Chapter - 1

EX: 1.1

1) i) Let $A = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$

$|A| = 40 - 42$
$= 2 \neq 0$

$\rho(A) = 2$

ii) $A = \begin{bmatrix} 1 & -1 \\ 3 & -6 \end{bmatrix}$

$|A| = -6 + 3$
$= -3 \neq 0$

$\rho(A) = 2$

iii) Let $A = \begin{bmatrix} 1 & 4 \\ 2 & 8 \end{bmatrix}$

$|A| = 8 - 8$
$= 0 = 0$

$\rho(A) = 1$

iv) Let $A = \begin{bmatrix} 2 & -1 & 1 \\ 3 & 1 & -5 \\ 1 & 1 & 1 \end{bmatrix}$

$|A| = 2(1+5) + 1(3+5) + 1(3-1)$
$= 2(6) + 1(8) + 1(2)$
$= 12 + 8 + 2$
$= 22 \neq 0$

$\rho(A) = 3$

v) $A = \begin{bmatrix} -1 & 2 & -2 \\ 4 & -3 & 4 \\ -2 & 4 & -4 \end{bmatrix}$

$|A| = -1(12-16) - 2(-16+8) - 2(16-6)$
$= -1(-4) - 2(-8) - 2(10)$
$= 4 + 16 - 20$
$= 0$

Minor of 2×2

$\begin{vmatrix} -1 & 2 \\ 4 & -3 \end{vmatrix} = 3 - 8$
$= -5 \neq 0$

$\rho(A) = 2$

vi) Let $A = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 2 & 4 & 1 & -2 \\ 3 & 6 & 3 & -7 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & 0 & 3 & -8 \\ 0 & 0 & 6 & -16 \end{bmatrix}$ $R_2 \to R_2 \to 2R_1$, $R_3 \to R_3 - 3R_1$

$\sim \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & 0 & 3 & -8 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $R_3 \to R_3 - 2R_2$

$\rho(A) = 2$

vii) Let $A = \begin{bmatrix} 3 & -1 & -5 & -1 \\ 1 & 2 & 1 & -5 \\ 1 & 5 & -7 & 2 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & -2 & 1 & -5 \\ 3 & 1 & -5 & -1 \\ 1 & 5 & -7 & 2 \end{bmatrix}$ $R_1 \leftrightarrow R_2$

$\sim \begin{bmatrix} 1 & -2 & 1 & -5 \\ 0 & 7 & -8 & 14 \\ 0 & 7 & -8 & 7 \end{bmatrix}$ $R_2 \to R_2 - 3R_1$, $R_3 \to R_3 - R_1$

$$\sim \begin{bmatrix} 1 & -2 & 1 & -5 \\ 0 & 7 & -8 & 14 \\ 0 & 0 & 0 & -7 \end{bmatrix} \quad R_3 \to R_3 - R_2$$

$$\rho(A) = 3$$

iii) $$\begin{bmatrix} 1 & -2 & 3 & 4 \\ -2 & 4 & -1 & -3 \\ -1 & 2 & 7 & 6 \end{bmatrix}.$$

$$\sim \begin{bmatrix} 1 & -2 & 3 & 4 \\ 0 & 0 & 5 & 5 \\ 0 & 0 & 10 & 10 \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 + 2R_1 \\ R_3 \to R_3 + R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & -2 & 3 & 3 \\ 0 & 0 & 5 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad R_3 \to R_3 - 2R_2$$

$$\rho(A) = 2.$$

2) $$A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -2 & 3 \\ -2 & 4 & -6 \\ 5 & 1 & -1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1-2-5 & -2+4+1 & 3-6+1 \\ 2+6+20 & -4-12+4 & 6+18-4 \\ 3+4+15 & -6-8+3 & 9+12-3 \end{bmatrix}$$

$$AB = \begin{bmatrix} -6 & 1 & -2 \\ 28 & -12 & 20 \\ 22 & -11 & 18 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -6 & -2 \\ -12 & 28 & 20 \\ -11 & 22 & 18 \end{bmatrix} \quad C_1 \leftrightarrow C_2$$

