Padasalai's
Telegram Groups!

(தலைப்பிற்கு கீழே உள்ள லிங்கை கிளிக் செய்து குழுவில் இணையவும்!)

- Padasalai's NEWS - Group
https://t.me/joinchat/NIfCqVRBNj9hhV4wu6_NqA

- Padasalai's Channel - Group
https://t.me/padasalaichannel

- Lesson Plan - Group
https://t.me/joinchat/NIfCqVWwo5iL-21gpzrXLw

- 12th Standard - Group
https://t.me/Padasalai_12th

- 11th Standard - Group
https://t.me/Padasalai_11th

- 10th Standard - Group
https://t.me/Padasalai_10th

- 9th Standard - Group
https://t.me/Padasalai_9th

- 6th to 8th Standard - Group
https://t.me/Padasalai_6to8

- 1st to 5th Standard - Group
https://t.me/Padasalai_1to5

- TET - Group
https://t.me/Padasalai_TET

- PGTRB - Group
https://t.me/Padasalai_PGTRB

- TNPSC - Group
https://t.me/Padasalai_TNPSC

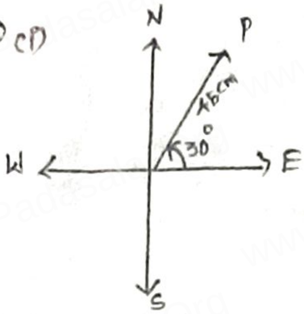
11

தொகுதி II

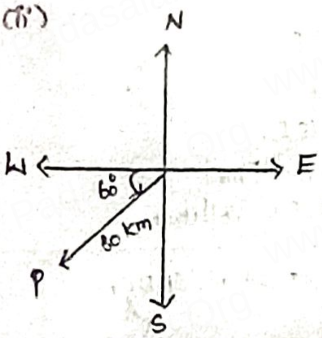
கணிதவியல்

பள்ளிக் கல்வித்துறை
தமிழ்நாடு அரசு

① (i)



(ii)



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② Given The Relation R is defined $\vec{a} R \vec{b}$ is $\vec{a} = \vec{b}$.

To Prove, R is Equivalence relation.

(i) Reflexive:-

We know that $\vec{a}$ is equal to $\vec{a}$

$\Rightarrow \vec{a} R \vec{a}$.

R is Reflexive.

(ii) Symmetric:-

Let $\vec{a} R \vec{b} \Rightarrow \vec{a}$ is equal to $\vec{b}$.

Then $\vec{b}$ is equal to $\vec{a}$

$\Rightarrow \vec{b} R \vec{a}$.

R is symmetric.

(iii) Transitive:

Let $\vec{a} R \vec{b} \Rightarrow \vec{a}$ is equal to $\vec{b}$

and $\vec{b} R \vec{c} \Rightarrow \vec{b}$ is equal to $\vec{c}$.

Then $\vec{a}$ is equal to $\vec{c}$.

$\Rightarrow \vec{a} R \vec{c}$.

R is Transitive.

From (i), (ii) & (iii) Hence The Relation R is equivalence relation.

(3) Let P and Q be the Point of trisection of AB.

Let $AP = PQ = QB = \lambda$ (say)

P divides AB in the ratio 1:2

Position Vector of $P = \vec{OP} = \frac{1\vec{OB} + 2\vec{OA}}{1+2}$

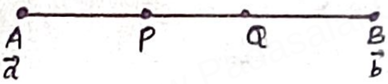
$$P = \frac{1.\vec{b} + 2\vec{a}}{1+2} = \frac{\vec{b}+2\vec{a}}{3}$$

$$\boxed{P = \frac{\vec{b}+2\vec{a}}{3}}$$

Q is the Point of PB.

$$\vec{OQ} = \frac{1.\vec{OP} + 1\vec{OB}}{1+1} = \frac{\frac{\vec{b}+2\vec{a}}{3} + \vec{b}}{2}$$

$$= \frac{\vec{b}+2\vec{a}+3\vec{b}}{6}$$



$$= \frac{2\vec{a}+4\vec{b}}{6}$$

$$= \frac{2(\vec{a}+2\vec{b})}{6}$$

$$= \frac{\vec{a}+2\vec{b}}{3}$$

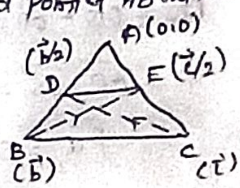
$$\boxed{\vec{OQ} = \frac{\vec{a}+2\vec{b}}{3}}$$

④ For our Convenience, We chose A as the orgin.

Let $\vec{b}$ and $\vec{c}$ are the Position Vectors of B and C.

Since D and E are the mid Point of AB and AC.

$\therefore D = \vec{b}/2$. $E = \vec{c}/2$.



To Prove, $\vec{BE} + \vec{DC} = \frac{3}{2}(\vec{BC})$.

$\vec{BE} = \vec{OE} - \vec{OB} = \vec{c}/2 - \vec{b} = \vec{c}/2 - \vec{b}$

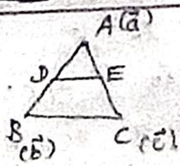
$\vec{DC} = \vec{OC} - \vec{OD} = \vec{c} - \vec{b}/2 = \vec{c} - \vec{b}/2$

$\vec{BE} + \vec{DC} = \vec{c}/2 - \vec{b} + \vec{c} - \vec{b}/2 = \frac{3}{2}\vec{c} - \frac{3}{2}\vec{b}$

$= \frac{3}{2}(\vec{c} - \vec{b})$

$$\boxed{\vec{BE} + \vec{DC} = \frac{3}{2}(\vec{BC})}$$

⑤ Let ABC be a triangle and let O be the orgin of reference.



