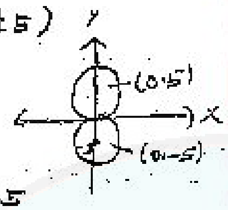
CHAPTER - V

TWO DIMENSIONAL ANALYTICAL GEOMETRY - II

EXERCISE - 5.1

1. Given $r=5$, Centre $(0, \pm 5)$



Equation of the circle

$(x-h)^2+(y-k)^2=r^2$

here $(h,k)=(0,\pm 5)$, $r=5$

equation:

$(x-0)^2+(y\pm 5)^2=5^2$

$x^2+y^2+25\pm 10y=25$

$\boxed{x^2+y^2\pm 10y=0}$

2. Equation of the circle

$(x-h)^2+(y-k)^2=r^2$

here $(h,k)=(2,-1)$

$(x-2)^2+(y+1)^2=r^2$

It passes through $(3,6)$

$(3-2)^2+(6+1)^2=r^2$

$1+49=r^2$, $\boxed{r^2=50}$

Equation of the circle

$\boxed{(x-2)^2+(y+1)^2=50}$

3. Equation of the circle

$(x-h)^2+(y-k)^2=r^2$

Centre is $(h,k)=(-r,-r)$

$(x+r)^2+(y+r)^2=r^2$ —— (1)

It is passes through $(-4,-2)$

$(-4+r)^2+(-2+r)^2=r^2$

$16+r^2-8r+4+r^2-4r=r^2$

$r^2-12r+20=0$

$(r-10)(r-2)=0$

$r=10, r=2$

When $r=10$

(1) ⇒

$(x+10)^2+(y+10)^2=10^2$

$(x^2+100+20x)+y^2+\cancel{100}+20y=\cancel{100}$

$\boxed{x^2+y^2+20x+20y+100=0}$

When $r=2$

(1) ⇒

$(x+2)^2+(y+2)^2=2^2$

$x^2+4+4x+y^2+4+4y=4$

$\boxed{x^2+y^2+4x+4y+4=0}$

4) Centre $(h,k)=(2,3)$

Equation of the circle

$(x-h)^2+(y-k)^2=r^2$

$(x-2)^2+(y-3)^2=r^2$ —— (1)

given lines: → (2)

$3x-2y-1=0$, $4x+y-27=0$ ↳ (3)

(3) × 2 ⇒ $8x+2y-54=0$

2 × (1) ⇒ $3x-2y-1=0$

$11x-55=0$

$11x=55$

$\boxed{x=5}$

(3) ⇒

$4(5)+y-27=0$

$20+y-27=0$ $\boxed{y=7}$

Point of inflection $(5,7)$

(1) ⇒ $(5-2)^2+(7-3)^2=r^2$

$9+16=r^2$

$\boxed{r^2=25}$

(1) ⇒ $(x-2)^2+(y-3)^2=25$

$x^2+4-4x+y^2+9-6y-25=0$

$\boxed{x^2+y^2-4x+6y-12=0}$

∴ The equation $\boxed{x^2+y^2-4x-6y-12=0}$

5.

Equation of the circle

$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$

Here $(x_1,y_1)=(3,4)$ $(x_2,y_2)=(2,-7)$

$(x-3)(x-2)+(y-4)(y+7)=0$

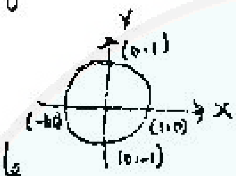
$x^2-2x-3x+6+y^2+7y-4y-28=0$

$\boxed{x^2+y^2-5x+3y-22=0}$

6. From the diagram

Centre $(0,0)$

radius : $r=1$



Equation of the circle

$(x-h)^2+(y-k)^2=r^2$

$\boxed{x^2+y^2=1}$

7. Area of the circle $=9\pi$

$\pi r^2=9\pi$

$\boxed{r^2=9}$

Two diameters are

$x+y=5$ —— (1)

$x-y=1$ —— (2)

(1) + (2) $\Rightarrow$ $2x=6$ $\Rightarrow$ $\boxed{x=3}$

(1) $\Rightarrow$ $3+y=5$ $\Rightarrow$ $\boxed{y=2}$

Centre $(h,k)=(3,2)$

Equation of the circle:

$(x-h)^2+(y-k)^2=r^2$

$(x-3)^2+(y-2)^2=9$

$x^2+9-6x+y^2+4-4y=9$

$\boxed{x^2+y^2-6x-4y+4=0}$

8. $y=2\sqrt{2}x+c$

$y=mx+c$

$m=2\sqrt{2}$

$m^2=(2\sqrt{2})^2=8$

$x^2+y^2=16$

$\boxed{a^2=16}$

Condition:

$c^2=a^2(1+m^2)$

$c^2=16(1+8)$

$=16(9)$

$c^2=144$

$\boxed{c=\pm 12}$

9.

$x^2+y^2-6x+6y-8=0$

Eqn of the tangent

$xx_1+yy_1-6\left(\frac{x+x_1}{2}\right)+6\left(\frac{y+y_1}{2}\right)-8=0$

$xx_1+yy_1-3(x+x_1)+3(y+y_1)-8=0$

Here $(x_1,y_1)=(2,2)$

$2x+2y-3(x+2)+3(y+2)-8=0$

$2x+2y-3x-6+3y+6-8=0$

$-x+5y-8=0$

$\boxed{x-5y-8=0}$

Eqn of the Normal:

$5x+y+k=0$

It passes through $(2,2)$

$5(2)+2+k=0$

$12+k=0$

$\boxed{k=-12}$

Equation of the normal is $\boxed{5x+y-12=0}$

10. $x^2+y^2-5x+2y-5=0$

at $(-2,1)$

(1) $\Rightarrow$ $4+1+10-2-5=12>0$

$(-2,1)$ lies outside the circle.

at $(0,0)$

(1) $\Rightarrow$ $0+0-0+0-5=-5<0$

$(0,0)$ lies inside the circle

at $(-4,-3)$

(1) $\Rightarrow$ $16+9+20-6-5=34>0$

$(-4,-3)$ lies outside of the circle.

11.

(i) $x^2+(y+2)^2=0$

$(x-h)^2+(y-k)^2=r^2$

Centre $(h,k)=(0,-2)$

radius $r=0$

(ii) $x^2+y^2+2gx+2fy+c=0$

$x^2+y^2+6x-4y+4=0$

$2g=6$ $\quad$ $2f=-4$

$\boxed{g=3}$ $\quad$ $\boxed{f=-2}$

$\boxed{c=4}$

Centre $(-g,-f)=(-3,2)$

radius $r=\sqrt{g^2+f^2-c}$

$=\sqrt{9+4-4}=\sqrt{9}=3$

$\boxed{r=3}$

(iii) $x^2+y^2-x+2y-3=0$

$2g=-1$ | $2f=2$ | $c=-3$

$g=-1/2$ | $f=1$ |

Centre $(-g,-f)=(1/2,-1)$

radius $r=\sqrt{g^2+f^2-c}=\sqrt{1/4+1+3}$

$\boxed{r=\frac{\sqrt{17}}{2}}$

(iv) $2x^2+2y^2-6x+4y+2=0$

$\div 2 \Rightarrow x^2+y^2-3x+2y+1=0$

$2g=-3$ | $2f=2$ | $c=1$

$g=-3/2$ | $f=1$ |

Centre $=(-g,-f)=(3/2,-1)$

radius; $r=\sqrt{g^2+f^2-c}=\sqrt{9/4+1-1}$

$\boxed{r=3/2}$

12)

$3x^2+(3-p)xy+qy^2-2px=8pq$ —(1)

Co-eff of x^2 = Co-eff of y^2

$3=q$

$\boxed{q=3}$

Again co-eff of $xy=0$

$3-p=0$

$\boxed{p=3}$

(1) $\Rightarrow$ $3x^2+3y^2-6x-72=0$

$\div 3 \Rightarrow$ $x^2+y^2-2x-24=0$

$2g=-2$ | $2f=0$ | $c=-24$

$g=-1$ | $f=0$ |

Centre $(-g,-f)=(1,0)$

radius $r=\sqrt{g^2+f^2-c}$

$=\sqrt{1+0+24}=\sqrt{25}$

$\boxed{r=5}$

EXERCISE : 5.3

1. $2x^2-y^2=7$

Compared with

$Ax^2+Bxy+Cy^2+Dx+Ey+F=0$

$A=2 \quad B=0 \quad C=-1$

$B^2-4AC = 0-4(2)(-1) = 8 > 0$

The given equation is a Hyperbola.

2. $3x^2+3y^2-4x+3y+10=0$

Compared with

$Ax^2+Bxy+Cy^2+Dx+Ey+F=0$

$A=3 \quad B=0 \quad C=3$

$A=C=3 \quad \& \quad B=0$

The given equation is a circle

3. $3x^2+2y^2=14$ Compared with

$Ax^2+Bxy+Cy^2+Dx+Ey+F=0$

$B^2-4AC = 0-4(3)(2) = -12 < 0$

The given equation an Ellipse

4. $x^2+y^2+x-y=0$

Compared with

$Ax^2+Bxy+Cy^2+Dx+Ey+F=0$

$A=1, \quad B=0 \quad C=1 \quad D=1 \quad E=-1$
$F=0$

Here $A=C=1$, $B=0$

The equation is a circle

5) $11x^2-25y^2-44x+50y-256=0$

Compared with

$Ax^2+Bxy+Cy^2+Dx+Ey+F=0$

$A=11 \quad B=0 \quad C=-25 \quad D=-44$

$E=50 \quad F=-256$

$B^2-4AC = 0-4(11)(-25)$

$= 1100 > 0$

The given equation is a Hyperbola

6) $y^2+4x+3y+4=0$

Compared with

$Ax^2+Bxy+Cy^2+Dx+Ey+F=0$

$A=0, B=0, C=1, D=4,$
$E=3, F=4$

$B^2-4AC = 0-4(0)(1) = 0$

The given equation is a Parabola

11) (i)

$x^2+(y+2)^2=0$

$(x-h)^2+(y-k)^2=r^2$

Centre $=(0,-2)$

radius $r=0$

(ii) $x^2+y^2+6x-4y+4=0$

$x^2+y^2+2gx+2fy+c=0$

$2g=6$, $g=3$ | $2f=-4$, $f=-2$ | $c=4$

Centre $(-g,-f)=(-3,2)$

radius $r=\sqrt{g^2+f^2-c}=\sqrt{9+4-4}=3$

(iii) $x^2+y^2-x+2y-3=0$

$2g=-1$, $g=-1/2$ | $2f=2$, $f=1$ | $c=-3$

Centre $(-g,-f)=(1/2,-1)$

radius $r=\sqrt{g^2+f^2-c}=\sqrt{1/4+1+3}$

$=\sqrt{\frac{1+4+12}{4}}$

$\boxed{r=\frac{\sqrt{17}}{2}}$

(iv) $2x^2+2y^2-6x+4y+2=0$

$\div 2 \Rightarrow$ $x^2+y^2-3x+2y+1=0$

$2g=-3$, $g=-3/2$ | $2f=2$, $f=1$ | $c=1$

Centre $(-g,-f)=(3/2,-1)$

radius $=\sqrt{g^2+f^2-c}=\sqrt{9/4+1-1}$

$\boxed{r=3/2}$

12)

$3x^2+(3-p)xy+qy^2-2px=8pq$ --- ①

Co-eff x^2 = Co-eff y^2

$\boxed{3=q}$

Again co-eff of $xy=0$

$3-p=0$

$\boxed{p=3}$

① $\Rightarrow$ $3x^2+3y^2-6x-72=0$

$\div 3 \Rightarrow$ $x^2+y^2-2x-24=0$

$2g=-2$, $g=-1$ | $2f=0$, $f=0$ | $c=-24$

Centre $=(-g,-f)=(1,0)$

radius $r=\sqrt{g^2+f^2-c}$

$=\sqrt{1+0+24}=\sqrt{25}$

$\boxed{r=5 \text{ units}}$

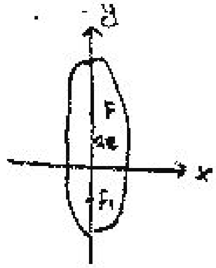
$C=(0,0)$

$F(0,4)$

$F_1(0,-4)$

$A=(0,5)$

$A'(0,-5)$



$ae=4 \quad a=5$

$5e=4$

$e=4/5 \qquad b^2=a^2-(ae)^2$

$=25-16=9$

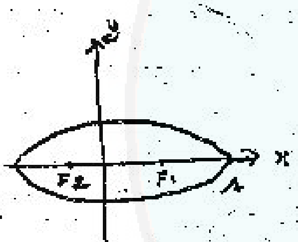
$\boxed{b^2=9}$

Equation of the ellipse

$$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$$

$$\frac{x^2}{9}+\frac{y^2}{25}=1$$

(iii)



Length of the latus rectum

$2b^2/a=8$

$2b^2=8a$

$\boxed{b^2=4a}$

$b^2=a^2(1-e^2)$

$4\cancel{a}=a^{\cancel{2}}[1-e^2]$

$4=a[1-9/25]$

$4=a[16/25]$

$\boxed{a=25/4}$

$b^2=4a=4\left(\frac{25}{4}\right)=25$

$b^2=25$

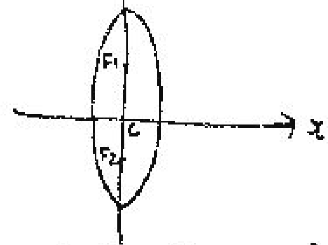
equation of an elipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$$\frac{x^2}{\left(\frac{625}{16}\right)}+\frac{y^2}{25}=1$$

$$\frac{16x^2}{625}+\frac{y^2}{25}=1$$

(iv)



Equation of the ellipses $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$

Length of the latus rectum $\frac{2b^2}{a}=4$

$2b^2=4a$

$\boxed{b^2=2a}$

distance between foci:

$2ae=4\sqrt{2}$

$\boxed{ae=2\sqrt{2}}$

$b^2=a^2-(ae)^2$

$2a=a^2(1-e^2)=a^2-a^2e^2$

$2a=a^2-(2\sqrt{2})^2$

$2a=a^2-8$

$a^2-2a-8=0$

$(a-4)(a+2)=0$

$a=4$ and $a=-2$ not possible

$b^2=2(4)=8$

equation of the ellipses

$$\frac{x^2}{8}+\frac{y^2}{16}=1$$

3

(i) Foc: $(\pm ae,0)=(\pm 2,0)$

$ae=2$

$e=3/2 \qquad a(3/2)=2$

$a=4/3$

$b^2=(ae)^2-a^2=4-16/9=\frac{36-16}{9}$

$b^2=20/9$

equation of the hyperbola

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

$$\frac{x^2}{16/9}-\frac{y^2}{20/9}=1$$

$$\boxed{\frac{9x^2}{16}-\frac{9y^2}{20}=1}$$

(iv.)

$x^2-2x+8y+17=0$

$x^2-2x+17=-8y$

$x^2-2x+1=-8y-16$

$(x-1)^2=-8(y+2)$

The Parabola open downward

Centre $(h,k)=(1,-2)$

Vertices $(0+h,-a+k)=(1,-4)$

Latus rectum : $y=-a$

$y+2=-2$

$\boxed{y=-4}$

Directrix $y=a$

$y+2=2$

$\boxed{y=0}$

Length of the Latus rectum :

$4a=4(2)=8$

(v) $y^2-4y-8x+12=0$

$y^2-4y+4=8x-8$

$(y-2)^2=8(x-1)$

$y^2=4ax$ $\boxed{a=2}$

The Parabola open rightward

vertices $(h,k)=(2,+2)$

Focus : $(h+a,0+k)=(3,2)$

Latus rectum $x=+a$

$x-1=2$

$x=1+2=3$

$\boxed{x=3}$

Directrix $x=-a$

$x-1=-2$

$\boxed{x=-1}$

Length of the Latus rectum :

$4a=4(2)=8$

5.

