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|---|---|---|---|---|---|---|
| **12th Standard** | Syllabus | Books | Study Materials – EM | Study Materials - TM | Practical | Online Test (EM & TM) |
| | Monthly Q&A | Mid Term Q&A | Revision Q&A | PTA Book Q&A | Centum Questions | Creative Questions |
| | Quarterly Exam | Half Yearly Exam | Public Exam | NEET | | |

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|---|---|---|---|---|---|---|
| **11th Standard** | Syllabus | Books | Study Materials – EM | Study Materials - TM | Practical | Online Test (EM & TM) |
| | Monthly Q&A | Mid Term Q&A | Revision Q&A | Centum Questions | Creative Questions | |
| | Quarterly Exam | Half Yearly Exam | Public Exam | NEET | | |

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|---|---|---|---|---|---|---|
| **10th Standard** | Syllabus | Books | Study Materials - EM | Study Materials - TM | Practical | Online Test (EM & TM) |
| | Monthly Q&A | Mid Term Q&A | Revision Q&A | PTA Book Q&A | Centum Questions | Creative Questions |
| | Quarterly Exam | Half Yearly Exam | Public Exam | NTSE | SLAS | |

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|---|---|---|---|---|---|---|
| **9th Standard** | Syllabus | Books | Study Materials | 1st Mid Term | 2nd Mid Term | 3rd Mid Term |
| | Quarterly Exam | Half Yearly Exam | Annual Exam | RTE | | |

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| **8th Standard** | Syllabus | Books | Study Materials | 1st Mid Term | 2nd Mid Term | 3rd Mid Term |
| | Term 1 | Term 2 | Term 3 | Public Model Q&A | NMMS | Periodical Test |

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| **7th Standard** | Syllabus | Books | Study Materials | 1st Mid Term | 2nd Mid Term | 3rd Mid Term |
| | Term 1 | Term 2 | Term 3 | Periodical Test | SLAS | |

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| **6th Standard** | Syllabus | Books | Study Materials | 1st Mid Term | 2nd Mid Term | 3rd Mid Term |
| | Term 1 | Term 2 | Term 3 | Periodical Test | SLAS | |

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| **1st to 5th Standard** | Syllabus | Books | Study Materials | Periodical Test | SLAS | |
| | Term 1 | Term 2 | Term 3 | Public Model Q&A | | |

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| **Exams** | TET | TNPSC | PGTRB | Polytechnic | Police | Computer Instructor |
| | DEO | BEO | LAB Asst | NMMS | RTE | NTSE |

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| **Portal** | Matrimony | Mutual Transfer | Job Portal |

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| **Volunteers** | Centum Team | Creative Team | Key Answer Team |

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# CHAPTER - IV

## INVERSE TRIGONOMETRIC FUNCTIONS

### EX – 4.1

1.

(i)

$-10\pi \le x \le 10\pi$ and $\sin x = 0$

Soln:

$\sin x = 0$ (given)

$x = n\pi$ [because $\sin n\pi = 0$]

where $n = 0, \pm 1, \pm 2, \ldots \pm 10$

(ii) $\sin x = -1$ (given)

$\sin x = \sin(-\pi/2)$

$x = -\pi/2, 3\pi/2, 7\pi/2, \ldots$

$x = (4n-1)\pi/2, n \in Z$

where $n = 0, \pm 1, \pm 2, \pm 3, 4$

2)

(i) $y = \sin 7x$

$y = c \sin ax$

$c = 1$  $a = 7$

Period of the function $\sin x$ is $2\pi$

Period of the function $\sin 7x$ is $\frac{2\pi}{7}$

Since $\sin 7(2\pi/7) = \sin 2\pi$

Amplitude of $\sin x = 1$

[Max hight sin curve is 1]

Amplitude of $\sin 7x$ is 1

Period $= \frac{2\pi}{|a|} = 2\pi/7$

amplitude $= |c| = 1$

(ii) $y = -\sin(\frac{1}{3}x)$

Period of the function $\sin x$ is $2\pi$

Period of the function $\sin(\frac{1}{3}x)$ is $6\pi$

Since $\sin(\frac{1}{3} 6\pi) = \sin 2\pi$

Amplitude of $\sin(\frac{1}{3}x)$ is 1

(iii) $y = 4\sin(-2x)$

$y = c \sin ax$

$c = 4$  $a = -2$

Period of the function $\sin x$ is $2\pi$

Period of the function $\sin 2x$ is $\pi$

Since $\sin 2(\pi) = \sin 2\pi$

Amplitude of $\sin 2x$ is 4

Period $= \frac{2\pi}{|-2|} = \pi$

amplitude $= 4$.

3) $y = \sin(\frac{1}{3}x)$ for $0 \le x \le 6\pi$

$a = 1/3$

Period $= \frac{2\pi}{1/3} = 6\pi$

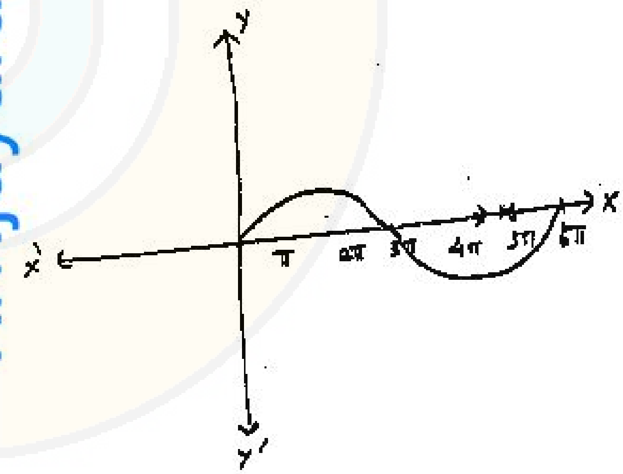
amplitude $= 1$

$y = c \sin(ax)$

Period $= \frac{2\pi}{|a|}$

amplitude $= |c|$

amplitude $= 1$




4)

