# Padasalai's Telegram Groups!

( தலைப்பிற்கு கீழே உள்ள லிங்கை கிளிக் செய்து குழுவில் இணையவும்! )

- Padasalai's NEWS - Group

https://t.me/joinchat/NIfCqVRBNj9hhV4wu6_NqA

- Padasalai's Channel - Group

https://t.me/padasalaichannel

- Lesson Plan - Group

https://t.me/joinchat/NIfCqVWwo5iL-21gpzrXLw

- 12th Standard - Group

https://t.me/Padasalai_12th

- 11th Standard - Group

https://t.me/Padasalai_11th

- 10th Standard - Group

https://t.me/Padasalai_10th

- 9th Standard - Group

https://t.me/Padasalai_9th

- 6th to 8th Standard - Group

https://t.me/Padasalai_6to8

- 1st to 5th Standard - Group

https://t.me/Padasalai_1to5

- TET - Group

https://t.me/Padasalai_TET

- PGTRB - Group

https://t.me/Padasalai_PGTRB

- TNPSC - Group

https://t.me/Padasalai_TNPSC

SURA'S

# Business Mathematics and Statistics

## 12th Std

Based on the New Syllabus and New Textbook

FREE Practice Workbook with Practice Model Question Papers

## Salient Features

- Prepared as per the New Textbook
- Exhaustive Additional Questions & Answers in all chapters.
- Model Question Papers 1 to 6 (PTA) : Questions are incorporated in the appropriate sections.
- Govt. Model Question Paper - 2019
- Common Quarterly Question Paper - 2019
- Common Half-yearly Question Paper - 2019

SURA PUBLICATIONS
Chennai

# CONTENTS

Chapter 1

# APPLICATIONS OF MATRICES AND DETERMINANTS

## CHAPTER SNAPSHOT

**Rank of a matrix :-**

The rank of a matrix A is the order of the largest non-zero minor of A and is denoted by $\rho$ (A).

**(i)** $\rho(A) \geq 0$.

**(ii)** If A is a matrix of order $m \times n$, then $\rho(A) \leq \min\{m, n\}$.

**(iii)** Rank of a zero matrix is 0.

**(iv)** The rank of a non - singular matrix of order $n \times n$ *is "n"*.

**Elementary transformations :**

**(i)** Interchange any two rows (or columns)

$R_i \leftrightarrow R_j$ $(C_i \leftrightarrow C_j)$

**(ii)** Multiplication of each element of a row (or column) by any non-zero scalar *k*.

$R_i \to k\,R_i$ (or $C_i \to k\,C_i$)

**(iii)** Addition to the elements of any row (or column) the same scalar multiples of corresponding elements of any other row (or column).

$R_i \to R_i + k\,R_j$ (or $C_i \to C_i + k\,C_j$)

**Equivalent matrices:**

Two matrices A and B are said to be equivalent if one is obtained from the another by applying a finite number of elementary transformations.

$A \cong b$

**Echelon form :**

A matrix A of order $m \times n$ is said to be in echelon form if

**(i)** Every row of a which has all its entries 0 occurs below every row which has a non - zero entry.

**(ii)** The number of zeros before the first non - zero element in a row is less than the number of such rows in the next row.

**Transition matrix :**

The transition probabilities $P_{jk}$ satisfy $P_{jk} > 0$ and $\sum_k P_{jk} = 1$ for all *j*

Given $x + y + z = 5000$ ----- (1)

Also $\frac{6x}{100} + \frac{7y}{100} + \frac{8z}{100} = 358$

$\because$ Interest $= \frac{PNR}{100} = \frac{x \times 1 \times 6}{100} = \frac{6x}{100}$

$\Rightarrow \frac{6x + 7y + 8z}{100} = 358$

$\Rightarrow 6x + 7y + 8z = 35800$ ----- (2)

Given that $\frac{6x}{100} + \frac{7y}{100} = 70 + \frac{8z}{100}$

$\Rightarrow \frac{6x + 7y}{100} = \frac{7000 + 8z}{100}$

$\Rightarrow 6x + 7y = 700 + 8z$

$\Rightarrow 6x + 7y - 8z = 7000$ ----- (3)

The matrix equation corresponding to the given system is.

