Padasalai's Telegram Groups!

(தலைப்பிற்கு கீழே உள்ள லிங்கை கிளிக் செய்து குழுவில் இணையவும்!)

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XII TH MATHS QUESTION BANK (VOLUME – I & II)

5 MARK QUESTIONS

I. APPLICATIONS OF MATRICES AND DETERMINANTS

1. If $F(\alpha) = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}$, show that $[F(\alpha)]^{-1} = F(-\alpha)$ [Ex: 1.1 –(3)]

2. If $A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$, show that $A^2-3A-7I_2 = O_2$. Hence find A^{-1} [Ex: 1.1 –(4)]

3. If $A = \begin{bmatrix} 8 & 6 & -2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$, verify that $A(\text{adj } A) = (\text{adj } A)A = |A|\, I_3$ [Eg: 1.1]

4. If $A = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$, find x and y such that $A^2+xA+yI_2 = O_2$. Hence, find A^{-1} [Eg: 1.10]

5. If $A = \frac{1}{7}\begin{bmatrix} 6 & -3 & a \\ b & -2 & 6 \\ 2 & c & 3 \end{bmatrix}$, is orthogonal, find a,band c , and hence A^{-1} [Eg: 1.12]

6. Decrypt the received encoded message [45 -28 23], [46 -18 3], [5 -5 5] with the encryption matrix $\begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ and the decryption matrix as its inverse, where the system of codes are described by the numbers 1-26 to the letters A - Z respectively, and the number 0 to a blank space. [page no : 15]

7. Show that the matrix $\begin{bmatrix} 3 & 1 & 4 \\ 2 & 0 & -1 \\ 5 & 2 & 1 \end{bmatrix}$ is non-singular and reduce it to the identity matrix by elementary row transformations [Eg: 1.19]

Find the inverse of each of the following by Gauss-Jordan method :

8. $\begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 6 & -2 & -3 \end{bmatrix}$ [Ex: 1.2 –(3)-(ii)] 9. $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$ [Ex. 1.2 –(3)-(iii)] 10. $\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$ [Eg: 1.21]

Solve the following system of linear equations by matrix inversion method:

11. $2x + 3y - z = 9,\ x + y + z = 9,\ 3x - y - z = -1$ [Ex: 1.3 –(1)-(iii)]
12. $x + y + z - 2 = 0,\ 6x - 4y + 5z - 31 = 0,\ 5x + 2y + 2z = 13$ [Ex: 1.3 –(1)-(iv)]
13. $2x_1 + 3x_2 + 3x_3 = 5,\ x_1 - 2x_2 + x_3 = -4,\ 3x_1 - x_2 - 2x_3 = 3$ [Eg: 1.23]
14. If $A = \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$, find the products *AB* and *BA* and hence solve the system of equations $x + y + 2z = 1,\ 3x + 2y + z = 7,\ 2x + y + 3z = 2$ [Ex: 1.3 –(2)]
15. If $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, find the products *AB* and *BA* and hence solve the system of equations $x - y + z = 4,\ x - 2y - 2z = 9,\ 2x + y + 3z = 1$ [Eg: 1.24]
16. The prices of three commodities *A*,*B* and *C* are Rs. *x*,*y* and *z* per units respectively. A person *P* purchases 4 units of *B* and sells two units of *A* and 5 units of *C* . Person *Q* purchases 2 units of *C* and sells 3 units of *A* and one unit of *B* . Person *R* purchases one unit of *A* and sells 3 unit of *B* and one unit of *C* . In the process, *P*,*Q* and *R* earn Rs. 15,000, Rs. 1,000 and Rs. 4,000 respectively. Find the prices per unit of *A*,*B* and *C* . (matrix inversion method) [Ex: 1.3 –(5)]

Solve the following systems of linear equations by Cramer's rule:

17. $3x + 3y - z = 11,\ 2x - y + 2z = 9,\ 4x + 3y + 3z = 25$ [Ex: 1.4 –(1)-(iii)]
18. $\frac{3}{x} - \frac{4}{y} - \frac{2}{z} - 1 = 0,\ \frac{1}{x} + \frac{2}{y} + \frac{1}{z} - 2 = 0,\ \frac{2}{x} - \frac{5}{y} - \frac{4}{z} + 1 = 0$ [Ex: 1.4 –(1)-(iv)]
19. $x_1 - x_2 = 3,\ 2x_1 + 3x_2 + 4x_3 = 17,\ x_2 + 2x_3 = 7$ [Eg: 1.25]

20. A family of 3 people went out for dinner in a restaurant. The cost of two dosai, three idlies and two vadais is Rs.150. The cost of the two dosai, two idlies and four vadais is Rs. 200. The cost of five dosai, four idlies and two vadais is Rs. 250. The family has Rs. 350 in hand and they ate 3 dosai and six idlies and six vadais. Will they be able to manage to pay the bill within the amount they had ? [Ex: 1.4 –(5)]
21. In a T20 match, Chennai Super Kings needed just 6 runs to win with 1 ball left to go in the last over. The last ball was bowled and the batsman at the crease hit it high up. The ball traversed along a path in a vertical plane and the equation of the path is $y = ax^2+bx+c$ with respect to a xy -coordinate system in the vertical plane and the ball traversed through the points(10,8), (20,16), (40,22), can you conclude that Chennai Super Kings won the match? Justify your answer. (All distances are measured in metres and the meeting point of the plane of the path with the farthest boundary line is (70,0)) [Eg: 1.26]

Solve the following systems of linear equations by Gaussian elimination method:

22. $2x - 3y + 3z = 2,\ x + 2y - z = 3,\ 3x - y + 2z = 1$ [Ex: 1.5 –(1)-(i)]
23. $2x + 4y + 6z = 22,\ 3x + 8y + 5z = 27,\ -x + y + 2z = 2$ [Ex: 1.5 –(1)-(ii)]
24. $4x + 3y + 6z = 25,\ x + 5y + 7z = 13,\ 2x + 9y + z = 1$ [Eg: 1.27]
25. If ax^2+bx+c is divided by x+3,x-5, and $x-1$, the remainders are 21,61, and 9 respectively. Find *a, b* and c. (Use Gaussian elimination method.) [Ex: 1.5 –(2)]
26. An amount of Rs. 65,000 is invested in three bonds at the rates of 6%, 8% and 10% per annum respectively. The total annual income is Rs. 4,800. The income from the third bond is Rs. 600 more than that from the second bond. Determine the price of each bond. (Use Gaussian elimination.) [Ex: 1.5 –(3)]
27. A boy is walking along the path $y = ax^2+bx+c$ through the points (-6,8),(-2,-12) and (3,8). He wants to meet his friend at $P(7,60)$. Will he meet his friend? (Use Gaussian elimination method.) [Ex: 1.5 –(4)]
28. The upward speed v (t)of a rocket at time t is approximated by $v(t) = at^2+bt+c,\ 0\le t\le 100$ where *a,b* and c are constants. It has been found that the speed at times $t=3,\ t=6,$ and $t = 9$ seconds are respectively, 64, 133, and 208 miles per second respectively. Find the speed at time $t=15$ seconds. (Use Gaussian elimination method.) [Eg: 1.28]

Test for consistency and if possible, solve the following systems of equations by rank method.

29. $x - y + 2z = 2,\ 2x + y + 4z = 7,\ 4x - y + z = 4$ [Ex: 1.6 –(1)-(i)]
30. $3x - y + z = 2,\ x - 3y + 2z = 1,\ 7x - y + 4z = 5$ [Ex: 1.6 –(1)-(ii)]
31. $2x - 2y + z = 5,\ x - y + z = 1,\ 3x + y + 2z = 4$ [Ex: 1.6 –(1)-(iii)]
32. $2x - y + z = 2,\ 6x - 3y + 3z = 6,\ 4x - 2y + 2z = 4$ [Ex: 1.6 –(1)-(iv)]
33. $x + 2y - z = 3,\ 3x - y + 2z = 1,\ x - 2y + 3z = 3,\ x - 2y + z + 1 = 0$ [Eg: 1.29]
34. $4x - 2y + 6z = 8,\ x + y - 3z = -1,\ 15x - 3y + 9z = 21$ [Eg: 1.30]
35. $x - y + z = -9,\ 2x - 2y + 2z = -18,\ 3x - 3y + 3z + 27 = 0$ [Eg: 1.31]
36. $x - y + z = -9,\ 2x - y + z = 4,\ 3x - y + z = 6,\ 4x - y + 2z = 7$ [Eg: 1.32]
37. Find the value of k for which the equations $kx - 2y + z = 1,\ x - 2ky + z = -2,\ x - 2y + kz = 1$ have (i) no solution (ii) unique solution (iii) infinitely many solution [Ex: 1.6 –(2)]
38. Investigate the values of λ and μ the system of linear equations $2x + 3y + 5z = 9,\ 7x + 3y - z = 8,\ 2x + 3y + \lambda z = \mu$, have (i) no solution (ii) a unique solution (iii) an infinite number of solutions. [Ex: 1.6 –(3)]
39. Find the condition on *a,b* and *c* so that the following system of linear equations has one parameter family of solutions : $x + y + z = a,\ x + 2y + 3z = b,\ 3x + y + 7z = c$ [Eg: 1.33]
40. Investigate for what values of λ and μ the system of linear equations $x + 2y + z = 7,\ x + y + \lambda z = \mu,\ x + 3y - 5z = 5$ has (i) no solution (ii) a unique solution (iii) an infinite number of solutions. [Eg: 1.34]

Solve the following system of homogenous equations.

41. $3x + 2y + 7z = 0,\ 4x - 3y - 2z = 0,\ 5x + 9y + 23z = 0$ [Ex: 1.7 –(1)-(i)]
42. $2x + 3y - z = 0,\ x - y - 2z = 0,\ 3x + y + 3z = 0$ [Ex: 1.7 –(1)-(ii)]
43. $x + 2y + 3z = 0,\ 3x + 4y + 4z = 0,\ 7x + 10y + 12z = 0$ [Eg: 1.35]
44. $x + 3y - 2z = 0,\ 2x - y + 4z = 0,\ x - 11y + 14z = 0$ [Eg: 1.36]
45. $x + y - 2z = 0,\ 2x - 3y + z = 0,\ 3x - 7y + 10z = 0,\ 6x - 9y + 10z = 0$ [Eg: 1.37]
46. Determine the values of λ for which the following system of equations $x+y+3z=0,\ 4x+3y+\lambda z=0,\ 2x+y+2z=0$ has (i) a unique solution (ii) a non-trivial solution. [Ex: 1.7 –(2)]
47. Determine the values of λ for which the following system of equations $(3\lambda-8)x + 3y+3z=0,\ 3x+(3\lambda-8)y+3z=0,\ 3x+3y+(3\lambda-8)z=0$ has a non-trivial solution. [Eg: 1.38]

48. If the system of equations px+by+cz=0, ax+qy+cz=0, ax+by+rz=0 has a non-trivial solution and $p\neq a$, $q\neq b$, $r\neq c$, prove that $\frac{p}{p-a}+\frac{q}{q-b}+\frac{r}{r-c}=2$ [Eg: 1.40]
49. By using Gaussian elimination method, balance the chemical reaction equation : $C_2H_6 + O_2 \rightarrow H_2O + CO_2$ [Ex: 1.7 –(3)]
50. By using Gaussian elimination method, balance the chemical reaction equation: $C_5H_8 + O_2 \rightarrow CO_2 + H_2O$ (The above is the reaction that is taking place in the burning of organic compound called isoprene.) [Eg: 1.39]