(i) The Principal value of

$\sin^{-1}(x) = [-\pi/2, \pi/2]$

But $2\pi/3 \notin [-\pi/2, \pi/2]$

$\sin(2\pi/3) = \sin(3\pi/3 - \pi/3)$  $[\sin(\pi - \theta) = \sin\theta]$

$= \sin(\pi - \pi/3) = \sin(\pi/3)$

$\pi/3 \in [-\pi/2, \pi/2]$

$\sin(2\pi/3) = \sin \pi/3$

$\sin^{-1}\left[\sin\left(\frac{2\pi}{3}\right)\right] = \sin^{-1}\left[\sin\left(\frac{\pi}{3}\right)\right]$

$= \frac{\pi}{3}$

$\left[\because \sin^{-1}(\sin(\theta)) = \theta\right]$

(ii) $\sin\left(\frac{5\pi}{4}\right) = \sin\left(\frac{4\pi}{4} + \frac{\pi}{4}\right)$

$= \sin\left(\pi + \frac{\pi}{4}\right)$

$= -\sin\left(\frac{\pi}{4}\right)$

$= \sin\left(-\frac{\pi}{4}\right)$

$[\because \sin(\pi+\theta) = -\sin\theta]$

$(\sin(-\theta) = -\sin\theta]$

$\sin^{-1}\left[\sin\left(\frac{5\pi}{4}\right)\right] = \sin^{-1}\left[\sin\left(-\frac{\pi}{4}\right)\right]$

$= -\frac{\pi}{4}$

$\left[\because \sin^{-1}[\sin(\theta)] = \theta\right]$

5. $\sin x = \sin^{-1} x$ is possible only when $x = 0$, $\because x \in R$

6.

(i) $f(x) = \sin^{-1}\left(\frac{x^2+1}{2x}\right)$

Soln:

The domain of $\sin^{-1} x = [-1, 1]$

$-1 \le x \le 1$

The domain of $\sin^{-1}\left(\frac{x^2+1}{2x}\right)$ is

$-1 \le \frac{x^2+1}{2x} \le 1$

$\frac{x^2+1}{2x} \ge -1$

$x^2+1 \ge -2x$

$x^2+2x+1 \ge 0$

$(x+1)^2 \ge 0$

$-1 \le x \le 1$

and $\frac{x^2+1}{2x} \le 1$

$x^2+1 \le 2x$

$x^2-2x+1 \le 0$

$(x-1)^2 \le 0$ which is impossible

(ii) $g(x) = 2\sin^{-1}(2x-1) - \frac{\pi}{4}$

The domain of $\sin^{-1}(x) = [-1, 1]$

$-1 \le x \le 1$

The domain of $\sin^{-1}(2x-1)$

$-1 \le 2x-1 \le 1$

Adding 1,

$-1+1 \le 2x - \cancel{1} + \cancel{1} \le 1+1$

$0 \le 2x \le 1+1$

$0 \le 2x \le 2$

$\div 2 \Rightarrow \quad 0 \le x \le 1$

7) $\sin^{-1}\left(\sin\frac{5\pi}{9}\cos\frac{\pi}{9} + \cos\frac{5\pi}{9}\sin\frac{\pi}{9}\right)$

Soln:

$\sin(A+B) = \sin A\cos B + \cos A \sin B$

$\left(\sin\frac{5\pi}{9}\cos\frac{\pi}{9} + \cos\frac{5\pi}{9}\sin\frac{\pi}{9}\right) = \sin\left(\frac{5\pi}{9} + \frac{\pi}{9}\right)$

$= \sin\left(\frac{6\pi}{9}\right) = \sin\left(\frac{2\pi}{3}\right)$

But $\frac{2\pi}{3} \notin \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\sin\left(\frac{2\pi}{3}\right) = \sin\left(\frac{3\pi}{3} - \frac{\pi}{3}\right) = \sin\left(\pi - \frac{\pi}{3}\right)$

$= \sin\left(\frac{\pi}{3}\right) \qquad [\sin(\pi-\theta) = \sin\theta]$

$\sin^{-1}\left[\sin\frac{5\pi}{9}\cos\frac{\pi}{9} + \cos\frac{5\pi}{9}\sin\frac{\pi}{9}\right]$

$= \sin^{-1}\left(\sin\left(\frac{\pi}{3}\right)\right) = \frac{\pi}{3}$

$= \frac{\pi}{3}$, $\frac{\pi}{3} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$[\sin^{-1}(\sin\theta)] = \theta$.

1.

(i)

$\cos x = 0$

$x = (2n+1) \pm \pi/2$

where $n = 0, \pm1, \pm2, \pm3, \pm4, \pm5$

(ii) $\cos x = 1$

$x = (2n+1)\pi$

$n = 0, \pm1, \pm2, -3$

2.

$\cos(-\pi/6) = \cos \pi/6$

Since $\cos(-\theta) = \cos\theta$

$\cos^{-1}[\cos(-\pi/6)] = \cos^{-1}[\cos(\pi/6)]$

$= \pi/6$

[Since $\cos^{-1}[\cos\theta] = \theta$]



$\therefore \cos^{-1}[\cos(-\pi/6)] \neq -\pi/6$

$-\pi/6 \notin [0, \pi]$, $-\pi/6$ not in the domain

3. Let $\theta = \cos^{-1}(-x)$

$\cos\theta = -x$

$\cos(\pi - \theta) = -x$

$\pi - \theta = \cos^{-1}(+x)$

$\theta = \pi - \cos^{-1}(x)$

$\theta = \pi - \cos^{-1}(x)$

$\cos^{-1}(-x) = \pi - \cos^{-1}(x)$

So it is true.

