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IMPORTANT 2 MARKS AND 3 MARKS

CHAPTER 1

Solve the following system: $x+2y+3z=0,\ 3x+4y+4z=0,\ 7x+10y+12z=0.$

Solve the following systems of linear equations by Cramer's rule: $5x-2y+16=0, x+3y-7=0$

Solve the following systems of linear equations by Cramer's rule: $\frac{3}{x}+2y=12, \frac{2}{x}+3y=13$

Solve the following system of linear equations, using matrix inversion method: $5x+2y=3,\ 3x+2y=5$.

Find the inverse of each of the following by Gauss-Jordan method: $\begin{bmatrix} 2 & -1 \\ 5 & -2 \end{bmatrix}$

Find the rank of the following matrices by minor method: $\begin{bmatrix} 1 & -2 & -1 & 0 \\ 3 & -6 & -3 & 1 \end{bmatrix}$

Find the rank of the following matrices by row reduction method: $\begin{bmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 1 & -2 & 3 \\ 1 & -1 & 1 \end{bmatrix}$

Find the rank of the matrix $\begin{bmatrix} 2 & -2 & 4 & 3 \\ -3 & 4 & -2 & -1 \\ 6 & 2 & -1 & 7 \end{bmatrix}$ by reducing it to an echelon form.

Define Rank of a matrix

Reduce the matrix $\begin{bmatrix} 3 & -1 & 2 \\ -6 & 2 & 4 \\ -3 & 1 & 2 \end{bmatrix}$ to a row-echelon form.

$A=\begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$, show that $A^T A^{-1}=\begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$.

Find the matrix A for which $A\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}=\begin{bmatrix} 14 & 7 \\ 7 & 7 \end{bmatrix}$

Given $A=\begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}$, $B=\begin{bmatrix} 3 & -2 \\ 1 & 1 \end{bmatrix}$ and $C=\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$, find a matrix X such that $AXB=C$.

Find the adjoint of the following: $\begin{bmatrix} -3 & 4 \\ 6 & 2 \end{bmatrix}$

Find the inverse (if it exists) of the following: $\begin{bmatrix} -2 & 4 \\ 1 & -3 \end{bmatrix}$

Prove that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal.

Verify the property $\left(A^T\right)^{-1} = \left(A^{-1}\right)^T$ with $A = \begin{bmatrix} 2 & 9 \\ 1 & 7 \end{bmatrix}$.

If A is symmetric, prove that adj A is also symmetric.

If adj $A = \begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, find A^{-1}.

If A is a non-singular matrix of odd order, prove that $|\text{adj } A|$ is positive.

State and prove inverse reversal Law

State and prove Double law of inverse

State and Prove Left and right cancellation law

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is non-singular, find A^{-1}.

CHAPTER 2

If $\omega \neq 1$ is a cube root of unity, show that $(1-\omega+\omega^2)^6 + (1+\omega-\omega^2)^6 = 128.$

If $z = 2-2i$, find the rotation of z by $\theta = \frac{2\pi}{3}$ in the counter clockwise direction about the origin

Find the value of $\sum_{k=1}^{8}\left(\cos\frac{2k\pi}{9} + i\sin\frac{2k\pi}{9}\right)$

Find all cube roots of $\sqrt{3}+i$.

Find the fourth roots of unity.

Find the cube roots of unity.

Simplify $(1+i)^{18}$

Simplify $\left(\sin\frac{\pi}{6} + i\cos\frac{\pi}{6}\right)^{18}$.

If $z=(\cos\theta+i\sin\theta)$, show that $z^n+\frac{1}{z^n}=2\cos n\theta$ and $z^n-\frac{1}{z^n}=2i\sin n\theta$

State Demovires theorem

Find the rectangular form of the complex numbers $\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)\left(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}\right)$

Find the rectangular form of the complex numbers $\dfrac{\cos\frac{\pi}{6}-i\sin\frac{\pi}{6}}{2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)}$.

Write in polar form of the following complex numbers $\dfrac{i-1}{\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}}$.

Find the quotient $\dfrac{2\left(\cos\frac{9\pi}{4}+i\sin\frac{9\pi}{4}\right)}{4\left(\cos\left(\frac{-3\pi}{2}\right)+i\sin\left(\frac{-3\pi}{2}\right)\right)}$ in rectangular form.