Let D and E be the midpoints of AB and AC

Let $\vec{OA} = \vec{a}$, $\vec{OB} = \vec{b}$, $\vec{OC} = \vec{c}$, $\vec{OD} = \frac{\vec{a}+\vec{b}}{2}$, $\vec{OE} = \frac{\vec{a}+\vec{c}}{2}$

$\vec{DE} = \vec{OE} - \vec{OD}$

$= \left(\frac{\vec{a}+\vec{c}}{2}\right) - \left(\frac{\vec{a}+\vec{b}}{2}\right)$

$= \frac{1}{2}(\vec{c} - \vec{b}) = \frac{1}{2}[\vec{OC} - \vec{OB}]$

$\vec{DE} = \frac{1}{2}(\vec{BC})$

$\vec{DE} = \frac{1}{2}\vec{BC} \Rightarrow DE \parallel BC$

Also, $\vec{DE} = \frac{1}{2}\vec{BC}$

$\Rightarrow |\vec{DE}| = \frac{1}{2}|\vec{BC}|$

$\boxed{DE = \frac{1}{2} BC}$

Hence $DE \parallel BC$ and $DE = \frac{1}{2} BC$

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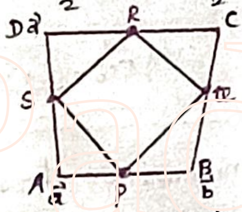
(6) Let ABCD be a quadrilateral and let P, Q, R, S be the midpoints of the sides AB, BC, CD and DA.

The position vectors of

$P = \frac{\vec{a}+\vec{b}}{2}$, $Q = \frac{\vec{b}+\vec{c}}{2}$

$R = \frac{\vec{c}+\vec{d}}{2}$ $S = \frac{\vec{d}+\vec{a}}{2}$



To Prove PQRS is a Parallelogram it is enough to Prove that $\vec{PQ} = \vec{SR}$ and $\vec{PS} = \vec{QR}$.

$\vec{PQ} = \vec{OQ} - \vec{OP} = \left(\frac{\vec{b}+\vec{c}}{2}\right) - \left(\frac{\vec{a}+\vec{b}}{2}\right) = \frac{\vec{c}-\vec{a}}{2}$

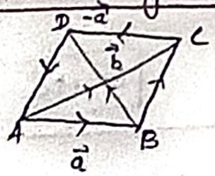
$\vec{SR} = \vec{OR} - \vec{OS} = \left(\frac{\vec{c}+\vec{d}}{2}\right) - \left(\frac{\vec{d}+\vec{a}}{2}\right) = \frac{\vec{c}-\vec{a}}{2}$

$\therefore \vec{PQ} = \vec{SR} \Rightarrow PQ \parallel SR$ and $PQ = SR$.

Similarly we can Prove that $PS = QR$ and $PS \parallel QR$

Hence PQRS is a Parallelogram.

(7) ABCD is a Parallelogram.



$\vec{AB} = +\vec{DC}$ and

$\vec{AD} = +\vec{BC}$. But

$\vec{AB} = \vec{a} \Rightarrow \vec{CD} = -\vec{a}$.

Since, $\vec{AC} = \vec{AB} + \vec{BC}$

$\vec{b} = \vec{a} + \vec{BC}$

$\boxed{\vec{BC} = \vec{b} - \vec{a}}$

$\vec{BC} = +\vec{DA} = -\vec{BC} = -(\vec{b} - \vec{a}) = \vec{a} - \vec{b}$.

(2)

$\boxed{\vec{DA} = \vec{a} - \vec{b}}$

$\vec{BD} = \vec{BC} + \vec{CD} = \vec{b} - \vec{a} - \vec{a}$

$\boxed{\vec{BD} = \vec{b} - 2\vec{a}}$

$\therefore$ The Required sides are

$\vec{CD} = -\vec{a}$, $\vec{BC} = \vec{b} - \vec{a}$

$\vec{DA} = \vec{a} - \vec{b}$

other diagonal $\vec{BD} = \vec{b} - 2\vec{a}$.

(8) Given

$\vec{PO} + \vec{OQ} = \vec{PQ}$.

$\vec{QO} + \vec{OR} = \vec{QR}$

Now $\vec{PQ} = \vec{QR}$

$\Rightarrow \vec{PQ} \parallel \vec{QR}$

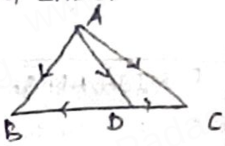
But Q is a Common Point

$\Rightarrow$ P, Q, R are collinear

(9) D is the midpoint of ΔABC,

$\Rightarrow \vec{BD} = \vec{DC}$

Join AD.



$\vec{AB} = \vec{AD} + \vec{DB}$

$\vec{AC} = \vec{AD} + \vec{DC}$.

LHS $\vec{AB} + \vec{AC} = \vec{AD} + \vec{DB} + \vec{AD} + \vec{DC}$

$= 2\vec{AD} + (\vec{DB} + \vec{DC})$

$= 2\vec{AD} + 0$ [$\vec{DB}$ and $\vec{DC}$ are equal and opposite]

$= 2\vec{AD}$

$=$ RHS.

(10) Let O be the origin.

We know That

$\vec{OG} = \frac{\vec{OA} + \vec{OB} + \vec{OC}}{3}$.

To Prove $\vec{GA} + \vec{GB} + \vec{GC} = \vec{0}$.

