9th Standard	Syllabus	Books	Study Materials	1st Mid Term	2nd Mid Term	3rd Mid Term
	Quarterly Exam	Half Yearly Exam	Annual Exam	RTE		

8th Standard	Syllabus	Books	Study Materials	1st Mid Term	2nd Mid Term	3rd Mid Term
	Term 1	Term 2	Term 3	Public Model Q&A	NMMS	Periodical Test

7th Standard	Syllabus	Books	Study Materials	1st Mid Term	2nd Mid Term	3rd Mid Term
	Term 1	Term 2	Term 3	Periodical Test	SLAS	

6th Standard	Syllabus	Books	Study Materials	1st Mid Term	2nd Mid Term	3rd Mid Term
	Term 1	Term 2	Term 3	Periodical Test	SLAS	

1st to 5th Standard	Syllabus	Books	Study Materials	Periodical Test	SLAS	
	Term 1	Term 2	Term 3	Public Model Q&A		

Exams	TET	TNPSC	PGTRB	Polytechnic	Police	Computer Instructor
	DEO	BEO	LAB Asst	NMMS	RTE	NTSE

Portal	Matrimony	Mutual Transfer	Job Portal

Volunteers	Centum Team	Creative Team	Key Answer Team

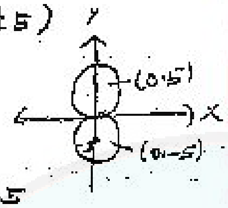
Downloads	LESSON PLAN	Department Exam	Income Tax	Forms & Proposals	Fonts	Downloads
	Proceedings	GO's	Regulation Orders	Pay Orders	Panel	

CHAPTER - V

TWO DIMENSIONAL ANALYTICAL GEOMETRY - II

EXERCISE - 5.1

1. Given $r = 5$, Centre $(0, \pm 5)$



Equation of the circle

$(x-h)^2+(y-k)^2=r^2$

here $(h,k)=(0,\pm 5)$, $r=5$

equation:

$(x-0)^2+(y\pm 5)^2=5^2$

$x^2+y^2+25\pm 10y=25$

$\boxed{x^2+y^2\pm 10y=0}$

2. Equation of the circle

$(x-h)^2+(y-k)^2=r^2$

here $(h,k)=(2,-1)$

$(x-2)^2+(y+1)^2=r^2$

It Passes Through $(3,6)$

$(3-2)^2+(6+1)^2=r^2$

$1+49=r^2$, $\boxed{r^2=50}$

Equation of the Circle

$\boxed{(x-2)^2+(y+1)^2=50}$

3. Equation of the circle

$(x-h)^2+(y-k)^2=r^2$

Centre is $(h,k)=(-r,-r)$

$(x+r)^2+(y+r)^2=r^2$ ——— ①

It is passes through $(-4,-2)$

$(-4+r)^2+(-2+r)^2=r^2$

$16+r^2-8r+4+r^2-4r=r^2$

$r^2-12r+20=0$

$(r-10)(r-2)=0$

$r=10$, $r=2$

When $r=10$

① ⇒

$(x+10)^2+(y+10)^2=10^2$

$(x^2+100+20x)+y^2+\cancel{100}+20y=\cancel{100}$

$\boxed{x^2+y^2+20x+20y+100=0}$

When $r=2$

① ⇒

$(x+2)^2+(y+2)^2=2^2$

$x^2+4+4x+y^2+4+4y=4$

$\boxed{x^2+y^2+4x+4y+4=0}$

4) Centre $(h,k)=(2,3)$

Equation of the circle

$(x-h)^2+(y-k)^2=r^2$

$(x-2)^2+(y-3)^2=r^2$ ——— ①

given lines: → ②

$3x-2y-1=0$, $4x+y-27=0$ → ③

③ × 2 ⇒ $8x+2y-54=0$

2 × ① ⇒ $3x-2y-1=0$

$11x-55=0$

$11x=55$

$\boxed{x=5}$

③ ⇒

$4(5)+y-27=0$

$20+y-27=0$ $\boxed{y=7}$

Point of intersection $(5,7)$

① ⇒ $(5-2)^2+(7-3)^2=r^2$

$9+16=r^2$

$\boxed{r^2=25}$

① ⇒ $(x-2)^2+(y-3)^2=25$

$x^2+4-4x+y^2+9-6y-25=0$

$\boxed{x^2+y^2-4x+6y-12=0}$

∴ The equation $\boxed{x^2+y^2-4x-6y-12=0}$

5.

Equation of the circle

$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$

Here $(x_1,y_1)=(3,4)$ $(x_2,y_2)=(2,-7)$

$(x-3)(x-2)+(y-4)(y+7)=0$

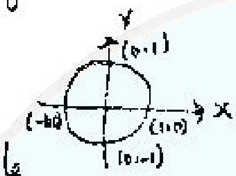
$x^2-2x-3x+6+y^2+7y-4y-28=0$

$\boxed{x^2+y^2-5x+3y-22=0}$

6. From the diagram

Centre $(0,0)$

radius : $r=1$



Equation of the circle

$(x-h)^2+(y-k)^2=r^2$

$\boxed{x^2+y^2=1}$

7. Area of the circle $=9\pi$

$\pi r^2=9\pi$

$\boxed{r^2=9}$

Two diameters are

$x+y=5$ —— ①

$x-y=1$ —— ②

① + ② ⇒ $2x=6$ ⇒ $\boxed{x=3}$

① ⇒ $3+y=5$ ⇒ $\boxed{y=2}$

Centre $(h,k)=(3,2)$

Equation of the circle:

$(x-h)^2+(y-k)^2=r^2$

$(x-3)^2+(y-2)^2=9$

$x^2+9-6x+y^2+4-4y=9$

$\boxed{x^2+y^2-6x-4y+4=0}$

8. $y=2\sqrt{2}x+c$

$y=mx+c$

$m=2\sqrt{2}$

$m^2=(2\sqrt{2})^2=8$

$x^2+y^2=16$

$\boxed{a^2=16}$

Condition:

$c^2=a^2(1+m^2)$

$c^2=16(1+8)$

$=16(9)$

$c^2=144$

$\boxed{c=\pm 12}$

9.

