₹1,000 and ₹3,000

EX 1.22 Solve the following system of linear equations, using matrix inversion method $5x+2y=3,\ 3x+2y=5$

Soln:
$$\begin{bmatrix}5 & 2\\ 3 & 2\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}3\\ 5\end{bmatrix}$$

$$AX=B$$

$$X=A^{-1}B \text{ ——— } (1)$$

$$|A|=\begin{vmatrix}5 & 2\\ 3 & 2\end{vmatrix}=10-6=4\neq 0$$

A is non-singular and hence A^{-1} exists

$$\text{adj}A=\begin{bmatrix}2 & -2\\ -3 & 5\end{bmatrix}$$

$$A^{-1}=\frac{1}{|A|}(\text{adj}A)$$

$$A^{-1}=\frac{1}{4}\begin{bmatrix}2 & -2\\ -3 & 5\end{bmatrix}$$

$$(1)\Rightarrow X=A^{-1}B$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{4}\begin{bmatrix}2 & -2\\ -3 & 5\end{bmatrix}\begin{bmatrix}3\\ 5\end{bmatrix}$$

$$=\frac{1}{4}\begin{bmatrix}6-10\\ -9+25\end{bmatrix}$$

$$=\frac{1}{4}\begin{bmatrix}-4\\ 16\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}-1\\ 4\end{bmatrix}$$

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$x=-1,\ y=4$

EX 1.23 Solve the following

System of equations, using matrix inversion method:

$2x_1+3x_2+3x_3=5$, $x_1-2x_2+x_3=-4$, $3x_1-x_2-2x_3=3$

Soln:

$$\begin{bmatrix}2&3&3\\1&-2&1\\3&-1&-2\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}5\\-4\\3\end{bmatrix}$$

$$AX=B$$

$$X=A^{-1}B \text{ ——— } (1)$$

$$|A|=\begin{vmatrix}2&3&3\\1&-2&1\\3&-1&-2\end{vmatrix}$$

$=2(4+1)-3(-2-3)+3(-1+6)$

$=10+15+15=40\neq 0$

A is non-singular and hence A^{-1} exists

$$[A_{ij}]=\begin{bmatrix}-2&1&1&-2\\-1&-2&3&-1\\3&3&2&3\\-2&1&1&-2\end{bmatrix}$$

$$[A_{ij}]=\begin{bmatrix}4+1&3+2&-1+6\\-3+6&-4-9&9+2\\3+6&3-2&-4-3\end{bmatrix}$$

$$[A_{ij}]=\begin{bmatrix}5&5&5\\3&-13&11\\9&1&-7\end{bmatrix}$$

$$\operatorname{adj}A=[A_{ij}]^T=\begin{bmatrix}5&3&9\\5&-13&1\\5&11&-7\end{bmatrix}$$

$$A^{-1}=\frac{1}{|A|}(\operatorname{adj}A)$$

$$A^{-1}=\frac{1}{40}\begin{bmatrix}5&3&9\\5&-13&1\\5&11&-7\end{bmatrix}$$

(1) $\Rightarrow$ $X=A^{-1}B$

$$\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\frac{1}{40}\begin{bmatrix}5&3&9\\5&-13&1\\5&11&-7\end{bmatrix}\begin{bmatrix}5\\-4\\3\end{bmatrix}$$

$$=\frac{1}{40}\begin{bmatrix}25-12+27\\25+52+3\\25-44-21\end{bmatrix}$$

$$=\frac{1}{40}\begin{bmatrix}40\\80\\-40\end{bmatrix}$$

$$\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}1\\2\\-1\end{bmatrix}$$

$x_1=1$, $x_2=2$, $x_3=-1$

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EX 1.24 If $A=\begin{bmatrix}-4&4&4\\-7&1&3\\5&-3&-1\end{bmatrix}$ and $B=\begin{bmatrix}1&-1&1\\1&-2&-2\\2&1&3\end{bmatrix}$, find the Products AB and BA and hence solve the system of equations $x-y+z=4$, $x-2y-2z=9$, $2x+y+3z=1$

Soln:

$$AB=\begin{bmatrix}-4&4&4\\-7&1&3\\5&-3&-1\end{bmatrix}\begin{bmatrix}1&-1&1\\1&-2&-2\\2&1&3\end{bmatrix}$$

$$AB=\begin{bmatrix}8&0&0\\0&8&0\\0&0&8\end{bmatrix}=8\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$AB=8I_3$$

$$BA=\begin{bmatrix}1&-1&1\\1&-2&-2\\2&1&3\end{bmatrix}\begin{bmatrix}-4&4&4\\-7&1&3\\5&-3&-1\end{bmatrix}$$

$$BA=\begin{bmatrix}8&0&0\\0&8&0\\0&0&8\end{bmatrix}=8\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$BA=8I_3$$

$\therefore AB=BA=8I_3$

$$\left(\frac{1}{8}A\right)B=B\left(\frac{1}{8}A\right)=I_3$$

$$B^{-1}=\frac{1}{8}A$$

The given System of equations in matrix form

$$\begin{bmatrix}1&-1&1\\1&-2&-2\\2&1&3\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}4\\9\\1\end{bmatrix}$$

$$BX=C$$

$$X=B^{-1}C$$

$X = \left(\frac{1}{8}A\right)C$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

$$= \frac{1}{8}\begin{bmatrix} -16+36+4 \\ -28+9+3 \\ 20-27-1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 24 \\ -16 \\ -8 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$$

$\therefore x = 3, y = -2, z = -1$

1.4.3(ii) Cramer's Rule

EXERCISE 1.4

1) Solve the following systems of linear equations by Cramer's rule:

(i) $5x - 2y + 16 = 0$, $x + 3y - 7 = 0$

Soln:

$$\Delta = \begin{vmatrix} 5 & -2 \\ 1 & 3 \end{vmatrix} = 15 + 2 = 17$$

$\Delta = 17$

$$\Delta_1 = \begin{vmatrix} -16 & -2 \\ 7 & 3 \end{vmatrix} = -48 + 14$$

$\Delta_1 = -34$

$$\Delta_2 = \begin{vmatrix} 5 & -16 \\ 1 & 7 \end{vmatrix} = 35 + 16$$

$\Delta_2 = 51$

By Cramer's Rule, we get

$$x = \frac{\Delta_1}{\Delta} = \frac{-\cancel{34}^{2}}{\cancel{17}} = -2$$

$$y = \frac{\Delta_2}{\Delta} = \frac{\cancel{51}^{3}}{\cancel{17}} = 3$$

$\therefore$ The Solution is $(x = -2, y = 3)$

(ii) $\frac{3}{x} + 2y = 12$, $\frac{2}{x} + 3y = 13$

Soln: Let $\frac{1}{x} = a$, $y = b$

$3a + 2b = 12$ and $2a + 3b = 13$

$$\Delta = \begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix} = 9 - 4 = 5$$

$\Delta = 5$

$$\Delta_1 = \begin{vmatrix} 12 & 2 \\ 13 & 3 \end{vmatrix} = 36 - 26$$

$\Delta_1 = 10$

$$\Delta_2 = \begin{vmatrix} 3 & 12 \\ 2 & 13 \end{vmatrix} = 39 - 24$$

$\Delta_2 = 15$

By Cramer's Rule, we get

$$a = \frac{\Delta_1}{\Delta} = \frac{\cancel{10}^{2}}{\cancel{5}} = 2$$

$$b = \frac{\Delta_2}{\Delta} = \frac{\cancel{15}^{3}}{\cancel{5}} = 3$$

When $a = 2$,

$\frac{1}{x} = a \Rightarrow \frac{1}{x} = 2$

$\boxed{x = \frac{1}{2}}$

When $b = 3$, $y = b$

$\Rightarrow \boxed{y = 3}$

$\therefore$ The Solution is $(x = \frac{1}{2}, y = 3)$

(iii) $3x + 3y - z = 11$, $2x - y + 2z = 9$, $4x + 3y + 2z = 25$

Soln:

$$\Delta = \begin{vmatrix} 3 & 3 & -1 \\ 2 & -1 & 2 \\ 4 & 3 & 2 \end{vmatrix}$$

$= 3(-2-6) - 3(4-8) - 1(6+4)$

$= 3(-8) - 3(-4) - 1(10)$

$= -24 + 12 - 10$

$\Delta = -22$

$$\Delta_1 = \begin{vmatrix} 11 & 3 & -1 \\ 9 & -1 & 2 \\ 25 & 3 & 2 \end{vmatrix}$$

$= 11(-2-6) - 3(18-50) - 1(27+25)$

$= 11(-8) - 3(-32) - 1(52)$

$= -88 + 96 - 52$

$\Delta_1 = -44$

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$$\Delta_2 = \begin{vmatrix} 3 & 11 & -1 \\ 2 & 9 & 2 \\ 4 & 25 & 2 \end{vmatrix}$$

$= 3(18-50) - 11(4-8) - 1(50-36)$

$= 3(-32) - 11(-4) - 1(14)$

$= -96 + 44 - 14$

$\Delta_2 = -66$

$$\Delta_3 = \begin{vmatrix} 3 & 3 & 11 \\ 2 & -1 & 9 \\ 4 & 3 & 25 \end{vmatrix}$$

$= 3(-25-27) - 3(50-36) + 11(6+4)$

$= 3(-52) - 3(14) + 11(10)$

$= -156 - 42 + 110$

$\Delta_3 = -88$

By Cramer's Rule, we get

$x = \frac{\Delta_1}{\Delta} = \frac{\cancel{+44}\,2}{\cancel{+22}} = 2$

$y = \frac{\Delta_2}{\Delta} = \frac{\cancel{+66}\,3}{\cancel{+22}} = 3$

$z = \frac{\Delta_3}{\Delta} = \frac{\cancel{+88}\,4}{\cancel{+22}} = 4$

$\therefore$ The Solution is

$(x = 2, y = 3, z = 4)$

(iv) $\frac{3}{x} - \frac{4}{y} - \frac{2}{z} - 1 = 0$,

$\frac{1}{x} + \frac{2}{y} + \frac{1}{z} - 2 = 0$,

$\frac{2}{x} - \frac{5}{y} - \frac{4}{z} + 1 = 0$

Soln:

Let $\frac{1}{x} = a$, $\frac{1}{y} = b$

and $\frac{1}{z} = c$

$3a - 4b - 2c = 1$

$a + 2b + c = 2$

$2a - 5b - 4c = -1$

$$\Delta = \begin{vmatrix} 3 & -4 & -2 \\ 1 & 2 & 1 \\ 2 & -5 & -4 \end{vmatrix}$$

$= 3(-8+5) + 4(-4-2) - 2(-5-4)$

$= 3(-3) + 4(-6) - 2(-9)$

$= -9 - 24 + 18$

$\Delta = -15$

$$\Delta_1 = \begin{vmatrix} 1 & -4 & -2 \\ 2 & 2 & 1 \\ -1 & -5 & -4 \end{vmatrix}$$

$= 1(-8+5) + 4(-8+1) - 2(-10+2)$

$= 1(-3) + 4(-7) - 2(-8)$

$= -3 - 28 + 16$

$\Delta_1 = -15$

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$$\Delta_2 = \begin{vmatrix} 3 & 1 & -2 \\ 1 & 2 & 1 \\ 2 & -1 & -4 \end{vmatrix}$$

$= 3(-8+1) - 1(-4-2) - 2(-1-4)$

$= 3(-7) - 1(-6) - 2(-5)$

$= -21 + 6 + 10$

$\Delta_2 = -5$

$$\Delta_3 = \begin{vmatrix} 3 & -4 & 1 \\ 1 & 2 & 2 \\ 2 & -5 & -1 \end{vmatrix}$$

$= 3(-2+10) + 4(-1-4) + 1(-5-4)$

$= 3(8) + 4(-5) + 1(-9)$

$= 24 - 20 - 9$

$\Delta_3 = -5$

By Cramer's Rule, we get

$a = \frac{\Delta_1}{\Delta} = \frac{\cancel{+15}}{\cancel{+15}} = 1$

$b = \frac{\Delta_2}{\Delta} = \frac{\cancel{+5}\,1}{\cancel{+15}\,3} = \frac{1}{3}$

$c = \frac{\Delta_3}{\Delta} = \frac{\cancel{+5}\,1}{\cancel{+15}\,3} = \frac{1}{3}$

$\frac{1}{x} = a \Rightarrow \frac{1}{x} = 1$

$\boxed{x = 1}$

$\frac{1}{y} = b \Rightarrow \frac{1}{y} = \frac{1}{3}$

$\boxed{y = 3}$

$\frac{1}{z} = c \Rightarrow \frac{1}{z} = \frac{1}{3}$

$\boxed{z = 3}$

$\therefore$ The Solution is

$(x = 1, y = 3, z = 3)$

2) In a Competitive examination, one mark is awarded for every correct answer while $\frac{1}{4}$ mark is deducted for every wrong answer. A student answered 100 questions and got 80 marks. How many questions did he answer correctly? (Use Cramer's rule to solve the problem)

