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(தலைப்பிற்கு கீழே உள்ள லிங்கை கிளிக் செய்து குழுவில் இணையவும்!)

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EX 2.8 CHAPTER-2 [COMPLEX NUMBERS] KEY POINTS

(4) $x = \cos\alpha + i\sin\alpha$

$y = \cos\beta + i\sin\beta$

(i) $\frac{x}{y} = \frac{\cos\alpha + i\sin\alpha}{\cos\beta + i\sin\beta}$

$\frac{x}{y} = \cos(\alpha-\beta) + i\sin(\alpha-\beta)$

$\text{lly}\quad \frac{y}{x} = \cos(\alpha-\beta) - i\sin(\alpha-\beta)$

now

$\frac{x}{y} + \frac{y}{x} = 2\cos(\alpha-\beta)$

(ii) $xy = (\cos\alpha + i\sin\alpha)(\cos\beta + i\sin\beta)$

$xy = \cos(\alpha+\beta) + i\sin(\alpha+\beta)$

$\text{lly}\quad \frac{1}{xy} = \cos(\alpha+\beta) - i\sin(\alpha+\beta)$

now $xy - \frac{1}{xy} = 2i\sin(\alpha+\beta)$

(iii) $x^m = (\cos\alpha + i\sin\alpha)^m$

$x^m = \cos m\alpha + i\sin m\alpha$

$y^n = (\cos\beta + i\sin\beta)^n$

$y^n = \cos n\beta + i\sin n\beta$

$\frac{x^m}{y^n} = \frac{\cos m\alpha + i\sin m\alpha}{\cos n\beta + i\sin n\beta}$

$\frac{x^m}{y^n} = \cos(m\alpha - n\beta) + i\sin(m\alpha - n\beta)$

$\text{lly}\quad \frac{y^n}{x^m} = \cos(m\alpha - n\beta) - i\sin(m\alpha - n\beta)$

now

$\frac{x^m}{y^n} - \frac{y^n}{x^m} = 2i\sin(m\alpha - n\beta)$

(iv) $x^m y^n = (\cos m\alpha + i\sin m\alpha)(\cos n\beta + i\sin n\beta)$

$x^m y^n = \cos(m\alpha + n\beta) + i\sin(m\alpha + n\beta)$

$\text{lly}\quad \frac{1}{x^m y^n} = \cos(m\alpha + n\beta) - i\sin(m\alpha + n\beta)$

now, $x^m y^n + \frac{1}{x^m y^n} = 2\cos(m\alpha + n\beta)$

---

(5) solve the equation $z^3 + 27 = 0$

$\rightarrow$

$z^3 = -27$

$z^3 = 27 \times -1$

$z = 27^{1/3} \times -1^{1/3}$

$z = 3[\cos\pi + i\sin\pi]^{1/3}$

$z = 3[\cos(2k\pi+\pi) + i\sin(2k\pi+\pi)]^{1/3}$

$z = 3[\cos(2k+1)\pi + i\sin(2k+1)\pi]^{1/3}$

$z = 3\left[\cos\frac{(2k+1)\pi}{3} + i\sin\frac{(2k+1)\pi}{3}\right]$

$k = 0, 1, 2$

$k=0,\quad z = 3[\cos\pi/3 + i\sin\pi/3]$

$k=1,\quad z = 3[\cos 3\pi/3 + i\sin 3\pi/3]$

$k=2,\quad z = 3[\cos 5\pi/3 + i\sin 5\pi/3]$

---

(6) given $(z-1)^3 + 8 = 0$

$(z-1)^3 = -8$

$(z-1)^3 = 8 \times -1$

$z-1 = 8^{1/3} \times (-1)^{1/3}$

$z-1 = 2[\cos\pi + i\sin\pi]^{1/3}$

$z-1 = 2[\cos(2k\pi+\pi) + i\sin(2k\pi+\pi)]^{1/3}$

$z-1 = 2[\cos(2k+1)\pi + i\sin(2k+1)\pi]^{1/3}$

$z-1 = 2\left[\cos\frac{(2k+1)\pi}{3} + i\sin\frac{(2k+1)\pi}{3}\right]$

$k = 0, 1, 2$

$k=0,\quad z-1 = 2[\cos\pi/_2 + i\sin\pi/_3]$

$\boxed{\cancel{z=-1}}\ \boxed{z = 1 - 2\omega^2}$

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$k=1$,

$$z-1 = 2\left[\cos 3\pi/3 + i\sin 3\pi/3\right]$$

$\boxed{\cancel{z=1-2\omega}}$ $\boxed{z=-1}$

$k=2$,

$$z-1 = 2\left[\cos 5\pi/3 + i\sin 5\pi/3\right]$$

$\boxed{z = 1-2\omega^2}$

(10) Prove that the values of $\sqrt[4]{-1}$ are $\pm\frac{1}{\sqrt{2}}(1\pm i)$

$\rightarrow$

$$\sqrt[4]{-1} = (-1)^{1/4}$$

$$= \left[\cos\pi + i\sin\pi\right]^{1/4}$$

$$= \left[\cos(2k\pi+\pi) + i\sin(2k\pi+\pi)\right]^{1/4}$$

$$= \cos\frac{(2k+1)\pi}{4} + i\sin\frac{(2k+1)\pi}{4}$$

$k = 0,1,2,3$

$k=0$, $\Rightarrow \cos\pi/4 + i\sin\pi/4$

$\Rightarrow \frac{1}{\sqrt{2}}(1+i)$

$k=1$, $\Rightarrow \cos 3\pi/4 + i\sin 3\pi/4$

$\Rightarrow -\frac{1}{\sqrt{2}}(1-i)$

$k=2$, $\Rightarrow \cos 5\pi/4 + i\sin 5\pi/4$

$\Rightarrow -\frac{1}{\sqrt{2}}(1+i)$

$k=3$, $\Rightarrow \cos 7\pi/4 + i\sin 7\pi/4$

$\Rightarrow \frac{1}{\sqrt{2}}(1-i)$

eg 2.34 solve the equation $z^3+8i=0$,

$\rightarrow$

$$z^3 = -8i$$

$$z = (-8)^{1/3} \times (i)^{1/3}$$

$$z = -2\left[\cos\pi/2 + i\sin\pi/2\right]^{1/3}$$

$$z = -2\left[\cos(2k\pi+\pi/2) + i\sin(2k\pi+\pi/2)\right]^{1/3}$$

$$z = -2\left[\cos\left(\frac{4k\pi+\pi}{2}\right) + i\sin\left(\frac{4k\pi+\pi}{2}\right)\right]^{1/3}$$

$$z = -2\left[\cos\frac{(4k+1)\pi}{6} + i\sin\frac{(4k+1)\pi}{6}\right]$$

$k=0,1,2$

$k=0$, $z = -2\left[\cos\pi/6 + i\sin\pi/6\right]$

$k=1$, $z = -2\left[\cos 5\pi/6 + i\sin 5\pi/6\right]$

$k=2$, $z = -2\left[\cos 9\pi/6 + i\sin 9\pi/6\right]$

eg 2.35 Find all cube roots of $\sqrt{3}+i$

$\rightarrow$ let $\sqrt{3}+i = r(\cos\theta + i\sin\theta) \rightarrow$ (1)

$r = \sqrt{(\sqrt{3})^2+1^2}$	$\theta = \tan^{-1}(y/x)$
$r = \sqrt{3+1} = \sqrt{4}$	$\theta = \tan^{-1}(1/\sqrt{3})$
$r = 2$	$\theta = \pi/6$

(1) $\Rightarrow \therefore \sqrt{3}+i = 2\left[\cos\pi/6 + i\sin\pi/6\right]$

$$(\sqrt{3}+i)^{3} = 2\left[\cos\pi/6 + i\sin\pi/6\right]$$

$$= 2^{1/3}\left[\cos\pi/6 + i\sin\pi/6\right]^{1/3}$$

$$= 2^{1/3}\left[\cos(2k\pi+\pi/6) + i\sin(2k\pi+\tfrac{\pi}{6})\right]^{1/3}$$

$$= 2^{1/3}\left[\cos\left(\frac{12k\pi+\pi}{6}\right) + i\sin\left(\frac{12k\pi+\pi}{6}\right)\right]^{1/3}$$

$$(\sqrt{3}+i)^{3} = 2^{1/3}\left[\cos\frac{(12k+1)\pi}{18} + i\sin\frac{(12k+1)\pi}{18}\right]$$

$k=0,1,2$

$k=0$, $z = 2^{1/3}\left[\cos\pi/18 + i\sin\pi/18\right]$

$k=1$, $z = 2^{1/3}\left[\cos 13\pi/18 + i\sin\frac{13\pi}{18}\right]$

$k=2$, $z = 2^{1/3}\left[\cos\frac{25\pi}{18} + i\sin\frac{25\pi}{18}\right]$

eg 2.31 solve $(-\sqrt{3}+3i)^{31}$

$\rightarrow$ let $-\sqrt{3}+3i = r(\cos\theta + i\sin\theta) \rightarrow$ (1)

$r = \sqrt{(\sqrt{3})^2+3^2}$	$\theta = \tan^{-1}(y/x)$
$= \sqrt{3+9}$	$\theta = \tan^{-1}(3/\sqrt{3})$
$= \sqrt{12}$	$\theta = 2\pi/3$
$r = 2\sqrt{3}$	

(1) $\Rightarrow -\sqrt{3}+3i = 2\sqrt{3}\left[\cos 2\pi/3 + i\sin 2\pi/3\right]$

$$(-\sqrt{3}+3i)^{31} = (2\sqrt{3})^{31}\left[\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right]^{31}$$

(2)

$$(-\sqrt{3}+3i)^{31} = (2\sqrt{3})^{31}\left[\cos\frac{62\pi}{3} + i\sin\frac{62\pi}{3}\right]$$

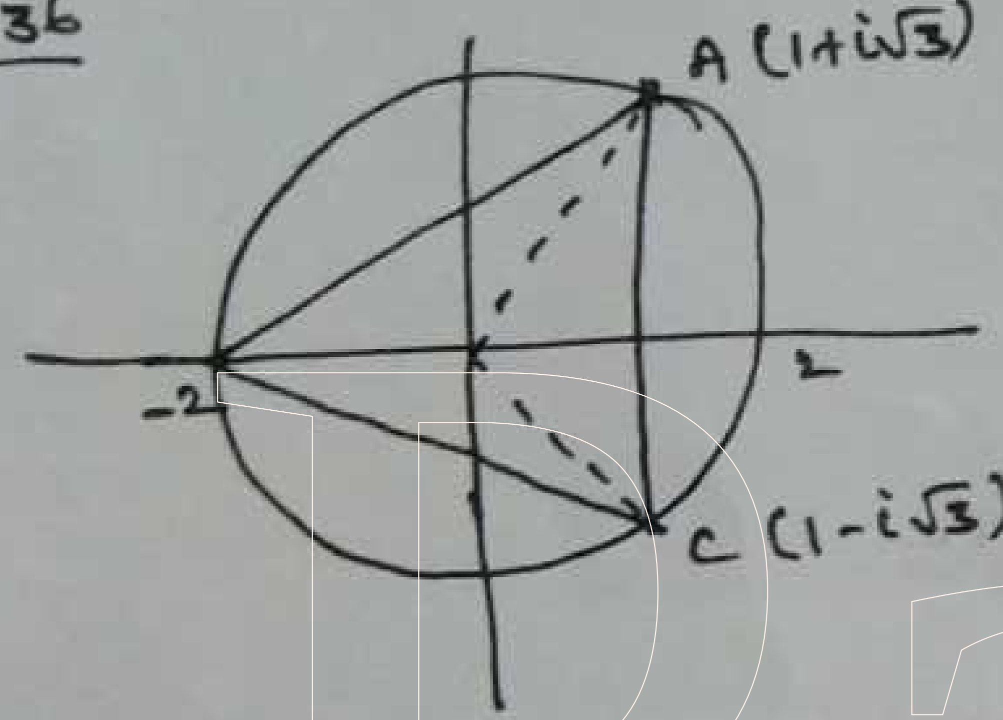
$$= (2\sqrt{3})^{31}\left[\cos\left(20\pi+\frac{2\pi}{3}\right) + i\sin\left(20\pi+\frac{2\pi}{3}\right)\right]$$

$$= (2\sqrt{3})^{31}\left[\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right]$$

~~$= (2\sqrt{3})^{31}\left[\cos(\pi-\pi/3) + i\sin\right.$~~

$$= (2\sqrt{3})^{31}\left[-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right]$$

Eg 2.36



$\vec{OA} = 1+i\sqrt{3}$

$\vec{OB} = z_1 \cdot e^{i2\pi/3}$

$= (1+i\sqrt{3}) \cdot e^{i2\pi/3}$

$= (1+i\sqrt{3})\left[\cos 2\pi/3 + i\sin 2\pi/3\right]$

$= -2$

$\vec{OC} = -2 \cdot e^{i2\pi/3}$

$= -2 \cdot \left[\cos 2\pi/3 + i\sin 2\pi/3\right]$

$= 1-i\sqrt{3}$

$z_2 = -2, \quad z_3 = 1-i\sqrt{3}$

Ex 2.7 (5)

$a = \cos\alpha + i\sin\alpha$

$b = \cos\beta + i\sin\beta$

$c = \cos\gamma + i\sin\gamma$

$a+b+c = 0$

we have $a^3+b^3+c^3 = 3abc$

$(\text{cis}\,\alpha)^3 + (\text{cis}\,\beta)^3 + (\text{cis}\,\gamma)^3 = 3\left[\text{cis}\,\alpha \cdot \text{cis}\,\beta \cdot \text{cis}\,\gamma\right]$

$\text{cis}\,3\alpha + \text{cis}\,3\beta + \text{cis}\,3\gamma = 3\left[\text{cis}\,(\alpha+\beta+\gamma)\right]$

Equating real and Im Part

(i) $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha+\beta+\gamma)$

(ii) $\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha+\beta+\gamma)$

---

eg 2.27 If $z = x+iy$, $\arg\frac{z-1}{z+1} = \pi/2$ then show that $x^2+y^2=1$.

$\rightarrow$

Put $z = x+iy$

$$\frac{z-1}{z+1} = \frac{x+iy-1}{x+iy+1}$$

$$= \frac{(x-1)+iy}{(x+1)+iy} \times \frac{(x+1)-iy}{(x+1)-iy}$$

$$\arg\left(\frac{z-1}{z+1}\right) = \pi/2$$

$$\tan^{-1}\left(\frac{2y}{x^2+y^2-1}\right) = \pi/2$$

$$x^2+y^2 = 1$$

~~Ex 2.7~~ **Ex 2.7** (6) If $z = x+iy$, $\arg\frac{z-i}{z+2} = \pi/4$ then show that $x^2+y^2+3x-3y+2=0$

$\rightarrow$

Put $z = x+iy$

$$\frac{z-i}{z+2} = \frac{x+iy-i}{x+iy+2}$$

$$= \frac{x+i(y-1)}{(x+2)+iy} \times \frac{(x+2)-iy}{(x+2)-iy}$$

$$\arg\left(\frac{z-i}{z+2}\right) = \pi/4$$

$$\tan^{-1}\left(\frac{2y-x-2}{x^2+y^2+2x-y}\right) = \pi/4$$

$$x^2+y^2+3x-3y+2 = 0$$

Ex 2.6 (2) If $z = x+iy$ is a complex number s.t $\text{Im}\left(\frac{2z+1}{iz+1}\right) = 0$, show that $2x^2+2y^2+x-2y=0$

$\rightarrow$

Put $z = x+iy$

$$\frac{2z+1}{iz+1} = \frac{2(x+iy)+1}{i(x+iy)+1}$$

$$= \frac{(2x+1)+i2y}{ix-y+1} = \frac{(2x+1)+i2y}{(1-y)+ix}$$

$$= \frac{(2x+1)+i2y}{(1-y)+ix} \times \frac{(1-y)-ix}{(1-y)-ix}$$

$$\text{Im}\left(\frac{2z+1}{iz+1}\right) = 0$$

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$$\Rightarrow \frac{-x(2x+1)+2y(1-y)}{(1-y)^2+x^2}=0$$

$$\Rightarrow 2x^2+2y^2+x-2y=0.$$

① Ex 2.5 Put $z=x+iy$

given $\left|\frac{z-4i}{z+4i}\right|=1$

$$\left|\frac{x+iy-4i}{x+iy+4i}\right|=1$$

$$\left|\frac{x+i(y-4)}{x+i(y+4)}\right|=1$$

$$\frac{\sqrt{x^2+(y-4)^2}}{\sqrt{x^2+(y+4)^2}}=1$$

Square on both sides

$$\frac{x^2+(y-4)^2}{x^2+(y+4)^2}=1$$

$$\boxed{y=0}$$

eg 2.15 If $z_1\,z_2\,z_3$ be complex number s.t $|z_1|=|z_2|=|z_3|=r$

P.t $\left|\frac{z_1z_2+z_2z_3+z_3z_1}{z_1+z_2+z_3}\right|=r$

$\longrightarrow$ $z_1=\frac{r^2}{\overline{z_1}}$

$z_2=\frac{r^2}{\overline{z_2}}$, $z_3=\frac{r^2}{\overline{z_3}}$

$$z_1+z_2+z_3=\frac{r^2}{\overline{z_1}}+\frac{r^2}{\overline{z_2}}+\frac{r^2}{\overline{z_3}}$$

$$=r^2\left[\frac{1}{\overline{z_1}}+\frac{1}{\overline{z_2}}+\frac{1}{\overline{z_3}}\right]$$

$$=r^2\left[\frac{\overline{z_2}\,\overline{z_3}+\overline{z_1}\,\overline{z_3}+\overline{z_1}\,\overline{z_2}}{\overline{z_1}\,\overline{z_2}\,\overline{z_3}}\right]$$

$$z_1+z_2+z_3=\cancel{r^2}\left[\frac{\overline{z_2z_3+z_1z_3+z_1z_2}}{r^{\cancel{3}}}\right]$$

$$|z_1+z_2+z_3|=\left|\frac{z_2z_3+z_1z_3+z_1z_2}{r}\right|$$

$$r=\left|\frac{z_2z_3+z_1z_3+z_1z_2}{z_1+z_2+z_3}\right|$$

eg 2.8 Show that (i) $(2+i\sqrt3)^{10}+(2-i\sqrt3)^{10}$ is real and (ii) $\left(\frac{19+9i}{5-3i}\right)^{15}-\left(\frac{8+i}{1+2i}\right)^{15}$ is Purely Imaginary

$\longrightarrow$ (i) let $z=(2+i\sqrt3)^{10}+(2-i\sqrt3)^{10}$

$$\bar z=\overline{(2+i\sqrt3)^{10}+(2-i\sqrt3)^{10}}$$

$$\bar z=(2-i\sqrt3)^{10}+(2+i\sqrt3)^{10}$$

$$\bar z=z$$

It is real

(ii) let $\frac{19+9i}{5-3i}=2+3i$, $\frac{8+i}{1+2i}=2-3i$

$$z=(2+3i)^{15}-(2-3i)^{15}$$

$$\bar z=\overline{(2+3i)^{15}-(2-3i)^{15}}$$

$$\bar z=(2-3i)^{15}-(2+3i)^{15}$$

$$\bar z=-\left[(2+3i)^{15}-(2-3i)^{15}\right]$$

$$\bar z=-z$$

$\therefore$ It is Purely Imaginary

Ex 2.4 ⑦ Show that $(2+i\sqrt3)^{10}-(2-i\sqrt3)^{10}$ is Purely Imaginary (ii) $\left(\frac{19-7i}{9+i}\right)^{12}+\left(\frac{20-5i}{7-6i}\right)^{12}$ is real

$\longrightarrow$

(i) $z=(2+i\sqrt3)^{10}-(2-i\sqrt3)^{10}$

$$\bar z=\overline{(2+i\sqrt3)^{10}-(2-i\sqrt3)^{10}}$$

$$=(2-i\sqrt3)^{10}-(2+i\sqrt3)^{10}$$

$$=-\left[(2+i\sqrt3)^{10}-(2-i\sqrt3)^{10}\right]$$

$$\bar z=-z$$

It is Purely Imaginary

(ii) $\frac{19-7i}{9+i}=2-i$, $\frac{20-5i}{7-6i}=2+i$

$$z=(2-i)^{12}+(2+i)^{12}$$

$$\bar z=\overline{(2-i)^{12}+(2+i)^{12}}$$

$$\bar z=(2+i)^{12}+(2-i)^{12}$$

$$\bar z=z$$

It is real.

eg 2.18 Given the complex number $z = 3+2i$ represent complex numbers $z, iz, z+iz$ in one argand diagram show that complex numbers is an isosceles right triangles.