(i) $\frac{x^2}{25}+\frac{y^2}{9}=1$

$a^2=25 \Rightarrow a=\pm 5$

$b^2=9 \Rightarrow b=\pm 3$

$(ae)^2=a^2-b^2=25-9=16$

$ae=4$

Centre $(h,k)=(0,0)$

Focus $(h\pm ae,k)=(\pm 4,0)$

vertices $(h\pm a,k)=(\pm 5,0)$

Directrix $x=\pm a/e$

$e=\sqrt{\frac{a^2-b^2}{a^2}}=\sqrt{\frac{25-16}{25}}=\sqrt{\frac{9}{25}}=3/5$

$x=\pm\frac{5}{3/5}=\pm 25/3$

5 (ii)

$\frac{x^2}{3}+\frac{y^2}{10}=1$

$a^2=10$ $b^2=3$

$(ae)^2=a^2-b^2=10-3=7$

$\boxed{ae=\sqrt{7}}$

Centre $(h,k)=(0,0)$

vertices $(h,\pm a+k),(0,\pm\sqrt{10})$

Focus $(h,\pm ae+k)=(0,\pm\sqrt{7})$

$e=\sqrt{\frac{a^2-b^2}{a^2}}=\sqrt{\frac{10-3}{10}}=\frac{\sqrt{7}}{\sqrt{10}}$

directrix $=\pm a/e=\frac{\pm\sqrt{10}}{\sqrt{7}/\sqrt{10}}$

$\boxed{y=10/\sqrt{7}}$

(ii)

Centre $(h,k) = (2,1)$

Foc : $(8,1)$

distance between Centre & Focus

$ae = \sqrt{(8-2)^2+(1-1)^2} = \sqrt{6^2} = 6$

$\boxed{ae = 6}$

corresponding directrix $x = 4$

$(4,1)$

distance between Centre and directrix

$a/e = \sqrt{(4-2)^2+(1-1)^2} = \sqrt{2^2}$

$a/e = 2$

$ae\,(a/e) = 6 \times 2 \Rightarrow \boxed{a^2 = 12}$

$(ae)^2 = 36$

$b^2 = (ae)^2 - a^2 = 36 - 12$

$\boxed{b^2 = 24}$

Eqn of the hyperbola

$$\frac{(x-2)^2}{a^2} - \frac{(y-1)^2}{b^2} = 1$$

$$\boxed{\frac{(x-2)^2}{12} - \frac{(y-1)^2}{24} = 1}$$

(iii)

Passing through $(x,y) = (5,-2)$

Length of tranverse axis, long x axis

Length 8 units

$2a = 8 \Rightarrow \boxed{a = 4}$

Centre $(h,k) = (0,0)$

Eqn of the hyperbola

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at $(5,-2)$ $a = 4$

$\frac{25}{16} - \frac{4}{b^2} = 1$

$\frac{25}{16} - 1 = \frac{4}{b^2}$

$\frac{25-16}{16} = \frac{4}{b^2}$

$\frac{9}{16} = \frac{4}{b^2}$

$b^2 = 64/9$

Eqn of the Hyperbola

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\frac{x^2}{16} - \frac{y^2}{64/9} = 1$

$$\boxed{\frac{x^2}{16} - \frac{9y^2}{64} = 1}$$

4 (i) $y^2 = 16x$. $y^2 = 4ax$

$\boxed{a = 4}$

The Parabola open rightward

vertices $(0,0)$

Focus : $(a,0) = (4,0)$

Latus rectum $= x = 4$. $[x = a]$

Directrix : $x = -a \Rightarrow x = -4$

Length of latus rectum $= 4(a) = 4(4) = 16$

(ii) $x^2 = 24y$

$x^2 = 4(6)y$ $\boxed{a = 6}$

The Parabola open upward

Vertices : $(0,0)$

Focus : $(0,a) = (0,6)$

Latus rectum : $y = a \Rightarrow y = 6$

Directrix : $y = -a \Rightarrow y = -6$.

(iii) $y^2 = -8x$ $y^2 = -4ax$

$y^2 = -4(2)x$ $\boxed{a = 2}$

The Parabola open leftward

vertices $(0,0) = (h,k)$

Focus $(-a,0) = (-2,0)$

~~Latus rectum~~

Directrix $x = +a \Rightarrow x = +2$

L.R $= x = -a \Rightarrow x = -2$

Length of L.R. $= 4a = 4(2) = 8$

6 (iii)

$\frac{x^2}{25} - \frac{y^2}{144} = 1$

$a^2 = 25 \quad b^2 = 144$

$(ae)^2 = a^2 + b^2 = 25 + 144 = 169$

$ae = 13$

Centre $(h,k) = (0,0)$

Focus $(h \pm ae, k) = (\pm 13, 0)$

Vertices $(h \pm a, k) = (\pm 5, 0)$

$e = \sqrt{\frac{a^2+b^2}{a^2}} = \sqrt{\frac{169}{25}} = 13/5$

Directrix $x = \pm a/e = \pm 5/\frac{13}{5}$

$\boxed{x = \pm 25/13}$

(iv) $\frac{y^2}{16} - \frac{x^2}{9} = 1$

$a^2 = 16 \quad b^2 = 9$

$(ae)^2 = a^2 + b^2 = 16 + 9 = 25$

$\boxed{ae = 5}$

Centre $(h,k) = (0,0)$

Focus $(h, k \pm ae) = (0, \pm 5)$

Vertices $(h, k \pm a) = (0, \pm 4)$

$e = \sqrt{\frac{a^2+b^2}{a^2}} = 5/4$

Directrix $y = \pm a/e = 4/\frac{5}{4} = 16/5$

$\boxed{y = \pm 16/5}$

8.

(i) $\frac{(x-3)^2}{225} + \frac{(y-4)^2}{289} = 1$

$b^2 = 225 \qquad a^2 = 289$

$(ae)^2 = a^2 - b^2 = 289 - 225$

$(ae)^2 = 64$

$\boxed{ae = 8}$

Centre $(h,k) = (3,4)$

Focus : $(h, k \pm ae) = (3, 4 \pm 8)$

$F_1(3,12) \quad F_2(3,-4)$

Vertices $(h, k \pm a) = (3, 4 \pm 17)$

$A(3,21) \quad A'(3,-13)$

$e = \sqrt{\frac{a^2-b^2}{a^2}} = 8/17$

Directrix $y - 4 = \pm a/e$

$y - 4 = \pm \frac{17}{8/17} = \pm \frac{289}{8}$

$y = 4 \pm 289/8$

(ii) $\frac{(x+1)^2}{100} + \frac{(y-2)^2}{64} = 1$

$a^2 = 100 \quad b^2 = 64$

$(ae)^2 = a^2 - b^2 = 100 - 64 = 36$

$\boxed{ae = 6}$

Centre $(h,k) = (-1,2)$

Focus $(h \pm ae, k) = (-1 \pm 6, 2)$

$F_1(5,2) \quad F_2(-7,2)$

Vertices $(h \pm a, k) = (-1 \pm 10, 2)$

$A(9,2) \quad A'(-11,2)$

$e = \sqrt{\frac{a^2-b^2}{a^2}} = 6/10 = 3/5$

$\boxed{e = 3/5}$

Eqn of Directrix $x = \pm a/e$

$x + 1 = \pm 10/\frac{3}{5}$

$x + 1 = \pm 50/3$

$x = -1 \pm 50/3$

(iii)
$$\frac{(x+3)^2}{225} - \frac{(y-4)^2}{64} = 1$$

$a^2 = 225 \quad b^2 = 64$

$(ae)^2 = a^2 + b^2 = 225 + 64 = 289$

$\boxed{ae = 17}$

Centre $(h,k) = (-3, 4)$

Focus $(h \pm ae) = (-3 \pm 17, 4)$

$F_1(14, 4) \quad F_2(-20, 4)$

Vertices $(h \pm a, k) = (-3 \pm 15, 4)$

$A(12, 4) \quad A'(-18, 4)$

Directrix $x = \pm a/e = \frac{15}{17/15} = \frac{225}{17}$

$x + 3 = \pm 225/17$

$\boxed{x = -3 \pm 225/17}$

---

6)

Soln

Let (ae, y_1) is a Point on the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{—— (1)}$$

Since $e^2 = \frac{a^2+b^2}{a^2}$

$(1) \Rightarrow \frac{\cancel{a^2}e^2}{\cancel{a^2}} - \frac{y_1^2}{b^2} = 1$

$\frac{a^2+b^2}{a^2} - \frac{y_1^2}{b^2} = 1$

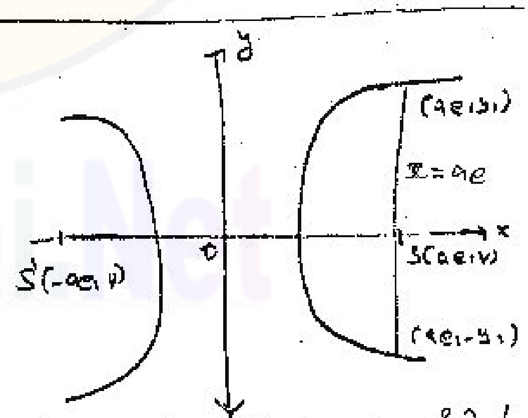
$\frac{a^2}{a^2} + \frac{b^2}{a^2} - 1 = \frac{y_1^2}{b^2}$

$\cancel{1} + b^2/a^2 - \cancel{1} = y_1^2/b^2$

$\frac{b^2}{a^2} = y_1^2/b^2$

$y_1^2 = b^4/a^2$

$\boxed{y_1 = \pm b^2/a}$



The end Points of Latus rectum $(ae, b^2/a), (ae, -b^2/a)$

length of LR $= b^2/a + b^2/a = 2b^2/a$

Hence the Proof.

7) By defn

$\frac{SP}{PM} = e$

$\boxed{SP = e\,PM}$ —(1)

$\frac{S'P}{PM'} = e$

$\boxed{S'P = e\,PM'}$ —(2)

$(1) - (2) \Rightarrow$

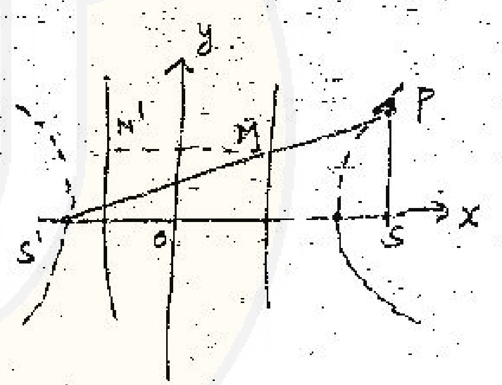
$SP - S'P = ePM - eP'M$

$= e(PM - PM')$

$= e\,MM'$

$= e(2a/e) = 2a$

$SP - S'P = 2a =$ length of the transverse axis



Ex. 5.2 8 — (v)

$$18x^2+12y^2-144x+48y+120=0$$

It is an ellipse

$$18x^2-144x+12y^2+48y=-120$$

$$18(x^2-8x)+12(y^2+4y)=-120$$

$$18(\underbrace{x^2-8x+4^2}-4^2)+12(y^2+4y+2^2-2^2)=-120$$

$$18((x-4)^2-16)+12((y+2)^2-4)=-120$$

$$18(x-4)^2-288+12(y+2)^2-48=-120$$

$$18(x-4)^2+12(y+2)^2=-120+288+48$$

$$18(x-4)^2+12(y+2)^2=216$$

$$\div 216 \Rightarrow \frac{18(x-4)^2}{216}+\frac{12(y+2)^2}{216}=\frac{216}{216}$$

$$\frac{(x-4)^2}{12}+\frac{(y+2)^2}{18}=1$$

Major axis Parallel to y axis.

$$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$$

$h=4$ $k=-2$

$b^2=12$ $a^2=18$

$a=\sqrt{18}=3\sqrt{2}$

$c^2=a^2-b^2=18-12=6$

$c^2=6$ $\boxed{c=\sqrt{6}}$

$$e=c/a=\sqrt{6}/\sqrt{18}=\sqrt{6/18}=1/\sqrt{3}$$

$$\boxed{e=1/\sqrt{3}}$$

$$a/e=\frac{\sqrt{18}}{1/\sqrt{3}}=\frac{\sqrt{18}\times\sqrt{3}}{1}=\sqrt{18\times 3}$$

$$\boxed{a/e=3\sqrt{6}}$$

	$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$	$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$	$\frac{(x-4)^2}{12}+\frac{(y+2)^2}{18}=1$
Centre	$(0,0)$	(h,k)	$C(4,-2)$
Vertices	$(0,\pm a)$	$(h,k\pm a)$	$4,-2\pm\sqrt{18}$; $A(4,-2+\sqrt{18})$, $A'(4,-2-\sqrt{18})$
Foci	$(0,\pm c)$	$(h,k\pm c)$	$4,-2\pm\sqrt{6}$; $S(4,-2+\sqrt{6})$, $S'(4,-2-\sqrt{6})$
Eqn of Directrix	$y=\pm a/e$	$y=k\pm a/e$	$y=-2\pm3\sqrt{6}$; $y=-2+3\sqrt{6}$, $y=-2-3\sqrt{6}$

(vi) $9x^2 - y^2 - 36x - 6y + 18 = 0$

It is a hyperbola

$9x^2 - 36x - y^2 - 6y = -18$

$9(x^2 - 4x) - (y^2 + 6y) = -18$

$9(x^2 - 4x + 2^2 - 2^2) - (y^2 + 6y + 3^2 - 3^2) = -18$

$9((x-2)^2 - 4) - ((y+3)^2 - 9) = -18$

$9(x-2)^2 - 36 - (y+3)^2 + 9 = -18$

$9(x-2)^2 - (y+3)^2 = -18 + 36 - 9$

$9(x-2)^2 - (y+3)^2 = 9$

$\div 9 \Rightarrow \dfrac{9(x-2)^2}{9} - \dfrac{(y+3)^2}{9} = \dfrac{9}{9}$

$\dfrac{(x-2)^2}{1} - \dfrac{(y+3)^2}{9} = 1.$

Transverse axis Parallel to x axis.

$\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = 1$

$h = 2$ $k = -3$

$a^2 = 1$ $b^2 = 9$

$\boxed{a = 1}$ $b = 3$

$c^2 = a^2 + b^2 = 1 + 9 = 10$

$\boxed{c = \sqrt{10}}$

$e = c/a = \sqrt{10}/1$

$\boxed{e = \sqrt{10}}$

$\boxed{a/e = 1/\sqrt{10}}$

	$\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$	$\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = 1$	$\dfrac{(x-2)^2}{1} - \dfrac{(y+3)^2}{9} = 1$
Centre	$C(0,0)$	$C(h,k)$	$C(2,-3)$
vertices	$(\pm a, 0)$	$(h \pm a, k)$	$(2 \pm 1, -3)$ $(3,-3)A$ $(1,-3)A'$
Foci	$(\pm c, 0)$	$(h \pm c, k)$	$(2 \pm \sqrt{10}, -3)$ $S(2+\sqrt{10}, -3)$ $S'(2-\sqrt{10}, -3)$
Directrix eqn	$x = \pm a/e$	$x = h \pm a/e$	$x = 2 \pm 1/\sqrt{10}$ $x = 2 + 1/\sqrt{10}$ $x = 2 - 1/\sqrt{10}$

Ex - 5.2 -- 8 (iv)

iv) $\frac{(y-2)^2}{25} - \frac{(x+1)^2}{16} = 1$

This is hyperbola

Transverse axis parallel to y axis.

$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

$h = -1 \quad k = 2$

$a^2 = 25 \quad b^2 = 16$

$a = 5$

$c^2 = a^2 + b^2 = 25 + 16 = 41$

$c = \sqrt{41}$

$e = c/a = \sqrt{41}/5$

$a/e = \frac{5}{\sqrt{41}/5} = \frac{25}{\sqrt{41}}$.

	$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$	$\frac{(y-2)^2}{25} - \frac{(x+1)^2}{16} = 1$
Centre	$C(0,0)$	(h,k)	$C(-1,2)$
Vertices	$(0, \pm a)$	$(h, k \pm a)$	$(-1, 2\pm 5)$ $(-1, 2+5) = (-1, 7)$ A $(-1, 2-5) = (-1, -3)$ A'
Foci	$(0, \pm c)$	$(h, k \pm c)$	$(-1, 2 \pm \sqrt{41})$ S $(-1, 2+\sqrt{41})$ S' $(-1, 2-\sqrt{41})$
Equation of Directrices	$y = \pm a/e$	$y = k \pm a/e$	$y = 2 \pm \frac{\sqrt{25}}{\sqrt{41}}$ $y = 2 + 25/\sqrt{41}$ $y = 2 - 25/\sqrt{41}$.

EXERCISE : 5.4

1.

$2x^2+7y^2=14$

$\div 14 \Rightarrow \frac{x^2}{7}+\frac{y^2}{2}=1$

$a^2=7 \quad b^2=2$

Any tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is of the form

$y=mx \pm \sqrt{a^2m^2+b^2}$ $\quad (5,2)$ $x \; y$

$2=m(5) \pm \sqrt{7m^2+2}$

$2-5m = \pm\sqrt{7m^2+2}$

Squaring on both side

$(2-5m)^2=(\sqrt{7m^2+2})^2$

$4-20m+25m^2=7m^2+2$

$18m^2-20m+2=0$

$\div 2 \Rightarrow 9m^2-10m+1=0$

$(9m-1)(m-1)=0$

$m=1/9$ (or) $m=1$

Tangent equation

when $m=1 \quad (x_1,y_1)=(5,2)$

$y-y_1=m(x-x_1)$

$y-2=1(x-5)$

$\boxed{x-y-3=0}$

when $m=1/9 \quad (x_1,y_1)=(5,2)$

$y-2=1/9(x-5)$

$9y-18=x-5$

$\boxed{x-9y+13=0}$

2) $\frac{x^2}{16}-\frac{y^2}{64}=1$, It is hyperbola

$a^2=16 \quad b^2=64$

slope of the tangent $m=-10/-3$

$\boxed{m=10/3}$

Any tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is of the form

$y=mx \pm \sqrt{a^2m^2-b^2}$

$y=\frac{10}{3}x \pm \sqrt{16\left(\frac{100}{9}\right)-64}$

$y=\frac{10}{3}x \pm \sqrt{\frac{1024}{9}}$

$y=\frac{10}{3}x \pm \frac{32}{3}$

$3y=10x \pm 32$

$10x-3y \pm 32=0$

Tangent eqns are

$10x-3y+32=0$

$10x-3y-32=0$

3) $x-y+4=0$

$y=x+4$

$y=mx+c$

$m=1 \quad c=4$

$x^2+3y^2=12$

$\div 12 \Rightarrow \frac{x^2}{12}+\frac{y^2}{4}=1$

$a^2=12 \quad b^2=4$

Condition for tangency

$c^2=a^2m^2+b^2$

$c^2=16 \quad a^2m^2+b^2=12(1)^2+4$

$=12+4=16$

The line is a tangent to the ellipse.