Find the product $\frac{3}{2}\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)\cdot 6\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right)$ in rectangular from.

Find the principal argument Arg z, when $z=\frac{-2}{1+i\sqrt{3}}$.

Represent the complex number (i) $-1-i$ (ii) $1+i\sqrt{3}$ in polar form.

Find the modulus and principal argument of the following complex numbers.
(i) $\sqrt{3}+i$ (ii) $-\sqrt{3}+i$ (iii) $-\sqrt{3}-i$ (iv) $\sqrt{3}-i$

Obtain the Cartesian equation for the locus of $z=x+iy$ in each of the following cases:
(i) $|z-4|=16$ (ii) $|z-4|^2-|z-1|^2=16$.

Show that the following equations represent a circle, and, find its centre and radius.
(i) $|z-2-i|=3$ (ii) $|2z+2-4i|=2$ (iii) $|3z-6+12i|=8$.

Obtain the Cartesian form of the locus of $z=x+iy$ in each of the following cases:

(i) $[\mathrm{Re}(iz)]^2=3$ (ii) $\mathrm{Im}[(1-i)z+1]=0$ (iii) $|z+i|=|z-1|$ (iv) $\bar{z}=z^{-1}$.

If $z=x+iy$ is a complex number such that $\left|\frac{z-4i}{z+4i}\right|=1$ show that the locus of z is real axis.

Show that $|z+2-i|<2$ represents interior points of a circle. Find its centre and radius.

Obtain the Cartesian form of the locus of z in each of the following cases. (i) $|z|=|z-i|$ (ii) $|2z-3-i|=3$

Show that $|3z-5+i|=4$ represents a circle, and, find its centre and radius.

Find the square roots of (i) $4+3i$ (ii) $-6+8i$ (iii) $-5-12i$.

If the area of the triangle formed by the vertices z, iz, and $z+iz$ is 50 square units, find the value of $|z|$

Which one of the points $10-8i$, $11+6i$ is closest to $1+i$.

If $|z|=3$, show that $7\leq|z+6-8i|\leq 13$.

If $|z|=1$, show that $2\leq|z^2-3|\leq 4$.

Find the modulus of the following complex numbers

(i) $\frac{2i}{3+4i}$ (ii) $\frac{2-i}{1+i}+\frac{1-2i}{1-i}$ (iii) $(1-i)^{10}$ (iv) $2i(3-4i)(4-3i)$.

Find the square root of $6-8i$.

Show that the equation $z^2=\overline{z}$ has four solutions.

If $|z|=2$ show that $3\leq|z+3+4i|\leq 7$

Which one of the points $i, -2+i$, and 3 is farthest from the origin?

Find the following (i) $\left|\frac{2+i}{-1+2i}\right|$ (ii) $\left|\overline{(1+i)}(2+3i)(4i-3)\right|$ (iii) $\left|\frac{i(2+i)^3}{(1+i)^2}\right|$

Prove the following properties:

(i) z is real if and only if $z=\overline{z}$ (ii) $\text{Re}(z)=\frac{z+\overline{z}}{2}$ and $\text{Im}(z)=\frac{z-\overline{z}}{2i}$

Find the least value of the positive integer n for which $\left(\sqrt{3}+i\right)^n$

(i) real (ii) purely imaginary.

Write the following in the rectangular form:

(i) $\overline{(5+9i)+(2-4i)}$ (ii) $\frac{10-5i}{6+2i}$ (iii) $\overline{3i}+\frac{1}{2-i}$

If $z_1=2-i$ and $z_2=-4+3i$, find the inverse of z_1z_2 and $\frac{z_1}{z_2}$.

If $z=x+iy$, find the following in rectangular form.

(i) $\text{Re}\left(\frac{1}{z}\right)$ (ii) $\text{Re}(i\,\overline{z})$ (iii) $\text{Im}(3z+4\overline{z}-4i)$

The complex numbers u, v, and w are related by $\frac{1}{u}=\frac{1}{v}+\frac{1}{w}$.

If $v=3-4i$ and $w=4+3i$, find u in rectangular form.

Show that (i) $\left(2+i\sqrt{3}\right)^{10}+\left(2-i\sqrt{3}\right)^{10}$ is real ;

Find z^{-1}, if $z=(2+3i)(1-i)$.