II. COMPLEX NUMBERS

1. Show that $\left(\frac{19-7i}{9+i}\right)^{12}+\left(\frac{20-5i}{7-6i}\right)^{12}$ is real. [Ex: 2.4 –(7)-(ii)]
2. Show that $\left(\frac{19+9i}{5-3i}\right)^{15}-\left(\frac{8+i}{1+2i}\right)^{15}$ is purely imaginary. [Eg: 2.8 –(ii)]
3. Let z_1, z_2 and z_3 be complex numbers such that $|z_1|=|z_2|=|z_3|=r>0$ and. $z_1+z_2+z_3\neq 0$ Prove that $\left|\frac{z_1z_2+z_2z_3+z_3z_1}{z_1+z_2+z_3}\right|=r$ [Eg: 2.15]
4. If z_1, z_2 and z_3 are three complex numbers such that $|z_1|=1$, $|z_2|=2$, $|z_3|=3$ and $|z_1+z_2+z_3|=1$ show that $|9z_1z_2+4z_1z_3+z_2z_3|=6$ [Ex: 2.5 –(7)]
5. If z=x+iy is a complex number such that $\text{Im}\left(\frac{2z+1}{iz+1}\right)=0$,show that the locus of z is $2x^2+2y^2+x-2y=0$ [Ex:2.6-(2)]
6. If z = x+ iy and arg $\left(\frac{z-i}{z+2}\right)=\frac{\pi}{4}$, then show that $x^2+y^2+3x-3y+2=0$ [Ex:2.7-(6)]
7. If z = x+ iy and arg $\left(\frac{z-1}{z+1}\right)=\frac{\pi}{2}$, then show that $x^2+y^2=1$ [Eg: 2.27]
8. Simplify : $(1+i)^{18}$ [Eg: 2.31-(i)] 9. Simplify : $(-\sqrt{3}+3i)^{31}$ [Eg: 2.31-(ii)]
10. Find all cube roots of $\sqrt{3}+i$ [Eg: 2.35] 11. Show that $\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5+\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^5=-\sqrt{3}$ [Ex:2.8-(2)]

If $2\cos\alpha = x+\frac{1}{x}$ and $2\cos\beta = y+\frac{1}{y}$, show that

12. $\frac{x}{y}+\frac{y}{x}=2\cos(\alpha-\beta)$ [Ex: 2.8 –(4)-(i)] 13. $xy-\frac{1}{xy}=2i\sin(\alpha+\beta)$ [Ex: 2.8 –(4)-(ii)]
14. $\frac{x^m}{y^n}-\frac{y^n}{x^m}=2i\sin(m\alpha-n\beta)$ [Ex: 2.8 –(4)-(iii)] 15. $x^my^n+\frac{1}{x^my^n}=2\cos(m\alpha+n\beta)$ [Ex: 2.8 -(4)-(iv)]
16. Solve the equation $z^3+27=0$ [Ex: 2.8 –(5)] 17. Solve the equation $z^3+8i=0$, where $z\in\mathbb{C}$. [Eg: 2.34]
18. Prove that the values of $\sqrt[4]{-1}$ are $\pm\frac{1}{\sqrt{2}}(1\pm i)$ [Ex: 2.8 –(10)]
19. Suppose z_1, z_2 ,and z_3 are the vertices of an equilateral triangle inscribed in the circle $|z|=2$. If $z_1=1+i\sqrt{3}$, then find z_2 and z_3 [Eg: 2.36]
20. Find the cube roots of unity. [Eg: 2.32] 21. Find the fourth roots of unity. [Eg: 2.33]

III. THEORY OF EQUATIONS

1. Solve the equation $3x^3-16x^2+23x-6=0$ if the product of two roots is 1. [Ex: 3.1 –(4)]
2. Solve the equation $x^3-9x^2+14x+24=0$ if it is given that two of its roots are in the ratio 3:2 [Ex: 3.1 –(6)]
3. Form the equation whose roots are the squares of the roots of the cubic equation $x^3+ax^2+bx+c=0$ [Eg: 3.6]
4. Find a polynomial equation of minimum degree with rational coefficients, having $\sqrt{5}-\sqrt{3}$ as a root. [Ex: 3.2 –(4)]
5. Determine *k* and solve the equation $2x^3-6x^2+3x+k=0$ if one of its roots is twice the sum of the other two roots. [Ex:3.3 (4)]
6. Find all zeros of the polynomial $x^6-3x^5-5x^4+22x^3-39x^2-39x+135$, if it is known that 1+ 2i and $\sqrt{3}$ are two of its zeros.[Ex:3.3 (5)]
7. If 2 + i and $3-\sqrt{2}$ are roots of the equation $x^6-13x^5+62x^4-126x^3+6x^2+127x-140=0$, find all roots. [Eg: 3.15]
8. Solve : (x-5)(x-7)(x+6)(x+4) = 504 [Ex: 3.4 –(1)-(i)] 9.Solve : (x-4)(x-7)(x-2)(x+1) = 16 [Ex: 3.4 –(1)-(ii)]
10. Solve : (2x-1)(x+3)(x-2)(2x+3) + 20 = 0 [Ex: 3.4 –(2)] 11.Solve : (x-2)(x-7)(x-3)(x+2) +19 = 0 [Eg: 3.23]
12.Solve : (2x-3)(6x-1)(3x-2)(x-12) -7 = 0 [Eg: 3.24]
13. Solve the equation $6x^4-5x^3-38x^2-5x+6=0$ if it is known that $\frac{1}{3}$ is a solution. [Ex: 3.5 –(7)]
14. Solve : $2\sqrt{\frac{x}{a}}+3\sqrt{\frac{a}{x}}=\frac{b}{a}+\frac{6a}{b}$ [Ex: 3.5 –(4)] 15. Solve the following equation: $x^4-10x^3+26x^2-10x+1=0$ [Eg: 3.28]
16. Solve the following equation : $6x^4-35x^3+62x^2-35x+6=0$ [Ex: 3.5 –(5)-(i)]
17. Discuss the maximum possible number of positive and negative zeros of the polynomials x^2-5x+6 and $x^2-5x+16$. Also draw rough sketch of the graphs. [Ex: 3.6 –(2)]

