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11. PROBABILITY DISTRIBUTIONS

Random Variable:-

A random variable x is a function defined on a sample space S into the real numbers R such that the inverse image of points or subset or interval of R is an event in S, for which probability is assigned.

11.1 / 180 Suppose two coins are tossed once. If x denotes the number of tails (i) write down the sample space. (ii) find the inverse image of 1. (iii) the values of the random variable and number of elements in its inverse images.

(i) Sample space:

$$S = \{HH, HT, TH, TT\}$$

(ii) x be a random variable of getting number of tails.

$x(TT) = 2$ $\quad x(TH) = 1$ $\quad x(HT) = 1$

$x(HH) = 0$

$\therefore$ The random variables are 0, 1, 2

(iii) Inverse of 1:

$$x^{-1}\{1\} = \{HT, TH\}$$

(iv) No of elements in inverse images:

Random variable	0	1	2	Total
Inverse images	1	2	1	4

11.2 / 181. Suppose a pair of unbiased dice is rolled once. If x denotes the total score of two dice, write down (i) the sample space (ii) the values taken by the random variable x (iii) the inverse image of 10. and (iv) the number of elements in inverse image of x.

$$S = \{1,2,3,4,5,6\}\{1,2,3,4,5,6\}$$

$$n(S) = 36.$$

$S = \{(1,1)\ (1,2),\ (1,3) \ldots (6,6)\}$

(ii) X be a randomvariable denoting the sum of the numbers on the dice.

$X = 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12$

$X(1,1) = 2 \qquad X(1,2) = X(2,1) = 3$

$X(3,1) = X(1,3) = X(2,2) = 4$

$X(1,4) = X(2,3) = X(3,2) = X(4,1) = 5$

$X(1,5) = X(2,4) = X(3,3) = X(4,2) = X(5,1) = 6$

$X(1,6) = X(2,5) = X(3,4) = X(4,3) = X(5,2) = X(6,1) = 7$

$X(2,6) = X(3,5) = X(4,4) = X(5,3) = X(6,2) = 8$

$X(3,6) = X(4,5) = X(5,4) = X(6,3) = 9$

$X(4,6) = X(5,5) = X(6,4) = 10$

$X(5,6) = X(6,5) = 11$

$X(6,6) = 12.$

(iii) The inverse images of 10

$X^{-1}(10) = \{(4,6)\ (5,5)\ (6,4)\}$

(iv) The number of inverse images:

values of Random variable	2	3	4	5	6	7	8	9	10	11	12
Inverse image	1	2	3	4	5	6	5	4	3	2	1

11.3
182

An urn contains 2 white balls and 3 red balls. A sample of 3 balls are chosen at random from the urn. If X denotes the number of red balls chosen. find the values taken by the random variable X and its number of inverse images.

Number of balls $= 2 + 3 = 5$

$n(S) = 5C_3 = 5C_2 = \frac{5 \times 4}{2 \times 1}$ $\quad [nC_r = nC_{n-r}]$

$\boxed{n(S) = 10}$

$S = \{w_1w_2r_1,\ w_1w_2r_2,\ w_1w_2r_3,\ w_1r_1r_2,\ w_1r_2r_3,\ w_1r_1r_3,\ w_2r_1r_2,\ w_2r_2r_3,\ w_2r_1r_3,\ r_1r_2r_3\}$

Let X be a random variable of getting number of red balls

$X = 0, 1, 2$

Values of the Random variable.	1	2	3	Total
No of elements in inverse images:	3	6	1	10.

11.4 / 183 Two balls are chosen randomly from an urn containing 6 white and 4 black balls. Suppose that we win ₹30 for each black ball selected and we lose ₹20 for each white ball selected. If X denotes the winning amount, find the values of X and number of points in its inverse images.

The possible events of selection are

X (Both Black balls) = ₹2(30) = ₹60

X (1 Black, 1 white) = ₹30 − ₹20 = ₹10

X (Both white balls) = ₹2 (−20) = −₹40

Let X denotes the winning amount

$X = 60, 10, -40$

Random variable :	60	10	−40	Total
inverse images :	6	24	15	45

when $X = 60$; No of inverse image $= 6C_0 \times 4C_2$
$= 1 \times 6 = 6$

when $X = 10$; No of inverse image $= 6C_1 \times 4C_1 = 24$

when $X = -40$ " " $= 6C_2 \times 4C_0 = 15$.

Ex : 11.1

1 / 184 Suppose X is the number of tails occurred when three fair coins are tossed once simultaneously. Find the values of the random variable X, and the number of points in its inverse images.

$S = \{H,T\} \times \{H,T\} \times \{H,T\}$

$n(S) = 2^3 = 8$

Let X be a random variable of getting the number of tails.

$X = 0, 1, 2, 3$

when $X = 0$, $X^{-1}(0) = \{HHH\}$

$X = 1$, $X^{-1}(1) = \{HHT, HTH, THH\}$

$X = 2$, $X^{-1}(2) = \{HTT, THT, TTH\}$

$X = 3$, $X^{-1}(3) = \{TTT\}$

Random variable :	0	1	2	3	Total
No of Inverse images :	1	3	3	1	8

$\frac{2}{184}$ In a Pack of 52 Playing cards, two cards are drawn at random simultaneously If the number of black cards drawn is a random variable, find the values of random variable and number of Points in its inverse images.

Let X be a random variable of getting a black card.

$$X = 0, 1, 2$$

$$n(S) = 52C_2 = \frac{\overset{26}{\cancel{52}} \times 51}{\cancel{2} \times 1} = 1326$$

when $X = 0$, $X^{-1}(0) = 26C_0 \times 26C_2 = 1 \times \frac{\overset{13}{\cancel{26}} \times 25}{\cancel{2} \times 1}$
$= 325$

when $X = 1$, $X^{-1}(1) = 26C_1 \times 26C_1 = 26 \times 26 = 676$

when $X = 2$ $X^{-1}(2) = 26C_2 \times 26C_0 = 325$

Random variable :	0	1	2	Total
No of Inverse images :	325	676	325	1326

$\frac{3}{184}$ An urn Contains 5 mangos and 4 apples Three fruits are taken at random. If the number of apples taken in a random variable then find the values of the random variable and the number of Points in its inverse images.

Let X be a random variable of getting the number of apples

$$X = 0, 1, 2, 3.$$

$$n(S) = 9C_3 = \frac{\overset{3}{\cancel{9}} \times \overset{4}{\cancel{8}} \times 7}{\cancel{3} \times \cancel{2} \times 1} = 84$$

when $X = 0$, $X^{-1}(0) = 4C_0 \times 5C_3 = 1 \times 10 = 10$

when $X = 1$ $X^{-1}(1) = 4C_1 \times 5C_2 = 4 \times 10 = 40$

when $X = 2$ $X^{-1}(2) = 4C_2 \times 5C_1 = 6 \times 5 = 30$

when $X = 3$ $X^{-1}(3) = 4C_3 \times 5C_0 = 4 \times 1 = 4$

Random variable :	0	1	2	3	Total
No of Inverse images :	10	40	30	4	84

$\frac{4}{184}$ Two balls are chosen randomly from an urn containing 6 red and 8 black balls. Suppose that we win ₹15 for each red balls selected and we lose ₹10 for each black ball selected. If X denotes the winning amount. find the values of X and number of Points in its inverse images.

The Possible events of selection are

X (Both are red balls) = ₹ 2(15) = ₹ 30

X (1 Red, 1 Black) = ₹15 − ₹10 = ₹5

X (Both are black balls) = ₹ 2(−10) = −₹ 20

Let X be a random variable denotes the winning amount

$n(S) = 14_{C_2} = \frac{14\times13}{2\times1} = 91$

$X = 30, 5, -20$

when $X = 30$ $X^{-1}(30) = 6_{C_2}\times 8_{C_0} = 15\times1 = 15$

when $X = 5$ $X^{-1}(5) = 6_{C_1}\times 8_{C_1} = 6\times8 = 48$

when $X = -20$ $X^{-1}(-20) = 6_{C_0}\times 8_{C_2} = 1\times28 = 28$

Random variable :	30	5	−20	total
No of Inverse images :	15	48	28	91.

$\frac{5}{184}$ A six sided die is marked '2' on one face. 3 on two of its faces. and 4 on remaining three faces. The die thrown twice. If X denotes the total score in two throws, find the values of the random variable and number of Points in its inverse images.

Since X denotes the total score in two throws, it takes on the values 4, 5, 6, 7, 8

+	2	3	3	4	4	4
2	4	5	5	6	6	6
3	5	6	6	7	7	7
3	5	6	6	7	7	7
4	6	7	7	8	8	8
4	6	7	7	8	8	8
4	6	7	7	8	8	8

No of the Random variable	4	5	6	7	8
No of elements in inverse images :	1	4	10	12	9

Types of Random variable :

* Discrete Random Variable
* Continuous Random variable.

Discrete Random variable :-

A random variable x is defined on a sample space S into the real numbers R is called discrete random variable, if the range of x is countable, that is it can assume only a finite or countably infinite number of values, where every value in the set S has positive probability with total ~~are~~ one.

Probability mass function :-

If x is a discrete random variable with discrete values $x_1, x_2, x_3 \dots x_n$ then the function denoted by $f(\cdot)$ or $P(\cdot)$ and defined by

$$f(x_k) = P(X = x_k) \quad \text{for } k = 0, 1, 2 \dots n$$

is called the probability mass function of x.

The function $f(x)$ is p.m.f if and only if satisfies the following properties for the set of real values $x_1, x_2 \dots x_n \dots$

(i) $f(x_k) \geq 0 \quad \forall k = 1, 2 \dots n$

(ii) $\sum_{k} f(x_k) = 1.$

Cumulative distribution function [c.d.f]

The cumulative distribution function $F(x)$ of a discrete random variable x, taking the values $x_1, x_2, x_3 \dots$ such that $x_1 < x_2 < x_3 \dots$ with p.m.f $f(x_i)$ is

$$F(X) = P(X \leq x) = \sum_{x_i < x} f(x_i) \quad x \in R.$$

Properties of c.d.f :-

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(i) $0 \le F(x) \le 1$

(ii) $F(x)$ is real valued non decreasing function
$(x < y$ then $F(x) \le F(y))$

(iii) $F(x)$ is right continuous function.
(ie) $\lim_{x \to a^+} F(x) = F(a)$.

(iv) $F(-\infty) = 0$

(v) $F(\infty) = 1$

(vi) $P(x_1 < x \le x_2) = F(x_2) - F(x_1)$

(vii) $P(X > x) = 1 - P(X \le x)$
$= 1 - F(x)$

(viii) $P(X = x_k) = F(x_k) - F(x_k^-)$

11.7
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If the p.m.f $f(x)$ of a random variable X is

x	1	2	3	4
$f(x)$	$\frac{1}{12}$	$\frac{5}{12}$	$\frac{5}{12}$	$\frac{1}{12}$

find (i) c.d.f (ii) $P(X \le 3)$ (iii) $P(X \ge 2)$

By the definition of C.D.F

$$F(x) = P(X \le x) = \sum_{x_i < x} P(X = x_i)$$

$F(1) = P(X \le 1) = P(X = 1) = \frac{1}{12}$

$F(2) = P(X \le 2) = P(X=1) + P(X=2) = \frac{1}{12} + \frac{5}{12} = \frac{6}{12} = \frac{1}{2}$

$F(3) = P(X \le 3)$
$= P(X=1) + P(X=2) + P(X=3)$
$= \frac{1}{12} + \frac{5}{12} + \frac{5}{12} = \frac{11}{12}$

$F(4) = P(X \le 4) = P(X=1) + P(X=2) + P(X=3) + P(X=4)$
$= \frac{1}{12} + \frac{5}{12} + \frac{5}{12} + \frac{1}{12} = \frac{12}{12} = 1$

X	1	2	3	4
$F(x)$	$\frac{1}{12}$	$\frac{1}{2}$	$\frac{11}{12}$	1

(ii) $P(X \le 3) = F(3) = \frac{11}{12}$

(iii) $P(X \ge 2) = 1 - P(X < 2) = 1 - P(X=1) = 1 - \frac{1}{12} = \frac{11}{12}$.

11.8
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A six sided die is marked 1 on one face, 2 on two of its faces, and 3 on remaining three faces. The die is rolled twice. If x denotes the total score in two throws.