LHS:

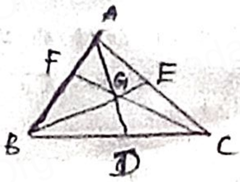
$\vec{GA} + \vec{GB} + \vec{GC} = (\vec{OA} - \vec{OG}) + (\vec{OB} - \vec{OG}) + (\vec{OC} - \vec{OG})$

$= \vec{OA} + \vec{OB} + \vec{OC} - 3\vec{OG}$

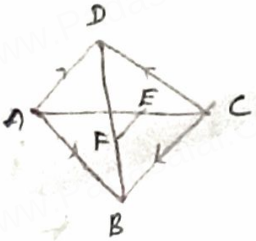
$= 3\left(\frac{\vec{OA} + \vec{OB} + \vec{OC}}{3}\right) - 3\vec{OG}$

$= 3\vec{OG} - 3\vec{OG}$

$= \vec{0}$.



(12) ABCD is a quadrilateral in which E and F are midpoints of AC and BD.



$\vec{AB} = \vec{AE} + \vec{EF} + \vec{FB}$

$\vec{AD} = \vec{AE} + \vec{EF} + \vec{FD}$

$\vec{CB} = \vec{CE} + \vec{EF} + \vec{FB}$

$\vec{CD} = \vec{CE} + \vec{EF} + \vec{FD}$

So, $\vec{AB} + \vec{AD} + \vec{CB} + \vec{CD} = 4\vec{EF} + 2(\vec{AE} + \vec{CE}) + 2(\vec{FB} + \vec{FD})$

$= 4\vec{EF} + \vec{0} + \vec{0}$

$= 4\vec{EF}$.

[$\because$ $\vec{AE}$ and $\vec{CE}$ are equal and opposite, $\vec{FB}$ and $\vec{FD}$ are equal and opposite vectors]

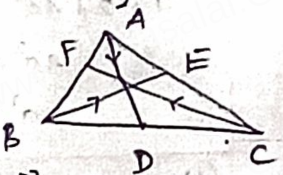
(11) In ΔABC, D, E, F are the midpoints of BC, CA and AB.

To Prove $\vec{AD} + \vec{BE} + \vec{CF} = \vec{0}$

Now, $\vec{AD} = \vec{AB} + \vec{BD} = \vec{AB} + \frac{1}{2}\vec{BC}$

$\vec{BE} = \vec{BC} + \vec{CE} = \vec{BC} + \frac{1}{2}\vec{CA}$

$\vec{CF} = \vec{CA} + \vec{AF} = \vec{CA} + \frac{1}{2}\vec{AB}$



$\vec{AD} + \vec{BE} + \vec{CF} = \frac{3}{2}(\vec{AB} + \vec{BC} + \vec{CA})$

$= \frac{3}{2}(\vec{AC} + \vec{CA})$

$= \frac{3}{2}(\vec{0}) = \vec{0}$

$\therefore \vec{AD} + \vec{BE} + \vec{CF} = \vec{0}$.

Hence Proved.

EXERCISE - 8.2

1) (i)

Sum of the squares of the Direction Cosine is 1.

(i) Given $\frac{1}{5}, \frac{3}{5}, \frac{4}{5}$

$(\frac{1}{5})^2 + (\frac{3}{5})^2 + (\frac{4}{5})^2 = \frac{1}{25} + \frac{9}{25} + \frac{16}{25}$

$= \frac{26}{25} \neq 1$. Thus Not direction Cosines

(3)

(ii) $(\frac{1}{\sqrt{2}})^2 + (\frac{1}{2})^2 + (\frac{1}{2})^2$

$= \frac{1}{2} + \frac{1}{4} + \frac{1}{4} = \frac{2+1+1}{4} = \frac{4}{4} = 1$

Thus, direction Cosines

(iii) $(\frac{4}{3})^2 + (0)^2 + (\frac{3}{4})^2$

$= \frac{16}{9} + 0 + \frac{9}{16} = \frac{337}{144} \neq 1$, Thus not direction Cosines

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2.

(i) 1, 2, 3

Given $(x, y, z) = (1, 2, 3)$

$r = \sqrt{x^2+y^2+z^2} = \sqrt{1+4+9} = \sqrt{14}$

$\boxed{\text{Direction Cosines are } \left(\frac{x}{r}, \frac{y}{r}, \frac{z}{r}\right)}$

$= \left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)$

(ii) 3, −1, 3.

Given $(x, y, z) = (3, -1, 3)$

$r = \sqrt{x^2+y^2+z^2} = \sqrt{(3)^2+(-1)^2+(3)^2}$

$= \sqrt{9+1+9} = \sqrt{19}$

Direction Cosine are $\left(\frac{3}{\sqrt{19}}, \frac{-1}{\sqrt{19}}, \frac{3}{\sqrt{19}}\right)$

(iii) $(x, y, z) = (0, 0, 7)$

$r = \sqrt{x^2+y^2+z^2} = \sqrt{0+0+7^2} = \sqrt{49} = 7$.

Direction Cosines are $\left(\frac{0}{7}, \frac{0}{7}, \frac{7}{7}\right)$

$= (0, 0, 1)$.

(3)

(i) $3\hat{i} - 4\hat{j} + 8\hat{k}$.