$$\begin{pmatrix} 1 & 1 & 1 \\ 6 & 7 & 8 \\ 6 & 7 & -8 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5000 \\ 35800 \\ 7000 \end{pmatrix}$$

| Augmented matrix [A, B] | Elementary Transformation |
|---|---|
| $\begin{pmatrix} 1 & 1 & 1 & 5000 \\ 6 & 7 & 8 & 35800 \\ 6 & 7 & -8 & 7000 \end{pmatrix}$ | |
| $\sim \begin{pmatrix} 1 & 1 & 1 & 5000 \\ 0 & 1 & 2 & 5800 \\ 0 & 1 & -14 & -23000 \end{pmatrix}$ | $R_2 \to R_2 - 6R_1$ <br> $R_3 \to R_3 - 6R_1$ |
| $\sim \begin{pmatrix} 1 & 1 & 1 & 5000 \\ 0 & 1 & 2 & 5800 \\ 0 & 0 & -16 & -28800 \end{pmatrix}$ | $R_3 \to R_3 - R_2$ |

The last equivalent matrix is in echelon form and $\rho(A) = \rho([A, B]) = 3$ = Number of unknowns.

Thus, the given system is consistent with unique solution. To find the solution, let us rewrite the above echelon form into the matrix form.

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & -16 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5000 \\ 5800 \\ -28800 \end{pmatrix}$$

$\Rightarrow x + y + z = 5000$ ----- (1)

$\Rightarrow y + 2z = 5800$ ----- (2)

$\Rightarrow -16z = -28800$ ----- (3)

(3)$\Rightarrow -16z = -28800$

$\Rightarrow z = -\frac{28800}{-16} = 1800$

Substituting $z = 1800$ in (2) we get,

$y + 2(1800) = 5800$

$\Rightarrow y + 3600 = 5800$

$\Rightarrow y = 5800 - 3600$

$\Rightarrow y = 2200$

Substituting $y = 2200$ and $z = 1800$ in (1) we get,

$x + 2200 + 1800 = 5000$

$\Rightarrow x + 4000 = 5000$

$\Rightarrow x = 5000 - 4000$

$\Rightarrow x = 1000$

Hence, the amount of investment in each bond is Rs. 1000, Rs.2200 and Rs. 1800 respectively.

## EXERCISE 1.2

**1. Solve the following equations by using Cramer's rule.**

**(i) $2x + 3y = 7$, $3x + 5y = 9$**

*Solution:*

$\Delta = \begin{vmatrix} 2 & 3 \\ 3 & 5 \end{vmatrix} = 10 - 9 = 1 \neq 0.$

Since $\Delta \neq 0$, we can apply Cramer's rule and the system is consistent with unique solution.

$\Delta x = \begin{vmatrix} 7 & 3 \\ 9 & 5 \end{vmatrix} = 7(5) - 9(3)$

$= 35 - 27 = 8$

$\Delta y = \begin{vmatrix} 2 & 7 \\ 3 & 9 \end{vmatrix} = 2(9) - 3(7)$

$= 18 - 21 = -3$

$\therefore x = \frac{\Delta x}{\Delta} = \frac{8}{1} = 8$

$y = \frac{\Delta y}{\Delta} = \frac{-3}{1} = -3$

$\therefore$ Solution set is {8, –3)

**(ii) $5x + 3y = 17$ ; $3x + 7y = 31$** *[HY-2019]*

*Solution:*

$\Delta = \begin{vmatrix} 5 & 3 \\ 3 & 7 \end{vmatrix} = 5(7) - 3(3)$

$= 35 - 9 = 26$

$$\therefore \quad B = 1 - A = 1 - 0.5625 = 0.4375 = 43.75\%$$

∴ Equilibrium is reached when A = 56.25% and B = 43.75%

**4. Two products A and B currently share the market with shares 50% and 50% each respectively. Each week some brand switching takes place. Of those who bought A the previous week, 60% buy it again whereas 40% switch over to B. Of those who bought B the previous week, 80% buy it again where as 20% switch over to A. Find their shares after one week and after two weeks. If the price war continues, when is the equilibrium reached?**

***Solution:***

Transition probability matrix

$$T = \begin{matrix} & \begin{matrix} A & B \end{matrix} \\ \begin{matrix} A \\ B \end{matrix} & \begin{pmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{pmatrix} \end{matrix}$$

By the given data

$$A = 50\% = 0.5$$
$$B = 50\% = 0.5$$

Shares after one week

$$\begin{pmatrix} 0.5 & 0.5 \end{pmatrix}\begin{pmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{pmatrix}$$
$$= \begin{pmatrix} (0.5)(0.6)+(0.5)(0.2) & 0.5(0.4)+0.5(0.8) \end{pmatrix}$$
$$= \begin{pmatrix} 0.30+0.10 & 0.20+0.40 \end{pmatrix} = \begin{pmatrix} 0.40 & 0.60 \end{pmatrix}$$

∴ Shares after one week for products A and B are 40% and 60% respectively.

