

CHAPTER I: APPLICATION OF MATRICES AND DETERMINANTS (1)

Formulae:-

1. $(AB)^{-1} = B^{-1}A^{-1}$ if A & B are non singular matrices of same order
2. $(A^{-1})^{-1} = A$ if A is non singular, A^{-1} is non singular
3. if A is non singular square matrix order n , then
 - (i) $(adj A)^{-1} = (adj A^{-1}) = \frac{1}{|A|} A$
 - (ii) $|adj A| = |A|^{n-1}$

$$(iii) adj(adj A) = |A|^{n-1} A \quad (iv) adj(\lambda A) = \lambda^{n-1} adj A, \lambda \text{ is nonzero scalar}$$

$$(v) |adj(adj A)| = |A|^{(n-1)^2} \quad (vi) (adj A)^T = adj(A^T)$$

$$4) adj(AB) = (adj B)(adj A)$$

5) A square matrix A is called orthogonal if $AA^T = A^T A = I$

$$6) (i) A^{-1} = \frac{1}{|A|} adj A \quad (ii) (A^{-1})^{-1} = A$$

$$7) (\lambda A)^{-1} = \frac{1}{\lambda} A^{-1} \quad (iii) A(adj A) = (adj A)A = |A| I$$

$$8) A^{-1} = \pm \frac{1}{\sqrt{|adj A|}} adj A \quad 9) A = \pm \frac{1}{\sqrt{|adj A|}} adj(adj A)$$

9) Methods to solve the system of linear equations $Ax = B$

(i) by matrix inversion method $X = A^{-1}B, |A| \neq 0$

(ii) by Cramer's rule $x = \frac{\Delta_x}{\Delta}, y = \frac{\Delta_y}{\Delta}, z = \frac{\Delta_z}{\Delta}, \Delta \neq 0$

(iii) by Gaussian elimination (Rank) method

10) (i) if $\rho(A) = \rho(A|B) < n$ Then the system has infinitely many solutions

(ii) if $\rho(A) = \rho(A|B) = n$ Then the system has unique solution

(iii) if $\rho(A) \neq \rho(A|B)$ then The system is inconsistent and has no solution.

11) The homogenous system of linear equation $Ax = 0$

(i) has the trivial solution if $|A| \neq 0$

(ii) has a non trivial solution if $|A| = 0$.

12) A matrix A is orthogonal iff A is non singular and $A^{-1} = A^T$.

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(2)

Exercise 1.1

1. Find the adjoint of the following.

(i) $\begin{bmatrix} -3 & 4 \\ 6 & 2 \end{bmatrix}$

Soln

$$\text{Let } A = \begin{bmatrix} -3 & 4 \\ 6 & 2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad \because \text{interchanging } a_{22} \text{ \& } a_{11}$$

Sign Changing $a_{12} \text{ \& } a_{21}$

$$\text{adj } A = \begin{bmatrix} 2 & -4 \\ -6 & -3 \end{bmatrix}$$

(ii)

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$$

$$\text{adj } A = \begin{bmatrix} 8-7 & 7-6 & 3-4 \\ 3-6 & 4-3 & 3-2 \\ 21-12 & 9-14 & 8-9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & -1 \\ -3 & 1 & 1 \\ 9 & -5 & -1 \end{bmatrix}$$

	c ₁	c ₂	c ₃	c ₄
R ₁	2	3	1	2
R ₂	3	4	1	3
R ₃	3	7	2	3
R ₄	2	3	1	2
R ₅	3	4	1	3

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(iii)

$$A = \frac{1}{3} \begin{bmatrix} 2 & 2 & 1 \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix}_{3 \times 3}$$

Soln

$$n = 3$$

$$\text{adj}(kA) = k^{n-1} \text{adj } A$$

$$\text{adj}(A) = \left(\frac{1}{3}\right)^{3-1} \begin{bmatrix} 2+4 & -2-4 & 4-1 \\ 2+4 & 4-1 & -2-4 \\ 4-1 & 2+4 & 2+4 \end{bmatrix}$$

$$= \frac{1}{9} \begin{bmatrix} 6 & -6 & 3 \\ 6 & 3 & -6 \\ 3 & 6 & 6 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 2 & -2 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 2 \end{bmatrix}$$

	c ₁	c ₂	c ₃	c ₄
R ₁	2	2	1	2
R ₂	-2	1	2	-2
R ₃	1	-2	2	1
R ₄	2	2	1	2
R ₅	-2	1	2	-2

2.

Find the inverse (if it exists) of the following

(i) $A = \begin{bmatrix} -2 & 4 \\ 1 & -3 \end{bmatrix}$

$$|A| = 6 - 4 = 2 \neq 0 \quad \therefore A^{-1} \text{ exists.}$$

A is non singular

$$\text{adj } A = \begin{bmatrix} -3 & -4 \\ -1 & -2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{2} \begin{bmatrix} -3 & -4 \\ -1 & -2 \end{bmatrix}$$

(3)

(ii) $A = \begin{bmatrix} 5 & 1 & 1 \\ 1 & 5 & 1 \\ 1 & 1 & 5 \end{bmatrix}$

Soln

$$|A| = 5(25-1) - 1(5-1) + 1(1-5) = 120 - 4 - 4 = 112 \neq 0$$

$$= 5(24) - 1(4) + 1(-4) \quad A^{-1} \text{ is exist}$$

$$\text{adj } A = \begin{pmatrix} 25-1 & 1-5 & 1-5 \\ 1-5 & 25-1 & 1-5 \\ 1-5 & 1-5 & 25-1 \end{pmatrix}$$

$$= \begin{pmatrix} 24 & -4 & -4 \\ -4 & 24 & -4 \\ -4 & -4 & 24 \end{pmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{112} \begin{pmatrix} 24 & -4 & -4 \\ -4 & 24 & -4 \\ -4 & -4 & 24 \end{pmatrix}$$

$$= \frac{1}{28} \begin{pmatrix} 6 & -1 & -1 \\ -1 & 6 & -1 \\ -1 & -1 & 6 \end{pmatrix}$$

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(iii) $A = \begin{pmatrix} 2 & 3 & 1 \\ 3 & 4 & 1 \\ 3 & 7 & 2 \end{pmatrix}$

Soln

$$|A| = 2(8-7) - 3(6-3) + 1(21-12) = 2 - 9 + 9 = 2 \neq 0$$

$$A^{-1} \text{ is exist}$$

$$\text{adj } A = \begin{bmatrix} 8-7 & 2-6 & 3-4 \\ 3-6 & 4-3 & 3-2 \\ 21-12 & 9-14 & 8-9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -4 & -1 \\ -3 & 1 & 1 \\ 9 & -5 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{2} \begin{bmatrix} 1 & -4 & -1 \\ -3 & 1 & 1 \\ 9 & -5 & -1 \end{bmatrix}$$

3)

If $F(\alpha) = \begin{bmatrix} \cos \alpha & 0 & \sin \alpha \\ 0 & 1 & 0 \\ -\sin \alpha & 0 & \cos \alpha \end{bmatrix}$ show that $[F(\alpha)]^{-1} = F(-\alpha)$

Soln

$$|F(\alpha)| = \cos \alpha (\cos \alpha - 0) - 0 + \sin \alpha (0 + \sin \alpha) = \cos^2 \alpha + \sin^2 \alpha = 1$$

$$|F(\alpha)| = 1 \neq 0$$

$$\text{adj } F(\alpha) = \begin{bmatrix} \cos \alpha & 0 & \sin \alpha \\ 0 & 1 & 0 \\ -\sin \alpha & 0 & \cos \alpha \end{bmatrix}$$

$$\begin{bmatrix} \cos \alpha & 0 & \sin \alpha \\ 0 & 1 & 0 \\ -\sin \alpha & 0 & \cos \alpha \end{bmatrix}$$

$$F(x)^{-1} = \frac{1}{|F(x)|} \text{adj } F(x) = \frac{1}{1} \begin{bmatrix} \cos x & 0 & -\sin x \\ 0 & 1 & 0 \\ -\sin x & 0 & \cos x \end{bmatrix} \quad (4)$$

$$= \begin{bmatrix} \cos x & 0 & +\sin x \\ 0 & 1 & 0 \\ -\sin x & 0 & \cos x \end{bmatrix} \quad - (1)$$

$$F(-x) = \begin{bmatrix} \cos(-x) & 0 & -\sin(-x) \\ 0 & 1 & 0 \\ \sin(-x) & 0 & \cos x \end{bmatrix} = \begin{bmatrix} \cos x & 0 & +\sin x \\ 0 & 1 & 0 \\ -\sin x & 0 & \cos x \end{bmatrix}$$

$$F(-x) = F(x)^{-1}$$

$$\begin{aligned} \because \cos(-0) &= \cos 0 \\ \sin(-0) &= -\sin 0 \end{aligned}$$

4) If $A = \begin{pmatrix} 5 & 3 \\ -1 & -2 \end{pmatrix}$ Show that $A^2 - 3A - 7I_2 = 0$ Hence find A^{-1}

Soln

$$A^2 = A \times A = \begin{pmatrix} 5 & 3 \\ -1 & -2 \end{pmatrix} \begin{pmatrix} 5 & 3 \\ -1 & -2 \end{pmatrix} = \begin{pmatrix} 25-3 & 15+6 \\ -5+2 & -3+4 \end{pmatrix} = \begin{pmatrix} 22 & 9 \\ -3 & 1 \end{pmatrix}$$

$$-3A = -3 \begin{pmatrix} 5 & 3 \\ -1 & -2 \end{pmatrix} = \begin{pmatrix} -15 & -9 \\ 3 & 6 \end{pmatrix}$$

$$-7I_2 = -7 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -7 & 0 \\ 0 & -7 \end{pmatrix}$$

$$A^2 - 3A - 7I_2 = \begin{pmatrix} 22 & 9 \\ -3 & 1 \end{pmatrix} + \begin{pmatrix} -15 & -9 \\ 3 & 6 \end{pmatrix} + \begin{pmatrix} -7 & 0 \\ 0 & -7 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\therefore A^2 - 3A - 7I_2 = 0$$

Post multiply this eqn by A^{-1}

$$A - 3I - 7A^{-1} = 0$$

$$A - 3I = 7A^{-1}$$

$$A^{-1} = \frac{1}{7} (A - 3I) = \frac{1}{7} \left[\begin{pmatrix} 5 & 3 \\ -1 & -2 \end{pmatrix} - 3 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right]$$

$$= \frac{1}{7} \left[\begin{pmatrix} 5 & 3 \\ -1 & -2 \end{pmatrix} + \begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix} \right] = \frac{1}{7} \begin{bmatrix} 2 & 3 \\ -1 & -5 \end{bmatrix}$$

5) If $A = \frac{1}{9} \begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}$ Prove that $A^{-1} = A^T$

Soln

R.H.S

$$A^T = \frac{1}{9} \begin{bmatrix} -8 & 4 & 1 \\ 1 & 4 & -8 \\ 4 & 7 & 4 \end{bmatrix}$$

$$\underline{\text{LHS}} \quad n=3 \quad |A| = \left(\frac{1}{9}\right)^3 \begin{bmatrix} -8(16+56) - 1(16-7) + 4(-32-4) \end{bmatrix} \quad \therefore |kA| = k^n |A| \quad (6)$$

$$= \frac{1}{729} \begin{bmatrix} -8(72) - 1(9) + 4(-36) \end{bmatrix} = \frac{1}{729} (-576 - 23 - 144)$$

$$= \frac{1}{729} (-729) = -1 \neq 0$$

$\therefore A^{-1}$ exist.

$$\text{adj} A = \left(\frac{1}{9}\right)^{3-1} \begin{bmatrix} 16+56 & -32-4 & 7-16 \\ 7-16 & -32-4 & 16+56 \\ -32-4 & 1-64 & -32-4 \end{bmatrix}$$

$$= \left(\frac{1}{9}\right)^2 \begin{bmatrix} 72 & -36 & -9 \\ -9 & -36 & 72 \\ -36 & -63 & -36 \end{bmatrix}$$

$$= \frac{1}{9} \begin{bmatrix} 8 & -4 & -1 \\ -1 & -4 & 8 \\ -4 & -7 & -4 \end{bmatrix}$$

$$| \text{adj}(kA) | = k^{n^2} | \text{adj} A |$$

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$$A^{-1} = \frac{1}{|A|} \text{adj} A = \frac{1}{-1} \left(\frac{1}{9} \begin{bmatrix} 8 & -4 & -1 \\ -1 & -4 & 8 \\ -4 & -7 & -4 \end{bmatrix} \right) = \frac{1}{9} \begin{bmatrix} -8 & 4 & 1 \\ 1 & 4 & 8 \\ 4 & 7 & 4 \end{bmatrix} \quad (7)$$

by ① & ②

$$A^T = A^{-1}$$

6) If $A = \begin{pmatrix} 8 & -4 \\ -5 & 3 \end{pmatrix}$ verify that $A(\text{adj} A) = (\text{adj} A)A = |A| I_2$

Soln

$$|A| = 24 - 20 = 4 \neq 0 \quad A^{-1} \text{ exist.}$$

$$\text{adj} A = \begin{pmatrix} 3 & 4 \\ 5 & 8 \end{pmatrix}$$

$$A(\text{adj} A) = \begin{pmatrix} 8 & -4 \\ -5 & 3 \end{pmatrix} \begin{pmatrix} 3 & 4 \\ 5 & 8 \end{pmatrix} = \begin{pmatrix} 24-20 & 32-32 \\ -15+15 & -20+24 \end{pmatrix} = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} \quad (1)$$

$$(\text{adj} A)A = \begin{pmatrix} 3 & 4 \\ 5 & 8 \end{pmatrix} \begin{pmatrix} 8 & -4 \\ -5 & 3 \end{pmatrix} = \begin{pmatrix} 24-20 & -12+12 \\ 40-40 & -20+24 \end{pmatrix} = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} \quad (2)$$

$$|A| I_2 = 4 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} \quad (3)$$

by ①, ② & ③ $A(\text{adj} A) = (\text{adj} A)A = |A| I_2$

7) If $A = \begin{pmatrix} 3 & 2 \\ 7 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} -1 & -3 \\ 5 & 2 \end{pmatrix}$ verify that $(AB)^{-1} = B^{-1}A^{-1}$ (7)

Soln

$$|A| = 15 - 14 = 1 \neq 0 \quad A^{-1} \text{ exist.}$$

$$\text{adj } A = \begin{pmatrix} 5 & -2 \\ 7 & 3 \end{pmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$= \frac{1}{1} \begin{pmatrix} 5 & -2 \\ 7 & 3 \end{pmatrix} = \begin{pmatrix} 5 & -2 \\ 7 & 3 \end{pmatrix}$$

$$|B| = -2 + 15 = 13 \neq 0$$

$$\text{adj } B = \begin{pmatrix} 2 & 3 \\ -5 & -1 \end{pmatrix}$$

$$B^{-1} = \frac{1}{|B|} \text{adj } B = \frac{1}{13} \begin{pmatrix} 2 & 3 \\ -5 & -1 \end{pmatrix}$$

RHS

$$B^{-1}A^{-1} = \frac{1}{13} \begin{pmatrix} 2 & 3 \\ -5 & -1 \end{pmatrix} \times \begin{pmatrix} 5 & -2 \\ 7 & 3 \end{pmatrix} = \frac{1}{13} \begin{pmatrix} 10 - 21 & -4 + 9 \\ -25 + 7 & 10 - 3 \end{pmatrix}$$

$$= \frac{1}{13} \begin{pmatrix} -11 & 5 \\ -18 & 7 \end{pmatrix}$$

LHS $AB = \begin{pmatrix} 3 & 2 \\ 7 & 5 \end{pmatrix} \begin{pmatrix} -1 & -3 \\ 5 & 2 \end{pmatrix} = \begin{pmatrix} -3 + 10 & -9 + 4 \\ -7 + 25 & -21 + 10 \end{pmatrix} = \begin{pmatrix} 7 & -5 \\ 18 & -11 \end{pmatrix}$

$$|AB| = -77 + 90 = 13 \neq 0$$

$$\text{adj } AB = \begin{pmatrix} -11 & 5 \\ +18 & 7 \end{pmatrix}$$

$$(AB)^{-1} = \frac{1}{|AB|} \text{adj } AB = \frac{1}{13} \begin{pmatrix} -11 & 5 \\ 18 & 7 \end{pmatrix}$$

8)

If $\text{adj } A = \begin{pmatrix} 2 & -4 & 2 \\ -3 & 12 & -7 \\ -2 & 0 & 2 \end{pmatrix}$ find A

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Soln

$$A = \pm \frac{1}{\sqrt{|\text{adj } A|}} \text{adj}(\text{adj } A)$$

$$|\text{adj } A| = 2(24 - 0) + 4(-6 - 14) + 2(0 + 24) = 48 - 80 + 48 = 16$$

$$\text{adj}(\text{adj } A) = \begin{pmatrix} 24 & 8 & 4 \\ 20 & 8 & 8 \\ 24 & 8 & 12 \end{pmatrix} = \frac{1}{4} \begin{pmatrix} 6 & 2 & 1 \\ 5 & 2 & 2 \\ 6 & 2 & 3 \end{pmatrix}$$

$$\sqrt{|\text{adj } A|} = \sqrt{16} = 4$$

$$A = \pm 4 \left(\frac{1}{4} \right) \begin{pmatrix} 6 & 2 & 1 \\ 5 & 2 & 2 \\ 6 & 2 & 3 \end{pmatrix} = \pm \begin{pmatrix} 6 & 2 & 1 \\ 5 & 2 & 2 \\ 6 & 2 & 3 \end{pmatrix}$$

$$\begin{vmatrix} 2 & -4 & 2 & 2 & -4 \\ -3 & 12 & -7 & -3 & 12 \\ -2 & 0 & 2 & -2 & 0 \\ 2 & -4 & 2 & 2 & -4 \\ -3 & 12 & -7 & -3 & 12 \end{vmatrix}$$

9) If $\text{adj } A = \begin{pmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{pmatrix}$ find A^{-1}

Soln
 $A^{-1} = \frac{1}{|\text{adj } A|} \text{adj } A$

$$|\text{adj } A| = 0 + 2(36 - 18) + 0 = 2(18) = 36.$$

$$A^{-1} = \frac{1}{36} \begin{pmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{pmatrix} = \frac{1}{6} \begin{pmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ 3 & 0 & 6 \end{pmatrix}$$

10) Find $\text{adj}(\text{adj } A)$ if $\text{adj } A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{pmatrix}$

Soln

$$\begin{aligned} \text{adj}(\text{adj } A) &= \begin{pmatrix} 2-0 & 0-0 & 0-2 \\ 0-0 & 1+1 & 0-0 \\ 0+2 & 0-0 & 2-0 \end{pmatrix} \\ &= \begin{pmatrix} 2 & 0 & -2 \\ 0 & 2 & 0 \\ 2 & 0 & 2 \end{pmatrix} \end{aligned}$$

$$\begin{vmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 2 & 0 & 0 & 2 \\ -1 & 0 & 1 & -1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 2 & 0 & 0 & 2 \end{vmatrix}$$

11) $A = \begin{pmatrix} 1 & \tan x \\ -\tan x & 1 \end{pmatrix}$ Show that $A^T A^{-1} = \begin{pmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{pmatrix}$

Soln

$$|A| = 1 + \tan^2 x = \sec^2 x$$

$$\text{adj } A = \begin{pmatrix} 1 & -\tan x \\ \tan x & 1 \end{pmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{\sec^2 x} \begin{pmatrix} 1 & -\tan x \\ \tan x & 1 \end{pmatrix}$$

$$A^T = \begin{pmatrix} 1 & -\tan x \\ \tan x & 1 \end{pmatrix}$$

$$A^T A^{-1} = \frac{1}{\sec^2 x} \begin{pmatrix} 1 & -\tan x \\ \tan x & 1 \end{pmatrix} \begin{pmatrix} 1 & -\tan x \\ \tan x & 1 \end{pmatrix}$$

$$= \frac{1}{\sec^2 x} \begin{pmatrix} 1 - \tan^2 x & -\tan x - \tan x \\ \tan x + \tan x & -\tan^2 x + 1 \end{pmatrix}$$

$$= \frac{1}{\sec^2 x} \begin{pmatrix} 1 - \tan^2 x & -2 \tan x \\ 2 \tan x & 1 - \tan^2 x \end{pmatrix} = \cos^2 x \begin{pmatrix} 1 - \frac{\sin^2 x}{\cos^2 x} & -2 \frac{\sin x}{\cos x} \\ 2 \frac{\sin x}{\cos x} & 1 - \frac{\sin^2 x}{\cos^2 x} \end{pmatrix}$$

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$$= \begin{pmatrix} \cos^2 x - \sin^2 x & -2\sin x \cos x \\ 2\sin x \cos x & \cos^2 x - \sin^2 x \end{pmatrix} = \begin{pmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{pmatrix} \quad (9)$$

= R.H.S.

Hence proved.

$$\therefore \begin{aligned} \cos 2A &= \cos^2 A - \sin^2 A \\ \sin 2A &= 2 \sin A \cos A \end{aligned}$$

- 12) Find the matrix A for which $A \begin{pmatrix} 5 & 3 \\ -1 & -2 \end{pmatrix} = \begin{pmatrix} 14 & 7 \\ 7 & 7 \end{pmatrix}$
soln order of A is 2×2 .

Let us take $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 5 & 3 \\ -1 & -2 \end{pmatrix} = \begin{pmatrix} 14 & 7 \\ 7 & 7 \end{pmatrix}$$

$$\begin{pmatrix} 5a-b & 3a-2b \\ 5c-d & 3c-2d \end{pmatrix} = \begin{pmatrix} 14 & 7 \\ 7 & 7 \end{pmatrix}$$

$$5a-b = 14 \quad \text{--- (1)}$$

$$3a-2b = 7 \quad \text{--- (2)}$$

$$5c-d = 7 \quad \text{--- (3)}$$

$$3c-2d = 7 \quad \text{--- (4)}$$

$$\begin{aligned} (2) - 2 \times (1) &\Rightarrow 3a-2b = 7 \\ &\quad -10a+2b = -28 \\ \hline &\quad -7a = -21 \end{aligned}$$

$$\boxed{a = 3}$$

$$(1) \Rightarrow 15-b = 14$$

$$15-14 = b$$

$$\boxed{b = 1}$$

$$\begin{aligned} (4) - 2 \times (3) &\Rightarrow 3c-2d = 7 \\ &\quad -10c+2d = -14 \\ \hline &\quad -7c = -7 \end{aligned}$$

$$\boxed{c = 1}$$

$$(3) \Rightarrow 5-d = 7$$

$$5-7 = d$$

$$d = -2$$

$$\therefore A = \begin{pmatrix} 3 & 1 \\ 1 & -2 \end{pmatrix}$$

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- 13) Given $A = \begin{pmatrix} 1 & -1 \\ 2 & 0 \end{pmatrix}$ $B = \begin{pmatrix} 3 & -2 \\ 1 & 1 \end{pmatrix}$ $C = \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix}$ find a matrix X such that $AXB = C$

soln

$$\begin{aligned} A \times B &= C \\ A^{-1} A (X) B B^{-1} &= A^{-1} C B^{-1} \\ X &= A^{-1} C B^{-1} \end{aligned}$$

(Pre & Post multiply
 by A^{-1} and B^{-1})

$$|A| = 0+2 = 2 \neq 0$$

$$\text{adj } A = \begin{pmatrix} 0 & 1 \\ -2 & 1 \end{pmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{2} \begin{pmatrix} 0 & 1 \\ -2 & 1 \end{pmatrix}$$

$$|B| = 3 + 2 = 5$$

$$\text{adj } B = \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix} \quad \therefore B^{-1} = \frac{1}{|B|} \text{adj } B = \frac{1}{5} \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}$$

$$X = A^{-1} C B^{-1}$$

$$= \frac{1}{2} \begin{pmatrix} 0 & 1 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix} \frac{1}{5} \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}$$

$$= \frac{1}{10} \begin{bmatrix} 0+2 & 0+2 \\ -2+2 & -2+2 \end{bmatrix} \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix} = \frac{1}{10} \begin{pmatrix} 2 & 2 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}$$

$$= \frac{1}{10} \begin{pmatrix} 2-2 & 4+6 \\ 0-0 & 0-0 \end{pmatrix} = \frac{1}{10} \begin{pmatrix} 0 & 10 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

14.

$$\text{If } A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \text{ show that } A^{-1} = \frac{1}{2} (A^2 - 3I)$$

soln

$$A^2 = A \times A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$$

$$A^2 - 3I = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix} - 3 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix} + \begin{pmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix} \quad \text{--- (i)}$$

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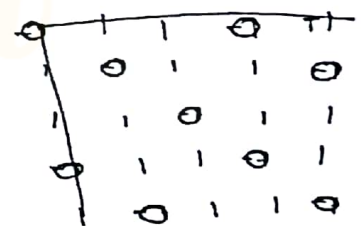
$$|A| = 0 - 1(0-1) + 1(1-0) = 1+1 = 2$$

$$\text{adj } A = \begin{pmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{2} \begin{pmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix}$$

$$A^{-1} = \frac{1}{2} (A^2 - 3I) \quad \text{using (i)}$$

Hence proved.



- 15) Decrypt the received encoded message $(2 \ -3) \ (20 \ 4)$ with the encryption matrix $\begin{pmatrix} -1 & -1 \\ 2 & 1 \end{pmatrix}$ and the decryption matrix as its inverse, where the system of codes are described by the numbers 1-26 to the letters A-Z respectively and the number 0 to a blank space.

Soln

Encoding matrix $\begin{pmatrix} -1 & -1 \\ 2 & 1 \end{pmatrix}$

un coded
row matrix

Encoding
matrix

coded row
matrix

$$(2 \ -3)$$

$$\begin{pmatrix} -1 & -1 \\ 2 & 1 \end{pmatrix}$$

$$= (-2-6 \ -2-3)$$

$$= (-8 \ -5)$$

$$(20 \ 4)$$

$$\begin{pmatrix} -1 & -1 \\ 2 & 1 \end{pmatrix}$$

$$= (-20+8 \ -20+4) = (-12 \ -16)$$

so the encoded message is $(-8 \ -5) \ (-12 \ -16)$

The receiver will decode the message by the inverse key - post multiplying by the inverse of A.

So the decoding matrix is $A^{-1} = \frac{1}{|A|} \text{adj } A$

$$|A| = -1+2 = 1$$

$$\text{adj } A = \begin{pmatrix} 1 & 1 \\ -2 & -1 \end{pmatrix}$$

$$\therefore A^{-1} = \begin{pmatrix} 1 & 1 \\ -2 & -1 \end{pmatrix}$$

The receiver decodes the coded message as follows

Coded row
matrix

Decoding
matrix

Decoded
row matrix

$$(-8 \ -5)$$

$$\begin{pmatrix} 1 & 1 \\ -2 & -1 \end{pmatrix}$$

$$= (-8+10 \ -8+5) = (2 \ -3)$$

$$(-12 \ -16)$$

$$\begin{pmatrix} 1 & 1 \\ -2 & -1 \end{pmatrix}$$

$$= (-12+32 \ -12+16) = (20 \ 4)$$

So the sequence of decoded row matrices is $(2, -3) \ (20, 4)$

Thus the receiver reads the message is

$$(-8 \ -5) \ (-12 \ -16)$$

H E L P

word is

HELP

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(12)

Exercise 1.2

1. Find the rank of the following matrices by minor method.

(i) $A = \begin{pmatrix} 2 & -4 \\ -1 & 2 \end{pmatrix}$

$$\begin{vmatrix} 2 & -4 \\ -1 & 2 \end{vmatrix} = 4 - 4 = 0.$$

Rank of A is 1

$$\rho(A) = 1$$

(ii) $A = \begin{pmatrix} -1 & 3 \\ 4 & -7 \\ 3 & -4 \end{pmatrix}$

$$\begin{vmatrix} -1 & 3 \\ 4 & -7 \end{vmatrix} = 7 - 12 = -5 \neq 0$$

$$\rho(A) = 2$$

(iii) $A = \begin{pmatrix} 1 & -2 & -1 & 0 \\ 3 & -6 & -3 & 1 \end{pmatrix}$

$$\begin{vmatrix} 1 & -2 \\ 3 & -6 \end{vmatrix} = 6 - 6 = 0$$

$$\begin{vmatrix} -1 & 0 \\ -3 & 1 \end{vmatrix} = -1 - 0 = -1 \neq 0$$

$$\rho(A) = 2$$

(iv) $A = \begin{pmatrix} 1 & -2 & 3 \\ 2 & 4 & -6 \\ 5 & 1 & -1 \end{pmatrix}$

$$|A| = 1(-4 + 6) + 2(-2 + 30) + 3(2 - 20)$$

$$= 1(-2) + 2(18) + 3(-18) = -2 + 36 - 54 = -20 \neq 0$$

$$\rho(A) = 3$$

(v) $A = \begin{pmatrix} 0 & 1 & 2 & 1 \\ 0 & 2 & 4 & 3 \\ 8 & 1 & 0 & 2 \end{pmatrix}$

$$\begin{vmatrix} 0 & 1 & 2 \\ 0 & 2 & 4 \\ 8 & 1 & 0 \end{vmatrix} = 8(4 - 2) = 8(2) = 16 \neq 0$$

$$\begin{vmatrix} 1 & 2 & 1 \\ 2 & 4 & 3 \\ 1 & 0 & 2 \end{vmatrix} = 1(8) - 2(4 - 3) + 1(0 - 4) = 8 - 2(1) - 4 = 1 \neq 0$$

$$\rho(A) = 3$$

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- 2) Find the rank of the following matrices by row reduction method. (12)

$$i) A = \begin{pmatrix} 1 & 1 & 1 & 3 \\ 2 & -1 & 3 & 4 \\ 5 & -1 & 7 & 11 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 1 & 1 & 3 \\ 0 & -6 & 2 & -4 \\ 0 & -3 & 1 & -2 \end{pmatrix}$$

$$R_2 \rightarrow R_2 - 2R_1$$

$$R_3 \rightarrow R_3 - 5R_1$$

$$\sim \begin{pmatrix} 1 & 1 & 1 & 3 \\ 0 & -3 & 1 & -2 \\ 0 & -6 & 2 & -4 \end{pmatrix}$$

$$R_3 \leftrightarrow R_2$$

$$\sim \begin{pmatrix} 1 & 1 & 1 & 3 \\ 0 & -3 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$R_3 \rightarrow R_3 - 2R_2$$

$$\rho(A) = 2$$

ii)

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 1 & -2 & 3 \\ 1 & -1 & 1 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 2 & -1 \\ 0 & -7 & 5 \\ 0 & -4 & 4 \\ 0 & -3 & 2 \end{pmatrix}$$

$$R_2 \rightarrow R_2 - 3R_1$$

$$R_3 \rightarrow R_3 - R_1$$

$$R_4 \rightarrow R_4 - R_1$$

$$\sim \begin{pmatrix} 1 & 2 & -1 \\ 0 & 7 & -5 \\ 0 & 4 & -4 \\ 0 & 3 & -2 \end{pmatrix}$$

$$R_2 \rightarrow (-1)R_2$$

$$R_3 \rightarrow (-1)R_3$$

$$R_4 \rightarrow (-1)R_4$$

$$\sim \begin{pmatrix} 1 & 2 & -1 \\ 0 & 7 & -5 \\ 0 & 0 & -8 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R_3 \rightarrow 7R_3 - 4R_2$$

$$R_4 \rightarrow 7R_4 - 3R_2$$

$$\sim \begin{pmatrix} 1 & 2 & -1 \\ 0 & 7 & -5 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R_3 \rightarrow \frac{R_3}{-8}$$

$$\sim \begin{pmatrix} 1 & 2 & -1 \\ 0 & 7 & -5 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$R_4 \rightarrow R_4 - R_3$$

$$\rho(A) = 3.$$

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(14)

$$A = \begin{pmatrix} 3 & -8 & 5 & 2 \\ 2 & -5 & 1 & 4 \\ -1 & 2 & 3 & -2 \end{pmatrix}$$

$$\sim \begin{pmatrix} -1 & 2 & 3 & -2 \\ 2 & -5 & 1 & 4 \\ 3 & -8 & 5 & 2 \end{pmatrix} \quad R_1 \leftrightarrow R_3$$

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$$\sim \begin{pmatrix} 1 & -2 & -3 & 2 \\ 2 & -5 & 1 & 4 \\ 3 & -8 & 5 & 2 \end{pmatrix} \quad R_1 \rightarrow 1 \rightarrow R_1$$

$$\sim \begin{pmatrix} 1 & -2 & -3 & 2 \\ 0 & -1 & 7 & 0 \\ 0 & -2 & 14 & -4 \end{pmatrix} \quad \begin{array}{l} R_2 \rightarrow R_2 - 2R_1 \\ R_3 \rightarrow R_3 - 3R_1 \end{array}$$

$$\sim \begin{pmatrix} 1 & -2 & -3 & 2 \\ 0 & -1 & 7 & 0 \\ 0 & 0 & 0 & -4 \end{pmatrix} \quad R_3 \rightarrow R_3 - 2R_1$$

$$\therefore \rho(A) = 3$$

3) Find the inverse of each of the following by Gauss Jordan method

i) $A = \begin{pmatrix} 2 & -1 \\ 5 & -2 \end{pmatrix}$

Soln
 $[A | I_2] = \left[\begin{array}{cc|cc} 2 & -1 & 1 & 0 \\ 5 & -2 & 0 & 1 \end{array} \right] \xrightarrow{R_1 \rightarrow \frac{R_1}{2}} \left[\begin{array}{cc|cc} 1 & -1/2 & 1/2 & 0 \\ 5 & -2 & 0 & 1 \end{array} \right]$

$$\xrightarrow{R_2 \rightarrow R_2 - 5R_1} \left[\begin{array}{cc|cc} 1 & -1/2 & 1/2 & 0 \\ 0 & 1/2 & -5/2 & 1 \end{array} \right] \xrightarrow{R_2 \rightarrow 2R_2} \left[\begin{array}{cc|cc} 1 & -1/2 & 1/2 & 0 \\ 0 & 1 & -5 & 2 \end{array} \right]$$

$$\xrightarrow{R_1 \rightarrow R_1 + \frac{1}{2}R_2} \left[\begin{array}{cc|cc} 1 & 0 & -2 & 1 \\ 0 & 1 & -5 & 2 \end{array} \right]$$

inverse of matrix A is $\begin{pmatrix} -2 & 1 \\ -5 & 2 \end{pmatrix}$

ii) $A = \begin{pmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 6 & -2 & -3 \end{pmatrix}$

Soln
 $[A | I_3] = \left(\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 1 & 0 \\ 6 & -2 & -3 & 0 & 0 & 1 \end{array} \right)$

(15)

$$\begin{array}{l} R_2 \rightarrow R_2 - R_1 \\ R_3 \rightarrow R_3 - 6R_1 \end{array} \rightarrow \left(\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 4 & -3 & -6 & 0 & 1 \end{array} \right)$$

$$R_3 \rightarrow R_3 - 4R_2 \rightarrow \left(\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & -2 & -4 & 1 \end{array} \right)$$

$$R_2 \rightarrow R_2 + R_3 \rightarrow \left(\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -3 & -3 & 1 \\ 0 & 0 & 1 & -2 & -4 & 1 \end{array} \right)$$

$$R_1 \rightarrow R_1 + R_2 \rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & -3 & 1 \\ 0 & 1 & 0 & -3 & -3 & 1 \\ 0 & 0 & 1 & -2 & -4 & 1 \end{array} \right)$$

inverse of A is $\left(\begin{array}{ccc} -2 & -3 & 1 \\ -3 & -3 & 1 \\ -2 & -4 & 1 \end{array} \right)$

ii)

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{pmatrix}$$

Soln

$$[A | I_3] = \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 5 & 3 & 0 & 1 & 0 \\ 1 & 0 & 8 & 0 & 0 & 1 \end{array} \right)$$

$$R_2 \rightarrow R_2 - 2R_1$$

$$R_3 \rightarrow R_3 - R_1$$

$$\rightarrow \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & -2 & 5 & -1 & 0 & 1 \end{array} \right)$$

$$R_3 \rightarrow R_3 + 2R_2$$

$$\rightarrow \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array} \right)$$

$$R_2 \rightarrow R_2 + 3R_3$$

$$\rightarrow \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & -5 & -3 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array} \right)$$

$$R_1 \rightarrow R_1 + R_2$$

$$\rightarrow \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 0 & 13 & -5 & -3 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array} \right)$$

$$R_1 \rightarrow R_1 - 2R_2$$

$$\rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 3 & -25 & 10 & 6 \\ 0 & 1 & 0 & 13 & -5 & -3 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array} \right)$$

$$R_1 \rightarrow R_1 - 3R_3$$

$$\rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -40 & 16 & 9 \\ 0 & 1 & 0 & 13 & -5 & -3 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array} \right)$$

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(16)

Inverse of matrix is $\begin{pmatrix} -40 & 16 & 9 \\ 13 & -5 & 3 \\ 5 & -2 & -1 \end{pmatrix}$

Exercise 1.3

1. Solve the following system of linear equation by matrix inversion method.

(i) $2x + 5y = -2$, $x + 2y = -3$

Soln $\begin{pmatrix} 2 & 5 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ -3 \end{pmatrix}$

$$AX = B$$

$$X = A^{-1}B$$

$$\therefore A = \begin{pmatrix} 2 & 5 \\ 1 & 2 \end{pmatrix}$$

$$|A| = 4 - 5 = -1$$

$$\text{adj } A = \begin{pmatrix} 2 & -5 \\ -1 & 2 \end{pmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{-1} \begin{pmatrix} 2 & -5 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} -2 & 5 \\ 1 & -2 \end{pmatrix}$$

$$X = A^{-1}B = \begin{pmatrix} -2 & 5 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} -2 \\ -3 \end{pmatrix} = \begin{pmatrix} 4 - 15 \\ -2 + 6 \end{pmatrix} = \begin{pmatrix} -11 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -11 \\ 4 \end{pmatrix}$$

$$\therefore \boxed{\begin{matrix} x = -11 \\ y = 4 \end{matrix}}$$

(ii) $2x - y = 8$, $3x - 2y = -2$

Soln $\begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ -2 \end{pmatrix}$ Hence $A = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix}$

$$AX = B$$

$$X = A^{-1}B$$

$$|A| = -4 + 3 = -1$$

$$\text{adj } A = \begin{pmatrix} -2 & 1 \\ -3 & 2 \end{pmatrix}$$

$$X = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} \begin{pmatrix} 8 \\ -2 \end{pmatrix}$$

$$= \begin{pmatrix} 16 + 2 \\ 24 + 4 \end{pmatrix} = \begin{pmatrix} 18 \\ 28 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 18 \\ 28 \end{pmatrix}$$

$$\boxed{\begin{matrix} x = 18 \\ y = 28 \end{matrix}}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{-1} \begin{pmatrix} -2 & 1 \\ -3 & 2 \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix}$$

(17)

(ii)

$$2x + 3y - z = 9, \quad x + y + z = 9, \quad 3x - y - z = -1$$

Soln

$$\begin{pmatrix} 2 & 3 & -1 \\ 1 & 1 & 1 \\ 3 & -1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 9 \\ 9 \\ -1 \end{pmatrix}$$

$$A X = B \quad \text{where } A = \begin{pmatrix} 2 & 3 & -1 \\ 1 & 1 & 1 \\ 3 & -1 & -1 \end{pmatrix}$$

$$X = A^{-1} B$$

$$|A| = 2(-1+1) - 3(-1-3) - 1(-1-3) = 0 + 12 + 4 = 16$$

$$\text{adj } A = \begin{pmatrix} -1+1 & 1+3 & 3+1 \\ 3+1 & -2+3 & -1-2 \\ -1-3 & 9+2 & 2-3 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 4 & 4 \\ 4 & 1 & -3 \\ -4 & 11 & -1 \end{pmatrix}$$

$$\begin{array}{c} \begin{array}{ccccccc} 2 & 3 & -1 & 2 & 3 \\ 1 & 1 & 1 & 1 & 1 \\ 3 & -1 & -1 & 3 & -1 \\ 2 & 3 & -1 & 2 & 3 \\ 1 & 1 & 1 & 1 & 1 \end{array} \end{array}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{16} \begin{pmatrix} 0 & 4 & 4 \\ 4 & 1 & -3 \\ -4 & 11 & -1 \end{pmatrix}$$

$$X = A^{-1} B = \frac{1}{16} \begin{pmatrix} 0 & 4 & 4 \\ 4 & 1 & -3 \\ -4 & 11 & -1 \end{pmatrix} \begin{pmatrix} 9 \\ 9 \\ -1 \end{pmatrix} = \frac{1}{16} \begin{pmatrix} 36-4 \\ 36+9+3 \\ -36+99+1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{16} \begin{pmatrix} 32 \\ 48 \\ 64 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$$

$$\boxed{\begin{array}{l} x = 2 \\ y = 3 \\ z = 4 \end{array}}$$

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(iii)

$$x + y + z - 2 = 0, \quad 6x - 4y + 5z - 31 = 0, \quad 5x + 2y + 2z = 13.$$

Soln

$$\begin{pmatrix} 1 & 1 & 1 \\ 6 & -4 & 5 \\ 5 & 2 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 31 \\ 13 \end{pmatrix}$$

$$A X = B$$

$$X = A^{-1} B.$$

$$\text{where } A = \begin{pmatrix} 1 & 1 & 1 \\ 6 & -4 & 5 \\ 5 & 2 & 2 \end{pmatrix}$$

$$|A| = 1(-8-10) - 1(12-25) + 1(12+20)$$

$$= -18 + 13 + 32 = 27.$$

$$\text{adj} A = \begin{pmatrix} -8-10 & 2-2 & 5+4 \\ 25-12 & 2-5 & 6-5 \\ 12+20 & 5-2 & -4-6 \end{pmatrix}$$

$$= \begin{pmatrix} -18 & 0 & 9 \\ 13 & -3 & 1 \\ 32 & 3 & -10 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 4 & -4 & 5 & 6 & -4 \\ 5 & 2 & 2 & 5 & 2 \\ 1 & 1 & 1 & 1 & 1 \\ 2 & -4 & 5 & 6 & -4 \end{pmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj} A = \frac{1}{27} \begin{pmatrix} -18 & 0 & 9 \\ 13 & -3 & 1 \\ 32 & 3 & -10 \end{pmatrix}$$

$$X = A^{-1}B = \frac{1}{27} \begin{pmatrix} -18 & 0 & 9 \\ 13 & -3 & 1 \\ 32 & 3 & -10 \end{pmatrix} \begin{pmatrix} 2 \\ 31 \\ 13 \end{pmatrix} = \frac{1}{27} \begin{pmatrix} -36+0+117 \\ 26-93+13 \\ 64+93-130 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{27} \begin{pmatrix} 81 \\ -54 \\ 27 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$$

$$\begin{cases} x=3 \\ y=-2 \\ z=1 \end{cases}$$

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2.

If $A = \begin{pmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{pmatrix}$ find the products

AB and BA and hence solve the system of equations

$$x+y+2z=1, \quad 3x+2y+z=7, \quad 2x+y+3z=2.$$

Soln

$$AB = \begin{pmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{pmatrix} = \begin{pmatrix} -5+3+6 & -5+2+3 & -10+1+9 \\ 7+3-10 & 7+2-5 & 14+1-15 \\ 1-3+2 & 1-2+1 & 2-1+3 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix} = 4 I_3$$

$$BA = \begin{pmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{pmatrix} \begin{pmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{pmatrix} = \begin{pmatrix} -5+7+2 & 1+1-2 & 3-5+2 \\ -15+14+1 & 3+2-1 & 9-10+1 \\ -10+7+3 & 2+1-3 & 6-5+3 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix} = 4 I_3.$$

$$AB = BA = 4 I_3$$

$$\left(\frac{1}{4}A\right)B = B\left(\frac{1}{4}A\right) = I \Rightarrow B^{-1} = \frac{1}{4}A.$$

matrix form $\begin{pmatrix} 1 & 2 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 7 \\ 2 \end{pmatrix}$

$A \cdot X = B$

(19)

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{4} A \begin{pmatrix} 1 \\ 7 \\ 2 \end{pmatrix} = \frac{1}{4} \begin{pmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 7 \\ 2 \end{pmatrix}$$

$$= \frac{1}{4} \begin{pmatrix} -5+7+6 \\ 7+7-10 \\ 1-7+2 \end{pmatrix} = \frac{1}{4} \begin{pmatrix} 8 \\ 4 \\ -4 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$$

$$\boxed{x=2, y=2, z=-1}$$

3

A man is appointed in a job with a monthly salary of certain amount and a fixed amount of annual increment. If his salary was ₹ 19800 per month at the end of the first month after 3 years of service and ₹ 23400 per month at the end of the first month after 9 years of service. find his starting salary and his annual increment.

Soln Let the starting month salary be ₹ x and increment be ₹ y .

after 3 years salary will be

$$x + 3y = 19800 \quad \text{--- (1)}$$

after 9 years salary will be

$$x + 9y = 23400 \quad \text{--- (2)}$$

$$\text{(2)} - \text{(1)} \quad 6y = 3600$$

$$y = 600$$

$$\text{(1)} \Rightarrow x + 1800 = 19800$$

$$x = 19800 - 1800 = 18000$$

$$x = 18000$$

Starting month salary = 18000
monthly increment = 6000

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- 4) 4 men and 4 women can finish a piece of work jointly in 3 days while 2 men and 5 women can finish the same work jointly in 4 days. Find the time taken by one man alone and that of one woman alone to finish the same work by using matrix inversion method?

Soln.

$$4x + 4y = \frac{1}{3}$$

$$2x + 5y = \frac{1}{4}$$

matrix form $\begin{pmatrix} 4 & 4 \\ 2 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1/3 \\ 1/4 \end{pmatrix}$
 $AX = B$ where $A = \begin{pmatrix} 4 & 4 \\ 2 & 5 \end{pmatrix}$
 $X = A^{-1}B$

$$|A| = 20 - 8 = 12, \quad \text{adj } A = \begin{pmatrix} 5 & -4 \\ -2 & 4 \end{pmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{12} \begin{pmatrix} 5 & -4 \\ -2 & 4 \end{pmatrix}$$

$$X = \frac{1}{12} \begin{pmatrix} 5 & -4 \\ -2 & 4 \end{pmatrix} \begin{pmatrix} 1/3 \\ 1/4 \end{pmatrix} = \frac{1}{12} \begin{pmatrix} 5/3 - 1 \\ -2/3 + 1 \end{pmatrix} = \frac{1}{12} \begin{pmatrix} 2/3 \\ 1/3 \end{pmatrix} = \begin{pmatrix} 1/18 \\ 1/36 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1/18 \\ 1/36 \end{pmatrix}$$

$\therefore$ one man can do 18 days
 one woman can do 36 days.

- 5) The prices of 3 commodities A, B and C are ₹ x, y & z per units respectively. A person P purchases 4 units of B and sells two units of A and 5 units of C. Person Q purchases 2 units of C and sells 3 units of A & one unit of B. In the process, P, Q & R earn ₹ 15000, ₹ 1000 and ₹ 4000 respectively. Find the prices per unit of A, B & C.

Soln let x, y, z are commodities of A, B & C.

$$2x + 4y + 5z = 15000 \quad \text{--- (1)}$$

$$3x + y + 2z = 1000 \quad \text{--- (2)}$$

$$x + 3y + z = 4000 \quad \text{--- (3)}$$

matrix form $\begin{pmatrix} 2 & 4 & 5 \\ 3 & 1 & 2 \\ 1 & 3 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 15000 \\ 1000 \\ 4000 \end{pmatrix}$

$$AX = B$$

$$X = A^{-1}B$$

$$|A| = 2(1-6) - 4(3-2) + 5(9-1)$$

$$= 2(-5) - 4(1) + 5(8) = -10 - 4 + 40 = 26$$

(21)

$$\text{adj } A = \begin{pmatrix} 1-6 & 15-4 & 8-5 \\ 2-3 & 2-5 & 15-4 \\ 9-1 & 4-6 & 2-12 \end{pmatrix}$$

$$= \begin{pmatrix} -5 & 11 & 3 \\ -1 & -3 & 11 \\ 8 & -2 & -10 \end{pmatrix}$$

$$\begin{array}{c} 2 \quad 4 \quad 5 \quad 2 \quad 4 \\ 3 \quad 1 \quad 2 \quad 3 \quad 1 \\ 1 \quad 3 \quad 1 \quad 1 \quad 3 \\ 2 \quad 4 \quad 5 \quad 2 \quad 4 \\ 3 \quad 1 \quad 2 \quad 3 \quad 1 \end{array}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{26} \begin{pmatrix} -5 & 11 & 3 \\ -1 & -3 & 11 \\ 8 & -2 & -10 \end{pmatrix}$$

$$X = A^{-1}B = \frac{1}{26} \begin{pmatrix} -5 & 11 & 3 \\ -1 & -3 & 11 \\ 8 & -2 & -10 \end{pmatrix} \begin{pmatrix} 15000 \\ 1000 \\ 4000 \end{pmatrix}$$

$$= \frac{1}{26} \begin{pmatrix} -75000 + 11000 + 12000 \\ -15000 - 3000 + 44000 \\ 120000 - 2000 + 40000 \end{pmatrix} = \frac{1}{26} \begin{pmatrix} 52000 \\ 26000 \\ 78000 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2000 \\ 1000 \\ 3000 \end{pmatrix}$$

$$\boxed{\begin{array}{l} x = 2000 \\ y = 1000 \\ z = 3000 \end{array}}$$

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75
23
52
47
21

(22)

Exercise 1.4.