$x^2+y^2-6x+6y-8=0$

Eqn of the tangent

$xx_1+yy_1-6\left(\frac{x+x_1}{2}\right)+6\left(\frac{y+y_1}{2}\right)-8=0$

$xx_1+yy_1-3(x+x_1)+3(y+y_1)-8=0$

Here $(x_1,y_1)=(2,2)$

$2x+2y-3(x+2)+3(y+2)-8=0$

$2x+2y-3x-6+3y+6-8=0$

$-x+5y-8=0$

$\boxed{x-5y-8=0}$

Eqn of the Normal:

$5x+y+k=0$

It passes through $(2,2)$

$5(2)+2+k=0$

$12+k=0$

$\boxed{k=-12}$

Equation of the normal is $\boxed{5x+y-12=0}$

10. $x^2+y^2-5x+2y-5=0$

at $(-2,1)$

① ⇒ $4+1+10-2-5=12>0$

$(-2,1)$ lies outside the circle.

at $(0,0)$

① ⇒ $0+0-0+0-5=-5<0$

$(0,0)$ lies inside the circle

at $(-4,-3)$

① ⇒ $16+9+20-6-5=34>0$

$(-4,-3)$ lies outside of the circle.

11.

(i) $x^2+(y+2)^2=0$

$(x-h)^2+(y-k)^2=r^2$

Centre $(h,k)=(0,-2)$

radius $r=0$

(ii) $x^2+y^2+2gx+2fy+c=0$

$x^2+y^2+6x-4y+4=0$

$2g=6$ $\quad$ $2f=-4$

$\boxed{g=3}$ $\quad$ $\boxed{f=-2}$

$\boxed{c=4}$

Centre $(-g,-f)=(-3,2)$

radius $r=\sqrt{g^2+f^2-c}$

$=\sqrt{9+4-4}=\sqrt{9}=3$

$\boxed{r=3}$

(iii) $x^2+y^2-x+2y-3=0$

$2g=-1$, $g=-1/2$ | $2f=2$, $f=1$ | $c=-3$

Centre $(-g,-f)=(1/2,-1)$

radius $r=\sqrt{g^2+f^2-c}=\sqrt{1/4+1+3}$

$\boxed{r=\frac{\sqrt{17}}{2}}$

(iv) $2x^2+2y^2-6x+4y+2=0$

$\div 2 \Rightarrow$ $x^2+y^2-3x+2y+1=0$

$2g=-3$, $g=-3/2$ | $2f=2$, $f=1$ | $c=1$

Centre $=(-g,-f)=(3/2,-1)$

radius ; $r=\sqrt{g^2+f^2-c}=\sqrt{9/4+1-1}$

$\boxed{r=3/2}$

12)

$3x^2+(3-p)xy+qy^2-2px=8pq$ — (1)

Co-eff of x^2 = Co-eff of y^2

$3=q$

$\boxed{q=3}$

Again Co-eff of $xy=0$

$3-p=0$

$\boxed{p=3}$

(1) $\Rightarrow$ $3x^2+3y^2-6x-72=0$

$\div 3 \Rightarrow$ $x^2+y^2-2x-24=0$

$2g=-2$, $g=-1$ | $2f=0$, $f=0$ | $c=-24$

Centre $(-g,-f)=(1,0)$

radius $r=\sqrt{g^2+f^2-c}$

$=\sqrt{1+0+24}=\sqrt{25}$

$\boxed{r=5}$

EXERCISE : 5.3

1. $2x^2-y^2=7$

Compared with

$Ax^2+Bxy+Cy^2+Dx+Ey+F=0$

$A=2 \quad B=0 \quad C=-1$

$B^2-4AC=0-4(2)(-1)=8>0$

The given equation is a Hyperbola.

2. $3x^2+3y^2-4x+3y+10=0$

Compared with

$Ax^2+Bxy+Cy^2+Dx+Ey+F=0$

$A=3 \quad B=0 \quad C=3$

$A=C=3 \quad \& \quad B=0$

The given equation is a circle

3. $3x^2+2y^2=14$ Compared with

$Ax^2+Bxy+Cy^2+Dx+Ey+F=0$

$B^2-4AC=0-4(3)(2)=-12<0$

The given equation an Ellipse

4. $x^2+y^2+x-y=0$

Compared with

$Ax^2+Bxy+Cy^2+Dx+Ey+F=0$

$A=1, \quad B=0 \quad C=1 \quad D=1 \quad E=-1$
$F=0$

Here $A=C=1$, $B=0$

The equation is a circle

5) $11x^2-25y^2-44x+50y-256=0$

Compared with

$Ax^2+Bxy+Cy^2+Dx+Ey+F=0$

$A=11 \quad B=0 \quad C=-25 \quad D=-44$

$E=50 \quad F=-256$

$B^2-4AC=0-4(11)(-25)$

$=1100>0$

The given equation is a Hyperbola

6) $y^2+4x+3y+4=0$

Compared with

$Ax^2+Bxy+Cy^2+Dx+Ey+F=0$

$A=0, B=0, C=1, D=4,$
$E=3, F=4$

$B^2-4AC=0-4(0)(1)=0$

The given equation is a Parabola

11) (i)

$x^2+(y+2)^2=0$

$(x-h)^2+(y-k)^2=r^2$

Centre $=(0,-2)$

radius $r=0$

(ii) $x^2+y^2+6x-4y+4=0$

$x^2+y^2+2gx+2fy+c=0$

$2g=6$, $g=3$ | $2f=-4$, $f=-2$ | $c=4$

Centre $(-g,-f)=(-3,2)$

radius $r=\sqrt{g^2+f^2-c}=\sqrt{9+4-4}=3$

(iii) $x^2+y^2-x+2y-3=0$

$2g=-1$, $g=-1/2$ | $2f=2$, $f=1$ | $c=-3$

Centre $(-g,-f)=(1/2,-1)$

radius $r=\sqrt{g^2+f^2-c}=\sqrt{1/4+1+3}$

$=\sqrt{\frac{1+4+12}{4}}$

$\boxed{r=\frac{\sqrt{17}}{2}}$

(iv) $2x^2+2y^2-6x+4y+2=0$

$\div 2 \Rightarrow x^2+y^2-3x+2y+1=0$

$2g=-3$, $g=-3/2$ | $2f=2$, $f=1$ | $c=1$

Centre $(-g,-f)=(3/2,-1)$

radius $=\sqrt{g^2+f^2-c}=\sqrt{9/4+1-1}$

$\boxed{r=3/2}$

12)