$$\sim \begin{bmatrix} 1 & -6 & -2 \\ 0 & -44 & -4 \\ 0 & -44 & -4 \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 + 12R_1 \\ R_3 \to R_3 + 11R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & -6 & -2 \\ 0 & -44 & -4 \\ 0 & 0 & 0 \end{bmatrix} \quad R_3 \to R_3 - R_2$$

$$\rho(AB) = 2.$$

$$BA = \begin{bmatrix} 1 & -2 & 3 \\ -2 & 4 & -6 \\ 5 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1-4+9 & 1+6-6 & -1-8+9 \\ -2+8-18 & -2-12+12 & 2+16-18 \\ 5+2-3 & 5-3+2 & -5+4-3 \end{bmatrix}$$

$$BA = \begin{bmatrix} 6 & 1 & 0 \\ -12 & -2 & 0 \\ 4 & 4 & -4 \end{bmatrix}$$

$$BA \sim \begin{bmatrix} 4 & 4 & -4 \\ -12 & -2 & 0 \\ 6 & 1 & 0 \end{bmatrix} \quad R_1 \leftrightarrow R_3$$

$$\sim \begin{bmatrix} 1 & 1 & -1 \\ -12 & -2 & 0 \\ 6 & 1 & 0 \end{bmatrix} \quad R_1 \to R_1/4$$

$$\sim \begin{bmatrix} 1 & 1 & -1 \\ 0 & 10 & -12 \\ 0 & -5 & 6 \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 + 12R_1 \\ R_3 \to R_3 - 6R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & -1 \\ 0 & 10 & -12 \\ 0 & 0 & 0 \end{bmatrix} \quad R_3 \to 2R_3 + R_2$$

$$\rho(BA) = 2$$

3) $x+y+z=9$

$2x+5y+7z=52$

$2x+y-z=0$

$AX=B$

$$\begin{bmatrix}1&1&1\\2&5&7\\2&1&-1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}9\\52\\0\end{bmatrix}$$

$$(A,B)\sim\begin{bmatrix}1&1&1&9\\2&5&7&52\\2&1&-1&0\end{bmatrix}$$

$$\sim\begin{bmatrix}1&1&1&9\\0&3&5&34\\0&-1&-3&-18\end{bmatrix}\begin{matrix}R_2\to R_2-2R_1\\R_3\to R_3-2R_1\end{matrix}$$

$$\sim\begin{bmatrix}1&1&1&9\\0&3&5&34\\0&0&-4&-20\end{bmatrix}R_3\to 3R_3+R_2$$

$\rho(A)=3,\quad \rho(A,B)=3$

$\rho(A)=\rho(A,B)=3.$

It is consistent. It has unique solution.

$AX=B$

$$\begin{bmatrix}1&1&1\\0&3&5\\0&0&-4\end{bmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{bmatrix}9\\34\\-20\end{bmatrix}$$

$x+y+z=9 \quad -(1)$

$3y+5z=30 \quad -(2)$

$-4z=-20$

$z=\frac{+20}{+4}=5$

Sub in (2)

$3y+5(5)=34$

$3y=34-25$

$3y=9$

$y=9/3$

$y=3$

Sub in (1)

$x+3+5=9$

$x+8=9$

$x=9-8$

$x=1$

The solution $x=1,\ y=3,\ z=5$

4) $5x+3y+7z=4$

$3x+26y+2z=9$

$7x+2y+10z=5$

$AX=B$

$$\begin{bmatrix}5&3&7\\3&26&2\\7&2&10\end{bmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}4\\9\\5\end{pmatrix}$$

$$(A,B)=\begin{bmatrix}5&3&7&4\\3&26&2&9\\7&2&10&5\end{bmatrix}$$

$$\sim\begin{bmatrix}1&3/5&7/5&4/5\\3&26&2&9\\7&2&10&5\end{bmatrix}R_1\to R_1/5$$

$$\sim\begin{bmatrix}1&3/5&7/5&4/5\\0&121/5&-11/5&33/5\\0&-11/5&1/5&-3/5\end{bmatrix}\begin{matrix}R_2\to R_2-3R_1\\R_3\to R_3-7R_1\end{matrix}$$

$$\sim\begin{bmatrix}1&3/5&7/5&4/5\\0&121&-11&33\\0&-11&1&-3\end{bmatrix}\begin{matrix}R_2\to 5R_2\\R_3\to 5R_3\end{matrix}$$

$$\sim \begin{bmatrix} 1 & 3/5 & 7/5 & 4/5 \\ 0 & 121 & -11 & 33 \\ 0 & 0 & 0 & 0 \end{bmatrix} R_3 \to 11R_3 + R_2$$

