XII-01

Application of Matrices and Determinants

Mathematics - Unit Test

Time: 1.15   Marks: 50

Part - A

Answer any five questions   **5 x 2 = 10**

1. $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$, show that $A^T A^{-1} = \begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$.

2. If $adj(A) = \begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$, find A^{-1}

3. Verify the property $\left(A^T\right)^{-1} = \left(A^{-1}\right)^T$ with $A = \begin{bmatrix} 2 & 9 \\ 1 & 7 \end{bmatrix}$.

4. Find the rank of the following matrices by minor method: $\begin{bmatrix} 1 & -2 & -1 & 0 \\ 3 & -6 & -3 & 1 \end{bmatrix}$

5. 4 men and 4 women can finish a piece of work jointly in 3 days while 2 men and 5 women can finish the same work jointly in 4 days. Find the time taken by one man alone and that of one woman alone to finish the same work by using matrix inversion method.

6. If A is a non-singular matrix of odd order, prove that $|adj\ A|$ is positive.

7. Solve the following system of homogenous equations.

   $2x + 3y - z = 0, \quad x - y - 2z = 0, \quad 3x + y + 3z = 0$

Part - B

Answer any five questions   **5 x 3 = 15**

8. If $A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$, show that $A^2 - 3A - 7I_2 = O_2$. Hence find A^{-1}

9. Decrypt the received encoded message $\begin{bmatrix} 2 & -3 \end{bmatrix}\begin{bmatrix} 20 & 4 \end{bmatrix}$ with the encryption matrix $\begin{bmatrix} -1 & -1 \\ 2 & 1 \end{bmatrix}$

and the decryption matrix as its inverse, where the system of codes are described by the numbers 1-26 to the letters $A - Z$ respectively, and the number 0 to a blank space.

10. Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 0 & 5 \end{bmatrix}$ by reducing it to a row-echelon form.

11. Find the inverse of $A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$ by Gauss-Jordan method.

12. In a competitive examination, one mark is awarded for every correct answer while $\dfrac{1}{4}$ mark is deducted for every wrong answer. A student answered 100 questions and got 80 marks. How many questions did he answer correctly ? (Use Cramer's rule to solve the problem).

13. Test for consistency and if possible, solve the following systems of equations by rank method.

$2x + 2y + z = 5, \quad x - y + z = 1, \quad 3x + y + 2z = 4$

14. Given $A = \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -2 \\ 1 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$, find a matrix X such that $AXB = C$.

Part - C

Answer any five questions   5 x 5 = 25

15. A boy is walking along the path $y = ax^2 + bx + c$ through the points $(-6, 8), (-2, -12)$, and $(3, 8)$. He wants to meet his friend at $P(7, 60)$. Will he meet his friend? (Use Gaussian elimination method.)

16. Test the consistency of the following system of linear equations

$x - y + z = -9, \quad 2x - y + z = 4, \quad 3x - y + z = 6, \quad 4x - y + 2z = 7.$

17. Find the value of k for which the equations $kx - 2y + z = 1, \quad x - 2ky + z = -2, \quad x - 2y + kz = 1$ have

(i) no solution   (ii) unique solution   (iii) infinitely many solution

18.   If the system of equations $px + by + cz = 0$, $ax + qy + cz = 0$, $ax + by + rz = 0$ has a non-trivial

solution and $p \neq a, q \neq b, r \neq c$, prove that $\dfrac{p}{p-a} + \dfrac{q}{q-b} + \dfrac{r}{r-c} = 2$.

19. By using Gaussian elimination method, balance the chemical reaction equation:
$C_2H_6 + O_2 \rightarrow H_2O + CO_2$

20. If $F(\alpha) = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}$, show that $\left[F(\alpha)\right]^{-1} = F(-\alpha)$.

21. A family of 3 people went out for dinner in a restaurant. The cost of two dosai, three idlies and two vadais is ₹ 150. The cost of the two dosai, two idlies and four vadais is ₹ 200. The cost of five dosai, four idlies and two vadais is ₹ 250. The family has ₹ 350 in hand and they ate 3 dosai and six idlies and six vadais. Will they be able to manage to pay the bill within the amount they had ?

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