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XI STANDARD PHYSICS

VOLUME-II/UNIT-7

PROPERITIES OF MATTER

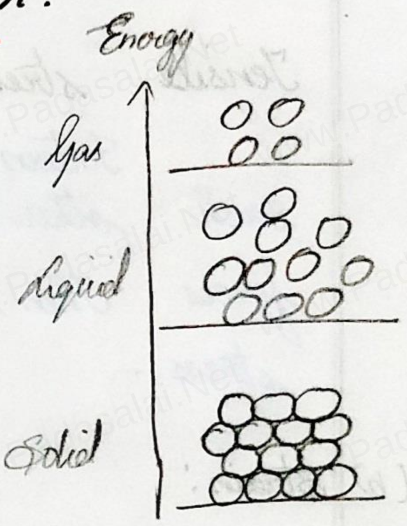
(STEP BY STEP SOLVED DERIVATIONS AND SOLVED PROBLEMS WITHOUT LOG BOOK)

(NEW SYLLABUS-2019)

(Easy Notes for XI Std State board Public Examination)

Unit-7, Properties of matter.

Microscopic understanding of various states of matter.



Elasticity:

A body regains its original shape and size after the removal of deforming force, it is said to be elastic and the property is called elasticity.

Deforming force:

The force which changes the size or shape of a body is called a deforming force.

Eg: Rubber, metals, steel ropes.

Plasticity:

If a body doesn't regain its original shape and size after removal of the deforming force, it is said to be a plastic body and the property is called plasticity.

Eg: Glass

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(a) Stress:

When a body is subjected to a deforming force, internal force is developed in it, called as restoring force. The force per unit area is called as stress. The SI unit of stress is Nm^{-2} or pascal (Pa) its dimension is $[ML^{-1}T^{-2}]$.

$$\text{Stress}, \sigma = \frac{\text{Force}}{\text{Area}} = \frac{F}{A}$$

(i) Longitudinal and shearing stress:

$$\left.\begin{array}{l}\text{Normal stress (or)}\\ \text{longitudinal stress}\end{array}\right\} \sigma_n = \frac{F_n}{\Delta A}$$

Illy.

$$\left.\begin{array}{l}\text{Tangential stress (or)}\\ \text{shearing stress}\end{array}\right\} \sigma_t = \frac{F_t}{\Delta A}$$

Tensile stress:

Internal forces on the two sides of ΔA may pull each other. it is stretched by equal and opposite forces. Then, the longitudinal stress is called tensile stress.

(b) Strain:

$$\text{Strain}, \varepsilon = \frac{\text{Change in size}}{\text{Original size}} = \frac{\Delta l}{l}$$

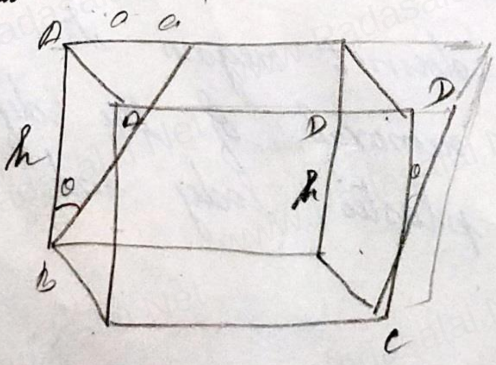
Longitudinal strain:

$$\varepsilon_l = \frac{\text{increase in length of the rod}}{\text{Original (or) natural length of the rod}} = \frac{\Delta l}{l}$$

It is divided into two types:

(i) Tensile strain
(ii) Compressive strain.

2. Shearing strain:



$$\varepsilon_s = \frac{AA'}{BA} = \frac{x}{h} = \tan\theta$$

$$\varepsilon_s = \frac{x}{h} = \theta = \text{Angle of shear}.$$

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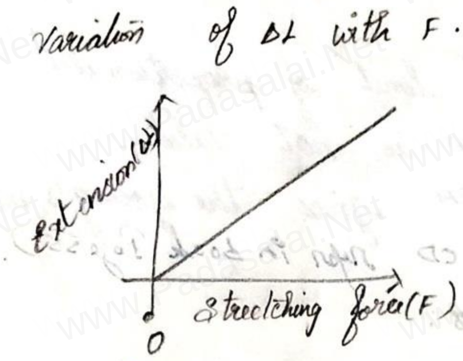
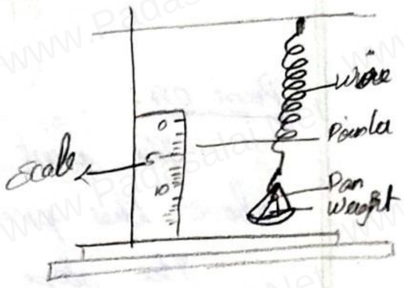
Elastic limit:

The maximum stress within which the body regains its original size and shape after the removal of deforming force is called the elastic limit.

Hook's law and its experimental verification:

Variation of ΔL with F.





From the figure,

$$\Delta L = (\text{slope})\, F \rightarrow (1)$$

We know that,

$$\text{Volume } (V) = AL \rightarrow (2)$$

multiply and divide equ(1) by volume $\longrightarrow F(\text{slope}) = \frac{V}{V}\Delta L$

from equ (2).

$$F(\text{slope}) = \frac{AL}{AL}\, \Delta L$$

Divide by A on both sides

$$\frac{F}{A} = \frac{\cancel{A}L}{\cancel{A}L \cdot A} \times \frac{\Delta L}{A}$$

$$\frac{F}{A} = \frac{L}{A(\text{slope})} \cdot \frac{\Delta L}{L}$$

$$\frac{F}{A} \propto \frac{\Delta L}{L}$$

We know that,

$$\frac{F}{A} = \text{stress } \sigma$$

$$\frac{\Delta L}{L} = \text{strain } \varepsilon$$

$$\sigma \propto \varepsilon$$

$\therefore$ The stress is proportional to the strain in the elastic limit.

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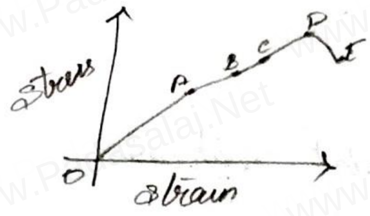
Stress - strain profile curve :



OA : Proportional limit
B : Elastic limit
D : Ultimate stress point
E : Breaking or rupture point.

Point OA :

The point A is called limit of proportionality because above this point Hook's law is not valid.

The slope of the line OA gives the young's modulus of the wire. (Portion AB, BC, CD refer in book Page 55).

Define : Breaking (or) tensile stress :

The maximum stress beyond which the wire breaks is called breaking stress or tensile strength.

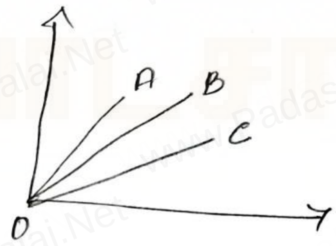
Moduli of elasticity :

young's modulus :

The young's modulus is defined as the ratio between tensile stress and tensile strain. The S.I unit for young's modulus is Nm^{-2} (or) pascal.

$$Y = \frac{\sigma_t}{\varepsilon_t} \quad (or) \quad Y = \frac{\sigma_c}{\varepsilon_c}.$$

Example 7.1



Stress in $\propto$ to strain (obey Hook's law).

slope of A > slope of B > slope of C.