$\longrightarrow$

$z = 3+2i$

$iz = 3i-2$

$z+iz = 3+2i+3i-2$

$z+iz = 1+5i$

$A(3+2i)$, $B(1+5i)$, $C(-2+3i)$

$AC^2 = |(3+2i)-(-2+3i)|^2$

$AC^2 = 26$

$BC^2 = |(-2+3i)-(1+5i)|^2$

$BC^2 = 13$

$BA^2 = |(1+5i)-(3+2i)|^2$

$BA^2 = 13$

$\therefore AB^2 + BC^2 = AC^2$

It is an isosceles right angle

Ex 2.8 ② show that $\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^5 = -\sqrt{3}$

$\longrightarrow$

$= \left(\cos \pi/6 + i\sin \pi/6\right)^5 + \left(\cos\pi/6 - i\sin\pi/6\right)^5$

$= \left(\cos 5\pi/6 + i\sin 5\pi/6\right) + \left(\cos 5\pi/6 - i\sin 5\pi/6\right)$

$= 2\cos 5\pi/6$

$= 2\cos(\pi - \pi/6)$

$= -2\cos\pi/6$

$= -\cancel{2}\times \sqrt{3}/\cancel{2}$

$= -\sqrt{3}$

eg 2.31 (i) simplify $(1+i)^{18}$

$\rightarrow$

$1+i = r(\cos\theta + i\sin\theta)$

$r = \sqrt{1+1}$	$\theta = \tan^{-1}(y/x)$
$r = \sqrt{2}$	$\theta = \tan^{-1}(1)$
	$\theta = \pi/4$

$1+i = \sqrt{2}\left(\cos\pi/4 + i\sin\pi/4\right)$

$(1+i)^{18} = (\sqrt{2})^{18}\left[\cos\pi/4 + i\sin\pi/4\right]^{18}$

$= (\sqrt{2})^{18}\left[\cos\frac{18\pi}{4} + i\sin\frac{18\pi}{4}\right]$

$= 2^9\left[\cos(4\pi+\pi/2) + i\sin(4\pi+\pi/2)\right]$

$= 2^9\left[\cos\pi/2 + i\sin\pi/2\right]$

$= 2^9(0+i(1))$

$= 512i$

Ex 2.4 ⑥ find the least value of the Positive Integer n for which $(\sqrt{3}+i)^n$. (i) real (ii) Purely Imaginary

$\longrightarrow$

$z = (\sqrt{3}+i)^n$

Put $n=1$, $z = (\sqrt{3}+i)$

Put $n=2$, $z = (\sqrt{3}+i)^2$

$z = 3 + i^2 + 2\sqrt{3}i$

$z = 2 + 2\sqrt{3}i$

Put $n=3$, $z = (\sqrt{3}+i)^3$

$= (\sqrt{3})^3 + i^3 + 3(\sqrt{3})^2 i + 3\sqrt{3}i^2$

$= 3\cancel{\sqrt{3}} - i + 9i - 3\cancel{\sqrt{3}}$

$= 8i$

which is Purely Imaginary when $n=3$

Put $n=6$,

$z = (\sqrt{3}+i)^6$

$= (\sqrt{3}+i)^3 \cdot (\sqrt{3}+i)^3$

$= 8i \times 8i$

$= -64$

which is Purely Real when $n=6$.

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CHAPTER-4 [INVERSE OF TRIGONOMETRIC FUNCTIONS]

Eg 4.22:

If $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=\pi$, show that $x^2+y^2+z^2+2xy=1$

$\rightarrow$

$\sin\alpha=\sqrt{1-x^2}$, $\sin\beta=\sqrt{1-y^2}$

$\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=\pi$

$\alpha+\beta+\cos^{-1}z=\pi$

$\boxed{\alpha+\beta=\pi-\cos^{-1}z}\rightarrow(1)$

now,

$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

$\cos(\alpha+\beta)=xy-\sqrt{1-x^2}\cdot\sqrt{1-y^2}\rightarrow(2)$

$(1)\Rightarrow\cos(\alpha+\beta)=\cos(\pi-\cos^{-1}z)$

$\cos(\alpha+\beta)=-z\rightarrow(3)$

(2) & (3) $xy-\sqrt{1-x^2}\cdot\sqrt{1-y^2}=-z$

Square on both sides we get

$x^2+y^2+z^2+2xy=1$

Eg 4.23

$\tan^{-1}\left(\frac{d}{1+a_1a_2}\right)=\tan^{-1}a_2-\tan^{-1}a_1$

lly $\tan^{-1}\left(\frac{d}{1+a_2a_3}\right)=\tan^{-1}a_3-\tan^{-1}a_2$

$\tan^{-1}\left(\frac{d}{1+a_{n-1}a_n}\right)=\tan^{-1}a_n-\tan^{-1}a_{n-1}$

Taking tan on both sides

$\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right)+\dots+\tan^{-1}\left(\frac{d}{1+a_{n-1}a_n}\right)\right]=$

$=\tan\left[\tan^{-1}a_n-\tan^{-1}a_1\right]$

$=\tan\left[\tan^{-1}\frac{a_n-a_1}{1+a_1a_n}\right]$

$=\frac{a_n-a_1}{1+a_1a_n}$

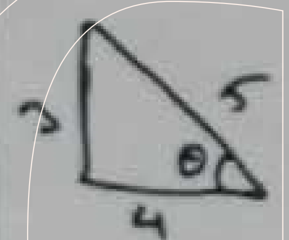
Eg 4.20 Evaluate $\sin\left[\sin^{-1}(3/5)+\sec^{-1}(5/4)\right]$

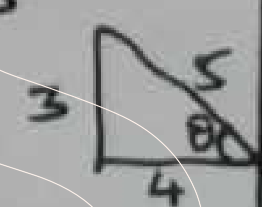
$\rightarrow$

$\sin A=3/5$ | $\sec B=5/4$

$\cos A=4/5$ | $\cos B=4/5$

$\sin B=3/5$





$\sin(A+B)=\sin A\cos B+\cos A\sin B$

$=\left(\frac{3}{5}\times\frac{4}{5}\right)+\left(\frac{4}{5}\times\frac{3}{5}\right)$

$=\frac{12}{25}+\frac{12}{25}$

$\sin\left(\sin^{-1}\frac{3}{5}+\sec^{-1}\frac{5}{4}\right)=\frac{24}{25}$

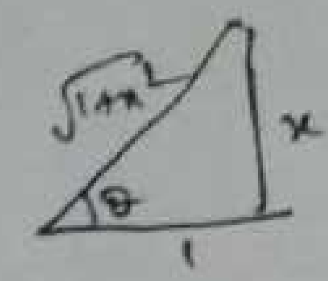
Eg 4.29: solve

$\cos\left(\sin^{-1}\frac{x}{\sqrt{1+x^2}}\right)=\sin\left(\cot^{-1}(3/4)\right)$

$\rightarrow$ L.H.S:

let $\theta=\sin^{-1}\frac{x}{\sqrt{1+x^2}}$

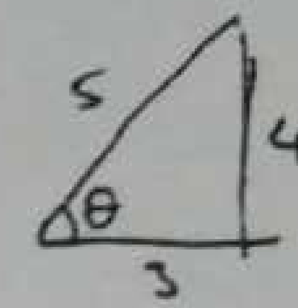
$\sin\theta=\frac{x}{\sqrt{1+x^2}}$



$\cos\theta=\frac{1}{\sqrt{1+x^2}}\rightarrow(1)$

R.H.S:

let $\theta=\cot^{-1}(3/4)$



$\cot\theta=3/4$

$\tan\theta=4/3$

$\sin\theta=4/5\rightarrow(2)$

$(1)=(2)$

$\frac{1}{\sqrt{1+x^2}}=\frac{4}{5}$

$\sqrt{1+x^2}=\frac{5}{4}$

Square on both sides

$1+x^2=\frac{25}{16}$

$x^2=\frac{25}{16}-\frac{1}{1}=\frac{25-16}{16}=\frac{9}{16}$

$\boxed{x=\pm 3/4}$

Eg 4.10

Find the values of $\tan^{-1}(-1)+\cos^{-1}(1/2)+\sin^{-1}(-1/2)$

$\rightarrow$

$\tan^{-1}(-1)=-\pi/4$

$\cos^{-1}(1/2)=\pi/3$

$\sin^{-1}(-1/2)=-\pi/6$

$\tan^{-1}(-1)+\cos^{-1}(1/2)+\sin^{-1}(-1/2)=-\pi/4+\pi/3-\pi/6$

$=-\pi/12$

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5) Prove that $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \tan^{-1}\left[\dfrac{x+y+z-xyz}{1-xy-yz-zx}\right]$

$\rightarrow$

$$\tan^{-1}\left(\frac{x+y}{1-xy}\right) + \tan^{-1}z$$

$$= \tan^{-1}\left[\frac{\left(\frac{x+y}{1-xy}\right)+z}{\frac{1}{1}-\left(\frac{x+y}{1-xy}\right)z}\right]$$

$$= \tan^{-1}\left[\frac{\frac{x+y+z(1-xy)}{\cancel{1-xy}}}{\frac{(1-xy)-(x+y)z}{\cancel{1-xy}}}\right]$$

$$= \tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right]$$

6) If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$, show that $x+y+z = xyz$

$\rightarrow$

$$\tan^{-1}\left(\frac{x+y}{1-xy}\right) + \tan^{-1}z = \pi$$

$$\tan^{-1}\left(\frac{\frac{x+y}{1-xy}+\frac{z}{1}}{\frac{1}{1}-\left(\frac{x+y}{1-xy}\right)z}\right) = \pi$$

$$\frac{\frac{x+y+z(1-xy)}{1-xy}}{\frac{(1-xy)-(x+y)z}{1-xy}} = \tan\pi$$

$$x+y+z-xyz = 0$$

10) Find the number of solution of the eq. $\tan^{-1}(x-1) + \tan^{-1}(x) + \tan^{-1}(x+1) = \tan^{-1}(3x)$

$\rightarrow$

$$\tan^{-1}\left[\frac{(x-1)+(x+1)}{1-(x-1)(x+1)}\right] + \tan^{-1}x = \tan^{-1}(3x)$$

$$\tan^{-1}\left[\frac{2x}{1-(x^2-1)}\right] + \tan^{-1}x = \tan^{-1}(3x)$$

$$\tan^{-1}\left(\frac{2x}{2-x^2}\right) + \tan^{-1}x = \tan^{-1}(3x)$$

$$\tan^{-1}\left(\frac{\frac{2x}{2-x^2}+x}{1-\left(\frac{2x}{2-x^2}\right)x}\right) = \tan^{-1}(3x)$$

$$\cancel{\tan^{-1}}\left(\frac{4x-x^3}{2-3x^2}\right) = \cancel{\tan^{-1}}(3x)$$

$$4x-x^3 = 3x(2-3x^2)$$

$$4x-x^3 = 6x-9x^3$$

$$8x^3-2x = 0$$

$$\boxed{x=0,\ x=\pm 1/2}$$

9(iv) solve $\cot^{-1}x - \cot^{-1}(x+2) = \pi/12$

$\rightarrow$

$$\tan^{-1}(1/x) - \tan^{-1}\left(\frac{1}{x+2}\right) = \pi/12$$

$$\tan^{-1}\left[\frac{1/x - \frac{1}{x+2}}{1+(1/x)\left(\frac{1}{x+2}\right)}\right] = \frac{180}{12}$$

$$\tan^{-1}\left[\frac{\frac{(x+2)-x}{x\cancel{(x+2)}}}{\frac{x(x+2)+1}{x\cancel{(x+2)}}}\right] = 15$$

$$\frac{2}{x^2+2x+1} = \tan 15^\circ$$

$$\frac{2}{(x+1)^2} = \tan(60^\circ-45^\circ)$$

$$\frac{2}{(x+1)^2} = \frac{\tan 60^\circ - \tan 45^\circ}{1+\tan 60^\circ\cdot\tan 45^\circ}$$

$$\frac{2}{(x+1)^2} = \frac{\sqrt{3}-1}{1+\sqrt{3}}$$

$$\frac{2}{(x+1)^2} = \frac{\sqrt{3}-1}{\sqrt{3}+1}\times\frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$\boxed{x=\sqrt{3}}$$

eg 4.28

$$\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \pi/4$$

$\rightarrow$

$$\tan^{-1}\left(\frac{\frac{x-1}{x-2}+\frac{x+1}{x+2}}{1-\left(\frac{x-1}{x-2}\right)\left(\frac{x+1}{x+2}\right)}\right) = \pi/4$$

$$\frac{\frac{(x-1)(x+2)+(x+1)(x-2)}{(x-2)\cancel{(x+2)}}}{\frac{(x-2)(x+2)-(x-1)(x+1)}{(x-2)\cancel{(x+2)}}} = \tan\pi/4$$

$$\boxed{x = \pm 1/\sqrt{2}}$$

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4.2 Solve $\tan^{-1}(2x) + \tan^{-1}(3x) = \pi/4$

$\rightarrow$

$$\tan^{-1}\left[\frac{2x+3x}{1-(2x)(3x)}\right] = \pi/4$$

$$\tan^{-1}\frac{5x}{1-6x^2} = \pi/4$$

$$\frac{5x}{1-6x^2} = \tan \pi/4$$

$$\frac{5x}{1-6x^2} = 1$$

$$x = -1, \quad x = \frac{1}{6}$$

$x = -1$ is not possible, $6x^2 < 1$

$$\boxed{x = \frac{1}{6}}$$

Ex 4.5 (4) P.T $\tan^{-1}x + \tan^{-1}\frac{2x}{1-x^2} = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$

$\rightarrow$

$$= \tan^{-1}\left(\frac{\frac{x}{1} + \frac{2x}{1-x^2}}{\frac{1}{1} - x \cdot \frac{2x}{1-x^2}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{x(1-x^2)+2x}{\cancel{1-x^2}}}{\frac{1-x^2-x(2x)}{\cancel{1-x^2}}}\right)$$

$$= \tan^{-1}\left(\frac{x(1-x^2)+2x}{1-x^2-2x^2}\right)$$

$$= \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$$

---

9) (i) Solve $\sin^{-1} 5/x + \sin^{-1}\frac{12}{x} = \pi/2$

$\rightarrow$

$$\sin\theta = \frac{12}{x}$$

$$\cos\theta = 5/x$$

we know that

$$\cos^2\theta + \sin^2\theta = 1$$

$$\left(\frac{5}{x}\right)^2 + \left(\frac{12}{x}\right)^2 = 1$$

$$\frac{25}{x^2} + \frac{144}{x^2} = 1$$

$$\frac{169}{x^2} = 1$$

$$x^2 = 169$$

$$\boxed{x = \pm 13}$$

(ii) P.T $2\tan^{-1}x = \cos^{-1}\left(\frac{1-a^2}{1+a^2}\right) - \cos^{-1}\left(\frac{1-b^2}{1+b^2}\right)$

$\rightarrow$

R.H.S :

$$\cos^{-1}\left(\frac{1-a^2}{1+a^2}\right) = 2\tan^{-1}a$$

$$\cos^{-1}\left(\frac{1-b^2}{1+b^2}\right) = 2\tan^{-1}b$$

$$\cos^{-1}\left(\frac{1-a^2}{1+a^2}\right) - \cos^{-1}\left(\frac{1-b^2}{1+b^2}\right) = 2\tan^{-1}a - 2\tan^{-1}b$$

$$= 2\left[\tan^{-1}a - \tan^{-1}b\right]$$

$$= 2\left[\tan^{-1}\left(\frac{a-b}{1+ab}\right)\right]$$

$$= 2\tan^{-1}x$$

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Ex 4.3 (4) (ii) Find the value of

$$\sin\left[\tan^{-1}(1/2) - \cos^{-1}(4/5)\right]$$

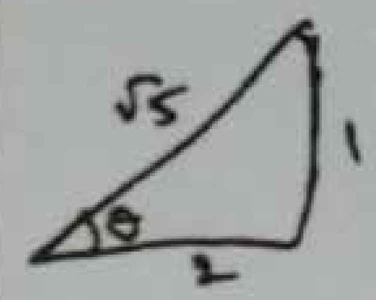
$\rightarrow$

$A = \tan^{-1} 1/2$

$\tan A = 1/2$

$\sin A = \frac{1}{\sqrt{5}}$

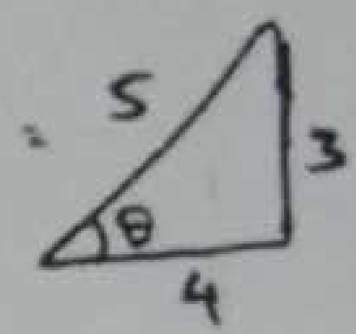
$\cos A = \frac{2}{\sqrt{5}}$



$B = \cos^{-1}(4/5)$

$\cos B = 4/5$

$\sin B = 3/5$



$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$= \frac{1}{\sqrt{5}} \cdot \frac{4}{5} - \frac{2}{\sqrt{5}} \cdot \frac{3}{5}$$

$$= \frac{4}{5\sqrt{5}} - \frac{6}{5\sqrt{5}} = \frac{-2}{5\sqrt{5}}$$

---

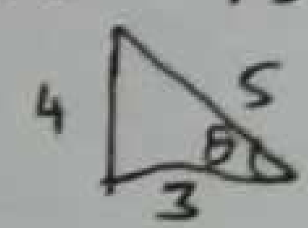
4) (iii) $\cos\left[\sin^{-1}(4/5) - \tan^{-1}(3/4)\right]$

$\rightarrow$

$A = \sin^{-1}(4/5)$

$\sin A = 4/5$

$\cos A = 3/5$

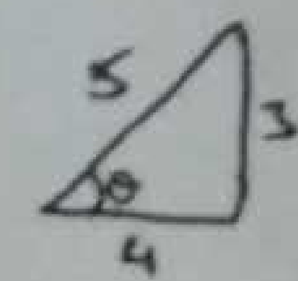


$B = \tan^{-1}(3/4)$

$\tan B = 3/4$

$\sin B = 3/5$

$\cos B = 4/5$



$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$= \frac{3}{5} \cdot \frac{4}{5} + \frac{4}{5} \cdot \frac{3}{5}$$

$$= \frac{12}{5} + \frac{12}{5} = 24/25$$

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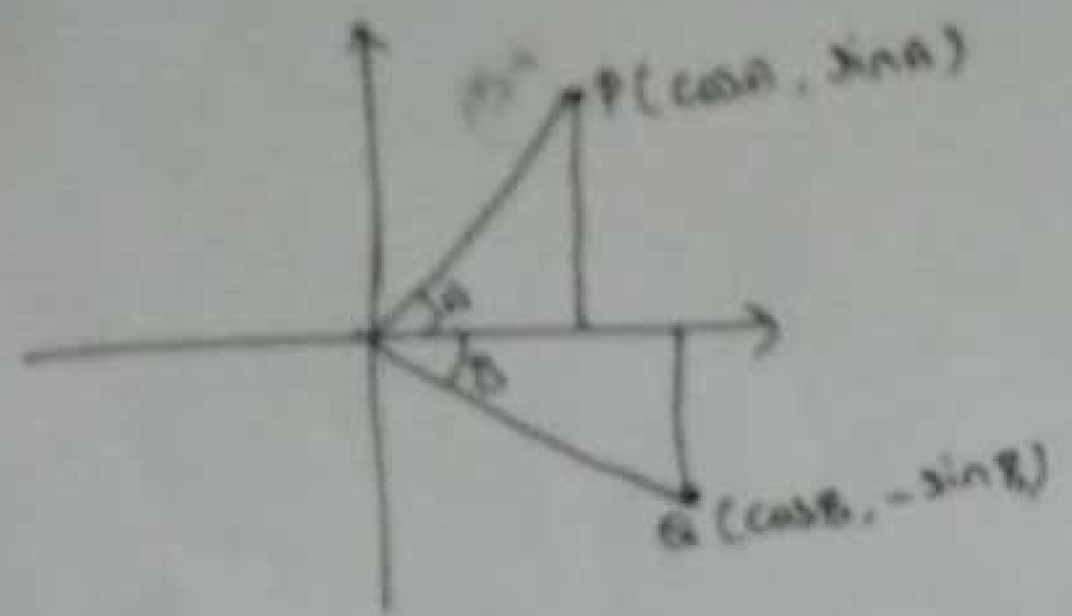
(1) P.T $\sin(A+B) = \sin A\cos B + \cos A\sin B$



