



Padalsalai's Telegram Groups!

(தலைப்பிற்கு கீழே உள்ள லிங்கை கிளிக் செய்து குழுவில் இணையவும்!)

- Padalsalai's NEWS - Group
https://t.me/joinchat/NIfCqVRBNj9hhV4wu6_NqA
- Padalsalai's Channel - Group
<https://t.me/padasalaichannel>
- Lesson Plan - Group
<https://t.me/joinchat/NIfCqVWwo5iL-21gpzrXLw>
- 12th Standard - Group
https://t.me/Padalsalai_12th
- 11th Standard - Group
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- 10th Standard - Group
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- 9th Standard - Group
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- 6th to 8th Standard - Group
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29/11/19.

chapter - 2.

2 - Integral calculus - I.

$$\int = \frac{1}{d}$$

$$d = \frac{1}{\int}$$

$$\left[\begin{array}{c} \text{In differentiation} \\ S \xrightarrow{(+)} C \\ \xleftarrow{(-)} \end{array} \right]$$

$$\text{definite } \int_a^b f(x) dx = [f(x)]_a^b$$

$$\left[\begin{array}{c} \text{In Integration} \\ C \xrightarrow{(+)} S \\ \xleftarrow{(-)} \end{array} \right]$$

$$= F(b) - F(a).$$

$$\text{Indefinite } \int f(x) dx = F(x) + c.$$

$$\Rightarrow \begin{array}{l} d(x^2+2) = 2x+0 \\ d(x^2) = 2x. \end{array}$$

$$\int 2x dx = \frac{2x^2}{2} = x^2 + c.$$

$$d(x^2-2) = 2x.$$

$$* \int k \cdot dx = kx.$$

$$* \int x dx = \frac{x^2}{2}$$

$$* \int x^n dx = \frac{x^{n+1}}{(n+1) \cdot 1} = \frac{x^{n+1}}{n+1}.$$

$$* \int e^x \cdot dx = \frac{e^x}{1}$$

$$* \int e^{ax} \cdot dx = \frac{e^{ax}}{a}$$

$$* \int \frac{1}{x} \cdot dx = \log x$$

$$* \int \sin x \cdot dx = -\cos x$$

$$* \int \cos x \cdot dx = \sin x.$$

$$* \int \frac{f'(x)}{f(x)} dx = \log(f(x))$$

$$\text{eg. } \int \frac{2x}{(x^2+3)} dx = \log(x^2+3)$$

$$* \int (f(x))^n \cdot f'(x) dx = \frac{(f(x))^{n+1}}{n+1}$$

$$\text{eg. } \int (x^2+3)^2 \cdot 2x \cdot dx = \frac{(x^2+3)^3}{3}$$

$$* \int \frac{f'(x)}{\sqrt{f(x)}} dx = 2\sqrt{f(x)} + c$$

$$\textcircled{1} \int \sqrt{3x+6} \, dx$$

$$= \int (3x+6)^{1/2} \cdot dx$$

$$= \frac{(3x+6)^{3/2}}{3/2 (3+0)} + C$$

$$= \boxed{\frac{2}{9} (3x+6)^{3/2} + C} //$$

$$\textcircled{2} \int \left(9x^2 - \frac{4}{x^2}\right)^2 dx$$

$$\textcircled{x} \boxed{(a-b)^2 = a^2 + b^2 - 2ab}$$

$$= \int \left[81x^4 + \frac{16}{x^4} - 72x^2 \cdot \frac{1}{x^2}\right] dx$$

$$= \int (81x^4 + 16x^{-4} - 72) dx$$

$$= \int \left[81 \frac{x^5}{5} + \frac{16x^{-3}}{-3} - 72x + C\right]$$

$$= \boxed{\frac{81}{5} x^5 - \frac{16}{3x^3} - 72x + C} //$$

$$\textcircled{3} \int (3+x)(2-5x) dx$$

$$= \int (6 - 15x + 2x - 5x^2) dx$$

$$= \boxed{6x - \frac{15x^2}{2} + \frac{2x^2}{2} - \frac{5x^3}{3} + C}$$

$$\textcircled{1} \int \sqrt{x} (x^3 - 2x + 3) dx$$

$$= \int x^{1/2} (x^3 - 2x + 3) dx$$

$$= \int (x^{7/2} - 2x^{3/2} + 3x^{1/2}) dx$$

$$= \int (x^{7/2} - 2x^{3/2} + 3x^{1/2}) dx$$

$$= \frac{x^{9/2}}{9/2} - \frac{2x^{5/2}}{5/2} + \frac{3x^{3/2}}{3/2} + C$$

$$= \boxed{\frac{2}{9} x^{9/2} - \frac{4}{5} x^{5/2} + 2x^{3/2} + C} //$$

$$\textcircled{2} \int \frac{8x+13}{\sqrt{4x+7}} dx$$

$$= \int \frac{(8x+14) - 1}{(4x+7)^{1/2}} \cdot dx$$

$$= \int \frac{(8x+14)}{(4x+7)^{1/2}} - \frac{1}{(4x+7)^{1/2}} \cdot dx$$

$$= \int \frac{2(4x+7)}{(4x+7)^{1/2}} - (4x+7)^{1/2} \cdot dx$$

$$= 2 \int (4x+7)^{3/2} \cdot dx - \int (4x+7)^{-1/2} \cdot dx$$

$$= \frac{2 (4x+7)^{5/2}}{\frac{5}{2} (4)^{3/2}} - \frac{(4x+7)^{1/2}}{\frac{1}{2} (4)^{1/2}}$$

$$= \boxed{\frac{(4x+7)^{5/2}}{3} - \frac{(4x+7)^{1/2}}{2} + C}$$

$$\textcircled{c} \int \frac{1}{\sqrt{x+1} \sqrt{x-1}} dx$$

$$= \int \frac{1}{\sqrt{x+1} + \sqrt{x-1}} \times \frac{\sqrt{x+1} - \sqrt{x-1}}{\sqrt{x+1} - \sqrt{x-1}} dx$$

$$= \int \frac{(x+1)^{1/2} - (x-1)^{1/2}}{(x+1) - (x-1)} dx \quad [(\sqrt{x})^2 = x]$$

$$= \frac{1}{2} \frac{(x+1)^{3/2}}{\frac{3}{2} (1)} - \frac{(x-1)^{3/2}}{\frac{3}{2} (1)} + C$$

$$= \boxed{\frac{(x+1)^{3/2}}{3} - \frac{(x-1)^{3/2}}{3} + C}$$

eg 2.1

$$\int \frac{ax^2+bx+c}{\sqrt{x}} dx$$

$$= \int \frac{ax^2+bx+c}{(x)^{1/2}} dx$$

$$= \int ax^2 \cdot x^{-1/2} + bx \cdot x^{-1/2} + cx^{-1/2} \cdot dx$$

$$= \int ax^{3/2} + bx^{1/2} + cx^{-1/2} \cdot dx$$

$$= a \int x^{3/2} \cdot dx + b \int x^{1/2} \cdot dx + c \int x^{-1/2} \cdot dx$$

$$= a \left[\frac{x^{5/2}}{5/2} \right] + b \left[\frac{x^{3/2}}{3/2} \right] + c \left[\frac{x^{1/2}}{1/2} \right] + C$$

$$= \boxed{\frac{2a}{5} x^{5/2} + \frac{2b}{3} x^{3/2} + 2c x^{1/2} + C}$$

eg 2.2

$$\int \sqrt{2x+1} \cdot dx$$

$$\int (2x+1)^{1/2} \cdot dx$$

$$= \frac{(2x+1)^{3/2}}{\frac{3}{2}} + C$$

$$= \boxed{\frac{(2x+1)^{3/2}}{3} + C}$$

eg 2.3

$$\int \frac{dx}{(2x+3)^2}$$

$$= \int (2x+3)^{-2}$$

$$= \frac{(2x+3)^{-1}}{2(-1)}$$

$$= \frac{(2x+3)^{-1}}{-2}$$

$$= \boxed{\frac{-1(2x+3)}{2} + C}$$

eg 2.4

$$\int \left(x + \frac{1}{x}\right)^2 \cdot dx$$

$$= \int \left(x^2 + \frac{1}{x^2} - 2x \cdot \frac{1}{x}\right)$$

$$= \int (x^2 + x^{-2} - 2)$$

$$= \frac{x^3}{3} + \frac{x^{-1}}{-1} + 2x + C$$

$$= \boxed{\frac{x^3}{3} - \frac{1}{x} + 2x + C}$$

eg 2.5

$$\int (x^2 + 7)(x - 4) \cdot dx$$

$$= \int (x^3 - 4x^2 + 7x - 28) \cdot dx$$

$$= \frac{x^4}{4} - \frac{4x^3}{3} + \frac{7x^2}{2} - 28x + C$$

$$= \boxed{\frac{x^4}{4} - \frac{4x^3}{3} + \frac{7x^2}{2} - 28x + C}$$

eg 2.6

$$\int \frac{2x^2 - 14x + 24}{x - 3} \cdot dx$$

$$= \int \frac{(x-3)(2x-8)}{(x-3)} \cdot dx$$

$$= \int (2x - 8) \cdot dx$$

$$= 2 \frac{x^2}{2} - 8x + C$$

$$= \boxed{x^2 - 8x + C}$$

$$\begin{array}{r} 148 \\ \times 2 \\ \hline 296 \\ \times 8 \\ \hline 1184 \\ \hline 1480 \\ \hline 14800 \\ \hline 148000 \\ \hline 1480000 \end{array}$$

Step method

d

$$d(x) = 1$$

$$d\left(\frac{1}{x}\right) = -\frac{1}{x^2}$$

$$d\left(\frac{1}{x^2}\right) = -\frac{2}{x^3}$$

$$d\left(\frac{2}{x^3}\right) = -\frac{6}{x^4}$$

$$d\left(\frac{8}{x^3}\right) = -\frac{24}{x^4}$$

I

$$\int \frac{1}{x} \cdot dx = \log x$$

$$\int \frac{1}{x^2} \cdot dx = -\frac{1}{(1)x^1}$$

$$\int \frac{2}{x^3} \cdot dx = \frac{-2}{x \cdot x^2} + C$$

$$\int e^{ax} \cdot dx = \frac{e^{ax}}{a}$$

$$\int a^x \cdot dx = \frac{a^x}{\log a}$$

$$\int a^{mx+n} \cdot dx = \frac{a^{mx+n}}{m \log a} \quad \left| \begin{array}{l} e^{\log} \\ \log \end{array} \right.$$

$$\log x^n = n \log x$$

$$\textcircled{P} \quad f'(x) = 8x^3 - 2x \quad f(2) = 8$$

$$\int f'(x) \cdot dx = \int (8x^3 - 2x) \cdot dx$$

$$f(x) = \frac{8x^4}{4} - \frac{2x^2}{2}$$

$$\boxed{f(x) = 2x^4 - x^2 + C} \rightarrow \textcircled{1}$$

$$f(2) = 2(2)^4 - (2)^2 + C$$

$$8 = 2(16) - 4 + C$$

$$8 = 32 - 4 + C$$

$$8 = 28 + C$$

$$C = -28 + 8$$

$$\boxed{C = -20}$$

$$\textcircled{1} \Rightarrow \boxed{2x^4 - x^2 - 20}$$

$$\textcircled{Q} \quad f'(x) = x + b, \quad f(1) = 5 \quad f(2) = 13$$

$$f'(x) = x + b$$

$$\int f'(x) \cdot dx = \int (x + b) \cdot dx$$

$$f(x) = \frac{x^2}{2} + bx + C \rightarrow \textcircled{1}$$

$$\boxed{f(1) = \frac{1}{2} + b + C}$$

$$\begin{aligned} ① \rightarrow 5 &= \frac{1}{2} + b + c \\ 5 - \frac{1}{2} &= b + c \\ \frac{9}{2} &= b + c \\ \boxed{9 = 2b + 2c} &\rightarrow ② \end{aligned}$$

$$\begin{aligned} ① \Rightarrow f(2) &= \frac{14}{2} + 2b + c \\ 13 &= 2 + 2b + c \\ \boxed{11 = 2b + c} &\rightarrow ③ \end{aligned}$$

$$\begin{aligned} ③ - ② \\ 11 &= 2b + c \\ 9 &= 2b + 2c \\ \hline 2 &= -c \\ \boxed{c = -2} \end{aligned}$$

$$3 \Rightarrow 11 = 2b - 2$$

$$2b = 13$$

$$\boxed{b = \frac{13}{2}}$$

$$\boxed{f(x) = \frac{x^2}{2} + \frac{13}{2}x - 2}$$

$$\int \frac{x+2}{\sqrt{2x+3}} dx$$

and $\div$ by 2

$$\begin{aligned} ② \text{ ③ } \int \frac{e^{2x} + e^{4x}}{e^{2x} + e^{-2x}} dx \\ = \int \frac{e^{2x}(1 + e^{4x})}{e^{2x} + \frac{1}{e^{2x}}} dx \\ = \int \frac{e^{2x}(1 + e^{4x})}{\frac{e^{4x} + 1}{e^{2x}}} dx \\ = \int e^{4x} dx = \frac{e^{4x}}{4} + c \end{aligned}$$

$$\begin{aligned} ⑥ \int (1 - \frac{1}{x^2}) e^{(x + \frac{1}{x})} dx \\ = \int d e^{(x + \frac{1}{x})} \end{aligned}$$

$$= \boxed{e^{(x + \frac{1}{x})} + c}$$

$$⑦ \int \frac{i}{x (\log x)^2} dx$$

$$t = \log x$$

$$dt = \frac{1}{x} dx$$

$$= \int \frac{1}{t^2} dt$$

$$= -\frac{1}{t} + c$$

$$= \boxed{-\frac{1}{\log x} + c}$$

$$\textcircled{x} a e^{ax} dx = \int d(e^{ax})$$

$$\textcircled{8} f'(x) = e^x$$

$$\int f'(x) dx = \int e^x dx$$

$$f(x) = e^x + c$$

$$f(0) = e^0 + c$$

$$2 = 1 + c$$

$$\boxed{c = 1}$$

$$\boxed{f(x) = e^x + 1}$$

$$\Rightarrow \int \frac{1}{x} \cdot dx = \log |x|$$

Type - I:

$$\frac{ax+b}{(x+a)(x+b)} = \frac{A}{(x+a)} + \frac{B}{(x+b)}$$

(diff)

Type - II:

$$\frac{ax+b}{(x+a)^2} = \frac{A}{(x+a)} + \frac{B}{(x+a)^2}$$

(same)

Type - III:

$$\frac{ax+b}{ax^2+bx+c} = \frac{Ax+B}{ax^2+bx+c}$$

(m. factor)

$$\textcircled{1} \int \left(\sqrt{2x} - \frac{1}{\sqrt{2x}} \right)^2 dx$$

$\frac{5x-2}{2}$

$$= \int \left(2x + \frac{1}{2x} - 2 \right) dx$$

$$= \left[\frac{2x^2}{2} + \frac{1}{2} \log |x| - 2x + C \right]$$

$$\textcircled{2} \int \frac{x^4 - x^2 + 2}{x-1} \cdot dx$$

$$= \int \frac{x^2(x^2-1)+2}{x-1} \cdot dx$$

$$= \int \frac{(x^2-1)(x+1) + 2}{x-1} + \frac{2}{(x-1)}$$

$$= \int \frac{(x^3+x^2) + 2}{(x-1)} dx$$

$$= \left[\frac{x^4}{4} + \frac{x^3}{3} + 2 \log |x-1| + C \right]$$

$$\textcircled{2} \int \frac{x^3}{x+2} \cdot dx$$

$$= \int \frac{x^3+2^3-2^3}{(x+2)} \cdot dx$$

$$= \int \frac{(x^3+2^3) - 8}{(x+2)} \cdot dx$$

$$= \frac{(x^3+2^3)}{(x+2)} - \frac{8}{(x+2)} \cdot dx$$

$$[\because a^3+b^3=(a+b)(a^2-ab+b^2)]$$

$$= \int \frac{(x+2)(x^2-2x+4) - 8}{(x+2)} \cdot dx$$

$$= \left[\frac{x^3}{3} + \frac{2x^2}{2} + 4x - 8 \log |x+2| + C \right]$$

$$\begin{array}{r} x^2 - 2x + 4 \\ \times \quad x+2 \\ \hline x^3 + 2x^2 - 2x^2 - 4x + 4x + 8 \\ \hline x^3 + 2x^2 - 4x + 8 \end{array}$$

$$⑧ \quad f'(x) = \frac{1}{x} \quad f(1) = \frac{11}{4}$$

$$\int f'(x) dx = \int \frac{1}{x} dx$$

$$f(x) = \log|x| + c$$

$$f(1) = \log(1) + c$$

$$\frac{11}{4} = 0 + c$$

$$\boxed{c = \frac{11}{4}}$$

$$\boxed{f(x) = \log|x| + \frac{11}{4}}$$

$$⑨ \quad \int \frac{3x+2}{(x-2)(x-3)} dx$$

$$\frac{3x+2}{(x-2)(x-3)} = \frac{A}{(x-2)} + \frac{B}{(x-3)}$$

$$\frac{3x+2}{(x-2)(x-3)} = \frac{A(x-3) + B(x-2)}{(x-2)(x-3)}$$

$$(3x+2) = A(x-3) + B(x-2)$$

$$\underline{x=3}$$

$$11 = 0 + B(1)$$

$$\boxed{B=11}$$

$$\underline{x=2}$$

$$8 = A(-1) + 0$$

$$\boxed{A=-8}$$

$$\frac{3x+2}{(x-2)(x-3)} = \frac{-8}{x-2} + \frac{11}{(x-3)}$$

$$\int \frac{3x+2}{(x-2)(x-3)} = \int \frac{-8}{(x-2)} + \frac{11}{(x-3)} dx$$

$$= \boxed{-8 \log|x-2| + 11 \log|x-3| + c}$$

$$④ \quad \frac{x^3 + 3x^2 - 7x + 11}{x+5}$$

$$\begin{array}{r} x^2 - 2x + 3 \\ x+5 \overline{) x^3 + 3x^2 - 7x + 11} \\ \underline{x^3 + 5x^2} \\ -2x^2 - 7x \\ \underline{-2x^2 - 10x} \\ 3x + 11 \\ \underline{3x + 15} \\ -4 \end{array}$$

$$= \int x^2 - 2x + 3 - \frac{4}{x+5}$$

$$= \frac{x^3}{3} - \frac{2x^2}{2} + 3x - 4 \log|x+5| + c$$

$$= \frac{x^3}{3} - x^2 + 3x - 4 \log|x+5| + c$$

9/12/19.

$$(6) \frac{4x^2 + 2x + 6}{(x+1)^2(x-3)}$$

$$\frac{4x^2 + 2x + 6}{(x+1)^2(x-3)} = \frac{A}{(x-3)} + \frac{B}{(x+1)} + \frac{C}{(x+1)^2}$$

$$\frac{4x^2 + 2x + 6}{(x+1)^2(x-3)} = \frac{A(x+1)^2 + B(x+1)(x-3) + C(x-3)}{(x+1)^2(x-3)}$$

$$4x^2 + 2x + 6 = A(x+1)^2 + B(x+1)(x-3) + C(x-3)$$

$$\text{Put } x = -1$$

$$4(-1)^2 + 2(-1) + 6 = 0 + 0 + (-4)$$

$$4 - 2 + 6 = -4C$$

$$8 = -4C$$

$$C = -2$$

$$\text{Put } x = 3$$

$$4(3)^2 + 2(3) + 6 = A(4)^2 + 0 + 0$$

$$36 + 6 + 6 = A(16)$$

$$48 = A(16)$$

$$A = 3$$

equating x^2 .

$$4 = A + B$$

$$4 = 3 + B$$

$$B = 1$$

$$\therefore \frac{4x^2 + 2x + 6}{(x+1)^2(x-3)} = \frac{3}{x-3} + \frac{1}{(x+1)} - \frac{2}{(x+1)^2}$$

$$\int \frac{4x^2 + 2x + 6}{(x+1)^2(x-3)} \cdot dx = \int \frac{3}{x-3} \cdot dx + \int \frac{1}{(x+1)} \cdot dx - \int \frac{2}{(x+1)^2} \cdot dx$$

$$= \left[3 \log|x-3| + \log|x+1| + \frac{2}{(x+1)} \right] + C$$

$$(7) \frac{3x^2 - 2x + 5}{(x-1)(x^2+5)} = \frac{A}{(x-1)} + \frac{Bx+C}{x^2+5}$$

$$\frac{3x^2 - 2x + 5}{(x-1)(x^2+5)} = \frac{A(x^2+5) + (Bx+C)(x-1)}{(x-1)(x^2+5)}$$

$$3x^2 - 2x + 5 = A(x^2+5) + (Bx+C)(x-1)$$

$$\text{Put } x = 1$$

$$3(1)^2 - 2(1) + 5 = A(1^2+5) + 0 + 0$$

$$3 - 2 + 5 = 6A$$

$$6 = 6A$$

$$A = 1$$

Equate x .

$$-2 = -B + C$$

$$-2 = -2 + C$$

$$C = 0$$

$$\frac{3x^2 - 2x + 5}{(x-1)(x^2+5)} = \frac{1}{(x-1)} + \frac{2x+0}{x^2+5}$$

$$\int \frac{3x^2 - 2x + 5}{(x-1)(x^2+5)} \cdot dx = \int \frac{1}{(x-1)} \cdot dx + \int \frac{2x}{x^2+5} \cdot dx$$

$$= \left[\log|x-1| + \log|x^2+5| + C \right]$$

Formula:

$$(i) \int \sin x \, dx = -\cos x + c \quad (ii) \int \cos x \, dx = \sin x + c$$

$$(iii) \int \sec^2 x \, dx = \tan x + c \quad (iv) \int \csc^2 x \, dx = -\cot x + c$$

$$(v) \int \sin(ax+b) \, dx = -\frac{1}{a} \cos(ax+b) + c$$

$$(vi) \int \cos(ax+b) \, dx = \frac{1}{a} \sin(ax+b) + c$$

$$(vii) \int \sec^2(ax+b) \, dx = \frac{1}{a} \tan(ax+b) + c$$

$$(viii) \int \csc^2(ax+b) \, dx = -\frac{1}{a} \cot(ax+b) + c$$

11/12/19

ex 2.11

$$(1) \int (2 \cos x - 3 \sin x + 4 \sec^2 x - 5 \csc^2 x) \, dx$$

$$= \boxed{2 \sin x + 3 \cos x + 4 \tan x + 5 \cot x + c}$$

$$(2) \int \sin^3 x \, dx$$

$$= \int \frac{1}{4} (3 \sin x - \sin 3x) \, dx$$

$$= \boxed{\frac{1}{4} \left[-3 \cos x + \frac{\cos 3x}{3} \right] + c}$$

$$\sin^3 A = 3 \sin A - 4 \sin^3 A$$

$$\cos^3 A = 4 \cos^3 A - 3 \cos A$$

$$\sin^3 A = \frac{1}{4} [3 \sin A - \sin 3A]$$

$$(3) \int \frac{\cos 2x + 2 \sin^2 x}{\cos^2 x} \, dx$$

$$= \int \frac{\cos 2x + 2 \left(\frac{1 - \cos 2x}{2} \right)}{\cos^2 x} \, dx$$

$$= \int \frac{1}{\cos^2 x} \, dx$$

$$= \boxed{\int \sec^2 x \, dx = \tan x + c}$$

$$\sin^2 A = \frac{1 - \cos 2A}{2}$$

$$\cos^2 A = \cos^2 A - \sin^2 A$$

$$\frac{1}{\cos^2 x} = \sec^2 x$$

$$(4) \int \frac{1}{\sin^2 x \cos^2 x} \, dx$$

$$= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} \, dx$$

$$= \int \left(\frac{\sin^2 x}{\sin^2 x \cos^2 x} + \frac{\cos^2 x}{\sin^2 x \cos^2 x} \right) \cdot dx$$

$$= \int (\sec^2 x + \operatorname{cosec}^2 x) \cdot dx$$

$$= \boxed{\tan x - \cot x + c} //$$

$$\frac{1}{\cos^2 x} = \sec^2 x$$

$$\frac{1}{\sin^2 x} = \operatorname{cosec}^2 x$$

$$\textcircled{5}. \int \sqrt{1 - \sin 2x} \cdot dx$$

$$= \int \sqrt{(\cos^2 x + \sin^2 x - 2 \cos x \sin x)} \cdot dx$$

$$= \int \sqrt{(\cos x - \sin x)^2} \cdot dx$$

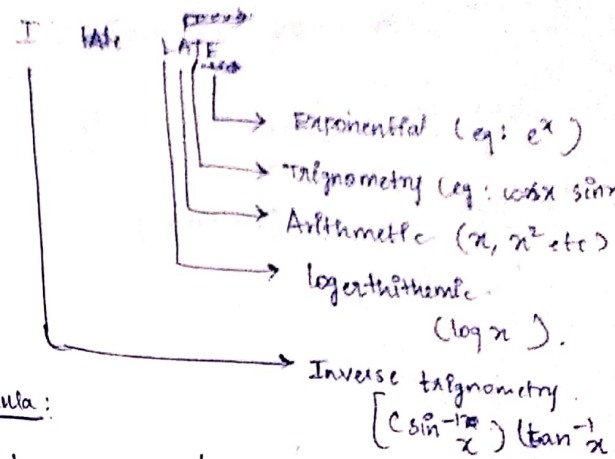
$$= \int (\cos x - \sin x) \cdot dx$$

$$= \boxed{\sin x + \cos x + c} //$$

$$1 = \cos^2 x + \sin^2 x$$

$$\sin 2A = 2 \cos A \sin A$$

Integration by parts :



Formula:

$$\int u dv = uv - \int v du$$

Selection of u by "I late":

eg: $\int x e^x \cdot dx$

$\downarrow$ $\downarrow$

A. e

$$\boxed{u = x} \quad \boxed{dv = e^x \cdot dx}$$

u = differentiate. $[u', u'', u''']$

dv = Integrate: $[v, v_1, v_2, v_3]$

Notes

IBT $\rightarrow \int u dv = uv - \int v du$

BT $\rightarrow \int u dv = uv - u'v_1 + u''v_2 - \dots$

$\int \frac{\log x \cdot dx}{x}$ $\int \frac{\tan x (\sin^{-1} x) \cdot dx}{x}$

$u = \log x$

$u = \tan x$

$\int \frac{x e^{\log x} \cdot dx}{x}$

$u = x$

En : 2.5

① $\int x e^{-x} \cdot dx$

$I = \int \frac{x e^{-x}}{x} \cdot dx$

$u = x$
 $u' = 1$
 $u'' = 0$

$dv = e^{-x} \cdot dx$
 $\int dv = \int e^{-x} \cdot dx$
 $v = \frac{e^{-x}}{-1} = -e^{-x}$
 $v_1 = e^{-x}$

$\int u dv = uv - u'v_1 + u''v_2 - \dots$

$I = -x e^{-x} - 1 e^{-x} + c$

$I = -e^{-x} [x+1] + c$

② $\int \frac{x^3 e^{3x}}{x} \cdot dx$

$u = x^3$
 $u' = 3x^2$
 $u'' = 6x$
 $u''' = 6$
 $u^{(4)} = 0$

$I = \int x^3 e^{3x} \cdot dx$

$\int u dv = uv - u'v_1 + u''v_2 - u'''v_3 + \dots$

$I = \frac{x^3 e^{3x}}{3} - 3x^2 \frac{e^{3x}}{9} + \frac{6x e^{3x}}{27} - \frac{6 e^{3x}}{81} + c$

$= \frac{e^{3x}}{3} \left[x^3 - x^2 + \frac{2x}{3} - \frac{2}{9} \right] + c$

③ $\int \log x \cdot dx$

$I = \log x \cdot dx$

$u = \log x$
 $u' = \frac{1}{x}$

$\int u dv = uv - \int v du$

$I = \log x (x) - \int x \frac{1}{x} dx$
 $= [x \log x - x + c]$

④ $\int x \log x$

$I = \int x \log x \cdot dx$

$u = \log x$
 $u' = \frac{1}{x}$

$dv = x \cdot dx$
 $\int dv = \int x \cdot dx$

$v = \frac{x^2}{2}$

$dv = e^{3x} \cdot dx$
 $\int dv = \int e^{3x} \cdot dx$
 $v = \frac{e^{3x}}{3}$

$v_1 = \frac{e^{3x}}{9}$

$v_2 = \frac{e^{3x}}{27}$

$v_3 = \frac{e^{3x}}{81}$

$dv = dx$
 $\int dv = \int dx$
 $v = x$

$$\int u dv = uv - \int v du$$

$$I = \log x \cdot \frac{x^2}{2} - \int \frac{x^2}{2} \cdot \frac{1}{x} dx$$

$$= \frac{x^2}{2} \log x - \frac{1}{2} \left(\frac{x^2}{2} \right) + C$$

$$= \boxed{\frac{x^2}{2} \log x - \frac{x^2}{4} + C}$$

$$\textcircled{5}. \int x^n \cdot \log x \cdot dx$$

$$I = \int x^n \log x \cdot dx$$

$$\begin{cases} u = \log x \\ u' = \frac{1}{x} \end{cases}$$

$$\begin{cases} dv = x^n \cdot dx \\ \int dv = \int x^n \cdot dx \\ v = \frac{x^{n+1}}{n+1} \end{cases}$$

$$\int u dv = uv - \int v du$$

$$I = \log x \left(\frac{x^{n+1}}{n+1} \right) - \int \frac{x^{n+1}}{n+1} \times \frac{1}{x} \cdot dx$$

$$= \left(\frac{x^{n+1}}{n+1} \right) \log x - \frac{x^{n+1}}{(n+1)(n+1)} + C$$

$$\textcircled{6} \int x^5 e^{x^2} \cdot dx$$

$$I = \int x^5 e^{x^2} \cdot dx$$

$$= \int x^4 \cdot x e^{x^2} dx = \int (x^2)^2 (x e^{x^2}) \cdot dx$$

$$\text{Put } t = x^2$$

$$dt = 2x \cdot dx$$

$$\boxed{\frac{dt}{2} = x \cdot dx}$$

$$I = \int t^2 e^t \frac{dt}{2}$$

$$I = \frac{1}{2} \int t^2 e^t dt$$

By BT

$$\int u dv = uv - u v_1 + u^2 v_2$$

$$I = \frac{1}{2} [t^2 e^t - 2t e^t + 2 e^t] + C$$

$$= \frac{e^t}{2} [t^2 - 2t + 2] + C$$

$$= \frac{e^{x^2}}{2} [x^4 - 2x^2 + 2] + C //$$

$$\begin{array}{l|l} u = t^2 & dv = e^t dt \\ u' = 2t & \int dv = \int e^t dt \\ u'' = 2 & v = e^t \\ u''' = 0 & v_1 = e^t \\ & v_2 = e^t \end{array}$$

eg: 2.98

$$\int (\log x)^2 dx$$

$$I = \int (\log x)^2 \cdot dx$$

By IBP:

$$\int u dv = uv - \int v du$$

$$I = (\log x)^2 (x) - \int x \times 2 \log x \times \frac{1}{x} \cdot dx$$

$$= x (\log x)^2 - 2 \int \log x \cdot dx \quad \text{--- (1)}$$

$$I_1 = \int \log x \cdot dx$$

By IBP:

$$\int u dv = uv - \int v du$$

$$I_1 = (\log x)(x) - \int x \times \frac{1}{x} \cdot dx$$

$$= x \log x - x$$

① $\Rightarrow$

$$I = x (\log x)^2 - 2(x \log x - x) + C$$

$$= x (\log x)^2 - 2 \log x + 2] + C$$

$$= x ((\log x)^2 - \log x^2 + 2) + C \quad \left(\because x \log x = \log x^n \right)$$

$$u = (\log x)^2$$

$$u' = 2 \log x \times \frac{1}{x}$$

$$dv = dx$$

$$\int dv = \int dx$$

$$v = x$$

$$u = \log x$$

$$u' = \frac{1}{x}$$

$$\int dv = \int dx$$

$$v = x$$

Formula :

Type - VI

$$1. \int [f(x)]^n f'(x) dx = \frac{[f(x)]^{n+1}}{n+1} + C, n \neq -1$$

$$2. \int \frac{f'(x)}{f(x)} \cdot dx = \log |f(x)| + C$$

$$3. \int \frac{f'(x)}{\sqrt{f(x)}} \cdot dx = 2\sqrt{f(x)} + C$$

$$4. \int e^x [f(x) + f'(x)] dx = e^x f(x) + C$$

$$5. \int e^{ax} [a f(x) + f'(x)] dx = e^{ax} f(x) + C$$

Ex. 2.6

$$① \int \frac{2x+5}{x^2+5x-7} \cdot dx$$

$$d(x^2+5x-7) = 2x+5$$

$$= \boxed{\log |x^2+5x-7| + C} //$$

$$② \int \frac{e^{3 \log x}}{x^4+1} \cdot dx$$

$$= \int \frac{e^{\log x^3}}{x^4+1} \cdot dx$$

$$= \frac{1}{4} \int \frac{4x^3 dx}{x^4+1} \quad [d(x^4+1) = 4x^3 + 0]$$

$$= \boxed{\frac{1}{4} \log |x^4+1| + C}$$

$$③ \int \frac{e^{2x}}{e^{2x}-2} \cdot dx$$

$$d(e^{2x}-2) = 2e^{2x}$$

$$= \frac{1}{2} \int \frac{2e^{2x}}{e^{2x}-2} \cdot dx$$

$$= \boxed{\frac{1}{2} \log |e^{2x}-2| + C} //$$

$$\textcircled{A} \int \frac{(\log x)^3}{x} \cdot dx.$$

$$= \int (\log x)^3 \cdot \frac{1}{x} \cdot dx$$

$$\Rightarrow \left[f(x)^n \cdot f'(x) \cdot dx = \frac{(f(x))^{n+1}}{n+1} \right]$$

$$= \frac{(\log x)^4}{4} + C$$

$$\textcircled{5} \int \frac{6x+7}{\sqrt{3x^2+7x-1}} \cdot dx.$$

$$d(3x^2+7x-1) = 6x+7$$

$$\int \frac{f'(x)}{\sqrt{f(x)}} \cdot dx = 2\sqrt{f(x)}$$

$$= 2\sqrt{3x^2+7x-1} + C$$

$$6. (4x+2) \sqrt{x^2+x+1} \cdot dx.$$

$$d(x^2+x+1) = (2x+1)$$

$$= 2 \int (2x+1) (x^2+x+1)^{1/2} dx.$$

$$\int f'(x) f(x)^n \cdot dx = \frac{f(x)^{n+1}}{n+1}$$

$$= 2 \frac{(x^2+x+1)^{3/2}}{3/2}$$

$$= \frac{4}{3} (x^2+x+1)^{3/2}$$

$$\textcircled{7} x^8 (1+x^9)^5.$$

$$d(1+x^9) = 0+9x^8$$

$$f'(x) + f(x)^n \cdot dx = \frac{f(x)^{n+1}}{n+1}$$

$$= \frac{1}{9} \int 9x^8 (1+x^9)^5 \cdot dx$$

$$= \frac{1}{9} \frac{(1+x^9)^6}{6} + C$$

$$= \left[\frac{(1+x^9)^6}{54} + C \right]$$

$$\textcircled{8} \int \frac{x^{e-1} + e^{x-1}}{x^e + e^x} \cdot dx.$$

$$x^n = nx^{n-1}$$

$$d(\ln e + e^x) = e x^{e-1} + e^x$$

$$= e \left[x^{e-1} + \frac{e^x}{e} \right]$$

$$= e \left[x^{e-1} + e^x \cdot e^{-1} \right]$$

$$= e \left[x^{e-1} + e^{x-1} \right]$$

$$= \frac{1}{e} \int \frac{e^{(n^e-1)} + e^{n-1}}{x^e + e^n} \cdot dn$$

$$= \frac{1}{e} \log |x^e + e^n| + c //$$

$$⑦. \int \frac{1}{x \log x} \cdot dx$$

$$\text{Put } t = \log x$$

$$\boxed{dt = \frac{1}{x} \cdot dx}$$

$$= \int \frac{1}{t} \cdot dt \quad \left| \int \frac{1}{x} \cdot dx = \log x \right.$$

$$= \log |t| + c$$

$$= \boxed{\log |\log x| + c} //$$

$$⑩. I = \int \frac{x}{2x^4 - 3x^2 - 2} \cdot dx = \int \frac{x \cdot dx}{2(x^2)^2 - 3(x^2) - 2}$$

$$\text{Put } t = x^2$$

$$dt = 2x \cdot dx$$

$$\boxed{\frac{dt}{2} = x \cdot dx}$$

$$= \int \frac{dt/2}{2t^2 - 3t - 2} = \frac{1}{2} \int \frac{dt}{2t^2 - 3t - 2}$$

$$\begin{array}{c} -4 \\ -4 \\ -3 \end{array}$$

$$= \int \frac{dt}{(t-2)(2t+1)} \rightarrow \textcircled{1}$$

$$\frac{1}{(t-2)(2t+1)} = \frac{A}{t-2} + \frac{B}{2t+1}$$

$$\frac{1}{(t-2)(2t+1)} = \frac{A(2t+1) + B(t-2)}{(t-2)(2t+1)}$$

$$1 = A(2t+1) + B(t-2)$$

$$\text{Put } \boxed{t=2}$$

$$1 = 5A + 0$$

$$\boxed{A = \frac{1}{5}}$$

$$t = -\frac{1}{2}$$

$$1 = 0 + B(-\frac{1}{2} - 2)$$

$$\boxed{B = -\frac{2}{5}}$$

$$\frac{1}{(t-2)(2t+1)} = \frac{1/5}{t-2} - \frac{2/5}{2t+1}$$

$$\textcircled{1} \Rightarrow I = \frac{1}{5} \int \left(\frac{1}{t-2} - \frac{2}{2t+1} \right) \cdot dt$$

$$d(t-2) = 1$$

$$d(2t+1) = 2$$

$$= \frac{1}{5} \left[\log |t-2| - \log |2t+1| \right] + c$$

$$= \frac{1}{5} (\log |x^2 - 2| - \log |2x^2 + 1|) + C$$

$$= \frac{1}{5} \left(\log \left| \frac{x^2 - 2}{2x^2 + 1} \right| \right) + C$$

$$(11) I = \int e^x (1+x) \log(xe^x) \cdot dx$$

$$\text{Put } \boxed{t = xe^x}$$

$$[d(uv) = uv' + vu']$$

$$dt = x d(e^x) + e^x(dx)$$

$$dt = [xe^x + e^x(1)] \cdot dx$$

$$= e^x(x+1) dx$$

$$I = \int \log t \cdot dt$$

$$\int \log x \cdot dx = x \log x - x + C$$

$$= t \log t - t + C$$

$$= \boxed{(xe^x) \log(xe^x) - (xe^x) + C}$$

Notes:



$$\frac{1}{x(x^2+1)} = \frac{1}{x} - \frac{x}{x^2+1}$$

$$\frac{1}{x(x+1)} = \frac{1}{x} - \frac{1}{x+1}$$

$$\frac{x}{(x+1)^2} = \frac{1}{(x+1)} - \frac{1}{(x+1)^2}$$

$$(12) I = \int \frac{1}{x(x^2+1)} \cdot dx$$

$$\frac{1}{x(x^2+1)} = \frac{1}{x} - \frac{x}{(x^2+1)}$$

$$= \int \left(\frac{1}{x} - \frac{x}{x^2+1} \right) \cdot dx$$

$$d(x^2+1) = 2x \cdot dx$$

$$= \int \left(\frac{1}{x} - \frac{1}{2} \frac{2x}{x^2+1} \right) \cdot dx$$

$$= \boxed{\log|x| - \frac{1}{2} \log|x^2+1| + C} //$$

$$(13) \quad I = \int e^x \left[\frac{1}{x^2} - \frac{2}{x^3} \right] \cdot dx \quad \left| d\left(\frac{1}{x^2}\right) = -\frac{2}{x^3} \right.$$

$$= \int e^x (f(x) + f'(x)) dx$$

$$= e^x f(x) + C$$

$$= \boxed{e^x \frac{1}{x^2} + C} //$$

$$(14) \quad I = \int e^x \left[\frac{x-1}{(x+1)^3} \right] dx$$

$$= \int e^x \left[\frac{x-1+1-1}{(x+1)^3} \right] \cdot dx$$

$$= \int e^x \left[\frac{(x+1)-2}{(x+1)^3} \right] \cdot dx$$

$$= \int e^x \left[\frac{(x+1)}{(x+1)^3} - \frac{2}{(x+1)^3} \right] \cdot dx$$

$$= \int e^x \left[\frac{1}{(x+1)^2} - \frac{2}{(x+1)^3} \right]$$

$$= \int e^x [f(x) + f'(x)] \cdot dx$$

$$= e^x f(x) + C$$

$$= e^x \frac{1}{(x+1)^2} + C$$

$$(15) \quad I = \int e^{3x} \left[\frac{3x-1}{9x^2} \right] dx$$

$$= \frac{1}{9} \int e^{3x} \left[\frac{3x}{x^2} - \frac{1}{x^2} \right] \cdot dx$$

$$= \frac{1}{9} \int e^{3x} \left[\frac{3}{x} - \frac{1}{x^2} \right] \cdot dx \quad \left| d\left(\frac{1}{x}\right) = -\frac{1}{x^2} \right|$$