Echelon form,

$$\rho(A) = \rho(A,B) = 2$$

It is consistent. It has infinitely many solutions.

$$AX = B$$

$$\begin{bmatrix} 1 & 3/5 & 7/5 \\ 0 & 121 & -11 \\ 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{bmatrix} 4/5 \\ 33 \\ 0 \end{bmatrix}$$

$$x + \tfrac{3}{5}y + \tfrac{7}{5}z = \tfrac{4}{5} \quad -(1)$$

$$121y - 11z = 33 \quad -(2)$$

Let $z = k, k \in R$

$$121y - 11k = 33$$

$$121y = 33 + 11k$$

$$y = \frac{\cancel{11}[3+k]}{\cancel{121}\ 11}$$

$$y = \frac{3+k}{11}$$

Sub in (1)

$$x + \tfrac{3}{5}\left[\frac{3+k}{11}\right] + \tfrac{7}{5}k = \tfrac{4}{5}$$

$$x + \frac{9+3k}{55} + \tfrac{7}{5}k = 4/5$$

$$55x + 9 + 3k + 77k = 44$$

$$55x = 44 - 9 - 80k$$

$$x = \frac{35 - 80k}{55}$$

$$x = \frac{\cancel{5}(7-16k)}{\cancel{55}\ 11}$$

$$x = \frac{7-16k}{11}$$

the solutions $\left(\frac{7-16k}{11}, \frac{3+k}{11}, k\right), k \in R$

b) $3x - y + \lambda z = 1$

$2x + y + z = 2$

$x + 2y - \lambda z = -1$

$$\begin{bmatrix} 3 & -1 & \lambda \\ 2 & 1 & 1 \\ 1 & 2 & -\lambda \end{bmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

$$AX = B$$

$$[A/B] = \begin{bmatrix} 3 & -1 & \lambda & 1 \\ 2 & 1 & 1 & 2 \\ 1 & 2 & -\lambda & -1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & -\lambda & -1 \\ 2 & 1 & 1 & 2 \\ 3 & -1 & \lambda & 1 \end{bmatrix} R_1 \leftrightarrow R_3$$

$$\sim \begin{bmatrix} 1 & 2 & -\lambda & -1 \\ 0 & -3 & 1+2\lambda & 4 \\ 0 & -7 & 4\lambda & 4 \end{bmatrix} \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 2 & -\lambda & -1 \\ 0 & -3 & 1+2\lambda & 4 \\ 0 & 0 & -1-2\lambda & -16 \end{bmatrix} R_3 \to 3R_3 - 7R_2$$

It is consistent, fail to have unique solution.

$$\rho(A) = \rho(A,B) < n$$

$$-7 - 2\lambda = 0$$

$$-7 = 2\lambda$$

$$-7/2 = \lambda$$

7) $2x+3y-6z=5000$

$3x-y+2z=2000$

$-x+3y+z=5500.$

$$\begin{bmatrix}2&3&-6\\3&-1&2\\-1&3&1\end{bmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{bmatrix}5000\\2000\\5500\end{bmatrix}$$

$AX=B.$

$$[A/B]=\begin{bmatrix}2&3&-6&5000\\3&-1&2&2000\\-1&3&1&5500\end{bmatrix}$$

$$\sim\begin{bmatrix}-1&3&1&5500\\3&-1&2&2000\\2&3&-6&5000\end{bmatrix} R_1\leftrightarrow R_3$$

$$\sim\begin{bmatrix}1&-3&-1&-5500\\3&-1&2&2000\\2&3&-6&5000\end{bmatrix} R_1\to -R_1$$

$$\sim\begin{bmatrix}1&-3&-1&-5500\\0&8&5&18500\\0&9&-4&16000\end{bmatrix}\begin{matrix}R_2\to R_2-3R_1\\R_3\to R_3-2R_1\end{matrix}$$

$$\sim\begin{bmatrix}1&-3&-1&-5500\\0&8&5&18500\\0&0&-77&-38500\end{bmatrix} R_3\to 8R_3-9R_2$$