(i) Find p.m.f (ii) Find the c.d.f

(iii) Find $P(3 \le x < 6)$ (iv) $P(x \ge 4)$

Since x denotes the total score in two throws it takes on the values 2, 3, 4, 5, 6.

$x = 2, 3, 4, 5, 6$

+	1	2	2	3	3	3
1	2	3	3	4	4	4
2	3	4	4	5	5	5
2	3	4	4	5	5	5
3	4	5	5	6	6	6
3	4	5	5	6	6	6
3	4	5	5	6	6	6

$P(x=2) = \frac{1}{36}$

$P(x=3) = \frac{4}{36}$

$P(x=4) = \frac{10}{36}$

$P(x=5) = \frac{12}{36}$

$P(x=6) = \frac{9}{36}$

(i) The p.m.f

x	2	3	4	5	6
$f(x)$	$\frac{1}{36}$	$\frac{4}{36}$	$\frac{10}{36}$	$\frac{12}{36}$	$\frac{9}{36}$

(ii) cumulative distributive function:-

$F(x) = P(X \le x)$

$F(2) = P(X \le 2) = P(X=2) = \frac{1}{36}$

$F(3) = P(X \le 3) = P(X=2) + P(X=3) = \frac{1}{36} + \frac{4}{36} = \frac{5}{36}$

$F(4) = P(X \le 4) = P(X=2) + P(X=3) + P(X=4)$

$= \frac{1}{36} + \frac{4}{36} + \frac{10}{36} = \frac{15}{36}$

$F(5) = P(X \le 5) = P(X=2) + P(X=3) + P(X=4) + P(X=5)$

$= \frac{1}{36} + \frac{4}{36} + \frac{10}{36} + \frac{12}{36} = \frac{27}{36}$

$F(6) = P(X \le 6) = \frac{36}{36} = 1.$

x	2	3	4	5	6
$F(x)$	$\frac{1}{36}$	$\frac{5}{36}$	$\frac{15}{36}$	$\frac{27}{36}$	1

(iii) $P(3 \le x < 6) = P(X=3) + P(X=4) + P(X=5)$

$= \frac{4}{36} + \frac{10}{36} + \frac{12}{36} = \frac{26}{36} = \frac{13}{18}$

(iv) $P(X \ge 4) = P(X=4) + P(X=5) + P(X=6)$

$= \frac{10}{36} + \frac{12}{36} + \frac{9}{36} = \frac{31}{36}$

11.10
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A random variable x has the following p.m.f

x	1	2	3	4	5	6
$f(x)$	k	$2k$	$6k$	$5k$	$6k$	$10k$

Find (i) $P(2<x<6)$ (ii) $P(2\le x<5)$
(iii) $P(x\le 4)$ (iv) $P(3<x)$

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Since $f(x)$ is a p

$\sum f(x)=1$

$k+2k+6k+5k+6k+10k=$

$30k=1$

$\boxed{k=1/30}$

x	1	2	3	4	5	6
$f(x)$	$\frac{1}{30}$	$\frac{2}{30}$	$\frac{6}{30}$	$\frac{5}{30}$	$\frac{6}{30}$	$\frac{10}{30}$

(i) $P(2<x<6)=P(x=3)+P(x=4)+P(x=5)$
$=\frac{6}{30}+\frac{5}{30}+\frac{6}{30}=\frac{17}{30}$

(ii) $P(2\le x<5)=P(x=2)+P(x=3)+P(x=4)$
$=\frac{2}{30}+\frac{6}{30}+\frac{5}{30}=\frac{13}{30}$

(iii) $P(x\le 4)=P(x=1)+P(x=2)+P(x=3)+P(x=4)$
$=\frac{1}{30}+\frac{2}{30}+\frac{6}{30}+\frac{5}{30}=\frac{14}{30}=\frac{7}{15}$

(iv) $P(3<x)=P(x>3)=P(x=4)+P(x=5)+P(x=6)$
$=\frac{5}{30}+\frac{6}{30}+\frac{10}{30}=\frac{21}{30}$.

Ex : 11.2

1
194

Three fair coins are tossed simultaneously Find the Probability mass function for number of heads occurred.

Let x be a random variable of getting a number of heads. $P(H)=\frac{1}{2}$ $P(T)=\frac{1}{2}$

$x=0,1,2,3$

$P(x=0)=P(TTT)=\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}=\frac{1}{8}$

$P(x=1)=P(TTH)+P(THT)+P(HTT)=\frac{3}{8}$

$P(x=2)=P(HHT)+P(HTH)+P(THH)=\frac{3}{8}$

$P(x=3)=P(HHH)=\frac{1}{8}$

x :	0	1	2	3
$f(x)$:	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

$\frac{2}{194}$ A six faced die is marked '1' on one face, '3' on two of its faces, and '5' on remaining three faces. The die is thrown twice. If x denotes the total score in two throws, find

(i) P.m.f (ii) C.D.F (iii) $P(4 \le x \le 10)$

(iv) $P(x \ge 6)$.

Since x denotes the total score in two throws.

$x = 2, 4, 6, 8, 10$

+	1	3	3	5	5	5
1	2	4	4	6	6	6
3	4	6	6	8	8	8
3	4	6	6	8	8	8
5	6	8	8	10	10	10
5	6	8	8	10	10	10
5	6	8	8	10	10	10

$P(x=2) = 1/36$

$P(x=4) = 4/36$

$P(x=6) = 10/36$

$P(x=8) = 12/36$

$P(x=10) = 9/36$

(i) P.m.f.

x :	2	4	6	8	10
$f(x)$:	$\frac{1}{36}$	$\frac{4}{36}$	$\frac{10}{36}$	$\frac{12}{36}$	$\frac{9}{36}$

(ii) C.D.F

$$F(x) = P(X \le x) = \sum_{x_i < x} P(X=x)$$

$F(2) = P(X \le 2) = P(X=2) = \frac{1}{36}$

$F(4) = P(X \le 4) = P(X=2) + P(X=4) = \frac{1}{36} + \frac{4}{36} = \frac{5}{36}$

$F(6) = P(X \le 6) = P(X=2) + P(X=4) + P(X=6)$

$= \frac{1}{36} + \frac{4}{36} + \frac{10}{36} = \frac{15}{36} = \frac{5}{12}$

$F(8) = P(X \le 8) = P(X=2) + P(X=4) + P(X=6) + P(X=8)$

$= \frac{1}{36} + \frac{4}{36} + \frac{10}{36} + \frac{12}{36} = \frac{27}{36} = \frac{9}{12} = \frac{3}{4}$

$F(10) = P(X \le 10) = 1$

x	2	4	6	8	10
F(x) :	$\frac{1}{36}$	$\frac{5}{36}$	$\frac{15}{36}$	$\frac{27}{36}$	1

(iii) $P(4 \le X < 10) = P(X=4) + P(X=6) + P(X=6)$

$$= \frac{4}{36} + \frac{10}{36} + \frac{12}{36} = \frac{26}{36} = \frac{13}{18}$$

(iv) $P(X \ge 6) = P(X=6) + P(X=8) + P(X=10)$

$$= \frac{10}{36} + \frac{12}{36} + \frac{9}{36} = \frac{31}{36}$$

3/194 Find the p.m.f and c.d.f of number of girl child in families with 4 children, assuming equal probabilities for boys and girls.

Let X be a random variable of getting a number of girl child.

$$X = 0, 1, 2, 3, 4.$$

$$n(S) = 2^4 = 16$$

$P(X=0) = \frac{4C_0}{16} = \frac{1}{16}$ $P(X=1) = \frac{4C_1}{16} = \frac{4}{16}$

$P(X=2) = \frac{4C_2}{16} = \frac{6}{16}$ $P(X=3) = \frac{4C_3}{16} = \frac{4}{16}$

$P(X=4) = \frac{4C_4}{16} = \frac{1}{16}$

x	0	1	2	3	4
$f(x)$	$\frac{1}{16}$	$\frac{4}{16}$	$\frac{6}{16}$	$\frac{4}{16}$	$\frac{1}{16}$

(ii) C.D.F:

$$F(x) = P(X \le x) = \sum_{x_i \le x} P(X=x)$$

$F(0) = P(X \le 0) = P(X=0) = \frac{1}{16}$

$F(1) = P(X \le 1) = \frac{5}{16}$

$F(2) = P(X \le 2) = \frac{11}{16}$

$F(3) = P(X \le 3) = \frac{15}{16}$

$F(4) = P(X \le 4) = \frac{16}{16} = 1.$

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x :	0	1	2	3	4
$F(x)$:	$\frac{1}{16}$	$\frac{5}{16}$	$\frac{11}{16}$	$\frac{15}{16}$	1

4/194. Suppose a discrete random variable can only take the values 0, 1, and 2. The Probability mass function is defined by

$$f(x) = \begin{cases} \frac{x^2+1}{k} & \text{for } x = 0, 1, 2 \\ 0 & \text{otherwise.} \end{cases}$$

find (i) the value of k (ii) C.D.F (iii) $P(X \geq 1)$

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$f(x) = \frac{x^2+1}{k}$

when $x = 0$, $f(x) = \frac{1}{k}$

$x = 1$ $f(x) = \frac{2}{k}$

$x = 2$ $f(x) = \frac{5}{k}$

x	0	1	2
$f(x)$:	$\frac{1}{k}$	$\frac{2}{k}$	$\frac{5}{k}$

Since $f(x)$ is a p.m.f

$\Sigma f(x) = 1$

$\frac{1}{k} + \frac{2}{k} + \frac{5}{k} = 1$

$\frac{8}{k} = 1 \Rightarrow \boxed{k = 8}$

(i) P.m.f

x	0	1	2
$f(x)$:	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{5}{8}$

(ii) C.D.F :-

$F(x) = P(X \leq x) = \sum_{x_i < x} P(X = x)$

$F(0) = P(X \leq 0) = \frac{1}{8}$

$F(1) = P(X \leq 1) = \frac{3}{8}$

$F(2) = P(X \leq 2) = \frac{8}{8} = 1.$

x :	0	1	2
$F(x)$:	$\frac{1}{8}$	$\frac{3}{8}$	1

(iii) $P(X \geq 1) = P(X = 1) + P(X = 2)$

$= \frac{2}{8} + \frac{5}{8} = \frac{7}{8}.$

$\frac{6}{194}$ A random variable x has the following p.m.f

x :	1	2	3	4	5
$f(x)$:	k^2	$2k^2$	$3k^2$	$2k$	$3k$

(i) Find the value of 'k' (ii) $P(2 \le X < 5)$

(iii) $P(3 < X)$

Since $f(x)$ is a p.m.f

$\Sigma f(x) = 1$

$k^2 + 2k^2 + 3k^2 + 2k + 3k = 1$

$6k^2 + 5k - 1 = 0$

$(6k-1)(k+1) = 0$

$\boxed{k = \frac{1}{6}}$ $\boxed{k \ne -1}$

-6: $\frac{-1}{6k}$, $\frac{\cancel{6}}{\cancel{6}k}$

(i) P.m.f

x :	1	2	3	4	5
$f(x)$:	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{2}{6}$	$\frac{3}{6}$

(ii) C.D.F :

$F(X) = P(X \le x) = \sum_{x_i < x} P(X = x)$

$F(1) = P(X \le 1) = \frac{1}{36}$

$F(2) = P(X \le 2) = \frac{3}{36} = \frac{1}{12}$

$F(3) = P(X \le 3) = \frac{6}{36} = \frac{1}{6}$

$F(4) = P(X \le 4) = \frac{1}{36} + \frac{2}{36} + \frac{3}{36} + \frac{12}{36} = \frac{18}{36} = \frac{1}{2}$

$F(5) = P(X \le 5) = \frac{1}{36} + \frac{2}{36} + \frac{3}{36} + \frac{12}{36} + \frac{18}{36} = \frac{36}{36} = 1$

x :	1	2	3	4	5
$F(x)$:	$\frac{1}{36}$	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{2}$	1.