$3x^2+(3-p)xy+qy^2-2px=8pq$ --- (1)

co-eff x^2 = co-eff y^2

$\boxed{3=q}$

Again co-eff of $xy=0$

$3-p=0$

$\boxed{p=3}$

(1) $\Rightarrow$ $3x^2+3y^2-6x-72=0$

$\div 3 \Rightarrow x^2+y^2-2x-24=0$

$2g=-2$, $g=-1$ | $2f=0$, $f=0$ | $c=-24$

Centre $=(-g,-f)=(1,0)$

radius $r=\sqrt{g^2+f^2-c}$

$=\sqrt{1+0+24}=\sqrt{25}$

$\boxed{r=5 \text{ units}}$

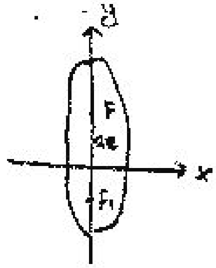
$C=(0,0)$

$F(0,1)$

$F_1(0,-3)$

$A=(0,4)$

$A'(0,-4)$



$ae=4 \quad a=5$

$5e=4$

$e=4/5$

$b^2=a^2-(ae)^2$

$=25-16=9$

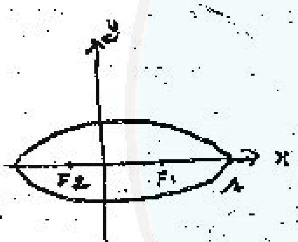
$\boxed{b^2=9}$

Equation of the ellipse

$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$

$\frac{x^2}{9}+\frac{y^2}{25}=1$

(iii)



Length of the latus rectum

$2b^2/a=8$

$2b^2=8a$

$\boxed{b^2=4a}$

$b^2=a^2(1-e^2)$

$4a=a^2[1-e^2]$

$4=a[1-9/25]$

$4=a[16/25]$

$\boxed{a=25/4}$

$b^2=4a=4\left(\frac{25}{4}\right)=25$

$b^2=25$

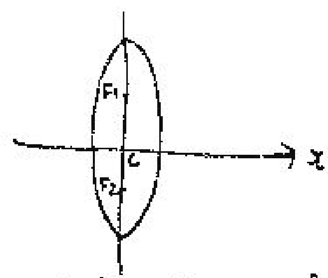
equation of an elipse

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$\frac{x^2}{\left(\frac{625}{16}\right)}+\frac{y^2}{25}=1$

$\frac{16x^2}{625}+\frac{y^2}{25}=1$

(iv)



Equation of the ellipses $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$

Length of the latus rectum $\frac{2b^2}{a}=4$

$2b^2=4a$

$\boxed{b^2=2a}$

distance between foci:

$2ae=4\sqrt{2}$

$\boxed{ae=2\sqrt{2}}$

$b^2=a^2-(ae)^2$

$2a=a^2(1-e^2)=a^2-a^2e^2$

$2a=a^2-(2\sqrt{2})^2$

$2a=a^2-8$

$a^2-2a-8=0$

$(a-4)(a+2)=0$

$a=4$ and $a=-2$ not Possible

$b^2=2(4)=8$

equation of the ellipses

$\frac{x^2}{8}+\frac{y^2}{16}=1$

3

(i) Foci: $(\pm ae,0)=(\pm 2,0)$

$ae=2$

$e=3/2 \quad a(3/2)=2$

$a=4/3$

$b^2=(ae)^2-a^2=4-16/9=\frac{36-16}{9}$

$b^2=20/9$

equation of the hyperbola

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$\frac{x^2}{16/9}-\frac{y^2}{20/9}=1$

$\boxed{\frac{9x^2}{16}-\frac{9y^2}{20}=1}$

(iv)

$x^2 - 2x + 8y + 17 = 0$

$x^2 - 2x + 17 = -8y$

$x^2 - 2x + 1 = -8y - 16$

$(x-1)^2 = -8(y+2)$

The Parabola open downward

Centre $(h,k) = (1,-2)$

Vertices $(0+h, -a+k) = (1,-4)$

Latus rectum : $y = -a$

$y+2 = -2$

$\boxed{y = -4}$

Directrix $y = a$

$y+2 = 2$

$\boxed{y = 0}$

Length of the latus rectum :

$4a = 4(2) = 8$

(v) $y^2 - 4y - 8x + 12 = 0$

$y^2 - 4y + 4 = 8x - 8$

$(y-2)^2 = 8(x-1)$

$y^2 = 4ax$ $\boxed{a = 2}$

The Parabola open rightward

vertices $(h,k) = (2,+2)$

Focus : $(h+a, 0+k) = (3,2)$

Latus rectum $x = +a$

$x - 1 = 2$

$x = 1 + 2 = 3$

$\boxed{x = 3}$

Directrix $x = -a$

$x - 1 = -2$

$\boxed{x = -1}$

Length of the Latus rectum :

$4a = 4(2) = 8$

5.

(i) $\frac{x^2}{25} + \frac{y^2}{9} = 1$

$a^2 = 25 \Rightarrow a = \pm 5$

$b^2 = 9 \Rightarrow b = \pm 3$

$(ae)^2 = a^2 - b^2 = 25 - 9 = 16$

$ae = 4$

Centre $(h,k) = (0,0)$

Focus $(h \pm ae, k) = (\pm 4, 0)$

vertices $(h \pm a, k) = (\pm 5, 0)$

Directrix $x = \pm a/e$

$e = \sqrt{\frac{a^2 - b^2}{a^2}} = \sqrt{\frac{25-16}{25}} = \sqrt{\frac{9}{25}} = 3/5$

$x = \pm \frac{5}{3/5} = \pm 25/3$

5 (ii)

$\frac{x^2}{3} + \frac{y^2}{10} = 1$

$a^2 = 10$ $b^2 = 3$

$(ae)^2 = a^2 - b^2 = 10 - 3 = 7$

$\boxed{ae = \sqrt{7}}$

Centre $(h,k) = (0,0)$

vertices $(h, \pm a + k), (0, \pm\sqrt{10})$

Focus $(h, \pm ae + k) = (0, \pm\sqrt{7})$

$e = \sqrt{\frac{a^2 - b^2}{a^2}} = \sqrt{\frac{10-3}{10}} = \frac{\sqrt{7}}{\sqrt{10}}$

directrix $= \pm a/e = \frac{\pm\sqrt{10}}{\sqrt{7}/\sqrt{10}}$

$\boxed{y = 10/\sqrt{7}}$

(ii)

Centre $(h,k) = (2,1)$

Foc: $(8,1)$

distance between Centre & Focus

$ae = \sqrt{(8-2)^2+(1-1)^2} = \sqrt{6^2} = 6$

$\boxed{ae = 6}$

corresponding directrix $x = 4$

$(4,1)$

distance between Centre and directrix

$a/e = \sqrt{(4-2)^2+(1-1)^2} = \sqrt{2^2}$

$a/e = 2$

$ae\,(a/e) = 6 \times 2 \Rightarrow \boxed{a^2 = 12}$

$(ae)^2 = 36$

$b^2 = (ae)^2 - a^2 = 36 - 12$

$\boxed{b^2 = 24}$

Eqn of the hyperbola

$$\frac{(x-2)^2}{a^2} - \frac{(y-1)^2}{b^2} = 1$$

$$\boxed{\frac{(x-2)^2}{12} - \frac{(y-1)^2}{24} = 1}$$

(iii)