It is echelon form

$\rho(A)=3,\ \rho(A,B)=3$

consistent, unique solution

$AX=B$

$$\begin{bmatrix}1&-3&-1\\0&8&5\\0&0&-77\end{bmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{bmatrix}-5500\\18500\\-38500\end{bmatrix}$$

$x-3y-z=-5500$ — ①

$8y+5z=18500$ — ②

$-77z=-38500.$

$z=\frac{-38500}{-77}$

$z=500$

Sub in ②

$8y+2500=18500$

$8y=16000$

$y=2000$

sub in ①

$x-6000-500=-5500.$

$x-6500=-5500$

$x=-5500+6500$

$x=1000.$

The solutions are $(1000, 2000, 500)$

8) Let x,y,z be the three different bonds

$x+y+z=5000$ — ①

$\frac{6}{100}x+\frac{7}{100}y+\frac{8}{100}z=358$

$(\times 100)$

$6x+7y+8z=35800$ — ②

$\frac{6}{100}x+\frac{7}{100}y=70+\frac{8}{100}z$

$(\times 100)$

$6x+7y-8z=7000$ — ③

$$\begin{bmatrix}1&1&1\\6&7&8\\6&7&-8\end{bmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{bmatrix}5000\\35800\\7000\end{bmatrix}$$

$AX=B.$

$$[A/B] = \begin{bmatrix} 1 & 1 & 1 & 5000 \\ 6 & 7 & 8 & 35800 \\ 6 & 7 & -8 & 7000 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 5000 \\ 6 & 7 & 8 & 35800 \\ 0 & 0 & -16 & -28800 \end{bmatrix} R_3 \to R_3 - R_2$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 5000 \\ 0 & 1 & 2 & 5800 \\ 0 & 0 & -16 & -28800 \end{bmatrix} R_2 \to R_2 - 6R_1$$

It is an echelon form

$\rho(A) = 3$, $\rho(A,B) = 3$

Consistent. It has unique solution

$AX = B$.

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & -16 \end{bmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{bmatrix} 5000 \\ 5800 \\ -28800 \end{bmatrix}$$

$x + y + z = 5000$ — (1)

$y + 2z = 5800$ — (2)

$-16z = -28800$.

$z = 1800$

Sub in (2)

$y + 3600 = 5800$

$y = 2200$.

Sub in (1)

$x + 2200 + 1800 = 5000$

$x = 1000$

Amt invested in 6% is ₹1000

Amt invested in 7% is ₹2200

Amt invested in 8% is ₹1800.

5) $x + y + z = 3$

$x + 2y + 3z = 4$

$x + 4y + 9z = 6$.

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 6 \end{bmatrix}$$

$$(A,B) \sim \begin{bmatrix} 1 & 1 & 1 & 3 \\ 1 & 2 & 3 & 4 \\ 1 & 4 & 9 & 6 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 3 & 8 & 3 \end{bmatrix} R_3 \to R_3 - R_1$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 2 & 0 \end{bmatrix} R_3 \to R_3 - 3R_2$$

$\rho(A) = 3$, $\rho(A,B) = 3$

$\rho(A) = \rho(A,B) = 3$

It is consistent. It has unique solution

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$$

$x + y + z = 3$ — (1)

$y + 2z = 1$ — (2)

$2z = 0$.

$z = 0$

Sub in (2)

$y + 2(0) = 1$

$y = 1$

Sub in (1)

