UNIT 5 MOTION OF SYSTEM OF PARTICLES AND RIGID BODIES

1.Define centre of mass.

The center of mass of a body is defined as a point where the entire mass of the body appears to be concentrated.

2.What is point mass?

A point mass is a hypothetical point particle which has nonzero mass and no size or shape.

3.What is the position of center of mass of two point masses based on the choice of the coordinate system?

- When the masses are on positive X-axis
- When the origin coincides with any one of the masses
- When the origin coincides with the center of mass itself

4.What are the effect of Center of mass in explosions?

If an explosion is caused by the internal forces in a body which is at rest or in motion, the state of the center of mass is not affected. It continues to be in the same state of rest or motion. But, the kinematic quantities of the fragments get affected.

If the explosion is caused by an external agency, then the kinematic quantities of the center of mass as well as the fragments get affected.

5.Define Torque

Torque is defined as the moment of the external applied force about a point or axis of rotation. The expression for torque is, $\tau = r \times F$

6.How to find the direction of torque?

The direction of torque is found using right hand rule. This rule says that if fingers of right hand are kept along the position vector with palm facing the direction of the force and when the fingers are curled the thumb points to the direction of the torque.

7.What is the reason why the children playing the traditional game will lean on to one side?

Naturally the body takes this position to balance the gravitational force (mg) and Normal force (N) acting on the body and to nullify the torque. Failing which, both these forces act along different lines leading to a net torque which makes one to fall.

8.What are the consideration for calculation of torques on rigid bodies?

- Consider only those forces that lie on planes perpendicular to the axis (and do not intersect the axis).
- Consider position vectors which are perpendicular to the axis.

9.Does angular momentum exists in linear motion.?

There is a misconception that the angular momentum is a quantity that is associated only with rotational motion. It is not true. The angular momentum is also associated with bodies in the linear motion. The angular momentum is associated with bodies with linear motion also. If the straight path of the particle passes through the origin, then the angular momentum is zero, which is also a constant.

10.Define couple.

A pair of forces which are equal in magnitude but opposite in direction and separated by a perpendicular distance so that their lines of action do not coincide that causes a turning effect is called a couple.

11.Define centre of gravity of a body.

The center of gravity of a body is the point at which the entire weight of the body acts irrespective of the position and orientation of the body.

12.What are the different values of torque?

- The torque is maximum when, $\vec{r}$ and $\vec{F}$ are perpendicular to each other. That is when θ = 90° and sin 90° = 1, Hence, τmax = rF.
- The torque is zero when $\vec{r}$ and $\vec{F}$ are parallel or antiparallel. If parallel, then θ = 0° and sin 0° = 0. If antiparallel, then θ = 180° and sin 180° = 0. Hence, τ = 0.
- The torque is zero if the force acts at the reference point. i.e. $\vec{r}$ = 0, τ = 0.

13. Define angular momentum

The angular momentum of a point mass is defined as the moment of its linear momentum. In other words, the angular momentum L of a point mass having a linear momentum p at position r with respect to a point or axis is mathematically written as, $\vec{L} = \vec{r} * \vec{p}$

14.State the condition for mechanical equilibrium of rigid body.

A rigid body is said to be in mechanical equilibrium when both its linear momentum and angular momentum remain constant.

15. Derive the relation between angular momentum and angular velocity.

$$L = rmv \sin 90°$$

$$L = rmv$$

The relation between linear velocity v and angular velocity ω in a circular motion is,

$$\mathbf{v = r\omega}$$

$$L = rmr\omega$$

$$L = mr^2 \omega$$

In vector notation,

$$\vec{L} = mr^2\vec{\omega}$$

$$I = \sum_{i=1}^{n} (m_i r^2_i)$$

L = Iω. where I is the moment of inertia

16.Define principle of moments.

Consider a light rod of negligible mass which is pivoted at a point along its length. Let two parallel forces F_1 and F_2 act at the two ends at distances d_1 and d_2 from the point of pivot and the normal reaction force N at the point of pivot. In this both the net force and net torque is zero. This is called principle of moments.

$$d_1F_1 = d_2F_2 \qquad \frac{F_1}{F_2} = \frac{d_2}{d_1}$$

17.What is the mechanical advantage of the simple lever?

$$\frac{F_1}{F_2} = \frac{d_2}{d_1}$$

The ratio (d_2/d_1) is called the mechanical advantage of the simple lever.

18.What is the condition for the bending of cyclists in curves?

A cyclist has to bend by an angle θ from vertical given by the below expression to stay in equilibrium

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$$

19.What are the significance of moment of inertia?

- The moment of inertia of a body is not an invariable quantity.
- It depends not only on the mass of the body, but also on the way the mass is distributed around the axis of rotation.

20.How could pure rolling problems can be solved?

The pure rolling problems could be solved by considering the motion as any one of the following two cases.

- The combination of translational motion and rotational motion about the center of mass.

(or)

- The momentary rotational motion about the point of contact.

21.What is the kinetic energy of pure rolling motion?

The total kinetic energy (KE) of pure rolling motion is the sum of kinetic energy due to translational motion (KE_{TRANS}) and kinetic energy due to rotational motion (KE_{ROT}).

22.How can a pure rolling motion be considered?

The pure rolling motion can be considered in two different ways.

- The combination of translational motion and rotational motion about the center of mass.

(or)

- The momentary rotational motion about the point of contact.

23.What is the Power Delivered by torque?

Power delivered is the work done per unit time.

$$P = \frac{dw}{dt} = \tau \frac{d\theta}{dt} \quad \because (dw = \tau d\theta)$$

$$P = \tau\omega$$

24.Derive the relation between rotational kinetic energy and angular momentum.

The angular momemtum of a rigid body is

$$L = I\omega$$

The rotational kinetic energy of a rigid body is

$$KE = \frac{1}{2} I\omega^2$$

$$KE = \frac{1}{2}\frac{I^2\omega^2}{I} = \frac{1}{2}\frac{(I\omega)^2}{I}$$

$$KE = \frac{L^2}{2I}$$

25.What is the workdone by the torque?

The work done by the force is

$$dw = F\,ds$$

$$ds = r\,d\theta$$

$$dw = F\,ds; \quad dw = F\,r\,d\theta$$

$$dw = \tau\,d\theta$$

26. State the law of conservation of angular momentum.

When no external torque acts on the body, the net angular momentum of a rotating rigid body remains constant. This is known as law of conservation of angular momentum.

$$\text{If } \tau = 0 \text{ then, } \frac{dL}{dt} = 0;\ L = \text{constant}$$

27.Define radius of gyration

The radius of gyration of an object is the perpendicular distance from the axis of rotation to an equivalent point mass, which would have the same mass as well as the same moment of inertia of the object.

28. State Perpendicular axis theorem

The theorem states that the moment of inertia of a plane laminar body about an axis perpendicular to its plane is equal to the sum of moments of inertia about two perpendicular axes lying in the plane of the body such that all the three axes are mutually perpendicular and have a common point.

29. State Parallel axis theorem

Parallel axis theorem states that the moment of inertia of a body about any axis is equal to the sum of its moment of inertia about a parallel axis through its center of mass and the product of the mass of the body and the square of the perpendicular distance between the two axes.