$\vec{OP} = \cos A\,\vec{i} + \sin A\,\vec{j}$

$\vec{OQ} = \cos B\,\vec{i} - \sin B\,\vec{j}$

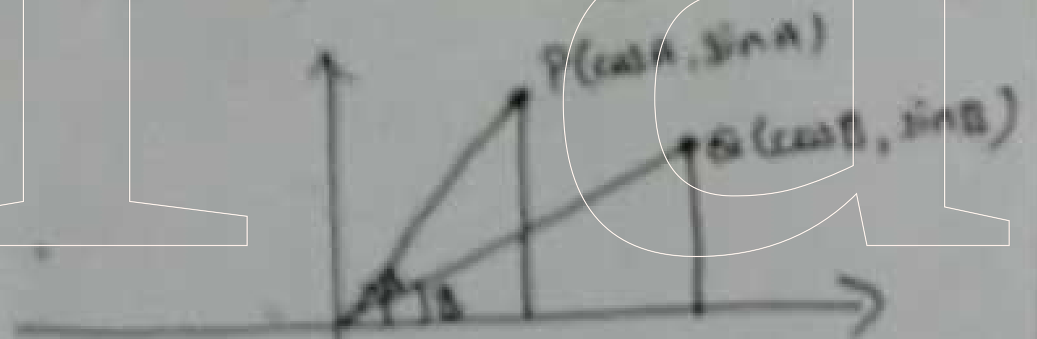
now, $\vec{OQ}\times\vec{OP} = \vec{k}\,(\sin A\cos B + \cos A\sin B) \to (1)$

by defn $\vec{OQ}\times\vec{OP} = \vec{k}\,\sin(A+B) \to (2)$

from (1) & (2)

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

eg 6.5 P.T $\sin(A-B) = \sin A\cos B - \cos A\sin B$



$\vec{OP} = \cos A\,\vec{i} + \sin A\,\vec{j}$

$\vec{OQ} = \cos B\,\vec{i} + \sin B\,\vec{j}$

now, $\vec{OQ}\times\vec{OP} = \vec{k}\,[\sin A\cos B - \cos A\sin B] \to (1)$

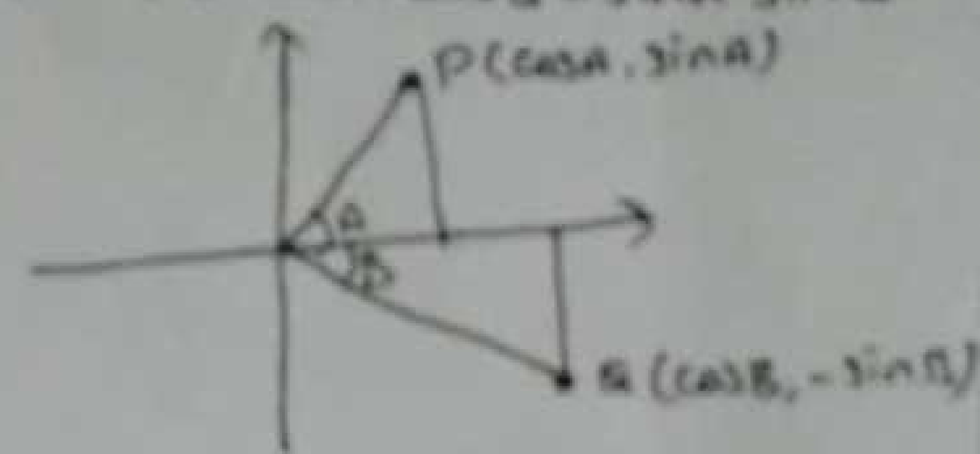
by defn $\vec{OQ}\times\vec{OP} = \vec{k}\,\sin(A-B) \to (2)$

from (1) & (2)

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

eg 6.3

P.T $\cos(A+B) = \cos A\cos B - \sin A\sin B$



$\vec{OP} = \cos A\,\vec{i} + \sin A\,\vec{j}$

$\vec{OQ} = \cos B\,\vec{i} - \sin B\,\vec{j}$

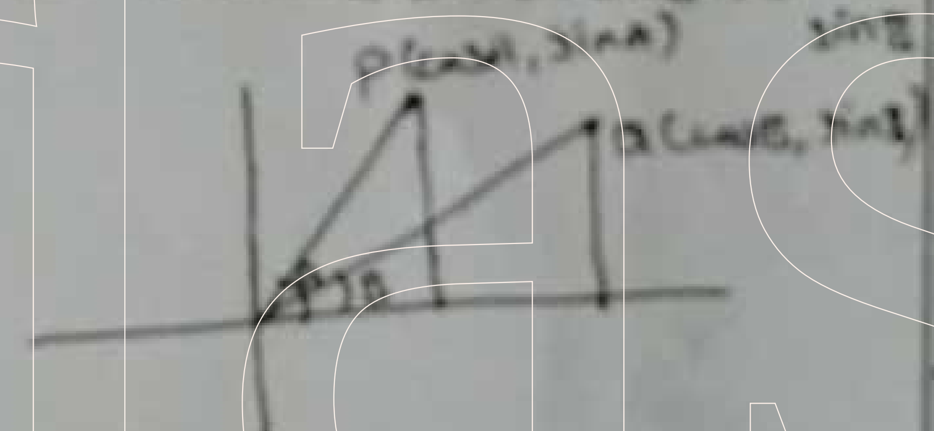
now, $\vec{OQ}\cdot\vec{OP} = [\cos A\cos B - \sin A\sin B] \to (1)$

by defn $\vec{OQ}\cdot\vec{OP} = \cos(A+B) \to (2)$

from (1) & (2)

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

Ex 6.1 (4) P.T $\cos(A-B) = \cos A\cos B + \sin A\sin B$



$\vec{OP} = \cos A\,\vec{i} + \sin A\,\vec{j}$

$\vec{OQ} = \cos B\,\vec{i} + \sin B\,\vec{j}$

now, $\vec{OQ}\cdot\vec{OP} = [\cos A\cos B + \sin A\sin B] \to (1)$

by defn $\vec{OQ}\cdot\vec{OP} = \cos(A-B) \to (2)$

from (1) & (2)

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

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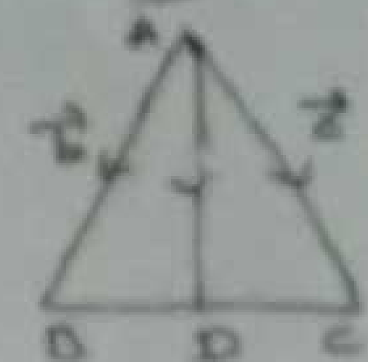
eg 6.9 S&P Apollonius theorem

Stmt: If D is the mid point of the side BC of a $\triangle ABC$, then $|\vec{AB}|^2 + |\vec{AC}|^2 = 2\left[|\vec{AD}|^2 + |\vec{BD}|^2\right]$



Proof:

$\vec{AD} = \frac{\vec{b}+\vec{c}}{2}$

R.H.S:

$|\vec{AD}|^2 = \vec{AD}\cdot\vec{AD}$

$= \frac{\vec{b}+\vec{c}}{2}\cdot\frac{\vec{b}+\vec{c}}{2}$

$|\vec{AD}|^2 = \frac{1}{4}\left[|\vec{b}|^2 + |\vec{c}|^2 + 2\vec{b}\cdot\vec{c}\right] \to (1)$

now, $\vec{BD} = \frac{\vec{c}-\vec{b}}{2}$

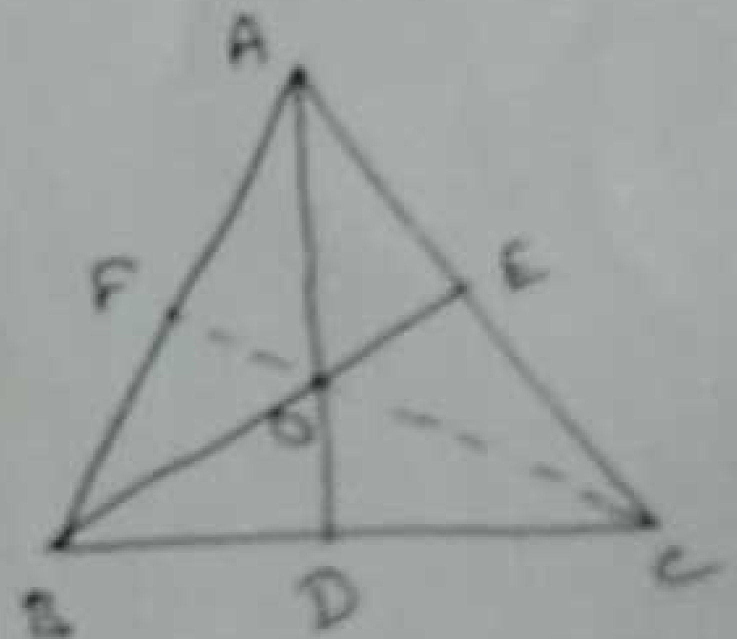
$|\vec{BD}|^2 = \frac{1}{4}\left[|\vec{b}|^2 + |\vec{c}|^2 - 2\vec{b}\cdot\vec{c}\right] \to (2)$

(1) + (2) $|\vec{AD}|^2 + |\vec{BD}|^2 = \frac{1}{4}\left[2\left(|\vec{b}|^2 + |\vec{c}|^2\right)\right]$

$2\left[|\vec{AD}|^2 + |\vec{BD}|^2\right] = |\vec{AB}|^2 + |\vec{AC}|^2$

eg 6.7 Prove by vector method perpendicular (altitudes) from the vertices to the opposite of $\triangle$ are concurrent.

→



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$\vec{OA}\cdot\vec{BC}=0$

$\vec{OA}\cdot(\vec{OC}-\vec{OB})=0$

$\vec{OA}\cdot\vec{OC}-\vec{OA}\cdot\vec{OB}=0 \to (1)$

now, $\vec{OB}\cdot\vec{CA}=0$

$\vec{OB}\cdot(\vec{OA}-\vec{OC})=0$

$\vec{OB}\cdot\vec{OA}-\vec{OB}\cdot\vec{OC}=0 \to (2)$

Add (1) & (2)

$\vec{OA}\cdot\vec{OC}-\vec{OB}\cdot\vec{OC}=0$

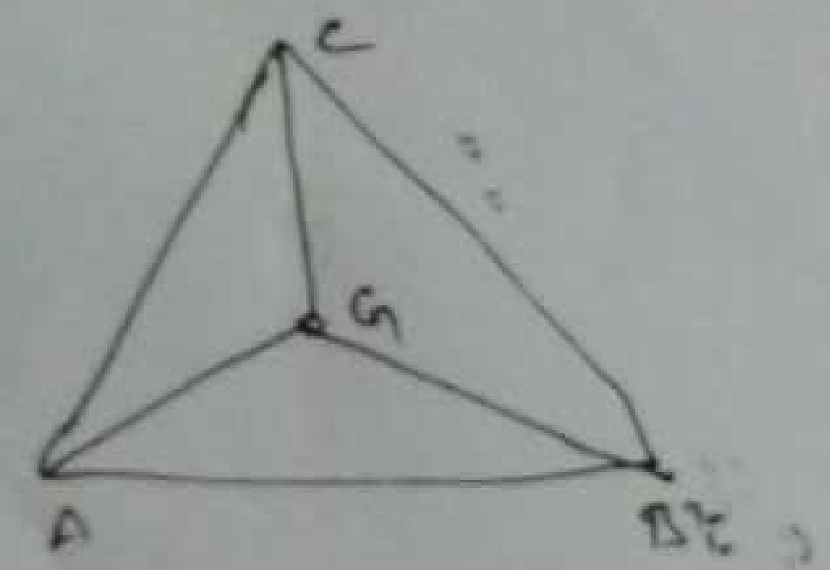
$\vec{BA}\cdot\vec{OC}=0$

$BA\perp\vec{OC}$

$\vec{BA}\perp\vec{CF}$

Ex 6.1 (8) If G is the centroid of a ΔABC, prove that area of ΔGAB = (area of ΔGBC) = area of ΔGCA) = $1/3$ (area of ΔABC)

$\rightarrow$



Area of $\Delta GAB = \frac{1}{2}|\vec{AB}\times\vec{AG}|$

$= \frac{1}{2}|\vec{AB}\times(\vec{OG}-\vec{OA})|$

$= \frac{1}{2}\left|\vec{AB}\times\left(\frac{\vec{OA}+\vec{OB}+\vec{OC}}{3}-\vec{OA}\right)\right|$

$= \frac{1}{2}\left|\vec{AB}\times\frac{\vec{OA}+\vec{OB}+\vec{OC}-3\vec{OA}}{3}\right|$

$= \frac{1}{2}\times\frac{1}{3}\left|\vec{AB}\times(\vec{OB}-\vec{OA})+(\vec{OC}-\vec{OA})\right|$

$= \frac{1}{3}\times\left[\frac{1}{2}|\vec{AB}\times\vec{BC}|\right]$

$= \frac{1}{3}$ Area of ΔABC

$||^{ly}$ Area of $\Delta GBC = \frac{1}{3}$ Area of ΔABC

Area of $\Delta GCA = 1/3$ Area of ΔABC

Ex 6.3 (4) If $\vec{a}=2\vec{i}+3\vec{j}-\vec{k}$, $\vec{b}=3\vec{i}+5\vec{j}+2\vec{k}$, $\vec{c}=-\vec{i}-2\vec{j}+3\vec{k}$, verify

(i) $(\vec{a}\times\vec{b})\times\vec{c}=(\vec{a}\cdot\vec{c})\vec{b}-(\vec{b}\cdot\vec{c})\vec{a}$

L.H.S:

$$\vec{a}\times\vec{b}=\begin{vmatrix}\vec{i}&\vec{j}&\vec{k}\\2&3&-1\\3&5&2\end{vmatrix}$$

$=\vec{i}(6+5)-\vec{j}(4+3)+\vec{k}(10-9)$

$\vec{a}\times\vec{b}=11\vec{i}-7\vec{j}+\vec{k}$

$$(\vec{a}\times\vec{b})\times\vec{c}=\begin{vmatrix}\vec{i}&\vec{j}&\vec{k}\\11&-7&1\\-1&-2&3\end{vmatrix}$$

$=\vec{i}(-21+2)-\vec{j}(33+1)+\vec{k}(-22-7)$

$(\vec{a}\times\vec{b})\times\vec{c}=-19\vec{i}-34\vec{j}-29\vec{k}$

R.H.S:

$\vec{a}\cdot\vec{c}=(2\vec{i}+3\vec{j}-\vec{k})\cdot(-\vec{i}-2\vec{j}+3\vec{k})$

$=-2-6-3$

$=-11$

$(\vec{a}\cdot\vec{c})\vec{b}=-11(3\vec{i}+5\vec{j}+2\vec{k})$

$=-33\vec{i}-55\vec{j}-22\vec{k}$

$\vec{b}\cdot\vec{c}=(3\vec{i}+5\vec{j}+2\vec{k})\cdot(-\vec{i}-2\vec{j}+3\vec{k})$

$=-3-10+6$

$=-7$

$(\vec{b}\cdot\vec{c})\vec{a}=-7(2\vec{i}+3\vec{j}-\vec{k})$

$=-14\vec{i}-21\vec{j}+7\vec{k}$

$(\vec{a}\cdot\vec{c})\vec{b}-(\vec{b}\cdot\vec{c})\vec{a}=-19\vec{i}-34\vec{j}-29\vec{k}$

$\therefore$ L.H.S = R.H.S

(ii) $\vec{a}\times(\vec{b}\times\vec{c})=(\vec{a}\cdot\vec{c})\vec{b}-(\vec{a}\cdot\vec{b})\vec{c}$

L.H.S:

$$\vec{b}\times\vec{c}=\begin{vmatrix}\vec{i}&\vec{j}&\vec{k}\\3&5&2\\-1&-2&3\end{vmatrix}$$

$=\vec{i}(15+4)-\vec{j}(9+2)+\vec{k}(-6+5)$

$=19\vec{i}-11\vec{j}-\vec{k}$

$$\vec{a}\times(\vec{b}\times\vec{c})=\begin{vmatrix}\vec{i}&\vec{j}&\vec{k}\\2&3&-1\\19&-11&-1\end{vmatrix}$$

$=\vec{i}(-3-11)-\vec{j}(-2+19)+\vec{k}(-22-57)$

$=-14\vec{i}-17\vec{j}-79\vec{k}$

R.H.S:

$\vec{a}\cdot\vec{c}=-11$

$(\vec{a}\cdot\vec{c})\vec{b}=-33\vec{i}-55\vec{j}-22\vec{k}$

$\vec{a}\cdot\vec{b}=19$

$(\vec{a}\cdot\vec{b})\vec{c}=-19\vec{i}-38\vec{j}+57\vec{k}$

$(\vec a\cdot\vec c)\vec b-(\vec a\cdot\vec b)\vec c=-14\vec i-17\vec j-79\vec k$

Eg 6.23 If $\vec a=\vec i-\vec j$, $\vec b=\vec i-\vec j-4\vec k$, $\vec c=3\vec j-\vec k$, $\vec d=2\vec i+5\vec j+\vec k$, verify

(i) $(\vec a\times\vec b)\times(\vec c\times\vec d)=[\vec a\ \vec b\ \vec d]\vec c-[\vec a\ \vec b\ \vec c]\vec d$

L.H.S:

$$\vec a\times\vec b=\begin{vmatrix}\vec i & \vec j & \vec k\\ 1 & -1 & 0\\ 1 & -1 & -4\end{vmatrix}$$

$=\vec i(4+0)-\vec j(-4-0)+\vec k(-1+1)$

$=4\vec i+4\vec j$

$$\vec c\times\vec d=\begin{vmatrix}\vec i & \vec j & \vec k\\ 0 & 3 & -1\\ 2 & 5 & 1\end{vmatrix}$$

$=\vec i(3+5)-\vec j(0+2)+\vec k(0-6)$

$=8\vec i-2\vec j-6\vec k$

$$(\vec a\times\vec b)\times(\vec c\times\vec d)=\begin{vmatrix}\vec i & \vec j & \vec k\\ 4 & 4 & 0\\ 8 & -2 & -6\end{vmatrix}$$

$=\vec i(-24+0)-\vec j(-24-0)+\vec k(-8-32)$

$=-24\vec i+24\vec j-40\vec k$

R.H.S:

$$[\vec a\ \vec b\ \vec d]=\begin{vmatrix}1 & -1 & 0\\ 1 & -1 & -4\\ 2 & 5 & 1\end{vmatrix}$$

$=1(-1+20)-1(1+8)$

$=19+9$

$=28$

$$[\vec a\ \vec b\ \vec c]=\begin{vmatrix}1 & -1 & 0\\ 1 & -1 & -4\\ 2 & 5 & 1\end{vmatrix}$$

$=1(1+12)+1(-1+0)$

$=12$

$[\vec a\ \vec b\ \vec d]\vec c=28(3\vec j-\vec k)$

$=84\vec j-28\vec k$

$[\vec a\ \vec b\ \vec c]\vec d=12(2\vec i+5\vec j+\vec k)$

$=24\vec i+60\vec j+12\vec k$

$[\vec a\ \vec b\ \vec d]\vec c-[\vec a\ \vec b\ \vec c]\vec d=-24\vec i+24\vec j-40\vec k$

Ex 6.5 (4) Show that the lines $\frac{x-3}{3}=\frac{y-3}{-1}$, $z-1=0$ and $\frac{x-6}{2}=\frac{z-1}{3}$, $y-2=0$ intersect. Also find the point of intersection.

$\rightarrow$ $(x_1, y_1, z_1)=(3,3,1)$

$(x_2\ y_2\ z_2)=(6,2,1)$

$(b_1, b_2\ b_3)=(3,-1,0)$

$(d_1\ d_2\ d_3)=(2,0,3)$

$$=\begin{vmatrix}x_2-x_1 & y_2-y_1 & z_2-z_1\\ b_1 & b_2 & b_3\\ d_1 & d_2 & d_3\end{vmatrix}$$

$$=\begin{vmatrix}6-3 & 2-3 & 0\\ 3 & -1 & 0\\ 2 & 0 & 3\end{vmatrix}$$

$$=\begin{vmatrix}3 & -1 & 0\\ 3 & -1 & 0\\ 2 & 0 & 3\end{vmatrix}\quad R_1\equiv R_2$$

$=0$

The lines are intersect.

Point of Intersection:

$\frac{x-3}{3}=\frac{y-3}{-1}=\lambda$, $z=1$

Point is $(3\lambda+3, -\lambda+3, 1)\rightarrow$ (1)

And $\frac{x-6}{2}=\frac{z-1}{3}=\mu$, $y=2$

Point is $(2\mu+6, 3\mu+1, 2)\rightarrow$ (2)

(1) = (2) we get $\boxed{\lambda=1}$

$\therefore$ Point is $(6,2,1)$

I. Eq. of a Plane given one Point and Parallel to two given vectors:

vector form:

$$\vec r=\vec a+s\vec u+t\vec v$$

cartesian form:

$$\begin{vmatrix}x-x_1 & y-y_1 & z-z_1\\ l_1 & m_1 & n_1\\ l_2 & m_2 & n_2\end{vmatrix}=0.$$

II. Eq. of a Plane Passing through two Points and Parallel to a vector.

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vector form :

$$\vec{r} = (1-s)\vec{a} + s\vec{b} + t\vec{u}$$

cartesian form :

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l & m & n \end{vmatrix} = 0$$

III Eq. of a Plane Passing through three given non-collinear Points.

vector form :

$$\vec{r} = (1-s-t)\vec{a} + s\vec{b} + t\vec{c}$$

cartesian form :

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

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CHAPTER-12 [DISCRETE MATHEMATICS]

Eg 12.19

using the equivalence properties, show that

$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

$\rightarrow$ w.k.t

$p \leftrightarrow q \equiv (\neg p \vee q) \wedge (\neg q \vee p)$

$\equiv (\neg p \vee q) \wedge (p \vee \neg q)$

$\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q) \vee (q \wedge p) \vee (q \wedge \neg q)$

$\equiv (\neg p \wedge \neg q) \vee (q \wedge p)$

$\equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

Eg 12.9

$Z_5 = \{[0], [1], [2], [3], [4]\}$

$+_5$	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

(i) closure axiom:

All the entries of the elements of Z_5.