1. Solve the following systems of linear equations by Cramer's rule.

i) $5x - 2y + 16 = 0 \quad x + 3y - 7 = 0 \Rightarrow 5x - 2y = -16$
 $x + 3y = 7$

$$\Delta = \begin{vmatrix} 5 & -2 \\ 1 & 3 \end{vmatrix} = 15 + 2 = 17$$

$$\Delta x = \begin{vmatrix} -16 & -2 \\ 7 & 3 \end{vmatrix} = -48 + 14 = -34$$

$$\Delta y = \begin{vmatrix} 5 & -16 \\ 1 & 7 \end{vmatrix} = 35 + 16 = 51$$

by Cramer's rule $x = \frac{\Delta x}{\Delta} = \frac{-34}{17} = -2$

$$y = \frac{\Delta y}{\Delta} = \frac{51}{17} = 3$$

$$\therefore \boxed{x = -2, y = 3}$$

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(ii)

$$\frac{3}{x} + 2y = 12, \quad \frac{2}{x} + 3y = 13.$$

Soln $\frac{3}{x} + 2y = 12$ — (1)
 $2y = 12 - \frac{3}{x}$
 $y = \frac{1}{2} \left(12 - \frac{3}{x} \right)$

$\frac{2}{x} + 3y = 13$ — (2)
 $\frac{2}{x} + 3 \left(\frac{1}{2} \right) \left(12 - \frac{3}{x} \right) = 13.$

$$\therefore \frac{2}{x} + \frac{3}{2} \left(12 - \frac{3}{x} \right) = 13.$$

$$\frac{2}{x} + 18 - \frac{9}{2x} = 13 \Rightarrow \frac{4-9}{2x} = 13 - 18$$

$$\frac{-5}{2x} = -5$$

$$\frac{1}{2x} = 1 \Rightarrow 2x = 1 \Rightarrow x = \frac{1}{2}.$$

(1) $\frac{3}{x} + 2y = 12$
 $6 + 2y = 12$
 $2y = 6$
 $y = 3$

$$\therefore \boxed{x = \frac{1}{2}, y = 3}$$

(11) $3x + 3y - z = 11$, $2x - y + 2z = 9$, $4x + 3y + 2z = 25$

Soln $D = \begin{vmatrix} 3 & 3 & -1 \\ 2 & -1 & 2 \\ 4 & 3 & 2 \end{vmatrix} = 3(-2-4) - 3(4-8) - 1(6+4)$
 $= 3(-8) - 3(-4) - 1(10) = -24 + 12 - 10$
 $= -22$

$\Delta_x = \begin{vmatrix} 11 & 3 & -1 \\ 9 & -1 & 2 \\ 25 & 3 & 2 \end{vmatrix} = 11(-2-6) - 3(18-50) - 1(27+25)$
 $= 11(-8) - 3(-32) - 1(52)$
 $= -88 + 96 - 52 = -44$

$\Delta_y = \begin{vmatrix} 3 & 11 & -1 \\ 2 & 9 & 2 \\ 4 & 25 & 2 \end{vmatrix} = 3(18-50) - 11(4-8) - 1(50-36)$
 $= 3(-32) - 11(-4) - 1(14)$
 $= -96 + 44 - 14 = -66$

$\Delta_z = \begin{vmatrix} 3 & 3 & 11 \\ 2 & -1 & 9 \\ 4 & 3 & 25 \end{vmatrix} = 3(-25-27) - 3(50-36) + 11(6+4)$
 $= 3(-52) - 3(14) + 11(10) = -156 - 42 + 110 = -88$

by Cramer's rule $x = \frac{\Delta_x}{D} = \frac{-44}{-22} = 2$

$y = \frac{\Delta_y}{D} = \frac{-66}{-22} = 3$

$z = \frac{\Delta_z}{D} = \frac{-88}{-22} = 4$

$\therefore x = 2$
 $y = 3$
 $z = 4$

(12) $\frac{3}{x} - \frac{4}{y} - \frac{2}{z} = 1$, $\frac{1}{x} + \frac{2}{y} + \frac{1}{z} - 2 = 0$, $\frac{2}{x} - \frac{5}{y} - \frac{4}{z} + 1 = 0$.

Soln Put $a = \frac{1}{x}$, $b = \frac{1}{y}$, $c = \frac{1}{z}$.

$3a - 4b - 2c = 1$ — (1)

$a + 2b + c = 2$ — (2)

$2a - 5b - 4c = -1$ — (3)

$D = \begin{vmatrix} 3 & -4 & -2 \\ 1 & 2 & 1 \\ 2 & -5 & -4 \end{vmatrix} = 3(-8+5) + 4(-4-2) - 2(-5-4)$
 $= -9 - 24 + 18 = -15$

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$$\Delta_a = \begin{vmatrix} 1 & -4 & -2 \\ 2 & 2 & 1 \\ -1 & -5 & -4 \end{vmatrix} = 1(-8+5) + 4(-8+1) - 2(-10+2) \quad (24)$$

$$= -3 - 28 + 16 = -15$$

$$\Delta_b = \begin{vmatrix} 3 & 1 & -2 \\ 1 & 2 & 1 \\ 2 & -1 & -4 \end{vmatrix} = 3(-8+1) - 1(-4-2) - 2(-1-4)$$

$$= 3(-7) - 1(-6) - 2(-5)$$

$$= -21 + 6 + 10 = -5$$

$$\Delta_c = \begin{vmatrix} 3 & -4 & 1 \\ 1 & 2 & 2 \\ 2 & -5 & -1 \end{vmatrix} = 3(-2+10) + 4(-1-4) + 1(-5-4)$$

$$= 24 - 20 - 9 = -5$$

$$a = \frac{\Delta_a}{\Delta} = \frac{-15}{-15} = 1$$

$$b = \frac{\Delta_b}{\Delta} = \frac{-5}{-15} = \frac{1}{3}$$

$$c = \frac{\Delta_c}{\Delta} = \frac{-5}{-15} = \frac{1}{3}$$

$$\begin{cases} x = 1 \\ y = 3 \\ z = 3 \end{cases}$$

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- 2) In a Competitive examination, one mark is awarded for every correct answer while $\frac{1}{4}$ mark is deducted for every wrong answer. A student answered 100 questions and got 80 marks. How many questions did he answer correctly?

Soln. Total number of questions 100.
Let the correct questions be x
and the wrong questions be y
 $x + y = 100$ — (1)

$$x - \frac{1}{4}y = 80 \quad \text{--- (2)}$$

$$\Delta = \begin{vmatrix} 1 & 1 \\ 1 & -\frac{1}{4} \end{vmatrix} = -\frac{1}{4} - 1 = -\frac{5}{4}$$

$$\Delta_x = \begin{vmatrix} 100 & 1 \\ 80 & -\frac{1}{4} \end{vmatrix} = -25 - 80 = -105$$

$$\Delta_y = \begin{vmatrix} 1 & 100 \\ 1 & 80 \end{vmatrix} = 80 - 100 = -20$$

$$x = \frac{\Delta_x}{\Delta} = \frac{-105}{-\frac{5}{4}} = 21 \times 4 = 84$$

$$y = \frac{\Delta_y}{\Delta} = \frac{-20}{-\frac{5}{4}} = 4 \times 4 = 16$$

$$\begin{array}{r} 100 \\ -20 \\ \hline 80 \\ 80 \times \frac{1}{4} \\ \hline 20 \\ 80 - 20 \\ \hline 60 \end{array}$$

corrected questions = 84.

wrong questions = 16.

(25)

3)

A chemist has one solution which is 50% acid and another solution which is 25% acid. How much each should be mixed to make 10 litres of a 40% acid solution?

Soln let two solutions be x & y .

$$x + y = 10 \quad \text{--- (1)}$$

$$0.25x + (0.50)y = (0.40) \times 10 \quad \text{--- (2)}$$

$$\textcircled{2} \times 100 \quad 25x + 50y = 400$$

$$\textcircled{2} \div 5 \quad 5x + 10y = 80 \quad \text{--- (3)}$$

$$\Delta = \begin{vmatrix} 1 & 1 \\ 5 & 10 \end{vmatrix} = 10 - 5 = 5$$

$$\Delta x = \begin{vmatrix} 40 & 1 \\ 80 & 10 \end{vmatrix} = 100 - 80 = 20$$

$$\Delta y = \begin{vmatrix} 1 & 10 \\ 5 & 80 \end{vmatrix} = 80 - 50 = 30$$

$$x = \frac{\Delta x}{\Delta} = \frac{20}{5} = 4 \text{ litres of } 25\% \text{ solution}$$

$$y = \frac{\Delta y}{\Delta} = \frac{30}{5} = 6 \text{ litres of } 50\% \text{ solution}$$

$$\boxed{\begin{matrix} x=4 \\ y=6 \end{matrix}}$$

4)

An amount of 65000 is invested in three bonds at the rate of 6%, 8% and 10% per annum respectively. The total annual income is ₹ 4800. The income from the third bond is ₹ 600 more than that from the second bond. determine the price of each bond.

Soln

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- 4) A fish tank can be filled in 10 min using both pumps A & B simultaneously pump B can pump water in or out at the same rate. If pump B is inadvertently run in reverse then the tank will be filled in 30 min. How long would it take each pump to fill the tank by itself?

Soln Pump A fills $(\frac{1}{20})^{th}$ of the tank in 1 hour

Pump B fills $(\frac{1}{30})^{th}$ of the tank in 1 hour.

both can filled $(\frac{1}{10})^{th}$ of the tank in 1 hour

$$\frac{1}{20} + \frac{1}{y} = \frac{1}{10}$$

Pump B filled in 30 min.

$$\frac{1}{20} + \frac{1}{30} = \frac{1}{10}$$

$$\frac{1}{20} = \frac{1}{10} - \frac{1}{30} = \frac{3-1}{30} = \frac{2}{30} = \frac{1}{15} \quad (\text{Pump A})$$

$$\frac{1}{y} = \frac{1}{30} \quad (\text{Pump B})$$

$\therefore$ Pump A takes 15 min

Pump B takes 30 min to fill the tank.

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- 5) A family of 3 people went out for dinner in a restaurant. The cost of two dosai, three idlies & two vadas is ₹150, The cost of the two dosai, two idlies and four vadas is ₹200. Cost of five dosai, four idlies and two vadas is ₹250. The family has ₹350 in hand and ate 3 dosai and six idlies and six vadas will they be able to manage to pay the bill within amount they had?

Soln let dosai, idlies & vadas be x, y, z (27) -

$$2x + 3y + 2z = 150$$

$$2x + 2y + 4z = 200$$

$$5x + 4y + 2z = 250$$

$$\Delta = \begin{vmatrix} 2 & 3 & 2 \\ 2 & 2 & 4 \\ 5 & 4 & 2 \end{vmatrix} = 2(4-16) - 3(4-20) + 2(8-10)$$

$$= 2(-12) - 3(-16) + 2(-2) = -24 + 48 - 4 = 20$$

$$\Delta x = \begin{vmatrix} 150 & 3 & 2 \\ 200 & 2 & 4 \\ 250 & 4 & 2 \end{vmatrix} = 150(4-16) - 3(400-1000) + 2(800-500)$$

$$= 150(-12) - 3(-600) + 2(300)$$

$$= -1800 + 1800 + 600 = 600$$

$$\Delta y = \begin{vmatrix} 2 & 150 & 2 \\ 2 & 200 & 4 \\ 5 & 250 & 2 \end{vmatrix} = 2(400-1000) - 150(4-20) + 2(500-1000)$$

$$= 2(-600) - 150(-16) + 2(-500)$$

$$= -1200 + 2400 - 1000 = 200$$

$$\Delta z = \begin{vmatrix} 2 & 3 & 150 \\ 2 & 2 & 200 \\ 5 & 4 & 250 \end{vmatrix} = 2(500-800) - 3(500-1000) + 150(8-10)$$

$$= 2(-300) - 3(-500) + 150(-2)$$

$$= -600 + 1500 - 300 = 600$$

$$x = \frac{\Delta x}{\Delta} = \frac{600}{20} = 30; \quad y = \frac{\Delta y}{\Delta} = \frac{200}{20} = 10; \quad z = \frac{\Delta z}{\Delta} = \frac{600}{20} = 30$$

$$x = 30, \quad y = 10, \quad z = 30.$$

They ate 3 dosai, 6 idlies & 6 vadas

$$3x + 6y + 6z = 3(30) + 6(10) + 6(30)$$

$$= 90 + 60 + 180 = 330$$

They had 350 spend 330.

They can eat within the amount

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Exercise 1.5

(28)

1. Solve the following systems of linear equations by Gaussian elimination method

(i) $2x - 2y + 3z = 2, \quad x + 2y - z = 3, \quad 3x - y + 2z = 1$

Soln Augmented matrix

$$[A|B] = \left[\begin{array}{ccc|c} 2 & -2 & 3 & 2 \\ 1 & 2 & -1 & 3 \\ 3 & -1 & 2 & 1 \end{array} \right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 2 & -2 & 3 & 2 \\ 3 & -1 & 2 & 1 \end{array} \right]$$

$$\begin{array}{l} R_2 \rightarrow R_2 - 2R_1 \\ R_3 \rightarrow R_3 - 3R_1 \end{array} \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & -6 & 5 & -4 \\ 0 & -7 & 5 & -8 \end{array} \right] \xrightarrow{C_1 \leftrightarrow C_3} \left[\begin{array}{ccc|c} 1 & -1 & 2 & 3 \\ 0 & 5 & -6 & -4 \\ 0 & 5 & -7 & -8 \end{array} \right]$$

$$R_3 \rightarrow R_3 - 5R_2 \rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 2 & 3 \\ 0 & 5 & -6 & -4 \\ 0 & 0 & -1 & -4 \end{array} \right]$$

Writing the equivalent equations from echelon form.

$$\begin{array}{l} -z = -4 \\ z = 4 \end{array} \quad \begin{array}{l} 5y - 6z = -4 \\ 5y - 24 = -4 \\ 5y = -4 + 24 \\ 5y = 20 \\ y = 4 \end{array} \quad \begin{array}{l} x - y + 2z = 3 \\ x - 4 + 8 = 3 \\ x = 3 + 4 - 8 \\ x = -1 \end{array}$$

$$\therefore x = -1, y = 4, z = 4$$

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(ii) $2x + 4y + 6z = 22, \quad 3x + 8y + 5z = 27, \quad -x + y + 2z = 2$

Soln Augmented matrix

$$[A|B] = \left[\begin{array}{ccc|c} 2 & 4 & 6 & 22 \\ 3 & 8 & 5 & 27 \\ -1 & 1 & 2 & 2 \end{array} \right] \xrightarrow{R_1 \leftrightarrow \frac{R_1}{2}} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 11 \\ 3 & 8 & 5 & 27 \\ -1 & 1 & 2 & 2 \end{array} \right]$$

$$\begin{array}{l} R_2 \rightarrow R_2 - 3R_1 \\ R_3 \rightarrow R_3 + R_1 \end{array} \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 11 \\ 0 & 2 & -4 & -6 \\ 0 & 3 & 5 & 13 \end{array} \right] \xrightarrow{R_2 \rightarrow \frac{R_2}{2}} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 11 \\ 0 & 1 & -2 & -3 \\ 0 & 3 & 5 & 13 \end{array} \right]$$

$$R_3 \rightarrow R_3 - 3R_2 \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 11 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 11 & 22 \end{array} \right]$$

Writing the equivalent equations from echelon form

(29)

$$11z = 22$$

$$z = 2$$

$$y - 2z = -3$$

$$y - 4 = -3$$

$$y = -3 + 4$$

$$y = 1$$

$$x + 2y + 3z = 11$$

$$x + 2(1) + 3(2) = 11$$

$$x + 2 + 6 = 11$$

$$x = 11 - 8 = 3$$

$$x = 3$$

$$\therefore \begin{cases} x = 3 \\ y = 1 \\ z = 2 \end{cases}$$

- 2) If $ax^2 + bx + c$ is divided by $x+3$, $x-5$, and $x-1$. The remainders are 21, 61 & 9 respectively. Find a , b & c .

Soln

$$f(x) = ax^2 + bx + c$$

$$f(-3) \Rightarrow 9a - 3b + c = 21 \quad \text{--- (1) } (x+3=0 \Rightarrow x=-3)$$

$$f(5) \Rightarrow 25a + 5b + c = 61 \quad \text{--- (2) } (x-5=0 \Rightarrow x=5)$$

$$f(1) \Rightarrow a + b + c = 9 \quad \text{--- (3) } (x-1=0 \Rightarrow x=1)$$

$$\text{matrix form } \begin{pmatrix} 9 & -3 & 1 \\ 25 & 5 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 21 \\ 61 \\ 9 \end{pmatrix}$$

$$A \times = B$$

$$(A/B) = \begin{pmatrix} 9 & -3 & 1 & 21 \\ 25 & 5 & 1 & 61 \\ 1 & 1 & 1 & 9 \end{pmatrix} \xrightarrow{R_1 \leftrightarrow R_3} \begin{pmatrix} 1 & -3 & 9 & 21 \\ 25 & 5 & 1 & 61 \\ 9 & -3 & 1 & 21 \end{pmatrix}$$

$$\begin{array}{l} R_2 \rightarrow R_2 - 25R_1 \\ R_3 \rightarrow R_3 - 9R_1 \end{array} \rightarrow \begin{pmatrix} 1 & -3 & 9 & 21 \\ 0 & -8 & -24 & 40 \\ 0 & -4 & -8 & -12 \end{pmatrix}$$

$$\begin{array}{l} R_2 \rightarrow \frac{R_2}{-8} \\ R_4 \rightarrow \frac{R_4}{-4} \end{array} \rightarrow \begin{pmatrix} 1 & -3 & 9 & 21 \\ 0 & 1 & 3 & -5 \\ 0 & 1 & 2 & -3 \end{pmatrix}$$

$$R_3 \rightarrow R_3 - R_2 \rightarrow \begin{pmatrix} 1 & -3 & 9 & 21 \\ 0 & 1 & 3 & -5 \\ 0 & 0 & -4 & -8 \end{pmatrix}$$

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$$\begin{array}{l} -4c = -8 \\ c = 2 \end{array} \quad \begin{array}{l} b + 2c = 5 \\ b + 4 = 5 \\ b = 5 - 4 = 1 \end{array}$$

$$\begin{array}{l} a + b + c = 9 \\ a + 1 + 2 = 9 \\ a = 9 - 3 \\ a = 6 \end{array}$$

$$\boxed{a = 6, b = 1, c = 2}$$

- 3) An amount of ₹65000 is invested in three bonds at the rates of 6%, 8% and 10% per annum respectively. The total annual income is ₹5000. The income from the ^{first} bond is ₹600, more than that from the second bond. Determine the price of each bond.

Soln Let the amount of 3 bonds be x, y, z .

$$x + y + z = 65000 \quad \text{--- (1)}$$

$$0.06x + 0.08y + 0.10z = 5000 \quad \text{--- (2) (Total income 5000)}$$

$$0.06x - 0.08y = 600 \quad \text{--- (3) (1st bond is 600 more than 2nd bond)}$$

matrix form

$$[A|B] \sim \begin{pmatrix} 1 & 1 & 1 & 65000 \\ 0.06 & 0.08 & 0.10 & 5000 \\ 0.06 & -0.08 & 0 & 600 \end{pmatrix}$$

$$R_2 \rightarrow R_2 - 0.06R_1$$

$$R_3 \rightarrow R_3 - 0.06R_1$$

$$\begin{pmatrix} 1 & 1 & 1 & 65000 \\ 0 & 0.02 & 0.04 & 1100 \\ 0 & -0.14 & -0.06 & -3300 \end{pmatrix}$$

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$$R_2 \rightarrow \frac{R_2}{2}$$

$$R_3 \rightarrow R_3 / -3$$

$$\begin{pmatrix} 1 & 1 & 1 & 65000 \\ 0 & 0.01 & 0.02 & 550 \\ 0 & 0.07 & 0.03 & 1650 \end{pmatrix}$$

$$R_2 \rightarrow R_2 \times 100$$

$$R_3 \rightarrow R_3 \times 100$$

$$\begin{pmatrix} 1 & 1 & 1 & 65000 \\ 0 & 1 & 2 & 55000 \\ 0 & 7 & 3 & 165000 \end{pmatrix}$$

$$R_3 \rightarrow R_3 - 7R_2$$

$$\begin{pmatrix} 1 & 1 & 1 & 65000 \\ 0 & 1 & 2 & 55000 \\ 0 & 0 & -11 & -220000 \end{pmatrix}$$

$$-11Z = -220000$$

$$Z = 20000$$

$$y + 2Z = 55000$$

$$y + 40000 = 55000$$

$$y = 15000$$

$$x + y + z = 65000$$

$$x + 20000 + 15000 = 65000$$

$$x = 30000$$

$$\boxed{\begin{matrix} x = 30000 \\ y = 15000 \\ z = 20000 \end{matrix}}$$

- 4) A boy is walking along the path $y = ax^2 + bx + c$ through the points $(-6, 8)$, $(-2, 12)$, and $(3, 8)$. He wants to meet his friend at $P(7, 60)$ will he meet his friend?

Soln $y = ax^2 + bx + c$

(3)

$$\text{At } (-6, 8) \Rightarrow 8 = 36a - 6b + c \quad \text{--- (1)}$$

$$\text{At } (-2, -12) \Rightarrow -12 = 4a - 2b + c \quad \text{--- (2)}$$

$$\text{At } (3, 8) \Rightarrow 8 = 9a + 3b + c \quad \text{--- (3)}$$

Matrix form $\begin{pmatrix} 36 & -6 & 1 & 8 \\ 4 & -2 & 1 & -12 \\ 9 & 3 & 1 & 8 \end{pmatrix}$

$$[A|B] = \begin{pmatrix} 36 & -6 & 1 & 8 \\ 4 & -2 & 1 & -12 \\ 9 & 3 & 1 & 8 \end{pmatrix} \xrightarrow{\substack{R_2 \rightarrow R_2 - R_1 \\ R_3 \rightarrow R_3 - R_1}} \begin{pmatrix} 36 & -6 & 1 & 8 \\ 0 & 12 & -8 & 116 \\ 6 & -18 & -3 & -24 \end{pmatrix}$$

$$\xrightarrow{\substack{R_2 \rightarrow R_2/4 \\ R_3 \rightarrow R_3/-3}} \begin{pmatrix} 36 & -6 & 1 & 8 \\ 0 & 3 & -2 & 29 \\ 0 & 6 & 1 & 8 \end{pmatrix} \xrightarrow{R_3 \rightarrow R_3 - 2R_2} \begin{pmatrix} 36 & -6 & 1 & 8 \\ 0 & 3 & -2 & 29 \\ 0 & 0 & 5 & -50 \end{pmatrix}$$

$$\begin{array}{l|l|l} 5c = -50 & +3b - 2c = 29 & 36a - 18 - 10 = 8 \\ c = -10 & 3b - 20 = 29 & 36a = 8 + 18 + 10 \\ & 3b = 49 & 36a = 36 \\ & b = 16\frac{1}{3} & a = 1 \end{array}$$

At $P(7, 60)$

$$y = ax^2 + bx + c$$

$$60 = 1(7^2) + 3(7) - 10$$

$$60 = 49 + 21 - 10$$

$$60 = 60$$

$\therefore$ he will meet his friend at $P(7, 60)$.

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Exercise 1.6

(32)

1. Test for consistency and if possible solve the following system of equations by rank method.

(i) $x - y + 2z = 2, \quad 2x + y + 4z = 7, \quad 4x - y + z = 4.$

Soln Augment matrix. $[A | B]$

$$\begin{pmatrix} 1 & -1 & 2 \\ 2 & 1 & 4 \\ 4 & -1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 7 \\ 4 \end{pmatrix}$$

$$A X = B$$

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$$[A | B] = \left(\begin{array}{ccc|c} 1 & -1 & 2 & 2 \\ 2 & 1 & 4 & 7 \\ 4 & -1 & 1 & 4 \end{array} \right)$$

$$R_2 \rightarrow R_2 - 2R_1$$

$$R_3 \rightarrow R_3 - 4R_1$$

$$\xrightarrow{\quad} \left(\begin{array}{ccc|c} 1 & -1 & 2 & 2 \\ 0 & 3 & 0 & 3 \\ 0 & 3 & -7 & -4 \end{array} \right) \xrightarrow{R_3 \rightarrow R_3 - R_2} \left(\begin{array}{ccc|c} 1 & -1 & 2 & 2 \\ 0 & 3 & 0 & 3 \\ 0 & 0 & -7 & -7 \end{array} \right)$$

$$\rho(A) = 3 \quad \rho(A|B) = 3.$$

The system is consistent.

$$\rho(A) = \rho(A|B) = 3 = n.$$

it has unique solution.

$$\begin{array}{c|c|c} -7z = -7 & 3y = 3 & x - y + 2z = 2 \\ \hline \boxed{z = 1} & \boxed{y = 1} & x - 1 + 2 = 2 \\ & & \boxed{x = 1} \end{array}$$

$\therefore$ Solution is $x=1, y=1, z=1.$

(ii) $3x + y + z = 2, \quad x - 3y + 2z = 1, \quad 7x - y + 4z = 5$

Soln matrix form. $\begin{pmatrix} 3 & 1 & 1 \\ 1 & -3 & 2 \\ 7 & -1 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}$

$$A X = B$$

Augmented matrix $[A | B] = \left(\begin{array}{ccc|c} 3 & 1 & 1 & 2 \\ 1 & -3 & 2 & 1 \\ 7 & -1 & 4 & 5 \end{array} \right)$

(23)

$$R_1 \leftrightarrow R_2 \rightarrow \begin{pmatrix} 1 & -3 & 2 & 1 \\ 3 & 1 & 1 & 2 \\ 7 & -1 & 4 & 5 \end{pmatrix} \xrightarrow{\substack{R_2 \rightarrow R_2 - 3R_1 \\ R_3 \rightarrow R_3 - 7R_1}} \begin{pmatrix} 1 & -3 & 2 & 1 \\ 0 & 10 & -5 & -1 \\ 0 & 20 & -10 & -2 \end{pmatrix}$$

$$R_3 \rightarrow R_3 - 2R_2 \rightarrow \begin{pmatrix} 1 & -3 & 2 & 1 \\ 0 & 10 & -5 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\rho(A) = 2 \quad \rho(A:B) = 2$$

$$\rho(A) = \rho(A:B) = 2 < n.$$

The system is consistent. It has infinitely many solution.

put $x = t$

$$10y - 5z = -1$$

$$10y = -1 + 5z = 5t - 1$$

$$y = \frac{5t - 1}{10}$$

$$x - 3y + 2z = 1$$

$$x - 3\left(\frac{5t - 1}{10}\right) + 2t = 1$$

$$x = 1 - 2t + 3\left(\frac{5t - 1}{10}\right) = \frac{10 - 20t + 15t - 3}{10}$$

$$x = \frac{7 - 5t}{10}$$

$$(x, y, z) = \left(\frac{7 - 5t}{10}, \frac{5t - 1}{10}, t \right) \quad \forall t \in \mathbb{R}.$$

(iii)

$$2x + 2y + z = 5, \quad x - y + z = 1, \quad 3x + y + 2z = 4.$$

Soln matrix form $\begin{pmatrix} 2 & 2 & 1 \\ 1 & -1 & 1 \\ 3 & 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \\ 4 \end{pmatrix}$

$$AX = B.$$

Augmented matrix $[A|B] = \begin{bmatrix} 2 & 2 & 1 & 5 \\ 1 & -1 & 1 & 1 \\ 3 & 1 & 2 & 4 \end{bmatrix}$

$$R_1 \leftrightarrow R_2 \rightarrow \begin{pmatrix} 1 & -1 & 1 & 1 \\ 2 & 2 & 1 & 5 \\ 3 & 1 & 2 & 4 \end{pmatrix} \xrightarrow{\substack{R_2 \rightarrow R_2 - 2R_1 \\ R_3 \rightarrow R_3 - 3R_1}} \begin{pmatrix} 1 & -1 & 1 & 1 \\ 0 & 4 & -1 & 3 \\ 0 & 4 & -1 & 1 \end{pmatrix}$$

$$R_3 \rightarrow R_3 - R_2 \rightarrow \begin{pmatrix} 1 & -1 & 1 & 1 \\ 0 & 4 & -1 & 3 \\ 0 & 0 & 0 & -2 \end{pmatrix}$$

$$\rho(A) = 2 \quad \rho(A:B) = 3$$

$$\rho(A) \neq \rho(A:B)$$

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$\therefore$ The system is inconsistent.
it has no solution.

(10) $2x - y + z = 2, \quad 6x - 3y + 3z = 6, \quad 4x - 2y + 2z = 4.$

Soln Matrix form $[A|B] = \begin{pmatrix} 2 & -1 & 1 & 2 \\ 6 & -3 & 3 & 6 \\ 4 & -2 & 2 & 4 \end{pmatrix}$

Augmented matrix.

$R_2 \rightarrow \frac{R_2}{3}$
 $R_3 \rightarrow R_3/2 \rightarrow \begin{pmatrix} 2 & -1 & 1 & 2 \\ 2 & -1 & 1 & 2 \\ 2 & -1 & 1 & 2 \end{pmatrix} \xrightarrow{\substack{R_2 \rightarrow R_2 - R_1 \\ R_3 \rightarrow R_3 - R_1}} \begin{pmatrix} 2 & -1 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

$\rho(A) = 1 \quad \rho(A:B) = 1$

$\rho(A) = \rho(A:B) = 1 < n.$

$\therefore$ The system reduces into single equation
 $\therefore$ it is consistent and has infinitely many solution.

put $y = s \quad z = t$

$2x - y + z = 2$

$2x - s + t = 2$

$2x = 2 + s - t$

$x = \frac{2 + s - t}{2}$

$(x, y, z) = \left(\frac{2 + s - t}{2}, s, t \right) \quad \forall s, t \in \mathbb{R}.$

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- 2) Find the value of k for which the equations
 $kx - 2y + z = 1, \quad x + 2ky + z = -2, \quad x - 2y + kz = 1$ have
(i) no solution (ii) unique solution (iii) infinitely many solution.

Soln Matrix form. $\begin{pmatrix} k & -2 & 1 \\ 1 & -2k & 1 \\ 1 & -2 & k \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$
 $A \cdot X = B.$

Augmented matrix
 $[A|B] = \begin{pmatrix} k & -2 & 1 & 1 \\ 1 & -2k & 1 & -2 \\ 1 & -2 & k & 1 \end{pmatrix}$

$$\begin{aligned} & 2 - k - k^2 \\ &= -(k^2 + k - 2) \\ &= -(k+2)(k-1) \\ &= (k+2)(1-k) \end{aligned}$$

Case (i) if $\lambda = 5$ $\mu = 9$
 $\rho(A) = \rho(A:B) = 2 < n$

(36)

The system is consistent it has infinitely many solutions

Case (ii) if $\lambda \neq 5$ $\mu = 9$

$$\rho(A) = 3 \quad \rho(A:B) = 3$$

The system is consistent it has unique solution

Case (iii) if $\lambda \neq 5$ $\mu \neq 9$

$$\rho(A) = 2 \quad \rho(A:B) = 3$$

The system is inconsistent it has no solution.

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Padasalai

பாடசாலை

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Exercise 1.7

1. Solve the following system of homogeneous equations
 (i) $3x + 2y + 7z = 0$, $4x + 3y - 2z = 0$, $5x + 9y + 23z = 0$

Soln Matrix form $\begin{pmatrix} 3 & 2 & 7 \\ 4 & -3 & -2 \\ 5 & 9 & 23 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$
 $AX = B$

Augmented matrix

$$[A|B] = \begin{pmatrix} 3 & 2 & 7 & 0 \\ 4 & -3 & -2 & 0 \\ 5 & 9 & 23 & 0 \end{pmatrix}$$

$$R_2 \rightarrow 3R_2 - 4R_1$$

$$R_3 \rightarrow 3R_3 - 5R_1$$

$$\begin{pmatrix} 3 & 2 & 7 & 0 \\ 0 & -17 & -34 & 0 \\ 0 & 17 & 34 & 0 \end{pmatrix} \xrightarrow{R_3 \rightarrow R_3 + R_2} \begin{pmatrix} 3 & 2 & 7 & 0 \\ 0 & -17 & -34 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

writing into equation form.

$$-17y - 34z = 0 \quad \text{--- (1)}$$

$$3x + 2y + 7z = 0 \quad \text{--- (2)}$$

$$\text{Put } z = t \quad -17y = 34z$$

$$y = \frac{34t}{-17} = -2t$$

$$\textcircled{1} \Rightarrow 3x + 2(-2t) + 7t = 0$$

$$3x - 4t + 7t = 0$$

$$3x + 3t = 0$$

$$3x = -3t$$

$$x = -t$$

$$(x, y, z) = (-t, -2t, t) \quad \forall t \in \mathbb{R}$$

- (ii) $2x + 3y - z = 0$, $x - y - 2z = 0$, $3x + y + 3z = 0$

Soln Matrix form. $\begin{pmatrix} 2 & 3 & -1 \\ 1 & -1 & -2 \\ 3 & 1 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$
 $AX = B$

Augmented matrix

$$[A|B] = \begin{pmatrix} 2 & 3 & -1 & 0 \\ 1 & -1 & -2 & 0 \\ 3 & 1 & 3 & 0 \end{pmatrix}$$

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$$R_1 \leftrightarrow R_2 \rightarrow \begin{pmatrix} 1 & -1 & -2 & 0 \\ 2 & 3 & -1 & 0 \\ 3 & 1 & 3 & 6 \end{pmatrix} \xrightarrow{\substack{R_2 \rightarrow R_2 - 2R_1 \\ R_3 \rightarrow R_3 - 3R_1}} \begin{pmatrix} 1 & -1 & -2 & 0 \\ 0 & 5 & 3 & 0 \\ 0 & 4 & 7 & 6 \end{pmatrix}$$

$$R_3 \rightarrow 5R_3 - 4R_2 \rightarrow \begin{pmatrix} 1 & -1 & -2 & 0 \\ 0 & 5 & 3 & 0 \\ 0 & 0 & 33 & 6 \end{pmatrix}$$

Convert into equation form.

$$\begin{array}{l|l} 33z = 0 & 5y + 3z = 0 \\ z = 0 & 5y = 0 \\ & y = 0 \end{array} \quad \left| \begin{array}{l} x - y - 2z = 0 \\ x - 0 - 0 = 0 \\ x = 0 \end{array} \right.$$

$$\therefore x = 0 \quad y = 0 \quad z = 0$$

$\therefore$ It has only trivial solution.

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2) Determine the values of λ for which the following system of equations

$x + y + 3z = 0$, $4x + 3y + \lambda z = 0$, $2x + y + 2z = 0$ has
i) unique solution ii) a non trivial solution.

Soln Matrix form $\begin{pmatrix} 1 & 1 & 3 \\ 4 & 3 & \lambda \\ 2 & 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$
 $AX = B$

$$[A/B] = \begin{pmatrix} 1 & 1 & 3 & 0 \\ 4 & 3 & \lambda & 0 \\ 2 & 1 & 2 & 0 \end{pmatrix} \xrightarrow{\substack{R_2 \rightarrow R_2 - 4R_1 \\ R_3 \rightarrow R_3 - 2R_1}} \begin{pmatrix} 1 & 1 & 3 & 0 \\ 0 & -1 & \lambda - 12 & 0 \\ 0 & -1 & -4 & 0 \end{pmatrix}$$

$$R_2 \leftrightarrow R_3 \rightarrow \begin{pmatrix} 1 & 1 & 3 & 0 \\ 0 & -1 & -4 & 0 \\ 0 & -1 & \lambda - 12 & 0 \end{pmatrix} \xrightarrow{R_3 \rightarrow R_3 - R_2} \begin{pmatrix} 1 & 1 & 3 & 0 \\ 0 & -1 & -4 & 0 \\ 0 & 0 & \lambda - 8 & 0 \end{pmatrix}$$

Case i) if $\lambda = 8$

$$\rho(A) = \rho(A:B) = 2$$

The system is consistent it has infinitely many solutions

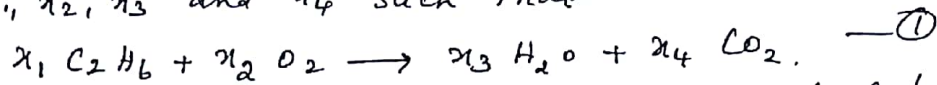
Case ii) if $\lambda \neq 8$.

$$\rho(A) = 3 \quad \rho(A:B) = 3$$

The system is consistent it has unique solution.
it has homogeneous eqn $\therefore$ it has solution $\begin{cases} x = 0 \\ y = 0 \\ z = 0 \end{cases}$

- 3) By using Gaussian elimination method, balance the chemical reaction equation (39)
- $$C_2H_6 + O_2 \rightarrow H_2O + CO_2.$$

Soln We are searching for positive integers x_1, x_2, x_3 and x_4 such that



The number of carbon atoms on the left hand side of (1) should be equal to the number of carbon atoms on the RHS of (1) so we get a linear homogenous equation

$$2x_1 = x_4 \Rightarrow 2x_1 - x_4 = 0 \quad \text{--- (2)}$$

$$6x_1 = 2x_3 \Rightarrow 6x_1 - 2x_3 = 0 \Rightarrow 3x_1 - x_3 = 0 \quad \text{--- (3)}$$

$$2x_2 = x_3 + 2x_4 \Rightarrow 2x_2 - x_3 - 2x_4 = 0 \quad \text{--- (4)}$$

eqn (2), (3) & (4) constitute a homogenous system of linear equations in four unknowns.

$$[A|B] = \begin{bmatrix} 2 & 0 & 0 & -1 & 0 \\ 3 & 0 & -1 & 0 & 0 \\ 0 & 2 & -1 & -2 & 0 \end{bmatrix}$$

By Gaussian elimination method, we get

$$[A|B] \xrightarrow{R_2 \rightarrow 2R_2 - 3R_1} \begin{pmatrix} 2 & 0 & 0 & -1 & 0 \\ 0 & 2 & -1 & -2 & 0 \\ 0 & 2 & -1 & -2 & 0 \end{pmatrix}$$

$R_2 \leftrightarrow R_3$

$$\longrightarrow \begin{pmatrix} 2 & 0 & 0 & -1 & 0 \\ 0 & 2 & -1 & -2 & 0 \\ 0 & 0 & -2 & 3 & 0 \end{pmatrix}$$

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$$\rho(A) = \rho(A|B) = 3 < 4.$$

The system is consistent and has infinitely number of solutions.

Writing the equations using the echelon form

We get $-2x_3 + 3x_4 = 0,$

$$2x_2 - x_3 - 2x_4 = 0$$

$$2x_1 - x_4 = 0.$$

So one of the unknowns should be chosen arbitrarily as a non zero real number

(40)

put $x_4 = t$.

$$\begin{array}{l|l}
 2x_1 = x_4 & -2x_3 + 3x_4 = 0 \\
 \boxed{x_1 = \frac{t}{2}} & -2x_3 = -3t \\
 & \boxed{x_3 = \frac{3}{2}t}
 \end{array}
 \quad
 \begin{array}{l}
 2x_2 - x_3 - 2x_4 = 0 \\
 2x_2 - \frac{3}{2}t - 2t = 0 \\
 2x_2 = \frac{3}{2}t + 2t = \frac{7t}{2} \\
 \boxed{x_2 = \frac{7t}{4}}
 \end{array}$$

$(x_1, x_2, x_3, x_4) = (\frac{t}{2}, \frac{7t}{4}, \frac{3t}{2}, t) \quad \forall t \in \mathbb{R}$
 let us choose $t = 4$.

$$\begin{array}{l|l}
 \text{we get } x_1 = \frac{4}{2} = 2 & x_2 = \frac{7 \cdot 4}{4} = 7 \\
 \boxed{x_1 = 2} & \boxed{x_2 = 7} \\
 x_3 = \frac{3}{2} (4) & x_4 = t \\
 \boxed{x_3 = 6} & x_4 = 4.
 \end{array}$$

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$\therefore x_1 = 2, x_2 = 7, x_3 = 6, x_4 = 4.$

So the balanced equation is



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Exercise 1.8

1) Choose the correct answer

ii) if $|\text{adj}(\text{adj} A)| = |A|^9$, then the order of the square matrix A is

$$\text{Soln } |\text{adj}(\text{adj} A)| = |A|^{(n-1)^2}$$

$$(n-1)^2 = 9 = 3^2$$

$$n-1 = 3$$

$$\boxed{n = 4}$$

Ans : (2) 4.

2) If A is a 3×3 non singular matrix such that $B = A^{-1} A^T$
 $AA^T = A^T A$ and $BB^T = ?$

$$\text{Soln } AA^T = A^T A \quad \& \quad B = A^{-1} A^T$$

$$B^T = (A^{-1})^T (A^T)^T = (A^{-1})^T A$$

$$BB^T = (A^{-1} A^T) (A^{-1})^T A = A^{-1} (A A^{-1})^T A$$

$$= A^{-1} (I)^T A = A^{-1} A = I$$

Ans (3) I 3) If $A = \begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix}$, $B = \text{adj} A$ and $C = 3A$ then $\frac{|\text{adj} B|}{|C|} = ?$

$$\text{Soln } \frac{|\text{adj} B|}{|C|} = \frac{|\text{adj}(\text{adj} A)|}{|3A|} = \frac{|A|^{n-1}}{3^n |A|} = \frac{|A|^{n-1}}{3^n |A|} = \frac{1}{9}$$

Here $n=2$ Ans (2) $\frac{1}{9}$.4) If $A \times \begin{pmatrix} 1 & -2 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}$ then $A = ?$ Soln order of A is 2×2 let A be $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 & -2 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}$$

$$\begin{array}{l|l} a+b=6 & c+d=0 \\ -2a+4b=0 & -2c+4d=6 \end{array}$$

Solve the equations we get

$$A = \begin{pmatrix} 4 & 2 \\ 1 & -1 \end{pmatrix}$$

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$$b=2, a=4, d=-1, c=-1$$

$$\text{Ans (3)} \begin{pmatrix} 4 & 2 \\ -1 & -1 \end{pmatrix}$$

5)

If $A = \begin{pmatrix} 7 & 3 \\ 4 & 2 \end{pmatrix}$ then $9I - A = ?$

(42)

Soln $|A| = 14 - 12 = 2$, $\text{adj } A = \begin{pmatrix} 2 & -3 \\ -4 & 7 \end{pmatrix}$, $A^{-1} = \frac{1}{|A|} \text{adj } A$

$$A^{-1} = \frac{1}{2} \begin{pmatrix} 2 & -3 \\ -4 & 7 \end{pmatrix} \Rightarrow 2A^{-1} = \begin{pmatrix} 2 & -3 \\ -4 & 7 \end{pmatrix} \quad \text{--- (1)}$$

$$9I - A = 9 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 7 & 3 \\ 4 & 2 \end{pmatrix} = \begin{pmatrix} 9 & 0 \\ 0 & 9 \end{pmatrix} - \begin{pmatrix} 7 & 3 \\ 4 & 2 \end{pmatrix} = \begin{pmatrix} 2 & -3 \\ -4 & 7 \end{pmatrix} \quad \text{--- (2)}$$

by (1) & (2) $9I - A = 2A^{-1}$

Ans: (4) $2A^{-1}$

6)

If $A = \begin{pmatrix} 2 & 0 \\ 1 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 4 \\ 2 & 0 \end{pmatrix}$ then $|\text{adj } (AB)| = ?$

Soln $AB = \begin{pmatrix} 2 & 0 \\ 1 & 5 \end{pmatrix} \begin{pmatrix} 1 & 4 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 8 \\ 11 & 4 \end{pmatrix}$

$$\text{adj } AB = \begin{pmatrix} 4 & -8 \\ -11 & 2 \end{pmatrix} \therefore |\text{adj } AB| = 8 - 88 = -80$$

Ans (2) -80 .

7.

If $P = \begin{pmatrix} 1 & x & 0 \\ 1 & 3 & 0 \\ 2 & 4 & -2 \end{pmatrix}$ is the adjoint of 3×3 matrix A and $|A| = 4$ then x is

$$|\text{adj } A| = |A|^{n-1}$$

$$n=3$$

$$1(-6+0) - x(-2) = 4^2$$

$$-6 + 2x = 16$$

$$2x = 16 + 6 = 22$$

$$\boxed{x = 11}$$

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Ans (4) 11

8)

If $A = \begin{pmatrix} 3 & 1 & -1 \\ 2 & -2 & 0 \\ 1 & 2 & -1 \end{pmatrix}$ and $A^{-1} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ thenthe value of a_{23} is

$$a_{23} \text{ in } A^{-1} \text{ is } - \begin{vmatrix} 3 & 1 \\ 2 & 0 \end{vmatrix} = -(0+2) = -2$$

$$|A| = 3(2) - 1(-2) - 1(4+2)$$

$$= 6 + 2 - 6 = 2$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A \quad a_{23} \text{ is } \frac{1}{2}(-2) = -1$$

Ans (4) -1

- 9) If A, B and C are invertible matrices of some order, (43)
Then which one of the following is not true.

Ans (2)

10. If $(AB)^{-1} = \begin{pmatrix} 12 & -17 \\ -19 & 27 \end{pmatrix}$ and $A^{-1} = \begin{pmatrix} 1 & -1 \\ -2 & 3 \end{pmatrix}$ then $B^{-1} = ?$

Soln

$$(AB)^{-1} = B^{-1} A^{-1}$$

$$(A^{-1})^{-1} = A$$

$$(AB)^{-1} A = B^{-1} (A^{-1}) A$$

$$|A^{-1}| = 3 - 2 = 1$$

$$\text{adj } A^{-1} = \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix}$$

$$(AB)^{-1} A = B^{-1}$$

$$A = \frac{1}{|A^{-1}|} \text{adj } A^{-1} = \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix}$$

$$B^{-1} = \begin{pmatrix} 12 & -17 \\ -19 & 27 \end{pmatrix} \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 36-34 & 12+17 \\ -57+54 & -19+27 \end{pmatrix} = \begin{pmatrix} 2 & -5 \\ -3 & 8 \end{pmatrix}$$

Ans (1) $\begin{pmatrix} 2 & -5 \\ -3 & 8 \end{pmatrix}$

- 11) If $A^T A^{-1}$ is symmetric then $A^2 = ?$

Soln

$$(A^T A^{-1})^T = A^T A^{-1}$$

$$(A^{-1})^T (A^T)^T = A^T A^{-1}$$

$$(A^{-1})^T A = A^T A^{-1}$$

$$A^T (A^{-1})^T A A = (A^T) A^T A^{-1} A$$

$$A^2 = (A^T)^2$$

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Pre & post multiply
by A^T and A

Ans (2) $(A^T)^2$

- 12) If A is non singular matrix such that $A^{-1} \begin{pmatrix} 5 & 3 \\ -2 & -1 \end{pmatrix}$
then $(A^T)^{-1} = ?$

Soln

$$A^{-1} = \begin{pmatrix} 5 & 3 \\ -2 & -1 \end{pmatrix} \quad |A^{-1}| = -5+6=1, \quad \text{adj } A^{-1} = \begin{pmatrix} -1 & -3 \\ 2 & 5 \end{pmatrix}$$

$$A = (A^{-1})^{-1} = \frac{1}{1} \begin{pmatrix} -1 & -3 \\ 2 & 5 \end{pmatrix} = \begin{pmatrix} -1 & -3 \\ 2 & 5 \end{pmatrix}$$

$$A^T = \begin{pmatrix} 5 & 3 \\ -2 & -1 \end{pmatrix}$$

$$(A^{-1})^{-1} = \frac{1}{-5+6} \begin{pmatrix} -1 & -3 \\ 2 & 5 \end{pmatrix} = \frac{1}{1} \begin{pmatrix} -1 & -3 \\ 2 & 5 \end{pmatrix} = A$$

Ans (3) $\begin{pmatrix} -1 & -3 \\ 2 & 5 \end{pmatrix}$

- 13) If $A = \begin{pmatrix} 3/5 & 4/5 \\ x & 3/5 \end{pmatrix}$ and $A^T = A^{-1}$ then the value of x is

Soln

$$A^T = A^{-1}$$

$$A A^T = I$$

$$\begin{pmatrix} 3/5 & 4/5 \\ x & 3/5 \end{pmatrix} \begin{pmatrix} 3/5 & x \\ 4/5 & 3/5 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$3/5 x + \frac{12}{25} = 0$$

$$\frac{3}{5} x = -\frac{12}{25}$$

$$x = -\frac{4}{5}$$

Ans (1) $\frac{4}{5}$.