IV. INVERSE TRIGONOMETRIC FUNCTIONS

1. Find the value of : $\sin\left(\tan^{-1}\left(\frac{1}{2}\right) - \cos^{-1}\left(\frac{4}{5}\right)\right)$ [Ex: 4.3 –(4)-(ii)]
2. Find the value of : $\cos\left(\sin^{-1}\left(\frac{4}{5}\right) - \tan^{-1}\left(\frac{3}{4}\right)\right)$ [Ex: 4.3 –(4)-(iii)]
3. Evaluate : $\sin\left(\sin^{-1}\left(\frac{3}{5}\right) + \sec^{-1}\left(\frac{5}{4}\right)\right)$ [Eg: 4.20] 4. Solve : $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$ [Eg: 4.28]
5. solve : $\cos\left(\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)\right) = \sin\left(\cot^{-1}\left(\frac{3}{4}\right)\right)$ [Eg: 4.29]
6. prove that : $\sin^{-1}\left(\frac{3}{5}\right) - \cos^{-1}\left(\frac{12}{13}\right) = \sin^{-1}\left(\frac{16}{65}\right)$ [Ex: 4.5 –(4)-(ii)]
7. prove that : $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right]$ [Ex: 4.5 –(5)]
8. If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$, show that x+y+z = xyz [Ex: 4.5 –(6)]
9. simplify : $\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\left(\frac{x-y}{x+y}\right)$ [Ex: 4.5 –(8)] 10. solve : $\sin^{-1}\left(\frac{5}{x}\right) + \sin^{-1}\left(\frac{12}{x}\right) = \frac{\pi}{2}$ [Ex: 4.3 –(9)-(i)]
11. solve : $\cot^{-1}x - \cot^{-1}(x+2) = \frac{\pi}{12}$, $x > 0$ [Ex: 4.3 –(9)-(iv)]
12. Find the number of solution of the equation $\tan^{-1}(x-1) + \tan^{-1}x + \tan^{-1}(x+1) = \tan^{-1}(3x)$ [Ex: 4.5 –(10)]
13. If $a_1, a_2, a_3, \ldots a_n$ is an arithmetic progression with common difference *d*, prove that

$\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{d}{1+a_2a_3}\right) + \cdots + \tan^{-1}\left(\frac{d}{1+a_na_{n-1}}\right)\right] = \frac{a_n - a_1}{1+a_1a_2}$ [Eg: 4.23]

V. TWO DIMENSIONAL ANALYTICAL GEOMETRY-II

1. Find the equation of circles that touch both the axes and pass through (-4,-2)in general form. [Ex: 5.1 –(3)]
2. Find the equation of the circle with centre (2,3) and passing through the intersection of the lines 3x-2y-1=0 and 4x+y-27=0 [Ex: 5.1 –(4)]
3. Find the equation of the circle through the points (1,0), (-1,0) and (0,1) [Ex: 5.1 –(6)]
4. Find the equation of the circle through the points (1,1), (2,-1) and (3,2) [Eg: 5.10]
5. Find the equation of the tangent and normal to the circle $x^2+y^2-6x+6y-8=0$ at (2,2) [Ex: 5.1 –(9)]
6. Determine whether the points (-2,1),(0,0) and (-4,-3) lie outside, on or inside the circle $x^2+y^2-5x+2y-5=0$ [Ex: 5.1 –(10)]
7. A road bridge over an irrigation canal have two semi circular vents each with a span of 20*m* and the supporting pillars of width 2*m*. write the equations of the arches. [Eg: 5.13]

Find the vertex, focus, equation of directrix and length of the latus rectum of the following:

8. $x^2-2x+8y+17=0$ [Ex: 5.2 –(4)-(iv)] 9. $y^3-4y-8x+12=0$ [Ex: 5.2 –(4)-(v)] 10. $x^2-4x-5y-1=0$ [Eg: 5.17]

Identify the type of conic and find centre, foci, vertices, and directrices of each of the following :

11. $18x^2+12y^2-144x+48y+120=0$ [Ex: 5.2 –(8)-(v)] 12. $9x^2-y^2-36x-6y+18=0$ [Ex: 5.2 –(8)-(vi)]
13. Find the equation of the ellipse whose eccentricity is $\frac{1}{2}$, one of the foci is (2,3) and a directrix is $x = 7$. Also find the length of the major and minor axes of the ellipse. [Eg: 5.19]
14. Find the foci, vertices and length of major and minor axis of the conic $4x^2+36y^2+40x-288y+532=0$ [Eg: 5.20]
15. For the ellipse$4x^2+y^2+24x-2y+21=0$, find the centre, vertices, and the foci. Also prove that the length of latus rectum is 2 . [Eg: 5.21]
16. Find the centre, foci, and eccentricity of the hyperbola $11x^2-25y^2-44x+50y-256=0$ [Eg: 5.24]
17. Find the equations of the two tangents that can be drawn from (5,2) to the ellipse $2x^2+7y^2=14$ [Ex: 5.4 –(1)]
18. Find the equations of tangents to the hyperbola $\frac{x^2}{16} - \frac{y^2}{64} = 1$ which are parallel to10x–3y +9=0 [Ex: 5.4 –(2)]
19. Show that the line x-y+4=0 is a tangent to the ellipse $x^2+3y^2=12$. Also find the coordinates of the point of contact. [Ex:5.4 –(3)]
20. A bridge has a parabolic arch that is 10*m* high in the centre and 30*m* wide at the bottom. Find the height of the arch 6*m* from the centre, on either sides. [Ex:5.5 –(1)]
21. A tunnel through a mountain for a four lane highway is to have a elliptical opening. The total width of the highway (not the opening) is to be 16*m*, and the height at the edge of the road must be sufficient for a truck 4*m* high to clear if the highest point of the opening is to be 5*m* approximately . How wide must the opening be? [Ex:5.5 –(2)]
22. At a water fountain, water attains a maximum height of 4*m* at horizontal distance of 0. 5 *m* from its origin. If the path of water is a parabola, find the height of water at a horizontal distance of 0.75*m* from the point of origin. [Ex:5.5 –(3)]