$$= \frac{1}{9} \int e^{ax} [a f(x) + f'(x)] dx$$

$$= \frac{1}{9} \times e^{ax} f(x) + C$$

$$= \frac{1}{9} e^{3x} \frac{1}{x} + C$$

$$= \boxed{\frac{e^{3x}}{9x} + C} //$$

eg 2:33

$$I = \int \frac{x^3}{(x^2+1)^3} \cdot dx$$

$$\text{Put } t = x^2 + 1 \Rightarrow \boxed{x^2 = t - 1}$$

$$dt = 2x \cdot dx \Rightarrow \frac{dt}{2} = x dx.$$

$$I = \int \frac{x^2 \cdot x dx}{(x^2 + 1)^3}$$

$$= \int \frac{t-1}{t^3} \cdot \frac{dt}{2}$$

$$= \frac{1}{2} \int \left(\frac{t}{t^3} - \frac{1}{t^3} \right) dt$$

$$= \frac{1}{2} \int \left(\frac{1}{t^2} - \frac{1}{t^3} \right) dt$$

$$= \frac{1}{2} \left[\frac{-1}{t^2} + \frac{1}{2t^2} \right] + c$$

$$= \frac{1}{2} \left[\frac{-1}{t} + \frac{1}{2t^2} \right] + c$$

$$= \frac{1}{2} \left[\frac{-1}{(x^2+1)} + \frac{1}{2(x^2+1)^2} \right] + c$$

eg 2.34

$$I = \int \frac{dx}{x(x^3+1)} \times \frac{x^2}{x^2}$$

$$= \int \frac{x^2 \cdot dx}{x^3(x^3+1)}$$

$$\begin{aligned} \text{Put } t &= x^3 \\ dt &= 3x^2 \cdot dx \\ \frac{dt}{3} &= x^2 \cdot dx \end{aligned}$$

$$= \int \frac{dt/3}{t(t+1)} \quad \left| \frac{1}{x(x+1)} = \frac{1}{x} - \frac{1}{x+1} \right|$$

$$= \frac{1}{3} \int \frac{dt}{t(t+1)}$$

$$= \frac{1}{3} \left[\frac{1}{t} - \frac{1}{t+1} \right] \cdot dx$$

$$= \frac{1}{3} [\log |t| - \log |t+1|] + c$$

$$= \frac{1}{3} [\log |x^3| - \log |x^3+1|] + c$$

$$= \boxed{\frac{1}{3} \log \left| \frac{x^3}{x^3+1} \right| + c}$$

eg: 2.37

$$I = \int \frac{x e^x}{(1+x)^2} \cdot dx \quad \left| \frac{x}{(1+x)^2} = \frac{1}{(1+x)} - \frac{1}{(1+x)^2} \right|$$

$$= \int e^x \left[\frac{1}{(1+x)} - \frac{1}{(1+x)^2} \right] \cdot dx$$

$$= \int e^x (f(x) + f'(x)) \cdot dx$$

$$= e^x f(x) + c$$

$$= \boxed{e^x \frac{1}{1+x} + c}$$

eg: 2.39

$$I = \int \frac{1}{(\log x)} - \frac{1}{(\log x)^2} \cdot dx$$

$$\text{Put } t = \log x$$

$$dt = \frac{1}{x} \cdot dx$$

$$\boxed{dx = x dt}$$

$$\begin{aligned} t &= \log x \\ \text{take 'e'} \\ e^t &= x \end{aligned}$$

$$= \int \left(\frac{1}{t} - \frac{1}{t^2} \right) e^t \cdot dt$$

$$= \int e^t (f(t) + f'(t)) \cdot dt$$

$$= e^t f(t) + c$$

$$= e^t \frac{1}{t} + c$$

$$= \frac{e^{\log x}}{\log x} + c$$

$$= \boxed{\frac{x}{\log x} + c}$$

Formula:

$$\textcircled{1} \int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + c$$

$a^2 - b^2 = (a+b)(a-b)$

$$\textcircled{2} \int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log \left| \frac{a-x}{a+x} \right| + c$$

$$\textcircled{3} \int \frac{dx}{\sqrt{x^2 - a^2}} = \log \left| x + \sqrt{x^2 - a^2} \right| + c$$

$$\textcircled{4} \int \frac{dx}{\sqrt{x^2 + a^2}} = \log \left| x + \sqrt{x^2 + a^2} \right| + c$$

$$\textcircled{5} \int \sqrt{x^2 - a^2} \cdot dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \log \left| x + \sqrt{x^2 - a^2} \right| + c$$

$$\textcircled{6} \int \sqrt{x^2 + a^2} \cdot dx = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \log \left| x + \sqrt{x^2 + a^2} \right| + c$$

$$\begin{aligned} \textcircled{3} \quad & \int \frac{1}{\sqrt{x^2-9}} \cdot dx \\ &= \int \frac{1}{(\sqrt{2x})^2 - 3^2} \cdot dx \quad \text{let } \boxed{x = \sqrt{2x}} \quad \boxed{a=3} \\ & \quad \boxed{\sqrt{2(\sqrt{2x})} = \sqrt{2}} \\ &= \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + C \\ &= \frac{1}{2 \times 3} \log \left| \frac{\sqrt{2x} - 3}{\sqrt{2x} + 3} \right| + C \\ &= \frac{1}{6\sqrt{2}} \log \left| \frac{\sqrt{2x} - 3}{\sqrt{2x} + 3} \right| + C \end{aligned}$$

$$\begin{aligned} \textcircled{17} \quad & \int \sqrt{x^2 - 2} \, dx \quad \text{let, } \boxed{x = \sqrt{2}u} \quad \left\{ \begin{array}{l} x = \sqrt{2}u \\ dx = \sqrt{2} du \end{array} \right. \\ & = \int \sqrt{x^2 - (\sqrt{2})^2} \cdot \sqrt{2} \, du \\ & = \int \sqrt{x^2 - a^2} \cdot du = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \log \left| x + \sqrt{x^2 - a^2} \right| \\ & = \frac{x}{2} \sqrt{x^2 - (\sqrt{2})^2} - \frac{(\sqrt{2})^2}{2} \log \left| x + \sqrt{x^2 - (\sqrt{2})^2} \right| + C \\ & = \boxed{\frac{x}{2} \sqrt{x^2 - 2} - \log |x + \sqrt{x^2 - 2}| + C} \end{aligned}$$

$$\begin{aligned}
 \textcircled{10} \int \sqrt{4x^2 - 5} \cdot dx & \quad \boxed{a = \sqrt{5}}, \quad \boxed{x = 2x} \\
 & \quad \boxed{d(2x) = 2} \\
 & = \int \sqrt{(2x)^2 - (\sqrt{5})^2} \cdot dx \\
 & = \int \sqrt{x^2 - a^2} \cdot dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \log |x + \sqrt{x^2 - a^2}| + C \\
 & \therefore = \frac{2x}{2} \sqrt{(2x)^2 - (\sqrt{5})^2} - \frac{(\sqrt{5})^2}{2} \log |2x + \sqrt{(2x)^2 - (\sqrt{5})^2}| + C \\
 & = \frac{1}{4} [2x \sqrt{4x^2 - 5} - 5 \log |2x + \sqrt{4x^2 - 5}|] + C
 \end{aligned}$$

eg 2.44

$$\begin{aligned}
 I & = \int \frac{x^2}{x^2 - 25} \cdot dx \\
 & = \int \frac{(x^2 - 25 + 25)}{x^2 - 25} \cdot dx \\
 & = \int \left(\frac{x^2 - 25}{x^2 - 25} + \frac{25}{x^2 - 25} \right) \cdot dx \\
 & = \int \left(1 + 25 \frac{1}{x^2 - 5^2} \right) \cdot dx \\
 & \quad \boxed{x = x}, \quad \boxed{a = 5} \quad (d(x) = 1)
 \end{aligned}$$

$$\begin{aligned}
 \int \frac{dx}{x^2 - a^2} & = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + C \\
 & = x + \frac{5}{2} \times \frac{1}{2 \times 5} \log \left| \frac{x-5}{x+5} \right| + C \\
 & = \boxed{x + \frac{1}{2} \log \left| \frac{x-5}{x+5} \right| + C}
 \end{aligned}$$

⑦

$$\begin{aligned}
 I & = \int \frac{e^x}{e^{2x} - 9} \cdot dx \\
 & = \int \frac{e^x}{(e^x)^2 - 3^2} \cdot dx
 \end{aligned}$$

$$\text{put } t = e^x$$

$$dt = e^x \cdot dx$$

$$= \int \frac{dt}{t^2 - 3^2}$$

$$= \int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + C$$

$$= \frac{1}{2 \times 3} \log \left| \frac{t-3}{t+3} \right| + C$$

$$= \boxed{\frac{1}{6} \log \left| \frac{e^x - 3}{e^x + 3} \right| + C}$$

$$\textcircled{11} \quad I = \int \frac{x^3}{\sqrt{x^8-1}} \cdot dx \quad \left| d(x^4) = 4x^3 \right.$$

$$= \int \frac{x^3}{\sqrt{(x^4)^2-1^2}} \cdot dx$$

$$\boxed{t = x^4}$$

$$dt = 4x^3 \cdot dx$$

$$\boxed{\frac{dt}{4} = x^3 \cdot dx}$$

$$I = \int \frac{dt/4}{\sqrt{t^2-1^2}}$$

$$= \frac{1}{4} \int \frac{dt}{\sqrt{t^2-1^2}}$$

$$= \frac{1}{4} \int \frac{dn}{\sqrt{n^2-a^2}}$$

$$\boxed{n=t, a=1}$$

$$\boxed{dt=1}$$

$$= \frac{1}{4} \log |n + \sqrt{n^2-a^2}| + C$$

$$= \frac{1}{4} \log |t + \sqrt{t^2-1^2}| + C$$

$$= \boxed{\frac{1}{4} \log |x^4 + \sqrt{x^8-1}| + C}$$

$$\textcircled{2} \quad \int \frac{1}{9-8x-x^2} \cdot dx$$

$$9-8x-x^2 = -[x^2+8x-9]$$

$$= \frac{2^2}{2} = 4 \rightarrow \boxed{b^2 = 16}$$

$$= -[x^2+8x+16-16-9]$$

$$x^2+8x+b^2 = (x+b)^2$$

$$= -[(x+4)^2 - 5^2]$$

$$= 5^2 - (x+4)^2$$

$$I = \int \frac{dx}{5^2 - (x+4)^2}$$

$$\boxed{x = x+4} \quad \boxed{a=5}$$

$$d(x+4) = 1$$

$$= \int \frac{dn}{a^2-n^2} = \frac{1}{2a} \log \left| \frac{a+n}{a-n} \right| + C$$

$$= \frac{1}{2 \times 5} \log \left| \frac{5+(x+4)}{5-(x+4)} \right| + C$$

$$= \boxed{\frac{1}{10} \log \left| \frac{5+(x+4)}{5-(x+4)} \right| + C}$$

$$\textcircled{6} I = \int \frac{1}{2x^2 + 3x - 4} \cdot dx$$

$$= \frac{1}{2} \int \frac{dx}{x^2 + 3x - 4}$$

$$\begin{aligned} (x^2 + 3x - 4) &= x^2 - 3x + \frac{9}{4} - \frac{9}{4} - 4 \\ + \frac{3}{2} &= b \\ \boxed{b^2 = \frac{9}{4}} \end{aligned}$$

$$\begin{aligned} &= \left(x + \frac{3}{2}\right)^2 - \left(\frac{7}{4}\right) \\ &= \left(x + \frac{3}{2}\right)^2 - \left(\frac{\sqrt{7}}{2}\right)^2 \end{aligned}$$

$$I = \frac{1}{2} \int \frac{dx}{\left(x + \frac{3}{2}\right)^2 - \left(\frac{\sqrt{7}}{2}\right)^2}$$

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + C$$

$$x = x + \frac{3}{2} \quad a = \frac{\sqrt{7}}{2}$$

$$\begin{aligned} &= \frac{1}{2} \left[\frac{1}{2 \times \frac{\sqrt{7}}{2}} \log \left| \frac{x + \frac{3}{2} - \frac{\sqrt{7}}{2}}{x + \frac{3}{2} + \frac{\sqrt{7}}{2}} \right| \right] + C \\ &= \frac{1}{2\sqrt{7}} \end{aligned}$$

$$= \frac{1}{2} \int \frac{dx}{x^2 + 3x - 4}$$

$$\begin{aligned} x^2 + 3x - 4 &= x^2 + 3x + \frac{9}{4} - \frac{9}{4} - 4 \\ + \frac{3}{2} &= b \\ b^2 &= \frac{9}{4} \end{aligned}$$

$$= \left(x + \frac{3}{2}\right)^2 - \frac{25}{4}$$

$$I = \frac{1}{2} \int \frac{dx}{\left(x + \frac{3}{2}\right)^2 - \left(\frac{5}{2}\right)^2}$$

$$\boxed{x = x + \frac{3}{2}}$$

$$\boxed{a = \frac{5}{2}}$$

$$d(x) = 1$$

$$= \int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + C$$

$$= \frac{1}{2} \times \frac{1}{2 \left(\frac{5}{2}\right)} \log \left| \frac{\left(x + \frac{3}{2}\right) - \frac{5}{2}}{\left(x + \frac{3}{2}\right) + \frac{5}{2}} \right| + C$$

$$= \frac{1}{2} \times \frac{1}{5} \log \left| \frac{2x + -2}{2x + 8} \right| + C$$

$$= \boxed{\frac{1}{10} \log \left| \frac{2x-2}{2x+8} \right| + C}$$

$$④ I = \int \frac{1}{x^2 - x - 2} \cdot dx$$

$$\begin{aligned} x^2 - x - 2 &= x^2 - x + \frac{1}{4} - \frac{1}{4} - 2 \\ -\frac{1}{2} &= b \\ \boxed{b^2 = \frac{1}{4}} \end{aligned} \quad \begin{aligned} &= \left(x - \frac{1}{2}\right)^2 - \left(\frac{3}{2}\right)^2 \end{aligned}$$

$$I = \int \frac{1}{\left(x - \frac{1}{2}\right)^2 - \left(\frac{3}{2}\right)^2} \cdot dx$$

$$= \int \frac{dt}{x^2 - a^2} = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + c$$

$$\boxed{x = x - \frac{1}{2}} \quad \boxed{a = \frac{3}{2}}$$

$$d\left(x - \frac{1}{2}\right) = 1$$

$$= \frac{1}{x \times \frac{3}{2}} \log \left| \frac{x - \frac{1}{2} - \frac{3}{2}}{x - \frac{1}{2} + \frac{3}{2}} \right| + c$$

$$= \frac{1}{3} \log \left| \frac{\frac{2x-4}{2}}{\frac{2x+2}{2}} \right| + c$$

$$= \boxed{\frac{1}{3} \log \left| \frac{2x-4}{2x+2} \right| + c} //$$

$$⑤ I = \int \frac{1}{x^2 + 3x + 2} \cdot dx$$

$$\begin{aligned} (x^2 + 3x + 2) &= x^2 + 3x + \frac{9}{4} - \frac{9}{4} + 2 \\ +\frac{3}{2} &= b \\ \boxed{b^2 = \frac{9}{4}} \end{aligned} \quad \begin{aligned} &= \left(x + \frac{3}{2}\right)^2 - \frac{1}{4} \\ &= \left(x + \frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2 \end{aligned}$$

$$I = \int \frac{dx}{\left(x + \frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2}$$

$$= \int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + c$$

$$\boxed{x = x + \frac{3}{2}} \quad \boxed{a = \frac{1}{2}}$$

$$= \frac{1}{x \times \frac{1}{2}} \log \left| \frac{x + \frac{3}{2} - \frac{1}{2}}{x + \frac{3}{2} + \frac{1}{2}} \right| + c$$

$$= \log \left| \frac{\frac{2x+2}{2}}{\frac{2x+4}{2}} \right|$$

$$= \boxed{\log \left| \frac{2x+2}{2x+4} \right| + c}$$

$$(6) \int \frac{1}{2x^2 + 6x - 8} \cdot dx$$

$$I = 2x^2 + 6x - 8$$

$$= \frac{1}{2} \int \frac{dx}{x^2 + 3x - 4}$$

$$(9) \int \frac{1}{\sqrt{x^2 + 6x + 13}} \cdot dx$$

$$I = \sqrt{x^2 + 6x + 13} = x^2 + 6x + 9 - 9 + 13$$

$$\frac{b}{2} = 3 = b$$

$$b^2 = 9$$

$$= (x+3)^2 + 4$$

$$= (x+3)^2 + (2)^2$$

$$I = \int \frac{dx}{\sqrt{(x+3)^2 + (2)^2}}$$

$$= \int \frac{dx}{\sqrt{x^2 + a^2}} = \log |x + \sqrt{x^2 + a^2}| + C$$

$$x = x+3, \quad a = 2$$

$$d(x+3) = 1$$

$$= \log |(x+3) + \sqrt{(x+3)^2 + (2)^2}| + C$$

$$(15) I = \int \sqrt{1+x+x^2} \cdot dx$$

$$1 + (x) + x^2 = x^2 + x + 1$$

$$\frac{1}{2} = b$$

$$b^2 = \frac{1}{4}$$

$$= x^2 + x + \frac{1}{4} - \frac{1}{4} + 1$$

$$= \left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$$

$$= \left(x + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2$$

$$I = \int \sqrt{\left(x + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} \cdot dx$$

$$= \int \sqrt{x^2 + a^2} \cdot dx = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \log |x + \sqrt{x^2 + a^2}| + C$$

$$x = x + \frac{1}{2}$$

$$a = \frac{\sqrt{3}}{2}$$

$$d\left(x + \frac{1}{2}\right) = 1$$

$$I = \frac{\left(x + \frac{1}{2}\right)}{2} \sqrt{\left(x + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} + \frac{\left(\frac{\sqrt{3}}{2}\right)^2}{2} \log \left(x + \frac{1}{2}\right) + \sqrt{\left(x + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} + C$$

$$= \frac{2x+1}{4} \sqrt{1+x+x^2} + \frac{3}{8} \log \left| \frac{2x+1}{2} + \sqrt{1+x+x^2} \right| + C$$

$$\begin{aligned}
 (15) \quad I &= \int \sqrt{2x^2 + 4x + 1} \cdot dx \\
 &= \int \sqrt{2(x^2 + 2x + \frac{1}{2})} \cdot dx \\
 &= \sqrt{2} \int \sqrt{x^2 + 2x + \frac{1}{2}} \cdot dx
 \end{aligned}$$

consider:

$$\begin{aligned}
 x^2 + 2x + \frac{1}{2} &= x^2 + 2x + 1 - 1 + \frac{1}{2} \\
 \frac{x}{2} = 1 = b & \quad \boxed{b^2 = 1} \\
 &= (x+1)^2 - \frac{1}{2} \\
 &= (x+1)^2 - \left(\frac{1}{\sqrt{2}}\right)^2
 \end{aligned}$$

$$\begin{aligned}
 I &= \sqrt{2} \int \sqrt{(x+1)^2 - \left(\frac{1}{\sqrt{2}}\right)^2} \cdot dx \\
 &= \sqrt{2} \int \sqrt{x^2 - a^2} \cdot dx \\
 &= \sqrt{2} \left[\frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \log |x + \sqrt{x^2 - a^2}| + c \right] \\
 &\quad \boxed{x = x+1, \quad a = \frac{1}{\sqrt{2}}} \\
 &\quad d(x+1) = 1
 \end{aligned}$$

$$\begin{aligned}
 &= \sqrt{2} \left[\frac{(x+1)}{2} \sqrt{(x+1)^2 - \left(\frac{1}{\sqrt{2}}\right)^2} - \frac{\left(\frac{1}{\sqrt{2}}\right)^2}{2} \log |(x+1) + \sqrt{(x+1)^2 - \left(\frac{1}{\sqrt{2}}\right)^2}| + c \right] \\
 &= \sqrt{2} \left[\frac{(x+1)}{2} \sqrt{x^2 + 2x + \frac{1}{2}} - \frac{1}{4} \log |(x+1) + \sqrt{x^2 + 2x + \frac{1}{2}}| + c \right]
 \end{aligned}$$

$$\begin{aligned}
 (16) \quad I &= \int \frac{1}{x + \sqrt{x^2 - 1}} \cdot dx \\
 &= \int \frac{1}{x + \sqrt{x^2 - 1}} \times \frac{x - \sqrt{x^2 - 1}}{x - \sqrt{x^2 - 1}} \cdot dx \\
 &= \int \frac{x - \sqrt{x^2 - 1}}{x^2 - (\sqrt{x^2 - 1})^2} \cdot dx \\
 &= \int \frac{x - \sqrt{x^2 - 1}}{x^2 - (x^2 - 1)} \cdot dx \\
 &= \int (x - \sqrt{x^2 - 1}) \cdot dx \\
 &= \left[\frac{x^2}{2} - \left[\frac{x}{2} \sqrt{x^2 - 1} - \frac{1}{2} \log |x + \sqrt{x^2 - 1}| + c \right] \right]
 \end{aligned}$$

Ex 2.8

$$I. \textcircled{1} \quad I = \int_0^1 e^{2x} \cdot dx$$

$$= \left[\frac{e^{2x}}{2} \right]_0^1$$

$$= \frac{e^2}{2} - \frac{e^0}{2}$$

$$= \frac{e^2}{2} - \frac{1}{2}$$

$$\boxed{I = \frac{1}{2}(e^2 - 1)}$$

$$\begin{array}{l}
 e^0 = 1 \\
 e^{\log e}
 \end{array}$$

$$\begin{aligned}
 \textcircled{2} \quad I &= \int_0^{1/4} \sqrt{1-4x} \cdot dx \\
 &= \int_0^{1/4} (1-4x)^{1/2} \cdot dx \\
 &= \left[\frac{(1-4x)^{3/2}}{\frac{3}{2} \times -4} \right]_0^{1/4} \\
 &= -\frac{1}{6} (1-4 \times \frac{1}{4})^{3/2} - (1-0)^{3/2}
 \end{aligned}$$

$$I = \frac{1}{6}$$

$$\begin{aligned}
 \textcircled{3} \quad I &= \int_1^2 \frac{x \cdot dx}{x^2+1} \\
 d(x^2+1) &= 2x \cdot dx \\
 &= \frac{1}{2} \int_1^2 \frac{2x \cdot dx}{x^2+1} \\
 &= \frac{1}{2} \left[\log |x^2+1| \right]_1^2 \\
 &= \frac{1}{2} [\log |5| - \log |2|]
 \end{aligned}$$

$$I = \frac{1}{2} \log \left(\frac{5}{2} \right)$$

$$\begin{aligned}
 \textcircled{4} \quad I &= \int_0^3 \frac{e^x \cdot dx}{1+e^x} \\
 d(1+e^x) &= 0+e^x \\
 &= \left[\log |1+e^x| \right]_0^3 \\
 &= \log |1+e^3| - \log |1+1|
 \end{aligned}$$

$$I = \log \left(\frac{1+e^3}{2} \right)$$

$$\textcircled{5} \quad I = \int_0^1 x e^{x^2} \cdot dx$$

Put $t = x^2$
 $dt = 2x \cdot dx$
 $\frac{dt}{2} = x \cdot dx$

x	0	1
$t=x^2$	0	1

$$= \int_0^1 e^t \cdot \frac{dt}{2}$$

$$= \frac{1}{2} (e^t)_0^1$$

$$= \frac{1}{2} (e^1 - e^0)$$

$$I = \frac{1}{2} [e-1]$$

$$(6) \int_1^e \frac{dx}{x(1+\log x)^3}$$

Put $t = 1 + \log x$

$$dt = \frac{1}{x} \cdot dx$$

$$= \int_1^2 \frac{dt}{t^3}$$

$$= \left[\frac{-1}{2t^2} \right]_1^2 = -\frac{1}{2} \left[\frac{1}{4} - \frac{1}{1} \right]$$

$$= -\frac{1}{2} \times -\frac{3}{4}$$

$$\boxed{I = \frac{3}{8}}$$

(7)

$$I = \int_{-1}^1 \frac{2x+3}{x^2+3x+7} \cdot dx$$

$$[d(x^2+3x+7) = 2x+3]$$

$$= \left[\log |x^2+3x+7| \right]_{-1}^1$$

$$= \log |1+3+7| - \log |1+1-3+7|$$

$$= \log |11| - \log |5|$$

$$= \boxed{\log \left[\frac{11}{5} \right]}$$

x	1	e
$t = 1 + \log x$	1	2

$$\left\{ \begin{array}{l} x=1, t=1+\log 1 = 1 \\ x=e, t=1+\log e = 2 \end{array} \right\}$$

$$(8) I = \int_0^{\pi/2} \sqrt{1+\cos x} \cdot dx$$

$$= \int_0^{\pi/2} \sqrt{2 \cos^2 \frac{x}{2}} \cdot dx$$

$$= \sqrt{2} \int_0^{\pi/2} \cos \frac{x}{2} \cdot dx$$

$$= \sqrt{2} \left[\frac{\sin \frac{x}{2}}{\frac{1}{2}} \right]_0^{\pi/2}$$

$$= 2\sqrt{2} \left[\sin \frac{\pi}{4} - \sin 0 \right]$$

$$= 2\sqrt{2} \left(\frac{1}{\sqrt{2}} - 0 \right)$$

$$= \boxed{2}$$

(9)

formula

$$\cos^2 A = \frac{1 + \cos 2A}{2}$$

$$1 + \cos A = \frac{2 \cos^2 \frac{A}{2}}{2}$$

$$\sin^2 A = \frac{1 - \cos 2A}{2}$$

$$1 - \cos A = \frac{2 \sin^2 \frac{A}{2}}{2}$$

$$\sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$\sin 0 = 0$$

(9)

$$I = \int_1^2 \frac{x-1}{x^2} \cdot dx$$

$$= \int_1^2 \left(\frac{x}{x^2} - \frac{1}{x^2} \right) \cdot dx$$

$$= \int_1^2 \left(\frac{1}{x} - \frac{1}{x^2} \right) \cdot dx$$

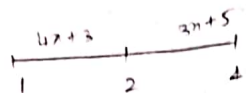
$$= \left[\log |x| + \frac{1}{x} \right]_1^2$$

$$= \left(\log |2| + \frac{1}{2} \right) - \left(\log |1| + \frac{1}{1} \right)$$

$$= \log 2 + \frac{1}{2} - \log 1 - 1$$

$$= \log 2 - \frac{1}{2} = \boxed{\frac{1}{2} (2 \log 2 - 1)}$$

$$\textcircled{1} I = \int_1^4 f(x) \cdot dx \quad f(x) = \begin{cases} 4x+3, & 1 \leq x \leq 2 \\ 3x+5, & 2 < x \leq 4 \end{cases}$$



$$I = \int_1^2 (4x+3) \cdot dx + \int_2^4 (3x+5) \cdot dx$$

$$= \left[\frac{4x^2}{2} + 3x \right]_1^2 + \left[\frac{3x^2}{2} + 5x \right]_2^4$$

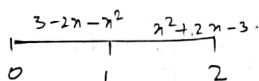
$$= \left(\frac{16}{2} + 6 \right) - \left(\frac{4}{2} + 3 \right) + \left(\frac{48}{2} + 20 \right) - \left(\frac{12}{2} + 10 \right)$$

$$= \frac{16}{2} + 6 - \frac{4}{2} - 3 + \frac{48}{2} + 20 - \frac{12}{2} - 10$$

$$= 58 - 21$$

$$= \boxed{37}$$

$$\textcircled{2} I = \int_0^2 f(x) \cdot dx = \begin{cases} 3-2x-x^2, & x \leq 1 \\ x^2+2x-3, & 1 < x \leq 2 \end{cases}$$



$$I = \int_0^1 (3-2x-x^2) \cdot dx + \int_1^2 (x^2+2x-3) \cdot dx$$

$$= \left[3x - \frac{2x^2}{2} - \frac{x^3}{3} \right]_0^1 + \left[\frac{x^3}{3} + \frac{2x^2}{2} - 3x \right]_1^2$$

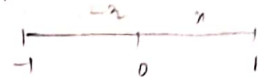
$$= \left(3 - 1 - \frac{1}{3} \right) - 0 + \left(\frac{8}{3} + 4 - 6 \right) - \left(\frac{1}{3} + 1 - 3 \right)$$

$$= 2 - \frac{1}{3} + \frac{8}{3} - 2 - \frac{1}{3} + 2$$

$$= 2 + \frac{6}{3} = \frac{10}{3} = 4 \frac{2}{3}$$

$$\textcircled{3} I = \int_{-1}^1 f'(x)$$

$$f'(x) = \begin{cases} -x & x \geq 0 \\ x & x < 0 \end{cases}$$



$$I = \int_{-1}^0 -x \cdot dx + \int_0^1 x \cdot dx$$

$$= \left[-\frac{x^2}{2} \right]_{-1}^0 + \left[\frac{x^2}{2} \right]_0^1$$

$$= \frac{1}{2} [0 - (-1)] + (1 - 0)$$

$$= \frac{1}{2} [1 + 1]$$

$$= \boxed{1}$$

$$\textcircled{4} f(x) = \begin{cases} cx & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\int_0^1 f(x) \cdot dx = 2$$

$$\int_0^1 cx \cdot dx = 2$$

$$\left[\frac{cx^2}{2} \right]_0^1 = 2$$

$$\left(\frac{cx^2}{2} \right)_0^1 = 4$$

$$c(1) - c(0) = 4$$

$$\boxed{c = 4}$$

$$\int \frac{dx}{16-x^2} \cdot dx$$

let $\boxed{x = 4}$ $\boxed{\gamma = x}$
 $d(\gamma) = 1$

$$= \frac{1}{2 \times 4} \log \left| \frac{4+x}{4-x} \right| + C.$$

eg 2.41

$$= \int \frac{1}{1^2 - (5x)^2}$$

$$\text{let } \boxed{a=1} \quad \boxed{x=5x}$$

$$\boxed{a(5a) = 5}$$

$$= \frac{1}{2 \times 1} \log \left| \frac{1+5x}{1-5x} \right| + C$$

$$= \frac{5}{10} \log \left| \frac{1+5x}{1-5x} \right| + C$$

92.92

$$\int \frac{dx}{2+x-x^2}$$

$$= -\frac{1}{2} = b$$

$$= -(\cancel{x^2} - \cancel{x} + \cancel{\frac{1}{4}} - \cancel{\frac{1}{4}} - 2)$$

$$= (x - \frac{1}{2})^2 - (\frac{3}{2})^2$$

$$I = \int \frac{1}{(x - \frac{1}{2})^2 - (\frac{3}{2})^2} \cdot dx$$

$$= \int \frac{dt}{x^2 - a^2} = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right|$$

eg 2.58

$$\begin{aligned}
 & \int_0^1 (e^x - 4a^x + 2 + \sqrt{x}) \cdot dx \\
 &= \left[e^x - \frac{4a^x}{\log a} + 2x + \frac{x^{3/2}}{3/2} \right]_0^1 \\
 &= \left(e^1 - \frac{4a}{\log a} + 2 + \frac{1}{3/2} \right) - \left(e^0 - \frac{4(a)^0}{\log a} + 0 + 0 \right) \\
 &= \left(e - \frac{4a}{\log a} + 2 + \frac{2}{3} - 1 + \frac{4}{\log a} \right) \\
 &= e + \frac{4}{\log a} (1-a) + \frac{7}{6}
 \end{aligned}$$

eg 2.65

$$I = \int_{-1}^1 x \sqrt{x+1} \cdot dx$$

Put $t = x+1 \Rightarrow x = t-1$

$$dt = dx$$

$$I = \int_0^2 (t-1)\sqrt{t} \cdot dt$$

$$= \int_0^2 (t^{3/2} - t^{1/2}) \cdot dt$$

$$= \int_0^2 (t^{3/2} - t^{1/2}) \cdot dt$$

$$= \left[\frac{t^{5/2}}{5/2} - \frac{t^{3/2}}{3/2} \right]_0^2$$

$$= \left[\frac{2}{5} t^{5/2} - \frac{2}{3} t^{3/2} \right]_0^2$$

$$= \frac{2}{5} (2)^{5/2} - \frac{2}{3} (2)^{3/2} - 0$$

x	-1	1
t = x+1	0	2

$$= 2(2)^{3/2} \left[\frac{2}{5} - \frac{1}{3} \right]$$

$$= 2(2\sqrt{2}) \frac{6-5}{15}$$

$$= \frac{4\sqrt{2}}{15} //$$

eg 2.66

$$\int_0^{\infty} e^{-x/2} \cdot dx$$

$$= \left(\frac{e^{-x/2}}{-1/2} \right)_0^{\infty}$$

$$= -2 \left(\frac{1}{e^{x/2}} \right)_0^{\infty}$$

$$= -2 \left(\frac{1}{e^{\infty}} - \frac{1}{e^0} \right)$$

$$= -2 \left(\frac{1}{\infty} - \frac{1}{1} \right)$$

$$= \boxed{2} //$$

$$\infty = \frac{1}{0}$$

$$\frac{1}{\infty} = \frac{1}{\frac{1}{0}} = 0$$

eg 2.67

$$I = \int_0^{\infty} x^2 e^{-x^3} \cdot dx$$

Put $t = x^3$

$$dt = 3x^2 \cdot dx$$

$$\boxed{\frac{dt}{3} = x^2 \cdot dx}$$

x	0	∞
t = x^3	0	∞

$$\begin{aligned}
 I &= \int_0^{\infty} e^t \cdot \frac{dt}{3} = \frac{1}{3} \left[\frac{e^t}{1} \right]_0^{\infty} \\
 &= -\frac{1}{3} \left[\frac{1}{e^t} \right]_0^{\infty} \\
 &= -\frac{1}{3} \left[\frac{1}{e^{\infty}} - \frac{1}{e^0} \right] \\
 &= -\frac{1}{3} [0 - 1] \\
 &= \boxed{\frac{1}{3}}
 \end{aligned}$$

eg: 2.70

$$I = \int_0^{\pi/2} x \cdot \sin x \cdot dx$$

$$\begin{aligned}
 u &= x \\
 u' &= 1 \\
 u'' &= 0
 \end{aligned}
 \quad \left| \quad \begin{aligned}
 dv &= \sin x \cdot dx \\
 \int dv &= \int \sin x \cdot dx \\
 v &= -\cos x \\
 v_1 &= -\sin x
 \end{aligned}
 \right.$$

$$\int u dv = (uv - u'v_1 + u''v_2 - \dots)$$

$$\begin{aligned}
 I &= [-x \cos x + 1 \sin x]_{\pi/2}^0 \\
 &= \left(-\pi/2 \cos \frac{\pi}{2} + \sin \frac{\pi}{2} \right) - (0 + \sin 0)
 \end{aligned}$$

$$= \boxed{1}$$

eg: 2.71

$$\begin{aligned}
 \int_1^a 3x^2 \cdot dx &= -1 \\
 \Rightarrow \left[\frac{3x^3}{3} \right]_1^a &= -1
 \end{aligned}$$

$$\Rightarrow a^3 - 1 = -1$$

$$a^3 = 0$$

$$\boxed{a = 0}$$

eg 2.72

$$\int_a^b dx = 1, \quad \int_a^b x \cdot dx = 1$$

$$\begin{aligned}
 [x]_a^b &= 1 \\
 [b-a] &= 1 \\
 \left[\frac{x^2}{2} \right]_a^b &= 1 \\
 [x^2]_a^b &= 2
 \end{aligned}$$

$$b^2 - a^2 = 2$$

$$\Rightarrow (b-a)(b+a) = 2$$

$$1(b+a) = 2$$

$$\boxed{b+a = 2}$$

$$b-a = 1$$

$$b+a = 2$$

$$2b = 3$$

$$\boxed{b = \frac{3}{2}} \quad \rightarrow \quad b+a = 2$$

$$\frac{3}{2} + a = 2$$

$$a = 2 - \frac{3}{2}$$

$$\boxed{a = \frac{1}{2}}$$

The value of

$$\boxed{a = \frac{1}{2}} \text{ and } \boxed{b = \frac{3}{2}}$$

eg 2.56

$$\int_0^1 (x^3 + 7x^2 - 5x) \cdot dx$$

$$= \left[\frac{x^4}{4} + \frac{7x^3}{3} - \frac{5x^2}{2} \right]_0^1$$

$$= \left[\frac{1 \times 6}{4 \times 6} + \frac{7 \times 8}{3 \times 6} - \frac{5 \times 12}{2 \times 6} \right]$$

$$= \frac{6 + 56 - 60}{24}$$

$$= \frac{+2}{24}$$

$$= \boxed{\frac{1}{12}}$$

C.B

$$\textcircled{1} \int_0^{\pi/2} \cos^9 x \, dx$$

$$\textcircled{2} \int_0^{\pi/2} \cos^6 x \, dx$$

$$\textcircled{3} \int_0^{\pi/2} \sin^5 x \, dx$$

$$\textcircled{4} \int_0^{\pi/2} \sin^8 x \, dx$$

Formula:

$$\int_a^b f(x) \cdot dx = - \int_b^a f(x) \cdot dx$$

$$\textcircled{*} \int_0^a f(x) \cdot dx = \int_0^a f(a-x) \cdot dx$$

$$\textcircled{*} \int_{-a}^a f(x) \cdot dx = 2 \int_0^a f(x) \cdot dx, \text{ if } f \text{ is even}$$

$$f(-x) = f(x),$$

$\hookrightarrow 0$, f is odd.

$$f(-x) = -f(x).$$

$$f(x) = x \Rightarrow f(-x) = -x = -f(x)$$

f is odd.

$$f(x) = \cos x \Rightarrow f(-x) = \cos(-x) = \cos x = f(x)$$

$$\therefore \int_a^b f(x) \cdot dx = \int_a^b f(a+b-x) \cdot dx$$

ex 2.9

$$\textcircled{1} \quad I = \int_{-\pi/4}^{\pi/4} x^3 \cdot \cos^3 x \cdot dx$$

$$f(x) = x^3 \cos^3 x$$

$$f(-x) = -x^3 \cos^3 x$$

$$= -f(x)$$

f is odd

$$\boxed{I = 0}$$

$$\textcircled{2} \quad I = \int_{-\pi/2}^{\pi/2} \sin^2 \theta \, d\theta$$

$$f(\theta) = \sin^2 \theta$$

$$f(-\theta) = f(\theta)$$

f is even

$$I = 2 \int_0^{\pi/2} \sin^2 \theta \, d\theta$$

$$= 2 \times \frac{1}{2} \times \pi/2$$

$$I = \boxed{\pi/2}$$

$$\textcircled{3} \quad I = \int_{-1}^1 \log \left(\frac{2-x}{2+x} \right) dx$$

$$I = \int_{-1}^1 \log(2-x) - \log(2+x) \cdot dx$$

$$f(x) = \log(2-x) - \log(2+x)$$

$$f(-x) = \log(2+x) - \log(2-x)$$

$$= -[\log(2-x) - \log(2+x)]$$

$$f(-x) = -f(x)$$

f is odd

$$\boxed{I = 0}$$

$$\textcircled{4} \quad I = \int_0^{\pi/2} \frac{\sin^7 x}{\sin^7 x + \cos^7 x} \cdot dx \rightarrow \textcircled{1}$$

$$x = a - x = \pi/2 - x$$

$$I = \int_0^{\pi/2} \frac{\cos^7 x}{\cos^7 x + \sin^7 x} \cdot dx \rightarrow \textcircled{2}$$

$$\textcircled{1} + \textcircled{2} \quad 2I = \int_0^{\pi/2} \frac{(\sin^7 x + \cos^7 x)}{(\sin^7 x + \cos^7 x)} \cdot dx$$

$$2I = [x]_0^{\pi/2} = \pi/2 - 0$$

$$\boxed{I = \pi/4}$$

$$\textcircled{5} \quad \int_0^1 \log \left(\frac{1}{x} - 1 \right) \cdot dx$$

$$I = \int_0^1 \log \left(\frac{1-x}{x} \right) \cdot dx \rightarrow \textcircled{1}$$

$$x = a - x = 1 - x$$

$$\begin{aligned} \sin \pi/2 - \theta &= \cos \theta \\ \cos \pi/2 - \theta &= \sin \theta \end{aligned}$$

$$I = \int_0^1 \log \left(\frac{x}{1-x} \right) \cdot dx \quad - (2)$$

① + ②

$$\begin{aligned} 2I &= \int_0^1 \left(\log \frac{1-x}{x} + \log \frac{x}{1-x} \right) \cdot dx \\ &= \int_0^1 \log \frac{1-x}{x} \times \frac{x}{1-x} \cdot dx \\ &= \int_0^1 0 \cdot dx = 0 \end{aligned}$$

$$2I = 0 \quad \boxed{I = 0}$$

⑥ $\int_0^1 \frac{x}{(1-x)^{3/4}} \cdot dx$

$$I = \int_0^1 \frac{x}{(1-x)^{3/4}} \cdot dx$$

$x = 1-x \Rightarrow 1-x = 1-x$

$$I = \int_0^1 \frac{1-x}{(x)^{3/4}} \cdot dx$$

$$= \int_0^1 (1-x) x^{-3/4} \cdot dx$$

$$= \int_0^1 (x^{-3/4} - x^{1/4}) \cdot dx$$

$$= \left(\frac{x^{1/4}}{1/4} - \frac{x^{5/4}}{5/4} \right) \Big|_0^1$$

$$x=1 \Rightarrow x=1-x=0$$

$$= \frac{1}{1/4} - \frac{1}{5/4} - 0$$

$$= \frac{4}{1} - \frac{4}{5}$$

$$= \frac{20-4}{5} = \boxed{\frac{16}{5}}$$

eg. 2.79

$$I = \int_2^5 \frac{\sqrt{x}}{\sqrt{x} + \sqrt{7-x}} \cdot dx \rightarrow (1)$$

$$x = a+b-x = 2+5-x$$

$$\boxed{x = 7-x}$$

$$I = \int_2^5 \frac{\sqrt{7-x}}{\sqrt{7-x} + \sqrt{x}} \cdot dx \rightarrow (2)$$

① + ②

$$2I = \int_2^5 \frac{(\sqrt{x} + \sqrt{7-x})}{\sqrt{7-x} + \sqrt{x}} \cdot dx$$

$$= [x]_2^5$$

$$2I = 5-2$$

$$\boxed{I = 3/2}$$

C.A
①

$$\int_0^{\pi/2} \frac{f(\sin x)}{f(\sin x) + f(\cos x)} \cdot dx \quad (3) \quad \int_0^3 \frac{\sqrt{x}}{\sqrt{x} + \sqrt{3-x}} \cdot dx$$