(iii) $P(2 \le x < 5) = P(X=2) + P(X=3) + P(X=4)$

$= \frac{2}{36} + \frac{3}{36} + \frac{12}{36} = \frac{17}{36}$

(iv) $P(3 < x) = P(x > 3) = P(X=4) + P(X=5)$

$= \frac{2}{6} + \frac{3}{6} = \frac{5}{6}$

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Probability mass function from Cumulative distribution function :-

Suppose X is a discrete random variable taking the values $x_1, x_2, x_3 \ldots$ such that $x_1 < x_2 < x_3 \ldots$ and $F(x_i)$ is the distribution function. Then the Probability mass function $f(x_i)$ is given by

$$f(x_i) = F(x_i) - F(x_{i-1}) \quad i = 1, 2, 3 \ldots$$

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Find the Probability mass function $f(x)$ of the discrete random variable X whose cumulative distribution function $F(x)$ is given by

$$F(x) = \begin{cases} 0 & -\infty < x < -2 \\ 0.25 & -2 \leq x < -1 \\ 0.60 & -1 \leq x < 0 \\ 0.90 & 0 \leq x < 1 \\ 1 & 1 \leq x < \infty \end{cases}$$

Also find (i) $P(X < 0)$ (ii) $P(X \geq -1)$

Since X is a discrete random variable, The random variables are

$X = -2, -1, 0, 1$

$f(x_i) = F(x_i) - F(x_{i-1})$

$f(-2) = P(X = -2) = F(-2) = 0.25$

$f(-1) = P(X = -1) = F(-1) - F(-2) = 0.6 - 0.25$
$= 0.35$

$f(0) = P(X = 0) = F(0) - F(-1) = 0.9 - 0.6$
$= 0.3$

$f(1) = P(X = 1) = F(1) - F(0)$
$= 1 - 0.9 = 0.1$

$\therefore$ The P.m.f is

x	-2	-1	0	1
$f(x)$	0.25	0.35	0.3	0.1

(i) $P(X<0) = P(X=-2)+P(X=-1)$

$= 0.25+0.35 = 0.6$

(ii) $P(X \geq -1) = P(X=-1)+P(X=0)+P(X=1)$

$= 0.35+0.3+0.1$

$= 0.75$

5/194 The C.D.F of a discrete random variable is given by

$$F(x) = \begin{cases} 0 & -\infty < x < -1 \\ 0.15 & -1 \leq x < 0 \\ 0.35 & 0 \leq x < 1 \\ 0.60 & 1 \leq x < 2 \\ 0.85 & 2 \leq x < 3 \\ 1 & 3 \leq x < \infty \end{cases}$$

Find (i) P.m.f (ii) $P(X<1)$ (iii) $P(X \geq 2)$

Since X is a discrete random variable, the random variables are

$X = -1, 0, 1, 2, 3$

$f(x_i) = F(x_i) - F(x_{i-1})$

$f(-1) = P(X=-1) = F(-1) = 0.15$

$f(0) = P(X=0) = F(0)-F(-1) = 0.35-0.15 = 0.20$

$f(1) = P(X=1) = F(1)-F(0) = 0.60-0.35 = 0.25$

$f(2) = P(X=2) = F(2)-F(1) = 0.85-0.60 = 0.25$

$f(3) = P(X=3) = F(3)-F(2) = 1-0.85 = 0.15$

∴ The p.m.f is

x :	-1	0	1	2	3
$f(x)$:	0.15	0.20	0.25	0.25	0.15

(ii) $P(X<1) = P(X=0)+P(X=-1)$

$= 0.20+0.15 = 0.35$

(iii) $P(X \geq 2) = P(X=2)+P(X=3)$

$= 0.25+0.15 = 0.40.$

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7/194. The C.D.F of a discrete random variable is given by

$$F(x) = \begin{cases} 0 & -\infty < x < 0 \\ 1/2 & 0 \le x < 1 \\ 3/5 & 1 \le x < 2 \\ 4/5 & 2 \le x < 3 \\ 9/10 & 3 \le x < 4 \\ 1 & 4 \le x < \infty \end{cases}$$

Find (i) P.m.f (ii) $P(X<3)$ (iii) $P(X \ge 2)$

Since X is a discrete random variable, the random variables are

$X = 0, 1, 2, 3, 4.$

$f(0) = P(X=0) = F(0) = \frac{1}{2}$

$f(1) = P(X=1) = F(1) - F(0) = \frac{3}{5} - \frac{1}{2} = \frac{6-5}{10} = \frac{1}{10}$

$f(2) = P(X=2) = F(2) - F(1) = \frac{4}{5} - \frac{3}{5} = \frac{1}{5}$

$f(3) = P(X=3) = F(3) - F(2) = \frac{9}{10} - \frac{4}{5} = \frac{9-8}{10} = \frac{1}{10}$

$f(4) = P(X=4) = F(4) - F(3) = 1 - \frac{9}{10} = \frac{1}{10}$

∴ The Pmf is

x :	0	1	2	3	4
$f(x)$:	$\frac{1}{2}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{10}$

(ii) $P(X<3) = P(X=0) + P(X=1) + P(X=2)$

$= \frac{1}{2} + \frac{1}{10} + \frac{1}{5} = \frac{5+1+2}{10} = \frac{8}{10} = \frac{4}{5}$

(iii) $P(X \ge 2) = P(X=2) + P(X=3) + P(X=4)$

$= \frac{1}{5} + \frac{1}{10} + \frac{1}{10} = \frac{2+1+1}{10} = \frac{4}{10} = \frac{2}{5}$

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Continuous Random Variable :-

Let S be a sample space and let a random variable $X : S \to R$ that takes on any value in a set I of R. Then X is called a continuous random variable if $P(X = x) = 0$ for every x in I.

Probability density function : (p.d.f)

A non-negative real valued function $f(x)$ is said to be a probability density function if for each possible outcome x of a continuous random variable $x \in [a, b]$ of a continuous random variable X having the property

$$P(a \le x \le b) = \int_a^b f(x)\, dx.$$

A function $f(\cdot)$ is p.d.f for some continuous random variable X if and only if it satisfies the following properties,

(i) $f(x) \ge 0$

(ii) $\int_{-\infty}^{\infty} f(x)\, dx = 1.$

Note :-

* If X is a continuous random variable

$$P(a \le x \le b) = \int_a^b f(x)\, dx$$ which means

that $$P(X = a) = \int_a^a f(x)\, dx = 0$$

That is probability when X takes on any one particular value is zero.

* In the discrete case $f(a) = P(X = a)$ is the probability that X takes the value a.

In the continuous case $f(x)$ at $x = a$ is not the probability that X takes the value 'a'. that is $f(a) \ne P(X = a)$

* when the random variable is continuous the summation used in discrete is replaced by integration.

* For the continuous random variable

$P(a<x<b) = P(a \le x \le b) = P(a < x \le b)$
$= P(a \le x < b)$

Cumulative Distribution Function : [C.D.F]

The distribution function or cumulative distribution function $F(x)$ of a continuous random variable X with Probability density $f(x)$ is

$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(u)\, du \qquad -\infty < u < \infty$$

Properties of distribution function :-

* $0 \le F(x) \le 1$
* $F(x)$ is a real valued non decreasing. [if $x < y$ then $F(x) \le F(y)$]
* $F(x)$ is continuous every where.
* $F(-\infty) = 0$
* $F(\infty) = 1$
* $P(X > x) = 1 - P(X \le x) = 1 - F(x)$
* $P(a < X < b) = F(b) - F(a)$
* $f(x) = \frac{d}{dx}(F(x))$ (ie) $f(x) = F'(x)$

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Find the constant 'c' such that the function $f(x) = \begin{cases} cx^2 & 1 < x < 4 \\ 0 & \text{otherwise} \end{cases}$ is a density function and compute

(i) $P(1.5 < X < 3.5)$ (ii) $P(X \le 2)$ (iii) $P(3 < X)$

Since $f(x)$ is a p.d.f

$$\int_{-\infty}^{\infty} f(x)\, dx = 1$$

$$\int_1^4 cx^2\,dx = 1$$

$$c\left[\frac{x^3}{3}\right]_1^4 = 1$$

$$c\left[\frac{64}{3} - \frac{1}{3}\right] = 1$$

$$\left[\frac{63}{3}\right]c = 1 \Rightarrow 21c = 1$$

$$\boxed{c = \frac{1}{21}}$$

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$$\therefore f(x) = \begin{cases} \frac{1}{21}x^2 & 1 < x < 4 \\ 0 & \text{otherwise.} \end{cases}$$

(i) $P(1.5 < x < 3.5) = \int_{3/2}^{7/2} \frac{1}{21}x^2\,dx = \frac{1}{21}\left[\frac{x^3}{3}\right]_{3/2}^{7/2}$

$$= \frac{1}{3\times21}\left[\frac{343}{8} - \frac{27}{8}\right] = \frac{1}{3(21)}\left[\frac{316}{8}\right] = \frac{1}{3(21)}\left(\frac{79}{2}\right)$$

$$= \frac{79}{126}$$

(ii) $P(x \le 2) = \int_1^2 \frac{1}{21}x^2.dx = \frac{1}{21}\left[\frac{x^3}{3}\right]_1^2$

$$= \frac{1}{21}\left[\frac{8}{3} - \frac{1}{3}\right] = \frac{1}{21}\left(\frac{7}{3}\right) = \frac{7}{63} = \frac{1}{9}$$

(iii) $P(3 < x) = P(x > 3) = \int_3^4 \frac{1}{21}x^2.dx$

$$= \frac{1}{21}\left[\frac{x^3}{3}\right]_3^4 = \frac{1}{21(3)}(64-27)$$

$$= \frac{37}{63}.$$

Ex : 11.3

$\frac{1}{202}$ The p.d.f of x is given by $f(x) = \begin{cases} kxe^{-2x} & x>0 \\ 0 & x \le 0 \end{cases}$

find 'k'

Since $f(x)$ is a p.d.f

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

Gamma integral
$$\int_0^\infty x^n e^{-ax}\,dx = \frac{n!}{a^{n+1}}$$

$$k\int_0^\infty x e^{-2x}\,dx = 1$$

$$k\left[\frac{1!}{2^{1+1}}\right] = 1$$

$$\frac{k}{4} = 1 \Rightarrow \boxed{k = 4}$$

$$\therefore f(x) = \begin{cases} 4xe^{-2x} & x>0 \\ 0 & x \le 0 \end{cases}$$

$\frac{2}{202}$ The P.d.f of x is $f(x) = \begin{cases} x & 0 < x < 1 \\ 2-x & 1 \le x < 2 \\ 0 & \text{otherwise.} \end{cases}$

Find (i) $P(0.2 \le x < 0.6)$

(ii) $P(1.2 \le x < 1.8)$

(iii) $P(0.5 \le x < 1.5)$

(i) $P(0.2 \le x < 0.6) = \int_{0.2}^{0.6} x\, dx$

$= \left[\frac{x^2}{2}\right]_{0.2}^{0.6} = \frac{1}{2}[0.36 - 0.04)$

$= \frac{1}{2}[0.32] = 0.16$

(ii) $P(1.2 \le x < 1.8) = \int_{1.2}^{1.8} (2-x)\, dx = \left[2x - \frac{x^2}{2}\right]_{1.2}^{1.8}$

$= \left[2(1.8) - \frac{1.8^2}{2}\right] - \left[2(1.2) - \frac{1.2^2}{2}\right]$

$= \left[3.6 - \frac{3.24}{2}\right] - \left[2.4 - \frac{1.44}{2}\right]$

$= 3.6 - 1.62 - 2.4 + 0.72$

$= 0.3.$

(iii) $P(0.5 \le x < 1.5) = \int_{0.5}^{1} f(x)dx + \int_{1}^{1.5} f(x)dx$

$= \int_{0.5}^{1} x\,dx + \int_{1}^{1.5} (2-x)dx$

$= \left[\frac{x^2}{2}\right]_{0.5}^{1} + \left[2x - \frac{x^2}{2}\right]_{1}^{1.5}$

$= \left[\frac{1}{2} - \frac{0.25}{2}\right] + \left[\left(2(1.5) - \frac{1.5^2}{2}\right) - \left(2 - \frac{1}{2}\right)\right]$

$= \frac{1}{2} - 0.125 + 3 - \frac{2.25}{2} - 2 + \frac{1}{2}$

$= 1 + 3 - 2 - 0.125 - 1.125 = 2 - 1.25 = 0.75.$

$\frac{3}{202}$ Suppose the amount of milk sold daily at a milk booth is distributed with a minimum of 200 litres and a maximum of 600 litres with P.d.f

$$f(x) = \begin{cases} k & 200 \le x \le 600 \\ 0 & \text{otherwise} \end{cases}$$

Find (i) the value of 'k'

(ii) The distribution function

(iii) The Probability that daily Sales will fall between 300 litres and 500 litres?