Passing through $(x,y) = (5,-2)$

Length of tranverse axis, long x axis

Length 8 units

$2a = 8 \Rightarrow \boxed{a = 4}$

Centre $(h,k) = (0,0)$

Eqn of the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{at } (5,-2)\ a = 4$$

$$\frac{25}{16} - \frac{4}{b^2} = 1$$

$$\frac{25}{16} - 1 = \frac{4}{b^2}$$

$$\frac{25-16}{16} = \frac{4}{b^2}$$

$$\frac{9}{16} = \frac{4}{b^2}$$

$b^2 = 64/9$

Eqn of the Hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{16} - \frac{y^2}{64/9} = 1$$

$$\boxed{\frac{x^2}{16} - \frac{9y^2}{64} = 1}$$

4 (i) $y^2 = 16x$. $y^2 = 4ax$

$\boxed{a = 4}$

The Parabola open rightward

vertices $(0,0)$

Focus : $(a,0) = (4,0)$

Latus rectum $= x = 4$. $[x = a]$

Directrix : $x = -a \Rightarrow x = -4$

Length of latus rectum $= 4(a) = 4(4) = 16$

(ii) $x^2 = 24y$

$x^2 = 4(6)y$ $\boxed{a = 6}$

The Parabola open upward

Vertices : $(0,0)$

Focus : $(0,a) = (0,6)$

Latus rectum : $y = a \Rightarrow y = 6$

Directrix : $y = -a \Rightarrow y = -6$.

(iii) $y^2 = -8x$ $y^2 = -4ax$

$y^2 = -4(2)x$ $\boxed{a = 2}$

The Parabola open leftward

vertices $(0,0) = (h,k)$

Focus $(-a,0) = (-2,0)$

~~Latus rectum~~

Directrix $x = +a \Rightarrow x = +2$

L.R $= x = -a \Rightarrow x = -2$

Length of L.R. $= 4a = 4(2) = 8$

6 (iii)

$\frac{x^2}{25} - \frac{y^2}{144} = 1$

$a^2 = 25 \quad b^2 = 144$

$(ae)^2 = a^2 + b^2 = 25 + 144 = 169$

$ae = 13$

Centre $(h, k) = (0, 0)$

Focus $(h \pm ae, k) = (\pm 13, 0)$

Vertices $(h \pm a, k) = (\pm 5, 0)$

$e = \sqrt{\frac{a^2+b^2}{a^2}} = \sqrt{\frac{169}{25}} = 13/5$

Directrix $x = \pm a/e = \pm 5/_{13/5}$

$\boxed{x = \pm 25/13}$

(iv) $\frac{y^2}{16} - \frac{x^2}{9} = 1$

$a^2 = 16 \quad b^2 = 9$

$(ae)^2 = a^2 + b^2 = 16 + 9 = 25$

$\boxed{ae = 5}$

Centre $(h, k) = (0, 0)$

Focus $(h, k \pm ae) = (0, \pm 5)$

Vertices $(h, k \pm a) = (0, \pm 4)$

$e = \sqrt{\frac{a^2+b^2}{a^2}} = 5/4$

Directrix $y = \pm a/e = 4/_{5/4} = 16/5$

$\boxed{y = \pm 16/5}$

8.

(i) $\frac{(x-3)^2}{225} + \frac{(y-4)^2}{289} = 1$

$b^2 = 225 \quad a^2 = 289$

$(ae)^2 = a^2 - b^2 = 289 - 225$

$(ae)^2 = 64$

$\boxed{ae = 8}$

Centre $(h, k) = (3, 4)$

Focus : $(h, k \pm ae) = (3, 4 \pm 8)$

$F_1(3, 12) \quad F_2(3, -4)$

Vertices $(h, k \pm a) = (3, 4 \pm 17)$

$A(3, 21) \quad A'(3, -13)$

$e = \sqrt{\frac{a^2-b^2}{a^2}} = 8/17$

Directrix $y - 4 = \pm a/e$

$y - 4 = \pm 17/_{8/17} = \pm \frac{289}{8}$

$y = 4 \pm 289/8$

(ii) $\frac{(x+1)^2}{100} + \frac{(y-2)^2}{64} = 1$

$a^2 = 100 \quad b^2 = 64$

$(ae)^2 = a^2 - b^2 = 100 - 64 = 36$

$\boxed{ae = 6}$

Centre $(h, k) = (-1, 2)$

Focus $(h \pm ae, k) = (-1 \pm 6, 2)$

$F_1(5, 2) \quad F_2(-7, 2)$

Vertices $(h \pm a, k) = (-1 \pm 10, 2)$

$A(9, 2) \quad A'(-11, 2)$

$e = \sqrt{\frac{a^2-b^2}{a^2}} = 6/10 = 3/5$

$\boxed{e = 3/5}$

Eqn of Directrix $x = \pm a/e$

$x + 1 = \pm 10/_{3/5}$

$x + 1 = \pm 50/3$

$x = -1 \pm 50/3$

(iii) $\frac{(x+3)^2}{225} - \frac{(y-4)^2}{64} = 1$

$a^2 = 225 \qquad b^2 = 64$

$(ae)^2 = a^2 + b^2 = 225 + 64 = 289$

$\boxed{ae = 17}$

Centre $(h, k) = (-3, 4)$

Focus $(h \pm ae) = (-3 \pm 17, 4)$

$F_1(14, 4) \qquad F_2(-20, 4)$

Vertices $(h \pm a, k) = (-3 \pm 15, 4)$

$A(12, 4) \qquad A'(-18, 4)$

Directrix $x = \pm a/e = \frac{15}{17/15} = \frac{225}{17}$

$x + 3 = \pm 225/17$

$\boxed{x = -3 \pm 225/17}$

---

7) By defn

$\frac{SP}{PM} = e$

$\boxed{SP = ePM}$ — ①

$\frac{S'P}{PM'} = e$

$\boxed{S'P = ePM'}$ — ②

① − ② ⇒

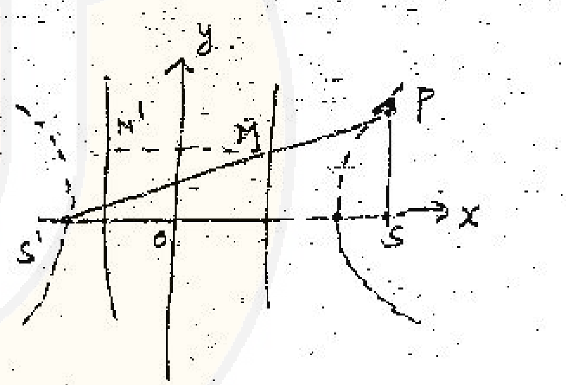
$SP - S'P = ePM - eP'M$

$= e(PM - PM')$

$= eMM'$

$= e(2a/e) = 2a$

$SP - S'P = 2a$ = length of the transverse axis



---

6) soln

Let (ae, y_1) is a point on the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{—} ①$$