Which implies

Y. modulus of C < Y. modulus of B < Y. modulus of A

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Example 7.2

$g = 10 ms^{-2}$

We know that,

$\frac{F}{A} = Y \times \frac{\Delta L}{L}$ $\longrightarrow$ $Y = \frac{\sigma_t}{\varepsilon_t}$

$Y = \frac{\frac{F}{A}}{\frac{\Delta L}{L}}$

$\Delta L = \left[\frac{F}{A}\right]\left[\frac{L}{Y}\right]$

$F = mg = 5 \times 10 = 50$

$= \left[\frac{\cancel{50}}{\cancel{1.25} \times 10^{-4}}\right]\left[\frac{\cancel{10}}{\cancel{4} \times 10^{10}}\right]$

$= 10^{6} \times 10^{-10}$

$= 10^{-4}\ m.$

Bulk modules:

Bulk modules is defined as the ratio of volume stress to the volume strain.

$$\text{Bulk modulus, } K = \frac{\text{Normal stress or Pressure}}{\text{Volume strain}}$$

The normal stress or pressure is, $\sigma_n = \frac{F_n}{\Delta A} = \Delta P$

The volume strain is, $\varepsilon_v = \frac{\Delta V}{V}$

Therefore,

$$K = -\frac{\sigma_n}{\varepsilon_v} = -\frac{\Delta P}{\frac{\Delta V}{V}}$$

(-) sign implies that when pressure is applied the body its volume decreases.

Example 7.3:

Bulk modules, $K = \frac{\frac{F}{A}}{\frac{\Delta V}{V}}$

here, $\frac{F}{A} = P$

$K = \frac{P}{\frac{\Delta V}{V}}$

$K = P \cdot \frac{V}{\Delta V}$

$K = \frac{10^6 \times 1}{1.5 \times 10^{-5}}$

$= \frac{10^6 \times 1}{15 \times 10^{-5} \times 10^{-1}}$

$= \frac{10^6 \times 1}{15 \times 10^{-4}}$

$= \frac{1}{15} \times 10^{6+4}$

$K = 0.067 \times 10^{10}\ Nm^{-2}$

$K = 6.67 \times 10^{10}\ Nm^{-2}$

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Compressibility:

The reciprocal of the bulk modulus is the compressibility. It is defined as the fractional change in volume per unit increases the pressure.

$$C = \frac{1}{K} = \frac{-\varepsilon_v}{\sigma_n} = \frac{\frac{\Delta V}{V}}{\Delta P}.$$

Rigidity modulus or shear modulus:

It is defined as the ratio of shearing stress to angle of shear (or) shearing strain. The S.I unit is Nm^{-2} (or) pascal.

$$\eta_R = \frac{\text{shearing stress}}{\text{angle of shear (or) shearing strain}}$$

The shearing stress,

$$\sigma_s = \frac{\text{tangential force}}{\text{area over which it is applied}}$$

$$= \frac{F_t}{\Delta A}.$$

The angle of shear or shearing strain,

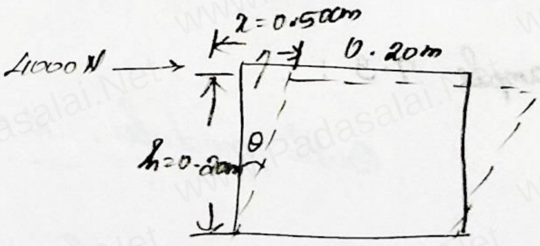
$$\varepsilon_s = \frac{x}{h} = \theta.$$

$\therefore$ Rigidity modulus is,

$$\eta_R = \frac{\sigma_s}{\varepsilon_s} = \frac{\frac{F_t}{\Delta A}}{x/h}$$

$$= \frac{\frac{F_t}{\Delta A}}{\theta}.$$

Example 7.4



$$\eta_R = \frac{\frac{F_t}{\Delta A}}{x/h}$$

$$= \frac{F_t}{\Delta A} \times \frac{h}{x}$$

here,

$h = L$

$$\eta_R = \frac{F_t}{\Delta A} \times \frac{L}{x}.$$

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$L = 0.20\,m$, $F = 4000\,N$, $x = 0.50\,cm = 0.005\,m$, $A = L^2 = (0.20)^2 = 0.04\,m^2$.

$$\eta_R = \frac{F}{A} \times \frac{L}{x}$$

$$= \frac{4000}{0.04} \times \frac{0.20}{0.005} = \frac{4000 \times 20 \times 10^{-1}}{4 \times 10^{-1} \times 5 \times 10^{-3}}$$

$$= 4 \times 10^6\,Nm^{-2}. \qquad = \frac{4 \times 10^3 \times 20 \times 10^{-1} \times 10^3 \times 10^1}{4 \times 5}$$

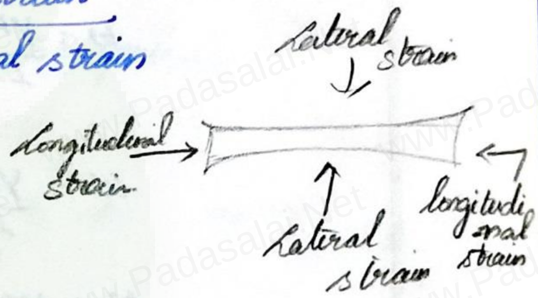
$$= 4 \times 10^7 \times 10^{-1} = 4 \times 10^6\,Nm^{-2}$$

Poisson's ratio:

It is defined as the ratio ~~which~~ is defined as the ratio of relative contraction (lateral strain) to relative expansion (longitudinal strain). It is denoted by the symbol μ.

$$\text{Poisson's ratio, } \mu = \frac{\text{lateral strain}}{\text{longitudinal strain}}$$

$$\mu = \frac{\frac{d}{D}}{\frac{l}{L}}$$



here,

L - length of wire.

D - Diameter

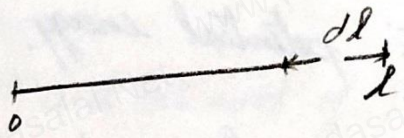
l - increase in length

d - decrease in diameter.

$$\mu = -\frac{L}{l} \times \frac{d}{D}.$$

[(-) sign indicates the elongation along longitudinal and contraction along lateral dimension].

Elastic Energy:



The work done in stretching the wire by dl.

$$dw = F\,dl \rightarrow (1).$$

The total work done in stretching the wire from 0 to l is

$$w = \int_0^l dw \rightarrow (2).$$

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sub. equ ① in ②.

$$W = \int_0^l F\, dl \rightarrow ③$$

from young's modulus,

$$Y = \frac{F}{A} \times \frac{L}{l}$$

$$F = \frac{YAl}{L} \rightarrow ④$$

sub. equ ④ in ③.

$$W = \int_0^l \frac{YAl}{L} . dl$$

Since, l is dummy variable. take it as l'.

$$W = \int_0^l \frac{YAl'}{L}\, dl'$$

$$W = \frac{YA}{L} \int_0^l l'\, dl'$$

$$W = \frac{YA}{L} \left[\frac{l'^{1+1}}{1+1} \right]_0^l$$

$$W = \frac{YA}{L} \left[\frac{l'^2}{2} \right]_0^l$$

$$W = \frac{YA}{L} \left[\frac{l^2}{2} - 0 \right].$$

$$W = \frac{1}{2} \left(\frac{YAl}{L} \right) l$$

from equ ④.

$$W = \frac{1}{2} F l$$

$\frac{1}{2} Fl = W =$ Elastic potential energy.

Energy per unit volume is called energy density

$$u = \frac{\text{Elastic Potential energy}}{\text{Volume}}$$

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$$u = \frac{\frac{1}{2} F.l}{AL}$$

$$\frac{1}{2}\frac{F}{A}\cdot\frac{l}{L} = \frac{1}{2}(\text{stress} \times \text{strain}).$$

$\left[\frac{F}{A} = \text{stress}, \ \frac{l}{L} = \text{strain}\right]$.