$x + 1 + 0 = 3$

$x = 2$

Solutions are $[2, 1, 0]$

1\. (i) $2x+3y=7$ — ①

$3x+5y=9$ — ②

$\Delta = \begin{vmatrix} 2 & 3 \\ 3 & 5 \end{vmatrix}$

$= 10-9$

$= 1$

$\Delta x = \begin{vmatrix} 7 & 3 \\ 9 & 5 \end{vmatrix}$

$= 35-27$

$= 8$

$\Delta y = \begin{vmatrix} 2 & 7 \\ 3 & 9 \end{vmatrix}$

$= 18-21$

$= -3$

$\Delta \neq 0$, cramers rule

$x = \frac{\Delta x}{\Delta} = \frac{8}{1} = 8$

$y = \frac{\Delta y}{\Delta} = \frac{-3}{1} = -3$

The solutions are $x=8, y=-3$

1\. (ii) $5x+3y=17$ — ①

$3x+7y=31$ — ②

$\Delta = \begin{vmatrix} 5 & 3 \\ 3 & 7 \end{vmatrix}$

$= 35-9$

$= 26$

$\Delta x = \begin{vmatrix} 17 & 3 \\ 31 & 7 \end{vmatrix}$

$= 119-93$

$= 26$

$\Delta y = \begin{vmatrix} 5 & 17 \\ 3 & 31 \end{vmatrix}$

$= 155-51$

$= 104$

$\Delta \neq 0$ cramers rule

$x = \frac{\Delta x}{\Delta} = \frac{26}{26} = 1$

$y = \frac{\Delta y}{\Delta} = \frac{104}{26} = 4$

The solutions are $x=1, \; y=4$.

1\. (iii) $2x+y-z=3$ — ①

$x+y+z=1$ — ②

$x-2y-3z=4$ — ③

$\Delta = \begin{vmatrix} 2 & 1 & -1 \\ 1 & 1 & 1 \\ 1 & -2 & -3 \end{vmatrix}$

$= 2(-3+2)-1(-3-1)-1(-2-1)$

$= 2(-1)-1(-4)-1(-3)$

$= -2+4+3$

$= 5.$

$$\Delta x = \begin{vmatrix} 3 & 1 & -1 \\ 1 & 1 & 1 \\ 4 & -2 & -3 \end{vmatrix}$$

$= 3(-3+2)-1(-3-4)-1(-2-4)$

$= 3(-1)-1(-7)-1(-6)$

$= -3+7+6$

$= 10$

$$\Delta y = \begin{vmatrix} 2 & 3 & -1 \\ 1 & 1 & 1 \\ 1 & 4 & -3 \end{vmatrix}$$

$= 2(-3-4)-3(-3-1)-1(4-1)$

$= 2(-7)-3(-4)-1(3)$

$= -14+12-3$

$= -5$

$$\Delta z = \begin{vmatrix} 2 & 1 & 3 \\ 1 & 1 & 1 \\ 1 & -2 & 4 \end{vmatrix}$$

$= 2(4+2)-1(4-1)+3(-2-1)$

$= 2(6)-1(3)+3(-3)$

$= 12-3-9$

$= 0$

$\Delta \neq 0$, cramers rule

$x = \frac{\Delta x}{\Delta} = \frac{10}{5} = 2$

$y = \frac{\Delta y}{\Delta} = \frac{-5}{5} = -1$

$z = \frac{\Delta z}{\Delta} = \frac{0}{5} = 0$

The solutions are $x=2$, $y=-1$, $z=0$

1. (iv) $x+y+z=6$ —— (1)

$2x+3y-z=5$ —— (2)

$6x-2y-3z=-7$ —— (3)

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & -1 \\ 6 & -2 & -3 \end{vmatrix}$$

$= 1(-9-2)-1(-6+6)+1(-4-18)$

$= 1(-11)-1(0)+1(-22)$

$= -11+0-22$

$= -33$

$$\Delta x = \begin{vmatrix} 6 & 1 & 1 \\ 5 & 3 & -1 \\ -7 & -2 & -3 \end{vmatrix}$$

$= 6(-9-2)-1(-15-7)+1(-10+21)$

$= 6(-11)-1(-22)+1(11)$

$= -66+22+11$

$= -33$

$$\Delta y = \begin{vmatrix} 1 & 6 & 1 \\ 2 & 5 & -1 \\ 6 & -7 & -3 \end{vmatrix}$$

$= 1(-15-7)-6(-6+6)+1(-14-30)$

$= 1(-22)-6(0)+1(-44)$

$= -22-44$

$= -66$

$$\Delta_z = \begin{vmatrix} 1 & 1 & 6 \\ 2 & 3 & 5 \\ 6 & -2 & -7 \end{vmatrix}$$

$$= 1(-21+10)-1(-14-30)$$
$$+(-4-18)$$
$$= 1(-11)-1(-44)+6(-22)$$
$$= -11+44-132$$
$$= -99$$