If $z_1=3-2i$ and $z_2=6+4i$, find $\frac{z_1}{z_2}$ in the rectangular form

If $\frac{z+3}{z-5i}=\frac{1+4i}{2}$, find the complex number z in the rectangular form

Simplify $\left(\frac{1+i}{1-i}\right)^3-\left(\frac{1-i}{1+i}\right)^3$. into rectangular form

Write $\frac{3+4i}{5-12i}$ in the $x+iy$ form, hence find its real and imaginary parts.

If $z_1=2+5i$, $z_2=-3-4i$, and $z_3=1+i$, find the additive and multiplicative inverse of z_1, z_2, and z_3.

Given the complex number $z=2+3i$, represent the complex numbers in Argand diagram.
(i) z, iz, and $z+iz$ (ii) z, $-iz$, and $z-iz$.

Simplify the following

(i) i^7 (ii) i^{1729} (iii) $i^{-1924}+i^{2018}$ (iv) $\sum_{n=1}^{102} i^n$ (v) $i\, i^2\, i^3 \cdots i^{40}$

CHAPTER 3

Show that the polynomial $9x^9+2x^5-x^4-7x^2+2$ has at least six imaginary roots.

Discuss the nature of the roots of the following polynomials:
(i) $x^{2018}+1947x^{1950}+15x^8+26x^6+2019$ (ii) $x^5-19x^4+2x^3+5x^2+11$

Find the exact number of real zeros and imaginary of the polynomial $x^9+9x^7+7x^5+5x^3+3x$.

Show that the equation $x^9-5x^5+4x^4+2x^2+1=0$ has atleast 6 imaginary solutions.

Determine the number of positive and negative roots of the equation $x^9-5x^8-14x^7=0$.

Discuss the maximum possible number of positive and negative roots of the polynomial equation $9x^9-4x^8+4x^7-3x^6+2x^5+x^3+7x^2+7x+2=0$.

Discuss the maximum possible number of positive and negative zeros of the polynomials x^2-5x+6 and $x^2-5x+16$. Also draw rough sketch of the graphs.

Solve the equation $7x^3-43x^2=43x-7$.

Find solution, if any, of the equation $2\cos^2 x-9\cos x+4=0$.

Note

Reciprocal equations of Type I correspond to those in which the coefficients from the beginning are equal to the coefficients from the end.

For instance, the equation $6x^5+x^4-43x^3-43x^2+x+6=0$ is of type I.

Reciprocal equations of Type II correspond to those in which the coefficients from the beginning are equal in magnitude to the coefficients from the end, but opposite in sign.

For instance, the equation $6x^5-41x^4+97x^3-97x^2+41x-6=0$ is of Type II.

- For an odd degree reciprocal equation of Type I, $x=-1$ must be a solution.
- For an odd degree reciprocal equation of Type II, $x=1$ must be a solution.
- For an even degree reciprocal equation of Type II, the middle term must be 0. Further $x=1$ and $x=-1$ are solutions.
- For an even degree reciprocal equation, by taking $x+\frac{1}{x}$ or $x-\frac{1}{x}$ as y, we can obtain a polynomial equation of degree one half of the degree of the given equation ; solving this polynomial equation, we can get the roots of the given polynomial equation.

Solve the equation $7x^3-43x^2=43x-7$.

Solve the equation $6x^6-35x^5+56x^4-56x^2+35x-6=0$

Solve the following equation: $x^4-10x^3+26x^2-10x+1=0$.

Solve the equation : $x^4-14x^2+45=0$.

Solve the cubic equations : (i) $2x^3-9x^2+10x=3$, (ii) $8x^3-2x^2-7x+3=0$.

Determine k and solve the equation $2x^3-6x^2+3x+k=0$ if one of its roots is twice the sum of the other two roots.

Solve the cubic equation : $2x^3-x^2-18x+9=0$ if sum of two of its roots vanishes.

Obtain the condition that the roots of $x^3+px^2+qx+r=0$ are in A.P.

Find the condition that the roots of $ax^3+bx^2+cx+d=0$ are in geometric progression

Solve the equation $2x^3+11x^2-9x-18=0$.

Solve the equation $x^3-3x^2-33x+35=0$.

Solve the equation $x^4-9x^2+20=0$.