Example 7.5

Given:

$l = 2m$, $A = 10^{-6} m^2$, $Y = 12 \times 10^{10} Nm^{-2}$.

$F = 980 N$

(i) stress $= \frac{F}{A}$

$= \frac{980}{10^{-6}} = \frac{98 \times 10^1 \times 10^6}{12}$

$= 98 \times 10^1 \times 10^6$

$= 98 \times 10^7 Nm^{-2}$.

(ii) strain $= \frac{\text{stress}}{Y} = \frac{98 \times 10^7}{12 \times 10^{10}}$

$= \frac{98 \times 10^7 \times 10^{-10}}{12}$

$= 8.1666 \times 10^{-3}$

$= 8.17 \times 10^{-3}$ (No unit)

98×2 = 196

98 × 8.17 = 800.66

(iii) Energy $= \frac{1}{2}(\text{stress} \times \text{strain}) \times \text{volume}$.

$= \frac{1}{2}(98 \times 10^7) \times (8.17 \times 10^{-3}) \times 2 \times 10^{-6}$

$= 80066 \times 10^7 \times 10^{-9}$

$= 800.66 \times 10^{-2}$

$= 8.0066$

$= 8$ joule.

Pressure of fluid:

$$P = \frac{F}{A}.$$

Pressure is a scalar quantity. It's S.I unit and dimensions are Nm^{-2} (or) pascal (Pa) and $[ML^{-1}T^{-2}]$.

Density of a Fluid:

The density of a fluid is defined as its mass per unit volume.

density, $\rho = \frac{m}{V}$.

The dimension and S.I unit of ρ are $[ML^{-3}]$ and kgm^{-3}.

It is a scalar quantity.

Example 7.6

Radius of sphere, $R = 1.5$ cm $= 1.5 \times 10^{-2}$

mass, $m = 0.038$ kg.

Density of sphere, $\rho = \frac{m}{V}$.

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Volume of sphere $= \frac{4}{3}\pi R^3$

$= \frac{4}{3} \times 3.14 \times (1.5 \times 10^{-2})^3$

$= \frac{4}{3} \times 3.14 \times 3.375 \times 10^{-6}$

$= 4 \times 3.14 \times 1.125 \times 10^{-6}$

$= 14.13000 \times 10^{-6}$

$= 1.413 \times 10^{-5}$

$\rho = \frac{0.038}{1.413 \times 10^{-5}}$

$= 0.02690 \times 10^{-3} \times 10^{3} \times 10^{5}$

$= 0.0269 \times 10^{-5}$

$= 269 \times 10^{-1}$

$\rho = 2690 \ kg m^{-3}$.

Hence, the specific gravity of the sphere $= \frac{2690}{1000}$

$= 2.69$

Pressure due to fluid column at rest :

Let

h_1 - depth from air-water interface to level 1 of the cylinder.

h_2 - depth from air-water interface to level 2 of the cylinder.

F_1 - the force acting downward on level 1

F_2 - the force acting upward on level 2.

here,

$F_1 = P_1 A \rightarrow (A)$

$F_2 = P_2 A \rightarrow (B)$

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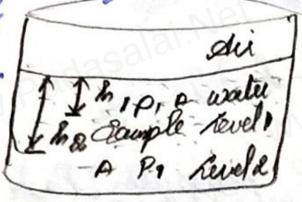
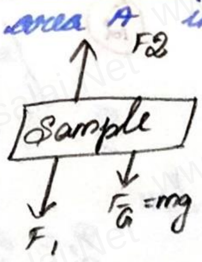
Case 1: (i) A sample of water with base area A in a static fluid with its forces in equilibrium





$$F_2 - F_1 = mg \rightarrow (1)$$

$$F_2 - F_1 = F_G \rightarrow (2).$$

where,
m - mass of water in sample element.
ρ - density of water.

We know that,

$$\text{density } \rho = \frac{m}{V}$$

$$m = \rho \times V \rightarrow (3).$$

$$\text{Volume}, V = A(h_2 - h_1) \rightarrow (4).$$

sub equ (4) in (3).

$$\text{mass}(m) = \rho \cdot A(h_2 - h_1) \rightarrow (5).$$

then,
gravitational force $(F_G) = mg$.

sub equ (5) in (6).

$$F_G = \rho A(h_2 - h_1)g \rightarrow (7)$$

from (1).

$$F_2 = F_1 + mg.$$

from equ (A) and (B).

$$P_2 A = P_1 A + mg. \rightarrow (8).$$

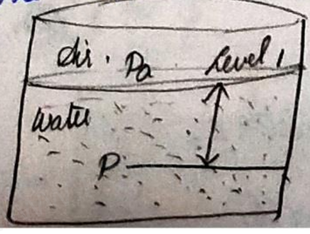
sub. equ (5) in equ (8).

$$P_2 A = P_1 A + \rho A(h_2 - h_1)g.$$

$$P_2 \cancel{A} = \cancel{A}(P_1 + \rho(h_2 - h_1)g).$$

$$\boxed{P_2 = P_1 + \rho(h_2 - h_1)g.} \rightarrow (C).$$

Case 2: The Pressure (P) at a depth (h) below the water surface.



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from eqn ①.

$P_2 = P_1 + \rho(h_2 - h_1)g$.

here,

P_1 = Atmospheric pressure (P_a).
P_2 = Pressure (P) at depth.
height (h_1) = 0.
(h_2) = h

Then,

$P = P_a + \rho g h \rightarrow$ ③.

We know that,

Atmospheric pressure (P_a) = 1.013×10^5 Pa

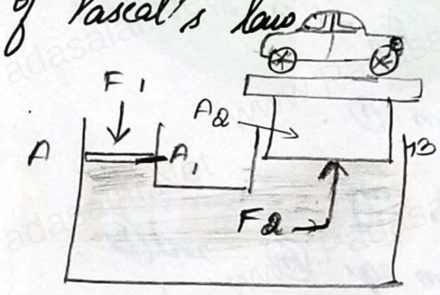
if atmospheric pressure (P_a) is neglected.

$P = \rho g h$

Pascal's law and its applications:

Pascal's law state's that, If the pressure in a liquid is changed at a particular point, the change is transmitted to the entire liquid without being dimished in magnitude.

Application of Pascal's law



Let,

A_1 - Cross sectional area of piston 1

A_2 - Cross sectional area of piston 2.

F_1 — force given at piston 1

F_2 — force transmitted at piston 2.

here, the pressure of the liquid at piston 1.

$(P) = \frac{F_1}{A_1} \rightarrow$ ①.

According to Pascal's law:

The upward force on piston B,

$F_2 = P \times A_2 \rightarrow$ ②.

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sub. equ (1) in (2).

$$F_2 = \frac{F_1}{A_1} \times A_2.$$

$$\boxed{F_2 = \frac{A_2}{A_1} \times F_1}$$

Where,

$\frac{A_2}{A_1}$ = mechanical advantage of the lift.

Example 7.7

Diameter = 60 cm & 5 cm

$F_1 = 50N$ $F_2 = ?$

$r = \frac{D}{2}$ Area $= \pi r^2$

Area of smaller piston, $A_1 = \pi \left(\frac{5}{2}\right)^2$

$= \pi \left(\frac{25}{4}\right)$.