Point of contact

$\left(\frac{-a^2m}{c},\frac{b^2}{c}\right)=\left(\frac{-12(1)}{4},\frac{4}{4}\right)$

$=(-3,1)$

4) $y^2 = 16x$

$4a = 16$

$a = 4$

Any line $\perp^{er}$ to $2x+2y+3=0$ is

$2x - 2y + k = 0$

$2x - 2y + k = 0$

$x - y + k/2 = 0$ --- ①

$y = x + k/2$

$m = 1 \quad c = k/2$

The condition for the line $y = mx + c$ be a tangent to the Parabola $y^2 = 4ax$ is $c = a/m$

$c = 4/1$

$k/2 = 4 \Rightarrow \boxed{k = 8}$

$k = 8$ in ①

$x - y + 4 = 0$

Hence the Required eqn.

5) $y^2 = 8x$

$4a = 8 \Rightarrow a = 2$

Here $t = 2 \quad a = 2$

The equation of tangent at 't' on the Parabola is

$yt = x + at^2$

$y(2) = x + 2(4)$

$2y = x + 8$

$\boxed{x - 2y + 8 = 0}$

6) $12x^2 - 9y^2 = 108$

$\div 108 \Rightarrow \frac{x^2}{9} - \frac{y^2}{12} = 1$

$a^2 = 9 \quad b^2 = 12$

$a = 3 \quad b = 2\sqrt{3}$

Tangent equation at (θ)

$\frac{x \sec\theta}{a} - \frac{y \tan\theta}{b} = 1$

$\theta = \pi/3$

$\frac{x \sec \pi/3}{3} - \frac{y \tan \pi/3}{2\sqrt{3}} = 1$

$\frac{2x}{3} - \frac{\sqrt{3}\,y}{2\sqrt{3}} = 1$

$\frac{2x}{3} - \frac{y}{2} = 1$

$\boxed{4x - 3y - 6 = 0}$

Normal equation at 'θ'

$\frac{ax}{\sec\theta} + \frac{by}{\tan\theta} = a^2 + b^2$

$\theta = \pi/3$

$\frac{3x}{2} + \frac{2\sqrt{3}\,y}{\sqrt{3}} = 9 + 12$

$\frac{3x}{2} + 2y = 21$

$\boxed{3x + 4y - 42 = 0}$

7) tangent eqn at t_1 is

$yt_1 = x + at_1^2$ —— ①

as t_2 is

$yt_2 = x + at_2^2$ —— ②

① − ② ⇒

$y(t_1 - t_2) = a(t_1^2 - t_2^2)$

$y = \frac{a(t_1+t_2)(t_1-t_2)}{(t_1-t_2)}$

$\boxed{y = a(t_1+t_2)}$

Using $y = a(t_1+t_2)$ in ① we get

$a(t_1+t_2)t_1 = x + at_1^2$

$at_1^2 + at_1t_2 = x + at_1^2$

$x = at_1t_2$

The Point of intersection is

$at_1t_2,\ a(t_1+t_2)$

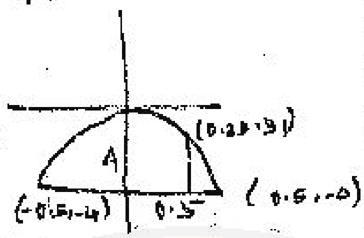
3) Eqn of the Parabola is

$x^2 = -4ay$ —— ①

It Passes through $(0.5, -4)$

$(0.5)^2 = -4a(-4)$

$a = \frac{0.25}{16}$



① ⇒ $x^2 = -4\left[\frac{0.25}{16}\right]y$

It Passes Through $(0.25, -y_1)$

$(0.25)^2 = -4\left[\frac{0.25}{16} \times (-y_1)\right]$

$y_1 = \frac{4 \times (0.25)^2}{0.25} = 4 \times 0.25$

$\boxed{y_1 = 1\,m}$

The Required height $= 4 - y_1 = 4 - 1 = 3m$

4) Eqn of Parabola is

(a) $y^2 = 4ax$

$a = 1.2$

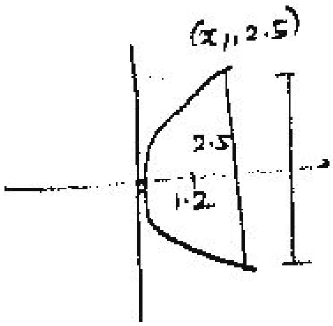
$y^2 = 4.8x$

b) It Passes Through $(x, 2.5)$

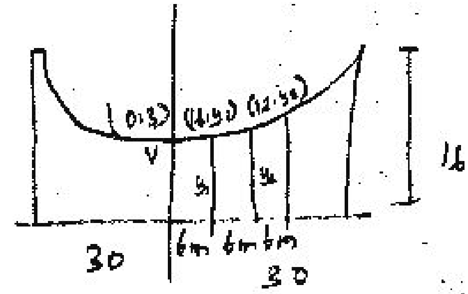
$(2.5)^2 = 4.8(x_1)$

$x_1 = \frac{2.5 \times 2.5}{4.8} = 1.3\,m$

The depth of the satelite dish at the vertex 1.2 m



5)



Eqn of the Parabola is

$(x-h)^2 = 4a(y-k)$

$(x-0)^2 = 4a(y-3)$

It Passes through $(30, 16)$

$30^2 = 4a(16-3)$

$a = \frac{30 \times 30}{13 \times 4}$

$x^2 = 4\left(\frac{30 \times 30}{13 \times 4}\right)(y-3)$

$x^2 = \frac{30 \times 30}{13}(y-3)$

It Passes Through $(6, y_1)$

$6^2 = \frac{900}{13}(y_1 - 3)$

$y_1 - 3 = \frac{36 \times 13}{900} = 0.52$

$y_1 = 3 + 0.52 = 3.52\,m$

It also Passes through $(12, y_2)$

$12^2 = \frac{900}{13}(y_2 - 3)$

$y_2 - 3 = \frac{144 \times 13}{900} = 2.08$

$y_2 = 3 + 2.08$

$\boxed{y_2 = 5.08\,m}$

8)

Normal equation of a Parabola at $P(at_1^2, 2at_1)$ is

$y + xt_1 = at_1^3 + 2at_1$

It meets the Parabola again at $Q(at_2^2, 2at_2)$

$2at_2 + (at_2^2)t_1 = at_1^3 + 2at_1$

$2at_2 + at_1t_2^2 = at_1^3 + 2at_1$

$2at_2 - 2at_1 = at_1^3 - at_1t_2^2$

$2a(t_2 - t_1) = at_1(t_1^2 - t_2^2)$

$2a(t_2 - t_1) = at_1(t_1 + t_2)(t_1 - t_2)$

$2\cancel{a}\cancel{(t_2 - t_1)} = -\cancel{a}t_1\cancel{(t_2 - t_1)}(t_1 + t_2)$

$-2 = t_1(t_1 + t_2)$

$t_1 + t_2 = -2/t_1$

$t_2 = -2/t_1 - t_1$

$t_2 = -2/t_1 - t_1$

$t_2 = -(t_1 + 2/t_1)$

Hence Proved.

EXERCISE 5.5

1. Eqn of Parabola is

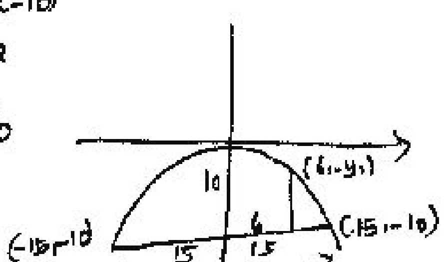
$x^2 = -4ay$

It Passes through $(15, -10)$

$15^2 = -4a(-10)$

$225 = 40a$

$a = 225/40$



① ⇒ $x^2 = -4\left(\frac{225}{40}\right)y$

It Passes through $(6, -y_1)$

$36 = -4\left(\frac{225}{40}\right)(-y_1)$

$\frac{36 \times 10}{225} = y_1$

$1.6 = y_1$

The required height $= 10 - y_1$

$= 10 - 1.6$

$= 8.4\text{ m}$

2)

$CA = a = 8$ $\quad CB = b = 5$

Eqn of ellipse is

$x^2/b^2 + y^2/a^2 = 1$

$x^2/25 + y^2/64 = 1$

It is passes through $(4, y_1)$

$\frac{16}{25} + \frac{y_1^2}{64} = 1$

$\frac{y_1^2}{64} = 1 - 16/25$

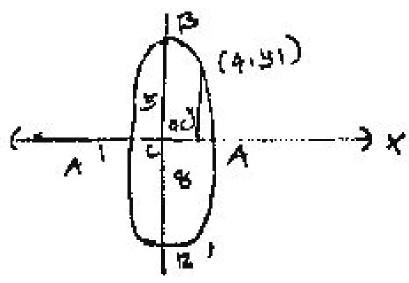
$y_1^2 = 64\left(9/25\right)$

$y_1 = 8 \times 3/5 = 24/5$

$\boxed{y_1 = 4.8\text{ m}}$

The Required wide for the opening is $2y = 2(4.8)$

$= 9.6\text{ m}$



6)
$3k = 150$

$k = 50$

$(x_1, 50)$ lies on the Parabola

$$\frac{x_1^2}{30^2} - \frac{50^2}{44^2} = 1$$

$$\frac{x_1^2}{30^2} = 1 + \frac{50^2}{44^2}$$

$$x_1^2 = \frac{30^2}{44^2}\left[1936 + 2500\right]$$

$$x_1 = \frac{30}{44}\sqrt{4436}$$

$$= \frac{30}{44}(66.60)$$

$$\boxed{x_1 = 45.40}$$

Radius of the top of the tower is 45.40m.

$(x_2, -100)$ lies on the hyperbola

$$\frac{x_2^2}{30^2} - \frac{100^2}{44^2} = 1$$

$$\frac{x_2^2}{30^2} = 1 + \frac{100^2}{44^2}$$

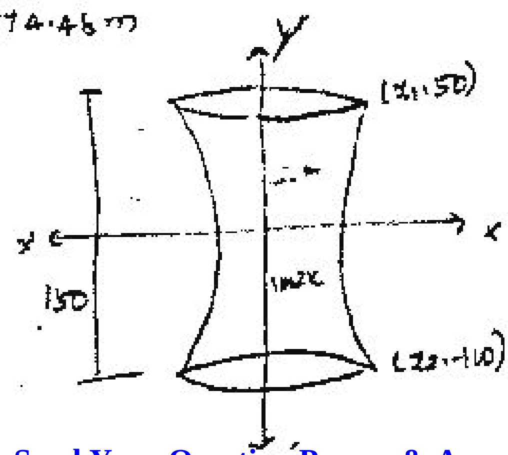
$$x_2^2 = \frac{30^2}{44^2}(1936 + 10000)$$

$$x_2 = \frac{30}{44}\sqrt{11936}$$

$$= \frac{30}{44} \times 109.25$$

$$x_2 = \frac{3277.5}{44} = 74.46\,m$$

Radius of the base of the tower is 74.46m



7)
$\triangle BAP \cong \triangle PCD$

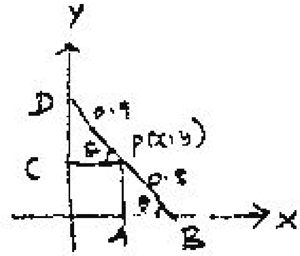
Let $\angle ABP = \angle CPD = \theta$

In $\triangle PAB$

$$\sin\theta = \frac{y}{0.3}$$

In $\triangle PCD$

$$\cos\theta = \frac{x}{0.9}$$



$$\sin^2\theta + \cos^2\theta = 1$$

$$\frac{x^2}{0.9^2} + \frac{y^2}{(0.3)^2} = 1$$

$$\frac{x^2}{(0.9)^2} + \frac{y^2}{(0.3)^2} = 1$$

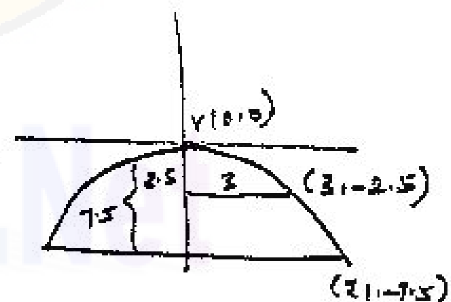
$$e = \sqrt{1 - b^2/a^2} = \sqrt{1 - \frac{0.3^2}{0.9^2}}$$

$$= \sqrt{1 - (1/3)^2} = \sqrt{1 - 1/9}$$

$$e = \sqrt{8/9}$$

$$\boxed{e = \frac{2\sqrt{2}}{3}\,m}$$

8)



The equation of the Parabola is

$$x^2 = -4ay$$

It Passes Through $(3, -2.5)$

$$3^2 = -4a(-2.5)$$

$$\boxed{a = 9/10}$$

$x^2 = -4\left(\frac{9}{10}\right)y$

$x^2 = -16/5\, y$

It Passes through $(x_1, -7.5)$

$x_1^2 = -16/5\,(-7.5)$

$x_1^2 = 27$

$x_1 = 3\sqrt{3}$ m

The water strikes the ground $3\sqrt{3}$ m beyond the vertical line.

9) The equation of the Parabola is $x^2 = -4ay$

It Passes through $(6, -4)$

$36 = 16a$

$a = 36/16 = 9/4$

$\boxed{a = 9/4}$

$x^2 = -4(9/4)y$

$\boxed{x^2 = -9y}$

Diff w.r.to x

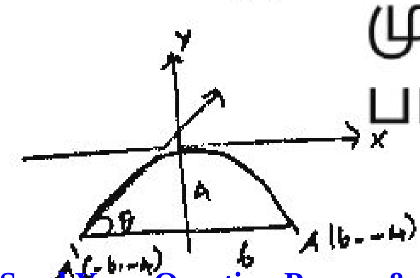
$2x = -9\, dy/dx$

$\frac{dy}{dx} = -2x/9$

at $(-6, -4)$

$\tan\theta = \frac{dy}{dx} = \frac{-2}{9}(-6) = 4/3$

$\boxed{\theta = \tan^{-1} 4/3}$



10)

$ae = 5$

$AP \sim BP = 2a = 6$

$\boxed{a = 3}$

$b^2 = a^2(e^2-1) = (ae)^2 - a^2$

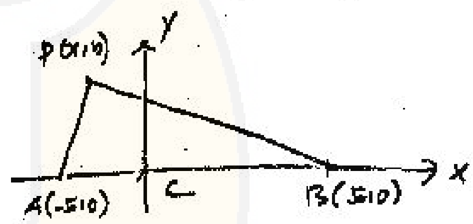
$= 25 - 9$

$\boxed{b^2 = 16}$

The equation of the required hyperbola

$\frac{x^2}{9} - \frac{y^2}{16} = 1$

The locus of p is hyperbola



பிழைகள் இருப்பின் மன்னிக்கவும்.

ரெ. விஜய...

17/6/19

ரெ. விஜயராகவன்,
முதுகலை ஆசிரியர்-கணிதம்,
பட்டுக்கோட்டை.

CHAPTER – VI

VECTOR ALGEBRA

EXERCISE – 6.1

1) Let $\vec{OA} = \vec{a}$, $\vec{OB} = \vec{b}$

$\vec{OD} = \frac{\vec{a}+\vec{b}}{2}$

$|\vec{OA}| = |\vec{OB}|$

$|\vec{a}| = |\vec{b}|$

To prove. $\vec{OD} \perp \vec{AB}$ (ie) $\vec{OD} \cdot \vec{AB} = 0$

$\vec{OD} \cdot \vec{AB} = \vec{OD} \cdot (\vec{OB} - \vec{OA})$

$= \left(\frac{\vec{a}+\vec{b}}{2}\right) \cdot (\vec{b} - \vec{a})$

$= \frac{1}{2}(\vec{b}+\vec{a}) \cdot (\vec{b} - \vec{a})$

$= \frac{1}{2}\left(|\vec{b}|^2 - |\vec{a}|^2\right)$ $\left[|\vec{a}| = |\vec{b}|\right]$

$= \frac{1}{2}\left(|\vec{b}|^2 - |\vec{b}|^2\right) = 0$

$= 0$

$\therefore \vec{OD} \cdot \vec{AB} = 0$ Thus

$\vec{OD} \perp \vec{AB}$ Hence Proved.

2. Let ABC be an isosceles Triangle with AB = AC

Let D be the midpoint of BC.