Gamma Integral: (Γ)
(+ve) $\rightarrow$ $(0 \text{ to } \infty)$

$$\Gamma(n) = \int_0^{\infty} x^{n-1} e^{-x} \cdot dx$$

$$\Gamma(n+1) = n!$$

$$\Gamma(n) = (n-1) \Gamma(n-1)$$

$$\Gamma(n+1) = n \Gamma(n)$$

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

$$\int_0^{\infty} x^n e^{-ax} \cdot dx = \frac{n!}{a^{n+1}}$$

$$\frac{e^x : 2.10}{=}$$

$$\textcircled{i)} \Gamma(4) = \Gamma(3+1) = 3! \\ = 3 \times 2 = \boxed{6} //$$

$$\textcircled{ii)} \Gamma\left(\frac{7}{2}\right) = \frac{7}{2} \times \frac{5}{2} \times \frac{3}{2} \times \frac{1}{2} \times \sqrt{\pi} \\ = \boxed{\frac{105}{16} \pi} //$$

$$\textcircled{iii)} I = \int_0^{\infty} e^{-nx} x^6 \cdot dx$$

By GI, ,

$$\int_0^{\infty} x^n e^{-ax} \cdot dx = \frac{n!}{a^{n+1}}$$

$$\boxed{a=n} \quad \boxed{n=6}$$

$$\boxed{I = \frac{6!}{n^7}}$$

$$\textcircled{iv)} I = \int_0^{\infty} e^{-x/2} x^5 \cdot dx$$

By GI,

$$\int_0^{\infty} x^n e^{-ax} \cdot dx = \frac{n!}{a^{n+1}}$$

$$\boxed{n=5} \quad \boxed{a=1/2}$$

$$I = \frac{5!}{\left(\frac{1}{2}\right)^6} = 2^6 (5!)$$

eg 2.80

$$\textcircled{v)} \int_0^{\infty} e^{-x^2} \cdot dx$$

$$\Gamma(n) = \int_0^{\infty} t^{n-1} \cdot e^{-t} \cdot dt$$

$$\boxed{t=x^2}, \quad dt = 2x \cdot dx$$

$$\Gamma(n) = \int_0^{\infty} (x^2)^{n-1} e^{-x^2} (2x \cdot dx)$$

$$= 2 \int_0^{\infty} x^{2n-2} e^{-x^2} \cdot x^1 \cdot dx$$

$$= 2 \int_0^{\infty} x^{2n-1} e^{-x^2} \cdot dx$$

Put $\boxed{n = \frac{5}{2}}$ $\left(\begin{array}{l} 2n-1=0 \\ 2n=1 \\ n=\frac{1}{2} \end{array} \right)$

$$\Gamma\left(\frac{1}{2}\right) = 2 \int_0^{\infty} x^0 e^{-x^2} \cdot dx \quad (x^0=1)$$

$$\sqrt{\pi} = 2 \int_0^{\infty} e^{-x^2} \cdot dx$$

$$\int_0^{\infty} e^{-x^2} \cdot dx = \boxed{\frac{\sqrt{\pi}}{2}}$$

② $f(x) = \begin{cases} x^2 e^{-2x}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}$

$$\int_0^{\infty} f(x) \cdot dx$$

$$\int_0^{\infty} x^2 e^{-2x} \cdot dx = \frac{n!}{a^{n+1}}$$

$$\boxed{a=2} \quad \boxed{n=2} = \frac{2!}{2^3} = \frac{2 \times 1}{2^3}$$

$$= \frac{2}{2^3} = \boxed{\frac{1}{4}}$$

eg: 2.80

(i) $\Gamma(6) = \Gamma(5+1)$

$$= \boxed{5!} = 5 \times 4 \times 3 \times 2 \times 1 = \boxed{120}$$

(ii) $\Gamma\left(\frac{5}{2}\right) = \frac{5}{2} \times \frac{3}{2} \times \frac{1}{2} \times \sqrt{\pi}$

$$= \boxed{\frac{15\sqrt{\pi}}{8}}$$

(iii) $\int_0^{\infty} e^{-2x} x^5 \cdot dx$

$$\Rightarrow \int_0^{\infty} e^{-ax} x^n \cdot dx = \frac{n!}{a^{n+1}} \quad \boxed{\begin{array}{l} n=5 \\ a=2 \end{array}}$$

$$= \frac{5!}{2^{5+1}}$$

$$= \boxed{\frac{5!}{2^6}}$$

Formula

$$\int_a^b f(x) \cdot dx = \lim_{n \rightarrow \infty} h [f(a) + f(a+h) + \dots]$$

$$= \lim_{n \rightarrow \infty} \sum_{r=1}^n h f(a+rh)$$

$$h = \frac{b-a}{n}$$

$$1+2+\dots+n = \frac{n(n+1)}{2} = \sum_{r=1}^n r$$

$$1^2+2^2+\dots+n^2 = \frac{n(n+1)(2n+1)}{6} = \sum_{r=1}^n r^2$$

$$1^3+2^3+\dots+n^3 = \left(\frac{n(n+1)}{2}\right)^2 = \sum_{r=1}^n r^3$$

eg: ex: 2.11

$$\textcircled{2} \int_1^3 x \cdot dx$$

$$a=1 \quad b=3 \quad f(x)=x$$

$$h = \frac{b-a}{n} = \frac{3-1}{n} = \frac{2}{n}$$

$$f(a+rh) = f\left(1+n\left(\frac{2}{n}\right)\right)$$

$$= f\left(1+\frac{2r}{n}\right) = f(x)$$

$$f\left(1+\frac{2r}{n}\right) = \left(1+\frac{2r}{n}\right)$$

$$\int_1^3 x \cdot dx = \lim_{n \rightarrow \infty} \sum_{r=1}^n h f(a+rh)$$

$$= \lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{2}{n} \cdot \left(1 + \frac{2r}{n}\right)$$

$$= \lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{2}{n} + \sum_{r=1}^n \frac{4r}{n^2}$$

$$= \lim_{n \rightarrow \infty} \frac{2}{n} \cdot n + \frac{4}{n^2} \cdot n \cdot n$$

$$= \lim_{n \rightarrow \infty} 2 + \frac{4}{n^2} \cdot n^2$$

$$= \lim_{n \rightarrow \infty} 2 + \frac{4}{n^2} \cdot \left(n^2 \cdot \left(1 + \frac{1}{n}\right)\right)$$

$$= \lim_{n \rightarrow \infty} 2 + \frac{4}{n^2} \cdot \left(n^2 \cdot \left(1 + \frac{1}{n}\right)\right)$$

$$= 2 + 2(1+0)$$

$$= 4$$

$$\int_1^3 x \cdot dx = \left(\frac{x^2}{2}\right) \Big|_1^3 = \frac{9}{2} - \frac{1}{2} = \frac{8}{2} = 4$$

$$\textcircled{1}. \int_0^1 (x+4) \cdot dx$$

$$\boxed{a=0} \quad \boxed{b=1} \quad f(x) = x+4$$

$$h = \frac{b-a}{n} = \frac{1-0}{n} = \frac{1}{n} \Rightarrow h$$

$$f(a+rh) = f\left(0 + r \cdot \frac{1}{n}\right)$$

$$= f\left(\frac{r}{n}\right)$$

$$= \left[\frac{r}{n} + 4\right]$$

$$\int_0^1 (x+4) \cdot dx = \lim_{n \rightarrow \infty} \sum_{r=1}^n h f(a+rh)$$

$$= \lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{1}{n} \left(\frac{r}{n} + 4\right)$$

$$= \lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{r}{n^2} + \sum_{r=1}^n \frac{4}{n}$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n^2} \sum_{r=1}^n r + \frac{4}{n} \sum_{r=1}^n 1$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n^2} \frac{n(n+1)}{2} + \frac{4}{n} \cdot n$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n^2} \frac{n^2(1+\frac{1}{n})}{2} + 4$$

$$= \frac{1+\frac{1}{\infty}}{2} + 4$$

$$= \frac{1}{2} + 4$$

$$\int_0^1 (x+4) \cdot dx = \left[\frac{x^2}{2} + 4x\right]_0^1$$

$$\textcircled{2} \int_0^1 x^2 \cdot dx$$

$$\boxed{a=0} \quad \boxed{b=1} \quad h = \frac{b-a}{n} = \frac{1}{n}$$

$$f(x) = x^2$$

$$f(a+rh) = \frac{r^2}{n^2}$$

$$\int_0^1 x^2 \cdot dx = \lim_{n \rightarrow \infty} \sum_{r=1}^n h f(a+rh)$$

$$= \lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{1}{n} \cdot \frac{r^2}{n^2}$$

$$= \lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{1}{n^3} r^2$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n^3} \sum_{r=1}^n r^2$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n^3} \frac{n(n+1)(2n+1)}{6}$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n^3} \frac{n^3(1+\frac{1}{n})(2+\frac{1}{n})}{6}$$

$$= \frac{(1+\frac{1}{\infty})(2+\frac{1}{\infty})}{6}$$

$$= \frac{1 \times 2}{6} = \frac{2}{6} = \frac{1}{3}$$

check:

$$\int_0^1 (x+4) \cdot dx = \left[\frac{x^2}{2} + 4x\right]_0^1 = \frac{1}{2} + 4 = \frac{9}{2}$$

check:

$$\int_0^1 x^2 \cdot dx = \left[\frac{x^3}{3}\right]_0^1 = \frac{1}{3}$$

$$\textcircled{3}. \int_1^3 (2x+3) \cdot dx$$

$$a=1$$

$$b=3$$

$$h = \frac{b-a}{n} = \frac{3-1}{n}$$

$$f(a+rh) = f\left(1+r\left(\frac{2}{n}\right)\right)$$

$$= f\left(1+\frac{2r}{n}\right)$$

$$= 2\left(1+\frac{2r}{n}\right) + 3$$

$$= \left(2 + \frac{4r}{n}\right) + 3$$

$$\int_1^3 (2x+3) \cdot dx = \lim_{n \rightarrow \infty} \sum_{r=1}^n hf(a+rh)$$

$$= \lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{2}{n} \left(2 + \frac{4r}{n}\right) + 3$$

$$= \lim_{n \rightarrow \infty} \sum_{r=1}^n \left(\frac{4}{n} + \frac{8r}{n^2}\right) + 3$$

$$= \lim_{n \rightarrow \infty} \frac{4}{n} \sum_{r=1}^n 1 + \frac{8}{n^2} \sum_{r=1}^n r + 3$$

$$= \lim_{n \rightarrow \infty} \frac{4}{n} \cdot n + \frac{8}{n^2} \cdot \frac{n(n+1)}{2} + 3$$

$$= \lim_{n \rightarrow \infty} 4 + \frac{8}{n^2} \cdot \frac{n^2(1+\frac{1}{n})}{2} + 3$$

$$= \lim_{n \rightarrow \infty} 4 + 8 \cdot \frac{(1+\frac{1}{n})}{2} + 3$$

$$= 4 + 8 \cdot \frac{(1+0)}{2} + 3$$

$$= 4 + \frac{8}{2} + 3$$

checking:

$$\left[\frac{2x^2}{2} + 3x \right]_1^3$$

$$= \left[(3)^2 + 3(3) \right] - \left[(1)^2 + 3(1) \right]$$

$$= 9 + 9 - 4$$

$$= 14$$

Chapter - 7

Probability Distribution.

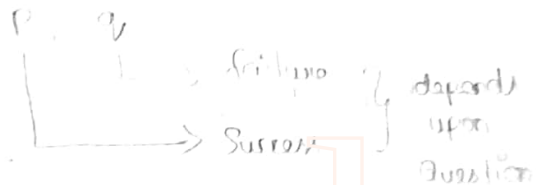
- 1- binomial
- 2- Poisson
- 3- Normal

Binomial ($n \leq 10$) $\Rightarrow$ Mean $>$ Variance

$$\text{Mean} = np$$

$$\text{Variance} = npq$$

$$\text{S.D} = \sqrt{npq}$$



$$p + q = 1$$

$$\Rightarrow p = 1 - q ; q = 1 - p$$

$$p, q \leq 1$$

$$P(x) = {}^n C_x p^x q^{n-x}, n = 0 \text{ to } x$$

$${}^n C_r = \frac{n!}{r!(n-r)!}$$

$$0! = 1$$

$$1! = 1$$

$${}^n C_0 = {}^n C_n = 1$$

$${}^n C_1 = n$$

$${}^n C_r = {}^n C_{n-r}$$

$${}^5 C_2 = {}^5 C_3$$

$$\frac{5!}{2!3!} = \frac{5 \times 4}{2 \times 1} \times \frac{3!}{3!}$$

$${}^5 C_2 = \frac{5 \times 4}{2 \times 1}$$

$${}^5 C_3 = \frac{5 \times 4}{2 \times 1}$$

Exercise 7.1

$$16 \quad \text{mean} = 5 \quad \text{S.D} = 2$$

$$np = 5 \quad \sqrt{npq} = 2$$

$$npq = 4$$

$$\frac{npq}{np} = \frac{4}{5}$$

$$q = \frac{4}{5} \quad , \quad p = \frac{1}{5}$$

$$np = 5$$

$$n \frac{1}{5} = 5$$

$$n = 25$$

$$P(x) = {}^nC_x p^x q^{n-x}$$

$$= {}^{25}C_x \left(\frac{1}{5}\right)^x \left(\frac{4}{5}\right)^{25-x}$$

$$6. \quad n=20, \quad p = 5\%$$

$$= \frac{5}{100}$$

$$p = 0.05 \quad ; \quad q = 0.95$$

$$P(x) = {}^nC_x (p)^x q^{n-x}$$

$$i) \quad P(x=3):$$

$$= {}^{20}C_3 (0.05)^3 (0.95)^{20-3}$$

$$= \frac{20 \times 19 \times 18}{3 \times 2 \times 1} (0.05)^3 (0.95)^{17}$$

$$= 1140 \times 0.0005226$$

$$= 0.059$$

$$ii) \quad P(x \geq 2) = 1 - P(x \leq 1)$$

$$= 1 - P(x=0) + P(x=1)$$

$$= 1 - \left[{}^{20}C_0 (0.05)^0 (0.95)^{20} + {}^{20}C_1 (0.05)^1 (0.95)^{19} \right]$$

$$= 1 - [1(0.3584) + 20(0.05)(0.373)]$$

$$= 1 - 0.7339$$

$$= 0.2643.$$

$$\text{iii) } P(X=4) = {}^{20}C_4 (0.05)^4 (0.95)^{16}$$

$$= \frac{20 \times 19 \times 18 \times 17}{4 \times 3 \times 2 \times 1} \times 0.0000625 \times 0.4401$$

$$= 0.0133.$$

$$\text{iv) mean} = np = 20 \times 0.05$$

$$= 1$$

$$\text{Variance} = npq = 1 \times 0.95$$

$$= 0.95.$$

15.

$\alpha = \text{doublet}$

$$n=4, \quad P(\text{doublet}) = \frac{6}{36} = \frac{1}{6} = p$$

$$q = \frac{5}{6}$$

$$P(x) = {}^nC_x p^x q^{n-x}$$

$$P(x=2) = {}^4C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2$$

$$= \frac{4 \times 3}{2 \times 1} \times \frac{5^2}{6^4}$$

$$= \frac{21}{216} \quad (\text{or}) \quad 0.115$$

$$17 \quad \text{mean} = 4, \quad \text{var} = 3$$

$$np = 4, \quad npq = 3$$

$$4q = 3$$

$$q = \frac{3}{4}; \quad p = \frac{1}{4}; \quad np = 4$$

$$n \frac{1}{4} = 4$$

$$n = 16$$

$$P(x) = {}^nC_x p^x q^{n-x}$$

$$= {}^{16}C_x \left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{16-x}$$

$$P(x=15) = {}^{16}C_{15} \left(\frac{1}{4}\right)^{15} \left(\frac{3}{4}\right)^1$$

$$= 16 \times \frac{3}{(4)^{16}}$$

$$= (4)^2 \times \frac{3}{(4)^{16}}$$

$$= \frac{3}{(4)^{14}}$$

10 x : defective.

$$n = 4.$$

$$p = 18\% = \frac{18}{100} = \frac{9}{50}$$

$$q = \frac{41}{50}$$

$$P(x) = {}^nC_x p^x q^{n-x}$$

$$i) P(x=1) = {}^4C_1 \left(\frac{9}{50}\right)^1 \left(\frac{41}{50}\right)^3$$

$$= 4 \times 0.18 \times 0.5514$$

$$= 0.3969.$$

$$ii) P(x=0) = {}^4C_0 \left(\frac{9}{50}\right)^0 \left(\frac{41}{50}\right)^4$$

$$= 0.4521$$

iii) almost 2 $P(X \leq 2) = P(X=2) + P(X=1) + P(X=0)$

$$= {}^4C_2 \left(\frac{9}{50}\right)^2 \left(\frac{41}{50}\right)^2 + 0.3969 + 0.4521$$

$$= \frac{4 \times 3}{2 \times 1} \times 0.0324 \times 0.6724$$

$$= 0.1307 + 0.849$$

$$= 0.9797$$

Eg: 7.2

coin tossed 6 times.

$$n=6, P=\frac{1}{2}, q=\frac{1}{2}$$

head:

$P(X=2)$:

$${}^nC_x P^x q^{n-x}$$

$$= {}^6C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{6-2}$$

$$= \frac{6 \times 5}{2 \times 1} \left(\frac{1}{4}\right) \left(\frac{1}{2}\right)^4$$

$$= 15 \left(\frac{1}{4}\right) \left(\frac{1}{16}\right)$$

$$P(X=2) = \frac{15}{64}$$

Eg: 7.1 3:2

$$\text{win} = p = \frac{3}{5}, q = \frac{2}{5}, n=5$$

$$P(x) = {}^nC_x P^x q^{n-x}$$

$$P(X \geq 3) = 1 - P(X < 3)$$

$$= 1 - (P(X=0) + P(X=1) + P(X=2))$$

$$= 1 - \left[{}^5C_0 \left(\frac{3}{5}\right)^0 \left(\frac{2}{5}\right)^5 + {}^5C_1 \left(\frac{3}{5}\right)^1 \left(\frac{2}{5}\right)^4 + {}^5C_2 \left(\frac{3}{5}\right)^2 \left(\frac{2}{5}\right)^3 \right]$$

$$= 1 - [0.01024 + 5(0.6)(0.0256)] +$$

$$= 1 - \left[\frac{2^5}{5^5} + 5 \times \frac{3}{5} \times \frac{2^4}{5^4} + \frac{5 \times 4}{2 \times 1} \cdot \frac{3^2}{5^2} \times \frac{2^3}{5^3} \right]$$

$$= 1 - \left[\frac{2^5}{5^5} + 5 \times \frac{3}{5} \times 2^4 + \frac{3^2}{5^3} \times 2^3 \right]$$

$$= 1 - \left[\frac{32 + 240 + 720}{3125} \right]$$

$$= 1 - \frac{992}{3125}$$

$$= 1 - 0.3174$$

$$P(x \geq 3) = 0.6826$$

Eg: 7.3

$$npq = 16$$

$$12q = 16$$

$$q = \frac{16}{12} \neq 1$$

$\therefore$ The Gn statement is wrong.

Eg: 7.5

$$n=5, P(\text{Head})$$

$$P(x) = \frac{1}{2}, q = \frac{1}{2}$$

$$P(x) = {}^n C_x \cdot p^x \cdot q^{n-x}$$

$$P(x=3) = {}^5 C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2$$

$$= \frac{5 \times 4}{3 \times 2} \cdot \frac{1}{2^3} \times \frac{1}{2^2}$$

$$= \frac{5 \times 4}{2 \times 1} \times \frac{1}{2^5}$$

$$= \frac{5}{16}$$

Eg: 7.6

$$\text{mean} = 20,$$

$$np = 20$$

$$\text{S.D} = (4)^2$$

$$npq = 16$$

$$20q = 16$$

$$q = \frac{16}{20}$$

$$q = \frac{4}{5}, p = \frac{1}{5}$$

$$np = 20$$

$$n \cdot \frac{1}{5} = 20 \quad n = 100$$

The Parameters are

$$n = 100, p = \frac{1}{5}, q = \frac{4}{5}$$

$$P(x) = {}^{100}C_x \left(\frac{1}{5}\right)^x \left(\frac{4}{5}\right)^{100-x}$$

Eg: 7.1

$$E(x) = 2$$

$$\downarrow$$

$$\text{mean} = 2$$

$$\downarrow$$

$$np = 2$$

$$\text{var}(x) = \frac{4}{3}$$

$$npq = \frac{4}{3}$$

$$2q = \frac{4}{3}$$

$$q = \frac{4}{3 \times 2}$$

$$q = \frac{2}{3}$$

$$q = \frac{2}{3}, p = \frac{1}{3}, np = 2$$

$$n\left(\frac{1}{3}\right) = 2$$

$$n = 6$$

$$P(x) = {}^nC_x p^x q^{n-x}$$

$$P(x=5) = {}^6C_5 \left(\frac{1}{3}\right)^5 \left(\frac{2}{3}\right)^1$$

$$= \frac{2}{6} \times \frac{1}{3^5} \times \frac{2}{3}$$

$$= \frac{4}{243}$$

$$= 0.016$$

Eg: 7.8

$$n = 7 \text{ (weak)}$$

$$p = \frac{3}{10}$$

$$p = \frac{3}{10}, q = \frac{7}{10}$$

$$P(x) = {}^nC_x p^x q^{n-x}$$

$$P(X \geq 2) = 1 - P(X < 2)$$

$$= 1 - P(X=0) + P(X=1)$$

$$= 1 - \left[{}^7C_0 \left(\frac{3}{10}\right)^0 \left(\frac{7}{10}\right)^7 + {}^7C_1 \left(\frac{3}{10}\right)^1 \left(\frac{7}{10}\right)^6 \right]$$

$$= 1 - [0.082 + 0.247]$$

$$= 1 - 0.329$$

$$= 0.671$$

Eg: 7.4

$$P = 0.4, \quad q = 0.6, \quad n = 5$$

$$i) P(X=1) = {}^5C_1 (0.4)^1 (0.6)^4$$

$$= 5 \times 0.4 \times 0.1296$$

$$= 0.2592$$

$$ii) P(X \geq 1) = 1 - P(X < 1)$$

$$= 1 - P(X=0)$$

$$= 1 - {}^5C_0 (0.4)^0 (0.6)^5$$

$$= 1 - 1(0.07776)$$

$$= 0.9222$$

Eg: 7.9

$$P(\text{Guessing Correctly}) = \frac{1}{2}$$

$$P = \frac{1}{2}, \quad q = \frac{1}{2}, \quad n = 10$$

$$P(X) = {}^nC_x P^x q^{n-x}$$

$$P(X \geq 6) = 1 - P(X < 6)$$

$$= 1 - P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5)$$

$$= 1 - {}^{10}C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{10} + {}^{10}C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^9 + {}^{10}C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^8 + {}^{10}C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^7 + {}^{10}C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^6 + {}^{10}C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^5$$

$$= 1 - \left[(1) \left(\frac{1}{2}\right)^{10} + 10 \left(\frac{1}{2}\right)^{10} + \frac{10 \times 9}{2 \times 1} \left(\frac{1}{2}\right)^{10} + \frac{10 \times 9 \times 8}{3 \times 2 \times 1} \left(\frac{1}{2}\right)^{10} + \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} \left(\frac{1}{2}\right)^{10} + \frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2 \times 1} \left(\frac{1}{2}\right)^{10} \right]$$

$$= 1 - \left(\frac{1}{2}\right)^{10} [1 + 10 + 45 + 120 + 210 + 252]$$

$$= 1 - (0.000976)(638)$$

$$0.000976(638)$$

$$= 1 - 0.62304$$

$$= 0.3769$$

8. $x = \text{girls}$

$$P(G) = \frac{1}{2}, \quad q = \frac{1}{2}, \quad n = 3$$

$$P(x) = {}^nC_x p^x q^{n-x}$$

$$P(x=2) = {}^3C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^1$$

$$= 3 \times \frac{1}{2} \times \frac{1}{2}$$

$$= 0.375$$

9. mean = 1.2

$$np = 1.2, \quad n = 6$$

$$6p = 1.2$$

$$p = \frac{1.2}{6}$$

$$p = 0.2, \quad q = 0.8$$

$$P(x) = {}^nC_x p^x q^{n-x}$$

$$P(x \leq 2) = P(x=0) + P(x=1)$$

$$= {}^6C_0 (0.2)^0 (0.8)^6 + {}^6C_1 (0.2)^1 (0.8)^5$$

$$= 0.262 + 0.3932$$

$$= 0.6552$$



$$P = 0.09, q = 0.91$$

$$P(x \geq 1) = \frac{1}{3}$$

$$1 - P(x < 1) = \frac{1}{3}$$

$$P(x < 1) = 1 - \frac{1}{3}$$

$$P(x=0) = \frac{2}{3}$$

$$P(x) = {}^nC_x p^x q^{n-x}$$

$${}^nC_0 (0.09)^0 (0.91)^n = \frac{2}{3}$$

$$(0.91)^n = 0.67$$

$$\therefore n = 5 \text{ or } n = 0.70$$

7. x = reading news paper

$$P = 40\%$$

$$= 0.4$$

$$q = 0.6, n = 9$$

$$P(x) = {}^nC_x p^x q^{n-x}$$

$$i) P(x=0) = \frac{9!}{0!} (0.4)^0 (0.6)^9$$

$$= 0.01007$$

$$ii) P(x=9) = \frac{9!}{9!} (0.4)^9 (0.6)^0$$

$$= 1 (0.000262)$$

$$= 0.000262$$

$$\text{iii) } P(x \geq \frac{2}{3} \times 9)$$

$$= P(x \geq 6)$$

$$= P(x=6) + P(x=7) + P(x=8) + P(x=9)$$

$$= {}^9C_6 (0.4)^6 (0.6)^3 + {}^9C_7 (0.4)^7 (0.6)^2 + {}^9C_8 (0.4)^8 (0.6)^1 + {}^9C_9 (0.4)^9 (0.6)^0$$

$$= \frac{9 \times 8 \times 7}{3 \times 2 \times 1} \times 0.00088 + \frac{9 \times 8}{2 \times 1} \times 0.000589 + 9 \times 0.00039 + 1 \times 0.00026$$

$$= 0.0739 + 0.0212 + 0.00351 + 0.00026$$

$$= 0.099$$

$$12. n = 28,$$

$$n = 5, P(\text{foreign}) = \frac{18}{28} = 0.64$$

$$q = 0.36$$

$$P(x) = {}^nC_x p^x q^{n-x}$$

$$P(x \geq 3) = 1 - P(x < 3)$$

$$= 1 - P(x=0) + P(x=1) + P(x=2)$$

$$= 1 - [{}^5C_0 (0.64)^0 (0.36)^5 + {}^5C_1 (0.64)^1 (0.36)^4 + {}^5C_2 (0.64)^2 (0.36)^3]$$

$$= 1 - [0.006047 + 5(0.64)(0.01679) + 10(0.4096)(0.0467)]$$

$$= 1 - [0.006047 + 0.05318 + 0.19128]$$

$$= 1 - 0.251048$$

$$= 0.748952$$

13. $P = \frac{1}{2}, Q = \frac{1}{2}, n = 4$

$P(x) = {}^nC_x P^x Q^{n-x}$

$$\begin{aligned} \text{i) } P(x \geq 1) &= 1 - P(x < 1) \\ &= 1 - P(x = 0) \\ &= 1 - {}^4C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^4 \\ &= 1 - 0.0625 \\ &= 0.9375 \end{aligned}$$

$$\begin{aligned} \text{out of } 750 &= 0.9375 \times 750 \\ &= 703 \text{ boys.} \end{aligned}$$

$$\begin{aligned} \text{ii) } P(\text{at most } 2) &= P(x \leq 2) \\ &= 1 - P(x > 2) \\ &= 1 - [P(x = 3) + P(x = 4)] \\ &= 1 - \left[{}^4C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^1 + {}^4C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^0 \right] \\ &= 1 - \left(4 \times \frac{1}{16} + \frac{1}{16} \right) \\ &= 1 - 0.3125 \\ &= 0.6875. \end{aligned}$$

$$\begin{aligned} \text{out of } 750 &= 0.6875 \times 750 \\ &= 516 \text{ girls.} \end{aligned}$$

$$\begin{aligned} \text{iii) } P(\text{both sexes}) &= 1 - [P(x = 0) + P(x = 4)] \\ &= 1 - \left[{}^4C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^4 + {}^4C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^0 \right] \\ &= 1 - [0.0625 + 0.0625] \\ &= 1 - 0.1250 \\ &= 0.8750 \end{aligned}$$

$$\begin{aligned} \text{out of } 750 &= 0.8750 \times 750 \\ &= 656 \text{ G \& Boys.} \end{aligned}$$

20

$$P = 2/3, n = 5$$



$$P = 2(1-P)$$

$$P = 2 - 2P$$

$$3P = 2$$

$$P = 2/3$$

$$q = 1/3$$

$$P(x) = {}^nC_x P^x q^{n-x}$$

$$i) P(x=3) = {}^5C_3 \left(\frac{2}{3}\right)^3 \left(\frac{1}{3}\right)^2$$

$$= \frac{10 \times 8}{243}$$

$$= 0.3292$$

$$ii) P(x \geq 3) = 1 - P(x=0) + P(x=1) + P(x=2)$$

$$= 1 - \left[{}^5C_0 \left(\frac{2}{3}\right)^0 \left(\frac{1}{3}\right)^5 + {}^5C_1 \left(\frac{2}{3}\right)^1 \left(\frac{1}{3}\right)^4 + {}^5C_2 \left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right)^3 \right]$$

$$= 1 - (0.0041 + 0.0411 + 0.1046)$$

$$= 1 - 0.2098$$

$$= 0.7902$$

$$14. n = 15, P = 40\%$$

$$P = 0.4, q = 0.6$$

$$P(x) = {}^nC_x P^x q^{n-x}$$

$$i) P(x=3) = {}^{15}C_3 (0.4)^3 (0.6)^{12}$$

$$= \frac{{}^{15}C_3}{3 \times 2 \times 1} (0.064)(0.00217)$$

$$= 0.0631$$

$$ii) P(x \geq 12)$$

$$= 1 - P(x = 3)$$

$$= 1 - {}^{15}C_3 (0.4)^3 (0.6)^{12}$$

$$= 1 - 0.0631$$

$$iii) P(x \geq 3) = 1 - P(x < 3)$$

$$= 1 - P(x=0) + P(x=1) + P(x=2)$$

$$= 1 - \left[{}^{15}C_0 (0.4)^0 (0.6)^{15} + {}^{15}C_1 (0.4)^1 (0.6)^{14} + {}^{15}C_2 (0.4)^2 (0.6)^{13} \right]$$

$$= 1 - \left[0.0004687 + 15(0.4)(0.0007812) + \frac{15 \times 14}{2 \times 1} (0.16)(0.001302) \right]$$

$$= 1 - [0.0004687 + 0.0046872 + 0.021874]$$

$$= 1 - 0.0270299$$

$$= 0.9729$$

$$18. P = \frac{3}{100} \quad p = 0.03, \quad q = 0.97, \quad n = 10$$

$$P(x) = {}^nC_x p^x q^{n-x}$$

$$P(x \geq 1) = 1 - P(x < 1)$$

$$= 1 - P(x=0)$$

$$= 1 - \left[{}^{10}C_0 (0.03)^0 (0.97)^{10} \right]$$

$$= 1 - 0.73742$$

$$= 0.2625$$

$$19. P = 0.13, \quad q = 0.27, \quad n = 5$$

$$P(x) = {}^nC_x p^x q^{n-x}$$

$$P(x \geq 3) = 1 - [P(x=0) + P(x=1) + P(x=2)]$$

$$= 1 - [P(x=0) + P(x=1) + P(x=2)]$$

$$= 1 - \left[{}^5C_0 (0.73)^0 (0.27)^5 + {}^5C_1 (0.73)^1 (0.27)^4 + {}^5C_2 (0.73)^2 (0.27)^3 \right]$$

$$= 1 - \left[0.00143 + 5(0.73)(0.00531) + \frac{5 \times 4}{2 \times 1} (0.5329)(0.0196) \right]$$

$$= 1 - [0.00143 + 0.01938 + 0.10444]$$

$$= 1 - 0.12525$$

$$= 0.874$$

Eg: 7.11



$$\text{mean} = np$$

$$\text{Variance} = npq$$

$$\text{Sum} \Rightarrow np + npq = 24 \quad (1)$$

$$\text{Product} \Rightarrow np \times npq = 128$$

$$npq = 24 - np$$

$$np(24 - np) = 128$$

$$24np - (np)^2 = 128$$

$$np^2 - 24np + 128 = 0$$

$np = 16$, $np = 8 \rightarrow$ gives value p above 1
so it is omitted.

$$(1) \Rightarrow np = 16$$

$$np + npq = 24$$

$$16 + 16q = 24$$

$$16q = 24 - 16$$

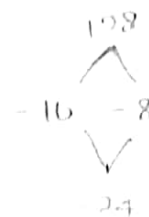
$$16q = 8 \quad q = \frac{8}{16}$$

$$q = \frac{1}{2}, \quad p = \frac{1}{2}$$

$$np = 16$$

$$n\left(\frac{1}{2}\right) = 16$$

$$n = 32$$



$$(1) \Rightarrow np = 8$$

$$np + npq = 128.24$$

$$8 + 8q = 24$$

$$8q = 24 - 8$$

$$8q = 16$$

$$q = 2 \neq 1 \quad \therefore np \neq 8$$

$$P(x) = {}^nC_x p^x q^{n-x}$$

$$= {}^{32}C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{32-x}$$

$$x = 0 \text{ to } 32$$

Eg: 7.10

$$P = 0.2, q = 0.8, n = 10.$$

$$P(x) = {}^nC_x p^x q^{n-x}$$

$$\begin{aligned} \text{i) } P(\text{at least } 1) &= P(x \geq 1) \\ &= 1 - P(x < 1) \\ &= 1 - P(x = 0) \\ &= 1 - \left[{}^{10}C_0 (0.2)^0 (0.8)^{10} \right] \\ &= 1 - [0.10737] \\ &= 0.8926. \end{aligned}$$

$$\begin{aligned} \text{ii) } P(x = 1) &= {}^{10}C_1 (0.2)^1 (0.8)^9 \\ &= 10 (0.2) (0.1342) \\ &= 0.2684. \end{aligned}$$

Eg: 7.12

A, B : $P = \frac{1}{2}, q = \frac{1}{2}$.

a) A beats B exactly 3 games out of 4.

$$n=4, x=3.$$

$$P(x) = {}^nC_x p^x q^{n-x}$$

$$= {}^4C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^1$$

$$= 4 \left(\frac{1}{2^4}\right)$$

$$= 4 \left(\frac{1}{16}\right)$$

$$= 0.25 = 25\%$$

b) A beats B exactly in 5 games out of 8.

$$n=8, x=5$$

$$P(x) = {}^nC_x p^x q^{n-x}$$

$$= {}^8C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^3$$

$$= \frac{8 \times 7 \times 6}{3 \times 2 \times 1} (0.03125)(0.125)$$

$$= 0.21875$$

$$= 21.875\%$$

$\therefore$ first event is more probable.

Exercise 7.2

Poisson Distribution

$$(e^{-\lambda})$$

$$\text{mean} = \text{var} = np = npq = \lambda$$

$$P(x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!}$$

$$6. \quad \lambda = 2.8 = np$$

$x = \text{death}$

$$P(x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!}$$

$$P(x=2) = \frac{e^{-2.8} \cdot (2.8)^2}{2!}$$

$$= \frac{0.06 \times 7.84}{2}$$

$$= 0.2352$$

$$10. \quad \text{mean} = 4 = \lambda$$

$$P(x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!}$$

$$P(x=0) = \frac{e^{-4} \times 4^0}{0!}$$

$$= \frac{0.0183 \times 1}{1}$$

$$= 0.0183$$

$$\text{out of } 100 = 0.0183 \times 100$$

$$= 1.83$$

$$= 2 \text{ days}$$

$$\begin{aligned}
 \text{ii) } P(X \geq 2) &= 1 - P(X < 2) \\
 &= 1 - P(X=0) - P(X=1) \\
 &= 1 - \left(\frac{e^{-4} 4^0}{0!} + \frac{e^{-4} (4)^1}{1!} \right) \\
 &= 1 - (0.0183 + 0.0183 \times 4) \\
 &= 1 - 0.0915 \\
 &= 0.9085.
 \end{aligned}$$

$$\begin{aligned}
 \text{out of } 100 &= 0.9085 \times 100 \\
 &= 90.8 = 91 \text{ days.}
 \end{aligned}$$

$$\begin{aligned}
 \text{iii) } P(X \leq 3) &= P(X=0) + P(X=1) + P(X=2) + P(X=3) \\
 &= 0.0915 + \frac{e^{-4} (4)^2}{2!} + \frac{e^{-4} (4)^3}{3!} \\
 &= 0.0915 + \frac{0.0183 \times 16}{2} + \frac{0.0183 (64)}{3 \times 2} \\
 &= 0.0915 + 0.1464 + 0.1952 \\
 &= 0.4331.
 \end{aligned}$$

$$\begin{aligned}
 \text{out of } 100 &= 0.4331 \times 100 \\
 &= 43 \text{ days.}
 \end{aligned}$$

$$7. \quad P = \frac{5}{100}, \quad n = 120$$

$$\lambda = np = \frac{5}{100} \times 120$$

$$\lambda = 6.$$

$$P(x) = \frac{e^{-\lambda} \lambda^x}{x!}$$

$$\begin{aligned}
 P(X=0) &= \frac{e^{-6} (6)^0}{0!} \\
 &= 0.0025
 \end{aligned}$$

8.

$$\text{mean} = 1.5 = \lambda$$

$$\lambda = 1.5$$

$$P(x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!}$$

$$\begin{aligned} \text{i) } P(x=0) &= \frac{e^{-1.5} \cdot (1.5)^0}{0!} \\ &= 0.2231 \end{aligned}$$

ii) P(some demand is refused)

$$P(x > 2) = 1 - P(x \leq 2)$$

$$= 1 - P(x=0) + P(x=1) + P(x=2)$$

$$= 1 - \left[\frac{e^{-1.5} \cdot (1.5)^0}{0!} + \frac{e^{-1.5} \cdot (1.5)^1}{1!} + \frac{e^{-1.5} \cdot (1.5)^2}{2!} \right]$$

$$= 1 - e^{-1.5} \left[(1.5)^0 + (1.5)^1 + \frac{(1.5)^2}{2} \right]$$

$$= 1 - 0.2231 [1 + 1.5 + 1.125]$$

$$= 1 - [0.2231 \times 3.625]$$

$$= 1 - 0.8087$$

$$= 0.1912$$

C.Q:

$$\text{mean} = 3$$

$$\lambda = 3$$

$$P(x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!}$$

out of 1000 drivers.

$$\text{P) } P(x=0) = \frac{e^{-3} \cdot (3)^0}{0!}$$

$$= 0.0498$$

$$\text{out of 1000} = 0.0498 \times 1000 = 49.8$$

$$ii) P(x \geq 3) = 1 - P(x < 3)$$

$$= 1 - [P(x=0) + P(x=1) + P(x=2)]$$

$$= 1 - \left[\frac{e^{-3} (3)^0}{0!} + \frac{e^{-3} (3)^1}{1!} + \frac{e^{-3} (3)^2}{2!} \right]$$

$$= 1 - e^{-3} \left[1 + 3 + \frac{9}{2} \right]$$

$$= 1 - 0.0498(8.5)$$

$$= 1 - 0.4233$$

$$= 0.5767$$

$$\text{out of } 1000 = 0.5767 \times 1000$$

$$= 576.7 = 577$$

C.Q:

$$P(x=2) = P(x=3) \quad \text{Find } P(x=5)$$

$$P(x) = \frac{e^{-\lambda} \lambda^x}{x!}$$

$$\frac{e^{-\lambda} \lambda^2}{2!} = \frac{e^{-\lambda} \lambda^3}{3!}$$

$$1 = \frac{\lambda}{3}$$

$$\lambda = 3$$

$$P(x=5) = \frac{e^{-3} (3)^5}{5!}$$

$$= \frac{e^{-3} \times 243}{5 \times 4 \times 3 \times 2 \times 1}$$

$$= \frac{0.0497 \times 243}{120}$$

$$= 0.100645 \quad (\text{or})$$

$$0.101$$

Q: In a busstand, the buses are comes inside the busstand per minute are given by Poisson distribution is

$$\lambda = 0.9 \text{ i.e. } 9 \text{ buses}$$

- i) exactly 9 buses comes inside within 5 minutes.
- ii) less than 10 buses comes inside within 8 minutes.
- iii) atleast 14 buses comes inside within 11 minutes

$$\text{i) } P(x=9) \quad \lambda = 0.9 \times 5$$

$$\lambda = 4.5$$

$$P(x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!}$$

$$= \frac{e^{-4.5} \cdot (4.5)^9}{9!}$$

$$\text{ii) } P(x \leq 10) = P(x = 0 \text{ to } 10)$$

$$= \sum_{x=0}^{10} \frac{e^{-7.2} \cdot (7.2)^x}{x!}$$

$$\lambda = 8 \text{ min}^{-1}$$

$$= 8 \times 0.9$$

$$\lambda = 7.2$$

$$\text{iii) } P(x \geq 14) = P(x = 0 \text{ to } 13)$$

$$= 1 - P(x < 14)$$

$$= 1 - P(x = 0 \text{ to } 13)$$

$$= 1 - \sum_{x=0}^{13} \frac{e^{-9.9} \cdot (9.9)^x}{x!}$$

$$\lambda = 11 \text{ min}$$

$$= 11 \times 0.9$$

$$\lambda = 9.9$$

9. $\lambda = 2.5$

$$P(x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!}$$

$$i) P(x=0) = \frac{e^{-2.5} \cdot (2.5)^0}{0!}$$

$$= 0.0821$$

$$ii) P(x=3) = \frac{e^{-2.5} \cdot (2.5)^3}{3!}$$

$$= \frac{0.0821 \times 15.625}{3 \times 2 \times 1}$$

$$= 0.2138$$

$$iii) P(x \geq 5) = 1 - P(x \leq 4)$$

$$= 1 - [P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4)]$$

$$= 1 - e^{-2.5} \left[\frac{(2.5)^0}{0!} + \frac{(2.5)^1}{1!} + \frac{(2.5)^2}{2!} + \frac{(2.5)^3}{3!} + \frac{(2.5)^4}{4!} \right]$$

$$= 1 - 0.0821 [1 + 2.5 + 3.125 + 2.604 + 1.627]$$

$$= 1 - 0.0821 \times 10.856$$

$$= 1 - 0.8912$$

$$= 0.1087$$

10. $\lambda = 0.25$

$$P(x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!}$$

$$P(x \geq 2) = 1 - P(x \leq 1)$$

$$= 1 - [P(x=0) + P(x=1)]$$

$$= 1 - e^{-0.25} \left[\frac{(0.25)^0}{0!} + \frac{(0.25)^1}{1!} \right]$$

$$= 1 - 0.7788 [1 + 0.25]$$

$$= 1 - 0.7788 (1.25) = 0.0265$$

12. $\lambda = 2.$

$$P(x) = \frac{e^{-\lambda} \lambda^x}{x!}$$

$$\begin{aligned} \text{i) } P(x=0) &= \frac{e^{-2} \times (2)^0}{0!} \\ &= 0.1353. \end{aligned}$$

$$\begin{aligned} \text{ii) } P(x \geq 3) &= 1 - P(x < 3) \\ &= 1 - [P(x=0) + P(x=1) + P(x=2)] \\ &= 1 - e^{-2} \left[\frac{(2)^0}{0!} + \frac{(2)^1}{1!} + \frac{(2)^2}{2!} \right] \\ &= 1 - 0.1353 [1 + 2 + 2] \\ &= 1 - 0.6765 \\ &= 0.3235. \end{aligned}$$