$\Delta \neq 0$ cramers rule

$$x = \frac{\Delta x}{\Delta} = \frac{-33}{-33} = 1$$

$$y = \frac{\Delta y}{\Delta} = \frac{-66}{-33} = 2$$

$$z = \frac{\Delta z}{\Delta} = \frac{-99}{-33} = 3$$

The solutions is $x=1, y=2, z=3$

1. (v) $x+4y+3z = 2$ — ①

$2x-6y+6z = -3$ — ②

$5x-2y+3z = -5$ — ③

$$\Delta = \begin{vmatrix} 1 & 4 & 3 \\ 2 & -6 & 6 \\ 5 & -2 & 3 \end{vmatrix}$$

$$= 1(-18+12)-4(6-30)$$
$$+3(-4+30)$$
$$= -6+96+78$$
$$= 168$$

$$\Delta x = \begin{vmatrix} 2 & 4 & 3 \\ -3 & -6 & 6 \\ -5 & -2 & 3 \end{vmatrix}$$

$$= 2(-18+12)-4(-9+30)+3(6-30)$$
$$= -12-84-72$$
$$= -168$$

$$\Delta y = \begin{vmatrix} 1 & 2 & 3 \\ 2 & -3 & 6 \\ 5 & -5 & 3 \end{vmatrix}$$

$$= 1(-9+30)-2(6-30)+3(-10+15)$$
$$= 21+48+15$$
$$= 84.$$

$$\Delta z = \begin{vmatrix} 1 & 4 & 2 \\ 2 & -6 & -3 \\ 5 & -2 & -5 \end{vmatrix}$$

$$= 1(30-6)-4(-10+15)+2(-4+30)$$
$$= 24-20+52$$
$$= 56$$

$\Delta \neq 0$ cramers rule.

$$x = \frac{\Delta x}{\Delta} = \frac{-168}{168} = \cancel{1}\ -1$$

$$y = \frac{\Delta y}{\Delta} = \frac{84}{168} = \frac{1}{2}$$

$$z = \frac{\Delta z}{\Delta} = \frac{56}{168} = \frac{1}{3}$$

The solutions are $x=-1, y=\frac{1}{2},$

$z = \frac{1}{3}$

2\. Let x be the labour and y be the capital per unit

$3x+2y=62$ — ①

$4x+y=56$ — ②

$$\Delta = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix} = 3-8 = -5$$

$$\Delta x = \begin{vmatrix} 62 & 2 \\ 56 & 1 \end{vmatrix} = 62-112 = -50$$

$$\Delta y = \begin{vmatrix} 3 & 62 \\ 4 & 56 \end{vmatrix} = 168-248 = -80$$

$\Delta \neq 0$, cramers rule

$$x = \frac{\Delta x}{\Delta} = \frac{-50}{-5} = 10$$

$$y = \frac{\Delta y}{\Delta} = \frac{-80}{-5} = 16$$

The cost of labour per unit = ₹ 10

The cost of capital per unit = ₹ 16

3\. Let x & y be the two accounts

$x+y=8600$ — ①

$$\frac{19\times1\times x}{4\times100} + \frac{13\times1\times y}{2\times100} = 431\cdot25$$

$(\times400)$

$19x+26y=172500$ — ②

$$\Delta = \begin{vmatrix} 1 & 1 \\ 19 & 26 \end{vmatrix} = 26-19 = 7$$

$$\Delta x = \begin{vmatrix} 8600 & 1 \\ 172500 & 26 \end{vmatrix} = 223600-172500 = 51100$$

$$\Delta y = \begin{vmatrix} 1 & 8600 \\ 19 & 172500 \end{vmatrix} = 172500-163400 = 9100$$