Soln:

Let x and y be the number of questions answered correctly and incorrectly

$$x+y=100 \quad \text{---} \; (1)$$

$$x-\frac{1}{4}y=80$$

$$4x-y=320 \quad \text{---} \; (2)$$

$$\Delta=\begin{vmatrix}1 & 1\\ 4 & -1\end{vmatrix}=-1-4=-5$$

$$\Delta=-5$$

$$\Delta_1=\begin{vmatrix}100 & 1\\ 320 & -1\end{vmatrix}=-100-320$$

$$\Delta_1=-420$$

$$\Delta_2=\begin{vmatrix}1 & 100\\ 4 & 320\end{vmatrix}=320-400$$

$$\Delta_2=-80$$

By Cramer's Rule, we get

$$x=\frac{\Delta_1}{\Delta}=\frac{\cancel{+420}^{84}}{\cancel{+5}}=84$$

$$y=\frac{\Delta_2}{\Delta}=\frac{\cancel{+80}^{16}}{\cancel{+5}}=16$$

$\therefore x=84, y=16$

The student has answered 84 questions correctly

3) A chemist has one solution which is 50% acid and another solution which is 25% acid. How much each should be mixed to make 10 litres of a 40% acid solution? (Use Cramer's rule to solve the problem)

Soln:

Let x litres of 50% acid and y litres of 25% acid solution to be mixed

$$x+y=10 \quad \text{---} \; (1)$$

$$50\%\,x+25\%\,y=(40\%)(10)$$

$$\frac{50}{100}x+\frac{25}{100}y=\frac{\cancel{40}^{4}}{\cancel{100}_{\cancel{10}}}(\cancel{10})$$

$$\frac{\cancel{25}^{1}}{\cancel{100}_{4}}(2x+y)=4$$

$$2x+y=16 \quad \text{---} \; (2)$$

$$\Delta=\begin{vmatrix}1 & 1\\ 2 & 1\end{vmatrix}=1-2=-1$$

$$\Delta=-1$$

$$\Delta_1=\begin{vmatrix}10 & 1\\ 16 & 1\end{vmatrix}=10-16=-6$$

$$\Delta_1=-6$$

$$\Delta_2=\begin{vmatrix}1 & 10\\ 2 & 16\end{vmatrix}=16-20=-4$$

$$\Delta_2=-4$$

By Cramer's Rule, we get

$$x=\frac{\Delta_1}{\Delta}=\frac{\cancel{+}6}{\cancel{+}1}=6$$

$$y=\frac{\Delta_2}{\Delta}=\frac{\cancel{+}4}{\cancel{+}1}=4$$

$\therefore x=6$ litres and $y=4$ litres

$\therefore$ 6 litres of 50% acid and 4 litres of 25% acid solution to be mixed to get 10 litres of 40% acid solution

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4) A fish tank can be filled in 10 minutes using both pumps A and B simultaneously. However pump B can pump water in or out at the same rate. If pump B is inadvertently run in reverse, then the tank will be filled in 30 minutes. How long would it take each pump to fill the tank by itself? (Use Cramer's rule to solve the problem)

Soln: Given: $\frac{1}{A}+\frac{1}{B}=\frac{1}{10}$

and $\frac{1}{A}-\frac{1}{B}=\frac{1}{30}$

Let $x=\frac{1}{A}$, $y=\frac{1}{B}$

$x+y=\frac{1}{10}$

$10x+10y=1$ —— (1)

$x-y=\frac{1}{30}$

$30x-30y=1$ —— (2)

$\Delta=\begin{vmatrix}10 & 10\\ 30 & -30\end{vmatrix}=-300-300$

$\Delta=-600$

$\Delta_1=\begin{vmatrix}1 & 10\\ 1 & -30\end{vmatrix}=-30-10$

$\Delta_1=-40$

$\Delta_2=\begin{vmatrix}10 & 1\\ 30 & 1\end{vmatrix}=10-30$

$\Delta_2=-20$

By Cramer's Rule, we get

$x=\frac{\Delta_1}{\Delta}=\frac{\cancel{+40}\,1}{\cancel{+600}\,15}=\frac{1}{15}$

$y=\frac{\Delta_2}{\Delta}=\frac{\cancel{+20}\,1}{\cancel{+600}\,30}=\frac{1}{30}$

$\therefore x=\frac{1}{15}$ and $y=\frac{1}{30}$

$\frac{1}{A}=\frac{1}{15}$ and $\frac{1}{B}=\frac{1}{30}$

A = 15 minutes and

B = 30 minutes

$\therefore$ Pumps A and B fill the tank by 15 minutes and 30 minutes respectively

5) A family of 3 people went out for dinner in a restaurant. The cost of two dosai, three idlies and two vadais is ₹150. The cost of the two dosai, two idlies and four vadais is ₹200. The cost of five dosai, four idlies and two vadais is ₹250. The family has ₹350 in hand and they ate 3 dosai and six idlies and six vadais. Will they be able to manage to pay the bill within the amount they had?

Soln:

Let the cost of one dosai, one idly and one vadai be ₹ x, y and z respectively.

$2x+3y+2z=150$,

$2x+2y+4z=200$,

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$5x+4y+2z=250$

$$\Delta = \begin{vmatrix} 2 & 3 & 2 \\ 2 & 2 & 4 \\ 5 & 4 & 2 \end{vmatrix}$$

$= 2(4-16)-3(4-20)+2(8-10)$

$= 2(-12)-3(-16)+2(-2)$

$= -24+48-4$

$\Delta = 20$

$$\Delta_1 = \begin{vmatrix} 150 & 3 & 2 \\ 200 & 2 & 4 \\ 250 & 4 & 2 \end{vmatrix}$$

$= 150(4-16)-3(400-1000)+2(800-500)$

$= 150(-12)-3(-600)+2(300)$

$= -\cancel{1800}+\cancel{1800}+600$

$\Delta_1 = 600$

$$\Delta_2 = \begin{vmatrix} 2 & 150 & 2 \\ 2 & 200 & 4 \\ 5 & 250 & 2 \end{vmatrix}$$

$= 2(400-1000)-150(4-20)+2(500-1000)$

$= 2(-600)-150(-16)+2(-500)$

$= -1200+2400-1000$

$\Delta_2 = 200$

$$\Delta_3 = \begin{vmatrix} 2 & 3 & 150 \\ 2 & 2 & 200 \\ 5 & 4 & 250 \end{vmatrix}$$

$= 2(500-800)-3(500-1000)+150(8-10)$

$= 2(-300)-3(-500)+150(-2)$

$= -600+1500-300$

$\Delta_3 = 600$

By Cramer's Rule, we get

$x = \dfrac{\Delta_1}{\Delta} = \dfrac{\cancel{600}^{30}}{\cancel{20}} = 30$

$y = \dfrac{\Delta_2}{\Delta} = \dfrac{\cancel{200}^{10}}{\cancel{20}} = 10$

$z = \dfrac{\Delta_3}{\Delta} = \dfrac{\cancel{600}^{30}}{\cancel{20}} = 30$

$\therefore x = 30,\ y = 10,\ z = 30$

The cost of one dosai is ₹30 and idly is ₹10 and vadai is ₹30

Also,

$3x+6y+6z = 3(30)+6(10)+6(30)$

$= 90+60+180$

$= 330$

$\therefore$ ₹330 < ₹350

Yes. they will be able to manage to pay the bill.

EX 1.25

Solve by Cramer's rule the system of equations $x_1 - x_2 = 3$, $2x_1+3x_2+4x_3=17$, $x_2+2x_3=7$

Soln:

$$\Delta = \begin{vmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{vmatrix}$$

$= 1(6-4)+1(4-0)+\cancel{0(2-0)}$

$= 2+4 = 6$

$\Delta = 6$

$$\Delta_1 = \begin{vmatrix} 3 & -1 & 0 \\ 17 & 3 & 4 \\ 7 & 1 & 2 \end{vmatrix}$$

$= 3(6-4)+1(34-28)+\cancel{0(17-21)}$

$= 3(2)+1(6)$

$= 6+6 = 12$

$\Delta_1 = 12$

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$$\Delta_2 = \begin{vmatrix} 1 & 3 & 0 \\ 2 & 17 & 4 \\ 0 & 7 & 2 \end{vmatrix}$$

$= 1(34-28) - 3(4-0) + \cancel{0(14-0)}$

$= 1(6) - 3(4) = 6 - 12$

$\Delta_2 = -6$

$$\Delta_3 = \begin{vmatrix} 1 & -1 & 3 \\ 2 & 3 & 17 \\ 0 & 1 & 7 \end{vmatrix}$$

$= 1(21-17) + 1(14-0) + 3(2-0)$

$= 1(4) + 1(14) + 3(2)$

$= 4 + 14 + 6 = 24$

$\Delta_3 = 24$

By cramer's Rule, we get

$x_1 = \frac{\Delta_1}{\Delta} = \frac{\cancel{12}^{2}}{\cancel{6}} = 2$

$x_2 = \frac{\Delta_2}{\Delta} = \frac{\cancel{-6}}{\cancel{-6}} = -1$

$x_3 = \frac{\Delta_3}{\Delta} = \frac{\cancel{24}^{4}}{\cancel{6}} = 4$

$\therefore$ The Solution is

$(x_1 = 2, x_2 = -1, x_3 = 4)$

EX 1.26

In a T20 match, Chennai Super Kings needed just 6 runs to win with 1 ball left to go in the last over. The last ball was bowled and the batsman at the crease hit it high up. The ball traversed along a path in a vertical plane and the equation of the path is $y = ax^2 + bx + c$ with respect to a xy-co-ordinate system in the vertical plane and the ball traversed through the points $(10,8), (20,16), (40,22)$, Can you conclude that Chennai Super Kings won the match? Justify your answer (All distances are measured in metres and the meeting point of the plane of the path with the farthest boundary line is $(70,0)$)

Soln: Given:

$y = ax^2 + bx + c$ and $(10,8), (20,16)$ and $(40,22)$

When $x = 10, y = 8$

$100a + 10b + c = 8$ —— ①

When $x = 20, y = 16$

$400a + 20b + c = 16$ —— ②

When $x = 40, y = 22$

$1600a + 40b + c = 22$ —— ③

By Cramer's Rule, we get

$$\Delta = \begin{vmatrix} 100 & 10 & 1 \\ 400 & 20 & 1 \\ 1600 & 40 & 1 \end{vmatrix}$$

$$= (100)(10) \begin{vmatrix} 1 & 1 & 1 \\ 4 & 2 & 1 \\ 16 & 4 & 1 \end{vmatrix}$$

$= 1000[1(2-4) - 1(4-16) + 1(16-32)]$

$= 1000[-2 + 12 - 16]$

$= 1000(-6) = -6000$

$\Delta = -6000$

$$\Delta_1 = \begin{vmatrix} 8 & 10 & 1 \\ 16 & 20 & 1 \\ 22 & 40 & 1 \end{vmatrix}$$

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$= (2)(10)\begin{vmatrix} 4 & 1 & 1 \\ 8 & 2 & 1 \\ 11 & 4 & 1 \end{vmatrix}$

$= 20[4(2-4) - 1(8-11) + 1(32-22)]$

$= 20[-8+3+10]$

$= 20(5) = 100$

$\Delta_1 = 100$

$\Delta_2 = \begin{vmatrix} 100 & 8 & 1 \\ 400 & 16 & 1 \\ 1600 & 22 & 1 \end{vmatrix}$

$= (100)(2)\begin{vmatrix} 1 & 4 & 1 \\ 4 & 8 & 1 \\ 16 & 11 & 1 \end{vmatrix}$

$= 200[1(8-11) - 4(4-16) + 1(44-128)]$

$= 200[-3+48-84]$

$= 200(-39) = -7800$

$\Delta_2 = -7800$

$\Delta_3 = \begin{vmatrix} 100 & 10 & 8 \\ 400 & 20 & 16 \\ 1600 & 40 & 22 \end{vmatrix}$

$= (100)(10)(2)\begin{vmatrix} 1 & 1 & 4 \\ 4 & 2 & 8 \\ 16 & 4 & 11 \end{vmatrix}$

$= 2000[1(22-32) - 1(44-128) + 4(16-32)]$

$= 2000[-10+84-64]$

$= 2000(10) = 20000$

$\Delta_3 = 20000$

By Cramer's Rule, we get

$a = \frac{\Delta_1}{\Delta} = \frac{\cancel{100}^{1}}{-\cancel{6000}_{60}} = \frac{-1}{60}$

$b = \frac{\Delta_2}{\Delta} = \frac{+\cancel{7800}^{13}}{+\cancel{6000}_{10}} = \frac{13}{10}$

$c = \frac{\Delta_3}{\Delta} = \frac{\cancel{20000}^{10}}{-\cancel{6000}_{3}} = \frac{-10}{3}$

$\therefore a = \frac{-1}{60}$, $b = \frac{13}{10}$

and $c = \frac{-10}{3}$

Given: The equation of Path is $y = ax^2 + bx + c$

$y = \frac{-1}{60}x^2 + \frac{13}{10}x - \frac{10}{3}$

when $x = 70$,

$y = \frac{-1}{60}(70)^2 + \frac{13}{10}(70) - \frac{10}{3}$

$= \frac{-4900}{60} + \frac{910}{10} - \frac{10}{3}$

$= \frac{-4900 + 5460 - 200}{60}$

$= \frac{\cancel{360}^{6}}{\cancel{60}} = 6$

$\boxed{y = 6}$

The ball went by 6 metres high over the boundary line and it is impossible for a fielder standing even just before the boundary line to jump and catch the ball. Hence the ball went for a Super Six and the Chennai Super Kings won the match

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1.4.3(iii) Gaussian Elimination Method

EXERCISE 1.5

1) Solve the following

Systems of linear equations by Gaussian elimination method.