Shares after two weeks

$$\begin{pmatrix} 0.4 & 0.6 \end{pmatrix}\begin{pmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{pmatrix}$$
$$= \begin{pmatrix} (0.4)(0.6)+(0.6)(0.2) & (0.4)(0.4)+0.6(0.8) \end{pmatrix}$$
$$= \begin{pmatrix} 0.24+0.12 & 0.16+0.48 \end{pmatrix}$$
$$= \begin{pmatrix} 0.36 & 0.64 \end{pmatrix}$$

∴ Shares after two week for products A and B are 36% and 64% respectively.

At equilibrium, we must have

$$\begin{pmatrix} A & B \end{pmatrix} T = \begin{pmatrix} A & B \end{pmatrix}$$

where $A + B = 1$

$$\begin{pmatrix} A & B \end{pmatrix}\begin{pmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{pmatrix} = \begin{pmatrix} A & B \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 0.6A+0.2B & 0.4A+0.8B \end{pmatrix} = \begin{pmatrix} A & B \end{pmatrix}$$

Equating the corresponding entries on both sides we get,

$$0.6A + 0.2B = A$$
$$\Rightarrow 0.6A + 0.2(1 - A) = A$$
$$\Rightarrow 0.6A + 0.2 - 0.2A = A$$
$$\Rightarrow 0.2 = A - 0.6A + 0.2A$$
$$\Rightarrow 0.2 = A(1 - 0.6 + 0.2)$$
$$\Rightarrow 0.2 = A(0.4 + 0.2)$$
$$\Rightarrow 0.2 = A(0.6)$$
$$\Rightarrow A = \frac{0.2}{0.6} = 0.33 \Rightarrow A = 33\%$$

and $B = 1 - A = 1 - 0.33 = 0.67 \Rightarrow B = 67\%$

∴ Equilibrium is reached when A = 33% and B = 67%

## EXERCISE 1.4

### Choose the correct answer

**1. If A = (1 2 3), then the rank of $AA^T$ is**

(a) 0 (b) 2 (c) 3 (d) 1

**[Ans: (d) 1]**

***Hint:***

$$AA^T = \begin{pmatrix} 1 & 2 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = (1 + 4 + 9) = 14$$

∴ Rank of $AA^T$ is 1

**2. The rank of $m \times n$ matrix whose elements are unity is**

(a) 0 (b) 1 (c) $m$ (d) $n$

**[Ans: (b) 1]**

***Hint:***

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & --- & 1 \\ 1 & 1 & 1 & 1 & 1 & --- & 1 \\ - & - & - & - & - & --- & - \\ - & - & - & - & - & --- & - \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$m$ rows and $n$ columns applying the elementary transformation.

$R_2 \to R_2 - R_1, R_3 \to R_1$ ----- $R_m \to R_m - R_1$ we get

$$A = \begin{pmatrix} 1 & 1 & 1 & --- & --- & 1 \\ 0 & 0 & 0 & --- & --- & 0 \\ 0 & 0 & 0 & --- & --- & 0 \\ - & - & - & --- & --- & - \\ - & - & - & - & - & - \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

∴ Rank is 1

$\therefore \begin{pmatrix} 0.4 & 0.6 \end{pmatrix}\begin{pmatrix} 0.6 & 0.4 \\ 0.25 & 0.75 \end{pmatrix}$

$= \begin{pmatrix} (0.4)(0.6)+(0.6)(0.25) & (0.4)(0.4)+(0.6)(0.75) \end{pmatrix}$

$= \begin{pmatrix} 0.24+0.15 & 0.16+0.45 \end{pmatrix} = \begin{pmatrix} 0.39 & 0.61 \end{pmatrix}$

$\therefore$ 39% of people who received the current letter can be expected to order a subscription.