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- 14) If $A = \begin{pmatrix} 1 & \tan \theta/2 \\ -\tan \theta/2 & 1 \end{pmatrix}$ and $AB = I$ then $B = ?$

Soln

$$AB = I \Rightarrow B = A^{-1} = \frac{1}{1 + \tan^2 \theta/2} \begin{pmatrix} 1 & -\tan \theta/2 \\ \tan \theta/2 & 1 \end{pmatrix}$$

$$= \frac{1}{\sec^2 \theta/2} \begin{pmatrix} 1 & -\tan \theta/2 \\ \tan \theta/2 & 1 \end{pmatrix} = \cos^2 \theta/2 A^T$$

Ans (2) $\cos^2 \theta/2 A^T$

- 15) If $A = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$ and $A(\text{adj } A) = \begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$ then $k = ?$

Soln

$$\text{adj } A = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$$

$$A(\text{adj } A) = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} = \begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$$

$$\cos^2 \theta + \sin^2 \theta = k$$

$$k = 1.$$

Ans (4) $k = 1$

- 16) If $A = \begin{pmatrix} 2 & 3 \\ 5 & -2 \end{pmatrix}$ be such that $\lambda A^{-1} = A$ then $\lambda = ?$

Soln

$$\lambda A^{-1} = A$$

$$\lambda I = A^2 = \begin{pmatrix} 2 & 3 \\ 5 & -2 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ 5 & -2 \end{pmatrix} = \begin{pmatrix} 4+15 & 6-6 \\ 10-10 & 15+4 \end{pmatrix}$$

$$\lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 19 & 0 \\ 0 & 19 \end{pmatrix}$$

$$\Rightarrow \lambda = 19.$$

Ans (3) 19

- 17) If $\text{adj } A = \begin{pmatrix} 2 & 3 \\ 4 & -1 \end{pmatrix}$ and $\text{adj } B = \begin{pmatrix} 1 & -2 \\ -3 & 1 \end{pmatrix}$ then $\text{adj } AB$ is 45
- Soln $\text{adj}(AB) = (\text{adj } A)(\text{adj } B) = \begin{pmatrix} 2 & 3 \\ 4 & -1 \end{pmatrix} \begin{pmatrix} 1 & -2 \\ -3 & 1 \end{pmatrix}$
 $= \begin{pmatrix} 2-9 & -4+3 \\ 4+3 & -8-1 \end{pmatrix} = \begin{pmatrix} -7 & -1 \\ 7 & -9 \end{pmatrix}$ Ans (1) $\begin{pmatrix} -7 & -1 \\ 7 & -9 \end{pmatrix}$

- 18) The rank of the matrix $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ -1 & -2 & -3 & -4 \end{pmatrix}$ is
- Soln
 $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ -1 & -2 & -3 & -4 \end{pmatrix} \xrightarrow[R_3 \rightarrow -R_3]{R_2 \rightarrow R_2 - 2R_1} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 \\ -1 & -2 & -3 & -4 \end{pmatrix} \xrightarrow[R_3 \rightarrow R_3 + R_1]{R_2 \rightarrow R_2 - R_1} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$
 $\rho(A) = 1$. Ans (1) 1

- 19) If $x^a y^b = e^m$, $x^c y^d = e^n$, $\Delta_1 = \begin{vmatrix} m & b \\ n & d \end{vmatrix}$, $\Delta_2 = \begin{vmatrix} a & m \\ c & n \end{vmatrix}$, $\Delta_3 = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$ Then the values of x and y are respectively
- Soln $x^a y^b = e^m$
 $a \log x + b \log y = m$ — (1)
 $x^c y^d = e^n$
 $c \log x + d \log y = n$ — (2)
 $\Delta_1 = \begin{vmatrix} m & b \\ n & d \end{vmatrix} = md - nb$
 $= ad \log x + bd \log y - bc \log x - bd \log y$
 $= (ad - bc) \log x$ — (1)
 $\Delta_2 = \begin{vmatrix} a & m \\ c & n \end{vmatrix} = an - cm$
 $= a c \log x + a d \log y - a c \log x - b c \log y$
 $= (ad - bc) \log y$ — (2)
 $\Delta_3 = (ad - bc)$
 $\frac{\Delta_1}{\Delta_3} = \log x$
 $e^{\Delta_1/\Delta_3} = x$
 $\frac{\Delta_2}{\Delta_3} = \log y$
 $e^{\Delta_2/\Delta_3} = y$
 Ans (4) $(e^{\Delta_1/\Delta_3}, e^{\Delta_2/\Delta_3})$

- 20) Which of the following is/are correct?
 (i) adjoint of diagonal matrix is also a diagonal matrix
 (iv) $A(\text{adj } A) = (\text{adj } A)A = |A|I$.
- Ans (4)

- 21) If $\rho(A) = \rho(A/B)$, then the system $AX = B$ of linear equation is consistent.
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- Ans (2)

- 22) If $0 \leq \theta \leq \pi$ and the system of equations
 $x + (\sin \theta) y - (\cos \theta) z = 0$, $\cos \theta x - y + z = 0$
 $(\sin \theta) x + y - z = 0$ has a non trivial solution then θ is

Soln If $AX=0$ has a non trivial solution if $|A|=0$. (46)

$$\begin{vmatrix} 1 & \sin \omega & -\cos \omega \\ \cos \omega & -1 & 1 \\ \sin \omega & 1 & -1 \end{vmatrix} = 0 \Rightarrow 1(1-1) - \sin \omega (-\cos \omega - \sin \omega) - \cos \omega (\cos \omega + \sin \omega) = 0$$

$$\Rightarrow + \sin \omega \cos \omega + \sin^2 \omega - \cos^2 \omega - \cos \omega \sin \omega = 0$$

$$\sin^2 \omega = \cos^2 \omega$$

$$\sin \omega = \cos \omega$$

$$\omega = 45^\circ$$

Ans (H) $\theta = 45^\circ$

23) The augmented matrix of a system of linear equations is $\begin{pmatrix} 1 & 2 & 7 & 3 \\ 0 & 1 & 4 & 6 \\ 0 & 0 & \lambda-7 & \mu+5 \end{pmatrix}$. The system has infinitely many solutions if

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Soln $\rho(A) = \rho(A:B) = 2$

$$\begin{aligned} \lambda-7 &= 0 & \mu+5 &= 0 \\ \lambda &= 7 & \mu &= -5 \end{aligned}$$

Ans (A) $\lambda=7 \mu=-5$

24) Let $A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$ & $AB = \begin{pmatrix} 3 & 1 & -1 \\ 1 & 3 & x \\ -1 & 1 & 3 \end{pmatrix}$ If B is the inverse of A . Then the value of x is

Soln $B = A^{-1}$

$$4A^{-1} = \begin{pmatrix} 3 & 1 & -1 \\ 1 & 3 & x \\ -1 & 1 & 3 \end{pmatrix} \Rightarrow A^{-1} = \frac{1}{4} \begin{pmatrix} 3 & 1 & -1 \\ 1 & 3 & x \\ -1 & 1 & 3 \end{pmatrix}$$

x is a minor matrix of A .

$$x = - \begin{vmatrix} 2 & 1 \\ -1 & -1 \end{vmatrix} = -(-2+1) = -(-1) = 1 \quad \text{Ans (H) 1}$$

25) If $A = \begin{pmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$ then $\text{adj}(\text{adj} A)$ is

Soln $\text{adj}(\text{adj} A) = |A|^{n-1} \cdot A = |A|^2 |A|$

$$|A| = 3(-3+4) + 3(2) + 4(-2) = 3+6-8 = 1.$$

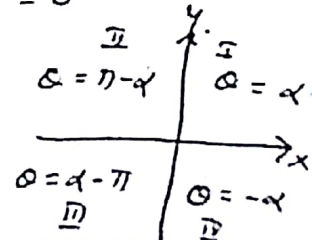
$$\text{adj}(\text{adj} A) = 1^2 \begin{pmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix} = \begin{pmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix} = A$$

Ans (1) $\begin{pmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$

CHAPTER 2: Complex Number

1. $\sqrt{-1} = i, i^2 = -1, i^3 = -i, i^4 = 1, i^{4n} = 1 \forall n \in \mathbb{N}$
2. $|z_1 + z_2| \leq |z_1| + |z_2|$ (Triangle inequality)
3. $|z_1 - z_2| \geq ||z_1| - |z_2||$
4. 1) $\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$ 2) $\overline{z_1 - z_2} = \overline{z_1} - \overline{z_2}$ 3) $\overline{z_1 z_2} = \overline{z_1} \cdot \overline{z_2}$
 4) $\left(\frac{z_1}{z_2}\right) = \frac{\overline{z_1}}{\overline{z_2}}, z_2 \neq 0$ 5) $\operatorname{Re}(z) = \frac{z + \overline{z}}{2}$ 6) $\operatorname{Im}(z) = \frac{z - \overline{z}}{2i}$
 7) $\overline{z^n} = (\overline{z})^n \forall n \in \mathbb{Z}$ 8) z is real iff $z = \overline{z}$
 9) z is purely imaginary iff $-z = \overline{z}$ 10) $\overline{\overline{z}} = z$.
- 5) 1) $|z| = |\overline{z}|$ 2) $|z_1 z_2| = |z_1| |z_2|$ 3) $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$
 4) $|z^n| = |z|^n$ 5) $\operatorname{Re}(z) \leq |z|$ 6) $\operatorname{Im}(z) \leq |z|$.
- 6) $z = x + iy \Rightarrow |z| = \sqrt{x^2 + y^2}$.
- 7) Square root of $a + ib$ is $\sqrt{a+ib} = \pm \left(\frac{\sqrt{|z|+a}}{2} + i \frac{b}{|b|} \sqrt{\frac{|z|-a}{2}} \right)$
 where $z = a + ib$, and $b \neq 0$.
- 8) 1) Polar form $z = r(\cos \theta + i \sin \theta)$
 2) $z^{-1} = \frac{1}{r} (\cos \theta - i \sin \theta)$
 3) $z_1 z_2 = r_1 r_2 [\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2)]$
 4) $\frac{z_1}{z_2} = \frac{r_1}{r_2} [\cos(\theta_1 - \theta_2) + i \sin(\theta_1 - \theta_2)]$
- 9) De Moivre's theorem: $(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta, \forall n \in \mathbb{Z}$.
- 10) n^{th} roots of a complex number $z = \cos \theta + i \sin \theta$
 $z^{1/n} = r^{1/n} \left[\cos\left(\frac{\theta + 2k\pi}{n}\right) + i \sin\left(\frac{\theta + 2k\pi}{n}\right) \right], k = 0, 1, 2, \dots, (n-1)$
- 11) 1) $1 = \cos 0 + i \sin 0$, 2) $i = \cos \frac{\pi}{2} + i \sin \frac{\pi}{2}$, 3) $-1 = \cos \pi + i \sin \pi$.
- 12) $\sin \theta + i \cos \theta = \cos(\frac{\pi}{2} - \theta) + i \sin(\frac{\pi}{2} - \theta)$
- 13) cube root of unity 1) $\omega^3 = 1$ 2) $1 + \omega + \omega^2 = 0$
 3) $\omega = \frac{-1 + i\sqrt{3}}{2}$ 4) $\omega^2 = \frac{-1 - i\sqrt{3}}{2}$.
- 14) n^{th} root of unity $1 + \omega + \omega^2 + \dots + \omega^{n-1} = 0$ & $\omega^n = 1$.
- 15) Principle argument of a complex number $\theta = \arg z$

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Exercise 2.1

1. Simplify the following.

$$\begin{aligned}
 i^{1947} + i^{1950} &= (i^4)^{486} \cdot i^3 + (i^4)^{487} \cdot i^2 \\
 &= i^3 + i^2 = -i - 1 \\
 &= -1 - i
 \end{aligned}$$

$$\begin{aligned}
 i^2 &= -1 \\
 i^3 &= -i \\
 i^4 &= 1 \\
 i^{4n} &= 1
 \end{aligned}$$

$$\begin{aligned}
 2) \quad i^{1948} - i^{-1869} &= (i^4)^{487} - (i^4)^{-467} \cdot i^3 \\
 &= 1 - (1)(-i) = 1 + i
 \end{aligned}$$

$$\begin{aligned}
 3) \quad \sum_{n=1}^{12} i^n &= i + i^2 + i^3 + i^4 + \dots + i^{12} \\
 &= (i + i^2 + i^3 + i^4) + (i^5 + i^6 + i^7 + i^8) \\
 &\quad + (i^9 + i^{10} + i^{11} + i^{12}) \\
 &= (i + i^2 + i^3 + i^4) + (i + i^2 + i^3 + i^4) + (i + i^2 + i^3 + i^4) \\
 &= 0 + 0 + 0 = 0.
 \end{aligned}$$

$$\begin{aligned}
 1 + i + i^2 + i^3 + i^4 &= 1 + i - 1 - i + 1 = 1
 \end{aligned}$$

$$\begin{aligned}
 4) \quad i^{59} + i^{1/59} &= i^{59} + i^{-59} = (i^4)^{14} i^3 + i^{-60} \cdot i \\
 &= -i + i = 0.
 \end{aligned}$$

$$\begin{aligned}
 5) \quad i \cdot i^2 \cdot i^3 \cdot \dots \cdot i^{2000} &= i^{1+2+3+\dots+2000} \\
 &= i^{\frac{2000 \times 2001}{2}} = i^{1000 \times 2001} \\
 &= i^{2001000} = (i^4)^{500250} = 1
 \end{aligned}$$

$$\begin{aligned}
 1 + 2 + \dots + n &= \frac{n(n+1)}{2}
 \end{aligned}$$

$$\begin{aligned}
 6) \quad \sum_{n=1}^{60} i^{n+50} &= i^{51} + i^{52} + i^{53} + \dots + i^{60} \\
 &= i^3 + i^2 + i + i^4 + i^3 + i^2 + i + i^4 + i^3 + i^2 + i + i^4 \\
 &= -i + i - 1 + 1 - i + i - 1 + 1 - i + i - 1 + 1 = 1 - i
 \end{aligned}$$

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Exercise 2.2

②

1.

Evaluate the following if $z = 5 - 2i$ and $w = -1 + 3i$

(i) $z + w = 5 - 2i - 1 + 3i = 4 + i$

(ii) $z - iw = (5 - 2i) - i(-1 + 3i) = 5 - 2i + i - 3i^2$

$$= 5 - 2i + i + 3 = 8 - i$$

(iii) $2z + 3w = 2(5 - 2i) + 3(-1 + 3i) = 10 - 4i - 3 + 9i$

$$= 7 + 5i$$

(iv) $zw = (5 - 2i)(-1 + 3i) = -5 + 15i + 2i - 6i^2$

$$= -5 + 17i - 6(-1) = -5 + 17i + 6 = 1 + 17i$$

(v) $z^2 + 2zw + w^2 = (5 - 2i)^2 + 2(5 - 2i)(-1 + 3i) + (-1 + 3i)^2$

$$= 25 + 4i^2 - 20i + 2(-5 + 15i + 2i - 6i^2) + 1 + 3i^2 - 6i$$

$$= 25 - 4 - 20i - 10 + 30i + 4i + 12 + 1 - 3 - 6i$$

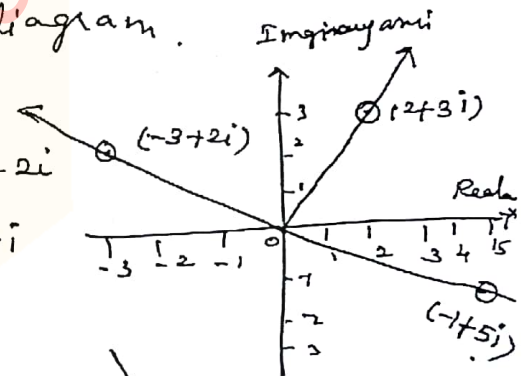
$$= 15 + 8i$$

2. Given the complex number $z = 2 + 3i$ represent the complex numbers in Argand diagram.(i) z , iz and $z + iz$.

$$z = 2 + 3i$$

$$iz = i(2 + 3i) = 2i + 3i^2 = -3 + 2i$$

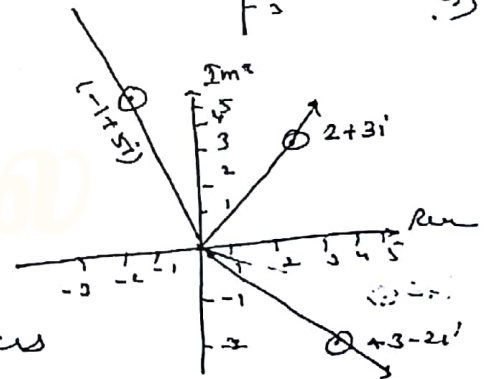
$$z + iz = (2 + 3i) - 3 + 2i = -1 + 5i$$

(ii) z , $-iz$, $z - iz$

$$z = 2 + 3i$$

$$-iz = -i(2 + 3i) = -2i - 3i^2 = +3 - 2i$$

$$z - iz = 2 + 3i - 3 + 2i = -1 + 5i$$



3)

Find the values of the real numbers x and y if the complex numbers. $(3-i)x - (2-i)y + 2i + 5$ and $2x + (-1+2i)y + 3 + 2i$ are equal.

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Soln

$$(3-i)x - (2-i)y + 2i + 5 = 2x + (-1+2i)y + 3 + 2i \quad (4)$$

$$3x - ix - 2y + iy + 2i + 5 = 2x - y + 2iy + 3 + 2i$$

$$(3x - 2y + 5) + i(-x + y + 2) = (2x - y + 3) + i(2y + 2)$$

Equating real & imaginary parts

$$(i) \quad 3x - 2y + 5 = 2x - y + 3$$

$$3x - 2y + 5 - 2x + y - 3 = 0$$

$$x - y + 2 = 0 \quad \text{--- (1)}$$

$$(ii) \quad -x + y + 2 = 2y + 2$$

$$x - y - 2 + 2y + 2 = 0$$

$$x + y = 0 \quad \text{--- (2)}$$

solve (1) + (2)

$$x - y = -2$$

$$x + y = 0$$

$$\hline 2x = -2$$

$$x = -1$$

Put in (2) $x + y = 0$

$$-1 + y = 0$$

$$y = 1$$

$$\boxed{\begin{matrix} \therefore x = -1 \\ y = 1 \end{matrix}}$$

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Exercise 2.3

(5)

1. If $z_1 = 1-3i$, $z_2 = -4i$ and $z_3 = 5$ show that

$$(i) (z_1 + z_2) + z_3 = z_1 + (z_2 + z_3)$$

$$\text{LHS: } (z_1 + z_2) + z_3 = (1-3i-4i) + (5) = (1-7i) + 5 = 6-7i$$

$$\text{RHS: } z_1 + (z_2 + z_3) = (1-3i) + (-4i+5) = 6-7i$$

$$\therefore \text{LHS} = \text{RHS.}$$

$$(ii) (z_1 z_2) z_3 = z_1 (z_2 z_3)$$

$$\text{LHS } z_1 z_2 = (1-3i)(-4i) = -4i + 12i^2 = -4i - 12$$

$$(z_1 z_2) z_3 = (-12-4i)5 = -60-20i$$

$$\text{RHS } (z_2 z_3) = (-4i)(5) = -20i$$

$$z_1 (z_2 z_3) = (1-3i)(-20i) = -20i + 60i^2 = -20i - 60 = -60 - 20i$$

$$\therefore \text{LHS} = \text{RHS.}$$

2. If $z_1 = 3$, $z_2 = -7i$ and $z_3 = 5+4i$ show that

$$(i) z_1 (z_2 + z_3) = z_1 z_2 + z_1 z_3$$

$$\text{LHS } z_1 (z_2 + z_3) = 3(-7i + 5 + 4i) = 3(5 - 3i) = 15 - 9i$$

$$\text{RHS } z_1 z_2 = 3(-7i) = -21i$$

$$z_1 z_3 = 3(5+4i) = 15+12i$$

$$z_1 z_2 + z_1 z_3 = -21i + 15 + 12i = 15 - 9i$$

$$\text{LHS} = \text{RHS.}$$

$$(ii) (z_1 + z_2) z_3 = z_1 z_3 + z_2 z_3$$

$$\text{LHS } (z_1 + z_2) z_3 = (3 - 7i)(5 + 4i) = 15 + 12i - 35i - 28i^2 = 15 - 23i + 28 = 43 - 23i$$

$$\text{RHS } z_1 z_3 = 3(5+4i) = 15+12i$$

$$z_2 z_3 = -7i(5+4i) = -35i - 28i^2 = -35i + 28$$

$$z_1 z_3 + z_2 z_3 = 15 + 12i - 35i + 28 = 43 - 23i$$

$$\therefore \text{LHS} = \text{RHS.}$$

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- 3) If $z_1 = 2+5i$, $z_2 = -3-4i$ and $z_3 = 1+i$ find the additive and multiplicative inverse of z_1, z_2 & z_3 .

Soln

i) $z_1 = 2+5i$

additive inverse $-z_1 = -2-5i$

multiplicative inverse $\frac{1}{z_1} = \frac{1}{2+5i} \cdot \frac{(2-5i)}{(2-5i)} = \frac{2-5i}{2^2+5^2}$

$= \frac{2-5i}{29} = \frac{2}{29} - \frac{5i}{29}$ $[\because (a+ib)(a-ib) = a^2+b^2]$

ii) $z_2 = -3-4i$

additive inverse $-z_2 = 3+4i$

multiplicative inverse $\frac{1}{z_2} = \frac{1}{(-3-4i)} \cdot \frac{(-3+4i)}{(-3+4i)}$

$= \frac{-3+4i}{(-3)^2+4^2} = \frac{-3+4i}{9+16} = \frac{-3+4i}{25} = \frac{-3}{25} + i \frac{4}{25}$

iii) $z_3 = 1+i$

additive inverse $-z_3 = -1-i$

multiplicative inverse $\frac{1}{z_3} = \frac{1}{1+i} \cdot \frac{(1-i)}{(1-i)}$

$= \frac{1-i}{1^2+1^2} = \frac{1-i}{2} = \frac{1}{2} - i \left(\frac{1}{2}\right)$

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Exercise 2.4

(7)

1. Write the following in the rectangular form.

$$(i) \overline{(5+9i)} + \overline{(2-4i)} = \overline{7+5i} = 7-5i$$

$$(ii) \frac{10-5i}{6+2i} = \frac{10-5i}{6+2i} \cdot \frac{(6-2i)}{(6-2i)} = \frac{60-20i-30i+10i^2}{6^2+2^2}$$

$$= \frac{60-50i-10}{36+4} = \frac{50-50i}{40} = \frac{50}{40} - \frac{50}{40}i$$

$$= \frac{5}{4} - \frac{5}{4}i$$

$$(iii) 3\bar{i} + \frac{1}{2-i} = -3i + \frac{1}{2-i} \left(\frac{2+i}{2+i} \right) = -3i + \frac{2+i}{2^2+1^2}$$

$$= -3i + \frac{2+i}{(4+1)} = -3i + \frac{2+i}{5}$$

$$= \frac{-15i+2+i}{5} = \frac{2-14i}{5} = \frac{2}{5} - \frac{14}{5}i$$

2) If $z = x+iy$ find the following in rectangular form.

$$(i) \operatorname{Re} \left(\frac{1}{z} \right)$$

$$\frac{1}{z} = \frac{1}{x+iy} = \frac{1}{x+iy} \cdot \frac{(x-iy)}{(x-iy)} = \frac{x-iy}{x^2+y^2} = \frac{x}{x^2+y^2} - i \frac{y}{x^2+y^2}$$

$$\operatorname{Re} \left(\frac{1}{z} \right) = \frac{x}{x^2+y^2}$$

$$(ii) \operatorname{Re} (i\bar{z})$$

$$z = x+iy$$

$$\bar{z} = x-iy$$

$$i\bar{z} = ix - i^2y = ix + y = y + ix$$

$$\operatorname{Re} (i\bar{z}) = y$$

$$(iii) \operatorname{Im} (3z + 4\bar{z} - 4i)$$

$$3z + 4\bar{z} - 4i = 3(x+iy) + 4(x-iy) - 4i$$

$$= 3x + i3y + 4x - i4y - 4i$$

$$= 7x + i(-y-4) = 7x + i(-y-4)$$

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(8)

$$\operatorname{Im}(3z + 4\bar{z} - 4i) = -7 - 4.$$

- 3) If $z_1 = 2 - i$ & $z_2 = -4 + 3i$ find the inverse of $z_1 z_2$ & $\frac{z_1}{z_2}$

Soln $z_1 z_2 = (2 - i)(-4 + 3i) = -8 + 6i + 4i - 3i^2$

$$= -8 + 10i + 3 = -5 + 10i$$

$$\frac{1}{z_1 z_2} = \frac{1}{-5 + 10i} = \frac{1}{-5 + 10i} \cdot \frac{(-5 - 10i)}{(-5 - 10i)} = \frac{-5 - 10i}{(-5)^2 + 10^2}$$

$$= \frac{-5 - 10i}{25 + 100} = \frac{-5 - 10i}{125} = \frac{-81}{125} - \frac{10^2}{125}i$$

$$= -\frac{1}{25} (1 + 2i)$$

$$\text{inverse of } z_1 z_2 = -\frac{1}{25} (1 + 2i)$$

1) Inverse of $\frac{z_1}{z_2}$ is $\frac{z_2}{z_1}$

$$\therefore \frac{z_2}{z_1} = \frac{-4 + 3i}{2 - i} = \frac{(-4 + 3i)}{(2 - i)} \times \frac{(2 + i)}{(2 + i)}$$

$$= \frac{-8 - 4i + 6i + 3i^2}{2^2 + 1^2} = \frac{-8 + 2i - 3}{5} = \frac{-11 + 2i}{5}$$

- 4) The complex numbers u, v and w are related by $\frac{1}{u} = \frac{1}{v} + \frac{1}{w}$. If $v = 3 - 4i$, and $w = 4 + 3i$ find u in rectangular form.

Soln. $\frac{1}{v} = \frac{1}{3 - 4i} \cdot \frac{(3 + 4i)}{(3 + 4i)} = \frac{3 + 4i}{3^2 + 4^2} = \frac{3 + 4i}{9 + 16} = \frac{3 + 4i}{25}$

$$\frac{1}{w} = \frac{1}{4 + 3i} \cdot \frac{(4 - 3i)}{(4 - 3i)} = \frac{4 - 3i}{4^2 + 3^2} = \frac{4 - 3i}{16 + 9} = \frac{4 - 3i}{25}$$

$$\frac{1}{u} = \frac{1}{v} + \frac{1}{w} = \frac{3 + 4i}{25} + \frac{4 - 3i}{25} = \frac{7 - i}{25}$$

- 5) Prove the following properties.

(i) z is real if and only if $z = \bar{z}$.

Soln z is real.

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(9)

$$\therefore y = 0$$

$$z = x + i0 = x$$

$$z = x$$

$$\Rightarrow \bar{z} = x$$

$$\therefore z = \bar{z}$$

$$\text{if } z = \bar{z}$$

$$z = x + iy \quad \bar{z} = x - iy$$

$$x + iy = x - iy$$

$$2iy = 0$$

$$\boxed{y = 0} \Rightarrow z \text{ is real.}$$

$$\therefore z \text{ is real iff } z = \bar{z}$$

$$(i) \quad \operatorname{Re}(z) = \frac{z + \bar{z}}{2}$$

$$z = x + iy$$

$$\bar{z} = x - iy$$

$$z + \bar{z} = 2x$$

$$\frac{\bar{z} + z}{2} = x = \operatorname{Re}(z)$$

$$(ii) \quad \operatorname{Im}(z) = \frac{z - \bar{z}}{2i}$$

$$z = x + iy, \quad \bar{z} = x - iy$$

$$z - \bar{z} = x + iy - x + iy$$

$$z - \bar{z} = 2iy$$

$$y = \frac{z - \bar{z}}{2i}$$

$$\operatorname{Im}(z) = \frac{z - \bar{z}}{2i}$$

- 4) Find the least value of the positive integer n for which $(\sqrt{3} + i)^n$ is real.

$$(\sqrt{3} + i)^2 = (\sqrt{3} + i)(\sqrt{3} + i) = 3 - 1 + 2\sqrt{3}i = 2 + 2\sqrt{3}i$$

$$= 2(1 + \sqrt{3}i)$$

$$(\sqrt{3} + i)^4 = 2(1 + \sqrt{3}i) \cdot 2(1 + \sqrt{3}i) = 4(1 - 3 + 2i\sqrt{3})$$

$$= 4(-2 + 2i\sqrt{3}) = 8(-1 + i\sqrt{3})$$

$$(\sqrt{3} + i)^6 = 8(-1 + i\sqrt{3}) \cdot 2(1 + \sqrt{3}i) = 16(\sqrt{3}^2 + 1^2)$$

$$= 16(3 + 1) = 16(4) = 32 = \text{purely real.}$$

$\therefore n=6$ is the least +ve integer for

$(\sqrt{3} + i)^n$ is purely real.

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$$(ii) (\sqrt{3}+i)^2 = 2(1+i\sqrt{3})$$

$$\begin{aligned} (\sqrt{3}+i)^3 &= 2(1+i\sqrt{3})(\sqrt{3}+i) = 2(\sqrt{3}+i+i\sqrt{3}+i^2\sqrt{3}) \\ &= 2(\sqrt{3}+i+i\sqrt{3}-\sqrt{3}) = 2(4i) = 8i \\ &= \text{purely imaginary} \end{aligned}$$

$\therefore n=3$ is the least +ve integer for purely imaginary.

7) Show that $(2+i\sqrt{3})^{10} - (2-i\sqrt{3})^{10}$ is purely imaginary.

(i) Soln z is purely imaginary $\bar{z} = -z$.

$$z = (2+i\sqrt{3})^{10} - (2-i\sqrt{3})^{10}$$

$$\bar{z} = \overline{(2+i\sqrt{3})^{10} - (2-i\sqrt{3})^{10}}$$

$$= (2-i\sqrt{3})^{10} - (2+i\sqrt{3})^{10}$$

$$= -[(2+i\sqrt{3})^{10} - (2-i\sqrt{3})^{10}]$$

$$\bar{z} = -z. \quad \therefore z \text{ is purely imaginary}$$

$\therefore (2+i\sqrt{3})^{10} - (2-i\sqrt{3})^{10}$ is purely imaginary.

(ii) $\left(\frac{19-7i}{9+i}\right)^{12} + \left(\frac{20-5i}{7-6i}\right)^{12}$ is real

$$\text{Soln } \frac{(19-7i)}{9+i} \times \frac{(9-i)}{9-i} = \frac{171-19i-63i-7}{81+1} = \frac{164-82i}{82} = 2-i$$

$$\frac{(20-5i)}{(7-6i)} \times \frac{(7+6i)}{(7+6i)} = \frac{140+120i-35i+30}{49+36} = \frac{170+85i}{85} = 2+i$$

$$z = \left(\frac{19-7i}{9+i}\right)^{12} + \left(\frac{20-5i}{7-6i}\right)^{12}$$

$$= (2-i)^{12} + (2+i)^{12}$$

$$\bar{z} = \overline{(2-i)^{12} + (2+i)^{12}}$$

$$= (2+i)^{12} + (2-i)^{12}$$

$$\bar{z} = z$$

$\therefore z$ is real.

$$\therefore \left(\frac{19-7i}{9+i}\right)^{12} + \left(\frac{20-5i}{7-6i}\right)^{12} \text{ is real.}$$

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Exercise 2.5.

1. Find the modulus of the following complex number. (1A)

Soln. $\frac{2i}{3+4i}$

$$\left| \frac{2i}{3+4i} \right| = ?$$

$$\frac{2i}{3+4i} \cdot \frac{(3-4i)}{(3-4i)} = \frac{6i-8i^2}{3^2+4^2} = \frac{6i+8}{16+9} = \frac{8+6i}{25}$$

$$\left| \frac{2i}{3+4i} \right| = \left| \frac{8+6i}{25} \right| = \frac{1}{25} \sqrt{8^2+6^2} = \frac{1}{25} \sqrt{64+36} = \frac{1}{25} \sqrt{100}$$

$$= \frac{1}{25} (10) = \frac{2}{5}$$

(ii) $\frac{2-i}{1+i} \cdot \frac{1-2i}{1-i} = \frac{(2-i)(1-i) + (1-2i)(1+i)}{(1+i)(1-i)}$

$$= \frac{2-2i-i+i^2 + 1+i-2i-2i^2}{1^2+1^2} = \frac{3-4i-1+2}{2}$$

$$= \frac{4-4i}{2} = \frac{2(2-2i)}{2} = 2-2i$$

$$|2-2i| = \sqrt{2^2+2^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

(iii) $2i(3-4i)(4-3i) = 2i(12-9i-16i-12)$

$$= 2i(-25i) = -50i^2 = 50$$

$$|50| = 50$$

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- (2) For any two complex numbers z_1 & z_2 such that $|z_1| = |z_2| = 1$ and $z_1 z_2 \neq -1$ then show that $\frac{z_1+z_2}{1+z_1 z_2}$ is a real number.

Soln.

$$|z_1| = |z_2| = 1$$

$$\left| \frac{z_1+z_2}{1+z_1 z_2} \right| = \frac{|z_1+z_2|}{|1+z_1 z_2|} \leq \frac{|z_1|+|z_2|}{|1+z_1 z_2|}$$

$$\leq \frac{1+1}{1+1} \leq 1$$

$$\left| \frac{z_1+z_2}{1+z_1 z_2} \right| \leq 1 \quad \text{lies between } [0,1]$$

$$\therefore \frac{z_1+z_2}{1+z_1 z_2} \text{ is a real number.}$$

(a) Aliter $|z_1| = 1 \Rightarrow |z_1|^2 = 1 \Rightarrow z_1 \bar{z}_1 = 1$

$$\bar{z}_1 = \frac{1}{z_1} \quad ||^y \quad \bar{z}_2 = \frac{1}{z_2}$$

$$\overline{\left(\frac{z_1 + z_2}{1 + z_1 z_2} \right)} = \frac{\overline{z_1 + z_2}}{\overline{1 + z_1 z_2}} \quad (z_1 z_2 \neq -1)$$

$$= \frac{\bar{z}_1 + \bar{z}_2}{1 + \bar{z}_1 \bar{z}_2} = \frac{\frac{1}{z_1} + \frac{1}{z_2}}{1 + \frac{1}{z_1} \cdot \frac{1}{z_2}} = \frac{\frac{z_1 + z_2}{z_1 z_2}}{\frac{1 + z_1 z_2}{z_1 z_2}}$$

$$\overline{\left(\frac{z_1 + z_2}{1 + z_1 z_2} \right)} = \frac{z_1 + z_2}{1 + z_1 z_2}$$

$$\Rightarrow \frac{z_1 + z_2}{1 + z_1 z_2} \text{ is real. (if } z \text{ is real iff } z = \bar{z})$$

3) which one of the points $10-8i$, $11+6i$ is closest to $1+i$

Soln

$$|10-8i| = \sqrt{10^2 + 8^2} = \sqrt{100 + 64} = \sqrt{164}$$

$$|11+6i| = \sqrt{121 + 36} = \sqrt{157}$$

$$|1+i| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$\therefore 11+6i$ is closest to $1+i$

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4) if $|z| = 3$ show that $7 \leq |z+6-8i| \leq 13$

Soln $|z+6-8i| \leq |z| + |6-8i|$

$$|z+z_2| \leq |z_1| + |z_2|$$

$$\leq 3 + \sqrt{6^2 + 8^2}$$

$$\leq 3 + \sqrt{36 + 64} = 3 + \sqrt{100} = 3 + 10 = 13$$

$$\leq 13 \quad - (1)$$

$$|z+6-8i| \geq ||z| - |6-8i||$$

$$\geq |3 - \sqrt{6^2 + 8^2}| = |3 - 10| = |-7|$$

$$\geq 7 \quad - (2)$$

by ① & ② $7 \leq |2+6-8i| \leq 13.$

5) If $|z|=1$ show that $2 \leq |z^2-3| \leq 4$

soln

$$\begin{array}{l|l} |z^2-3| \geq ||z|^2-3| & |z^2-3| \leq |z^2|+|-3| \\ \geq |1-3| & \leq 1+3 \\ \geq |-2| & \\ \geq 2 & -① \end{array} \quad \begin{array}{l} \\ \\ \\ |z^2-3| \leq 4. -② \end{array}$$

by ① & ② $2 \leq |z^2-3| \leq 4.$

6) If $|z - \frac{2}{z}| = 2$ show that the greatest and least value of $|z|$ are $\sqrt{3}+1$ & $\sqrt{3}-1$ respectively.

soln

$$\begin{aligned} |z| &= |z - \frac{2}{z} + \frac{2}{z}| \\ &\leq |z - \frac{2}{z}| + \frac{2}{|z|} \\ |z| &\leq 2 + \frac{2}{|z|} \end{aligned}$$

$$|z| - 2 - \frac{2}{|z|} \leq 0$$

$$\frac{|z|^2 - 2|z| + 2}{|z|} \leq 0$$

$$|z|^2 - 2|z| - 2 \leq 0$$

$$|z| = \frac{+2 \pm \sqrt{4 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1} = \frac{2 \pm \sqrt{4+8}}{2} = \frac{2 \pm \sqrt{12}}{2}$$

$$= \frac{2 \pm 2\sqrt{3}}{2} = \frac{2(1 \pm \sqrt{3})}{2} = 1 \pm \sqrt{3}$$

$$= \sqrt{3}+1, \sqrt{3}-1.$$

$\therefore$ least & greatest values are $\sqrt{3}-1$ & $\sqrt{3}+1$.

7) If z_1, z_2, z_3 are three complex numbers such that $|z_1|=1, |z_2|=2, |z_3|=3$ and $|z_1+z_2+z_3|=1$ show that $|9z_1z_2+4z_1z_3+z_2z_3|=6$

Soln

$$|z_1| = 1 \quad |z_2| = 2 \quad |z_3| = 3, \quad |z_3|^2 = 9$$

$$|z_1|^2 = 1 \quad |z_2|^2 = 4 \quad z_3 \bar{z}_3 = 3^2$$

$$z_1 \bar{z}_1 = 1 \quad z_2 \bar{z}_2 = 2 \quad z_3 = \frac{9}{\bar{z}_3}$$

$$z_1 = \frac{1}{(\bar{z}_1)} \quad z_2 = \frac{4}{\bar{z}_2} \quad z_3 = \frac{9}{\bar{z}_3}$$

$$z_1 + z_2 + z_3 = \frac{1}{\bar{z}_1} + \frac{4}{\bar{z}_2} + \frac{9}{\bar{z}_3}$$

$$|z_1 + z_2 + z_3| = \left| \frac{\bar{z}_2 \bar{z}_3 + 4 \bar{z}_3 \bar{z}_1 + 9 \bar{z}_1 \bar{z}_2}{\bar{z}_1 \bar{z}_2 \bar{z}_3} \right|$$

$$= \frac{|9 \bar{z}_1 \bar{z}_2 + 4 \bar{z}_2 \bar{z}_1 + \bar{z}_2 \bar{z}_3|}{|\bar{z}_1| |\bar{z}_2| |\bar{z}_3|}$$

$$= \frac{|9 z_1 z_2 + 4 z_3 z_1 + z_2 z_3|}{1 \cdot 2 \cdot 3}$$

$$6 = |9 z_1 z_2 + 4 z_1 z_3 + z_2 z_3|$$

$$\therefore |9 z_1 z_2 + 4 z_1 z_3 + z_2 z_3| = 6$$

Hence proved

- 8) If the area of the triangle formed by the vertices $z, iz, \text{ and } z+iz$ is 50 square units. find the value of $|z|$

Soln let A, B, C be $z, iz, z+iz$.

$$\text{Area of a triangle} = \frac{1}{2} \times AB \times PC$$

$$AB = |z - iz|$$

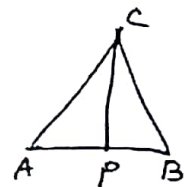
$$P \text{ is midpoint of } AB = \frac{A+B}{2} = \left[\frac{z+iz}{2} \right]$$

$$PC = \left| (z+iz) - \left(\frac{z+iz}{2} \right) \right| = \left| \frac{z+iz}{2} \right|$$

$$\text{Area of triangle} = 50$$

$$\frac{1}{2} AB \times PC = 50$$

$$\frac{1}{2} |z - iz| \left| \frac{z+iz}{2} \right| = 50$$



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$$\Rightarrow \frac{1}{4} |z^2 + z^2| = 50$$

$$\frac{2|z|^2}{4} = 50$$

$$\frac{|z|^2}{2} = 50$$

$$|z|^2 = 100$$

$$\boxed{|z| = 10}$$

(15)

(9) Show that the equation $z^3 + 2\bar{z} = 0$ has five solutions

$$z^3 = -2\bar{z}$$

$$z^2(z^2) = -2z\bar{z}$$

$$z^4 = -2|z|^2$$

$$|z|^4 + 2|z|^2 = 0$$

$$|z|^2 (|z|^2 + 2) = 0$$

$$|z|^2 = 0$$

$$|z|^2 = -2$$

$$|z| = 0$$

$$z = 0$$

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$$z^3 = -2\bar{z} \Rightarrow z^3 = -2 \frac{1}{z}$$

$$z^4 = -2$$

it has four solutions including $z=0$

$z^3 + 2\bar{z} = 0$ It has five solutions.

10) Find the square roots of $4+3i$

$$\underline{\text{Soln}} \quad \sqrt{a+ib} = \pm \left[\sqrt{\frac{|z|+a}{2}} + i \frac{b}{|b|} \sqrt{\frac{|z|-a}{2}} \right]$$

$$|z| = |4+3i| = \sqrt{4^2+3^2} = \sqrt{16+9} = \sqrt{25} = 5$$

$$a=4 \quad b=3$$

$$\sqrt{4+3i} = \pm \left(\sqrt{\frac{5+4}{2}} + i \frac{3}{|3|} \sqrt{\frac{5-4}{2}} \right)$$

$$= \pm \left(\sqrt{\frac{1}{2}} + i \sqrt{\frac{1}{2}} \right) = \pm \left(\frac{1}{\sqrt{2}} + i \frac{1}{\sqrt{2}} \right)$$

$$= \pm \left(\frac{1+i}{\sqrt{2}} \right)$$

(ii) $-6+8i$

$$|z| = \sqrt{(-6)^2 + 8^2} = \sqrt{36+64} = \sqrt{100} = 10$$

$$\sqrt{-6+8i} = \pm \left(\sqrt{\frac{10-6}{2}} + i \frac{8}{8} \sqrt{\frac{10+6}{2}} \right) = \pm (\sqrt{2} + i\sqrt{8})$$

$$= \pm (\sqrt{2} + i 2\sqrt{2})$$

(iii) $-5-12i$

$$|z| = \sqrt{(-5)^2 + (-12)^2} = \sqrt{25+144} = \sqrt{169} = 13$$

$$\sqrt{-5-12i} = \pm \left(\sqrt{\frac{13-5}{2}} - i \frac{12}{12} \sqrt{\frac{13+5}{2}} \right)$$

$$= \pm (\sqrt{4} - i\sqrt{9}) = \pm (2 - i3)$$

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Exercise 2.6

1. If $z = x + iy$ is a complex number such that $\left| \frac{z-4i}{z+4i} \right| = 1$ (11)
Show that the locus of z is real axis

Soln

$$\left| \frac{z-4i}{z+4i} \right| = 1$$

$$z = x + iy$$

$$\left| \frac{x+iy-4i}{x+iy+4i} \right| = 1$$

$$\left| \frac{x+i(y-4)}{x+i(y+4)} \right| = 1$$

$$|x+i(y-4)| = |x+i(y+4)|$$

$$\sqrt{x^2 + (y-4)^2} = \sqrt{x^2 + (y+4)^2}$$

$$x^2 + 16 - 8y = x^2 + 16 + 8y$$

$$16y = 0$$

$y = 0$ which is real axis.

$\therefore$ The locus of z is real axis

- 2) If $z = x + iy$ is a complex number such that $\operatorname{Im} \left(\frac{2z+1}{iz+1} \right) = 0$ show that the locus of z is $2x^2 + 2y^2 + x - 2y = 0$.

Soln

$$\frac{2z+1}{iz+1} = \frac{2(x+iy)+1}{i(x+iy)+1} = \frac{2x+i2y+1}{ix-y+1}$$

$$\text{put } z = x + iy$$

$$= \frac{(1+2x)+i2y}{(1-y)+ix} \times \frac{[(1-y)-ix]}{[(1-y)-ix]}$$

$$= \frac{(1+2x)(1-y) - ix(1+2x) + i2y(1-y) + 2xy}{(1-y)^2 + x^2}$$

$$\operatorname{Im} \left(\frac{2z+1}{iz+1} \right) = \frac{-x(1+2x) + 2y(1-y)}{(1-y)^2 + x^2}$$

(18)

$$-2x^2 - x + 2y - 2y^2 = 0$$

$2x^2 + 2y^2 - 2y + x = 0$ is the locus of z .

3) Obtain the cartesian form of the locus of $z = x + iy$ in each of the following cases.

(i) $[\operatorname{Re}(iz)]^2 = 3$

$$iz = i(x + iy) = ix - y = -y + ix.$$

$$\operatorname{Re}(iz) = -y$$

$$[\operatorname{Re}(iz)]^2 = 3$$

$$(-y)^2 = 3$$

$$y^2 - 3 = 0$$

(ii) $\operatorname{Im}[(1-i)z + 1] = 0$

$$\begin{aligned} (1-i)z + 1 &= (1-i)(x + iy) + 1 \\ &= x + iy - ix + y + 1 \\ &= (x + y + 1) + i(y - x) \end{aligned}$$

$$\operatorname{Im}[(1-i)z + 1] = 0$$

$$y - x = 0$$

$$x - y = 0$$

(iii) $|z + i| = |z - 1|$

$$|x + iy + i| = |x + iy - 1|$$

$$|x + i(y + 1)| = |(x - 1) + iy|$$

$$\sqrt{x^2 + (y + 1)^2} = \sqrt{(x - 1)^2 + y^2}$$

$$x^2 + y^2 + 2y + 1 = x^2 + 1 - 2x + y^2$$

$$2x + 2y = 0$$

$$x + y = 0$$

(iv) $\bar{z} = z^{-1}$

$$z = x + iy$$

$$\bar{z} = x - iy$$

$$\begin{aligned} \bar{z} &= \frac{1}{z} = \frac{1}{x + iy} \\ &= \frac{x - iy}{x^2 + y^2} \end{aligned}$$

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$$(iv) \bar{z} = z^{-1}$$

$$\bar{z} = \frac{1}{z}$$

$$z\bar{z} = 1$$

$$(x+iy)(x-iy) = 1$$

$$x^2 + y^2 = 1$$

$x^2 + y^2 - 1 = 0$ is the locus of z .

4. Show that the following equations represent a circle, and find its centre and radius.

$$(i) |z - 2 - i| = 3$$

$$\text{soln } |z - (2+i)| = 3$$

$$\text{radius} = 3$$

$$\text{Centre} = 2+i$$

$$(ii) |2z + 2 - 4i| = 2$$

$$2|z + 1 - 2i| = 2$$

$$2|z + (1 - 2i)| = 2$$

$$\text{radius} = 1 \quad \times \quad \text{Centre } (1 - 2i)$$

$$(iii) |3z - 6 + 12i| = 8$$

$$3|z - 2 + 4i| = 8$$

$$|z - (2 - 4i)| = 8/3$$

$$\text{radius} = 8/3 \quad \text{Centre} = 2 - 4i$$

5) Obtain the cartesian equation for the locus of $z = x+iy$ in each of the following cases

$$(i) |z - 4| = 16$$

$$z = x+iy$$

$$|x+iy - 4| = 16$$

$$|(x-4) + iy| = 16$$

$$\sqrt{(x-4)^2 + y^2} = 16$$

$$(x^2 + 8x + 16) + y^2 = 256$$

$$x^2 + y^2 + 8x - 24 = 0 \text{ is the locus of } z.$$

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(ii)

$$|z-4|^2 - |z-1|^2 = 16$$

$$(z-4)(\overline{z-4}) - (z-1)(\overline{z-1}) = 16$$

$$|z|^2 = z\overline{z}$$

$$z = x+iy$$

$$(x+iy-4)(\overline{x+iy-4}) - (x+iy-1)(\overline{x+iy-1}) = 16$$

$$(x+iy-4)(x-4-iy) - [(x-1)+iy][(x-1)-iy] = 16$$

$$[(x-4)^2 + y^2] - [(x-1)^2 + y^2] = 16$$

$$(x^2 - 8x + 16 + y^2) - (x^2 - 2x + 1 + y^2) = 16$$

$$x^2 - 8x + 16 + y^2 - x^2 + 2x - 1 - y^2 - 16 = 0$$

$$-6x - 1 = 0$$

$6x + 1 = 0$ is the locus of z .