4. Let $\cos^{-1}(1/2) = \theta$

$\cos\theta = 1/2$

$\cos\theta = \cos \pi/3$

$\theta = \pi/3 \in [0, \pi]$

$\therefore \cos^{-1}(1/2) = \pi/3$

5)

$\cos^{-1}(1/2) = \theta$

$\cos\theta = 1/2 = \cos \pi/3$

$\theta = \pi/3 \in [0, \pi]$

$\boxed{\cos^{-1}(1/2) = \pi/3}$

If $\sin^{-1}(1/2) = \theta$

$\sin\theta = 1/2 = \sin \pi/6$

$\theta = \pi/6$

$\boxed{\sin^{-1}(1/2) = \pi/6}$

$2\cos^{-1}(1/2) + \sin^{-1}(1/2) = 2(\pi/3) + \pi/6$

$= \frac{4\pi + \pi}{6} = 5\pi/6$

(ii)

Let $\cos^{-1}(1/2) = \theta$

$\cos\theta = 1/2 = \cos \pi/3$

$\theta = \pi/3 \in [0, \pi]$

$\boxed{\cos^{-1}(1/2) = \pi/3}$

If $\sin^{-1}(-1) = \theta$

$\sin\theta = -1 = \sin(-\pi/2)$

$\theta = -\pi/2$

$\boxed{\sin^{-1}(-1) = -\pi/2}$

$\cos^{-1}(1/2) + \sin^{-1}(-1) = \pi/3 - \pi/2 = \frac{2\pi - 3\pi}{6} = -\pi/6$

(iii) W.K.T

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\cos \pi/7 \cos \pi/17 - \sin \pi/7 \sin \pi/17 = \cos(\pi/7 + \pi/17)$

$= \cos\left(\frac{17\pi + 7\pi}{119}\right)$

$= \cos\left(\frac{24\pi}{119}\right)$

$\cos^{-1}\left(\cos\frac{\pi}{7}\cos\frac{\pi}{17} - \sin\frac{\pi}{7}\sin\frac{\pi}{17}\right)$

$= \cos^{-1}\cos\left(\frac{24\pi}{119}\right)$

$= \frac{24\pi}{119}$.

6)

$f(x) = \sin^{-1}\left(\frac{|x|-2}{3}\right) + \cos^{-1}\left(\frac{1-|x|}{4}\right)$

Soln

$f(x) = \sin^{-1}\left(\frac{|x|-2}{3}\right) + \cos^{-1}\left(\frac{1-|x|}{4}\right)$

$g(x) = \sin^{-1}\left(\frac{|x|-2}{3}\right)$

$-1 \le \frac{|x|-2}{3} \le 1$ [Since domain of $\sin^{-1}x, \cos^{-1}x$ is $[-1,1]$]

$-3 \le |x|-2 \le 3$

Add +2

$-3+2 \le |x|-2+2 \le 3+2$

$\left.\begin{array}{l}-1 \le |x| \le 5\\ |x| \le 5\end{array}\right\}$ —— (1)

Let $h(x) = \cos^{-1}\left(\frac{1-|x|}{4}\right)$

$-1 \le \left(\frac{1-|x|}{4}\right) \le 1$

$-4 \le 1-|x| \le 4$

Sub (-1)

$-4-1 \le 1-1-|x| \le 4-1$

$-5 \le |x| \le 3$

$\times$ ply $(-)$ $5 \ge |x| \ge -3$

$-3 \le |x| \le 5$

$|x| \le 5$ —— (2).

$\therefore$ The domain $-5 \le x \le 5$

(ie) $\therefore [-5, 5]$.

(ii) $g(x) = \sin^{-1}x + \cos^{-1}x$

domain of $\sin^{-1}x + \cos^{-1}x$ is $[-1, 1]$

[Since domain of $\sin^{-1}x, \cos^{-1}x$ is $[-1,1]$]

7)

$\frac{\pi}{2} < \cos^{-1}(3x-1) < \pi$

$\cos\frac{\pi}{2} < 3x-1 < \cos\pi$

$0 < 3x-1 < -1$

Add 1

$0+1 < 3x-1+1 < -1+1$

$1 < 3x < 0$

$\div 3 \Rightarrow \frac{1}{3} < x < 0$

This inequality holds only if

$x < 0$ or $x > \frac{1}{3}$

8)

(i) W.K.T

$\cos^{-1}(x) + \sin^{-1}(x) = \frac{\pi}{2}$

$\cos^{-1}\left(\frac{4}{5}\right) + \sin^{-1}\left(\frac{4}{5}\right) = \frac{\pi}{2}$

$\cos\left[\cos^{-1}\left(\frac{4}{5}\right) + \sin^{-1}\left(\frac{4}{5}\right)\right] = \cos\frac{\pi}{2}$

$= 0$

(ii) $\cos\left(\frac{4\pi}{3}\right) = \cos\left(\frac{3\pi}{3} + \frac{\pi}{3}\right)$

$= \cos\left(\pi + \frac{\pi}{3}\right)$

$\cos\left(\pi + \frac{\pi}{3}\right) = -\cos\left(\frac{\pi}{3}\right)$ $(\cos(\pi+\theta) = -\cos\theta)$

$\cos\left(\frac{4\pi}{3}\right) = -\cos\frac{\pi}{3}$

$\cos^{-1}\left(\cos\left(\frac{4\pi}{3}\right)\right) = \cos^{-1}\left(-\cos\frac{\pi}{3}\right)$

$\boxed{\cos^{-1}\left[\cos\left(\frac{4\pi}{3}\right)\right] = -\frac{\pi}{3}}$

$\cos\left(\frac{5\pi}{4}\right) = \cos\left(\frac{4\pi}{4} + \frac{\pi}{4}\right)$

$= -\cos\left(+\frac{\pi}{4}\right) = -\cos\left(\frac{\pi}{4}\right)$

$\cos^{-1}\left[\cos\left(\frac{5\pi}{4}\right)\right] = -\cos^{-1}\left(\cos\left(\frac{\pi}{4}\right)\right)$

$\boxed{\cos^{-1}\left[\cos\left(\frac{5\pi}{4}\right)\right] = -\frac{\pi}{4}}$

$\cos^{-1}\left[\cos\left(\frac{4\pi}{3}\right)\right] + \cos^{-1}\left[\cos\left(\frac{5\pi}{4}\right)\right]$

$= -\frac{\pi}{3} - \frac{\pi}{4} = -\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$

$= -\frac{7\pi}{12}$

**Ex — 4.3**

1. Let $f(x) = \tan^{-1}\left(\sqrt{9-x^2}\right)$

$\sqrt{9-x^2} \in R$

But $\sqrt{9-x^2} \geq 0$

$9 - x^2 \geq 0$

$x^2 - 9 \leq 0$

$(x+3)(x-3) \leq 0$

Domain is $[-3, 3]$

(ii)

$\tan^{-1}(x)$ is function with $(-\infty, \infty)$ as domain.