$|\vec{AB}|^2 = |\vec{AC}|^2$

$|\vec{AD} + \vec{DB}|^2 = |\vec{AD} + \vec{DC}|^2$ $\left[\vec{DC} = -\vec{DB}\right]$

$|\vec{AD} + \vec{DB}|^2 = |\vec{AD} - \vec{DB}|^2$

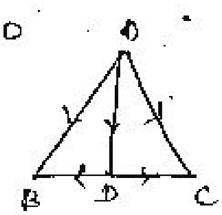
$\cancel{\vec{AD}^2} + \cancel{\vec{DB}^2} + 2\vec{AD} \cdot \vec{DB} = \cancel{\vec{AD}^2} + \cancel{\vec{DB}^2} - 2\vec{AD} \cdot \vec{DB}$

$2\vec{AD} \cdot \vec{DB} + 2\vec{AD} \cdot \vec{DB} = 0$

$4\vec{AD} \cdot \vec{DB} = 0$

$\vec{AD} \cdot \vec{DB} = 0$

$\vec{AD} \perp \vec{DB}$



Iiiy $\vec{AD} \perp \vec{DC}$

$\therefore$ median to the base of an isosceles triangle is perpendicular to the base.

3) Let AB be the diameter of the circle with centre at O

Let P be any point on the circle.

To Prove $\angle APB = 90^\circ$

We know That

$OA = OB = OP$ (radii)

Now $\vec{PA} = \vec{PO} + \vec{OA}$

$\vec{PB} = \vec{PO} + \vec{OB}$

$= \vec{PO} - \vec{OA}$ $\left[\vec{OB} = -\vec{OA}\right]$

$\vec{PA} \cdot \vec{PB} = (\vec{PO} + \vec{OA}) \cdot (\vec{PO} - \vec{OA})$

$= (\vec{PO})^2 - (\vec{OA})^2$

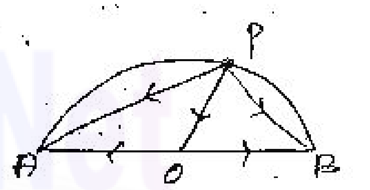
$= (PO)^2 - (OA)^2$

$= 0$ $[PO = OA]$

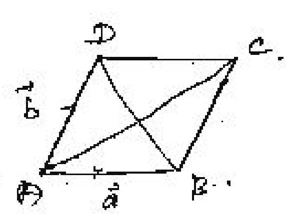
$\vec{PA} \cdot \vec{PB} = 0$

$\therefore \vec{PA}$ is $\perp$ to $\vec{PB}$

$\therefore \angle APB = 90^\circ$



4)



Let ABCD be the Rhombus

Let $\vec{AB} = \vec{a}$

$\vec{AD} = \vec{b}$

We have $AB = BC = CD = DA$

$\therefore |\vec{a}| = |\vec{b}|$ —— (1)

Now

$\vec{AC} = \vec{AB} + \vec{BC}$

$= \vec{a} + \vec{b}$

$\vec{BD} = \vec{BA} + \vec{AD}$

$= \vec{b} - \vec{a}$

$\therefore \vec{AC} \cdot \vec{BD} = (\vec{a}+\vec{b}) \cdot (\vec{b} - \vec{a})$

$= (\vec{b}+\vec{a}) \cdot (\vec{b} - \vec{a})$

$= (\vec{b})^2 - (\vec{a})^2$

$= 0 \quad (\because |\vec{a}| = |\vec{b}|)$

Thus $\vec{AC} \cdot \vec{BD} = 0$

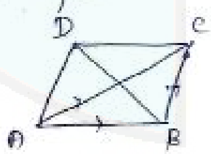
(ie) $\vec{AC} \perp \vec{BD}$

Hence the diagonals of a rhombus are at right angles.

5)

Let ABCD be a Parallelogram.

To Prove ABCD is a rectangle if the diagonals are equal.



Now. $\vec{AC} = \vec{AB} + \vec{BC}$

$\vec{BD} = \vec{BC} + \vec{CD}$

$= \vec{BC} - \vec{AB}$

But $(\vec{AC})^2 = (\vec{BD})^2$

$(\vec{AB} + \vec{BC})^2 = (\vec{BC} - \vec{AB})^2$

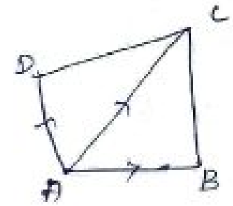
$\cancel{\vec{AB}^2} + \cancel{\vec{BC}^2} + 2\,\vec{AB} \cdot \vec{BC} = \cancel{(\vec{BC})^2} + \cancel{(\vec{AB})^2} - 2\vec{AB} \cdot \vec{BC}$

$4\,\vec{AB} \cdot \vec{BC} = 0$

(ie) $\vec{AB} \perp \vec{BC}$

(ie) ABCD is a rectangle.

6)



vector area of quadrilateral ABCD = Vector area of ΔABC + Vector area of ΔACD

$= \frac{1}{2}(\vec{AB} \times \vec{AC}) + \frac{1}{2}(\vec{AC} \times \vec{AD})$

$= -\frac{1}{2}(\vec{AC} \times \vec{AB}) + \frac{1}{2}(\vec{AC} \times \vec{AD})$

$= \frac{1}{2}(\vec{AC}) \times (-\vec{AB} + \vec{AD})$

$= \frac{1}{2}(\vec{AC}) \times (\vec{BA} + \vec{AD})$

$= \frac{1}{2}|\vec{AC} \times \vec{BD}|$

vector area of quadrilateral $ABCD = \frac{1}{2}|\vec{AC} \times \vec{BD}|$.

7)

Let ABCD and ABC'D' be two Parallelograms on the same base AB and between the Same Parallel lines

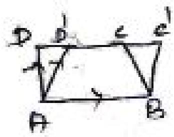
To Prove

Area of ABCD = Area of ABC'D'

Area of $ABC'D' = |\vec{AB} \times \vec{AD'}|$

$= |\vec{AB} \times (\vec{AD} + \vec{DD'})|$

$= |(\vec{AB} \times \vec{AD}) + (\vec{AB} \times \vec{DD'})|$

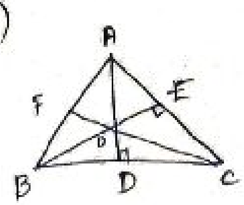


$= |(\vec{AB} \times \vec{AD})| + 0$

$= |\vec{AB} \times \vec{AD}|$

= Area of ABCD

8)



Area of $\Delta GAB = \frac{1}{2}(\vec{AB} \times \vec{AG})$

$= \frac{1}{2}\left[(\vec{OB} - \vec{OA}) \times (\vec{OG} - \vec{OA})\right]$

$= \frac{1}{2}\left[(\vec{b} - \vec{a}) \times \left(\frac{\vec{a}+\vec{b}+\vec{c}}{3} - \vec{a}\right)\right]$

$= \frac{1}{2}\left[(\vec{b} - \vec{a}) \times \left(\frac{\vec{a}+\vec{b}+\vec{c}-3\vec{a}}{3}\right)\right]$

$= \frac{1}{2}\left[(\vec{b} - \vec{a}) \times \left(\frac{\vec{b}+\vec{c}-2\vec{a}}{3}\right)\right]$

$= \frac{1}{6}\left[\vec{b}\times\vec{b} + \vec{b}\times\vec{c} - \vec{b}\times 2\vec{a} - \vec{a}\times\vec{b} - \vec{a}\times\vec{c} + \vec{a}\times 2\vec{a}\right]$

$= \frac{1}{6}\left[\vec{b}\times\vec{c} - 2\vec{b}\times\vec{a} - \vec{a}\times\vec{b} - \vec{a}\times\vec{c}\right]$

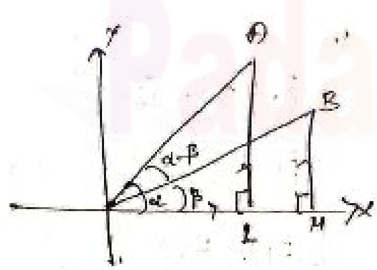
$= \frac{1}{6}\left[\vec{b}\times\vec{c} + 2\vec{a}\times\vec{b} - \vec{a}\times\vec{b} + \vec{c}\times\vec{a}\right]$

$= \frac{1}{6}\left[\vec{b}\times\vec{c} + \vec{a}\times\vec{b} + \vec{c}\times\vec{a}\right]$

$= \frac{1}{3} \times \frac{1}{2}\left[\vec{b}\times\vec{c} + \vec{a}\times\vec{b} + \vec{c}\times\vec{a}\right]$

$= \frac{1}{3}$ [area of ΔABC]

9)



Take two Points A and B on the unit circle with Centre as origin O.

So $|\vec{OA}| = |\vec{OB}| = 1$

$\angle AOX = \alpha$ and $\angle BOX = \beta$

$\angle AOB = \alpha - \beta$

Let $\hat{i}$ and $\hat{j}$ be the unit vectors along x, y axis.

Draw AL and BM Perpendicular to x axis

Then A $(\cos\alpha, \sin\alpha)$ and B$(\cos\beta, \sin\beta)$ are the Co-ordinates.

$\vec{OA} = \vec{OL} + \vec{LA}$

$= \cos\alpha\,\hat{i} + \sin\alpha\,\hat{j}$

$\vec{OB} = \vec{OM} + \vec{MB}$

$= \cos\beta\,\hat{i} + \sin\beta\,\hat{j}$

By defn

$\vec{OA}\cdot\vec{OB} = |\vec{OA}||\vec{OB}|\cos(\alpha-\beta)$

$= 1(1)\cos(\alpha-\beta)$

$\vec{OA}\cdot\vec{OB} = \cos(\alpha-\beta)$ —— (1)

$\vec{OA}\cdot\vec{OB} = (\cos\alpha\,\hat{i} + \sin\alpha\,\hat{j})\cdot(\cos\beta\,\hat{i} + \sin\beta\,\hat{j})$

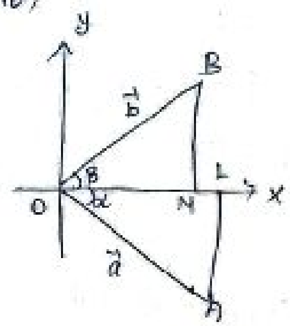
$= \cos\alpha\cos\beta + \sin\alpha\sin\beta$ —— (2)

From (1) & (2)

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

Hence Proved.

10)



Take two points A and B on the unit circle with centre as origin O.

So $|\vec{OA}| = |\vec{OB}| = 1$

$\angle AOX = \alpha$ and $\angle BOX = \beta$

$\angle AOB = \alpha + \beta$

Let $\hat{i}$ and $\hat{j}$ be the unit vectors along x, y axis.

Draw AL and BM perpendicular to x axis.

Then $A(\cos\alpha, -\sin\alpha)$ and $B(\cos\beta, \sin\beta)$ are the co-ordinates.

$\vec{OA} = \vec{OL} + \vec{LA}$

$= \cos\alpha \hat{i} - \sin\alpha \hat{j}$

$\vec{OB} = \vec{ON} + \vec{MB}$

$= \cos\beta \hat{i} + \sin\beta \hat{j}$

By defn

$\vec{OB} \times \vec{OA} = |\vec{OB}||\vec{OA}| \sin(\alpha+\beta)\hat{k}$

$= 1(1) \sin(\alpha+\beta)\hat{k}$

$\vec{OB} \times \vec{OA} = \sin(\alpha+\beta)\hat{k}$ —— (1)

By value

$$\vec{OB} \times \vec{OA} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \cos\beta & \sin\beta & 0 \\ \cos\alpha & -\sin\alpha & 0 \end{vmatrix}$$

$= \pm \hat{k} (\sin\alpha\cos\beta + \cos\alpha\sin\beta)$ —— (2)

From (1) & (2)

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

Hence proved.

11) Given

$\vec{F_1} = 8\hat{i} + 2\hat{j} - 6\hat{k}$

$\vec{F_2} = 6\hat{i} + 2\hat{j} - 2\hat{k}$

Resultant Force $\vec{F} = \vec{F_1} + \vec{F_2}$

$\vec{F} = 14\hat{i} + 4\hat{j} - 8\hat{k}$

Let A and B be the points (1,2,3) and (5,4,1)

displacement

$\vec{d} = \vec{OB} - \vec{OA}$

$= (5\hat{i} + 4\hat{j} + \hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k})$

$\vec{d} = 4\hat{i} + 2\hat{j} - 2\hat{k}$

$\boxed{\text{Workdone} = \vec{F} \cdot \vec{d}}$

$= (14\hat{i} + 4\hat{j} - 8\hat{k}) \cdot (4\hat{i} + 2\hat{j} - 2\hat{k})$

$= 56 + 8 + 16$

$= 80$

$\boxed{\text{Workdone } W = 80 \text{ units}}$

12)

$\vec{F_1} = 3\hat{i} + 4\hat{j} + 5\hat{k}$

$|\vec{F_1}| = \sqrt{3^2 + 4^2 + 5^2} = \sqrt{50}$

$\boxed{|\vec{F_1}| = 5\sqrt{2}}$

Unit Force $\hat{F_1} = \dfrac{\vec{F_1}}{|\vec{F_1}|} =$

$5\sqrt{2}\,\hat{F_1} = 5\sqrt{2}\left(\dfrac{3\hat{i} + 4\hat{j} + 5\hat{k}}{5\sqrt{2}}\right)$

First Force $= 3\vec{i}+4\vec{j}+5\vec{k}$

$\vec{F_2} = 10\vec{i}+6\vec{j}-8\vec{k}$

$|\vec{F_2}| = \sqrt{10^2+6^2+8^2} = \sqrt{200}$

$\boxed{|\vec{F_2}| = 10\sqrt{2}}$

Unit Force $\hat{F_2} = \dfrac{\vec{F_2}}{|\vec{F_2}|}$

$10\sqrt{2}\ \hat{F_2} = \left(\dfrac{10\vec{i}+6\vec{j}-8\vec{k}}{10\sqrt{2}}\right)10\sqrt{2}$

Second Force $= 10\vec{i}+6\vec{j}-8\vec{k}$

Resultant Force

$\vec{F} = 3\vec{i}+4\vec{j}+5\vec{k}+10\vec{i}+6\vec{j}-8\vec{k}$

$\vec{F} = 13\vec{i}+10\vec{j}-3\vec{k}$

Then the displacement

$\vec{d} = \vec{OB}-\vec{OA}$

$= (6\vec{i}+\vec{j}-3\vec{k})-(4\vec{i}-3\vec{j}-2\vec{k})$

$= 6\vec{i}+\vec{j}-3\vec{k}-4\vec{i}+3\vec{j}+2\vec{k}$

$\boxed{\vec{d} = 2\vec{i}+4\vec{j}-\vec{k}}$

Work done $W = \vec{F}\cdot\vec{d}$

$= (13\vec{i}+10\vec{j}-3\vec{k})\cdot(2\vec{i}+4\vec{j}-\vec{k})$

$= 26+40+3$

$\boxed{W = 69 \text{ units}}$

13) $\vec{F} = 3\vec{i}+4\vec{j}-5\vec{k}$

$\vec{r} =$ (through the point $-$ about the point)

$\vec{r} = 4\vec{i}+2\vec{j}-3\vec{k}-(2\vec{i}-3\vec{j}+4\vec{k})$

$\boxed{\vec{r} = 2\vec{i}+5\vec{j}-7\vec{k}}$

torque $= \vec{t} = \vec{r}\times\vec{F}$

$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 5 & 7 \\ 3 & 4 & -5 \end{vmatrix}$

$= \vec{i}(-25+28)-\vec{j}(-10+21)+\vec{k}(8-15)$

$= 3\vec{i}-11\vec{j}-7\vec{k}$

Magnitude $|\vec{t}| = \sqrt{9+121+49}$

$= \sqrt{179}$

direction cosines are

$\dfrac{3}{\sqrt{179}}, \dfrac{-11}{\sqrt{179}}, \dfrac{-7}{\sqrt{179}}$.