$\therefore$ closure axiom is true.

(ii) Associative ~~commutative~~ axiom:

$+_5$ is always associative for Z_5.

$\therefore$ Associative axiom is true.

(iii) commutative axiom:

$+_5$ is always commutative for Z_5

$\therefore$ commutative axiom is true.

(iv) Identity axiom:

clearly, 0 is the Identity element of Z_5.

$\therefore$ Identity axiom is true.

(v) Inverse axiom:

Inverse of 1 is 4
2 is 3
3 is 2
4 is 1

$\therefore$

Eg 12.10

$A = \{1, 3, 4, 5, 9\}$

$\times_{11}$	1	3	4	5	9
1	1	3	4	5	9
3	3	9	1	4	5
4	4	1	5	9	3
5	5	4	9	3	1
9	9	5	3	1	4

(i) closure axiom:

All the entries of the element of A.

$\therefore$ closure axiom is true.

(ii) Associative axiom:

$\times_{11}$ is always associative in A.

$\therefore$ Associative axiom is true.

(iii) commutative axiom:

$\times_{11}$ is always commutative in A.

$\therefore$ commutative axiom is true.

(iv) Identity axiom:

clearly, 1 is an Identity element

$\therefore$ Identity axiom is true.

(v) Inverse axiom:

Inverse of 1 is 1
3 is 4
4 is 3
5 is 9
9 is 5

$\therefore$ Inverse axiom is true.

Ex 12.1

(5) (i) closure axiom:

let $a, b \in Q$

$a * b = \frac{a+b}{2} \in Q$

$\therefore$ closure axiom is true.

commutative axiom:

$a * b = \frac{a+b}{2}$

$b * a = \frac{b+a}{2}$

$\therefore a * b = b * a$

$\therefore$ commutative axiom is true.

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Associative axiom:

$(a*b)*c = a*(b*c)$

$\left(\frac{a+b}{2}\right)*c = a*\left(\frac{b+c}{2}\right)$

$\frac{a+b+2c}{4} \neq \frac{2a+b+c}{4}$

$*$ is not associative

) Identity axiom:

$a*e = a$

$\frac{a+e}{2} = a$

$a+e = 2a$

$\boxed{e=a}$

It is not Possible, Identity is not satisfied. and so Inverse is not satisfied.

---

9) (i) let $A = \begin{pmatrix} x & x \\ x & x \end{pmatrix}$, $B = \begin{pmatrix} y & y \\ y & y \end{pmatrix}$

Closure axiom:

$AB = \begin{pmatrix} x & x \\ x & x \end{pmatrix}\begin{pmatrix} y & y \\ y & y \end{pmatrix}$

$AB = \begin{pmatrix} 2xy & 2xy \\ 2xy & 2xy \end{pmatrix} \in M$

$\therefore$ closure axiom is true.

Commutative axiom:

$AB = \begin{pmatrix} 2xy & 2xy \\ 2xy & 2xy \end{pmatrix}$

$BA = \begin{pmatrix} 2xy & 2xy \\ 2xy & 2xy \end{pmatrix}$

$AB = BA$

$\therefore$ commutative axiom is true.

Associative axiom:

let $A = \begin{pmatrix} x & x \\ x & x \end{pmatrix}$, $B = \begin{pmatrix} y & y \\ y & y \end{pmatrix}$, $C = \begin{pmatrix} z & z \\ z & z \end{pmatrix}$

$(AB)C = A(BC)$

$\begin{pmatrix} 2xy & 2xy \\ 2xy & 2xy \end{pmatrix}\begin{pmatrix} z & z \\ z & z \end{pmatrix} = \begin{pmatrix} x & x \\ x & x \end{pmatrix}\begin{pmatrix} 2yz & 2yz \\ 2yz & 2yz \end{pmatrix}$

$\begin{pmatrix} 4xyz & 4xyz \\ 4xyz & 4xyz \end{pmatrix} = \begin{pmatrix} 4xyz & 4xyz \\ 4xyz & 4xyz \end{pmatrix}$

$\therefore$ Associative axiom is true.

(ii) closure axiom:

$AB = \begin{pmatrix} 2xy & 2xy \\ 2xy & 2xy \end{pmatrix} \in M$

$\therefore$ closure axiom is true.

Identity axiom:

let $E = \begin{pmatrix} e & e \\ e & e \end{pmatrix}$

$AE = A$

$\begin{pmatrix} x & x \\ x & x \end{pmatrix}\begin{pmatrix} e & e \\ e & e \end{pmatrix} = \begin{pmatrix} x & x \\ x & x \end{pmatrix}$

$\begin{pmatrix} 2xe & 2xe \\ 2xe & 2xe \end{pmatrix} = \begin{pmatrix} x & x \\ x & x \end{pmatrix}$

now, $2\cancel{x}e = \cancel{x}$

$\boxed{e = 1/2}$

$\therefore E = \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}$

Inverse axiom:

let $A^{-1} = \begin{pmatrix} y & y \\ y & y \end{pmatrix}$

$AA^{-1} = E$

$\begin{pmatrix} x & x \\ x & x \end{pmatrix}\begin{pmatrix} y & y \\ y & y \end{pmatrix} = \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}$

$\begin{pmatrix} 2xy & 2xy \\ 2xy & 2xy \end{pmatrix} = \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}$

$2xy = 1/2$

$\boxed{y = 1/4x}$

$A^{-1} = \begin{pmatrix} 1/4x & 1/4x \\ 1/4x & 1/4x \end{pmatrix}$

10) given $x*y = x+y-xy$

we Assume $x*y = 1$

$x+y-xy = 1$

$y-xy = 1-x$

$y\cancel{(1-x)} = \cancel{(1-x)}$

$y = 1$

~~It is a~~ our assumption is wrong

$\therefore x*y \neq 1$

$\therefore$ closure axiom is true.

Commutative axiom:

$x*y = x+y-xy$

$y*x = y+x-xy$

$\therefore x*y = y*x$

$\therefore$ commutative axiom is true.

Associative axiom:

$x*(y*z) = (x*y)*z$

$x+y+z-xy-yz-zx+xyz = x+y+z-xy-yz-zx+xyz$

$\therefore$ Associative axiom is true.

(ii) Identity axiom :

$$a * e = a$$

$$\cancel{a} + e - ae = \cancel{a}$$

$$e(1-a) = 0$$

$$e = 0 \in G.$$

$\therefore$ Identity axiom is true.

Inverse axiom :

$$a * a^{-1} = e$$

$$a + a^{-1} - aa^{-1} = 0$$

$$a^{-1}(1-a) = -a$$

$$a^{-1} = \frac{-a}{1-a} \in G.$$

$\therefore$ Inverse axiom is true.

Truth table for AND ($\wedge$)

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Truth table for OR ($\vee$)

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Truth Table for $p \to q$

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Truth Table for $p \leftrightarrow q$

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Truth Table for $p \bar{\vee} q$

p	q	$p \bar{\vee} q$
T	T	F
T	F	T
F	T	T
F	F	F

Refer all truth table sums

eg 12.7 given $m * n = m + n - mn$

(i) closure axiom :

$m * n = m + n - mn$ is clearly an Integer

$\therefore$ closure axiom is true.

(ii) Associative axiom :

$$(m * n) * p = m * (n * p)$$

$$m + n + p - mn - np - pm + mnp = m + n + p - mn - np - pm + mnp$$

$\therefore$ Associative axiom is true.

(iii) Identity axiom :

$$m * e = m$$

$$\cancel{m} + e - me = \cancel{m}$$

$$e(1-m) = 0$$

$$e = 0 \in Z$$

$\therefore$ Identity axiom is true.

(iv) Inverse axiom :

$$m * m^{-1} = e$$

$$m + m^{-1} - mm^{-1} = 0$$

$$m^{-1}(1-m) = -m$$

$$m^{-1} = \frac{-m}{1-m} \in Z$$

case (i) : If $m = 2$

$m^{-1} = 2 \in Z$

$\therefore$ Inverse axiom is exist.

case (ii) : If $m \neq 2$

$m^{-1} \notin Z$

Inverse axiom does not exist

(v) commutative axiom :

$$m * n = m + n - mn$$

$$n * m = n + m - nm$$

$$m * n = n * m$$

$\therefore$ commutative axiom is true.

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2 and 3 marks

① In an algebraic structure the Identity element must be unique.

$\rightarrow$

let e_1 and e_2 be two Identity element

Treat e_1 as the Identity

$$e_1 * e_2 = e_2 \rightarrow ①$$

Treat e_2 as the Identity

$$e_1 * e_2 = e_1 \rightarrow ②$$

from ① & ②

$$e_1 = e_2$$

② In an algebraic structure the Inverse of an element must be unique.

$\rightarrow$

let a_1 and a_2 be two Inverse element

Treat a_1 as the ~~Identity~~ Inverse

$$a * a_1 = e \rightarrow ①$$

Treat a_2 as the Inverse

$$a * a_2 = e \rightarrow ②$$

now,

$$a_1 = a_1 * e = a_1 * (a * a_2) = \boxed{(a_1 * a) * a_2} = a_2$$

Eg 12.6

given $a * b = a^b$

(i) closure axiom:

$a, b \in N$

$a * b = a^b \in N$

$\therefore$ closure axiom is true.

(ii) commutative axiom:

$$a * b = a^b$$
$$b * a = b^a$$
$$\therefore a * b \neq b * a$$

It is not commutative.

(iii) associative axiom:

$$a * (b * c) = (a * b) * c$$
$$a^{(b^c)} \neq a^{bc}$$

$\therefore$ It is not associative.

Eg 12.8 $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$$A \vee B = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \vee \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$A \wedge B = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \wedge \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$$

Ex 12.1

① (i) $a, b \in R$,

$\therefore a|b| \in R$

$\therefore a * b \in R$

$*$ is binary on R.

(ii) $a, b \in R$

$\min(a, b) \in R$

$\therefore a * b \in R$

$*$ is binary on R.

(iii) $a, b \in R$

$\sqrt{b} \notin R$

$\therefore a\sqrt{b} \notin R$

$\therefore a * b \notin R$

$*$ is not a binary on R.

② Consider $m = -3, n = 2$ $m, n \in Z$

$$m \circledast n = m^n + n^m$$
$$= (-3)^2 + (2)^{-3}$$
$$= 9 + \frac{1}{8} = \frac{73}{8} \notin Z$$

$\circledast$ is not a binary operation on Z.

③ given $a * b = a + b + ab - 7$

let $a, b \in R$, then $ab \in R$ also $a + b \in R$

so. $a + b + ab - 7 \in R$.

$\therefore a * b \in R$

$\therefore *$ is binary on R.

now,

$$3 * \left(-\frac{7}{15}\right) = 3 + \left(-\frac{7}{15}\right) + (3)\left(-\frac{7}{15}\right) - 7$$
$$= 3 - \frac{7}{15} - \frac{21}{15} - 7 = -\frac{4}{1} - \frac{28}{15}$$
$$= -\frac{88}{15}$$

⑦ $b * c = b$ [from table]

$c * b = d$

$\therefore$ It is not commutative

now,

$$a * (b * c) = (a * b) * c$$
$$a * b = c * c$$
$$a \neq c$$

$\therefore$ It is not associative.

Refer all truth table sums

and

Refer eg 12.2, 12.3, 12.5 and Ex 12.1 $\rightarrow$ 4, 6

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5 MARKS

CHAPTER - 5
ANALYTICAL GEOMETRY

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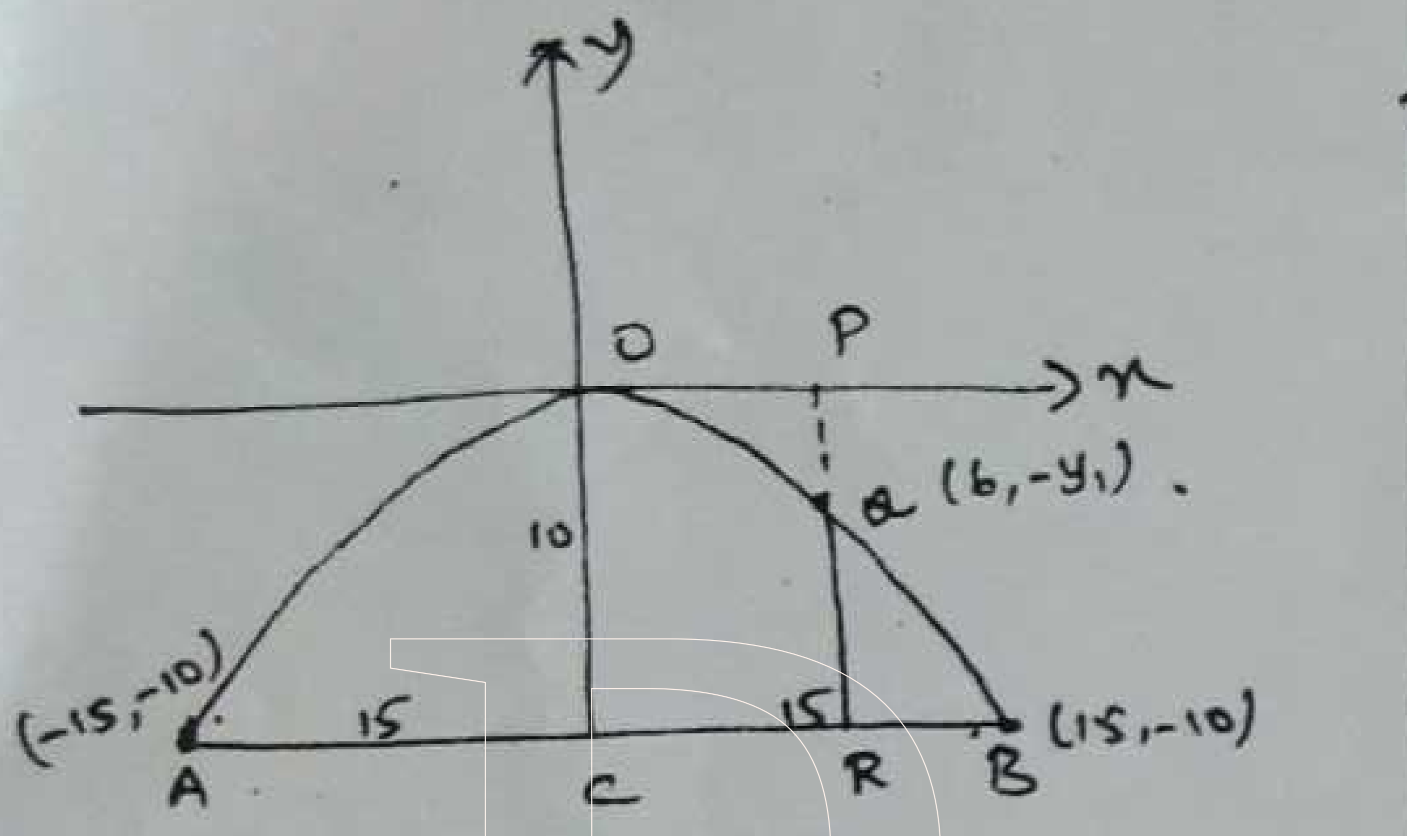
Ex 5.5