Area of larger piston $A_2 = \pi \left(\frac{60}{2}\right)^2$

$= \pi \left(\frac{3600}{4}\right)$.

$$F_2 = \frac{A_2}{A_1} \times F_1$$

$$= \frac{\pi \times \frac{3600}{4}}{\pi \times \frac{25}{4}} \times 50$$

$$= \frac{\cancel{\pi} \times 3600}{\cancel{4}} \times \frac{\cancel{4}}{\cancel{\pi} \times \cancel{25}} \times \cancel{50}^{2}$$

$$= 3600 \times 2$$

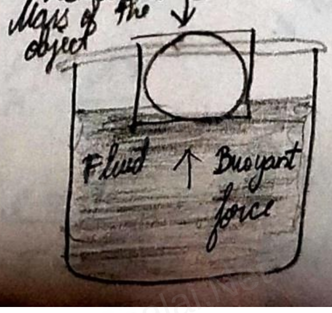
$$\boxed{F_2 = 7200N}$$

$\therefore$ The force of 50 N, the force of 7200 N can be lifted.

Buoyancy:

The upward force exerted by a fluid that opposes the weight of an immersed object in a fluid is called upthrust or buoyant force and the phenomenon is called buoyancy.

V.V.V.I (2) Archimedes Principle:



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(14) It states that when a body is partially or wholly immersed in a fluid, it experiences an upward thrust equal to weight of the fluid displaced by it and its upthrust acts through the centre of gravity of the liquid displaced.

Example 7.8

Let each side of the cube be 'l'.

Volume is 3 cm depth of cube.

$V = (3\,cm) \times l^2$

$= 3l^2\,cm. = 3l^2 \times 10^{-2}\,m.$

According to principle of floatation:

$V\rho g = mg$

$V\rho = m.$

ρ is density of water $= 1000\,kgm^{-3}$.

$(3l^2 \times 10^{-2}\,m) \times (1000\,kgm^{-3}) = 300 \times 10^{-3}$.

$l^2 = \dfrac{\cancel{300}^{100} \times 10^{-3}}{\cancel{3} \times 10^{-2} \times 1000}$

$= \dfrac{100 \times 10^{-3}}{10^{-2} \times 10^{3}}$

$l^2 = 100 \times 10^{-3} \times 10^{-3} \times 10^{2}$

$= 100 \times 10^{-6+2}$

$l^2 = 100 \times 10^{-4}$

$l = \sqrt{100 \times 10^{-4}}$

$= \sqrt{10 \times 10 \times 10^{-2} \times 10^{-2}}$

$= 10 \times 10^{-2}\,m.$

$l = 10\,cm.$

$\therefore$ Volume of cube, $V = l^3$

$= (10)^3$

$V = 1000\,cm^3$

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Viscosity:

It is defined as the property of a fluid to oppose the relative motion between its layers.

Co-efficient of viscosity:

$$F \propto A \rightarrow (1)$$

$$F \propto \frac{dv}{dx} \rightarrow (2)$$

from equ (1) & (2).

$$F \propto A\frac{dv}{dx}$$

$$F = -\eta A\frac{dv}{dx}$$

η - Co-efficient of viscosity.

$$F = -\eta A\cdot\frac{dv}{dx}$$

$$-\eta = \frac{F}{A}\cdot\frac{dx}{dv}$$

$$\eta = -\left(\frac{F}{A}\cdot\frac{dx}{dv}\right).$$

The dimensional formula for co-efficient of viscosity is $[ML^{-1}T^{-1}]$.

Example 7.9:

Given:

Area $(A) = 2.5\times10^{-4}$

$dx = 0.25\times10^{-3}m$

$F = 2.5N$

$dv = 8\times10^{-2}ms^{-1}$

$$F = -\eta A\frac{dv}{dx}$$

$$\eta = \frac{F}{A}\cdot\frac{dx}{dv}$$

$$= \left(\frac{2.5N}{2.5\times10^{-4}m^{2}}\right)\cdot\left(\frac{0.25\times10^{-3}m}{8\times10^{-2}ms^{-1}}\right)$$

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$$= \frac{0.25 N \times 10^{-3} \times 10^{4} \times 10^{2} m^{-2}}{3 s^{-1}}$$

$$= \frac{0.25 \times 10^{-3+6} Nm^{-2} s}{3}$$

$$= 0.083 \times 10^{3} Nm^{-2} s.$$

Streamlined flow:

When a liquid flows such that each particle of the liquid passing through a point moves along the same path with the same velocity as its predecessor then the flow of liquid is said to be a streamlined flow.

Turbulent flow:

Any pattern of fluid motion carotarised by chaotic changes in pressure and flow velocity.

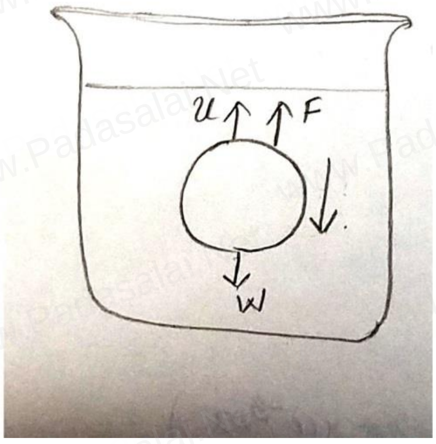
Critical velocity:

The flow of fluid is streamlined up to a certain velocity is called critical velocity.

Law of similarity:

It states that when there are two geometrically similar flow, both are essentially equal to each other as long as they embrace the same reynold's number.

Terminal Velocity:



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Forces acting on the sphere are.

(i) F_G - gravitational force of sphere acting vertically downwards.

(ii) U - Upthrust due to buoyancy.

(iii) F - Viscous force.

Gravitational force acting on the sphere.

$$F_G = mg.$$

$$\text{mass } (m) = \text{volume} \times \text{density}$$

$$= \frac{4}{3}\pi r^3 \times \rho$$

$$F_G = \frac{4}{3}\pi r^3 \rho g \rightarrow (1).$$

Upthrust, $U = \frac{4}{3}\pi r^3 \sigma g \rightarrow (2)$.

Viscous Force, $F = 6\pi \eta r v_t \rightarrow (3)$.

At terminal velocity, v_t

downward force = upward force.

$$F_G = U + F$$

$$F_G - U = F$$

$$F = F_G - U \rightarrow (4).$$

sub. equ (1), (2), & (3) in (4).

$$6\pi \eta r v_t = \frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \sigma g$$

$$6\pi \eta r v_t = 4/3\, \pi r^3 g\, (\rho - \sigma).$$

$$v_t = \frac{\frac{4}{3}\cancel{\pi} r^3 g\, (\rho - \sigma)}{6\cancel{\pi}\, \eta\, \cancel{r}}$$

$$= \frac{\overset{2}{\cancel{4}}\, r^2 g\, (\rho - \sigma)}{3 \times \underset{3}{\cancel{6}}\, \eta}$$

$$= \frac{2}{9} \times \frac{r^2 (\rho - \sigma) g}{\eta} \rightarrow (5)$$

rer,
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here constant,

$$k = \frac{2}{9}\frac{r^2(\rho-\sigma)g}{\eta}$$

$v_t \propto r^2$ [Radius of sphere].

from equ ⑤.

if $\sigma > \rho$

$(\rho - \sigma)$ becomes negative.

Stoke's law and its application:

Viscous force (F) acting on a spherical body of radius r depends directly on.

(i) Radius (r) of the sphere.
(ii) Velocity (v) of the sphere
(iii) (η) co-efficient of viscosity of the liquid.

$$F \propto \eta^x r^y v^z$$

$$F = K \eta^x r^y v^z$$

Where,
K - dimensional constant.

Using dimensions,

$$[MLT^{-2}] = K\,[ML^{-1}T^{-1}]^x \times [L]^y \times [LT^{-1}]^z.$$

$$M^1L^1T^{-2} = K\,[M^xL^{-x}T^{-x}]\cdot[L^y]\cdot[L^zT^{-z}]$$

$$= K\,[M^x L^{-x+y+z} T^{-x-z}]$$

$x = 1$

$-x+y+z = 1$

$-1+y+1 = 1$

$\boxed{y = +1}$

$-x-z = -2$

$-1-z = -2$

$-z = -2+1$

$-z = -1$

$\boxed{z = 1}$

$x = 1,\ y = 1,\ z = 1$

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$F = K\eta^1 r^1 v^1$

$F = K\eta r v$

here, constant, $K = 6\pi$

$F = 6\pi\eta r v$

This relation is known as stoke's law.