Since $e^2 = \frac{a^2 + b^2}{a^2}$

$$① \Rightarrow \frac{a^2 e^2}{a^2} - \frac{y_1^2}{b^2} = 1$$

$$\frac{a^2 + b^2}{a^2} - \frac{y_1^2}{b^2} = 1$$

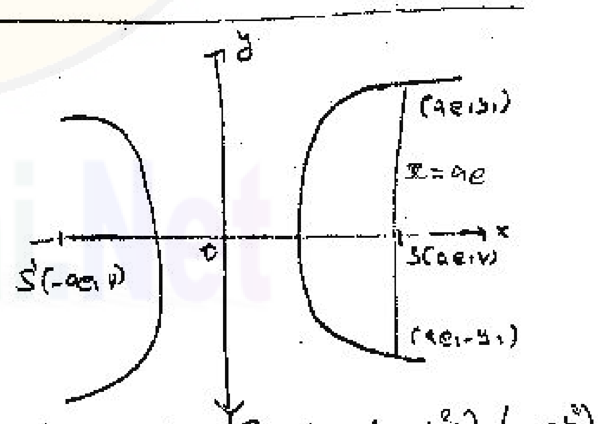
$$\frac{a^2}{a^2} + \frac{b^2}{a^2} - 1 = \frac{y_1^2}{b^2}$$

$$\cancel{1} + b^2/a^2 - \cancel{1} = y_1^2/b^2$$

$$\frac{b^2}{a^2} = y_1^2/b^2$$

$$y_1^2 = b^4/a^2$$

$$\boxed{y_1 = \pm b^2/a}$$



The end points of latus rectum $(ae, b^2/a), (ae, -b^2/a)$

length of LR $= b^2/a + b^2/a = 2b^2/a$

Hence the proof.

Ex. 5.2 8 --- (v)

$$18x^2+12y^2-144x+48y+120=0$$

It is an ellipse

$$18x^2-144x+12y^2+48y=-120$$

$$18(x^2-8x)+12(y^2+4y)=-120$$

$$18(\underbrace{x^2-8x+4^2}-4^2)+12(y^2+4y+2^2-2^2)=-120$$

$$18\left((x-4)^2-16\right)+12\left((y+2)^2-4\right)=-120$$

$$18(x-4)^2-288+12(y+2)^2-48=-120$$

$$18(x-4)^2+12(y+2)^2=-120+288+48$$

$$18(x-4)^2+12(y+2)^2=216$$

$$\div 216 \Rightarrow \frac{18(x-4)^2}{216}+\frac{12(y+2)^2}{216}=\frac{216}{216}$$

$$\frac{(x-4)^2}{12}+\frac{(y+2)^2}{18}=1$$

Major axis Parallel to y axis.

$$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$$

$h=4$ $\quad k=-2$

$b^2=12$ $\quad a^2=18$

$a=\sqrt{18}=3\sqrt{2}$

$c^2=a^2-b^2=18-12=6$

$c^2=6$ $\quad \boxed{c=\sqrt{6}}$

$$e=c/a=\sqrt{6}/\sqrt{18}=\sqrt{6/18}=1/\sqrt{3}$$

$$\boxed{e=1/\sqrt{3}}$$

$$a/e=\frac{\sqrt{18}}{1/\sqrt{3}}=\frac{\sqrt{18}\times\sqrt{3}}{1}=\sqrt{18\times 3}$$

$$\boxed{a/e=3\sqrt{6}}$$

	$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$	$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$	$\frac{(x-4)^2}{12}+\frac{(y+2)^2}{18}=1$
Centre	$(0,0)$	(h,k)	$C(4,-2)$
vertices	$(0,\pm a)$	$(h,k\pm a)$	$4,-2\pm\sqrt{18}$ $\quad A(4,-2+\sqrt{18})$ $\quad A'(4,-2-\sqrt{18})$
Foci	$(0,\pm c)$	$(h,k\pm c)$	$4,-2\pm\sqrt{6}$ $\quad S(4,-2+\sqrt{6})$ $\quad S'(4,-2-\sqrt{6})$
Eqn of Directrix	$y=\pm a/e$	$y=k\pm a/e$	$y=-2\pm 3\sqrt{6}$ $\quad y=-2+3\sqrt{6}$ $\quad y=-2-3\sqrt{6}$

(vi) $9x^2 - y^2 - 36x - 6y + 18 = 0$

It is a hyperbola

$9x^2 - 36x - y^2 - 6y = -18$

$9(x^2 - 4x) - (y^2 + 6y) = -18$

$9(x^2 - 4x + 2^2 - 2^2) - (y^2 + 6y + 3^2 - 3^2) = -18$

$9((x-2)^2 - 4) - ((y+3)^2 - 9) = -18$

$9(x-2)^2 - 36 - (y+3)^2 + 9 = -18$

$9(x-2)^2 - (y+3)^2 = -18 + 36 - 9$

$9(x-2)^2 - (y+3)^2 = 9$

$\div 9 \Rightarrow \frac{9(x-2)^2}{9} - \frac{(y+3)^2}{9} = \frac{9}{9}$

$\frac{(x-2)^2}{1} - \frac{(y+3)^2}{9} = 1$.

$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

$h = 2$ $\quad k = -3$

$a^2 = 1$ $\quad b^2 = 9$

$\boxed{a = 1}$ $\quad b = 3$

$c^2 = a^2 + b^2 = 1 + 9 = 10$

$\boxed{c = \sqrt{10}}$

$e = c/a = \sqrt{10}/1$

$\boxed{e = \sqrt{10}}$

$\boxed{a/e = 1/\sqrt{10}}$

Transverse axis Parallel to x axis.