(i) $2x-2y+3z=2$, $x+2y-z=3$, $3x-y+2z=1$

Soln:

$$[A|B]=\left[\begin{array}{ccc|c}2&-2&3&2\\1&2&-1&3\\3&-1&2&1\end{array}\right]$$

$$\sim\left[\begin{array}{ccc|c}1&2&-1&3\\2&-2&3&2\\3&-1&2&1\end{array}\right] R_1\leftrightarrow R_2$$

$$\sim\left[\begin{array}{ccc|c}1&2&-1&3\\0&-6&5&-4\\0&-7&5&-8\end{array}\right] \begin{array}{l}R_2\to R_2-2R_1\\R_3\to R_3-3R_1\end{array}$$

$$\sim\left[\begin{array}{ccc|c}1&2&-1&3\\0&-6&5&-4\\0&0&-5&-20\end{array}\right] R_3\to 6R_3-7R_2$$

$x+2y-z=3$ — (1)

$-6y+5z=-4$ — (2)

$-5z=-20$ — (3)

(3) $\Rightarrow$ $+\cancel{5}z=+\cancel{20}\,4$

$\boxed{z=4}$

(2) $\Rightarrow$ $-6y+5(4)=-4$

$-6y+20=-4$

$-6y=-4-20$

$+\cancel{6}y=+\cancel{24}\,4$

$\boxed{y=4}$

(1) $\Rightarrow$ $x+2(4)-4=3$

$x+8-4=3$

$x+4=3$

$x=3-4$

$\boxed{x=-1}$

$\therefore$ The Soln is $x=-1$, $y=4$, $z=4$

(ii) $2x+4y+6z=22$, $3x+8y+5z=27$, $-x+y+2z=2$

Soln:

$$[A|B]=\left[\begin{array}{ccc|c}2&4&6&22\\3&8&5&27\\-1&1&2&2\end{array}\right]$$

$$\sim\left[\begin{array}{ccc|c}1&2&3&11\\3&8&5&27\\-1&1&2&2\end{array}\right] R_1\to\frac{R_1}{2}$$

$$\sim\left[\begin{array}{ccc|c}1&2&3&11\\0&2&-4&-6\\0&3&5&13\end{array}\right] \begin{array}{l}R_2\to R_2-3R_1\\R_3\to R_3+R_1\end{array}$$

$$\sim\left[\begin{array}{ccc|c}1&2&3&11\\0&1&-2&-3\\0&3&5&13\end{array}\right] R_2\to\frac{R_2}{2}$$

$$\sim\left[\begin{array}{ccc|c}1&2&3&11\\0&1&-2&-3\\0&0&11&22\end{array}\right] R_3\to R_3-3R_2$$

$x+2y+3z=11$ — (1)

$y-2z=-3$ — (2)

$11z=22$ — (3)

(3) $\Rightarrow$ $\cancel{11}z=\cancel{22}\,2$

$\boxed{z=2}$

(2) $\Rightarrow$ $y-2(2)=-3$

$y-4=-3$

$y=4-3$

$\boxed{y=1}$

(1) $\Rightarrow$ $x+2(1)+3(2)=11$

$x+2+6=11$

$x+8=11$

$x=11-8$

$\boxed{x=3}$

$\therefore$ The Soln is $x=3$, $y=1$, $z=2$

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2) If ax^2+bx+c is divided by $x+3$, $x-5$ and $x-1$, the remainders are 21, 61 and 9 respectively. Find a, b and c (Use Gaussian elimination method)

Soln: Let $f(x)=ax^2+bx+c$

$f(-3)=21$, $f(5)=61$ and $f(1)=9$

when $x=-3$,

$a(-3)^2+b(-3)+c=f(-3)$

$9a-3b+c=21$ — (1)

when $x=5$,

$a(5)^2+b(5)+c=f(5)$

$25a+5b+c=61$ — (2)

when $x=1$,

$a(1)^2+b(1)+c=9$

$a+b+c=9$ — (3)

$$[A|B]=\left[\begin{array}{ccc|c}9&-3&1&21\\25&5&1&61\\1&1&1&9\end{array}\right]$$

$$\sim\left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 25 & 5 & 1 & 61 \\ 9 & -3 & 1 & 21 \end{array}\right] R_1 \leftrightarrow R_3$$

$$\sim\left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & -20 & -24 & -164 \\ 0 & -12 & -8 & -60 \end{array}\right] \begin{array}{l} R_2 \to R_2 - 25R_1 \\ R_3 \to R_3 - 9R_1 \end{array}$$

$$\sim\left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & 5 & 6 & 41 \\ 0 & 3 & 2 & 15 \end{array}\right] \begin{array}{l} R_2 \to \frac{R_2}{-4} \\ R_3 \to \frac{R_3}{-4} \end{array}$$

$$\sim\left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & 5 & 6 & 41 \\ 0 & 0 & -8 & -48 \end{array}\right] R_3 \to 5R_3 - 3R_2$$

$a+b+c=9$ — ①

$5b+6c=41$ — ②

$-8c=-48$ — ③

③ ⇒ $+\cancel{8}c=+\cancel{48}\,6$

$\boxed{c=6}$

② ⇒ $5b+6(6)=41$

$5b+36=41$

$5b=41-36$

$\cancel{5}b=\cancel{5}$

$\boxed{b=1}$

① ⇒ $a+1+6=9$

$a+7=9$

$a=9-7$

$\boxed{a=2}$

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∴ The Soln is $a=2, b=1, c=6$

3) An amount of ₹65,000 is invested in three bonds at the rates of 6%, 8% and 9% per annum respectively. The total annual income is ₹4,800. The income from the third bond is ₹600 more than that from the second bond. Determine the price of each bond (Use Gaussian elimination method)

Soln: From the given information,

$x+y+z=65000$ — ①

$6\%x+8\%y+9\%z=4800$

$\frac{6}{100}x+\frac{8}{100}y+\frac{9}{100}z=4800$

$\frac{1}{100}(6x+8y+9z)=4800$

$6x+8y+9z=480000$ — ②

$9\%z=8\%y+600$

$\frac{9}{100}z=\frac{8}{100}y+600$

$\frac{9z}{\cancel{100}}=\frac{8y+60000}{\cancel{100}}$

$9z=8y+60000$

$-8y+9z=60000$ — ③

From ①, ② and ③ we get,

$$[A|B]=\left[\begin{array}{ccc|c} 1 & 1 & 1 & 65000 \\ 6 & 8 & 9 & 480000 \\ 0 & -8 & 9 & 60000 \end{array}\right]$$

$$\sim\left[\begin{array}{ccc|c} 1 & 1 & 1 & 65000 \\ 0 & 2 & 3 & 90000 \\ 0 & -8 & 9 & 60000 \end{array}\right] R_2 \to R_2 - 6R_1$$

$$\sim\left[\begin{array}{ccc|c} 1 & 1 & 1 & 65000 \\ 0 & 2 & 3 & 90000 \\ 0 & 0 & 21 & 420000 \end{array}\right] R_3 \to R_3 + 4R_2$$

$x+y+z=65000$ — ①

$2y+3z=90000$ — ②

$21z=420000$ — ③

③ ⇒ $\cancel{21}z=\cancel{420000}\,20000$

$\boxed{z=20000}$

② ⇒ $2y+3(20000)=90000$

$2y+60000=90000$

$2y=90000-60000$

$\cancel{2}y=\cancel{30000}\,15000$

$\boxed{y=15000}$

① ⇒ $x+15000+20000=65000$

$x+35000=65000$

$x=65000-35000$

$\boxed{x=30000}$

The Price of each bonds

are ₹30000, ₹15000 and ₹20000

4) A boy is walking along the path $y=ax^2+bx+c$ through the points $(-6,8)$, $(-2,-12)$ and $(3,8)$. He wants to meet his friend at $P(7,60)$. Will he meet his friend? (Use Gaussian elimination method)

Soln: Given:

$$y=ax^2+bx+c$$

When $x=-6$, $y=8$

$$a(-6)^2+b(-6)+c=8$$

$$36a-6b+c=8 \quad \text{--- (1)}$$

When $x=-2$, $y=-12$

$$a(-2)^2+b(-2)+c=-12$$

$$4a-2b+c=-12 \quad \text{--- (2)}$$

When $x=3$, $y=8$

$$a(3)^2+b(3)+c=8$$

$$9a+3b+c=8 \quad \text{--- (3)}$$

From (1), (2) and (3) we get,

$$[A|B]=\left[\begin{array}{ccc|c}36 & -6 & 1 & 8\\ 4 & -2 & 1 & -12\\ 9 & 3 & 1 & 8\end{array}\right]$$

$$\sim\left[\begin{array}{ccc|c}4 & -2 & 1 & -12\\ 36 & -6 & 1 & 8\\ 9 & 3 & 1 & 8\end{array}\right] R_1 \leftrightarrow R_2$$

$$\sim\left[\begin{array}{ccc|c}1 & -\frac{1}{2} & \frac{1}{4} & -3\\ 36 & -6 & 1 & 8\\ 9 & 3 & 1 & 8\end{array}\right] R_1 \to \frac{R_1}{4}$$

$$\sim\left[\begin{array}{ccc|c}1 & -\frac{1}{2} & \frac{1}{4} & -3\\ 0 & 12 & -8 & 116\\ 0 & \frac{15}{2} & -\frac{5}{4} & 35\end{array}\right] \begin{array}{l} \\ R_2\to R_2-36R_1\\ R_3\to R_3-9R_1\end{array}$$

$$\sim\left[\begin{array}{ccc|c}1 & -\frac{1}{2} & \frac{1}{4} & -3\\ 0 & 12 & -8 & 116\\ 0 & 1 & -\frac{1}{6} & \frac{14}{3}\end{array}\right] R_3\to\left(\frac{2}{15}\right)R_3$$

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$$\sim\left[\begin{array}{ccc|c}1 & -\frac{1}{2} & \frac{1}{4} & -3\\ 0 & 12 & -8 & 116\\ 0 & 0 & 6 & -60\end{array}\right] R_3\to 12R_3-R_2$$

$$a-\frac{1}{2}b+\frac{1}{4}c=-3 \quad \text{--- (1)}$$

$$12b-8c=116 \quad \text{--- (2)}$$

$$6c=-60 \quad \text{--- (3)}$$

$$(3) \Rightarrow \cancel{6}c=-\cancel{60}^{10}$$

$$\boxed{c=-10}$$

$$(2) \Rightarrow 12b-8(-10)=116$$

$$12b+80=116$$

$$12b=116-80$$

$$\cancel{12}b=\cancel{36}\,3$$

$$\boxed{b=3}$$

$$(1) \Rightarrow a-\frac{1}{2}(3)+\frac{1}{4}(-10)=-3$$

$$a-\frac{3}{2}-\frac{\cancel{10}^{5}}{\cancel{4}_{2}}=-3$$

$$a-\frac{\cancel{8}^{4}}{\cancel{2}}=-3$$

$$a=4-3 \Rightarrow \boxed{a=1}$$

∴ The Soln is $a=1$, $b=3$ and $c=-10$

Given: $y=ax^2+bx+c$

$$y=(1)x^2+3x-10$$

$$y=x^2+3x-10$$

The given eqn passes through $P(7,60)$ we get,

$$60=(7)^2+3(7)-10$$

$$60=49+21-10$$

$$60=60$$

Yes, he meet his friend.