## PTA Question & Answers

### 1 Marks

**1. If O(A) = 3 × 3 and ρ(A) = 2 then ρ(adjA) is________.** *[PTA-1]*

(a) 1 (b) 2 (c) 3 (d) 0

**[Ans: (a) 1]**

**2. If A is matrix [A,B] is the augmented matrix then which of the following is true?** *[PTA-2]*

(a) ρ([A,B]) = ρ(A)

(b) ρ([A,B]) ≥ ρ(A)

(c) ρ([A,B]) = ρ(A)>n

(d) ρ([A,B]) < ρ(A) **[Ans: (b) ρ([A,B]) ≥ ρ(A)]**

**3.** If $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ then the rank of $AA^T$ is : *[PTA-4]* *[HY-2019]*

(a) 1 (b) 2 (c) 3 (d) 0

**[Ans: (a) 1]**

**4. If A is matrix of order 4 and |A| = –2 then the value of |adj (A)| is ______** *[PTA-6]*

(a) –4 (b) 4 (c) –8 (d) 8

**[Ans: (c) A adj (A) = I]**

**Hint:** $|\text{adj } A| = |A|^{n-1} = (-2)^{4-1} = (-2)^3 = -8$

### 2 Marks

**1.** **If** $A = \begin{bmatrix} x & x & x \\ 4 & -2 & 1 \\ 2 & 3 & 4 \end{bmatrix}$ **find x if ρ(A) = 3** *[PTA-3]*

**Solution:** $\begin{vmatrix} x & x & x \\ 4 & -2 & 1 \\ 2 & 3 & 4 \end{vmatrix}$

$= x\,[-8-3] - x\,[16-2] + x\,[12+4]$

$= -11x - 14x + 16x = -9x$

$-9x \neq 0$

$x \neq 0$

**2. Find the rank of the matrix** $\begin{bmatrix} 2 & -4 & x \\ 4 & -8 & 2x \\ 5 & 7 & 1 \end{bmatrix}$ *[PTA-4]*

**Solution:** $\begin{bmatrix} 2 & -4 & x \\ 4 & -8 & 2x \\ 5 & 7 & 1 \end{bmatrix} \xrightarrow{R1 \to \frac{R_1}{2}} \begin{bmatrix} 1 & -2 & \frac{x}{2} \\ 4 & -8 & 2x \\ 5 & 7 & 1 \end{bmatrix}$

$\xrightarrow[R_3 \to R_3 - 5R_1]{R_2 \to R_2 - 4R1} \begin{bmatrix} 1 & -2 & \frac{x}{2} \\ 0 & 0 & 0 \\ 0 & 17 & \frac{2-5x}{2} \end{bmatrix}$

Rank 2

**3. Parithi is either Sad (S) or happy (H) each day. He is happy in one day, he is sad on the next day by four times out of five. If he is sad on one day, he his happy on the next day by two times out of three. Over a long run, what are the chances that Parithi is happy on any given day?** *[PTA-5;QY-2019]*

**Solution:** The transition probability matrix is

$$T = \begin{pmatrix} \frac{4}{5} & \frac{1}{5} \\ \frac{2}{3} & \frac{1}{3} \end{pmatrix}$$

At equilibrium, $(S\ H)\begin{pmatrix} \frac{4}{5} & \frac{1}{5} \\ \frac{2}{3} & \frac{1}{3} \end{pmatrix} = (S\ H)$

where S + H = 1

$$\frac{4}{5}S + \frac{2}{3}H = S$$

$$\frac{4}{5}S + \frac{2}{3}(1-S) = S$$

On solving this, we get

$$S = \frac{10}{13} \text{ and } H = \frac{3}{13}$$

In the long run, on a randomly selected day, his chances of being happy is $\frac{10}{13}$

$$y = \frac{\Delta y}{\Delta} = \frac{-3}{1} = -3$$

∴ Solution set is {8, –3}

**2. If the rank of the matrix $\begin{bmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & K \\ 9 & 10 & 11 & 12 \end{bmatrix}$ is 2. Find the value of 'K'**

*Solution:* $\begin{vmatrix} 2 & 3 & 5 \\ 3 & 4 & 6 \\ 4 & 5 & k \end{vmatrix} = 0 \Rightarrow k = 7$ *[HY-2019]*