	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	$\frac{(x-2)^2}{1} - \frac{(y+3)^2}{9} = 1$
Centre	$c(0,0)$	$c(h,k)$	$c(2,-3)$
vertices	$(\pm a, 0)$	$(h \pm a, k)$	$(2 \pm 1, -3)$ $(3,-3)A$ $(1,-3)A'$
Foci	$(\pm c, 0)$	$(h \pm c, k)$	$(2 \pm \sqrt{10}, -3)$ $S(2+\sqrt{10}, -3)$ $S'(2-\sqrt{10}, -3)$
Directrix eqn	$x = \pm a/e$	$x = h \pm a/e$	$x = 2 \pm 1/\sqrt{10}$ $x = 2 + 1/\sqrt{10}$ $x = 2 - 1/\sqrt{10}$

Ex - 5.2 -- 8 (iv)

iv) $\frac{(y-2)^2}{25} - \frac{(x+1)^2}{16} = 1$

This is hyperbola

Transverse axis parallel to y axis.

$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

$h = -1 \quad k = 2$

$a^2 = 25 \quad b^2 = 16$

$a = 5$

$c^2 = a^2 + b^2 = 25 + 16 = 41$

$c = \sqrt{41}$

$e = c/a = \sqrt{41}/5$

$a/e = \frac{5}{\sqrt{41}/5} = \frac{25}{\sqrt{41}}$.

Centre	$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$	$\frac{(y-2)^2}{25} - \frac{(x+1)^2}{16} = 1$
Centre	$c(0,0)$	(h,k)	$c(-1,2)$
Vertices	$(0, \pm a)$	$(h, k \pm a)$	$(-1, 2 \pm 5)$ $(-1, 2+5) = (-1, 7)$ A $(-1, 2-5) = (-1, -3)$ A'
Foci	$(0, \pm c)$	$(h, k \pm c)$	$(-1, 2 \pm \sqrt{41})$ S $(-1, 2+\sqrt{41})$ S' $(-1, 2-\sqrt{41})$
Equation of Directrices	$y = \pm a/e$	$y = k \pm a/e$	$y = 2 \pm \frac{25}{\sqrt{41}}$ $y = 2 + \frac{25}{\sqrt{41}}$ $y = 2 - \frac{25}{\sqrt{41}}$.

EXERCISE : 5.4

1.

$2x^2+7y^2=14$

$\div 14 \Rightarrow \frac{x^2}{7}+\frac{y^2}{2}=1$

$a^2=7 \quad b^2=2$

Any tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is of the form

$y=mx \pm \sqrt{a^2m^2+b^2}$ $\quad (5,2)$

$2=m(5) \pm \sqrt{7m^2+2}$

$2-5m = \pm\sqrt{7m^2+2}$

Squaring on both side

$(2-5m)^2=(\sqrt{7m^2+2})^2$

$4-20m+25m^2=7m^2+2$

$18m^2-20m+2=0$

$\div 2 \Rightarrow 9m^2-10m+1=0$

$(9m-1)(m-1)=0$

$m=1/9$ (or) $m=1$

Tangent equation

when $m=1$ $\quad (x_1,y_1)=(5,2)$

$y-y_1=m(x-x_1)$

$y-2=1(x-5)$

$\boxed{x-y-3=0}$

when $m=1/9$ $\quad (x_1,y_1)=(5,2)$

$y-2=1/9(x-5)$

$9y-18=x-5$

$\boxed{x-9y+13=0}$

2) $\frac{x^2}{16}-\frac{y^2}{64}=1$, It is hyperbola

$a^2=16 \quad b^2=64$

slope of the tangent $m=-10/-3$

$\boxed{m=10/3}$

Any tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is of the form

$y=mx \pm \sqrt{a^2m^2-b^2}$

$y=\frac{10}{3}x \pm \sqrt{16\left(\frac{100}{9}\right)-64}$

$y=\frac{10}{3}x \pm \sqrt{\frac{1024}{9}}$

$y=\frac{10}{3}x \pm \frac{32}{3}$

$3y=10x \pm 32$

$10x-3y \pm 32=0$

Tangent eqns are

$10x-3y+32=0$

$10x-3y-32=0$

3) $x-y+4=0$

$y=x+4$

$y=mx+c$

$m=1 \quad c=4$

$x^2+3y^2=12$

$\div 12 \Rightarrow \frac{x^2}{12}+\frac{y^2}{4}=1$

$a^2=12 \quad b^2=4$

Condition for tangency

$c^2=a^2m^2+b^2$

$c^2=16 \quad a^2m^2+b^2=12(1)^2+4$

$=12+4=16$

The line is a tangent to the ellipse.

Point of Contact

$\left(\frac{-a^2m}{c}, \frac{b^2}{c}\right)=\left(\frac{-12(1)}{4}, \frac{4}{4}\right)$

$=(-3,1)$

4) $y^2 = 16x$

$4a = 16$

$a = 4$

Any line $\perp^{er}$ to $2x+2y+3=0$ is

$2x - 2y + k = 0$

$2x - 2y + k = 0$

$x - y + k/2 = 0$ --- ①

$y = x + k/2$

$m = 1 \quad c = k/2$

The condition for the line $y = mx + c$ be a tangent to the Parabola $y^2 = 4ax$ is $c = a/m$

$c = 4/1$

$k/2 = 4 \Rightarrow \boxed{k = 8}$

$k = 8$ in ①

$x - y + 4 = 0$

Hence the Required eqn.

5) $y^2 = 8x$

$4a = 8 \Rightarrow a = 2$

Here $t = 2 \quad a = 2$

The equation of tangent at 't' on the Parabola is

$yt = x + at^2$

$y(2) = x + 2(4)$

$2y = x + 8$

$\boxed{x - 2y + 8 = 0}$

6) $12x^2 - 9y^2 = 108$

$\div 108 \Rightarrow \frac{x^2}{9} - \frac{y^2}{12} = 1$

$a^2 = 9 \qquad b^2 = 12$

$a = 3 \qquad b = 2\sqrt{3}$

Tangent equation at (θ)

$\frac{x\sec\theta}{a} - \frac{y\tan\theta}{b} = 1$

$\theta = \pi/3$

$\frac{x\sec\pi/3}{3} - \frac{y\tan\pi/3}{2\sqrt{3}} = 1$

$\frac{2x}{3} - \frac{\sqrt{3}y}{2\sqrt{3}} = 1$

$\frac{2x}{3} - \frac{y}{2} = 1$

$\boxed{4x - 3y - 6 = 0}$

Normal equation at 'θ'