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Exercise 2.7.
(i) write in polar form of the following Complex numbers (21)

(i) $2 + i2\sqrt{3} = r(\cos\theta + i\sin\theta)$

$$r = \sqrt{2^2 + (2\sqrt{3})^2} = \sqrt{4 + 12} = \sqrt{16} = 4$$

$$r\cos\theta = 2$$

$$r\sin\theta = 2\sqrt{3}$$

$$\cos\theta = \frac{2}{4} = \frac{1}{2}$$

$$\sin\theta = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$$

$\theta = \pi/3$ which is in I quadrant

$$\theta = \alpha = \pi/3$$

$$\begin{cases} \sin\theta = +ve \\ \cos\theta = +ve \end{cases}$$

$$2 + i2\sqrt{3} = 4 (\cos\pi/3 + i\sin\pi/3)$$

$$= 4 [\cos(2k\pi + \pi/3) + i\sin(2k\pi + \pi/3)] \quad \forall k \in \mathbb{Z}$$

(ii) $3 - i\sqrt{3} = r(\cos\theta + i\sin\theta)$

$$r = \sqrt{3^2 + (-\sqrt{3})^2} = \sqrt{9 + 3} = \sqrt{12} = 2\sqrt{3}$$

$$r\cos\theta = 3$$

$$r\sin\theta = -\sqrt{3}$$

$$\cos\theta = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$$

$$\sin\theta = \frac{-\sqrt{3}}{2\sqrt{3}} = -\frac{1}{2}$$

$\theta = \pi/6$ which is in IV quadrant

$$\theta = -\alpha = -\pi/6$$

$$\begin{cases} \sin\theta = -ve \\ \cos\theta = +ve \end{cases}$$

$$3 - i\sqrt{3} = 2\sqrt{3} [\cos(-\pi/6) + i\sin(-\pi/6)]$$

$$= 2\sqrt{3} [\cos(2k\pi - \pi/6) + i\sin(2k\pi - \pi/6)] \quad \forall k \in \mathbb{Z}$$

(iii) $-2 - i2 = r(\cos\theta + i\sin\theta)$

$$r = \sqrt{(-2)^2 + (-2)^2} = \sqrt{4 + 4} = \sqrt{8} = 2\sqrt{2}$$

$$r\cos\theta = -2$$

$$r\sin\theta = -2$$

$$\cos\theta = \frac{-2}{2\sqrt{2}} = -\frac{1}{\sqrt{2}}$$

$$\sin\theta = \frac{-2}{2\sqrt{2}} = -\frac{1}{\sqrt{2}}$$

$\theta = \pi/4$ which is in III quadrant

$$\theta = \alpha - \pi$$

$$= \pi/4 - \pi = -\frac{3\pi}{4}$$

$$\begin{cases} \sin\theta = -ve \\ \cos\theta = -ve \end{cases}$$

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$$\begin{aligned}
 -2-iz &= 2\sqrt{2} [\cos(-3\pi/4) + i\sin(-3\pi/4)] \\
 &= 2\sqrt{2} [\cos(2k\pi - 3\pi/4) + i\sin(2k\pi - 3\pi/4)] \quad \forall k \in \mathbb{Z}
 \end{aligned}$$

(iv)

$$\frac{i-1}{\cos \pi/3 + i\sin \pi/3}$$

$$-1+i = r(\cos \theta + i\sin \theta)$$

$$r = \sqrt{(-1)^2 + 1^2} = \sqrt{1+1} = \sqrt{2}$$

$$r \cos \theta = -1$$

$$r \sin \theta = 1$$

$$\cos \theta = -\frac{1}{\sqrt{2}}$$

$$\sin \theta = 1/\sqrt{2}$$

$\theta = \pi/4$ which is II quadrant.

$$\sin \theta = +ve$$

$$\cos \theta = -ve$$

$$\theta = \pi - \alpha$$

$$= \pi - \pi/4$$

$$= 3\pi/4$$

$$-1+i = \sqrt{2} (\cos 3\pi/4 + i\sin 3\pi/4)$$

$$\frac{i-1}{\cos \pi/3 + i\sin \pi/3} = \frac{\sqrt{2} (\cos 3\pi/4 + i\sin 3\pi/4)}{\cos (\pi/3) + i\sin \pi/3} = \sqrt{2} \frac{e^{i3\pi/4}}{e^{i\pi/3}}$$

$$= \sqrt{2} e^{i(3\pi/4 - \pi/3)} = \sqrt{2} e^{i(9\pi/12 - 4\pi/12)}$$

$$= \sqrt{2} e^{i5\pi/12}$$

$$= \sqrt{2} \left[\cos \frac{5\pi}{12} + i\sin \frac{5\pi}{12} \right]$$

$$= \sqrt{2} \left[\cos \left(2k\pi + \frac{5\pi}{12} \right) + i\sin \left(2k\pi + \frac{5\pi}{12} \right) \right] \quad \forall k \in \mathbb{Z}$$

2. Find the rectangular form of the complex numbers

$$1) (\cos \pi/6 + i\sin \pi/6) (\cos \pi/12 + i\sin \pi/12)$$

$$= \cos (\pi/6 + \pi/12) + i\sin (\pi/6 + \pi/12)$$

$$= \cos \left(\frac{3\pi}{12} \right) + i\sin \left(\frac{3\pi}{12} \right) = \cos \pi/4 + i\sin \pi/4$$

$$= \frac{1}{\sqrt{2}} + i \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} (1+i)$$

2. (11)

$$\frac{\cos \pi/6 - i \sin \pi/6}{2 (\cos \pi/3 + i \sin \pi/3)} = \frac{1}{2} \frac{e^{-i\pi/6}}{e^{i\pi/3}} = \frac{1}{2} e^{-i\pi/6 - i\pi/3} \quad (23)$$

$$= \frac{1}{2} e^{-i(\pi/6 + \pi/3)} = \frac{1}{2} e^{-i\pi/2}$$

$$= \frac{1}{2} (\cos \pi/2 - i \sin \pi/2)$$

$$= \frac{1}{2} (0 - i) = -i/2.$$

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3)

If $(x_1 + iy_1)(x_2 + iy_2)(x_3 + iy_3) + \dots + (x_n + iy_n) = a + ib$
show that (i) $(x_1^2 + y_1^2)(x_2^2 + y_2^2) + \dots + (x_n^2 + y_n^2) = a^2 + b^2$
(ii) $\sum_{r=1}^n \tan^{-1} \left(\frac{y_r}{x_r} \right) = \tan^{-1} \left(\frac{b}{a} \right) + 2k\pi \quad \forall k \in \mathbb{Z}.$

Soln. $(x_1 + iy_1)(x_2 + iy_2)(x_3 + iy_3) + \dots + (x_n + iy_n) = a + ib \quad (1)$

Take modulus on both sides

$$|(x_1 + iy_1)(x_2 + iy_2)(x_3 + iy_3) + \dots + (x_n + iy_n)| = |a + ib|$$

$$|x_1 + iy_1| |x_2 + iy_2| |x_3 + iy_3| \dots |x_n + iy_n| = |a + ib|$$

$$\sqrt{x_1^2 + y_1^2} \cdot \sqrt{x_2^2 + y_2^2} \cdot \sqrt{x_3^2 + y_3^2} \dots \sqrt{x_n^2 + y_n^2} = \sqrt{a^2 + b^2}$$

Take square on both sides.

$$(x_1^2 + y_1^2)(x_2^2 + y_2^2)(x_3^2 + y_3^2) \dots (x_n^2 + y_n^2) = (a^2 + b^2)$$

In (i) Take argument on both sides.

$$\arg[(x_1 + iy_1)(x_2 + iy_2) + \dots + (x_n + iy_n)] = \arg(a + ib)$$

$$\Rightarrow \arg(x_1 + iy_1) + \arg(x_2 + iy_2) \quad [\because \arg(z_1 \cdot z_2) = \arg z_1 + \arg z_2]$$

$$+ \dots + \arg(x_n + iy_n) = \arg(a + ib) \quad [\because \arg(x + iy) = \tan^{-1} \frac{y}{x}]$$

$$\tan^{-1} \frac{y_1}{x_1} + \tan^{-1} \frac{y_2}{x_2} + \dots + \tan^{-1} \frac{y_n}{x_n} = \tan^{-1} \frac{b}{a}$$

In general $\sum_{r=1}^n \tan^{-1} \frac{y_r}{x_r} = 2k\pi + \tan^{-1} \left(\frac{b}{a} \right) \quad \forall k \in \mathbb{Z}$

Hence proved.

4) If $\frac{1+z}{1-z} = \cos 2\theta + i \sin 2\theta$ show that $z = i \tan \theta$. (24)

$$\begin{aligned}\frac{1+z}{1-z} &= (\cos \theta + i \sin \theta)^2 \\ (1+z)(1-z) &= (\cos \theta + i \sin \theta)^2 \\ (1+z)^2 &= (\cos \theta + i \sin \theta)^2 \\ 1+z &= \cos \theta + i \sin \theta \\ 1+z &= \frac{\cos \theta}{\cos \theta} + \frac{i \sin \theta}{\cos \theta} \\ 1+z &= 1 + i \tan \theta \\ \therefore z &= i \tan \theta\end{aligned}$$

Hence proved.

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5) If $\cos \alpha + \cos \beta + \cos \gamma = \sin \alpha + \sin \beta + \sin \gamma = 0$ show that
(i) $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3 \cos(\alpha + \beta + \gamma)$ and
(ii) $\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3 \sin(\alpha + \beta + \gamma)$

Soln Let us take

$$a = \cos \alpha + i \sin \alpha$$

$$b = \cos \beta + i \sin \beta$$

$$c = \cos \gamma + i \sin \gamma$$

$$\begin{aligned}a+b+c &= (\cos \alpha + \cos \beta + \cos \gamma) + i (\sin \alpha + \sin \beta + \sin \gamma) \\ &= 0 + i \cdot 0 \\ &= 0\end{aligned}$$

$$a+b+c = 0$$

if $a+b+c = 0$ we know that

$$a^3 + b^3 + c^3 = 3abc$$

$$\begin{aligned}(\cos \alpha + i \sin \alpha)^3 + (\cos \beta + i \sin \beta)^3 + (\cos \gamma + i \sin \gamma)^3 \\ = 3 [(\cos \alpha + i \sin \alpha) (\cos \beta + i \sin \beta) (\cos \gamma + i \sin \gamma)]\end{aligned}$$

$$[(\cos 3\alpha + i \sin 3\alpha) + (\cos 3\beta + i \sin 3\beta) + (\cos 3\gamma + i \sin 3\gamma)] \quad (25)$$

$$= 3 [\cos(\alpha + \beta + \gamma) + i \sin(\alpha + \beta + \gamma)]$$

$$(\cos 3\alpha + \cos 3\beta + \cos 3\gamma) + i(\sin 3\alpha + \sin 3\beta + \sin 3\gamma)$$

$$= 3 \cos(\alpha + \beta + \gamma) + i 3 \sin(\alpha + \beta + \gamma)$$

Equate real & imaginary parts on both sides

$$\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3 \cos(\alpha + \beta + \gamma)$$

$$\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3 \sin(\alpha + \beta + \gamma)$$

Hence proved.

6) If $z = x + iy$ and $\arg\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$ then show that $x^2 + y^2 + 3x - 3y + 2 = 0$.

Soln $z = x + iy$.

$$\arg\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$$

$$\left[\because \arg\left(\frac{z_1}{z_2}\right) = \arg z_1 - \arg z_2 \right]$$

$$\arg(z-i) - \arg(z+2) = \frac{\pi}{4}$$

$$\left[\because \arg(x+iy) = \tan^{-1} \frac{y}{x} \right]$$

$$\arg(x+iy-i) - \arg(x+iy+2) = \frac{\pi}{4}$$

$$\arg(x+i(y-1)) - \arg((x+2)+iy) = \frac{\pi}{4}$$

$$\tan^{-1}\left(\frac{y-1}{x}\right) - \tan^{-1}\frac{y}{x+2} = \frac{\pi}{4}$$

$$\left[\because \tan^{-1} A - \tan^{-1} B = \tan^{-1} \left(\frac{A-B}{1+AB} \right) \right]$$

$$\tan^{-1} \left[\frac{\frac{y-1}{x} - \frac{y}{x+2}}{1 + \left(\frac{y-1}{x}\right)\left(\frac{y}{x+2}\right)} \right] = \frac{\pi}{4}$$

$$\frac{(y-1)(x+2) - xy}{x(x+2) + y(y-1)} = \tan \frac{\pi}{4} = 1$$

$$xy + 2y - x - 2 - xy = x^2 + 2x + y^2 - y$$

$$x^2 + y^2 + 2x - y - 2y + x + 2 = 0$$

$$x^2 + y^2 + 3x - 3y + 2 = 0 \text{ is the locus of } z.$$

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Exercise 2.8

1. If $\omega \neq 1$ is a cube root of unity, then show that

$$\frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2} + \frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2} = -1.$$

Soln

$$\frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2} = \dots$$

$$= \frac{a+b\omega+c\omega^2}{\omega(b\omega+c\omega^2+a)} = \omega$$

$$\frac{1}{\omega} (b\omega+c\omega^2+a)$$

$$\frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2} = \frac{1}{\omega} \frac{(a\omega+b\omega^2+c)}{(c+a\omega+b\omega^2)} = \frac{1}{\omega} \times \frac{\omega^2}{\omega^2} = \frac{\omega^2}{\omega^3} = \omega^2$$

$$\text{LHS} = \omega + \omega^2 = -1$$

$$\left[\because 1 + \omega + \omega^2 = 0 \right]$$

$$\left[\because \omega^3 = 1 \right]$$

2. Show that $\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^5 = -\sqrt{3}$

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Soln

$$\frac{\sqrt{3}}{2} + \frac{i}{2} = r(\cos \theta + i \sin \theta)$$

$$r = \sqrt{\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \sqrt{\frac{3}{4} + \frac{1}{4}} = \sqrt{\frac{4}{4}} = \sqrt{1} = 1$$

$$r \cos \theta = \frac{\sqrt{3}}{2} \quad r \sin \theta = \frac{1}{2}$$

$$\sin \theta = \frac{1}{2}$$

$$\cos \theta = \frac{\sqrt{3}}{2}$$

$$\theta = \pi/6$$

$$\theta = \alpha = \pi/6.$$

[which is I quadrant.
 $\sin \theta = +ve, \cos \theta = +ve$]

$$\frac{\sqrt{3}}{2} + \frac{i}{2} = 1 (\cos \pi/6 + i \sin \pi/6)$$

$$\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 = \cos 5\pi/6 + i \sin 5\pi/6$$

Similarly $\left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^5 = \cos 5\pi/6 - i \sin 5\pi/6.$

$$\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^5 = 2 \cos 5\pi/6 = 2 \cos (\pi - \pi/6)$$

$$= -2 \cos \pi/6 = -2 \left(\frac{\sqrt{3}}{2}\right)$$

$$= -\sqrt{3} \text{ // Proved.}$$

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Find the value of $\left(\frac{1 + \sin \frac{\pi}{10} + i \cos \frac{\pi}{10}}{1 + \sin \frac{\pi}{10} - i \cos \frac{\pi}{10}} \right)^{10}$

Soln

$$z = \cos \frac{\pi}{10} + i \sin \frac{\pi}{10} = i (\sin \frac{\pi}{10} - i \cos \frac{\pi}{10})$$

$$\frac{1}{z} = \cos \frac{\pi}{10} - i \sin \frac{\pi}{10}$$

$$\left(\frac{1 + \sin \frac{\pi}{10} + i \cos \frac{\pi}{10}}{1 + \sin \frac{\pi}{10} - i \cos \frac{\pi}{10}} \right)^{10} = \left(\frac{1 + i (\cos \frac{\pi}{10} - i \sin \frac{\pi}{10})}{1 - i (\cos \frac{\pi}{10} + i \sin \frac{\pi}{10})} \right)^{10}$$

$$= \left(\frac{1 + i \frac{1}{z}}{1 - i z} \right)^{10} = \left(\frac{1 - \frac{1}{zi}}{1 - iz} \right)^{10} = \left(\frac{zi - 1}{zi} \right)^{10} \quad i \times \frac{i}{i} = \frac{-1}{i}$$

$$= \left(\frac{-\frac{1}{zi} (1 - zi)}{(1 - iz)} \right)^{10} = \left(\frac{-\frac{1}{iz}}{1} \right)^{10} = \frac{1}{i^{10}} \left(\frac{1}{z} \right)^{10}$$

$$= \frac{1}{-1} (\cos \frac{\pi}{10} - i \sin \frac{\pi}{10}) \quad i^{10} = i^2 = -1$$

$$= -1 (\cos \pi + i \sin (-\pi))$$

$$= -1 (-1 + i(0)) = (-1)(-1) = 1.$$

4) If $2 \cos \alpha = x + \frac{1}{x}$ and $2 \cos \beta = y + \frac{1}{y}$ show that

i) $\frac{x}{y} + \frac{y}{x} = 2 \cos(\alpha - \beta)$

Soln $2 \cos \alpha = x + \frac{1}{x}$

$$2 \cos \alpha = \frac{x^2 + 1}{x}$$

$$x^2 + 1 = 2x \cos \alpha$$

$$x^2 - 2x \cos \alpha + 1 = 0$$

$$x = \frac{2 \cos \alpha \pm \sqrt{4 \cos^2 \alpha - 4}}{2 \cdot 1} = \frac{2 \cos \alpha \pm 2 \sqrt{\cos^2 \alpha - 1}}{2}$$

$$= \frac{2(\cos \alpha \pm \sqrt{-\sin^2 \alpha})}{2} = \cos \alpha \pm i \sin \alpha$$

$$x = \cos \alpha \pm i \sin \alpha.$$

$$\begin{aligned} \because \sin^2 \alpha + \cos^2 \alpha &= 1 \\ \cos^2 \alpha - 1 &= -\sin^2 \alpha \end{aligned}$$

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Similarly $y = \cos \beta + i \sin \beta$

$$x = \cos \alpha + i \sin \alpha \quad y = \cos \beta + i \sin \beta$$

$$\frac{x}{y} = \frac{\cos \alpha + i \sin \alpha}{\cos \beta + i \sin \beta} = \cos(\alpha - \beta) + i \sin(\alpha - \beta)$$

$$\frac{y}{x} = \frac{1}{\frac{x}{y}} = \cos(\alpha - \beta) - i \sin(\alpha - \beta)$$

$$\frac{x}{y} + \frac{y}{x} = 2 \cos(\alpha - \beta)$$

$$(i) \quad xy = (\cos \alpha + i \sin \alpha)(\cos \beta + i \sin \beta) \\ = \cos(\alpha + \beta) + i \sin(\alpha + \beta)$$

$$\frac{1}{xy} = \cos(\alpha + \beta) - i \sin(\alpha + \beta)$$

$$xy + \frac{1}{xy} = 2i \sin(\alpha + \beta)$$

$$(iv) \quad x^m = (\cos \alpha + i \sin \alpha)^m = \cos m\alpha + i \sin m\alpha \\ y^n = (\cos \beta + i \sin \beta)^n = \cos n\beta + i \sin n\beta \\ x^m y^n = (\cos m\alpha + i \sin m\alpha)(\cos n\beta + i \sin n\beta) \\ = \cos(m\alpha + n\beta) + i \sin(m\alpha + n\beta)$$

$$\frac{1}{x^m y^n} = \frac{1}{\cos(m\alpha + n\beta) + i \sin(m\alpha + n\beta)} \\ = \cos(m\alpha + n\beta) - i \sin(m\alpha + n\beta)$$

$$x^m y^n + \frac{1}{x^m y^n} = 2 \cos(m\alpha + n\beta)$$

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$$(ii) \quad \frac{x^m}{y^n} = \frac{\cos(m\alpha) + i \sin m\alpha}{\cos n\beta + i \sin n\beta} \\ = \cos(m\alpha - n\beta) + i \sin(m\alpha - n\beta)$$

$$\frac{y^n}{x^m} = \cos(m\alpha - n\beta) - i \sin(m\alpha - n\beta)$$

$$\frac{x^m}{y^n} - \frac{y^n}{x^m} = 2i \sin(m\alpha - n\beta)$$

5) Solve the equation $z^3 + 27 = 0$

(29)

Soln

$$z^3 + 27 = 0$$

$$z^3 = -27$$

$$z^3 = 27(-1)$$

$$z = (27)^{1/3} (-1)^{1/3} = (3^3)^{1/3} [\cos \pi + i \sin \pi]^{1/3}$$

$$= 3 [\cos (2k\pi + \pi) + i \sin (2k\pi + \pi)]^{1/3}$$

$$= 3 \left[\cos \left(\frac{2k+1}{3} \pi \right) + i \sin \left(\frac{2k+1}{3} \pi \right) \right] \quad k = 0, 1, 2$$

$$k=0, \quad z = 3(\cos \pi + i \sin \pi) = 3 \operatorname{cis} \pi$$

$$k=1, \quad z = 3(\cos 3\pi + i \sin 3\pi) = 3(-1 + i0) = -3$$

$$k=2, \quad z = 3(\cos 5\pi + i \sin 5\pi) = 3 \operatorname{cis} 5\pi.$$

6) If $\omega \neq 1$ is a cube root of unity, show that the roots of the equation $(z-1)^3 + 8 = 0$ are $-1, 1-2\omega, 1-2\omega^2$.

Soln $(z-1)^3 + 8 = 0$

$$(z-1)^3 = -8 = 8(-1)$$

$$z-1 = 8^{1/3} (-1)^{1/3}$$

$$z-1 = (2^3)^{1/3} [\cos \pi + i \sin \pi]^{1/3}$$

$$= 2 [\cos (2k\pi + \pi) + i \sin (2k\pi + \pi)]^{1/3}$$

$$= 2 \left[\cos \left(\frac{2k+1}{3} \pi \right) + i \sin \left(\frac{2k+1}{3} \pi \right) \right] \quad k = 0, 1, 2.$$

$$k=0, \quad z-1 = 2 \operatorname{cis} \pi = 2 \left[\frac{1}{2} + i \frac{\sqrt{3}}{2} \right] = -2 \left(\frac{-1}{2} - i \frac{\sqrt{3}}{2} \right)$$

$$k=1, \quad z-1 = 2 \operatorname{cis} 3\pi = 2[-1 + i0] = -2$$

$$k=2, \quad z-1 = 2 \operatorname{cis} 5\pi = 2[\cos(\pi - \pi/3) + i \sin(\pi - \pi/3)]$$

$$= 2[-\cos \pi/3 + i \sin \pi/3] = 2 \left[-\frac{1}{2} + i \frac{\sqrt{3}}{2} \right]$$

$$z-1 = -2\omega$$

$$z = 1 - 2\omega^2$$

$z-1 = -2$	$z-1 = -2\omega$
$z = -2+1$	$z = 1-2\omega$
$= -1$	

Ans: $z = -1, 1-2\omega, 1-2\omega^2$.

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7) Find the value of $\sum_{k=1}^8 \cos\left(\frac{2k\pi}{9}\right) + i \sin\left(\frac{2k\pi}{9}\right)$ (3d)

Soln

$$\begin{aligned} & \sum_{k=1}^8 \left[\cos\left(\frac{2k\pi}{9}\right) + i \sin\left(\frac{2k\pi}{9}\right) \right] \\ &= \text{cis } \frac{2\pi}{9} + \text{cis } \frac{4\pi}{9} + \text{cis } \frac{6\pi}{9} + \text{cis } \frac{8\pi}{9} + \dots + \text{cis } \frac{16\pi}{9} \\ &= \text{cis } \left(\frac{2\pi}{9} + \frac{4\pi}{9} + \frac{6\pi}{9} + \dots + \frac{16\pi}{9} \right) \\ &= \text{cis } \left[\frac{2\pi}{9} (1+2+3+\dots+8) \right] = \text{cis } \frac{2\pi}{9} \left(\frac{8 \times 9}{2} \right) \\ &= \text{cis } 8\pi = \cos 8\pi + i \sin 8\pi \\ &= (-1)^8 + i(0) = 1. \end{aligned}$$

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show that.

8) If $\omega \neq 1$ is the cube root of unity

(i) $(1-\omega+\omega^2)^6 + (1+\omega-\omega^2)^6 = 128$

Soln

$$\begin{aligned} & (1-\omega+\omega^2)^6 + (1+\omega-\omega^2)^6 \\ &= (-\omega-\omega)^6 + (-\omega^2-\omega^2)^6 \\ &= (-2\omega)^6 + (-2\omega^2)^6 \\ &= 2^6 \omega^6 + 2^6 \omega^{12} = 2^6 [\omega^6 + \omega^{12}] \\ &= 2^6 (1+1) = 2^6 \cdot 2 = 2^7 = 128 \end{aligned}$$

$$\therefore 1+\omega+\omega^2 = 0$$

$$1+\omega = -\omega^2$$

$$1+\omega^2 = -\omega$$

$$\omega^3 = 1$$

(ii) $(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)\dots(1+\omega^{2^{11}}) = 1$

Soln

$$\begin{aligned} & (1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)\dots(1+\omega^{2^{10}})(1+\omega^{2^{11}}) \\ &= (1+\omega)(1+\omega^2)(1+\omega)(1+\omega^2)\dots(1+\omega)(1+\omega^2) \\ &= (-\omega^2)(-\omega)(-\omega^2)(-\omega)\dots(-\omega^2)(-\omega) \quad \left[\begin{array}{l} \because 1+\omega = -\omega^2 \\ 1+\omega^2 = -\omega \end{array} \right] \\ &= \omega^3 \cdot \omega^3 \dots \omega^3 = 1 \cdot 1 \cdot 1 \dots 1 = 1 \quad \text{R.H.S.} \quad \omega^3 = 1 \end{aligned}$$

(9)

If $z = 2-2i$ find the rotation of z by θ radians in the counter clockwise direction about the origin when (i) $\theta = \frac{\pi}{3}$ (ii) $\theta = \frac{2\pi}{3}$ (iii) $\theta = \frac{3\pi}{2}$.

Soln $z = 2 - 2i$

The rotation of z by θ in clockwise direction about the origin is $ze^{i\theta}$.

$$z = 2 - 2i = 2(1 - i) = r \cos \alpha + i r \sin \alpha$$

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$$r = \sqrt{2} \quad r \cos \alpha = 1 \quad r \sin \alpha = -1$$

$$\cos \alpha = \frac{1}{\sqrt{2}} \quad \sin \alpha = -\frac{1}{\sqrt{2}}$$

$$\alpha = \frac{7\pi}{4} \quad \therefore \theta = -\alpha = -\frac{7\pi}{4}$$

$$z = 2\sqrt{2} (\cos(\frac{7\pi}{4}) + i \sin(\frac{7\pi}{4})) = e^{-i\frac{7\pi}{4}}$$

i) $\theta = \frac{\pi}{3}$

$$ze^{i\theta} = 2\sqrt{2} e^{-i\frac{7\pi}{4}} \cdot e^{i\frac{\pi}{3}} = 2\sqrt{2} e^{i(-\frac{7\pi}{4} + \frac{\pi}{3})} = 2\sqrt{2} e^{i(-\frac{21\pi + 4\pi}{12})}$$

$$= 2\sqrt{2} e^{i\frac{17\pi}{12}}$$

ii) $\theta = \frac{2\pi}{3}$

$$ze^{i\theta} = 2\sqrt{2} e^{-i\frac{7\pi}{4}} \cdot e^{i\frac{2\pi}{3}} = 2\sqrt{2} e^{i(-\frac{7\pi}{4} + \frac{2\pi}{3})} = 2\sqrt{2} e^{i(-\frac{21\pi + 8\pi}{12})}$$

$$= 2\sqrt{2} e^{i\frac{29\pi}{12}}$$

iii) $\theta = \frac{3\pi}{2}$

$$ze^{i\theta} = 2\sqrt{2} e^{-i\frac{7\pi}{4}} \cdot e^{i\frac{3\pi}{2}} = 2\sqrt{2} e^{i(-\frac{7\pi}{4} + \frac{3\pi}{2})} = 2\sqrt{2} e^{i(-\frac{7\pi + 6\pi}{4})}$$

$$= 2\sqrt{2} e^{i\frac{13\pi}{4}}$$

10) Prove that the values of $\sqrt[4]{-1}$ are $\pm \frac{1}{\sqrt{2}} (1 \pm i)$

Soln $-1 = \cos \pi + i \sin \pi$

$$(-1)^{1/4} = (\cos \pi + i \sin \pi)^{1/4} = [\cos (2k\pi + \pi) + i \sin (2k\pi + \pi)]^{1/4}$$

$$= \cos \left(\frac{2k+1}{4} \pi \right) + i \sin \left(\frac{2k+1}{4} \pi \right) \quad k = 0, 1, 2, 3.$$

$k=0$ $\sqrt[4]{-1} = \cos \frac{\pi}{4} + i \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}} + i \frac{1}{\sqrt{2}}$

$k=1$ $\sqrt[4]{-1} = \cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} = \cos(\pi - \frac{\pi}{4}) + i \sin(\pi - \frac{\pi}{4})$

$$= -\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} = -\frac{1}{\sqrt{2}} + i \left(\frac{1}{\sqrt{2}} \right)$$

$k=2$ $\sqrt[4]{-1} = \cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4} = \cos(\pi + \frac{\pi}{4}) + i \sin(\pi + \frac{\pi}{4})$

$$= -\cos \frac{\pi}{4} - i \sin \frac{\pi}{4} = -\frac{1}{\sqrt{2}} - i \frac{1}{\sqrt{2}}$$

$k=3$ $\sqrt[4]{-1} = \cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4} = \cos(2\pi - \frac{\pi}{4}) + i \sin(2\pi - \frac{\pi}{4})$

$$= \cos \frac{\pi}{4} - i \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}} - i \frac{1}{\sqrt{2}}$$

$\therefore$ Roots are $\pm \frac{1}{\sqrt{2}} \pm i \frac{1}{\sqrt{2}} = \pm \frac{1}{\sqrt{2}} (1 \pm i)$ Ans.

Exercise 2.9

1. Choose the correct or the most suitable answer from the given four alternatives.

1) $i^n + i^{n+1} + i^{n+2} + i^{n+3}$ is
 $i^n (1 + i + i^2 + i^3) = i^n (1 + i - 1 - i) = i^n (0) = 0$
 Ans (1) 0

2) The value of $\sum_{i=1}^{13} (i^n + i^{n-1})$ is
 Soln $(i^1 + i^0) + (i^2 + i^1) + (i^3 + i^2) + \dots + (i^{13} + i^{12})$
 ~~$i^1 + i^0 + i^2 + i^1 + i^3 + i^2 + \dots + i^{13} + i^{12}$~~
 $= i^0 + 2(i^1 + i^2 + i^3 + i^4 + i^5 + i^6 + i^7 + \dots + i^{12}) + i^{13}$
 $= 1 + 2(i - i - i + i + i - i - i + i + i - i - i + i) + i$
 $= 1 + 2(0) + i = 1 + i$
 Ans (1) $1 + i$

- 3) The area of the triangle formed by the complex numbers z , iz , and $z+iz$ in the Argand's diagram is

Soln $|A-B| = |z-iz|$

P is mid point of AB $= \frac{A+B}{2} = \frac{z+iz}{2}$

$PC = |z+iz - (\frac{z+iz}{2})| = |\frac{z+iz}{2}|$

Area of the triangle $= \frac{1}{2} AB \times PC = \frac{1}{2} |z-iz| |\frac{z+iz}{2}|$

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$= \frac{1}{4} |z^2 + z^2| = \frac{1}{4} 2|z|^2 = \frac{|z|^2}{2}$
 Ans (1) $\frac{|z|^2}{2}$

- 4) The conjugate of the complex number is $\frac{1}{i-2}$ Then the complex number is

Soln $\frac{1}{i-2} \times \frac{-i-2}{-i-2} = \frac{1}{-i-2} = \frac{-1}{2+i}$
 Ans (2) $\frac{-1}{i+2}$

- 5) If $z = \frac{(3+4i)^3 (3i+4)^2}{(8+6i)^2}$ then $|z|$ is equal to.

Soln $|z| = \frac{|3+4i|^3 |3i+4|^2}{|8+6i|^2} = \frac{(\sqrt{3^2+4^2})^3 (\sqrt{3^2+4^2})^2}{(\sqrt{8^2+6^2})^2} = \frac{2^3 (5)^2}{10^2}$
 $= \frac{8 \times 25}{100} = 2$
 Ans (3) 2

- 6) If z is a non zero complex number, such that $2iz^2 = \bar{z}$ then $|z|$ is (33)
- Soln $2iz^2 = \bar{z}$
 $2iz = \frac{\bar{z}}{z} = \frac{1}{z}$
 $2iz = \frac{1}{z} \times \frac{i}{-i} = \frac{-i}{2}$
 $|z| = \sqrt{\frac{1}{4}} = \frac{1}{2}$ Ans ① $\frac{1}{2}$
- 7) If $|z - 2 + i| \leq 2$ then the greatest value of $|z|$ is
- Soln $|z - 2 + i| = |z + (-2 + i)| \leq |z| + |-2 + i|$
 $\leq |z| + \sqrt{2^2 + 1^2}$
 $2 \geq |z| + \sqrt{5}$
 $2 + \sqrt{5} \geq |z|$
 $|z| \leq 2 + \sqrt{5}$ Ans ④ $\sqrt{5} + 2$
- 8) If $|z - \frac{3}{z}| = 2$ then the least value of $|z|$ is
- Soln $|z| = |z - \frac{3}{z} + \frac{3}{z}| \leq |z - \frac{3}{z}| + |\frac{3}{z}|$
 $|z| \leq 2 + \frac{3}{|z|}$
 $|z| - 2 - \frac{3}{|z|} \leq 0$
 $|z|^2 - 2|z| - 3 \leq 0$
 $|z|$ is $+3$ or -1
least value is 1 Ans ⑤ 1
- 9) If $|z| = 1$ then the value of $\frac{1+z}{1+\bar{z}}$ is
- Soln $|z| = 1$
 $x^2 + y^2 = 1$
 $\frac{1+z}{1+\bar{z}} = \frac{(1+x) + iy}{(1+x) - iy} \times \frac{(1+x) + iy}{(1+x) + iy}$
 $= \frac{(1+x)^2 - y^2 + 2iy(1+x)}{(1+x)^2 + y^2} = \frac{1+x^2+2x+y^2+2iy(1+x)}{1+x^2+2x+y^2}$
 $\frac{1+z}{1+\bar{z}} = \frac{1+z}{1+\frac{1}{z}} = \frac{1+z}{\frac{z+1}{z}} = z \frac{(1+z)}{(1+z)} = z$
Ans (z)

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-3
 $\begin{matrix} -3 & \wedge & 1 \end{matrix}$

10) The solution of the equation $|z| - z = 1 + 2i$ is

Soln $|z| - 1 - 2i = z$

$$|z|^2 = z \bar{z} = (|z| - 1 - 2i)(|z| - 1 + 2i)$$

$$= |z|^2 + |z|(-1 + 2i - 1 - 2i) + (-1 - 2i)(-1 + 2i)$$

$$|z|^2 = |z|^2 - 2|z| + 5 \Rightarrow 2|z| = 5$$

$$|z| = \frac{5}{2}$$

put in ①

$$z = \frac{5}{2} - 1 - 2i = \frac{3}{2} - 2i$$

Ans $\frac{3}{2} - 2i$

11) The solution if $|z_1| = 1$, $|z_2| = 2$, $|z_3| = 3$ and $|9z_1z_2 + 4z_1z_3 + z_2z_3| = 12$ then the value of $|z_1 + z_2 + z_3|$ is

Soln $|z_1| = 1$, $|z_2| = 2$, $|z_3| = 3$

$$|z_1|^2 = 1, |z_2|^2 = 4, |z_3|^2 = 9$$

$$z_1 \bar{z}_1 = 1, z_2 \bar{z}_2 = 4, z_3 \bar{z}_3 = 9$$

$$z_1 = \frac{1}{\bar{z}_1}, z_2 = \frac{4}{\bar{z}_2}, z_3 = \frac{9}{\bar{z}_3}$$

$$z_1 + z_2 + z_3 = \frac{1}{\bar{z}_1} + \frac{4}{\bar{z}_2} + \frac{9}{\bar{z}_3} = \frac{z_2 \bar{z}_3 + 4 \bar{z}_1 \bar{z}_3 + 9 \bar{z}_1 \bar{z}_2}{z_1 \bar{z}_2 \bar{z}_3}$$

$$|z_1 + z_2 + z_3| = \frac{|9z_1z_2 + 4z_1z_3 + z_2z_3|}{|z_1z_2z_3|} = \frac{12}{1 \cdot 2 \cdot 3} = 1$$

$$|z_1 + z_2 + z_3| = 1$$

Ans. 1

(12) If z is a complex number such that $z \in \mathbb{C} / \mathbb{R}$ and $z + \frac{1}{z} \in \mathbb{R}$ then $|z|$ is

$$z = x + iy$$

$$\frac{1}{z} = x - iy$$

$$z + \frac{1}{z} =$$

(35)

- 13) z_1, z_2 and z_3 are complex numbers such that $z_1 + z_2 + z_3 = 0$ and $|z_1| = |z_2| = |z_3| = 1$ then $z_1^2 + z_2^2 + z_3^2$ is

Soln $|z_1| = 1 \Rightarrow z_1 \bar{z}_1 = 1 \Rightarrow \bar{z}_1 = \frac{1}{z_1}$ $|z_2| = 1 \Rightarrow \bar{z}_2 = \frac{1}{z_2}$ $|z_3| = 1 \Rightarrow \bar{z}_3 = \frac{1}{z_3}$

$$(z_1 + z_2 + z_3) = \frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} \Rightarrow \frac{z_2 z_3 + z_1 z_3 + z_1 z_2}{|z_1 z_2 z_3|}$$

$$0 = \frac{z_1 z_2 + z_3 z_2 + z_1 z_3}{|z_1 z_2 z_3|} \Rightarrow |z_1 z_2 + z_2 z_3 + z_3 z_1| = 0$$

$$\therefore (z_1 + z_2 + z_3)^2 = z_1^2 + z_2^2 + z_3^2 + 2(z_1 z_2 + z_3 z_2 + z_3 z_1)$$

$$0 = z_1^2 + z_2^2 + z_3^2 + 2(0)$$

$$\therefore z_1^2 + z_2^2 + z_3^2 = 0$$

- 14) If $\frac{z-1}{z+1}$ is purely imaginary then $|z|$ is

$$\frac{z-1}{z+1} = \frac{x+iy-1}{x+iy+1} = \frac{(x-1)+iy}{(x+1)+iy} \cdot \frac{(x+1)-iy}{(x+1)-iy}$$

$$= \frac{(x^2-1) - iy(x-1) + iy(x+1) + y^2}{(x+1)^2 + y^2}$$

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it is purely imaginary $\text{Re} \left(\frac{z-1}{z+1} \right) = 0$

$$\frac{x^2-1+y^2}{(x+1)^2+y^2} = 0$$

$$x^2+y^2-1=0$$

$$x^2+y^2=1$$

$$|z|^2=1$$

$$|z|=1$$

Ans 1

- 15) if $z = x+iy$ is a complex number such that $|z+2| = |z-2|$ then the locus of z is

Soln $|z+2| = |z-2|$

$$|x+iy+2| = |x+iy-2|$$

$$\sqrt{(x+2)^2+y^2} = \sqrt{(x-2)^2+y^2}$$

$$x^2+4x+y^2 = x^2-4x+y^2$$

$$8x = 0$$

$x = 0$ which is imaginary axis

Ans (2)

(36)

16) The principal argument of $\frac{3}{-1+i}$ is

Soln $\frac{3}{-1+i} \frac{(-1-i)}{(-1-i)} = \frac{3(-1-i)}{-1^2+1^2} = \frac{3}{2} (-1-i)$

$$-1-i = r(\cos\theta + i\sin\theta)$$

$$r = \sqrt{2} \quad \cos\theta = -\frac{1}{\sqrt{2}} \quad \sin\theta = -\frac{1}{\sqrt{2}}$$

$\theta = \theta_4$ which is III quadrant

$$\theta = \theta - \pi = \theta_4 - \pi = -3\theta_4$$

$$-1-i = \sqrt{2} [\cos(-3\theta_4) + i\sin(-3\theta_4)]$$

$$= \frac{3}{2} \sqrt{2} (\cos(-3\theta_4) + i\sin(-3\theta_4))$$

argument is $-3\theta_4$ Ans (3) $-3\theta_4$

17) The principal argument of $(\sin 40^\circ + i\cos 40^\circ)^5$ is

$$\begin{aligned} (\sin 40^\circ + i\cos 40^\circ)^5 &= (\cos 50^\circ + i\sin 50^\circ)^5 \\ &= (\cos 250^\circ + i\sin 250^\circ) \\ &= \cos(360^\circ - 110^\circ) + i\sin(360^\circ - 110^\circ) \end{aligned}$$

Principal argument is (-110°)

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Ans -110°

18) If $(1+i)(1+2i)(1+3i)\dots(1+ni) = x+iy$ then
 $2 \cdot 5 \cdot 10 \dots (1+n^2)$ is

Soln Take modulus on both sides

$$\sqrt{1^2+1^2} \sqrt{1^2+2^2} \sqrt{1^2+3^2} \dots \sqrt{1+n^2} = \sqrt{x^2+y^2}$$

$$(2)(5)(10)\dots(1+n^2) = x^2+y^2$$

Ans (3) x^2+y^2

19) If $w \neq 1$ is a cube root of unity and $(1+w)^7 = A+Bw$
Then $(A+B)$ equals.

Soln

$$(1+\omega)^2 = 1 + \omega^2 + 2\omega = -\omega + 2\omega = \omega$$

$$[(1+\omega)^2]^3 = (1+\omega)^6 = \omega^6 = 1$$

$$(1+\omega)^6 (1+\omega) = 1+\omega = A+B\omega$$

$$\therefore A=1 \quad B=1.$$

Ans (4) (1,1)

$$(A,B) = (1,1)$$

20) The principal argument of the Complex number

$$\frac{(1+i\sqrt{3})^2}{4i(1-i\sqrt{3})}$$
 is

$$\arg \left(\frac{(1+i\sqrt{3})^2}{4i(1-i\sqrt{3})} \right) = \arg(1+i\sqrt{3})^2 - \arg 4i - \arg(1-i\sqrt{3})$$

$$= 2 \arg(1+i\sqrt{3}) - \arg(4i) - \arg(1-i\sqrt{3})$$

$$= 2 \tan^{-1} \sqrt{3} - \tan^{-1} \left(\frac{4}{0} \right) - \tan^{-1} \left(-\frac{\sqrt{3}}{1} \right)$$

$$= 2 \cdot \frac{\pi}{3} - \frac{\pi}{2} + \frac{\pi}{3} = \pi - \frac{\pi}{2}$$

$$= \frac{\pi}{2}$$

Ans $\frac{\pi}{2}$.

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21) If α & β are the roots of $x^2+x+1=0$ then $\alpha^{2020} + \beta^{2020}$ is

$$\text{Soln } x^2+x+1=0.$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1} = \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm i\sqrt{3}}{2}$$

$$\alpha = \frac{-1+i\sqrt{3}}{2}, \quad \beta = \frac{-1-i\sqrt{3}}{2}$$

$$\alpha = \omega$$

$$\beta = \omega^2$$

$$\alpha^{2020} + \beta^{2020}$$

$$= \omega^{2020} + (\omega^2)^{2020}$$

$$= \omega^{2020} + \omega^{4040}$$

$$= \omega^1 + \omega^2 = \omega + \omega^2$$

$$= -1$$

Ans (A) (-1)

$$\begin{array}{r} 2020 \quad 2020 \\ 3 \overline{) 12} \\ \underline{9} \quad 22 \\ \underline{21} \quad 28 \\ \underline{27} \quad 10 \\ \underline{9} \quad 1 \end{array}$$

$$\begin{array}{r} \omega^3 = 1 \\ 3 \overline{) 134} \\ \underline{9} \quad 40 \\ \underline{3} \quad 10 \\ \underline{9} \quad 14 \\ \underline{9} \quad 5 \end{array}$$

22) The product of all four values of $(\cos \theta_3 + i \sin \theta_3)^{3/4}$ is

Soln $(\cos \theta_3 + i \sin \theta_3)^{3/4} = (\cos \theta + i \sin \theta)^{1/4}$ (38)

$$= \cos \frac{(2k+1)\pi}{4} + i \sin \frac{(2k+1)\pi}{4} \quad k = 0, 1, 2, 3$$

$k=0, \quad \text{cis } \pi/4, \quad k=2 \Rightarrow \text{cis } 5\pi/4$

$k=1 \quad \text{cis } 3\pi/4 \quad k=3 \Rightarrow \text{cis } 7\pi/4$

Product of four values $\text{cis} (\pi/4 + 3\pi/4 + 5\pi/4 + 7\pi/4)$

$\text{cis} (16\pi/4) = \text{cis } 4\pi = (-1)^4 = 1$
Ans (3) 1

23) if $w \neq 1$ is a cubic root of unity and $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -w^2-1 & w^2 \\ 1 & w^2 & w^7 \end{vmatrix} = 31k$
then k is equal to.

Soln $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -w^2-1 & w^2 \\ 1 & w^2 & w^7 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & w & w^2 \\ 1 & w^2 & w \end{vmatrix} = 31k$

$\because 1+w+w^2=0$
 $w = -1-w^2$
 $\because w^3=1$
 $w^4=1$

$1(w^2-w) - 1(w-w^2) + 1(w^2-w) = 31k$

$w^2-w - w + w^2 + w^2 - w = 31k$

$3w^2 - 3w = 31k$

$w^2 - w = k$

$k = \left(\frac{-1 - i\sqrt{3}}{2} \right) - \left(\frac{-1 + i\sqrt{3}}{2} \right) = \frac{-1 - i\sqrt{3} + 1 - i\sqrt{3}}{2}$

$= \frac{-2i\sqrt{3}}{2} = -i\sqrt{3}$

Ans (4) $-i\sqrt{3}$

24) The value of $\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}} \right)^{10}$ is

Soln $1+i\sqrt{3} = r(\cos \theta + i \sin \theta)$

$r=2 \quad \cos \theta = 1/2 \quad \sin \theta = \sqrt{3}/2$

$\therefore \theta = \pi/3$

$1+i\sqrt{3} = 2(\cos \pi/3 + i \sin \pi/3) = 2e^{i\pi/3}$

$1-i\sqrt{3} = 2e^{-i\pi/3}$

$\frac{1+i\sqrt{3}}{1-i\sqrt{3}} = \frac{2e^{i\pi/3}}{2e^{-i\pi/3}} = e^{i2\pi/3}$

$\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}} \right)^{10} = (e^{i2\pi/3})^{10}$

Ans (5) $e^{i2\pi/3}$

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(39)

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25) If $\omega = \text{cis } 2\pi/3$ then the number of distinct roots of

$$\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0.$$

Soln

$$\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0$$

$$[\because \omega^3 = 1 \\ 1 + \omega + \omega^2 = 0]$$

$$(z+1)(z^2 + z\omega + \omega^2 z + 1 - 1) - \omega(z\omega + \omega^2 - \omega^2) + \omega^2(\omega - z\omega\omega) = 0$$

$$z^3 + z^2\omega + z^2\omega^2 + z^2 + z\omega + \omega^2 z - z\omega^2 - z\omega = 0$$

$$z^2(z + \omega + \omega^2 + 1) = 0$$

$$z^2(z + 0) = 0$$

$$z^3 = 0$$

$$z = 0, 0, 0$$

The number of distinct roots is 1

CHAPTER 5 THEOREY OF EQUATIONS

1. $ax^2 + bx + c = 0$
 $\alpha + \beta = -\frac{b}{a}$ $\alpha\beta = \frac{c}{a}$ $\alpha^2 - (a+\beta)\alpha + \alpha\beta = 0$
2. $ax^2 + bx^2 + c = 0$
 $\alpha + \beta + \gamma = -\frac{b}{a}$, $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$ and $\alpha\beta\gamma = -\frac{c}{a}$.
3. If z_0 is a root of a polynomial equation with real coefficients then its complex conjugate $\bar{z}_0$ is also a root.
4. Let p and q be rational numbers such that $\sqrt{q}$ is irrational. If $p + \sqrt{q}$ is a root of a quadratic equation then $p - \sqrt{q}$ is also a root.
5. $ax^4 + bx^3 + cx^2 + dx + e = 0$
 $\alpha + \beta + \gamma + \delta = -\frac{b}{a}$, $\alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta = \frac{c}{a}$
 $\alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta = -\frac{d}{a}$ & $\alpha\beta\gamma\delta = \frac{e}{a}$.
6. $\sqrt{p} + \sqrt{q}$ is a root of a polynomial then $\pm\sqrt{p} \pm \sqrt{q}$ is also a root of a polynomial.
- 7) Nature of a roots $ax^2 + bx + c = 0$
 $D = b^2 - 4ac < 0$ The roots are imaginary.
 $D = 0$ The roots are equal.
 $D > 0$ The roots are distinct and real.
8. If $p(x) = 0$ is a polynomial equation such that whenever α is a root, $\frac{1}{\alpha}$ is also a root then the polynomial equation $p(x) = 0$ must be a reciprocal equation.
9. Rational root theorem: Let $ax^n + \dots + a_1x + a_0$ with $a_n \neq 0$ $a_0 \neq 0$ be a polynomial with integer coefficients. If $\frac{p}{q}$ with $(p, q) = 1$ is a root of the polynomial then p is a factor of a_0 and q is a factor of a_n .
- 10) Descartes Rule: If p is a number of positive roots of a polynomial $p(x)$ and s is the number of sign changes in coefficients of $p(x)$ then $s - p$ is a non negative even integer.