## 3 Marks

**1. Find the rank of the matrix $A = \begin{pmatrix} 1 & 2 & -4 & 5 \\ 2 & -1 & 3 & 6 \\ 8 & 1 & 9 & 7 \end{pmatrix}$**

*Solution:* The order of A is 3 × 4

∴ $\rho(A) \leq \min(3, 4)$ *[Govt.MQP-2019]*

$\rho(A) \leq 3$

Consider the third order minor,

$$\begin{vmatrix} 1 & 2 & -4 \\ 2 & -1 & 3 \\ 8 & 1 & 9 \end{vmatrix} = 1\begin{vmatrix} -1 & 3 \\ 1 & 9 \end{vmatrix} - 2\begin{vmatrix} 2 & 3 \\ 8 & 9 \end{vmatrix} - 4\begin{vmatrix} 2 & -1 \\ 8 & 1 \end{vmatrix}$$

$= 1(-9-3) - 2(18-24) - 4(2+8)$

$= 1(-12) - 2(-6) - 4(10)$

$= -12 + 12 - 40 = -40 \neq 0.$

There is a minor of order 3, which is not zero

∴ $\rho(A) = 3.$

**2.** Show that the equations $x + y = 5$, $2x + y = 8$ are consistent and solve them. *[QY-2019]*

*Solution:* The matrix equation corresponding to the given system is

$$\begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 8 \end{pmatrix}$$

| Matrix A | Augment matrix [A, B] | Elementary Transformation |
|---|---|---|
| $\begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}$ $\sim\begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}$ | $\begin{pmatrix} 1 & 1 & 5 \\ 2 & 1 & 8 \end{pmatrix}$ $\sim\begin{pmatrix} 1 & 1 & 5 \\ 1 & -1 & -2 \end{pmatrix}$ | $R_2 \to R_2 - 2R_1$ |
| ρ (A) = 2 | ρ([A,B]) = 2 | |

Number of non-zero rows is 2.

ρ(A) = ρ([A,B]) = 2 = Number of unknowns.

The given system is consistent and has unique solution.

Now, the given system is transformed into

$$\begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$$

$x + y = 5$

$y = 2$

$\therefore (1)\ x + 2 = 5$

$x = 3$

Solution is $x = 3$, $y = 2$

**3. Show that the equations $x - 4y + 7z = 14$, $3x + 8y - 2z = 13$, $7x - 8y + 26z = 5$ are inconsistent.** *[HY-2019]*

*Solution:* $[A, B] = \begin{bmatrix} 1 & -4 & 7 & 14 \\ 3 & 8 & -2 & 13 \\ 7 & -8 & 26 & 5 \end{bmatrix}$

$$= \begin{bmatrix} 1 & -4 & 7 & 14 \\ 0 & 20 & -23 & -29 \\ 0 & 0 & 0 & 64 \end{bmatrix} \begin{matrix} R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 - 7R_1 \\ R_3 \to R_3 - R_2 \end{matrix}$$

$\to \rho(A) = 2, \rho(A, B) = 3 \Rightarrow \rho(A) \neq \rho(A, B)$

The system is inconsistent and had no solution.

## Additional Question

### I. Choose the correct answer :

**1. If the minor of $a_{23}$ = the co-factor of $a_{23}$ in $|a_{ij}|$ then the minor of $a_{23}$ is.**

(a) 1 (b) 2 (c) 0 (d) 3

**[Ans: (c) 0]**

**2. If AB = BA = |A| I then the matrix B is the.**

(a) inverse of A (b) Transpose of A
(c) Adjoint of A (d) 2A

**[Ans: (c) Adjoint of A]**

**3. If A is a square matrix of order 3, then |adj A| is**

(a) $|A|^2$ (b) $|A|$ (c) $|A|^3$ (d) $|A|^4$

**[Ans: (a) $|A|^2$]**

**4. If |A| = 0, then |adj A| is.**

(a) 0 (b) 1 (c) –1 (d) ± 1

**[Ans: (a) 0]**

Since $\Delta \neq 0$, Cramer's rule can be applied and the system has unique solution.