① Given $AB = 30M$

$AC = BC = 15M$

$OC = 10M$

Point $B(15, -10)$



Eq. of Parabola is $x^2 = -4ay \rightarrow$ ①

The Point is $(15, -10)$

$(15)^2 = -4a(-10)$

$225 = 40a$

$a = \frac{225}{40}$ in ①

$x^2 = -4\left(\frac{225}{40}\right)y$

$x^2 = -\frac{225}{10}y$

The Point is $(6, -y_1)$

$36 = -\frac{225}{10} \times -y_1$

$\frac{36 \times 10}{225} = y_1$

$\boxed{\frac{8}{5} = y_1}$

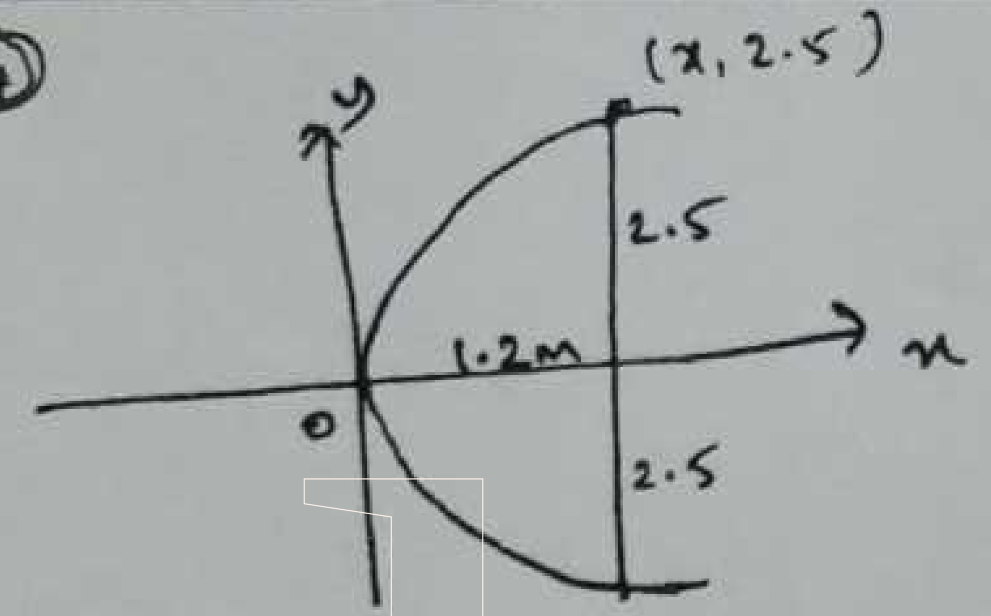
$\therefore QR = PR - PQ$

$= 10 - 8/5$

$= 42/5$

$\boxed{QR = 8.4M}$

④



Eq. of Parabola $y^2 = 4ax \rightarrow$ ①

The Point is $(1.2, 0)$

$a = 1.2$ in ①

$y^2 = 4(1.2)x$

$y^2 = 4.8x$

The Point is $(x_1, 2.5)$ lies

$(2.5)^2 = 4.8x_1$

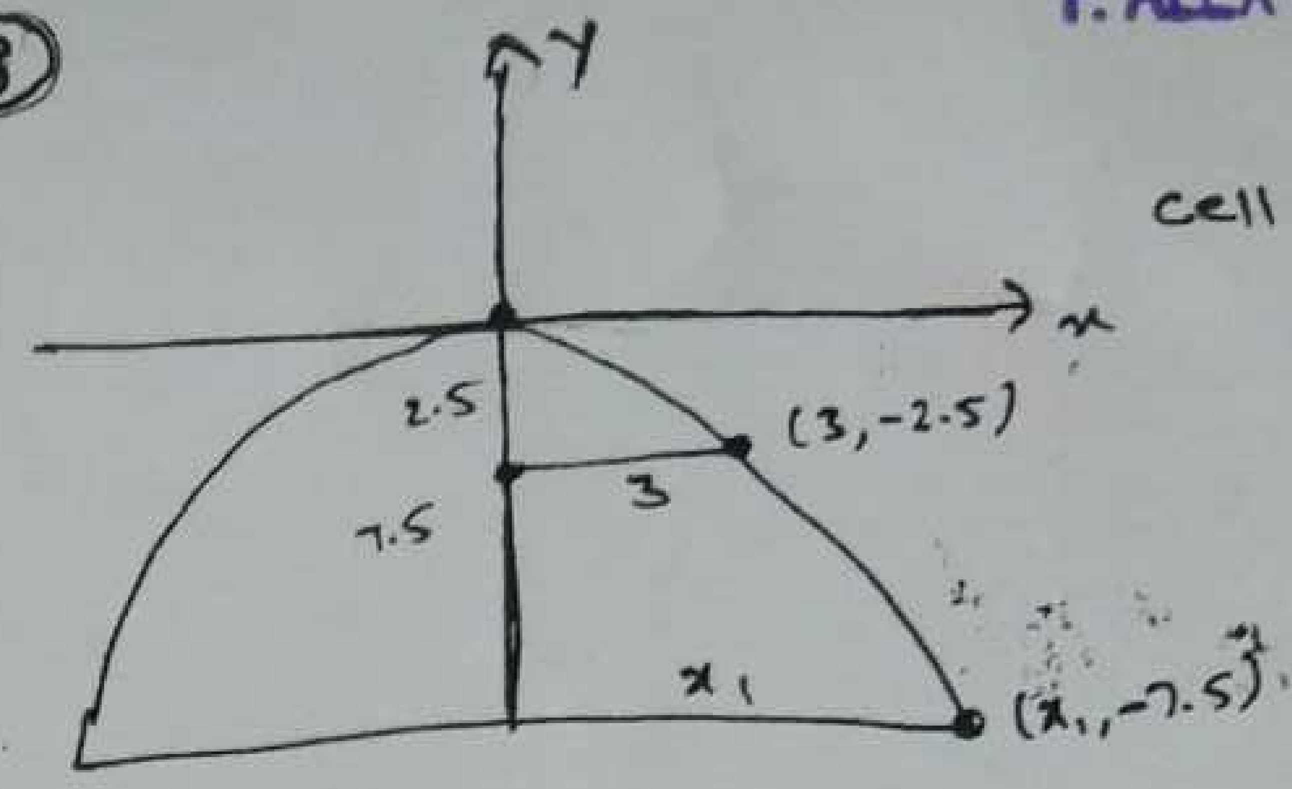
$\frac{2.5 \times 2.5}{4.8} = x_1$

$\boxed{x_1 = 1.3}$

(a) Eq. of Parabola $y^2 = 4.8x$

(b) Depth $= 1.3M$

⑧



Eq. of Parabola $x^2 = -4ay \rightarrow$ ①

The Point is $(3, -2.5)$

$(3)^2 = -4a(-2.5)$

$9 = 10a$

$\boxed{a = 9/10}$ in ①

$x^2 = -4 \times \frac{9}{10}y$

The Point is $(x_1, -7.5)$

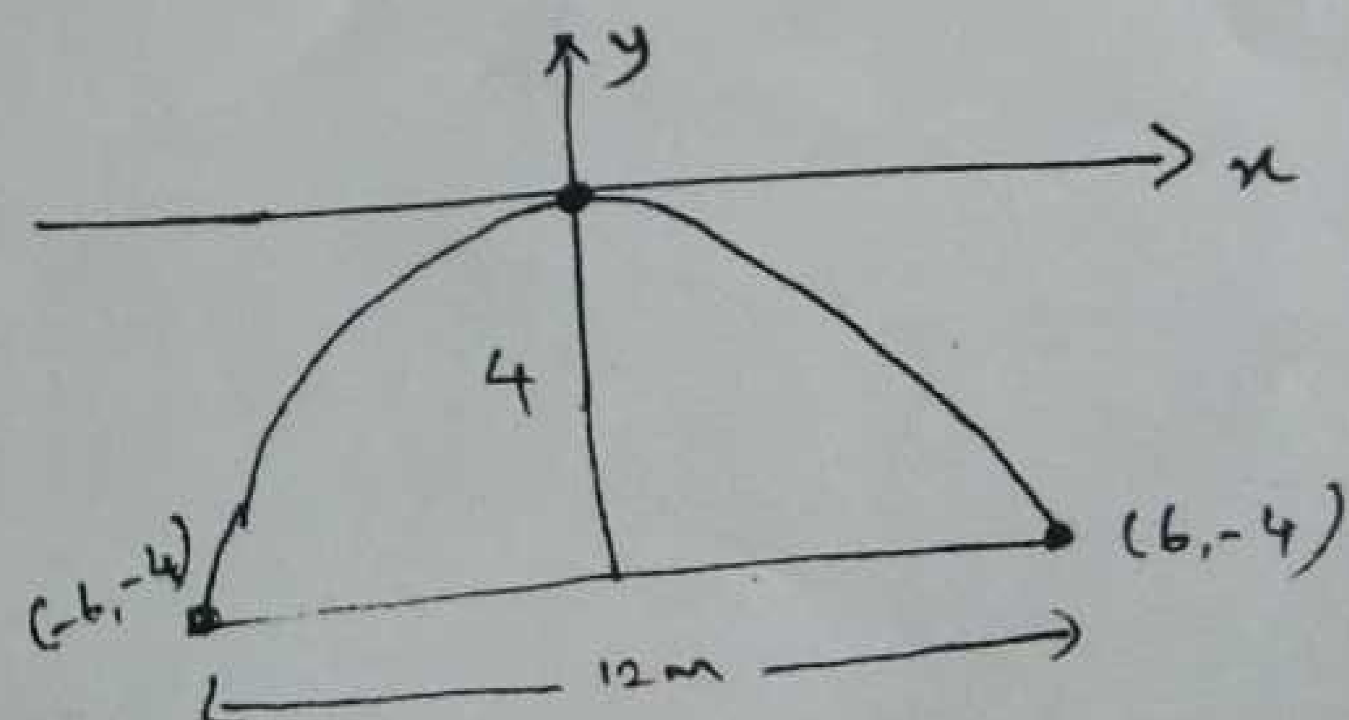
$x_1^2 = -4 \times \frac{9}{10} \times -7.5$

$= 30 \times \frac{9}{10}$

$x_1^2 = 27$

$x_1 = 3\sqrt{3}\,M$

⑨



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Eq. of Parabola $x^2 = -4ay \rightarrow (1)$

the Point is $(6, -4)$

$$(6)^2 = -4a(-4)$$
$$36 = 16a$$
$$\boxed{a = 9/4} \text{ in } (1)$$
$$x^2 = -\cancel{4} \times \frac{9}{\cancel{4}} y$$
$$x^2 = -9y$$

~~The Point is~~

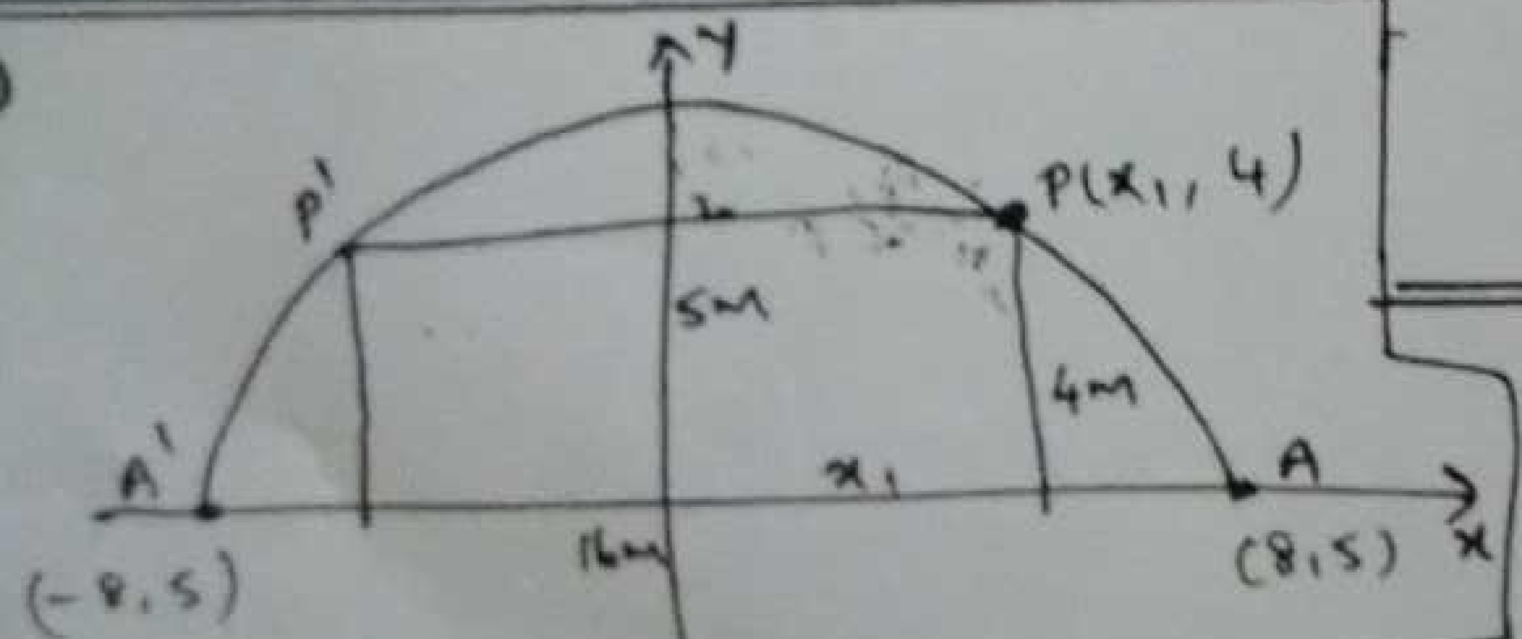
Angle of Projection: Diff. w.r.to 'x'

$$2x = -9 \cdot \frac{dy}{dx}$$

At $(-6, -4)$

$$2(-6) = -9 \times \frac{dy}{dx}$$
$$\frac{12}{9} = \frac{dy}{dx}$$
$$m = 4/3$$
$$\tan\theta = 4/3$$
$$\therefore \boxed{\theta = \tan^{-1}(4/3)}$$

(2)



$$AA' = 16$$
$$2a = 16$$
$$\boxed{a = 8}, \boxed{b = 5}$$

Eq. of ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$
$$\frac{x^2}{64} + \frac{y^2}{25} = 1$$

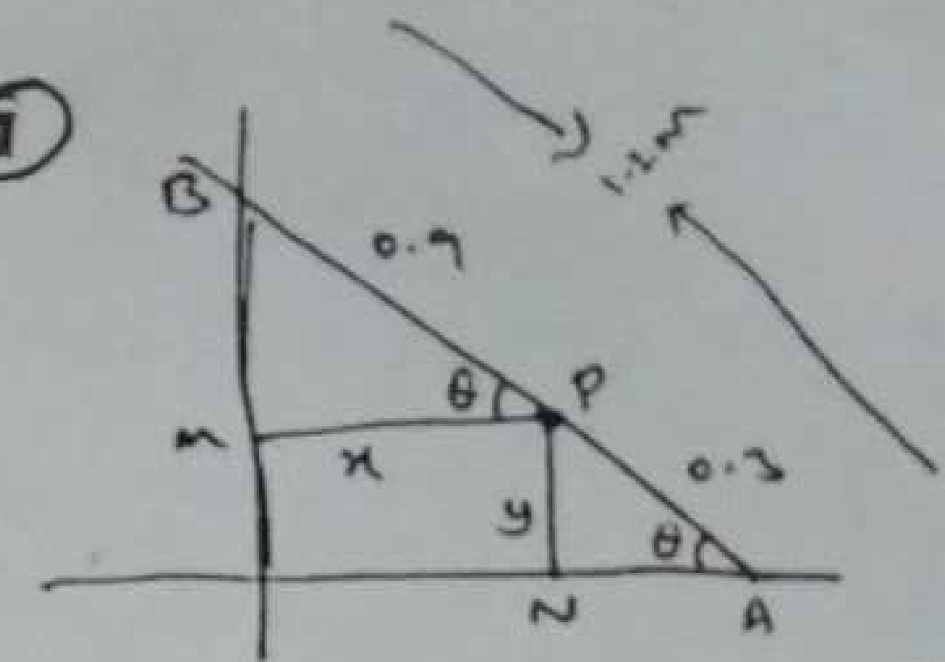
The Point is $(x_1, 4)$

$$\frac{x_1^2}{64} + \frac{16}{25} = 1$$
$$\frac{x_1^2}{64} = 1 - \frac{16}{25}$$
$$x_1^2 = \frac{9}{25} \times 64$$
$$x_1 = \frac{3}{5} \times 8$$
$$x_1 = 24/5$$
$$\boxed{x_1 = 4.8}$$

Required wide $= 2x_1$

$$= 2(4.8)$$
$$PP' = 9.6\,m$$

(7)



In Δ ANP $\quad \sin\theta = \frac{y}{0.3}$

In Δ PmB $\quad \cos\theta = \frac{x}{0.9}$

w.k.T $\quad \sin^2\theta + \cos^2\theta = 1$

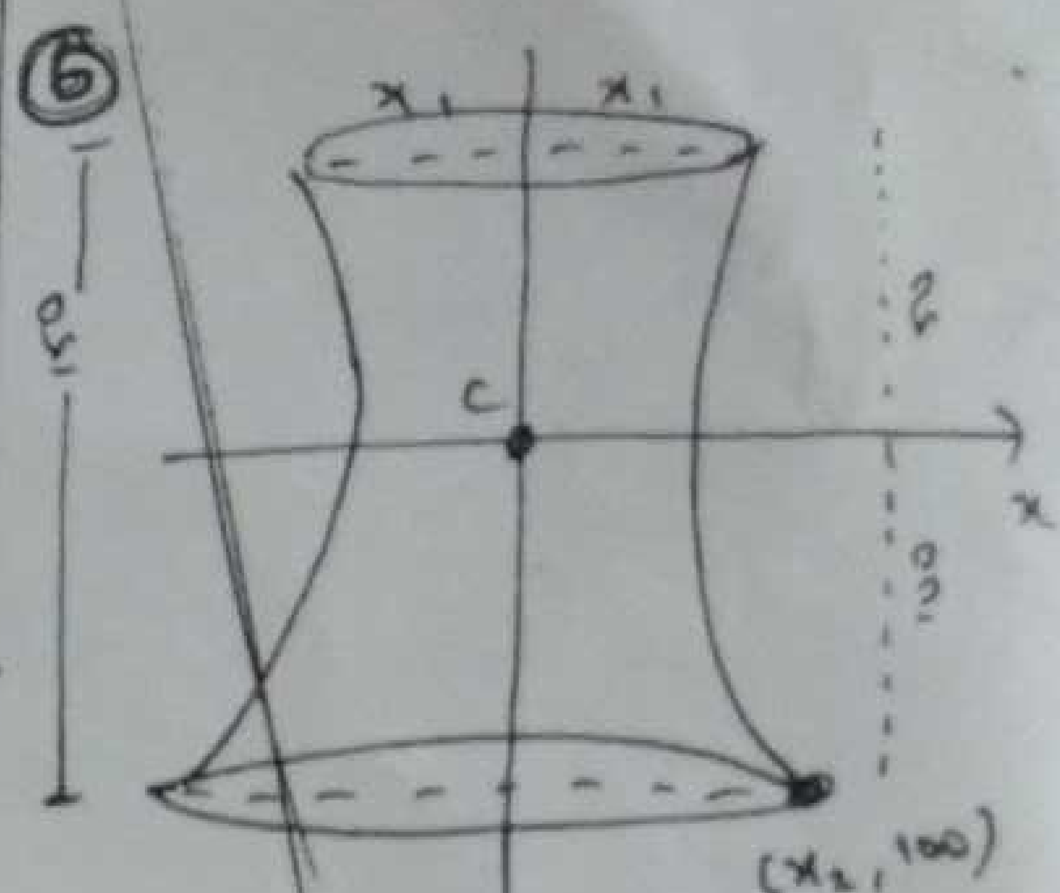
$$\frac{y^2}{0.09} + \frac{x^2}{0.81} = 1$$

which is Required equation

now,

$$e = \sqrt{1 - b^2/a^2}$$
$$= \sqrt{1 - \frac{0.09}{0.81}}$$
$$= \sqrt{1 - \frac{\cancel{9}}{\cancel{81}_9}}$$
$$= \sqrt{1 - 1/9}$$
$$= \sqrt{8/9}$$
$$\boxed{e = \frac{2\sqrt{2}}{3}\,m}$$

(6)



Eq. of hyperbola

$$\frac{x^2}{30^2} - \frac{y^2}{44^2} = 1$$

The Point is $(x, 50)$

$$\frac{x_1^2}{30^2} - \frac{50^2}{44^2} = 1$$
$$\frac{x_1^2}{30^2} = 1 + \frac{50^2}{44^2}$$
$$= \frac{(44)^2 + (50)^2}{(44)^2}$$
$$x_1^2 = 30^2 \left[\frac{(44)^2 + (50)^2}{(44)^2}\right]$$
$$x_1 = \frac{30}{44}\left[\sqrt{(44)^2 + (50)^2}\right]$$
$$x_1 = 45.41\,m$$

diameter $= 2x_1$

$$= 2(45.41)$$
$$= 90.82\,m$$

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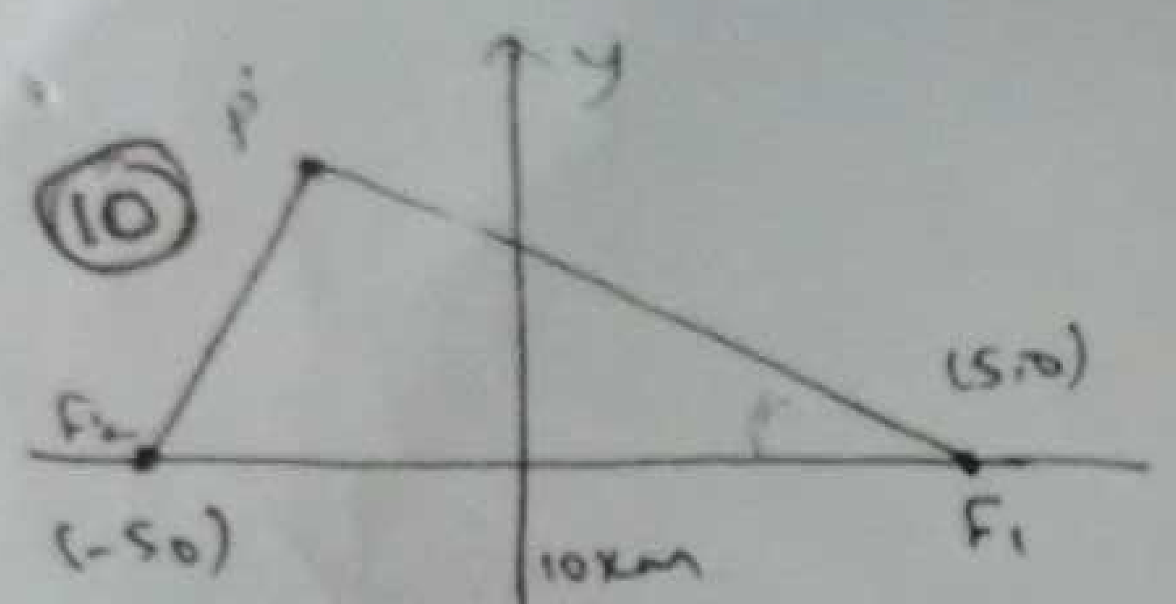


From diagram

$F_1P - F_2P = 6$

$2a = 6$

$\boxed{a = 3}$

now, $ae = 5 \Rightarrow 3e = 5$

$\boxed{e = 5/3}$

now,

$b^2 = a^2(e^2 - 1)$

$= 9\left[\frac{25}{9} - 1\right]$

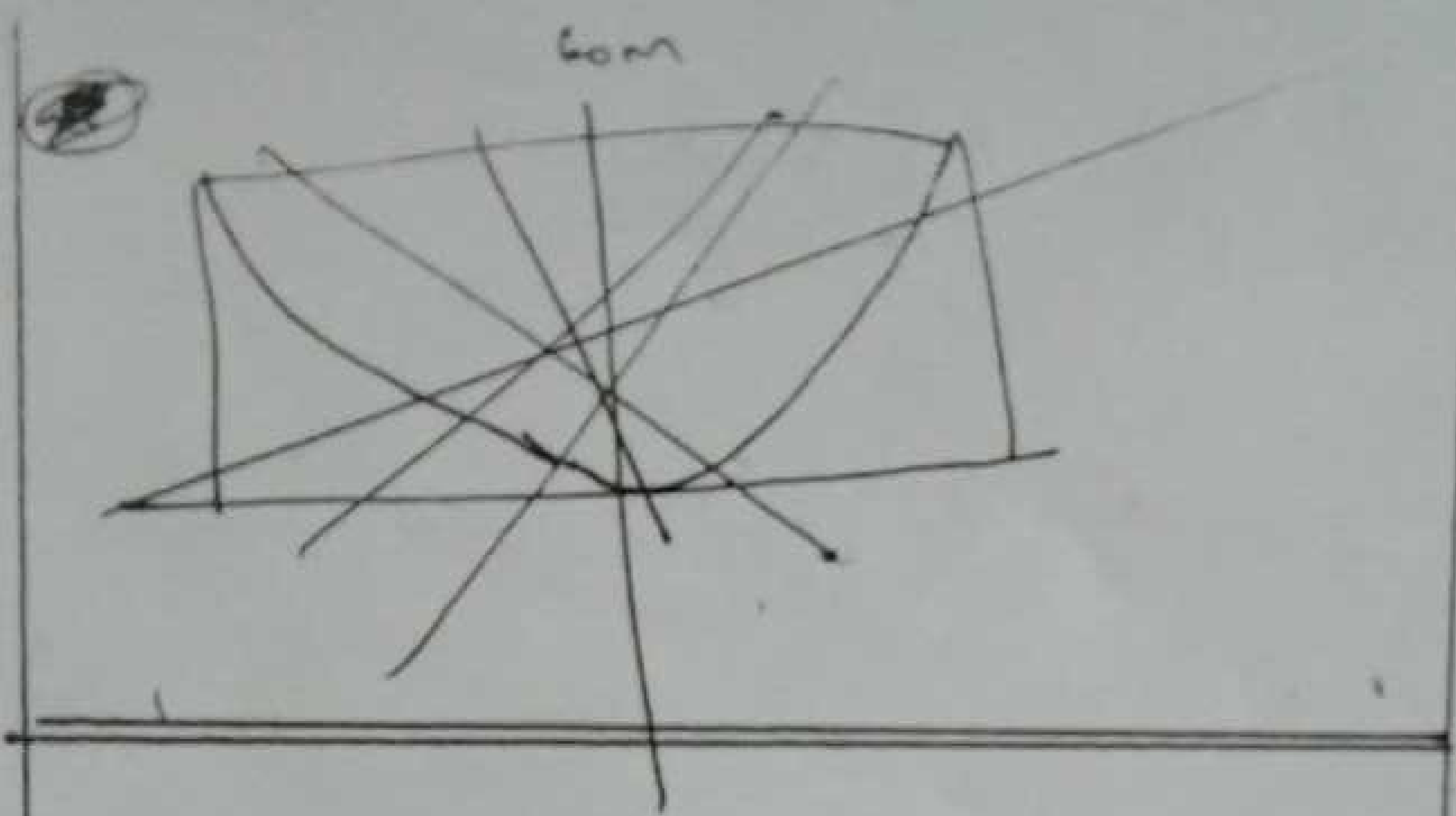
$= \cancel{9}\left[\frac{16}{\cancel{9}}\right]$