Poiseulle's equation:

Let, $v = \left[\frac{V}{t}\right]$

here,

η - Co-efficient of viscosity of liquid

r - Radius of the tube.

$\left(\frac{P}{l}\right)$ - Pressure gradient.

$$v \propto \eta^a r^b \left(\frac{P}{l}\right)^c$$

$$v = K\eta^a r^b \left(\frac{P}{l}\right)^c \rightarrow (1)$$

Where, K is a dimensionless constant,

therefore,

$$v = \frac{\text{Volume}}{\text{time}}$$

$$= \frac{\text{Area} \times \text{Length}}{\text{time}}$$

$$= \frac{L^2 \times L}{T}$$

$$= \frac{L^3}{T}$$

$$v = [L^3 T^{-1}] \rightarrow (2)$$

$\frac{dP}{dx} = \frac{\text{Pressure}}{\text{distance}} = \frac{\text{Force / Area}}{\text{distance}}$

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$$= \frac{\text{mass} \times \text{acceleration}/\text{area}}{\text{distance}}$$

$$= \frac{M \times \frac{\text{Velocity}}{\text{time}}}{L \times L \times \frac{\text{Time}}{\text{distance}}}$$

$$= \frac{M \times L}{L \times L \times T \times T \times L}$$

$$= \frac{M \times \cancel{L}}{\cancel{L^3}\, T^2}$$

$$= \frac{M}{L^2 T^2}$$

$$= [ML^{-2}T^{-2}] \rightarrow (3)$$

$$\eta = [ML^{-1}T^{-1}] \rightarrow (4) \text{ and } r = [L] \rightarrow (5)$$

sub. equ (2), (3), (4) & (5) in (1).

$$[L^3T^{-1}] = [ML^{-1}T^{-1}]^a [L]^b [ML^{-2}T^{-2}]^c$$

$$M^0L^3T^{-1} = [M^aL^{-a}T^{-a}][L^b][M^cL^{-2c}T^{-2c}]$$

$$M^0L^3T^{-1} = M^{a+c} L^{-a+b-2c} T^{-a-2c}$$

$a+c=0$

$\boxed{c = -a}$

$c = -(-1)$

$\boxed{c=1}$

$-a+b-2c = 3$

$-a+b-2 = 3$

$-a+b = 5$

$b = 5+a$

$b = 5-1$

$\boxed{b=4}$

$-a-2c = -1$

$-a-2(-a) = -1$

$-a+2a = -1$

$1a = -1$

$\boxed{a=-1}$

$a=-1,\ b=4,\ c=1.$

$$V = k\, \eta^a r^b \left(\frac{P}{l}\right)^c$$

$$V = K\, \eta^{-1} r^4 \left(\frac{P}{l}\right)^1$$

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here. $k = \frac{\pi}{8}$

$$v = \frac{\pi}{8} \times \frac{1}{\eta} \times a^4 \times \frac{P}{l}$$

$$v = \frac{\pi a^4 P}{8 \eta l}$$

Surface Tension:

2m Liquid have no definite shape but have a definite volume. Hence, they acquire a free surface when poured into a container. Therefore, the surfaces have some additional energy called as surface energy. The phenomenon behind the above fact is called surface tension. 2m

Factors affecting surface tension:

Temperature:

Surface tension at T_t at $t°c$.

$$T_t = T_0 (1 - \alpha t).$$

Where,

T_0 - surface tension at temperature $0°c$

α - temperature co-efficient of surface tension.

2 [Critical temp. of water is 374°c] 2

Vanderwalls equation:

$$T_t = T_0 \left[1 - \frac{t}{t_c}\right]^{3/2}$$

By generalising.

$$T_t = T_0 \left[1 - \frac{t}{t_c}\right]^{n}$$

here.

n - Varies value of different liquids

t & t_c - temperature & critical temperature in absolute scale.

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Example 7.10

$\Delta A = A_2 - A_1$

$= 2(100-50)\times 10^{-4} m^2$

$= (200-100)\times 10^{-4} m^2$

$= 100\times 10^{-4} m^2$

Work done, $W = T\times \Delta A$

$T = \frac{W}{\Delta A}$

$= \frac{2.4\times 10^{-4} J}{100\times 10^{-4} m^2}$

$= 2.4\times 10^{-4}\times 10^{4}\times 10^{-2}$

$= 2.4\times 10^{-2} Nm^{-1}$.

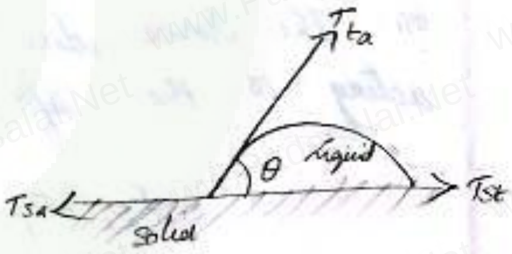
Angle of contact:

The angle between the tangent to the liquid surface at the point of contact and the solid surface inside the liquid is known as angle of contact between the solid and liquid.

T_{SA} - solid - air interface

T_{LA} - liquid - air interface

T_{SL} - solid - liquid interface.



$T_{SA} = T_{LA}\cos\theta + T_{SL}$

$T_{SA} - T_{SL} = T_{LA}\cos\theta$.

$\frac{T_{SA}-T_{SL}}{T_{LA}} = \cos\theta$.

$$\boxed{\cos\theta = \frac{T_{SA}-T_{SL}}{T_{LA}}}$$

(i) If $T_{SA} > T_{SL}$ and $T_{SA} - T_{SL} > 0$. Therefore $\cos\theta$ is positive.

(ii) If $T_{SA} < T_{SL}$ and $T_{SA} - T_{SL} < 0$. Therefore $\cos\theta$ is negative.

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Work done, $W = T \times \Delta A$

$T = \frac{W}{\Delta A}$

$= \frac{2.4 \times 10^{-4} J}{100 \times 10^{-4} m^2}$

$= 2.4 \times 10^{-4} \times 10^{4} \times 10^{-2}$

$= 2.4 \times 10^{-2} Nm^{-1}$.

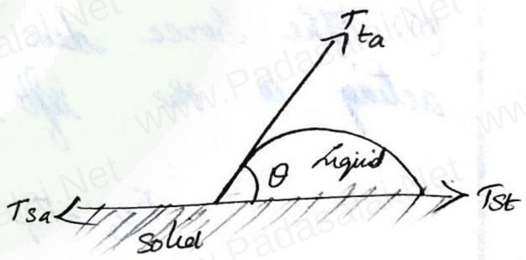
Angle of contact:

The angle between the tangent to the liquid surface at the point of contact and the solid surface inside the liquid is known as angle of contact between the solid and liquid.

T_{sa} - solid - air interface

T_{la} - liquid - air interface

T_{sl} - solid - liquid interface.



$T_{sa} = T_{la} \cos\theta + T_{sl}$

$T_{sa} - T_{sl} = T_{la} \cos\theta$.

$\frac{T_{sa} - T_{sl}}{T_{la}} = \cos\theta$.