The direction ratios of $3\hat{i} - 4\hat{j} + 8\hat{k}$ are $(3, -4, 8)$

Direction Cosines are $\left(\frac{x}{r}, \frac{y}{r}, \frac{z}{r}\right)$

$r = \sqrt{x^2+y^2+z^2} = \sqrt{(3)^2+(-4)^2+(8)^2} = \sqrt{9+16+64}$

$= \sqrt{89}$

Direction Cosine are $\left(\frac{3}{\sqrt{89}}, \frac{-4}{\sqrt{89}}, \frac{8}{\sqrt{89}}\right)$

4

(ii) $3\hat{i}+\hat{j}+\hat{k}$

The Direction Ratio of $3\hat{i}+\hat{j}+\hat{k}$ are $(3,1,1)$

$r=\sqrt{x^2+y^2+z^2}=\sqrt{3^2+1^2+1^2}$

$=\sqrt{9+1+1}=\sqrt{11}$

Direction Cosines are

$\left(\frac{3}{\sqrt{11}},\frac{1}{\sqrt{11}},\frac{1}{\sqrt{11}}\right)$

(iii) $\vec{j}$. It can be written as $0\hat{i}+\hat{j}+0\hat{k}$.

$\therefore$ Direction Ratio of $\vec{j}$ are $(0,1,0)$.

$r=\sqrt{0^2+1^2+0^2}=\sqrt{1}=1$.

$\therefore$ Direction Cosines are $(0,1,0)$.

(iv) $5\hat{i}-3\hat{j}-48\hat{k}$.

The Direction Ratio of $5\hat{i}-3\hat{j}-48\hat{k}$ are $(5,-3,-48)$.

$r=\sqrt{5^2+(-3)^2+(-48)^2}=\sqrt{25+9+}$

$=\sqrt{2338}$

Direction Cosines are

$\left(\frac{5}{\sqrt{2338}},\frac{-3}{\sqrt{2338}},\frac{-48}{\sqrt{2338}}\right)$

(v) $3\hat{i}+4\hat{j}-3\hat{k}$.

The Direction Ratio of $3\hat{i}-3\hat{j}+4\hat{k}$ are $(3,4,-3)$.

$r=\sqrt{(3)^2+(4)^2+(-3)^2}=\sqrt{9+16+9}$

$=\sqrt{34}$.

Direction Cosines are $\left(\frac{3}{\sqrt{34}},\frac{4}{\sqrt{34}},\frac{-3}{\sqrt{34}}\right)$

(vi) $\hat{i}-\hat{k}$.

It can be written $\hat{i}+0\hat{j}-\hat{k}$.

$\therefore$ Direction Ratio of $\vec{i}-\vec{k}$ are $(1,0,-1)$.

$r=\sqrt{1^2+(0)^2+(-1)^2}=\sqrt{2}$.

The Direction Cosines are $\left(\frac{1}{\sqrt{2}},0,\frac{-1}{\sqrt{2}}\right)$

4

VECTOR ALGEBRA

5

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மேல்நிலை ஆசிரியர் – கணிதம்

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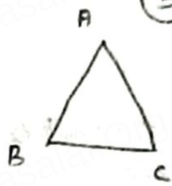
4) Let A, B, C be the vertices of the triangle ABC.

(5)



D, E, F be the midpoints of the sides BC, CA and BA respectively.

Take A(1, 0, 0) B(0, 1, 0) and C(0, 0, 1).

midpoint of BC

$$D = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$\boxed{D = (0, \tfrac{1}{2}, \tfrac{1}{2})}$$

midpoint of CA:

$$E = \left(\tfrac{1}{2}, 0, \tfrac{1}{2}\right)$$

midpoint of AB:

$$F = \left(\tfrac{1}{2}, \tfrac{1}{2}, 0\right).$$

$\therefore$ Median $\vec{AD} = \vec{OD} - \vec{OA} = (0\vec{i} + \tfrac{1}{2}\vec{j} + \tfrac{1}{2}\vec{k}) - (\vec{i} + 0\vec{j} + 0\vec{k})$

$$\boxed{\vec{AD} = -\vec{i} + \tfrac{1}{2}\vec{j} + \tfrac{1}{2}\vec{k}}$$

$$r = \sqrt{x^2+y^2+z^2} = \sqrt{(-1)^2 + (\tfrac{1}{2})^2 + (\tfrac{1}{2})^2} = \sqrt{1 + \tfrac{1}{4} + \tfrac{1}{4}} = \frac{\sqrt{6}}{2}$$

$$\boxed{r = \sqrt{6}/2}$$

Direction Cosines of $\vec{AD}$ are $\dfrac{-1}{\sqrt{6}/2}, \dfrac{1}{2 \times \frac{\sqrt{6}}{2}}, \dfrac{1}{2 \times \frac{\sqrt{6}}{2}}$

$$\boxed{\text{D'C of AD are } = \left(\frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)}$$

Median $\vec{BE} = \vec{OE} - \vec{OB}$

$$= \left(\tfrac{1}{2}\vec{i} - 0\vec{j} + \tfrac{1}{2}\vec{k}\right) - (0\vec{i} + \vec{j} + 0\vec{k})$$

$$= \tfrac{1}{2}\vec{i} - \vec{j} + \tfrac{1}{2}\vec{k}.$$

$$r = \sqrt{(\tfrac{1}{2})^2 + (-1)^2 + (\tfrac{1}{2})^2} = \sqrt{\tfrac{1}{4} + 1 + \tfrac{1}{4}} = \sqrt{6}/2 \qquad \boxed{r = \sqrt{6}/2}$$

Direction Cosines of $\vec{BE}$ are

$$\frac{1}{2 \times \sqrt{6}/2}, \frac{-1}{\sqrt{6}/2}, \frac{1}{2 \times \sqrt{6}/2}$$

$$\boxed{\text{D'C of BE are } \left(\tfrac{1}{\sqrt{6}}, -\tfrac{2}{\sqrt{6}}, \tfrac{1}{\sqrt{6}}\right)}$$

6

Median $\vec{CF} = \vec{OF} - \vec{OC}$

$= (\frac{1}{2}\vec{i} + \frac{1}{2}\vec{j} + 0\vec{k}) - (0\vec{i} + 0\vec{j} + \vec{k})$