EX 1.27 Solve the following system of linear equations, by Gaussian elimination Method:

$4x+3y+6z=25$, $x+5y+7z=13$, $2x+9y+z=1$

Soln:

$$[A|B] = \left[\begin{array}{ccc|c} 4 & 3 & 6 & 25 \\ 1 & 5 & 7 & 13 \\ 2 & 9 & 1 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 5 & 7 & 13 \\ 4 & 3 & 6 & 25 \\ 2 & 9 & 1 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 5 & 7 & 13 \\ 0 & -17 & -22 & -27 \\ 0 & -1 & -13 & -25 \end{array}\right] \begin{array}{l} \\ R_2 \to R_2 - 4R_1 \\ R_3 \to R_3 - 2R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 5 & 7 & 13 \\ 0 & -17 & -22 & -27 \\ 0 & 0 & -199 & -398 \end{array}\right] \begin{array}{l} \\ \\ R_3 \to 17R_3 - R_2 \end{array}$$

$x + 5y + 7z = 13$ — (1)

$-17y - 22z = -27$ — (2)

$-199z = -398$ — (3)

(3) $\Rightarrow +199z = +398$ 2

$\boxed{z = 2}$

(2) $\Rightarrow -17y - 22(2) = -27$

$-17y - 44 = -27$

$-17y = 44 - 27$

$-\cancel{17}y = \cancel{17}$

$\boxed{y = -1}$

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(1) $\Rightarrow x + 5(-1) + 7(2) = 13$

$x - 5 + 14 = 13$

$x + 9 = 13$

$x = 13 - 9$

$\boxed{x = 4}$

$\therefore$ The Soln is $x = 4$, $y = -1$, $z = 2$

EX 1.28

The upward speed $v(t)$ of a rocket at time t is approximated by $v(t) = at^2 + bt + c$, $0 \le t \le 100$ where a, b and c are constants. It has been found that the speed at times $t = 3$, $t = 6$ and $t = 9$ seconds are respectively 64, 133 and 208 miles per second respectively. Find the speed at time $t = 15$ seconds (Use Gaussian elimination method)

Soln: Given: $v(t) = at^2 + bt + c$

Since $v(3) = 64$, $v(6) = 133$ and $v(9) = 208$

when $t = 3$,

$9a + 3b + c = 64$

when $t = 6$,

$36a + 6b + c = 133$

when $t = 9$,

$81a + 9b + c = 208$

$$[A|B] = \left[\begin{array}{ccc|c} 9 & 3 & 1 & 64 \\ 36 & 6 & 1 & 133 \\ 81 & 9 & 1 & 208 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & \frac{1}{3} & \frac{1}{9} & \frac{64}{9} \\ 36 & 6 & 1 & 133 \\ 81 & 9 & 1 & 208 \end{array}\right] \begin{array}{l} R_1 \to \frac{R_1}{9} \\ \\ \\ \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & \frac{1}{3} & \frac{1}{9} & \frac{64}{9} \\ 0 & -6 & -3 & -123 \\ 0 & -18 & -8 & -368 \end{array}\right] \begin{array}{l} \\ R_2 \to R_2 - 36R_1 \\ R_3 \to R_3 - 81R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & \frac{1}{3} & \frac{1}{9} & \frac{64}{9} \\ 0 & -6 & -3 & -123 \\ 0 & 0 & 1 & 1 \end{array}\right] \begin{array}{l} \\ \\ R_3 \to R_3 - 3R_2 \end{array}$$

$a + \frac{1}{3}b + \frac{1}{9}c = \frac{64}{9}$ — (1)

$-6b - 3c = -123$ — (2)

$c = 1$ — (3)

(3) $\Rightarrow \boxed{c = 1}$

(2) $\Rightarrow -6b - 3(1) = -123$

$-6b - 3 = -123$

$-6b = 3 - 123$

$+\cancel{6}b = +\cancel{120}$ 20

$\boxed{b = 20}$

(1) $\Rightarrow a + \frac{1}{3}(20) + \frac{1}{9}(1) = \frac{64}{9}$

$a + \frac{20}{3} = \frac{64}{9} - \frac{1}{9}$

$a + \frac{20}{3} = \frac{\cancel{63}^{7}}{\cancel{9}}$

$a + \frac{20}{3} = 7$

$a = 7 - \frac{20}{3}$

$a = \frac{21-20}{3}$

$\boxed{a = \frac{1}{3}}$

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Given: $V(t) = at^2 + bt + c$

$V(t) = \frac{1}{3}t^2 + 20t + 1$

when $t = 15$,

$V(15) = \frac{1}{3}(15)^2 + 20(15) + 1$

$= \frac{1}{\cancel{3}}(\cancel{225}^{75}) + 300 + 1$

$= 75 + 300 + 1$

$V(15) = 376$

At 15 Seconds, The Speed is 376 miles Per Seconds

1.5 Applications of Matrices: Consistency of System of linear equations by rank Method

1.5.1 Non-homogeneous linear Equations

Ex 1.29

Test for Consistency of the following System of linear equations and if Possible Solve:

$x + 2y - z = 3$, $3x - y + 2z = 1$, $x - 2y + 3z = 3$, $x - y + z + 1 = 0$

Soln:

$$\begin{bmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 1 & -2 & 3 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 3 \\ -1 \end{bmatrix}$$

$AX = B$

$$[A|B] = \left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 3 & -1 & 2 & 1 \\ 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & -1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & -7 & 5 & -8 \\ 0 & -4 & 4 & 0 \\ 0 & -3 & 2 & -4 \end{array}\right] \begin{array}{l} \\ R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 - R_1 \\ R_4 \to R_4 - R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & -7 & 5 & -8 \\ 0 & 0 & 8 & 32 \\ 0 & 0 & -4 & -16 \end{array}\right] \begin{array}{l} \\ \\ R_3 \to 7R_3 - 4R_2 \\ R_4 \to 4R_4 - 3R_3 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & -7 & 5 & -8 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & -4 & -16 \end{array}\right] R_3 \to \frac{R_3}{8}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & -7 & 5 & -8 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right] R_4 \to R_4 + 4R_3$$

$\therefore \rho[A] = \rho[A|B] = 3$

The given System is Consistent and has a unique Solution

$x + 2y - z = 3$ —— (1)

$-7y + 5z = -8$ —— (2)

$z = 4$ —— (3)

(3) $\Rightarrow$ $\boxed{z = 4}$

(2) $\Rightarrow$ $-7y + 5(4) = -8$

$-7y + 20 = -8$

$-7y = -20 - 8$

$+\cancel{7}y = +\cancel{28}^{4}$

$\boxed{y = 4}$

(1) $\Rightarrow$ $x + 2(4) - 4 = 3$

$x + 8 - 4 = 3$

$x + 4 = 3$

$x = 3 - 4$

$\boxed{x = -1}$

$\therefore$ The Soln is $x = -1$, $y = 4$, $z = 4$

EXERCISE 1.6

1) Test for Consistency and if possible, Solve the System of equations by rank method

(i) $x - y + 2z = 2$, $2x + y + 4z = 7$, $4x - y + z = 4$

Soln:

$$\begin{bmatrix}1 & -1 & 2\\ 2 & 1 & 4\\ 4 & -1 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}2\\ 7\\ 4\end{bmatrix}$$

$AX = B$

$$[A|B]=\left[\begin{array}{ccc|c}1 & -1 & 2 & 2\\ 2 & 1 & 4 & 7\\ 4 & -1 & 1 & 4\end{array}\right]$$

$$\sim\left[\begin{array}{ccc|c}1 & -1 & 2 & 2\\ 0 & 3 & 0 & 3\\ 0 & 3 & -7 & -4\end{array}\right]\begin{array}{l}\\ R_2\to R_2-2R_1\\ R_3\to R_3-4R_1\end{array}$$

$$\sim\left[\begin{array}{ccc|c}1 & -1 & 2 & 2\\ 0 & 3 & 0 & 3\\ 0 & 0 & -7 & -7\end{array}\right]\begin{array}{l}\\ \\ R_3\to R_3-R_2\end{array}$$

$\rho[A]=\rho[A|B]=3=$ number of unknowns

The given system is consistent and has a unique solution

$x-y+2z=2$ —— ①

$3y=3$ —— ②

$-7z=-7$ —— ③

③ $\Rightarrow$ $+\cancel{7}z=+\cancel{7}$

$\boxed{z=1}$

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② $\Rightarrow$ $\cancel{3}y=\cancel{3}$

$\boxed{y=1}$

① $\Rightarrow$ $x-1+2(1)=2$

$x-1+2=2$

$x+1=2$

$x=2-1$

$\boxed{x=1}$

$\therefore$ The Soln is $x=1, y=1, z=1$

EX 1.30

Test for consistency of the following system of linear equations and if possible solve:

$4x-2y+6z=8,\ x+y-3z=-1,\ 15x-3y+9z=21$

Soln:

$$\begin{bmatrix}4 & -2 & 6\\ 1 & 1 & -3\\ 15 & -3 & 9\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}8\\ -1\\ 21\end{bmatrix}$$

$AX=B$

$$[A|B]=\left[\begin{array}{ccc|c}4 & -2 & 6 & 8\\ 1 & 1 & -3 & -1\\ 15 & -3 & 9 & 21\end{array}\right]$$

$$\sim\left[\begin{array}{ccc|c}1 & 1 & -3 & -1\\ 4 & -2 & 6 & 8\\ 15 & -3 & 9 & 21\end{array}\right]R_1\leftrightarrow R_2$$

$$\sim\left[\begin{array}{ccc|c}1 & 1 & -3 & -1\\ 0 & -6 & 18 & 12\\ 0 & -18 & 54 & 36\end{array}\right]\begin{array}{l}\\ R_2\to R_2-4R_1\\ R_3\to R_3-15R_1\end{array}$$

$$\sim\left[\begin{array}{ccc|c}1 & 1 & -3 & -1\\ 0 & -6 & 18 & 12\\ 0 & 0 & 0 & 0\end{array}\right]\begin{array}{l}\\ \\ R_3\to R_3-3R_2\end{array}$$

$\rho[A]=\rho[A|B]=2<3=$ number of unknowns

The given system is consistent and has infinitely many solutions

$x+y-3z=-1$ —— ①

$-6y+18z=12$ —— ②

Take $z=t,\ t\in R$

② $\Rightarrow$ $-6y+18t=12$

$6y=18t-12$

$\div 6$, $\boxed{y=3t-2}$

① $\Rightarrow$ $x+\cancel{3t}-2-\cancel{3t}=-1$

$x=2-1$

$\boxed{x=1}$

$\therefore$ The Soln is $x=1$, $y=3t-2$, $z=t$, $t\in R$

EXERCISE 1.6

1(ii) Test for consistency and if possible, solve the system of equations by rank method

$3x+y+z=2,\ x-3y+2z=1,\ 7x-y+4z=5$

Soln:

$$\begin{bmatrix}3 & 1 & 1\\ 1 & -3 & 2\\ 7 & -1 & 4\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}2\\ 1\\ 5\end{bmatrix}$$

$AX = B$

$$[A|B] = \left[\begin{array}{ccc|c} 3 & 1 & 1 & 2 \\ 1 & -3 & 2 & 1 \\ 7 & -1 & 4 & 5 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & -3 & 2 & 1 \\ 3 & 1 & 1 & 2 \\ 7 & -1 & 4 & 5 \end{array}\right] R_1 \leftrightarrow R_2$$

S. RAJAN PGT IN MATHS

$$\sim \left[\begin{array}{ccc|c} 1 & -3 & 2 & 1 \\ 0 & 10 & -5 & -1 \\ 0 & 20 & -10 & -2 \end{array}\right] \begin{array}{l} R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 - 7R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & -3 & 2 & 1 \\ 0 & 10 & -5 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right] R_3 \to R_3 - 2R_2$$

$\rho[A] = \rho[A|B] = 2 < 3 =$ number of unknowns

The given system is consistent and has infinitely many solutions

$x - 3y + 2z = 1$ —— ①

$10y - 5z = -1$ —— ②

Take $z = t,\ t \in R$

② $\Rightarrow 10y - 5t = -1$

$10y = 5t - 1$

$$\boxed{y = \frac{1}{10}(5t - 1)}$$

① $\Rightarrow x - 3\left[\frac{1}{10}(5t - 1)\right] + 2t = 1$

Multiple by 10,

$10x - 15t + 3 + 20t = 10$

$10x + 5t + 3 = 10$

$10x = 10 - 3 - 5t$

$10x = 7 - 5t$

$$\boxed{x = \frac{1}{10}(7 - 5t)}$$

$\therefore$ The Soln is $x = \frac{1}{10}(7 - 5t)$, $y = \frac{1}{10}(5t - 1)$, $z = t$, $t \in R$

EX 1.31 Test for consistency of the following system of linear equations and if possible solve: $x - y + z = -9$, $2x - 2y + 2z = -18$, $3x - 3y + 3z + 27 = 0$

Soln:

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & -3 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -9 \\ -18 \\ -27 \end{bmatrix}$$

$AX = B$

$$[A|B] = \left[\begin{array}{ccc|c} 1 & -1 & 1 & -9 \\ 2 & -2 & 2 & -18 \\ 3 & -3 & 3 & -27 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & -1 & 1 & -9 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{array}$$

$\rho[A] = \rho[A|B] = 1 < 3 =$ number of unknowns

The given system is consistent and has infinitely many solutions.