Chapter: 3

Theory of Equations

(2)

Exercise 3.1

1.

If the sides of a cubic box are increased by 1, 2, 3 units respectively to form a cuboid, then the volume is increased by 52 cubic units. Find the volume of the cuboid.

Soln let the sides of cubic boxes are x, x, x .

$$\text{volume of a cube} = x^3$$

it is increased by 1, 2, 3.

$$\text{cuboid sides be } (x+1), (x+2), (x+3)$$

$$\text{volume of a cuboid}$$

$$(x+1)(x+2)(x+3) = x^3 + 52.$$

$$(x^2 + 3x + 2)(x+3) = x^3 + 52$$

$$x^3 + 3x^2 + 2x + 3x^2 + 9x + 6 = x^3 + 52$$

$$6x^2 + 11x + 6 - 52 = 0$$

$$6x^2 + 11x - 46 = 0$$

$$6x^2 + 12x + 23x - 46 = 0$$

$$6x(x+2) + 23(x-2) = 0$$

$$x+2 = 0 \quad 6x+23 = 0$$

$$x = -2 \quad 6x = -23$$

$$x = -\frac{23}{6} \text{ (is not possible)}$$

$$\therefore x = 2.$$

$$\text{volume of a cube} = x^3 = 2^3 = 8.$$

2. Construct a cubic equation with roots.

(i) 1, 2 and 3

$$\alpha = 1, \beta = 2, \gamma = 3$$

$$\alpha + \beta + \gamma = 6$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = 2 + 6 + 3 = 11$$

$$\alpha\beta\gamma = 6.$$

$$x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma = 0$$

$$x^3 - 6x^2 + 11x - 6 = 0$$

(ii) 1, 1 and -2.

$$\alpha = 1, \beta = 1, \gamma = -2.$$

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(3)

$$\alpha + \beta + \gamma = 1 + 1 - 2 = 0$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = 1 - 2 - 2 = -3$$

$$\alpha\beta\gamma = 1(1)(-2) = -2$$

$$x^3 - 0x^2 - 3x + 2 = 0$$

$$x^3 - 3x + 2 = 0$$

(ii)

2, -2 and 4.

$$\alpha = 2 \quad \beta = -2 \quad \gamma = 4$$

$$\alpha + \beta + \gamma = 2 - 2 + 4 = 4$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = -4 - 8 + 8 = -4$$

$$\alpha\beta\gamma = 2(-2)(4) = -16$$

$$x^3 - 4x^2 - 4x + 16 = 0$$

3

If α, β and γ are the roots of the cubic equation $x^3 + 2x^2 + 3x + 4 = 0$ form a cubic equation whose roots are

(i) $2\alpha, 2\beta, 2\gamma$.

$$x^3 + 2x^2 + 3x + 4 = 0$$

$$\alpha + \beta + \gamma = -\frac{b}{a} = -\frac{2}{1} = -2$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} = \frac{3}{1} = 3$$

$$\alpha\beta\gamma = -\frac{d}{a} = -4$$

$$(i) \quad 2\alpha + 2\beta + 2\gamma = 2(\alpha + \beta + \gamma) = 2(-2) = -4$$

$$(2\alpha)(2\beta) + (2\beta)(2\gamma) + (2\gamma)(2\alpha) = 4\alpha\beta + 4\beta\gamma + 4\gamma\alpha$$

$$= 4(\alpha\beta + \beta\gamma + \gamma\alpha) = 4(3) = 12$$

$$2\alpha(2\beta)(2\gamma) = 8(\alpha\beta\gamma) = 8(-4) = -32$$

$$x^3 - (-4)x^2 + 12x - (-32) = 0$$

$$x^3 + 4x^2 + 12x + 32 = 0$$

4. Solve the equation $3x^3 - 16x^2 + 23x - 6 = 0$ if the product of two roots is 1

Soln let the roots be α, β, γ .

$$\text{Given } \alpha\beta = 1$$

$$\beta = \frac{1}{\alpha}$$

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Sum of the roots $\alpha + \frac{1}{\alpha} + \gamma = \frac{16}{3}$ — (1)

(4)

Product of the roots $\alpha \cdot \frac{1}{\alpha} \cdot \gamma = \frac{6}{3} = 2$
 $\gamma = 2$

Put in (1)

$$\alpha + \frac{1}{\alpha} + 2 = \frac{16}{3} \Rightarrow \alpha + \frac{1}{\alpha} = \frac{16}{3} - 2 = \frac{16-6}{3} = \frac{10}{3}$$

$$\frac{\alpha^2 + 1}{\alpha} = \frac{10}{3} \Rightarrow 3\alpha^2 + 3 = 10\alpha \Rightarrow 3\alpha^2 - 10\alpha + 3 = 0$$

$$(3\alpha - 1)(\alpha - 3) = 0$$

$$\alpha = 3 \quad \alpha = \frac{1}{3}$$

$$\begin{array}{r} -10 \\ 3 \overline{) 39} \\ \underline{39} \\ 0 \\ \underline{0} \\ 0 \end{array}$$

If $\alpha = 3 \quad \beta = \frac{1}{3}, \gamma = 2$

(ii) if $\alpha = \frac{1}{3} \quad \beta = 3, \gamma = 2$

5. Find the Sum of squares of roots of the equation

$$2x^4 - 8x^3 + 6x^2 - 3 = 0$$

soln

$$\alpha + \beta + \gamma + \delta = \frac{8}{2} = 4$$

$$\alpha\beta + \beta\gamma + \gamma\alpha + \delta\alpha + \beta\delta + \gamma\delta = \frac{6}{2} = 3$$

$$(\alpha + \beta + \gamma + \delta)^2 = \alpha^2 + \beta^2 + \gamma^2 + \delta^2 + 2(\alpha\beta + \beta\gamma + \gamma\alpha + \delta\alpha + \beta\delta + \gamma\delta)$$

$$\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = (\alpha + \beta + \gamma + \delta)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha + \delta\alpha + \beta\delta + \gamma\delta)$$

$$= 4^2 - 2(3) = 16 - 6 = 10$$

6. Solve the equation $x^3 - 9x^2 + 14x + 24 = 0$ if it is given that two of its roots are in the ratio 3:2.

soln. let the roots be $3\alpha, 2\alpha, \beta$.

Sum of the roots: $3\alpha + 2\alpha + \beta = +9$
 $5\alpha + \beta = 9$ — (1)

Product of two roots: $(3\alpha)(2\alpha) + 2\alpha(\beta) + 3\alpha(\beta) = 14$
 $6\alpha^2 + 5\alpha\beta = 14$ — (2)

Product of 3 roots: $(3\alpha)(2\alpha)\beta = -24$
 $\alpha^2\beta = -4$ — (3)

(1) $\Rightarrow \beta = 9 - 5\alpha$

(2) $\Rightarrow 6\alpha^2 + 5\alpha(9 - 5\alpha) = 14$
 $6\alpha^2 + 45\alpha - 25\alpha^2 = 14$

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 கோவில்

$$-19x^2 + 45x - 14 = 0$$

$$19x^2 - 45x + 14 = 0$$

$$(x-2)(x-\frac{7}{19}) = 0$$

$$x=2 \quad x=\frac{7}{19}$$

If $x=2$, $\beta = 9 - 5(x) = 9 - 5(2) = 9 - 10 = -1$

Roots are $3x, 2x, \beta$

$$3(2), 2(2), -1 \quad (\text{i.e.}) \quad 6, 4, -1$$

If $x=\frac{7}{19}$, $\beta = 9 - 5(\frac{7}{19}) = \frac{136}{19}$

Roots are $3x, 2x, \beta$ (i.e.) $\frac{21}{19}, \frac{14}{19}, \frac{136}{19}$

7) If α, β and γ are the roots of the polynomial equation $ax^3 + bx^2 + cx + d = 0$ find the value of $\sum \frac{x}{\beta\gamma}$ in terms of the coefficients

Soln

$$\sum \frac{x}{\beta\gamma} = \frac{x}{\beta\gamma} + \frac{\beta}{\alpha\gamma} + \frac{\gamma}{\alpha\beta} = \frac{x^2 + \beta^2 + \gamma^2}{\alpha\beta\gamma}$$

$$\alpha + \beta + \gamma = -\frac{b}{a}, \quad \alpha\beta\gamma = -\frac{d}{a}, \quad \alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$$

$$x^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)$$

$$= \left(-\frac{b}{a}\right)^2 - 2\left(\frac{c}{a}\right) = \frac{b^2}{a^2} - \frac{2c}{a}$$

$$\sum \frac{x}{\beta\gamma} = \frac{\frac{b^2}{a^2} - \frac{2c}{a}}{-\frac{d}{a}} = \frac{b^2 - 2ac}{a^2} \times \frac{a}{-d} = \frac{b^2 - 2ac}{-ad}$$

$$= \frac{2ac - b^2}{ad}$$

8) If α, β, γ and δ are the roots of the polynomial equation $2x^4 + 5x^3 - 7x^2 + 8 = 0$. Find a quadratic equation with integer coefficients whose roots are $\alpha\beta\gamma + \delta$ and $\alpha\beta\gamma\delta$.

Soln

$$p: \alpha + \beta + \gamma + \delta = -\frac{5}{2}$$

$$q: \alpha\beta\gamma\delta = -\frac{8}{2} = -4$$

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$$p+q = -\frac{5}{2} - 4 = -\frac{5-8}{2} = -\frac{13}{2} \quad (6)$$

$$pq = -\frac{5}{2}(-4) = +10$$

$$x^2 - (p+q)x + pq = 0$$

$$x^2 + \frac{13}{2}x + 10 = 0$$

$$2x^2 + 13x + 20 = 0$$

9) If p and q are the roots of the equation $lx^2 + mx + n = 0$ show that $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = 0$.

Soln. p & q are the roots of the equation

$$p+q = -\frac{n}{l}, \quad pq = \frac{n}{l}$$

$$\text{L.H.S.} \quad \sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = \frac{\sqrt{p}}{\sqrt{q}} + \frac{\sqrt{q}}{\sqrt{p}} + \frac{\sqrt{n}}{\sqrt{l}} = \frac{p+q}{\sqrt{p}\sqrt{q}} + \frac{\sqrt{n}}{\sqrt{l}}$$

$$= \frac{-\frac{n}{l}}{\sqrt{pq}} + \frac{\sqrt{n}}{\sqrt{l}} = \frac{-\frac{n}{l}}{\sqrt{\frac{n}{l}}} + \frac{\sqrt{n}}{\sqrt{l}}$$

$$= \frac{-\frac{n}{l} + \frac{n}{l}}{\sqrt{\frac{n}{l}}} = 0 = R.H.S.$$

10)

If the equations $x^2 + px + q = 0$ and $x^2 + p'x + q' = 0$ have a common root, show that it must be equal to $\frac{pq' - p'q}{q - q'}$ or $\frac{q - q'}{p' - p}$

Soln. Let α be common roots of $x^2 + px + q = 0$ and $x^2 + p'x + q' = 0$

$$(i) \quad \alpha^2 + p\alpha + q = 0 \quad \text{and} \quad \alpha^2 + p'\alpha + q' = 0$$

$$\frac{\alpha^2}{pq' - p'q} = \frac{\alpha}{q - q'} = \frac{1}{p' - p}$$

$$\alpha^2 = \frac{pq' - p'q}{p' - p}$$

$$\alpha = \frac{q - q'}{p' - p}$$

$$\alpha = \frac{\alpha^2}{\alpha} = \frac{\frac{pq' - p'q}{p' - p}}{\frac{q - q'}{p' - p}} = \frac{pq' - p'q}{q - q'}$$

$$\begin{array}{cccc} x^2 & \alpha & 1 \\ 1 & p & q & 1 \\ 1 & p' & q' & 1 \end{array}$$

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Hence proved.

- 11) Formulate into a mathematical problem to find a number such that when its cube root is added to it, the result is 6. (7)

Soln. Let the number be x
its cube root be $\sqrt[3]{x}$.

$$x + \sqrt[3]{x} = 6$$

$$\sqrt[3]{x} = 6 - x$$

$$x^{1/3} = 6 - x \Rightarrow x = (6 - x)^3$$

$$x = 6^3 - x^3 + 3(6^2)x - 3(6^2)x$$

$$216 - x^3 + 18x^2 - 108x - x = 0$$

$$-x^3 + 18x^2 - 109x + 216 = 0$$

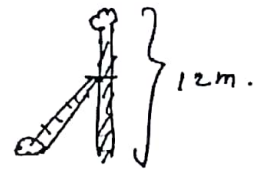
$$x^3 - 18x^2 + 109x - 216 = 0$$

- 12) A 12 metre tall tree was broken into two parts. It was found that the height of the part which was left standing was the cube root of the length of the part that was cut away. Formulate this into a mathematical problem to find the height of the part which was cut away.

Soln Let the height of tree = 12.

length of cut part = x^3

length of left out part = $\sqrt[3]{x^3} = x$.



$$x + x^3 = 12$$

$$x^3 + x - 12 = 0$$

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Exercise 3.2

(8)

1. If k is real, discuss the nature of the roots of the polynomial equation $2x^2 + kx + k = 0$ in terms of k

Soln

$$\Delta = b^2 - 4ac$$

$$a = 2 \quad b = k \quad c = k$$

$$\Delta = k^2 - 4 \cdot 2(k)$$

$$\Delta = k^2 - 8k$$

If $k < 0$, The polynomial has real roots ($\Delta > 0$)

If $\Delta = 0 \quad k^2 - 8k = 0$
 $k(k - 8) = 0$

$$k = 0 \text{ or } k = 8$$

$\therefore k = 0$ or $k = 8$ The roots are real & equal

If $0 < k < 8 \quad \Delta < 0$ The roots are imaginary

If $k > 8$ The roots are real and distinct.

- 2) Find a polynomial equation of minimum degree with rational coefficients having $2 + i\sqrt{3}$ as a root.

Soln

Let the roots be $2 + i\sqrt{3}$

another roots be $2 - i\sqrt{3}$.

$$\text{Sum of the roots} = 2 + i\sqrt{3} + 2 - i\sqrt{3} = 4$$

$$\text{Product of the roots} = (2 + i\sqrt{3})(2 - i\sqrt{3}) = 2^2 + (\sqrt{3})^2$$

$$= 4 + 3 = 7$$

$$x^2 - (SR)x + PR = 0$$

$$\boxed{x^2 - 4x + 7 = 0}$$

- 3) Find a polynomial equation of minimum degree with rational coefficients having $2i + 3$ as a root

Soln

Let the roots be $3 + 2i$

another root be $3 - 2i$

$$SR = 3 + 2i + 3 - 2i = 6$$

$$PR = (3 + 2i)(3 - 2i) = 3^2 + 2^2 = 9 + 4 = 13$$

$$x^2 - (SR)x + PR = 0$$

$$\boxed{x^2 - 6x + 13 = 0}$$

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4. Find a polynomial equation of minimum degree with rational coefficients having $\sqrt{5}-\sqrt{3}$ as a root. (9)

Soln Let the root be $\sqrt{5}-\sqrt{3}$
another root is $\sqrt{5}+\sqrt{3}$

$$SR = \sqrt{5}-\sqrt{3} + \sqrt{5}+\sqrt{3} = 2\sqrt{5}$$

$$PR = (\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3}) = \sqrt{5}^2 - \sqrt{3}^2 = 5-3=2$$

$$x^2 - (SR)x + PR = 0$$

$$x^2 - 2\sqrt{5}x + 2 = 0 \text{ which is not rational coefficient.}$$

To make rational coefficient.

$$(x^2 - 2\sqrt{5}x + 2)(x^2 + 2\sqrt{5}x + 2) = 0$$

$$(x^2 + 2)^2 - (2\sqrt{5}x)^2 = 0$$

$$(2\sqrt{5})^2 = 4 \times 5 = 20$$

$$x^4 + 4 + 4x^2 - 20x^2 = 0$$

$$\boxed{x^4 - 16x^2 + 4 = 0}$$

is a rational coefficients polynomial equation.

- 5) Prove that a straight line and parabola cannot intersect at more than two points.

Soln, let the equation of st. line be $y = mx + c$

let the equation of parabola $y^2 = 4ax$

$$(mx + c)^2 = 4ax$$

$$m^2x^2 + c^2 + 2mcx - 4ax = 0$$

$$m^2x^2 + (2c - 4a)x + c^2 = 0$$

which is a quadratic equation.

$\therefore$ This equation can't have more than two solution.

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4)

Exercise 3.3

(10)

1. Solve the cubic equation $2x^3 - x^2 - 18x + 9 = 0$ if sum of two of its roots vanishes

soln $2x^3 - x^2 - 18x + 9 = 0$

let the roots be α, β, γ .

Given $\alpha + \beta = 0$.

$\alpha + \beta + \gamma = \frac{1}{2}$

$\gamma = \frac{1}{2}$.

$\gamma = \frac{1}{2}$.

$\alpha + \beta = 0 \quad x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$x^2 - 0x + p = 0$

$(x^2 + p) = 0$

$(x - \frac{1}{2}) = 0$

$2x - 1 = 0$.

$\therefore (x^2 + p)(2x - \frac{1}{2}) = 2x^3 - x^2 - 18x + 9$.

$2x^3 - x^2 + 2px - p = 2x^3 - x^2 - 18x + 9$.

$-p = 9$

$p = -9$

$x^2 - 9 = 0$

$x^2 = 9$

$x = \pm 3$

Roots are 3, -3, $\frac{1}{2}$.

2. Solve the equation $9x^3 - 36x^2 + 44x - 16 = 0$ if the roots form an arithmetic progression.

soln. let the roots are in A.P

let us take $a-d, a, a+d$.

Sum of the roots $= a-d + a + a+d = \frac{36}{9} = 4$

$3a = 4$

$a = \frac{4}{3}$

Product of the roots $= (a-d)a(a+d) = (a^2 - d^2)a = \frac{16}{9}$

$(\left(\frac{4}{3}\right)^2 - d^2) \left(\frac{4}{3}\right) = \frac{16}{9}$

$\frac{16}{9} - d^2 = \frac{4}{3}$

$\frac{16}{9} - \frac{4}{3} = d^2$

(10)

$$d^2 = \frac{16-12}{9} = \frac{4}{9}$$

$$d = \frac{2}{3}$$

$$\therefore a-d = \frac{4}{3} - \frac{2}{3} = \frac{2}{3}$$

$$a+d = \frac{4}{3} + \frac{2}{3} = \frac{6}{3} = 2$$

Roots are $a-d, a, a+d$ be $\frac{2}{3}, \frac{4}{3}, 2$.

3) Solve the equation $3x^3 - 26x^2 + 52x - 24 = 0$ if its roots form a geometric progression.

Soln. Let the roots form a G.P. be $\frac{a}{\lambda}, a, a\lambda$

Product of the roots $\frac{a}{\lambda} \times a \times a\lambda = \frac{24}{3} = 8$
 $a^3 = 8$

Sum of the roots $\frac{a}{\lambda} + a + a\lambda = \frac{26}{3}$
 $a = 2$

$$a \left(\frac{1}{\lambda} + 1 + \lambda \right) = \frac{26}{3}$$

$$\left(\frac{1}{\lambda} + 1 + \lambda \right) = \frac{26}{3} \times \frac{1}{2} = \frac{13}{3}$$

$$3 + 3\lambda + 3\lambda^2 = 13\lambda$$

$$3\lambda^2 + 3\lambda - 13\lambda + 3 = 0$$

$$3\lambda^2 - 10\lambda + 3 = 0$$

$$(\lambda - 3)(3\lambda - 1) = 0$$

$$\lambda = 3 \text{ (or) } \lambda = \frac{1}{3}$$

If $\lambda = 3$

$$\frac{a}{\lambda} = \frac{2}{3}, \quad a\lambda = 2(3) = 6$$

If $\lambda = \frac{1}{3}$

$$\frac{a}{\lambda} = \frac{2}{\frac{1}{3}} = 6, \quad a\lambda = 2\left(\frac{1}{3}\right) = \frac{2}{3}$$

Roots are $\frac{a}{\lambda}, a, a\lambda$

if $\lambda = 3$ roots are $\frac{2}{3}, 2, 6$.

if $\lambda = \frac{1}{3}$ roots are $6, 2, \frac{2}{3}$.

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$$\begin{array}{r} 9 \\ 3 \overline{) 27} \\ \underline{- 27} \\ 0 \end{array}$$

- 4) Determine k and solve the equation $2x^3 - 6x^2 + 3x + k = 0$ if one of its roots is twice the sum of the other two roots. (12)

Soln. $2x^3 - 6x^2 + 3x + k = 0$ let the roots be α, β, γ

Given $\alpha = 2(\beta + \gamma)$

$\alpha + \beta + \gamma = 3$ $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{3}{2}$ $\alpha\beta\gamma = -\frac{k}{2}$

① $\therefore 2(\beta + \gamma) + (\beta + \gamma) = 3$

$3(\beta + \gamma) = 3$

$(\beta + \gamma) = 1$

$\therefore \alpha = 2(\beta + \gamma) = 2(1) = 2$

$\gamma = 1 - \beta = 1 - \left(\frac{1 + \sqrt{3}}{2}\right) = \frac{2 - 1 - \sqrt{3}}{2} = \frac{1 - \sqrt{3}}{2}$

⑥ $\Rightarrow \alpha\beta\gamma = -\frac{k}{2}$
 $2\beta\left(\frac{1}{\beta}\right) = -\frac{k}{2}$
 $4 = -k$
 $k = -4$

② $\Rightarrow 2\beta + \beta\left(\frac{1}{\beta}\right) + \frac{1}{\beta}(2) = \frac{3}{2}$
 $2\beta + 1 + \frac{2}{\beta} = \frac{3}{2}$

② $\Rightarrow 2\beta + \beta(1 - \beta) + (1 - \beta)2 = \frac{3}{2}$

$\alpha\beta + \beta - \beta^2 + 2 - 2\beta = \frac{3}{2}$

$-\beta^2 + \beta + 2 = \frac{3}{2}$

$-2\beta^2 + 2\beta + 4 - 3 = 0$

$2\beta^2 - 2\beta - 1 = 0$

$\beta = \frac{2 \pm \sqrt{4 + 8}}{4} = \frac{2 \pm \sqrt{12}}{4} = \frac{2 \pm 2\sqrt{3}}{4}$

$\beta = \frac{1 \pm \sqrt{3}}{2}$

$\therefore \gamma = \frac{1 \pm \sqrt{3}}{2}$

③ $\alpha\beta\gamma = -\frac{k}{2}$

$2\left(\frac{1 + \sqrt{3}}{2}\right)\left(\frac{1 - \sqrt{3}}{2}\right) = -\frac{k}{2}$
 $-k = 1 - 3 \Rightarrow -k = -2 \Rightarrow k = 2$

- 5) Find all zeros of the polynomial $x^6 - 3x^5 - 5x^4 + 22x^3 - 39x^2 + 135$, if it is known that $1 + 2i$ and $\sqrt{3}$ are two of its zeros.

Soln Given roots $1 + 2i, \sqrt{3}$

i) Another roots be $1 - 2i, -\sqrt{3}$

SR of roots $= 1 + 2i + 1 - 2i = 2$

PR of roots $= (1 + 2i)(1 - 2i) = 1^2 + 2^2 = 1 + 4 = 5$

$x^2 - 2x + 5 = 0$

ii) SR of roots $= \sqrt{3} - \sqrt{3} = 0$

PR of roots $= (\sqrt{3})(-\sqrt{3}) = -3$

$x^2 - 0x - 3 = 0$

$x^2 - 3 = 0$

$$(x^2 - 2x + 5)(x^2 - 3) = x^4 - 2x^3 + 2x^2 + 6x - 15 \quad (13)$$

$$x^6 - 3x^5 - 5x^4 + 22x^3 - 39x^2 - 39x + 135 = 0$$

$$= (x^4 - 2x^3 + 2x^2 + 6x - 15)(x^2 + px - 9)$$

Equate coefficient of x on both sides.

$$-39 = -54 - 15p$$

$$-39 + 54 = -15p$$

$$15 = -15p$$

$$\boxed{p = -1}$$

$$\therefore x^2 - x - 9 = 0$$

$$x = \frac{1 \pm \sqrt{1 - 4 \cdot 1 \cdot (-9)}}{2 \cdot 1} = \frac{1 \pm \sqrt{1+36}}{2} = \frac{1 \pm \sqrt{37}}{2}$$

Roots are $\frac{1+\sqrt{37}}{2}, \frac{1-\sqrt{37}}{2}, 1+2i, 1-2i, \sqrt{3}, -\sqrt{3}$.

b) Solve the cubic equations

i) $2x^3 - 9x^2 + 10x = 3$

$$2x^3 - 9x^2 + 10x - 3 = 0$$

Sum of the coefficients $2 - 9 + 10 - 3 = 0$

$\therefore x = 1$ is one of the roots

$$\begin{array}{r|rrrr} 1 & 2 & -9 & 10 & -3 \\ & 0 & 2 & -7 & 3 \\ \hline & 2 & -7 & 3 & 0 \end{array}$$

$$2x^2 - 7x + 3 = 0$$

$$(x-3)(2x-1) = 0$$

$$x = 3 \text{ or } x = \frac{1}{2}$$

Roots are $3, \frac{1}{2}, 1$.

ii) Solve the equation $8x^3 - 2x^2 - 7x + 3 = 0$.

Sum of alternative coefficients are equal

$$8 - 7 = -2 + 3$$

$$1 = 1$$

$\therefore x+1$ is a factor.

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$$\begin{array}{r|rrrr} -1 & 8 & -2 & -7 & 3 \\ & 0 & -8 & 10 & -3 \\ \hline & 8 & -10 & 3 & 0 \end{array}$$

$$8x^2 - 10x + 3 = 0$$

$$(4x-3)(2x-1) = 0$$

$$4x = 3$$

$$x = 3/4$$

$$2x = 1$$

$$x = 1/2$$

Roots are $3/4, 1/2, -1$.

(14)

$$\begin{array}{r} 24 \\ \times 2 \\ \hline 48 \\ \times 2 \\ \hline 96 \\ \times 2 \\ \hline 192 \end{array}$$

7) Solve the equation: $x^4 - 14x^2 + 45 = 0$

Sol put $t = x^2$

$$t^2 - 14t + 45 = 0$$

$$(t-9)(t-5) = 0$$

$$t-9 = 0$$

$$t = 9$$

$$x^2 = 9$$

$$x = \pm 3$$

$$t-5 = 0$$

$$t = 5$$

$$x^2 = 5$$

$$x = \pm \sqrt{5}$$

Roots are $3, -3, \sqrt{5}, -\sqrt{5}$.

$$\begin{array}{r} 45 \\ \times 5 \\ \hline 225 \\ \times 5 \\ \hline 1125 \end{array}$$

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Exercise 3.4

(15)

1. Solve (i) $(x-5)(x-7)(x+6)(x+4) = 504$.

Soln $(x-5)(x+4)(x-7)(x+6) - 504 = 0$

$$(x^2 - x - 20)(x^2 - x - 42) - 504 = 0$$

Put $y = x^2 - x$.

$$(y-20)(y-42) - 504 = 0$$

$$y^2 - 42y - 20y + 840 - 504 = 0$$

$$y^2 - 62y + 336 = 0$$

$$(y-56)(y-6) = 0$$

$$y = 56 \quad y = 6$$

$$x^2 - x = 56 \quad \text{or} \quad x^2 - x = 6$$

$$x^2 - x - 56 = 0$$

$$(x-8)(x+7) = 0$$

$$x = 8, -7$$

$$(x-2)(x+3) = 0$$

$$x = 2, -3$$

Roots are $-2, 3, 8$ and -7 .

2) $(x-4)(x-7)(x-2)(x+1) = 16$.

Soln $(x-4)(x-2)(x-7)(x+1) - 16 = 0$

$$(x^2 - 6x + 8)(x^2 - 6x - 7) - 16 = 0$$

Put $y = x^2 - 6x$

$$(y+8)(y-7) - 16 = 0$$

$$y^2 - 7y + 8y - 56 - 16 = 0$$

$$y^2 + y - 72 = 0$$

$$(y+9)(y-8) = 0$$

$$y = -9 \quad y = 8$$

$$x^2 - 6x = -9$$

$$x^2 - 6x + 9 = 0$$

$$(x-3)^2 = 0$$

$$x = 3, 3$$

$$x^2 - 6x = 8$$

$$x^2 - 6x - 8 = 0$$

$$x = \frac{6 \pm \sqrt{36 - 4 \cdot 1 \cdot (-8)}}{2 \cdot 1} = \frac{6 \pm \sqrt{36 + 32}}{2}$$

$$= \frac{6 \pm \sqrt{68}}{2} = \frac{6 \pm 2\sqrt{17}}{2}$$

$$= \cancel{\frac{6}{2}} \left(\frac{3 \pm \sqrt{17}}{1} \right) = 3 \pm \sqrt{17}$$

Roots are $3, 3, 3 \pm \sqrt{17}$.

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2. Solve : $(2x-1)(x+3)(x-2)(2x+3)+20=0$. (16)

Solve

$$(x+3)(x-2)(2x-1)(2x+3)+20=0$$

$$(x^2+x-6)(4x^2-6x-2x-3)+20=0$$

$$(x^2+x-6)(4x^2+4x-3)+20=0$$

$$4(x^2+x-6)(x^2+x-\frac{3}{4})+20=0$$

$$(\div 4) (x^2+x-6)(x^2+x-\frac{3}{4}+\frac{20}{4})=0$$

Put $y = x^2+x$.

$$(y-6)(y-\frac{3}{4}+\frac{20}{4})=0$$

$$y^2-\frac{3}{4}y-6y+\frac{9}{2}+\frac{20}{4}=0$$

$$y^2-y(\frac{27}{4})+\frac{38}{4}=0$$

(x4)

$$4y^2-27y+38=0$$

$$4y^2-8y-19y+38=0$$

$$(4y-19)(y-2)=0$$

$$y=2 \quad y=\frac{19}{4}$$

$$x^2+x=2$$

$$x^2+x=\frac{19}{4}$$

$$x^2+x-2=0$$

$$4x^2+4x-19=0$$

$$(x+2)(x-1)=0$$

$$4x^2+4x-19=0$$

$$x=-2 \quad x=1$$

$$x = \frac{-4 \pm \sqrt{16-4 \cdot 1 \cdot (-19)}}{2 \cdot 4}$$

$$= \frac{-4 \pm \sqrt{16+76}}{16}$$

$$= \frac{-4 \pm \sqrt{92}}{8} = \frac{-4 \pm 2\sqrt{23}}{8}$$

$$= \boxed{\frac{-2 \pm \sqrt{23}}{4}}$$

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Exercise 3.5

(17)

1. Solve the following equation.

i) $\sin^2 x - 5 \sin x + 4 = 0$

put $\sin x = t$

$t^2 - 5t + 4 = 0$

$(t-4)(t-1) = 0$

$t = 4 \quad \vee \quad t = 1$

$\sin x = 4$

(is not possible)

$\sin x = 1$

$\sin x = \sin \frac{\pi}{2}$

$x = 2n\pi + \frac{\pi}{2} \quad \forall n \in \mathbb{Z}$

2) $12x^3 + 8x = 29x^2 - 4$

$12x^3 - 29x^2 + 8x - 4 = 0$

$$\begin{array}{r|rrrr} 2 & 12 & -29 & 8 & -4 \\ & 0 & 24 & -10 & -4 \\ \hline & 12 & -5 & -2 & 0 \end{array}$$

$12x^2 - 5x - 2 = 0$

$12x^2 - 8x + 3x - 2 = 0$

$4x(3x-2) + 1(3x-2) = 0$

$(3x-2)(4x+1) = 0$

$3x = 2$

$x = \frac{2}{3}$

$4x = -1$

$x = -\frac{1}{4}$

Roots are $\frac{2}{3}, 2, -\frac{1}{4}$

3) Examine for the rational roots.

$2x^3 - x^2 - 1 = 0$

Soln

Sum of the coeffts

 $x = 1$ is a root

$2 - 1 - 1 = 0$

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$$\begin{array}{c|cccc} 1 & 2 & -1 & 0 & -1 \\ & 0 & 2 & 1 & 1 \\ \hline & 2 & 1 & 1 & 0 \end{array}$$

(18)

$$2x^2 + x + 1 = 0$$

$$x = \frac{-1 \pm \sqrt{1 - 4 \cdot 2 \cdot 1}}{2 \cdot 2} = \frac{-1 \pm \sqrt{1-8}}{2} = \frac{-1 \pm i\sqrt{7}}{2}$$

Which is imaginary roots

$\therefore x=1$ is rational roots.

(ii)

$$x^2 - 3x + 1 = 0.$$

$$\text{Soln } a_n = 1 \quad a_0 = 1.$$

If $\frac{p}{q}$ is a root of the polynomial. (Pr. 2)
by rational root theorem.
it have rational roots.

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$$3) \text{ Solve } 8x^{3/2n} - 8x^{-3/2n} = 63.$$

$$\text{put } k = \frac{3}{2n} \quad \therefore 8x^k - 8x^{-k} = 63$$

$$8x^k - 8/x^k = 63$$

$$8x^{2k} - 8 = 63x^k$$

$$8x^{2k} - 63x^k - 8 = 0$$

$$(x^k - 8)(8x^k + 1) = 0.$$

$$x^k - 8 = 0$$

$$x^k = 8$$

$$x^{3/2n} = 8$$

$$x = 8^{2n/3}$$

$$= (2^3)^{2n/3} = (2^2)^n$$

$$x = 4^n.$$

$\therefore x = 4^n$ is a root of the equation.

(19)

4.

$$\text{Solve } 2\sqrt{\frac{x}{a}} + 3\sqrt{\frac{a}{x}} = \frac{b}{a} + \frac{6a}{b}$$

$$\left(2\sqrt{\frac{x}{a}} + 3\sqrt{\frac{a}{x}}\right)^2 = \left(\frac{b}{a} + \frac{6a}{b}\right)^2$$

$$4\frac{x}{a} + 9\frac{a}{x} + 12 = \frac{b^2}{a^2} + \frac{36a^2}{b^2} + 12$$

multiply this eqn by $\frac{a^2}{4x}$

$$\left(\frac{2x}{a}\right)\left(\frac{4}{x}\right) + \left(\frac{9a}{x}\right)\left(\frac{a^2}{4x}\right) = \left(\frac{a}{4}\right)\frac{b^2}{a^2} + \left(\frac{9}{4}\right)\frac{6^2a^2}{b^2}$$

$$x^2 + \frac{9a^2}{4} = \frac{b^2x}{4a} + \frac{9 \cdot 36a^3x}{4b^2}$$

$$x^2 - \frac{b^2x}{4a} + \frac{9a^2}{4} - \frac{9 \cdot 36a^3x}{4b^2} = 0$$

$$x\left(x - \frac{b^2}{4a}\right) - \frac{9 \cdot 36a^3}{b^2}\left(x - \frac{b^2}{4a}\right) = 0$$

$$\left(x - \frac{b^2}{4a}\right)\left(x - \frac{9 \cdot 36a^3}{b^2}\right) = 0$$

$$x = \frac{b^2}{4a} \quad \text{or} \quad x = \frac{9 \cdot 36a^3}{b^2}$$

$$\text{Ans } \frac{b^2}{4a}, \frac{9 \cdot 36a^3}{b^2}$$

5) solve the equations

$$(i) 6x^4 - 35x^2 + 62x^2 - 35x + 6 = 0$$

Soln This equation is Type I even degree reciprocal equation. Hence it can be rewritten as

$$6\left(x^2 + \frac{1}{x^2}\right) - 35\left(x + \frac{1}{x}\right) + 62 = 0 \quad (1)$$

$$y = x + \frac{1}{x}$$

$$y^2 = x^2 + \frac{1}{x^2} + 2$$

$$y^2 - 2 = x^2 + \frac{1}{x^2}$$

$$(2) \Rightarrow 6(y^2 - 2) - 35y + 62 = 0$$

$$6y^2 - 12 - 35y + 62 = 0$$

$$6y^2 - 35y + 50 = 0$$

$$(y - 5/2)(y - 10/3) = 0$$

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12. 25. 12
2 (12)
11.50 10.30
-30, -40

150
57
-15 - 20
42 43

(20)

$$y = 5/2$$

$$y = \frac{10}{3}$$

$$x + \frac{1}{x} = 5/2$$

$$x + 1/x = \frac{10}{3}$$

$$\frac{x^2 + 1}{x} = 5/2$$

$$\frac{x^2 + 1}{x} = \frac{10}{3}$$

$$2x^2 + 2 - 5x = 0$$

$$3x^2 + 3 = 10x$$

$$2x^2 - 5x + 2 = 0$$

$$3x^2 - 10x + 3 = 0$$

$$x = 2, 1/2$$

$$x = 3, 1/3$$

Roots are $2, 1/2, 3, 1/3$.

(10)

$$x^4 + 3x^3 - 3x - 1 = 0$$

Sol This equation is Type I even degree reciprocal equation. Hence it can be rewritten as

$$\left(x^2 + \frac{1}{x^2}\right) + 3\left(x + \frac{1}{x}\right) + 0 = 0 \quad \text{--- (6)}$$

$$y = x + \frac{1}{x}$$

$$\Rightarrow y^2 = x^2 + \frac{1}{x^2} + 2$$

$$y^2 + 2 = x^2 + \frac{1}{x^2}$$

(7)

$$(y^2 + 2) + 3y = 0$$

$$y^2 + 3y + 2 = 0$$

$$(y + 2)(y + 1) = 0$$

$$y = -2, y = -1$$

$$x + \frac{1}{x} = -2$$

$$x + \frac{1}{x} = -1$$

$$\frac{x^2 - 1}{x} = -2$$

$$\frac{x^2 - 1}{x} = -1$$

$$x^2 - 1 = -2x$$

$$x^2 - 1 = -x$$

$$x^2 + x - 1 = 0$$

$$x^2 + 2x + 1 = 0$$

$$(x + 1)^2 = 0$$

$$x = -1, -1$$

$$x = \frac{-1 \pm \sqrt{1 - 4 \cdot 1 \cdot (-1)}}{2 \cdot 1} = \frac{-1 \pm \sqrt{5}}{2}$$

$$= \frac{-1 \pm \sqrt{5}}{2}$$

Roots are $-1, -1, \frac{-1 \pm \sqrt{5}}{2}$.

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7)

Solve the equation $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$ if it is known that $\frac{1}{3}$ is a solution.

(21)

Soln $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$

$$6\left(x^2 + \frac{1}{x^2}\right) - 5\left(x + \frac{1}{x}\right) - 38 = 0$$

$$x + \frac{1}{x} = y ; y^2 - 2 = x^2 + \frac{1}{x^2}$$

$$6(y^2 - 2) - 5y - 38 = 0$$

$$6y^2 - 12 - 5y - 38 = 0$$

$$6y^2 - 5y - 50 = 0$$

$$6y^2 - 20y + 15y - 50 = 0$$

$$2y(3y - 10) + 5(3y - 10) = 0$$

$$(3y - 10)(2y + 5) = 0$$

$$3y = 10$$

$$2y = -5$$

$$y = \frac{10}{3}$$

$$y = -\frac{5}{2}$$

$$x + \frac{1}{x} = \frac{10}{3}$$

$$\frac{x^2 + 1}{x} = \frac{10}{3}$$

$$3x^2 + 3 = 10x$$

$$3x^2 - 10x + 3 = 0$$

$$(x - 3)(3x - 1) = 0$$

$$x = 3$$

$$x = \frac{1}{3}$$

$$x + \frac{1}{x} = -\frac{5}{2}$$

$$\frac{x^2 + 1}{x} = -\frac{5}{2}$$

$$2x^2 + 2 = -5x$$

$$2x^2 - 5x + 2 = 0$$

$$(x - 2)\left(x - \frac{1}{2}\right) = 0$$

$$x = 2, x = \frac{1}{2}$$

Roots are $3, \frac{1}{3}, 2, \frac{1}{2}$

b) Find all real numbers satisfying

$$4^x - 3(2^{x+2}) + 2^5 = 0$$

$$(2^2)^x - 3(2^x \cdot 2^2) + 2^5 = 0$$

$$(2^x)^2 - 12(2^x) + 2^5 = 0$$

put $2^x = t$ $t^2 - 12t + 32 = 0$

$$(t - 8)(t - 4) = 0$$

$$t = 8$$

$$t = 4$$

$$2^x = 8 = 2^3$$

$$x = 3$$

$$2^x = 4 = (2^2)^2$$

$$x = \pm 2$$

Exercise 3.6

1. Discuss the maximum possible number of positive and negative roots of the polynomial equation
 $9x^9 - 4x^8 + 4x^7 - 3x^6 + 2x^5 + x^3 + 7x^2 + 7x + 2 = 0$.

Soln. clearly there are 4 sign changes for the given polynomial $p(x)$. and hence number of positive roots of $p(x)$ can't be more than four
 $p(-x) = -9x^9 - 4x^8 - 4x^7 - 3x^6 - 2x^5 - x^3 + 7x^2 - 7x + 2 = 0$
 There is two sign changes. hence the number of negative roots can't be more than two.
 $\therefore$ it has at most 4 positive roots and at most two negative roots.

2. Discuss the maximum possible number of positive and negative roots of the polynomial equations $x^2 - 5x + 6$ and $x^2 - 5x + 16$. Also draw rough sketch of the graphs.

$$p(x) = (x^2 - 5x + 6)(x^2 - 5x + 16)$$

$$= x^4 - 5x^3 + 16x^2 - 5x^3 + 25x^2 - 80x + 6x^2 - 30x + 96 = 0$$

$$x^4 - 10x^3 + 21x^2 - 110x + 96 = 0$$

~~it~~ it has two sign change
 $\therefore$ it has two +ve real roots

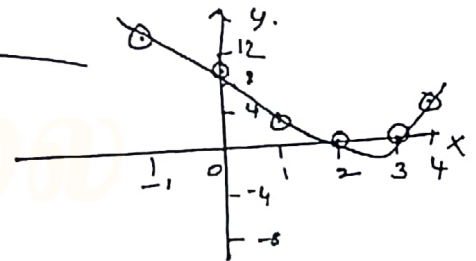
$$p(-x) = x^4 + 10x^3 + 21x^2 + 110x + 96$$

it has no sign change.
 no -ve real roots.

$$y = x^2 - 5x + 6$$

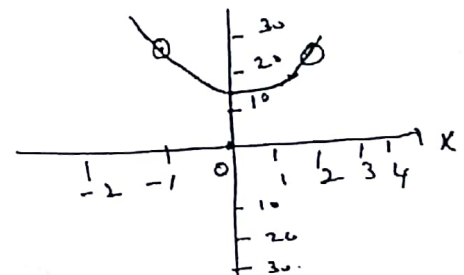
$x=1$	$y = 1 - 5 + 6 = 2$
$x=2$	$y = 4 - 10 + 6 = 0$
$x=0$	$y = 6$
$x=3$	$y = 9 - 15 + 6 = 0$
$x=-1$	$y = 1 + 5 + 6 = 12$
$x=4$	$y = 16 - 20 + 6 = 2$

$(1, 2), (0, 6), (-1, 12)$



$$y = x^2 - 5x + 16$$

x	0	1	-1	2	4
y	16	12	23	16	.



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3. Show that the equation $x^9 - 5x^5 + 4x^4 + 2x^2 + 1 = 0$ (23)
has at least 6 imaginary solutions.

Soln $P(x)$ has only one sign change

it has at most one +ve roots

$$P(-x) = -x^9 + 5x^5 + 4x^4 + 2x^2 + 1 = 0$$

it has only one sign changes

it has at most one -ve roots

Clearly 0 is not a roots.

So maximum number of real roots is 3

and hence there are at least six imaginary solutions

4. Determine the number of positive & negative roots of

the equation $x^9 - 5x^8 - 14x^7 = 0$

Soln $P(x) = x^9 - 5x^8 - 14x^7$. it has only one sign change

$\therefore$ it has one +ve roots.

$$P(-x) = -x^9 - 5x^8 + 14x^7$$

it has one -ve roots.

$\therefore$ It has one +ve & one -ve roots.

- 5) Find the exact number of real roots & imaginary of the equation $x^9 + 9x^7 + 7x^5 + 5x^3 + 3x$.

Soln $P(x) = x^9 + 9x^7 + 7x^5 + 5x^3 + 3x$.

it has no sign change. So it has no +ve real roots.

$$P(-x) = -x^9 - 9x^7 - 7x^5 - 5x^3 - 3x$$

it has no sign changes.

So it has no -ve real roots.

it has no positive real roots and no negative real roots.

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Exercise 3.7

(24)

1. Choose the most suitable answer

1. A zero of $x^3 + 64$ is

$$x^3 = -64$$

$$x^3 = -4 \times -4 \times -4 = (-4)^3$$

$$x = -4$$

Ans (4) -4

2. If f and g are polynomials of degree m and n respectively and if $h(x) = (f \circ g)(x)$ then the degree of h is

Soln For example $f(x) = x^m$ $g(x) = x^n$
 degree = m degree = n
 $(f \circ g)(x) = f(g(x)) = f(x^n) = (x^n)^m = x^{mn}$
 degree = mn .

Ans (1) mn .3. A polynomial equation in x of degree n always has
 Ans (3) n imaginary roots.4) If α, β and γ are the roots $x^3 + px^2 + qx + r$
 then $\sum \frac{1}{\alpha}$ is

$$\alpha + \beta + \gamma + r = -p, \quad \alpha\beta + \beta\gamma + \gamma\alpha = q, \quad \alpha\beta\gamma = -r.$$

$$\sum \frac{1}{\alpha} = \frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma} = \frac{q}{-r}$$

Ans (1) $-\frac{q}{r}$.(5) According to the rational root theorem which number is not possible rational root of $4x^7 + 2x^4 - 10x^3 - 5$?
 By rational root theorem. $\frac{p}{q}$ is a root of a polynomial $a_0 = -5$ $a_n = 4$ $(4, -5)$ p must divide 5 & q must divide 4.Possible values of p are $+1, -1, +5, -5$.Possible values of q are $+1, -1, -5/4, 5/4$ $5/4$ is not possible rational rootsAns (2) $5/4$.

- 6) The polynomial $x^3 - kx^2 + 9x$ has three real roots iff k satisfies.

$$\Delta \geq 0 \quad k^2 - 4 \cdot 1 \cdot 9 \geq 0$$

$$k^2 \geq 36$$

$$|k| \geq 6$$

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Ans. (4) $|k| \geq 6$

- 7) The number of real number of $[0, 2\pi]$ satisfying $\sin^4 x - 2\sin^2 x + 1 = 0$ is

put $\sin^2 x = t$

$$t^2 - 2t + 1 = 0 \Rightarrow (t-1)^2 = 0$$

$$t = 1, \quad t = 1$$

$$\sin^2 x = 1$$

$$\sin x = \pm 1$$

$$\therefore \sin x = 1$$

$$\sin x = \sin \frac{\pi}{2}$$

$$x = \frac{\pi}{2}$$

$$\sin x = -1$$

$$\sin x = \sin(\pi + \frac{\pi}{2})$$

$$x = \frac{3\pi}{2}$$

Number of real numbers in $[0, 2\pi]$ is 2.

- 8) If $x^3 + 12x^2 + 10ax + 1999$ definitely has a +ve root if it is a +ve root need to minimum one sign change $\therefore a \leq 0$ Then only we can get one sign change

Ans (3) $a \leq 0$

- 10) The polynomial $x^3 + 2x + 3$ has.
 $P(x) = x^3 + 2x + 3$, no sign changes $\Rightarrow$ no +ve real roots
 $P(-x) = -x^3 - 2x + 3$, one sign change $\Rightarrow$ one -ve real root
 $\therefore$ it has one negative & two imaginary roots

- 11) The number of +ve roots of the polynomial

$$\sum_{j=0}^n n C_j (-1)^j x^j$$

$$\sum_{j=0}^n n C_j (-1)^j x^j = x^n - n C_1 x^{n-1} + n C_2 x^{n-2} - \dots + (-1)^n$$

$P(x)$ has 'n' changes $\therefore$ it has n +ve roots

$P(-x)$ has no changes

$\therefore$ No -ve roots.