$\therefore \frac{1}{2}\tan^{-1}(1-x^2) - \frac{\pi}{4}$ is $R$

2.

(i) $\tan^{-1}\left(\tan \frac{5\pi}{4}\right)$

$\frac{5\pi}{4} \notin \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\tan\left(\frac{5\pi}{4}\right) = \tan\left(\frac{4\pi}{4} + \frac{\pi}{4}\right)$

$= \tan\left(\pi + \frac{\pi}{4}\right)$

$= \tan\left(\frac{\pi}{4}\right)$

$\tan^{-1}\left(\tan \frac{5\pi}{4}\right) = \tan^{-1}\tan\left(\frac{\pi}{4}\right)$

$= \frac{\pi}{4} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(ii) $\tan^{-1}\left(\tan\left(-\frac{\pi}{6}\right)\right)$

$= -\frac{\pi}{6} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

3

(i) W.K.T

$\tan^{-1}(\tan x) = x, x \in R$

$\tan^{-1}\left[\tan\left(\frac{7\pi}{4}\right)\right] = \frac{7\pi}{4}$

(ii) $\tan\left[\tan^{-1}(-0.2021)\right]$

W.K.T $\tan(\tan^{-1}x) = x, x \in R$

$\tan(\tan^{-1}(-0.2021)) = -0.2021$

(iii) $\tan\left[\tan^{-1}(1947)\right]$

W.K.T $\tan\left[\tan^{-1}x\right] = x, x \in R$

$\tan\left[\tan^{-1}(1947)\right] = 1947$

4) Let $\cos^{-1}\left(\frac{1}{2}\right) = y$

$\cos y = \frac{1}{2}$

$\cos y = \cos\frac{\pi}{3}$

$\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} \in [0, \pi]$

$\sin^{-1}\left(-\frac{1}{2}\right) = y$

$\sin y = -\frac{1}{2} = -\sin\frac{\pi}{6}$

$\sin y = \sin\left(-\frac{\pi}{6}\right)$

$\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\cos^{-1}\left(\frac{1}{2}\right) - \sin^{-1}\left(-\frac{1}{2}\right) = \frac{\pi}{3} - \left(-\frac{\pi}{6}\right)$

$= \frac{\pi}{3} + \frac{\pi}{6} = \frac{2\pi + \pi}{6} = \frac{3\pi}{6} = \frac{\pi}{2}$

$\tan\left[\cos^{-1}\left(\frac{1}{2}\right) - \sin^{-1}\left(-\frac{1}{2}\right)\right] = \tan\left(\frac{\pi}{2}\right)$

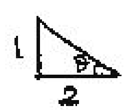
$= \infty$

(ii)

$\sin\left[\tan^{-1}\left(\frac{1}{2}\right) - \cos^{-1}\left(\frac{4}{5}\right)\right]$

Let $\tan^{-1}\left(\frac{1}{2}\right) = A$

$\tan A = \frac{1}{2}$




By using Pythagoras Theorem

$hyp^2 = op^2 + adj^2$

$= 1^2 + 2^2 = 5$

$\boxed{hyp = \sqrt{5}}$

$\sin A = \frac{opp}{hyp} = \frac{1}{\sqrt{5}}$

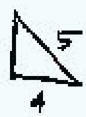
$\cos A = \frac{adj}{hyp} = 2/\sqrt{5}$

Similarly

Let $\cos^{-1}(4/5) = B$

$\cos B = 4/5$

$\frac{adj}{hyp} = 4/5$




By using Pythagoras Theorem

$hyp^2 = op^2 + adj^2$

$25 = op^2 + 16$

$\boxed{op = 3}$

$\sin B = \frac{op}{hyp} = 3/5$

$\sin\left[\tan^{-1}(1/2) - \cos^{-1}(4/5)\right] = \sin(A - B)$

W.K.T

$\sin(A-B) = \sin A \cos B - \cos A \sin B$

$= 1/\sqrt{5}\left(4/5\right) - \frac{2}{\sqrt{5}}\left(3/5\right)$

$= \frac{4}{5\sqrt{5}} - \frac{6}{5\sqrt{5}}$

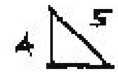
$= -2/5\sqrt{5}$

$\boxed{\sin\left[\tan^{-1}(1/2) - \cos^{-1}(4/5)\right] = -2/5\sqrt{5}}$

(iii) $\cos\left[\sin^{-1}(4/5) - \tan^{-1}(3/4)\right]$

Let $\sin^{-1}(4/5) = A$

$\sin A = 4/5 = \frac{O.P}{Hyp}$




By using Pythagoras Theorem

$hyp^2 = op^2 + adj^2$

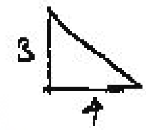
$25 = 16 + adj^2$

$\boxed{adj = 3}$

$\boxed{\cos A = \frac{adj}{hyp} = 3/5}$

Let $\tan^{-1}(3/4) = B$

$\tan B = 3/4$




$\tan B = \frac{O.P}{adj} = 3/4$

By using Pythagoras Theorem

$hyp^2 = op^2 + adj^2$

$= 3^2 + 4^2 = 25$

$\boxed{hyp = 5}$

$\sin B = \frac{op}{hyp} = 3/5$ $\boxed{\sin B = 3/5}$

$\cos B = \frac{adj}{hyp} = 4/5$

$\cos\left[\sin^{-1}(4/5) - \tan^{-1}(3/4)\right] = \cos(A - B)$

W.K.T $\cos(A-B) = \cos A \cos B + \sin A \sin B$

$= 3/5\left(4/5\right) + 4/5\left(3/5\right)$

$= 12/25 + 12/25$

$= 24/25$

$\boxed{\cos\left[\sin^{-1}(4/5) - \tan^{-1}(3/4)\right] = \frac{24}{25}}$

Ex - 4.4 (EXERCISE-4.4)

1.