Eg: 7.14

$$P(x) = \frac{e^{-\lambda} \lambda^x}{x!}$$

first $P(x=0) = 0.2725$

$$\frac{e^{-\lambda} \lambda^0}{0!} = 0.2725$$

$$e^{-\lambda} = 0.2725.$$

$$e^{\lambda} = \frac{1}{0.2725} = 3.6697$$

$$\lambda = 1.30$$

$$= 1.3$$

$$\begin{aligned} \text{Next } P(x=1) &= \frac{e^{-1.3} (1.3)^1}{1!} \\ &= 0.2725 \times 1.3 \\ &= 0.3543. \end{aligned}$$

Eg: 7.15

$$\lambda = \text{mean} = \frac{390}{520} = 0.75.$$

$$P(x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!}$$

$$\begin{aligned} P(x=0) &= \frac{e^{-0.75} \times (0.75)^0}{0!} \\ \text{no employ} &= 0.4723. \end{aligned}$$

$$\begin{aligned} \text{for 5 page} &= (0.4723)^5 \\ &= 0.0235. \end{aligned}$$

Eg: 7.16

$$\lambda = 10.$$

$$P(x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!}$$

$$P(x \leq 5) = P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4) + P(x=5)$$

$$= e^{-10} \left[\frac{(10)^0}{0!} + \frac{(10)^1}{1!} + \frac{(10)^2}{2!} + \frac{(10)^3}{3!} + \frac{(10)^4}{4!} + \frac{(10)^5}{5!} \right]$$

$$= 0.000045 \left[1 + 10 + \frac{100}{2} + \frac{1000}{3 \times 2 \times 1} + \frac{10000}{4 \times 3 \times 2 \times 1} + \frac{100000}{5 \times 4 \times 3 \times 2 \times 1} \right]$$

$$= 0.000045 [1 + 10 + 50 + 166.67 + 416.67 + 833.33]$$

$$= 0.000045 [1477.64]$$

$$= 0.0665.$$

Eg 7.17

$$\lambda = \frac{1}{\frac{5}{100}}$$

$$\lambda = \frac{1}{500} = 0.002 \times 10$$

$$\lambda = 0.02$$

$$P(x) = \frac{e^{-\lambda} \lambda^x}{x!}$$

$$i) P(x=0) = \frac{e^{-0.02} \cdot (0.02)^0}{0!}$$

$$= 0.9802$$

$$\begin{aligned} \text{In } 10 \text{ packets} &= 0.9802 \times 10 \\ 10000 &= 98020 \end{aligned}$$

$$ii) P(x=1) = \frac{e^{-0.02} \cdot (0.02)^1}{1!}$$

$$= 0.9802 \times 0.02$$

$$= 0.0196$$

$$\begin{aligned} \text{In } 10 \text{ packets} &= 1960 \\ 10000 & \end{aligned}$$

$$iii) P(x=2) = \frac{e^{-0.02} \cdot (0.02)^2}{2!}$$

$$= \frac{0.9802 \times 0.0004}{2}$$

Eg: 7.18

$$\lambda = 20000 \times 0.0001$$

$$\lambda = 2$$

$$P(x) = \frac{e^{-\lambda} \lambda^x}{x!}$$

$$\begin{aligned}
 \text{i) } P(x=3) &= \frac{e^{-2} (2)^3}{3!} \\
 &= \frac{0.1353 \times 8}{3 \times 2 \times 1} \\
 &= 0.1804
 \end{aligned}$$

$$\begin{aligned}
 \text{ii) } P(x \geq 2) &= 1 - P(x \leq 2) \\
 &= 1 - [P(x=0) + P(x=1) + P(x=2)] \\
 &= 1 - e^{-2} \left[\frac{(2)^0}{0!} + \frac{(2)^1}{1!} + \frac{(2)^2}{2!} \right] \\
 &= 1 - 0.1353 [1 + 2 + 2] \\
 &= 1 - 0.1353(5) \\
 &= 0.3235
 \end{aligned}$$

Eg: 7.19

$$\lambda = 8$$

$$P(x) = \frac{e^{-\lambda} \lambda^x}{x!}$$

$$\begin{aligned}
 \text{i) } P(x=5) &= \frac{e^{-8} (8)^5}{5!} \\
 &= \frac{0.000335 \times 32768}{5 \times 4 \times 3 \times 2 \times 1} \\
 &= 0.0915 \times 100 \\
 &= 9.16\%
 \end{aligned}$$

Eg: 7.20

$$P = \frac{1}{80} \quad \lambda = \frac{1}{80} \times 30$$

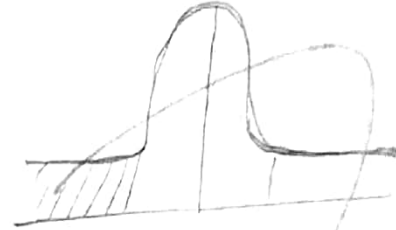
$$\lambda = 0.375$$

$$\begin{aligned}
 P(x \geq 2) &= 1 - P(x < 2) \\
 &= 1 - [P(x=0) + P(x=1)] \\
 &= 1 - e^{-0.375} \left[\frac{(0.375)^0}{0!} + \frac{(0.375)^1}{1!} \right] \\
 &= 0.6813 (1 + 0.375) = 0.0556
 \end{aligned}$$

Exercise 7.3

Normal Distribution.

$$P(Z < 2) = 0.9772$$



+2

$$P(Z < 2) = 0.5 + P(Z < 2)$$

$$= 0.5 + P(Z < 2)$$

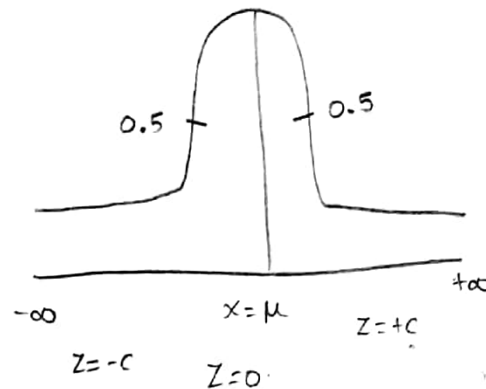
$$f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2} \left(\frac{x-\mu}{\sigma} \right)^2}$$

$$\mu = \text{Mean} = np = npq = \lambda.$$

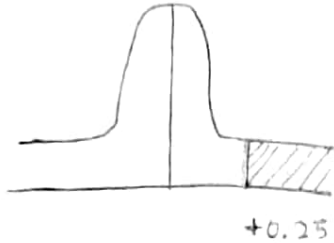
σ = Standard deviation

$$\Rightarrow z = \frac{x-\mu}{\sigma}$$

$$f(z) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2} z^2}$$



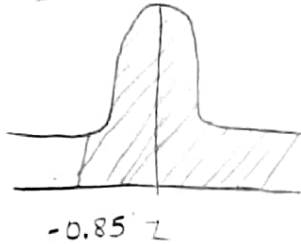
$$P(Z > c) = 0.25$$



$$P(Z > c) = 0.5 - P(Z = c)$$



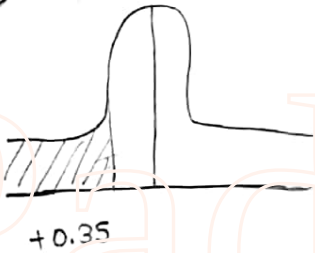
$$P(Z < c) = 0.85$$



$$P(Z > c) = 0.5 + P(Z = c)$$

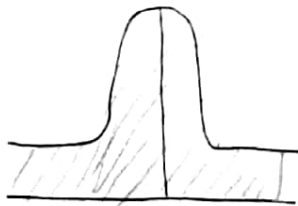
$$P(Z < c) = 0.5 + P(Z = c)$$

$$P(Z < c) = 0.35$$

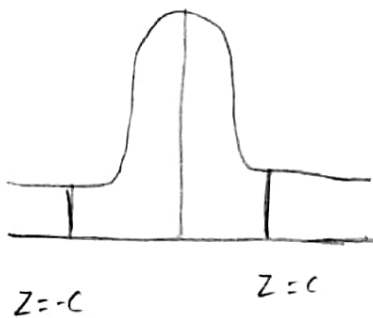


$$P(Z < c) = 0.5 - P(Z = c)$$

$$P(Z < c) = 0.99$$



$$P(Z < c) = 0.5 + P(Z = c)$$



$$P(-c < Z < c)$$

$$= 2P(0 < Z < c)$$

$$(or)$$

$$= 2P(Z = c)$$

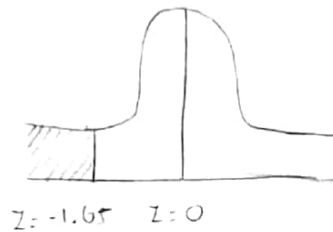
Eg: 1.21

i) $P(Z > 1.09)$



$$\begin{aligned} P(Z > 1.09) &= 0.5 - P(Z = 1.09) \\ &= 0.5 - 0.3621 \\ &= 0.1379 \end{aligned}$$

ii) $P(Z < -1.65)$



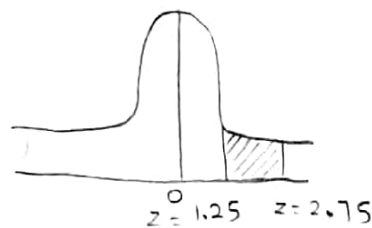
$$\begin{aligned} P(Z < -1.65) &= 0.5 - P(Z = 1.65) \\ &= 0.5 - 0.4505 \\ &= 0.0495 \end{aligned}$$

iii) $P(-1.00 < Z < 1.96)$



$$\begin{aligned} P(-1.00 < Z < 1.96) &= P(Z = 1.00) + P(Z = 1.96) \\ &= 0.3413 + 0.4750 \\ &= 0.8163 \end{aligned}$$

iv) $P(1.25 < Z < 2.75)$



$$\begin{aligned} P(1.25 < Z < 2.75) &= P(Z = 2.75) - P(Z = 1.25) \\ &= 0.4970 - 0.3944 \\ &= 0.1026 \end{aligned}$$

Eg: 7.22

$$\mu = 30, \sigma = 5.$$

$$i) 26 \leq x \leq 40$$

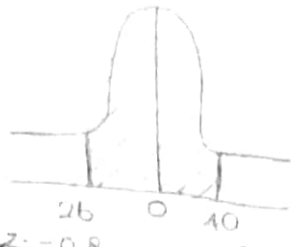
$$x = 26$$

$$z = \frac{x - \mu}{\sigma}$$

$$= \frac{26 - 30}{5}$$

$$= -\frac{4}{5}$$

$$z = -0.8$$



$$x = 40$$

$$= \frac{40 - 30}{5}$$

$$= \frac{10}{5}$$

$$z = 2.$$

$$P(26 \leq x \leq 40) = P(-0.8 \leq z \leq 2)$$

$$= P(z \leq 2) - P(z \leq -0.8)$$

$$= 0.9772 - 0.2081$$

$$= 0.7691.$$

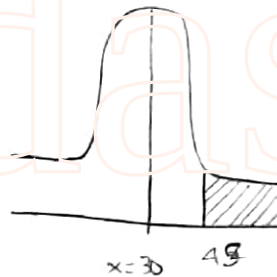
$$ii) x > 45$$

$$x = 45$$

$$z = \frac{45 - 30}{5}$$

$$= \frac{15}{5}$$

$$z = 3$$



$$P(x > 45) = P(z > 3)$$

$$= 0.5 - P(z \leq 3)$$

$$= 0.5 - 0.9987$$

$$= 0.0013.$$

5. / 14

5. $\mu = 2040$, $\sigma = 60$

$x = \text{bulb}$

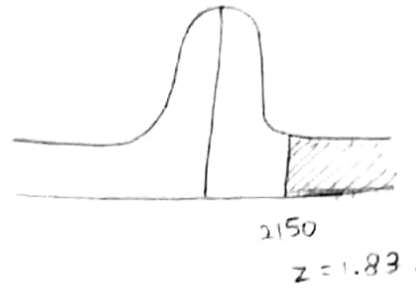
i) $P(x > 2150)$

$$z = \frac{x - \mu}{\sigma}$$

$$= \frac{2150 - 2040}{60}$$

$$= \frac{110}{60}$$

$$z = 1.83$$



$$P(x > 1.83) = 0.5 - P(z = 1.83)$$

$$= 0.5 - 0.4664$$

$$= 0.0336$$

out of 2000 = 0.0336×2000

$$= 67.2$$

(or) 67 bulbs

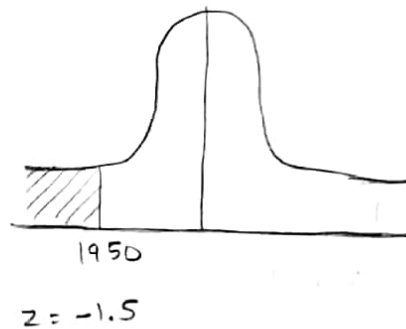
ii) $P(x < 1950)$

$$x = 1950$$

$$z = \frac{1950 - 2040}{60}$$

$$= \frac{-90}{60}$$

$$z = -1.5$$



$$P(x < 1950) = 0.5 - P(z = 1.5)$$

$$= 0.5 - 0.4332$$

$$= 0.0668$$

out of 2000 = 0.0668×2000

$$= 133.6$$

(or) 134 bulbs

$$\text{iii) } P(1920 \leq x \leq 2100)$$

$$x = 1920$$

$$x = 2100$$

$$z = \frac{1920 - 2040}{60}$$

$$= \frac{-120}{60}$$

$$z = -2$$

$$z = \frac{2100 - 2040}{60}$$

$$= \frac{60}{60}$$

$$z = 1$$



$$\begin{aligned} P(1920 \leq x \leq 2100) &= P(z = -2) + P(z = 1) \\ &= 0.4772 + 0.3413 \\ &= 0.8185. \end{aligned}$$

$$\begin{aligned} \text{out of } 2000 &= 0.8185 \times 2000 \\ &= 1637. \end{aligned}$$

$$8. \mu = 68, \sigma = 3.$$

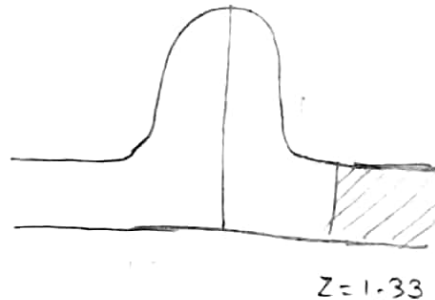
$$1) P(x > 72).$$

$$x = 72$$

$$z = \frac{x - \mu}{\sigma}$$

$$= \frac{72 - 68}{3}$$

$$z = 1.33$$



$$\begin{aligned} P(x > 72) &= 0.5 - P(z = 1.33) \\ &= 0.5 - 0.4082 \\ &= 0.0918 \end{aligned}$$

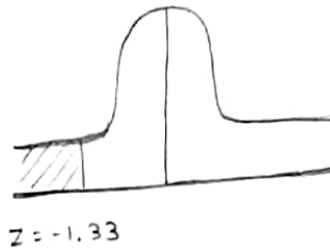
$$\begin{aligned} \text{out of } 500 &= 0.0918 \times 500 \\ &= 45.9 \\ &= 46. \end{aligned}$$

ii) $P(x \leq 64)$

$$x = 64$$

$$z = \frac{64 - 68}{3}$$

$$z = -1.33$$



$$P(x \leq 64) = 0.5 - P(z = 1.33)$$

$$= 0.5 - 0.4082$$

$$= 0.0918$$

$$\text{out of } 500 = 0.0918 \times 500$$

$$= 45.9$$

$$\text{or } (46)$$

iii) $P(65 < x < 71)$

$$x = 65$$

$$x = 71$$

$$z = \frac{65 - 68}{3}$$

$$z = \frac{71 - 68}{3}$$

$$z = -1$$

$$z = 1$$



$$P(65 < x < 71) = P(z = 1) + P(z = -1)$$

$$= 0.3413 + 0.3413$$

$$= 0.6826$$

$$\text{out of } 500 = 0.6826 \times 500$$

$$= 341.3$$

$$\approx 341$$

$$= P(-1 < z < 1)$$

$$= 2P(z \leq 1)$$

$$= 2 \times 0.3413$$

$$= 0.6826$$

$$\text{out of } 500 = 0.6826 \times 500$$

$$= 341.3$$

$$= 341$$

8. $\mu =$

7. $\mu = 12, \sigma = 4$

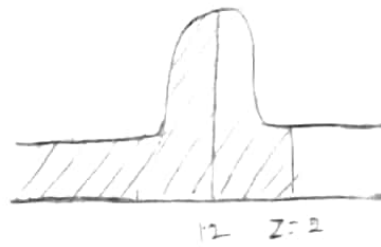
i) $P(X \leq 20)$

$x = 20$

$z = \frac{x - \mu}{\sigma}$

$= \frac{20 - 12}{4}$

$z = 2$



$P(X \leq 20) = 0.5 + P(Z = 2)$

$= 0.5 + 0.4712$

$= 0.9712$

$= 0.9712$

ii) $P(0 \leq X \leq 12)$

$x = 0$

$x = 12$

$z = \frac{0 - 12}{4}$

$z = \frac{12 - 12}{4}$

$z = -3$

$z = 0$



$P(0 \leq X \leq 12) = P(X = 3) + P(X = 0)$

$= 0.4987 + 0.000$

$= 0.4987$

9. $\mu = 16.28, \sigma = 0.12$

i) $P(X \leq 16.35)$

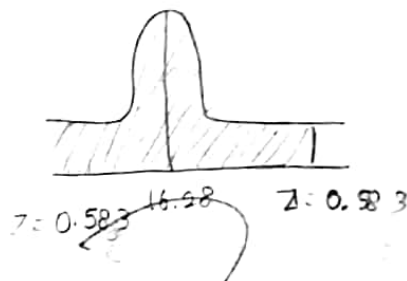
$x = 16.35$

$z = \frac{x - \mu}{\sigma}$

$= \frac{16.35 - 16.28}{0.12}$

$= \frac{0.07}{0.12}$

$z = 0.583$



$$P(x < 16.35) = 0.5 + P(Z < 0.0058)$$

$$= 0.5 + 0.2190$$

$$= 0.719$$

Eg: 7.23 $\mu = 550$ $\sigma = 15000$

i) $P(1,25,000 < x < 1,45,000)$

$$x = 1,25,000$$

$$x = 1,45,000$$

$$Z = \frac{x - \mu}{\sigma}$$

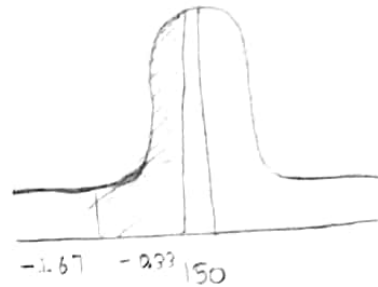
$$Z = \frac{x - \mu}{\sigma}$$

$$= \frac{1,25,000 - 1,50,000}{15000}$$

$$= \frac{1,45,000 - 1,50,000}{15000}$$

$$Z = -1.67$$

$$Z = -0.33$$



$$P(1,25,000 < x < 1,45,000) = P(Z = -0.33)$$

$$= P(Z = 1.67) - P(Z = 0.33)$$

$$= 0.4525 - 0.1293$$

$$= 0.3232$$

out of 550 =
 0.3232×550
 $= 178$

ii) $P(1,40,000 < x < 1,60,000)$

$$x = 1,40,000$$

$$x = 1,60,000$$

$$Z = \frac{x - \mu}{\sigma}$$

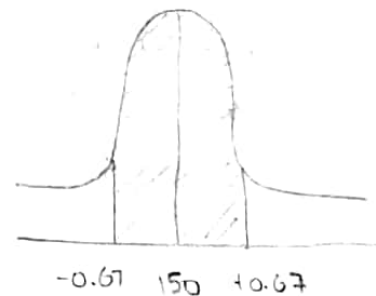
$$Z = \frac{x - \mu}{\sigma}$$

$$= \frac{1,40,000 - 1,50,000}{15000}$$

$$= \frac{1,60,000 - 1,50,000}{15000}$$

$$Z = -0.67$$

$$Z = 0.67$$



$$P(1,40,000 < x < 1,60,000) = P(Z = 0.67) + P(Z = 0.67)$$

$$= 0.2486 + 0.2486$$

$$= 0.4972$$

out of 550 = $0.4972 \times 550 = 273$

Eg: 7.24

$$\mu = 69.25, \text{ Variance} = 10.8$$

$$S.D = \sqrt{\text{Variance}}$$

$$= \sqrt{10.8}$$

$$\sigma = 3.286$$

$$i) P(x > 74)$$

$$x = 74$$

$$Z = \frac{x - \mu}{\sigma}$$

$$= \frac{74 - 69.25}{3.286}$$

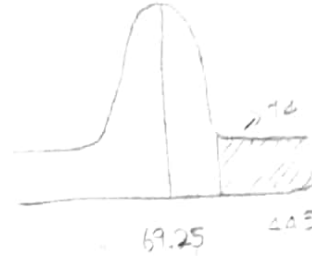
$$Z = 1.445$$

$$P(x > 74) = 0.5 - P(Z = 1.445)$$

$$= 0.5 - 0.4251$$

$$= 0.0749$$

$$\text{out of 1200 children} = 0.0749 \times 1200 = 90$$



Eg: 7.25.

$$\mu = 45, \sigma = 10$$

$$i) P(x < 35)$$

$$x = 35$$

$$Z = \frac{x - \mu}{\sigma}$$

$$= \frac{35 - 45}{10}$$

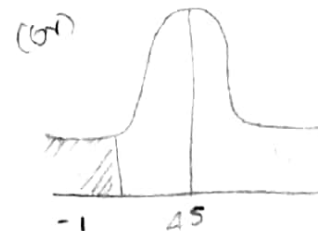
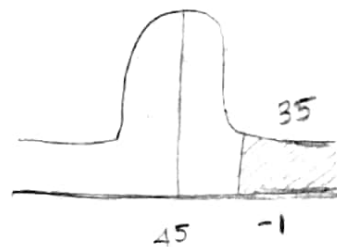
$$Z = -1$$

$$P(x < 35) = 0.5 - P(Z = 1)$$

$$= 0.5 - 0.3413$$

$$= 0.1587$$

$$\text{out of 1300} = 0.1587 \times 1300 = 206$$



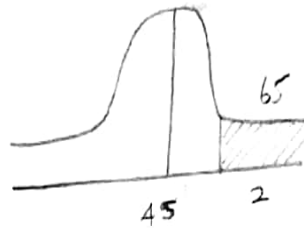
ii) $P(x > 65)$

$$x = 65$$

$$Z = \frac{x - \mu}{\sigma}$$

$$= \frac{65 - 45}{10}$$

$$Z = 2.$$



$$P(x > 65) = 0.5 - P(Z = 2)$$

$$= 0.5 - 0.4772$$

$$= 0.0228$$

out of 1300 students = 30.

Eg: 7.26

$$\mu = 125, \sigma = 18.$$

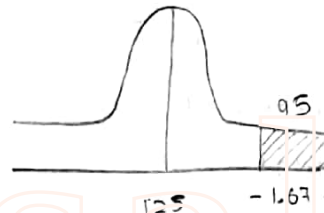
i) $P(x < 95)$

$$x = 95.$$

$$Z = \frac{x - \mu}{\sigma}$$

$$= \frac{95 - 125}{18}$$

$$Z = -1.67.$$



$$P(x < 95) = 0.5 - P(Z = 1.67)$$

$$= 0.5 - 0.4525$$

$$= 0.0475$$

$$\text{out of } 900 = 0.0475 \times 900 = 43.$$

Eg: 7.27

$$\mu = 69.25.$$

$$\text{variance} = 9.8$$

$$S.D = \sqrt{\text{variance}}$$

$$\sigma = 3.13.$$

i) $P(x > 72)$

$$x = 72.$$

$$x \leq 6.$$

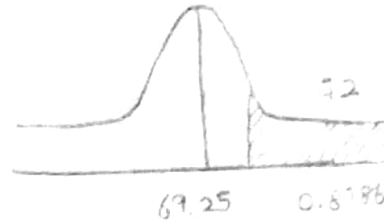
Soldier (6 × 12)

$$x = 72$$

$$= \frac{76 - 69.25}{3.13}$$

$$28 + 20.21$$

$$z = 0.8786$$



$$P(x > 72) = 0.5 - P(z = 0.8786)$$

$$= 0.5 - 0.3078$$

$$= 0.1922$$

$$\text{out of } 6000 = 0.1922 \times 6000$$

$$0.1922 \times 6000$$

$$= 1153.2$$

Eg: 7.28

$$\mu = 5, \quad \sigma = 0.6$$

i) $P(x < 6)$

$$x = 6$$

$$z = \frac{x - \mu}{\sigma}$$

$$= \frac{6 - 5}{0.6}$$

$$= 1.67$$



$$P(x < 6) = 0.5 - P(z = 1.67)$$

$$= 0.5 - 0.4525$$

$$= 0.0475$$

ii) $P(3.5 < x < 6.5)$

$$x = 3.5$$

$$x = 6.5$$

$$z = \frac{x - \mu}{\sigma}$$

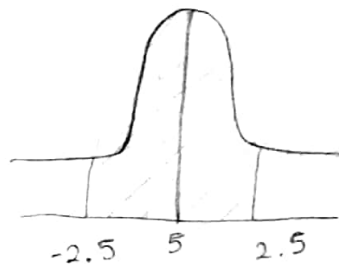
$$z = \frac{x - \mu}{\sigma}$$

$$= \frac{3.5 - 5}{0.6}$$

$$= \frac{6.5 - 5}{0.6}$$

$$z = -2.5$$

$$z = 2.5$$



$$\begin{aligned}
 P(8.5 < x < 6.5) &= P(z = 2.5) + P(z = 2.5) \\
 &= 0.4918 + 0.4918 \\
 &= 0.9836
 \end{aligned}$$

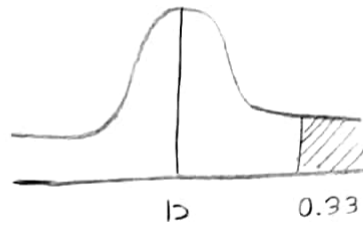
Eg: 7.29

$$\mu = 12, \sigma = 3$$

i) $P(x > 13)$

$$x = 13$$

$$\begin{aligned}
 z &= \frac{x - \mu}{\sigma} \\
 &= \frac{13 - 12}{3} \\
 &= 0.33
 \end{aligned}$$



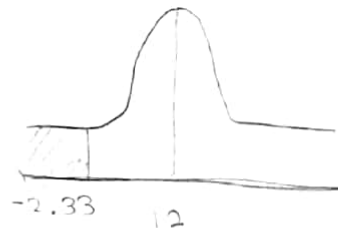
$$\begin{aligned}
 P(x > 13) &= 0.5 - P(z = 0.33) \\
 &= 0.5 - 0.1293 \\
 &= 0.3707
 \end{aligned}$$

$$\text{out of } 125 = 0.3707 \times 125 = 46.3375$$

ii) $P(x < 5)$

$$x = 5$$

$$\begin{aligned}
 z &= \frac{x - \mu}{\sigma} \\
 &= \frac{5 - 12}{3} \\
 &= -2.33
 \end{aligned}$$



$$\begin{aligned}
 P(x < 5) &= 0.5 - P(z = 2.33) \\
 &= 0.5 - 0.4901 \\
 &= 0.0099
 \end{aligned}$$

$$\text{out of } 125 = 0.0099 \times 125 = 1.2375$$

iii) $P(9 < x < 14)$

$$x = 9, \quad x = 14$$

$$z = \frac{x - \mu}{\sigma}$$

$$= \frac{9 - 12}{3}$$

$$= -1$$

$$z = \frac{x - \mu}{\sigma}$$

$$= \frac{14 - 12}{3}$$

$$= 0.667$$

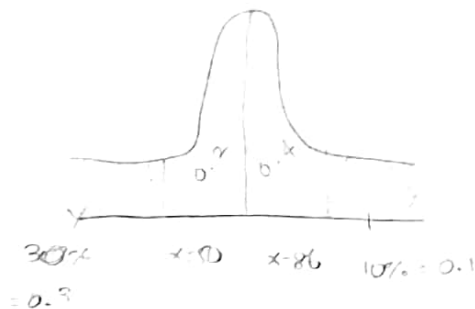


$$P(9 < x < 14) = P(z = -1) + P(z = 0.667)$$

$$= 0.3413 + 0.2486$$

$$= 0.5899$$

6.



$$P(x = 50) = 0.2$$

$$P(z = c) = 0.2$$

$$z = -c = -0.52$$

$$\frac{x - \mu}{\sigma} = -0.52$$

$$\frac{50 - \mu}{\sigma} = -0.52$$

$$50 - \mu = -0.52\sigma$$

$$\mu - 0.52\sigma = 50 \dots (1)$$

$$P(x = 86) = 0.4$$

$$P(z = c) = 0.4$$

$$z = c = 1.28$$

$$\frac{x - \mu}{\sigma} = 1.28$$

$$\frac{86 - \mu}{\sigma} = 1.28$$

$$86 - \mu = 1.28\sigma$$

$$\mu + 1.28\sigma = 86 \quad (2)$$

$$(2) \rightarrow \mu + 1.28\sigma = 86$$

$$(1) \rightarrow \begin{array}{r} \mu - 0.5\sigma = 50 \\ (-) \quad (+) \quad (-) \\ \hline \end{array}$$

$$1.8\sigma = 36$$

$$\sigma = \frac{36}{1.8}$$

$$\sigma = 20$$

$$(1) \Rightarrow \mu - 0.52\sigma = 50$$

$$\mu - 0.52(20) = 50$$

$$\mu - 10.4 = 50$$

$$\mu = 50 + 10.4$$

$$\mu = 60.4$$

$$10. \mu = 450, \sigma = 150$$

$$x = 450 + p$$

$$i) \text{ Penalty} = \frac{200000}{10000} = 20$$

$$P(X \geq 450 + 20)$$

$$= P(X \geq 470)$$

$$x = 470$$

$$Z = \frac{x - \mu}{\sigma}$$

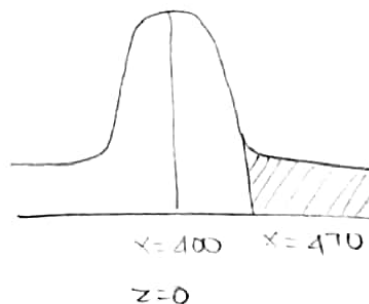
$$= \frac{470 - 450}{100}$$

$$= \frac{20}{100}$$

$$Z = 0.2$$

$$P(X \geq 470) = P(Z \geq 0.2)$$

$$= 0.5 - P(Z = 0.2)$$



$$= 0.5 - 0.2580$$

$$= 0.242$$

$$\text{ii) } P(x \leq 500)$$

$$x = 50$$

$$z = \frac{500 - 400}{100}$$

$$= \frac{100}{100}$$

$$z = 1$$



$$\begin{aligned} P(x \leq 500) &= 0.5 + P(z=1) \\ &= 0.5 + 0.3413 \\ &= 0.8413 \end{aligned}$$

Eg: 7.30

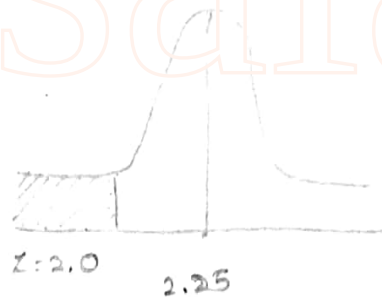
$$\mu = 2.25, \sigma = 0.25$$

$$x = 2.0$$

$$z = \frac{x - \mu}{\sigma}$$

$$= \frac{2.0 - 2.25}{0.25}$$

$$= -1$$



$$\begin{aligned} P(z < 1.0) &= 0.5 - P(z = 1.0) \\ &= 0.5 - 0.3413 \\ &= 0.1587 \end{aligned}$$

Eg: 7.31

$$\sigma = 100, \mu = 800$$

$$P(800 < x < 1000)$$

$$x = 800$$

$$x = 1000$$

$$Z = \frac{x - \mu}{\sigma}$$

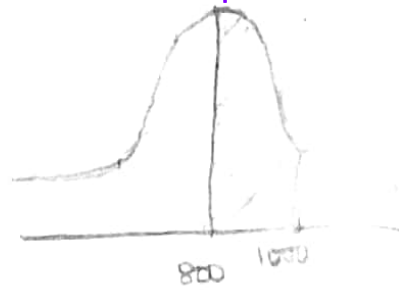
$$= \frac{800 - 800}{100}$$

$$Z = 0$$

$$Z = \frac{x - \mu}{\sigma}$$

$$= \frac{1000 - 800}{100}$$

$$Z = 2$$



$$P(800 < X < 1000) = 0.5 - \cdot$$

$$= P(Z=2) - P(Z=0)$$

$$= 0.4772 - 0$$

$$= 0.4772$$

Padasalai

Chapter 9

Applied Statistics

$$\text{Sum} : Y = T + S + C + J$$

$$\text{Product} : Y = TSCJ$$

Least Square:

$$Y = a + bx, \quad a, b \text{ Constant}$$

Y - actual value
 x - Time.

$$\sum Y = na + b \sum x.$$

$$\sum xy = a \sum x + b \sum x^2.$$

$$a = \frac{\sum y}{n} = \bar{y}, \quad b = \frac{\sum xy}{\sum x^2}.$$

Methods of Simple Average:

Seasonal Indices : S.I.

$$S.I. = \frac{SA}{GA} \times 100.$$

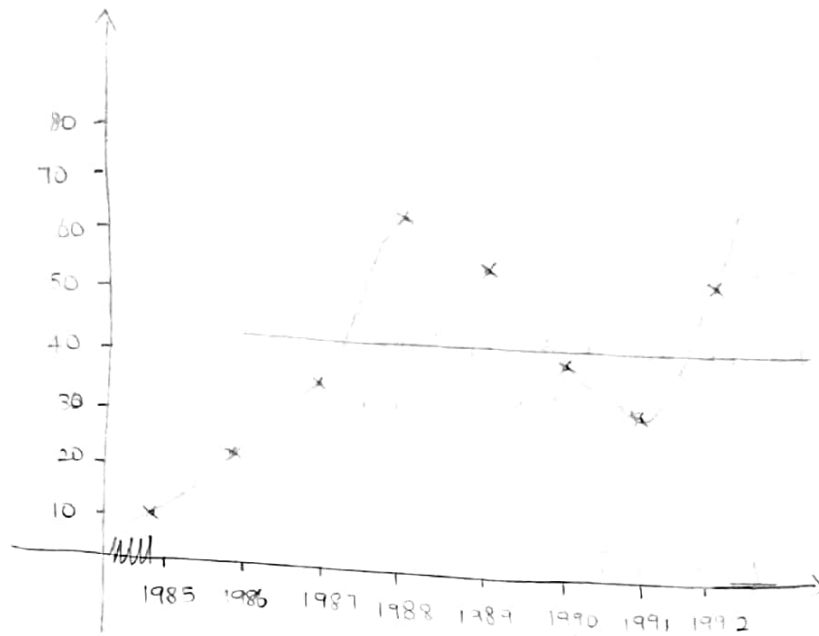
$$\text{Month} \cdot S.I. = \frac{SA(\text{each})}{GA} \times 100.$$

$$\text{Year} S.I. = \frac{SA(\text{each})}{GA} \times 100$$

$$\text{Quarterly} S.I. = \frac{SA(\text{each})}{GA} \times 100$$

16.

Y	1985	1986	1987	1988	1989	1990	1991	1992	1993
Sale	10	22	36	62	35	40	34	50	



15.

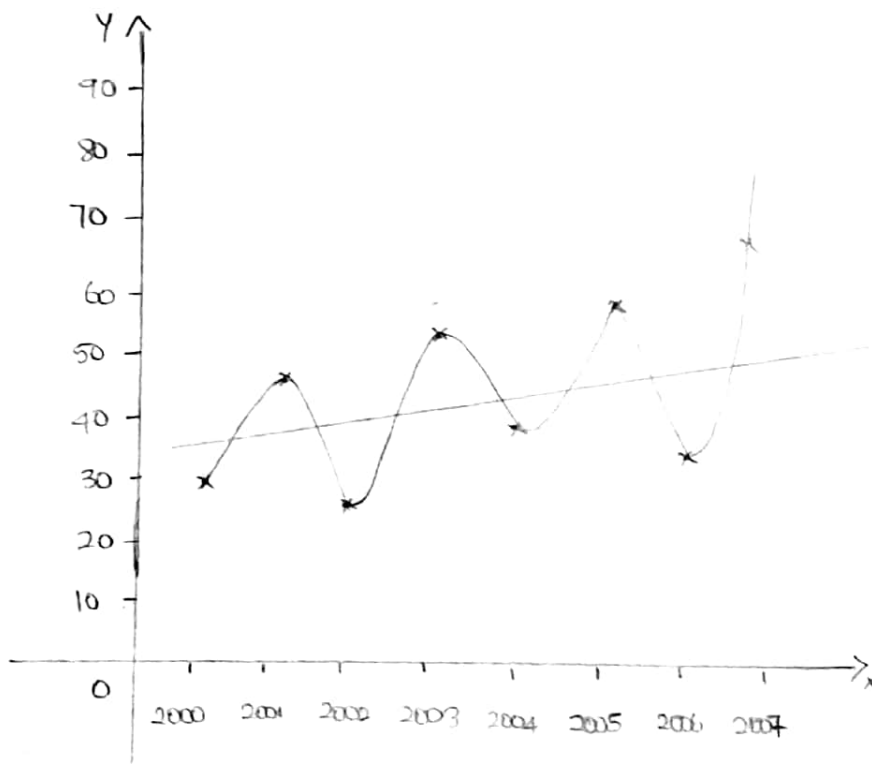
Y	Production	5 Year	5Y Average
1979	126	-	-
1980	123	-	-
1981	117	619	123.8
1982	128	617	123.4
1983	125	624	124.8
1984	124	621	124.2
1985	130	615	123.0
1986	114	609	121.8
1987	112	619	123.8
1988	129	613	122.6
1989	118	606	121.2
1990	123	-	-

14.

y	Profit	3 Years	3y Average
1986	15,420	-	-
1987	15,470	46,410	15470
1988	15,520	52010	1733.61
1989	21,020	63040	21013.33
1990	26,500	79,470	26490
1991	31,950	94,050	31350
1992	35,600	67,550	22516.67
1993	34,900	-	-

Eg: 9.1

Year	2000	2001	2002	2003	2004	2005	2006	2007
Sale	30	46	25	54	40	60	38	65



Eg: 9.4

Year	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004
No. of Stu	332	317	357	392	402	405	410	427	435	438

Year	No. of Student	3 Year	3 Y Average
1995	332	-	-
1996	317	1006	335.33
1997	357	1066	355.33
1998	392	1151	383.67
1999	402	1199	399.67
2000	405	1217	405.67
2001	410	1242	414
2002	427	1272	424
2003	435	1300	433.33
2004	438	-	-

17.

Year	1995	1996	1997	1998	1999	2000	2001
No. of Production centres	155	162	171	180	188	180	178

X	Y	$x = x - A$ $x - 1998$	x^2	xy	Σy Y_t
1995	155	-3	9	-465	159.57
1996	162	-2	4	-324	162.85
1997	171	-1	1	-171	166.19
1998	182	0	0	0	169.42
1999	158	1	1	158	172.71
2000	180	2	4	360	175.99
2001	178	3	9	534	179.28
$n=7$	$\Sigma y = 1186$	$\Sigma x = 0$	$\Sigma x^2 = 28$	$\Sigma xy = 92$	

$$a = \frac{\Sigma y}{n} = \frac{1186}{7} = 169.42 \quad b = \frac{\Sigma xy}{\Sigma x^2} = \frac{92}{28} = 3.285$$

$$y = a + bx$$

$$y = 169.42 + 3.285(x - 1998)$$

$$y_t = 159.565$$

18.