Ans (2) n

CHAPTER: 4 INVERSE TRIGONOMETRIC FUNCTIONS

1.

Inverse function	$\sin^{-1}x$	$\cos^{-1}x$	$\tan^{-1}x$	$\operatorname{cosec}^{-1}x$	$\sec^{-1}x$	$\cot^{-1}x$
Domain	$[-1, 1]$	$[-1, 1]$	$\mathbb{R}$	$[-1, -1] \cup [1, \infty)$	$(-\infty, -1] \cup [1, \infty)$	$\mathbb{R}$
Range	$[-\frac{\pi}{2}, \frac{\pi}{2}]$	$[0, \pi]$	$(-\frac{\pi}{2}, \frac{\pi}{2})$	$[-\frac{\pi}{2}, \frac{\pi}{2}] - \{0\}$	$(0, \pi) - \{\frac{\pi}{2}\}$	$(0, \pi)$

2.

Property I

- (i) $\sin^{-1}(\sin \theta) = \theta$ if $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ (ii) $\cos^{-1}(\cos \theta) = \theta$ if $\theta \in [0, \pi]$
 (iii) $\tan^{-1}(\tan \theta) = \theta$ if $\theta \in (-\frac{\pi}{2}, \frac{\pi}{2})$ (iv) $\operatorname{cosec}^{-1}(\operatorname{cosec} \theta) = \theta$ if $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}] - \{0\}$
 (v) $\sec^{-1}(\sec \theta) = \theta$ if $\theta \in [0, \pi] - \{\frac{\pi}{2}\}$
 (vi) $\cot^{-1}(\cot \theta) = \theta$ if $\theta \in (0, \pi)$

3.

Property II

- (i) $\sin(\sin^{-1}x) = x$ if $x \in [-1, 1]$ (ii) $\cos(\cos^{-1}x) = x$ if $x \in [-1, 1]$
 (iii) $\tan(\tan^{-1}x) = x$ if $x \in \mathbb{R}$ (iv) $\operatorname{cosec}(\operatorname{cosec}^{-1}x) = x$ if $x \in \mathbb{R} - \{0\}$
 (v) $\sec(\sec^{-1}x) = x$ if $x \in \mathbb{R} - \{0\}$ (vi) $\cot(\cot^{-1}x) = x$ if $x \in \mathbb{R}$

4.

Property III

1.

- $\sin^{-1}(\frac{1}{x}) = \operatorname{cosec}^{-1}x$ if $x \in \mathbb{R} - \{0\}$ (ii) $\cos^{-1}(\frac{1}{x}) = \sec^{-1}x$ if $x \in \mathbb{R} - \{0\}$
 (iii) $\tan^{-1} \frac{1}{x} = \begin{cases} \cot^{-1}x & \text{if } x > 0 \\ -\pi + \cot^{-1}x & \text{if } x < 0 \end{cases}$

5.

Property IV (Reflection identities)

- (i) $\sin^{-1}(-x) = -\sin^{-1}x$ if $x \in [-1, 1]$
 (ii) $\tan^{-1}(-x) = -\tan^{-1}x$ if $x \in \mathbb{R}$
 (iii) $\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}x$ if $|x| \geq 1$ or $x \in \mathbb{R} - \{0\}$
 (iv) $\cos^{-1}(-x) = \pi - \cos^{-1}x$ if $x \in [-1, 1]$
 (v) $\sec^{-1}(-x) = \pi - \sec^{-1}x$ if $|x| \geq 1$ or $x \in \mathbb{R} - \{0\}$
 (vi) $\cot^{-1}(-x) = \pi - \cot^{-1}x$ if $x \in \mathbb{R}$

6.

Property V (Complementary inverse identities)

- (i) $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$, $x \in [-1, 1]$ (ii) $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$, $x \in \mathbb{R}$
 (iii) $\operatorname{cosec}^{-1}x + \sec^{-1}x = \frac{\pi}{2}$, $x \in \mathbb{R} - \{0\}$ or $|x| \geq 1$

7.

Property VI

- (i) $\sin^{-1}x + \sin^{-1}y = \sin^{-1}(x\sqrt{1-y^2} + y\sqrt{1-x^2})$

where either $x^2 + y^2 \leq 1$ or $xy < 0$.

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$$(i) \sin^{-1} x + \sin^{-1} y = \sin^{-1} [x\sqrt{1-y^2} + y\sqrt{1-x^2}] \text{ where } x^2 + y^2 \leq 1 \text{ or } xy > 0$$

$$(ii) \cos^{-1} x + \cos^{-1} y = \cos^{-1} [xy - \sqrt{1-x^2}\sqrt{1-y^2}] \text{ if } x+y \geq 0$$

$$(iv) \cos^{-1} x - \cos^{-1} y = \cos^{-1} [xy + \sqrt{1-x^2}\sqrt{1-y^2}] \text{ if } x-y \geq 0$$

$$(v) \tan^{-1} x + \tan^{-1} y = \tan^{-1} \left(\frac{x+y}{1-xy} \right) \text{ if } xy < 1$$

$$(vi) \tan^{-1} x - \tan^{-1} y = \tan^{-1} \left(\frac{x-y}{1+xy} \right) \text{ if } xy > -1$$

Property VII

$$(i) 2\tan^{-1} x = \tan^{-1} \left(\frac{2x}{1-x^2} \right), |x| < 1$$

$$(ii) 2\tan^{-1} x = \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right), x \geq 0$$

$$(iii) 2\tan^{-1} x = \sin^{-1} \left(\frac{2x}{1+x^2} \right), |x| \leq 1$$

Property VIII

$$(i) \sin^{-1} (2x\sqrt{1-x^2}) = 2\sin^{-1} x \text{ if } |x| \leq \frac{1}{\sqrt{2}} \text{ or } \frac{-1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}}$$

$$(ii) \sin^{-1} (2x\sqrt{1-x^2}) = 2\cos^{-1} x \text{ if } \frac{1}{\sqrt{2}} \leq x \leq 1$$

Property IX

$$(i) \sin^{-1} x = \cos^{-1} \sqrt{1-x^2} \text{ if } 0 \leq x \leq 1 \quad (ii) \sin^{-1} x = -\cos^{-1} \sqrt{1-x^2} \text{ if } -1 \leq x < 0$$

$$(iii) \sin^{-1} x = \tan^{-1} \left(\frac{x}{\sqrt{1-x^2}} \right) \text{ if } -1 < x < 1 \quad (iv) \cos^{-1} x = \sin^{-1} \sqrt{1-x^2} \text{ if } 0 \leq x \leq 1$$

$$(v) \cos^{-1} x = \pi - \sin^{-1} \sqrt{1-x^2} \text{ if } -1 \leq x < 0 \quad (vi) \tan^{-1} x = \sin^{-1} \frac{x}{\sqrt{1+x^2}} = \cos^{-1} \frac{1}{\sqrt{1+x^2}} \text{ if } x > 0$$

Property X

$$(i) 3\sin^{-1} x = \sin^{-1} (3x - 4x^3), x \in [-\frac{1}{2}, \frac{1}{2}]$$

$$(ii) 3\cos^{-1} x = \cos^{-1} (4x^3 - 3x), x \in [\frac{1}{2}, 1]$$

Trigonometric function	Sine	Cosine	tangent	Cosecant	secant	Cotangent
Domain	$[-\frac{\pi}{2}, \frac{\pi}{2}]$	$[0, \pi]$	$(-\frac{\pi}{2}, \frac{\pi}{2})$	$[-\frac{\pi}{2}, \frac{\pi}{2}] \setminus \{0\}$	$[0, \pi] \setminus \{\frac{\pi}{2}\}$	$(0, \pi)$
Range	$[-1, 1]$	$[-1, 1]$	$\mathbb{R}$	$\mathbb{R} \setminus (-1, 1)$	$\mathbb{R} \setminus (-1, 1)$	$\mathbb{R}$

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CHAPTER: 4 INVERSE TRIGONOMETRIC FUNCTIONS

(2)

Exercise 4.1

1. Find all the values of x such that

(i) $-10\pi \leq x \leq 10\pi$ and $\sin x = 0$

Soln:-

$$\sin x = 0$$

$$\sin x = \sin 0$$

$$x = n\pi + (-1)^n(0)$$

$$= n\pi, n = 0, \pm 1, \pm 2, \dots, \pm 10.$$

$$\sin 0 = \sin \theta$$

$$0 = n\pi + (-1)^n \theta$$

$$n \in \mathbb{R}$$

(ii) $-8\pi \leq x \leq 8\pi$ and $\sin x = -1$

Soln $\sin x = -1$

$$\sin x = -\sin \pi/2 = \sin(-\pi/2)$$

$$x = (4n-1)\pi/2, n = 0, \pm 1, \pm 2, \pm 3, 4$$

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2. Find the period and amplitude of-

(i) $y = \sin 7x$.

For amplitude use the form $y = a \sin(bx - c) + d$.

$$\text{amplitude} = |a|$$

$$a = 1 \quad \therefore |a| = 1$$

$$\text{Period using the formula } \frac{2\pi}{|b|} = \frac{2\pi}{|7|} = \frac{2\pi}{7}$$

$$\therefore \text{amplitude} = 1$$

$$\text{period} = \frac{2\pi}{7}$$

(ii) $y = -\sin(\frac{1}{3}x)$

$$a = -1 \quad b = \frac{1}{3}$$

$$\therefore \text{amplitude } |a| = |-1| = 1$$

$$\text{period} = \frac{2\pi}{|b|} = \frac{2\pi}{|1/3|} = 3(2\pi) = 6\pi$$

(iii) $y = 4 \sin(-2x)$

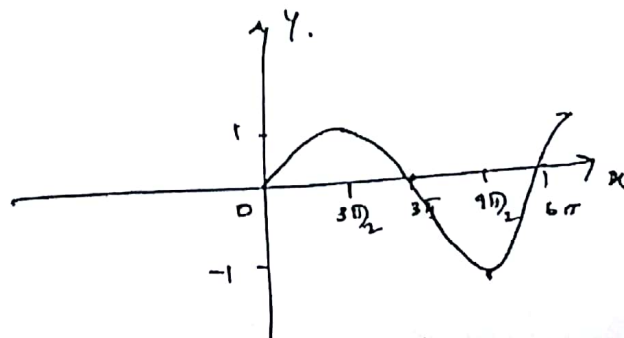
$$\text{Here } a = 4 \quad b = -2$$

$$\therefore \text{amplitude} = |4| = 4$$

$$\text{period} = \frac{2\pi}{|b|} = \frac{2\pi}{|-2|} = \frac{2\pi}{2} = \pi$$

- 3) Sketch the graph of $y = \sin(\frac{1}{3}x)$ for $0 \leq x \leq 6\pi$.

x	$f(x)$
0	0
3π	1
6π	0
9π	-1
12π	0



4. Find the value of (i) $\sin^{-1}(\sin \frac{2\pi}{3})$

(2)

Soln
 $y = \sin^{-1}(\sin 2\theta_3)$

$$= \sin^{-1}(\sin (\pi - \theta_3)) = \sin^{-1}(\sin \theta_3) = \theta_3$$

(ii) $\sin^{-1}(\sin (5\pi/4))$

$$y = \sin^{-1}(\sin (5\pi/4)) = \sin^{-1}[\sin(\pi + \theta_4)] = \sin^{-1}(-\sin \theta_4)$$

$$= \sin^{-1}(\sin(-\theta_4)) = -\theta_4$$

5. For what value of x does $\sin x = \sin^{-1} x$?

Soln $\sin 0 = 0$

$$\sin^{-1}(0) = 0$$

$$\therefore x = 0$$

b) Find the domain of the following.

(i) $y(x) = \sin^{-1}\left(\frac{x^2+1}{2x}\right)$

Soln. Range of $\sin^{-1} x$ is $[-1, 1]$.

$$-1 \leq \frac{x^2+1}{2x} \leq 1$$

Multiply by $2x^2 \geq 0$.

$$-2x^2 \leq x(x^2+1) \leq 2x^2$$

$$-2x^2 \leq x(x^2+1)$$

$$x(x^2+1) + 2x^2 \geq 0$$

$$x(x^2+1+2x) \geq 0$$

$$x(x+1)^2 \geq 0$$

$$x \geq 0 \quad (x+1)^2 \geq 0$$

$$x = -1$$

$$x(x^2+1) - 2x^2 \leq 0$$

$$x[x^2+1-2x] \leq 0$$

$$x(x-1)^2 \leq 0$$

$$x \leq 0 \quad x = 1$$

$x=0$ doesn't play with us
 this time

$\therefore$ solution is $\{-1, 1\}$

(ii) $g(x) = 2\sin^{-1}(2x-1) - \pi/4$

Soln Range of $\sin^{-1} x$ is $[-1, 1]$.

$$-1 \leq 2x-1 \leq 1$$

$$-1 \leq 2x-1$$

$$2x-1 \geq -1$$

$$\Rightarrow 2x \geq -1+1$$

$$2x \geq 0$$

$$\Rightarrow x \geq 0$$

$$2x-1 \leq 1$$

$$2x \leq 2$$

$$x \leq 1$$

$x \geq 0$ & $x \leq 1$ $\therefore$ solution is $[0, 1]$.

1) Find the value of $\sin^{-1}(\sin \frac{5\pi}{9} \cos \frac{\pi}{9} + \cos \frac{5\pi}{9} \sin \frac{\pi}{9})$

Soln.

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$A = \frac{5\pi}{9} \quad B = \frac{\pi}{9}$$

$$= \sin^{-1} \left[\sin \left(\frac{5\pi}{9} + \frac{\pi}{9} \right) \right]$$

$$= \sin^{-1} \left[\sin \left(\frac{6\pi}{9} \right) \right] = \sin^{-1} \left(\sin \left(\frac{2\pi}{3} \right) \right)$$

$$= \sin^{-1} \left(\sin \left(\pi - \frac{\pi}{3} \right) \right)$$

$$= \sin^{-1} \left(\sin \frac{\pi}{3} \right) = \frac{\pi}{3}$$

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(5)

Exercise 4.2

1. Find all values of x such that (i) $-6\pi \leq x \leq 6\pi$ and $\cos x = 0$

Soln $\cos x = 0$
 $\cos x = \cos \pi/2$

$$x = (2n+1)\pi/2, \quad n = 0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5.$$

- (ii) $-5\pi \leq x \leq 5\pi$, and $\cos x = 1$.

Soln $\cos x = 1$
 $\cos x = \cos 0$

$$x = (2n+1)\pi, \quad n = 0, \pm 1, \pm 2, -3.$$

2. State the reason for $\cos^{-1}[\cos(-\pi/6)] \neq -\pi/6$

Soln $\cos^{-1}[\cos(-\pi/6)] = \cos^{-1}[\cos \pi/6]$
 $= \pi/6 \neq -\pi/6. \quad [\because \cos(-\theta) = \cos \theta]$

3. Is $\cos^{-1}(-x) = \pi - \cos^{-1} x$ true? Justify your answer.

Soln. $\cos^{-1}(-x) = \pi - \cos^{-1} x.$

Take $x = \cos \theta \Rightarrow \theta = \cos^{-1} x$
 $\cos^{-1}(-x) = \pi - \theta$

$$\cos(\pi - \theta) = -\cos \theta = -x \in [-1, 1]$$

$$\pi - \cos^{-1} x = \pi - \cos^{-1} x = \pi - \theta.$$

$$\pi - \cos^{-1} x = \cos^{-1}(-x) \text{ is true.}$$

- 4) Find the principal value of $\cos^{-1}(1/2)$

Soln $y = \cos^{-1}(1/2)$

$$\cos y = 1/2 = \cos \pi/3.$$

$$y = \pi/3 \in [0, \pi]$$

Principal value is $\pi/3$

- 5) Find the value of (i) $2\cos^{-1}(1/2) + \sin^{-1}(1/2)$

Soln $y = \cos^{-1}(1/2)$

$$\cos y = 1/2 = \cos \pi/3$$

$$y = \pi/3 \in [0, \pi]$$

$$x = \sin^{-1}(1/2)$$

$$\sin x = 1/2 = \sin \pi/6$$

$$x = \pi/6 \in [0, \pi/2]$$

$$2 \cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(\frac{1}{2}\right) = 2(\pi/3) + (\pi/6) = \frac{2\pi}{3} + \frac{\pi}{6} = \frac{4\pi + \pi}{6} = \frac{5\pi}{6} \quad (6)$$

(ii) $\cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}(-1)$

$$\begin{array}{l|l} x = \cos^{-1}\left(\frac{1}{2}\right) & y = \sin^{-1}(-1) \\ \cos x = \frac{1}{2} = \cos \pi/3 & \sin y = -1 = \sin(-\pi/2) \\ x = \pi/3 & y = -\pi/2 \end{array}$$

$$\cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}(-1) = \pi/3 - \pi/2 = \frac{2\pi - 3\pi}{6} = -\pi/6$$

(iii) $\cos^{-1}\left[\cos \pi/7 \cos \pi/17 - \sin \pi/7 \sin \pi/17\right]$

$$= \cos^{-1}\left[\cos\left(\frac{\pi}{7} + \frac{\pi}{17}\right)\right] \quad [\because \cos(A+B) = \cos A \cos B - \sin A \sin B]$$

$$= \cos^{-1}\left[\cos\left(\frac{17\pi + 7\pi}{119}\right)\right]$$

$$= \cos^{-1}\left[\cos\left(\frac{24\pi}{119}\right)\right] = \frac{24\pi}{119}$$

6) Find the domain of $f(x) = \sin^{-1}\left(\frac{|x|-2}{3}\right) + \cos^{-1}\left(\frac{1-|x|}{4}\right)$

Soln $-1 \leq \sin^{-1} \leq 1$

$$-1 \leq \frac{|x|-2}{3} \leq 1$$

$$-3 \leq |x|-2 \leq 3$$

$$-3+2 \leq |x| \leq 3+2$$

$$-1 \leq |x| \leq 5$$

$$|x| \leq 5$$

$$-5 \leq x \leq 5$$

$$\therefore x \in [-5, 5]$$

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Since $-1 \leq |x|$ is not possible

$$1) g(x) = \sin^{-1} x + \cos^{-1} x.$$

(7)

Range of $\sin x$ and $\cos x$ is $[-1, 1]$

$$-1 \leq x \leq 1.$$

$$\therefore x \in [-1, 1].$$

7) For what value of x the inequality $\frac{\pi}{2} < \cos^{-1}(3x-1) < \pi$ holds

Soln

$$\frac{\pi}{2} < \cos^{-1}(3x-1) < \pi$$

$$\cos \frac{\pi}{2} < (3x-1) < \cos \pi$$

$$0 < 3x-1 < -1$$

$$0+1 < 3x < -1+1$$

$$1 < 3x < 0$$

$$0 < 3x < 1$$

$$\frac{1}{3} < x < \frac{1}{3}$$

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8) Find the value of (i) $\cos(\cos^{-1}(\frac{4}{5}) + \sin^{-1}(\frac{4}{5}))$

We know that $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$.

$$\cos(\cos^{-1}(\frac{4}{5}) + \sin^{-1}(\frac{4}{5})) = \cos \frac{\pi}{2} = 0.$$

$$(ii) \cos^{-1}(\cos \frac{4\pi}{3}) + \cos^{-1}(\cos \frac{5\pi}{4})$$

$$\cos \frac{4\pi}{3} = \cos(\pi + \frac{\pi}{3}) = -\cos(\frac{\pi}{3}) = \cos(-\frac{\pi}{3}) = \cos \frac{\pi}{3}$$

$$\cos \frac{5\pi}{4} = \cos(\pi + \frac{\pi}{4}) = -\cos(\frac{\pi}{4}) = \cos(-\frac{\pi}{4}) = \cos \frac{\pi}{4}$$

$$\text{Ans: } \cos^{-1}(\cos \frac{4\pi}{3}) + \cos^{-1}(\cos \frac{5\pi}{4})$$

$$= \cos^{-1}(\cos(\frac{\pi}{3})) + \cos^{-1}(\cos(\frac{\pi}{4})) = \frac{\pi}{3} + \frac{\pi}{4}$$

$$= \frac{4\pi + 3\pi}{12} = \frac{7\pi}{12}$$

8

Exercise 4.3

1. Find the domain of following functions
 (i) $\tan^{-1} \sqrt{9-x^2}$ (ii) $\frac{1}{2} \tan^{-1} (1-x^2) - \pi/4$.

Soln

$$9-x^2 \geq 0$$

$$9 \geq x^2$$

$$x^2 \leq 9$$

$$x \leq \pm 3.$$

$$\text{domain } [-3, 3]$$

- (ii) Range of $\tan^{-1} x$ is $\mathbb{R}$

$$-\infty < 1-x^2 < \infty$$

$$-\infty < -x^2 < \infty$$

$$-\infty < x < \infty$$

$$x \in \mathbb{R}.$$

$$\text{domain} = \mathbb{R}$$

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- 2) Find the value of (i) $\tan^{-1} (\tan 5\pi/4)$.

$$\begin{aligned} \text{Soln } \tan^{-1} (\tan (5\pi/4)) &= \tan^{-1} [\tan (\pi + \pi/4)] \\ &= \tan^{-1} [\tan \pi/4] = \pi/4. \end{aligned}$$

$$(ii) \tan^{-1} (\tan (-\pi/6)) = -\pi/6.$$

- 3) Find the value of

$$(i) \tan [\tan^{-1} (7\pi/4)] = 7\pi/4.$$

$$\text{We know that } \tan (\tan^{-1} (x)) = x.$$

$$(ii) \tan [\tan^{-1} (1947)] = 1947.$$

$$(iii) \tan [\tan^{-1} (-0.2021)] = -0.2021.$$

- 4) Find the value of

$$(i) \tan [\cos^{-1} (1/2) - \sin^{-1} (-1/2)]$$

Soln $\tan [\cos^{-1}(\frac{1}{2}) - \sin^{-1}(\frac{-1}{2})]$

$$= \tan [\cos^{-1} \frac{1}{2} + \sin^{-1} \frac{1}{2}]$$

$$= \tan (90^\circ) = \infty.$$

(9)

$$[\because \sin(-x) = -\sin x]$$

$$[\sin^{-1} x + \cos^{-1} x = 90^\circ]$$

(ii) $\sin [\tan^{-1}(\frac{1}{2}) - \cos^{-1}(\frac{4}{5})]$

$a = \tan^{-1} \frac{1}{2}$

$\tan a = \frac{1}{2} > 0$. (lies in I & III quadrant)

$b = \cos^{-1}(\frac{4}{5})$

$\cos b = \frac{4}{5} > 0$. (lies in I & IV quadrant only).

$\sin [\tan^{-1}(\frac{1}{2}) - \cos^{-1}(\frac{4}{5})] = \sin^{\#}(a-b)$

$= \sin a \cos b - \cos a \sin b$

$[\because \sin(A-B) = \sin A \cos B - \cos A \sin B]$

$= \frac{1}{\sqrt{5}} (\frac{4}{5}) - \frac{2}{\sqrt{5}} \cdot \frac{3}{5}$

$= \frac{4}{5\sqrt{5}} - \frac{6}{5\sqrt{5}} = -\frac{2}{5\sqrt{5}}$

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(iii) $\cos [\sin^{-1}(\frac{4}{5}) - \tan^{-1}(\frac{3}{4})]$

Soln $a = \sin^{-1}(\frac{4}{5})$

$\sin a = \frac{4}{5} > 0$ (lies in I & II quadrant only)

$\cos a = \pm \frac{3}{5}$

$b = \tan^{-1}(\frac{3}{4}) \Rightarrow \tan b = \frac{3}{4}$

$\sin b = \frac{3}{5}$

opp = 4

Hyp = 5

Adj = $\sqrt{5^2 - 4^2} = \sqrt{25 - 16}$
 $= \sqrt{9} = 3$

opp = 3

adj = 4

Hyp = $\sqrt{4^2 + 3^2} = \sqrt{16 + 9}$
 $= \sqrt{25} = 5$

$\cos [\sin^{-1}(\frac{4}{5}) - \tan^{-1}(\frac{3}{4})]$

$= \cos(a-b) = \cos a \cos b + \sin a \sin b$

$= (\frac{3}{5})(\frac{4}{5}) + (\frac{4}{5})(\frac{3}{5}) = \frac{12}{25} + \frac{12}{25} = \frac{24}{25}$

(10)

Exercise 4.4

1. Find the principal value of

(i) $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$

Soln $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = \alpha$

$$\sec \alpha = \frac{2}{\sqrt{3}} = \sec \pi/6$$

$$\alpha = \pi/6.$$

(ii) $\cot^{-1}(\sqrt{3})$

Soln $\cot^{-1}(\sqrt{3}) = \alpha$

$$\sqrt{3} = \cot \alpha$$

$$\cot \pi/6 = \cot \alpha$$

$$\alpha = \pi/6$$

(iii) $\operatorname{cosec}^{-1}(-\sqrt{2})$

Soln $\operatorname{cosec}^{-1}(-\sqrt{2}) = \alpha$

$$\operatorname{cosec} \alpha = -\sqrt{2} = -\operatorname{cosec}(\pi/4) = \operatorname{cosec}(-\pi/4)$$

$$\alpha = -\pi/4.$$

2) Find the value of

(i) $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$

Soln $x = \tan^{-1}(\sqrt{3})$

$$\tan x = \sqrt{3} = \tan \pi/6$$

$$x = \pi/6$$

$$y = \sec^{-1}(-2)$$

$$\sec y = -2 = -\sec \pi/3$$

$$\sec y = \sec(-\pi/3)$$

$$\sec y = \sec(\pi + \pi/3)$$

$$y = \pi + \pi/3$$

$$\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2) = \pi/6 - \pi/3$$

$$= \frac{\pi - 2\pi}{6} = -\pi/6.$$

$$\left[\begin{array}{l} \because \sec(-\theta) \\ = \sec \theta \end{array} \right]$$

(ii) $\sin^{-1}(-1) + \cos^{-1}\left(\frac{1}{2}\right) + \cot^{-1}(2).$

$$x = \sin^{-1}(-1)$$

$$\sin x = -1 = \sin(-\pi/2)$$

$$x = -\pi/2.$$

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$$y = \cos^{-1}\left(\frac{1}{2}\right)$$

$$\cos y = \frac{1}{2} = \cos \pi_3$$

$$y = \pi_3$$

$$z = \cot^{-1}(2) \quad (11)$$

$$\cot z = 2$$

$$z = \cot^{-1}(2), \text{ is constant.}$$

$$\sin^{-1}(-1) + \cos^{-1}\left(\frac{1}{2}\right) + \cot^{-1}(2)$$

$$= -\pi_2 + \pi_3 + \cot^{-1}(2)$$

$$= \frac{-3\pi + 2\pi}{6} + \cot^{-1}(2) = -\frac{\pi}{6} + \cot^{-1}(2)$$

$$3) \cot^{-1}(1) + \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) - \sec^{-1}(-\sqrt{2})$$

$$\text{soln} \quad x = \cot^{-1}(1)$$

$$\cot x = 1 = \cot \pi_4$$

$$x = \pi_4$$

$$y = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$$

$$\sin y = -\frac{\sqrt{3}}{2} = \sin(-\pi_3)$$

$$y = -\pi_3$$

$$z = \sec^{-1}(-\sqrt{2})$$

$$\sec z = -\sqrt{2}$$

$$\sec z = -\sec \pi_4 = \sec(-\pi_4) = \sec \pi_4$$

$$z = \pi_4$$

$$(\sec \pi - \pi) = \sec \pi$$

$$\cot^{-1}(1) + \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) - \sec^{-1}(-\sqrt{2})$$

$$= \pi_4 - \pi_3 - \pi_4 = -\pi_3$$

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Exercise 4.5

1. Find the value if it exists if not, give the reason (12)
for non existence

(i) $\sin^{-1}(\cos \pi)$ (ii) $\tan^{-1}(\sin(-5\pi/2))$ (iii) $\sin^{-1}(\sin 5)$

Soln
(i) $\sin^{-1}(\cos \pi) = \sin^{-1}(-1) = -\sin^{-1}(1) = -\pi/2$

(ii) $\tan^{-1}(\sin(-5\pi/2)) = \tan^{-1}[-\sin 5\pi/2]$
 $= \tan^{-1}[-\sin(2\pi + \pi/2)] = \tan^{-1}(-\sin \pi/2)$
 $= \tan^{-1}(-1) = -\tan^{-1}(1) = -\pi/4$

(iii) $\sin^{-1}(\sin 5)$
 $-\pi/2 \leq \sin^{-1} x \leq \pi/2$ ~~$\sin(5) \notin [-1, 1]$~~
 $-3\pi/2 \leq 5 \leq 2\pi$
 $-\pi/2 \leq 5 - 2\pi \leq 0 \leq \pi/2$
 $\sin(5 - 2\pi) = \sin 5$
 $\sin^{-1}(\sin 5) = 5 - 2\pi$

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- 2) Find the value of the expression in terms of x
with the help of a reference triangle

(i) $\sin(\cos^{-1}(1-x))$
 $= \sin[\cos^{-1}(\text{side of adj} / \text{side of Hyp})]$ $\left\{ \begin{array}{l} \therefore \cos(\frac{\text{adj}}{\text{Hyp}}) = \frac{1-x}{1} \\ \text{adj} = 1-x \quad \text{Hyp} = 1 \\ \text{opp} = \sqrt{1^2 - (1-x)^2} \end{array} \right.$
 $= \frac{\text{opp}}{\text{Hyp}} = \frac{\sqrt{2x-x^2}}{1} = \sqrt{2x-x^2}$ $= \sqrt{1 - (1-x)^2}$
 $= \sqrt{1 - 1 + 2x - x^2}$
 $= \sqrt{2x-x^2}$

(ii) $\cos(\tan^{-1}(3x-1))$
 $= \cos[\text{side of opp} / \text{side of hyp}]$
 $= \frac{\text{Adj}}{\text{Hyp}} = \frac{1}{\sqrt{9x^2 - 6x + 2}}$

$\left\{ \begin{array}{l} \therefore \text{opp} = 3x-1 \\ \text{adj} = 1 \\ \text{Hyp} = \sqrt{(3x-1)^2 + 1^2} \\ = \sqrt{9x^2 - 6x + 2} \end{array} \right.$

(iii) $\tan(\sin^{-1}(x+1/2)) = \tan(\sin^{-1}(\frac{2x+1}{2}))$
 $\tan[\text{side of opp} / \text{side of Hyp}]$
 $= \frac{\text{opp}}{\text{adj}} = \frac{2x+1}{\sqrt{3-4x-4x^2}}$

$\left\{ \begin{array}{l} \therefore \text{opp} = 2x+1 \quad \text{Hyp} = 2 \\ \text{adj} = \sqrt{2^2 - (2x+1)^2} \\ = \sqrt{4 - (4x^2 + 4x + 1)} \\ = \sqrt{3 - 4x - 4x^2} \end{array} \right.$

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(13)

$$(iii) 2 \tan^{-1}(\cos x) = \tan^{-1}(2 \operatorname{cosec} x).$$

Soln w.k.T $2 \tan^{-1} x = \tan^{-1} \frac{2x}{1-x^2}$

$$2 \tan^{-1}(\cos x) = \tan^{-1} \left(\frac{2 \cos x}{1 - \cos^2 x} \right) = \tan^{-1} \left(\frac{2 \cos x}{\sin^2 x} \right)$$

$$= \tan^{-1} \left(2 \frac{\cos x}{\sin^2 x} \right) = \tan^{-1}(2 \operatorname{cosec} x)$$

$$\frac{2 \cos x}{\sin^2 x} = 2 \operatorname{cosec} x$$

$$\frac{\cos x}{\sin^2 x} = \frac{1}{\sin x}$$

$$\cot x = 1 = \cot 45^\circ$$

$$x = 45^\circ$$

$$(iv) \cot^{-1} x - \cot^{-1}(x+2) = 15^\circ, \quad x > 0$$

Soln $\tan^{-1} \frac{1}{x} - \tan^{-1} \frac{1}{x+2} = \frac{\pi}{12} = \frac{180}{12} = 15^\circ$

$$\tan^{-1} \left(\frac{\frac{1}{x} - \frac{1}{x+2}}{1 + \frac{1}{x} \cdot \frac{1}{x+2}} \right) = 15^\circ$$

$$\tan^{-1} \frac{\frac{(x+2) - (x)}{x(x+2)}}{\frac{x(x+2) + 1}{x(x+2)}} = 15^\circ$$

$$\frac{x+2-x}{x^2+2x+1} = \tan 15^\circ = \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$\frac{2}{(x+1)^2} = \frac{\sqrt{3}-1}{\sqrt{3}+1} \cdot \frac{(\sqrt{3}+1)}{(\sqrt{3}+1)} = \frac{(\sqrt{3}-1)^2}{(\sqrt{3}+1)^2}$$

$$\frac{2}{(x+1)^2} = \frac{2}{(\sqrt{3}+1)^2}$$

Compare both sides

$$\boxed{x = \sqrt{3}}$$

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3) Find the value of (i) $\sin^{-1} [\cos (\sin^{-1} (\frac{\sqrt{3}}{2}))]$ (10)

$$\begin{aligned} \text{Soln} \quad \sin^{-1} [\cos (\sin^{-1} \frac{\sqrt{3}}{2})] &= \sin^{-1} [\cos \pi/3] = \sin^{-1} [\frac{1}{2}] \\ &= \frac{\pi}{6} \end{aligned}$$

$$(ii) \cot [\sin^{-1} \frac{3}{5} + \sin^{-1} \frac{4}{5}]$$

$$\cot [\sin^{-1} \frac{3}{5} + \cos^{-1} \frac{4}{5}]$$

$$\cot (\theta_2) = 0 \quad \because \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$$

$$\begin{aligned} (iii) \quad \sin^{-1} \frac{3}{5} &= a \quad \& \quad \cot^{-1} \frac{3}{2} = b \\ \sin a &= 3/5 \quad \& \quad \cot b = 3/2 \Rightarrow \tan b = \frac{2}{3} \\ \cos a &= \sqrt{1 - \sin^2 a} = \sqrt{1 - 9/25} = \sqrt{\frac{16}{25}} = \frac{4}{5} \end{aligned}$$

$$(iii) \tan (\sin^{-1} \frac{3}{5} + \cot^{-1} \frac{3}{2}) = \tan (a+b)$$

$$= \frac{\tan a + \tan b}{1 - \tan a \tan b} = \frac{3/4 + \frac{2}{3}}{1 - \frac{3}{4} \times \frac{2}{3}}$$

$$= \frac{\frac{9+8}{12}}{\frac{12-6}{12}} = \frac{17}{6}$$

$$\begin{aligned} \therefore \tan a &= \frac{\sin a}{\cos a} \\ &= \frac{3/5}{4/5} = \frac{3}{4} \end{aligned}$$

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4) Prove that (i) $\tan^{-1} \frac{2}{11} + \tan^{-1} \frac{7}{24} = \tan^{-1} \frac{1}{2}$

$$\text{Soln} \quad \tan^{-1} A + \tan^{-1} B = \tan^{-1} \left(\frac{A+B}{1-AB} \right)$$

$$\tan^{-1} \left(\frac{2}{11} \right) + \tan^{-1} \left(\frac{7}{24} \right) = \tan^{-1} \left(\frac{\frac{2}{11} + \frac{7}{24}}{1 - \frac{2}{11} \cdot \frac{7}{24}} \right)$$

$$= \tan^{-1} \left(\frac{\frac{48+77}{11 \times 24}}{\frac{264-14}{11 \times 24}} \right) = \tan^{-1} \left(\frac{125}{250} \right)$$

$$= \tan^{-1} \left(\frac{1}{2} \right)$$

$$(11) \sin^{-1} \frac{3}{5} - \cos^{-1} \frac{12}{13} = \sin^{-1} \frac{16}{65}$$

$$\begin{aligned} \text{Soln. } x &= \sin^{-1} \frac{3}{5} & y &= \cos^{-1} \frac{12}{13} \\ \sin x &= \frac{3}{5} & \cos y &= \frac{12}{13} \\ \cos x &= \frac{4}{5} & \sin y &= \frac{5}{13} \end{aligned}$$

$$\begin{aligned} (15) \quad \sin x &= \frac{3}{5} = \frac{\text{opp}}{\text{hyp}} \\ \text{adj} &= \sqrt{5^2 - 3^2} = \sqrt{25 - 9} \\ &= \sqrt{16} = 4 \\ \cos y &= \frac{12}{13} = \frac{\text{adj}}{\text{hyp}} \\ \text{opp} &= \sqrt{13^2 - 12^2} = \sqrt{169 - 144} \\ &= \sqrt{25} = 5 \end{aligned}$$

$$\sin^{-1} \left(\frac{3}{5} \right) - \cos^{-1} \left(\frac{12}{13} \right) =$$

$$\begin{aligned} \text{Let } \sin(x-y) &= \sin x \cos y - \cos x \sin y \\ &= \frac{3}{5} \left(\frac{12}{13} \right) - \frac{4}{5} \left(\frac{5}{13} \right) = \frac{36}{65} - \frac{20}{65} = \frac{16}{65} \end{aligned}$$

$$\sin(x-y) = \frac{16}{65}$$

$$(x-y) = \sin^{-1} \left(\frac{16}{65} \right)$$

$$5) \text{ Prove that } \tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \tan^{-1} \left(\frac{x+y+z-xyz}{1-xy-yz-zx} \right)$$

$$\text{Soln } \tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \tan^{-1} \left(\frac{x+y}{1-xy} \right) + \tan^{-1} z$$

$$= \tan^{-1} \left(\frac{\frac{x+y}{1-xy} + z}{1 - \left(\frac{x+y}{1-xy} \right) z} \right)$$

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$$= \tan^{-1} \left(\frac{\frac{x+y+z(1-xy)}{1-xy}}{\frac{(1-xy) - (x+y)z}{1-xy}} \right) = \tan^{-1} \left(\frac{x+y+z-xy}{1-xy-yz-zx} \right)$$

$$6) \text{ If } \tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi \text{ show that } x+y+z = xyz.$$

$$\text{Soln } \tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$$

$$\tan^{-1} \left(\frac{x+y}{1-xy} \right) + \tan^{-1} z$$

$$\tan^{-1} \left(\frac{\frac{x+y}{1-xy} + z}{1 - \left(\frac{x+y}{1-xy} \right) z} \right) = \pi$$

$$\tan^{-1} \left(\frac{\frac{x+y+z(1-xy)}{1-xy}}{\frac{1-xy-(xz+yz)}{1-xy}} \right) = \pi \quad (14)$$

$$\frac{x+y+z-xyz}{1-xy-xz-yz} = \tan \pi = 0.$$

$$x+y+z-xyz = 0$$

$$\boxed{x+y+z = xyz}$$

7) Prove that $\tan^{-1} x + \tan^{-1} \frac{2x}{1-x^2} = \tan^{-1} \left(\frac{3x-x^3}{1-3x^2} \right)$ if $|x| < \frac{1}{\sqrt{3}}$.

Soln

$$\tan^{-1} x + \tan^{-1} \left(\frac{2x}{1-x^2} \right)$$

$$= \tan^{-1} \left(\frac{x + \frac{2x}{1-x^2}}{1 - x \left(\frac{2x}{1-x^2} \right)} \right) = \tan^{-1} \left(\frac{\frac{x(1-x^2)+2x}{1-x^2}}{\frac{1-x^2-2x^2}{1-x^2}} \right)$$

$$= \tan^{-1} \left(\frac{x-x^3+2x}{1-3x^2} \right) \quad \text{if } \begin{matrix} 3x^2 < 1 \\ x^2 < \frac{1}{3} \\ |x| < \frac{1}{\sqrt{3}} \end{matrix}$$

$$= \tan^{-1} \left(\frac{3x-x^3}{1-3x^2} \right)$$

8) Simplify $\tan^{-1} \frac{x}{y} - \tan^{-1} \left(\frac{x-y}{x+y} \right)$

Soln

$$\tan^{-1} \left(\frac{x}{y} \right) - \tan^{-1} \left(\frac{x-y}{x+y} \right)$$

$$= \tan^{-1} \left(\frac{\frac{x}{y} - \frac{x-y}{x+y}}{1 + \frac{x}{y} \cdot \left(\frac{x-y}{x+y} \right)} \right) = \tan^{-1} \left(\frac{\frac{x(x+y) - y(x-y)}{y(x+y)}}{\frac{y(x+y) + x(x-y)}{y(x+y)}} \right)$$

$$= \tan^{-1} \left(\frac{x^2+xy-xy+y^2}{xy+y^2+x^2-xy} \right) = \tan^{-1} \left(\frac{x^2+y^2}{x^2+y^2} \right) = \tan^{-1}(1)$$

$$= \frac{\pi}{4}$$

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9) Solve: i) $\sin^{-1} \frac{5}{x} + \sin^{-1} \frac{12}{x} = \frac{\pi}{2}$.

(17)

Soln $\sin^{-1} \frac{5}{x} = \frac{\pi}{2} - \sin^{-1} \frac{12}{x}$

$$\frac{\pi}{2} + (\cos^{-1} \frac{5}{x}) = \frac{\pi}{2} + \sin^{-1} \frac{12}{x}$$

$$\cos^{-1} \frac{5}{x} = \sin^{-1} \frac{12}{x}$$

$$\cos \theta = \frac{5}{x} \quad \sin \theta = \frac{12}{x}$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\left(\frac{5}{x}\right)^2 + \left(\frac{12}{x}\right)^2 = 1$$

$$\frac{25}{x^2} + \frac{144}{x^2} = 1$$

$$\frac{169}{x^2} = 1$$

$$x^2 = 169$$

ii) $x = \pm 13$.

$$2 \tan^{-1} x = \cos^{-1} \left(\frac{1-a^2}{1+a^2} \right) - \cos^{-1} \left(\frac{1-b^2}{1+b^2} \right) \quad a > 0.$$

Soln
W.K.T $2 \tan^{-1} x = \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right)$

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$$\cos^{-1} \left(\frac{1-a^2}{1+a^2} \right) = 2 \tan^{-1} a$$

$$\cos^{-1} \left(\frac{1-b^2}{1+b^2} \right) = 2 \tan^{-1} b$$

RHS

$$\begin{aligned} \cos^{-1} \left(\frac{1-a^2}{1+a^2} \right) - \cos^{-1} \left(\frac{1-b^2}{1+b^2} \right) &= 2 \tan^{-1} a - 2 \tan^{-1} b \\ &= 2 [\tan^{-1} a - \tan^{-1} b] \\ &= 2 \left(\tan^{-1} \frac{a-b}{1+ab} \right) \end{aligned}$$

$$LHS = RHS$$

$$2 \tan^{-1} \left(\frac{a-b}{1+ab} \right) = 2 \tan^{-1} x$$

$$x = \frac{a-b}{1+ab}$$

$$a > 0.$$

16) Find the number of solution of the equation (18)

$$\tan^{-1}(n-1) + \tan^{-1}n + \tan^{-1}(n+1) = \tan^{-1}3n.$$

Soln $\Rightarrow \tan^{-1}(n-1) + \tan^{-1}n + \tan^{-1}(n+1)$

$$= \tan^{-1}(n-1) + \tan^{-1}(n+1) + \tan^{-1}n$$

$$= \tan^{-1}\left(\frac{n-1+n+1}{1-(n-1)(n+1)}\right) + \tan^{-1}n$$

$$= \tan^{-1}\left(\frac{2n}{1-(n^2-1)}\right) + \tan^{-1}n$$

$$= \tan^{-1}\left(\frac{2n}{1-n^2+1}\right) + \tan^{-1}n = \tan^{-1}\left(\frac{2n}{2-n^2}\right) + \tan^{-1}n$$

$$= \tan^{-1}\left(\frac{\frac{2n}{2-n^2} + n}{1 - \frac{2n}{2-n^2} \cdot n}\right) = \tan^{-1}\left(\frac{\frac{2n+2n-n^3}{2-n^2}}{\frac{2-n^2-2n^2}{2-n^2}}\right)$$

$$= \tan^{-1}\left(\frac{4n-n^3}{2-3n^2}\right)$$

Given LHS = RHS $\Rightarrow$
 $\tan^{-1}\left(\frac{4n-n^3}{2-3n^2}\right) = \tan^{-1}3n$

$$\frac{4n-n^3}{2-3n^2} = 3n$$

$$4n-n^3 = 6n-9n^3$$

$$8n^3 = 2n$$

$$8n^3 - 2n = 0$$

$$2n(n^2-1) = 0$$

$$n^2 = 1$$

$$n = 0$$

$$n = \pm 1.$$

number of solutions are 0, 1, -1.

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Exercise 4.6

Choose the correct or the most suitable answer from the given four alternatives. (19)

1. The value of $\sin^{-1}(\cos x)$ $0 \leq x \leq \pi$ is
Soln $\sin^{-1}(\cos x) = \sin^{-1}(\sin(\frac{\pi}{2} - x)) = \frac{\pi}{2} - x$
 Ans (3) $\frac{\pi}{2} - x$.

2. If $\sin^{-1}x + \sin^{-1}y = \frac{2\pi}{3}$ Then $\cos^{-1}x + \cos^{-1}y$ is equal to.

Soln $\sin^{-1}x + \cos^{-1}x + \cos^{-1}y + \sin^{-1}y = \frac{\pi}{2} + \frac{\pi}{2} = \pi$
 $= 2 \cdot \frac{\pi}{2} + \cos^{-1}x + \cos^{-1}y = \pi$
 $\cos^{-1}x + \cos^{-1}y = \pi - 2 \cdot \frac{\pi}{2} = \frac{3\pi - 2\pi}{2} = \frac{\pi}{2}$
 Ans (2) $\frac{\pi}{2}$

3. $\sin^{-1} \frac{3}{5} - \cos^{-1} \frac{12}{13} + \sec^{-1} \frac{5}{3} - \operatorname{cosec}^{-1} \frac{13}{12}$ is equal to.

$= \sin^{-1} \frac{3}{5} - \sec^{-1} \frac{13}{12} + \cos^{-1} \frac{3}{5} - \operatorname{cosec}^{-1} \frac{13}{12}$
 $= (\sin^{-1} \frac{3}{5} + \cos^{-1} \frac{3}{5}) - (\sec^{-1} \frac{13}{12} + \operatorname{cosec}^{-1} \frac{13}{12})$
 $= \frac{\pi}{2} - \frac{\pi}{2} = 0$

Ans (3) 0.

4. If $\sin^{-1}x = 2\sin^{-1}a$ has a solution then.

$-\frac{\pi}{2} \leq 2\sin^{-1}a \leq \frac{\pi}{2}$

$-\frac{\pi}{4} \leq \sin^{-1}a \leq \frac{\pi}{4}$

$\sin(-\frac{\pi}{4}) \leq a \leq \sin \frac{\pi}{4}$

$-\frac{1}{\sqrt{2}} \leq a \leq \frac{1}{\sqrt{2}}$

$|a| \leq \frac{1}{\sqrt{2}}$

Ans (1) $|a| \leq \frac{1}{\sqrt{2}}$

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- 5) $\sin^{-1}(\cos x) = \frac{\pi}{2} - x$ is valid for

$\cos x = \sin \frac{\pi}{2} - x$

$\cos x \in [0, 1] \therefore 0 \leq x \leq \pi$

Ans (2) $0 \leq x \leq \pi$

6) If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$. The value of

$$x^{2017} + y^{2018} + z^{2019} - \frac{9}{x^{101} + y^{101} + z^{101}} \text{ is}$$

Given $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$ ($\because \sin^{-1}(1) = \frac{\pi}{2}$)
 $\therefore x = y = z = 1$ ($\frac{\pi}{2} + \frac{\pi}{2} + \frac{\pi}{2} = \frac{3\pi}{2}$)

$$x^{2017} + y^{2018} + z^{2019} - \frac{9}{x^{101} + y^{101} + z^{101}}$$

$$= 1 + 1 + 1 - \frac{9}{1+1+1} = 3 - \frac{9}{3} = 3 - 3 = 0$$

Ans (1) 0

7) If $\cot^{-1}x = \frac{2\pi}{5}$ for some $x \in \mathbb{R}$ The value of $\tan^{-1}x$ is

Soln $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$

$$\tan^{-1}x = \frac{\pi}{2} - \cot^{-1}x = \frac{\pi}{2} - \frac{2\pi}{5} = \frac{5\pi - 4\pi}{10} = \frac{\pi}{10}$$

Ans (3) $\frac{\pi}{10}$

8) The domain of the function defined by $f(x) = \sin^{-1}\sqrt{x-1}$ is

Soln $f(x) = \sin^{-1}\sqrt{x-1}$

Soln $\sqrt{x-1} \geq 0 \quad -1 \leq \sqrt{x-1} \leq 1$

$$\therefore 0 \leq \sqrt{x-1} \leq 1$$

$$0 \leq x-1 \leq 1$$

$$1 \leq x \leq 2$$

$$x \in [1, 2]$$

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Ans (1) $[1, 2]$

9) If $x = \frac{1}{5}$ The value of $\cos(\cos^{-1}x + 2\sin^{-1}x)$ is

Soln $\cos[\cos^{-1}x + \sin^{-1}x + \sin^{-1}x] = \cos(\frac{\pi}{2} + \sin^{-1}x)$

$$= -\sin(\sin^{-1}x)$$

$$= -x = -\frac{1}{5}$$

($\because \cos(90^\circ + \theta) = -\sin\theta$)
Ans (4) $-\frac{1}{5} = x$

10) $\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{2}{9}$ is equal to.