(i) Let $\sec^{-1}(2/\sqrt{3}) = \theta$

$\sec\theta = 2/\sqrt{3}$

and $\theta \in [0,\pi] \setminus \{\pi/2\}$

$\cos\theta = \sqrt{3}/2$

$\cos y = \sqrt{3}/2 = \cos \pi/6$

and $\pi/6 \in [0,\pi] \setminus \{\pi/2\}$

$\therefore$ The Principal value of $\sec^{-1}(2/\sqrt{3}) = \pi/6$

(ii) $\cot^{-1}\sqrt{3}$

Let $\cot^{-1}(\sqrt{3}) = \theta$

$\cot\theta = \sqrt{3}$

$\tan\theta = 1/\sqrt{3} = \tan \pi/6$

$\pi/6 \in [0,\pi]$

The Principal value of $\cot^{-1}(\sqrt{3}) = \pi/6$

(iii) $\operatorname{cosec}^{-1}(-\sqrt{2})$

Let $\operatorname{cosec}^{-1}(\sqrt{2}) = \theta$

$\operatorname{cosec}\theta = -\sqrt{2}$

$\sin\theta = -1/\sqrt{2}$

$\sin\theta = -1/\sqrt{2} = \sin(-\pi/4)$

$\theta = -\pi/4$, $-\pi/4 \in [-\pi/2, \pi/2] \setminus 0$

$\therefore$ The Principal value of $\operatorname{cosec}^{-1}(-\sqrt{2}) = -\pi/4$

2.

(i) $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$

Let $\tan^{-1}(\sqrt{3}) = \theta$

$\tan\theta = \sqrt{3} = \tan \pi/3$

$\pi/3 \in [0,\pi]$

$$\boxed{\tan^{-1}(\sqrt{3}) = \pi/3}$$

Let $\sec^{-1}(-2) = y$

$\sec y = -2$

$\cos y = -1/2 = \cos(-\pi/3)$

and $y = -\pi/3 \notin [0,\pi] \setminus \{\pi/2\}$

$\therefore \cos(-\pi/3) = \cos\left(\frac{3\pi}{3} - \pi/3\right)$

$= \cos(2\pi/3)$ and $\frac{2\pi}{3} \in [0,\pi] \setminus \{\pi/2\}$

$\therefore$ The Principal value of $\sec^{-1}(-2) = 2\pi/3$

$\therefore \tan^{-1}(\sqrt{3}) - \sec^{-1}(-2) = \pi/3 - 2\pi/3 = -\pi/3$

(ii) $\sin^{-1}(-1) + \cos^{-1}(1/2) + \cot^{-1}(2)$

Soln

Let $\sin^{-1}(-1) = y$

$\sin y = -1 = \sin(-\pi/2)$

$$\boxed{\sin^{-1}(-1) = -\pi/2 \in [-\pi/2, \pi/2]}$$

Let $\cos^{-1}(1/2) = y$

$\cos y = 1/2 = \cos \pi/3$

$$\boxed{\cos^{-1}(1/2) = \pi/3 \in [0,\pi]}$$

$\therefore \sin^{-1}(-1) + \cos^{-1}(1/2) + \cot^{-1}(2)$

$= -\pi/2 + \pi/3 + \cot^{-1}(2)$

$= \frac{-3\pi + 2\pi}{6} + \cot^{-1}(2)$

$= -\pi/6 + \cot^{-1}(2)$ //

(iii)

$\cot^{-1}(1)+\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)-\sec^{-1}(-\sqrt{2})$

Soln:

Let $\cot^{-1}(1)=\theta$

$\cot\theta=1$

$\tan\theta=1$

$\tan\pi/4=1$ and $\pi/4\in[0,\pi]$

$\boxed{\cot^{-1}(1)=\pi/4}$

$\sin^{-1}\left(-\sqrt{3}/2\right)=y$

$\sin y=-\sqrt{3}/2$

$\sin y=-\sin\pi/3=\sin(-\pi/3)$

$\boxed{\sin^{-1}(-\sqrt{3}/2)=-\pi/3\in[-\pi/2,\pi/2]}$

Let $\sec^{-1}(-\sqrt{2})=y$

$\sec y=-\sqrt{2}$

$\cos y=-1/\sqrt{2}=\cos(-\pi/4)$

and $-\pi/4\notin[0,\pi]\setminus\{\pi/2\}$

But $\cos(-\pi/4)=\cos(\pi-\pi/4)$

$=\cos(3\pi/4)$

and $3\pi/4\in[0,\pi]\setminus\{\pi/2\}$

$\therefore$ The principal value of

$\boxed{\sec^{-1}(-\sqrt{2})=3\pi/4}$

$\cot^{-1}(1)+\sin^{-1}(-\sqrt{3}/2)-\sec^{-1}(-\sqrt{2})=$

$\pi/4-\pi/3-3\pi/4$

$=\frac{3\pi-4\pi-9\pi}{12}=\frac{-10\pi}{12}$

$=-5\pi/6$

EXERCISE - 4.5

1

(i) $\sin^{-1}(\cos\pi)$

Soln:

$\sin^{-1}(\cos\pi)=\sin^{-1}(-1)$

$=-\pi/2$

$\sin^{-1}(-1)=-\pi/2$

$\sin(-\pi/2)=-1$.

(ii) $\tan^{-1}\left[\sin\left(-\frac{5\pi}{2}\right)\right]$

Soln:

$\sin(-5\pi/2)=-\sin(5\pi/2)$

$=-\sin(2\pi+\pi/2)$

$=-\sin(\pi/2)=-1$

$\tan^{-1}[\sin(-5\pi/2)]=\tan^{-1}(-1)$

We know that $\tan^{-1}(-1)=-\tan^{-1}(1)$

$=-\pi/4$

So $\tan^{-1}[\sin(-5\pi/2)]=\tan^{-1}[(-\pi/4)]=-\tan^{-1}(1)$

$=-\pi/4$

(iii) $\sin^{-1}(\sin 5)$

Soln

$\sin^{-1}(\sin\theta)=\theta$ as $\theta\in[-\pi/2,\pi/2]$

So, 5 is not in the interval, but

$5-2\pi\in[-\pi/2,\pi/2]$

$\sin 5=\sin(2\pi+(5-2\pi))$

$=\sin(5-2\pi)$

$\sin^{-1}[\sin 5]=\sin^{-1}[\sin(5-2\pi)]$

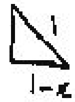
$=5-2\pi$

2. $\sin[\cos^{-1}(1-x)]$

Soln:

Let $\cos^{-1}(1-x)=\theta$

$\cos\theta = 1-x$




$\cos\theta = \frac{adj}{hyp} = \frac{1-x}{1}$

Pythagoras Theorem

$hyp^2 = op^2 + adj^2$

$1^2 = op^2 + (1-x)^2$

$op^2 = 1 - x^2 + 2x - 1$

$\boxed{opp = \sqrt{2x-x^2}}$

$\sin\theta = \frac{op}{hyp} = \frac{\sqrt{2x-x^2}}{1}$

$\boxed{\sin\theta = \sqrt{2x-x^2}}$

$\sin(\cos^{-1}[1-x]) = \sin(\theta)$

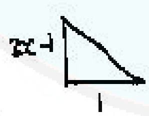
$= \sqrt{2x-x^2}$

(ii) $\cos[\tan^{-1}[3x-1]]$

Soln:

Let $\tan^{-1}(3x-1)=\theta$

$\tan\theta = 3x-1$




$\tan\theta = \frac{opp}{adj} = \frac{3x-1}{1}$

By Pythagoras Theorem

$hyp^2 = op^2 + adj^2$

$= (3x-1)^2 + 1^2$

$= 9x^2 + 1 - 6x + 1$

$hyp = \sqrt{9x^2-6x+2} = \sqrt{9x^2-6x+2}$

$\cos\theta = \frac{adj}{hyp} = \frac{1}{\sqrt{9x^2-6x+2}}$

$\cos[\tan^{-1}(3x-1)] = \cos\theta$

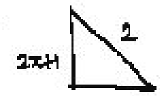
$= \frac{1}{\sqrt{9x^2-6x+2}}$

(iii) $\tan[\sin^{-1}(x+\frac{1}{2})]$

Soln:

Let $\sin^{-1}(x+\frac{1}{2}) = \theta$

$\sin\theta = x + \frac{1}{2} = \frac{2x+1}{2}$




$\sin\theta = \frac{2x+1}{2} = \frac{opp}{hyp}$

By using Pythagoras Theorem

$hyp^2 = opp^2 + adj^2$

$2^2 = (2x+1)^2 + adj^2$

$4 = 4x^2 + 1 + 4x + adj^2$

$adj^2 = 4 - 4x^2 - 1 - 4x = 3 - 4x - 4x^2$

$\boxed{adj = \sqrt{(3-4x-4x^2)}}$

$\tan\theta = \frac{opp}{adj} = \frac{2x+1}{\sqrt{(3-4x-4x^2)}}$

$\tan[\sin^{-1}(x+\frac{1}{2})] = \tan[\theta]$

$= \frac{2x+1}{\sqrt{3-4x-4x^2}}$

3)

(i) $\sin^{-1}\left(\cos\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)\right)$

Soln:

$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$

$\cos\left(\sin\left(\frac{\sqrt{3}}{2}\right)\right) = \cos\left(\frac{\pi}{3}\right)$

$= \frac{1}{2}$

Now

$\sin^{-1}\left(\cos\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)\right) = \sin^{-1}\left(\frac{1}{2}\right)$

$= \frac{\pi}{6}$

(ii) $\cot\left(\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{4}{5}\right)$

Soln:

Let $\sin^{-1}\left(\frac{3}{5}\right) = A$

$\sin A = \frac{3}{5} = \frac{opp}{hyp}$

By using Pythagoras Theorem

$hyp^2 = opp^2 + adj^2$

$5^2 = 3^2 + adj^2$

$adj^2 = 25 - 9 = 16$

$\boxed{adj = 4}$

$\tan A = \frac{opp}{adj} = \frac{3}{4}$

If $\sin^{-1}(4/5) = B$

$\sin B = 4/5 = \frac{opp}{hyp}$

By using Pythagoras Theorem

$hyp^2 = opp^2 + adj^2$

$5^2 = 4^2 + adj^2$

$adj^2 = 25 - 16 = 9$

$\boxed{adj = 3}$

$\tan B = \frac{opp}{adj} = 4/3$

$\cot(\sin^{-1} 3/5 + \sin^{-1} 4/5) = \cot(A+B)$

$= \frac{1}{\tan(A+B)}$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$= \frac{3/4 + 4/3}{1 - (3/4)(4/3)}$

$= \frac{\frac{9+16}{12}}{1 - 12/12}$

$= \frac{25/12}{0}$

$\frac{1}{\tan(A+B)} = 0$

$\frac{1}{\tan(A+B)} = 0$

$\cot(\sin^{-1} 3/5 + \sin^{-1} 4/5) \doteq \cot(A+B)$

$= \frac{1}{\tan(A+B)} = 0.$

(iii)

$\tan(\sin^{-1} 3/5 + \cot^{-1} 3/2)$

Soln:

Let $\sin^{-1}(3/5) = A$

$\sin A = 3/5 = \frac{opp}{hyp}$

By using Pythagoras Theorem

$hyp^2 = opp^2 + adj^2$

$5^2 = 3^2 + adj^2$

$25 = 9 + adj^2$

$adj = \sqrt{16} = 4$ $\boxed{adj = 4}$

$\tan A = \frac{opp}{adj} = 3/4$

If $\cot^{-1}(3/2) = B$

$\cot B = 3/2$

$\tan B = 2/3$

$\tan(\sin^{-1} 3/5 + \cot^{-1} 3/2) = \tan(A+B)$

$= \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$= \frac{3/4 + 2/3}{1 - (3/4)(2/3)}$

$= \frac{\frac{9+8}{12}}{1 - 6/12} = \frac{17/12}{6/12}$

$\boxed{\tan(\sin^{-1} 3/5 + \cot^{-1} 3/2) = 17/6}$

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பட்டுக்கோட்டை.