Since $f(x)$ is a p.d.f

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{200}^{600} k\,dx = 1 \Rightarrow k[x]_{200}^{600} = 1$$

$$k[600-200] = 1$$

$$400k = 1$$

$$\boxed{k = 1/400}$$

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$$\therefore f(x) = \begin{cases} \frac{1}{400} & 200 \le x \le 600 \\ 0 & \text{otherwise.} \end{cases}$$

Distribution function :-

$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(u)\,du.$$

$(\ )[\ ](\ )$: $-\infty$, 200, 600, ∞

(i) when $x \in (-\infty, 200)$

$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(u)\,du = 0$$

(ii) when $x \in [200, 600]$

$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(u)\,du$$

$$= \int_{-\infty}^{200} f(u)\,du + \int_{200}^{x} f(u)\,du$$

$$= 0 + \int_{200}^{x} \frac{1}{400}\,du = \frac{1}{400}[u]_{200}^{x}$$

$$= \frac{1}{400}(x-200)$$

(iii) when $x \in (600, \infty)$

$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(u)\,du$$

$$= \int_{-\infty}^{200} f(u)\,du + \int_{200}^{600} f(u)\,du + \int_{600}^{x} f(u)\,du$$

$$= 0 + \frac{1}{400}(u)_{200}^{600} + 0 = \frac{1}{400}(400) = 1$$

$$F(x) = \begin{cases} 0 & -\infty < x < 200 \\ \frac{1}{400}(x-200) & 200 \le x \le 600 \\ 1 & x > 600 \end{cases}$$

(iii) $P(300 < X < 500) = F(500) - F(300)$

$$= \frac{1}{400}(500-200) - \frac{1}{400}(300-200)$$

$$= \frac{1}{400}(300-100) = \frac{200}{400} = \frac{1}{2}.$$

$\frac{4}{202}$ The P.d.f of x is given by

$$f(x) = \begin{cases} k e^{-x/3} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

Find (i) value of 'k' (ii) the distribution fun. (iii) $P(X < 3)$ (iv) $P(5 \leq X)$ (v) $P(X \leq 4)$.

Since $f(x)$ is a P.d.f

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$k\int_{0}^{\infty} e^{-x/3}\,dx = 1$$

$$k\left[\frac{e^{-x/3}}{-1/3}\right]_0^{\infty} = 1 \Rightarrow -3k\left[e^{-\infty} - e^{0}\right] = 1$$

$$3k = 1$$

$$\boxed{k = 1/3}$$

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$$f(x) = \begin{cases} \frac{1}{3} e^{-x/3} & x > 0 \\ 0 & x \leq 0 \end{cases}$$

(ii) Distribution function :-

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(u)\,du$$

(i) when $x \in (-\infty, 0]$

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(u)\,du = 0$$

(ii) when $x \in (0, \infty)$

$$F(x) = P(X \leq x) = \int_{-\infty}^{0} f(u)\,du + \int_{0}^{x} f(u)\,du$$

$$= 0 + \int_0^x \frac{1}{3} e^{-u/3}\,du$$

$$= \frac{1}{3}\left[\frac{e^{-u/3}}{-1/3}\right]_0^x = (-1)\left\{e^{-x/3} - 1\right\}$$

$$= 1 - e^{-x/3}$$

$$F(x) = \begin{cases} 1 - e^{-x/3} & x > 0 \\ 0 & x \leq 0 \end{cases}$$

(iii) $P(X < 3) = P(X \leq 3) = F(3)$

$$= 1 - e^{-3/3} = 1 - e^{-1} = 1 - \frac{1}{e} = \frac{e-}{e}$$

(iv) $P(5 \leq X) = P(X \geq 5) = \frac{1}{3}\int_5^x e^{-x/3}\,dx$

$$= \frac{1}{3}\left\{\frac{e^{-x/3}}{-1/3}\right\}_5^x$$

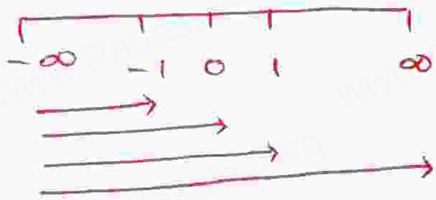
$$= (-1)\left\{e^{-x/3} - e^{-5/3}\right\} = e^{-5/3} - e^{-x/3}$$

(v) $P(X \leq 4) = F(4) = 1 - e^{-4/3}$

$\frac{5}{203}$ If x is the random variable with p.d.f $f(x)$ given by

$$f(x) = \begin{cases} x+1 & -1 \leq x < 0 \\ -x+1 & 0 \leq x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

then find (i) The distribution function.
ii) $P(-0.5 \leq X \leq 0.5)$



Distribution function :-

$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(u)\,du.$

(i) when $x \in (-\infty, -1)$

$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(u)\,du = 0$

(ii) when $x \in [-1, 0)$

$F(x) = P(X \leq x) = \int_{-\infty}^{-1} f(u)\,du + \int_{-1}^{x} f(u)\,du$

$= 0 + \int_{-1}^{x} (u+1)\,du = \left[\frac{u^2}{2} + u\right]_{-1}^{x}$

$= \left(\frac{x^2}{2} + x\right) - \left(\frac{1}{2} - 1\right) = \frac{x^2}{2} + x + \frac{1}{2}$

$= \frac{1}{2}(x^2 + 2x + 1) = \frac{1}{2}(x+1)^2$

(iii) when $x \in [0, 1)$

$F(x) = \int_{-\infty}^{-1} f(u)\,du + \int_{-1}^{0} f(u)\,du + \int_{0}^{x} f(u)\,du$

$= 0 + \int_{-1}^{0} (u+1)\,du + \int_{0}^{x} (-u+1)\,du$

$= \left[\frac{u^2}{2} + u\right]_{-1}^{0} + \left[-\frac{u^2}{2} + u\right]_{0}^{x}$

$= 0 - \left(\frac{1}{2} - 1\right) + \left(-\frac{x^2}{2} + x\right) - 0$

$= \frac{1}{2} - \frac{x^2}{2} + x = \frac{-1}{2}(x^2 - 2x - 1)$

(iv) when $x \in [1, \infty)$

$F(x) = P(X \leq x) = \int_{-\infty}^{-1} f(u)\,du + \int_{-1}^{0} f(u)\,du + \int_{0}^{1} f(u)\,du + \int_{1}^{x} f(u)\,du$

$= 0 + \int_{-1}^{0} (u+1)\,du + \int_{0}^{1} (-u+1)\,du + 0$

$= \left(\frac{u^2}{2} + u\right)_{-1}^{0} + \left(-\frac{u^2}{2} + u\right)_{0}^{1} = 0 - \left(\frac{1}{2} - 1\right) + \left(-\frac{1}{2} + 1\right)$

$$\therefore F(x) = \begin{cases} 0 & x \le -1 \\ \frac{1}{2}(x+1)^2 & -1 \le x < 0 \\ \frac{-1}{2}(x^2-2x-1) & 0 \le x < 1 \\ 1 & x \ge 1 \end{cases}$$

(ii) $P(-0.5 \le X \le 0.5) = F(0.5) - F(-0.5)$

$$= -\frac{1}{2}\left\{0.5^2 - 2(0.5) - 1\right\} - \left\{\frac{1}{2}(-0.5+1)^2\right\}$$

$$= -\frac{1}{2}\left[0.25 - 1 - 1\right] - \frac{1}{2}(0.25)$$

$$= -\frac{1}{2}\left[0.25 - 2 + 0.25\right]$$

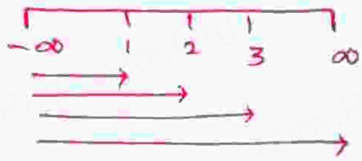
$$= -\frac{1}{2}\left[0.5 - 2\right] = -\frac{1}{2}\left[-1.5\right] = 0.75.$$

$P(-0.5 \le X \le 0.5) = 0.75.$

11.12 / 198 If X is the random variable with P.d.f $f(x)$ is given by

$$f(x) = \begin{cases} x-1 & 1 \le x < 2 \\ -x+3 & 2 \le x < 3 \\ 0 & \text{otherwise.} \end{cases}$$

find (i) The distribution function.
(ii) $P(1.5 \le X \le 2.5)$



Distribution function :-

$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(u)\,du.$$

(i) when $x \in (-\infty, 1)$

$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(u)\,du = 0$$

(ii) when $x \in [1, 2)$

$$F(x) = P(X \le x) = \int_{-\infty}^{1} f(u)\,du + \int_{1}^{x} f(u)\,du.$$

$$= 0 + \int_{1}^{x} (u-1)\,du = \left[\frac{u^2}{2} - u\right]_1^x$$

$$= \left(\frac{x^2}{2} - x\right) - \left(\frac{1}{2} - 1\right) = \frac{x^2}{2} - x + \frac{1}{2}$$

$$= \frac{1}{2}(x^2 - 2x + 1) = \frac{1}{2}(x-1)^2$$

(iii) when $x \in [2,3)$

$$F(x) = P(X \le x) = \int_{-\infty}^{1} f(u)du + \int_{1}^{2} f(u)du + \int_{2}^{x} f(u)du$$

$$= 0 + \int_{1}^{2} (u-1)\,du + \int_{2}^{x} (3-u)\,du$$

$$= \left[\frac{u^2}{2} - u\right]_1^2 + \left[3u - \frac{u^2}{2}\right]_2^x$$

$$= (2-2) - \left(\frac{1}{2} - 1\right) + \left[3x - \frac{x^2}{2}\right] - \left[6-2\right]$$

$$= \frac{1}{2} + 3x - \frac{x^2}{2} - 4 = 3x - \frac{x^2}{2} - \frac{7}{2}$$

$$= -\frac{1}{2}\left[x^2 - 6x - 7\right]$$

(iv) when $x \in [3, \infty)$

$$F(x) = P(X \le x) = \int_{-\infty}^{1} f(u)du + \int_{1}^{2} f(u)du + \int_{2}^{3} f(u)du + \int_{3}^{x} f(u)d$$

$$= 0 + \int_{1}^{2} (u-1)du + \int_{2}^{3} (3-u)du + 0$$

$$= \left[\frac{u^2}{2} - u\right]_1^2 + \left[3u - \frac{u^2}{2}\right]_2^3$$

$$= \left[(2-2) - \left(\frac{1}{2} - 1\right)\right] + \left[\left(9 - \frac{9}{2}\right) - (6-2)\right]$$

$$= \frac{1}{2} + \frac{9}{2} - 4 = 5 - 4 = 1$$

$$\therefore F(x) = \begin{cases} 0 & x < 1 \\ \frac{1}{2}(x-1)^2 & 1 \le x < 2 \\ -\frac{1}{2}(x^2 - 6x - 7) & 2 \le x < 3 \\ 1 & x \ge 3 \end{cases}$$

(ii) $P(1.5 \le X \le 2.5) = F(2.5) - F(1.5)$

$$= -\frac{1}{2}\left\{(2.5)^2 - 6(2.5) - 7\right\} - \frac{1}{2}\left\{(1.5-1)^2\right\}$$

$$= -\frac{1}{2}\left[6.25 - 15 + 7\right\} - \frac{1}{2}(0.59)^2$$

$$= -\frac{1}{2}\left[6.25 - 8 + 0.25\right]$$

$$= -\frac{1}{2}(6.50 - 8) = -\frac{1}{2}(-1.5) = 0.75$$

$P(1.5 \le X \le 2.5) = 0.75.$

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11.14 / 200 The p.d.f of random variable X is given,

$$f(x) = \begin{cases} k & 1 \le x \le 5 \\ 0 & \text{otherwise} \end{cases}$$

find (i) k (ii) Distribution function (iii) $P(X < 3)$ (iv) $P(2 < X < 4)$ (v) $P(3 \le X)$

Since $f(x)$ is a p.d.f

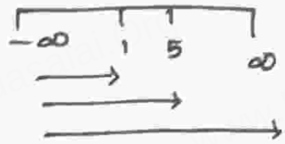
$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_1^5 k\,dx = 1 \Rightarrow k[x]_1^5 = 1$$

$$k(5-1) = 1$$

$$\boxed{k = \frac{1}{4}}$$

$$\therefore f(x) = \begin{cases} \frac{1}{4} & 1 \le x \le 5 \\ 0 & \text{otherwise.} \end{cases}$$