$\boxed{\cos\theta = \frac{T_{sa} - T_{sl}}{T_{la}}}$

(i) If $T_{sa} > T_{sl}$ and $T_{sa} - T_{sl} > 0$. Therefore $\cos\theta$ is positive

(ii) If $T_{sa} < T_{sl}$ and $T_{sa} - T_{sl} < 0$. Therefore $\cos\theta$ is negative

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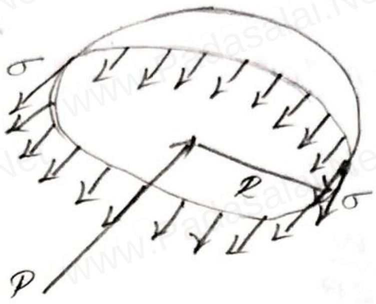
(iii) If $T_{sa} > T_{la} + T_{sl}$ there will be no equilibrium and liquid will spread over the solid.

Excess of pressure inside air bubble in a liquid:



Air bubble.

P_1 & P_2 - pressure outside and inside air bubble.

Then, excess pressure, $\Delta P = P_1 - P_2$

(i) Force due to surface tension acting towards right around the rim of length $2\pi R$ is $F_T = 2\pi RT$ → ①

(ii) The Force due to outside pressure P_1 is to the right acting across a cross sectional area of πR^2 is

$$F_{P_1} = P_1 \pi R^2 . \rightarrow ②$$

(iii) The Force due to Pressure P_2 inside the bubble, acting to the left is $F_{P_2} = P_2 \pi R^2 . \rightarrow ③$.

$$F_{P_2} = F_T + F_{P_1} \rightarrow ④$$

sub. equ ① ②, ③ in ④.

$$P_2 \pi R^2 = 2\pi RT + P_1 \pi R^2$$

$$P_2 \pi R^2 - P_1 \pi R^2 = 2\pi RT$$

$$\pi R^2 (P_2 - P_1) = 2\pi RT$$

$$P_2 - P_1 = \frac{2\pi RT}{\pi R^2}$$

$$P_2 - P_1 = \frac{2T}{R}.$$

∴ Excess Pressure is $\Delta P = P_2 - P_1 = \frac{2T}{R}$.

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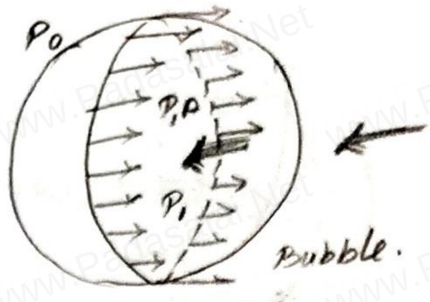
Excess Pressure inside a soap bubble. 25



Soap bubble.

R - Radius of soap bubble.
T - Surface tension of soap bubble.

$T = 2 \times 2\pi R T$

$T = 4\pi R T$

(i) Force due to surface tension $F_T = 4\pi R T \rightarrow (1)$. towards right.

(ii) Force due to outside pressure, $F_{P_1} = P_1 \pi R^2 \rightarrow (2)$ towards right.

(iii) Force due to inside pressure, $F_{P_2} = P_2 \pi R^2 \rightarrow (3)$ toward left.

As the bubble is in equilibrium,

$F_{P_2} = F_T + F_{P_1} \rightarrow (4)$.

sub. equ (1) (2) & (3) in (4).

$P_2 \pi R^2 = 4\pi R T + P_1 \pi R^2$

$P_2 \pi R^2 - P_1 \pi R^2 = 4\pi R T$

$\pi R^2 (P_2 - P_1) = 4\pi R T$

$P_2 - P_1 = \dfrac{4\pi R T}{\pi R^2}$

Excess Pressure, $\Delta P = P_2 - P_1 = \dfrac{4T}{R}$

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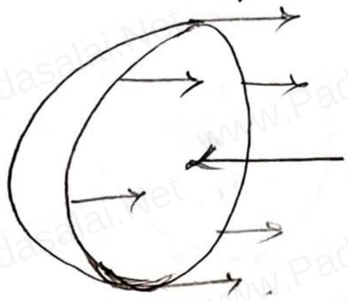
Excess Pressure inside the liquid drop:



here,

T - surface tension of liquid.

The various forces acting on the liquid are.

(i) Force due to surface Tension. $F_T = 2\pi R T \rightarrow (1)$.
towards right.

(ii) Force due to outside pressure, $F_{P_1} = P_1 \pi R^2 \rightarrow (2)$
towards right

(iii) Force due to inside pressure, $F_{P_2} = P_2 \pi R^2 \rightarrow (3)$
towards left.

$$F_{P_2} = F_T + F_{P_1} \rightarrow (4).$$

sub. eqn (1). (2) & (3) in (4).

$$P_2 \pi R^2 = 2\pi R T + P_1 \pi R^2$$

$$P_2 \pi R^2 - P_1 \pi R^2 = 2\pi R T$$

$$\pi R^2 (P_2 - P_1) = 2\pi R T$$

$$P_2 - P_1 = \frac{2\pi R T}{\pi R^2}$$

$$P_2 - P_1 = \frac{2T}{R}.$$

Example 7.11:

$h = 4\,mm$

$= 4 \times 10^{-3}\,m$

$R = 2.0\,cm$

$= 2 \times 10^{-2}$

$\Delta P = P_2 - P_1$

$= \frac{4T}{R}.$

$\rho g h = \frac{4T}{R}$

$T = \frac{\rho g h R}{4}$

The specific gravity of oil $= 0.8$

specific gravity of oil $= \frac{\text{density of substance}}{\text{density of water}}$

density of substance, ρ = specific gravity × density of water

$= 0.8 \times 997\ kg/m^3$

$\rho = 797.6$

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$P_2 - P_1 = \rho g h$. $\rho g h = \frac{4T}{R} \Rightarrow T = \frac{\rho g h R}{4}$

Surface Tension, $T = \frac{\rho g h R}{4}$

$= \frac{800 \times 9.8 \times \cancel{4}\times 10^{-3} \times 2\times 10^{-2}}{\cancel{4}}$

$= 8\times 10^{2} \times 9.8 \times 10^{-3} \times 2 \times 10^{-2}$

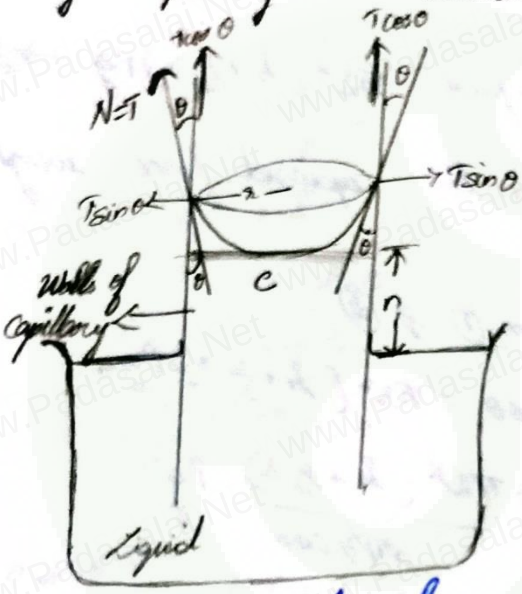
$= 8 \times 9.8 \times 2 \times 10^{2} \times 10^{-5}$

$= 1568 \times 10^{-3}$

$= 15.68 \times 10^{-2}\ Nm^{-1}$.

10^{1}
98×2
196×8
1568

Surface Tension by capillary rise method:



F_T - surface tension acts along tangent at the point of contact downwards.

$T\sin\theta$ - Horizontal component.

$T\cos\theta$ - Vertical component.

Total upward force $= (T\cos\theta)(2\pi r)$

$= 2\pi r T\cos\theta_c \rightarrow (1)$.

Where,

θ - angle of contact.

r - radius of tube.

ρ - density of water.

h - height to which the liquid rise inside the tube.