14)

$\vec{F_1} = -3\vec{i}+6\vec{j}-3\vec{k}$

$\vec{F_2} = 4\vec{i}-10\vec{j}+12\vec{k}$

Force $\vec{F_3} = 4\vec{i}+7\vec{j}$

Resultant force $\vec{F} = \vec{F_1}+\vec{F_2}+\vec{F_3}$

$= 5\vec{i}+3\vec{j}+9\vec{k}$

$\vec{r}$ = (at the point — about the point)

$\vec{r} = (8\vec{i}-6\vec{j}-4\vec{k}) - (18\vec{i}+3\vec{j}-9\vec{k})$

$\vec{r} = -10\vec{i}-9\vec{j}+5\vec{k}$

torque $= \vec{r}\times\vec{F}$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -10 & -9 & 5 \\ 5 & 3 & 9 \end{vmatrix}$$

$= \vec{i}(-81-15)-\vec{j}(-90-25)+\vec{k}(-30+45)$

$= \vec{i}(-96)-\vec{j}(-115)+\vec{k}(15)$

$\boxed{\vec{t} = -96\vec{i}+115\vec{j}+15\vec{k}}$

EXERCISE – 6.2

1) $\vec{a} = \vec{i}-2\vec{j}+3\vec{k}$

$\vec{b} = 2\vec{i}+\vec{j}-2\vec{k}$

$\vec{c} = 3\vec{i}+2\vec{j}+\vec{k}$

$$\vec{a}\cdot(\vec{b}\times\vec{c}) = \begin{vmatrix} 1 & -2 & 3 \\ 2 & 1 & -2 \\ 3 & 2 & 1 \end{vmatrix}$$

$= 1(1+4)+2(2+6)+3(4-3)$

$= 5+16+3$

$= 24$

$\boxed{\vec{a}\cdot(\vec{b}\times\vec{c}) = 24}$

2)

$\vec{a} = -6\vec{i}+14\vec{j}+10\vec{k}$

$\vec{b} = 14\vec{i}-10\vec{j}-6\vec{k}$

$\vec{c} = 2\vec{i}+4\vec{j}-2\vec{k}$

Volume of the Parallelopiped $= [\vec{a}\ \vec{b}\ \vec{c}]$

$$= \begin{vmatrix} -6 & 14 & 10 \\ 14 & -10 & -6 \\ 2 & 4 & -2 \end{vmatrix}$$

$$= (2)(2)(2)\begin{vmatrix} -3 & 7 & 5 \\ 7 & -5 & -3 \\ 1 & 2 & -1 \end{vmatrix}$$

$= 8\{-3(5+6)-7(-7+3)+5(14+5)\}$

$= 8\{-33+28+95\}$

$= 8(90)$

$\boxed{\text{Volume} = 720 \text{ cu. units}}$

3)

$\vec{a} = 7\vec{i}+\lambda\vec{j}-3\vec{k}$

$\vec{b} = \vec{i}+2\vec{j}-\vec{k}$

$\vec{c} = -3\vec{i}+7\vec{j}+5\vec{k}$

Volume of a Parallelopiped

$[\vec{a}\ \vec{b}\ \vec{c}] = 90$ cu. units

$$\begin{vmatrix} 7 & \lambda & -3 \\ 1 & 2 & -1 \\ -3 & 7 & 5 \end{vmatrix} = 90$$

$7(10+7)-\lambda(5-3)-3(7+6) = 90$

$119-2\lambda-39 = 90$

$-2\lambda = 90+39-119$

$-2\lambda = 10$

$\boxed{\lambda = -5}$

4)

volume of a Parallelopiped

$[\vec{a}\ \vec{b}\ \vec{c}] = \pm 4$

$(\vec{a}+\vec{b})\cdot(\vec{b}\times\vec{c}) + (\vec{b}+\vec{c})\cdot(\vec{c}\times\vec{a}) + (\vec{c}+\vec{a})\cdot(\vec{a}\times\vec{b})$

$= \vec{a}\cdot(\vec{b}\times\vec{c}) + \vec{b}\cdot(\vec{b}\times\vec{c}) + \vec{b}\cdot(\vec{c}\times\vec{a}) + \vec{c}\cdot(\vec{c}\times\vec{a}) + \vec{c}\cdot(\vec{a}\times\vec{b}) + \vec{a}\cdot(\vec{a}\times\vec{b})$

$= [\vec{a}\ \vec{b}\ \vec{c}] + 0 + [\vec{a}\ \vec{b}\ \vec{c}] + 0 + [\vec{a}\ \vec{b}\ \vec{c}] + 0$

$= 3[\vec{a}\ \vec{b}\ \vec{c}] = 3(\pm 4) = \pm 12$.

5) $\vec{a} = -2\vec{i}+5\vec{j}+3\vec{k}$

$\vec{b} = \vec{i}+3\vec{j}-2\vec{k}$

$\vec{c} = -3\vec{i}+\vec{j}+4\vec{k}$

volume of the Parallelopiped

$$[\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} -2 & 5 & 3 \\ 1 & 3 & -2 \\ -3 & 1 & 4 \end{vmatrix}$$

$= 12$

Vector area of the base Parallelogram $= |\vec{b}\times\vec{c}|$

$$\vec{b}\times\vec{c} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 3 & -2 \\ -3 & 1 & 4 \end{vmatrix}$$

$= 14\vec{i}+2\vec{j}+10\vec{k}$

Area of the Parallelogram $= |\vec{b}\times\vec{c}|$

$|\vec{b}\times\vec{c}| = \sqrt{14^2+2^2+10^2} = \sqrt{300} = 10\sqrt{3}$

Volume of the Parallelopiped $=$ base area $\times$ height (altitude)

$12 = 10\sqrt{3} \times$ altitude

altitude $= \frac{12}{10\sqrt{3}} = \frac{6}{5\sqrt{3}}$

$= \frac{6}{5\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{2\sqrt{3}}{5}$

6) Let $\vec{a} = 2\vec{i}+3\vec{j}+\vec{k}$

$\vec{b} = \vec{i}-2\vec{j}+2\vec{k}$

$\vec{c} = 3\vec{i}+\vec{j}+3\vec{k}$

$$[\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} 2 & 3 & 1 \\ 1 & -2 & 2 \\ 3 & 1 & 3 \end{vmatrix}$$

$= 2(-6-2)-3(3-6)+1(1+6)$

$= -16+9+7 = 0$

$[\vec{a}\ \vec{b}\ \vec{c}] = 0$

$\vec{a}, \vec{b}, \vec{c}$ are Coplanar.

7)

$c_1 = 1 \quad c_2 = 2$

$\vec{a} = \vec{i}+\vec{j}+\vec{k}$

$\vec{b} = \vec{i}$

$\vec{c} = c_1\vec{i}+c_2\vec{j}+c_3\vec{k}$

Since $\vec{a}, \vec{b}, \vec{c}$ are Coplanar

$[\vec{a}\ \vec{b}\ \vec{c}] = 0$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0$$

$-1(c_3-c_2) = 0$

$-c_3+c_2 = 0$

$c_3 = c_2 = 2$

$\boxed{c_3 = 2}$

8) $\vec{a} = \hat{i} - \hat{k}$

$\vec{b} = x\hat{i} + \hat{j} + (1-x)\hat{k}$

$\vec{c} = y\hat{i} + x\hat{j} + (1+x-y)\hat{k}$

$$[\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} 1 & 0 & -1 \\ x & 1 & 1-x \\ y & x & 1+x-y \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 0 & -1 \\ 0 & 1 & 1-x \\ 1 & x & 1+x-y \end{vmatrix} \quad C_1 \to C_1 - C_2 + C_3$$

$= 0 - 0 - 1(0-1) = 1$

$[\vec{a}\ \vec{b}\ \vec{c}] = 1$

$[\vec{a}\ \vec{b}\ \vec{c}]$ depends on neither x nor y.

9) Let $\vec{p} = a\hat{i} + a\hat{j} + c\hat{k}$

$\vec{q} = \hat{i} + \hat{k}$

$\vec{r} = c\hat{i} + c\hat{j} + b\hat{k}$

Since $\vec{p}$, $\vec{q}$ and $\vec{r}$ are Coplanar

$[\vec{p}\ \vec{q}\ \vec{r}] = 0$

$$\begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix} = 0$$

$a(0-c) - a(b-c) + c(c-0) = 0$

$-ac - ab + ac + c^2 = 0$

$-ab + c^2 = 0$

$c^2 = ab$

$c = \sqrt{ab}$

$\boxed{c = \pm\sqrt{ab}}$

C is geometric mean of a and b

Hence Proved.

10) Given $\vec{c}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$.

So $\vec{c}$ is parallel to $\vec{a} \times \vec{b}$

$[\vec{a}\ \vec{b}\ \vec{c}] = \vec{a} \cdot (\vec{b} \times \vec{c})$

$|[\vec{a}\ \vec{b}\ \vec{c}]| = |\vec{a}|\,|\vec{b} \times \vec{c}|$

$= |\vec{a}|\,|\vec{b}|\,|\vec{c}| \sin\frac{\pi}{4}$

$= |\vec{a}|\,|\vec{b}| \cos(\frac{1}{2})$

$= |\vec{a}|\,|\vec{b}|\,(\frac{1}{2})$

$\boxed{[\vec{a}\ \vec{b}\ \vec{c}]^2 = \frac{1}{4}|\vec{a}|^2|\vec{b}|^2}$

Hence Proved.

EXERCISE – 6.3

Vector triple Product :-

$\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c}$

$(\vec{a}\times\vec{b})\times\vec{c} = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{b}\cdot\vec{c})\vec{a}$

Lagrange's Identity :-

$$(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d}) = \begin{vmatrix} \vec{a}\cdot\vec{c} & \vec{a}\cdot\vec{d} \\ \vec{b}\cdot\vec{c} & \vec{b}\cdot\vec{d} \end{vmatrix}$$

1)

$\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$

$\vec{b} = 2\hat{i} + \hat{j} - 2\hat{k}$

$\vec{c} = 3\hat{i} + 2\hat{j} + \hat{k}$

(i) $(\vec{a}\times\vec{b})\times\vec{c}$:

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 2 & 1 & -2 \end{vmatrix}$$

$= \hat{i}(4-3) - \hat{j}(-2-6) + \hat{k}(1+4)$

$\boxed{\vec{a}\times\vec{b} = \hat{i} + 8\hat{j} + 5\hat{k}}$

$$(\vec{a}\times\vec{b})\times\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 8 & 5 \\ 3 & 2 & 1 \end{vmatrix}$$

$= \hat{i}(8-10) - \hat{j}(1-15) + \hat{k}(2-24)$

$\boxed{(\vec{a}\times\vec{b})\times\vec{c} = -2\hat{i} + 14\hat{j} - 22\hat{k}}$

(ii) $\vec{a}\times(\vec{b}\times\vec{c})$

$$\vec{b}\times\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 3 & 2 & 1 \end{vmatrix}$$

$= \hat{i}(1+4) - \hat{j}(2+6) + \hat{k}(4-3)$

$\boxed{\vec{b}\times\vec{c} = 5\hat{i} - 8\hat{j} + \hat{k}}$

$$\vec{a}\times(\vec{b}\times\vec{c}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 5 & -8 & 1 \end{vmatrix}$$

$= \hat{i}(-2+24) - \hat{j}(1-15) +$
$\hat{k}(-8+10)$

$\boxed{\vec{a}\times(\vec{b}\times\vec{c}) = 22\hat{i} + 14\hat{j} + 2\hat{k}}$

2) Let $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$

$\hat{i}\times(\vec{a}\times\hat{i}) = (\hat{i}\cdot\hat{i})\vec{a} - (\hat{i}\cdot\vec{a})\hat{i}$

$= \vec{a} - x\hat{i}$

$\hat{j}\times(\vec{a}\times\hat{j}) = (\hat{j}\cdot\hat{j})\vec{a} - (\hat{j}\cdot\vec{a})\hat{j}$

$= \vec{a} - y\hat{j}$

$\hat{k}\times(\vec{a}\times\hat{k}) = (\hat{k}\cdot\hat{k})\vec{a} - (\hat{k}\cdot\vec{a})\hat{k}$

$= \vec{a} - z\hat{k}$

LHS

$\hat{i}\times(\vec{a}\times\hat{i}) + \hat{j}\times(\vec{a}\times\hat{j}) + \hat{k}\times(\vec{a}\times\hat{k})$

$= \vec{a} - x\hat{i} + \vec{a} - y\hat{j} + \vec{a} - z\hat{k}$

$= 3\vec{a} - (x\hat{i} + y\hat{j} + z\hat{k})$

$= 3\vec{a} - \vec{a}$

$= 2\vec{a}$

$=$ RHS

Hence Proved.

3).

$[\vec{a}-\vec{b} \quad \vec{b}-\vec{c} \quad \vec{c}-\vec{a}]$

$= (\vec{a}-\vec{b})\cdot[(\vec{b}-\vec{c})\times(\vec{c}-\vec{a})]$

$= (\vec{a}-\vec{b})\cdot[(\vec{b}\times\vec{c}) - (\vec{b}\times\vec{a}) - (\vec{c}\times\vec{c})$
$+ (\vec{c}\times\vec{a})]$

$= (\vec{a}-\vec{b})\cdot[(\vec{b}\times\vec{c}) - (\vec{b}\times\vec{a}) + (\vec{c}\times\vec{a})]$

$\because \vec{c}\times\vec{c} = \vec{0}$

$= \vec{a}\cdot(\vec{b}\times\vec{c}) - \vec{a}\cdot(\vec{b}\times\vec{a}) + \vec{a}\cdot(\vec{c}\times\vec{a}) - \vec{b}\cdot(\vec{b}\times\vec{c}) + \vec{b}\cdot(\vec{b}\times\vec{a}) - \vec{b}\cdot(\vec{c}\times\vec{a})$

$= [\vec{a}\ \vec{b}\ \vec{c}] - [\vec{a}\ \vec{b}\ \vec{a}] + [\vec{a}\ \vec{c}\ \vec{a}] - [\vec{b}\ \vec{b}\ \vec{c}] + [\vec{b}\ \vec{b}\ \vec{a}] - [\vec{b}\ \vec{c}\ \vec{a}]$

$= [\vec{a}\ \vec{b}\ \vec{c}] - 0 + 0 - 0 + 0 - [\vec{a}\ \vec{b}\ \vec{c}]$

$= 0$

$[\vec{a}-\vec{b}\quad \vec{b}-\vec{c}\quad \vec{c}-\vec{a}] = 0$

Hence Proved.

4)
$\vec{a} = 2\vec{i}+3\vec{j}-\vec{k}$
$\vec{b} = 3\vec{i}+5\vec{j}+2\vec{k}$
$\vec{c} = -\vec{i}-2\vec{j}+3\vec{k}$

To verify

$(\vec{a}\times\vec{b})\times\vec{c} = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{b}\cdot\vec{c})\vec{a}$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 3 & -1 \\ 3 & 5 & 2 \end{vmatrix}$$

$= \vec{i}(6+5) - \vec{j}(4+3) + \vec{k}(10-9)$

$= 11\vec{i} - 7\vec{j} + \vec{k}$

$$(\vec{a}\times\vec{b})\times\vec{c} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 11 & -7 & 1 \\ -1 & -2 & 3 \end{vmatrix}$$

$= \vec{i}(-21+2) - \vec{j}(33+1) + \vec{k}(-22-7)$

$= -19\vec{i} - 34\vec{j} - 29\vec{k}$ —— (1)

$(\vec{a}\cdot\vec{c})\vec{b} = (-2-6-3)(3\vec{i}+5\vec{j}+2\vec{k})$

$= -33\vec{i} - 55\vec{j} - 22\vec{k}$

$(\vec{b}\cdot\vec{c})\vec{a} = (-3-10+6)(2\vec{i}+3\vec{j}-\vec{k})$

$= -14\vec{i} - 21\vec{j} + 7\vec{k}$

$(\vec{a}\cdot\vec{c})\vec{b} - (\vec{b}\cdot\vec{c})\vec{a} = -33\vec{i} - 55\vec{j} - 22\vec{k} + 14\vec{i} + 21\vec{j} - 7\vec{k}$

$= -19\vec{i} - 34\vec{j} - 29\vec{k}$ —— (2)

From (1) & (2)

$(\vec{a}\times\vec{b})\times\vec{c} = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{b}\cdot\vec{c})\vec{a}$

Hence Proved.

(ii) $$\vec{b}\times\vec{c} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 5 & 2 \\ -1 & -2 & 3 \end{vmatrix}$$

$= 19\vec{i} - 11\vec{j} - \vec{k}$

$$\vec{a}\times(\vec{b}\times\vec{c}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 3 & -1 \\ 19 & -11 & -1 \end{vmatrix}$$

$= -14\vec{i} - 17\vec{j} - 79\vec{k}$ —— (1)

$(\vec{a}\cdot\vec{c})\vec{b} = -33\vec{i} - 55\vec{j} - 22\vec{k}$

$(\vec{a}\cdot\vec{b})\cdot\vec{c} = (6+15-2)(-\vec{i}-2\vec{j}+3\vec{k})$

$= -19\vec{i} - 38\vec{j} + 57\vec{k}$

$(\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c} =$

$-33\vec{i} - 55\vec{j} - 22\vec{k} + 19\vec{i} + 38\vec{j} - 57\vec{k}$

$= -14\vec{i} - 17\vec{j} - 79\vec{k}$ —— (2)

From (1) & (2)

$\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c}$

Hence Proved.

5)
$\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$
$\vec{b} = \hat{i} + 2\hat{j} - 4\hat{k}$
$\vec{c} = \hat{i} + \hat{j} + \hat{k}$

$$(\vec{a}\times\vec{b})\cdot(\vec{a}\times\vec{c}) = \begin{vmatrix} \vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{c} \\ \vec{b}\cdot\vec{a} & \vec{b}\cdot\vec{c} \end{vmatrix}$$

$$= \begin{vmatrix} 4+9+1 & 2+3-1 \\ -2+6+4 & -1+2-4 \end{vmatrix}$$

$$= \begin{vmatrix} 14 & 4 \\ 8 & -3 \end{vmatrix}$$

$$= -42-32$$

$$\boxed{(\vec{a}\times\vec{b})\cdot(\vec{a}\times\vec{c}) = -74}$$

6) Given $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are Coplanar vectors.

$\vec{a}, \vec{b}, \vec{c}$ are Coplanar or $\vec{a}, \vec{b}, \vec{d}$ are also Coplanar

$\Rightarrow [\vec{a}\ \vec{b}\ \vec{c}] = 0$, $[\vec{a}\ \vec{b}\ \vec{d}] = 0$

$$(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d}) = [\vec{a}\ \vec{b}\ \vec{d}]\vec{c} - [\vec{a}\ \vec{b}\ \vec{c}]\vec{d}$$

$$= (0)\vec{c} - (0)\vec{d}$$

$$= \vec{0}$$

$$\boxed{(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d}) = \vec{0}}$$

Hence Proved.