Distribution function :-

$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(u)\,du.$$

when $x \in (-\infty, 1)$

$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(u)\,du = 0$$

when $x \in [1, 5]$

$$F(x) = P(X \le x) = \int_{-\infty}^{1} f(u)\,du + \int_1^x f(u)\,du$$

$$= 0 + \frac{1}{4}\int_1^x du = \frac{1}{4}[u]_1^x$$

$$= \frac{1}{4}[x-1]$$

when $x \in (5, \infty)$

$$F(x) = P(X \le x) = \int_{-\infty}^{1} f(u)\,du + \int_1^5 f(u)\,du + \int_5^x f(u)\,du.$$

$$= 0 + \frac{1}{4}(u)_1^5 + 0$$

$$= \frac{1}{4}(5-1) = \frac{1}{4}(4) = 1$$

$$\therefore F(x) = \begin{cases} 0 & x \le 1 \\ \frac{1}{4}(x-1) & 1 \le x \le 5 \\ 1 & x > 5 \end{cases}$$

(ii) $P(X < 3) = P(X \le 3) = F(3)$

$$= \frac{1}{4}(3-1) = \frac{2}{4} = \frac{1}{2}$$

(iii) $P(2 < X < 4) = F(4) - F(2)$

$$= \frac{1}{4}(4-1) - \frac{1}{4}(2-1)$$

$$= \frac{1}{4}(3) - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$$

(iv) $P(3 \le X) = P(X \ge 3)$

$$= 1 - P(X < 3)$$

$$= 1 - P(X \le 3)$$

$$= 1 - \frac{1}{4}(3-1) = 1 - \frac{1}{2} = \frac{1}{2}.$$

$\frac{11.15}{201}$ Let X be a random variable denoting the life time of an electrical equipment having p.d.f

$$f(x) = \begin{cases} k e^{-2x} & x > 0 \\ 0 & x \le 0 \end{cases}$$

Find (i) Value of k (ii) Distribution function iii) $P(X < 2)$ (iv) calculate the Probability that X is atleast for four unit of time.

(v) $P(X = 3)$

Since $f(x)$ is a p.d.f

(i) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

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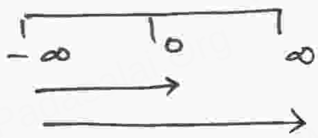
$$k\int_0^{\infty} e^{-2x}\,dx = 1$$

$$k\left[\frac{e^{-2x}}{-2}\right]_0^{\infty} = 1$$

$$-\frac{k}{2}\left[e^{-\infty} - e^{0}\right] = 1$$

$$\frac{k}{2} = 1 \Rightarrow \boxed{k = 2}$$

$$\therefore f(x) = \begin{cases} 2e^{-2x} & x > 0 \\ 0 & x \le 0 \end{cases}$$



(ii) Distribution function :-

$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(u)\,du$$

when $x \in (-\infty, 0]$

$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(u)\,du = 0.$$

(ii) when $x \in (0, \infty)$

$$F(x) = P(X \leq x) = \int_{-\infty}^{0} f(u)\,du + \int_{0}^{x} f(u)\,du$$

$$= 0 + \int_{0}^{x} 2e^{-2u}\,du = 2\left[\frac{e^{-2u}}{-2}\right]_0^x$$

$$= (-1)\left[e^{-2x} - 1\right] = 1 - e^{-2x}$$

$$\therefore F(x) = \begin{cases} 1 - 2e^{-2x} & x > 0 \\ 0 & x \leq 0 \end{cases}$$

(iii) $P(X < 2) = P(X \leq 2) = F(2)$

$$= 1 - e^{-4} = 1 - \frac{1}{e^4} = \frac{e^4 - 1}{e^4}$$

(iv) $P(X \geq 4) = 1 - P(X < 4)$

$$= 1 - P(X \leq 4)$$

$$= 1 - \{1 - e^{-8}\} = e^{-8}$$

(v) $P(X = 3) = ?$

In the continuous random variable,

$$P(X = a) = 0$$

$$\therefore P(X = 3) = 0.$$

Probability density function from Probability distribution function :-

11.13 / 199 If X is the random variable with distribution function $F(x)$ given by

$$F(x) = \begin{cases} 0 & x < 0 \\ x & 0 \leq x < 1 \\ 1 & x \geq 1 \end{cases}$$

then find (i) p.d.f (ii) $P(0.2 \leq X \leq 0.7)$

$$f(x) = \frac{d}{dx}\left[F(x)\right]$$

$$f(x) = F'(x)$$

$$f(x) = \begin{cases} 0 & x < 0 \\ 1 & 0 \leq x < 1 \\ 0 & x \geq 1 \end{cases}$$

$$f(x) = \begin{cases} 1 & 0 \le x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

(ii) $P(0.2 \le x \le 0.7) = F(0.7) - F(0.2)$

$= 0.7 - 0.2 = 0.5.$

6/203 If x is the random variable with distribution function $F(x)$ given by

$$F(x) = \begin{cases} 0 & -\infty < x < 0 \\ \frac{1}{2}(x^2+x) & 0 \le x < 1 \\ 1 & 1 \le x < \infty \end{cases}$$

then find (i) p.d.f (ii) $P(0.3 \le x \le 0.6)$

$$f(x) = \frac{d}{dx}\{F(x)\}$$

$$f(x) = F'(x)$$

$$f(x) = \begin{cases} 0 & -\infty < x < 0 \\ \frac{1}{2}(2x+1) & 0 \le x < 1 \\ 0 & 1 \le x < \infty \end{cases}$$

$$f(x) = \begin{cases} 0 & \text{otherwise.} \\ \frac{1}{2}(2x+1) & 0 \le x < 1 \end{cases}$$

(ii) $P(0.3 \le x \le 0.6) = F(0.6) - F(0.3)$

$= \frac{1}{2}(0.6^2+0.6) - \frac{1}{2}(0.3^2+0.3)$

$= \frac{1}{2}[0.36+0.6-0.09-0.3]$

$= \frac{1}{2}[0.96 - 0.39]$

$= \frac{1}{2}[0.57] = 0.285$

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Mathematical Expectation:

Suppose x is a random variable with P.m.f (or) P.d.f f(x). The expected value or mean or mathematical Expectation of x denoted by E(x) or μ is

$$E(X) = \begin{cases} \sum x f(x) & x \text{ is discrete} \\ \int_{-\infty}^{\infty} x f(x)\, dx & x \text{ is Continuous.} \end{cases}$$

Suppose x is a random variable with P.m.f or P.d.f f(x). The excepted value of the function g(x), a new random variable is

$$E(g(x)) = \begin{cases} \sum g(x) f(x) & g(x) \text{ is discrete} \\ \int_{-\infty}^{\infty} g(x) f(x)\, dx & g(x) \text{ is continuous.} \end{cases}$$

If $g(x) = x^k$ the above theorem yield the expected value called the k^{th} moment about the Origin of the random variable x.

$$E(x^k) = \begin{cases} \sum x^k f(x) & x \text{ is discrete} \\ \int_{-\infty}^{\infty} x^k f(x)\, dx & x \text{ is Continuous} \end{cases}$$

Variance :-

The variance of a random variable x denoted by Var(x) or V(x) or σ^2 is

$$V(x) = E(x^2) - [E(x)]^2$$
$$= E(x-\mu)^2$$

Properties of Mathematical Expectation and variance :-

(i) $E(ax+b) = aE(x)+b$ where a and b are constants.

Proof :-

Let x be a discrete random variable

$$E(ax+b) = \sum_{i=1}^{\infty} (ax_i+b) f(x_i)$$

$$= \sum_{i=1}^{\infty} [a x_i f(x_i) + b f(x_i)]$$

$$= a\sum_{i=1}^{\infty} x_i f(x_i) + b \sum f(x_i)$$

$$= aE(x) + b(1) \quad [\because \sum f(x_i) = 1]$$

$$= aE(x) + b.$$

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* $E(ax) = aE(x)$
* $E(b) = b$

(ii) $\boxed{Var(x) = E(x^2) - [E(x)]^2}$

Proof :-

$$Var(x) = E(x-\mu)^2$$

$$= E(x^2 - 2x\mu + \mu^2)$$

$$= E(x^2) - 2\mu E(x) + \mu^2$$

$$= E(x^2) - 2\mu.\mu + \mu^2$$

$$= E(x^2) - 2\mu^2 + \mu^2$$

$$= E(x^2) - \mu^2$$

$$= E(x^2) - [E(x)]^2$$

$$\therefore \sigma^2 = Var(x) = E(x^2) - [E(x)]^2.$$

(iii) $Var(ax+b) = a^2 Var(x)$ where a and b are constants.

$$V(ax+b) = E(ax+b - E(ax+b)]^2$$

$$= E[ax+\cancel{b} - aE(x) - \cancel{b}]^2$$

$$= E(ax - aE(x)]^2$$

$$= a^2 E(x - E(x))^2$$

$$= a^2 E(x-\mu)^2$$

$$V(ax+b) = a^2 Var(x).$$

* $var(ax) = a^2 \, var\, x$
* $var(b) = 0$

11.16
207.

Suppose $f(x)$ given below represents a P.m.f.

x :	1	2	3	4	5	6
$f(x)$:	c^2	$2c^2$	$3c^2$	$4c^2$	c	$2c$

Find (i) the value of c

(ii) Mean and Variance.

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Since $f(x)$ is a P.m.f

$$\sum f(x) = 1$$

$$c^2 + 2c^2 + 3c^2 + 4c^2 + c + 2c = 1$$

$$10c^2 + 3c - 1 = 0$$

$$(5c-1)(2c+1) = 0$$

$$\boxed{c = 1/5} \qquad \boxed{c \neq -\tfrac{1}{2}}$$

-10; $\frac{-2}{10c}$, $\frac{5}{10c}$

∴ P.m.f is

x	1	2	3	4	5	6
$f(x)$	$\frac{1}{25}$	$\frac{2}{25}$	$\frac{3}{25}$	$\frac{4}{25}$	$\frac{1}{5}$	$\frac{2}{5}$

$$\text{mean} = E(x) = \sum x f(x)$$

$$= 1\left(\tfrac{1}{25}\right) + 2\left(\tfrac{2}{25}\right) + 3\left(\tfrac{3}{25}\right) + 4\left(\tfrac{4}{25}\right) + 5\left(\tfrac{1}{5}\right) + 6\left(\tfrac{2}{5}\right)$$

$$= \frac{1}{25}\left[1 + 4 + 9 + 16 + 25 + 60\right] = \frac{115}{25} = 4.6$$

$$E(x^2) = \sum x^2 f(x)$$

$$= 1\left(\tfrac{1}{25}\right) + 4\left(\tfrac{2}{25}\right) + 9\left(\tfrac{3}{25}\right) + 16\left(\tfrac{4}{25}\right) + 25\left(\tfrac{1}{5}\right) + 36\left(\tfrac{2}{5}\right)$$

$$= \frac{1}{25}\left[1 + 8 + 27 + 64 + 125 + 360\right] = \frac{585}{25}$$

$$\text{variance} = E(x^2) - [E(x)]^2$$

$$= \frac{585}{25} - \left(\frac{23}{5}\right)^2 = \frac{585}{25} - \frac{529}{25} = \frac{56}{25}$$

$$= 2.24.$$

11.17 / 208

Two balls are chosen randomly from an urn containing 8 white and 4 black balls. Suppose that we win ₹20 for each black ball selected and we lose ₹10 for each white ball selected and find the expected winning amount and variance.

Let X be a random variable denote the winning amount.