$b^2 = 16$

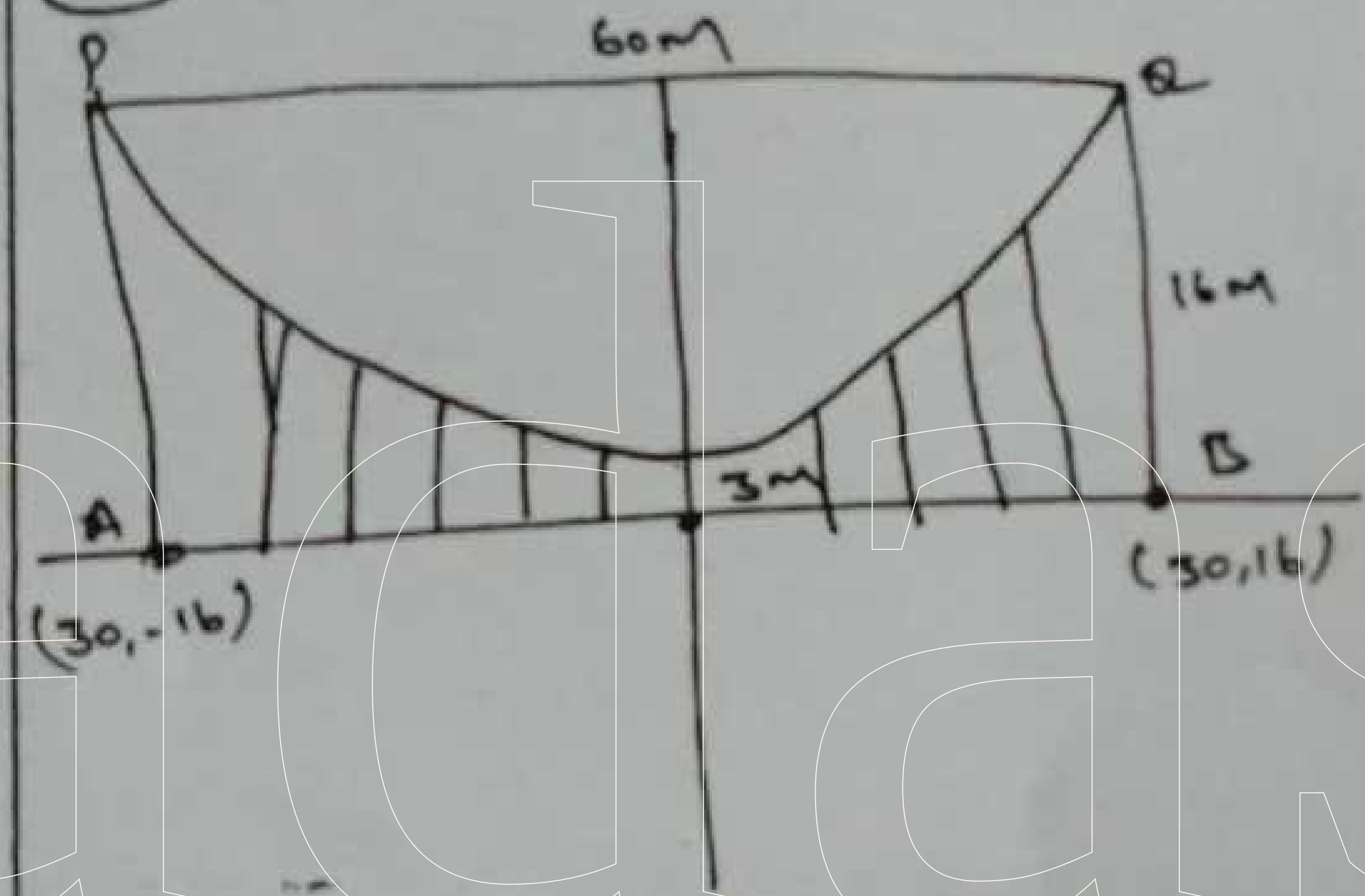
Required Equation is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$



5



$2a = 60$, $(h, k) = (0, 3)$

$a = 30$

Eq. of Parabola $(x-h)^2 = 4a(y-k)$

$(x-0)^2 = 4a(y-3)$

$x^2 = 4a(y-3) \rightarrow (1)$

The Point is $(30, 16)$

$30^2 = 4a(16-3)$

$30^2 = 4a \times 13$

$a = \frac{30 \times 30}{4 \times 13}$ in (1)

$x^2 = \cancel{4}\,\frac{30 \times 30}{\cancel{4} \times 13}(y-3)$

$x^2 = \frac{30 \times 30}{13}(y-3)$

when $x = 6$:

$\frac{36 \times 13}{30 \times 30} = y - 3$

$\frac{52}{100} = y - 3$

$0.52 + 3 = y$

$\boxed{y = 3.52\,m}$

when $x = 12$

$(12)^2 = \frac{4 \times 30 \times 30}{4 \times 13}(y-3)$

$\frac{144 \times 13}{30 \times 30} = y - 3$

$\frac{208}{100} = y - 3$

$2.08 + 3 = y$

$\boxed{y = 5.08\,m}$

length of 1^{st} cable $= 3.52\,m$

" " 2^{nd} cable $= 5.08\,m$

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Show that the line $5x+12y=9$ is tangent to the ~~line ellipse~~ hyperbola $x^2-9y^2=9$. Find point of contact.

~~Ex 5.4~~ CREATED

(1) given $5x+12y=9$

$$12y = 9-5x$$
$$y = -\frac{5}{12}x + \frac{9}{12}$$
$$y = -\frac{5}{12}x + \frac{3}{4}$$

(compare to $y=mx+c$)

$m = -5/12,\ c = 3/4$

given $x^2-9y^2=9$

$\div 9$ $\quad \frac{x^2}{9} - \frac{y^2}{1} = 1$

$a^2=9,\ b^2=1$

condition

$$c^2 = a^2m^2 - b^2$$
$$\frac{9}{16} = \cancel{9}\times\frac{25}{\cancel{144}_{16}} - 1$$
$$= \frac{25}{16} - 1$$
$$\frac{9}{16} = 9/16$$

Point of contact:

$$P = \left(-\frac{a^2m}{c},\ -\frac{b^2}{c}\right)$$
$$= \left(-\frac{\cancel{9}^{3}\times -5/\cancel{12}_{4}}{3/4},\ \frac{-1}{3/4}\right)$$
$$= \left(\frac{4}{\cancel{3}}\times\frac{\cancel{3}\times 5}{\cancel{4}},\ -4/3\right)$$
$$P = (5, -4/3)$$

(3) Ex 5.4 – X

given $x-y+4=0$

$$-y = -x-4$$
$$y = x+4$$

$m=1,\ c=4$

given $x^2+3y^2=12$

$\div 12$ $\quad \frac{x^2}{12} + \frac{y^2}{4} = 1$

$a^2=12,\ b^2=4$

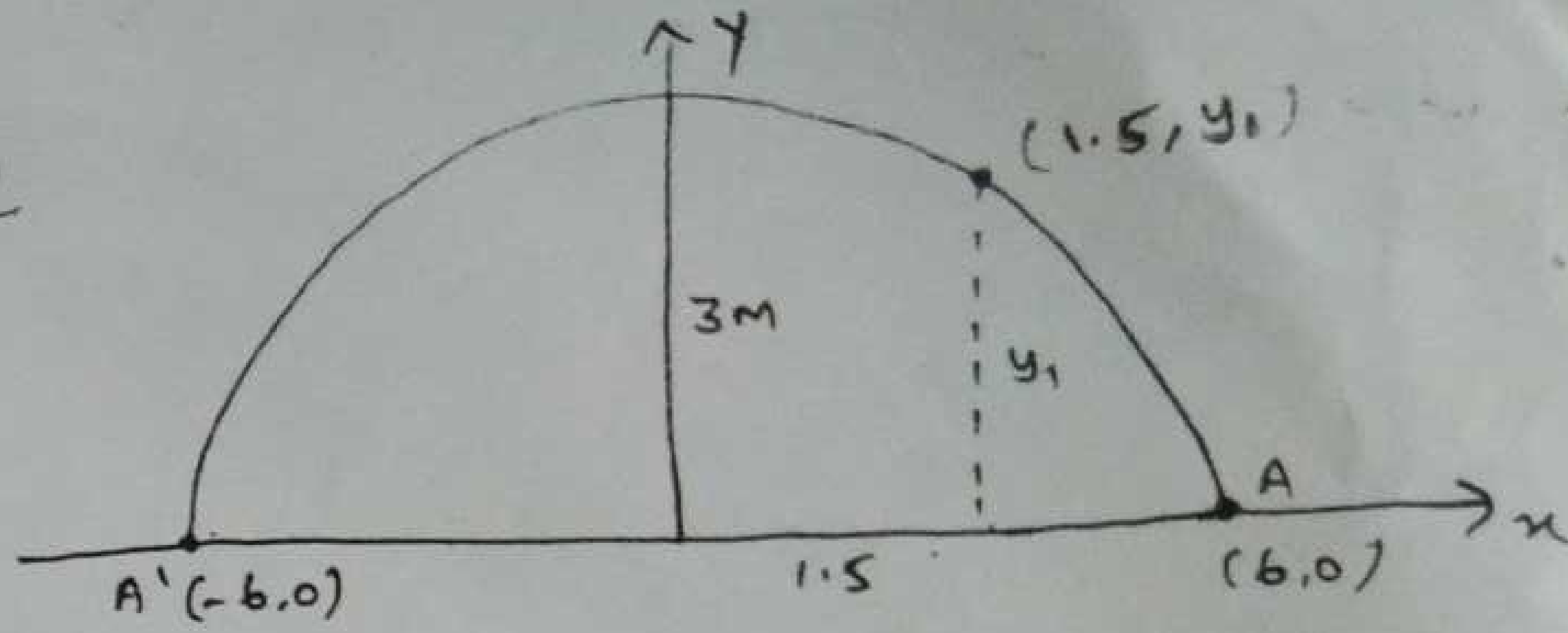
condition

$$c^2 = a^2m^2+b^2$$
$$16 = (12)(1)+(4)$$
$$16 = 16$$

Point of contact:

$$P = \left(-\frac{a^2m}{c},\ \frac{b^2}{c}\right)$$
$$= \left(\frac{-12\times 1}{4},\ \frac{4}{4}\right)$$
$$P = (-3, 1)$$

Eg 5.30



$AA' = 12$

$2a = 12$

$\boxed{a=6}$, $\boxed{b=3}$

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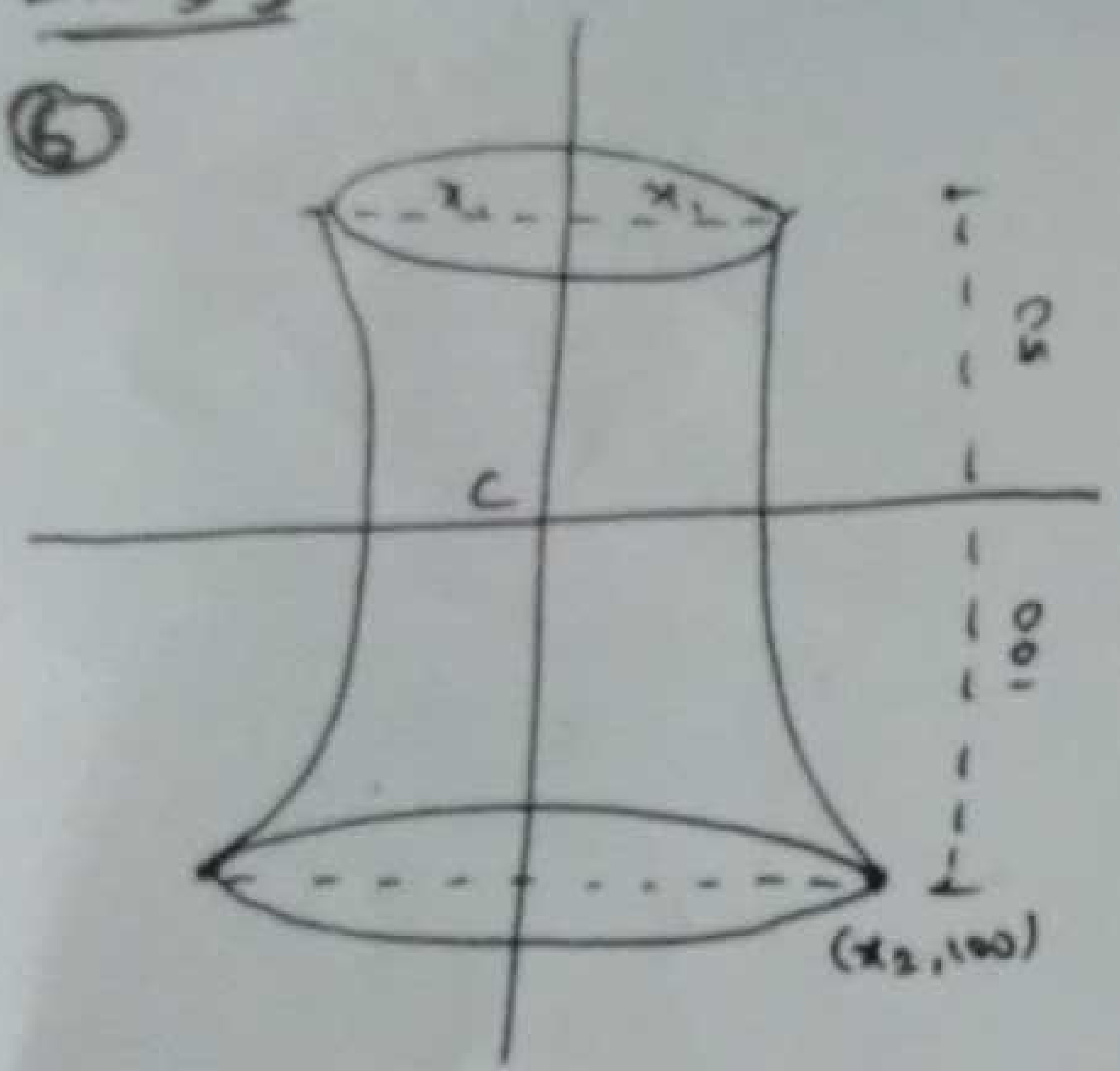
Eq. of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

The point is $(1.5, y_1)$

$$\frac{(1.5)^2}{36} + \frac{y_1^2}{9} = 1$$
$$\frac{2.25}{36} + \frac{y_1^2}{9} = 1$$
$$\frac{y_1^2}{9} = 1 - \frac{2.25}{36}$$
$$\frac{y_1^2}{9} = \left(\frac{36-2.25}{36}\right)$$
$$y_1^2 = \cancel{9}\left(\frac{36-2.25}{\cancel{36}_4}\right)$$
$$y_1^2 = \frac{33.75}{4} = 8.44$$
$$y_1 = 2.90 \text{ (app)}$$

Height of arch way 1.5m from the centre is app 2.90m, Truck's height = 2.7m, Ex 5.5

(6)



Eq. of hyperbola $\frac{x^2}{30^2} - \frac{y^2}{44^2} = 1$

The Point is $(x_1, 50)$

$$\frac{x_1^2}{30^2} - \frac{50^2}{44^2} = 1$$

$$\frac{x_1^2}{30^2} = 1 + \frac{50^2}{44^2}$$

$$\frac{x_1^2}{30^2} = \frac{(44)^2 + 50^2}{44^2}$$

$$x_1^2 = 30^2\left[\frac{44^2+50^2}{44^2}\right]$$

$$x_1 = \cancel{45.24\,m}\ 45.41\,m$$

The point is $(x_1, 100)$

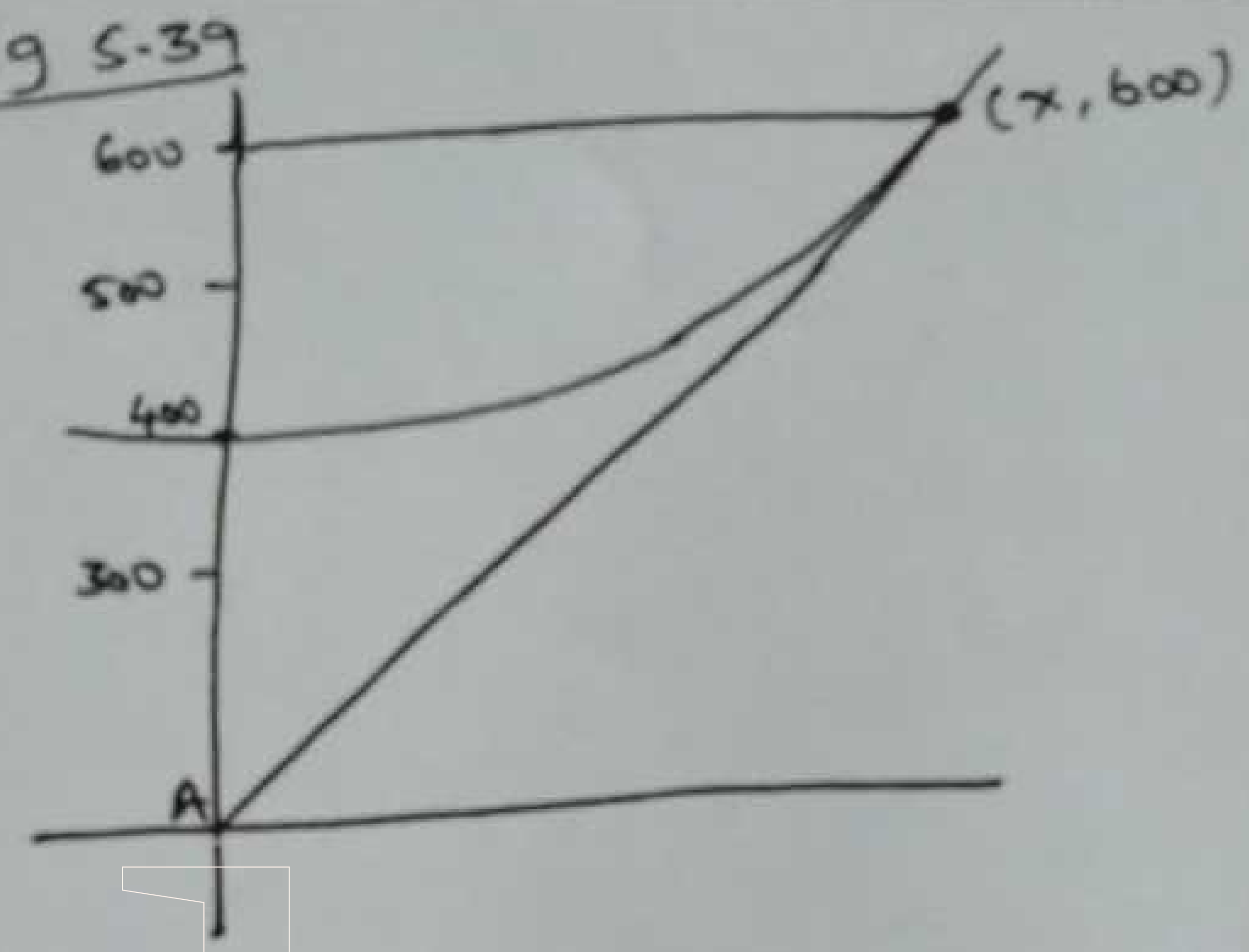
$$\frac{x_1^2}{30^2} - \frac{100^2}{44^2} = 1$$

$$\frac{x_1^2}{30^2} = 1 + \frac{100^2}{44^2}$$

$$x_1^2 = 30^2\left(\frac{44^2+100^2}{44^2}\right)$$

$$x_1 = 74.45\,m$$

Eg 5.39



Eq. of hyperbola

$$\frac{(y-300)^2}{a^2} - \frac{(x-0)^2}{b^2} = 1 \to (1)$$

The point is (0,400)

$$\frac{(400-300)^2}{a^2} - \frac{0^2}{b^2} = 1$$

$$\frac{100^2}{a^2} = 1$$

$$a^2 = 100^2$$

$$\boxed{a^2 = 10000}$$

by Pythagoras theorem

$$600^2 + x^2 = (x+200)^2$$

$$360000 + \cancel{x^2} = \cancel{x^2} + 400x + 40000$$

$$360000 - 40000 = 400x$$

$$32,000 = 400x$$

$$x = \frac{32000}{400} = 800$$

now (800, 600) lies on hyperbola

$$(1) \Rightarrow \frac{(600-300)^2}{10000} - \frac{(800-0)^2}{b^2} = 1$$

$$\frac{(300)^2}{10000} - \frac{640000}{b^2} = 1$$

$$\frac{90000}{10000} - 1 = \frac{640000}{b^2}$$

$$8 = \frac{640000}{b^2}$$

$$b^2 = \frac{640000}{8}$$

$$\boxed{b^2 = 80000}$$

∴ Required Equation is

$$\frac{(y-300)^2}{10000} - \frac{x^2}{80000} = 1$$

Ex 5.1

(6) Eq. of circle

$$x^2+y^2+2gx+2fy+c=0 \to (1)$$

(1,0) Passing through

$$(1)^2+(0)^2+2g(1)+2f(0)+c=0$$

$$1+2g+c=0$$

$$2g+c=-1 \to (2)$$

(-1,0) Passing through

$$(-1)^2+(0)^2+2g(-1)+2f(0)+c=0$$

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$1 - 2g + c = 0$

$-2g + c = -1 \rightarrow (3)$

(0,1) Passing through

$(0)^2 + (1)^2 + 2g(0) + 2f(1) + c = 0$

$1 + 2f + c = 0$

$2f + c = -1 \rightarrow (4)$

solve (1) and (2)

$$\begin{aligned} \cancel{2g} + c &= -1 \\ -\cancel{2g} + c &= -1 \\ \hline 2c &= -2 \\ \boxed{c = -1} \end{aligned}$$