$= \frac{1}{2}\vec{i} + \frac{1}{2}\vec{j} - \vec{k}$

$r = \sqrt{(\frac{1}{2})^2 + (\frac{1}{2})^2 + (-1)^2} = \sqrt{\frac{1}{4} + \frac{1}{4} + 1} = \frac{\sqrt{6}}{2}$

$\boxed{r = \sqrt{6}/2}$

Direction Cosines of CF are $\frac{1}{2 \times \sqrt{6}/2}, \frac{1}{2 \times \sqrt{6}/2}, \frac{-1}{\sqrt{6}/2}$

$\boxed{D'C \text{ of } CF \text{ are } = \left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}\right)}$

$\therefore$ Direction Cosines of Medians are $\left(\frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$

$\left(\frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$ and

$\left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}\right)$.

5) Given the Direction Cosines are $\frac{1}{2}, \frac{1}{\sqrt{2}}, a$.

We know That Sum of the square of the Direction Cosines is equal to 1.

$\therefore (\frac{1}{2})^2 + (\frac{1}{\sqrt{2}})^2 + (a)^2 = 1$

$\frac{1}{4} + \frac{1}{2} + a^2 = 1$

$\frac{3}{4} + a^2 = 1$

$3 + 4a^2 = 4$

$4a^2 = 4 - 3$

$a^2 = \frac{1}{4}$

$\boxed{a = \pm \frac{1}{2}}$

6) Given Points are $A(1, 0, 0)$ and $B(0, 1, 0)$ are set of Direction ratio $a, a+b, a+b+c$.

(i) $\vec{AB} = \vec{OB} - \vec{OA} = (0\vec{i} + \vec{j} + 0\vec{k}) - (\vec{i} + 0\vec{j} + 0\vec{k})$

$= -\vec{i} + \vec{j}$

D'R of the lines $\vec{AB}$ are $(-1, 1, 0)$.

Given $(a, a+b, a+b+c) = (-1, 1, 0)$

$\boxed{a = -1}$ | $a + b = 1$, $-1 + b = 1$, $\boxed{b = 2}$ | $a + b + c = 0$, $-1 + 2 + c = 0$, $1 + c = 0$, $\boxed{c = -1}$

(ii) $\vec{BA} = \vec{OA} - \vec{OB}$

$= \vec{i} + 0\vec{j} + 0\vec{k} - (0\vec{i} + \vec{j} + 0\vec{k})$

$= \vec{i} - \vec{j}$.

D'R of the line $\vec{BA}$ are $(1, -1, 0)$

Given $(a, a+b, a+b+c) = (1, -1, 0)$

$\boxed{a = 1}$ | $a + b = -1$, $\boxed{b = -2}$ | $a + b + c = 0$, $1 - 2 + c = 0$, $-1 + c = 0 \Rightarrow \boxed{c = 1}$

7

7) Let the given vectors be
$\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = 3\hat{i} - 4\hat{j} - 4\hat{k}$
and $\vec{c} = \hat{i} - 3\hat{j} - 5\hat{k}$

$\vec{a} + \vec{c} = (2\hat{i} - \hat{j} + \hat{k}) + (\hat{i} - 3\hat{j} - 5\hat{k})$
$= 3\hat{i} - 4\hat{j} - 4\hat{k}$
$= \vec{b}$
$\boxed{\vec{a} + \vec{c} = \vec{b}}$

Now $|\vec{a}| = |2\hat{i} - \hat{j} + \hat{k}|$
$= \sqrt{(2)^2 + (-1)^2 + (1)^2}$
$= \sqrt{4+1+1}$
$\boxed{|\vec{a}| = \sqrt{6}} \Rightarrow \boxed{|\vec{a}|^2 = 6}$

$|\vec{b}| = \sqrt{(3)^2 + (-4)^2 + (-4)^2}$
$= \sqrt{9+16+16}$
$\boxed{|\vec{b}| = \sqrt{41}} \Rightarrow \boxed{|\vec{b}|^2 = 41}$

$|\vec{c}| = \sqrt{(-1)^2 + (-3)^2 + (-5)^2} = \sqrt{1+9+25}$
$|\vec{c}| = \sqrt{35}$
$|\vec{c}| = \sqrt{35} \Rightarrow \boxed{|\vec{c}|^2 = 35}$

$|\vec{b}|^2 = |\vec{a}|^2 + |\vec{c}|^2 = 41$ $= 6 + 35$
$= 41$

$\because$ $\vec{a} + \vec{c} = \vec{b}$ and
$|\vec{b}|^2 = |\vec{a}|^2 + |\vec{c}|^2$, Thus
Given vectors form the sides of a right angled triangle.

8) Given the vectors
$\vec{a} = 3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\vec{b} = \hat{i} + \lambda\hat{j} + 3\hat{k}$.
We know that if $\vec{a}$ and $\vec{b}$ are Parallel,
$a_1 = b_1$, $a_2 = b_2$, $a_3 = b_3$.
$\vec{a} = 3\hat{i} + 2\hat{j} + 9\hat{k}$ | $\vec{b} = \hat{i} + \lambda\hat{j} + 3\hat{k}$
$\vec{a} = \hat{i} + \frac{2}{3}\hat{j} + 3\hat{k}$

Now, $a_1 = b_1$, $a_2 = b_2$, $a_3 = b_3$
equating components of $\hat{j}$ from $\vec{a}$ and $\vec{b}$, we get
$\frac{2}{3} = \lambda$
$\boxed{\lambda = 2/3}$