The possible outcomes are

X(both are black) $= (₹20)2 = ₹40$

X(1 Black, 1 white) $= ₹20 - ₹10 = ₹10$

X(both are white) $= ₹(-20) = -₹20$

∴ The random variables are 40, 10, −20

$$n(S) = 12C_2 = \frac{12\times 11}{2\times 1} = 66$$

$$P(X=40) = \frac{4C_2}{12C_2} = \frac{6}{66} = \frac{1}{11}$$

$$P(X=10) = \frac{4C_1 \times 8C_1}{12C_2} = \frac{4\times 8}{66} = \frac{32}{66}$$

$$P(X=-20) = \frac{4C_0 \times 8C_2}{12C_2} = \frac{1\times \frac{\cancel{8}^4\times 7}{\cancel{2}}}{66} = \frac{28}{66}$$

x	40	10	−20
$f(x)$	$\frac{6}{66}$	$\frac{32}{66}$	$\frac{28}{66}$

$$\text{mean} = E(X) = \Sigma x f(x)$$

$$= 40\left(\frac{6}{66}\right) + 10\left(\frac{32}{66}\right) - 20\left(\frac{28}{66}\right)$$

$$= \frac{1}{66}[240 + 320 - 560] = \frac{0}{66} = 0$$

$$\text{mean} = E(X) = 0$$

$$E(X^2) = \Sigma x^2 f(x)$$

$$= 1600\left(\frac{6}{66}\right) + 100\left(\frac{32}{66}\right) + 400\left(\frac{28}{66}\right)$$

$$= \frac{1}{66}[9600 + 3200 + 11200]$$

$$= \frac{1}{66}(24000) = \frac{4000}{11}$$

$$\text{variance} = E(X^2) - [E(X)]^2$$

$$= \frac{4000}{11} - 0$$

$$= \frac{4000}{11}$$

Ex : 11.4

$\frac{1}{210}$ For the random variable x the given P.m.f as below, find the mean and Variance

(i) $f(x) = \begin{cases} \frac{1}{10} & x = 2, 5 \\ \frac{1}{5} & x = 0, 1, 3, 4 \end{cases}$

x :	0	1	2	3	4	5
f(x) :	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{10}$

$\text{mean} = E(x) = \sum x f(x)$

$= 0\left(\frac{1}{5}\right) + 1\left(\frac{1}{5}\right) + 2\left(\frac{1}{10}\right) + 3\left(\frac{1}{5}\right) + 4\left(\frac{1}{5}\right) + 5\left(\frac{1}{10}\right)$

$= \frac{1}{10}\left[0 + 2 + 2 + 6 + 8 + 5\right] = \frac{23}{10} = 2.3$

$E(x^2) = \sum x^2 f(x)$

$= 0\left(\frac{1}{5}\right) + 1\left(\frac{1}{5}\right) + 4\left(\frac{1}{10}\right) + 9\left(\frac{1}{5}\right) + 16\left(\frac{1}{5}\right) + 25\left(\frac{1}{10}\right)$

$= \frac{1}{10}\left[0 + 2 + 4 + 18 + 32 + 25\right]$

$= \frac{81}{10} = 8.1$

$\text{Variance} = E(x^2) - [E(x)]^2$

$= \frac{81}{10} - \left(\frac{23}{10}\right)^2 = \frac{810}{100} - \frac{529}{100} = \frac{281}{100}$

$= 2.81.$

(ii) $f(x) = \begin{cases} \frac{4-x}{6} & x = 1, 2, 3 \\ 0 & \end{cases}$

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x :	1	2	3
f(x) :	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$

$E(x) = \sum x f(x) = 1\left(\frac{1}{2}\right) + 2\left(\frac{1}{3}\right) + 3\left(\frac{1}{6}\right)$

$= \frac{1}{2} + \frac{2}{3} + \frac{1}{2} = 1 + \frac{2}{3} = \frac{5}{3} = 1.67$

$E(x^2) = \sum x^2 f(x) = 1\left(\frac{1}{2}\right) + 4\left(\frac{1}{3}\right) + 9\left(\frac{1}{6}\right)$

$= \frac{1}{2} + \frac{4}{3} + \frac{3}{2} = 2 + \frac{4}{3} = \frac{10}{3}$

$$\text{Variance} = E(x^2) - [E(x)]^2$$

$$= \frac{10}{3} - \left(\frac{5}{3}\right)^2 = \frac{30}{9} - \frac{25}{9} = \frac{5}{9}$$

$$= 0.56$$

mean $= 1.67$ Variance $= 0.56$.

$\frac{2}{210}$ Two balls are drawn in succession without replacement from an urn containing four red balls and three black balls. Let X be the possible outcomes drawing red balls. Find the p.m.f and mean of X.

Let X be a random variable of getting a red ball.

B	R
3	4

$X = 0, 1, 2$

$$P(X=0) = P(BB) = \frac{3C_2 \times 4C_0}{7C_2} = \frac{3\times 1}{21} = \frac{3}{21} = \frac{1}{7}$$

$$P(X=1) = \frac{3C_1 \times 4C_1}{7C_2} = \frac{3\times 4}{21} = \frac{12}{21} = \frac{4}{7}$$

$$P(X=2) = \frac{3C_0 \times 4C_2}{7C_2} = \frac{1\times 6}{21} = \frac{6}{21} = \frac{2}{7}$$

x	0	1	2
$f(x)$	$\frac{1}{7}$	$\frac{4}{7}$	$\frac{2}{7}$

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$$\text{mean} = E(X) = \sum x f(x)$$

$$= 0\left(\frac{1}{7}\right) + 1\left(\frac{4}{7}\right) + 2\left(\frac{2}{7}\right)$$

$$= \frac{4+4}{7} = \frac{8}{7}$$

$\frac{3}{210}$ If μ and σ^2 are the mean and variance of the discrete random variable X, and $E(X+3) = 10$ and $E(X+3)^2 = 116$ find μ and σ^2

$$* \; E(ax+b) = a\,E(x) + b$$

$$E(x+3) = 10$$
$$E(x) + E(3) = 10$$
$$E(x) + 3 = 10$$
$$E(x) = 10-3$$
$$E(x) = 7.$$

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$$E(x+3)^2 = 116$$
$$E(x^2+6x+9) = 116$$
$$E(x^2)+6E(x)+9 = 116$$
$$E(x^2)+6(7)+9 = 116$$
$$E(x^2) = 116-9-42$$
$$E(x^2) = 65$$
$$\text{variance} = E(x^2)-[E(x)]^2$$
$$= 65-7^2 = 65-49$$
$$\text{variance} = 16$$

$\frac{4}{210}$ Four coins are tossed once. Find the p.m.f, mean and variance for number of heads occurred.

Let X be a random variable of getting the number of heads occurred.

$$x = 0, 1, 2, 3, 4$$
$$n(S) = 2^4 = 16$$

$P(x=0) = \frac{4C_0}{16} = \frac{1}{16}$ $P(x=1) = \frac{4C_1}{16} = \frac{4}{16}$

$P(x=2) = \frac{4C_2}{16} = \frac{6}{16}$ $P(x=3) = \frac{4C_3}{16} = \frac{4}{16}$

$P(x=4) = \frac{4C_4}{16} = \frac{1}{16}$

x	0	1	2	3	4
$f(x)$	$\frac{1}{16}$	$\frac{4}{16}$	$\frac{6}{16}$	$\frac{4}{16}$	$\frac{1}{16}$

$$\text{mean} = E(x) = \sum x f(x)$$

$$= 0\left(\frac{1}{16}\right)+1\left(\frac{4}{16}\right)+2\left(\frac{6}{16}\right)+3\left(\frac{4}{16}\right)+4\left(\frac{1}{16}\right)$$

$$= \frac{1}{16}\left[4+12+12+4\right] = \frac{32}{16} = 2$$

$$E(x^2) = \sum x^2 f(x)$$

$$= 0\left(\frac{1}{16}\right)+1\left(\frac{4}{16}\right)+4\left(\frac{6}{16}\right)+9\left(\frac{4}{16}\right)+16\left(\frac{1}{16}\right)$$

$$= \frac{1}{16}\left[4+24+36+16\right] = \frac{80}{16} = 5$$

$$\text{variance} = E(x^2) - [E(x)]^2$$

$$= 5 - 2^2 = 5-4 = 1$$

$\therefore$ mean $= 2$ variance $= 1$

8/210 A lottery with 600 tickets gives one Prize of ₹200, four Prizes of ₹100 and six Prizes of ₹50. If the ticket costs is ₹2. find the expected winning amount of a ticket.

Let X be a random variable of winning amount of a ticket.

$X =$ ₹200, ₹100, ₹50, 0

$P(X=200) = \frac{1}{600}$ $P(X=100) = \frac{4}{600}$

$P(X=50) = \frac{6}{600}$ $P(X=0) = \frac{589}{600}$

x :	200	100	50	0
$f(x)$:	$\frac{1}{600}$	$\frac{4}{600}$	$\frac{6}{600}$	$\frac{589}{600}$

$$E(X) = 200\left(\frac{1}{600}\right)+100\left(\frac{4}{600}\right)+50\left(\frac{6}{600}\right)+0\left(\frac{589}{600}\right)$$

$$= \frac{1}{600}(200+400+300) = \frac{900}{600} = \frac{3}{2} = 1.50$$

Expected winning amount = Amount won − cost of ticket

$= 1.50 - 2$

$= -0.50$

$\therefore$ Loss RS 0.50

Continuous Random variable :-

$\frac{1}{210}$. For the random variable x with the given Probability density function as below. find the mean and variance.

(iii) $f(x) = \begin{cases} 2(x-1) & 1 < x \leq 2 \\ 0 & \text{otherwise.} \end{cases}$

$$\text{mean} = E(X) = \int_{-\infty}^{\infty} x\, f(x)\, dx$$

$$= 2\int_1^2 x(x-1)\, dx = 2\int_1^2 (x^2 - x)\, dx$$

$$= 2\left[\frac{x^3}{3} - \frac{x^2}{2}\right]_1^2 = 2\left\{\left(\frac{8}{3} - \frac{4}{2}\right) - \left(\frac{1}{3} - \frac{1}{2}\right)\right\}$$

$$= 2\left[\frac{7}{3} - \frac{3}{2}\right] = 2\left[\frac{14-9}{6}\right] = \frac{2(5)}{6} = \frac{5}{3}$$

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx$$

$$= 2\int_1^2 x^2(x-1)\, dx = 2\int_1^2 (x^3 - x^2)\, dx$$

$$= 2\left[\frac{x^4}{4} - \frac{x^3}{3}\right]_1^2 = 2\left[\left(\frac{16}{4} - \frac{8}{3}\right) - \left(\frac{1}{4} - \frac{1}{3}\right)\right]$$

$$= 2\left[\frac{15}{4} - \frac{7}{3}\right] = 2\left[\frac{45-28}{12}\right] = 2\left(\frac{17}{12}\right) = \frac{17}{6}$$

$$\text{variance} = E(X^2) - [E(X)]^2$$

$$= \frac{17}{6} - \left(\frac{5}{3}\right)^2 = \frac{17}{6} - \frac{25}{9}$$

$$= \frac{153-150}{54} = \frac{3}{54} = \frac{1}{18}$$

mean $= \frac{5}{3}$ variance $= \frac{1}{18}$.

(iv) $f(x) = \begin{cases} \frac{1}{2} e^{-x/2} & x > 0 \\ 0 & \text{otherwise.} \end{cases}$

$$E(X) = \int_{-\infty}^{\infty} x\, f(x)\, dx$$

$$= \int_0^\infty \frac{1}{2} x e^{-x/2} \, dx$$

Gamma Integral

$$\int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$$

$n = 1$

$a = \frac{1}{2}$

$$= \frac{1}{2}\left[\frac{1!}{\left(\frac{1}{2}\right)^{1+1}}\right] = \frac{4}{2} = 2$$

mean = 2

$$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x) dx$$

$$= \frac{1}{2}\int_0^\infty x^2 e^{-x/2} dx$$

$n = 2$

$a = \frac{1}{2}$

$$= \frac{1}{2}\left[\frac{2!}{\left(\frac{1}{2}\right)^{2+1}}\right] = \frac{1}{2}(2 \cdot 2^3) = 8$$

$$\text{variance} = E(x^2) - [E(x)]^2$$

$$= 8 - 2^2 = 8 - 4 = 4$$

$\therefore$ mean = 2 variance = 4.

$\frac{5}{210}$. A commuter train arrives punctually at a station every half hour. Each morning, a student leaves his house to the train station. Let x denote the amount of time, in minutes. That the student waits for the train from the time he reaches the train station. It is known that the p.d.f of x is

$$f(x) = \begin{cases} \frac{1}{30} & 0 < x < 30 \\ 0 & \text{otherwise.} \end{cases}$$

obtain and interpret the expected value of the random variable x.

$$E(x) = \int_{-\infty}^{\infty} x f(x) dx$$

$$= \frac{1}{30}\int_0^{30} x \, dx = \frac{1}{30}\left[\frac{x^2}{2}\right]_0^{30}$$

$$= \frac{1}{30}\left(\frac{900}{2} - 0\right) = \frac{1}{30}\left(\frac{900}{2}\right) = 15$$

mean = 15 min.