23. An engineer designs a satellite dish with a parabolic cross section. The dish is 5*m* wide at the opening, and the focus is placed 1.2 *m* from the vertex
    (a) Position a coordinate system with the origin at the vertex and the *x* -axis on the parabola's axis of symmetry and find an equation of the parabola. (b) Find the depth of the satellite dish at the vertex. [Ex:5.5 –(4)]
24. Parabolic cable of a 60*m* portion of the roadbed of a suspension bridge are positioned as shown below. Vertical Cables are to be spaced every 6*m* along this portion of the roadbed. Calculate the lengths of first two of these vertical cables from the vertex. [Ex:5.5 –(5)]
25. Cross section of a Nuclear cooling tower is in the shape of a hyperbola with equation $\frac{x^2}{30^2} - \frac{y^2}{44^2} = 1$.The tower is 150*m* tall and the distance from the top of the tower to the centre of the hyperbola is half the distance from the base of the tower to the centre of the hyperbola. Find the diameter of the top and base of the tower. [Ex:5.5 –(6)]
26. A rod of length 1.2 *m* moves with its ends always touching the coordinate axes. The locus of a point *P* on the rod, which is 0.3 *m* from the end in contact with *x* -axis is an ellipse. Find the eccentricity. [Ex:5.5 –(7)]
27. Assume that water issuing from the end of a horizontal pipe, 7.5*m* above the ground, describes a parabolic path. The vertex of the parabolic path is at the end of the pipe. At a position 2 5 . *m* below the line of the pipe, the flow of water has curved outward 3*m* beyond the vertical line through the end of the pipe. How far beyond this vertical line will the water strike the ground? [Ex:5.5 –(8)]
28. On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of 4*m* when it is 6*m* away from the point of projection. Finally it reaches the ground 12*m* away from the starting point.
    Find the angle of projection. [Ex:5.5 –(9)]
29. Points *A* and *B* are 10*km* apart and it is determined from the sound of an explosion heard at those points at different times that the location of the explosion is 6 *km* closer to *A* than *B* . Show that the location of the explosion is restricted to a particular curve and find an equation of it. [Ex:5.5 –(10)]
30. A semielliptical archway over a one-way road has a height of 3*m* and a width of 12*m*. The truck has a width of 3*m* and a height of 2.7*m*. Will the truck clear the opening of the archway? [Eg:5.30]
31. The maximum and minimum distances of the Earth from the Sun respectively are 152 $\times 10^6$ km and 9.45$\times 10^6$ km. The Sun is at one focus of the elliptical orbit. Find the distance from the Sun to the other focus [Eg:5.31]
32. Two coast guard stations are located 600 km apart at points *A*(0,0)and *B*(0,600). A distress signal from a ship at *P* is received at slightly different times by two stations. It is determined that the ship is 200 km farther from station *A* than it is from station *B* . Determine the equation of hyperbola that passes through the location of the ship. [Eg:5.39]
33. Certain telescopes contain both parabolic mirror and a hyperbolic mirror. In the telescope the parabola and hyperbola share focus F_1 which is 14*m* above the vertex of the parabola. The hyperbola's second focus F_2 is 2*m* above the parabola's vertex. The vertex of the hyperbolic mirror is 1*m* below F_1 . Position a coordinate system with the origin at the centre of the hyperbola and with the foci on the *y* -axis. Then find the equation of the hyperbola. [Eg:5.40]

VI. APPLICATIONS OF VECTOR ALGEBRA

1. By vector method, prove that $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ [Eg:6.3]
2. Using vector method, prove that $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$ [Ex:6.1 –(9)]
3. Prove by vector method that $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$ [Eg:6.5]
4. Prove by vector method that $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ [Ex:6.1 –(10)]
5. Prove by vector method that the perpendiculars (attitudes) from the vertices to the opposite sides of a triangle are concurrent. [Eg:6.7]
6. If $\vec{a} = -2\hat{i}+3\hat{j}-2\hat{k}$, $\vec{b} = 3\hat{i}-\hat{j}+3\hat{k}$, $\vec{c} = 2\hat{i}-5\hat{j}+\hat{k}$. find $(\vec{a}\times\vec{b})\times\vec{c}$ and $\vec{a}\times(\vec{b}\times\vec{c})$ State whether they are equal. [Eg:6.22]
7. If $\vec{a} = \hat{i}-\hat{j}$, $\vec{b} = \hat{i}-\hat{j}-4\hat{k}$, $\vec{c} = 3\hat{j}-\hat{k}$ and $\vec{d} = 2\hat{i}+5\hat{j}+\hat{k}$, verify that $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d}) = [\vec{a},\vec{b},\vec{d}]\vec{c} - [\vec{a},\vec{b},\vec{c}]\vec{d}$ [Eg: 6.23-(i)]
8. If $\vec{a} = \hat{i}-\hat{j}$, $\vec{b} = \hat{i}-\hat{j}-4\hat{k}$, $\vec{c} = 3\hat{j}-\hat{k}$ and $\vec{d} = 2\hat{i}+5\hat{j}+\hat{k}$, verify that $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d}) = [\vec{a},\vec{c},\vec{d}]\vec{b} - [\vec{b},\vec{c},\vec{d}]\vec{a}$ [Eg: 6.23-(ii)]
9. If $\vec{a} = 2\hat{i}+3\hat{j}-\hat{k}$, $\vec{b} = 3\hat{i}+5\hat{j}+2\hat{k}$, $\vec{c} = -\hat{i}-2\hat{j}+3\hat{k}$, verify that $(\vec{a}\times\vec{b})\times\vec{c} = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{b}\cdot\vec{c})\vec{a}$ [Ex:6.3 –(4)-(i)]
10. If $\vec{a} = 2\hat{i}+3\hat{j}-\hat{k}$, $\vec{b} = 3\hat{i}+5\hat{j}+2\hat{k}$, $\vec{c} = -\hat{i}-2\hat{j}+3\hat{k}$, verify that $\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c}$ [Ex:6.3 –(4)-(ii)]
11. Find the vector equation in parametric form and Cartesian equations of a straight passing through the points (-5,7,-4) and (13,-5, 2). Find the point where the straight line crosses the *xy* -plane. [Eg:6.27]