Soln $\tan^{-1}\left(\frac{\frac{1}{4} + \frac{2}{9}}{1 - \frac{1}{4} \cdot \frac{2}{9}}\right) = \tan^{-1}\left(\frac{\frac{9+8}{36}}{\frac{36-2}{36}}\right) = \tan^{-1}\left(\frac{17}{34}\right)$

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$$= \tan^{-1}\left(\frac{1}{2}\right)$$

$$\text{Ans } (4) \tan^{-1} \frac{1}{2} \quad (21)$$

11) If the function $y(x) = \sin^{-1}(x^2 - 3)$ then x belongs to

Soln

$$-1 \leq x^2 - 3 \leq 1$$

$$-1 \leq x^2 - 3 \leq 1$$

$$-1 + 3 \leq x^2 \leq 1 + 3 \Rightarrow 2 \leq x^2 \leq 4$$

$$\pm\sqrt{2} \leq x \leq \pm 2$$

$$\therefore [-2, -\sqrt{2}] \cup [\sqrt{2}, 2]$$

Ans (3)

12) If $\cot^{-1} 2$ and $\cot^{-1} 3$ are two angles of a triangle then the third angle is

Soln

$$A + B + C = \pi \quad (\text{triangle})$$

$$\cot^{-1} 2 + \cot^{-1} 3 + C = \pi$$

$$\cot^{-1} \left(\frac{1 - 2(3)}{2 + 3} \right) + C = \pi$$

$$\cot^{-1} \left(\frac{1 - 6}{5} \right) + C = \pi$$

$$\cot^{-1} \left(\frac{-5}{5} \right) + C = \pi$$

$$\Rightarrow \cot^{-1}(-1) + C = \pi$$

$$\pi/4 + C = \pi$$

$$C = \pi - \pi/4 = 3\pi/4$$

Ans (2)

13) $\sin^{-1}(\tan \pi/4) - \sin^{-1}(\sqrt{3}/2) = \pi/6$ Then x is a root of the equation.

Soln

$$\sin^{-1}(1) - \sin^{-1}(\sqrt{3}/2) = \pi/6$$

$$\pi/2 - \sin^{-1}(\sqrt{3}/2) = \pi/6$$

$$\cos^{-1}(\sqrt{3}/2) = \pi/6$$

$$\sqrt{3}/2 = \cos \pi/6 = \frac{1}{2}$$

$$\frac{\sqrt{3}}{\sqrt{x}} = \frac{1}{2}$$

$$\sqrt{x} = 2\sqrt{3}$$

$$x = 4(3) = 12$$

$$x = 12$$

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14) $\sin^{-1}(2\cos^2 x - 1) + \cos^{-1}(1 - 2\sin^2 x) = \sin^{-1}(2\cos^2 x - 1) + \cos^{-1}(1 - \sin^2 x - \sin^2 x)$
 $= \sin^{-1}(2\cos^2 x - 1) + \cos^{-1}(\cos^2 x - (1 - \cos^2 x))$ (22)
 $= \sin^{-1}(2\cos^2 x - 1) + \cos^{-1}(\cos^2 x - 1 + \cos^2 x)$
 $= \sin^{-1}(2\cos^2 x - 1) + \cos^{-1}(2\cos^2 x - 1)$
 $= \pi/2$ $\because \sin^{-1} x + \cos^{-1} x = \pi/2$
 Ans (3)

15) If $\cot^{-1} \sqrt{\sin x} + \tan^{-1} \sqrt{\sin x} = u$ then $\cos 2u$ is equal to
Soln $\cot^{-1} x + \tan^{-1} x = \pi/2$
 $\therefore u = \pi/2$
 $\cos 2u = \cos 2(\pi/2) = \cos \pi = -1$ Ans (5)

16) If $|x| \leq 1$ Then $2\tan^{-1} x - \sin^{-1} \frac{2x}{1+x^2}$ is equal to
Soln $\sin^{-1} \frac{2x}{1+x^2} = 2\tan^{-1} x$ Ans (3)
 $\therefore 2\tan^{-1} x - 2\tan^{-1} x = 0$

17) The eqn $\tan^{-1} x - \cot^{-1} x = \tan^{-1} \left(\frac{1}{\sqrt{3}} \right) = \pi/6$
Soln $\tan^{-1} x + \cot^{-1} x = \pi/2$
 Add (2) & (1) $2\tan^{-1} x = \pi/6 + \pi/2 = 2\pi/3$
 $\tan^{-1} x = \pi/3$ Ans (2)
 $x = \sqrt{3}$
 which is unique solution.

18) If $\sin^{-1} x + \cot^{-1} \left(\frac{1}{2} \right) = \pi/2$ Then x is equal to.

$\sin^{-1} x + \cos^{-1} \left(\frac{1}{\sqrt{5}} \right) = \pi/2$ $\cot^{-1} \frac{1}{2} = \frac{\text{adj}}{\text{hyp}}$
 $\text{Hyp} = \sqrt{2^2 + 1^2} = \sqrt{5}$
 $\cos^{-1} \frac{\text{adj}}{\text{Hyp}} = \frac{1}{\sqrt{5}}$
 w.k.T $\sin^{-1} x + \cos^{-1} x = \pi/2$
 $\therefore x = \frac{1}{\sqrt{5}}$
 Ans. $\frac{1}{\sqrt{5}}$

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19) If $\sin^{-1} \frac{n}{5} + \operatorname{cosec}^{-1} \frac{n}{4} = \frac{\pi}{2}$ Then the values of n . (23)!

Soln $\sin^{-1} \frac{n}{5} + \cos^{-1} \frac{3}{5} = \frac{\pi}{2}$

$= \sin^{-1} \frac{n}{5} + \frac{\pi}{2} - \sin^{-1} \frac{3}{5} = \frac{\pi}{2}$

$\sin^{-1} \frac{n}{5} = \sin^{-1} \frac{3}{5}$

$\frac{n}{5} = \frac{3}{5} \Rightarrow n = 3$ Ans (4)

$\operatorname{cosec} a = \frac{\text{Hyp}}{\text{opp.}} = \frac{5}{4}$

$\text{adj} = \sqrt{5^2 - 4^2}$

$= \sqrt{25 - 16} = \sqrt{9} = 3$

20) $\sin(\tan^{-1} x)$, $|x| < 1$ is equal to.

Soln ~~$\sin(\tan^{-1} x) = \sin(\frac{\pi}{2} - \frac{\pi}{2}) = \sin(\frac{\pi}{2}) = 1$~~ Let $a = \tan^{-1} x$

$\tan a = x$

w.k.T $1 + \tan^2 a = \sec^2 a$

$1 + x^2 = \sec^2 a$

$\sec a = \sqrt{1 + x^2}$

$\frac{1}{\cos a} = \sqrt{1 + x^2}$

$\cos a = \frac{1}{\sqrt{1 + x^2}}$

$\sin a = \sqrt{1 - \cos^2 a} = \sqrt{1 - \frac{1}{1 + x^2}} = \sqrt{\frac{1 + x^2 - 1}{1 + x^2}}$

$= \frac{x}{\sqrt{1 + x^2}}$

Ans (4)

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CHAPTER 5: Two Dimensional Analytical Geometry II ①

1. Eqn of the circle $(x-h)^2 + (y-k)^2 = r^2$
Center (h, k) and radius $= r$.
2. Eqn of a circle in general form $x^2 + y^2 + 2gx + 2fy + c = 0$
center $(-g, -f)$ & radius $= \sqrt{g^2 + f^2 - c}$.
3. Equation of a circle with $(x_1, y_1), (x_2, y_2)$ as extremities of one of the diameter is $(x-x_1)(x-x_2) + (y-y_1)(y-y_2) = 0$
- 4) Equation of tangent and normal } at (x_1, y_1) on circle $x^2 + y^2 + 2gx + 2fy + c = 0$
is $xx_1 + yy_1 + g(x+x_1) + f(y+y_1) + c = 0$
and $yx_1 - xy_1 + g(y-y_1) - f(x-x_1) = 0$.

5) Condition for the line $y = mx + c$ to be a tangent to the conics.

Conic	Equation	Condition to be tangent	Point of Contact	Equation of tangent
1- circle	$x^2 + y^2 = a^2$	$c^2 = a^2(1+m^2)$	$(\frac{-am}{\sqrt{1+m^2}}, \frac{\pm a}{\sqrt{1+m^2}})$	$y = mx \pm \frac{a}{\sqrt{1+m^2}}$
2. Parabola	$y^2 = 4ax$	$c = \frac{a}{m}$	$(\frac{a}{m^2}, \frac{2a}{m})$	$y = mx + \frac{a}{m}$
3) Ellipse	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$c^2 = a^2 m^2 + b^2$	$(-\frac{a^2 m}{c}, \frac{b^2}{c})$	$y = mx \pm \sqrt{a^2 m^2 + b^2}$
4) Hyperbola	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$c^2 = a^2 m^2 - b^2$	$(-\frac{a^2 m}{c}, -\frac{b^2}{c})$	$y = mx \pm \sqrt{a^2 m^2 - b^2}$

6) Tangent and Normal.

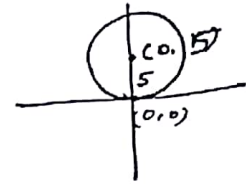
Curve	Equation	Equation of tangent cartesian form	Equation of normal
i) Circle	$x^2 + y^2 = a^2$	i) $xx_1 + yy_1 = a^2$ ii) Parametric form $x \cos \theta + y \sin \theta = a$	i) $xy_1 - yx_1 = 0$ ii) $x \sin \theta - y \cos \theta = 0$
ii) Parabola	$y^2 = 4ax$	i) $yy_1 = 2a(x+x_1)$ ii) $yt = x + at^2$	i) $xy_1 + 2y = 2ax_1 + x_1 y_1$ ii) $y + xt = a(t^3 + 2at)$
iii) Ellipse	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	i) $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$ ii) $\frac{x \cos \theta}{a} + \frac{y \sin \theta}{b} = 1$	i) $\frac{a^2 x}{x_1} + \frac{b^2 y}{y_1} = a^2 - b^2$ ii) $\frac{ax}{\cos \theta} - \frac{by}{\sin \theta} = a^2 - b^2$
iv) Hyperbola	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	i) $\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$ ii) $\frac{x \sec \theta}{a} - \frac{y \tan \theta}{b} = 1$	i) $\frac{a^2 x}{x_1} + \frac{b^2 y}{y_1} = a^2 + b^2$ ii) $\frac{ax}{\sec \theta} + \frac{by}{\tan \theta} = a^2 + b^2$

Chapter 5: Two Dimensional Analytical Geometry - II ②

Exercise 5.1

1. Obtain the equation of the circle with radius 5 cm and touching x axis at the origin in general form.

Soln radius = 5 cm
 Passing through point (0,0)
 Centre at y axis be (0,k) = (0,±5)
 Equation of circle be
 $(x-h)^2 + (y-k)^2 = r^2$ — ①
 $(x-0)^2 + (y-k)^2 = r^2 = 5^2$
 At (0,0)
 $x^2 + (y-k)^2 = 25$
 $0 + (0-k)^2 = 25$
 $k^2 = 25$
 $k = \pm 5$



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(0,k) = (0, ±5)
 $\therefore \textcircled{1} \Rightarrow (x-0)^2 + (y \pm 5)^2 = r^2$
 $x^2 + y^2 \pm 10y + 25 = r^2$
 $x^2 + y^2 \pm 10y = 0$

2. Find the equation of the circle with centre (2,-1) and passing through the point (3,6) in standard form.

Soln Centre = (2,-1) = (h,k)
 Passing through the point (3,6)
 Eqn of the circle $(x-h)^2 + (y-k)^2 = r^2$ — ①
 $(3-2)^2 + (6+1)^2 = r^2$
 $1^2 + 7^2 = r^2$
 $1 + 49 = r^2$
 $r^2 = 50$

① $\Rightarrow (x-2)^2 + (y+1)^2 = 50$

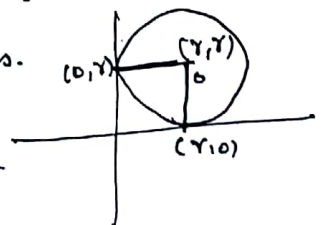
3. Find the equation of circles that touch both the axes and pass through (-4,-2) in general form.

Soln Let the circles touch both the axes.
 Centre be (r,r) = (h,k)

Eqn of the circle $(x-h)^2 + (y-k)^2 = r^2$
 $(x-r)^2 + (y-r)^2 = r^2$ — ①

Passing through (-4,-2)

$(-4-r)^2 + (-2-r)^2 = r^2$



$$4x^2 + 8x + 4 + x^2 + 4x = 2x^2$$

$$x^2 + 12x + 20 = 0$$

$$(x+10)(x+2) = 0$$

$$x = -10, x = 2.$$

$$i) x = -10 \text{ in } ①$$

$$(x+10)^2 + (y+10)^2 = 10^2$$

$$x^2 + 100 + 20x + y^2 + 100 + 20y = 100$$

$$x^2 + y^2 + 20x + 20y + 100 = 0$$

- 4) Find the equation of the circle with centre (2, 3) and passing through the intersection of the lines $3x - 2y - 1 = 0$ and $4x + y - 27 = 0$

Soln Centre (2, 3) = (h, k)

Point of intersection :-

$$\text{Solve } 3x - 2y - 1 = 0 \quad \text{--- (1)}$$

$$4x + y - 27 = 0 \quad \text{--- (2)}$$

$$① \Rightarrow 3x - 2y = 1$$

$$② \times 2 \Rightarrow 8x + 2y = 54$$

$$11x = 55$$

$$x = 5 \text{ put in } ①$$

$$15 - 2y - 1 = 0$$

$$14 = 2y$$

$$y = 7.$$

Passing through point is (5, 7)

Eqn of circle be $(x-h)^2 + (y-k)^2 = r^2$ --- (3)

$$(5-2)^2 + (7-3)^2 = r^2$$

$$3^2 + 4^2 = r^2$$

$$r^2 = 25$$

$$\therefore ③ \Rightarrow (x-2)^2 + (y-3)^2 = 25.$$

$$x^2 - 4x + 4 + y^2 - 6y + 9 - 25 = 0$$

$$x^2 + y^2 - 4x - 6y - 12 = 0$$

- 5) Obtain the equation of the circle for which (3, 4) and (2, -7) are the ends of a diameter.

Soln $(x-x_1)(x-x_2) + (y-y_1)(y-y_2) = 0$

$$(x-3)(x-2) + (y-4)(y+7) = 0$$

$$x^2 - 5x + 6 + y^2 + 3y - 28 = 0$$

$$x^2 + y^2 - 5x + 3y - 28 = 0.$$

$$(x_1, y_1) = (3, 4)$$

$$(x_2, y_2) = (2, -7)$$

- 6) Find the equation of the circle through the points (1,0) (-1,0) and (0,1) (4)

Soln Let the points $(1,0)$ $(-1,0)$ & $(0,1)$
 x_1, y_1 x_2, y_2 x_3, y_3 .

If two points lie anywhere on the circumference but diametrically opposite each other. Then the radius of the circle won't be minimum.

$\therefore (1,0)$ & $(-1,0)$ form the end points of diameter

$$\frac{(x-x_1)(x-x_2) + (y-y_1)(y-y_2)}{(x_1-x_2)^2 + (y_1-y_2)^2} = \frac{(x-1)(x+1) + (y-0)(y-0)}{(1-(-1))^2 + (0-0)^2} = \frac{(x^2-1) + y^2}{4}$$

Radius be unit circle ($r=1$) centre (0,0)

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x-0)^2 + (y-0)^2 = 1^2$$

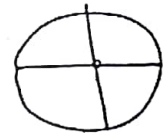
$$x^2 + y^2 = 1$$

$(-1,0)$ also pass through this circle. $(-1)^2 + 0^2 = 1^2$.

$\therefore x^2 + y^2 = 1$ be the equation of the circle through the points (1,0) & (-1,0) & (0,1)

- 7) A circle of area 9π square units has two of its diameters along the lines $x+y=5$ & $x-y=1$. Find the equation of the circle.

Soln area of a circle $\pi r^2 = 9\pi$
 $r^2 = 9$



Solve $x+y=5$ — (1)

$x-y=1$ — (2) we get centre of the circle.

Add (1) & (2)

$$2x = 6$$

$$x = 3$$

(1) $\Rightarrow$

$$3+y=5$$

$$y=2$$

centre $(h,k) = (3,2)$ & $r^2 = 9$

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x-3)^2 + (y-2)^2 = 9$$

$$x^2 + 9 - 6x + y^2 - 4y + 4 = 9$$

$$x^2 - 6x + y^2 - 4y + 4 = 0$$

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- 8) If $y = 2\sqrt{2}x + c$ is a tangent to the circle $x^2 + y^2 = 16$ find the value of c . (5)

Soln The condition of the line $y = mx + c$ to be a tangent to the circle $x^2 + y^2 = a^2$ is

$$c^2 = a^2(1 + m^2)$$

$$a^2 = 16$$

$$m = 2\sqrt{2}$$

$$m^2 = 4 \times 2 = 8$$

$$c^2 = 16(1 + 8)$$

$$c^2 = 16(9)$$

$$c = \pm 4 \times 3 = \pm 12$$

$$\therefore c = \pm 12$$

9. Find the equation of the tangent and normal to the circle $x^2 + y^2 - 6x + 6y - 8 = 0$ at $(2, 2)$.

Soln Eqn of the tangent be

$$xx_1 + yy_1 - \frac{3}{2}(x+x_1) + \frac{3}{2}(y+y_1) - 8 = 0$$

$$\text{At } (2, 2) \quad 2x + 2y - 3(x+2) + 3(y+2) - 8 = 0$$

$$2x + 2y - 3x - 6 + 3y + 6 - 8 = 0$$

$$-x + 5y - 8 = 0$$

$$x - 5y + 8 = 0$$

Eqn of the normal be

$$yx_1 - xy_1 + 9(y-y_1) - f(x-x_1) = 0$$

$$\text{At } (2, 2) \quad y(2) - x(2) - 3(y-2) + 3(x-2) = 0$$

$$2y - 2x - 3y + 6 - 3x + 6 = 0$$

$$-5x - y + 12 = 0$$

$$5x + y - 12 = 0$$

$$2y = -6$$

$$y = -3$$

$$2x = 6$$

$$x = 3$$

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Another method for normal

Eqn of tangent is $\perp$ to normal.

$$x - 5y + 8 = 0$$

$\perp$ eqn be $5x + y + k = 0$

$$10 + 2 + k = 0$$

$$\text{At } (2, 2)$$

$$k = -12$$

$\therefore 5x + y - 12 = 0$ is eqn of normal

- $$At(-2,1) \quad (-2)^2 + 1^2 - 5(-2) + 2(1) - 5$$
- $$= 4 + 1 + 10 + 2 - 5 = 12 > 0.$$

$\therefore (-2, 1)$ lies outside the circle

(17) At (0,0) $0+0-0+0-5 = -5 < 0$
(0,0) lies inside the circle

$$\text{(iv)}^{At} (-4, -3) \quad (-4)^2 + (-3)^2 - 5(-4) + 2(-3) - 5$$

$$= 16 + 9 + 20 + 6 - 5 = 54 > 0$$

$(-4, -3)$ lies outside the circle.

- ii) Find the centre and radius of the following circles

$$(1) \quad x^2 + (y+2)^2 = 0.$$

1) $x^2 + (y+2)^2 = 0$.
Compare with $(x-h)^2 + (y-k)^2 = r^2$.

$$h=0 \quad k=-2 \quad r^2=0$$

Centre $(h, k) = (0, -2)$

$r = 0.5 \text{ rad/s}$

(ii) $x^2 + y^2 + 6x - 4y + 4 = 0$

$$\begin{array}{lll} 2g = 6 & 2f = -4 & c = 4. \\ g = 3 & f = -2 & \end{array}$$

Center $(-g, -f) = (-8, 2)$

radius $r = \sqrt{g^2 + f^2 - c} = \sqrt{9 + 16 - 4} = 3$.

(iii) $x^2 + y^2 - x + 2y - 3 = 0$.

$$2g = -1$$

$$2f = 2$$

02-2.

$$g = -1/2$$

$$f = 1$$

Centre $(-g, -f) = (1/2, -1)$

Centre $(-3, -3) = (-\frac{1}{2}, -1)$
 Radius $\sqrt{g^2 + f^2 - c} = \sqrt{\frac{1}{4} + 1 + 3} = \sqrt{\frac{1+4+12}{4}} = \sqrt{\frac{17}{4}} = \frac{\sqrt{17}}{2}$

$$(iv) \quad 2x^2 + 2y^2 - 6x - 4y + 2 = 0$$

$$(\div 2) \quad x^2 + y^2 - 3x + 2y + 1 = 0$$

c 2 1

$$29 = 13$$

$$2p = 2$$

$$g = -3/2$$

$$f = 1$$

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$$\text{Centre } (-g, -f) = (3/2, -1)$$

(7)

$$r = \sqrt{g^2 + f^2 - c} = \sqrt{\frac{9}{4} + 1 - 1} = \frac{3}{2}$$

- 12) If the equation $3x^2 + (3-p)xy + 2y^2 - 2px = 8p$ represents a circle find p & q . Also determine the centre and radius of the circle.

Soln $3x^2 + (3-p)xy + 2y^2 - 2px = 8p$ represent a circle means

$$\text{coefft of } x^2 = \text{coefft of } y^2$$

$$3 = 2$$

$$\therefore q = 3$$

$$\text{coefft of } xy = 0$$

$$3-p = 0$$

$$p = 3$$

$$3x^2 + 3y^2 - 6x = 8(3)(3)$$

$$3x^2 + 3y^2 - 6x - 72 = 0$$

$$(\div 3) \quad x^2 + y^2 - 2x - 24 = 0$$

$$2g = -2 \quad 2f = 0 \quad c = -24$$

$$g = -1$$

$$f = 0$$

$$\text{Centre } (-g, -f) = (1, 0)$$

$$\text{radius} = \sqrt{g^2 + f^2 - c} = \sqrt{1 + 0 + 24} = \sqrt{25} = 5$$

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Exercise 5.2

1. Find the equation of the parabola in each of the cases given below.

(i) focus (4,0) and direction $x = -4$.

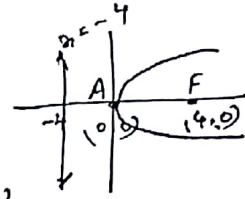
Equation of a parabola be

$$y^2 = 4ax$$

$$a = 4. \quad (\text{Distance AF} = 4 \text{ unit})$$

$$\therefore y^2 = 4(4)x$$

$$y^2 = 16x.$$



(ii) passes through (2,-3) and symmetric about y axis

Soln Eqn of a parabola be

$$x^2 = -4ay$$

At (2,-3)

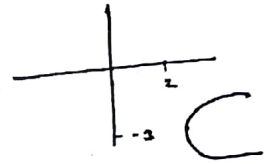
$$4 = -4a(-3)$$

$$1 = 3a$$

$$\Rightarrow a = \frac{1}{3}$$

$$x^2 = -4\left(\frac{1}{3}\right)y$$

$$3x^2 = -4y$$



(iii) vertex (1,-2) and focus (4,-2)

Soln $a = \text{Distance VF} = 3 \text{ units.}$

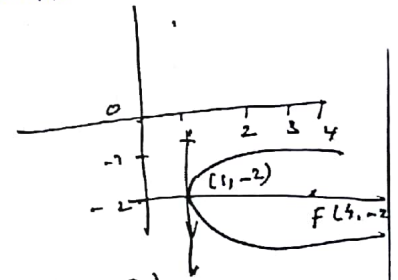
Eqn of a parabola be

$$(y-k)^2 = 4a(x-h)$$

$$(y+2)^2 = 4(3)(x-1)$$

$$(y+2)^2 = 12(x-1)$$

$$(h,k) = (1,-2)$$



(iv) end points of latus rectum (4,-8) and (4,8)

Soln

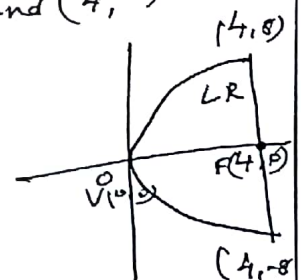
Distance between VF is 4 unit.

$$a = 4.$$

$$y^2 = 4ax$$

$$y^2 = 4(4)x$$

$$y^2 = 16x$$



- 2) Find the equation of the ellipse in each of the cases given below

(i) foci $(\pm 3, 0)$, $e = \frac{1}{2}$

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Soln $ae = 3$

$e = 1/2$

$a(1/2) = 3$

$a = 6$

$\therefore a^2 = 36$

$b^2 = a^2(1 - e^2)$

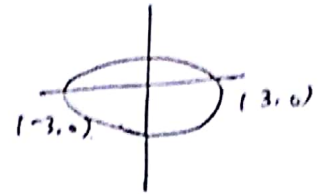
$b^2 = 36(1 - 1/4) = 36(3/4) = 27$

$b^2 = 27$

Eqn of the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\frac{x^2}{36} + \frac{y^2}{27} = 1$

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(ii) foci $(0, \pm 4)$ and end points of major axis are $(0, \pm 5)$

Soln $ae = 4$

$5e = 4$

$e = \frac{4}{5}$

$2a = 10$

$a = 5$

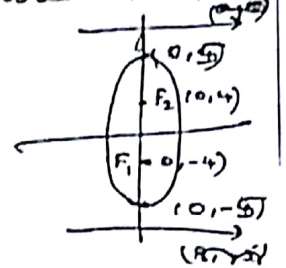
$a^2 = 25$

$b^2 = a^2(1 - e^2) = 25(1 - \frac{16}{25}) = 25(\frac{9}{25})$

$b^2 = 9$

Eqn of the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\frac{x^2}{25} + \frac{y^2}{9} = 1$



(iii) length of latus rectum 8, eccentricity $= \frac{3}{5}$ and major axis on x axis

Soln $e = \frac{3}{5}$

Latus rectum $\frac{2b^2}{a} = 8$

$\frac{2b^2}{a} = 8$

$b^2 = \frac{4a}{2} = 4a = 4 \times \left(\frac{25}{4}\right) = 25$

$b^2 = a^2(1 - e^2)$

$4a = a^2(1 - \frac{9}{25})$

$4 = a(\frac{16}{25}) \Rightarrow \frac{25}{4} = a$

$\therefore a^2 = \frac{625}{16}$

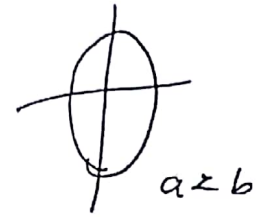
$\therefore \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{\frac{625}{16}} + \frac{y^2}{25} = 1$

$\frac{16x^2}{625} + \frac{y^2}{25} = 1$

- 14) length of latus rectum 4, distance between foci $(2\sqrt{2})$ and major axis as y axis

Soln

$$\begin{aligned} 2ae &= 4\sqrt{2} \\ ae &= 2\sqrt{2} \end{aligned} \quad \left| \quad \begin{aligned} \frac{b^2}{a} &= 4 \\ b^2 &= 2a \end{aligned} \right.$$



$$b^2 = a^2(1 - e^2)$$

$$2a = a^2 - a^2e^2$$

$$2a = a^2 - (2\sqrt{2})^2 \Rightarrow a^2 - 8 - 2a = 0$$

$$(a-4)(a+2) = 0$$

$$a = 4 \quad a = -2 \text{ (is not possible)}$$

$$\therefore a = 4 \Rightarrow a^2 = 16$$

$$b^2 = 2a = 2(4) = 8$$

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

$$\frac{x^2}{8} + \frac{y^2}{16} = 1$$

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- 3) Find the equation of the hyperbola in each of the cases given below.

- i) foci $(\pm 2, 0)$ eccentricity $= 3/2$.

Soln

$$ae = 2$$

$$e = 3/2$$

$$a(3/2) = 2$$

$$a = 4/3$$

$$a^2 = 16/9$$

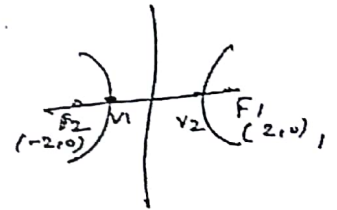
$$b^2 = a^2(e^2 - 1) = \frac{16}{9} \left(\frac{9}{4} - 1 \right) = \frac{16}{9} \left(\frac{5}{4} \right)$$

$$= \frac{20}{9}$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{16/9} - \frac{y^2}{20/9} = 1$$

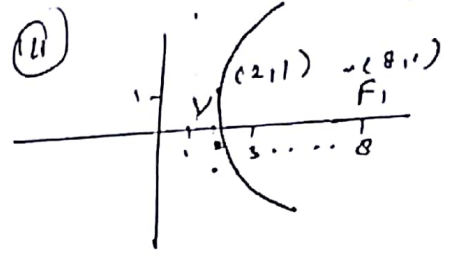
$$9x^2/16 - 9y^2/20 = 1$$



- ii) Centre $(2, 1)$ one of the foci $(8, 1)$ and corresponding direction $x = 4$.

Soln Distance $VF = ae = 6$ — (1)

Direction $\frac{a}{e} = 4$ — (2)



(1) $\times$ (2) $a \times \frac{4}{e} = 24$
 $a^2 = 24$

$\frac{(1)}{(2)} = \frac{ae}{a/e} = \frac{b}{4}$
 $c^2 = \frac{3}{2}$

$e = \frac{\sqrt{3}}{\sqrt{2}}$

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$b^2 = a^2 (e^2 - 1)$

$b^2 = 24 \left(\frac{3}{2} - 1 \right) = 24 \left(\frac{1}{2} \right) = 12$

$\therefore \frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

$(h, k) = (2, 1)$

$\frac{(x-2)^2}{24} - \frac{(y-1)^2}{12} = 1$

(iii) passing through $(5, -2)$ and length of the transverse axis along x-axis and length 8 units

Soln Transverse length $2a = 8$
 $a = 4$

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ — (1)

at $(5, -2)$ $\frac{25}{16} - \frac{4}{b^2} = 1$

$\frac{25}{16} - 1 = \frac{4}{b^2}$

$\frac{25-16}{16} = \frac{4}{b^2} \Rightarrow \frac{9}{16} = \frac{4}{b^2}$

$b^2 = \frac{16 \times 4}{9} = \frac{64}{9}$

(1) $\Rightarrow \frac{x^2}{16} - \frac{y^2}{\frac{64}{9}} = 1$

$\boxed{\frac{x^2}{16} - \frac{9y^2}{64} = 1}$

- 4) Find the vertex, focus, equation of directrix and length of the latus rectum of the following.

Soln: $y^2 = 16x = 4(4)x$

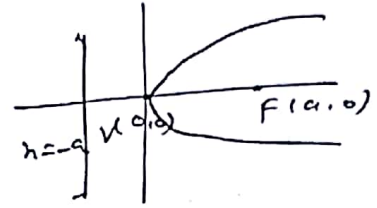
$a = 4.$

i) Vertex $V(0,0)$

ii) Focus $F(a,0) = F(4,0)$

iii) Eqn of the directrix $x = -a$
 $x = -4 \Rightarrow x + 4 = 0$

iv) Length of the latus rectum $= 4a = 4(4) = 16.$



(ii) $x^2 = 24y$

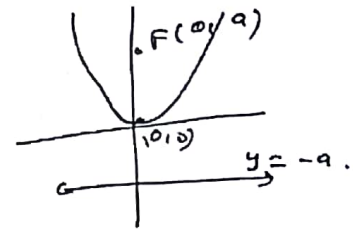
$4a = 24$

$a = 6.$

i) Vertex $V(0,0)$, ii) Focus $F(0,a) = F(0,6)$

iii) Eqn of the directrix $y = -a = -6$
 $\Rightarrow y + 6 = 0$

iv) Length of the latus rectum $= 4a = 4(6) = 24$



(iii) $y^2 = -8x$

$4a = 8$

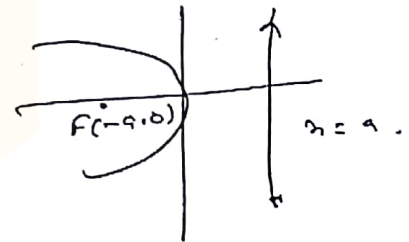
$a = 2$

i) Vertex $(0,0) = (0,0)$

ii) Focus $(-a,0) = (-2,0)$

iii) Eqn of the directrix $x = a = 2$
 $x - 2 = 0$

iv) Length of the latus rectum $4a = 8.$



(iv) $x^2 - 2x + 8y + 17 = 0$

$x^2 - 2x = -8y - 17$

$(x-2)^2 = -8y - 17 + 1$

$(x-2)^2 = -8y - 16 = -8(y+2)$

$X^2 = -8Y$

$4a = 8$

$a = 2$

$X = x - 2$

$x = X + 2$

$Y = y + 2$

$y = Y - 2$

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i) Vertex be $(1, -2) = (h, k)$

ii) Focus $(0+h, 0+k)$
 $= (0+1, 0-2) = (1, -2)$

iii) Eqn of the directrix $= y+k+a=0$

$$y-2+2=0$$

$$y=0$$

iv) Length of latus rectum is $4a$

$$= 4 \times 2 = 8 \text{ units}$$

(i) $y^2 - 4y - 8x + 12 = 0$

$$y^2 - 4y = 8x - 12$$

$$(y-2)^2 = 8x - 12 + 4 = 8x - 8 = 8(x-1)$$

$$(y-2)^2 = 8(x-1)$$

$$y^2 = 8x$$

$$y = y-2 \quad x = x-1$$

$$y = y+2$$

$$x = x+1$$

$$4a = 8$$

$$a = 2$$

Vertex $(1, 2) = (h, k)$

ii) Focus $(a+h, 0+k) = (2+1, 0+2) = (3, 2)$

iii) Eqn of the directrix $x = -a+h = -2+1 = -1$

$$\boxed{x+1=0}$$

iv) Length of latus rectus is $4a = 4 \times 2 = 8 \text{ units}$

5) Identify the type of conic and find centre, foci, vertices and directrices of each of the following.

i) $\frac{x^2}{25} + \frac{y^2}{9} = 1$

it is ellipse.

$$a^2 = 25$$

$$a = 5$$

$$b^2 = 9$$

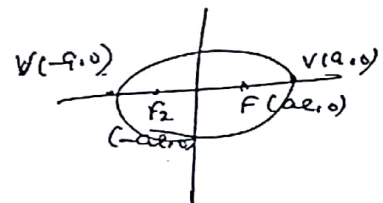
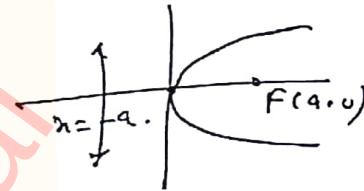
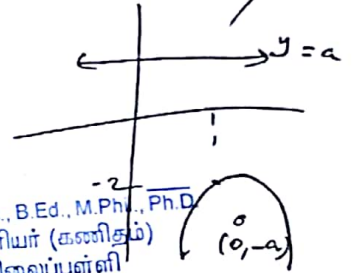
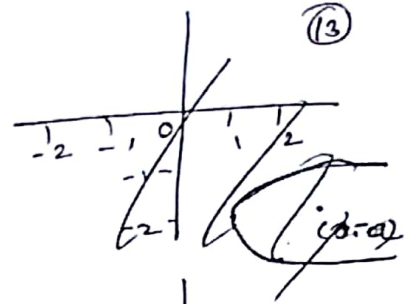
$$e^2 = \left(1 - \frac{b^2}{a^2}\right) = \left(1 - \frac{9}{25}\right) = \frac{25-9}{25} = \frac{16}{25}$$

$$e = \frac{4}{5}$$

$$ae = 5\left(\frac{4}{5}\right) = 4$$

Centre $= (0, 0)$, Vertex $(\pm a, 0) = (\pm 5, 0)$

Foci $(\pm ae, 0) = (\pm 4, 0)$



Eqn of direction $x = \pm a/e = \pm \frac{5}{4/5} = \pm \frac{25}{4}$ (14)

$$x = \pm \frac{25}{4}$$

(i) $\frac{x^2}{3} + \frac{y^2}{10} = 1$ $a^2 < b^2$ it is ellipse.

$$a^2 = 10 \quad b^2 = 3.$$

$$a = \pm \sqrt{10} \quad e^2 = \left(1 - \frac{b^2}{a^2}\right) = \left(1 - \frac{3}{10}\right) = \frac{7}{10}$$

$$e^2 = \frac{7}{10} \quad \therefore e = \sqrt{\frac{7}{10}}.$$

$$ae = \sqrt{10} \cdot \frac{\sqrt{7}}{\sqrt{10}} = \sqrt{7}.$$

(i) centre $(0, 0)$

(ii) vertex $(0, \pm a) = (0, \pm \sqrt{10})$

(iii) Foci $(0, \pm ae) = (0, \pm \sqrt{7})$

(iv) Eqn of the direction $y = \pm a/e = \pm \frac{\sqrt{10}}{\sqrt{7}} \cdot \sqrt{10} = \pm \frac{10}{\sqrt{7}}$

$$y = \pm \frac{10}{\sqrt{7}}.$$

(ii) $\frac{x^2}{25} - \frac{y^2}{144} = 1.$

it is Hyperbola.

$$a^2 = 25$$

$$b^2 = 144$$

$$e^2 = \left(1 + \frac{b^2}{a^2}\right) = \left(1 + \frac{144}{25}\right) = \left(\frac{169}{25}\right)$$

$$e^2 = \frac{169}{25} \quad \therefore e = \frac{13}{5}$$

$$a = 5 \quad ae = 5 \left(\frac{13}{5}\right) = 13$$

(i) centre $(0, 0)$

(ii) vertex $(\pm a, 0) = (\pm 5, 0)$

(iii) Foci $(\pm ae, 0) = (\pm 13, 0)$

(iv) Eqn of the direction $x = \pm a/e = \pm \frac{5}{13/5} = \pm \frac{25}{13}$

$$x = \pm \frac{25}{13}.$$

(iv)

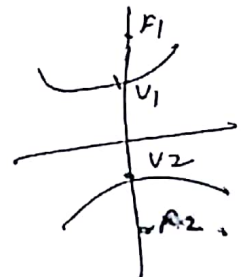
$$\frac{y^2}{16} - \frac{x^2}{9} = 1.$$

it is Hyperbola

$$a^2 = 16$$

$$b^2 = 9.$$

about y axis

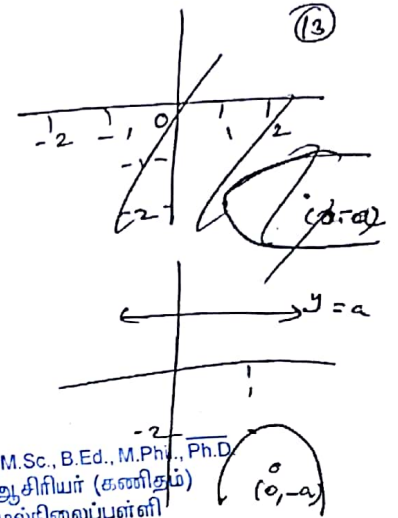


i) Vertex be $(1, -2) = (h, k)$

ii) Focus $(0+h, 0+k)$
 $= (0+1, 0-2) = (1, -2)$

iii) Eqn of the directrix $= y+k+a = 0$
 $y-2+2 = 0$
 $y = 0$

iv) Length of latus rectum is $4a$
 $= 4 \times 2 = 8 \text{ units}$



v) $y^2 - 4y - 8x + 12 = 0$

$$y^2 - 4y = 8x - 12$$

$$(y-2)^2 = 8x - 12 + 4 = 8x - 8 = 8(x-1)$$

$$(y-2)^2 = 8(x-1)$$

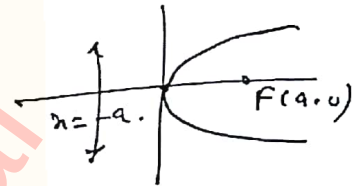
$$y^2 = 8x$$

$$y = y-2 \quad x = x-1$$

$$y = y+2 \quad x = x+1$$

$$4a = 8$$

$$a = 2$$



Vertex $(1, 2) = (h, k)$

ii) Focus $(a+h, 0+k) = (2+1, 0+2) = (3, 2)$

iii) Eqn of the directrix $x = -a+h = -2+1 = -1$
 $x+1 = 0$

iv) Length of latus rectus is $4a = 4 \times 2 = 8 \text{ units}$

5) Identify the type of conic and find centre, foci, vertices and directrices of each of the following.

i) $\frac{x^2}{25} + \frac{y^2}{9} = 1$

it is ellipse.

$$a^2 = 25$$

$$b^2 = 9$$

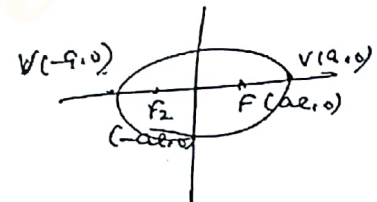
$$e^2 = \left(1 - \frac{b^2}{a^2}\right) = \left(1 - \frac{9}{25}\right) = \frac{25-9}{25} = \frac{16}{25}$$

$$e = \frac{4}{5}$$

$$ae = 5\left(\frac{4}{5}\right) = 4$$

Centre $= (0, 0)$, vertex $(\pm a, 0) = (\pm 5, 0)$

Foci $(\pm ae, 0) = (\pm 4, 0)$



$$a = 4, \quad e^2 = \left(1 + \frac{b^2}{a^2}\right) = \left(1 + \frac{4}{16}\right) = \frac{25}{16}$$

$$e = \frac{5}{4}$$

$$ae = 4 \left(\frac{5}{4}\right) = 5$$

(i) Centre (0,0)

(ii) Vertices (0, $\pm a$) = (0, ± 4)

(iii) Foci (0, $\pm ae$) = (0, ± 5)

(iv) Egn of the direction $y = \pm \frac{a}{e} = \pm \frac{4}{5/4} = \pm \frac{16}{5}$
 $y = \pm \frac{16}{5}$

b) Prove that the length of the latus rectum of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $\frac{2b^2}{a}$.

Soln The latus rectum LL' of an hyperbola

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ passes through (ae, y_1) .

$$\frac{a^2}{a^2} - \frac{y_1^2}{b^2} = 1$$

$$e^2 - 1 = \frac{y_1^2}{b^2}$$

$$(\because e^2 = 1 + \frac{b^2}{a^2})$$

$$y_1^2 = b^2 (e^2 - 1)$$

$$= b^2 \left(1 + \frac{b^2}{a^2} - 1\right)$$

$$y_1^2 = \frac{b^4}{a^2}$$

$$y_1 = \pm \frac{b^2}{a}$$

end points of latus rectums are

$$(ae, \frac{b^2}{a}) \text{ and } (ae, -\frac{b^2}{a}) \therefore LL' = \frac{b^2}{a} + \frac{b^2}{a}$$

$$LL' = 2 \frac{b^2}{a}. \text{ Hence proved.}$$

7) Show that the absolute value of difference of the focal distances of any point P on the hyperbola is the length of its transverse axis.

8. Identify the type of conic and find centre, foci, vertices and directrices of each of the following.

$$(i) \frac{(x-3)^2}{225} + \frac{(y-4)^2}{289} = 1$$

$$a^2 < b^2$$

it is ellipse.