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$\frac{6}{210}$ The time to failure in thousands of hours on an electronic equipment used in a manufactured computer has a p.d.f

$$f(x) = \begin{cases} 3e^{-3x} & x > 0 \\ 0 & \text{elsewhere.} \end{cases}$$

Find the expected life of this electronic equipment.

Gamma integral $\int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$

$$E(x) = \int_{-\infty}^{\infty} x f(x)\, dx$$

$$= 3\int_0^\infty x e^{-3x} dx$$

$$= 3\left[\frac{1!}{3^{1+1}}\right] = \frac{3}{9} = \frac{1}{3}$$

$$\text{mean} = \frac{1}{3}$$

$$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx$$

$$= 3\int_0^\infty x^2 e^{-3x} dx = 3\left[\frac{2!}{3^{2+1}}\right]$$

$$= \frac{3(2)}{3^3} = \frac{2}{9}$$

$$\text{Variance} = E(x^2) - [E(x)]^2$$

$$= \frac{2}{9} - \left(\frac{1}{3}\right)^2 = \frac{2}{9} - \frac{1}{9} = \frac{1}{9}.$$

$\frac{7}{210}$ The p.d.f of the random variable x is given by

$$f(x) = \begin{cases} 16x e^{-4x} & x > 0 \\ 0 & x \le 0 \end{cases}$$

find the mean and variance of x.

$$E(x) = \int_{-\infty}^{\infty} x f(x)\, dx$$

$$= 16\int_0^\infty x \cdot x e^{-4x} dx$$

$$= 16\int_0^\infty x^2 e^{-4x} dx$$

$$= 16\left[\frac{2!}{4^{2+1}}\right] = \frac{\cancel{16}(2)}{\cancel{64}_{4}} = \frac{2}{4} = \frac{1}{2}$$

$$\therefore \text{mean} = \frac{1}{2}.$$

$$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx$$

$$= 16\int_0^{\infty} x^3 . e^{-4x} . dx$$

$$= 16\left[\frac{3!}{4^{3+1}}\right] = \frac{16(6)}{16 \times 16} = \frac{3}{8}$$

$$\text{variance} = E(x^2) - [E(x)]^2$$

$$= \frac{3}{8} - \left(\frac{1}{2}\right)^2 = \frac{3}{8} - \frac{1}{4}$$

$$= \frac{3-2}{8} = \frac{1}{8}$$

$\therefore$ mean $= \frac{1}{2}$ variance $= \frac{1}{8}$.

11.18
209

Find the mean and variance of a random variable x, whose P.d.f is

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

$$E(x) = \int_{-\infty}^{\infty} x f(x)\, dx$$

Gamma integral

$$\int_0^{\infty} x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$$

$n = 1$
$a = \lambda$

$$= \lambda \int_0^{\infty} x\, e^{-\lambda x} . dx$$

$$= \lambda\left[\frac{1!}{\lambda^{1+1}}\right] = \frac{\lambda}{\lambda^2} = \frac{1}{\lambda}$$

$$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx$$

$$= \lambda \int_0^{\infty} x^2 e^{-\lambda x} . dx = \lambda\left[\frac{2!}{\lambda^{2+1}}\right] = \frac{2}{\lambda^2}$$

$$\text{variance} = E(x^2) - [E(x)]^2$$

$$= \frac{2}{\lambda^2} - \left(\frac{1}{\lambda}\right)^2 = \frac{2}{\lambda^2} - \frac{1}{\lambda^2} = \frac{1}{\lambda^2}$$

$\therefore$ mean $= \frac{1}{\lambda}$ variance $= \frac{1}{\lambda^2}$.

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Theoretical Distributions :-

* One Point distribution.
* Two Points distribution
* Bernoulli distribution.
* Binomial distribution.

one Point distribution :-

The random variable x has a one Point distribution if there exists a Point x_0 such that, the P.m.f $f(x)$ is defined as

$$f(x) = P(X = x_0) = 1$$

That is the Probability mass is concentrated at one Point.

cumulative distributive function :-

$$F(x) = \begin{cases} 0 & -\infty < x < x_0 \\ 1 & x_0 \leq x < \infty \end{cases}$$

Mean $= x_0$ Variance $= 0$.

Two Point distribution :-

* Unsymmetrical case
* Symmetrical case

→ unsymmetrical case :

The random Variable x has a two Point distribution if there exists two values x_1 and x_2 Such That

$$f(x) = \begin{cases} p & \text{for } x = x_1 \\ 1-p & \text{for } x = x_2 \end{cases} \qquad \boxed{0 < p < 1}$$

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Cubmulative distributive function :-

$$F(x) = \begin{cases} 0 & x < x_1 \\ p & x_1 \le x < x_2 \\ 1 & x_2 \le x < \infty \end{cases}$$

mean $= px_1 + qx_2$ where $q = 1-p$

variance $= pq(x_2 - x_1)^2$.

→ Symmetrical case :

when $p = q = \frac{1}{2}$ the two point distribution becomes

$$f(x) = \begin{cases} \frac{1}{2} & \text{for } x = x_1 \\ \frac{1}{2} & \text{for } x = x_2 \end{cases}$$

Cumulative distribution function :

$$F(x) = \begin{cases} 0 & x < x_1 \\ \frac{1}{2} & x_1 \le x < x_2 \\ 1 & x \ge x_2 \end{cases}$$

mean $= \frac{x_1 + x_2}{2}$ variance $= \frac{(x_2 - x_1)^2}{4}$

Bernoulli distribution :-

Let x be a random variable associated with a Bernoulli trial by defining it as

x (success) = 1 x (failure) = 0 such that

$$f(x) = \begin{cases} p & x = 1 \\ q = 1-p & x = 0 \end{cases} \quad \text{where } 0 < p < 1$$

then x is called a Bernoulli random variable and f(x) is called the Bernoulli distribution

P.m.f : $f(x) = p^x (1-p)^{1-x}$ $x = 0, 1$

cumulative distributive function :

$$F(x) = \begin{cases} 0 & x < 0 \\ q = 1-p & 0 \le x < 1 \\ 1 & x \ge 1 \end{cases}$$

Mean $= p$ variance $= pq$

when $p = q = \frac{1}{2}$ the Bernoulli distribution become

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$$f(x) = \begin{cases} \frac{1}{2} & x = 0 \\ \frac{1}{2} & x = 1. \end{cases}$$

C.D.F

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{2} & 0 \le x < 1 \\ 1 & x \ge 1 \end{cases}$$

mean $= \frac{1}{2}$ variance $= \frac{1}{4}.$

Binomial distribution :-

The binomial random variable X equals the number of successes with Probability p for a success and $q = 1-p$ for a failure in n-independent trials has a binomial distribution denoted by $X \sim B(n, p)$

The Probability mass function of X is

$$f(x) = nC_x\, p^x (1-p)^{n-x} \qquad x = 0, 1, 2 \ldots n$$

(or)

$$f(x) = nC_x\, p^x q^{n-x} \qquad x = 0, 1, 2 \ldots n$$

mean $= np$ variance $= npq$ $S.D = \sqrt{npq}.$

In a binomial distribution mean is always greater than variance

(ie) $np > npq$

11.19 / 215 Find the binomial distribution for each of the following:

(i) Five fair coins are tossed and x denotes the number of heads.

$n = 5 \quad p = \frac{1}{2} \quad q = \frac{1}{2}$

$P(X = x) = nC_x \, p^x q^{n-x} \quad x = 0, 1, 2 \dots n$

$P(X = x) = 5C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{5-x} \quad x = 0, 1, 2, 3, 4, 5.$

(ii) A fair die is rolled 10 times and x denotes the number of times 4 appeared.

Let x be a random variable denotes the number of times 4 appeared.

$n = 10 \quad p = \frac{1}{6} \quad q = \frac{5}{6}$

$P(X = x) = nC_x \, p^x q^{n-x} \quad x = 0, 1, 2, \dots n$

$P(X = x) = 10C_x \left(\frac{1}{6}\right)^x \left(\frac{5}{6}\right)^{10-x} \quad x = 0, 1, 2 \dots 10.$

11.20 / 215 A multiple choice examination has ten questions, each question has four distractors with exactly one correct answer. Suppose a student answers by guessing and if x denote the number of correct answer, find

(i) binomial distribution

(ii) Probability that the student will get seven correct answers.

(iii) The probability of getting atleast one correct answer.

Let x be a random variable denotes the number of correct answers.

$n = 10 \quad p = \frac{1}{4} \quad q = 1 - \frac{1}{4} = \frac{3}{4}$

(i) Binomial distribution:-

$P(X = x) = nC_x \, p^x q^{n-x} \quad x = 0, 1, 2, \dots n$

$P(X = x) = 10C_x \left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{10-x} \quad x = 0, 1, 2 \dots 10.$

(ii) Probability for Seven correct answers:

$$P(X=7) = 10C_7 \left(\frac{1}{4}\right)^7 \left(\frac{3}{4}\right)^{10-7}$$

$$= 10C_3 \left(\frac{1}{4}\right)^7 \left(\frac{3}{4}\right)^3 \qquad [10C_7 = 10C_3]$$

$$= \frac{10 \times 9 \times 8}{3 \times 2 \times 1} \cdot \frac{3^3}{4^{10}}$$

$$P(x=7) = 210\left(\frac{3^3}{4^{10}}\right)$$

(iii) Probability for atleast one correct answer:

$$P(X \geq 1) = 1 - P(X < 1)$$

$$= 1 - P(X=0)$$

$$= 1 - 10C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^{10-0}$$

$$= 1 - \left(\frac{3}{4}\right)^{10}.$$

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11.21 / 216. The mean and variance of a binomial variate X are respectively 2 and 1.5 find (i) $P(X=0)$ (ii) $P(X=1)$ (iii) $P(X \geq 1)$

mean $= np = 2$ variance $= npq = 1.5$

$np = 2$ $npq = \frac{3}{2}$

$2q = \frac{3}{2}$

$\boxed{q = 3/4}$.

$p = 1 - q = 1 - \frac{3}{4} = \frac{1}{4}$

$\boxed{p = \frac{1}{4}}$

$np = 2$

$n\left(\frac{1}{4}\right) = 2$ $\boxed{n = 8}$

$\therefore$ The binomial distribution is

$$P(X=x) = 8C_x \left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{8-x} \qquad x = 0, 1, 2 \cdots 8$$

(i) $P(X=0) = 8C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^8$

$$= \left(\frac{3}{4}\right)^8$$

(ii) $P(x=1) = 8C_1 \left(\frac{1}{4}\right)^1 \left(\frac{3}{4}\right)^{8-1}$

$= 8\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^7$

$= 2\left(\frac{3}{4}\right)^7$

(iii) $P(x \geq 1) = 1 - P(x < 1)$

$= 1 - P(x=0)$

$= 1 - 8C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^8 = 1 - \left(\frac{3}{4}\right)^8$

11.22
217.

On the average, 20% of the products manufactured by ABC Company are found to be defective. If we select 6 of these products at random and x denotes the number of defective products find the probability that

(i) Two products are defective
(ii) atmost one product is defective
(iii) atleast two products are defective.

Let x be a random variable denoting the number of defective products.

$p = 20\% = \frac{20}{100} = \frac{1}{5}$ $\quad q = 1 - \frac{1}{5} = \frac{4}{5}$ $\quad n = 6$

$P(X=x) = nC_x\, p^x q^{n-x} \quad x = 0, 1, 2 \ldots n.$

(i) Two products are defective:

$P(x=2) = 6C_2 \left(\frac{1}{5}\right)^2 \left(\frac{4}{5}\right)^{6-2}$

$= \frac{\cancel{6}^{3} \times 5}{\cancel{2} \times 1} \left(\frac{1}{5}\right)^2 \left(\frac{4}{5}\right)^4 = 15\left(\frac{4^4}{5^6}\right)$

(ii) Atmost one product is defective

$P(x \leq 1) = P(x=0) + P(x=1)$

$= 6C_0 \left(\frac{1}{5}\right)^0 \left(\frac{4}{5}\right)^6 + 6C_1 \left(\frac{1}{5}\right)^1 \left(\frac{4}{5}\right)^5$

$= \left(\frac{4}{5}\right)^6 + \frac{6}{5}\left(\frac{4}{5}\right)^5 = \left(\frac{4}{5}\right)^5 \left\{\frac{4}{5} + \frac{6}{5}\right\}$

$= 2\left(\frac{4}{5}\right)^5$.