If k is real, discuss the nature of the roots of the polynomial equation $2x^2+kx+k=0$, in terms of k.

Prove that a straight line and parabola cannot intersect at more than two points.

Prove that a line cannot intersect a circle at more than two points.

If $x^2+2(k+2)x+9k=0$ has equal roots, find k.

Show that, if p, q, r are rational, the roots of the equation $x^2 - 2px + p^2 - q^2 + 2qr - r^2 = 0$ are rational.

Show that the equation $2x^2 - 6x + 7 = 0$ cannot be satisfied by any real values of x.

Find a polynomial equation of minimum degree with rational coefficients, having $2 - \sqrt{3}$ as a root.

Find the monic polynomial equation of minimum degree with real coefficients having $2 - \sqrt{3}\, i$

Formalate into a mathematical problem to find a number such that when its cube root is added to it, the result is 6.

A 12 metre tall tree was broken into two parts. It was found that the height of the part which was left standing was the cube root of the length of the part that was cut away. Formulate this into a mathematical problem to find the height of the part which was cut away.

Find the sum of squares of roots of the equation $2x^4 - 8x^3 + 6x^2 - 3 = 0$.

Solve the equation $3x^3 - 16x^2 + 23x - 6 = 0$ if the product of two roots is 1.

Construct a cubic equation with roots (i) $1, 2$, and 3 (ii) $1, 1$, and -2 (iii) $2, \frac{1}{2}$ and 1.

If α, β, and γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, find the value of $\sum \frac{1}{\beta\gamma}$ in

If α and β are the roots of the quadratic equation $2x^2 - 7x + 13 = 0$, construct a quadratic equation whose roots are α^2 and β^2.

If α and β are the roots of the quadratic equation $17x^2 + 43x - 73 = 0$, construct a quadratic equation whose roots are $\alpha + 2$ and $\beta + 2$

CHAPTER 4

Find the principal value of $\sin^{-1}\left(\sin\left(-\frac{\pi}{3}\right)\right)$

Find all the values of x such that $-10\pi \le x \le 10\pi$ and $\sin x = 0$.

Find all the values of x such that $-8\pi \le x \le 8\pi$ and $\sin x = -1$.

Find the period and amplitude of $y = \sin 7x$

Find the period and amplitude of $y = -\sin\left(\frac{1}{3}x\right)$

Find the period and amplitude of $y = 4\sin(-2x)$

Find the value of $\sin^{-1}\left(\sin\left(\frac{5\pi}{4}\right)\right)$

For what value of x does $\sin x = \sin^{-1} x$?

Find all the values of x such that $-6\pi \le x \le 6\pi$ and $\cos x = 0$.

Find all the values of x such that $-5\pi \le x \le 5\pi$ and $\cos x = 1$.

Find the value of $2\cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(\frac{1}{2}\right)$

Find the value of $\cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}(-1)$

Find the value of $\tan^{-1}(-1) + \cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$

Find the value of $\tan\left(\tan^{-1}\left(\frac{7\pi}{4}\right)\right)$

Find the value of $\sin^{-1}\left(\cos\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)\right)$

Find the value of $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$

If $\cot^{-1}\left(\frac{1}{7}\right) = \theta$, find the value of $\cos\theta$

Prove that $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} = \frac{\pi}{4}$

Find the principal value of $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$

Find the principal value of $\cot^{-1}(\sqrt{3})$

Find the principal value of $\operatorname{cosec}^{-1}(-\sqrt{2})$

Simplify : $\sec^{-1}\left(\sec\left(\frac{5\pi}{3}\right)\right)$

Find the value of $\sec^{-1}\left(-\frac{2\sqrt{3}}{3}\right)$

CHAPTER 5

Find the general equation of a circle with centre $(-3,-4)$ and radius 3 units.

Find the equation of the circle described on the chord $3x + y + 5 = 0$ of the circle $x^2 + y^2 = 16$ as diameter.

Determine whether $x + y - 1 = 0$ is the equation of a diameter of the circle $x^2 + y^2 - 6x + 4y + c = 0$ for all possible values of c.

Find the general equation of the circle whose diameter is the line segment joining the points $(-4,-2)$ and $(1,1)$

Examine the position of the point $(2,3)$ with respect to the circle $x^2+y^2-6x-8y+12=0$.