7) Given $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$
$\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$
$\vec{c} = 3\hat{i} + 2\hat{j} + \hat{k}$

$$\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c}$$

$$= (3+4+3)\vec{b} - (2-2+3)\vec{c}$$

$$= 10\vec{b} - 3\vec{c}$$

$$l\vec{a} + m\vec{b} + n\vec{c} = 10\vec{b} - 3\vec{c}$$

Equating like terms we get $l = 0$ $m = 10$ $n = -3$

8) Let $\hat{a}, \hat{b}$ and $\hat{c}$ are three unit vectors.

$|\hat{a}| = |\hat{b}| = |\hat{c}| = 1$

Given

$$\hat{a}\times(\hat{b}\times\hat{c}) = \frac{1}{2}\hat{b}$$

$$(\hat{a}\cdot\hat{c})\hat{b} - (\hat{a}\cdot\hat{b})\hat{c} = \frac{1}{2}\hat{b} + 0\hat{c}$$

equating like terms

$\hat{a}\cdot\hat{c} = \frac{1}{2}$ $\hat{a}\cdot\hat{b} = 0$

$$|\hat{a}||\hat{c}|\cos\theta = \frac{1}{2}$$

$$(1)(1)\cos\theta = \frac{1}{2}$$

$$\theta = \cos^{-1}\left(\frac{1}{2}\right)$$

$$\boxed{\theta = \frac{\pi}{3}}$$

The angle between $\hat{a}$ and $\hat{c}$ is $\pi/3$.

Exercise – 6.4

A Point on the straight line and the direction of the straight line are given

a) Parametric form:

$\vec{r} = \vec{a} + t\vec{b}$

b) Non Parametric form:

$(\vec{r} - \vec{a}) \times \vec{b} = \vec{0}$

c) Cartesian form:

$\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3}$

straight line Passing through two given Points

(i) Parametric form:

$\vec{r} = \vec{a} + t(\vec{b} - \vec{a}), t \in R$

(ii) Non - Parametric form:

$(\vec{r} - \vec{a}) \times (\vec{b} - \vec{a}) = \vec{0}$

(iii) Cartesian form:

$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$

Angle between two straight line:

(i) Vector form:

$\vec{r} = \vec{a} + s\vec{b}$ $\vec{r} = \vec{c} + t\vec{d}$

$\theta = \cos^{-1}\left(\frac{|\vec{b} \cdot \vec{d}|}{|\vec{b}||\vec{d}|}\right)$

Cartesian form:

$\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3}$ and

$\frac{x-x_1}{d_1} = \frac{y-y_1}{d_2} = \frac{z-z_1}{d_3}$

$\theta = \cos^{-1}\left[\frac{|b_1d_1 + b_2d_2 + b_3d_3|}{\sqrt{b_1^2+b_2^2+b_3^2}\sqrt{d_1^2+d_2^2+d_3^2}}\right]$

6.4 - EXERCISE

1) Let $\vec{a} = 4\hat{i} + 3\hat{j} - 7\hat{k}$

$\vec{b} = 2\hat{i} - 6\hat{j} + 7\hat{k}$

Non Parametric form of vector equation:

$(\vec{r} - \vec{a}) \times \vec{b} = \vec{0}$

$(\vec{r} - (4\hat{i} + 3\hat{j} - 7\hat{k})) \times (2\hat{i} - 6\hat{j} + 7\hat{k}) = \vec{0}$

Cartesian form:

$(x_1, y_1, z_1) = (4, 3, -7)$

$(b_1, b_2, b_3) = (2, -6, 7)$

eqn:

$\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3}$

$\frac{x-4}{2} = \frac{y-3}{-6} = \frac{z+7}{7}$

2) Let $\vec{a} = -2\hat{i}+3\hat{j}+4\hat{k}$

$\vec{b} = -4\hat{i}+5\hat{j}-6\hat{k}$

Parametric form of vector eqn:

$\vec{r} = \vec{a} + t\vec{b}$

$\vec{r} = (-2\hat{i}+3\hat{j}+4\hat{k}) + t(-4\hat{i}+5\hat{j}-6\hat{k})$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3}$

$(x_1, y_1, z_1) = (-2, 3, 4)$

$(b_1, b_2, b_3) = (-4, 5, -6)$

$\frac{x+2}{-4} = \frac{y-3}{5} = \frac{z-4}{-6}$

3) The direction ratios of the straight line joining two points are $(2, -3, 5)$

Cartesian equation:

$\frac{x-6}{2} = \frac{y-7}{-3} = \frac{z-4}{5}$ (or)

$\frac{x-8}{2} = \frac{y-4}{-3} = \frac{z-9}{5}$

The straight line cuts the xz plane

(ie) $y = 0$

$\frac{x-6}{2} = \frac{7}{3} = \frac{z-4}{5}$

$\frac{x-6}{2} = 7/3$ | $\frac{z-4}{5} = 7/3$

$x-6 = 14/3$ | $\frac{z-4}{5} = 7/3$

$x = 14/3 + 6$ | $z-4 = 35/3$

$\boxed{x = 32/3}$ | $z = 4 + 35/3$

$\boxed{z = 47/3}$

The Required Point is $(32/3, 0, 47/3)$

4) The straight line passes through the points $(5, 6, 7)$ and $(7, 9, 13)$

The direction ratio's are $2, 3, 6$.

Let $\vec{r} = 2\hat{i}+3\hat{j}+6\hat{k}$

$|\vec{r}| = \sqrt{4+9+36} = \sqrt{49} = 7$

direction cosines $\left(2/7, 3/7, 6/7\right)$

Parametric form:

Let $\vec{a} = 5\hat{i}+6\hat{j}+7\hat{k}$

$\vec{b} = 7\hat{i}+9\hat{j}+13\hat{k}$

$\vec{r} = \vec{a} + t(\vec{b}-\vec{a})$

$\vec{r} = (5\hat{i}+6\hat{j}+7\hat{k}) + t(2\hat{i}+3\hat{j}+6\hat{k})$

Cartesian form:

$(x_1, y_1, z_1) = (5, 6, 7)$

$(x_2, y_2, z_2) = (7, 9, 13)$

$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$

$\frac{x-5}{2} = \frac{y-6}{3} = \frac{z-7}{6}$

5)

(i) $\vec{b} = \hat{i}+2\hat{j}-2\hat{k}$

$\vec{d} = -\hat{i}-2\hat{j}+2\hat{k}$

$\vec{b}\cdot\vec{d} = -1-4-4 = -9$

$|\vec{b}| = \sqrt{1^2+2^2+2^2} = 3$

$|\vec{d}| = \sqrt{1^2+2^2+2^2} = 3$

$\theta = \cos^{-1}\left[\frac{|\vec{b}\cdot\vec{d}|}{|\vec{b}||\vec{d}|}\right]$

$\theta = \cos^{-1}\left[9/9\right] = \cos^{-1}(1)$

$\boxed{\theta = 0°}$

(ii)

$\vec{b} = 3\vec{i}+4\vec{j}+5\vec{k}$

$\vec{d} = 2\vec{i}+\vec{j}+\vec{k}$

$\theta = \cos^{-1}\left[\dfrac{|6+4+5|}{\sqrt{9+16+25}\sqrt{4+1+1}}\right]$

$\theta = \cos^{-1}\left[\dfrac{15}{\sqrt{50}\sqrt{6}}\right]$

$\theta = \cos^{-1}\left[\dfrac{15}{5\sqrt{2}\sqrt{6}}\right]$

$\boxed{\theta = \cos^{-1}\left[\dfrac{3}{2\sqrt{3}}\right]}$

(iii) $2x = 3y = -z$

$\dfrac{x}{1/2} = \dfrac{y}{1/3} = \dfrac{z}{-1}$

$\dfrac{x-0}{1/2} = \dfrac{y-0}{1/3} = \dfrac{z-0}{-1}$

$\boxed{\vec{b} = \frac{1}{2}\vec{i}+\frac{1}{3}\vec{j}-\vec{k}}$

$6x = -y = -4z$

$\dfrac{x-0}{1/6} = \dfrac{y-0}{-1} = \dfrac{z-0}{(-1/4)}$

$\boxed{\vec{d} = \frac{1}{6}\vec{i}-\vec{j}-\frac{1}{4}\vec{k}}$

$\theta = \cos^{-1}\left[\dfrac{|\vec{b}\cdot\vec{d}|}{|\vec{b}||\vec{d}|}\right]$

$= \cos^{-1}\left[\dfrac{\frac{1}{12}-\frac{1}{3}+\frac{1}{4}}{\sqrt{\frac{1}{4}+\frac{1}{9}+1}\sqrt{\frac{1}{36}+1+\frac{1}{16}}}\right]$

$= \cos^{-1}(0)$

$\boxed{\theta = \pi/2}$

b)

Let $\vec{OA} = 7\vec{i}+2\vec{j}+\vec{k}$

$\vec{OB} = 6\vec{i}+3\vec{k}$

$\vec{OC} = 4\vec{i}+2\vec{j}+4\vec{k}$

$\vec{AB} = \vec{OB}-\vec{OA}$

$\boxed{\vec{AB} = -\vec{i}-2\vec{j}+2\vec{k}}$

$\vec{BC} = \vec{OC}-\vec{OB}$

$\boxed{\vec{BC} = -2\vec{i}+2\vec{j}+\vec{k}}$

$\vec{AB}\cdot\vec{BC} = 2-4+2 = 0$

$\vec{AB}\cdot\vec{BC} = 0$

$\vec{AB} \perp \vec{BC}$

$\boxed{\angle ABC = \pi/2}$

7) Let $\vec{OA} = 2\vec{i}+\vec{j}+4\vec{k}$

$\vec{OB} = (a-1)\vec{i}+4\vec{j}-\vec{k}$

$\vec{OC} = 2\vec{j}+(b-1)\vec{k}$

$\vec{OD} = 5\vec{i}+3\vec{j}-2\vec{k}$

$\vec{AB} = \vec{OB}-\vec{OA}$

$\boxed{\vec{AB} = (a-3)\vec{i}+3\vec{j}-5\vec{k}}$

$\vec{CD} = \vec{OD}-\vec{OC}$

$\boxed{\vec{CD} = 5\vec{i}+\vec{j}-(b+1)\vec{k}}$

Given $\vec{AB}$ $\parallel^{el}$ to $\vec{CD}$

$3\vec{CD} = 15\vec{i}+3\vec{j}-3(b+1)\vec{k}$

($\because \vec{j}$ component same)

$\vec{AB} = 3\,\vec{CD}$

$(a-3)\vec{i} + 3\vec{j} - 5\vec{k} = 15\vec{i} + 3\vec{j} - 3(b+1)\vec{k}$

equating like term

$a-3=15$ | $5 = 3(b+1)$

$\boxed{a=18}$ | $3b = 2$

$\boxed{b = 2/3}$

8) The given equation can be written as

$\frac{x-5}{5m+2} = \frac{y-2}{-5} = \frac{z-1}{1}$ and

$\frac{x}{1} = \frac{y+\frac{1}{2}}{2m} = \frac{z-1}{3}$

Let $\vec{b} = (5m+2)\vec{i} - 5\vec{j} + \vec{k}$

$\vec{d} = \vec{i} + 2m\vec{j} + 3\vec{k}$

Given straight line are perpendicular to each other

$\vec{b}\cdot\vec{d} = 0$

$(5m+2) - 10m + 3 = 0$

$-5m + 5 = 0$

$5m = 5$

$\boxed{m=1}$

9)

Let $\vec{OA} = 2\vec{i} + 3\vec{j} + 4\vec{k}$

$\vec{OB} = -\vec{i} + 4\vec{j} + 5\vec{k}$

$\vec{OC} = 8\vec{i} + \vec{j} + 2\vec{k}$

$\vec{AB} = \vec{OB} - \vec{OA}$

$\boxed{\vec{AB} = -3\vec{i} + \vec{j} + \vec{k}}$

$\vec{AC} = \vec{OC} - \vec{OA}$

$\boxed{\vec{AC} = 6\vec{i} + 2\vec{j} - 2\vec{k}}$

$\vec{AC} = -2(-3\vec{i} + \vec{j} + \vec{k})$

$\vec{AC} = -2(\vec{AB})$

$\vec{AB} \parallel \vec{AC}$

A is a common point

$\therefore$ The given points are collinear

EXERCISE - 6.5

(1) Let $\vec{a} = 5\hat{i}+2\hat{j}+8\hat{k}$

$\vec{b} = 2\hat{i} - 2\hat{j}+\hat{k}$

$\vec{d} = \hat{i}+2\hat{j}+2\hat{k}$

$$\vec{b}\times\vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -2 & 1 \\ 1 & 2 & 2 \end{vmatrix}$$

$= \hat{i}(-4-2)-\hat{j}(4-1)+\hat{k}(4+2)$

$= -6\hat{i}-3\hat{j}+6\hat{k}$

$\vec{b}\times\vec{d} = -3(2\hat{i}+\hat{j}-2\hat{k})$

Let $\vec{c} = 2\hat{i}+\hat{j}-2\hat{k}$

Vector equation:

$\vec{r} = \vec{a}+t\vec{c}$

$\vec{r} = (5\hat{i}+2\hat{j}+8\hat{k})+t(2\hat{i}+\hat{j}-2\hat{k})$

Cartesian equation:-

$$\frac{x-x_1}{c_1} = \frac{y-y_1}{c_2} = \frac{z-z_1}{c_3}$$

$$\frac{x-5}{2} = \frac{y-2}{1} = \frac{z-8}{-2}$$

(2) Let $\vec{a} = 6\hat{i}+\hat{j}+2\hat{k}$

$\vec{b} = \hat{i}+2\hat{j}-3\hat{k}$

$\vec{c} = 3\hat{i}+2\hat{j}-2\hat{k}$

$\vec{d} = 2\hat{i}+4\hat{j}-5\hat{k}$

$\vec{c}-\vec{a} = -3\hat{i}+\hat{j}-4\hat{k}$

$$[(\vec{c}-\vec{a})\ \vec{b}\ \vec{d}] = \begin{vmatrix} -3 & 1 & -4 \\ 1 & 2 & -3 \\ 2 & 4 & -5 \end{vmatrix}$$

$= -3(-10+12)-1(-5+6)+4(4-4)$

$= -6-1-0 = -7 \neq 0$

Given lines are Skew lines

$$\vec{b}\times\vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 2 & 4 & -5 \end{vmatrix}$$

$= \hat{i}(-10+12)-\hat{j}(-5+6)+\hat{k}(4-4)$

$\vec{b}\times\vec{d} = 2\hat{i}-\hat{j}$

$|\vec{b}\times\vec{d}| = \sqrt{4+1} = \sqrt{5}$

The shortest distance between the skew lines

$$d = \frac{|[(\vec{c}-\vec{a})\ \vec{b}\ \vec{d}]|}{|\vec{b}\times\vec{d}|}$$

$= |-7|/\sqrt{5}$

$\boxed{d = 7/\sqrt{5} \text{ units}}$

3. Condition is

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ b_1 & b_2 & b_3 \\ d_1 & d_2 & d_3 \end{vmatrix} = 0$$

Here $(x_1,y_1,z_1) = (1,-1,1)$

$(x_2,y_2,z_2) = (3,m,0)$

$(b_1,b_2,b_3) = (2,3,4)$

$(d_1,d_2,d_3) = (1,2,1)$

$$\begin{vmatrix} 2 & m+1 & -1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

$2(3-8)-(m+1)(2-4)-1(4-3)=0$

$2(-5)-(m+1)(-2)-1(1)=0$

$-10+2m+2-1=0$

$2m-9=0$

$2m=9$

$\boxed{m=9/2}$

4)

$\frac{x-3}{3} = \frac{y-3}{-1}$, $z-1=0 \Rightarrow z=1$

$\frac{x-6}{2} = \frac{z-1}{3}$, $y-2=0 \Rightarrow y=2$

Here $(x_1,y_1,z_1) = (3,3,1)$

$(x_2,y_2,z_2) = (6,2,1)$

$(b_1,b_2,b_3) = (3,-1,0)$

$(d_1,d_2,d_3) = (2,0,3)$

Condition for intersecting

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ b_1 & b_2 & b_3 \\ d_1 & d_2 & d_3 \end{vmatrix} = 0$$

$$\begin{vmatrix} 3 & -1 & 0 \\ 3 & -1 & 0 \\ 2 & 0 & 3 \end{vmatrix} = 0 \ (R_1 \equiv R_2)$$

The given lines are intersecting

Let $\frac{x-3}{3} = \frac{y-3}{-1} = \lambda$ and $z=1$

any point on this line is

$(3\lambda+3, -\lambda+3, 1)$

Let $\frac{x-6}{2} = \frac{z-1}{3} = k$ and $y=2$

Any point on this line is

$(2k+6, 2, 3k+1)$

for some λ, k

$(3\lambda+3, -\lambda+3, 1) = (2k+6, 2, 3k+1)$

$-\lambda+3=2$

$\boxed{\lambda=1}$

$3k+1=1$

$3k = 1-1 = 0$

$\boxed{k=0}$

The Required points of intersection

$(3(1)+3, -1+3, 1) = (6,2,1)$

5)