$\Delta \neq 0$, cramers rule

$$x = \frac{\Delta x}{\Delta} = \frac{51100}{7} = 7300$$

$$y = \frac{\Delta y}{\Delta} = \frac{9100}{7} = 1300$$

Invest amount of $4\frac{3}{4}\%$ is ₹ 7300

Invest amount of $6\frac{1}{2}\%$ is ₹ 1300

4. Let x be the houses of horse riding and y be the houses of quad bike riding.

$3x + 4y = 780$ — ①

$2x + 3y = 560$ — ③

$$\Delta = \begin{vmatrix} 3 & 4 \\ 2 & 3 \end{vmatrix}$$

$= 9-8$

$= 1$

$$\Delta x = \begin{vmatrix} 780 & 4 \\ 560 & 3 \end{vmatrix}$$

$= 2340 - 2240$

$= 100$

$$\Delta y = \begin{vmatrix} 3 & 780 \\ 2 & 560 \end{vmatrix}$$

$= 1680 - 1560$

$= 120$

$\Delta \neq 0$ cramers rule

$x = \frac{\Delta x}{\Delta} = \frac{100}{1} = 100$

$y = \frac{\Delta x}{\Delta} = \frac{120}{1} = 120$

1 hour charge in horse riding = ₹100

1 hour charge in quad riding = ₹120

5. Let x, y, z be the three commodities A, B & C.

$x + 2y + 3z = 11$ — ①

$2x + 4y + 5z = 21$ — ②

$3x + 5y + 6z = 27$ — ③

$$\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{vmatrix}$$

$= 1(24-25) - 2(12-15) + 3(10-12)$

$= -1 + 6 - 6$

$= -1$

$$\Delta x = \begin{vmatrix} 11 & 2 & 3 \\ 21 & 4 & 5 \\ 27 & 5 & 6 \end{vmatrix}$$

$= 11(24-25) - 2(126-135) + 3(105-108)$

$= -11 + 18 - 9$

$= -2$

$$\Delta y = \begin{vmatrix} 1 & 11 & 3 \\ 2 & 21 & 5 \\ 3 & 27 & 6 \end{vmatrix}$$

$= 1(126-135) - 11(12-15) + 3(54-63)$

$= -9 + 33 - 27$

$= -3$

$$\Delta z = \begin{vmatrix} 1 & 2 & 11 \\ 2 & 4 & 21 \\ 3 & 5 & 27 \end{vmatrix}$$

$= 1(108-105)-2(54-63)+11(10-12)$

$= 3+18-22$

$= -1$

$\Delta \neq 0$ cramers rule.

$x = \frac{\Delta x}{\Delta} = \frac{-2}{-1} = 2$

$y = \frac{\Delta y}{\Delta} = \frac{-3}{-1} = 3$

$z = \frac{\Delta z}{\Delta} = \frac{-1}{-1} = 1$

The weight of A commodity $= 2$

The weight of B commodity $= 3$

The weight of C commodity $= 1$

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6. Let x, y, z be the three accounts.

$x+y+z = 8500$ — (1)

$\frac{2\times1\times x}{100} + \frac{3\times1\times y}{100} + \frac{6\times1\times z}{100} = 380$

$(\times 100)$

$2x+3y+6z = 38000$ — (2)

$z = x+y$

$x+y-z = 0$ — (3)

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 6 \\ 1 & 1 & -1 \end{vmatrix}$$

$= 1(-3-6)-1(-2-6)+1(2-3)$

$= -9+8-1$

$= -2$

$$\Delta x = \begin{vmatrix} 8500 & 1 & 1 \\ 38000 & 3 & 6 \\ 0 & 1 & -1 \end{vmatrix}$$

$= 8500(-3-6)-1(-38000-0)+1(38000-0)$

$= -76500+38000+38000$

$= -500$

$$\Delta y = \begin{vmatrix} 1 & 8500 & 1 \\ 2 & 38000 & 6 \\ 1 & 0 & -1 \end{vmatrix}$$

$= 1(-38000-0)-8500(-2-6)+1(0-38000)$

$= -38000+68000-38000$

$= -8000$

$$\Delta I = \begin{vmatrix} 1 & 1 & 8500 \\ 2 & 3 & 38000 \\ 1 & 1 & 0 \end{vmatrix}$$

$= 1(0-38500)-1(0-38000)+8500(2-3)$

$= -38500+38500-8500$

$= -8500$

$\Delta \neq 0$ cramers rule.

$x = \frac{\Delta x}{\Delta} = \frac{-500}{-2} = 250$

$y = \frac{\Delta y}{\Delta} = \frac{-8000}{-2} = 4000$

$I = \frac{\Delta I}{\Delta} = \frac{-8500}{-2} = 4250$

Invest amount of 2% is ₹ 250

Invest amount of 3% is ₹ 4000

Invest amount of 6% is ₹ 4250

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(4)

Ex 1.3

1. $T = \begin{array}{c|cc} & A & B \\ A & 0.45 & 0.55 \\ B & 0.7 & 0.3 \end{array}$

$\begin{array}{cc} A & B \end{array}$
$[0.4 \quad 0.6] = \begin{bmatrix} 0.45 & 0.55 \\ 0.7 & 0.3 \end{bmatrix}$

$= [0.18+0.42 \quad 0.22+0.18]$

$= [0.6 \quad 0.4]$

60% receiving the current letter can be expected to order a subscription.