$\left[\text{The volume of liquid column in the tube, } V\right] = \left[\text{volume of the liquid column of radius } r \text{ height } h\right] + \left[\text{volume of liquid of radius and height } r - \text{volume of the hemisphere of radius } r\right]$

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$$V = \pi r^2 h + (\pi r^2 \times r) - \frac{2}{3}\pi r^3]$$

$$= \pi r^2 h + (\pi r^3 - \frac{2}{3}\pi r^3)$$

$$= \pi r^2 h + \pi r^3 (1 - 2/3)$$

$$= \pi r^2 h + \pi r^3 \left(\frac{3-2}{3}\right)$$

$$= \pi r^2 h + \pi r^3 (1/3)$$

Volume $(V) = \pi r^2 h + \frac{1}{3}\pi r^3 \rightarrow (2)$

mass of the liquid = density × volume.

$$= [(\pi r^2 h) + \frac{1}{3}\pi r^3] \times \rho$$

Weight of the liquid $= \pi r^2 (h + 1/3\, r)\rho g \rightarrow (3)$

At equilibrium, the upward force weight of the liquid column.

from (1) & (3).

$$2\pi r T \cos\theta = \pi r^2 (h + \frac{1}{3} r)\rho g$$

$$T = \frac{\pi r^2 (h + 1/3\, r)\rho g}{2\pi r \cos\theta}$$

$$T = \frac{r(h + 1/3\, r)\rho g}{2\cos\theta}$$

radius is very small hence neglected $r/3$

$$T = \frac{r h \rho g}{2\cos\theta}$$

$$T = \frac{r \rho g h}{2\cos\theta} \rightarrow (4)$$

Liquid rises through a height h.

$$h = \frac{2T\cos\theta}{r\rho g} \rightarrow (5)$$

$$h \propto \frac{1}{r}$$

Eg: 7.12

Radius of 1st capillary tube $= r \rightarrow (A)$

Radius of 2nd capillary tube $= r/3 \rightarrow (B)$

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We know that.

$h \propto \frac{1}{r}$

$hr = \text{constant}$

for first tube, constant $= h_1 r_1 \rightarrow ①$.

for second tube, constant $= h_2 r_2 \rightarrow ②$.

from ① & ②.

$h_1 r_1 = h_2 r_2$.

from Ⓐ & Ⓑ.

$h_1 r = h_2 (r/3)$

$h_2 = \frac{h_1 r}{r/3}$

$= \frac{3 h_1 r}{r}$

$h_2 = 3 h_1$

$= 3 \times (2 \times 10^{-2} m)$

$h_2 = 6 \times 10^{-2} m$

$\boxed{h_2 = 6 cm}$

Example 7.13

$T = 0.456 \ Nm^{-1}$,

$\rho = 13.6 \times 10^3 \ kg m^{-3}$

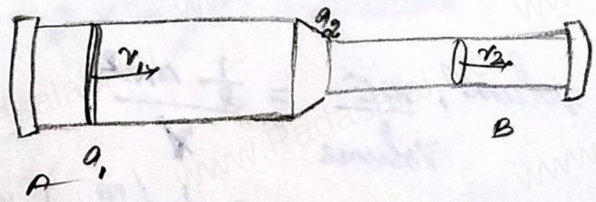
$r = 2 mm = 2 \times 10^{-3} m$

$\theta = 140°$

$h = \frac{2T\cos\theta}{r\rho g} = \frac{2 \times 0.456 \ Nm^{-1} \times \cos 140°}{2 \times 10^{-3} \times 13.6 \times 10^3 \times 9.8}$

$h = -6.89 \times 10^{-4} m$.

Bernoulli's theorem:



Let, m_1 - be the mass of fluid through section A in time Δt

$m_1 = (a_1 v_1 \Delta t)\rho \rightarrow ①$.

Let, m_2 - be the mass of fluid flowing through section B in time Δt,

$m_2 = (a_2 v_2 \Delta t)\rho \rightarrow ②$.

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For an incompressible liquid.

$$m_1 = m_2$$

sub equ ① & ②.

$$(a_1 v_1 \delta t)\rho = (a_2 v_2 \delta t)\rho$$

$$a_1 v_1 = a_2 v_2$$

$$av = \text{constant}.$$

Eg: 7.14

$$a_1 v_1 = a_2 v_2.$$

$$r_1 = 0.8\ cm$$
$$= 0.8 \times 10^{-2} m$$
$$r_2 = 0.4\ cm$$
$$= 0.4 \times 10^{-2} m$$

$$\pi a_2 r_2^2 = 30 \pi r_2^2 v_2$$

$$v_2 = \frac{1}{30}\left(\frac{r_1}{r_2}\right)^2 v_1$$

$$= \frac{1}{30} \times \left[\frac{0.8 \times 10^{-2}}{0.4 \times 10^{-2}}\right] \times 0.33\ ms^{-1}.$$

$$\frac{1}{30} \times 0.4 \times 0.33 = 0.44 \times 10^{-1}$$

$$= \frac{1}{30} \times 0.4 \times 0.33$$

$$= 0.44 \times 10^{-1}$$

$$v_2 = 0.044\ ms^{-1}$$

Pressure, Kinetic energy, potential energy of liquid:

i) Kinetic energy:

$$KE = \frac{1}{2} mv^2$$

K.E per unit mass $= \frac{KE}{m} = \frac{\frac{1}{2} mv^2}{m}$

$$= \frac{1}{2} v^2 \rightarrow ①$$

K.E per unit volume, $\frac{KE}{\text{Volume}} = \frac{\frac{1}{2} mv^2}{V}$

$$= \frac{1}{2}\left(\frac{m}{V}\right) v^2$$

$$= \frac{1}{2} \rho v^2.$$

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(ii) Potential energy:

$PE = mgh$

P.E per unit mass, $\frac{PE}{m} = \frac{mgh}{m}$

$= gh \rightarrow (2)$.

P.E per unit volume, $\frac{PE}{volume} = \frac{mgh}{V}$

$= \left(\frac{m}{V}\right) gh = \rho gh$.

(iii) Pressure energy:

Pressure = $\frac{Force}{Area}$

Force = Pressure × Area.

$F \times d = P(A \times d)$

$W = PV$

Pressure energy, $E_p = PV$

P energy per unit mass, $\frac{E_p}{m} = \frac{PV}{m}$

$= \frac{P}{\left(\frac{m}{V}\right)}$

$= \frac{P}{\rho} \rightarrow (3)$.

P. energy per unit volume, $\frac{E_p}{volume} = \frac{PV}{V}$

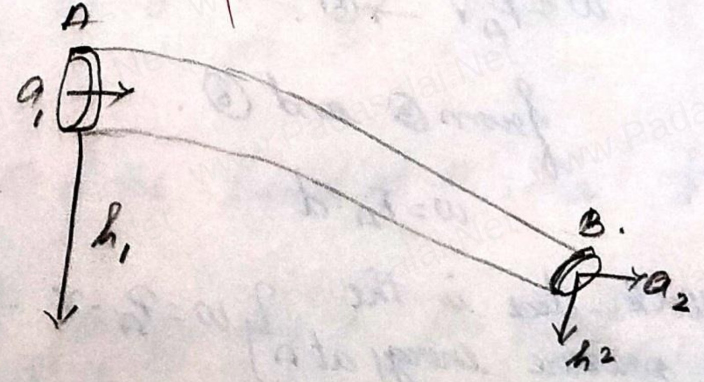
$= P$

Bernoulli's theorem & its application:

from eqn (1), (2) & (3).

$\frac{P}{\rho} + \frac{1}{2}V^2 + gh = constant$.