9)
(i) Let
$\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$
$\vec{b} = -2\hat{i} + 3\hat{j} - 4\hat{k}$
$\vec{c} = 0\hat{i} - \hat{j} + 2\hat{k}$

We know that, when we are able to write one vector as a linear combination of the other two vectors, then the given vectors are called Coplanar vectors

Let
$\vec{a} = s\vec{b} + t\vec{c}$.
$\hat{i} - 2\hat{j} + 3\hat{k} = s(-2\hat{i} + 3\hat{j} - 4\hat{k})$
$+ t(0\hat{i} - \hat{j} + 2\hat{k})$

Equating $\hat{i}$, $\hat{j}$ and $\hat{k}$ components.
$1 = -2s$ ——— (1)
$-2 = 3s - t$ ——— (2)
$3 = -4s + 2t$ ——— (3)

(1) $\Rightarrow$
$-2s = 1$
$\boxed{s = -1/2}$ in (2)

$3(-1/2) - t = -2$
$-3/2 - t = -2$
$-t = -2 + 3/2$
$-t = \frac{-4+3}{2} = -1/2$
$\boxed{t = 1/2}$

Substituting the values of s and t in (3)

8

From (3)

LHS = 3

RHS $= -4s + 2t$

$= -4(-\frac{1}{2}) + 2(\frac{1}{2})$

$= 2 + 1$

$= 3$

LHS = RHS

one vector is a linear Combination of other two vectors,

Thus $\vec{a}, \vec{b}, \vec{c}$ are coplanar.

(ii)

Let $\vec{a} = 5\hat{i} + 6\hat{j} + 7\hat{k}$

$\vec{b} = 7\hat{i} - 8\hat{j} + 9\hat{k}$

$\vec{c} = 3\hat{i} + 20\hat{j} + 5\hat{k}$.

We are able two write

$\vec{a} = s\vec{b} + t\vec{c}$ (s, t are scalars)

$5\hat{i} + 6\hat{j} + 7\hat{k} = s(7\hat{i} - 8\hat{j} + 9\hat{k}) + t(3\hat{i} + 20\hat{j} + 5\hat{k})$

Equating $\hat{i}, \hat{j}, \hat{k}$ Components

$5 = 7s + 3t$ ——— (1)

$6 = -8s + 20t$ ——— (2)

$7 = 9s + 5t$ ——— (3)

$\frac{(2)}{2} \Rightarrow 3 = -4s + 10t$ ——— (4)

Solving (1) & (4)

(1) $\times 4 \Rightarrow 28s + 12t = 20$

(4) $\times 7 \Rightarrow -28s + 70t = 21$

$82t = 41$

$\boxed{t = \frac{1}{2}}$

$t = \frac{1}{2}$ in (1) we get

$\boxed{s = \frac{1}{2}}$

$s = \frac{1}{2}, t = \frac{1}{2}$ in (3)

From (3) $\Rightarrow$

LHS : 7

RHS $9(\frac{1}{2}) + 5(\frac{1}{2})$

$= \frac{9}{2} + \frac{5}{2} = \frac{14}{2} = 7$

$\boxed{\text{LHS} = \text{RHS}}$

$\Rightarrow \therefore$ one vector is a linear Combination of other two vectors.

$\Rightarrow \vec{a}, \vec{b}, \vec{c}$ are Coplanar.

10)

Let the given Points A, B, C and D.

To Prove that the Points A, B, C and D are coplanar.

We have to Prove that $\vec{AB}, \vec{AC}$ and $\vec{AD}$ are coplanar.

$\vec{OA} = 4\hat{i} + 5\hat{j} + \hat{k}$ $\quad \vec{OB} = -\hat{j} - \hat{k}$

$\vec{OC} = 3\hat{i} + 9\hat{j} + 4\hat{k}$ $\quad \vec{OD} = -4\hat{i} + 4\hat{j} + 4\hat{k}$

$\vec{AB} = \vec{OB} - \vec{OA} = (-\hat{j} - \hat{k}) - (4\hat{i} + 5\hat{j} + \hat{k})$

$\boxed{\vec{AB} = -4\hat{i} - 6\hat{j} - 2\hat{k} = \vec{a}}$

$\vec{AC} = \vec{OC} - \vec{OA} = (3\hat{i} + 9\hat{j} + 4\hat{k}) - (4\hat{i} + 5\hat{j} + \hat{k})$

$\boxed{\vec{b} = \vec{AC} = -\hat{i} + 4\hat{j} + 3\hat{k}}$

$\vec{AD} = \vec{OD} - \vec{OA} = (-4\hat{i} + 4\hat{j} + 4\hat{k}) - (4\hat{i} + 5\hat{j} + \hat{k})$

$\boxed{\vec{c} = \vec{AD} = -8\hat{i} - \hat{j} + 3\hat{k}}$

To Prove $\vec{a}, \vec{b}, \vec{c}$ are coplanar.

we have to Prove that

$\vec{a} = s\vec{b} + t\vec{c}$.

Let

$-4\hat{i} - 6\hat{j} - 2\hat{k} = s(-\hat{i} + 4\hat{j} + 3\hat{k}) + t(-8\hat{i} - \hat{j} + 3\hat{k})$

Equating i, j, k Components

$-4 = -s - 8t$

$s + 8t = 4$ ——— (1)

$-6 = 4s - t$

$4s - t = -6$ ——— (2)

9

$3s+3t=-2$ —— (3)

solving (1) & (2)

$s+8t=4$

(2) $\times 4 \Rightarrow 32s-8t=-48$

$33s=-44$

$s=\frac{-44}{33}$

$\boxed{s=-4/3}$

$s=-4/3$ in (1)

$-4/3+8t=4$

$8t=4+4/3$

$8t=\frac{12+4}{3}$

$8t=16/3$

$t=16/3 \times 1/8$

$\boxed{t=2/3}$

$s=-4/3$ and $t=2/3$ in (3)

LHS -2

RHS

$3(-4/3)+3(2/3)$

$=-4+2$

$=-2$

LHS = RHS

we are able to one vector as a linear combination of the other two vectors.