12. Find the point of intersection of the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-4}{5}=\frac{y-1}{2}=z$ [Eg:6.33]
13. Find the parametric form of vector equation of a straight line passing through the point of intersection of the straight lines $\vec{r}=(\hat{i}+3\hat{j}-\hat{k})+t(2\hat{i}+3\hat{j}+2\hat{k})$ and $\frac{x-2}{1}=\frac{y-4}{2}=\frac{z+3}{4}$, and perpendicular to both straight lines. [Eg:6.34]
14. Show that the lines $\frac{x-3}{3}=\frac{y-3}{-1}$, z-1=0 and $\frac{x-6}{2}=\frac{z-1}{3}$, y-2=0 intersect. Also find the point of intersection. [Ex:6.5 –(4)]
15. Find the non-parametric form of vector equation, and Cartesian equation of the plane passing through the point (2,3,6) and parallel to the straight lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-3}{1}$ and $\frac{x+3}{2}=\frac{y-3}{-5}=\frac{z+1}{-3}$ [Ex:6.7 –(1)]
16. Find the parametric form of vector equation, and Cartesian equations of the plane passing through the points (2,2,1), (9,3,6) and perpendicular to the plane 2x+6y+6z=9 [Ex:6.7 –(2)]
17. Find parametric form of vector equation and Cartesian equations of the plane passing through the points (2, 2,1), (1,-2,3) and parallel to the straight line passing through the points (2,1,-3) and (-1,5,-8) [Ex:6.7 –(3)]
18. Find the non-parametric form of vector equation of the plane passing through the point (1,-2, 4) and perpendicular to the plane x+2y-3z=11 and parallel to the line $\frac{x+7}{3}=\frac{y+3}{-1}=\frac{z}{1}$ [Ex:6.7 –(4)]
19. Find the parametric form of vector equation, and Cartesian equations of the plane containing the line $\vec{r}=(\hat{i}-\hat{j}+3\hat{k})+t(2\hat{i}-\hat{j}+4\hat{k})$ and perpendicular to plane $\vec{r}\cdot(\hat{i}+2\hat{j}+\hat{k})=8$ [Ex:6.7 –(5)]
20. Find the parametric vector, non-parametric vector and Cartesian form of the equations of the plane passing through the points (3,6,-2), (-1,-2,6),and (6,-4,-2) [Ex:6.7 –(6)]
21. Find the non-parametric form of vector equation, and Cartesian equations of the plane $\vec{r}=(6\hat{i}-\hat{j}+\hat{k})+s(-\hat{i}+2\hat{j}+\hat{k})+t(-5\hat{i}-4\hat{j}-5\hat{k})$ [Ex:6.7 –(7)]
22. Find the non-parametric form of vector equation, and Cartesian equation of the plane passing through the point (0,1,-5) and parallel to the straight lines $\vec{r}=(\hat{i}+2\hat{j}-4\hat{k})+s(2\hat{i}+3\hat{j}+6\hat{k})$ and $\vec{r}=(\hat{i}-3\hat{j}+5\hat{k})+t(\hat{i}+\hat{j}-\hat{k})$ [Eg:6.43]
23. Find the vector parametric, vector non-parametric and Cartesian form of the equation of the plane passing through the points (-1,2,0), (2,2,-1) and parallel to the straight line $\frac{x-1}{1}=\frac{2y+1}{2}=\frac{z+1}{-1}$ [Eg:6.44]
24. Show that the straight lines $\vec{r}=(5\hat{i}+7\hat{j}-3\hat{k})+s(4\hat{i}+4\hat{j}-5\hat{k})$ and $\vec{r}=(8\hat{i}+4\hat{j}+5\hat{k})+t(7\hat{i}+\hat{j}+3\hat{k})$ are coplanar. Find the vector equation of the plane in which they lie. [Ex:6.8 –(1)]
25. Show that the lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{3}$ and $\frac{x-1}{-3}=\frac{y-4}{2}=\frac{z-5}{1}$ are coplanar.Also find the plane containing these lines.[Ex:6.8(2)
26. If the straight lines $\frac{x-1}{2}=\frac{y+1}{\lambda}=\frac{z}{2}$ and $\frac{x+1}{5}=\frac{y+1}{2}=\frac{z}{\lambda}$ are coplanar, find λ and equations of the planes containing these two lines. [Ex:6.8-(4)]
27. Show that the lines $\vec{r}=(-\hat{i}-3\hat{j}-5\hat{k})+s(3\hat{i}+5\hat{j}+7\hat{k})$ and $\vec{r}=(2\hat{i}+4\hat{j}+6\hat{k})+t(\hat{i}+4\hat{j}+7\hat{k})$ are coplanar. Also,find the non-parametric form of vector equation of the plane containing these lines. [Eg:6.46]
28. Find the equation of the plane passing through the line of intersection of the planes x+2y+3z=2 and x-y+z+11=3, and at a distance $\frac{2}{\sqrt{3}}$ from the point (3,1,-1) [Ex:6.9-(2)]
29. Find the coordinates of the foot of the perpendicular and length of the perpendicular from the point (4,3,2) to the plane x+2y+3z=2 [Ex:6.9-(8)]