$x - y + z = -9$

Take $\boxed{y = s}$, $\boxed{z = t}$, $s, t \in R$

$x - s + t = -9$

$$\boxed{x = s - t - 9}$$

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$\therefore$ The Soln is $x = s - t - 9$, $y = s$, $z = t$, $s, t \in R$

EXERCISE 1.6

①(iv) Test for consistency and if possible, solve the sytem of equations by rank method $2x - y + z = 2$, $6x - 3y + 3z = 6$, $4x - 2y + 2z = 4$

Soln:

$$\begin{bmatrix} 2 & -1 & 1 \\ 6 & -3 & 3 \\ 4 & -2 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \\ 4 \end{bmatrix}$$

$AX = B$

$$[A|B] = \left[\begin{array}{ccc|c} 2 & -1 & 1 & 2 \\ 6 & -3 & 3 & 6 \\ 4 & -2 & 2 & 4 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & -\frac{1}{2} & \frac{1}{2} & 1 \\ 1 & -\frac{1}{2} & \frac{1}{2} & 1 \\ 1 & -\frac{1}{2} & \frac{1}{2} & 1 \end{array}\right] \begin{array}{l} R_1 \to \frac{R_1}{2} \\ R_2 \to \frac{R_2}{6} \\ R_3 \to \frac{R_3}{4} \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & -\frac{1}{2} & \frac{1}{2} & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{array}$$

$\rho[A] = \rho[A|B] = 1 < 3 =$ number of unknowns

The given System is Consistent and has infinitely many Solutions.

$$x - \frac{1}{2}y + \frac{1}{2}z = 1$$

Take $y = s, z = t, s, t \in R$

$$x - \frac{1}{2}s + \frac{1}{2}t = 1$$

Multiple by 2,

$$2x - s + t = 2$$

$$2x = s - t + 2$$

$$\boxed{x = \frac{1}{2}(s - t + 2)}$$

$\therefore$ The Soln is

$x = \frac{1}{2}(s - t + 2)$, $y = s$, $z = t$, $s, t \in R$

EX 1.32 Test the Consistency of the following System of linear equations

$x - y + z = -9$, $2x - y + z = 4$, $3x - y + z = 6$, $4x - y + 2z = 7$

Soln:

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 1 \\ 3 & -1 & 1 \\ 4 & -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -9 \\ 4 \\ 6 \\ 7 \end{bmatrix}$$

$$AX = B$$

$$[A|B] = \left[\begin{array}{ccc|c} 1 & -1 & 1 & -9 \\ 2 & -1 & 1 & 4 \\ 3 & -1 & 1 & 6 \\ 4 & -1 & 2 & 7 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & -1 & 1 & -9 \\ 0 & 1 & -1 & 22 \\ 0 & 2 & -2 & 33 \\ 0 & 3 & -2 & 43 \end{array}\right] \begin{array}{l} \\ R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \\ R_4 \to R_4 - 4R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & -1 & 1 & -9 \\ 0 & 1 & -1 & 22 \\ 0 & 0 & 0 & -11 \\ 0 & 0 & 1 & -23 \end{array}\right] \begin{array}{l} \\ \\ R_3 \to R_3 - 2R_2 \\ R_4 \to R_4 - 3R_2 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & -1 & 1 & -9 \\ 0 & 1 & -1 & 22 \\ 0 & 0 & 1 & -23 \\ 0 & 0 & 0 & -11 \end{array}\right] R_3 \leftrightarrow R_4$$

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$\rho[A] = 3$, $\rho[A|B] = 4$

$\therefore \rho[A] \neq \rho[A|B]$

The given System is inconsistent and has no Solution.

EXERCISE 1.6

1)(iii) Test for Consistency and if possible solve the following system of equations by rank method.

$2x + 2y + z = 5$, $x - y + z = 1$, $3x + y + 2z = 4$

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Soln:

$$\begin{bmatrix} 2 & 2 & 1 \\ 1 & -1 & 1 \\ 3 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ 4 \end{bmatrix}$$

$$AX = B$$

$$[A|B] = \left[\begin{array}{ccc|c} 2 & 2 & 1 & 5 \\ 1 & -1 & 1 & 1 \\ 3 & 1 & 2 & 4 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 2 & 2 & 1 & 5 \\ 3 & 1 & 2 & 4 \end{array}\right] R_1 \leftrightarrow R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 4 & -1 & 3 \\ 0 & 4 & -1 & 1 \end{array}\right] \begin{array}{l} \\ R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 4 & -1 & 3 \\ 0 & 0 & 0 & -2 \end{array}\right] R_3 \to R_3 - R_2$$

$\rho[A] = 2$ and $\rho[A|B] = 3$

$\therefore \rho[A] \neq \rho[A|B]$

The given System is inconsistent and has no Solution

2) Find the value of k for which the equations $kx - 2y + z = 1$, $x - 2ky + z = -2$, $x - 2y + kz = 1$ have (i) no Solution (ii) unique Solution (iii) infinitely many Solution

Soln:

$$\begin{bmatrix} K & -2 & 1 \\ 1 & -2K & 1 \\ 1 & -2 & K \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

$$AX = B$$

$$[A|B] = \left[\begin{array}{ccc|c} K & -2 & 1 & 1 \\ 1 & -2K & 1 & -2 \\ 1 & -2 & K & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & -2 & K & 1 \\ 1 & -2K & 1 & -2 \\ K & -2 & 1 & 1 \end{array}\right] \quad R_1 \leftrightarrow R_3$$

$$\sim \left[\begin{array}{ccc|c} 1 & -2 & K & 1 \\ 0 & 2-2K & 1-K & -3 \\ 0 & 2K-2 & 1-K^2 & 1-K \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - KR_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & -2 & K & 1 \\ 0 & 2-2K & 1-K & -3 \\ 0 & 0 & 2-K-K^2 & -2-K \end{array}\right]$$

$R_3 \to R_3 + R_2$

$$\sim \left[\begin{array}{ccc|c} 1 & -2 & K & 1 \\ 0 & 2(1-K) & 1-K & -3 \\ 0 & 0 & (2+K)(1-K) & -(2+K) \end{array}\right]$$

Case(i) If $K=1$ then

$\rho[A]=1$ and $\rho[A|B]=3$

$\rho[A] \neq \rho[A|B]$

The given System is inconsistent and has no Solution

Case(ii) If $K \neq 1, K \neq -2$ then $\rho[A]=3$ and

$\rho[A|B]=3$

$\rho[A]=\rho[A|B]=3=$ number of unknowns

The given System is Consistent and has a unique Solution

Case (iii) If $K=-2$ then

$\rho[A]=2$ and $\rho[A|B]=2$

$\rho[A]=\rho[A|B]=2<3=$ num-ber of unknowns

The given System is Consistent and has an infinitely many Solution.

3) Investigate the values of λ and μ the System of linear equations $2x+3y+5z=9$, $7x+3y-5z=8$, $2x+3y+\lambda z=\mu$, have (i) no Solution (ii) a unique Solution (iii) infinite number of Solutions

Soln:

$$\begin{bmatrix} 2 & 3 & 5 \\ 7 & 3 & -5 \\ 2 & 3 & \lambda \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 8 \\ \mu \end{bmatrix}$$

$$AX = B$$

$$[A|B] = \begin{bmatrix} 2 & 3 & 5 & 9 \\ 7 & 3 & -5 & 8 \\ 2 & 3 & \lambda & \mu \end{bmatrix}$$

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$$\sim \left[\begin{array}{ccc|c} 1 & \frac{3}{2} & \frac{5}{2} & \frac{9}{2} \\ 7 & 3 & -5 & 8 \\ 2 & 3 & \lambda & \mu \end{array}\right] \quad R_1 \to \frac{R_1}{2}$$

$$\sim \left[\begin{array}{ccc|c} 1 & \frac{3}{2} & \frac{5}{2} & \frac{9}{2} \\ 0 & -\frac{15}{2} & -\frac{45}{2} & -\frac{47}{2} \\ 0 & 0 & \lambda-5 & \mu-9 \end{array}\right]$$

S.RAJAN PGT IN MATHS

$R_2 \to R_2 - 7R_1$

$R_3 \to R_3 - 2R_1$

Case(i) If $\lambda=5$ and $\mu \neq 9$ then $\rho[A]=2$ and $\rho[A|B]=3$

$\rho[A] \neq \rho[A|B]$

The given system is inconsistent and has no Solution.

Case(ii) If $\lambda \neq 5$ and $\mu \in R$ then $\rho[A]=3$ and $\rho[A|B]=3$

$\rho[A]=\rho[A|B]=3=$ num-ber of unknowns

The given System is Consistent and has a unique Solution

Case(iii) If $\lambda=5$ and $\mu=9$ then $\rho[A]=2$ and $\rho[A|B]=2$

$\rho[A]=\rho[A|B]=2<3=$ number of unknowns

The given System is Consistent and has infinite number of Solutions

EX 1.34 Investigate for what values of λ and μ the System of linear equations $x+2y+z=7$, $x+y+\lambda z=\mu$, $x+3y-5z=5$ has (i) no Solution (ii) a unique Solution (iii) an infinite number of Solutions

Soln:

$$\begin{bmatrix}1&2&1\\1&1&\lambda\\1&3&-5\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}7\\\mu\\5\end{bmatrix}$$

$$AX=B$$

$$[A|B]=\left[\begin{array}{ccc|c}1&2&1&7\\1&1&\lambda&\mu\\1&3&-5&5\end{array}\right]$$

$$\sim\left[\begin{array}{ccc|c}1&2&1&7\\1&3&-5&5\\1&1&\lambda&\mu\end{array}\right] R_2\leftrightarrow R_3$$

$$\sim\left[\begin{array}{ccc|c}1&2&1&7\\0&1&-6&-2\\0&-1&\lambda-1&\mu-7\end{array}\right] \begin{array}{l}R_2\to R_2-R_1\\R_3\to R_3-R_1\end{array}$$

$$\sim\left[\begin{array}{ccc|c}1&2&1&7\\0&1&-6&-2\\0&0&\lambda-7&\mu-9\end{array}\right] R_3\to R_3+R_2$$

Case(i) If $\lambda=7$ and $\mu\neq 9$ then $\rho[A]=2$ and $\rho[A|B]=3$

$\rho[A]\neq\rho[A|B]$

The given System is inconsistent and has no Solution

Case(ii) If $\lambda\neq 7$ and $\mu\in R$ then $\rho[A]=3$ and $\rho[A|B]=3$

$\rho[A]=\rho[A|B]=3=$ number of unknowns

The given System is Consistent and has a unique Solution

Case(iii) If $\lambda=7$ and $\mu=9$ then $\rho[A]=2$ and $\rho[A|B]=2$

$\rho[A]=\rho[A|B]=2<3=$ number of unknowns

The given System is consistent and has infinite number of Solutions

EX 1.33 Find the condition on a, b and c so that the following System of linear equations has one parameter family of Solutions: $x+y+z=a$, $x+2y+3z=b$, $3x+5y+7z=c$

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Soln:

$$\begin{bmatrix}1&1&1\\1&2&3\\3&5&7\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}a\\b\\c\end{bmatrix}$$

$$AX=B$$

$$[A|B]=\left[\begin{array}{ccc|c}1&1&1&a\\1&2&3&b\\3&5&7&c\end{array}\right]$$

$$\sim\left[\begin{array}{ccc|c}1&1&1&a\\0&1&2&b-a\\0&2&4&c-3a\end{array}\right] \begin{array}{l}R_2\to R_2-R_1\\R_3\to R_3-3R_1\end{array}$$

$$\sim\left[\begin{array}{ccc|c}1&1&1&a\\0&1&2&b-a\\0&0&0&(c-3a)-2(b-a)\end{array}\right] R_3\to R_3-2R_2$$

$$\sim\left[\begin{array}{ccc|c}1&1&1&a\\0&1&2&b-a\\0&0&0&c-2b-a\end{array}\right]$$