The line $3x+4y-12=0$ meets the coordinate axes at A and B. Find the equation of the circle drawn on AB as diameter.

Find the centre and radius of the circle $3x^2+(a+1)y^2+6x-9y+a+4=0$.

Find the equations of the tangent and normal to the circle $x^2+y^2=25$ at $P(-3,4)$.

If $y=4x+c$ is a tangent to the circle $x^2+y^2=9$, find c.

If $y=2\sqrt{2}x+c$ is a tangent to the circle $x^2+y^2=16$, find the value of c.

If the equation $3x^2+(3-p)xy+qy^2-2px=8pq$ represents a circle,
find p and q. Also determine the centre and radius of the circle.

Find the equation of the circle with centre $(2,-1)$ and passing through the point $(3,6)$ in standard form.

Obtain the equation of the circle for which $(3,4)$ and $(2,-7)$ are the ends of a diameter.

Find the length of Latus rectum of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

Prove that the sum of the focal distances of an point on the ellipse is equal to length of the major axis

Find the equation of the parabola with focus $(-\sqrt{2},0)$ and directrix $x=\sqrt{2}$.

Find the equation of the parabola whose vertex is $(5,-2)$ and focus $(2,-2)$.

Find the equation of the ellipse with foci $(\pm 2,0)$, vertices $(\pm 3,0)$.

Find the equation of the hyperbola with vertices $(0,\pm 4)$ and foci $(0,\pm 6)$.

Find the vertices, foci for the hyperbola $9x^2-16y^2=144$.

Find the length of Latus rectum of the parabola $y^2=4ax$.

The orbit of Halley's Comet is an ellipse 36.18 astronomical units long and by 9.12
astronomical units wide. Find its eccentricity.

Show that the absolute value of difference of the focal distances of any point P on the hyperbola is the length of its transverse axis.

Identify the type of the conic for the following equations:

(1) $16y^2=-4x^2+64$ (2) $x^2+y^2=-4x-y+4$

(3) $x^2-2y=x+3$ (4) $4x^2-9y^2-16x+18y-29=0$

Find the equations of tangent and normal to the parabola $x^2+6x+4y+5=0$ at $(1,-3)$.

Find the equations of tangent and normal to the ellipse $x^2+4y^2=32$ when $\theta=\frac{\pi}{4}$.

A line $3x+4y+10=0$ cuts a chord of length 6 units on a circle with centre of the circle $(2,1)$. Find the equation of the circle in general form.

Find the equations of tangent and normal to the parabola $x^2+6x+4y+5=0$ at $(1,-3)$.

Find the equations of tangent and normal to the ellipse $x^2+4y^2=32$ when $\theta=\frac{\pi}{4}$

Find the equation of the tangent at $t=2$ to the parabola $y^2=8x$. (Hint: use parametric form)

Find the equation of the tangent to the parabola $y^2=16x$ perpendicular to $2x+2y+3=0$.

The maximum and minimum distances of the Earth from the Sun respectively are 152×10^6 km and 94.5×10^6 km. The Sun is at one focus of the elliptical orbit. Find the distance from the Sun to the other focus.

A concrete bridge is designed as a parabolic arch. The road over bridge is $40m$ long and the maximum height of the arch is $15m$. Write the equation of the parabolic arch.

The parabolic communication antenna has a focus at $2m$ distance from the vertex of the antenna. Find the width of the antenna $3m$ from the vertex.

Reflective property of parabola

The light or sound or radio waves originating at a parabola's focus are reflected parallel to the parabola's axis and conversely the rays arriving parallel to the axis are directed towards the focus

The equation $y=\frac{1}{32}x^2$ models cross sections of parabolic mirrors that are used for solar energy. There is a heating tube located at the focus of each parabola; how high is this tube located above the vertex of the parabola?

A search light has a parabolic reflector (has a cross section that forms a 'bowl'). The parabolic bowl is $40\,cm$ wide from rim to rim and $30\,cm$ deep. The bulb is located at the focus.

(1) What is the equation of the parabola used for reflector?

(2) How far from the vertex is the bulb to be placed so that the maximum distance covered?