X	Y	$x = x - A$ $= x - 2002$	x^2	xy	Y_t
2000	35	-2	4	-70	43.2
2001	36	-1	1	-36	48.6
2002	79	0	0	0	54.00
2003	80	1	1	80	59.4
2004	46	2	4	80	64.8
$n=5$	$\Sigma y = 270$	$\Sigma x = 0$	$\Sigma x^2 = 10$	$\Sigma xy = 54$	

$$a = \frac{\Sigma y}{n} = \frac{270}{5} = 54 \quad b = \frac{\Sigma xy}{\Sigma x^2} = \frac{54}{10} = 5.4$$

$$y = a + bx$$

$$y = 5.4 + 5.4(x - 2000)$$

$$x = 2000$$

$$y_t = 43.2$$

Ex: 9.7

x	y	$x = \frac{x - A}{h} = \frac{x - 1998.5}{0.5}$	xy	x^2	y_t
1995	6.7	-7	-46.9	49	5.6166
1996	5.3	-5	-26.5	25	5.7190
1997	4.3	-3	-12.9	9	5.8214
1998	6.1	-1	-6.1	1	5.9238
1998.5					
1999	5.6	1	5.6	1	6.0261
2000	7.9	3	23.7	9	6.1285
2001	5.8	5	29.0	25	6.2309
2002	6.1	7	42.7	49	6.333
n=8	$\sum y = 47.8$	$\sum x = 0$	$\sum xy = 8.6$	$\sum x^2 = 148$	$\sum y_t = 50.0238$

$$a = \frac{\sum y}{n} = \frac{47.8}{8} = 5.975, \quad b = \frac{\sum xy}{\sum x^2} = \frac{8.6}{148} = 0.0581$$

$$y = a + bx$$

$$y = 5.975 + 0.0581(x - 1998.5)$$

Ex: $x = 1995, y_t = 5.61$

Eg: 9.6

Year	Sugarcane	$x = x - n$ $= x - 2003$	x^2	xy	y
2000	40	-3	9	-120	42.034
2001	45	-2	4	-90	43.07
2002	46	-1	1	-46	44.11
2003	42	0	0	0	45.143
2004	47	1	1	47	46.179
2005	50	2	4	100	47.22
2006	46	3	9	138	48.251
$n = 7$	$\sum y = 316$	$\sum x = 0$	$\sum x^2 = 28$	$\sum xy = 29$	

$$a = \frac{\sum y}{n} = \frac{316}{7} = 45.143, \quad b = \frac{\sum xy}{\sum x^2} = \frac{29}{28} = 1.0357$$

$$y = a + bx$$

$$y = 45.143 + 1.0357(x - 2003)$$

$$= 42.034$$

a.

x	y	$x = x - 1994$	x^2	xy
1992	46	-2	4	-92
1993	48	-1	1	-48
1994	42	0	0	0
1995	56	1	1	56
1996	52	2	4	104
$n = 5$	$\sum y = 244$	$\sum x = 0$	$\sum x^2 = 10$	$\sum xy = 20$

$$a = \frac{\sum y}{n} = \frac{244}{5} = 48.8, \quad b = \frac{\sum xy}{\sum x^2} = \frac{20}{10} = 2$$

$$Y = a + bx$$

$$Y = 48.8 + 2(x - 1994)$$

$$\text{at } x = 1997$$

$$Y = 48.8 + 2(1997 - 1994)$$

$$= 48.8 + 6$$

$$= 54.8$$

13.

Year	Q I	Q II	Q III	Q IV
2002	3.5	3.8	3.7	3.5
2003	3.6	4.2	3.4	4.1
2004	3.4	3.9	3.7	4.2
2005	4.2	4.5	3.8	4.4
2006	3.9	4.4	4.2	4.6
Total	18.6	20.8	18.8	20.8
Average	3.72	4.16	3.76	4.16

$$\text{Grand Average} = \frac{3.72 + 4.16 + 3.76 + 4.16}{4}$$

$$= \frac{15.8}{4} = 3.95$$

$$\text{S.I for Q I} = \frac{3.72}{3.95} \times 100 = 94.17$$

$$\text{S.I for Q II} = \frac{4.16}{3.95} \times 100 = 105.32$$

$$\text{S.I for Q III} = \frac{3.76}{3.95} \times 100 = 95.18$$

$$\text{S.I for Q IV} = \frac{4.16}{3.95} \times 100 = 105.32$$

20.

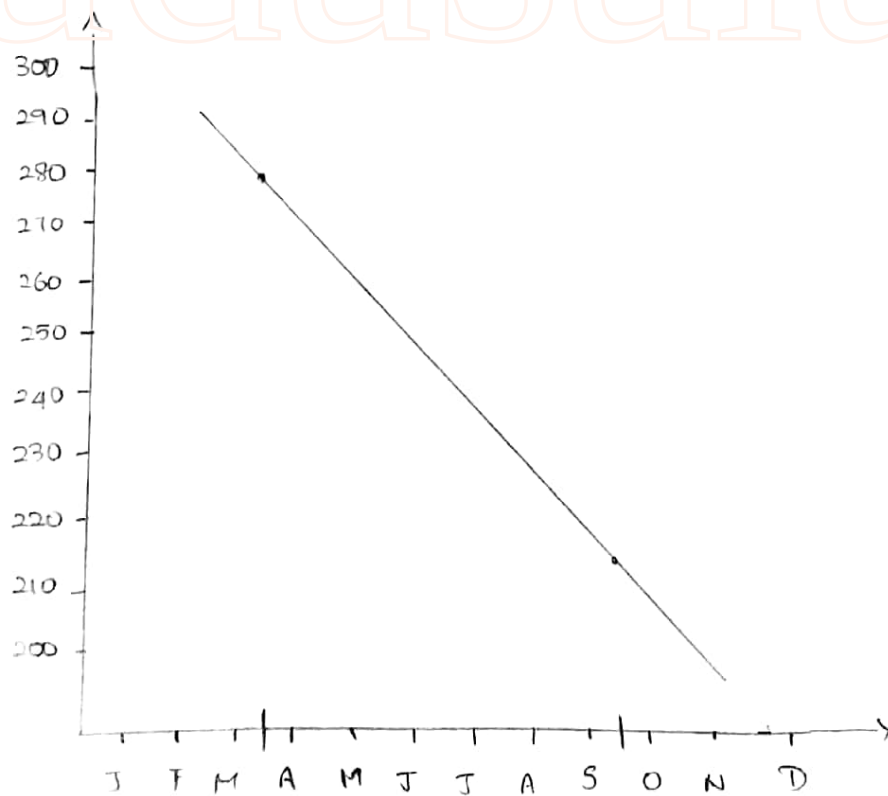
x.	J	F	M	A	M	J	J	A	S	O	N	D.
2002	15	18	17	19	16	20	21	18	17	15	14	18
2003	20	18	16	13	12	15	22	16	18	20	17	15
2004	18	25	21	11	14	16	19	20	17	16	18	20
Total	53	61	54	43	42	51	62	54	52	51	49	53
Average	17.67	20.33	18	14.33	14	17	20.67	18	17.33	17	16.33	17.67

$$\text{Grand Average} = \frac{\text{Total Average}}{\text{Total Number}}$$

$$= \frac{208.33}{12} = 17.36$$

Padasalai

19. Month	Sale	Average
Jan	280	
Feb	240	
Mar	270	
Apr	300	$\frac{1660}{6} = 276.67$
May	280	
Jun	290	
Jul	210	
Aug	200	
Sep	230	$\frac{1280}{6} = 213.33$
Oct	200	
Nov	230	
Dec	210	



Eg: 9.5

Four Yearly Centred Moving Average

Y	Sales	Four Yearly	4 Y Average	4 Y Centred Average
2001	124			
2002	120			
2003	135	519	129.75	132.37
2004	140	540	135.00	139.75
2005	145	578	144.5	147.81
2006	158	605	151.25	155.00
2007	162	635	158.75	
2008	170			

Eg: 9.2.

Y	Production	Average
2000	105	
2001	115	$\rightarrow \frac{340}{3} = 113.33$
2002	120	
2003	100	
2004	110	
2005	125	$\rightarrow \frac{370}{3} = 123.33$
2006	135	

Exercise 9.2

1. Price Index method:

0 - base Year.

1 - current Year

$$\text{Price Index } P_{01} = \frac{\sum P_1 W}{\sum P_0 W} \times 100.$$

$W \rightarrow$ weight, P_1 & P_0 = current + base Price.

q_1 & q_0 = current + base Quantity.

Laspeyre's PIN

$$P_{01}^L = \frac{\sum P_1 q_0}{\sum P_0 q_0} \times 100.$$

~~Laspeyre's~~ Paasche's PIN:

$$P_{01}^P = \frac{\sum P_1 q_1}{\sum P_0 q_1} \times 100.$$

Fisher's PIN:

$$P_{01}^F = \sqrt{P_{01}^L \times P_{01}^P}$$

$$= \sqrt{\frac{\sum P_1 q_0}{\sum P_0 q_0} \times \frac{\sum P_1 q_1}{\sum P_0 q_1}} \times 100.$$

2. Test's:

1. Time Reversal test: (TRT).

$$P_{01}^F = \sqrt{\frac{\sum P_1 q_0}{\sum P_0 q_0} \times \frac{\sum P_1 q_1}{\sum P_0 q_1}} \times 100.$$

$$P_{10}^F = \sqrt{\frac{\sum P_0 Q_1}{\sum P_1 Q_1} \times \frac{\sum P_0 Q_0}{\sum P_1 Q_0}} \times 100$$

$$TRT = P_{01}^F \times P_{10}^F = 1$$

Factorial Reversal test:

(FRT):

$$P_{01} \times Q_{01} = \frac{\sum P_1 Q_1}{\sum P_0 Q_0}$$

$$P_{01} = \sqrt{\frac{\sum P_1 Q_0}{\sum P_0 Q_0} \times \frac{\sum P_1 Q_1}{\sum P_0 Q_1}} \times 100, \quad Q_{01} = \sqrt{\frac{\sum Q_1 P_0}{\sum Q_0 P_0} \times \frac{\sum Q_1 P_1}{\sum Q_0 P_1}} \times 100$$

$P_{01} \rightarrow$ change in price of $Q_{01} \rightarrow$ change in quantity.

3. Cost of Living Index Number:

LIN's:

i) Aggregate Expenditure (or) Weighted Aggregate method:

$$\text{Cost of LIN} = \frac{\sum P_1 Q_0}{\sum P_0 Q_0} \times 100$$

ii) family budget Method:

weight $\rightarrow V \Rightarrow \sum P_0 Q_0$

$$\text{Cost of LIN} = \frac{\sum PV}{\sum V}, \quad P = \frac{P_1}{P_0} \times 100$$

4.

Commodity	base		Current		P_{010}	P_{090}	P_{091}	P_{091}
	P_0	Q_0	P_1	Q_1				
A	10	20	16	10	320	200	160	100
B	38	34	48	42	612	408	756	504
C	15	30	20	26	600	450	520	390
					1532	1058	1436	994

$$\text{Laspeyres' } P_{01}^L = \frac{\sum P_1 Q_0}{\sum P_0 Q_0} \times 100$$

$$= \frac{1532}{1058} \times 100$$

$$= 144.80$$

$$\text{Paasche's } P_{01}^P = \frac{\sum P_1 Q_1}{\sum P_0 Q_1} \times 100$$

$$= \frac{1436}{994} \times 100$$

$$= 144.46$$

$$\text{Fisher's } P_{01}^F = \sqrt{\frac{\sum P_1 Q_0}{\sum P_0 Q_0} \times \frac{\sum P_1 Q_1}{\sum P_0 Q_1}} \times 100$$

$$= \sqrt{144.80 \times 144.46}$$

$$= 144.62$$

Increased by $P_{01}^L = 45\%$, $P_{01}^P = 44\%$, $P_{01}^F = 45\%$.

15.

Commodity	1995		2005		$\sum P_0 Q_0$	$\sum P_1 Q_1$		
	P_0	Q_0	P_1	Q_1		$P_0 Q_0$	$P_1 Q_1$	$P_0 Q_1$
A	5	60	15	10	900	300	1050	350
B	4	20	8	35	160	80	280	140
C	3	15	6	20	90	45	120	60
					1150	425	1450	550

a) Laspeyres's:

$$L_{P_{01}} = \frac{\sum P_1 Q_0}{\sum P_0 Q_0} \times 100$$

$$= \frac{1150}{425} \times 100$$

$$= 270.58$$

b) Paasche's:

$$P_{P_{01}} = \frac{\sum P_1 Q_1}{\sum P_0 Q_1} \times 100$$

$$= \frac{1450}{550} \times 100$$

$$= 263.63$$

Increased by.

$$P_{01}^L = 171\%$$

$$P_{01}^P = 164\%$$

16. Commodity Price Quantity.

	P_0	P_1	Q_0	Q_1	$P_0 Q_0$	$P_0 Q_1$	$P_1 Q_1$	$P_0 Q_1$
A	12	14	18	16	252	216	224	192
B	15	16	20	15	320	300	240	225
C	14	15	24	20	360	336	300	280
D	12	12	29	23	348	348 276	276	216
					1280	1200 1040	1040	913

a) Laspeyres's

$$P_{01}^L = \frac{\sum P_1 Q_0}{\sum P_0 Q_0} \times 100$$

$$= \frac{1280}{1200} \times 100$$

$$= 106.67$$

b) Paasche's:

$$P_{01}^P = \frac{\sum P_1 Q_1}{\sum P_0 Q_1} \times 100$$

$$= \frac{1040}{973} \times 100$$

$$= 106.89$$

c) Fisher's:

$$P_{01}^F = \sqrt{\frac{\sum P_1 Q_0}{\sum P_0 Q_0} \times \frac{\sum P_1 Q_1}{\sum P_0 Q_1}} \times 100$$

$$= \sqrt{106.67 \times 106.89}$$

$$= 106.779$$

Increased: $P_{01}^L = 7\%$, $P_{01}^P = 7\%$, $P_{01}^F = 7\%$.

19. Commodity	2016		2017		$P_1 Q_0$	$P_0 Q_0$	$P_1 Q_1$	$P_0 Q_1$
	P_0	Q_0	P_1	Q_1				
Food	40	12	65	14	780	480	910	560
Fuel	12	14	78	20	1092	1008	1560	1440
Clothing	36	10	36	15	360	360	540	540
Wheat	20	6	42	4	252	120	168	80
Others	46	8	52	6	416	368	312	276
					2900	2336	3490	2896

FRT:

Fisher's $P_{01} = \sqrt{\frac{\sum P_1 q_0}{\sum P_0 q_0} \times \frac{\sum P_0 q_1}{\sum P_1 q_1}} \times 100$

$$= \sqrt{\frac{2900}{2336} \times \frac{3490}{2896}} \times 100$$

$$= \sqrt{1.2414 \times 1.2051} \times 100$$

$$= 122.286$$

TRI:

$$\frac{F}{P_{01}} \times \frac{F}{P_{10}} = \sqrt{\frac{\sum P P_0 q_0}{\sum P_0 q_0} \times \frac{\sum P P_1 q_1}{\sum P_1 q_1} \times \frac{\sum P P_0 q_1}{\sum P_1 q_1} \times \frac{\sum P P_1 q_0}{\sum P_0 q_0}}$$

$$= \frac{2900}{2336} \times \frac{3490}{2896} \times \frac{2896}{3490} \times \frac{2336}{2900}$$

$$= 1$$

Ex 9.10FRT:

Commodity.	Price		Quantity		$\sum P_0 q_0$	$\sum P_0 q_1$	$\sum P_1 q_0$	$\sum P_1 q_1$
	P_0	P_1	q_0	q_1				
Rice	38	35	6	7	228	266	210	245
Wheat	12	18	7	10	84	120	126	180
Rent	10	15	10	15	100	150	150	225
Fuel	25	30	12	16	300	400	360	480
Miscellaneous	30	33	8	10	240	300	264	330
					952	1236	1110	1460

Laplace's:

$$P_{01}^L = \frac{\sum P_1 q_0}{\sum P_0 q_0} \times 100$$

$$= \frac{1110}{952} \times 100 = 116.6$$

Paasche's

$$P_{01}^P = \frac{\sum P_1 q_1}{\sum P_0 q_1} \times 100$$

$$= \frac{1460}{1236} \times 100$$

$$= 118.12$$

Fisher's :

$$P_{01}^F = \sqrt{\frac{\sum P_1 q_0}{\sum P_0 q_0} \times \frac{\sum P_1 q_1}{\sum P_0 q_1}} \times 100$$

$$= \sqrt{116.6 \times 118.12} \times 100$$

$$= 117.36$$

Increased:

$$P_{01}^L = 17\%, \quad P_{01}^P = 18\%, \quad P_{01}^F = 17\%$$

18.

Commodity

Commodity	P_0	q_0	P_1	q_1	$P_1 q_0$	$P_0 q_0$	$P_1 q_1$	$P_0 q_1$
A	5	10	4	12	40	50	48	60
B	8	6	7	7	42	48	49	56
C	6	3	5	4	15	18	20	24
					97	112	117	140

Fisher's :

$$P_{01}^F = \sqrt{\frac{\sum P_1 q_0}{\sum P_0 q_0} \times \frac{\sum P_1 q_1}{\sum P_0 q_1}} \times 100$$

$$= \sqrt{\frac{97}{112} \times \frac{117}{140}} \times 100$$

$$= 106.1\%$$

Increased - 6%.

20.

Commodity	(IN) P	(W) V	Pv.
Food	2450	48	11600
Fuel	1240	20	24800
clothes	3250	12	39000
Rent	3750	15	56250
Miscell	4190	10	41900
		105	279550

∴ Family budget:

$$\text{Cost of LN} = \frac{\sum P_v}{\sum V} = \frac{279550}{105}$$

$$= 2662.380$$

21.

Commodity	Price		Weight	$P = \frac{P_1}{P_0} \times 100$	Pv.
	P_0	P_1	q.		
R	250	280	10	112.0	1120
W	70	85	5	121.4	607
C	150	170	6	113.3	679.8
O	25	35	4	140.0	560
D	85	90	3	105.9	317.7
			<u>28</u>		<u>3284.5</u>

Family budget

$$\text{Cost of LIN} : \frac{\sum P_1 V}{\sum V}$$

$$= \frac{3284.5}{28}$$

$$= 117.30$$

Increased by 17%.

22. Commodity	Price $P_0 P_1$	weight Q_0	$P_1 Q_0$	$P_0 Q_0$
P	22	25	2000	1760
Q	30	45	380	1350
R	42	50	1250	2100
S	25	35	1400	875
T	36	50	2600	1820
			8600	7905

Aggregate Expenditure

$$\text{Cost of LIN} : \frac{\sum P_1 Q_0}{\sum P_0 Q_0} \times 100$$

$$= \frac{8600}{7905} \times 100$$

$$= 108.79$$

Increased by 9%.

Fig: 9.1)

Commodity	Base		Current		$P_1 Q_0$	$P_0 Q_0$	$P_1 Q_1$	$P_0 Q_1$
	P_0	Q_0	P_1	Q_1				
Rice	15	5	16	8	80	75	128	120
wheat	10	6	18	9	108	60	162	90
Rent	8	7	15	8	105	56	120	64
Fuel	9	5	12	6	60	45	72	54
Transport	11	4	11	7	44	44	77	77
Mixell	16	6	15	10	90	96	150	160
					487	376	709	565

Lapayre's:

$$P_{01}^L = \frac{\sum P_1 Q_0}{\sum P_0 Q_0} \times 100$$

$$= \frac{487}{376} \times 100$$

$$= 129.52$$

Paasche's:

$$P_{01}^P = \frac{\sum P_1 Q_1}{\sum P_0 Q_1} \times 100$$

$$= \frac{709}{565} \times 100$$

$$= 125.48$$

Fisher's:

$$F_{01} = \sqrt{\frac{\sum P_1 Q_0}{\sum P_0 Q_0} \times \frac{\sum P_1 Q_1}{\sum P_0 Q_1}} \times 100$$

$$= \sqrt{129.52 \times 125.48}$$

$$= 127.48$$

Increased by:

$$30\%, P_{01}^P = 25\%, P_{01}^F = 27\%$$

Eg: 9.12 T.R.T.

Commodity	Price		Quantity		$P_0 Q_0$	$P_0 Q_1$	$P_1 Q_1$	$P_0 Q_1$
	P_0	P_1	Q_0	Q_1				
Rice	10	13	4	6	52	40	78	60
Wheat	15	18	7	8	126	105	144	120
Fuel	21	14	8	10	112	88	140	110
Misc	14	17	6	7	102	84	119	98
Rent	25	29	5	9	145	125	261	225
					537	442	742	613

Fisher:

$$P_{01}^F = \sqrt{\frac{\sum P_1 Q_0}{\sum P_0 Q_0} \times \frac{\sum P_0 Q_1}{\sum P_1 Q_1}} \times 100$$

$$= \sqrt{\frac{537}{442} \times \frac{742}{613}} \times 100$$

$$= 121.33 \quad 1.211$$

TRI:

$$P_1 \times P_0 = \sqrt{\frac{\sum P_1 Q_0}{\sum P_0 Q_0} \times \frac{\sum P_0 Q_1}{\sum P_1 Q_1} \times \frac{\sum P_1 Q_1}{\sum P_0 Q_1} \times \frac{\sum P_0 Q_0}{\sum P_1 Q_0}}$$

$$= \sqrt{\frac{537}{442} \times \frac{742}{613} \times \frac{613}{742} \times \frac{442}{537}}$$

$$= 1$$

FRI:

$$P_1 \times Q_0 = \frac{\sum P_1 Q_1}{\sum P_0 Q_0}$$

$$P_1 \times Q_0 = \sqrt{\frac{\sum P_1 Q_0}{\sum P_0 Q_0} \times \frac{\sum P_0 Q_1}{\sum P_1 Q_1} \times \frac{\sum P_1 Q_1}{\sum P_0 Q_1} \times \frac{\sum P_0 Q_0}{\sum P_1 Q_0}}$$

$$= \sqrt{\frac{537}{442} \times \frac{742}{613} \times \frac{613}{742} \times \frac{442}{537}}$$

$$= \sqrt{\frac{742 \times 742}{442 \times 442}}$$

$$= \frac{742}{442}$$

$$P_{01} \times Q_{01} = \frac{\sum P_1 Q_1}{\sum P_0 Q_0}$$

Eg: 9.13

Commodity	Base Year		Current		I	P ₀ Q ₀	P ₁ Q ₁	P ₀ Q ₁
	P ₀	Q ₀	P ₁	Q ₁				
Rice	10	5	11	6	55	50	66	60
Wheat	12	6	13	4	48	72	52	48
Rent	14	8	15	7	120	112	105	98
Fuel	16	9	17	8	153	144	136	128
Trans	18	7	19	5	133	126	95	90
Misc	20	4	21	3	84	80	63	60
					623	584	517	484

Fisher:

$$P_{01}^F = \sqrt{\frac{\sum P_1 Q_0}{\sum P_0 Q_0} \times \frac{\sum P_1 Q_1}{\sum P_0 Q_1}} \times 100$$

$$= \sqrt{\frac{623}{584} \times \frac{517}{484}} \times 100$$

$$= 106.74$$

TRI:

$$P_{01} \times P_{10} = \sqrt{\frac{\sum P_1 Q_0}{\sum P_0 Q_0} \times \frac{\sum P_1 Q_1}{\sum P_0 Q_1} \times \frac{\sum P_0 Q_1}{\sum P_1 Q_1} \times \frac{\sum P_0 Q_0}{\sum P_1 Q_0}}$$

$$= \sqrt{\frac{623}{584} \times \frac{517}{484} \times \frac{484}{517} \times \frac{584}{623}}$$

$$= 1$$

FRT:

$$\begin{aligned}
 P_{01} \times Q_{01} &= \sqrt{\frac{\sum P_1 Q_0}{\sum P_0 Q_0} \times \frac{\sum P_1 Q_1}{\sum P_0 Q_1} \times \frac{\sum P_1 P_0}{\sum P_0 P_1} \times \frac{\sum P_1 P_1}{\sum P_0 P_1}} \\
 &= \sqrt{\frac{623}{584} \times \frac{517}{484} \times \frac{484}{584} \times \frac{517}{623}} \\
 &= \sqrt{\frac{517 \times 517}{584 \times 584}} = \frac{517}{584}
 \end{aligned}$$

$$P_{01} \times Q_{01} = \frac{\sum P_1 Q_1}{\sum P_0 Q_0}$$

Eg: 9.14.

Commodity	Base		Current		$P_1 Q_0$	$P_0 Q_0$	$P_1 Q_1$	$P_0 Q_1$
	P_0	Q_0	P_1	Q_1				
Rice	40	5	43	4	240	200	192	160
Wheat	45	2	42	3	84	90	126	135
Rent	90	4	95	5	380	360	570	540
Fuel	85	3	80	2	240	225	160	170
Trains	50	5	65	8	325	250	520	400
Mixco	65	1	72	3	72	65	216	195
					1341	1220	1784	1600

Fisher:

$$P_{01}^F = \sqrt{\frac{\sum P_1 Q_0}{\sum P_0 Q_0} \times \frac{\sum P_1 Q_1}{\sum P_0 Q_1} \times 100}$$

$$= \sqrt{\frac{1341}{1220} \times \frac{1784}{1600} \times 100}$$

$$= 110.71$$

TRT:

$$P_{01} \times P_{10} = \sqrt{\frac{\sum P_1 Q_0}{\sum P_0 Q_0} \times \frac{\sum P_1 Q_1}{\sum P_0 Q_1} \times \frac{\sum P_0 Q_1}{\sum P_1 Q_1} \times \frac{\sum P_0 Q_0}{\sum P_1 Q_0}}$$

$$= \sqrt{\frac{1341}{1220} \times \frac{1784}{1600} \times \frac{1600}{1784} \times \frac{1220}{1341}}$$

$$= 1$$

FRT:

$$P_{01} \times Q_{01} = \frac{\sum P_1 Q_1}{\sum P_0 Q_0}$$

$$P_{01} \times Q_{01} = \sqrt{\frac{\sum P_1 Q_0}{\sum P_0 Q_0} \times \frac{\sum P_1 Q_1}{\sum P_0 Q_1} \times \frac{\sum Q_1 P_0}{\sum Q_0 P_0} \times \frac{\sum Q_1 P_1}{\sum Q_0 P_1}}$$

$$= \sqrt{\frac{1341}{1220} \times \frac{1784}{1600} \times \frac{1600}{1220} \times \frac{1784}{1341}}$$

$$= \sqrt{\frac{1784}{1220}}$$

$$= \frac{1784}{1220}$$

$$P_{01} \times Q_{01} = \frac{\sum P_1 Q_1}{\sum P_0 Q_0}$$

Eg: 9, 18

Commodity.	Price		weight		
	P ₀	P ₁		P ₁ Q ₀	P ₀ Q ₀
A	7	9	10	90	70
B	6	8	12	96	72
C	10	15	17	255	170
D	14	16	19	304	266
E	12	17	15	255	180
				1000	758

$$\text{Cost of LIN} = \frac{\sum P_1 Q_0}{\sum P_0 Q_0} \times 100 = \frac{1000}{758} \times 100 = 131.93$$

Eg: 9.16

Commodity	Price		weight	P ₀ q ₀	
	P ₀	P ₁		P ₀ q ₀	P ₁ q ₀
Rice	1500	1700	5	7500	8500
Sugar	1100	1200	3.5	3850	4200
Pulses	800	950	3	2400	2850
cloth	1200	1550	2	2400	3100
Ghee	550	700	0.75	412.5	525
Rent	2500	3000	12	30000	36000
Fuel	750	600	8	6000	4800
Misc	3200	3500	10	32000	35000
				84562.5	95225

$$\text{Cost of L.I.N} = \frac{\sum P_1 q_0}{\sum P_0 q_0} \times 100$$

$$= \frac{95225}{84562.5} \times 100$$

$$= 112.61$$

Increased by 13%.

Eg: 9.17

Commodity	Price		Quantity	P ₁ q ₀	
	P ₀	P ₁		P ₁ q ₀	P ₀ q ₀
Rice	32	48	25	1200	800
Sugar	25	42	10	420	250
oil	54	85	6	510	324
coffee	250	460	1	460	250
Tea	175	275	2	550	350
				3140	1974

$$\text{Cost of LIN} = \frac{\sum P_1 Q_0}{\sum P_0 Q_0} \times 100$$

$$= \frac{3140}{1974} \times 100$$

$$= 159.06$$

Increased by 59%.

Eg: 9.18

Commodity	Price P_0	P_1	weight (V)	$P = \frac{P_1}{P_0} \times 100$	P_v
A	350	400	40	114.28	4571.4
B	175	250	35	142.85	4999.99
C	100	115	15	115.00	1725
D	75	105	20	140.00	2800
E	60	80	25	133.33	3333.33
			135		16909.73

Family budget

Method:

$$\text{Cost of LIN} = \frac{\sum P_v}{\sum V}$$

$$= \frac{16909.73}{135}$$

$$= 125.26$$

Increased by 25%.

Exercise 9.3CL $\rightarrow$ Centre lineUCL $\rightarrow$ Upper Control lineLCL $\rightarrow$ Lower Control line.

Charts

Mean $\bar{x}$ Range $\bar{R}$

$$\bar{x} = \frac{\sum x}{n}$$

$$\bar{x} = \frac{\sum x}{n}$$

$$\bar{R} = \frac{\sum R}{n}$$

$$R = x_{\max} - x_{\min}$$

 $\bar{x}$
Case I in σ

$$UCL = \bar{x} + 3\sigma$$

$$CL = \bar{x}$$

$$LCL = \bar{x} - 3\sigma$$

Case II

 σ & n not given.

$$UCL = \bar{x} + A_2 \bar{R}$$

$$CL = \bar{x}$$

$$LCL = \bar{x} - A_2 \bar{R}$$

 R :Case I: σ

$$UCL = \bar{R} + 3\sigma$$

$$CL = \bar{R}$$

$$LCL = \bar{R} - 3\sigma$$

Case II not given σ .

$$UCL = D_4 \bar{R}$$

$$CL = \bar{R}$$

$$LCL = D_3 \bar{R}$$

16.

Sample	x_1	x_2	x_3	$\bar{x}$	R.
1	32	36	42	36.7	43-32=10
2	28	32	40	33.3	12
3	39	52	28	39.7	24
4	50	42	31	41.0	19
5	42	45	34	40.3	11
6	50	29	21	33.3	29
7	44	52	35	43.7	17
8	22	35	44	33.7	22

301.7

144

$$\bar{\bar{x}} = \frac{\sum \bar{x}}{n} = \frac{301.7}{8} = 37.71$$

$$\bar{R} = \frac{\sum R}{n} = \frac{144}{8} = 18$$

14.

S	$\bar{x}$	R.
1	15	7
2	17	7
3	15	4
4	18	9
5	17	8
6	14	7
7	18	12
8	15	4
9	17	11
10	16	5
	162	74

$$\bar{\bar{x}} = \frac{\sum \bar{x}}{n} = \frac{162}{10} = 16.2$$

$$\bar{R} = \frac{\sum R}{n} = \frac{74}{10} = 7.4$$

Mean $\bar{x}$

$$UCL = \bar{x} + A_2 \bar{R}$$

$$= 16.2 + 0.58 \times 7.4$$

$$UCL = 20.5$$

$$CL = \bar{x}$$

$$= 16.2$$

$$LCL = \bar{x} - A_2 \bar{R}$$

$$= 16.2 - 0.58 \times 7.4$$

$$LCL = 11.9$$

Range $\bar{R}$

$$UCL = D_4 \bar{R}$$

$$= 2.115 \times 7.4$$

$$= 15.65$$

$$CL = \bar{R}$$

$$= 7.4$$

$$LCL = D_3 \bar{R}$$

$$= 0 \times 7.4$$

$$= 0$$

21.

Time	weights in ml.					$\bar{x}$	$\bar{R}$
8:00 AM	43	(41)	42	(43)	41	42	43-41=2
9:00 AM	40	(39)	40	39	(44)	40.4	5
10:00 AM	42	42	(43)	(48)	40	41	5
11:00 AM	(39)	(43)	40	39	42	106	4
						164	16 ✓

$n=5$; $A_2 = 0.58$, $D_3 = 0$, $D_4 = 2.115$.
(Sample Size)

Mean $\bar{x}$:

$$UCL = \bar{x} + A_2 \bar{R}$$

$$= 42 + 0.58(4)$$

$$= 44.32$$

$$CL = \bar{x}$$

$$= 42$$

$$LCL = \bar{x} - A_2 \bar{R}$$

$$= 42 - 0.58(4)$$

$$= 38.68$$

Range ($\bar{R}$) :

$$UCL = D_4 \bar{R}$$

$$= 2.115 \times 4$$

$$= 8.46$$

$$CL = \bar{R}$$

$$= 4$$

$$LCL = D_3 \bar{R}$$

$$= 0(4)$$

$$= 0$$

$$\bar{x} = \frac{\sum x}{n}$$

$$= \frac{164}{4}$$

$$= 41$$

$$\bar{R} = \frac{\sum R}{n}$$

$$= \frac{16}{4}$$

$$= 4$$

Sample	Mean $\bar{x}$	Range (R)
1	11.2	7
2	11.8	4
3	10.8	8
4	11.6	5
5	11.0	7
6	9.6	4
7	10.4	8
8	9.6	4
9	10.6	7
10	10.0	9
	106.6	63

$$\begin{aligned}\bar{\bar{x}} &= \frac{\sum \bar{x}}{n} \\ &= \frac{106.6}{10} \\ &= 10.66\end{aligned}$$

$$\bar{R} = \frac{\sum R}{n} = \frac{63}{10} = 6.3$$

Mean $\bar{\bar{x}}$:

$$\begin{aligned}UCL &= \bar{\bar{x}} + A_2 \bar{R} \\ &= 10.67 + 0.58(6.3) \\ &= 14.324\end{aligned}$$

$$CL = \bar{\bar{x}} = 10.67$$

$$\begin{aligned}LCL &= \bar{\bar{x}} - A_2 \bar{R} \\ &= 10.67\end{aligned}$$

Range:

$$\begin{aligned}UCL &= D_4 \bar{R} \\ &= 2.115(6.3) \\ &= 13.32\end{aligned}$$

$$\begin{aligned}CL &= \bar{R} \\ &= 6.3\end{aligned}$$

$$\begin{aligned}LCL &= D_3 \bar{R} \\ &= 0\end{aligned}$$

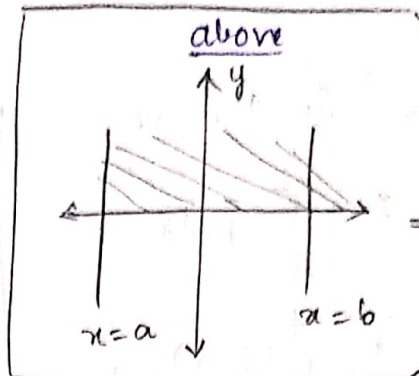
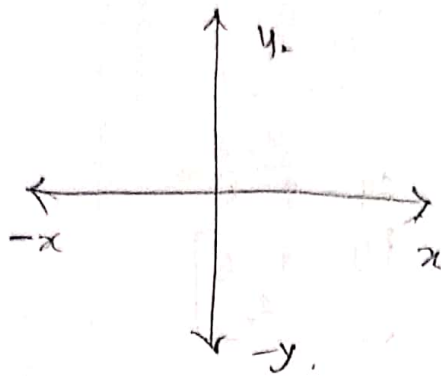
Sample	$\bar{x}$	R
1	29	39
2	26	10
3	37	39
4	34	17
5	14	12
6	45	20
7	39	5
8	20	21
9	34	23
10	23	15
	301	201

$$\bar{\bar{x}} = \frac{\sum \bar{x}}{n} = \frac{301}{10} = 30.1$$

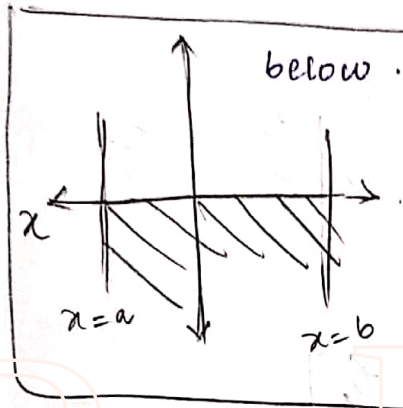
$$\bar{R} = \frac{\sum R}{n} = \frac{201}{10} = 20.1$$

Chapter - 3

Integral calculus - II.

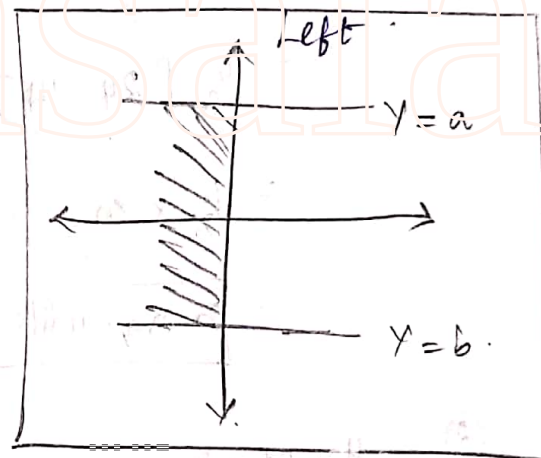
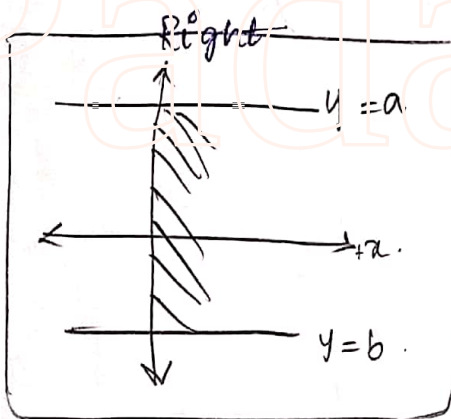


$$A = \int_a^b y \cdot dx$$



$$A = \int_a^b -y \cdot dx$$

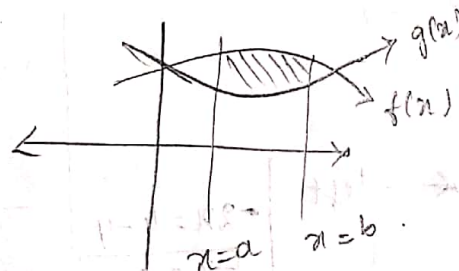
x-axis



$$A = \int_a^b x \cdot dy$$

y-axis

$$A = \int_a^b -x \cdot dy$$



$$A = \int_a^b f(x) - g(x) \cdot dx$$

①. $2y + x = 8$ above $x=2$ $x=4$

x -axis $\uparrow$ above.

$$A = \int_a^b y \cdot dx$$

$$A = \int_2^4 \frac{(8-x)}{2} \cdot dx$$

$$= \frac{1}{2} \left[8x - \frac{x^2}{2} \right]_2^4$$

$$= \frac{1}{2} \left[32 - \frac{16}{2} \right] - \left[16 - \frac{4}{2} \right]$$

$$= \frac{1}{2} [24 - 14]$$

$$= \frac{1}{2} \times 10$$

$$= 5 \text{ sq. units}$$

②. $y - 2x - 4 = 0$

y -axis $\leftarrow$ left

$$A = \int_a^b -x \cdot dy$$

$$-2x = 4 - y$$

$$-x = \frac{4-y}{2}$$

$$x = -\frac{4-y}{2}$$

$$A = \int_1^3 -\left(-\frac{4-y}{2}\right) dy$$

$$= \frac{1}{2} \int_1^3 (4-y) dy$$

$$= \frac{1}{2} \left[4y - \frac{y^2}{2} \right]_1^3$$

$$= \frac{1}{2} \left[12 - \frac{9}{2} \right] - \left[4 - \frac{1}{2} \right]$$

$$= \frac{1}{2} \left[\frac{24-9}{2} - \frac{8-1}{2} \right]$$

$$= \frac{1}{2} \left[\frac{15}{2} - \frac{7}{2} \right]$$

$$= \frac{1}{2} \times \frac{8}{2}$$

$$= 2 \text{ sq. units}$$

$$= \int_1^3 \left(\frac{4-y}{2} \right) dy$$

$$= \frac{1}{2} \left[4y - \frac{y^2}{2} \right]_1^3$$

$$= \frac{1}{2} \left((12 - \frac{9}{2}) - (4 - \frac{1}{2}) \right)$$

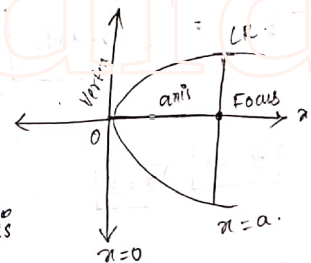
$$= \frac{1}{2} (12 - \frac{9}{2} - 4 + \frac{1}{2})$$

$$= \frac{1}{2} (8 - 4)$$

$$= \frac{1}{2} \times 4$$

$$= 2 \text{ sq. units}$$

③. $y^2 = 4ax$ & Latus Rectum.



$$A = 2 \int_0^a y \cdot dx$$

$$= 2 \int_0^a 2a^{1/2} \cdot x^{1/2} \cdot dx$$

$$= 4a^{1/2} \left(\frac{x^{3/2}}{3/2} \right)_0^a$$

$$= \frac{8}{3} a^{1/2} \times (a^{3/2} - 0)$$

$$y^2 = 4ax$$

$$y = \sqrt{4ax}$$

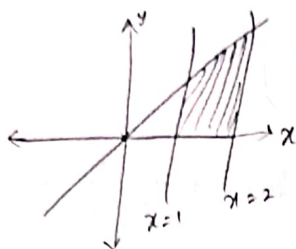
$$y = 2a^{1/2} \cdot x^{1/2}$$

$$= \frac{8a^2}{3} \text{ sq. units}$$

④

$$y = x$$

$$x=1, x=2$$



x -axis $\uparrow$ above.

$$A = \int_a^b y \cdot dx$$

$$A = \int_1^2 x \cdot dx$$

$$= \left(\frac{x^2}{2} \right)_1^2$$

$$= \left[\frac{4}{2} - \frac{1}{2} \right] = \left[\frac{3}{2} \text{ sq. units} \right]$$

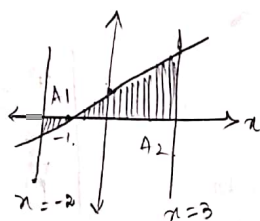
⑤

⑥

$$y-1=x$$

$$x=-2$$

$$x=3$$



x -axis

$$A = A_1 + A_2$$

$$= \int_{-2}^{-1} -y \cdot dx + \int_{-1}^3 y \cdot dx$$

$$= \int_{-2}^{-1} -(x+1) \cdot dx + \int_{-1}^3 (x+1) \cdot dx$$

$$= -\left[\frac{x^2}{2} + x \right]_{-2}^{-1} + \left[\frac{x^2}{2} + x \right]_{-1}^3$$

$$= -\left[\left(\frac{1}{2} - 1 \right) - \left(\frac{4}{2} - 2 \right) \right] + \left[\left(\frac{9}{2} + 3 \right) - \left(\frac{1}{2} - 1 \right) \right]$$

$$= \left[-\frac{1}{2} + 1 + \frac{4}{2} - 2 \right] + \left[\frac{9}{2} + 3 - \frac{1}{2} + 1 \right]$$

$$= \frac{11}{2} + 3$$

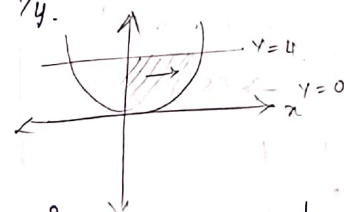
$$= \left[\frac{17}{2} \text{ sq. units} \right]$$

⑥

$$y = 4x^2$$

→ It shows that it is not x -axis
 $x=0$ $y=0$ $y=4$

$$x^2 = \frac{y}{4}$$



y -axis $\rightarrow$

$$A = \int_0^4 x \cdot dy$$

$$x^2 = \frac{y}{4}$$

$$x = \frac{y^{1/2}}{2}$$

$$= \int_0^4 \frac{y^{1/2}}{2} \cdot dy$$

$$= \frac{1}{2} \left[\frac{y^{3/2}}{3/2} \right]_0^4$$

$$= \frac{1}{3} [4^{3/2} - 0]$$

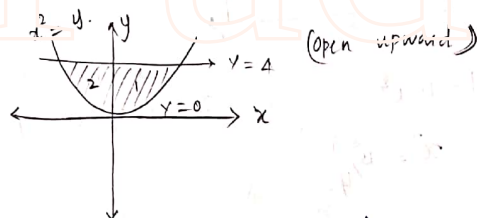
$$= \frac{1}{3} \times (2^3)^{3/2}$$

$$= \frac{1 \times 8}{3}$$

$$= \boxed{\frac{8}{3} \text{ sq. units}}$$

⑦.

$$f = x^2 \quad \& \quad [y = 4]$$



y-axis $\rightarrow$ (Right)

$$A = 2 \int_a^b x \cdot dy$$

$$= 2 \int_0^4 y^{1/2} \cdot dy$$

$$\begin{cases} y = x^2 \\ x = y^{1/2} \end{cases}$$

$$\begin{aligned} 4^{3/2} &= (2^2)^{3/2} \\ &= 8 \end{aligned}$$

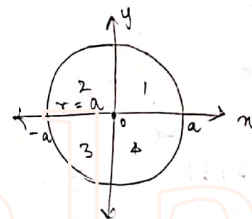
$$= 2 \left(\frac{y^{3/2}}{3/2} \right)_0^4$$

$$= \frac{4}{3} (4^{3/2} - 0)$$

$$= \frac{4}{3} \times (8 - 0)$$

$$= \boxed{\frac{32}{3} \text{ sq. units}}$$

eg: 2.7



$$x^2 + y^2 = a^2$$

$$\text{Put } y = 0 \Rightarrow x^2 = a^2$$

$$\boxed{x = \pm a}$$

x-axis

$$A = 4 \int_a^b y \cdot dx$$

$$A = 4 \int_0^a \sqrt{a^2 - x^2} \cdot dx$$

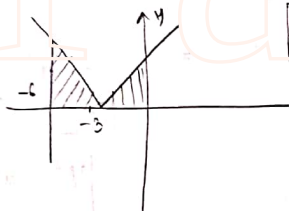
$$\left[\int \sqrt{a^2 - x^2} \cdot dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} \right]$$

$$\begin{aligned}
 &= 4 \left[\frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} \right]_0^a \\
 &= 4 \left(\frac{a}{2} \times 0 + \frac{a^2}{2} \sin^{-1} \frac{a}{a} - 0 \right) \\
 &= 4 \left(\frac{a^2}{2} \times \frac{\pi}{2} \right) \quad \left| \begin{array}{l} \sin^{-1} \frac{\pi}{2} = 1 \\ \pi = \sin^{-1}(1) \end{array} \right. \\
 &= \boxed{\pi a^2 \text{ sq. units}}
 \end{aligned}$$

eg 3.6

$$y = |x+3| \cdot \int_{-6}^0 |x+3| \cdot dx$$

$$y = \begin{cases} +(x+3) & x \geq -3 \\ -(x-3) & x < -3 \end{cases}$$



x	-3	-2	-1	0
y	0	1	2	3

x	-2	-1	0
y	1	2	3

π-ans:

$$\begin{aligned}
 \int_{-6}^0 |x+3| dx &= \int_{-6}^{-3} -(x+3) dx + \int_{-3}^0 (x+3) dx \\
 &= -\left(\frac{x^2}{2} + 3x\right)_{-6}^{-3} + \left(\frac{x^2}{2} + 3x\right)_{-3}^0 \\
 &= -\left(\frac{9}{2} + 9\right) + \left(\frac{9}{2} + 9\right) = 0
 \end{aligned}$$

$$\begin{aligned}
 &= -\left(\left(\frac{9}{2} - 9\right) - \left(\frac{36}{2} - 18\right)\right) + 0 - \left(\frac{9}{2} - 9\right) \\
 &= -\frac{9}{2} + 9 - \frac{36}{2} + 18 - \frac{9}{2} + 9 \\
 &= 36 - \frac{54}{2} \\
 &= 36 - 27 = \boxed{9 \text{ sq. units}} //
 \end{aligned}$$

Ex: 3.2

Formula:

$$\boxed{MC = C'(x)}$$

- ① Marginal cost $MC = \frac{dc}{dn}$, ($c \Rightarrow \text{cost}$)
- ② $c = \int (MC) dn + k$, ($k \rightarrow \text{fixed cost}$)
- ③ Average cost $AC = \frac{c}{n}$
- ④ Marginal Revenue function $MR = \frac{dR}{dn}$
[$R \rightarrow \text{total revenue function}$]
- ⑤ $R = \int (MR) \cdot dn + k$
- ⑥ Demand function $p = \frac{R}{n}$
- ⑦ If $P \rightarrow \text{Profit}$, $P = \int (MR - MC) \cdot dn + k$
- ⑧ Total Inventory cost $= C_1 \int_0^T I(n) \cdot dn$

C_1 = Holding cost, T = time period.