$\frac{ax}{\sec\theta} + \frac{by}{\tan\theta} = a^2 + b^2$

$\theta = \pi/3$

$3x/2 + \frac{2\sqrt{3}y}{\sqrt{3}} = 9 + 12$

$\frac{3x}{2} + 2y = 21$

$\boxed{3x + 4y - 42 = 0}$

7) tangent eqn at t_1 is

$yt_1 = x + at_1^2$ ——— ①

as t_2 is

$yt_2 = x + at_2^2$ ——— ②

① − ② ⇒

$y(t_1 - t_2) = a(t_1^2 - t_2^2)$

$y = \frac{a(t_1+t_2)(t_1-t_2)}{(t_1-t_2)}$

$\boxed{y = a(t_1+t_2)}$

Using $y = a(t_1+t_2)$ in ① we get

$a(t_1+t_2)t_1 = x + at_1^2$

$at_1^2 + at_1t_2 = x + at_1^2$

$x = at_1t_2$

The Point of intersection is

$at_1t_2,\ a(t_1+t_2)$

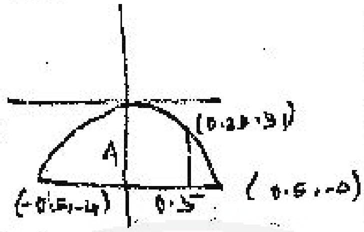
3) Eqn of the Parabola is

$x^2 = -4ay$ —— ①

It Passes through $(0.5, -4)$

$(0.5)^2 = -4a(-4)$

$a = \frac{0.25}{16}$



① ⇒ $x^2 = -4\left[\frac{0.25}{16}\right]y$

It Passes Through $(0.25, -y_1)$

$(0.25)^2 = -4\left[\frac{0.25}{16} \times (-y_1)\right]$

$y_1 = \frac{4 \times (0.25)^2}{0.25} = 4 \times 0.25$

$\boxed{y_1 = 1\,m}$

The Required height $= 4 - y_1 = 4 - 1 = 3m$

4) (a) Eqn of Parabola is

$y^2 = 4ax$

$a = 1.2$

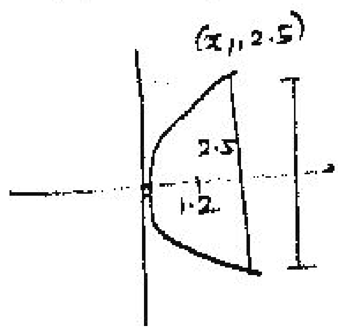
$y^2 = 4.8x$

b) It passes Through $(x, 2.5)$

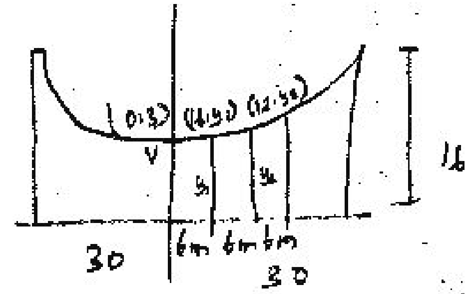
$(2.5)^2 = 4.8(x_1)$

$x_1 = \frac{2.5 \times 2.5}{4.8} = 1.3\,m$

The depth of the satellite dish at the vertex 1.3 m



5)



Eqn of the Parabola is

$(x-h)^2 = 4a(y-k)$

$(x-0)^2 = 4a(y-3)$

It Passes through $(30, 16)$

$30^2 = 4a(16-3)$

$a = \frac{30 \times 30}{13 \times 4}$

$x^2 = 4\left(\frac{30 \times 30}{13 \times 4}\right)(y-3)$

$x^2 = \frac{30 \times 30}{13}(y-3)$

It Passes Through $(6, y_1)$

$6^2 = \frac{900}{13}(y_1 - 3)$

$y_1 - 3 = \frac{36 \times 13}{900} = 0.52$

$y_1 = 3 + 0.52 = 3.52\,m$

It also Passes through $(12, y_2)$

$12^2 = \frac{900}{13}(y_2 - 3)$

$y_2 - 3 = \frac{144 \times 13}{900} = 2.08$

$y_2 = 3 + 2.08$

$\boxed{y_2 = 5.08\,m}$

8)

Normal equation of a Parabola at $P(at_1^2, 2at_1)$ is

$y + xt_1 = at_1^3 + 2at_1$

It meets the Parabola again at $Q(at_2^2, 2at_2)$

$2at_2 + (at_2^2)t_1 = at_1^3 + 2at_1$

$2at_2 + at_1t_2^2 = at_1^3 + 2at_1$

$2at_2 - 2at_1 = at_1^3 - at_1t_2^2$

$2a(t_2 - t_1) = at_1(t_1^2 - t_2^2)$

$2a(t_2 - t_1) = at_1(t_1 + t_2)(t_1 - t_2)$

$2\cancel{a}\cancel{(t_2 - t_1)} = -\cancel{a}t_1\cancel{(t_2 - t_1)}(t_1 + t_2)$

$-2 = t_1(t_1 + t_2)$

$t_1 + t_2 = -2/t_1$

$t_2 = -2/t_1 - t_1$

$t_2 = -2/t_1 - t_1$

$t_2 = -(t_1 + 2/t_1)$

Hence Proved.

EXERCISE 5.5

1. Eqn of Parabola is

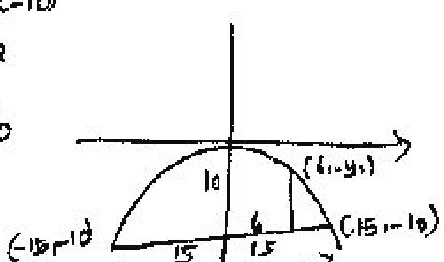
$x^2 = -4ay$

It Passes through $(15, -10)$

$15^2 = -4a(-10)$

$225 = 40a$

$a = 225/40$



① ⇒ $x^2 = -4\left(\frac{225}{40}\right)y$

It Passes through $(6, -y_1)$

$36 = -4\left(\frac{225}{40}\right)(-y_1)$

$\frac{36 \times 10}{225} = y_1$

$1.6 = y_1$

The required height $= 10 - y_1$

$= 10 - 1.6$

$= 8.4\,m$

2)