$x+1 = 2y = -12z$

$\frac{x+1}{1} = \frac{y-0}{1/2} = \frac{z-0}{-1/12}$

$x = y+2 = 6z-6$

$\frac{x-0}{1} = \frac{y+2}{1} = \frac{z-1}{1/6}$

Let $(x_1,y_1,z_1) = (1,0,0)$

$(x_2,y_2,z_2) = (0,-2,1)$

$(b_1,b_2,b_3) = (1, \frac{1}{2}, -\frac{1}{12})$

$(d_1,d_2,d_3) = (1,1,\frac{1}{6})$

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ b_1 & b_2 & b_3 \\ d_1 & d_2 & d_3 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & -2 & 1 \\ 1 & \frac{1}{2} & -\frac{1}{12} \\ 1 & 1 & \frac{1}{6} \end{vmatrix}$$

$= 1\left(\frac{1}{12}+\frac{1}{12}\right) + 2\left(\frac{1}{6}+\frac{1}{12}\right) + 1\left(1-\frac{1}{2}\right)$

$= \frac{2}{12} + 2\left(\frac{3}{12}\right) + 1\left(\frac{1}{2}\right)$

$= \frac{1}{6} + \frac{3}{6} + \frac{1}{2} = \frac{7}{6} \neq 0$

The given lines are skew lines.

$$\vec{b} \times \vec{d} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & \frac{1}{2} & -\frac{1}{12} \\ 1 & 1 & \frac{1}{6} \end{vmatrix}$$

$\vec{b} \times \vec{d} = \vec{i}\left(\frac{1}{12}+\frac{1}{12}\right) - \vec{j}\left(\frac{1}{6}+\frac{1}{12}\right) + \vec{k}\left(1-\frac{1}{2}\right)$

$= \vec{i}\,\frac{1}{6} - \vec{j}\,\frac{1}{4} + \hat{k}\,\frac{1}{2}$

$\vec{b} \times \vec{d} = \frac{1}{6}\vec{i} - \frac{1}{4}\vec{j} + \frac{1}{2}\vec{k}$

$|\vec{b} \times \vec{d}| = \sqrt{\frac{1}{36}+\frac{1}{16}+\frac{1}{4}}$

$= \sqrt{\frac{49}{144}} = \frac{7}{12}$

$\boxed{|\vec{b} \times \vec{d}| = \frac{7}{12}}$

The shortest distance between skew lines

$$d = \left[\frac{|(\vec{c}-\vec{a})\ \vec{b}\ \vec{d}|}{|\vec{b}\times\vec{d}|} \right]$$

$$= \frac{7/6}{7/12} = 2$$

$\boxed{d = 2 \text{ units}}$

6)

Let $\vec{a} = -\vec{i}+2\vec{j}+\vec{k}$

$\vec{b} = \vec{i}-2\vec{j}+\vec{k}$

$\vec{r} = \vec{a} + t\,\vec{b}$

$\vec{r} = (-\vec{i}+2\vec{j}+\vec{k}) + t(\vec{i}-2\vec{j}+\vec{k})$

Parallel to the straight line

$\vec{r} = (2\vec{i}+3\vec{j}-\vec{k}) + t(\vec{i}-2\vec{j}+\vec{k})$

Here $\vec{c} = 2\vec{i}+3\vec{j}-\vec{k}$

$\vec{c}-\vec{a} = 3\vec{i}+\vec{j}-2\vec{k}$

$$(\vec{c}-\vec{a})\times\vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 1 & -2 \\ 1 & -2 & 1 \end{vmatrix}$$

$$= -3\vec{i}-5\vec{j}-7\vec{k}$$

$|(\vec{c}-\vec{a})\times\vec{b}| = \sqrt{9+25+49} = \sqrt{83}$

$|\vec{b}| = \sqrt{1+4+1} = \sqrt{6}$

The distance between the parallel lines

$$d = \frac{|(\vec{c}-\vec{a})\times\vec{b}|}{|\vec{b}|} = \frac{\sqrt{83}}{\sqrt{6}} \text{ units.}$$

(7)

$\vec{b} = 2\vec{i}+3\vec{j}-\vec{k}$

$\vec{OD} = 5\vec{i}+4\vec{j}+2\vec{k}$

Let $\frac{x+1}{2} = \frac{y-3}{3} = \frac{z-1}{-1} = \lambda$

$F[(2\lambda-1), (3\lambda+2), (-\lambda+1)]$ — (1)

$\vec{OF} = (2\lambda-1)\vec{i}+(3\lambda+2)\vec{j}+(-\lambda+1)\vec{k}$

$\vec{DF} = \vec{OF} - \vec{OD}$

$= (2\lambda-1-5)\vec{i} + (3\lambda+3-4)\vec{j}$
$+ (-\lambda+1-2)\vec{k}$

$= (2\lambda-6)\vec{i} + (3\lambda-1)\vec{j} + (-\lambda-1)\vec{k}$

$\vec{DF} \perp \vec{b}$

$\Rightarrow \vec{DF}\cdot\vec{b} = 0$

$2(2\lambda-6)+3(3\lambda-1)-1(-\lambda-1) = 0$

$4\lambda-12+9\lambda-3+\lambda+1 = 0$

$4\lambda-12+9\lambda-3+\lambda+1 = 0$

$14\lambda-14 = 0$

$\boxed{\lambda = 1}$

From (1)

F is $(1, 5, 0)$

Equation of DF is

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$

$$\frac{x-5}{-4} = \frac{y-4}{2} = \frac{z-2}{-2}$$

EXERCISE – 6.6

① Given

$p = 7$

$\vec{d} = 3\hat{i} - 4\hat{j} + 5\hat{k}$

$|\vec{d}| = \sqrt{9+16+25} = \sqrt{50}$

$|\vec{d}| = 5\sqrt{2}$

Vector equation: $\left[\hat{d} = \frac{\vec{d}}{|\vec{d}|}\right]$

$\vec{r} \cdot \hat{d} = p$

$\vec{r} \cdot \left(\frac{3\hat{i} - 4\hat{j} + 5\hat{k}}{5\sqrt{2}}\right) = 7$

$\vec{r} \cdot (3\hat{i} - 4\hat{j} + 5\hat{k}) = 35\sqrt{2}$

② Equation of the plane is

$12x + 3y - 4z = 65$

Parametric form

$\vec{r} \cdot (12\hat{i} + 3\hat{j} - 4\hat{k}) = 65$. $[\vec{r} \cdot \vec{d} = q]$

$\vec{d} = 12\hat{i} + 3\hat{j} - 4\hat{k}$ $q = 65$

$p = \frac{q}{|\vec{d}|} = \frac{65}{\sqrt{144+9+16}} = \frac{65}{\sqrt{169}}$

$= \frac{65}{13} = 5$

$\boxed{p = 5}$

③ Given

$\vec{a} = 2\hat{i} + 6\hat{j} + 3\hat{k}$

$\vec{n} = \hat{i} + 3\hat{j} + 5\hat{k}$

Vector equation:

$\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$

$\vec{r} \cdot (\hat{i} + 3\hat{j} + 5\hat{k}) = (2\hat{i} + 6\hat{j} + 3\hat{k}) \cdot (\hat{i} + 3\hat{j} + 5\hat{k})$

$\vec{r} \cdot (\hat{i} + 3\hat{j} + 5\hat{k}) = 35$ ——(1)

(ii) Cartesian equation:

substituting $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

① ⇒

$(x\hat{i} + y\hat{j} + z\hat{k}) \cdot (\hat{i} + 3\hat{j} + 5\hat{k}) = 35$

$\boxed{x + 3y + 5z = 35}$

④

Given

magnitude $= 3\sqrt{3}$

Passes through the point

$\vec{a} = -\hat{i} + \hat{j} + 2\hat{k}$

The normal makes an equal angle with the coordinate axes.

If α, β, γ are the angles then

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\alpha = \beta = \gamma$

$3\cos^2\alpha = 1$

$\cos^2\alpha = \frac{1}{3}$ $\cos\alpha = \frac{1}{\sqrt{3}}$

$\vec{n} = 3\sqrt{3}\left(\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}\right)$

$= \frac{3\sqrt{3}}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$

$\vec{n} = 3(\hat{i} + \hat{j} + \hat{k}) = 3\hat{i} + 3\hat{j} + 3\hat{k}$

Vector equation:

$\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$

$\vec{r} \cdot (3\hat{i} + 3\hat{j} + 3\hat{k}) = (-\hat{i} + \hat{j} + 2\hat{k}) \cdot (3\hat{i} + 3\hat{j} + 3\hat{k})$

$\boxed{\vec{r} \cdot (3\hat{i} + 3\hat{j} + 3\hat{k}) = 6}$ —— (1)

Cartesian equation:

$(x\hat{i} + y\hat{j} + z\hat{k}) \cdot (3\hat{i} + 3\hat{j} + 3\hat{k}) = 6$

$3x + 3y + 3z = 6$

$\div 3 \Rightarrow$ $\boxed{x + y + z = 2}$

(5)

Given

$\vec{r}\cdot(6\hat{i}+4\hat{j}-3\hat{k})=12$

Comparing with

$\vec{r}\cdot\vec{n}=q$

$\vec{n}=6\hat{i}+4\hat{j}-3\hat{k}$ $q=12$

The eqn of the plane having intercepts a, b, c on the x, y, z axes

$\frac{q}{a}=6$ $\frac{q}{b}=4$ $\frac{q}{c}=-3$

$q=12$ in $q/a=6$

$\frac{12}{a}=6 \Rightarrow \boxed{a=2}$

$\frac{12}{b}=4 \Rightarrow 12=4b \Rightarrow \boxed{b=3}$

$\frac{12}{c}=-3 \Rightarrow -3c=12 \Rightarrow \boxed{c=-4}$

x intercept $=2$

y intercept $=3$

z intercept $=-4$

(6)

Let $A(a,0,0)$, $B(a,0,0)$ and $C(a,0,0)$ be the vertices of ΔABC

Centroid of $\Delta=\left(\frac{a}{3},\frac{b}{3},\frac{c}{3}\right)$

Given Centroid (u,v,w)

$\left(\frac{a}{3},\frac{b}{3},\frac{c}{3}\right)=(u,v,w)$

Equating $a/3=u \Rightarrow a=3u$

$b/3=v \Rightarrow b=3v$

$c/3=w \Rightarrow c=3w$

Equation: $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

$\frac{x}{3u}+\frac{y}{3v}+\frac{z}{3w}=1$

$\boxed{\frac{x}{u}+\frac{y}{v}+\frac{z}{w}=3}$

(7) EX — 6.7

Equation of a plane passing through three given non linear points

(i) Parametric form of vector equation:-

$\vec{r}=\vec{a}+s(\vec{b}-\vec{a})+t(\vec{c}-\vec{a}),\ s,t\in R$

(or)

$\vec{r}=(1-s-t)\vec{a}+s\vec{b}+t\vec{c},\ s,t\in R$

(ii) Non parametric form of vector equation:-

$(\vec{r}-\vec{a})\cdot[(\vec{b}-\vec{a})\times(\vec{c}-\vec{a})]=0$

(or)

$[\vec{r}-\vec{a},\ \vec{b}-\vec{a},\ \vec{c}-\vec{a}]=0$

(iii) Cartesian form of equation:-

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix}=0$$

Equation of a plane passing through two given distinct points and is parallel to a non zero vector:

(i) Parametric form:

$\vec{r}=\vec{a}+s(\vec{b}-\vec{a})+t\vec{c},\ s,t\in R$

(or)

$\vec{r}=(1-s)\vec{a}+s\vec{b}+t\vec{c},\ s,t\in R$

(ii) non parametric form:

$(\vec{r}-\vec{a})\cdot[(\vec{b}-\vec{a})\times\vec{c}]=0$

(iii) Cartesian form:

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ c_1 & c_2 & c_3 \end{vmatrix}=0$$

EXERCISE – 6.7

(1) Let $\vec{a} = 2\hat{i}+3\hat{j}+6\hat{k}$

$\vec{b} = 2\hat{i}+3\hat{j}+\hat{k}$

$\vec{c} = 2\hat{i}-5\hat{j}-3\hat{k}$

$\vec{r} = x\hat{i}+y\hat{j}+z\hat{k}$

Non Parametric form:

$(\vec{r}-\vec{a})\cdot(\vec{b}\times\vec{c})=0$ —(1)

$$\vec{b}\times\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 2 & -5 & -3 \end{vmatrix}$$

$\vec{b}\times\vec{c} = -4\hat{i}+8\hat{j}-16\hat{k}$

$[(x\hat{i}+y\hat{j}+z\hat{k})-(2\hat{i}+3\hat{j}+6\hat{k})]\cdot(-4\hat{i}+8\hat{j}-16\hat{k})=0$

Cartesian equation:

$[(x-2)\hat{i}+(y-3)\hat{j}+(z-6)\hat{k}]\cdot[-4\hat{i}+8\hat{j}-16\hat{k}]=0$

$-4(x-2)+8(y-3)-16(z-6)=0$

$-4x+8+8y-24-16z+96=0$

$-4x+8y-16z+80=0$

$\div -4$ $\boxed{x-2y+4z-20=0}$

(2) Let $\vec{a} = 2\hat{i}+2\hat{j}+\hat{k}$

$\vec{b} = 9\hat{i}+3\hat{j}+6\hat{k}$

$\vec{c} = 2\hat{i}+6\hat{j}+6\hat{k}$

Parametric form:

$\vec{r} = (1-s)\vec{a}+s\vec{b}+t\vec{c}$

$\vec{r} = (1-s)(2\hat{i}+2\hat{j}+\hat{k})+s(9\hat{i}+3\hat{j}+6\hat{k})$
$+t(2\hat{i}+6\hat{j}+6\hat{k})$

Cartesian form:

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ c_1 & c_2 & c_3 \end{vmatrix}=0$$

$$\begin{vmatrix} x-2 & y-2 & z-1 \\ 7 & 1 & 5 \\ 2 & 6 & 6 \end{vmatrix}=0$$

$(x-2)(6-30)-(y-2)(42-10)+(z-1)(42-2)=0$

$(x-2)(-24)-32(y-2)+40(z-1)=0$

$-24x+48-32y+64+40z-40=0$

$-24x-32y+40z+72=0$

$\div -8 \Rightarrow$

$3x+4y-5z-9=0$

(3) Let $\vec{a} = 2\hat{i}+2\hat{j}+\hat{k}$

$\vec{b} = \hat{i}-2\hat{j}+3\hat{k}$

Equation of line passing through (x_1,y_1,z_1), (x_2,y_2,z_2) is

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$

$(x_1,y_1,z_1) = (2,1,-3)$

$(x_2,y_2,z_2) = (-1,5,-8)$

$$\frac{x-2}{-3} = \frac{y-1}{4} = \frac{z+3}{-5}$$

Let $\vec{c} = -3\hat{i}+4\hat{j}-5\hat{k}$

Parametric form:

$\vec{r} = (1-s)\vec{a}+s\vec{b}+t\vec{c}$

$\vec{r} = (1-s)(2\hat{i}+2\hat{j}+\hat{k})+s(\hat{i}-2\hat{j}+3\hat{k})$
$+t(-3\hat{i}+4\hat{j}-5\hat{k})$

Cartesian form:

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ c_1 & c_2 & c_3 \end{vmatrix}=0$$

$$\begin{vmatrix} x-2 & y-2 & z-1 \\ -1 & -4 & 2 \\ -3 & 4 & -5 \end{vmatrix}=0$$

$(x-2)(20-8)-(y-2)(5+6)+(z-1)(-4+12)=0$

$12(x-2)-11(y-2)-16(z-1)=0$

$12x-24-11y+22-16z+16=0$

$\boxed{12x-11y-16z+14=0}$

(4) let $\vec{a}=\hat{i}-2\hat{j}+4\hat{k}$

$\vec{b}=\hat{i}+2\hat{j}-3\hat{k}$

$\vec{c}=3\hat{i}-\hat{j}+\hat{k}$

Non Parametric form:

$(\vec{r}-\vec{a})\cdot(\vec{b}\times\vec{c})=0$

$$\vec{b}\times\vec{c}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & -3\\ 3 & -1 & 1\end{vmatrix}$$

$=\hat{i}(2-3)-\hat{j}(1+9)+\hat{k}(-1-6)$

$=-\hat{i}-10\hat{j}-7\hat{k}$

$\vec{r}\cdot(\vec{b}\times\vec{c})-\vec{a}\cdot(\vec{b}\times\vec{c})=0$

$\vec{r}\cdot(-\hat{i}-10\hat{j}-7\hat{k})-(-1+20-28)=0$

$\vec{r}\cdot(-\hat{i}-10\hat{j}-7\hat{k})+9=0$

$\vec{r}\cdot(-\hat{i}-10\hat{j}-7\hat{k})=-9$

Cartesian form:

let $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$

$(x\hat{i}+y\hat{j}+z\hat{k})\cdot(-\hat{i}-10\hat{j}-7\hat{k})=-9$

$-x-10y-7z=-9$

$\boxed{x+10y+7z=9}$

(5) $\vec{a}=\hat{i}-\hat{j}+3\hat{k}$

$\vec{b}=2\hat{i}-\hat{j}-4\hat{k}$

$\vec{c}=\hat{i}+2\hat{j}+\hat{k}$

Parametric form:

$\vec{r}=\vec{a}+s\vec{b}+t\vec{c}$

$\vec{r}=(\hat{i}-\hat{j}+3\hat{k})+s(2\hat{i}-\hat{j}+4\hat{k})$
$+t(\hat{i}+2\hat{j}+\hat{k})$

Cartesian form:

$$\begin{vmatrix}x-x_1 & y-y_1 & z-z_1\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{vmatrix}=0$$

$$\begin{vmatrix}x-1 & y+1 & z-3\\ 2 & -1 & 4\\ 1 & 2 & 1\end{vmatrix}=0$$

$(x-1)(-1-8)-(y+1)(2-4)+(z-3)(4+1)=0$

$-9x+9+2y+2+5z-15=0$

$-9x+2y+5z-4=0$

$\boxed{9x-2y-5z+4=0}$

(6) let $\vec{a}=3\hat{i}+6\hat{j}-2\hat{k}$

$\vec{b}=-\hat{i}-2\hat{j}+6\hat{k}$

$\vec{c}=6\hat{i}-4\hat{j}-2\hat{k}$

Parametric form:

$\vec{r}=(1-s-t)\vec{a}+s\vec{b}+t\vec{c}$

$\vec{r}=(1-s-t)(3\hat{i}+6\hat{j}-2\hat{k})+s(-\hat{i}-2\hat{j}+6\hat{k})$
$+t(6\hat{i}-4\hat{j}-2\hat{k})$

Non Parametric form:

$(\vec{r}-\vec{a})\cdot[(\vec{b}-\vec{a})\times(\vec{c}-\vec{a})]=0$

$[\vec{r}-(3\hat{i}+6\hat{j}-2\hat{k})][(-4\hat{i}-8\hat{j}+8\hat{k})\times$
$(-3\hat{i}-10\hat{j})]=0$

Cartesian form:

$$\begin{vmatrix}x-x_1 & y-y_1 & z-z_1\\ x_2-x_1 & y_2-y_1 & z_2-z_1\\ x_3-x_1 & y_3-y_1 & z_3-z_1\end{vmatrix}=0$$

$$\begin{vmatrix}x-3 & y-6 & z+2\\ -4 & -8 & 8\\ 3 & -10 & 0\end{vmatrix}=0$$

$(x-3)(0+80)-(y-6)(c-24)$
$+(z+2)(40+24)=0$

$80x-240+24y-144+64z+128=0$

$80x+24y+64z-256=0$

$\div 8 \Rightarrow$ $\boxed{10x+3y+8z-32=0}$

(7)

Let $\vec{a}=6\vec{i}-\vec{j}+\vec{k}$

$\vec{b}=-\vec{i}+2\vec{j}+\vec{k}$

$\vec{c}=-5\vec{i}-4\vec{j}-5\vec{k}$

Non Parametric form:-

$(\vec{r}-\vec{a})\cdot(\vec{b}\times\vec{c})=0$

$$\vec{b}\times\vec{c}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ -1 & 2 & 1\\ -5 & -4 & -5\end{vmatrix}$$

$=\vec{i}(-10+4)-\vec{j}(5+5)+\vec{k}(4+10)$

$=-6\vec{i}-10\vec{j}+14\vec{k}$

$\vec{r}\cdot(\vec{b}\times\vec{c})-\vec{a}\cdot(\vec{b}\times\vec{c})=0$

$\vec{r}\cdot(-6\vec{i}-10\vec{j}+14\vec{k})-[-36+10+14]=0$

$\vec{r}\cdot(-6\vec{i}-10\vec{j}+14\vec{k})=-12$

Cartesian form:

Let $\vec{r}=x\vec{i}+y\vec{j}+z\vec{k}$

$(x\vec{i}+y\vec{j}+z\vec{k})\cdot(-6\vec{i}-10\vec{j}+14\vec{k})=-12$

$-6x-10y+14z=-12$

$\div -2$

$\boxed{3x+5y-7z=6}$

EXERCISE - 6.8

(1)

Let $\vec{a}=5\vec{i}+7\vec{j}-3\vec{k}$

$\vec{b}=4\vec{i}+4\vec{j}-5\vec{k}$

$\vec{c}=8\vec{i}+4\vec{j}+5\vec{k}$

$\vec{d}=7\vec{i}+\vec{j}+3\vec{k}$

Condition for Coplanarity:

$(\vec{c}-\vec{a})\cdot(\vec{b}\times\vec{d})=0$

$$\vec{b}\times\vec{d}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 4 & 4 & -5\\ 7 & 1 & 3\end{vmatrix}$$

$=\vec{i}(12+5)-\vec{j}(12+35)+\vec{k}(4-28)$

$\boxed{\vec{b}\times\vec{d}=17\vec{i}-47\vec{j}-24\vec{k}}$

$\boxed{\vec{c}-\vec{a}=3\vec{i}-3\vec{j}+8\vec{k}}$

$(\vec{c}-\vec{a})\cdot(\vec{b}\times\vec{d})=$

$(3\vec{i}-3\vec{j}+8\vec{k})\cdot(17\vec{i}-47\vec{j}-24\vec{k})$

$=51+141-192$

$=0$

The given two lines are Coplanar.

Non Parametric Vector equation:

$(\vec{r}-\vec{a})\cdot(\vec{b}\times\vec{d})=0$

$[\vec{r}-(5\vec{i}+7\vec{j}-3\vec{k})]\cdot(17\vec{i}-47\vec{j}-24\vec{k})=0$

$\vec{r}\cdot(\vec{b}\times\vec{d})-\vec{a}\cdot(\vec{b}\times\vec{d})=0$

$\vec{r}\cdot(17\vec{i}-47\vec{j}-24\vec{k})-(85-329+72)=0$

$\vec{r}\cdot(17\vec{i}-47\vec{j}-24\vec{k})+172=0$

(2)

Let $\vec{a} = 2\vec{i}+3\vec{j}+4\vec{k}$

$\vec{b} = \vec{i}+\vec{j}+3\vec{k}$

$\vec{c} = \vec{i}+4\vec{j}+5\vec{k}$

$\vec{d} = -3\vec{i}+2\vec{j}+\vec{k}$

Condition for Coplanarity :-

$(\vec{c}-\vec{a})\cdot(\vec{b}\times\vec{d})=0$

$\vec{c}-\vec{a} = -\vec{i}+\vec{j}+\vec{k}$

$$\vec{b}\times\vec{d} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 3 \\ -3 & 2 & 1 \end{vmatrix}$$

$= \vec{i}(1-6)-\vec{j}(1+9)+\vec{k}(2+3)$

$\vec{b}\times\vec{d} = -5\vec{i}-10\vec{j}+5\vec{k}$

$(\vec{c}-\vec{a})\cdot(\vec{b}\times\vec{d}) = 5-10+5 = 0$

Given lines are Coplanar.

(2) Non Parametric Vector equation:

$(\vec{r}-\vec{a})\cdot(\vec{b}\times\vec{d})=0$

$[\vec{r}-(2\vec{i}+3\vec{j}+4\vec{k})]\cdot[-5\vec{i}-10\vec{j}+5\vec{k}]=0$

$(\vec{r}-\vec{a})\cdot(\vec{b}\times\vec{d})=0$

$\vec{r}\cdot(\vec{b}\times\vec{d}) - \vec{a}\cdot(\vec{b}\times\vec{d})=0$

$[\vec{r}-(2\vec{i}+3\vec{j}+4\vec{k})] - (-5\vec{i}-10\vec{j}+5\vec{k})=0$

$\vec{r}\cdot(-5\vec{i}-10\vec{j}+5\vec{k}) - (-10-30+20)=0$

$-5x-10y+5z+20=0$

$\div -5$

$\Rightarrow \boxed{x+2y-z-4=0}$

(3)

Let $\vec{a} = \vec{i}+2\vec{j}+3\vec{k}$

$\vec{b} = \vec{i}+2\vec{j}+m^2\vec{k}$

$\vec{c} = 3\vec{i}+2\vec{j}+\vec{k}$

$\vec{d} = \vec{i}+m^2\vec{j}+2\vec{k}$

Condition for Coplanarity

$(\vec{c}-\vec{a})\cdot(\vec{b}\times\vec{d})=0$

$\vec{c}-\vec{a} = 2\vec{i}-2\vec{k}$

$$\vec{b}\times\vec{d} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & m^2 \\ 1 & m^2 & 2 \end{vmatrix}$$

$= \vec{i}(4-m^4)-\vec{j}(2-m^2)+\vec{k}(m^2-2)$

$(\vec{c}-\vec{a})\cdot(\vec{b}\times\vec{d})=0$

$(2\vec{i}-2\vec{k})\cdot[(4-m^4)\vec{i}-(2-m^2)\vec{j}+(m^2-2)\vec{k}]=0$

$2(4-m^4)-2(m^2-2)=0$

$2[4-m^4-m^2+2]=0$

$m^4+m^2-6=0$

Let $m^2=r$

$r^2+r-6=0$

$(r+3)(r-2)=0$

$r=-3$ (or) $r=2$

$m^2=-3$ $m^2=2$

not possible

So $m^2=2$

$\boxed{m=\pm\sqrt{2}}$

(A)

$(x_1, y_1, z_1) = (1, -1, 0)$

$(x_2, y_2, z_2) = (-1, -1, 0)$

$(b_1, b_2, b_3) = (2, \lambda, 2)$

$(d_1, d_2, d_3) = (5, 2, \lambda)$

Condition for Collinearity:

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ b_1 & b_2 & b_3 \\ d_1 & d_2 & d_3 \end{vmatrix} = 0$$

$$\begin{vmatrix} -2 & 0 & 0 \\ 2 & \lambda & 2 \\ 5 & 2 & \lambda \end{vmatrix} = 0$$

$-2(\lambda^2 - 4) = 0$

$\lambda^2 - 4 = 0$

$\lambda^2 = 4$

$\boxed{\lambda = \pm 2}$

(i) If $\lambda = 2$

eqn of plane is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ b_1 & b_2 & b_3 \\ d_1 & d_2 & d_3 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-1 & y+1 & z \\ 2 & 2 & 2 \\ 5 & 2 & 2 \end{vmatrix} = 0$$

$(x-1)(0) - (y+1)(4-10) + z(4-10) = 0$

$6(y+1) - 6z = 0$

$6y + 6 - 6z = 0$

$\boxed{y - z + 1 = 0}$

(ii) If $\lambda = -2$

eqn of plane is

$$\begin{vmatrix} x-1 & y+1 & z \\ 2 & -2 & 2 \\ 5 & 2 & -2 \end{vmatrix} = 0$$

$(x-1)(0) - (y+1)(-4-10) + z(14) = 0$

$14y + 14 + 14z = 0$

$\boxed{y + z + 1 = 0}$

EXERCISE – 6.9

① The Required equation of the plane is

$(2x-7y+4z-3)+\lambda(3x-5y+4z+1)=0 \longrightarrow ①$

④ Passing through $(-2,1,3)$

$(-4-7+12-3)+\lambda(-6-5+12+1)=0$

$-2+12\lambda=0$

$12\lambda=2$

$\boxed{\lambda=1/6}$

① ⇒

$(2x-7y+4z-3)+\frac{1}{6}(3x-5y+4z+1)=0$

$12x-42y+24z-18+3x-5y+4z+1=0$

$\boxed{15x-47y+28z-7=0}$

② The Required equation of the plane is

$(x+2y+3z-2)+\lambda(x-y+z-3)=0 \longrightarrow ①$

$(1+\lambda)x+(2-\lambda)y+(3+\lambda)-2-3\lambda=0$

Distance from $(3,1,-1)$ to the plane $=2/\sqrt{3}$

$$\frac{3(1+\lambda)+1(2-\lambda)-1(3+\lambda)-2-3\lambda}{\sqrt{(1+\lambda)^2+(2-\lambda)^2+(3+\lambda)^2}}=\frac{2}{\sqrt{3}}$$

$$\frac{\cancel{3}+\cancel{3\lambda}+\cancel{2}-\lambda-\cancel{3}-\lambda-\cancel{2}-\cancel{3\lambda}}{\sqrt{1+\lambda^2+2\lambda+4+\lambda^2-4\lambda+9+\lambda^2+6\lambda}}=\frac{2}{\sqrt{3}}$$

$-2\lambda\sqrt{3}=2\sqrt{3\lambda^2+4\lambda+14}$

Squaring on both sides

$12\lambda^2=4(3\lambda^2+4\lambda+14)$

$16\lambda=-56$

$\lambda=-56/16=-7/2$

$\lambda=-7/2$ in ①

$(x+2y+3z-2)-7/2(x-y+z-3)=0$

$2x+4y+6z-4-7x+7y-7z+21=0$

$-5x+11y-z+17=0$

$\boxed{5x-11y+z-17=0}$

③ Let $\vec{b}=\vec{i}+2\vec{j}-2\vec{k}$

$\vec{n}=6\vec{i}+3\vec{j}+2\vec{k}$

$$\theta=\sin^{-1}\left[\frac{\vec{b}\cdot\vec{n}}{|\vec{b}|\,|\vec{n}|}\right]$$

$$=\sin^{-1}\left[\frac{|6+6-4|}{\sqrt{1+4+4}\sqrt{36+9+4}}\right]$$

$$=\sin^{-1}\left[\frac{8}{3\times7}\right]$$

$$\boxed{\theta=\sin^{-1}\left(\frac{8}{21}\right)}$$

④ Let $\vec{n_1}=\vec{i}+\vec{j}-2\vec{k}$

$\vec{n_2}=2\vec{i}-2\vec{j}+\vec{k}$

Angle between the planes are

$$\theta=\cos^{-1}\left(\frac{|\vec{n_1}\cdot\vec{n_2}|}{|\vec{n_1}|\,|\vec{n_2}|}\right)$$

$$=\cos^{-1}\left(\frac{|2-2-2|}{\sqrt{1+1+4}\sqrt{4+4+1}}\right)$$

$$=\cos^{-1}\left(\frac{2}{\sqrt{6}\times3}\right)$$

$$\boxed{\theta=\cos^{-1}\left(\frac{2}{3\sqrt{6}}\right)}$$

5)

Equation of the parallel plane is $2x-3y+5z+k=0$

It passes through $(3,4,-1)$

$6-12-5+k=0$

$-11+k=0$

$\boxed{k=11}$

$2x-3y+5z+11=0$ is the required eqn of the plane.

Distance between the two parallel planes

$$= \frac{|d_1-d_2|}{\sqrt{a^2+b^2+c^2}}$$

$$= \left|\frac{7-11}{\sqrt{4+9+25}}\right|$$

$$= \frac{4}{\sqrt{38}} \text{ units}$$

6) Length of the perpendicular from (x_1,y_1,z_1) to the plane $ax+by+cz-p$ is

$$\delta = \left|\frac{ax_1+by_1+cz_1-p}{\sqrt{a^2+b^2+c^2}}\right|$$

Length of the perpendicular from $(1,-2,3)$ to the plane $x-y+z-5=0$ is

$$\delta = \left|\frac{1+2+3-5}{\sqrt{1+1+1}}\right|$$

$$\boxed{\delta = 1/\sqrt{3} \text{ units}}$$

7)

Let $\frac{x-1}{1} = \frac{y}{2} = \frac{z+1}{1} = t$

$(x,y,z) = (1+t, 2t, t-1)$ —— (1)

It lies on $2x-y+2z=2$

$2(1+t)-2t+2(t-1)=2$

$\cancel{2}+2t-\cancel{2t}+2t-\cancel{2}=2$

$\boxed{t=1}$

(1) ⇒

Required point

$(1+1, 2(1), 1-1)$

$=(2,2,0)$

Let $\vec{b} = \vec{i}+2\vec{j}+\vec{k}$

$\vec{n} = 2\vec{i}-\vec{j}+2\vec{k}$

$$\theta = \sin^{-1}\left[\frac{|\vec{b}\cdot\vec{n}|}{|\vec{b}||\vec{n}|}\right]$$

$$= \sin^{-1}\left[\frac{|2-2+2|}{\sqrt{1+4+1}\sqrt{4+1+4}}\right]$$

$$\boxed{\theta = \sin^{-1}\left(\frac{2}{3\sqrt{6}}\right)}$$

(ii)

Cartesian equation of the plane is $\frac{x-4}{1} = \frac{y-3}{2} = \frac{z-2}{3} = t$

$(x, y, z) = (t+4, 2t+3, 3t+2)$

The Point lies on $x+2y+3z=2$

$(t+4)+2(2t+3)+3(3t+2)=2$

$t+4+4t+6+9t+6=2$

$14t = 2-16$

$14t = -14$

$\boxed{t=-1}$

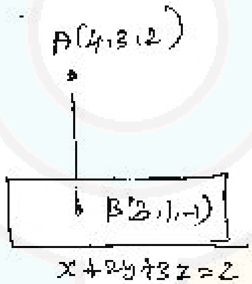
$(x, y, z) = (3, 1, -1)$ is the coordinates of the point of the perpendicular.

Distance between $(4,3,2)$ and $(3,1,-1)$

$d = \sqrt{(-1)^2+(-2)^2+(-3)^2}$

$= \sqrt{1+4+9}$

$\boxed{d = \sqrt{14}}$



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அன்புடன்

17/06/19

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