$I(n)$ = Inventory on Hand

(9) Amount of annuity after N payment.

$$A = \int_0^N P e^{rt} \cdot dt \quad \boxed{r = \text{interest}}$$

(10) Total sale = $\int_0^T f(t) \cdot dt$ $r = \text{time}$

(11) Elasticity of demand

(12) $\eta_d = \frac{-P}{x} \cdot \frac{dx}{dP}$

(12) $\frac{E_d}{E_y} = \frac{x}{y} \cdot \frac{dy}{dx}$

(13) Consumer Surplus

$$CS = \int_0^{x_0} f(x) \cdot dx - x_0 p_0$$

$f(x) = \text{demand}$

(14) Producer's Surplus

$$PS = x_0 p_0 - \int_0^{x_0} g(x) \cdot dx$$

$g(x) = \text{supply}$

Ex: 3.2

(1) Total cost

$r = 300$

$$\begin{aligned} &= 10,000 + \int_0^r f(n) \cdot dn \\ &= 10,000 + \int_0^{300} (2n - 240) \cdot dn \\ &= 10,000 + \left[\frac{2n^2}{2} - 240n \right]_0^{300} \\ &= 10,000 + (90,000 - 72,000) - 0 \\ &= 10,000 + 18,000 \\ &= \boxed{28,000} // \end{aligned}$$

(3) $\frac{dC}{dx} = 100 - 10x + 0.1x^2 = MC$

$C = \int MC \cdot dx + k$ $k = 500$

$C = \int (100 - 10x + 0.1x^2) + 500$

↓ No output
 $x=0$
 $C = 100x - \frac{10x^2}{2} + \frac{0.1x^3}{3} + 500$

$C=500$
 $\therefore k=500$
 $C(x) = 100x - 5x^2 + \frac{0.1x^3}{3} + 500$

$AC = \frac{C}{x} = \frac{100 - 5x + \frac{0.1x^2}{3} + \frac{500}{x}}$

Fixed cost $[x=0]$ $[C=0]$

⑦. $MC = 300x^{2/5}$ $\therefore [K=0]$

$$C = \int (MC) dx + K$$

$$= \int 300x^{2/5} + 0$$

$$= 300 \frac{x^{7/5}}{7/5}$$

$$C(x) = \frac{1500}{7} x^{7/5}$$

$$AC = \frac{C}{x} = \frac{1500}{7} x^{2/5}$$

$$AC = \frac{1500}{7} x^{2/5}$$

⑧ $MC = \frac{a}{\sqrt{ax+b}}$ $\lim_{x \rightarrow 0} C = 0, x=0$

$$C = \int (MC) dx + K$$

$$C = \int \frac{a}{\sqrt{ax+b}} dx + K$$

$$= a \int (ax+b)^{-1/2} dx + K$$

$$= a \left[\frac{(ax+b)^{1/2}}{1/2 \times (a+0)} \right] + K$$

$$C = 2\sqrt{ax+b} + K$$

$\lim_{x \rightarrow 0} [C=0] [x=0]$

$$0 = 2\sqrt{a+b} + K$$

$$K = -2\sqrt{b}$$

$$\therefore C = 2\sqrt{ax+b} - 2\sqrt{b}$$

⑨

$$C'(x) = \frac{x^2}{200} + 4 = MC$$

Integrate on both side

$$C = \int_a^b (MC) \cdot dx$$

$$C = \int_0^{200} \left(\frac{x^2}{200} + 4 \right) dx$$

$$= \left[\frac{x^3}{600} + 4x \right]_0^{200}$$

$$= \frac{80,00,000}{600} + 800 - 0$$

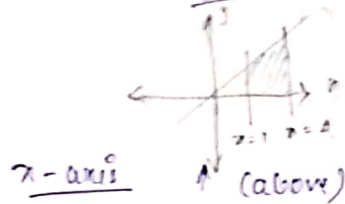
$$= 13333.3 + 800$$

$$= \boxed{14133.3}$$

eg 3.1

$$y = 4x + 3$$

$$x=1 \quad x=4$$



$$A = \int_a^b y \cdot dx$$

$$= \int_1^4 4x + 3 \cdot dx$$

$$= \left[\frac{4x^2}{2} + 3x \right]_1^4$$

$$= 2(4)^2 + 3(4) - (2(1) + 3)$$

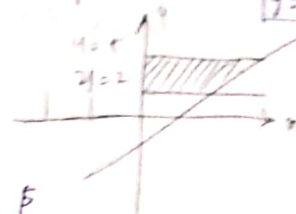
$$= 2(16) + 12 - 2 - 3$$

$$= 32 + 12 - 5$$

$$= \boxed{39 \text{ sq. units.}}$$

eg 3.2

$$x - 2y - 12 = 0$$



$$= \int_2^5 (3y + 12) \cdot dx$$

$$= \left[\frac{3y^2}{2} + 12x \right]_2^5$$

$$= (25 + 60) - (4 + 24)$$

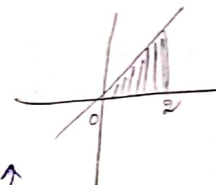
$$= 85 - 28$$

$$= 57 \text{ sq. units}$$

eg 3.3

$$y = 4 - x^2$$

$$x=0 \quad x=2$$



x-axis ↑

$$A = \int_a^b y \cdot dx$$

$$= \int_0^2 4 - x^2 \cdot dx$$

$$= \left[4x - \frac{x^3}{3} \right]_0^2$$

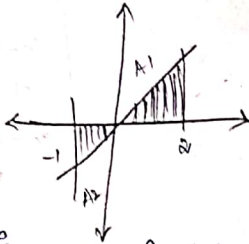
$$= \left(4(2) - \frac{(2)^3}{3} \right) - 0$$

$$= 8 - \frac{8}{3}$$

$$A = 16/3 \text{ sq. units}$$

eg 8.4

$$y = x \quad x = -1 \quad x = 2$$



$-x$ -axis, x -axis $\uparrow \downarrow$

$$= A_1 + A_2$$

$$= \int_{-1}^0 -x \cdot dy + \int_0^2 x \cdot dy$$

$$= \int_{-1}^0 -x \cdot dy + \int_0^2 x \cdot dy$$

$$= -\left[\frac{x^2}{2}\right]_{-1}^0 + \left[\frac{x^2}{2}\right]_0^2$$

$$= \left[0 - \left(-\frac{1}{2}\right)^2\right] + \left[\frac{(2)^2}{2} - 0\right]$$

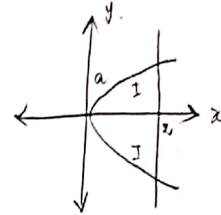
$$= -\left(-\frac{1}{2}\right) + \frac{4}{2}$$

$$= \frac{1}{2} + \frac{4}{2}$$

$$A = \frac{5}{2} \text{ Sq. units.}$$

eg 3.5

$$y^2 = 8x \quad \text{LR}$$



x -axis $\uparrow$

$$A = \int_0^2 y \cdot dx$$

$$= \int_0^2 2\sqrt{x} \cdot x^{1/2} \cdot dx$$

$$= 4\sqrt{2} \left[\frac{x^{3/2}}{3/2} \right]_0^2$$

$$= 4\sqrt{2} \times \frac{2}{3} \left[(2)^{3/2} - 0 \right]$$

$$= \frac{8}{3} (2)^{1/2} \cdot (2)^{3/2}$$

$$= \frac{8}{3} (2)^2$$

$$A = \frac{32}{3} \text{ Sq. units.}$$

$$y^2 = 8ax$$

$$y^2 = 8x$$

$$4a = 8$$

$$a = \frac{8}{4}$$

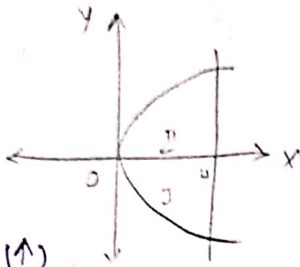
$$a = 2$$

$$y = \sqrt{2x} \quad x=0 \quad x=2$$

$$= 2\sqrt{2} x^{1/2}$$

ex 3.8

$$y^2 = 16x \quad x=0 \quad x=4$$



x-axis (↑)

$$A = 2 \int_0^4 y \cdot dx$$

$$= 2 \int_0^4 (x)^{1/2} \cdot dx$$

$$= 8 \left[\frac{x^{3/2}}{3/2} \right]_0^4$$

$$= 8 \times \frac{2}{3} [(4)^{3/2} - (0)^{3/2}]$$

$$= \frac{16}{3} (4)^{3/2}$$

$$A = \frac{128}{3} \text{ sq. units.}$$

$$\begin{aligned} y^2 &= 16x \\ y &= \sqrt{16x} \\ y &= 4x^{1/2} \end{aligned}$$

ex 3.2

$$(10) MC = 5 + 3e^{-0.03x}$$

$$R = \int (MR) \cdot dx$$

$$R = \int_0^{100} (5 + 3e^{-0.03x}) \cdot dx$$

$$= \left(5x + \frac{3e^{-0.03x}}{-0.03} \right)_0^{100}$$

$$= 500 + \frac{e^{-3}}{-0.01} - \left(0 + \frac{e^0}{-0.01} \right)$$

$$= 500 + \frac{0.05}{-0.01} + \frac{1}{0.01} \times \frac{100}{100}$$

$$= 500 - \frac{5}{1} + \frac{100}{1}$$

$$= 595$$

$$\text{Total revenue} = 595 \times 1000$$

$$= \boxed{595000}$$

(11)

$$MR = 9 - 4x^2$$

$$R = \int (MR) dx + K$$

$$R = \int (9 - 4x^2) \cdot dx + K$$

$$R = 9x - \frac{4x^3}{3} + K$$

$$x=0, R=0 \Rightarrow \boxed{K=0}$$

$$R = qn - \frac{11n^3}{3}$$

demand function $P, R/n$

$$P = q - \frac{11n^2}{3}$$

$$(1) \quad MR = \frac{1}{(2n+3)^2} - 1$$

$$R = \int \left(\frac{1}{(2n+3)^2} - 1 \right) \cdot dn + k \quad \left| \int \frac{1}{n^2} \cdot dn = -\frac{1}{n} \right.$$

$$= \int \left(\frac{1}{(2n+3)^2} - 1 \right) \cdot dn + k$$

$$= \left[\frac{1}{1(2n+3)} \cdot \frac{1}{2} - n \right] + k$$

$$R = \frac{-2}{2n+3} - n + k$$

$$n=0 \quad R=0$$

$$0 = \left(\frac{-2}{(0+3)} - 0 \right) + k$$

$$k = \frac{2}{3}$$

$$R = \frac{-2}{(2n+3)} - n + \frac{2}{3}$$

$$= \frac{-2}{(2n+3)} - n + \frac{2}{3}$$

$$AR = \frac{R}{n} = \frac{-2}{n(2n+3)} - 1 + \frac{2}{3n}$$

$$= \frac{-2}{n(2n+3)} + \frac{2}{3n} - 1$$

$$= \frac{-6n + 2n(2n+3)}{3n^2(2n+3)} - 1$$

$$= \frac{-6n + 4n^2 + 6n}{3n^2(2n+3)} - 1$$

$$= \frac{4}{(6n+9)} - 1$$

$$(12) \quad MR = 20e^{-n/10} \left(1 - \frac{n}{10} \right)$$

$$R = \int (MR) \cdot dn + k$$

$$R = \int 20e^{-n/10} \left(\frac{-n}{10} - 1 \right) \cdot dn + k$$

$$\left[\int e^{ax} (a f(x) + f'(x)) \cdot dx = e^{ax} f(x) \right]$$

$$= 20 \int e^{-n/10} \left(\frac{-1}{10}(n) + 1 \right) \cdot dn + k$$

$$b(n) = n \quad f'(x) = 1$$

$$R = 20e^{-n/10} (n) + k$$

$$n=0 \Rightarrow R=0$$

$$0 = 20e^0(0) + k$$

$$k=0$$

$$R = 20ne^{-n/10}$$

$$\text{Demand function } P = R/n = 20e^{-n/10}$$

$$(19) \quad MR = 20 - 5x + 3x^2$$

$$\int MR = \int 20 - 5x + 3x^2$$

$$R = 20x - \frac{5x^2}{2} + \frac{3x^3}{3} + K$$

$$R = 20x - \frac{5x^2}{2} + x^3 + K$$

$$x=0 \quad R=0 \quad K=0$$

$$\boxed{R = 20x - \frac{5x^2}{2} + x^3}$$

$$(20) \quad MR = 14 - 6x + 9x^2$$

$$\int MR = \int 14 - 6x + 9x^2$$

$$R = 14x - \frac{6x^2}{2} + \frac{9x^3}{3} + K$$

$$R = 14x - 3x^2 + 3x^3 + K$$

$$\text{When } x=0 \quad R=0 \quad K=0$$

$$\boxed{R = 14x - 3x^2 + 3x^3}$$

$$\text{Demand function } P = \frac{R}{x}$$

$$\boxed{P = 14 - 3x + 3x^2}$$

$$(19) \quad c'(x) = 5 + 0.13x$$

$$\int c'(x) = \int (5 + 0.13x) \cdot dx$$

$$\boxed{C = 5x + \frac{0.13x^2}{2} + K}$$

$$\boxed{x=0}, \quad \boxed{C=120}$$

$$120 = 0 + 0 + K$$

$$K_1 = 120$$

$$\boxed{C = 5x + \frac{0.13x^2}{2} + 120}$$

Given

$$R'(x) = 180$$

$$R = 18x + K_2$$

$$x=0$$

$$R=0$$

$$\boxed{R = 18x}$$

$$\boxed{K_2 = 0}$$

$$\text{Profit } f(x) = MR - MC$$

$$= R - C$$

$$= 18x - 5x - \frac{0.13x^2}{2} - 120$$

$$= \boxed{13x - 0.065x^2 - 120}$$

$$(15) R'(x) = 1500 - 4x - 3x^2$$

$$R = 1500x - \frac{4x^2}{2} - \frac{3x^3}{3} + k$$

$$R = 1500x - 2x^2 - x^3 + k$$

$$x=0 \Rightarrow R=0 \Rightarrow \boxed{k=0}$$

$$\therefore R = 1500x - 2x^2 - x^3$$

$$AR = \frac{R}{x}$$

$$= \boxed{1500 - 2x - x^2}$$

$$(16) MR = 10 - 3x - x^2$$

$$\int MR = \int 10 - 3x - x^2$$

$$R = 10x + \frac{3x^2}{2} - \frac{x^3}{3} + k$$

$$R=0, x=0, k=0$$

$$= 10x + \frac{3x^2}{2} - \frac{x^3}{3}$$

Demand function

$$P = \frac{R}{x}$$

$$= \frac{10x}{x} + \frac{3x^2}{2x} - \frac{x^3}{3x}$$

$$\boxed{P = 10 + \frac{3x}{2} - \frac{x^2}{3}}$$

$$(17) MC = \frac{14000}{\sqrt{7x+4}}$$

Fixed cost
FC = 18000

$$C = \int \frac{14000}{\sqrt{7x+4}} dx + k$$

$$d(7x+4) = 7(1)+0 = 7$$

$$= 2000 \int \frac{7}{\sqrt{7x+4}} 2x + k$$

$$= 2000 \times 2 \sqrt{7x+4} + k$$

$$= 4000 \sqrt{7x+4} + k$$

$$x=0 \Rightarrow C = 18000$$

$$18000 = 4000 \sqrt{4} + k$$

$$18000 = 8000 + k$$

$$\boxed{k = 10,000}$$

$$C = 4000 \sqrt{7x+4} + 10,000$$

$$A.C = \frac{C}{x}$$

$$= \frac{4000}{x} \sqrt{7x+4} + \frac{10,000}{x}$$

18) ~~19~~ ~~20~~

$$MC \propto x$$

$$\frac{dc}{dx} = k_1 x$$

$$dc = k_1 x \cdot dx$$

∫ on both side.

$$C = k_1 \frac{x^2}{2} + k_2$$

$$x=0 \quad \boxed{C = 5,000}$$

$$5000 = k_2$$

$$C = k_1 \frac{x^2}{2} + 5000$$

$$x=50 \quad C=5625$$

$$5625 = k_1 \frac{(50)^2}{2} + 5000$$

$$5625 - 5000 = k_1 \frac{(2500)}{2}$$

$$\frac{625}{1250} = k_1$$

$$\boxed{k_1 = \frac{1}{2}}$$

$$\boxed{C = \frac{x^2}{4} + 5000}$$

$$⑤ \quad P=1000 \quad r=5\% = 0.05 \quad n=5$$

$$\text{Amount after annuity } N = \int_0^N P e^{rt} \cdot dt$$

$$= \int_0^5 1000 e^{0.05t} \cdot dt$$

$$= 1000 \left(\frac{e^{0.05t}}{0.05} \right)_0^5$$

$$= \frac{1000}{0.05} (e^{0.25} - e^0)$$

$$= \frac{1000}{0.05} (1.284 - 1)$$

$$= \frac{1000}{0.05} (0.284)$$

$$= \frac{284}{0.05}$$

$$= \boxed{5680}$$

$$④ \quad f(x) = 500 - 0.03x^2$$

$$\text{Holding cost } C_1 = 0.3 \quad t = 30$$

Total inventory cost

$$TIC = C_1 \int_0^t I(x) \cdot dx$$

$$\begin{aligned}
 &= 0.3 \int_0^{30} (500 - 0.03x^2) \cdot dx \\
 &= 0.3 \left[500x - 0.03 \frac{x^3}{3} \right]_0^{30} \\
 &= 0.3 [15000 - 0.01 \times (30)^2 - 0] \\
 &= 0.3 [15000 - 270] \\
 &= 0.3 \times 14730 \\
 &= \boxed{4419}
 \end{aligned}$$

$$(3) \quad \eta_d = \frac{4-x}{x}$$

$$\frac{-P}{x} \cdot \frac{dx}{dP} = \frac{4-x}{x}$$

(x) f(x)

$$\frac{P}{x} \cdot \frac{dx}{dP} = \frac{x-4}{x}$$

$$\frac{dx}{x-4} = \frac{dP}{P}$$

$$d(x-4) = 1$$

J on bs.

$$\int \frac{dx}{x-4} = \int \frac{dP}{P}$$

$$\log |x-4| = \log |P| + \log |C|$$

$$\log |x-4| = \log |PC|$$

$$|x-4| = |PC|$$

$$\boxed{x-4 = PC}$$

$$P=4, \quad x=2$$

$$x-4 = 4C$$

$$C = -\frac{2}{4} = -\frac{1}{2}$$

$$x-4 = Px - \frac{1}{2}$$

$$\underline{\underline{(x)-2}}$$

$$-2x + 8 = P$$

$$\boxed{P = 8 - 2x}$$

$$\boxed{R = Px = 8x - 2x^2}$$

Consumer Surplus (demand) $f(x)$

$$CS = \int_0^{x_0} f(x) dx - x_0 p_0 \quad [\text{equilibrium } P_d = P_s]$$

Producer's Surplus (Supply) $f(x)$

$$PS = x_0 p_0 - \int_0^{x_0} f(x) dx$$

Exercise 3.3

① $P = 50 - 2x$, $x = 20$

$$P = 50 - 40 = 10$$

$$P = 10$$

$$\begin{aligned} CS &= \int_0^{20} f(x) dx - x_0 p_0 \\ &= \int_0^{20} (50 - 2x) dx - 20 \cdot 10 \\ &= \left(50x - \frac{2x^2}{2} \right)_0^{20} - 200 \\ &= 1000 - 400 - 200 \end{aligned}$$

$$P_d = 400$$

③ $P_d = 85 - 5x$, $P_s = 3x - 35$

Given equilibrium

$$P_d = P_s$$

$$85 - 5x = 3x - 35$$

$$85 + 35 = 5x + 3x$$

$$120 = 8x$$

$$x = 15$$

$$P = 85 - 5(15) = 10$$

$$\begin{aligned} CS &= \int_0^{20} f(x) dx - x_0 p_0 \\ &= \int_0^{15} (85 - 5x) dx - 150 \\ &= \left(85x - \frac{5x^2}{2} \right)_0^{15} - 150 \end{aligned}$$

$$\begin{aligned} &= 85(15) - \frac{5 \times 225}{2} - 0 - 150 \\ &= 1275 - 562.5 - 0 - 150 \end{aligned}$$

$$CS = 562.5$$

④ $P = e^{-x}$, $P = 0.5 = \frac{1}{2}$

$$0.5 = e^{-x}$$

$$\frac{1}{2} = e^{-x} = \frac{1}{e^x}$$

$$2 = e^x$$

taking log

$$x = \log 2$$

$$\begin{aligned}
 C_s &= \int_0^{x_0} f(x) dx - x_0 p_0 \\
 &= \int_0^{\log 2} e^{-x} \cdot dx - \log 2 \times \frac{1}{2} \\
 &= \left[\frac{e^{-x}}{-1} \right]_0^{\log 2} - \frac{1}{2} \log 2 \\
 &= -1 [e^{-\log 2} - e^0] - \frac{1}{2} \log 2 \\
 &= -\left(\frac{1}{e^{\log 2}} - 1\right) - \frac{1}{2} \log 2 \\
 &= -\left(\frac{1}{2} - 1\right) - \frac{1}{2} \log 2 \\
 &= \frac{1}{2} - \frac{1}{2} \log 2 \\
 &= \boxed{\frac{1}{2} (1 - \log 2)}
 \end{aligned}$$

⑤. $P = 7+x$ $x=5$

$$\begin{aligned}
 P &= 12 \\
 P_s &= x_0 p_0 - \int_0^{x_0} f(x) dx \\
 &= 60 - \int_0^5 (7+x) dx \\
 &= 60 - \left(7x + \frac{x^2}{2}\right)_0^5 \\
 &= 60 - \left(35 + \frac{25}{2}\right) - 0 \\
 &= 60 - (35 + 12.5)
 \end{aligned}$$

$$\begin{aligned}
 &= 60 - 47.5 \\
 &= \boxed{12.5}
 \end{aligned}$$

⑥. $P = \frac{36}{x+4}$, $P=6$

$$K = \frac{36}{x+4}$$

$$x+4 = 6$$

$$\boxed{x=2}$$

$$\begin{aligned}
 \therefore C_s &= \int_0^{x_0} f(x) dx - x_0 p_0 \\
 &= \int_0^2 \left(\frac{36}{x+4}\right) dx - 12 \\
 &= 36 \int_0^2 \left(\frac{1}{x+4}\right) dx - 12 \\
 &\quad d(x+4) = 1 \\
 &= 36 [\log |x+4|]_0^2 - 12 \\
 &= 36 \log |2+4| - \log |0+4| - 12 \\
 &= 36 \log 6 - \log 4 - 12 \\
 &= 36 \log \frac{6}{4} - 12 \\
 &= 36 \log \frac{3}{2} - 12
 \end{aligned}$$

$$⑧ \quad P_d = 1600 - x^2 \quad P_s = 2x^2 + 400$$

Qn equilibrium.

$$P_d = P_s$$

$$1600 - x^2 = 2x^2 + 400$$

$$1600 - 400 = 2x^2 + x^2$$

$$3x^2 = 1200$$

$$x^2 = 400$$

$$\boxed{x = \pm 20}$$

$$P = 1600 - 20^2 = 1600 - 400$$

$$\boxed{P = 1200}$$

$$P_s = 24000 - \int_0^{20} f(x) \cdot dx$$

$$= 24000 - \int_0^{20} (2x^2 + 400) \cdot dx$$

$$= 24000 - \left(\frac{2x^3}{3} + 400x \right)_0^{20}$$

$$= 24000 - \left(\frac{2(20)^3}{3} + 400(20) \right) - 0$$

$$= 24000 - (5333.3 + 8000)$$

$$= 24000 - 13333.3$$

$$= \boxed{10666.7}$$

$$⑨ \quad P_d = \frac{8}{x+1} - 2, \quad P_s = \frac{x+3}{2}$$

$$\text{equl} \quad P_d = P_s$$

$$\frac{8}{x+1} - 2 = \frac{x+3}{2}$$

$$\frac{8-2x-2}{x+1} = \frac{x+3}{2}$$

$$\frac{6-2x}{x+1} = \frac{x+3}{2}$$

$$\frac{6-2x}{x+1} = \frac{x+3}{2}$$

$$12 - 4x = x^2 + 3x + x + 3$$

$$x^2 + 4x - 9 = 0$$

$$\boxed{x=1} \quad \boxed{x=-9}$$

$$P = \frac{1+3}{2} = \frac{4}{2} = 2$$

$$C_s = \int_0^{20} f(x) \cdot dx - x \cdot P_0$$

$$= \int_0^1 \left(\frac{8}{x+1} - 2 \right) \cdot dx - 2$$

$$= (8 \log |x+1| - 2x)_0^1 - 2$$

$$= (8 \log 2 - 2) - (8 \log 1 - 0) - 2$$

$$\begin{array}{c} -9 \\ \wedge \\ -1 \quad 7 \\ \vee \\ 8 \end{array}$$

$$= 8 \log 2 - 2 - 2$$

$$C_5 = 8 \log 2 - 4$$

$$P_5 = x_0 p_0 - \int_0^{x_0} f(x) dx$$

$$= 2 - \int_0^1 \frac{x+3}{2} dx$$

$$= 2 - \frac{1}{2} \left[\frac{x^2}{2} + 3x \right]_0^1$$

$$= 2 - \frac{1}{2} \left(\frac{1}{2} + 3 \right) - 0$$

$$= 2 - \frac{1}{2} \left(\frac{7}{2} \right)$$

$$= 2 - \frac{7}{4}$$

$$\boxed{P_5 = \frac{1}{4}}$$

$$x = \sqrt{100 - p} \quad \left| \quad x = \frac{p}{2} - 10 \right.$$

$$\frac{p}{2} - 10 = \sqrt{100 - p}$$

square:

$$\left(\frac{p}{2} - 10 \right)^2 = 100 - p$$

$$\frac{p^2}{4} + 100 - 2 \times \frac{p}{2} \times 10 = 100 - p \Rightarrow 0$$

$$\frac{p^2}{4} - 9p = 0$$

$$p^2 - 36p = 0$$

$$p(p - 36) = 0$$

$$\boxed{p = 0} \quad \boxed{p = 36}$$

$$x = \sqrt{100 - 36} = \sqrt{64}$$

$$x = \pm 8$$

$$\boxed{x = 8}$$

$$x = \sqrt{100 - p}$$

$$x^2 = 100 - p$$

$$\boxed{p = 100 - x^2}$$

$$x = \frac{p}{2} - 10$$

$$\frac{p}{2} = x + 10$$

$$\boxed{p = 2x + 20}$$

$$C_5 = \int_0^{x_0} f(x) dx - x_0 p_0$$

$$= \int_0^8 (100 - x^2) dx - 288$$

$$= \left[100x - \frac{x^3}{3} \right]_0^8 - 288$$

$$= 800 - \frac{512}{3} - 288$$

$$= 512 - \frac{512}{3}$$

$$= \frac{1024}{3}$$

$$\boxed{C_5 = 341.3}$$

$$\begin{aligned}
 P_s &= 20 p_0 - \int_0^{20} f(x) dx \\
 &= 288 - \int_0^{20} (2x + 20) dx \\
 &= 288 - \left(\frac{2x^2}{2} + 20x \right)_0^{20} \\
 &= 288 - (64 + 160 - 0) \\
 &= 288 - 224 \\
 &= \boxed{64} //
 \end{aligned}$$

⑪

$$\begin{aligned}
 P_d &= 25 - 3x \\
 P_s &= 5 + 2x \\
 \text{equi } P_d &= P_s \\
 25 - 3x &= 5 + 2x \\
 25 - 5 &= 2x + 3x \\
 20 &= 5x \\
 x &= \frac{20}{5} \\
 x &= 4 \\
 &= 5 + 2(4) \\
 &= 5 + 8 \\
 P &= \boxed{13}
 \end{aligned}$$

$$\begin{aligned}
 C_s &= \int_0^{20} f(x) dx - x_0 p_0 \\
 &= \int_0^{20} (25 - 3x) dx - 52 \\
 &= \left(25x - \frac{3x^2}{2} \right)_0^{20} - 52 \\
 &= \left(100 - \frac{600}{2} \right) - 52 \\
 &= 76 - 52
 \end{aligned}$$

$$C_s = 24 \text{ units.}$$

$$\begin{aligned}
 P_s &= x_0 p_0 - \int_0^{20} f(x) \cdot dx \\
 &= 52 - \int_0^{20} (5 + 2x) \cdot dx \\
 &= 52 - \left(5x + \frac{2x^2}{2} \right)_0^{20} \\
 &= 52 - [20 + 16] \\
 &= 52 - 36
 \end{aligned}$$

$$P_s = 16 \text{ units}$$

Chapter - 5.

Numerical Method

Aim to find approximate values.

x -variable.

$f(x) = y$, next term add h with x .

$\Rightarrow x+h$.

Δ = Del.

∇ = Tel (alpha).

E = Shifting operator.

Forward Diff. operator.

(FDO) $\Rightarrow \Delta$

$$\Delta y_n = y_{n+1} - y_n$$

First Diff = Δ^1

$$\text{eg: } \Delta y_0 = y_1 - y_0$$

$$(*) f(x) = f(x+h) - f(x)$$

$$\text{If } \Delta c = 0, \Delta (cf(x)) = c \Delta f(x).$$

$$* \Delta^m \Delta^n f(x) = \Delta^{m+n} f(x)$$

$$* \Delta (f(x) \cdot g(x)) = f(x) \Delta g(x) + g(x) \Delta f(x).$$

$$* \Delta \frac{f(x)}{g(x)} = \frac{g(x) \Delta f(x) - f(x) \Delta g(x)}{g(x) g(x+h)}.$$

$$\Delta^2 y_n = \Delta y_{n+1} - \Delta y_n.$$

$$\Delta^3 y_n = \Delta^2 y_{n+1} - \Delta^2 y_n.$$

:

:

$$\Delta^{k+1} y_n = \Delta^k y_{n+1} - \Delta^k y_n$$

x	$y = f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
x_0	y_0					
x_1	y_1	Δy_0	$\Delta^2 y_0$	$\Delta^3 y_0$	$\Delta^4 y_0$	$\Delta^5 y_0$
x_2	y_2	Δy_1	$\Delta^2 y_1$	$\Delta^3 y_1$	$\Delta^4 y_1$	$\Delta^5 y_1$
x_3	y_3	Δy_2	$\Delta^2 y_2$	$\Delta^3 y_2$	$\Delta^4 y_2$	$\Delta^5 y_2$
x_4	y_4	Δy_3	$\Delta^2 y_3$	$\Delta^3 y_3$	$\Delta^4 y_3$	$\Delta^5 y_3$
x_5	y_5	Δy_4	$\Delta^2 y_4$	$\Delta^3 y_4$	$\Delta^4 y_4$	$\Delta^5 y_4$

Backward diff. operation.

$$\nabla y_n = y_n - y_{n-1}$$

$$\nabla y_{n+1} = y_{n+1} - y_n$$

$$\nabla^2 y_1 = y_1 - y_0$$

$$\nabla^2 y_1 = y_1 - y_0$$

$$\nabla^3 y_1 = \nabla^2 y_1 - \nabla^2 y_0$$

$$\nabla f(x+nh) = \nabla^n f(x)$$

Shifting operator E

(or) Displacement operator

$$E(f(x_0)) = f(x_0+h)$$

$$E f(x) = f(x+h)$$

$$E^n f(x) = f(x+nh)$$

$$E^{-n} f(x) = f(x-nh)$$

$$E y_0 = y_1$$

$$E y_{n-1} = y_n$$

$$E^2 y_0 = E(E y_0) = E y_1 = y_2$$

$$\therefore E^n y_0 = y_n$$

$$i) \Delta = E - 1$$

$$ii) E \Delta = \Delta E$$

$$iii) \nabla \equiv \frac{E-1}{E}$$

Eg: 5.1

FDO Table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
0	0	0.174	-0.001	-0.001
10	0.174	0.173	-0.002	-0.002
20	0.347	0.171	-0.002	-0.002
30	0.518			

Eg: 5.2:

$$y = f(x) = x^3 + 2x + 1, \quad x = 1, 2, 3, 4, 5$$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1	4	9	12	6	0
2	13	21	18	6	
3	34	39	24		
4	73	63			
5	136				

Eg: 5.3

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
1	8					
2	12	4				
3	19	7	3			
4	29	10	3	0		
5	42	13	3	0		
6	K	K-42	K-55			

Gn 2nd is Constant.

$$K - 55 = 3$$

$$K = 58$$

$\therefore 6^{\text{th}}$ term is 58.

1. $\Delta (\log ax)$

$$\Delta f(x) = f(x+h) - f(x)$$

$$\Delta (\log ax) = \log(ax+h) - \log(ax)$$

$$= \log \frac{ax+h}{ax}$$

$$= \log \left(\frac{ax}{ax} + \frac{h}{ax} \right)$$

$$= \log \left(1 + \frac{h}{ax} \right)$$

$$4. f(x) = x^2 + 3x \quad \therefore \Delta f(x) = 2x + 4$$

$$f(x) = x^2 + 3x$$

$$\Delta f(x) = \Delta (x^2 + 3x)$$

$$\therefore [\Delta f(x) = f(x+h) - f(x)]$$

$$= (x+h)^2 + 3(x+h) - (x^2 + 3x)$$

$$= x^2 + h^2 + 2xh + 3x + 3h - x^2 - 3x$$

$$= h^2 + 2xh + 3h$$

$$h=1$$

$$= 1 + 2x + 3(1)$$

$$= 2x + 4$$

$\therefore$ Hence Proved.

$$3. h=1 \quad ; \quad P_t = (E^{-1} \Delta) x^3 = 3x^2 - 3x + 1$$

$$E^{-n} \Delta^n f(x) = \Delta^n f(x+nh)$$

$$E^{-3} \Delta^3 f(x^3) = \Delta^3 (x-1)^3$$

$$\rightarrow f(x) = f(x+h) - f(x)$$

$$= (x-1+1) - (x-1)^3$$

$$= x^3 - (1 \times x^3 - 3x^2 \times 1 + 3x \times 1^2 - 1 \times 1^3)$$

$$= x^3 - (x^3 - 3x^2 + 3x - 1)$$

$$= x^3 - x^3 + 3x^2 - 3x + 1$$

$$= 3x^2 - 3x + 1$$

$$\therefore L.H.S = R.H.S$$

Proved.