$\Rightarrow$ The given vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar.

$\Rightarrow$ The given points A, B, C, D are coplanar.

11 Given

$\vec{a}=2\hat{i}+3\hat{j}-4\hat{k}$

$\vec{b}=3\hat{i}-4\hat{j}-5\hat{k}$

$\vec{c}=-3\hat{i}+2\hat{j}+3\hat{k}$

(i) $\vec{a}+\vec{b}+\vec{c}=$

$(2\hat{i}+3\hat{j}-4\hat{k})+(3\hat{i}-4\hat{j}-5\hat{k})+(-3\hat{i}+2\hat{j}+3\hat{k})$

$=2\hat{i}+\hat{j}-6\hat{k}$

$\boxed{\vec{a}+\vec{b}+\vec{c}=2\hat{i}+\hat{j}-6\hat{k}}$

magnitude :-

$|\vec{a}+\vec{b}+\vec{c}|=|2\hat{i}+\hat{j}-6\hat{k}|$

$=\sqrt{(2)^2+(1)^2+(-6)^2}$

$=\sqrt{4+1+36}$

$\boxed{|\vec{a}+\vec{b}+\vec{c}|=\sqrt{41}}$

Direction cosines are $\left(\frac{2}{\sqrt{41}}, \frac{1}{\sqrt{41}}, \frac{-6}{\sqrt{41}}\right)$

(ii)

$\vec{a}=2\hat{i}+3\hat{j}-4\hat{k}$

$\boxed{3\vec{a}=6\hat{i}+9\hat{j}-12\hat{k}}$

$\vec{b}=3\hat{i}-4\hat{j}-5\hat{k}$

$\boxed{-2\vec{b}=-6\hat{i}+8\hat{j}+10\hat{k}}$

$\vec{c}=-3\hat{i}+2\hat{j}+3\hat{k}$

$\boxed{5\vec{c}=-15\hat{i}+10\hat{j}+15\hat{k}}$

$3\vec{a}-2\vec{b}+5\vec{c}=$

$(6\hat{i}+9\hat{j}-12\hat{k})+(-6\hat{i}+8\hat{j}+10\hat{k})+(-15\hat{i}+10\hat{j}+15\hat{k})$

$=6\hat{i}+9\hat{j}-12\hat{k}-6\hat{i}+8\hat{j}+10\hat{k}-15\hat{i}+10\hat{j}+15\hat{k}$

$=-15\hat{i}+27\hat{j}+13\hat{k}$

$\boxed{3\vec{a}-2\vec{b}+5\vec{c}=-15\hat{i}+27\hat{j}+13\hat{k}}$

Magnitude :-

$|3\vec{a}-2\vec{b}+5\vec{c}|=\sqrt{(-15)^2+(27)^2+(13)^2}$

$|3\vec{a}-2\vec{b}+5\vec{c}|=\sqrt{1123}$

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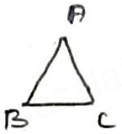
Direction Cosines are

$$\left(\frac{-15}{\sqrt{1123}}, \frac{27}{\sqrt{1123}}, \frac{13}{\sqrt{1123}}\right)$$

12) Let A, B, C be the Position Vectors. (ie)



$\vec{OA} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{OB} = 3\hat{i} - 4\hat{j} + 5\hat{k}$

$\vec{OC} = -2\hat{i} + 3\hat{j} - 7\hat{k}$

$\vec{AB} = \vec{OB} - \vec{OA}$

$= (3\hat{i} - 4\hat{j} + 5\hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k})$

$\boxed{\vec{AB} = 2\hat{i} - 6\hat{j} + 2\hat{k}} \Rightarrow \boxed{|AB| = \sqrt{44}}$

$\vec{BC} = \vec{OC} - \vec{OB}$

$= -2\hat{i} + 3\hat{j} - 7\hat{k} - (3\hat{i} - 4\hat{j} + 5\hat{k})$

$= -2\hat{i} + 3\hat{j} - 7\hat{k} - 3\hat{i} + 4\hat{j} - 5\hat{k}$

$\boxed{\vec{BC} = -5\hat{i} + 7\hat{j} - 12\hat{k}}$

$|\vec{BC}| = \sqrt{(-5)^2 + (7)^2 + (-12)^2}$

$= \sqrt{25 + 49 + 144}$

$\boxed{|\vec{BC}| = \sqrt{218}}$

$\vec{CA} = \vec{OA} - \vec{OC}$

$= \hat{i} + 2\hat{j} + 3\hat{k} - (-2\hat{i} + 3\hat{j} - 7\hat{k})$

$= \hat{i} + 2\hat{j} + 3\hat{k} + 2\hat{i} - 3\hat{j} + 7\hat{k}$

$\boxed{\vec{CA} = 3\hat{i} - \hat{j} + 10\hat{k}}$

$|\vec{CA}| = \sqrt{(3)^2 + (-1)^2 + (10)^2}$

$= \sqrt{9 + 1 + 100} = \sqrt{110}$

$\boxed{|\vec{CA}| = \sqrt{110}}$

Perimeter of the triangle $=$

$|\vec{AB}| + |\vec{BC}| + |\vec{CA}|$

$= \sqrt{44} + \sqrt{218} + \sqrt{110}$.