4

(i) $\tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24} = \tan^{-1}\frac{1}{2}$

Soln:

We know that

$$\tan^{-1}A + \tan^{-1}B = \tan^{-1}\left(\frac{A+B}{1-AB}\right)$$

LHS:

$$\tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24} = \tan^{-1}\left(\frac{\frac{2}{11}+\frac{7}{24}}{1-\frac{2}{11}\cdot\frac{7}{24}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{48+77}{11\times 24}}{1-\frac{14}{11\times 24}}\right)$$

$$= \tan^{-1}\left(\frac{125/264}{250/264}\right)$$

$$= \tan^{-1}\left(\frac{125}{250}\right) = \tan^{-1}\left(\frac{1}{2}\right)$$

$$= RHS$$

Hence the Proof.

(ii) $\sin^{-1}\left(\frac{3}{5}\right) - \cos^{-1}\left(\frac{12}{13}\right) = \sin^{-1}\left(\frac{16}{65}\right)$

Soln: Let $\sin^{-1}\left(\frac{3}{5}\right) = A$

$\cos^{-1}\left(\frac{12}{13}\right) = B$

$\sin^{-1}\left(\frac{3}{5}\right) - \cos^{-1}\left(\frac{12}{13}\right) = A - B$

To Prove $A - B = \sin^{-1}\left(\frac{16}{65}\right)$

Given $\sin^{-1}\left(\frac{3}{5}\right) = A$

$\sin A = \frac{3}{5} = \frac{opp}{hyp}$

Using Pythagoras Theorem

$hyp^2 = opp^2 + adj^2$

$5^2 = 3^2 + adj^2$

$adj^2 = 25 - 9 = 16$

$\boxed{adj = 4}$

$\boxed{\cos A = \frac{4}{5}}$

and $\cos^{-1}\left(\frac{12}{13}\right) = B$

$\cos B = \frac{12}{13} = \frac{adj}{hyp}$

Using Pythagoras Theorem

$hyp^2 = opp^2 + adj^2$

$13^2 = opp^2 + 12^2$

$opp^2 = 169 - 144 = 25$

$\boxed{opp = 5}$

$\sin B = \frac{opp}{hyp} = \frac{5}{13}$

We know that

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

$= \frac{3}{5}\left(\frac{12}{13}\right) - \frac{4}{5}\left(\frac{5}{13}\right)$

$= \frac{36-20}{65}$

$\sin(A-B) = \frac{16}{65}$

$\boxed{A - B = \sin^{-1}\left(\frac{16}{65}\right)}$

Hence Proved.

5)

We know that

$$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$$

$$= \tan^{-1}(A)$$

where $A = \frac{x+y}{1-xy}$

$\tan^{-1}x + \tan^{-1}y + \tan^{-1}z =$

$\tan^{-1}A + \tan^{-1}z$

$$= \tan^{-1}\left(\frac{A+z}{1-Az}\right)$$

$$= \tan^{-1}\left(\frac{\frac{x+y}{1-xy} + z}{1-\left(\frac{x+y}{1-xy}\right)z}\right)$$

$$= \tan^{-1}\left(\frac{\frac{x+y+z(1-xy)}{1-xy}}{1-\left(\frac{xz+yz}{1-xy}\right)}\right)$$

$$= \tan^{-1}\left(\frac{\frac{x+y+z-xyz}{1-xy}}{\frac{1-xy-xz-yz}{1-xy}}\right)$$

$$= \tan^{-1}\left(\frac{x+y+z-xyz}{1-xy} \times \frac{1-xy}{1-xy-xz-yz}\right)$$

$$= \tan^{-1}\left(\frac{x+y+z-xyz}{1-xy-xz-yz}\right)$$

Hence Proved.

6) Already we have to Proved that

$$\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-xz-yz}\right]$$

$$\pi = \tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-xz-yz}\right]$$

$$\tan\pi = \frac{x+y+z-xyz}{1-xy-xz-yz}$$

$$0 = \frac{x+y+z-xyz}{1-xy-xz-yz}$$

$$x+y+z-xyz = 0$$

$$\boxed{x+y+z = xyz}$$ Hence Proved

7) $\tan^{-1}x + \tan^{-1}\frac{2x}{1-x^2} = \tan^{-1}\frac{3x-x^3}{1-3x^2}$

Soln:

We know That

$$\tan^{-1}A + \tan^{-1}B = \tan^{-1}\left(\frac{A+B}{1-AB}\right)$$

$$\tan^{-1}x + \tan^{-1}\frac{2x}{1-x^2} = \tan^{-1}\left(\frac{x+\frac{2x}{1-x^2}}{1-(x)\left(\frac{2x}{1-x^2}\right)}\right)$$

$$= \tan^{-1}\left(\frac{\frac{x(1-x^2)+2x}{1-x^2}}{\frac{1-x^2-2x^2}{1-x^2}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{3x-x^3}{\cancel{1-x^2}}}{\frac{1-3x^2}{\cancel{1-x^2}}}\right)$$

$$= \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$$

Hence Proved

8. $\tan^{-1}\frac{x}{y} - \tan^{-1}\frac{x-y}{x+y}$

Soln:

We know That

$$\tan^{-1}A - \tan^{-1}B = \tan^{-1}\left(\frac{A-B}{1+AB}\right)$$

$$\tan^{-1}\frac{x}{y} - \tan^{-1}\frac{x-y}{x+y} = \tan^{-1}\left[\frac{x/y - \left(\frac{x-y}{x+y}\right)}{1 - x/y\left(\frac{x-y}{x+y}\right)}\right]$$

$$= \tan^{-1}\left[\frac{\frac{x(x+y)-y(x-y)}{x+y}}{\frac{y(x+y)+x(x-y)}{x+y}}\right]$$

$$= \tan^{-1}\left[\frac{x^2+y^2/x+y}{x^2+y^2/x+y}\right]$$

$$= \tan^{-1}(1)$$

$$= \pi/4$$

$$\tan^{-1}x/y - \tan^{-1}\left(\frac{x-y}{x+y}\right) = \pi/4$$

Hence Proved.

9.