$$\Delta a = \begin{vmatrix} 0 & 1 & 1 \\ -2 & 2 & 1 \\ -6 & 3 & 1 \end{vmatrix} = 0 - 1(-2+6) + 1(-6+12) = -4+6 = 2$$

$$\Delta b = \begin{vmatrix} 1 & 0 & 1 \\ 4 & -2 & 1 \\ 9 & -6 & 1 \end{vmatrix} = 1(-2+6) + 0 + 1(-24+18) = 4-6 = -2$$

$$\Delta c = \begin{vmatrix} 1 & 1 & 0 \\ 4 & 2 & -2 \\ 9 & 3 & -6 \end{vmatrix} = 1(-12+6) - 1(-24+18) + 0 = -6+6 = 0$$

$$a = \frac{\Delta a}{\Delta} = \frac{2}{-2} = -1$$

$$b = \frac{\Delta b}{\Delta} = \frac{-2}{-2} = 1$$

$$c = \frac{\Delta c}{\Delta} = \frac{0}{-2} = 0$$

$f(n) = (-1)x^2 + 1(x) + 0 \Rightarrow f(x) = x^2 + x.$

**5. A new transit system has just gone into operation in a city. Of those who use the transit system this year, 10% will switch over to using their own car next year and 90% will continue to use the transit system. Of those who use their cars this year, 80% will continue to use their cars next year and 20% will switch over to the transit system. Suppose the population of the city remains constant and that 50% of the commuters use the transit system and 50% of the commuters use their own car this year,**

**(i) What percent of commuters will be using the transit system after one year?**

**(ii) What percent of commuters will be using the transit system in the long run?**

***Solution:***

Let A represents the percent of commuters who use the transit system and B represents the percent of commuters who use their own car.

Transition probability matrix

$$T = \begin{matrix} & A & B \\ A & 0.9 & 0.1 \\ B & 0.2 & 0.8 \end{matrix}$$

Given 50% of commuters use the transit system and 50% of the commuters use their own car this year.

(i) Percentage of commuters after one year

$$\begin{pmatrix} 0.5 & 0.5 \end{pmatrix}\begin{pmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{pmatrix}$$

$= \begin{pmatrix} 0.5\times0.9+0.5\times0.2 & 0.5\times0.1+0.5\times0.8 \end{pmatrix}$

$= \begin{pmatrix} 0.45+0.10 & 0.05+0.40 \end{pmatrix}$

$= \begin{pmatrix} 0.55 & 0.45 \end{pmatrix}$

A = 55% and B = 45%

(ii) Equilibrium will be reached in the long run at equilibrium, we must have

$(A \quad B)T = (A \quad B)$ where $A + B = 1$

$\Rightarrow (A \quad B)\begin{pmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{pmatrix} = (A \quad B)$

$(0.9A + 0.2B \quad 0.1A + 0.8B) = (A \quad B)$

Equating the corresponding entries on both sides we get,

$0.9A + 0.2B = A \Rightarrow 0.9A + 0.2(1-A) = A$

[Since $A + B = 1 \Rightarrow B = 1 - A$]

$\Rightarrow 0.9A + 0.2 - 0.2A = A$

$\Rightarrow 0.2 = A - 0.9A + 0.2A$

$\Rightarrow 0.2 = A(1 - 0.9 + 0.2)$

$\Rightarrow 0.2 = A(0.3)$

$\Rightarrow A = \frac{0.2}{0.3} = 0.666 \Rightarrow A = 67\%$

$\therefore$ 67% of the commuters will be using the transit system in the long run.

***

Chapter
2

# INTEGRAL CALCULUS- I

## FORMULAE TO REMEMBER

**(i)** Integration is the reverse process of differentiation

**(ii)** $\int k\, f(x)\, dx = k \int f(x)\, dx$ where $k$ is a constant.

**(iii)** $\int [\, f(x) \pm g(x)]\, dx = \int f(x)dx \pm \int g(x)\, dx$

**(iv)** The following are the four principal methods of integration

(i) Integration by decomposition

(ii) Integration by Parts

(iii) Integration by Substitution

(iv) Integration by successive reduction

**First fundamental theorem of integral calculus :**

If $f(x)$ is a continuous function and $F(x) = \int_a^x f(t)dt$, then $F'(x) = f(x)$.

**Second fundamental theorem of integral calculus :**

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

**(i)** $\int_a^b f(x)\,dx$ is a definite constant, whereas $\int_a^x f(t)\,dt$ is a function of the variable $x$

**Indefinite integral :-**

An integral function which is expressed without limits, and so containing an containing an arbitrary constant.

**Proper definite integral :-**

An integral function which has both the limits. $a$ and $b$ are finite.

**Improper definite integral :-**

An integral function, in which the limits either $a$ or $b$ or both are infinite.