XII. DISCRETE MATHEMATICS

1. Verify the (i) closure property (ii) commutative property (iii) associative property (iv) existence of identity and (v) existence of inverse for the arithmetic operation + on Z . [Eg:12.2]
2. Verify the (i) closure property, (ii) commutative property, (iii) associative property (iv) existence of identity and (v) existence of inverse for the arithmetic operation - on Z . [Eg:12.3]
3. Verify the (i) closure property, (ii) commutative property, (iii) associative property (iv) existence of identity and (v) existence of inverse for the arithmetic operation + on Z_e= the set of all even integers. [Eg:12.4]
4. Verify the (i) closure property, (ii) commutative property, (iii) associative property (iv) existence of identity and (v) existence of inverse for the arithmetic operation + on Z_0 = the set of all odd integers.
5. Verify (i) closure property, (ii) commutative property, (iii) associative property, (iv) existence of identity, and (v) existence of inverse for following operation on the given set. m∗n = m+n-mn. m,n ∈ Z [Eg:12.7]
6. Verify (i) closure property, (ii) commutative property, (iii) associative property, (iv) existence of identity, and (v) existence of inverse for the operation $+_5$ on Z_5 using table corresponding to addition modulo 5. [Eg:12.9]

7. Verify (i) closure property, (ii) commutative property, (iii) associative property, (iv) existence of identity, and (v) existence of inverse for the operation x_{11} on a subset *A* = {1,3,4,5,9} of the set of remainders {0,1,2,3,4,5,6,7,8,9,10}. [Eg:12.10]
8. Define an operation $*$ on Q as follows: $a*b = \frac{a+b}{2}$; a,b ∈ Q. Examine the closure, commutative, associative, existence of identity and the existence of inverse properties for the operation $*$ on Q. [Ex:12.1-(5)-(i&ii)]
9. Let M = $\left\{\begin{pmatrix} x & x \\ x & x \end{pmatrix} : x \in R - \{0\}\right\}$ and let $*$ be the matrix multiplication. Determine whether *M* is closed under $*$. If so, examine the commutative, associative, existence of identity, existence of inverse properties for the operation $*$ on *M* . [Ex:12.1-(9)-(i&ii)]
10. Let *A* be Q \ {1}. Define $*$ on *A* by $x*y = x + y - xy$. Is $*$ binary on *A* ? If so, examine the commutative ,associative , existence of identity, existence of inverse properties for the operation $*$ on *A* . [Ex:12.1-(10)-(i&ii)]
11. Using the equivalence property, show that $p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \vee \sim q)$. [Ex:12.19]

VII. APPLICATIONS OF DIFFERENTIAL CALCULUS

1. A camera is accidentally knocked off an edge of a cliff 400 ft high. The camera falls a distance of $s = 16t^2$ in *t* seconds. (i) How long does the camera fall before it hits the ground?(ii) What is the average velocity with which the camera falls during the last 2 seconds? (iii) What is the instantaneous velocity of the camera when it hits the ground? [Ex:7.1-(2)]
2. A particle moves along a line according to the law $s(t) = 2t^3 - 9t^2 + 12t - 4$, where $t \geq 0$.(i) At what times the particle changes direction? (ii) Find the total distance travelled by the particle in the first 4 seconds. (iii) Find the particle's acceleration each time the velocity is zero. [Ex:7.1-(3)]
3. A beacon makes one revolution every 10 seconds. It is located on a ship which is anchored 5 km from a straight shore line. How fast is the beam moving along the shore line when it makes an angle of 45° with the shore? [Ex:7.1-(7)]
4. A conical water tank with vertex down of 12 metres height has a radius of 5 metres at the top. If water flows into the tank at a rate 10 cubic m/min, how fast is the depth of the water increases when the water is 8 metres deep? [Ex:7.1-(8)]
5. A ladder 17 metre long is leaning against the wall. The base of the ladder is pulled away from the wall at a rate of 5 m/s. When the base of the ladder is 8 metres from the wall. (i) How fast is the top of the ladder moving down the wall? (ii) At what rate, the area of the triangle formed by the ladder, wall, and the floor, is changing? [Ex:7.1-(9)]
6. A police jeep, approaching an orthogonal intersection from the northern direction, is chasing a speeding car that has turned and moving straight east. When the jeep is 0.6 km north of the intersection and the car is 0.8 km to the east. The police determine with a radar that the distance between them and the car is increasing at 20 km/hr. If the jeep is moving at 60 km/hr at the instant of measurement, what is the speed of the car? [Ex:7.1-(10)]
7. A particle moves along a horizontal line such that its position at any time $t \geq 0$ is given by $s(t) = t^3 - 6t^2 + 9t + 1$, where *s* is measured in metres and *t* in seconds?(i) At what time the particle is at rest? (ii) At what time the particle changes direction? (iii) Find the total distance travelled by the particle in the first 2 seconds. [Eg:7.6]
8. If we blow air into a balloon of spherical shape at a rate of 1000 cm^3 per second. At what rate the radius of the baloon changes when the radius is 7cm? Also compute the rate at which the surface area changes. [Eg:7.7]
9. Salt is poured from a conveyer belt at a rate of 30 cubic metre per minute forming a conical pile with a circular base whose height and diameter of base are always equal. How fast is the height of the pile increasing when the pile is 10 metre high? [Eg:7.9]
10. A road running north to south crosses a road going east to west at the point *P* . Car *A* is driving north along the first road, and car *B* is driving east along the second road. At a particular time car *A* is 10 kilometres to the north of *P* and travelling at 80 km/hr, while car *B* is 15 kilometres to the east of *P* and travelling at 100 km/hr. How fast is the distance between the two cars changing? [Eg:7.10]
11. A garden is to be laid out in a rectangular area and protected by wire fence. What is the largest possible area of the fenced garden with 40 metres of wire. [Ex:7.8-(4)]
12. A rectangular page is to contain 24 cm^2 of print. The margins at the top and bottom of the page are 1.5 cm and the margins at other sides of the page is 1 cm. What should be the dimensions of the page so that the area of the paper used is minimum. [Ex:7.8-(5)]
13. A farmer plans to fence a rectangular pasture adjacent to a river. The pasture must contain 1,80,000 sq.mtrs in order to provide enough grass for herds. No fencing is needed along the river. What is the length of the minimum needed fencing material? [Ex:7.8-(6)]
14. Find the dimensions of the rectangle with maximum area that can be inscribed in a circle of radius 10 cm. [Ex:7.8-(7)]
15. Prove that among all the rectangles of the given perimeter, the square has the maximum area. [Ex:7.8-(8)]
16. Find the dimensions of the largest rectangle that can be inscribed in a semi circle of radius *r* cm. [Ex:7.8-(9)]