Proof:



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V - volume of the liquid enters A and volume of liquid leaving B.

a_1, a_2 - area of crossection at point A and B.

v_A, v_B - velocity of liquid at point A and B.

P_A, P_B - liquid pressure at point A and B.

The force excerted by the liquid at A is $\Big\}$ $F_A = P_A a_A \rightarrow (1)$.

Distance travelled by the liquid in time t is $\Big\}$ $d = v_A t \rightarrow (2)$.

Then, work done, $W = \text{Force} \times \text{displacement}$

$W = F_A \times d \rightarrow (3)$.

sub. equ (1) & (2) in (3)

$$W = P_A a_A \times v_A t$$

$$W = P_A a_A v_A t \rightarrow (4).$$

here,

from (2).

$$a_A v_A t = a_A d.$$

$a_A d$ - volume of liquid entering at A.

So,

$$a_A v_A t = V \text{ (volume)} \rightarrow (5).$$

sub. equ (5) in (4).

$$W = P_A V \rightarrow (6).$$

$W = F_A d$

$W = P_A V.$

from (3) and (6).

$$W = F_A \cdot d$$

Work done is the pressure energy at A $\Big\}$ $W = P_A \cdot V \longrightarrow (7).$

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sub. (13), (14) & (15) in (16).

$$E_A = m\frac{P_A}{\rho} + \frac{1}{2}mv_A^2 + mgh_A \rightarrow (17).$$

similarly,

the total energy due to flow of liquid at B $\Big\} E_B = m\frac{P_B}{\rho} + \frac{1}{2}mv_B^2 + mgh_B$

from the law of conservation of energy.

$$E_A = E_B.$$

from (17) & (18).

$$m\frac{P_A}{\rho} + \frac{1}{2}mv_A^2 + mgh_A = \frac{mP_B}{\rho} + \frac{1}{2}mv_B^2 + mgh_B.$$

$$\cancel{m}\left[\frac{P_A}{\rho} + \frac{1}{2}v_A^2 + gh_A\right] = \cancel{m}\left[\frac{P_B}{\rho} + \frac{1}{2}v_B^2 + gh_B\right].$$

then,

$$\frac{P_A}{\rho} + \frac{1}{2}v_A^2 + gh_A = \frac{P_B}{\rho} + \frac{1}{2}v_B^2 + gh_B = \text{constant}$$

divide by g on both sides.

$$\frac{\frac{P_A}{\rho} + \frac{1}{2}v_A^2 + gh_A}{g} = \frac{\frac{P_B}{\rho} + \frac{1}{2}v_B^2 + gh_B}{g} = \text{constant}$$

$$\frac{P_A}{\rho g} + \frac{1}{2}\frac{v_A^2}{g} + \frac{\cancel{g}h_A}{\cancel{g}} = \frac{P_B}{\rho g} + \frac{1}{2}\frac{v_B^2}{g} + \frac{\cancel{g}h_B}{\cancel{g}} = \text{constant}.$$

$$\frac{P_A}{\rho g} + \frac{1}{2}\frac{v_A^2}{g} + h_A = \frac{P_B}{\rho g} + \frac{1}{2}\frac{v_B^2}{g} + h_B = \text{constant}.$$

the above equation can be written as.

$$\boxed{\frac{P}{\rho g} + \frac{1}{2}\frac{v^2}{g} + h = \text{constant}.}$$

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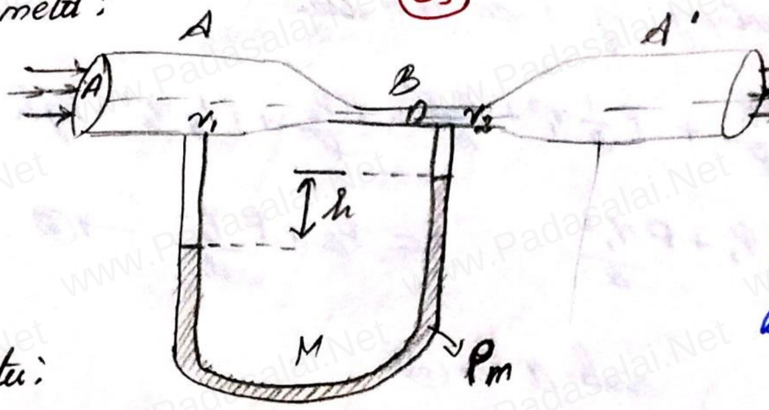
Venturi meter:



It is used to measure the rate of flow of incompressible fluid flowing through a pipe. It works for Bernoulli's theorem.

manometer:

It is in the form of U-shaped tube is attached between the wide & narrow tubes in a pipe line. It contains a liquid of density P_m.

Let,

P_1 & P_2 - Pressure of fluid at wider and narrow region of the tube.

ρ - density of fluid.

v_1 & v_2 - speed of fluid at point A and B.

then, A - area of cross section of point A.

a - area of cross section at point B.

The pressure difference b/w tubes A and B. $\Delta P = P$.

from the equation of continuity,

$$a_1 v_1 = a_2 v_2$$

here,

$$a_1 = A$$
$$a_2 = a$$

then,

$$A v_1 = a v_2$$

$$a v_2 = A v_1$$

$$v_2 = \frac{A v_1}{a}$$

from Bernoulli's equation

$$\frac{P}{\rho} + \frac{1}{2} v^2 + gh = \text{Constant} \quad (a).$$

$$\frac{P_1}{\rho} + \frac{1}{2} v_1^2 + g h_1 = \frac{P_2}{\rho} + \frac{1}{2} v_2^2 + g h_2.$$

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multiply by ρ on both sides

$$\frac{\rho P_1}{\rho} + \rho\frac{1}{2}v_1^2 + \rho g h = \frac{\rho P_2}{\rho} + \rho\frac{1}{2}v_2^2 + \rho g h$$

$$P_1 + \rho\frac{v_1^2}{2} = P_2 + \rho\frac{v_2^2}{2} \rightarrow (2)$$

sub. (1) in (2)

$$P_1 + \rho\frac{v_1^2}{2} = P_2 + \rho\times\frac{1}{2}\left(\frac{Av_1}{a}\right)^2$$

$$P_1 - P_2 = \frac{\rho}{2}\left(\frac{A^2v_1^2}{a^2}\right) - \rho\frac{v_1^2}{2}$$

$$P_1 - P_2 = \frac{\rho v_1^2}{2}\left[\frac{A^2}{a^2} - 1\right]$$

$$P_1 - P_2 = \frac{\rho}{2}v_1^2\left[\frac{A^2 - a^2}{a^2}\right]$$

here $P_1 - P_2 = \Delta P$

$$\Delta P = \frac{\rho}{2}v_1^2\left[\frac{A^2 - a^2}{a^2}\right]$$

$$\Delta P = \frac{\rho v_1^2(A^2 - a^2)}{2a^2}$$

$$2a^2\times\Delta P = \rho v_1^2(A^2 - a^2)$$

$$\rho v_1^2(A^2 - a^2) = 2a^2\times\Delta P$$

$$v_1^2 = \frac{2a^2\times\Delta P}{\rho(A^2 - a^2)}$$

$$v_1^2 = \frac{2(\Delta P)a^2}{\rho(A^2 - a^2)}$$

$$v_1 = \sqrt{\frac{2(\Delta P)a^2}{\rho(A^2 - a^2)}} \rightarrow (3)$$

The volume of liquid flowing out per second is,

$$V = Av_1 \rightarrow (4)$$

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sub. eqn (3) in (4).

$$v = A\sqrt{\frac{2(\Delta P)a^2}{P(A^2-a^2)}}$$

$$v = Aa\sqrt{\frac{2(\Delta P)}{P(A^2-a^2)}}.$$

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