An equation of the elliptical part of an optical lens system is $\frac{x^2}{16}+\frac{y^2}{9}=1$. The parabolic part of the system has a focus in common with the right focus of the ellipse .The vertex of the parabola is at the origin and the parabola opens to the right. Determine the equation of the parabola.

A room $34m$ long is constructed to be a whispering gallery. The room has an elliptical ceiling, If the maximum height of the ceiling is $8m$, determine where the foci are located.

If the equation of the ellipse is $\frac{(x-11)^2}{484}+\frac{y^2}{64}=1$ (x and y are measured in centimeters) where to the nearest centimeter, should the patient's kidney stone be placed so that the reflected sound hits the kidney stone?

CHAPTER 6

Find the angle between the planes $\vec{r}\cdot(\hat{i}+\hat{j}-2\hat{k})=3$ and $2x-2y+z=2$.

Find the length of the perpendicular from the point $(1,-2,3)$ to the plane $x-y+z=5$.

Find the point of intersection of the line $x-1=\frac{y}{2}=z+1$ with the plane $2x-y+2z=2$. Also, find the angle between the line and the plane.

Find the equation of the plane which passes through the point $(3,4,-1)$ and is parallel to the plane $2x-3y+5z+7=0$. Also, find the distance between the two planes.

Find the angle between the line $\vec{r}=(2\hat{i}-\hat{j}+\hat{k})+t(\hat{i}+2\hat{j}-2\hat{k})$ and the plane $\vec{r}\cdot(6\hat{i}+3\hat{j}+2\hat{k})=8$

Find the equation of the plane passing through the line of intersection of the planes $\vec{r}\cdot(2\hat{i}-7\hat{j}+4\hat{k})=3$ and $3x-5y+4z+11=0$, and the point $(-2,1,3)$.

Find the coordinates of the point where the straight line $\vec{r}=(2\hat{i}-\hat{j}+2\hat{k})+t(3\hat{i}+4\hat{j}+2\hat{k})$ intersects the plane $x-y+z-5=0$

Find the image of the point whose position vector is $\hat{i}+2\hat{j}+3\hat{k}$ in the plane $\vec{r}\cdot(\hat{i}+2\hat{j}+4\hat{k})=38$

Find the equation of the plane passing through the intersection of the planes $2x+3y-z+7=0$ and $x+y-2z+5=0$ and is perpendicular to the plane $x+y-3z-5=0$

Find the equation of the plane passing through the intersection of the planes $\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})+1=0$ and $\vec{r}\cdot(2\hat{i}-3\hat{j}+5\hat{k})=2$ and the point $(-1,2,1)$

Find the distance between the planes $\vec{r}\cdot(2\hat{i}-\hat{j}-2\hat{k})=6$ and $\vec{r}\cdot(6\hat{i}-3\hat{j}-6\hat{k})=27$

Find the distance of the point $(5,-5,-10)$ from the point of intersection of a straight line passing through the points $A(4,1,2)$ and $B(7,5,4)$ with the plane $x-y+z=5$.

Find the distance of a point $(2,5,-3)$ from the plane $\vec{r}\cdot(6\hat{i}-3\hat{j}+2\hat{k})=5$.

Find the angle between the straight line $\vec{r}=(2\hat{i}+3\hat{j}+\hat{k})+t(\hat{i}-\hat{j}+\hat{k})$ and the plane $2x-y+z=5$.

Find the acute angle between the planes $\vec{r}\cdot(2\hat{i}+2\hat{j}+2\hat{k})=11$ and $4x-2y+2z=15$

Find the vector and Cartesian form of the equations of a plane which is at a distance of 12 units from the origin and perpendicular to $6\hat{i}+2\hat{j}-3\hat{k}$.

Find the direction cosines of the normal to the plane and length of the perpendicular from the origin to the plane $\vec{r}\cdot(3\hat{i}-4\hat{j}+12\hat{k})=5$.

Find the vector and Cartesian equations of the plane passing through the point with position vector $4\hat{i}+2\hat{j}-3\hat{k}$ and normal to vector $2\hat{i}-\hat{j}+\hat{k}$

If a plane meets the coordinate axes at A,B,C such that the centriod of the triangle ABC is the point (u,v,w), find the equation of the plane.

Find the intercepts cut off by the plane $\vec{r}\cdot(6\hat{i}+4\hat{j}-3\hat{k})=12$ on the coordinate axes.