17. A manufacturer wants to design an open box having a square base and a surface area of 108 sq.cm. Determine the dimensions of the box for the maximum volume. [Ex:7.8-(10)]
18. The volume of a cylinder is given by the formula $V=\pi r^2 h$. Find the greatest and least values of V if $r+h=6$.[Ex:7.8-(11)]
19. A hollow cone with base radius a cm and height b cm is placed on a table. Show that the volume of the largest cylinder that can be hidden underneath is $\frac{4}{9}$ times volume of the cone. [Ex:7.8-(12)]
20. We have a 12 square unit piece of thin material and want to make an open box by cutting small squares from the corners of our material and folding the sides up. The question is, which cut produces the box of maximum volume? [Eg:7.62]
21. Find the points on the unit circle $x^2+y^2=1$ nearest and farthest from (1,1) . [Eg:7.63]
22. A steel plant is capable of producing x tonnes per day of a low-grade steel and y tonnes per day of a high-grade steel, where $y=\frac{40-5x}{10-x}$. If the fixed market price of low-grade steel is half that of high-grade steel, then what should be optimal productions in low-grade steel and high-grade steel in order to have maximum receipts. [Eg:7.64]
23. Prove that among all the rectangles of the given area square has the least perimeter. [Eg:7.65]

Find intervals of concavity and points of inflexion for the following functions:

24. $f(x)=x(x-4)^3$ [Ex:7.7-(1)-(i)]
25. $f(x)=\sin x+\cos x,\ 0<x<2\pi$ [Ex:7.7-(1)-(ii)]
26. $f(x)=\frac{1}{2}(e^x-e^{-x})$ [Ex:7.7-(1)-(iii)]
27. $f(x)=(x-1)^3(x-5),\ x\in R$ [Eg:7.57]
28. $y=3+\sin x$ [Eg:7.58]

Find the local extrema for the following functions using second derivative test :

29. $f(x)=-3x^5+5x^3$ [Ex:7.7-(2)-(i)]
30. $f(x)=x\log x$ [Ex:7.7-(2)-(ii)]
31. $f(x)=x^2e^{-2x}$ [Ex:7.7-(2)-(iii)]
32. $f(x)=4x^6-6x^4$ [Eg:7.60]
33. For the function $f(x)=4x^3+3x^2-6x+1$ find the intervals of monotonicity, local extrema, intervals of concavity and points of inflection. [Ex:7.7-(3)]
34. Find the local maximum and minimum of the function x^2y^2 on the line $x+y=10$. [Eg:7.61]

Find the intervals of monotonicities and hence find the local extremum for the following functions:

35. $f(x)=2x^3+3x^2-12x$ [Ex:7.6-(2)-(i)]
36. $f(x)=\frac{x^3}{3}-\log x$ [Ex:7.6-(2)-(iv)]
37. $f(x)=\sin x\cos x+5,\ x\in(0,2\pi)$ [Ex:7.6-(2)-(v)]
38. $f(x)=x^2-4x+4$ [Eg:7.50]
39. $f(x)=x\log x+3x$. [Eg:7.54]
40. $f(x)=\frac{1}{1+x^2}$ [Eg:7.55]
41. $f(x)=\frac{x}{1+x^2}$ [Eg:7.56]
42. Discuss the monotonicity and local extrema of the function $f(x)=\log(1+x)-\frac{x}{1+x}$, $x>-1$ and hence find the domain where, $\log(1+x)>\frac{x}{1+x}$ [Eg:7.53]

Find the tangent and normal to the following curves at the given points on the curve.

43. $y=x\sin x$ at $\left(\frac{\pi}{2},\frac{\pi}{2}\right)$ [Ex:7.2-(5)-(iii)]
44. $x=\cos t,\ y=2\sin^2 t$ at $t=\frac{\pi}{3}$ [Ex:7.2-(5)-(iv)]
45. Find the equations of the tangents to the curve $y=1+x^3$ for which the tangent is orthogonal with the line $x+12y=12$. [Ex:7.2-(6)]
46. Find the equations of the tangents to the curve $y=\frac{x+1}{x-1}$ which are parallel to the line $x+2y=6$. [Ex:7.2-(7)]
47. Find the equation of tangent and normal to the curve given by $x=7\cos t$ and $y=2\sin t$, $t\in R$ at any point on the curve. [Ex:7.2-(8)]
48. Find the angle between the rectangular hyperbola $xy=2$ and the parabola $x^2+4y=0$. [Ex:7.2-(9)]
49. Find the equation of the tangent and normal to the Lissajous curve given by $x=2\cos 3t$ and $y=3\sin 2t$, $t\in R$. [Eg:7.13]
50. Find the acute angle between $y=x^2$ and $y=(x-3)^2$. [Eg:7.14]
51. Find the acute angle between the curves $y=x^2$ and $x=y^2$ at their points of intersection (0,0)and (1,1) . [Eg:7.15]
52. If the curves $ax^2+by^2=1$ and $cx^2+dy^2=1$ intersect each other orthogonally then,show that $\frac{1}{a}-\frac{1}{b}=\frac{1}{c}-\frac{1}{d}$ [Eg:7.17]
53. Prove that the ellipse $x^2+4y^2=8$ and the hyperbola $x^2-2y^2=4$ intersect orthogonally. [Eg:7.18]
54. Evaluate : $\lim_{x\to\infty}(1+2x)^{\frac{1}{2\log x}}$ [Eg:7.44]
55. $\lim_{x\to\frac{\pi}{2}}(\sin x)^{\tan x}$ [Ex:7.5-(10)]
56. $\lim_{x\to 0^+}(\cos x)^{\frac{1}{x^2}}$ [Ex:7.5-(11)]