$= \log|x^3 - x^2 + 5x - 5| + c$

$$\frac{3x^2-2x+5}{(x-1)(x^2+5)} = \frac{A}{(x-1)} + \frac{Bx+c}{(x^2+5)}$$

$\Rightarrow 3x^2 - 2x + 5 = A(x^2+5) + (Bx+C)(x-1)$

Putting $x = 1$,

$3 - 2 + 5 = A(1+5)$

$\Rightarrow 6 = A(6) \Rightarrow$ $A = 1$

Putting $x = 0$,

$5 = 5A - C$

$\Rightarrow 5 = 5 - C$ $[\because A = 1]$

$\Rightarrow C = 5 - 5 \Rightarrow$ $C = 0$

Putting $x = -1$,

$3 + 2 + 5 = A(6) + (C-B)(-2)$

$\Rightarrow 10 = 6A + 2B - 2C$

$\Rightarrow 10 = 6 + 2B + 0$

$\Rightarrow 10 - 6 = 2B \Rightarrow 4 = 2B$

$\Rightarrow$ $B = 2$

**8. If $f'(x) = \frac{1}{x}$ and $f(1) = \frac{\pi}{4}$, then find $f(x)$.** [PTA-2]

***Sol.***

Given $f'(x) = \frac{1}{x}$

$\Rightarrow \int f'(x)\,dx = \int \frac{1}{x}\,dx$

$\Rightarrow f(x) = \log|x| + c$ ---(1)

Also, $f(1) = \frac{\pi}{4}$

$\Rightarrow \frac{\pi}{4} = \log|1| + c$

$\Rightarrow \frac{\pi}{4} = c$ $[\because \log 1 = 0]$

Substituting $c = \frac{\pi}{4}$ in (1) we get,

$f(x) = \log|x| + \frac{\pi}{4}$

## EXERCISE 2.3

**Integrate the following with respect to $x$**

**1. $e^{x\log a} + e^{a\log a} - e^{n\log x}$**

***Sol.***

$\int(e^{x\log a} + e^{a\log a} - e^{n\log x})\,dx$

$= \int\left(e^{\log a^x} + e^{\log a^a} - e^{\log x^n}\right)dx$ $[\because m\log n = \log n^m]$

$= \int(a^x + a^a - x^n)\,dx$ $[\because e^{\log x} = x]$

$= \left[\frac{a^x}{\log a}\right] + a^a(x) - \frac{x^{n+1}}{n+1} + c$ $\left[\because \int a^x = \frac{a^x}{\log a}\right]$

**2. $\frac{a^x - e^{x\log b}}{e^{x\log a}\,b^x}$**

***Sol.*** $\int \frac{a^x - e^{x\log b}}{e^{x\log a}\,b^x}\,dx$

$= \int \frac{a^x - e^{\log b^x}}{e^{\log a^x}\,b^x}\,dx$ $[\because m\log n = \log n^m]$

$= \int \frac{a^x - b^x}{a^x \cdot b^x}\,dx$ $[\because e^{\log x} = x]$

$= \int \frac{a^x}{a^x b^x}\,dx - \int \frac{b^x}{a^x b^x}\,dx$

$= \int \frac{1}{b^x}\,dx - \int \frac{1}{a^x}\,dx$

$= \int b^{-x}\,dx - \int a^{-x}\,dx$ $\left[\because \int a^{-x}\,dx = \frac{a^{-x}}{-\log a} + c\right]$

$= \frac{b^{-x}}{-\log b} - \frac{a^{-x}}{-\log a} + c$

$= -\frac{b^{-x}}{\log b} + \frac{a^{-x}}{\log a} + c$

$= \frac{-1}{b^x\log b} + \frac{1}{\log a \cdot a^x} + c$

$= \frac{1}{a^x\log a} - \frac{-1}{b^x\log b} + c$

**3. $(e^x + 1)^2 e^x$**

***Sol.***

$\int(e^x+1)^2 e^x\,dx$

$= \int[(e^x)^2 + 2(e^x)(1) + 1^2]\,e^x\,dx$ $[\because (a+b)^2 = a^2 + 2ab + b^2]$