A plane passes through the point $(-1,1,2)$ and the normal to the plane of magnitude $3\sqrt{3}$ makes equal acute angles with the coordinate axes. Find the equation of the plane.

Find a parametric form of vector equation of a plane which is at a distance of 7 units from the origin having $3,-4,5$ as direction ratios of a normal to it.

A variable plane moves in such a way that the sum of the reciprocals of its intercepts on the coordinate axes is a constant. Show that the plane passes through a fixed point

Determine whether the pair of straight lines $\vec{r}=(2\hat{i}+6\hat{j}+3\hat{k})+t(2\hat{i}+3\hat{j}+4\hat{k})$, $\vec{r}=(2\hat{j}-3\hat{k})+s(\hat{i}+2\hat{j}+3\hat{k})$ are parallel. Find the shortest distance between them.

Find the point of intersection of the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-4}{5}=\frac{y-1}{2}=z$.

Show that the points $(2,3,4),(-1,4,5)$ and $(8,1,2)$ are collinear.

The vertices of ΔABC are $A(7,2,1), B(6,0,3)$, and $C(4,2,4)$. Find $\angle ABC$.

. If the straight lines $\frac{x-5}{5m+2}=\frac{2-y}{5}=\frac{1-z}{-1}$ and $x=\frac{2y+1}{4m}=\frac{1-z}{-3}$ are perpendicular to each other, find the value of m

Find the acute angle between the following lines. $2x=3y=-z$ and $6x=-y=-4z$.

Find the acute angle between the following lines. $\frac{x+4}{3}=\frac{y-7}{4}=\frac{z+5}{5}$, $\vec{r}=4\hat{k}+t(2\hat{i}+\hat{j}+\hat{k})$.

Find the parametric form of vector equation and Cartesian equations of the straight line passing through the point $(-2,3,4)$ and parallel to the straight line $\frac{x-1}{-4}=\frac{y+3}{5}=\frac{8-z}{6}$.

Find the points where the straight line passes through $(6,7,4)$ and $(8,4,9)$ cuts the xz and yz planes.

If the straight line joining the points $(2,1,4)$ and $(a-1,4,-1)$ is parallel to the line joining the points $(0,2,b-1)$ and $(5,3,-2)$, find the values of a and b.

Find the direction cosines of the straight line passing through the points $(5,6,7)$ and $(7,9,13)$. Also, find the parametric form of vector equation and Cartesian equations of the straight line passing through two given points.

Show that the lines $\frac{x-1}{4}=\frac{2-y}{6}=\frac{z-4}{12}$ and $\frac{x-3}{-2}=\frac{y-3}{3}=\frac{5-z}{6}$ are parallel.

Show that the straight line passing through the points $A(6,7,5)$ and $B(8,10,6)$ is perpendicular to the straight line passing through the points $C(10,2,-5)$ and $D(8,3,-4)$.

Find the acute angle between the straight lines $\frac{x-4}{2}=\frac{y}{1}=\frac{z+1}{-2}$ and $\frac{x-1}{4}=\frac{y+1}{-4}=\frac{z-2}{2}$ and state whether they are parallel or perpendicular.

Find the acute angle between the lines $\vec{r}=(\hat{i}+2\hat{j}+4\hat{k})+t(2\hat{i}+2\hat{j}+\hat{k})$ and the straight line passing through the points $(5,1,4)$ and $(9,2,12)$.

Find the angles between the straight line $\frac{x+3}{2}=\frac{y-1}{2}=-z$ with coordinate axes.

Find the vector equation in parametric form and Cartesian equations of a straight passing through the points $(-5,7,-4)$ and $(13,-5,2)$. Find the point where the straight line crosses the xy-plane.

If $\vec{a},\vec{b},\vec{c},\vec{d}$ are coplanar vectors, show that $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=\vec{0}$.

Prove that $[\vec{a}-\vec{b},\ \vec{b}-\vec{c},\ \vec{c}-\vec{a}]=0$.