(iii) Atleast 2 products are defective :-

$P(x \geq 2) = 1 - P(x < 2) = 1 - \{P(x=0) + P(x=1)\}$

$= 1 - 2\left(\frac{4}{5}\right)^5$

Ex : 11.5

$\frac{1}{218}$ Compute $P(X=x)$ for the binomial distribution

(i) $n=6$ $\quad p=\frac{1}{3}$ $\quad k=3$

$q=1-p=1-\frac{1}{3}=\frac{2}{3}$

$P(X=x)=n_{C_x}\,p^x q^{n-x}$ $\quad x=0,1,2\ldots n$

$P(X=3)=6_{C_3}\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^{6-3}$

$=\frac{\cancel{6}\times5\times4}{\cancel{3}\times\cancel{2}\times1}\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^3$

$=\frac{20\,(2^3)}{3^6}=\frac{20(8)}{729}=\frac{160}{729}$

(ii) $n=10$ $\quad p=\frac{1}{5}$ $\quad k=4$

$q=1-p=1-\frac{1}{5}=\frac{4}{5}$

$P(X=x)=n_{C_x}\,p^x q^{n-x}$ $\quad x=0,1,2,3\ldots n$

$P(X=4)=10_{C_4}\left(\frac{1}{5}\right)^4\left(\frac{4}{5}\right)^{10-4}$

$=\frac{10\times\cancel{9}^{3}\times\cancel{8}\times7}{\cancel{4}\times\cancel{3}\times\cancel{2}\times1}\cdot\frac{4^6}{5^{10}}=210\left(\frac{4^6}{5^{10}}\right)$

(iii) $n=9$ $\quad p=\frac{1}{2}$ $\quad k=7$

$q=1-p=1-\frac{1}{2}=\frac{1}{2}$

$P(X=x)=n_{C_x}\,p^x q^{n-x}$ $\quad x=0,1,2\ldots n$

$P(X=7)=9_{C_7}\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^{9-7}$

$=9_{C_2}\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^2$

$=\frac{9\times\cancel{8}^{4}}{\cancel{2}\times1}\left(\frac{1}{2^9}\right)=36\left(\frac{1}{2^9}\right)$

$=9\left(\frac{1}{2^7}\right)=\frac{9}{128}$

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$\frac{2}{218}$ The Probability that Mr Q hits a target at any trail is $\frac{1}{4}$. Suppose he tries at the target 10 times. Find the Probability that he hits the target

(i) Exactly 4 times (ii) atleast one time.

Let x denotes he hits the target.

$P = \frac{1}{4}$ $\quad q = 1 - P = 1 - \frac{1}{4} = \frac{3}{4}$ $\quad n = 10$

$P(X = x) = n_{C_x} P^x q^{n-x}$ $\quad x = 0, 1, 2, \dots n$

(i) Exactly 4 times :-

$$P(X = 4) = 10_{C_4} \left(\frac{1}{4}\right)^4 \left(\frac{3}{4}\right)^{10-4}$$

$$= 10_{C_4} \left(\frac{1}{4}\right)^4 \left(\frac{3}{4}\right)^6$$

$$= \frac{10 \times \overset{3}{\cancel{9}} \times \cancel{8} \times 7}{\cancel{4} \times \cancel{3} \times \cancel{2} \times 1} \left(\frac{3^6}{4^{10}}\right) = 210 \left(\frac{3^6}{4^{10}}\right)$$

(ii) Atleast one time :-

$$P(X \geq 1) = 1 - P(X < 1)$$

$$= 1 - P(X = 0)$$

$$= 1 - 10_{C_0} \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^{10} = 1 - \left(\frac{3}{4}\right)^{10}.$$

$\frac{3}{218}$ Using binomial distribution find the mean and variance of X for the following experiments.

(i) A fair coin is tossed 100 times and X denotes the number of heads.

X denotes the number of heads

$P = \frac{1}{2}$ $\quad q = 1 - \frac{1}{2} = \frac{1}{2}$ $\quad n = 100$

mean $= nP = 100 \cdot \frac{1}{2} = 50$

variance $= npq = 100 \cdot \frac{1}{2} \cdot \frac{1}{2} = 25.$

(ii) A die is 240 times thrown, and X denote the number of times that four appeared.

X denotes the number of times 4 appeared.

$P = \frac{1}{6}$ $\quad q = 1 - \frac{1}{P} = 1 - \frac{1}{6} = \frac{5}{6}$ $\quad n = 240$

mean $= nP = \frac{1}{6}(240) = 40$

variance $= npq = 40\left(\frac{5}{6}\right) = \frac{100}{3}$

$\frac{4}{218}$ The Probability that a certain kind of Component will survive a electrical test is $\frac{3}{4}$. Find the Probability that exactly 3 of the 5 components tested survive.

$p = \frac{3}{4} \quad q = 1-p = 1-\frac{3}{4} = \frac{1}{4} \quad n=5$

$P(X=x) = n_{C_x} p^x q^{n-x} \quad x = 0,1,2,\dots n$

$P(X=3) = 5_{C_3}\left(\frac{3}{4}\right)^3\left(\frac{1}{4}\right)^{5-3}$

$= 5_{C_2}\left(\frac{3}{4}\right)^3\left(\frac{1}{4}\right)^2$

$= 10\left(\frac{3^3}{4^5}\right) = \frac{10(27)}{1024} = \frac{270}{1024}$

$= \frac{135}{512}$

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$\frac{5}{218}$ A retailer purchases a certain kind of electronic device from a manufracturer. The manufracturer indicates that the defective rate of the divice is 5%. The inspector of the retailer randomly picks 10 items from a shipment. what is the Probability that there will be

(i) atleast one defective item.

(ii) Exactly two defective item

$p = 5\% = \frac{5}{100} = \frac{1}{20} \quad q = 1-p = 1-\frac{1}{20} = \frac{19}{20}$

$n = 10$

$P(X=x) = n_{C_x} p^x q^{n-x} \quad x = 0,1,2\dots n$

(i) Atleast one defective item:

$P(X \geq 1) = 1 - P(X<1)$

$= 1 - P(X=0)$

$= 1 - 10_{C_0}\left(\frac{1}{20}\right)^0\left(\frac{19}{20}\right)^{10}$

$= 1 - \left(\frac{19}{20}\right)^{10}$

(ii) Exactly 2 defective item :-

$$P(X=2) = 10C_2 \left(\frac{1}{20}\right)^2 \left(\frac{19}{20}\right)^8$$

$$= 45 \left(\frac{19^8}{20^{10}}\right)$$

7/218 The mean and SD of a binomial variate x are respectively 6 and 2.
Find the P.m.f (ii) $P(X=3)$ (iii) $P(X \geq 2)$

mean $= np = 6$ S.D $= \sqrt{npq} = 2$

$npq = 4$

$$6q = 4$$

$$\boxed{q = \frac{2}{3}}$$

$$p = 1-q = 1-\frac{2}{3}$$

$$\boxed{p = \frac{1}{3}}$$

$$n\left(\frac{1}{3}\right) = 6$$

$$\boxed{n = 18}$$

$$\therefore P(X=x) = nC_x p^x q^{n-x} \quad x = 0,1,2\ldots n$$

$$P(X=x) = 18C_x \left(\frac{1}{3}\right)^x \left(\frac{2}{3}\right)^{18-x} \quad x = 0,1,2,3\ldots n$$

(ii) $$P(X=3) = 18C_3 \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^{15}$$

(iii) $$P(X \geq 2) = 1 - P(X<2)$$

$$= 1 - \{P(X=1) + P(X=0)\}$$

$$= 1 - \left\{18C_0 \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^{18} + 18C_1 \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^{17}\right\}$$

$$= 1 - \left[\left(\frac{2}{3}\right)^{18} + \frac{18}{3}\left(\frac{2}{3}\right)^{17}\right]$$

$$= 1 - \left(\frac{2}{3}\right)^{17}\left[\frac{2}{3} + 6\right]$$

$$= 1 - \frac{20}{3}\left(\frac{2}{3}\right)^{17}$$

8/218 If $X \sim B(n,p)$ such that $4P(X=4) = P(X=2)$ and $n=6$. Find the distribution, mean and standard deviation of X.

$$P(X=x) = nC_x p^x q^{n-x} \quad x = 0,1,2\ldots n$$

$$4P(X=4) = P(X=2) \qquad \boxed{n=6}$$

$$4\{\cancel{6C_4} p^4 q^2\} = \{\cancel{6C_2} p^2 q^4\} \qquad [nC_r = nC_{n-r}]$$

$$4p^2 = q^2$$

$$2p = q$$

$$2p = 1-p$$

$$3p = 1 \qquad \boxed{p = \frac{1}{3}}$$

$q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3}$

$$\boxed{q = \frac{2}{3}}$$

$P(X = x) = n_{C_x} \, p^x \, q^{n-x}$ $\quad x = 0, 1, 2, 3 \ldots n$

$P(X = x) = 6_{C_x} \left(\frac{1}{3}\right)^x \left(\frac{2}{3}\right)^{6-x}$

$x = 0, 1, 2, 3, 4, 5, 6$

mean $= np = 6\left(\frac{1}{3}\right) = 2$

variance $= npq = 2\left(\frac{2}{3}\right) = \frac{4}{3}$

$S.D = \sqrt{npq} = \sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}}$

$\frac{9}{218}$. In a binomial distribution consisting of 5 independent trails, the probability of 1 and 2 successes are 0.4096 and 0.2048 respectively. find the mean and variance of the random variable.

$P(X = 1) = 0.4096$

$P(X = 2) = 0.2048$ $\quad \boxed{n = 5}$

$P(X=1) = 2P(X=2)$

$\cancel{5_{C_1}} \, p^1 q^4 = 2\{\cancel{5_{C_2}} \, p^2 q^3\}$

$pq^4 = 2p^2q^3$

$q = 2p$

$1 - p = 2p$

$3p = 1$

$$\boxed{p = \frac{1}{3}}$$

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$q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3}$ $\quad \boxed{q = \frac{2}{3}}$

mean $= np = 5\left(\frac{1}{3}\right) = \frac{5}{3}$

variance $= npq = \frac{5}{3} \cdot \frac{2}{3} = \frac{10}{9}$

$S.D = \sqrt{npq} = \sqrt{\frac{10}{9}} = \frac{\sqrt{10}}{3}$

$\frac{6}{218}$ If the Probability that a fluorescent light has a useful life of atleast 600 hrs is 0.9 find the Probabilities that among 12 such lights

(i) Exactly 10 will have a useful life of atleast 600 hrs.

(ii) Atleast 11 will have a useful life of atleast 600 hrs.

(iii) Atleast 2 will not have a useful life of atleast 600 hrs.

Let X denotes the light has a useful life of atleast 600 hrs.

$P = 0.9 = \frac{9}{10}$ $\quad q = 1-P = \frac{1}{10}$ $\quad n = 12$

(i) Exactly 10 will have a useful life of atleast 600 hrs.

$$P(X=x) = n_{C_x} P^x q^{n-x} \quad x = 0, 1, 2 \cdots n$$

$$P(X=10) = 12_{C_{10}} \left(\frac{9}{10}\right)^{10} \left(\frac{1}{10}\right)^2$$

$$= \frac{12 \times 11}{2 \times 1} \left(\frac{9}{10}\right)^{10} \left(\frac{1}{10}\right)^2$$

$$= \frac{66}{100} \left(\frac{9}{10}\right)^{10} = 0.66 (0.9)^{10}$$

(ii) Atleast 11 will have a useful life of atleast 600 hrs.

$$P(X \geq 11) = P(X=11) + P(X=12$$

$$= 12_{C_{11}} \left(\frac{9}{10}\right)^{11} \left(\frac{1}{10}\right) + 12_{C_{12}} \left(\frac{9}{10}\right)^{12} \left(\frac{1}{10}\right)^0$$

$$= 12 \left(\frac{9}{10}\right)^{11} \left(\frac{1}{10}\right) + \left(\frac{9}{10}\right)^{12}$$

$$= \left(\frac{9}{10}\right)^{11} \left[\frac{12}{10} + \frac{9}{10}\right] = \left(\frac{9}{10}\right)^{11} \left(\frac{21}{10}\right)$$

$$= 2.1 (0.9)^{11}$$

(iii) Atleast 2 will not have a useful life of atleast 600 hrs.

$$(P(X<11) = 1 - P(X \geq 11) = 1 - \{P(X=11) + P(X=12)\}$$

$$= 1 - (2.1)(0.9)^{11}$$