Ex: 5.4

$$(i) \Delta e^{ax} \quad \therefore f(x) = f(x+h) - f(x)$$

$$= e^{a(x+h)} - e^{ax}$$

$$= e^{ax} \cdot e^h - e^{ax}$$

$$= e^{ax} (e^h - 1)$$

$$(ii) \Delta^2 e^x = \Delta [\Delta e^x] \quad f(x) = f(x+h) - f(x)$$

$$= \Delta [e^{x+h} - e^x]$$

$$= \Delta [e^x e^h - e^x]$$

$$= \Delta e^x [e^h - 1]$$

$$\Delta e^x = [e^{x+h} - e^x]$$

$$f(x) = f(x+h) - f(x) = [e^h - 1] [e^h - 1] e^x$$

$$= (e^h - 1)^2 e^x$$

$$(iii) \Delta \log x = \log(x+h) - \log x$$

$$= \log \frac{x+h}{x}$$

$$= \log \left(1 + \frac{h}{x} \right)$$

$$= \log \left[1 + \frac{h}{x} \right]$$

Ex: 5.5

$$u_0 = 1, u_1 = 11, u_2 = 21, u_3 = 28, u_4 = 29, \text{ find } \Delta^4 u_0$$

$$\Delta^4 u_0 =$$

$$(E-1)^4 u_0 = 0$$

$$= (1 \times E^4 - 4E^3 + 6E^2 - 4E + 1) u_0$$

$$= E^4 u_0 - 4E^3 u_0 + 6E^2 u_0 - 4E u_0 + u_0$$

$$= u_4 - 4u_3 + 6u_2 - 4u_1 + u_0$$

$$= 29 - 112 + 126 - 44 + 1$$

$$\Delta^4 u_0 = 0$$

6)

x	0	1	2	3	4
y _x	1	3	9	-	81

$$\Delta^4 y_0 = 0$$

$$\Delta^4 y_0 = 0$$

$$(E-1)^4 y_0 = 0$$

$$(E^4 - 4E^3 + 6E^2 - 4E + 1) y_0 = 0$$

$$E^4 y_0 - 4E^3 y_0 + 6E^2 y_0 - 4E y_0 + y_0 = 0$$

$$y_4 - 4y_3 + 6y_2 - 4y_1 + y_0 = 0$$

$$y_4 - 4y_3 + 6y_2 - 4y_1 + y_0 = 0$$

$$81 - 4y_3 + (6 \times 9) - (4 \times 3) + 1 = 0$$

$$4y_3 = 124$$

$$y_3 = \frac{124}{4}$$

$$y_3 = 31$$

8.	x	0	1	2	3	4	5
y=f(x)	0	-	8	15	-	-	35

$$\Delta^4 y_k = 0.$$

$$(E-1)^4 y_k = 0.$$

$$(E^4 - 4E^3 + 6E^2 - 4E + 1)y_k = 0.$$

$K=0$

$$(E^4 - 4E^3 + 6E^2 - 4E + 1)y_0 = 0$$

$$E^4 y_0 - 4E^3 y_0 + 6E^2 y_0 - 4E y_0 + y_0 = 0$$

$$(E^4 y_0 = y_4)$$

$$y_4 - 4y_3 + 6y_2 - 4y_1 + y_0 = 0.$$

(15x4) (4x2).

$$y_4 - 60 + 48 - 4y_1 + 0 = 0$$

$$y_4 - 4y_1 = 12 \dots (1).$$

$K=1$

$$(E^4 - 4E^3 + 6E^2 - 4E + 1)y_1 = 0.$$

$$E^4 y_1 - 4E^3 y_1 + 6E^2 y_1 - 4E y_1 + y_1 = 0$$

$$(E^4 y_1 = y_{4+1})$$

$$y_5 - 4y_4 + 6y_3 - 4y_2 + y_1 = 0$$

(6x15) (4x8)

$$35 - 4y_4 + 90 - 32 + y_1 = 0$$

$$-4y_4 + y_1 = -93 \dots (2)$$

$$(2) \rightarrow -4y_4 + y_1 = 93$$

$$4 \times (2) \rightarrow -16y_4 + 4y_1 = 372$$

$$-15y_1 = -45 \dots (3)$$

$$y_1 = 3.$$

$$(2) \Rightarrow y_4 - 4y_1 = 12.$$

$$y_4 - 4(3) = 12$$

$$y_4 - 12 = 12$$

$$y_4 = 24.$$

Eg: 5.11.

Year	1961	1962	1963	1964	1965	1966	1967
Production (4)	200	220	260	-	350	-	430.

5 entries = 0.

$$\Delta^5 y_k = 0$$

$$(E-1)^5 y_k = 0.$$

$$(E^5 - 5E^4 + 10E^3 - 10E^2 + 5E - 1)y_k = 0.$$

$K=0$

$$(E^5 y_0 - 5E^4 y_0 + 10E^3 y_0 - 10E^2 y_0 + 5E y_0 - y_0) = 0$$

$$y_5 - 5y_4 + 10y_3 - 10y_2 + 5y_1 - y_0 = 0$$

$$y_5 - 5(350) + 10y_3 - 10(260) + 5(220) - 200 = 0$$

$$y_5 + 10y_3 = 3450 \dots (1).$$

$K=1$

$$(E^5 y_1 - 5E^4 y_1 + 10E^3 y_1 - 10E^2 y_1 + 5E y_1 - y_1) = 0$$

$$y_6 - 5y_5 + 10y_4 - 10y_3 + 5y_2 - y_1 = 0$$

$$430 - 5y_5 + 10(350) - 10y_3 + 5(260) - 220 = 0$$

$$-5y_5 - 10y_3 = -5010 \dots (2).$$

$$\begin{aligned}
 (2) \rightarrow & -545 - 1043 = -5010 \\
 (1) \times 5 \rightarrow & \frac{545 + 5043}{1043 = 12240} \\
 & \boxed{43 = 306}
 \end{aligned}$$

$$\begin{aligned}
 (1) \rightarrow & 45 + 1043 = 3450 \\
 & 45 + 10(306) = 3450 \\
 & 45 + 3060 = 3450 \\
 & \boxed{45 = 390}
 \end{aligned}$$

Eg: 5.10

	0	1	2	3	4
x	2	3	4	5	6
$f(x)$	45.0	49.2	54.1	-	67.4

1 entries = 0

$$\begin{aligned}
 \Delta^4 y_0 &= 0 \\
 (E-1)^4 y_0 &= 0 \\
 (E^4 - 4E^3 + 6E^2 - 4E + 1)y_0 &= 0 \\
 E^4 y_0 - 4E^3 y_0 + 6E^2 y_0 - 4E y_0 + y_0 &= 0 \\
 E^4 y_0 &= y_n
 \end{aligned}$$

$$\begin{aligned}
 44 - 4y_3 + 6y_2 - 4y_1 + y_0 &= 0 \\
 67.4 - 4y_3 + 6(54.1) - 4(49.2) + 45.0 &= 0 \\
 67.4 - 4y_3 + 324.6 - 196.8 + 45.0 &= 0 \\
 -4y_3 &= -240.2
 \end{aligned}$$

$$\boxed{43 = 60.05}$$

Eg: 5.9

$y_3 = 2, y_4 = -6, y_5 = 8, y_6 = 9, y_7 = 17$, Calculate $\Delta^4 y_3$.

$$\begin{aligned}
 \Delta^4 y_3 &= 0 \\
 (E-1)^4 y_3 &= 0 \\
 (E^4 - 4E^3 + 6E^2 - 4E + 1)y_3 &= 0 \\
 (E^4 y_3 - 4E^3 y_3 + 6E^2 y_3 - 4E y_3 + y_3) &= 0
 \end{aligned}$$

$$\begin{aligned}
 E^4 y_3 - 4E^3 y_3 + 6E^2 y_3 - 4E y_3 + y_3 &= 0 \\
 y_7 - 4y_6 + 6y_5 - 4y_4 + y_3 &= 0 \\
 = 17 - 4(9) + 6(8) - 4(-6) + 2 &= 0 \\
 = 17 - 36 + 48 + 24 + 2 &= 55
 \end{aligned}$$

5. $\Delta \left(\frac{1}{(x+1)(x+2)} \right)$

$$\begin{aligned}
 \frac{1}{(x+1)(x+2)} &= \frac{1}{(x+1)} - \frac{1}{(x+2)} \\
 \Delta \left(\frac{1}{(x+1)(x+2)} \right) &= \Delta \left(\frac{1}{x+1} \right) - \Delta \left(\frac{1}{x+2} \right) \\
 &= \left(\frac{1}{x+1+1} - \frac{1}{x+1} \right) - \left(\frac{1}{x+1+2} - \frac{1}{x+2} \right) \\
 &= \frac{1}{x+2} - \frac{1}{x+1} - \frac{1}{x+3} + \frac{1}{x+2} \\
 &= \frac{2}{x+2} - \frac{1}{x+1} - \frac{1}{x+3} \\
 &= \frac{2(x+1)(x+3) - 1(x+2)(x+3) - 1(x+1)(x+2)}{(x+1)(x+2)(x+3)}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{2(x^2+4x+3) - 1(x^2+5x+6) - 1(x^2+8x+2)}{(x+1)(x+2)(x+3)} \\
 &= \frac{2x^2+8x+6 - x^2-5x-6 - x^2-8x-2}{(x+1)(x+2)(x+3)} \\
 &= \frac{-2}{(x+1)(x+2)(x+3)}
 \end{aligned}$$

Eg: 5.6

$$\begin{aligned}
 \Delta^2\left(\frac{1}{x}\right) &= \Delta\left(\Delta\left(\frac{1}{x}\right)\right) \\
 &= \Delta\left(\frac{1}{x+1} - \frac{1}{x}\right) \\
 &= \Delta\frac{1}{x+1} - \Delta\frac{1}{x} \\
 &= \frac{1}{x+1+1} - \frac{1}{x+1} - \left(\frac{1}{x+1} - \frac{1}{x}\right) \\
 &= \frac{1}{x+2} - \frac{1}{x+1} - \frac{1}{x+1} + \frac{1}{x} \\
 &= \frac{-2}{x+1} + \frac{1}{x+2} - \frac{1}{x} \\
 &= \frac{-2(x)(x+2) + 1(x)(x+1) - 1(x+1)(x+2)}{(x+1)(x+2)(x+3)} \\
 &= \frac{-2x^2-4x+ x^2+x+ x^2+x+2}{(x)(x+1)(x+2)} \\
 &= \frac{2}{(x)(x+1)(x+2)}
 \end{aligned}$$

Eg: 5.7

$$f(x) = f(x) + \Delta f(x) + \Delta^2 f(x) + \Delta^3 f(x)$$

$$\Delta f(x) = f(x+h) - f(x)$$

$$f(x+h) = f(x) + \Delta f(x)$$

L.H.S

$$f(4) = f(3) + \Delta f(3)$$

$$(x=3, h=1) = f(3) + \Delta(f(3))$$

$$= f(3) + \Delta f(3) + \Delta^2 f(3)$$

$$= f(3) + \Delta f(3) + \Delta^2(f(3) + \Delta f(3))$$

$$= f(3) + \Delta f(3) + \Delta^2 f(3) + \Delta^3 f(3)$$

L.H.S = R.H.S.

Eg: 5.5

$$\Delta \left[\frac{5x+12}{x^2+5x+6} \right]$$

$$(x+3)(x+2)$$

$$\frac{5x+12}{x^2+5x+6} = \frac{A}{(x+3)} + \frac{B}{(x+2)}$$

$$= \Delta \left[\frac{-3}{(x+3)} + \frac{2}{(x+2)} \right]$$

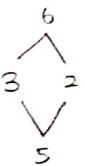
$$= \Delta \frac{-3}{(x+3)} + \Delta \frac{2}{(x+2)}$$

$$= \frac{-3}{x+1+3} - \frac{3}{x+3} + \frac{2}{x+1+2} - \frac{2}{x+2}$$

$$= \frac{-3}{x+4} - \frac{3}{x+3} + \frac{2}{x+3} - \frac{2}{x+2}$$

$$= \frac{-2}{x+2} - \frac{1}{x+3} + \frac{3}{x+4}$$

$$f(x) = f(x+h) - f(x)$$



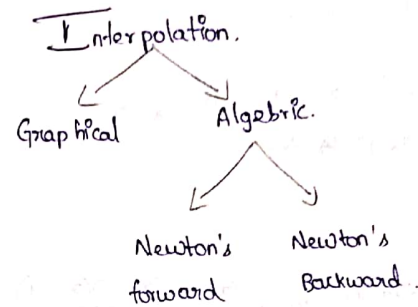
$$= \frac{-2(x+3)(x+4) - 1(x+2)(x+4) + 3(x+2)(x+3)}{(x+2)(x+3)(x+4)}$$

$$= \frac{-2(x^2+12x+12) - (x^2+8x+8) + 3(x^2+6x+6)}{(x+2)(x+3)(x+4)}$$

$$= \frac{-2x^2 - 24 - 14x - x^2 - 8 - 6x + 3x^2 + 18 + 15x}{(x+2)(x+3)(x+4)}$$

$$= \frac{-5x - 14}{(x+2)(x+3)(x+4)}$$

Exercise 5.2



Forward:

$$f(x_0 + nh) = f(x_0) + \frac{n}{1!} \Delta f(x_0) + \frac{n(n-1)}{2!} \Delta^2 f(x_0) + \dots$$

(or)

$$y(x_0 + nh) = y_0 + \frac{n}{1!} \Delta y_0 + \frac{n(n-1)}{2!} \Delta^2 y_0 + \dots$$

$$n = \frac{x - x_0}{h}$$

Backward:

$$f(x_n + nh) = f(x_n) + \frac{n}{1!} \Delta f(x_n) + \frac{n(n-1)}{2!} \Delta^2 f(x_n) + \dots$$

(or)

$$y(x_n + nh) = y_n + \frac{n}{1!} \Delta y_n + \frac{n(n-1)}{2!} \Delta^2 y_n + \dots$$

$$n = \frac{x - x_n}{h}$$

Lagrange's Interpolation formula:

$$y = f(x) = \frac{(x-x_1)(x-x_2)\dots(x-x_n)}{(x_0-x_1)(x_0-x_2)\dots(x_0-x_n)} y_0 + \frac{(x-x_0)(x-x_2)\dots(x-x_n)}{(x_1-x_0)(x_1-x_2)\dots(x_1-x_n)} y_1 + \dots + \frac{(x-x_0)(x-x_1)\dots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\dots(x_n-x_{n-1})} y_n$$

12.

x	3	7	11	19
$f(x)$	42	43	47	60

$$x_0 = 3, x_1 = 7, x_2 = 11, x_3 = 19 \quad \text{Gn. } x = 15.$$

$$y_0 = 42, y_1 = 43, y_2 = 47, y_3 = 60$$

Lagrange's:

$$= \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$

$$= \frac{(8)(-4)(-4)}{(4)(-8)(-16)} (42) + \frac{(12)(4)(-4)}{(4)(-1)(-12)} (43) + \frac{(12)(8)(4)}{(8)(4)(-8)} (47) + \frac{(12)(8)(4)}{(16)(2)(8)} (60)$$

$$= +10.5 + 43 + 70.5 + 15$$

$$= 53$$

$$\text{When } x=15, y=53.$$

x	1982	1983	1984	1985
-----	------	------	------	------

$f(x)$	150	235	365	525
--------	-----	-----	-----	-----

$$x_0 = 1982, x_1 = 1983, x_2 = 1984, x_3 = 1985 \quad \text{Gn. } x = 1986.$$

$$y_0 = 150, y_1 = 235, y_2 = 365, y_3 = 525$$

Lagrange's:

$$= \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} (y_0) + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} (y_1) +$$

$$\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} (y_2) + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} (y_3)$$

$$= \frac{(2)(-1)(-1)}{(4)(-2)(-4)} (150) + \frac{(3)(1)(-1)}{(1)(-1)(-3)} (235) + \frac{(3)(5)(4)}{(2)(8)(-2)} (365)$$

$$+ \frac{(3)(2)(1)}{(4)(3)(2)} (525)$$

$$= 593.75$$

$$10. \quad x = 26, x_0 = 15, x_1 = 25, x_2 = 30, x_3 = 35.$$

$$y_0 = 30, y_1 = 40, y_2 = 45, y_3 = 48.$$

Lagrange's:

$$= \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} (y_0) + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} (y_1) +$$

$$\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} (y_2) + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} (y_3)$$

$$= \frac{(1)(-4)(-9)}{(10)(-15)(-20)} (30) + \frac{(21)(-4)(-9)}{(10)(-5)(-10)} (40) + \frac{(11)(1)(-4)}{(15)(5)(-5)} (45)$$

$$+ \frac{(11)(1)(-4)}{(20)(10)(5)} (48)$$

$$= 53.336$$

9. $x = 1986$, $x_0 = 1974$, $x_1 = 1978$, $x_2 = 1982$, $x_3 = 1990$
 $y_0 = 25$, $y_1 = 60$, $y_2 = 80$, $y_3 = 170$.

log range:

$$= \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} (y_0) + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} (y_1)$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} (y_2) + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} (y_3)$$

$$= \frac{(8)(4)(-4)}{(4)(-4)(-16)} (25) + \frac{(12)(4)(-4)}{(4)(-4)(-12)} (60) + \frac{(12)(8)(-4)}{(8)(4)(-8)} (80)$$

$$+ \frac{(15)(8)(4)}{(16)(12)(8)} (170)$$

$$= 25 - 60 + 120 + 42.5$$

$$= 127.5$$

Eg: 5.22

$x = 10$, $x_0 = 5$, $x_1 = 6$, $x_2 = 9$, $x_3 = 11$
 $y_0 = 12$, $y_1 = 13$, $y_2 = 14$, $y_3 = 16$.

log range:

$$= \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} (y_0) + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} (y_1)$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} (y_2) + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} (y_3)$$

$$= \frac{(4)(1)(-1)}{(1)(-3)(-5)} (12) + \frac{(5)(1)(-1)}{(1)(-3)(-5)} (13) + \frac{(5)(4)(-1)}{(4)(3)(-2)} (14) +$$

$$\frac{(5)(4)(1)}{(5)(5)(2)} (16)$$

$$= 2 - 4.33 + 11.67 + 5.33$$

$$= 14.67$$

3.	x	0	1	2	3
	f(x)	1	2	1	10

$x = 1$, $x_0 = 0$, $h = 1$, $n = \frac{x-x_0}{h}$
 $= \frac{1-0}{1} = 1$ m n = 1

$$y(x) = y_0 + \frac{n}{1!} \Delta y_0 + \frac{n(n-1)}{2!} \Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!} \Delta^3 y_0 + \dots$$

2	y(x)	Δ	Δ ²	Δ ³
0	1			
1	2	1		
2	1	-1	-2	
3	10	9	10	12

$$y(x) = 1 + \frac{x}{1} (1) + \frac{x(x-1)}{2!} (-2) + \frac{x(x-1)(x-2)}{3!} (12)$$

$$= 1 + \frac{x(-2x^2+2x)}{2} + \frac{(x^3-2x^2-x+2x)}{6}$$

$$= \frac{x-2x^2+2x+x^3-2x^2-x+2x}{12}$$

$$= \frac{x^3-2x^2+2x}{12}$$

6. x 30 35 40 45 50
 $f(x)$ 15.9 14.9 14.1 13.9 12.5

forward.

$$x = 32, x_0 = 30, h = 5, n = \frac{x - x_0}{h} = \frac{32 - 30}{5} = 0.4$$

x	$f(x)$	Δ	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
30	15.9	-1	-0.2	-0.2	0.2
35	14.9	-0.8	+0	0	
40	14.1	-0.8	+0		
45	13.3	-0.8			
50	12.5				

$$y_{(32)} = y_0 + \frac{n}{1!} \Delta y_0 + \frac{n(n-1)}{2!} \Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!} \Delta^3 y_0 + \frac{n(n-1)(n-2)(n-3)}{4!} \Delta^4 y_0$$

$$= 15.9 + \frac{0.4}{1} (-1) + \frac{(0.4)(0.4-1)}{2!} (-0.2) + \frac{(0.4)(0.4-1)(0.4-2)}{3!} (-0.2) + \frac{(0.4)(0.4-1)(0.4-2)(0.4-3)}{4!} (0.2)$$

$$= 15.9 - 0.4 + -0.024 - 0.0128 - 0.00832$$

$$= 15.45$$

7. x 40 50 60 70 80 90
 T 180 204 226 250 276 304

backward.
 $x = 84, x_n = 90, h = 10, n = \frac{x - x_n}{h} = \frac{84 - 90}{10} = -0.6$

x	$f(x)$	Δ	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
40	180	24				
50	204	22	-2			
60	226	22	2	-4		
70	250	24	2	0	4	
80	276	26	2	0	0	
90	304	28	2			

$$y_{(84)} = y_n + \frac{n}{1!} \Delta y_n + \frac{n(n-1)}{2!} \Delta^2 y_n + \frac{n(n-1)(n-2)}{3!} \Delta^3 y_n + \frac{n(n-1)(n-2)(n-3)}{4!} \Delta^4 y_n + \dots$$

$$= 304 + \frac{-0.6}{1!} (28) + \frac{(-0.6)(-0.6-1)}{2!} (2) + 0 + 0 + \dots$$

$$= 304 - 16.8 - 0.24 - 0.0914$$

$$= 286.86$$

8. x 0 1 2 3
 $f(x)$ 1 2 11 34

backward.
 $x_n = 2.8$

$$x = 2.8, x_n = 3, h = 1, n = \frac{x - x_n}{h} = \frac{2.8 - 3}{1} = -0.2$$

x	f(x)	Δ	$\Delta^2 y$	$\Delta^3 y$
0	1			
1	2	1		
2	11	9	8	
3	34	23	14	6

$$y(2.8) = y_n + \frac{n}{1!} \Delta y_n + \frac{n(n-1)}{2!} \Delta^2 y_n + \frac{n(n-1)(n-2)}{3!} \Delta^3 y_n$$

$$= 34 + \frac{(-0.2)}{1!} (23) + \frac{(-0.2)(-0.2+1)}{2!} (14) + \frac{(-0.2)(-0.2+1)(-0.2+2)}{3!} (6)$$

$$= 34 - 4.6 - 1.12 - 0.288$$

$$= 27.992$$

Eg: 5.14.

x	0	1	2	3	4	5	6
y	2	4	10	16	20	24	38

$h = 1, x_0 = 3, n = \frac{x - x_0}{h} = \frac{3.2 - 3}{1} = 0.2$

x	f(x)	Δ	$\Delta^2 y$	$\Delta^3 y$
3	16			
4	20	4		
5	24	4	0	
6	38	14	10	10

$$y(3.2) = y_0 + \frac{n}{1!} \Delta y_0 + \frac{n(n-1)}{2!} \Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!} \Delta^3 y_0 + \dots$$

$$= 16 + \frac{0.2}{1} (4) + 0 + \frac{(0.2)(0.2-1)(0.2-2)}{3!} (10)$$

$$= 16 + 0.8 + 0.48$$

$$= 17.28$$

Eg: 5.15.

Mark	30-40	40-50	50-60	60-70	70-80	frequency
No. of Student	31	42	51	35	31	

$$x = 45, x_0 = 40, h = 10, n = \frac{x - x_0}{h}$$

$$= \frac{45 - 40}{10} = 0.5$$

Less than 40

x	f(x)	Δ	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
40	31				
50	73	42			
60	124	51	9		
70	159	35	-16	-25	
80	190	31	-4	12	37

$$y(45) = y_0 + \frac{n}{1!} \Delta y_0 + \frac{n(n-1)}{2!} \Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!} \Delta^3 y_0 + \dots$$

$$= 31 + \frac{0.5}{1} (42) + \frac{0.5(0.5-1)}{2!} (9) + \frac{0.5(0.5-1)(0.5-2)}{3!} (-25) + \frac{0.5(0.5-1)(0.5-2)(0.5-3)}{4!} (37)$$

$$= 314.214 - 1.125 - 1.5625 - 1.4453$$

$$= 47.861$$

Eg: 5.16

Weight	20-40	40-60	60-80	80-100	100-120
No. of Student	250	120	100	70	50

Forward

$$x = 70, x_0 = 40, h = 20, n = \frac{x - x_0}{h}$$

$$= \frac{70 - 40}{20} = 1.5$$

x	f(x)	Δ	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
below					
40	250				
60	370	120			
80	470	100	-20		
100	540	70	-30	-10	
120	590	50	-20	10	20

$$y_{(10)} = y_0 + \frac{n}{1!} \Delta y_0 + \frac{n(n-1)}{2!} \Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!} \Delta^3 y_0 + \dots$$

$$= 250 + \frac{1.5}{1} (120) + \frac{1.5(1.5-1)}{2!} (-20) + \frac{1.5(1.5-1)(1.5-2)}{3!} (-10)$$

$$+ \frac{1.5(1.5-1)(1.5-2)(1.5-3)}{4!} (20)$$

$$= 250 + 180 - 7.5 + 0.6875 + 0.46875$$

$$= 423.59$$

Eg: 5.17

Year	1891	1901	1911	1921	1931	Forward
Population	98152	132,285	1,68,016	1,95,610	2,46,050	

$$x = 1905, x_0 = 1891, h = 10, n = \frac{x - x_0}{h}$$

$$= \frac{1905 - 1891}{10} = 1.4$$

x	f(x)	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1891	98152				
1901	1,32,285	33533			
1911	1,68,016	35731	2258		
1921	1,95,610	27594	-8137		
1931	2,46,050	50380	22786	30982	

$$y_{(1905)} = y_0 + \frac{n}{1!} \Delta y_0 + \frac{n(n-1)}{2!} \Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!} \Delta^3 y_0 + \dots$$

$$= 98152 + \frac{1.4}{1} (33533) + \frac{1.4(1.4-1)}{2!} (2258) + \frac{1.4(1.4-1)(1.4-2)}{3!} (-8137)$$

$$+ \frac{1.4(1.4-1)(1.4-2)(1.4-3)}{4!} (30982)$$

$$= 98152 + 46946.2 + 632.24 + 585.48 + 928.212$$

$$= 141844$$

Eg: 5.17.

Forward

x	1941	1951	1961	1971	1981	1991
y	20	24	29	36	46	51

$$x = 1946, x_0 = 1941, h = 10, n = \frac{x - x_0}{h} = \frac{1946 - 1941}{10} = 0.5$$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
1941	20	4				
1951	24	5	1			
1961	29	7	2	1		
1971	36	10	3	1	-9	
1981	46	5	-5	-8	-9	-9
1991	51					

$$y(1946) = y_0 + \frac{n}{1!} \Delta y_0 + \frac{n(n-1)}{2!} \Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!} \Delta^3 y_0 + \dots$$

$$= 20 + \frac{0.5}{1} (4) + \frac{0.5(0.5-1)}{2!} (1) + \frac{0.5(0.5-1)(0.5-2)}{3!} (1) + 0$$

$$+ \frac{0.5(0.5-1)(0.5-2)(0.5-3)}{4!} (-9)$$

$$= 20 + 2 - 0.125 + 0.0625 + 0 - 0.24609375$$

$$= 21.69140625$$

Eg: 5.18

Backward

x	140	150	160	170	180
f(x)	3.685	4.854	6.302	8.016	10.225

$$x = 180, x_0 = 175, h = 10, n = \frac{x - x_0}{h} = \frac{180 - 175}{10} = 0.5$$

x	f(x)	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
140	3.685	1.169			
150	4.854	1.448	0.279		
160	6.302	1.774	0.326	0.047	
170	8.016	2.149	0.375	0.049	0.002
180	10.225				

$$y(180) = y_n + \frac{n}{1!} \Delta y_n + \frac{n(n-1)}{2!} \Delta^2 y_n + \frac{n(n-1)(n-2)}{3!} \Delta^3 y_n + \dots \quad (0.049)$$

$$= 10.225 + \frac{(-0.5)}{1} (2.149) + \frac{(-0.5)(-0.5-1)}{2!} (0.375) + \frac{(-0.5)(-0.5-1)(-0.5-2)}{3!} (0.047)$$

$$+ \frac{(-0.5)(-0.5-1)(-0.5-2)(-0.5-3)}{4!} (0.002)$$

$$= 10.225 - 1.0745 - 0.046875 - 0.000078125$$

$$= 9.1$$

Eg: 5.19

x	1	2	3	4	5	6	7	8
y	1	8	27	64	125	216	343	512

$$x_n = 8, x_a = 7.5, h = 1, n = \frac{x - x_n}{h} = \frac{-8 + 7.5}{1} = -0.5$$

x	f(x)	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1	1				
2	8	7			
3	27	19	12		
4	64	37	18	6	
5	125	61	24	6	0
6	216	91	30	6	0
7	343	127	36	6	0
8	512	169	42	6	0

$$y(7.5) = y_n + \frac{n}{1!} \Delta y_n + \frac{n(n+1)}{2!} \Delta^2 y_n + \frac{n(n+1)(n+2)}{3!} \Delta^3 y_n + \dots$$

$$= 512 + \frac{(-0.5)(169)}{1} + \frac{(-0.5)(-0.5+1)}{2!} 42 + \frac{(-0.5)(-0.5+1)(-0.5+2)}{3!} 6$$

$$= 512 - 84.5 - 5.25 - 0.375$$

$$= 421.875$$

Eg: 5.20

Age	45	50	55	60	65	backward
Premium	114.84	96.16	83.32	74.48	68.48	

$$x = 63, x_n = 65, h = 5, n = \frac{x - x_n}{h} = \frac{63 - 65}{5} = -0.4$$

x	f(x)	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
45	114.84				
50	96.16	-18.68			
55	83.32	-12.84	5.84		
60	74.48	-8.84	-4	-1.84	
65	68.48	-6	2.84	-1.16	0.68

$$y(63) = \Delta y_n + \frac{n}{1!} \Delta y_n + \frac{n(n+1)}{2!} \Delta^2 y_n + \frac{n(n+1)(n+2)}{3!} \Delta^3 y_n + \dots$$

$$= 68.48 + \frac{-0.4(-6)}{1} + \frac{(-0.4)(-0.4+1)}{2!} (-2.84) + \frac{(-0.4)(-0.4+1)(-0.4+2)}{3!} (-1.16)$$

$$+ \frac{(-0.4)(-0.4+1)(-0.4+2)(-0.4+3)}{4!} (0.68)$$

$$= 68.48 + 2.4 - 0.3408 + 0.07424 - 0.02828$$

$$= 70.585$$

Eg: 5.2)

x	0	1	2	3	4	5	6	7
y	1	2	4	7	11	16	22	29

Polynomial.

$$x = x, x_0 = 7, h = 1, n = \frac{x - x_0}{h}$$

$$= \frac{x-7}{1} = x-7$$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
0	1			
1	2	1		
2	4	2	1	
3	7	3	1	
4	11	4	1	
5	16	5	1	
6	22	6	1	
7	29	7	1	0

$$y(x) = y_0 + \frac{n}{1!} \Delta y_0 + \frac{n(n-1)}{2!} \Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!} \Delta^3 y_0 + \dots$$

$$= 29 + \frac{x-7}{1} (7) + \frac{(x-7)(x-7+1)}{2!} (1)$$

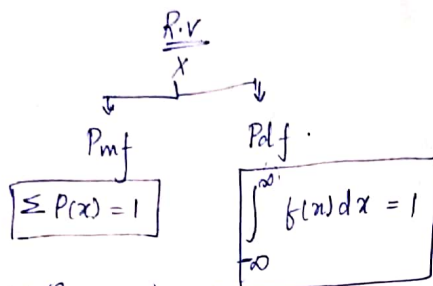
$$= 29 + 7x - 49 + \frac{1}{2} [(x-7)(x-6)]$$

$$= \frac{1}{2} [58 + 14x - 49 + x^2 + 42 - 13x]$$

$$= \frac{1}{2} [x^2 + x + 2]$$

2/01/2020

chapter - 6

Random variable & Mathematical Expectation.

Cumulative distribution function:

$$F(x) = P(X \leq x) = \int_{-\infty}^x f(t) dt.$$

$$f(x) = \frac{dF(x)}{dx} = f(x).$$

$$P(a \leq X \leq b) = F(b) - F(a).$$

$$P(X < a) = 1 - P(X \geq a)$$

$$\begin{array}{|c|c|c|} \hline 0 & a & 1 \\ \hline \end{array} = 1$$

$$P(X > a) = 1 - P(X \leq a)$$

$$P(X \leq a) = 1 - P(X > a)$$

$$P(X \geq a) = 1 - P(X < a)$$

eg 6.1

Lot	0	1	2	3	4
How sold	30	320	380	190	80

= 1000

x	0	1	2	3	4
P(x)	0.03	0.32	0.38	0.19	0.08

= 1

$$\Rightarrow P(x) > 0$$

$$\sum P(x) = 1 \Rightarrow \text{It is pmf.}$$

eg 6.2

x	0	1	2	3	4	5	6	7
P(x)	0	a	2a	2a	3a	a^2	2a^2	7a^2 + a

Qn:

$$(i) \text{ If } \sum P(x) = 1$$

$$0 + a + 2a + 2a + 3a + a^2 + 2a^2 + 7a^2 + a = 1$$

$$9a + 10a^2 = 1$$

$$10a^2 + 9a - 1 = 0$$

$$\boxed{a = \frac{1}{10}} \quad a = -1$$

$$\begin{array}{r} -10 \\ \sqrt{10} \end{array} \begin{array}{r} 10 \\ 9 \end{array}$$

$$(ii) P(X < 3) = 0 + 1 + 2$$

$$= 3a = \boxed{\frac{3}{10}}$$

$$(iii) P(X > 2) = 1 - P(X \leq 2)$$

$$= 1 - (0 + 1 + 2)$$

$$= 1 - \frac{3}{10} = \boxed{\frac{7}{10}}$$

$$\begin{aligned}
 \text{(iv)} \quad P(2 < X \leq 5) &= 3 + 4 + 5 \\
 &= 5a + a^2 \\
 &= \frac{5}{10} + \frac{1}{100} \\
 &= \frac{50 + 1}{100} \\
 &= \frac{51}{100}
 \end{aligned}$$

eg 6.3.

$$P(x) = \begin{cases} \frac{x}{20}, & x = 0, 1, 2, 3, 4, 5 \\ 0, & \text{otherwise.} \end{cases}$$

$$\begin{aligned}
 \text{(i)} \quad P(X < 3) &= 0 + 1 + 2 \\
 &= \frac{0}{20} + \frac{1}{20} + \frac{2}{20} \\
 &= \frac{3}{20}
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad P(2 < X \leq 4) &= 3 + 4 = \frac{3}{20} + \frac{4}{20} \\
 &= \frac{7}{20}
 \end{aligned}$$

eg 6.4 :

3 wires:

$$h(s) = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$$

$$= 2^3 = 8$$

$$X = \text{head} = 0, 1, 2, 3$$

X	0	1	2	3
P(x)	1/8	3/8	3/8	1/8

$$P(x) \geq 0$$

$$\sum P(x) = 1 \quad \text{It is pmf.}$$

eg 6.5

$$X = \text{sum of upturn } (d_1, d_2)$$

X	2	3	4	5	6	7	8	9	10	11	12
P(x)	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

$$\text{eg 6.6} \quad 6 = (1,5) (5,1) (2,4) (4,2) (3,3) = \frac{5}{36}$$

①

X	0	1	2	3
P(x)	0.3	0.2	0.4	0.1

$$F(x) = P(X \leq x)$$

$$F(0) = P(X \leq 0) = P(X=0) = 0.3$$

$$F(1) = P(X \leq 1) = 0 + 1 = 0.3 + 0.2 = 0.5$$

$$F(2) = P(X \leq 2) = 0 + 1 + 2 = 0.9$$

$$F(3) = P(X \leq 3) = 0 + 1 + 2 + 3 = 1.0 = 1$$

X	0	1	2	3
P(X ≤ x)	0.3	0.5	0.9	1

$$(2). P(x) = \begin{cases} 0.3 & x=3 \\ 0.2 & x=5 \\ 0.3 & x=8 \\ 0.2 & x=10 \\ 0 & \text{otherwise.} \end{cases}$$

$$F(x) = \begin{cases} 0 & \text{if } x < 3 \\ 0.3 & \text{if } 3 \leq x < 5 \\ 0.5 & \text{if } 5 \leq x < 8 \\ 0.8 & \text{if } 8 \leq x < 10 \\ 1 & \text{if } x \leq 10. \end{cases}$$

$$(3) P(X=x) = \begin{cases} kx & x=2,4,6 \\ k(x-2) & x=8 \\ 0 & \text{otherwise} \end{cases}$$

cm pf

x	2	4	6	8
P(x)	2k	4k	6k	6k

$$\sum P(x) = 1$$

$$2k + 4k + 6k + 6k = 1$$

$$18k = 1$$

$$k = \frac{1}{18}$$

value of x = x	0	1	2	3	4	5	6	7
P(x)	0	k	2k	2k	3k	k ²	2k ²	7k ² +k

(b) find k.

$$\sum P(x) = 1$$

$$\Rightarrow 0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$$

$$9k + 10k^2 = 1$$

$$10k^2 + 9k - 1 = 0$$

$$(k+1)(k - \frac{1}{10}) = 0$$

$$k = -1$$

$$k = \frac{1}{10}$$

(Hv) values

$$\therefore k = \frac{1}{10}$$

$$(ii) P(x \leq 6) = 0 + 1 + 2 + 3 + 4 + 5$$

$$= 0 + k + 2k + 2k + 3k + k^2$$

$$= 8k + k^2$$

$$= 8\left(\frac{1}{10}\right) + \left(\frac{1}{100}\right)$$

$$= \frac{80+1}{100} = \frac{81}{100}$$

$$P(x \geq 6) = 6 + 7$$

$$= 2k^2 + 7k^2 + k$$

$$= 9k^2 + k = \frac{9}{100} + \frac{1}{10} = \frac{9+10}{100} = \frac{19}{100}$$

$$= \frac{9}{100} + \frac{1}{10}$$

$$= \frac{9+10}{100}$$

$$= \boxed{\frac{19}{100}}$$

$$\begin{aligned} P(0 < X < 5) &= 1+2+3+4 \\ &= k+2k+3k+4k \\ &= 8k \\ &= \boxed{\frac{8}{10}} \end{aligned}$$

$$(ii) P(X \leq n) > \frac{1}{2}$$

$$P(X \leq 0) = 0 \times \frac{1}{2}$$

$$P(X \leq 1) = k = \frac{1}{10} \times \frac{1}{2}$$

$$P(X \leq 2) = 3k = \frac{3}{10} \times \frac{1}{2}$$

$$P(X \leq 3) = 5k = \frac{5}{10} \times \frac{1}{2}$$

$$P(X \leq 4) = 8k = \frac{8}{10} > \frac{1}{2}$$

$$\boxed{X=4}$$

$$(7) \int_{-3}^3 f(x) \cdot dx = 1$$

$$\text{Eg. } f(x) = \begin{cases} \frac{1}{16} (9+x^2+6x), & -3 \leq x \leq -1 \\ \frac{1}{16} (6-2x^2), & -1 \leq x \leq 1 \\ \frac{1}{16} (9+x^2-6x), & 1 \leq x \leq 3. \end{cases}$$

(L.H.S):

$$\begin{aligned} \int_{-3}^3 f(x) \cdot dx &= \int_{-3}^{-1} f(x) \cdot dx + \int_{-1}^1 f(x) \cdot dx + \int_1^3 f(x) \cdot dx \\ &= \frac{1}{16} \int_{-3}^{-1} (9+x^2+6x) \cdot dx + \int_{-1}^1 (6-2x^2) \cdot dx \\ &\quad + \int_1^3 (9+x^2-6x) \cdot dx \end{aligned}$$

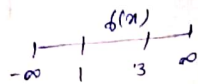
$$\begin{aligned} &= \frac{1}{16} \left[\left(9x + \frac{x^3}{3} + \frac{6x^2}{2} \right)_{-3}^{-1} + \left(6x - \frac{2x^3}{3} \right)_{-1}^1 + \left(9x + \frac{x^3}{3} - \frac{6x^2}{2} \right)_1^3 \right] \\ &= \frac{1}{16} \left[\left(9 \cdot \frac{1}{2} + \frac{1}{2} + \frac{3}{2} \right) - \left(-27 - \frac{27}{3} + \frac{54}{2} \right) + \left(6 - \frac{2}{3} \right) \right. \\ &\quad \left. - \left(-6 + \frac{2}{3} \right) + \left(27 + \frac{27}{3} - \frac{54}{2} \right) - \left(9 + \frac{1}{3} - \frac{6}{2} \right) \right] \\ &= \frac{1}{16} \left[-9 - \frac{1}{3} + 3 + 27 + 9 - 27 + 6 - \frac{2}{3} + 6 - \frac{2}{3} \right. \\ &\quad \left. + 27 + 9 - 27 - 9 - \frac{1}{3} + 3 \right] \\ &= \frac{1}{16} \left[\frac{-6}{3} + 18 \right] \\ &= \frac{1}{16} (16) = \boxed{1} \quad \therefore \text{R.H.S} \end{aligned}$$

$$(8). F(x) = \begin{cases} 0 & , x \leq 1 \\ 16(x-1)^4 & , 1 < x \leq 3 \\ 0 & , x > 3 \end{cases}$$

$$F'(x) = f(x)$$

$$f(x) = d(F(x)) = \begin{cases} 0 & , x \leq 1 \\ 4k(x-1)^3 & , 1 < x \leq 3 \\ 0 & , x > 3 \end{cases}$$

$$(i) \int_{-\infty}^{\infty} f(x) dx = 1$$



$$\int_1^3 4k(x-1)^3 dx = 1$$

$$4k \left[\frac{(x-1)^4}{4} \right]_1^3 = 1$$

$$k(16-0) = 1$$

$$k = \frac{1}{16}$$

(ii) Pdf.