$\sin^{-1}\frac{5}{x}+\sin^{-1}\frac{12}{x}=\pi/2$

Soln:

$\sin^{-1}5/x=\pi/2-\sin^{-1}12/x$

$\sin^{-1}5/x=\cos^{-1}12/x$

Let $\sin^{-1}5/x=\theta$

$\sin\theta=5/x$

IIly $\cos^{-1}12/x=\theta$

$\cos\theta=12/x$

We know That

$\sin^2\theta+\cos^2\theta=1$

$(5/x)^2+(12/x)^2=1$

$\frac{25}{x^2}+\frac{144}{x^2}=1$

$\frac{25+144}{x^2}=1$

$169=x^2$

$\boxed{x=13}$

(ii) $2\tan^{-1}x=\cos^{-1}\frac{1-a^2}{1+a^2}-\cos^{-1}\frac{1-b^2}{1+b^2}$

Soln

Let $a=\tan\theta_1$

$\frac{1-a^2}{1+a^2}=\frac{1-(\tan\theta_1)^2}{1+(\tan\theta_1)^2}$

$=\cos 2\theta_1$

since $\cos 2\theta=\frac{1-(\tan\theta)^2}{1+(\tan\theta)^2}$

Then $\tan^{-1}b=\theta_2$

$\frac{1-b^2}{1+b^2}=\frac{1-(\tan\theta_2)^2}{1+(\tan\theta_2)^2}$

$=\cos 2\theta_2$

$2\tan^{-1}x=\cos^{-1}\frac{1-a^2}{1+a^2}-\cos^{-1}\frac{1-b^2}{1+b^2}$

$=\cos^{-1}(\cos 2\theta_1)-\cos^{-1}(\cos 2\theta_2)$

$2\tan^{-1}x=2\theta_1-2\theta_2$

$=\theta_1-\theta_2 \qquad (\div 2)$

$\tan^{-1}x=\theta_1-\theta_2$

$=\tan^{-1}a-\tan^{-1}b$

$\cancel{\tan^{-1}}x=\cancel{\tan^{-1}}\left(\frac{a-b}{1+ab}\right)$

$\boxed{x=\frac{a-b}{1+ab}}$

(iii)

$2\tan^{-1}(\cos x)=\tan^{-1}(2\operatorname{cosec}x)$

Soln:

We know That

$\tan^{-1}A+\tan^{-1}B=\tan^{-1}\left(\frac{A+B}{1-AB}\right)$

$2\tan^{-1}(\cos x)=\tan^{-1}(\cos x)+\tan^{-1}(\cos x)$

$=\tan^{-1}\left(\frac{\cos x+\cos x}{1-\cos x\cdot\cos x}\right)$

$=\tan^{-1}\left(\frac{2\cos x}{1-\cos^2x}\right)$

$=\tan^{-1}\left(\frac{2\cos x}{\sin^2x}\right)$

Given $2\tan^{-1}(\cos x)=\tan^{-1}(2\operatorname{cosec}x)$

$\cancel{\tan^{-1}}\left(\frac{2\cos x}{\sin^2x}\right)=\cancel{\tan^{-1}}(2\operatorname{cosec}x)$

$\frac{\cancel{2}\cos x}{\sin x\cdot\sin x}=\cancel{2}\operatorname{cosec}x$

$\frac{\cos x}{\sin x\cdot\cancel{\sin x}}=\frac{1}{\cancel{\sin x}}$

$\frac{\cos x}{\sin x}=1$

$\cos x=\sin x$

$\boxed{x=\pi/4}$

(iv)

$\cot^{-1}x+\cot^{-1}(x+2)=\pi/12$

$\tan^{-1}(1/x)-\tan^{-1}\left(\frac{1}{x+2}\right)=\pi/12$

$\tan^{-1}\left[\frac{1/x-1/(x+2)}{1+1/x\cdot 1/(x+2)}\right]=\pi/12$

$\left[\frac{\frac{x+2-x}{x(x+2)}}{\frac{x(x+2)+1}{x(x+2)}}\right]=\tan(\pi/12)$

$\frac{2}{x^2+2x+1}=\tan\frac{180^\circ}{12}$

$\frac{2}{x^2+2x+1}=\tan 15^\circ$

$\frac{2}{x^2+2x+1}=2-\sqrt{3}$

$\frac{2}{2-\sqrt{3}}=x^2+2x+1$

$x^2+2x+1=\frac{2}{2-\sqrt{3}}\times\frac{2+\sqrt{3}}{2+\sqrt{3}}$

$=\frac{2(2+\sqrt{3})}{4-3}=\frac{2(2+\sqrt{3})}{4-3}$

$=4+2\sqrt{3}$

$=1+3+2\sqrt{3}$

$(x+1)^2=(1+\sqrt{3})^2$

$x+1=1+\sqrt{3}$

$\boxed{x=\sqrt{3}}$

10) $\tan^{-1}(x-1)+\tan^{-1}(x)+\tan^{-1}(x+1)=\tan^{-1}(3x)$

$\tan^{-1}(x-1)+\tan^{-1}(x+1)=\tan^{-1}(3x)-\tan^{-1}(x)$

we know that

$\tan^{-1}A+\tan^{-1}B=\tan^{-1}\left(\frac{A+B}{1-AB}\right)$

LHS:

$\tan^{-1}(x-1)+\tan^{-1}(x+1)$

$=\tan^{-1}\left(\frac{x-1+x+1}{1-(x-1)(x+1)}\right)$

$=\tan^{-1}\left(\frac{2x}{1-x^2+1}\right)$

$=\tan^{-1}\left(\frac{2x}{2-x^2}\right)$ —— (1)

RHS:

$\tan^{-1}A-\tan^{-1}B=\tan^{-1}\left(\frac{A-B}{1+AB}\right)$

$\tan^{-1}(3x)-\tan^{-1}(x)=\tan^{-1}\left(\frac{3x-x}{1+(3x)(x)}\right)$

$=\tan^{-1}\left(\frac{2x}{1+3x^2}\right)$ —— (2)

From (1) & (2)

RHS = LHS

$\tan^{-1}\left(\frac{2x}{2-x^2}\right)=\tan^{-1}\left(\frac{2x}{1+3x^2}\right)$

$\frac{2x}{2-x^2}=\frac{2x}{1+3x^2}$

$2x(1+3x^2)=2x(2-x^2)$

$2x+6x^3=4x-2x^3$

$6x^3+2x^3=4x-2x$

$8x^3=2x$

$\div 2x\Rightarrow 4x^2=1$

$x^2=1/4$

$\boxed{x=\pm 1/2}$

17/6/2019