For any vector $\vec{a}$, prove that $\hat{i}\times(\vec{a}\times\hat{i})+\hat{j}\times(\vec{a}\times\hat{j})+\hat{k}\times(\vec{a}\times\hat{k})=2\vec{a}$

If $\vec{a}=\hat{i}-2\hat{j}+3\hat{k},\ \vec{b}=2\hat{i}+\hat{j}-2\hat{k},\ \vec{c}=3\hat{i}+2\hat{j}+\hat{k}$, find $(\vec{a}\times\vec{b})\times\vec{c}$

$\vec{a}=2\hat{i}+3\hat{j}-\hat{k},\ \vec{b}=-\hat{i}+2\hat{j}-4\hat{k},\ \vec{c}=\hat{i}+\hat{j}+\hat{k}$ then find the value of $(\vec{a}\times\vec{b})\cdot(\vec{a}\times\vec{c})$.

If $\vec{a}=\hat{i}-2\hat{j}+3\hat{k},\ \vec{b}=2\hat{i}+\hat{j}-2\hat{k},\ \vec{c}=3\hat{i}+2\hat{j}+\hat{k}$, find $\vec{a}\cdot(\vec{b}\times\vec{c})$.

Find the volume of the parallelepiped whose coterminous edges are represented by the vectors $-6\hat{i}+14\hat{j}+10\hat{k},\ 14\hat{i}-10\hat{j}-6\hat{k}$ and $2\hat{i}+4\hat{j}-2\hat{k}$.

The volume of the parallelepiped whose coterminus edges are $7\hat{i}+\lambda\hat{j}-3\hat{k},\ \hat{i}+2\hat{j}-\hat{k}$, $-3\hat{i}+7\hat{j}+5\hat{k}$ is 90 cubic units. Find the value of λ.

If $\vec{a},\vec{b},\vec{c}$ are three non-coplanar vectors represented by concurrent edges of a parallelepiped of volume 4 cubic units, find the value of $(\vec{a}+\vec{b})\cdot(\vec{b}\times\vec{c})+(\vec{b}+\vec{c})\cdot(\vec{c}\times\vec{a})+(\vec{c}+\vec{a})\cdot(\vec{a}\times\vec{b})$.

Find the altitude of a parallelepiped determined by the vectors $\vec{a}=-2\hat{i}+5\hat{j}+3\hat{k},\ \vec{b}=\hat{i}+3\hat{j}-2\hat{k}$ and $\vec{c}=-3\vec{i}+\vec{j}+4\vec{k}$ if the base is taken as the parallelogram determined by $\vec{b}$ and $\vec{c}$.

Determine whether the three vectors $2\hat{i}+3\hat{j}+\hat{k},\ \hat{i}-2\hat{j}+2\hat{k}$ and $3\hat{i}+\hat{j}+3\hat{k}$ are coplanar.

Let $\vec{a}=\hat{i}+\hat{j}+\hat{k},\ \vec{b}=\hat{i}$ and $\vec{c}=c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$. If $c_1=1$ and $c_2=2$, find c_3 such that $\vec{a},\vec{b}$ and $\vec{c}$ are coplanar.

If $2\hat{i}-\hat{j}+3\hat{k},\ 3\hat{i}+2\hat{j}+\hat{k},\ \hat{i}+m\hat{j}+4\hat{k}$ are coplanar, find the value of m

A particle acted on by constant forces $8\hat{i}+2\hat{j}-6\hat{k}$ and $6\hat{i}+2\hat{j}-2\hat{k}$ is displaced from the point $(1,2,3)$ to the point $(5,4,1)$. Find the total work done by the forces.

Find the magnitude and direction cosines of the torque of a force represented by $3\hat{i}+4\hat{j}-5\hat{k}$ about the point with position vector $2\hat{i}-3\hat{j}+4\hat{k}$ acting through a point whose position vector is $4\hat{i}+2\hat{j}-3\hat{k}$.

Prove by vector method that an angle in a semi-circle is a right angle.

Prove by vector method that the parallelograms on the same base and between the same parallels are equal in area.

Prove by vector method that the area of the quadrilateral $ABCD$ having diagonals AC and BD is $\frac{1}{2}|\overline{AC}\times\overline{BD}|$.

Prove by vector method that the diagonals of a rhombus bisect each other at right angles.

A particle is acted upon by the forces $3\hat{i}-2\hat{j}+2\hat{k}$ and $2\hat{i}+\hat{j}-\hat{k}$ is displaced from the point $(1,3,-1)$ to the point $(4,-1,\lambda)$. If the work done by the forces is 16 units, find the value of λ.

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