13) $\vec{a} = 3\hat{i} - \hat{j} - 4\hat{k}$

$\vec{b} = -2\hat{i} + 4\hat{j} - 3\hat{k}$

$\vec{c} = \hat{i} + 2\hat{j} - \hat{k}$

$3\vec{a} - 2\vec{b} + 4\vec{c} = 3(3\hat{i} - \hat{j} - 4\hat{k})$

$-2(-2\hat{i} + 4\hat{j} - 3\hat{k}) + 4(\hat{i} + 2\hat{j} - \hat{k})$

$= 9\hat{i} - 3\hat{j} - 12\hat{k} + 4\hat{i} - 8\hat{j} + 6\hat{k}$
$+ 4\hat{i} + 8\hat{j} - 4\hat{k}$

$= 17\hat{i} - 3\hat{j} - 10\hat{k}$

Let $\vec{r} = 3\vec{a} - 2\vec{b} + 4\vec{c}$

Then $\vec{r} = 17\hat{i} - 3\hat{j} - 10\hat{k}$

Unit vectors Parallel to $\vec{r}$ is $\frac{\vec{r}}{|\vec{r}|}$.

$|\vec{r}| = \sqrt{17^2 + (-3)^2 + 10^2} = \sqrt{289 + 9 + 100}$

$\boxed{|\vec{r}| = \sqrt{398}}$

Unit vectors Parallel to $\vec{r} = \pm \frac{17\hat{i} - 3\hat{j} - 10\hat{k}}{\sqrt{398}}$.

14) Given

$2\vec{a} - 7\vec{b} + 5\vec{c} = 0$

$2\vec{a} + 5\vec{c} = 7\vec{b}$

$\div 2 \Rightarrow \frac{2}{7}\vec{a} + \frac{5}{7}\vec{c} = \vec{b}$.

$s\vec{a} + t\vec{c} = \vec{b}$.

$\therefore \vec{b}$ is a linear Combination of $\vec{a}$ and $\vec{c}$.

$\therefore \vec{a}, \vec{b}, \vec{c}$ are Collinear.

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15)

Given

$\vec{OP} = \hat{i} + \hat{j} + \hat{k}$

$\vec{OQ} = 2\hat{i} + 5\hat{k}$

$\vec{OR} = 3\hat{i} + 2\hat{j} - 3\hat{k}$

$\vec{OS} = \hat{i} - 6\hat{j} - \hat{k}$

$\vec{PQ} = \vec{OQ} - \vec{OP}$

$= (2\hat{i} + 5\hat{j}) - (\hat{i} + \hat{j} + \hat{k})$

$= 2\hat{i} + 5\hat{j} - \hat{i} - \hat{j} - \hat{k}$

$\boxed{\vec{PQ} = \hat{i} + 4\hat{j} - \hat{k}}$

$\vec{RS} = \vec{OS} - \vec{OR}$

$= \hat{i} - 6\hat{j} - \hat{k} - (3\hat{i} + 2\hat{j} - 3\hat{k})$

$= \hat{i} - 6\hat{j} - \hat{k} - 3\hat{i} - 2\hat{j} + 3\hat{k}$

$= -2\hat{i} - 8\hat{j} + 2\hat{k}$

$= -2(\hat{i} + 4\hat{j} + \hat{k})$

$= -2(\vec{PQ})$

$\vec{RS} = -2(\vec{PQ})$

Hence $PQ \parallel RS$.

16)

Let $\vec{a} = m(\hat{i} + \hat{j} + \hat{k})$

$|\vec{a}| = m\sqrt{1^2 + 1^2 + 1^2}$

$|\vec{a}| = m\sqrt{3}$

To make $\vec{a}$ as a unit vector

Thus $|\vec{a}| = 1$

$\therefore m\sqrt{3} = \pm 1$

$\boxed{m = \pm 1/\sqrt{3}}$

17)

Let $\vec{OA} = \hat{i} + \hat{j} + \hat{k}$

$\vec{OB} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{OC} = 2\hat{i} - \hat{j} + \hat{k}$

$\vec{AB} = \vec{OB} - \vec{OA}$

$= (\hat{i} + 2\hat{j} + 3\hat{k}) - (\hat{i} + \hat{j} + \hat{k})$

$\vec{AB} = \hat{j} + 2\hat{k}$

$|\vec{AB}| = \sqrt{1^2 + 2^2} = \sqrt{5}$

$\boxed{|\vec{AB}| = \sqrt{5}}$

$\vec{BC} = \vec{OC} - \vec{OB}$

$= 2\hat{i} - \hat{j} + \hat{k} - (\hat{i} + 2\hat{j} + 3\hat{k})$

$\vec{BC} = \hat{i} - 3\hat{j} - 2\hat{k}$

$|\vec{BC}| = \sqrt{1^2 + (-3)^2 + (-2)^2}$

$= \sqrt{1 + 9 + 4}$

$\boxed{|\vec{BC}| = \sqrt{14}}$

$\vec{CA} = \vec{OA} - \vec{OC}$

$= (\hat{i} + \hat{j} + \hat{k}) - (2\hat{i} - \hat{j} + \hat{k})$

$\vec{CA} = -\hat{i} + 2\hat{j}$

$|\vec{CA}| = \sqrt{(-1)^2 + (2)^2} = \sqrt{1 + 4} = \sqrt{5}$

$\boxed{|\vec{CA}| = \sqrt{5}}$

$|\vec{AB}| = |\vec{CA}|$ Therefore

The given Points form an Isoceles triangle.

VECTOR ALGEBRA – [8.1 & 8.2]

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PG ASST IN MATHEMATICS

PATTUKKOTTAI.