Put $c = -1$ in (2)

$2g - 1 = -1$

$2g = -1 + 1$

$\boxed{g = 0}$

Put $c = -1$ in (4)

$2f - 1 = -1$

$2f = 0$

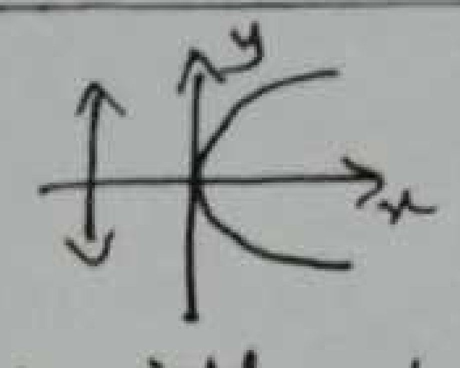
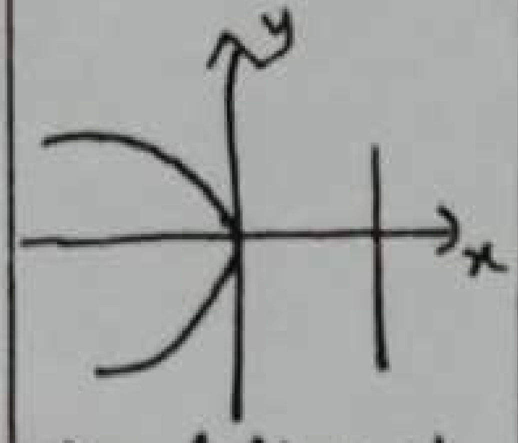
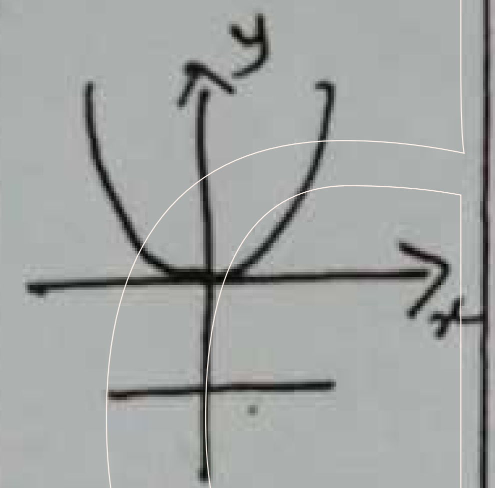
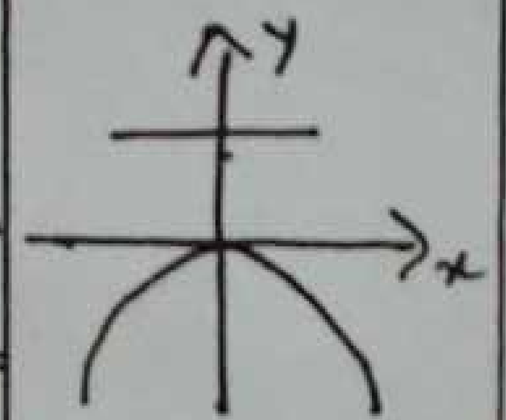
$\boxed{f = 0}$

Eq. of circle

$x^2 + y^2 + 2(0)x + 2(0)y - 1 = 0$

$$\boxed{x^2 + y^2 = 1}$$

Refer eg 5.10

Diagram and Type	Axis	vertex	Focus	Directrix	Eq. of latus rectum	length of l. Rectum
 Open rightward $y^2 = 4ax$	$y=0$	$(0,0)$	$(a,0)$	$x = -a$	$x = a$	$4a$
 Open leftward $y^2 = -4ax$	$y=0$	$(0,0)$	$(-a,0)$	$x = a$	$x = -a$	$4a$
 $x^2 = 4ay$	$x=0$	$(0,0)$	$(0,a)$	$y = -a$	$y = a$	$4a$
 $x^2 = -4ay$	$x=0$	$(0,0)$	$(0,-a)$	$y = a$	$y = -a$	$4a$

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Ex 5.2 Parabola

④ (v) $y^2-4y-8x+12=0$

$\cancel{y^2-4y+2}$

$y^2-4y = 8x-12$

$y^2-4y+2^2 = 8x-12+2^2$

$(y-2)^2 = 8x-8$

$(y-2)^2 = 8(x-1)$

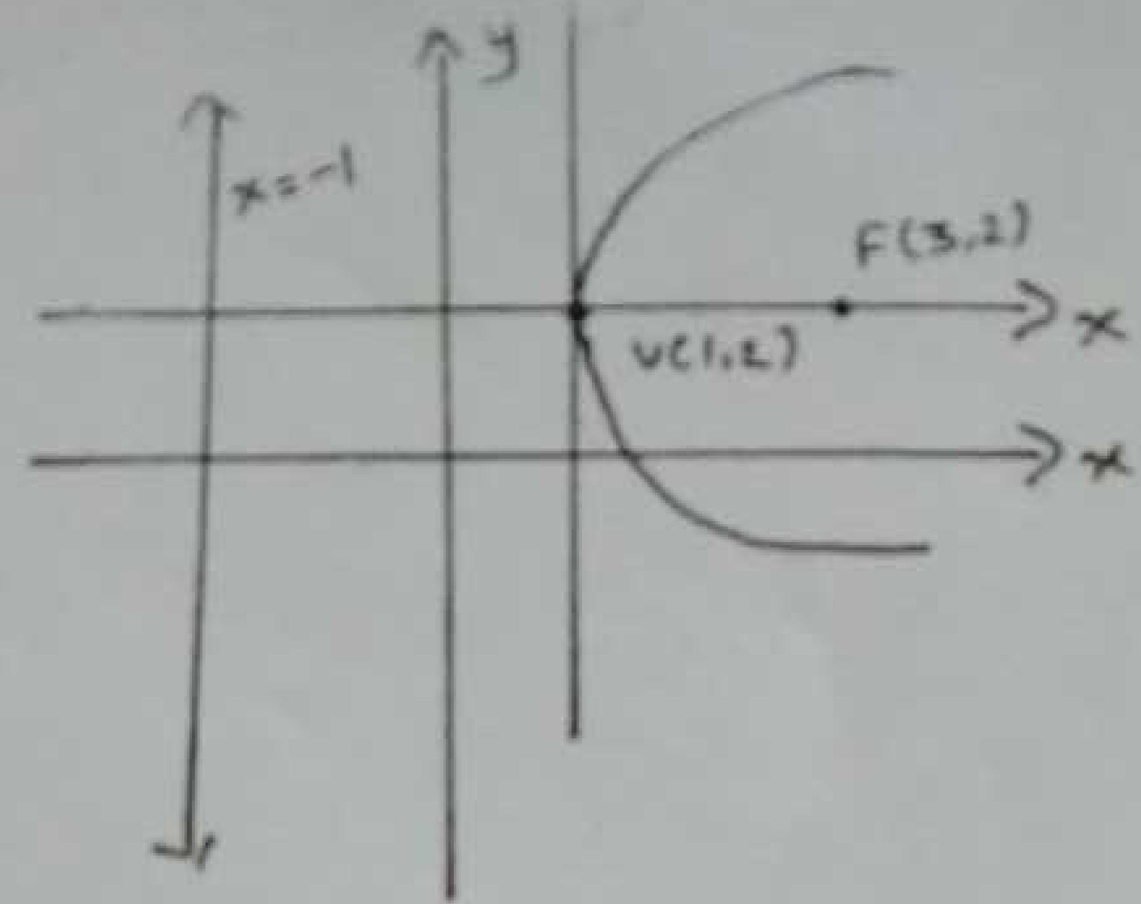
compare to $(y-k)^2 = 4a(x-h)$

$\therefore X = x-1$
$Y = y-2$ [$\because$ It is open rightward]

and $4a = 8$

$\boxed{a=2}$

	Referred to X, Y	Referred to x, y $X=x-1,\ Y=y-2$
(i) vertex	(0,0) X Y	$0=x-1$, $x=1$; $0=y-2$, $y=2$; v(1,2)
(ii) Focus	(a,0) (2,0) X Y	$2=x-1$, $x=3$; $0=y-2$, $y=2$; F(3,2)
(iii) Eq. of directrix	$X=-a$, $X=-2$	$-2=x-1$, $-2+1=x$, $x=-1$, $\boxed{x+1=0}$
(iv) length of latus rectum	4a	⇒ 4(2) ⇒ 8



(iv) $x^2-2x+8y+17=0$.

$x^2-2x = -8y-17$

$x^2-2x+1^2 = -8y-17+1^2$

$(x-1)^2 = -8y-16$

$(x-1)^2 = -8(y+2)$

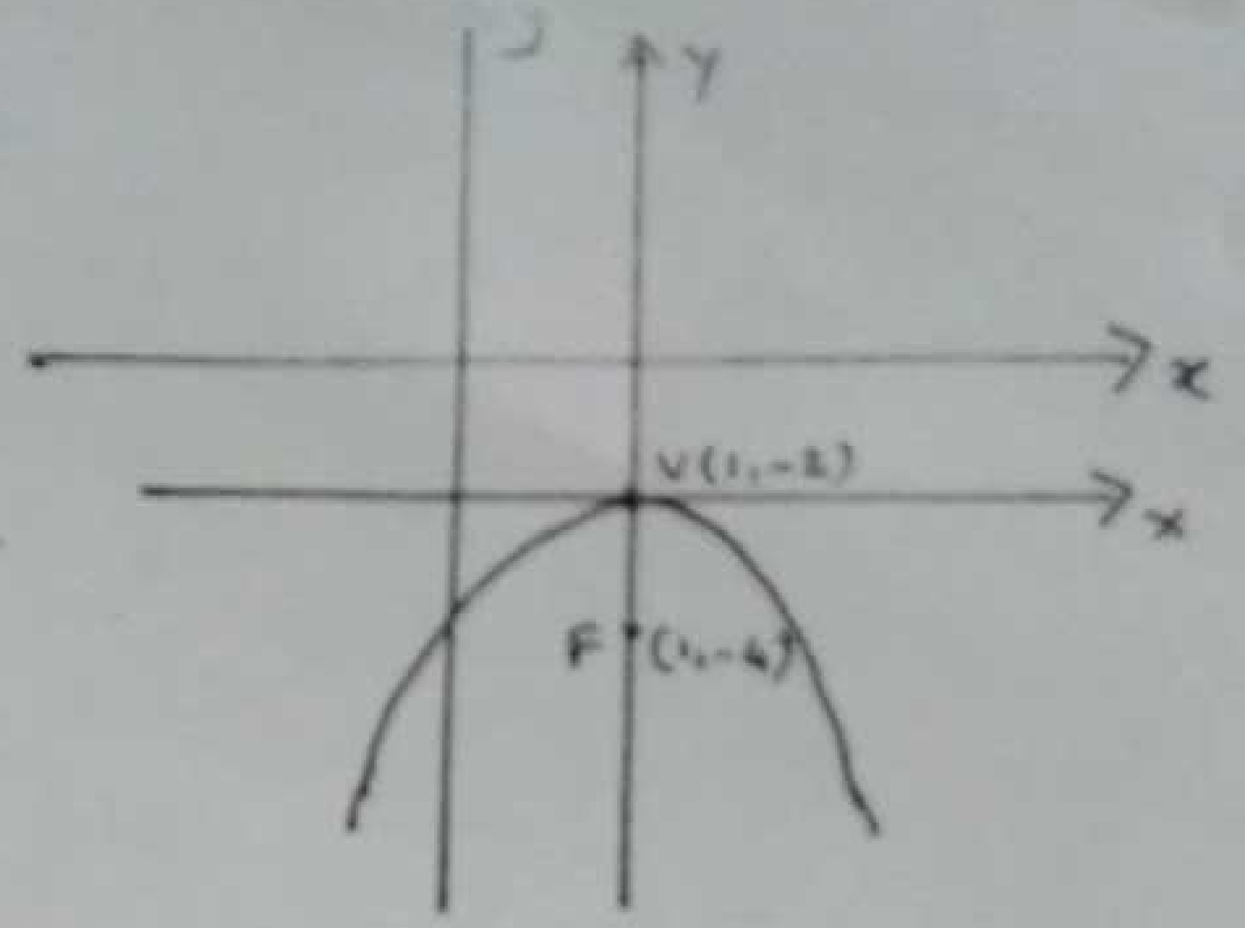
Compare to $(x-h)^2 = -4a(y-k)$

$\therefore X = x-1$
$Y = y+2$ [$\because$ It is open downward]

and $-4a = -8$

$\boxed{a=2}$

	Referred to X, Y	Referred to x, y $X=x-1,\ Y=y+2$
(i) vertex	(0,0) X Y	$0=x-1$, $x=1$; $0=y+2$, $y=-2$; V(1,-2)
(ii) Focus	(0,-a) (0,-2) X Y	$0=x-1$, $x=1$; $-2=y+2$, $y=-4$; F(1,-4)
(iii) Eq. of directrix	$Y=a$, $Y=2$	$2=y+2$, $y=2-2$, $y=0$
(iv) length of latus rectum	~~latus Rectum~~ 4a	4(2) = 8



Refer eg 5.17

Ex 5.2 Ellipse

8) (i) $\dfrac{(x-3)^2}{225} + \dfrac{(y-4)^2}{289} = 1$

Comparing $\dfrac{x^2}{b^2} + \dfrac{y^2}{a^2} = 1$

$a^2 = 289$
$a = 17$
$b^2 = 225$
$b = 15$

now $e = \sqrt{1-b^2/a^2}$

$= \sqrt{1-\frac{225}{289}} = \sqrt{\frac{289-225}{289}}$

$= \sqrt{64/289}$

$\boxed{e = 8/17}$

$ae = \cancel{17} \times 8/\cancel{17}$

$\boxed{ae = 8}$

now, $a/e = \dfrac{17}{8/17}$

$= 17 \times \dfrac{17}{8}$

$a/e = \dfrac{289}{8}$

$\therefore$ It is equation of ellipse ~~[illegible]~~ Parallel to y-axis

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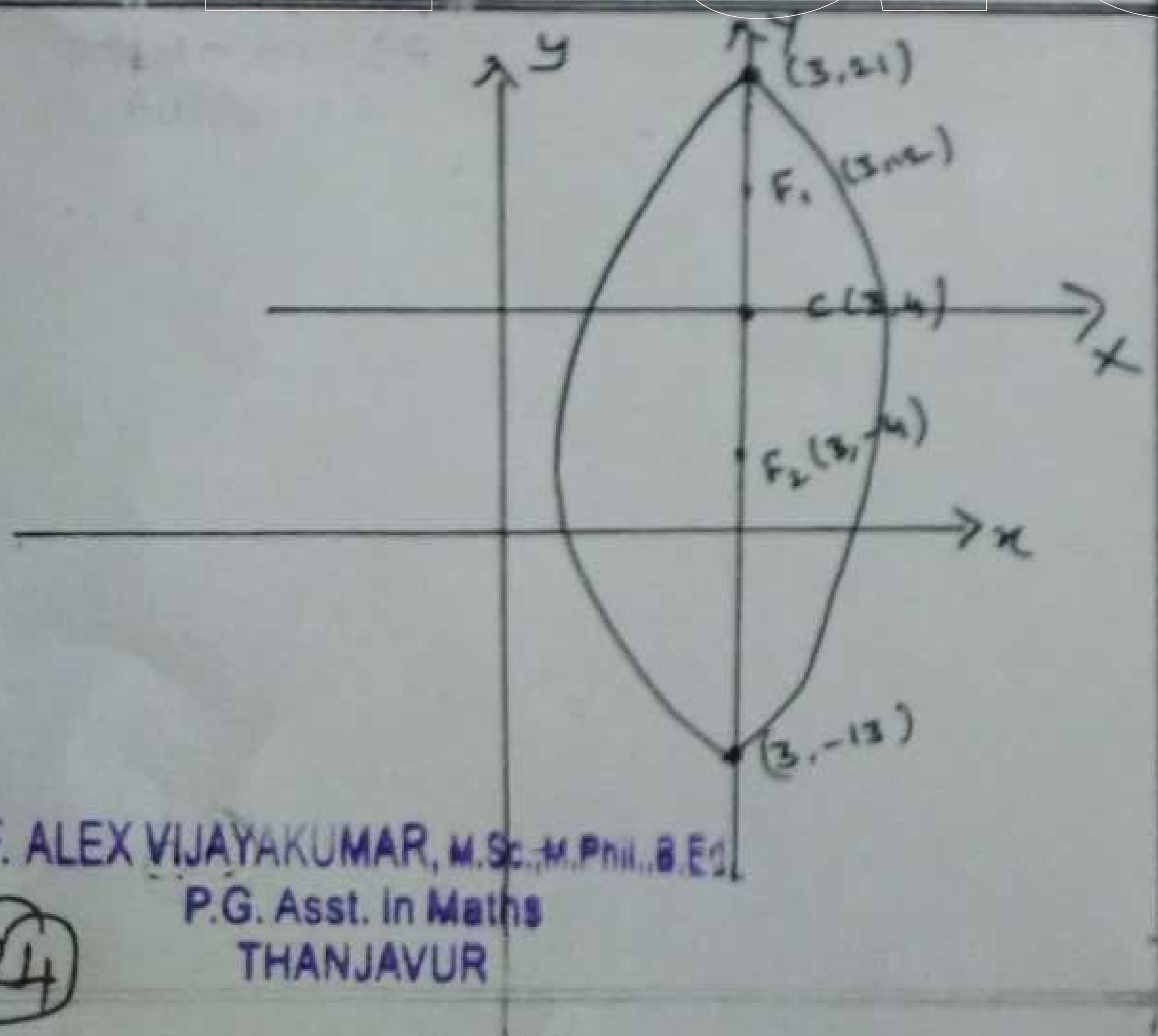
		Referred to X, Y	Referred to x, y: $X = x-3$, $Y = y-4$
(i)	Centre	$(0,0)$ X Y	$0 = x-3$, $x = 3$; $0 = y-4$, $y = 4$; $C(3,4)$
(ii)	Foci	$(0, \pm ae)$ $(0, \pm 8)$ X Y	$0 = x-3$, $x = 3$; $-8 = y-4$, $-8+4 = y$, $y = -4$; $8 = y-4$, $8+4 = y$, $y = 12$; $F_1(3,-4)$, $F_2(3,12)$
(iii)	Vertices	$(0, \pm a)$ $(0, \pm 17)$ X Y	$0 = x-3$, $x = 3$; $-17 = y-4$, $-17+4 = y$, $y = -13$; $17 = y-4$, $17+4 = y$, $y = 21$; $A(3,-13)$, $A'(3,21)$
(iv)	Directrix	$Y = \pm a/e$, $Y = \pm \frac{289}{8}$	$\frac{289}{8} = y-4$, $\frac{289}{8}+4 = y$, $y = \frac{321}{8}$; $-\frac{289}{8} = y-4$, $-\frac{289}{8}+4 = y$, $y = -\frac{257}{8}$