$CA = a = 8$ $\quad CB = b = 5$

Eqn of ellipse is

$x^2/b^2 + y^2/a^2 = 1$

$x^2/25 + y^2/64 = 1$

It is Passes through $(4, y_1)$

$\frac{16}{25} + \frac{y_1^2}{64} = 1$

$\frac{y_1^2}{64} = 1 - 16/25$

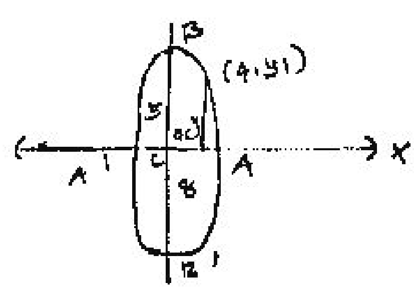
$y_1^2 = 64\,(9/25)$

$y_1 = 8 \times 3/5 = 24/5$

$\boxed{y_1 = 4.8\,m}$

The Required wide for the opening is $2y = 2(4.8)$

$= 9.6\,m$



6)
$3k = 150$

$k = 50$

$(x_1, 50)$ lies on the Parabola

$$\frac{x_1^2}{30^2} - \frac{50^2}{44^2} = 1$$

$$\frac{x_1^2}{30^2} = 1 + \frac{50^2}{44^2}$$

$$x_1^2 = \frac{30^2}{44^2}[1936+2500]$$

$$x_1 = \frac{30}{44}\sqrt{4436}$$

$$= \frac{30}{44}(66.60)$$

$$\boxed{x_1 = 45.40}$$

Radius of the top of the tower is 45.40m.

$(x_2, -100)$ lies on the hyperbola

$$\frac{x_2^2}{30^2} - \frac{100^2}{44^2} = 1$$

$$\frac{x_2^2}{30^2} = 1 + \frac{100^2}{44^2}$$

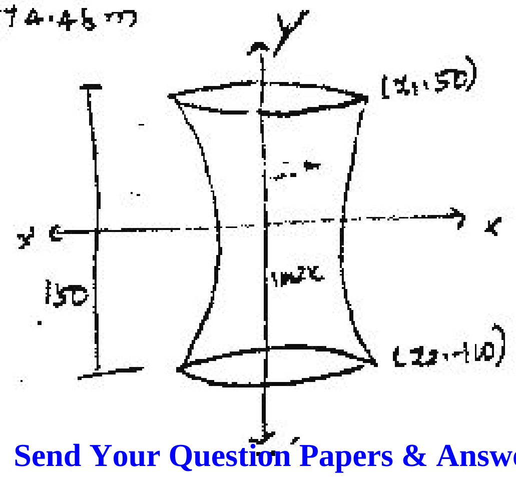
$$x_2^2 = \frac{30^2}{44^2}(1936+10000)$$

$$x_2 = \frac{30}{44}\sqrt{11936}$$

$$= \frac{30}{44} \times 109.25$$

$$x_2 = \frac{3277.5}{44} = 74.46m$$

Radius of the base of the tower is 74.46m



7)
$\Delta AP \cong PCD$

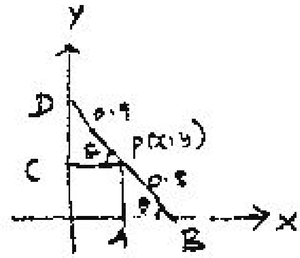
Lt $\angle ABP = \angle CPD = \theta$

In ΔPAB

$$\sin\theta = \frac{y}{0.3}$$

In ΔPCD

$$\cos\theta = x/0.9$$



$$\sin^2\theta + \cos^2\theta = 1$$

$$\frac{x^2}{0.9^2} + \frac{y^2}{(0.3)^2} = 1$$

$$\frac{x^2}{(0.9)^2} + \frac{y^2}{(0.3)^2} = 1$$

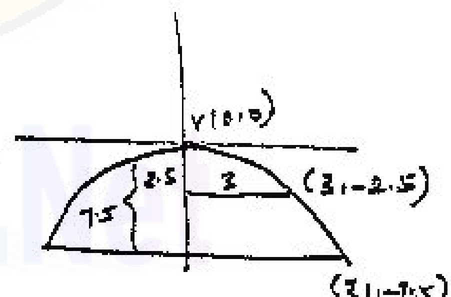
$$e = \sqrt{1 - b^2/a^2} = \sqrt{1 - \frac{0.3^2}{0.9^2}}$$

$$= \sqrt{1-(1/3)^2} = \sqrt{1 - 1/9}$$

$$e = \sqrt{8/9}$$

$$\boxed{e = \frac{2\sqrt{2}}{3} m}$$

8)



The equation of the Parabola is

$$x^2 = -4ay$$

It Passes Through $(3, -2.5)$

$$3^2 = -4a(-2.5)$$

$$\boxed{a = 9/10}$$

$x^2 = -4\left(\frac{9}{10}\right)y$

$x^2 = -\frac{16}{5}y$

It Passes through $(x_1, -7.5)$

$x_1^2 = -\frac{16}{5}(-7.5)$

$x_1^2 = 27$

$x_1 = 3\sqrt{3}$ m

The water strikes the ground $3\sqrt{3}$ m beyond the vertical line.

9) The equation of the Parabola is $x^2 = -4ay$

It Passes through $(6, -4)$

$36 = 16a$

$a = \frac{36}{16} = \frac{9}{4}$

$\boxed{a = 9/4}$

$x^2 = -4(9/4)y$

$\boxed{x^2 = -9y}$

Diff w.r.to x

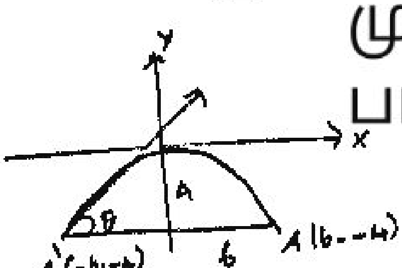
$2x = -9\,\frac{dy}{dx}$

$\frac{dy}{dx} = -\frac{2x}{9}$

at $(-6, -4)$

$\tan\theta = \frac{dy}{dx} = \frac{-2}{9}(-6) = \frac{4}{3}$

$\boxed{\theta = \tan^{-1}\frac{4}{3}}$



10)

$ae = 5$

$AP \sim BP = 2a = 6$

$\boxed{a = 3}$

$b^2 = a^2(e^2-1) = (ae)^2 - a^2$

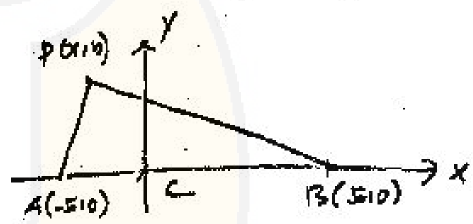
$= 25 - 9$

$\boxed{b^2 = 16}$

The equation of the required hyperbola

$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$

The locus of p is hyperbola



பிழைகள் இருப்பின் மன்னிக்கவும்.

ரெ. விஜய் 17/6/19

ரெ. விஜயராகவன்,
முதுகலை ஆசிரியர்-கணிதம்,
பட்டுக்கோட்டை.