The System Should have one parameter family of Solutions

$\rho[A]=\rho[A|B]=2$

$\therefore c-2b-a=0$

$\boxed{c=a+2b}$

1.5.2 Homogeneous System of linear equations

EX 1.36

Solve the System:
$x+3y-2z=0$, $2x-y+4z=0$, $x-11y+14z=0$

Soln:

$$\begin{bmatrix}1&3&-2\\2&-1&4\\1&-11&14\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$AX=B$$

$$[A|B]=\left[\begin{array}{ccc|c}1&3&-2&0\\2&-1&4&0\\1&-11&14&0\end{array}\right]$$

$$\sim\left[\begin{array}{ccc|c}1&3&-2&0\\0&-7&8&0\\0&-14&16&0\end{array}\right]\begin{array}{l}\\R_2\to R_2-2R_1\\R_3\to R_3-R_1\end{array}$$

$$\sim\left[\begin{array}{ccc|c}1&3&-2&0\\0&-7&8&0\\0&0&0&0\end{array}\right]\begin{array}{l}\\\\R_3\to R_3-2R_2\end{array}$$

$\rho[A]=\rho[A|B]=2<3=$ number of unknowns

The given System is Consistent and has an infinitely many non-trivial Solution

$$x+3y-2z=0 \quad \text{—(1)}$$
$$-7y+8z=0 \quad \text{—(2)}$$

Take $z=t$, $t\in R$

(2) $\Rightarrow -7y+8t=0$

$7y=8t$

$$\boxed{y=\frac{8t}{7}}$$

(1) $\Rightarrow x+3\left(\frac{8t}{7}\right)-2t=0$

$$x+\frac{24t-14t}{7}=0$$

$$x+\frac{10t}{7}=0$$

$$\boxed{x=-\frac{10t}{7}}$$

$\therefore$ The Soln is $x=-\frac{10t}{7}$, $y=\frac{8t}{7}$, $z=t$, $t\in R$

EXERCISE 1.7

1)(i) Solve the following System of homogeneous equations $3x+2y+7z=0$, $4x-3y-2z=0$, $5x+9y+23z=0$

Soln:

$$\begin{bmatrix}3&2&7\\4&-3&-2\\5&9&23\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$AX=B$$

$$[A|B]=\left[\begin{array}{ccc|c}3&2&7&0\\4&-3&-2&0\\5&9&23&0\end{array}\right]$$

$$\sim\left[\begin{array}{ccc|c}1&\frac{2}{3}&\frac{7}{3}&0\\4&-3&-2&0\\5&9&23&0\end{array}\right]\begin{array}{l}R_1\to \frac{R_1}{3}\\\\\\\end{array}$$

$$\sim\left[\begin{array}{ccc|c}1&\frac{2}{3}&\frac{7}{3}&0\\0&-\frac{17}{3}&-\frac{34}{3}&0\\0&\frac{17}{3}&\frac{34}{3}&0\end{array}\right]\begin{array}{l}\\R_2\to R_2-4R_1\\R_3\to R_3-5R_1\end{array}$$

$$\sim\left[\begin{array}{ccc|c}1&\frac{2}{3}&\frac{7}{3}&0\\0&-\frac{17}{3}&-\frac{34}{3}&0\\0&0&0&0\end{array}\right]\begin{array}{l}\\\\R_3\to R_3+R_2\end{array}$$

$\rho[A]=\rho[A|B]=2<3=$ number of unknowns

The given System is Consistent and has an infinitely many non-trivial Solution.

$$x+\frac{2}{3}y+\frac{7}{3}z=0 \quad \text{—(1)}$$
$$-\frac{17}{3}y-\frac{34}{3}z=0 \quad \text{—(2)}$$

Take $z=t$, $t\in R$

(2) $\Rightarrow -\frac{17}{3}y-\frac{34}{3}t=0$

$$\frac{17}{\cancel{3}}y=-\frac{\cancel{34}^{\,2}}{\cancel{3}}t$$

$$\boxed{y=-2t}$$

(1) $\Rightarrow x+\frac{2}{3}(-2t)+\frac{7}{3}t=0$

$$x+\left(\frac{7t-4t}{3}\right)=0$$

$$x+\frac{\cancel{3}t}{\cancel{3}}=0$$

$$\boxed{x=-t}$$

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$\therefore$ The Soln is $x=-t$, $y=-2t$, $z=t$, $t\in R$

(ii) Solve the following system

of homogeneous equations: $2x+3y-z=0$, $x-y-2z=0$, $3x+y+3z=0$

Soln:

$$\begin{bmatrix} 2 & 3 & -1 \\ 1 & -1 & -2 \\ 3 & 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$AX=B$$

$$[A|B]=\left[\begin{array}{ccc|c} 2 & 3 & -1 & 0 \\ 1 & -1 & -2 & 0 \\ 3 & 1 & 3 & 0 \end{array}\right]$$

$$\sim\left[\begin{array}{ccc|c} 1 & -1 & -2 & 0 \\ 2 & 3 & -1 & 0 \\ 3 & 1 & 3 & 0 \end{array}\right] R_1 \leftrightarrow R_2$$

$$\sim\left[\begin{array}{ccc|c} 1 & -1 & -2 & 0 \\ 0 & 5 & 3 & 0 \\ 0 & 4 & 9 & 0 \end{array}\right] \begin{array}{l} R_2 \to R_2-2R_1 \\ R_3 \to R_3-3R_1 \end{array}$$

$$\sim\left[\begin{array}{ccc|c} 1 & -1 & -2 & 0 \\ 0 & 5 & 3 & 0 \\ 0 & 0 & 33 & 0 \end{array}\right] R_3 \to 5R_3-4R_2$$

$\rho[A]=\rho[A|B]=3=$ number of unknowns

The given system is consistent and has a unique trivial solution

$x-y-2z=0$ —— (1)

$5y+3z=0$ —— (2)

$33z=0$ —— (3)

(3) $\Rightarrow$ $33z=0 \Rightarrow \boxed{z=0}$

(2) $\Rightarrow$ $5y+\cancel{3(0)}=0$

$5y=0 \Rightarrow \boxed{y=0}$

(1) $\Rightarrow$ $x-0-\cancel{2(0)}=0$

$\boxed{x=0}$

$\therefore$ It has only trivial solution.

(ie) $x=0,\ y=0,\ z=0$

EX 1.35

Solve the following system:

$x+2y+3z=0$, $3x+4y+4z=0$, $7x+10y+12z=0$

Soln:
$$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 4 \\ 7 & 10 & 12 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$AX=B$$

$$[A|B]=\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 3 & 4 & 4 & 0 \\ 7 & 10 & 12 & 0 \end{array}\right]$$

$$\sim\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -2 & -5 & 0 \\ 0 & -4 & -9 & 0 \end{array}\right] \begin{array}{l} R_2 \to R_2-3R_1 \\ R_3 \to R_3-7R_1 \end{array}$$

$$\sim\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -2 & -5 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] R_3 \to R_3-2R_2$$

$\rho[A]=\rho[A|B]=3=$ number of unknowns

The given system is consistent and has a unique trivial solution.

$x+2y+3z=0$ —— (1)

$-2y-5z=0$ —— (2)

$z=0$ —— (3)

(3) $\Rightarrow$ $\boxed{z=0}$

(2) $\Rightarrow$ $-2y-\cancel{5(0)}=0$

$-2y=0 \Rightarrow \boxed{y=0}$

(1) $\Rightarrow$ $x+\cancel{2(0)}+\cancel{3(0)}=0$

$\boxed{x=0}$

$\therefore$ It has only trivial solution.

(ie) $x=0,\ y=0,\ z=0$

EX 1.37 Solve the system:

$x+y-2z=0$, $2x-3y+z=0$, $3x-7y+10z=0$, $6x-9y+10z=0$

Soln:

$$\begin{bmatrix} 1 & 1 & -2 \\ 2 & -3 & 1 \\ 3 & -7 & 10 \\ 6 & -9 & 10 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$AX=B$$

$$[A|B]=\left[\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 2 & -3 & 1 & 0 \\ 3 & -7 & 10 & 0 \\ 6 & -9 & 10 & 0 \end{array}\right]$$

$$\sim\left[\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 0 & -5 & 5 & 0 \\ 0 & -10 & 16 & 0 \\ 0 & -15 & 22 & 0 \end{array}\right] \begin{array}{l} R_2 \to R_2-2R_1 \\ R_3 \to R_3-3R_1 \\ R_4 \to R_4-6R_1 \end{array}$$

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$$\sim \left[\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 0 & -5 & 5 & 0 \\ 0 & 0 & 6 & 0 \\ 0 & 0 & 7 & 0 \end{array}\right] \begin{array}{l} \\ \\ R_3 \to R_3 - 2R_2 \\ R_4 \to R_4 - 3R_2 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 0 & -5 & 5 & 0 \\ 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] R_4 \to 6R_4 - 7R_3$$

$\rho[A] = \rho[A|B] = 3 =$ number of unknowns

The given System is Consistent and has a unique trivial Solution.

$x + y - 2z = 0$ —①

$-5y + 5z = 0$ —②

$6z = 0$ —③

③ $\Rightarrow 6z = 0 \Rightarrow \boxed{z = 0}$

② $\Rightarrow -5y + 5(0) = 0$

$-5y = 0 \Rightarrow \boxed{y = 0}$

① $\Rightarrow x + 0 - 2(0) = 0$

$\boxed{x = 0}$

It has only trivial Solution

(ie) $x = 0, y = 0, z = 0$

EXERCISE 1.7

2) Determine the values of λ for which the following System of equations $x + y + 3z = 0$, $4x + 3y + \lambda z = 0$, $2x + y + 2z = 0$ has (i) a unique Solution (ii) a non-trivial Solution

Soln:

$$\begin{bmatrix} 1 & 1 & 3 \\ 4 & 3 & \lambda \\ 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$AX = B$

$$[A|B] = \left[\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 4 & 3 & \lambda & 0 \\ 2 & 1 & 2 & 0 \end{array}\right]$$

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$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 2 & 1 & 2 & 0 \\ 4 & 3 & \lambda & 0 \end{array}\right] R_2 \leftrightarrow R_3$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 0 & -1 & -4 & 0 \\ 0 & -1 & \lambda - 12 & 0 \end{array}\right] \begin{array}{l} \\ R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 4R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 0 & -1 & -4 & 0 \\ 0 & 0 & \lambda - 8 & 0 \end{array}\right] R_3 \to R_3 - R_2$$

Case(i) If $\lambda \neq 8$ then

$\rho[A] = 3$, $\rho[A|B] = 3$

$\rho[A] = \rho[A|B] = 3 =$ number of unknowns

The given System is Consistent and has a unique Solution (ie) Trivial Solution

Case(ii) If $\lambda = 8$ then

$\rho[A] = 2$, $\rho[A|B] = 2$

$\rho[A] = \rho[A|B] = 2 < 3 =$ number of unknowns

The given System is Consistent and has an infinitely many non-trivial Solution.

3) By using Gaussian elimination method, balance the chemical reaction equation:

$C_2H_6 + O_2 \to H_2O + CO_2$

Soln:

Let x_1, x_2, x_3 and x_4 be the positive integers.

$x_1 C_2H_6 + x_2 O_2 \to x_3 H_2O + x_4 CO_2$

Equating Carbon, Hydrogen, and oxygen atoms respectively we get,

$2x_1 = x_4 \Rightarrow 2x_1 - x_4 = 0$ —①

$6x_1 = 2x_3 \Rightarrow 6x_1 - 2x_3 = 0$

$\div 2$, $3x_1 - x_3 = 0$ —②

$2x_2 = x_3 + 2x_4$

$2x_2 - x_3 - 2x_4 = 0$ —③

$$\begin{bmatrix} 2 & 0 & 0 & -1 \\ 3 & 0 & -1 & 0 \\ 0 & 2 & -1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$AX = B$

$$[A|B] = \left[\begin{array}{cccc|c} 2 & 0 & 0 & -1 & 0 \\ 3 & 0 & -1 & 0 & 0 \\ 0 & 2 & -1 & -2 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{cccc|c} 2 & 0 & 0 & -1 & 0 \\ 0 & 0 & -2 & 3 & 0 \\ 0 & 2 & -1 & -2 & 0 \end{array}\right] \quad R_2 \to 2R_2 - 3R_1$$

$$\sim \left[\begin{array}{cccc|c} 2 & 0 & 0 & -1 & 0 \\ 0 & 2 & -1 & -2 & 0 \\ 0 & 0 & -2 & 3 & 0 \end{array}\right] \quad R_2 \leftrightarrow R_3$$

$\rho[A] = \rho[A|B] = 3 < 4 =$ number of unknowns

The given System is Consistent and has infinite number of Solutions

$2x_1 - x_4 = 0$ — (1)

$2x_2 - x_3 - 2x_4 = 0$ — (2)

$-2x_3 + 3x_4 = 0$ — (3)