Refer Ex 5.2 → 8(iii)



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(v) $18x^2 + 12y^2 - 144x + 48y + 120 = 0$

$18x^2 - 144x + 12y^2 + 48y = -120$

$18(x^2 - 8x) + 12(y^2 + 4y) = -120$

$18(x^2 - 8x + 4^2 - 4^2) + 12(y^2 + 4y + 2^2 - 2^2) = -120$

$18\left[(x-4)^2 - 16\right] + 12\left[(y+2)^2 - 4\right] = -120$

$18(x-4)^2 - 288 + 12(y+2)^2 - 48 = -120$

$18(x-4)^2 + 12(y+2)^2 = -120 + 288 + 48$

$18(x-4)^2 + 12(y+2)^2 = 216$

$\div 216$

$$\frac{(x-4)^2}{12} + \frac{(y+2)^2}{18} = 1$$

major axis is Parallel to y-axis

$X = x-4$, $Y = y+2$

$\cancel{b^2 = 12}$, $\cancel{b = 3\sqrt{2}}$ | $\cancel{a^2 = 48}$, $\cancel{a = 3\sqrt{2}}$

$a^2 = 18$, $a = 3\sqrt{2}$ | $b^2 = 12$, $b = 2\sqrt{3}$

now,

$e = \sqrt{1 - \frac{b^2}{a^2}}$

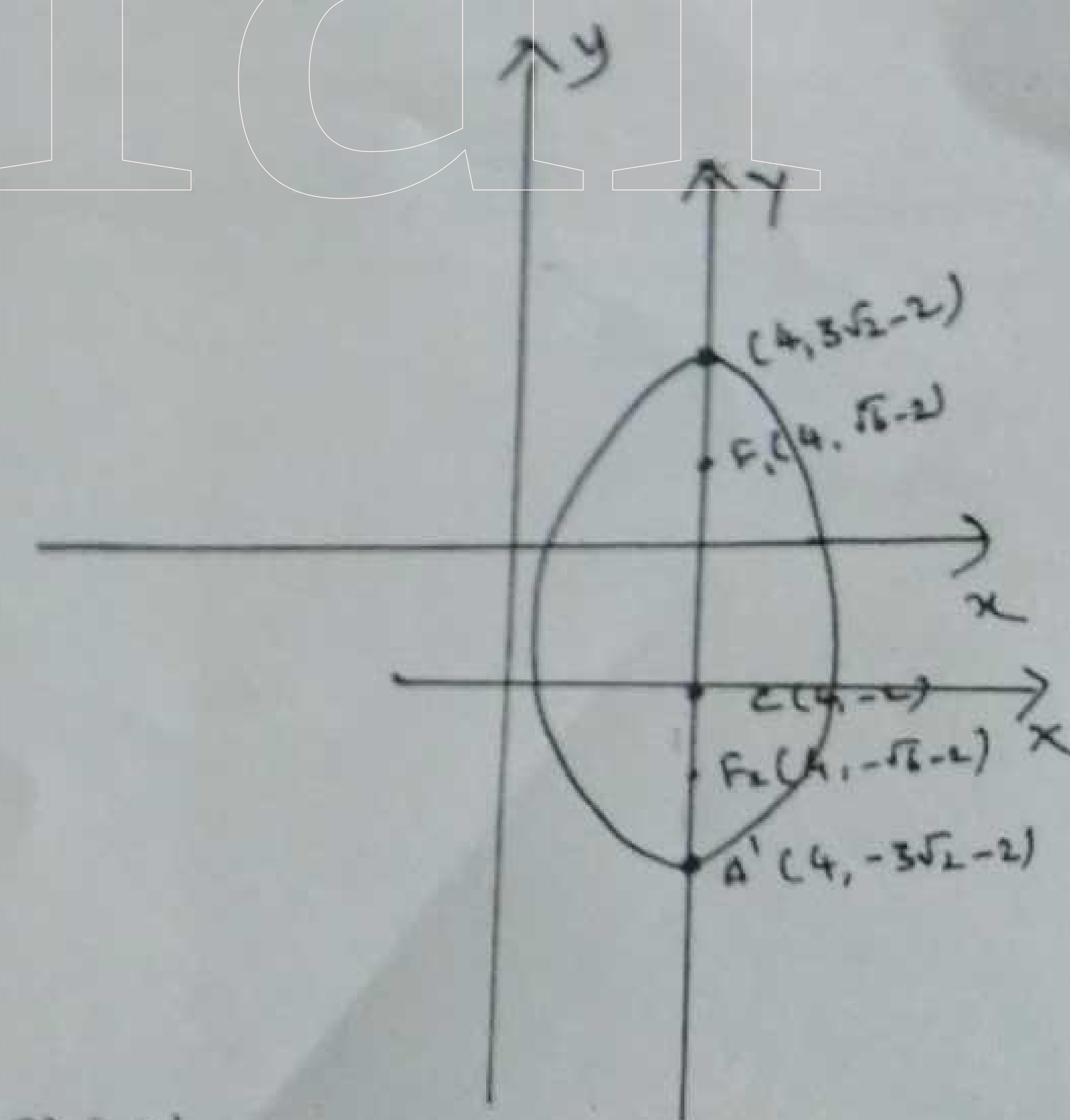
$= \sqrt{1 - \frac{12}{18}}$

$= \sqrt{\frac{6}{18}}$

$\boxed{e = \frac{1}{\sqrt{3}}}$

$ae = 3\sqrt{2} \times \frac{1}{\sqrt{3}} = \frac{\sqrt{3}\cancel{\sqrt{3}}\sqrt{2}}{\cancel{\sqrt{3}}} = \sqrt{6}$

		Referred to X, Y	Referred to x, y: $X = x-4$, $Y = y+2$
(i)	Centre	$(0,0)$ X Y	$0 = x-4$, $x = 4$; $0 = y+2$, $y = -2$; $C(4,-2)$
(ii)	Foci	$(0, \pm ae)$ $(0, \pm\sqrt{6})$ X Y	$0 = x-4$, $x = 4$; $\sqrt{6} = y+2$, $y = \sqrt{6}-2$; $-\sqrt{6} = y+2$, $y = -\sqrt{6}-2$; $F_1(4, \sqrt{6}-2)$, $F_2(4, -\sqrt{6}-2)$
(iii)	Vertices	$(0, \pm a)$ $(0, \pm 3\sqrt{2})$ X Y	$0 = x-4$, $x = 4$; $3\sqrt{2} = y+2$, $y = 3\sqrt{2}-2$; $-3\sqrt{2} = y+2$, $y = -3\sqrt{2}-2$; $A(4, 3\sqrt{2}-2)$, $A'(4, -3\sqrt{2}-2)$



Refer eg 5.21

Ex 5.2 Hyperbola

8(iii) $\frac{(x+3)^2}{225} - \frac{(y-4)^2}{64} = 1$

$\longrightarrow$

$a^2 = 225$ | $b^2 = 64$ | $X = x+3$
$a = 15$ | $b = 8$ | $Y = y-4$

now,

$$\frac{X^2}{225} - \frac{Y^2}{64} = 1$$

which is hyperbola.

Transverse axis is along x-axis

now,

$$e = \sqrt{1 + b^2/a^2}$$

$$= \sqrt{1 + 64/225}$$

$$= \sqrt{\frac{225+64}{225}}$$

$$= \sqrt{\frac{289}{225}}$$

$$\boxed{e = 17/15}$$

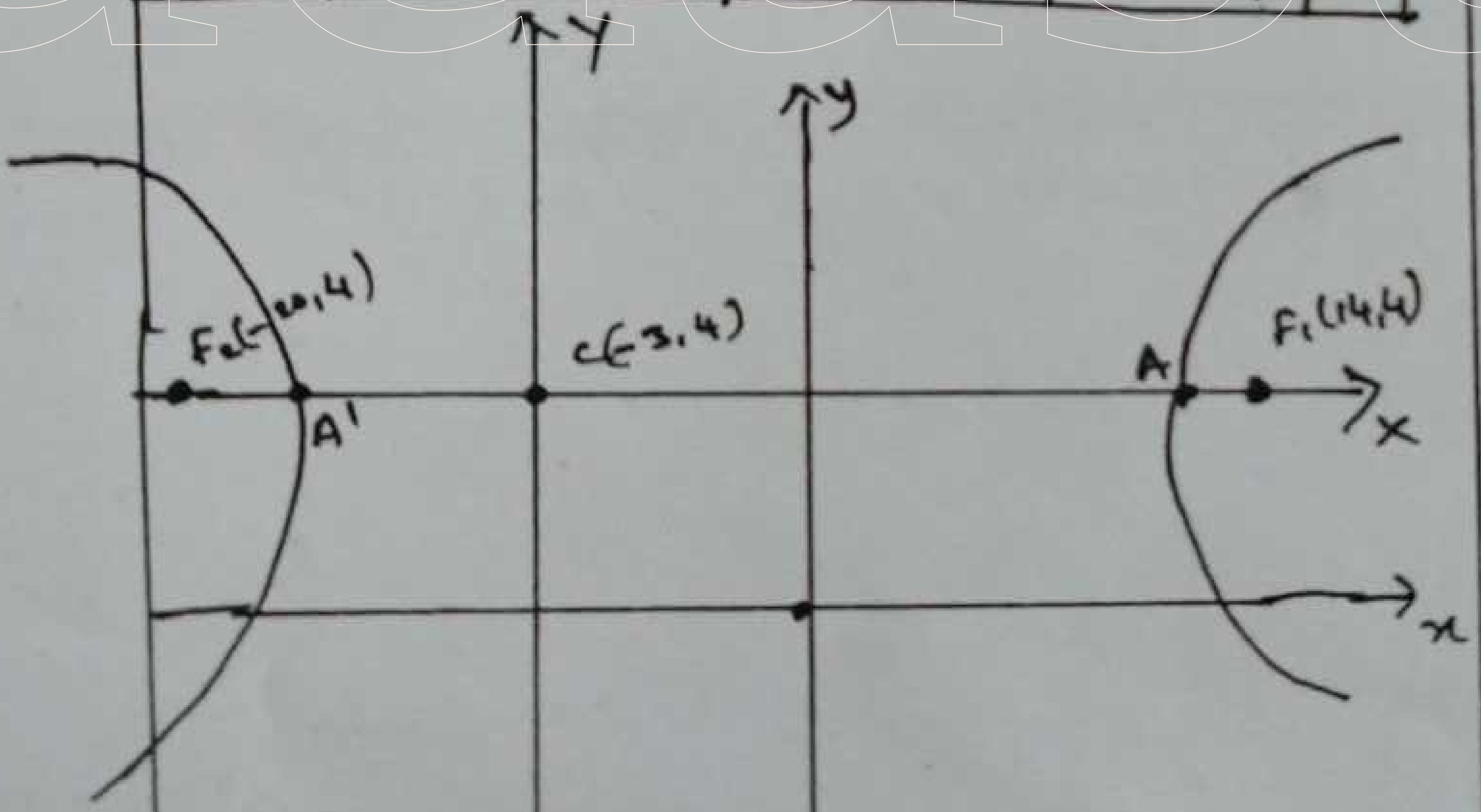
And $ae = \cancel{15} \times 17/\cancel{15}$

$$\boxed{ae = 17}$$

And $a/e = \frac{15}{17/15} = \frac{225}{17}$

$$\boxed{a/e = \frac{225}{17}}$$

	Referred to x, y	Referred to X, Y $X = x+3$, $Y = y-4$
Centre	$(0,0)$ X Y	$0 = x+3$, $x = -3$ | $0 = y-4$, $y = 4$ $C(-3,4)$
Foci	$(\pm ae, 0)$ $(\pm 17, 0)$ X , Y	$17 = x+3$, $17-3 = x$, $x = 14$ | $-17 = x+3$, $-17-3 = x$, $x = -20$ | $0 = y-4$, $y = 4$ $F_1(14,4)$, $F_2(-20,4)$
vertices	$(\pm a, 0)$ $(\pm 15, 0)$ X Y	$15 = x+3$, $15-3 = x$, $x = 12$ | $-15 = x+3$, $-15-3 = x$, $x = -18$ | $0 = y-4$, $y = 4$ $A(12,4)$, $A'(-18,4)$
Eq. of Directrix	$X = \pm a/e$ $X = \pm \frac{225}{17}$	$\frac{225}{17} = x+3$, $\frac{225}{17} - 3 = x$, $x = \frac{174}{17}$ | $-\frac{225}{17} = x+3$, $-\frac{225}{17} - 3 = x$, $x = -\frac{276}{17}$ | –



Refer Ex 5.2 → 8(iv), eg 5.24

8(iv) $9x^2 - y^2 - 36x - 6y + 18 = 0.$

$$9x^2 - 36x - (y^2 + 6y) = -18$$

$$9(x^2 - 4x) - (y^2 + 6y) = -18$$

$$9(x^2 - 4x + 4 - 4) - (y^2 + 6y + 9 - 9) = -18$$

$$9[(x-2)^2 - 4] - [(y+3)^2 - 9] = -18$$

$$9(x-2)^2 - 36 - (y+3)^2 + 9 = -18$$

$$9(x-2)^2 - (y+3)^2 = -18 + 36 - 9$$

$$9(x-2)^2 - (y+3)^2 = 9$$

$\div 9$ $$\frac{(x-2)^2}{1} - \frac{(y+3)^2}{9} = 1$$

Transverse axis is along x-axis

$a^2 = 1$ | $b^2 = 9$ | $X = x-2$
$a = 1$ | $b = 3$ | $Y = y+3$

$$e = \sqrt{1 + b^2/a^2} = \sqrt{1 + 9/1} = \sqrt{10}$$

$$\boxed{e = \sqrt{10}}$$

and $\boxed{ae = \sqrt{10}}$

$$\boxed{a/e = 1/\sqrt{10}}$$

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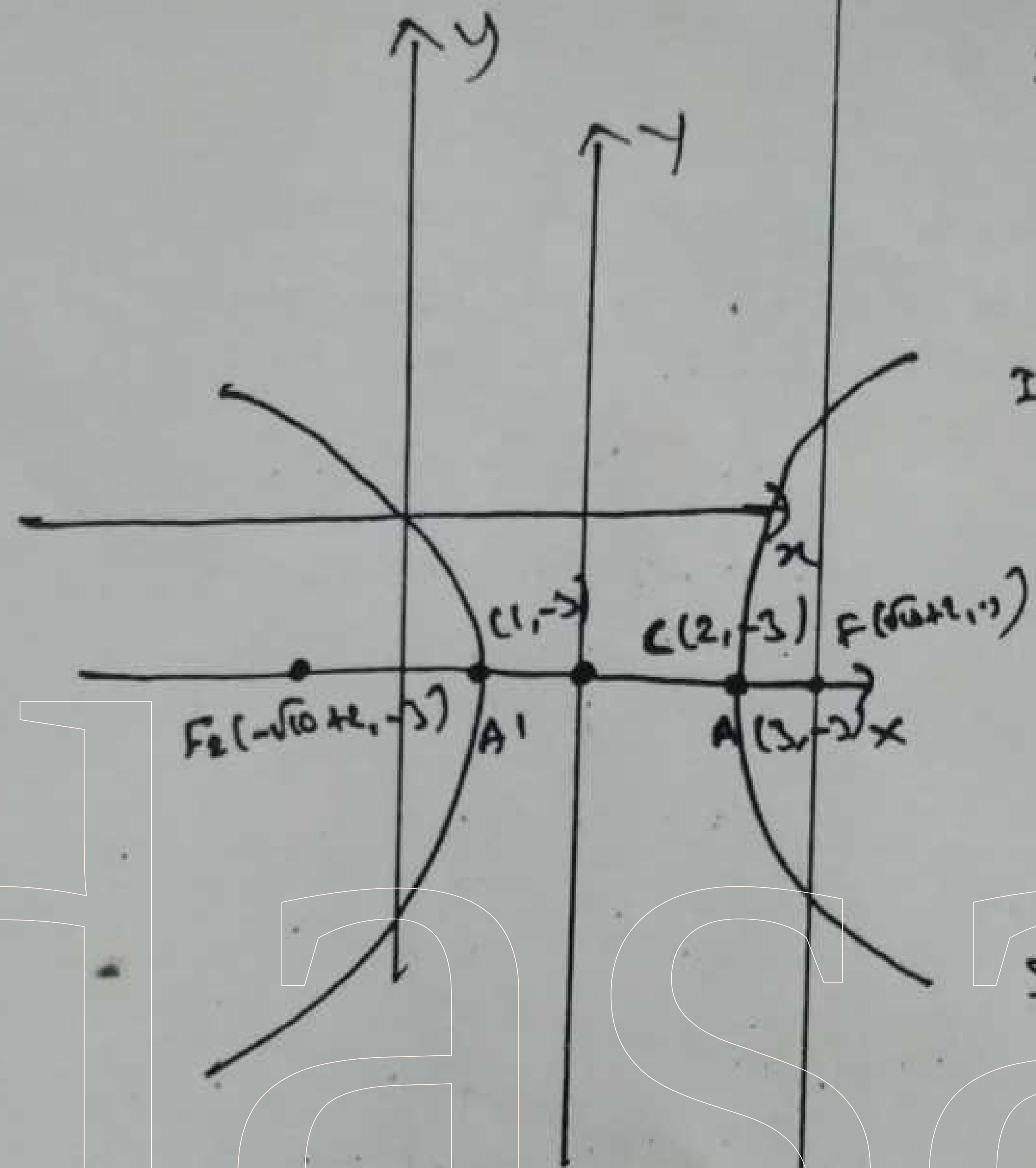
	Referred X, Y	Referred to x, y $X = x-2$	$Y = y+3$
Centre	$(0,0)$ X Y	$0 = x-2$, $x = 2$	$0 = y+3$, $y = -3$ $C(2,-3)$
Foci	$(\pm ae, 0)$ $(\pm\sqrt{10}, 0)$ X, Y	$\sqrt{10} = x-2$, $x = \sqrt{10}+2$ And $-\sqrt{10} = x-2$, $x = -\sqrt{10}+2$ $F_1(\sqrt{10}+2, -3)$ $F_2(-\sqrt{10}+2, -3)$	$0 = y+3$, $y = -3$
vertices	$(\pm a, 0)$ $(\pm 1, 0)$ X Y	$1 = x-2$, $x = 3$ and $-1 = x-2$, $x = 1$ $A(3,-3)$, $A'(1,-3)$	$0 = y+3$, $y = -3$
Eq. of Directrix	$X = \pm a/e$ $X = \pm 1/\sqrt{10}$	$1/\sqrt{10} = x-2$, $x = 1/\sqrt{10}+2$ and $-\frac{1}{\sqrt{10}} = x-2$, $x = 2-1/\sqrt{10}$	



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Ex 5.1

(6) given $x^2+y^2+2gx+2fy+c=0 \rightarrow (A)$

It passes $(1,0)$

$$1+0+2g(1)+2f(0)+c=0$$

$$2g+c=-1 \rightarrow (1)$$

It passes $(-1,0)$

$$1+0+2g(-1)+2f(0)+c=0$$

$$-2g+c=-1 \rightarrow (2)$$

It passes $(0,1)$

$$0+1+2g(0)+2f(1)+c=0$$

$$2f+c=-1 \rightarrow (3)$$

solve (1) (2) (3)

$$\cancel{2g}+c=-1$$
$$\cancel{-2g}+c=-1$$
$$2c=-2$$
$$\boxed{c=-1}$$

$c=-1$ in (1)

$$2g-1=-1$$
$$2g=0$$
$$\boxed{g=0}$$

$c=-1$ in (3)

$$2f-1=-1$$
$$2f=0$$
$$\boxed{f=0}$$

Required eq is

$$x^2+y^2-1=0 \quad \therefore \boxed{x^2+y^2=1}$$

Refer eg 5.10