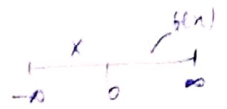
$$f(x) = \begin{cases} \frac{1}{16} (x-1)^3 & , 1 < x \leq 3 \\ 0 & , \text{otherwise.} \end{cases}$$

$$(9). f(x) = \begin{cases} Ae^{-x/5} & , x \geq 0 \\ 0 & , \text{otherwise.} \end{cases}$$

(i) A:

$$\text{pdf} \int_{-\infty}^{\infty} f(x) dx = 1$$

$$\Rightarrow \int_0^{\infty} Ae^{-x/5} dx = 1$$



$$A \left(\frac{e^{-x/5}}{-1/5} \right)_0^{\infty} = 1$$

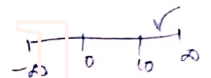
$$-5A(e^{-\infty} - e^{-0}) = 1$$

$$-5A(0-1) = 1$$

$$A = \frac{1}{5}$$

$$\begin{aligned} e^{\infty} &= \infty \\ e^{-\infty} &= 0 \\ e^0 &= e^0 = 1 \end{aligned}$$

(ii) (a) more than 10 min



$$P(x > 10) = \int_{10}^{\infty} \frac{1}{5} e^{-x/5} dx$$

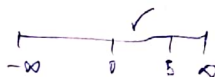
$$= \frac{1}{5} \left(\frac{e^{-x/5}}{-1/5} \right)_{10}^{\infty}$$

$$= - \left(e^{-\infty} - e^{-10/5} \right)$$

$$= -(-e^{-2})$$

$$= \frac{1}{e^2}$$

(b) less than 5



$$P(x < 5) = \int_0^5 \frac{1}{5} e^{-x/5} dx$$

$$\begin{aligned}
 &= \frac{1}{5} \left(\frac{e^{-x/5}}{-1/5} \right)_0^5 \\
 &= \frac{1}{5} - (e^{-5/5} - e^0) \\
 &= -\left(e^{-1} - 1\right) = -\left(\frac{1}{e} - 1\right) \\
 &= \left(1 - \frac{1}{e}\right) \\
 &= \boxed{\frac{e-1}{e}} //
 \end{aligned}$$

(C) 5 to 10

$$\begin{aligned}
 P(5 < x < 10) &= \int_5^{10} \frac{1}{5} e^{-x/5} dx \\
 &= \frac{1}{5} \left(\frac{e^{-x/5}}{-1/5} \right)_5^{10} \\
 &= -\left(e^{-10/5} - e^{-5/5}\right) \\
 &= -(e^{-2} - e^{-1}) \\
 &= -\left(\frac{1}{e^2} - \frac{1}{e}\right) \\
 &= \frac{1}{e} - \frac{1}{e^2} \\
 &= \boxed{\frac{e-1}{e^2}} //
 \end{aligned}$$

$$(10). F(x) = \begin{cases} 0 & x \leq 0 \\ x/2, & 0 \leq x < 1 \\ 1/2, & 1 \leq x < 2 \\ x/4, & 2 \leq x < 4 \\ 1, & x \geq 4. \end{cases}$$

(i) Yes,

$$\text{Pdf} \Rightarrow f(x) = \begin{cases} 0, & x \leq 0 \\ 1/2, & 0 \leq x < 1 \\ 0, & 1 \leq x < 2 \\ 1/4, & 2 \leq x < 4 \\ 0, & x \geq 4. \end{cases}$$

(ii) (a) more than 3

$$\begin{aligned}
 P(x > 3) &= \int_3^4 \frac{1}{4} dx \\
 &= \frac{1}{4} (x)_3^4 = \frac{1}{4} (4-3) = \boxed{\frac{1}{4}}
 \end{aligned}$$

(b) less than 3

$$\begin{aligned}
 P(x < 3) &= \int_0^3 f(x) dx \\
 &= \int_0^1 \frac{1}{2} dx + \int_2^3 \frac{1}{4} dx \\
 &= \frac{1}{2} (x)_0^1 + \frac{1}{4} (x)_2^3 \\
 &= \frac{1}{2} (1-0) + \frac{1}{4} (3-2) \\
 &= \frac{1}{2} + \frac{1}{4} = \frac{3}{4} //
 \end{aligned}$$

eq: 6.11

$$f(x) = \begin{cases} Ax, & \text{for } 0 \leq x < 10 \\ A(20-x), & \text{for } 10 \leq x < 20 \\ 0, & \text{otherwise} \end{cases}$$

a) find the A.

$$\int_{-\infty}^{\infty} f(x) dx = 1.$$

$$= \int_0^{10} Ax \cdot dx + \int_{10}^{20} A(20-x) \cdot dx = 1$$

$$= A \left\{ \left(\frac{x^2}{2} \right)_0^{10} + \left(20x - \frac{x^2}{2} \right)_{10}^{20} \right\} \cdot dx = 1$$

$$= A \left(\left(\frac{x^2}{2} \right)_0^{10} + \left(20x - \frac{x^2}{2} \right)_{10}^{20} \right) dx = 1$$

$$= A \left(\frac{100}{2} - 0 \right) + \left(\left(20(20) - \frac{400}{2} \right) - \left(200 - \frac{100}{2} \right) \right) = 1$$

$$= A(50 + 400 - 200 - 200 + 50) = 1$$

$$= A(100) = 1$$

$$\boxed{A = \frac{1}{100}}$$

(b) (i) $P(10 \leq X \leq 20)$

$$= \int_{10}^{20} A(20-x) \cdot dx$$

$$\begin{aligned} &= \int_{10}^{20} \frac{1}{100} (20-x) \cdot dx \\ &= \frac{1}{100} \left(20x - \frac{x^2}{2} \right)_{10}^{20} \\ &= \frac{1}{100} \left(400 - \frac{400}{2} - \left(200 - \frac{100}{2} \right) \right) \\ &= \frac{1}{100} (400 - 200 - 200 + 50) \\ &= \frac{1}{100} (50) \\ &= \boxed{0.5} \end{aligned}$$

$$(ii) P(0 \leq X < 10) = \int_0^{10} Ax \cdot dx$$

$$= \int_0^{10} \frac{1}{100} x \cdot dx$$

$$= \frac{1}{100} \int_0^{10} x \cdot dx$$

$$= \frac{1}{100} \left(\frac{x^2}{2} \right)_0^{10} dx$$

$$= \frac{1}{100} \left(\frac{100}{2} - 0 \right)$$

$$= \frac{1}{100} (50)$$

$$= \boxed{0.5}$$

$S = \{(HHH), (HHT), (HTH), (HTT), (THH), (THT), (TTH), (TTT)\}$

eg. 6.6

eg. 6.6

X
 $P(X)$

Cumulative

x	0	1	2	3
$P(x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$
$F(x)$	$\frac{1}{8}$	$\frac{4}{8}$	$\frac{7}{8}$	1

$$f(x) = \begin{cases} 0, & \text{for } x < 0 \\ \frac{1}{8}, & 0 \leq x < 1 \\ \frac{4}{8}, & 1 \leq x < 2 \\ \frac{7}{8}, & 2 \leq x < 3 \\ 1, & x \leq 3 \end{cases}$$

eg. 6.7

$$f(x) = 5x^4 \quad 0 \leq x \leq 1$$

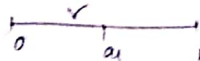
$$(i) P(X \leq a_1) = P(X > a_1)$$

$$P(X \leq a_1) = 1 - P(X > a_1)$$

$$= [P(X > a_1) = 1 - P(X \leq a_1)] \text{ formula}$$

$$2P(X \leq a_1) = 1$$

$$P(X \leq a_1) = \frac{1}{2}$$



$$\int_0^{a_1} f(x) dx = \frac{1}{2}$$

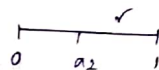
$$\int_0^{a_1} 5x^4 dx = \frac{1}{2}$$

$$5 \left(\frac{x^5}{5} \right) \Big|_0^{a_1} = \frac{1}{2}$$

$$a_1^5 - 0 = \frac{1}{2}$$

$$a_1 = \left(\frac{1}{2} \right)^{1/5}$$

$$(ii) P(X > a_2) = 0.05 = \frac{5}{100} = \frac{1}{20}$$



$$\int_{a_2}^1 f(x) dx = \frac{1}{20}$$

$$5 \left(\frac{x^5}{5} \right) \Big|_{a_2}^1 = \frac{1}{20}$$

$$1 - (a_2)^5 = \frac{1}{20}$$

$$(a_2)^5 = 1 - \frac{1}{20} = \frac{19}{20}$$

$$a_2 = \left(\frac{19}{20}\right)^{1/5}$$

$$a_2 = (0.95)^{1/5}$$

ex. 6.2

discrete

$$E(X) = \sum x p(x)$$

$$\text{Mean} = \mu'_1$$

$$E(X^2) = \sum x^2 p(x)$$

$$\mu'_2$$

$$\text{Var}(X) = E(X^2) - (E(X))^2$$

$$\mu_2 = \mu'_2 - (\mu'_1)^2$$

$$(i) E(a) = a, E(ax) = a E(X)$$

$$\text{Var}(a) = 0$$

$$\text{Var}(ax) = a^2 \text{Var}(X)$$

$$S.D. = \sqrt{\text{Var}(X)}$$

continuous

$$E(X) = \int_{-\infty}^{\infty} x f(x) dx$$

Mean

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x) dx$$

$$(1) X = 1, 2, 3, 4, 5, 6$$

x	1	2	3	4	5	6
p(x)	1/6	1/6	1/6	1/6	1/6	1/6

$$E(X) = \sum (x) p(x)$$

$$= \frac{1}{6} + \frac{2}{6} + \frac{3}{6} + \frac{4}{6} + \frac{5}{6} + \frac{6}{6}$$

$$= \frac{21}{6}$$

$$E(X) = 3.5$$

(2)

x	0	1	2	3
p(x)	0.2	0.1	0.4	0.3

$$E(X) = \sum x p(x)$$

$$= 0 + 0.1 + 0.8 + 0.9$$

$$E(X) = 1.8$$

(3)

x	3	4	5
p(x)	0.1	0.1	0.2

$$E(X) = \sum x p(x) = 0.3 + 0.4 + 1.0$$

$$= 1.7$$

$$E(X^2) = \sum x^2 p(x)$$

$$= 9 \times 0.1 + 16 \times 0.1 + 25 \times 0.2$$

$$= 0.9 + 1.6 + 5.0$$

$$E(X^2) = 7.5$$

$$\text{Var}(X) = E(X^2) - (E(X))^2$$

$$= 7.5 - (1.7)^2$$

$$= 7.5 - 2.89$$

$$\text{Var}(X) = 4.61$$

$$\text{S.D} = \sqrt{\text{Var}(X)}$$

$$= \sqrt{4.61}$$

$$= 2.147$$

$$\text{S.D} = 2.15$$

X	5000	-8000
P(X)	0.62	0.38

$$⑥. E(X) = 5000 \times 0.62 - 8000 \times 0.38$$

$$= 3100 - 3040$$

$$= 60$$

$$⑫. E(X) = 51$$

⑪.

X	4	-2
P(X)	1/2	1/2

$$E(X) = \sum x p(x)$$

$$= 4 \times \frac{1}{2} - 2 \times \frac{1}{2} = \frac{2}{2} = 1$$

$$E(X^2) = \sum x^2 p(x)$$

$$= 16 \times \frac{1}{2} + 4 \times \frac{1}{2}$$

$$= 8 + 2 = 10$$

$$\text{Var}(X) = E(X^2) - (E(X))^2$$

$$= 10 - 1 = 9$$

$$⑨. f(x) = \begin{cases} 2x & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

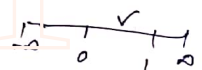
$$E(X) = \int_{-\infty}^{\infty} x f(x) dx$$

$$= \int_0^1 x (2x) dx$$

$$= 2 \left(\frac{x^3}{3} \right)_0^1$$

$$= \frac{2}{3} (1 - 0)$$

$$E(X) = \frac{2}{3}$$



⑧. $f(x) = \begin{cases} \frac{3}{x^4}, & x \geq 1 \\ 0, & \text{otherwise} \end{cases}$



$$E(x) = \int_{-\infty}^{\infty} x f(x) \cdot dx$$

$$= \int_1^{\infty} x \cdot \frac{3}{x^4} dx$$

$$= 3 \int_1^{\infty} \frac{1}{x^3} dx$$

$$= 3 \left[-\frac{1}{2x^2} \right]_1^{\infty}$$

$$= -\frac{3}{2} \left[\frac{1}{\infty} - \frac{1}{1} \right]$$

$$= -\frac{3}{2} (0 - 1)$$

$$\boxed{E(x) = \frac{3}{2}} \rightarrow \text{mean.}$$

$$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x) dx$$

$$= \int_1^{\infty} x^2 \left(\frac{3}{x^4} \right) dx$$

$$= 3 \int_1^{\infty} \frac{1}{x^2} dx$$

$$= 3 \left[-\frac{1}{x} \right]_1^{\infty}$$

$$= -3 \left[\frac{1}{\infty} - \frac{1}{1} \right]$$

$$= -3(0 - 1)$$

$$\boxed{E(x^2) = 3}$$

$$\text{Var}(x) = E(x^2) - (E(x))^2$$

$$= 3 - \frac{9}{4}$$

$$= \boxed{\frac{3}{4}}$$

⑨

$$Y = 2x + 1$$

Take var.

$$\text{Var } Y = \text{Var}(2x + 1)$$

$$= \text{Var}(2x) + \text{Var}(1)$$

$$= 2^2 \text{Var}(x) + 0$$

$$= 4(5)$$

$$\boxed{\text{Var } Y = 20}$$

eg 6.24

$$f(x) = k e^{-|x|}, \quad -\infty < x < \infty$$

Q1:

pdf:

$$\int_{-\infty}^{\infty} f(x) dx = 1$$

$$\int_{-\infty}^{\infty} k e^{-|x|} dx = 1$$

$$|x| = +x$$

$$= -x$$

$$2 \int_0^{\infty} k e^{-x} dx = 1$$

$$2 \int_0^{\infty} k e^{-x} dx = 1$$

$$2k \left[\frac{e^{-x}}{-1} \right]_0^{\infty} = 1$$

$$-2k [e^{-\infty} - e^0] = 1$$

$$\Rightarrow 2k (0 - 1) = 1$$

$$k = \frac{1}{2}$$

$$f(x) = \int_{-\infty}^{\infty} f(x) dx$$

$$= \int_{-\infty}^{\infty} x k e^{-|x|} dx$$

$$= \frac{1}{2} \int_{-\infty}^{\infty} x e^{-|x|} dx$$

$$= \frac{1}{2} \times 0 \quad [f(x) = x e^{-|x|} = \text{odd function}]$$

Mean = 0

$$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x) dx$$

$$= \int_{-\infty}^{\infty} x^2 k e^{-|x|} dx$$

$$= 2k \int_0^{\infty} x^2 e^{-x} dx$$

$$= 2 \times \frac{1}{2} \int_0^{\infty} x^2 e^{-x} dx$$

$$\left[\int_0^{\infty} x^n e^{-ax} dx = \frac{n!}{a^{n+1}} \right]$$

$$a=1 \quad n=2$$

$$= \frac{2!}{1!^{2+1}} = \frac{2!}{1^3} = 2$$

$$\text{Var}(x) = E(x^2) - (E(x))^2$$

$$= 2 - 0$$

$$= 2$$

$$(13) f(x) = \begin{cases} \frac{1}{30} e^{-x/30}, & x > 0 \\ 0, & x \leq 0 \end{cases}$$

$$E(x) = \int_{-\infty}^{\infty} x f(x) dx$$

$$= \int_0^{\infty} x \frac{1}{30} e^{-x/30} dx$$

$$\frac{1}{30} \int_0^{\infty} x e^{-x/30} dx$$

$$a = \frac{1}{30} \quad n = 1$$

$$= \frac{1}{30} \frac{1!}{\left(\frac{1}{30}\right)^2}$$

$$= \frac{1}{30} (30)^2$$

$$= 30 \times 1000$$

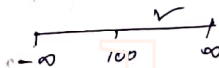
$$= 30,000 \text{ miles}$$

eg 6.25

$$f(x) = \begin{cases} e^{-x/100} & , x \geq 100 \\ 0 & , x < 100 \end{cases}$$

$$E(x) = \int_{-\infty}^{\infty} x f(x) dx$$

$$= \int_{100}^{\infty} x e^{-x/100} dx$$



$$\begin{aligned} u &= x \\ u' &= 1 \\ u'' &= 0 \end{aligned}$$

$$dv = e^{-x/100} dx$$

$$v = \frac{e^{-x/100}}{-1/100} = -100 e^{-x/100}$$

$$v_1 = -100 e^{-x/100}$$

$$v_1 = 10000 e^{-x/100}$$

$$\int u dv = uv - \int v du$$

$$\begin{aligned} \int_{100}^{\infty} x e^{-x/100} dx &= \left[x \cdot 10000 e^{-x/100} - \int 10000 e^{-x/100} dx \right]_{100}^{\infty} \\ &= (20000 \times 0 - 10000 \times 0) \\ &\quad - (100 \times 10000 e^{-100/100} - 10000 e^{-100/100}) \end{aligned}$$

$$= +10000 e^{-1} + 10000 e^{-1}$$

$$= 2 \times 10000 \times e^{-1}$$

$$= 20000 \times \frac{1}{2.718}$$

$$= 20000 \times 0.3679$$

$$= 7358 \text{ hours}$$

eg 6.7

$X=x$	1	2	3	4	5	6	7
$P(x)$	0.10	0.12	0.20	0.30	0.15	0.08	0.05

$$P(1) = P(x \leq 1) = P(1) = 0.10$$

$$P(2) = 0.10 + 0.12 = 0.22$$

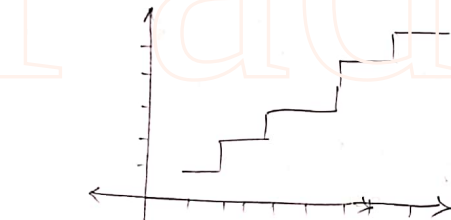
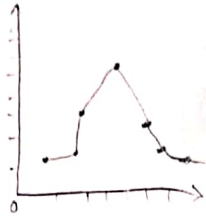
$$P(3) = 0.22 + 0.20 = 0.42$$

$$P(4) = 0.42 + 0.30 = 0.72$$

$$P(5) = 0.72 + 0.15 = 0.87$$

$$P(6) = 0.87 + 0.08 = 0.95$$

$$P(7) = 0.95 + 0.05 = 1.00$$



$$f(x) = \begin{cases} 0 & \text{if } x < 1 \\ 0.10 & \text{if } x \leq 1 \\ 0.22 & \text{if } x \leq 2 \\ 0.42 & \text{if } x \leq 3 \\ 0.72 & \text{if } x \leq 4 \\ 0.87 & \text{if } x \leq 5 \\ 0.95 & \text{if } x \leq 6 \\ 1 & \text{if } x \leq 7 \end{cases}$$

eg 6.8

Qn pdf.

$$f(x) = ax, \quad 0 \leq x \leq 1$$

$$\int_0^1 f(x) dx = 1$$

$$\int_0^1 ax dx = 1$$

$$a \int_0^1 x dx = 1$$

$$a \left(\frac{x^2}{2} \right)_0^1 = 1$$

$$\frac{a}{2} (1-0) = 1$$

$$a = 2$$

$$P\left(x \leq \frac{1}{2}\right) =$$

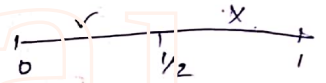
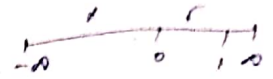
$$= \int_0^{\frac{1}{2}} ax dx$$

$$= \left(\frac{1}{2} \right) \int_0^{\frac{1}{2}} 2x dx$$

$$= \left[x^2 \right]_0^{\frac{1}{2}}$$

$$= \frac{1}{4} - 0$$

$$= \frac{1}{4}$$



eg 6.10

$$f(x) = \begin{cases} \frac{100}{x^2}, & x \geq 100 \\ 0, & x < 100 \end{cases}$$

$$F(x) = f(x)$$

$$f(x) = \int_{100}^x \frac{100}{n^2} \cdot dn$$

$$= \left[\frac{-100}{n} \right]_{100}^x$$

$$= \frac{-100}{x} - \left(\frac{-100}{100} \right)$$

$$= \frac{-100}{x} + 1$$

$$= \left[1 - \frac{100}{x} \right] \Rightarrow x \geq 100$$

eg 6.12

$X=x$	1	2	3	4	5	6	7	8	9	10
$P(x)$	0.15	0.10	0.10	0.01	0.08	0.01	0.05	0.02	0.28	0.20

$$E(x) = \sum xP(x)$$

$$= (1 \times 0.15) + (2 \times 0.10) + (3 \times 0.10) + (4 \times 0.01) \\ + (5 \times 0.08) + (6 \times 0.01) + (7 \times 0.05) + (8 \times 0.02) \\ + (9 \times 0.28) + (10 \times 0.20)$$

$$E(x) = 6.56$$

$$E(x^2) = \sum x^2 P(x)$$

$$= (1^2 \times 0.15) + (2^2 \times 0.10) + (3^2 \times 0.10) \\ + (4^2 \times 0.01) + (5^2 \times 0.08) + (6^2 \times 0.01) + (7^2 \times 0.05) \\ + (8^2 \times 0.02) + (9^2 \times 0.28) + (10^2 \times 0.20)$$

$$= (1 \times 0.15) + (4 \times 0.10) + (9 \times 0.10) + (16 \times 0.01) \\ + (25 \times 0.08) + (36 \times 0.01) + (49 \times 0.05) \\ + (64 \times 0.02) + (81 \times 0.28) + (100 \times 0.20)$$

$$= 50.38$$

$$\text{Var}(x) = E(x^2) - (E(x))^2$$

$$= 50.38 - (6.56)^2$$

$$= 7.35$$

eg 6.13

$$E(x) = \sum x P(x)$$

$$= \left(0 \times \frac{2}{11} \right) + \left(1 \times \frac{5}{11} \right) + \left(2 \times \frac{4}{11} \right)$$

$$= \frac{13}{11}$$

12/2/2020

Chapter - 8Sampling Techniques & Statistical Inference.Standard Errors (SE)

n = sample size. σ = Standard error
 σ^2 = variance ($\sigma + \sigma$)

Standard error.

① sample mean ($\bar{x}$)

$$\frac{\sigma}{\sqrt{n}}$$

②. Observed sample proportion (p)

$$\sqrt{\frac{pq}{n}}$$

③ Sample standard deviation (s)

$$\sqrt{\frac{\sigma^2}{2n}} \text{ (or) } \frac{\sigma}{\sqrt{2n}}$$

④ sample variance (s^2)

$$\sigma^2 \sqrt{\frac{2}{n}}$$

⑤ sample quartiles

$$1.36263$$

$$\frac{\sigma}{\sqrt{n}}$$

⑥ sample median

$$1.25331$$

$$\frac{\sigma}{\sqrt{n}}$$

⑦ sample correlation coefficient (r)

$$(1 - r^2)$$

$$\sqrt{n}$$

eg 8.6

$$\bar{x} = 20/\text{min}$$

$$Var = \sigma^2 = 4 \Rightarrow \sigma = 2$$

$$n = 1 \quad h\bar{x} = 60 \text{ min}$$

$$\text{Standard error} = \frac{\sigma}{\sqrt{n}} = \frac{2}{\sqrt{60}}$$

$$= \frac{2}{7.745} = 0.258$$

eg 8.7

$$n = ? \quad \sigma = 10$$

$$S.E = \frac{\sigma}{\sqrt{n}}$$

$$3 = \frac{10}{\sqrt{n}}$$

$$\sqrt{n} = \frac{10}{3}$$

Take square:

$$n = \frac{100}{9}$$

$$n = 11.11$$

$$n = 11$$

eg 8.8

$$n = 9000$$

$$P = \frac{3240}{9000} = 0.36$$

Probability:

$$P = 3(\sigma) +$$

$$= \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = 0.33$$

$$Q = 0.67$$

$$S.E = \sqrt{\frac{PQ}{n}}$$

$$= \sqrt{\frac{0.33 \times 0.67}{9000}}$$

$$= 0.00496$$

(5)

$$n = 600$$

$$P = 4\% = 0.04$$

$$Q = 0.96$$

$$S.E (P) = \sqrt{\frac{PQ}{n}}$$

$$= \sqrt{\frac{0.04 \times 0.96}{600}}$$

$$= 0.008$$

(18)

$$n = 1000$$

$$\sigma = 30$$

$$S.E. = \frac{\sigma}{\sqrt{n}}$$

$$= \frac{30}{\sqrt{1000}}$$

$$= 0.948$$

(19)

$$n = 60$$

$$\sigma = 3$$

$$S.E. = \frac{\sigma}{\sqrt{n}}$$

$$= \frac{3}{\sqrt{60}}$$

$$= \frac{1}{2 \times 60}$$

$$= 0.2738$$

(20)

$$n = 400$$

$$p = \frac{230}{400} = 0.575$$

$$S.E. (p) = \sqrt{\frac{pq}{n}}$$

$$= \sqrt{\frac{0.575 \times 0.425}{400}}$$

$$= 0.0248$$

$$p = 0.58$$

$$q = 0.42$$

$$eg 8.9$$

$$n = 50$$

$$\sigma = 6$$

$$S.E. = \frac{\sigma}{\sqrt{n}}$$

$$= \frac{6}{\sqrt{50}}$$

$$= \sqrt{\frac{36}{100}}$$

$$= \sqrt{0.36}$$

$$= 0.6$$

12/02/2020

Statistical Inference :

- (i) Estimation
- (ii) Testing Hypothesis.

Estimation :

Population mean = μ

$$S.D = \sigma$$

$$\text{mean} = \bar{x}$$

confidence Interval :

$$\bar{x} - Z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

Z_{α} table :

Critical Values	Z_{α}	Level of Significance (α)			
		1%	2%	5%	10%
Two tailed Test	$ Z_{\alpha} $	2.58	2.33	1.96	1.645
Right tailed Test	Z_{α}	2.33	2.055	1.645	1.28
Left tailed Test	$-Z_{\alpha}$	-2.33	-2.055	-1.645	-1.28

eg 8.11

$$\boxed{\sigma = 1.6} \quad \boxed{n = 64} \quad \boxed{\bar{x} = 90}$$

$$[Z_{\alpha/2} = 1.96 \text{ (5\%)}]$$

$$\bar{x} - Z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$$= 90 - 1.96 \times 1.6 \times \frac{1.6}{\sqrt{64}} \leq \mu \leq 90 + 1.96 \times 1.6 \times \frac{1.6}{\sqrt{64}}$$

$$= 90 - 1.96 \times 0.2 \leq \mu \leq 90 + 1.96 \times 0.2$$

$$= 90 - 0.392 \leq \mu \leq 90 + 0.392$$

$$= 89.608 \leq \mu \leq 90.392$$

$\therefore$ fall in the interval (89.6, 90.392) at 95%.

eg 8.12

$$\boxed{n = 100} \quad \boxed{\bar{x} = 7.4} \quad \boxed{\sigma = 1.2}$$

$$[Z_{\alpha/2} = 1.96 \text{ (5\%)}]$$

$$\bar{x} - Z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$$= 7.4 - 1.96 \times 1.2 \times \frac{1.2}{\sqrt{100}} \leq \mu \leq 7.4 + 1.96 \times 1.2 \times \frac{1.2}{10}$$

$$= 7.4 - 0.2352 \leq \mu \leq 7.4 + 0.2352$$

$$= 7.1648 \leq \mu \leq 7.6352$$

fall in the interval

(7.16, 7.63) at 95%.

eg 8.13

$$\boxed{n = 169} \quad \boxed{\bar{x} = 1350} \quad \boxed{\sigma = 100}$$

$$[Z_{\alpha/2} = 1.645 \text{ (10\%)}]$$

$$\bar{x} - Z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$$= 1350 - 1.645 \times \frac{100}{\sqrt{169}} \leq \mu \leq 1350 + 1.645 \times \frac{100}{13}$$

$$= 1350 - 12.6538 \leq \mu \leq 1350 + 12.6538$$

$$= 1337.3462 \leq \mu \leq 1362.6538$$

fall in the interval.

$$1337.35 \leq \mu \leq 1362.65$$

at 90%.

19/09/2020

Test of HypothesisZ = test.

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1)$$

Null hypothesis: $H_0: \mu = \mu_0$ accepted.Alternate hypothesis: $H_1: \mu \neq \mu_0$ rejected.

(two tail)

 $H_1: \mu > \mu_0$ Right tail. $H_1: \mu < \mu_0$ Left tail.Test of Hypothesis:Type I: The error of rejecting H_0 when it is true.Type II: The error of accepting H_0 when it is false.

Critical region (or) significant values:

$$Z = \frac{t - E(t)}{\sqrt{\text{Var}(t)}} = \frac{t - E(t)}{0.2} \sim N(0,1)$$

Conclusion:(i) $|Z| < Z_{\alpha/2}$, H_0 accepted(ii) $|Z| > Z_{\alpha/2}$, H_0 rejected.

(14)

$$[n=100] \quad [\mu=4] \quad [\sigma=3] \quad [\alpha=3.5]$$

Null hypothesis: $H_0: \mu = 4$, accepted.

Alternative hypothesis:

 $H_1: \mu \neq 4$, rejected.

level of significance 5%.

Z-test

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{35 - 4}{\frac{3}{\sqrt{100}}} = \frac{-0.5}{\frac{3}{10}}$$

$$= \frac{-0.5}{0.3} = [-1.667]$$

Conclusion:

$$[|Z| = 1.667]$$

$$|Z| = 1.66 < 1.96 = 5\%$$

$$|Z| < Z_{\alpha/2} \text{ and } H_0 \text{ accepted.}$$

eg 8.14

$$[n=50]$$

$$[\sigma=3.5]$$

$$[\alpha=10]$$

$$[\mu=9.5]$$

Null hypothesis: $H_0: \mu = 9.5$, accepted.

Alternative hypothesis:

 $H_1: \mu \neq 9.5$ rejected.

level of sign - 5%.

$$Z\text{-test } Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{10 - 9.5}{\frac{3.5}{\sqrt{50}}} = \frac{0.5}{\frac{3.5}{7.07}} = \frac{0.5}{0.495} = 1.01$$

Conclusion:

$$|Z| = 1.01 < 1.96 = 5\% \quad H_0 \text{ accepted}$$

(15)

$$n = 400$$

$$\bar{x} = 67.47$$

$$\mu = 67.39$$

$$\sigma = 1.30$$

Null hypothesis:

$$H_0: \mu = 67.39 \text{ accepted.}$$

Alternative hypothesis:

$$H_1: \mu \neq 67.39 \text{ rejected.}$$

level of sign = 5%.

Z-test:

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{67.47 - 67.39}{\frac{1.30}{\sqrt{400}}}$$

$$= \frac{0.08}{\frac{1.30}{20}} = \frac{0.08}{0.065} = 1.231$$

Conclusion:

$$|Z| = 1.231 < 1.96 = 5\% \text{ } H_0 \text{ accepted.}$$

$$|Z| > Z_{\alpha/2}, H_0 \text{ accepted.}$$

(16)

$$n = 100$$

$$\bar{x} = 78$$

$$\mu = 76$$

$$\sigma = 8$$

Null hypothesis:

$$H_0: \mu = 76 \text{ accepted.}$$

Alternative hypothesis:

$$H_1: \mu \neq 76 \text{ rejected}$$

level of sign = 5%.

(Two tail)

Z-test

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{78 - 76}{\frac{8}{\sqrt{100}}}$$

$$= \frac{-4}{\frac{8}{10}} = \frac{-4}{0.8}$$

Conclusion:

$$|Z| = 5 > 1.96 = 5\%.$$

$$|Z| > Z_{\alpha/2}, H_0 \text{ rejected.}$$

(17)

$$n = 50$$

$$\bar{x} = 1850$$

$$\mu = 1800$$

$$\sigma = 100$$

Null hypothesis:

$$H_0: \mu = 1800 \text{ accepted.}$$

Alternative hypothesis:

$$H_1: \mu \neq 1800 \text{ rejected}$$

(Two tail)

level of sign = 1%.

Z-test

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{1850 - 1800}{\frac{100}{\sqrt{50}}}$$

$$= \frac{50}{\frac{100}{7.07}} = \frac{50}{14.14} = 3.536$$

Conclusion:

$$|Z| = 3.536 > 2.58 = 1\%$$

$$|Z| > Z_{\alpha/2}, H_0 \text{ rejected.}$$

eg 8.15

$$\sigma = 20 \quad \bar{x} = 390 \quad \mu = 400 \quad n = 100$$

Null hypothesis:

$$H_0: \mu = 400 \text{ accepted.}$$

Alternative hypothesis:

$$H_1: \mu \neq 400 \text{ rejected.}$$

level of significance: 1%. (2.58)

Z-test:

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{390 - 400}{\frac{20}{\sqrt{100}}}$$

$$= \frac{-10}{\frac{20}{10}} = \frac{-10}{2}$$

$$\boxed{\bar{x} = -5}$$

$$\boxed{|Z| = 5}$$

Conclusion:

$$|Z| = 5$$

 $|Z| > Z_{\alpha/2}$ H_0 rejected.

eg: 8.16.

$$\sigma = 2.61, \quad \mu = 3.25 \quad \bar{x} = 3.4 \quad n = 900$$

(i) Null hypothesis:

$$H_0: \mu = 3.25 \text{ accepted.}$$

Alternative hypothesis:

$$H_1: \mu \neq 3.25 \text{ rejected.}$$

level of significance: 5%. (1.96)

Z-test:

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{3.4 - 3.25}{\frac{2.61}{\sqrt{900}}}$$

$$= \frac{0.15}{0.087}$$

$$\boxed{Z = 1.724}$$

Conclusion:

$$|Z| = 1.724$$

 $|Z| < Z_{\alpha/2}$ H_0 accepted.

(ii) Confidential limits:

(a) 95%. (5% (1.96))

$$\sigma = 2.61, \quad \bar{x} = 3.4, \quad n = 900$$

$$\bar{x} - Z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$$3.4 - 1.96(0.087) \leq \mu \leq 3.4 + 1.96(0.087)$$

$$3.2214 \leq \mu \leq 3.5705$$

$$\text{at } 95\% \quad (3.22, 3.57).$$

(b) 99%. (2% (2.33))

$$\bar{x} - Z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$$3.4 - 2.33(0.087) \leq \mu \leq 3.4 + 2.33(0.087)$$

$$3.197 \leq \mu \leq 3.6027$$

$$\text{at } 99\% \quad (3.197, 3.60).$$

Q. 8.13

$$n = 400$$

$$\sigma = 14.3$$

$$\bar{Y} = 153.3$$

$$\mu = 145.3$$

Null hypothesis:

$$H_0: \mu = 146.3 \text{ accepted}$$

Alternative hypothesis:

$$H_1: \mu \neq 146.3 \text{ rejected}$$

Level of significance:

 $\alpha = 0.05$ (1-tail)

$$Z = \frac{\bar{Y} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$$= \frac{153.3 - 146.3}{\frac{14.3}{\sqrt{400}}}$$

$$= \frac{7.0}{0.86}$$

$$|Z| = 8.60$$

Conclusion:

$$|Z| = 8.60$$

 $|Z| > Z_{\alpha/2}$ No rejected

Q. 8.18

$$n = 50$$

$$s.d. = 25$$

$$SD = \sqrt{var}$$

$$= \sqrt{625}$$

$$\sigma = 25$$

$$\mu = 52$$

$$\bar{X} = \frac{\text{Total Marks}}{n}$$

$$= \frac{2550}{50}$$

$$\bar{X} = 51$$

Null hypothesis:

$$H_0: \mu = 52 \text{ accepted}$$

Alternative hypothesis:

$$H_1: \mu \neq 52 \text{ rejected}$$

Level of significance:

 $\alpha = 0.05$ (2-tail) $\alpha = 0.025$

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$$= \frac{51 - 52}{\frac{25}{\sqrt{50}}}$$

$$= \frac{-1}{0.3535}$$

$$= -2.828$$

$$Z = -1.414$$

$$|Z| = 1.414$$

Conclusion:

$$|Z| = 1.414$$

 $|Z| < Z_{\alpha/2}$ No accepted

eg 8.19

$$n=50 \quad \bar{x} = 9.3 \quad \mu = 8.9 \quad \sigma = 1.6$$

Null hypothesis;

$$H_0: \mu = 8.9 \quad \text{Accepted}$$

Alternative hypothesis;

$$H_1: \mu \neq 8.9 \quad \text{Rejected}$$

level of significance;

$$5\% (1.96)$$

Z-test

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{9.3 - 8.9}{\frac{1.6}{\sqrt{50}}}$$

$$|Z| = 1.9676$$

 $|Z| < Z_{\alpha/2}$ H_0 accepted.

Auxiliary Constant:

Exercise 4.1

(3)

$$(i) y = cx + c - c^3$$

$$y' = c \quad + 0 - 0$$

$$y = y'x + y' - (y')^3$$

$$\frac{xy'}{y} = c^2$$

$$[xy' + y = 0]$$

(iv)

$$x^2 + y^2 = a^2$$

$$(22) \quad 2x + 2yy' = 0$$

$$[x + yy' = 0]$$

(ii)

$$y = c(x-c)^2 \rightarrow (1)$$

$$d(x^n) = nx^{n-1}$$

$$y' = 2c(x-c)^1 \cdot (1-0)$$

$$y' = 2c(x-c) \rightarrow (2)$$

$$\frac{y}{y'} = \frac{y}{2c(x-c)} = \frac{c(x-c)^2}{2c(x-c)}$$

$$\frac{2y}{y'} = x - c$$

$$c = x - \frac{2y}{y'}$$

$$[c = xy' - 2y]$$

$$y = \left(\frac{xy' - 2y}{y'} \right) \left(\frac{x - \frac{xy' - 2y}{y'}}{y'} \right)^2$$

$$= \left(\frac{xy' - 2y}{y'} \right) \left(\frac{xy' - xy' + 2y}{y'} \right)^2$$

$$y' = \left(\frac{xy' - 2y}{y'} \right) \frac{2y}{(y')^2}$$

$$(y')^3 = 2xy' - 2y^2$$

$$\boxed{(y')^3 - 2xy' + 2y^2 = 0}$$

Q 4.5

$$y = e^x (a \cos x + b \sin x)$$

(2 times diff)

$$y' = e^x (-a \sin x + b \cos x) + (a \cos x + b \sin x) e^x$$

$$y' = e^x (-a \sin x + b \cos x) + y$$

$$y' - y = e^x (-a \sin x + b \cos x)$$

$$-y'' - y' = e^x (-a \cos x - b \sin x)$$

$$+ (-a \sin x + b \cos x) e^x$$

$$y'' - y' = -y + y' - y$$

$$\boxed{y'' - 2y' + 2y = 0}$$

$$(2) (x-\alpha)^2 + (y-\beta)^2 = r^2 \rightarrow (1) \text{ is a circle}$$

$$2(x-\alpha)(1-0) + 2(y-\beta)(y'-0) = 0$$

(-2)

$$(x-\alpha) + (y-\beta)(y') = 0$$

$$(1-0) + (y-\beta)y'' + y'(y'-0) = 0$$

$$(y-\beta)y'' = -(y')^2 - 1$$

$$(y-\beta) = -\frac{(1+(y')^2)}{y''}$$

$$(x-\alpha) = \frac{(1+(y')^2)}{y''} y' = 0$$

$$(x-\alpha) = \frac{(1+(y')^2)}{y''} y'$$

$$\textcircled{1} y = mx + c \text{ at origin } (0,0)$$

$$0 = 0 + c$$

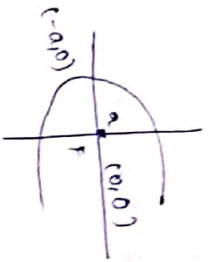
$$\boxed{c=0}$$

$$\boxed{y = mx}$$

$$y' = m$$

$$\textcircled{2} \Rightarrow \boxed{y = y'm}$$

(7)



$$(y-k)^2 = 4a(x-h)$$

(1, 2a)

$$(h, k) = (-a, 0)$$

$$y^2 = 4a(x+a) \rightarrow (1)$$

$$xyy' = x^2(1+0)$$

$$\boxed{\frac{yy'}{x} = a}$$

$$(1) \rightarrow y^2 = x^2 \left(\frac{yy'}{x} \right) \left(x + \frac{yy}{x} \right)$$

$$y^2 = \cancel{x} y y' \left(\frac{2x + yy'}{x} \right)$$

$$\boxed{y = 2xy' + y(yy')^2}$$

eg: 4.4

$$y = ae^{4x} + be^{-x}$$

(constant)

(1)

$$y' = 4ae^{4x} - be^{-x}$$

(2)

$$y'' = 16ae^{4x} + be^{-x}$$

(3)

$$(1) + (2) \rightarrow y + y' = 5ae^{4x} + 0$$

(4)

$$(2) + (3) \rightarrow y' + y'' = 20ae^{4x} + 0$$

$$y' + y'' = 4x \cdot 5ae^{4x}$$

$$y' + y'' = 4(y + y')$$

$$y' + y'' = 4y + 4y'$$

$$y'' - 3y' - 4y = 0$$

(5)

$$(x-a)^2 + (y-b)^2 = r^2 \rightarrow (1)$$

$$2(x-a)(1-0) + 2(y-b)(y'-0) = 0$$

(2)

$$(x-a) + (y-b)y' = 0$$

$$(x-0) + (y-b)y'' + y'(y'-0) = 0$$

$$(y-b)y'' = -(y')^2 - 1$$

$$\boxed{(y-b) = -\frac{(1+(y')^2)^2}{y''}}$$

$$(x-a) = \frac{(1+(y')^2)^2}{y''}$$

$$y' = 0$$

$$\boxed{(x-a) = \frac{(1+(y')^2)^2}{y''}}$$

(3)

$$\frac{(1+(y')^2)^2}{(y'')^2} + \frac{(1+(y')^2)^2}{(y'')^2} = r^2$$

$$= r^2$$

$$\frac{(1+(y')^2)^2}{(y'')^2} = (1+(y')^2)$$

$$= r^2$$

$$\boxed{\frac{(1+(y')^2)^3}{(y'')^2} = r^2}$$

eg of parametric

⑤. $(y-k)^2 = 4a(x-h) \rightarrow$ ①

$2a(y-k)y' = 4a(1-0)$

$(y-k)y' = 2a. \rightarrow$ ②

$(y-k)y'' + y'(y'-0) = 0$

$(y-k)y'' = -(y')^2$

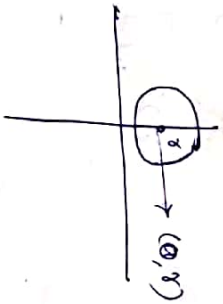
$(y-k) = \frac{-(y')^2}{y''}$

modul
 $y = mx + c$
 $y' = m$
 $y'' = 0$

③ $\rightarrow \frac{-(y')^2}{y''} y' = 2a$

$-(y')^3 = 2ay''$

$2ay'' + (y')^3 = 0$



$(x-h)^2 + (y-k)^2 = a^2$

$(h,k) = (0,0)$

$x^2 + (y-0)^2 = r^2 \rightarrow$ ⑦

Circle

$2x + 2(y-0)(y'-0) = 0$

$(\frac{1}{2}) 2x + (y-0)y' = 0$

$(y-x)y' = -x$

$y-x = \frac{-x}{y'}$

$x = y + \frac{x}{y'}$

⑧ $\Rightarrow x^2 + \frac{x^2}{(y')^2} = (y + \frac{x}{y'})^2$

$x^2 + \frac{x^2}{(y')^2} = y^2 + \frac{x^2}{(y')^2} + \frac{2xy}{y'}$

$x^2 = y^2 + \frac{2xy}{y'}$

(xy')

$x^2 y' = y^2 y' + 2xy$

eg 4.3
 $y = \frac{a}{x} + b$

$y' = \frac{-a}{x^2} + 0$

$d(\frac{1}{x}) = \frac{-1}{x^2}$

$x^2 y' = -a$

$x^2 y'' + 2xy' = 0$

$\frac{1}{x}$

$xy'' + 2y' = 0$