2. Transition propability matrix

$T = \begin{array}{cc} A & B \\ \end{array}$ $\begin{bmatrix} 0.7 & 0.3 \\ 0.3 & 0.7 \end{bmatrix}$

After one year

$\begin{array}{cc} A & B \end{array}$
$[0.6 \quad 0.4] \begin{bmatrix} 0.7 & 0.3 \\ 0.3 & 0.7 \end{bmatrix} =$

$= [0.42+0.12 \quad 0.18+0.28]$

$= [0.54 \quad 0.46]$

(i) commuters will be using the transit system after one year 54% & 46%.

(ii) at equillibrium

$(A\ B)T = A\ B$ where $A+B=1$

$$[A\ \ B]\begin{bmatrix}0.7 & 0.3\\ 0.3 & 0.7\end{bmatrix} = [A\ \ B]$$

$0.7A + 0.3B = A$

$0.7A + 0.3(1-A) = A$

$0.7A + 0.3 - 0.3A = A$

$0.4A + 0.3 = A$

$0.6A = 0.3$

$A = \dfrac{0.3}{0.6}$

$A = 0.5$

Transit system in the long run is 50%.

3. $T = \begin{bmatrix}0.65 & 0.35\\ 0.45 & 0.55\end{bmatrix}$

after one year

$$[0.15\ \ 0.85]\begin{bmatrix}0.65 & 0.35\\ 0.45 & 0.55\end{bmatrix} =$$

$$= [0.0975+0.3825\ \ \ 0.0525+0.4675]$$

$= [0.48\ \ \ 0.52]$

Market shares after one year $=$ $[0.48\ \ 0.52]$

A share's is 48%.

B share's is 52%.

at equillibrium

$[A\ B]T = [A, B]$ where $A+B=1$

$$[A\ \ B]\begin{bmatrix}0.65 & 0.35\\ 0.45 & 0.55\end{bmatrix} = [A\ \ B]$$

$0.65A + 0.45B = A$

$0.65A + 0.45(1-A) = A$

$0.65A + 0.45 - 0.45A = A$

$0.2A + 0.45 = A$

$0.45 = 0.8A$

$\dfrac{0.45}{0.8} = A$

$A = 0.5625$

$B = 1 - 0.5625$

$B = 0.4375$

equillibrium reached A share's is 56.25% and B share's is 43.75%.

4. $T = \begin{array}{c} \\ A \\ B \end{array}\begin{array}{c} \begin{array}{cc} A & B \end{array} \\ \begin{bmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{bmatrix} \end{array}$

after one year

$\begin{bmatrix} 0.5 & 0.5 \end{bmatrix}\begin{bmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{bmatrix} =$

$= \begin{bmatrix} 0.30+0.10 & 0.20+0.40 \end{bmatrix}$

$= \begin{bmatrix} 0.40 & 0.60 \end{bmatrix}$

A share = 40% , B share = 60%.

after two week

$\begin{bmatrix} 0.40 & 0.60 \end{bmatrix}\begin{bmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{bmatrix} =$

$= \begin{bmatrix} 0.24+0.12 & 0.16+0.48 \end{bmatrix}$

$= \begin{bmatrix} 0.36 & 0.64 \end{bmatrix}$

A share is 36%., B share is 64%.

at equillibrium

$[A \quad B]\, T = [A \quad B]$ where $A+B=1$

$\begin{bmatrix} A & B \end{bmatrix}\begin{bmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{bmatrix} = \begin{bmatrix} A & B \end{bmatrix}$

$0.6A + 0.2B = A$

$0.6A + 0.2(1-A) = A$

$0.6A + 0.2 - 0.2A = A$

$0.4A + 0.2 = A$

$0.2 = A - 0.4A$

$0.2 = 0.6A$

$\frac{0.2}{0.6} = A$

$A = 0.33$

$B = 1 - 0.33$

$= 0.67$

equillibrium reached

A shares = 33%.

B shares = 67%.