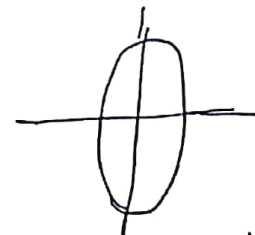
$$a^2 = 289$$

$$b^2 = 225$$

$$a = \pm 17$$

$$b = \pm 15$$

$$ae = 17 \left(\frac{8}{17} \right) = 8, \quad a/e = \frac{17}{8/17} = \frac{289}{8}$$



$$e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{225}{289}} = \sqrt{\frac{289-225}{289}} = \sqrt{\frac{64}{289}} = \frac{8}{17}$$

$$(i) \text{ Centres } (3, 4) = (h, k)$$

$$(ii) \text{ Vertices } (h \pm a, k) = (3, \pm 17 + 4)$$

$$= (3, 21) \text{ \& } (3, -13)$$

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$$(iii) \text{ Foci } (h \pm ae, k) = (3, \pm 8 + 4)$$

$$= (3, 12) \text{ \& } (3, -4)$$

$$(iv) \text{ Directrices } y = \pm \frac{a}{e} + k = \pm \frac{289}{8} + 4$$

$$= \frac{289}{8} + 4 \text{ \& } -\frac{289}{8} + 4$$

$$= \frac{289+32}{8} \text{ or } -\frac{289+32}{8} = \frac{321}{8} \text{ or } -\frac{257}{8} \quad (17)$$

(ii)

$$\frac{(x+1)^2}{100} + \frac{(y-2)^2}{64} = 1$$

it is ellipse $a^2 > b^2$

$$a^2 = 100$$

$$b^2 = 64$$

$$a = 10$$

$$e^2 = \left(1 - \frac{b^2}{a^2}\right) = 1 - \frac{64}{100} = \frac{36}{100}$$

$$e = \frac{3}{5}$$

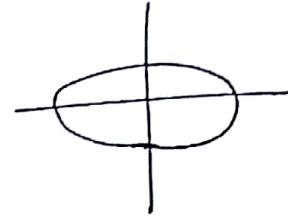
$$ae = 10\left(\frac{3}{5}\right) = 6 \quad ; \quad a/e = \frac{10}{3/5} = \frac{50}{3}$$

$$X = x+1$$

$$Y = y-2$$

$$x = X-1$$

$$y = Y+2$$



	Referred to $X \& Y$	Referred to $x \& y$ $x = X-1$ & $y = Y+2$
i) centre	(0, 0)	(-1, 2)
ii) Vertices	$V = (\pm a, 0)$ $V_1 = (+10, 0)$ $V_2 = (-10, 0)$	$V_1 (+9, 2)$ $V_2 (-11, 2)$
iii) Focus	$F (\pm ae, 0)$ $F_1 (+6, 0)$ $F_2 (-6, 0)$	$F_1 (5, 2)$ $F_2 (-7, 2)$
iv) Directrices	$X = \pm a/e = \pm \frac{50}{3}$	$x = \frac{50}{3} - 1 = \frac{47}{3}$ $x = -\frac{50}{3} - 1 = -\frac{53}{3}$

(iii)

$$\frac{(x+3)^2}{225} - \frac{(y-4)^2}{64} = 1$$

$$a^2 = 225 \quad b^2 = 64$$

$$a = \pm 15$$

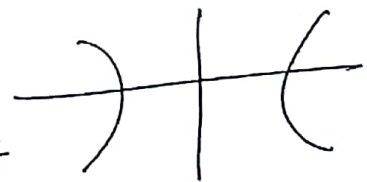
$$e^2 = 1 + \frac{b^2}{a^2} = 1 + \frac{64}{225} = \frac{225+64}{225} = \frac{289}{225}$$

$$e^2 = \frac{289}{225} \quad \therefore e = \frac{17}{15}$$

$$ae = 15\left(\frac{17}{15}\right) = 17$$

$$a/e = \frac{15}{17/15} = \frac{225}{17}$$

it is hyperbola on x axis



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(18)

$X = x+3$ $x = X-3$	$y = y-4$ $y = Y+4$	
	Referred to X & Y	Referred to x & y $x = X-3$ $y = Y+4$
i) centre	$C(0,0)$	$C(-3, 4)$
ii) Vertices	$V(\pm a, 0)$ $V_1(+15, 0)$ $V_2(-15, 0)$	$V_1(12, 4)$ $V_2(-18, 4)$
iii) Foci	$F(\pm ae, 0)$ $F_1(17, 0)$ $F_2(-17, 0)$	$F_1(14, 4)$ $F_2(-20, 4)$
iv) Direction	$X = \pm a/e = \pm \frac{225}{17}$	$x = \frac{225}{17} - 3 = \frac{174}{17}$ $x = -\frac{225}{17} - 3 = -\frac{276}{17}$

(19)

$$\frac{(y-2)^2}{25} - \frac{(x+1)^2}{16} = 1$$

it is hyperbola on y axis

$$a^2 = 25 \quad b^2 = 16$$

$$a = \pm 5$$

$$e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{16}{25}} = \sqrt{\frac{41}{25}} = \frac{\sqrt{41}}{5}$$

$$ae = 5\left(\frac{\sqrt{41}}{5}\right) = \sqrt{41}$$

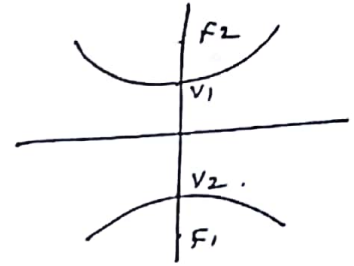
$$a/e = 5 / \frac{\sqrt{41}}{5} = \frac{25}{\sqrt{41}}$$

$$X = x+1$$

$$x = X-1$$

$$Y = y-2$$

$$y = Y+2$$



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	Referred to X & Y	Referred to x & y $x = X-1$ $y = Y+2$
i) centre	$C(0,0)$	$(-1, 2)$
ii) Vertices (0, $\pm a$)	$V_1(0, 5)$ $V_2(0, -5)$	$V_1(-1, 7)$ $V_2(-1, -3)$
iii) Foci (0, $\pm ae$)	$F_1(0, \sqrt{41})$ $F_2(0, -\sqrt{41})$	$F_1(-1, \sqrt{41}+2)$ $F_2(-1, -\sqrt{41}+2)$
iv) Direction	$Y = \pm a/e = \pm \frac{25}{\sqrt{41}}$	$y = \frac{25}{\sqrt{41}} + 2$ $y = -\frac{25}{\sqrt{41}} + 2$

(v)

$$18x^2 + 12y^2 - 144x + 48y + 120 = 0$$

$$18x^2 - 144x + 12y^2 + 48y = -120$$

$$18(x^2 - 8x) + 12(y^2 + 4y) = -120$$

$$18(x-4)^2 + 12(y+2)^2 = -120 + 288 + 48$$

$$18(x-4)^2 + 12(y+2)^2 = 216$$

$$\frac{18(x-4)^2}{216} + \frac{12(y+2)^2}{216} = 1$$

$$\frac{(x-4)^2}{12} + \frac{(y+2)^2}{18} = 1$$

it is ellipse on y axis

$$a^2 = 18 \quad b^2 = 12$$

$$a = \sqrt{18} = 3\sqrt{2}$$

$$e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{12}{18}} = \sqrt{1 - \frac{2}{3}}$$

$$= \sqrt{\frac{1}{3}} = \frac{1}{\sqrt{3}}$$

$$ae = 3\sqrt{2} \cdot \frac{1}{\sqrt{3}} = \sqrt{6}$$

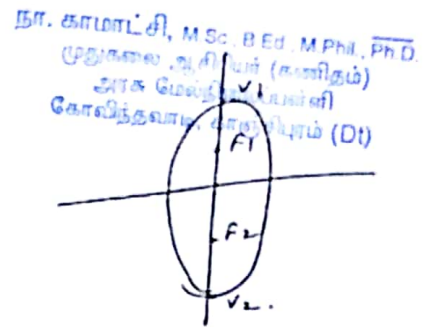
$$a/e = \frac{3\sqrt{2}}{1/\sqrt{3}} = 3\sqrt{6}$$

$$X = x - 4$$

$$x = X + 4$$

$$Y = y + 2$$

$$y = Y - 2$$



	Referred to x & y	Referred to x & y
i) Centre	C (0, 0)	(4, -2)
ii) Vertices (0, ±a)	V1 (0, +3√2) V2 (0, -3√2)	V1 (4, -2 + 3√2) V2 (4, -2 - 3√2)
iii) Foci (0, ±ae)	F1 (0, √6) F2 (0, -√6)	F1 (4, -2 + √6) F2 (4, -2 - √6)
iv) Directric Y = ±a/e	Y = ±3√6	Y = 3√6 - 2 Y = -3√6 - 2

(vi)

$$9x^2 - y^2 - 36x - 6y + 18 = 0$$

$$9x^2 - 36x - y^2 - 6y = -18$$

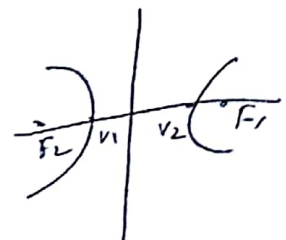
$$9(x^2 - 4x) - (y^2 + 6y) = -18$$

$$9(x-2)^2 - (y+3)^2 = -18 + 36 - 9$$

$$9(x-2)^2 - (y+3)^2 = 9$$

$$\frac{(x-2)^2}{1} - \frac{(y+3)^2}{9} = 1$$

it is hyperbola on x axis



$$a^2 = 9 \quad b^2 = 1$$

$$a = 3$$

$$ae = 3 \times \frac{\sqrt{10}}{3} = \sqrt{10}$$

$$\frac{a}{e} = \frac{3}{\sqrt{10}/3} = \frac{9}{\sqrt{10}}$$

$$c = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{1}{9}} = \sqrt{\frac{10}{9}} = \frac{\sqrt{10}}{3} \quad (2a)$$

$$X = x - 2$$

$$x = X + 2$$

$$y = y + 3$$

$$y = y - 3$$

	Referred to $x + y$	Referred to $x + y$ $x = x + 2 \quad y = y - 3$
i) Centre	$C(0, 0)$	$(2, -3)$
ii) Vertices ($\pm a, 0$)	$V_1(3, 0)$ $V_2(-3, 0)$	$V_1(5, -3)$ $V_2(-1, -3)$
iii) Foci ($\pm ae, 0$)	$F_1(\sqrt{10}, 0)$ $F_2(-\sqrt{10}, 0)$	$F_1(2 + \sqrt{10}, -3)$ $F_2(2 - \sqrt{10}, -3)$
iv) directrix	$x = \pm \frac{a}{e} = \pm \frac{9}{\sqrt{10}}$	$x = \frac{9}{\sqrt{10}} + 2$ (or) $= -\frac{9}{\sqrt{10}} + 2$

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(2)

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

which is a circle $(x-h)^2 + (y-k)^2 = r^2$.

(ii) $A \neq c$ & A and c are the same signs it is ellipse
 (iii) $A \neq c$ & A and c are the opposite signs it is hyperbola

(iv) $A \neq c$ & A and c are of opposite signs. it is hyperbola

(v) $A = c$ and $B = D = E = F = 0$ it is $x^2 + y^2 = 0$.

(vi) $A = C = F$ & $B = D = E = 0$ it is $x^2 + y^2 + z^2 = 0$. Q.S

There is no real solution.

There is no real solution.
(vi) $A=0$ or $C \neq 0$ and others are zeros the coordinate axes.

General equation yield coordinate axes.

(viii) $A = -c$ and rest are zero. $\Rightarrow x^2 - y^2 = 0$

Equation	Condition	Type of the conic.
$2x^2 - y^2 = 7$	4	Hyperbola
$3x^2 + 3y^2 - 4x + 3y + 10 = 0$	1	Circle.
$3x^2 + 2y^2 = 14$	3	ellipse
$x^2 + y^2 + x - y = 0$	1	circle.
$11x^2 - 25y^2 - 44x + 50y - 256 = 0$	4	hyperbola.
$y^2 + 4x + 3y + k = 0$	2	Parabola

Exercise 5.4

(22)

1. Find the equation of the two tangents that can be drawn from (5, 2) to the ellipse $2x^2 + 7y^2 = 14$.

Soln $2x^2 + 7y^2 = 14$

$$\frac{x^2}{7} + \frac{y^2}{2} = 1$$

$$y = mx + \sqrt{a^2m^2 + b^2}$$

$$a^2 = 7 \quad b^2 = 2$$

It passes through (5, 2)

$$2 = 5m + \sqrt{7m^2 + 2}$$

$$2 - 5m = \sqrt{7m^2 + 2}$$

$$(2 - 5m)^2 = (7m^2 + 2)$$

$$4 + 25m^2 - 20m = 7m^2 + 2$$

$$18m^2 - 20m + 2 = 0$$

$$9m^2 - 10m + 1 = 0$$

$$(9m - 1)(m - 1) = 0$$

$$m = 1 \quad \text{or} \quad m = \frac{1}{9}$$

Eqn of tangent

(i) $m = 1 \quad y - 2 = 1(x - 5)$

$$\boxed{x - y - 3 = 0}$$

(ii) $m = \frac{1}{9} \quad y - 2 = \frac{1}{9}(x - 5)$

$$9y - 18 = x - 5$$

$$\boxed{x - 9y + 13 = 0}$$

$\therefore$ Eqn of tangent be $x - y - 3 = 0$ & $x - 9y + 13 = 0$.

2. Find the equation of tangents to the hyperbola $\frac{x^2}{16} - \frac{y^2}{64} = 1$ which is parallel to $10x - 3y + 9 = 0$.

Soln $\frac{x^2}{16} - \frac{y^2}{64} = 1$

$$a^2 = 16 \quad b^2 = 64$$

$$10x - 3y + 9 = 0$$

$$m = \frac{-10}{-3} = \frac{10}{3}$$

$$c = \frac{9}{10}$$

$$\sqrt{a^2m^2 - b^2} = \sqrt{16\left(\frac{10}{3}\right)^2 - 64} = \sqrt{\frac{1600}{9} - 64} = \sqrt{\frac{1600 - 576}{9}}$$

$$= \sqrt{\frac{1024}{9}} = \frac{32}{3}$$

Eqn of tangent to the hyperbola be

$$y = mx \pm \sqrt{a^2m^2 - b^2}$$

$$y = \frac{10}{3}x \pm \frac{32}{3}$$

$$y = \frac{10}{3}x + \frac{32}{3}$$

$$3y = 10x + 32$$

$$\boxed{10x - 3y + 32 = 0}$$

or

$$y = \frac{10}{3}x - \frac{32}{3}$$

$$3y = 10x - 32$$

$$\boxed{10x - 3y - 32 = 0}$$

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- 4) Find the equation of the tangent to the parabola $y^2 = 16x$ perpendicular to $2x + 2y + 3 = 0$. (23)

Soln $y^2 = 16x$ $2x + 2y + 3 = 0$
 $4a = 16$ $x + y + \frac{3}{2} = 0$
 $a = 4$

The equation of the line which is perpendicular to $x + y + \frac{3}{2} = 0$ is

$x - y + 4 = 0$ - (1)
 $m = \frac{-1}{-1} = 1$ $C = \frac{a}{m} = \frac{4}{1} = 4$

$\therefore \textcircled{1} \Rightarrow \boxed{x - y + 4 = 0}$

- 3) Show that the line $x - y + 4 = 0$ is a tangent to the ellipse $x^2 + 3y^2 = 12$. Also find the co-ordinates of the point of contact.

Soln $y = mx + c$ be tangent to $x^2 + 3y^2 = 12$

The condition is $c^2 = a^2m^2 + b^2$.

$x - y + 4 = 0$ $x^2 + 3y^2 = 12$
 $x + 4 = y$ $\frac{x^2}{12} + \frac{y^2}{4} = 1$
 $m = 1$ $c = 4$ $a^2 = 12$ $b^2 = 4$

$c^2 = a^2m^2 + b^2$

$4^2 = 12(1)^2 + 4$

$16 = 16$

$\therefore$ Given line touches tangent $x^2 + 3y^2 = 12$.

Point of contact $\left(-\frac{a^2m}{c}, -\frac{b^2}{c}\right)$

$= \left(-\frac{12(1)}{4}, \frac{4}{4}\right) = (-3, 1)$

- 5) Find the equation of the tangent and normal to hyperbola $12x^2 - 9y^2 = 108$ at $Q = (9, 3)$

(Hint use parametric form)

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Soln Egn of the tangent to hyperbola be (24)

$$x \frac{\sec \alpha}{a} - y \frac{\tan \alpha}{b} = 1$$

Given. $12x^2 - 9y^2 = 108$

$$\frac{12x^2}{108} - \frac{9y^2}{108} = 1$$

$$\frac{x^2}{9} - \frac{y^2}{12} = 1$$

$$a^2 = 9 \quad b^2 = 12 \quad e = \frac{11}{3}$$

$$a = 3 \quad b = 2\sqrt{3}$$

$$\therefore x \frac{\sec \frac{11}{3}}{3} - y \frac{\tan \frac{11}{3}}{2\sqrt{3}} = 1$$

$$\frac{x(2)}{3} - \frac{y\sqrt{3}}{2\sqrt{3}} = 1$$

$$\frac{2x}{3} - \frac{y}{2} = 1$$

$$\frac{4x - 3y}{6} = 1 \Rightarrow \boxed{4x - 3y = 6}$$

i) Egn of the normal to hyperbola be

$$\frac{ax}{\sec \alpha} + \frac{by}{\tan \alpha} = a^2 + b^2$$

$$\frac{3x}{\sec \frac{11}{3}} + \frac{2\sqrt{3}y}{\tan \frac{11}{3}} = 9 + 12$$

$$\frac{3x}{2} + \frac{2\sqrt{3}y}{\sqrt{3}} = 21$$

$$\frac{3x + 4y}{2} = 21$$

$$\boxed{3x + 4y = 42}$$

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5) Find the equation of the tangent at $t = 2$ to the parabola $y^2 = 8x$ (Hint use parametric form).

Soln Egn of a tangent be

$$yt = x + at^2$$

Here $t = 2$

$$y^2 = 8x \quad \therefore 4a = 8$$

$$a = 2$$

(25)

$$y(2) = x + 2(4)$$

$$2y = x + 8$$

$$\boxed{x - 2y + 8 = 0}$$

- 7) Prove that the point of intersection of the tangents at t_1 and t_2 on the parabola $y^2 = 4ax$ is $(at_1t_2, a(t_1+t_2))$

Soln. Eqn of the tangent of parabola $y^2 = 4ax$ be

$$yt = x + at^2 \quad \text{--- (1)}$$

at t_1

$$yt_1 = x + at_1^2 \quad \text{--- (2)}$$

at t_2

$$yt_2 = x + at_2^2 \quad \text{--- (3)}$$

① - ③

$$y(t_1 - t_2) = a(t_1^2 - t_2^2)$$

$$y(t_1 - t_2) = a(t_1 + t_2)(t_1 - t_2)$$

$$y = a(t_1 + t_2)$$

$$\text{①} \Rightarrow t_1 a(t_1 + t_2) = x + at_1^2$$

$$x = at_1^2 + at_1t_2 - at_1^2$$

$$x = at_1t_2$$

$\therefore$ Point of intersection be $(at_1t_2, a(t_1+t_2))$

- 8) If the normal at the point t_1 on the parabola $y^2 = 4ax$ meets the parabola again at the point t_2 then prove that $t_2 = -(t_1 + \frac{2}{t_1})$

Soln

$$y^2 = 4ax$$

$$yy_1 = 2a(x+x_1)$$

$$yy_1 = 2a(x+x_1)$$

$$\text{at } (at_1^2, 2at_1) \Rightarrow y_1(2at_1) = 2a(x+at_1^2)$$

$$y = \frac{1}{t_1}x + at_1$$

$\therefore$ slope of tangent be $\frac{1}{t_1}$

Slope of normal be $-\frac{1}{1/t_1} = -t_1$

Eqn of normal at $(at_1^2, 2at_1)$ is

(2b)

$$\frac{y - 2at_1}{x - at_1^2} = -t_1$$

$$y - 2at_1 = -t_1(x - at_1^2)$$

$(at_2^2, 2at_2)$ lies on parabola.

$$2at_2 - 2at_1 = -t_1(at_2^2 - at_1^2)$$

$$2a(t_2 - t_1) = -t_1a(t_2^2 - t_1^2)$$

$$2(t_2 - t_1) = -t_1(t_2 - t_1)(t_1 + t_2)$$

$$2(t_2 - t_1) + (t_2 - t_1)t_1(t_1 + t_2) = 0$$

$$(t_2 - t_1)(2 + t_1(t_1 + t_2)) = 0$$

$$t_1 \neq t_2 \quad \therefore 2 + t_1(t_1 + t_2) = 0$$

$$t_1(t_1 + t_2) = -2$$

$$t_1 + t_2 = -\frac{2}{t_1}$$

$$t_2 = -\frac{2}{t_1} - t_1$$

$$t_2 = -\left(\frac{2}{t_1} + t_1\right)$$

Hence proved.

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Exercise 5.5

1. A bridge has a parabolic arch that is 10m high in the centre and 30m wide at the bottom. Find the height of the arch 6m from the centre on either sides.

Soln

$$PQ = 2a = 30m$$

$$a = 15m.$$

Point Q be $(15, -10)$.

Eqn of the parabola

$$x^2 = -4ay. \quad \text{--- (1)}$$

Q lies on parabola.

$$15^2 = -4a(-10)$$

$$\frac{225}{40} = a \Rightarrow a = \frac{225}{40}$$

$$\textcircled{1} \Rightarrow x^2 = -4 \left(\frac{225}{40} \right) y = -22.5y$$

$$x^2 = -\frac{225}{10}y$$

Let $B(6, y_1)$ lies on parabola

$$6^2 = -\frac{225}{10}y_1$$

$$y_1 = \frac{36 \times 10^2}{-225} = -\frac{8}{5} = \frac{8}{5} \text{ m.}$$

$$AB = AC - BC = 10 - \frac{8}{5} = \frac{50 - 8}{5} = \frac{42}{5}$$

$$AB = 8.4m$$

$\therefore$ The height of the arch 6m from the centre is 8.4m.

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2. A tunnel through a mountain for a four lane highway is to have a elliptical opening. The total width of the highway (not the opening) is to be 16m. and the height at the edge of the road must be sufficient for a truck 4m high to clear if the highest point of the opening is to be 5m approximately. How wide must the opening be?

Soln

$$AB = 2a = 16$$

$$a = 8$$

$$b = 5$$

Eqn of the ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{64} + \frac{y^2}{25} = 1$$

Let $P(x_1, 4)$ lies on ellipse.

$$\frac{x_1^2}{64} + \frac{4^2}{25} = 1$$

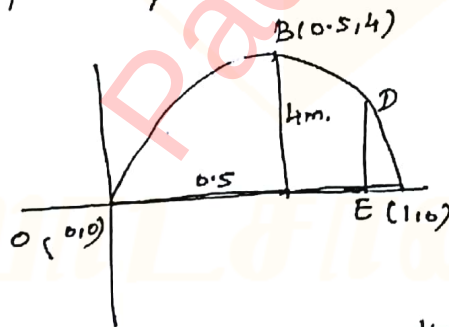
$$\frac{x_1^2}{64} = 1 - \frac{16}{25} = \frac{25-16}{25} = \frac{9}{25}$$

$$x_1^2 = 64 \times \frac{9}{25}$$

$$x_1 = \frac{8 \times 3}{5} = \frac{24}{5} = 4.8$$

$$\text{width} = 2x_1 = 2 \times 4.8 = 9.6 \text{ m.}$$

- 3) At a water fountain, water attains a maximum height of 4m at horizontal distance of 0.5m from its origin. If the path of water is a parabola. find the height of water at a horizontal distance of 0.75m from the point of origin.

Soln

$y = ax^2 + bx + c$. it passes through $O(0,0)$, $B(0.5, 4)$

and $E(1,0)$.(i) At $O(0,0)$ (ii) At $E(1,0)$ (iii) At $B(0.5, 4)$

$$c = 0$$

$$a + b + c = 0$$

$$a + b = 0 \quad \text{--- (1)}$$

$$4 = 0.25a + 0.5b + c = 0$$

$$0.25a + 0.5b = 4 \quad \text{--- (2)}$$

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(29)

$$\begin{aligned}
 \textcircled{1} \times 0.5 \Rightarrow & 0.5a + 0.5b = 4 \\
 & 0.25a + 0.5b = 4 \\
 & \quad \quad \quad (-) \quad \quad \quad (-) \quad \quad \quad (-) \\
 \hline
 -0.25a & = -4 \\
 a & = \frac{4}{0.25} = \frac{400}{25} = 16
 \end{aligned}$$

$$\begin{aligned}
 a + b & = 0 \\
 b & = -a = -16.
 \end{aligned}$$

$$\therefore c = 0 \quad a = +16, \quad b = -16.$$

$$\therefore y = -16x^2 + 16x.$$

$$\begin{aligned}
 \text{When } x & = 0.75 \quad y = -16(0.75)^2 + 16(0.75) \\
 & = -16(0.5625) + 12
 \end{aligned}$$

$$y = -9 + 12$$

$$y = 3m.$$

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Height of water at a horizontal distance of 0.75m from the point of origin is 3m.

- 4) An engineer designs a satellite dish with a parabolic cross section. The dish is 5m wide at the opening and the focus is placed 1.2m from the vertex.
- a) Position a co-ordinate system with the origin at the vertex and the x-axis on the parabola's axis of symmetry and find an equation of the parabola
- b) Find the depth of the satellite dish at the vertex.

Soln. Eqn of the parabola be

$$y^2 = 4ax \quad \text{--- (1)}$$

$$a = 1.2.$$

$$\therefore y^2 = 4 \times 1.2 x$$

$$\Rightarrow y^2 = 4.8x$$

$$AB = 5$$

$$AF = \frac{5}{2} = 2.5$$

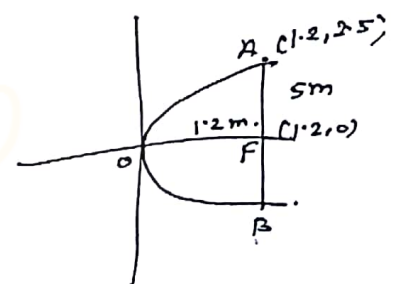
Point A be (1.2, 2.5) lies on parabola.

$$(2.5)^2 = 4(1.2)x$$

$$\frac{2.5 \times 2.5}{4 \times 1.2} = x$$

$$\Rightarrow x = \frac{2.5 \times 2.5}{4 \times 1.2} = 1.3m.$$

$$\begin{aligned}
 \therefore x & = 1.3m \\
 y^2 & = 4.8x
 \end{aligned}$$



- 5) Parabolic cable of a 60m portion of the roadbed of a suspension bridge are positioned as shown below. Vertical cables are to be spaced every 6m along this portion of the roadbed calculate the length of first two of these vertical cables from the vertex.

Soln vertex be $V(0, 3) = (h, k)$

$$2a = 60m$$

$$a = 30.$$

$\therefore$ let A & B be $(30, -16)$ and $(30, 16)$.

Egn of the parabola be

$$(x-h)^2 = 4a(y-k) \quad \text{--- (1)}$$

B(30, 16) lies on (1)

$$(30-0)^2 = 4a(16-3)$$

$$30^2 = 4a(13)$$

$$a = \frac{30 \times 30}{4 \times 13}$$

(i)

When $x = 6$

$$(x-h)^2 = 4a(y-k)$$

$$(6-0)^2 = 4 \left(\frac{30 \times 30}{4 \times 13} \right) (y-3)$$

$$\frac{36 \times 13}{30 \times 20} = y - 3$$

$$\frac{52}{100} + 3 = y$$

$$\therefore y = 0.52 + 3 = 3.52m.$$

$$\therefore y = 3.52m$$

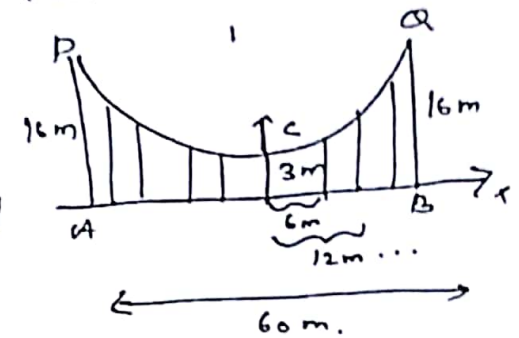
(ii) When $x = 12$

$$(12-0)^2 = 4a(y-3)$$

$$\frac{144}{\frac{48}{49 \times 13}} = y - 3$$

$$\frac{208}{100} + 3 = y \Rightarrow 2.08 + 3 = y$$

$$\therefore y = 5.08m$$



- 6) Cross section of a Nuclear cooling tower is in the shape of a hyperbola with equation $\frac{x^2}{30^2} - \frac{y^2}{44^2} = 1$. The tower is 150m tall and the distance from the top of the tower to the centre of the hyperbola is

half the distance from the base of the tower to the centre of the hyperbola. Find the diameter of the top and base of the tower. (31)

Soln Given. $p + 2p = 150$

$$3p = 150$$

$$p = 50.$$

Distance from the top of the tower
= 50 m.

Distance from the base of the tower be 100.

Eqn of hyperbola is

$$\frac{x^2}{30^2} - \frac{y^2}{44^2} = 1$$

i) $y = 50$ m Then $\frac{x^2}{30^2} - \frac{50^2}{44^2} = 1$

$$\frac{x^2}{30^2} = 1 + \frac{50^2}{44^2} = 1 + 1.29$$

$$\frac{x^2}{30^2} = 2.29$$

$$x^2 = 30^2 (2.29)$$

$$x = 30 \sqrt{2.29} = 45.35$$

$$\boxed{x = 45.35 \text{ m}}$$

(ii) When $y = -100$.

$$\frac{x^2}{30^2} - \frac{(-100)^2}{44^2} = 1$$

$$\frac{x^2}{30^2} = 1 + \frac{(100)^2}{44^2} = 1 + 5.17 = 6.17$$

$$x^2 = 30^2 (6.17)$$

$$x = 30 \sqrt{6.17}$$

$$\boxed{x = 74.51 \text{ m}}$$

7)

A rod of length 1.2 m moves with its ends always touching the coordinate axes. The locus of a point P on the rod, which is 0.3 m from the end in contact with x axis is an ellipse. Find the eccentricity.

Soln. Length of rod BD = 1.2 m

$$PB = 0.3 \text{ m}$$

$$PD = 1.2 - 0.3 = 0.9 \text{ m.}$$

$\triangle PAB$ & $\triangle PCD$ are similar triangles

In $\triangle PAB$ $\sin \theta = \frac{y}{0.3}$

In $\triangle PCD$ $\cos \theta = \frac{x}{0.9}$

We know that $\sin^2 \theta + \cos^2 \theta = 1$

$$\frac{y^2}{0.3^2} + \frac{x^2}{0.9^2} = 1$$

$$\frac{x^2}{0.9^2} + \frac{y^2}{0.3^2} = 1$$

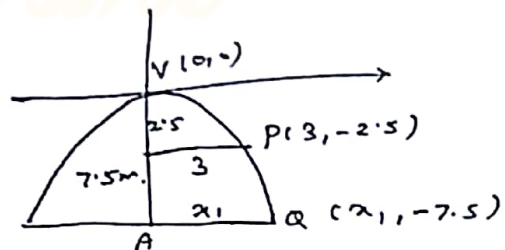
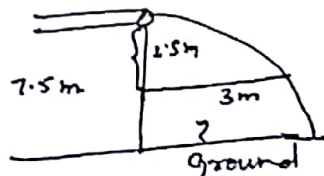
$$a^2 = 0.9^2 \quad b^2 = 0.3^2$$

$$\text{Eccentricity } e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{0.3^2}{0.9^2}} = \sqrt{1 - \left(\frac{1}{3}\right)^2}$$

$$= \sqrt{1 - \frac{1}{9}} = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3} \approx$$

- 8) Assume that water issuing from the end of a horizontal pipe, 7.5 m above the ground, describes a parabolic path. The vertex of the parabola path is at the end of the pipe. At a position 2.5 m below the line of the pipe. The flow of water has curved outward 3 m beyond the vertical line through the end of the pipe. How far beyond this vertical line will the water strike the ground?

Soln



Eqn of the parabola be $x^2 = -4ay$

Let $P(3, -2.5)$ lies on parabola.

$$3^2 = -4(a)(-2.5)$$

$$9 = 10a.$$

(33)

$$a = \frac{9}{10}$$

$$\therefore x^2 = -4\left(\frac{9}{10}\right)y \quad \text{--- (1)}$$

Let $Q(x_1, -7.5)$ also lies on parabola

$$x_1^2 = -4\left(\frac{9}{10}\right)(-7.5)$$

$$x_1^2 = 4 \times \frac{9}{10} \times 7.5$$

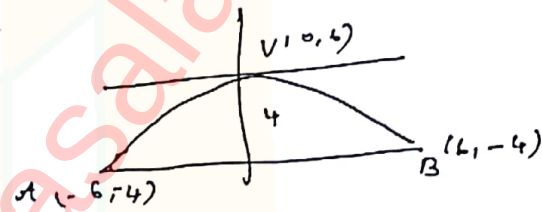
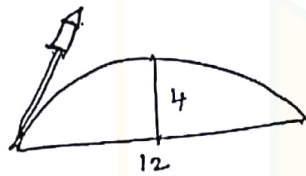
$$x_1^2 = 27$$

$$x = 3\sqrt{3} \text{ m.}$$

The water strike the ground at $3\sqrt{3} \text{ m.}$

- 9) on lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of 4m. when it is 6m away from the point of projection. Finally it reaches the ground 12m away from the starting point. Find the angle of projection.

Soln.



Eqn of the parabola be $x^2 = -4ay$ --- (1)

$B(6, -4)$ lies on parabola.

$$6^2 = -4a(-4)$$

$$\frac{36}{16} = a \quad \therefore a = \frac{9}{4}$$

$$\textcircled{1} \Rightarrow x^2 = -4\left(\frac{9}{4}\right)y$$

$$x^2 = -9y \quad \text{--- (2)}$$

Now need to find slope at $(-6, -4)$

Diff (2) w.r.to x

$$2x = -9 \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{2x}{-9}$$

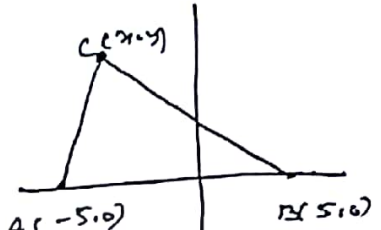
$$\text{At } (-6, -4) \quad \frac{dy}{dx} = \frac{2(-6)}{-9} = \frac{12}{9} = \frac{4}{3}$$

$$\tan \theta = \frac{4}{3}$$

$$\alpha = \tan^{-1} \left(\frac{4}{3} \right)$$

- 10) Points A and B are 10km apart and it is determined from the sound of an explosion heard at those points at different times that the location of the explosion is 6km closer to A than B. Show that the location of the explosion is restricted to a particular curve and find an equation of it.

Soln As shown in fig. A & B are on both sides of x axis at Co-ordinates $(-5,0)$ and $(5,0)$



The distance between A & B is 10. A point C is on the graph at Co-ordinates (x,y) . C is 6km closer to A than B.

$$\text{Distance between C \& A} = \sqrt{(x+5)^2 + y^2}$$

$$\text{Distance between C \& B} = \sqrt{(x-5)^2 + y^2}$$

Given distance between C & B is longer by 6

$$\text{Hence } \sqrt{(x-5)^2 + y^2} - \sqrt{(x+5)^2 + y^2} = 6$$

$$\sqrt{(x-5)^2 + y^2} = 6 + \sqrt{(x+5)^2 + y^2}$$

Squaring on both sides we get.

$$(x-5)^2 + y^2 = 36 + (x+5)^2 + y^2 + 12(\sqrt{(x+5)^2 + y^2})$$

$$x^2 + 2x - 10x + y^2 = x^2 + 10x + y^2 + 36 + 25 + 12\sqrt{(x+5)^2 + y^2}$$

$$-20x - 36 = 12\sqrt{(x+5)^2 + y^2}$$

$$-5x - 9 = 3\sqrt{(x+5)^2 + y^2}$$

Squaring both sides we get.

$$25x^2 + 81 + 90x = 9(x^2 + 25 + 10x + y^2)$$

$$25x^2 + 81 + 90x - 9x^2 - 90x - 9y^2 - 225 = 0$$

$$16x^2 - 9y^2 - 144 = 0$$

$$16x^2 - 9y^2 = 144$$

$$(\div 144) \quad \frac{x^2}{9} - \frac{y^2}{16} = 1 \quad \text{is the required eqn. of hyperbola.}$$

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Exercise 5-6

(35)

2. Choose the most appropriate answer.
 1. The equation of the circle passing through (1,5) and (4,1) and touching y-axis is $x^2 + y^2 - 5x - 6y + 9 + \lambda(4x + 3y - 19) = 0$ where λ is equal to.

2. The eccentricity of the hyperbola whose latus rectum is 8 and conjugate axis is equal to half the distance between the foci is

Soln $\frac{2b^2}{a} = 8 \quad b^2 = 4a$

$2b = \frac{1}{e} (2ae) \Rightarrow b = \frac{ae}{e}$

$\therefore \frac{2}{a} \left(\frac{ae}{e} \right)^2 = 8$

$\frac{2}{e} \left(\frac{a^2 e^2}{4e^2} \right) = 8 \Rightarrow ae^2 = 16$

$\therefore ae^2 = 16$

$e^2 = \frac{16}{12} = \frac{4}{3}$

$\therefore e = \frac{2}{\sqrt{3}}$

Ans (2)

- 3) The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points if.

Soln $\therefore 3x - 4y = m$

$\therefore x^2 + y^2 = 4x + 8y + 5 = 0$

$(x-2)^2 + (y-4)^2 = 25$

$C(2,4) \quad r=5$

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Distance from centre $< r$

(36)



$$\left| \frac{ah + bk + c}{\sqrt{a^2 + b^2}} \right| < r$$

$$\left| \frac{3 \times 2 - 4 \times 4 - 10}{\sqrt{9 + 16}} \right| < r \Rightarrow \left| \frac{-10 - m}{5} \right| < 5$$

$$-5 < \frac{10 + m}{5} < 5$$

$$-25 < 10 + m < 25 \Rightarrow -25 - 10 < m < 25 - 10$$

$$-35 < m < 15$$

Ans (4)

- 4) The length of the diameter of the circle which touches the x-axis at the point (1, 0) and passes through the point (2, 3)

Soln

$$CA = CB$$

$$CA^2 = CB^2$$

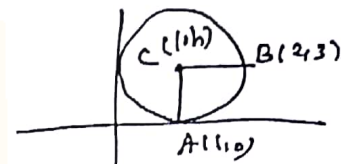
$$[1 - 1]^2 + [h - 0]^2 = [1 - 2]^2 + [h - 3]^2$$

$$h^2 = 1 + h^2 + 9 - 6h$$

$$6h = 10$$

$$h = \frac{10}{6} = \frac{5}{3}$$

$$\text{Diameter is } 2h = 2 \left(\frac{5}{3} \right) = \frac{10}{3}$$



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Ans (B)

- 5) The radius of the circle $3x^2 + 6y^2 + 4bx - 6by + 6^2 = 0$ is

Soln

$$b = 3$$

$$4b = 12$$

$$6b = 18$$

$$b^2 = 9$$

$$\therefore \text{Circle is } 3x^2 + 3y^2 + 12x - 18y + 9 = 0$$

$$(\div 3)$$

$$x^2 + y^2 + 4x - 6y + 3 = 0$$

$$2g = 4$$

$$2f = -6$$

$$c = 3$$

$$g = 2$$

$$f = -3$$

$$r = \sqrt{g^2 + f^2 - c} = \sqrt{4 + 9 - 3} = \sqrt{10}$$

Ans (C)

- 6) The centre of the circle inscribed in a square formed by the lines $x^2 - 8x - 12 = 0$ and $y^2 - 14y + 45 = 0$

- 7) The equation of the normal to the circle $x^2 + y^2 - 2x - 4y + 1 = 0$ which is parallel to the line $2x + 4y = 3$ is

Soln parallel line be $2x + 4y + \lambda = 0$

Centre be $(-g, -f) = (1, 1)$

which lies on line

$$2 + 4 + \lambda = 0$$

$$\lambda = -6$$

$$\therefore 2x + 4y - 6 = 0 \Rightarrow x + 2y = 3$$

Ans (P).



$$2g = -2$$

$$g = -1$$

$$2f = -4$$

$$f = -2$$

- 8) If $P(x, y)$ be any point on $16x^2 + 25y^2 = 400$ with foci $F_1(3, 0)$ and $F_2(-3, 0)$ then $PF_1 + PF_2$.

Soln $16x^2 + 25y^2 = 400$

$$\frac{x^2}{25} + \frac{y^2}{16} = 1$$

$$a^2 = 25 \therefore a = \pm 5$$

Ans (2)

$$PF_1 + PF_2 = \text{major axis} = 2a = 2 \times 5 = 10$$

- 9) The radius of the circle passing through the point $(6, 2)$ two of whose diameters are $x + y = 6$ and $x + 2y = 4$ is

Soln Solve $x + y = 6$ — (1)
 $x + 2y = 4$ — (2)

$$\textcircled{1} - \textcircled{2} \quad -y = 2$$

$$y = -2$$

$$\therefore x - 2 = 6$$

$$x = 8$$

Point be $(8, -2)$

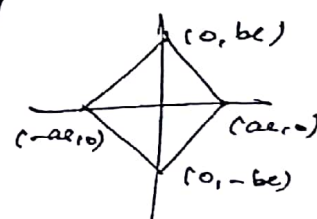
Another point $(6, 2)$

$$\text{Radius} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(8 - 6)^2 + (-2 - 2)^2} = \sqrt{2^2 + 4^2}$$

$$= \sqrt{4 + 16} = \sqrt{20} = 2\sqrt{5}$$

- 10) The area of quadrilateral formed with foci of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$

Soln



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$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Focus $(\pm ae, 0)$

$$e = \sqrt{1 + \frac{b^2}{a^2}}$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$$

Focus $(0, \pm be)$

$$e = \sqrt{1 + \frac{a^2}{b^2}}$$

(38)

quadrilateral $\square ABCD = \frac{1}{2} AC \times BD = \frac{1}{2} 2ae \times 2be$

$$= a \sqrt{1 + \frac{b^2}{a^2}} \times 2b \sqrt{1 + \frac{a^2}{b^2}} = a \frac{\sqrt{a^2 + b^2}}{a} \times 2b \frac{\sqrt{b^2 + a^2}}{b}$$

$$= 2(a^2 + b^2)$$

Ans ②

- 11) If the normals of the parabola $y^2 = 4x$ drawn at the end points of its latus rectum are tangents to the circle $(x-3)^2 + (y+2)^2 = r^2$ Then the value of r^2 is

Soln Egn of normal at points $(1, \pm 2)$

$$y = -x + 3$$

$$x + y - 3 = 0$$

$$y = x + 3$$

$$x - y - 3 = 0$$

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$$\left| \frac{3-2-3}{\sqrt{1+1}} \right| = r$$

$$\left| \frac{-2}{\sqrt{2}} \right| = r$$

$$\Rightarrow \frac{r}{\sqrt{2}} = r$$

$$r = \sqrt{2}$$

$$r^2 = 2$$

Ans ①.

- 12) If $x+y=k$ is normal to the parabola $y^2 = 12x$ then the value of k is

Soln $y^2 = 12x \Rightarrow 4a = 12 \Rightarrow a = 3$

$$y = mx + c \quad \therefore x + y = k \Rightarrow y = -x + k$$

$$\therefore m = -1 \quad c = k$$

$$c = -2am - am^2 \Rightarrow k = -2a(-1) - a(-1)^2$$

$$k = -6(-1) - 3(-1) = 6 + 3 = 9$$

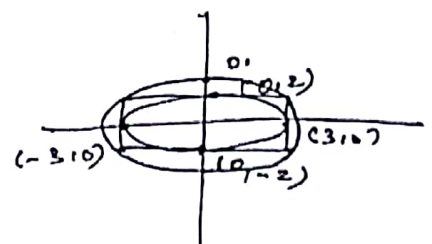
$$k = 9$$

Ans ④

- 13) The ellipse $E_1: \frac{x^2}{9} + \frac{y^2}{4} = 1$ is inscribed in a rectangle R whose sides are parallel to the coordinate axes. Another ellipse E_2 passing through the point $(0, 4)$ circumscribes the rectangle R . The eccentricity of the ellipse is

Soln. Egn be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\frac{9}{a^2} + \frac{4}{16} = 1$$



$$\frac{x^3}{a^2} = 1 - \frac{1}{4} = \frac{3}{4}$$

$$a^2 = 12 \Rightarrow b^2 = 16.$$

$$c^2 = 1 - \frac{a^2}{b^2} = 1 - \frac{12}{16} = 1 - \frac{3}{4} = \frac{1}{4}$$

Ans (3)

$$e = \frac{1}{2}$$

- 14) Tangents are drawn to the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ parallel to the straight line $2x - y = 1$ one of the points of contact of tangents on the hyperbola is

Soln $a^2 = 9$ $b^2 = 4$. $2x - y = 1$
 $y = 2x - 1$
 $m = 2$

$$c = \sqrt{a^2 m^2 + b^2}$$

$$= \sqrt{9(4) + 4} = \sqrt{36 + 4} = \sqrt{40} = 2\sqrt{10}$$

Point of contact $\left(\frac{a^2 m}{c}, \frac{b^2}{c}\right) = \left(\frac{9 \times 2}{2\sqrt{10}}, \frac{4}{2\sqrt{10}}\right)$
 $= \left(\frac{9}{\sqrt{10}}, \frac{1}{\sqrt{10}}\right)$
 Ans (3)

- 15) The equation of the circle passing through the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ having centre at $(0, 3)$ is

Soln $a^2 = 16$ $b^2 = 9$ $(h, k) = (0, 3)$

$$e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{9}{16}} = \sqrt{\frac{7}{16}} = \frac{\sqrt{7}}{4}$$

$$ae = 4 \times \frac{\sqrt{7}}{4} = \sqrt{7}$$

F($\sqrt{7}, 0$) lies on circle.

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x-0)^2 + (y-3)^2 = r^2 \Rightarrow \sqrt{7}^2 + 3^2 = r^2$$

$$7 + 9 = r^2 \Rightarrow r^2 = 16$$

$$\therefore (x-0)^2 + (y-3)^2 = 16$$

$$x^2 + y^2 - 6y + 9 = 16$$

$$x^2 + y^2 - 6y - 7 = 0$$

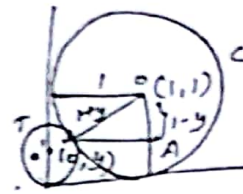
Ans. (1)

- 16) Let C be the circle with centre at $(1, 1)$ and radius = 1. If T is a circle-centred at $(0, 9)$ passing through the origin and touching the circle C externally. Then the radius T is equal to.

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Soln $\Delta OOA'$
 $(1+y)^2 = (1-y)^2 + 1$
 $1+y^2+2y = 1+y^2-2y+1$
 $4y = 1$
 $y = 1/4$

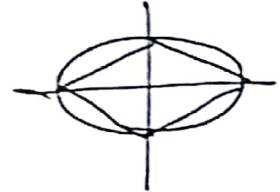
(4)



Ans (4)

- 17) Consider an ellipse whose centre is of the origin and its major axis is along x-axis. If its eccentricity is $\frac{3}{5}$ and the distance between its foci is 6. Then the area of the quadrilateral inscribed in the ellipse with diagonals as major & minor axes of the ellipse is

Soln $e = 3/5$ $2ae = 6$
 $2a(\frac{3}{5}) = 6$
 $a = 5$
 $b = 4$

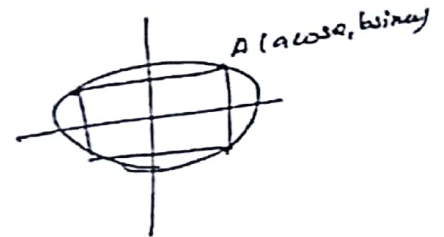


Area ABCD = $\frac{1}{2} \times \frac{1}{2} \times a \times b = 2ab = 2 \times 5 \times 4$
 $= 40$

Ans (4)

- 18) Area of the greatest rectangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

Soln. $x = a \cos \alpha$ $y = b \sin \alpha$
length = $2a \cos \alpha$ breadth = $2b \sin \alpha$
 $A = l \times b = 4ab \sin \alpha \cos \alpha$
 $A = 2ab \sin 2\alpha$



$\frac{dA}{d\alpha} = 2ab \cos 2\alpha (2) = 4ab \cos 2\alpha$

$\frac{dA}{d\alpha} = 0$ $\cos 2\alpha = 0$
 $2\alpha = 90^\circ$
 $\alpha = 45^\circ$

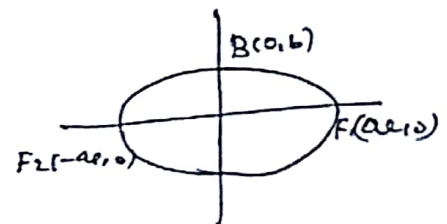
$\therefore A = 2ab \sin 2(45^\circ)$
 $= 2ab \sin 90^\circ$

$A = 2ab$

Ans (1)

- 19) An ellipse has OB as semi minor axes F and F' its foci and the angle FBF' is a right angle. Then the eccentricity of the ellipse is

Soln $\angle FBF' = 90^\circ$
 $(\sqrt{a^2e^2 + b^2})^2 + (\sqrt{a^2e^2 + b^2})^2 = (2ae)^2$
 $2(a^2e^2 + b^2) = 4a^2e^2$
 $2a^2e^2 + 2b^2 = 4a^2e^2$
 $2a^2e^2 = 2b^2$
 $c^2 = b^2/a^2$



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WKT $e^2 = 1 - \frac{b^2}{a^2} = 1 - e^2 \Rightarrow 2e^2 = 1 \Rightarrow e^2 = \frac{1}{2}$ (41)
 $e = \frac{1}{\sqrt{2}}$ Ans ①

20) The eccentricity of the ellipse $(x-3)^2 + (y-4)^2 = \frac{y^2}{9}$ is

Soln $PF = e^2 PS$
 $(x-h)^2 + (y-k)^2 = e^2 \left(\frac{4x+6y+c}{\sqrt{a^2+b^2}} \right)^2$

$(h,k) = (3,4) \quad a=0 \quad c=0$ Ans ②
 $e^2 = \frac{1}{9}$
 $e = \frac{1}{3}$

21) If the two tangents drawn from a point P to the Parabola $y^2 = 4x$ at right angles then the locus of P is

Soln Locus of P = Direction of $y^2 = 4x$ $\therefore a=4$
 $\therefore a=1$
 $\therefore n = -a = -1$ Ans ②
 $\therefore n = -1$

22) The circle passing through $(1,-2)$ and touching the axis of x at $(3,0)$ passing through the point

Soln $(x-3)^2 + (y-0)^2 + \lambda y = 0$
 At $(1,-2)$ $(1-3)^2 + (-2-0)^2 + \lambda y = 0$
 $4 + 4 - 2\lambda = 0$
 $8 = 2\lambda$
 $\lambda = 4$
 $x^2 - 6x + 9 + y^2 + 4y = 0$
 Apply all the points which satisfy that passes through the circle.
 At $(+5, 2)$
 $25 - 30 + 9 + 4 - 8 = 0$
 Ans ③

23) The locus of a point whose distance from $(-2,0)$ is $\frac{2}{3}$ times its distance from the line $x = -\frac{9}{2}$ is

Soln $P(h,k)$ $Q(-2,0)$
 $x = -\frac{9}{2}$ $PQ = \frac{2}{3} PM$
 $2x + 9 = 0$ $\sqrt{(h+2)^2 + k^2} = \frac{1}{3} \left| \frac{2h+9}{2} \right|$
 $(h+2)^2 + k^2 = \frac{1}{9} (2h+9)^2$
 $h^2 + 4 + 4h + k^2 = \frac{1}{9} (4h^2 + 36h + 81)$
 $9h^2 + 36 + 36h + 9k^2 = 4h^2 + 36h + 81$
 $5h^2 + 9k^2 = 45$
 $\frac{h^2}{9} + \frac{k^2}{5} = 1$

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$$\frac{x^2}{9} + \frac{y^2}{5} = 1 \text{ which is ellipse Ans (3) (2)}$$

- 24) The values of m for which the line $y = mx + 2\sqrt{5}$ touches the hyperbola $16x^2 - 9y^2 = 144$ are the roots of $x^2 - (a+b)x - 4 = 0$. Then the value of $(a+b)$ is

Soln $a^2 = 9$ $b^2 = 16$
 $a = 3$ $b = 4$

$$c^2 = a^2 m^2 - b^2$$

$$(2\sqrt{5})^2 = 9m^2 - 16$$

$$20 + 16 = 9m^2$$

$$m^2 = \frac{36}{9}$$

$\therefore m = 2$ which is roots of

$$x^2 - (a+b)x - 4 = 0$$

$$x^2 - (a+b)x + 4 = 0$$

$$a+b = 0$$

Ans (3)

- 25) If the coordinates at one end of a diameter of the circle $x^2 + y^2 - 8x - 4y + c = 0$ are $(11, 2)$ the co-ordinates of the other end are

Soln $2g = -8$ $2g = -4$
 $g = -4$ $f = -2$
 $C(-g, -f) = (4, 2)$

$$\frac{x_1 + x_2}{2} = 4$$

$$\frac{y_1 + y_2}{2} = 2$$

$$\frac{x_1 + 11}{2} = 4$$

$$\frac{y_1 + 2}{2} = 2$$

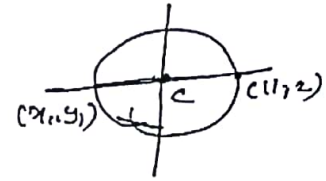
$$x_1 = 8 - 11$$

$$y_1 = 4 - 2$$

$$x_1 = -3$$

$$y_1 = 2$$

$\therefore$ other end be $(-3, 2)$



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