Take $x_4 = t$, $t \in R - \{0\}$

(3) $\Rightarrow -2x_3 + 3t = 0$

$2x_3 = 3t$

$\boxed{x_3 = \frac{3t}{2}}$

(2) $\Rightarrow 2x_2 - \frac{3t}{2} - 2t = 0$

Multiple by 2,

$4x_2 - 3t - 4t = 0$

$4x_2 - 7t = 0$

$4x_2 = 7t$

$\boxed{x_2 = \frac{7t}{4}}$

(1) $\Rightarrow 2x_1 - t = 0$

$2x_1 = t$

$\boxed{x_1 = \frac{t}{2}}$

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Take $t = 4$,

$x_1 = \frac{\cancel{4}^2}{\cancel{2}} \Rightarrow \boxed{x_1 = 2}$

$x_2 = \frac{7(\cancel{4})}{\cancel{4}} \Rightarrow \boxed{x_2 = 7}$

$x_3 = \frac{3(\cancel{4})^2}{\cancel{2}} \Rightarrow \boxed{x_3 = 6}$

and $\boxed{x_4 = 4}$

$\therefore$ The balanced equation is $2C_2H_6 + 7O_2 \rightarrow 6H_2O + 4CO_2$

EX 1.39

By using Gaussian elimination Method, balance the chemical reaction equation:

$C_5H_8 + O_2 \rightarrow CO_2 + H_2O$

(The above is the reaction that is taking place in the burning of organic Compound called isoprene)

Soln:

Let x_1, x_2, x_3 and x_4 be the positive integers

$x_1 C_5H_8 + x_2 O_2 \rightarrow x_3 CO_2 + x_4 H_2O$

Equating Carbon, hydrogen and oxygen atoms respectively we get,

$5x_1 = x_3 \Rightarrow 5x_1 - x_3 = 0$ — (1)

$8x_1 = 2x_4 \Rightarrow 8x_1 - 2x_4 = 0$

$\div 2$, $4x_1 - x_4 = 0$ — (2)

$2x_2 = 2x_3 + x_4$

$2x_2 - 2x_3 - x_4 = 0$ — (3)

$$\begin{bmatrix} 5 & 0 & -1 & 0 \\ 4 & 0 & 0 & -1 \\ 0 & 2 & -2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$AX = B$

$$[A|B] = \left[\begin{array}{cccc|c} 5 & 0 & -1 & 0 & 0 \\ 4 & 0 & 0 & -1 & 0 \\ 0 & 2 & -2 & -1 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{cccc|c} 4 & 0 & 0 & -1 & 0 \\ 5 & 0 & -1 & 0 & 0 \\ 0 & 2 & -2 & -1 & 0 \end{array}\right] \quad R_1 \leftrightarrow R_2$$

$$\sim \left[\begin{array}{cccc|c} 4 & 0 & 0 & -1 & 0 \\ 0 & 2 & -2 & -1 & 0 \\ 5 & 0 & -1 & 0 & 0 \end{array}\right] \quad R_2 \leftrightarrow R_3$$

$$\sim \left[\begin{array}{cccc|c} 4 & 0 & 0 & -1 & 0 \\ 0 & 2 & -2 & -1 & 0 \\ 0 & 0 & -4 & 5 & 0 \end{array}\right] \quad R_3 \to 4R_3 - 5R_1$$

$\rho[A] = \rho[A|B] = 3 < 4 =$ number of unknowns

The given System is consistent and has infinite number of Solutions.

$4x_1 - x_4 = 0$ —— ①

$2x_2 - 2x_3 - x_4 = 0$ —— ②

$-4x_3 + 5x_4 = 0$ —— ③

Take $\boxed{x_4 = t}$, $t \in R - \{0\}$

③ $\Rightarrow -4x_3 + 5t = 0$

$4x_3 = 5t$

$\boxed{x_3 = \frac{5t}{4}}$

② $\Rightarrow 2x_2 - \cancel{2}\left(\frac{5t}{\cancel{4}_2}\right) - t = 0$

Multiple by 2,

$4x_2 - 5t - 2t = 0$

$4x_2 - 7t = 0$

$4x_2 = 7t$

$\boxed{x_2 = \frac{7t}{4}}$

① $\Rightarrow 4x_1 - t = 0$

$4x_1 = t$

$\boxed{x_1 = \frac{t}{4}}$

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Take $t = 4$,

$x_1 = \frac{\cancel{4}}{\cancel{4}} \Rightarrow \boxed{x_1 = 1}$

$x_2 = \frac{7(\cancel{4})}{\cancel{4}} \Rightarrow \boxed{x_2 = 7}$

$x_3 = \frac{5(\cancel{4})}{\cancel{4}} \Rightarrow \boxed{x_3 = 5}$

and $\boxed{x_4 = 4}$

The balance eqn is

$C_5H_8 + 7O_2 \rightarrow 5CO_2 + 4H_2O$

Ex 1.38

Determine the values of λ for which the following System of equations $(3\lambda-8)x + 3y + 3z = 0$, $3x + (3\lambda-8)y + 3z = 0$, $3x + 3y + (3\lambda-8)z = 0$ has a non-trivial Solution

Soln: The given System is Consistent and has a non-trivial Solution

$\rho[A] = \rho[A|B] = 2 < 3 =$ number of unknowns

$$\begin{bmatrix} 3\lambda-8 & 3 & 3 \\ 3 & 3\lambda-8 & 3 \\ 3 & 3 & 3\lambda-8 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$AX = B$

$\therefore |A| = 0$

$$\begin{vmatrix} 3\lambda-8 & 3 & 3 \\ 3 & 3\lambda-8 & 3 \\ 3 & 3 & 3\lambda-8 \end{vmatrix} = 0$$

$$\begin{vmatrix} 3\lambda-2 & 3\lambda-2 & 3\lambda-2 \\ 3 & 3\lambda-8 & 3 \\ 3 & 3 & 3\lambda-8 \end{vmatrix} = 0$$

$R_1 \rightarrow R_1 + R_2 + R_3$

$$(3\lambda-2)\begin{vmatrix} 1 & 1 & 1 \\ 3 & 3\lambda-8 & 3 \\ 3 & 3 & 3\lambda-8 \end{vmatrix} = 0$$

$$(3\lambda-2)\begin{vmatrix} 1 & 1 & 1 \\ 0 & 3\lambda-11 & 0 \\ 0 & 0 & 3\lambda-11 \end{vmatrix} = 0$$

$R_2 \rightarrow R_2 - 3R_1$

$R_3 \rightarrow R_3 - 3R_1$

$(3\lambda-2)(3\lambda-11)^2 = 0$

$3\lambda - 2 = 0$, $3\lambda - 11 = 0$ (twice)

$\boxed{\lambda = \frac{2}{3}}$, $\boxed{\lambda = \frac{11}{3}}$ (twice)

Ex 1.40 If the System of equations $px + by + cz = 0$, $ax + qy + cz = 0$, $ax + by + rz = 0$ has a non-trivial Solution and $p \neq a$, $q \neq b$, $r \neq c$ Prove that

$$\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} = 2$$

Soln: Assume that the given System has a non-trivial Solution

$$\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$$

$$\begin{vmatrix} p & b & c \\ a-p & q-b & 0 \\ a-p & 0 & r-c \end{vmatrix} = 0 \quad \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$

$$\begin{vmatrix} p & b & c \\ -(p-a) & q-b & 0 \\ -(p-a) & 0 & r-c \end{vmatrix} = 0$$

$$(p-a)(q-b)(r-c)\begin{vmatrix} \frac{p}{p-a} & \frac{b}{q-b} & \frac{c}{r-c} \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} = 0$$

Since $p \neq a, q \neq b, r \neq c$

$$\frac{p}{p-a}(1-0) - \frac{b}{q-b}(-1+0) + \frac{c}{r-c}(0+1) = 0$$

$$\frac{p}{p-a} + \frac{b}{q-b} + \frac{c}{r-c} = 0$$

$$\frac{p}{p-a} + \frac{q-(q-b)}{q-b} + \frac{r-(r-c)}{r-c} = 0$$

$$\frac{p}{p-a} + \frac{q}{q-b} - \cancel{\frac{q-b}{q-b}} + \frac{r}{r-c} - \cancel{\frac{r-c}{r-c}} = 0$$

$$\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} - 1 - 1 = 0$$

$$\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} - 2 = 0$$

$$\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} = 2$$

EXERCISE 1.8

Choose the correct Answer:

1) If $|adj(adj A)| = |A|^9$, then the order of the square matrix A is

Soln: Given:

$$|adj(adj A)| = |A|^9 = |A|^{(n-1)^2}$$

$\therefore (n-1)^2 = 9 \Rightarrow n-1 = 3$ $\boxed{n=4}$

$\therefore$ order of A is 4

Ans: (2) 4

2) If A is a 3×3 non-singular matrix such that $AA^T = A^TA$ and $B = A^{-1}A^T$, then $BB^T =$

Soln:

$$BB^T = (A^{-1}A^T)(A^{-1}A^T)^T$$
$$= (A^TA^{-1})((A^T)^T(A^{-1})^T)$$
$$= (A^TA^{-1})(A(A^T)^{-1})$$
$$= A^T(A^{-1}A)(A^T)^{-1}$$
$$= A^T I (A^T)^{-1} = A^T(A^T)^{-1}$$
$$= I$$

$BB^T = I$

Ans: (3) I

3) If $A = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$, $B = adj A$ and $C = 3A$ then $\frac{|adj B|}{|C|} =$

Soln:

$$B = \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix} = adj A$$

$$adj B = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$$

$$|adj B| = \begin{vmatrix} 3 & 5 \\ 1 & 2 \end{vmatrix} = 6 - 5 = 1$$

$$C = \begin{bmatrix} 9 & 15 \\ 3 & 6 \end{bmatrix}$$

$$|C| = \begin{vmatrix} 9 & 15 \\ 3 & 6 \end{vmatrix} = 54 - 45 = 9$$

$$\frac{|adj B|}{|C|} = \frac{1}{9}$$

Ans: (2) $\frac{1}{9}$

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4) If $A\begin{bmatrix}1 & -2\\ 1 & 4\end{bmatrix}=\begin{bmatrix}6 & 0\\ 0 & 6\end{bmatrix}$ then $A=$

Soln: Given:

$A\begin{bmatrix}1 & -2\\ 1 & 4\end{bmatrix}=6\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

Compare,

$A(\text{adj}A)=|A|I_2$

$\text{adj}A=\begin{bmatrix}1 & -2\\ 1 & 4\end{bmatrix}$

$A=\begin{bmatrix}4 & 2\\ -1 & 1\end{bmatrix}$

Ans: (3) $\begin{bmatrix}4 & 2\\ -1 & 1\end{bmatrix}$

5) If $A=\begin{bmatrix}7 & 3\\ 4 & 2\end{bmatrix}$ then $9I-A=$

Soln:

$9I-A=9\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}-\begin{bmatrix}7 & 3\\ 4 & 2\end{bmatrix}$

$=\begin{bmatrix}9 & 0\\ 0 & 9\end{bmatrix}-\begin{bmatrix}7 & 3\\ 4 & 2\end{bmatrix}$

$9I-A=\begin{bmatrix}2 & -3\\ -4 & 7\end{bmatrix}$

$|A|=\begin{vmatrix}7 & 3\\ 4 & 2\end{vmatrix}=14-12=2$

$\text{adj}A=\begin{bmatrix}2 & -3\\ -4 & 7\end{bmatrix}=9I-A$

$A^{-1}=\frac{1}{|A|}(\text{adj}A)$

$A^{-1}=\frac{1}{2}(9I-A)$

$9I-A=2A^{-1}$

Ans: (4) $2A^{-1}$

6) If $A=\begin{bmatrix}2 & 0\\ 1 & 5\end{bmatrix}$ and $B=\begin{bmatrix}1 & 4\\ 2 & 0\end{bmatrix}$ then $|\text{adj}(AB)|=$

Soln:

$AB=\begin{bmatrix}2 & 0\\ 1 & 5\end{bmatrix}\begin{bmatrix}1 & 4\\ 2 & 0\end{bmatrix}$

$AB=\begin{bmatrix}2 & 8\\ 11 & 4\end{bmatrix}$

$\text{adj}(AB)=\begin{bmatrix}4 & -8\\ -11 & 2\end{bmatrix}$

$|\text{adj}(AB)|=\begin{vmatrix}4 & -8\\ -11 & 2\end{vmatrix}$

$=8-88=-80$

$|\text{adj}(AB)|=-80$

Ans: (2) -80

7) If $P=\begin{bmatrix}1 & x & 0\\ 1 & 3 & 0\\ 2 & 4 & -2\end{bmatrix}$ is the adjoint of 3x3 matrix A and $|A|=4$ then x is

Soln: Given:

$P=\text{adj}A=\begin{bmatrix}1 & x & 0\\ 1 & 3 & 0\\ 2 & 4 & -2\end{bmatrix}$

$|\text{adj}A|=|A|^{n-1}$

$\begin{vmatrix}1 & x & 0\\ 1 & 3 & 0\\ 2 & 4 & -2\end{vmatrix}=(4)^{3-1}$

$1(-6-0)-x(-2-0)+\cancel{0(4-6)}=(4)^2$

$-6+2x=16$

$2x=16+6=22$

$\cancel{2}x=\cancel{22}\,11$

$\boxed{x=11}$

Ans: (4) 11

8) If $A=\begin{bmatrix}3 & 1 & -1\\ 2 & -2 & 0\\ 1 & 2 & -1\end{bmatrix}$ and $A^{-1}=\begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{bmatrix}$ then the value of a_{23} is

Soln: $|A|=\begin{vmatrix}3 & 1 & -1\\ 2 & -2 & 0\\ 1 & 2 & -1\end{vmatrix}$

$=3(2-0)-1(-2-0)-1(4+2)$

$=\cancel{6}+2-\cancel{6}=2$

$|A|=2$

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$$[A_{ij}] = \begin{bmatrix} -2 & 0 & 2 & -2 \\ 2 & -1 & 1 & 2 \\ 1 & -1 & 3 & 1 \\ -2 & 0 & 2 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 2-0 & 0+2 & 4+2 \\ -2+1 & -3+1 & 1-6 \\ 0-2 & -2-0 & -6-2 \end{bmatrix}$$

$$[A_{ij}] = \begin{bmatrix} 2 & 2 & 6 \\ -1 & -2 & -5 \\ -2 & -2 & -8 \end{bmatrix}$$

$$\text{adj} A = [A_{ij}]^T = \begin{bmatrix} 2 & -1 & -2 \\ 2 & -2 & -2 \\ 6 & -5 & -8 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}(\text{adj} A)$$

$$= \frac{1}{2}\begin{bmatrix} 2 & -1 & -2 \\ 2 & -2 & -2 \\ 6 & -5 & -8 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1 & -\frac{1}{2} & -1 \\ 1 & -1 & -1 \\ 3 & -\frac{5}{2} & -4 \end{bmatrix}$$

The value of $a_{23} = -1$

Ans: (4) -1

9) If A, B and C are invertible matrices of some order, then which one of the following is not true?

Soln: From the option,

$\text{adj}(AB) = (\text{adj} A)(\text{adj} B)$ is not correct

[Since $\text{adj}(AB) = (\text{adj} B)(\text{adj} A)$ is correct]

Ans: (2) $\text{adj}(AB) = (\text{adj} A)(\text{adj} B)$

10) If $(AB)^{-1} = \begin{bmatrix} 12 & -17 \\ -19 & 27 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$ then $B^{-1} =$

Soln: $(AB)^{-1} = B^{-1}A^{-1}$

$$\begin{bmatrix} 12 & -17 \\ -19 & 27 \end{bmatrix} = B^{-1}\begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 12 & -17 \\ -19 & 27 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 12 & -17 \\ -19 & 27 \end{bmatrix} = \begin{bmatrix} a-2b & -a+3b \\ c-2d & -c+3d \end{bmatrix}$$

$\cancel{a} - 2b = 12$ — (1)

$-\cancel{a} + 3b = -17$ — (2)

$b = -5$

(1) $\Rightarrow a - 2(-5) = 12$

$a + 10 = 12$

$a = 12 - 10$

$\boxed{a = 2}$

$\cancel{c} - 2d = -19$

$-\cancel{c} + 3d = 27$

$d = 8$

(3) $\Rightarrow c - 2(8) = -19$

$c - 16 = -19$

$c = 16 - 19$

$\boxed{c = -3}$

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$$\therefore B^{-1} = \begin{bmatrix} 2 & -5 \\ -3 & 8 \end{bmatrix}$$

Ans: (1) $\begin{bmatrix} 2 & -5 \\ -3 & 8 \end{bmatrix}$

11) If $A^T A^{-1}$ is symmetric then $A^2 =$

Soln: Given:

$(A^T A^{-1})^T = (A^T A^{-1})$ is symmetric

$(A^{-1})^T (A^T)^T = A^T A^{-1}$

$(A^T)^{-1} A = A^T A^{-1}$ [using property]

Post multiple by A,

$((A^T)^{-1} A)A = (A^T A^{-1})A$

$(A^T)^{-1} A^2 = A^T (A^{-1} A)$

$(A^T)^{-1} A^2 = A^T I$

$(A^T)^{-1} A^2 = A^T$

Pre multiple by A^T,

$A^T (A^T)^{-1} A^2 = A^T A^T$

$$I A^2 = (A^T)^2$$

$$A^2 = (A^T)^2$$

Ans: (2) $(A^T)^2$

12) If A is a non-singular matrix such that $A^{-1} = \begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$ then $(A^T)^{-1} =$

Soln: By Property,

$$(A^T)^{-1} = (A^{-1})^T$$

$$= \begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}^T$$

$$(A^T)^{-1} = \begin{bmatrix} 5 & -2 \\ 3 & -1 \end{bmatrix}$$

Ans: (4) $\begin{bmatrix} 5 & -2 \\ 3 & -1 \end{bmatrix}$

13) If $A = \begin{bmatrix} 3/5 & 4/5 \\ x & 3/5 \end{bmatrix}$ and $A^T = A^{-1}$, then the value of x is

Soln:

Given: $\boxed{A^T = A^{-1}}$

$$AA^{-1} = I \Rightarrow AA^T = I$$

$$\begin{bmatrix} 3/5 & 4/5 \\ x & 3/5 \end{bmatrix}\begin{bmatrix} 3/5 & x \\ 4/5 & 3/5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & \frac{3}{5}x + \frac{12}{25} \\ \frac{3}{5}x + \frac{12}{25} & x^2 + \frac{9}{25} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\frac{3}{5}x + \frac{12}{25} = 0$$

$$\frac{\cancel{3}}{\cancel{5}}x = \frac{-\cancel{12}\,4}{\cancel{25}\,5}$$

$$\boxed{x = -\frac{4}{5}}$$

Ans: (1) $-\frac{4}{5}$

14) If $A = \begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}$ and $AB = I$ then $B =$

Soln: Given: $AB = I$

$$B = A^{-1} \quad \text{—(1)}$$

$$|A| = \begin{vmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{vmatrix}$$

$$|A| = 1 + \tan^2\frac{\theta}{2} = \sec^2\frac{\theta}{2}$$

$$(1) \Rightarrow B = A^{-1} = \frac{1}{|A|}(\text{adj}A)$$

$$= \frac{1}{\sec^2\frac{\theta}{2}}\begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix}$$

$$B = \left(\cos^2\frac{\theta}{2}\right)A^T$$

Ans: (2) $\left(\cos^2\frac{\theta}{2}\right)A^T$

15) If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ and $A(\text{adj}A) = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$ then $k =$

Soln: Given:

$$A(\text{adj}A) = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$$

$$|A|I = k\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$|A|I = kI \Rightarrow |A| = k$$

$$k = \begin{vmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{vmatrix}$$

$$k = \cos^2\theta + \sin^2\theta = 1$$

$$k = 1$$

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Ans: (4) 1

16) If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $\lambda A^{-1} = A$ then λ is

Soln: $|A| = \begin{vmatrix} 2 & 3 \\ 5 & -2 \end{vmatrix} = -4-15$

$$|A| = -19$$

$$A^{-1} = \frac{1}{|A|}(\text{adj}A)$$

$$= \frac{1}{-19}\begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{19}\begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$$

$$19A^{-1} = A$$

Compare, $\lambda A^{-1} = A$

$\therefore \boxed{\lambda = 19}$ Ans: (3) 19

17) If $\text{adj}A = \begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$ and $\text{adj}B = \begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix}$ then $\text{adj}(AB)$ is

Soln:

$\text{adj}(AB) = (\text{adj}B)(\text{adj}A)$

$= \begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$

$= \begin{bmatrix} -6 & 5 \\ -2 & -10 \end{bmatrix}$

Ans: (2) $\begin{bmatrix} -6 & 5 \\ -2 & -10 \end{bmatrix}$

18) The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ -1 & -2 & -3 & -4 \end{bmatrix}$ is

Soln:

Let $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ -1 & -2 & -3 & -4 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} \\ R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 + R_1 \end{matrix}$

$\rho(A) = 1$, Ans: (1) 1

19) If $x^a y^b = e^m$, $x^c y^d = e^n$, $\Delta_1 = \begin{vmatrix} m & b \\ n & d \end{vmatrix}$, $\Delta_2 = \begin{vmatrix} a & m \\ c & n \end{vmatrix}$, $\Delta_3 = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$, then the values of x and y are respectively

Soln: Given:

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$x^a y^b = e^m$

$\log(x^a y^b) = \log e^m$

$\log x^a + \log y^b = \log e^m$

$a\log x + b\log y = m$ —— (1)

IIIly $c\log x + d\log y = n$ —— (2)

By Cramer's Rule, we get

$\log x = \frac{\Delta_1}{\Delta_3} \Rightarrow \boxed{x = e^{\left(\frac{\Delta_1}{\Delta_3}\right)}}$

$\log y = \frac{\Delta_2}{\Delta_3} \Rightarrow \boxed{y = e^{\left(\frac{\Delta_2}{\Delta_3}\right)}}$

Ans: (4) $e^{\left(\frac{\Delta_1}{\Delta_3}\right)}, e^{\left(\frac{\Delta_2}{\Delta_3}\right)}$

20) Which of the following is/are correct?

Soln: Since statement (iii) is incorrect

Remaining statements are correct

Ans: (4) (i), (ii) and (iv)

21) If $\rho[A] = \rho[A|B]$ then the system $AX = B$ of linear equations is

Soln:

Ans: (2) Consistent

22) If $0 \le \theta \le \pi$ and the system of equations $x + (\sin\theta)y - (\cos\theta)z = 0$, $(\cos\theta)x - y + z = 0$, $(\sin\theta)x + y - z = 0$ has a non-trivial solution then θ is

Soln: Given: $|A| = 0$

$\begin{vmatrix} 1 & \sin\theta & -\cos\theta \\ \cos\theta & -1 & 1 \\ \sin\theta & 1 & -1 \end{vmatrix} = 0$

$1\cancel{(1-1)} - \sin\theta(-\cos\theta - \sin\theta)$

$-\cos\theta(\cos\theta + \sin\theta) = 0$

$\cancel{\sin\theta\cos\theta} + \sin^2\theta - \cos^2\theta$

$-\cancel{\sin\theta\cos\theta} = 0$

$\sin^2\theta = \cos^2\theta$

$\frac{\sin^2\theta}{\cos^2\theta} = 1$

$\tan^2\theta = 1$

$\tan\theta = 1$

$\theta = \tan^{-1}(1)$

$\theta = \pi/4$

Ans: (4) $\frac{\pi}{4}$

23) The augmented matrix of a system of linear equations is

$$\begin{bmatrix} 1 & 2 & 7 & 3 \\ 0 & 1 & 4 & 6 \\ 0 & 0 & \lambda-7 & \mu+5 \end{bmatrix}$$

The system has infinitely many solutions if

Soln: If $\lambda=7$, $\mu=-5$

then $\rho[A]=\rho[A|B]=2<3$ = number of unknowns

The given system is consistent and has an infinitely many solutions

Ans: (4) $\lambda=7$, $\mu=-5$

24) Let $A=\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and $4B=\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & x \\ -1 & 1 & 3 \end{bmatrix}$. If B is the inverse of A, then the value of x is

Soln: $|A|=\begin{vmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{vmatrix}$

$=2(4-1)+1(-2+1)+1(1-2)$

$=2(3)+1(-1)+1(-1)$

$=6-1-1=4$

$|A|=4$

$$[A_{ij}]=\begin{bmatrix} 2 & -1 & -1 & 2 \\ -1 & 2 & 1 & -1 \\ -1 & 1 & 2 & -1 \\ 2 & -1 & -1 & 2 \end{bmatrix}$$

$$=\begin{bmatrix} 4-1 & -1+2 & 1-2 \\ -1+2 & 4-1 & -1+2 \\ 1-2 & -1+2 & 4-1 \end{bmatrix}$$

$$=\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

$\text{adj}A=[A_{ij}]^T=\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$

$A^{-1}=\frac{1}{|A|}(\text{adj}A)$

$A^{-1}=\frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$

$4A^{-1}=\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$

Given: $4B=\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & x \\ -1 & 1 & 3 \end{bmatrix}$

B is the inverse of A

$\therefore x=1$

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Ans: (4) 1

25) If $A=\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ then $\text{adj}(\text{adj}A)$ is

Soln: $|A|=\begin{vmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{vmatrix}$

$=3(-3+4)+3(2-0)+4(-2-0)$

$=3+6-8=1$

$|A|=1$ and order $n=3$

By Property,

$\text{adj}(\text{adj}A)=|A|^{n-2}A$

$=(1)^{3-2}A$

$=(1)A=A=\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$

$\text{adj}(\text{adj}A)=\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$

Ans: (1) $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$

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