$= \int(e^{2x} + 2e^x + 1)\,e^x\,dx$

$= \int(e^{3x} + 2e^{2x} + e^x)\,dx$ $[a^m \cdot a^n = a^{m+n}]$

$$= \sqrt{2}\left[\frac{x+1}{2}\sqrt{x^2+2x+\frac{1}{2}} - \frac{1}{2(2)}\log\left|(x+1)+\sqrt{x^2+2x+\frac{1}{2}}\right|\right]+c$$

$$= \frac{x+1}{\sqrt{2}}\frac{\sqrt{2x^2+4x+1}}{\sqrt{2}} - \frac{\sqrt{2}}{4}\log\left|\sqrt{2}(x+1)+\sqrt{2x^2+4x+1}\right|+c$$

$$= \frac{x+1}{\sqrt{2}}\sqrt{2x^2+4x+1} - \frac{\sqrt{2}}{4}\log\left|\sqrt{2}(x+1)+\sqrt{2x^2+4x+1}\right|+c$$

**16.** $\dfrac{1}{x+\sqrt{x^2-1}}$

**Sol.** $\int\dfrac{dx}{x+\sqrt{x^2-1}}$

Multiply and divide by the conjugate of the denominator we get,

$$\int\frac{1}{x+\sqrt{x^2-1}}\times\frac{x-\sqrt{x^2-1}}{x-\sqrt{x^2-1}}\,dx$$

$$= \int\frac{\left(x-\sqrt{x^2-1}\right)dx}{x^2-\left(x^2-1\right)} \quad [\because (a+b)(a-b)=a^2-b^2]$$

$$= \int\frac{\left(x-\sqrt{x^2-1}\right)dx}{x^2-x^2-1}$$

$$= \int\left(x-\sqrt{x^2-1}\right)dx$$

$$= \int x\,dx - \int\sqrt{x^2-1}\,dx$$

$$= \frac{x^2}{2} - \left[\frac{x}{2}\sqrt{x^2-1} - \frac{1}{2}\log\left|x+\sqrt{x^2-1}\right|+c\right]$$

$$\left[\because \int\sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+c\right]$$

$$= \frac{x^2}{2} - \frac{x}{2}\sqrt{x^2-1} + \frac{1}{2}\log\left|x+\sqrt{x^2-1}\right|+c$$

## EXERCISE 2.8

**I. Using second fundamental theorem. evaluate the following**

**1.** $\int_0^1 e^{2x}dx$

**Sol.** $\int_0^1 e^{2x}dx$

$$= \left[\frac{e^{2x}}{2}\right]_0^1 = \frac{1}{2}[e^{2(1)} - e^{2(0)}]$$

$$= \frac{1}{2}[e^2 - e^0] = \frac{1}{2}[e^2 - 1]$$

**2.** $\int_0^{\frac{1}{4}}\sqrt{1-4x}\,dx$

**Sol.** $\int_0^{\frac{1}{4}}\sqrt{1-4x}\,dx = \int_0^{\frac{1}{4}}(1-4x)^{\frac{1}{2}}\,dx$

$$= \left[\frac{(1-4x)^{\frac{1}{2}+1}}{-4\left(\frac{1}{2}+1\right)}\right]_0^{\frac{1}{4}} = \left[\frac{(1-4x)^{\frac{3}{2}}}{-4\left(\frac{3}{2}\right)}\right]_0^{\frac{1}{4}}$$

$$= \left[\frac{(1-4x)^{\frac{3}{2}}}{-6}\right]_0^{\frac{1}{4}}$$

$$= \frac{-1}{6}\left[\left(1-4\left(\frac{1}{4}\right)\right)^{\frac{3}{2}} - (1-4(0))^{\frac{3}{2}}\right]$$

$$= \frac{-1}{6}\left[0^{\frac{3}{2}} - 1^{\frac{3}{2}}\right]$$

$$= \frac{-1}{6}(0-1) = \frac{1}{6}$$

**3.** $\int_1^2\dfrac{x\,dx}{x^2+1}$ *[Govt.MQP-2019]*

**Sol.** Let I $= \int_1^2\dfrac{x\,dx}{x^2+1}$

put $t = x^2+1$

$\Rightarrow dt = 2x\,dx$

$\Rightarrow \dfrac{dt}{2} = x\,dx$

$\therefore$ I $= \dfrac{1}{2}\int_2^5\dfrac{dt}{t}$

| |
|---|
| When $x = 1, t = 1^2+1 = 2$ |
| When $x = 2, t = 2^2+1 = 5$ |

**24.** $\int_0^{\pi/3} \tan x\, dx$ **is**

(a) log 2 (b) 0
(c) $\log\sqrt{2}$ (d) 2 log 2

**[Ans: (a) log 2]**

*Hint:* $I = \int_0^{\pi/3} \tan x\, dx = \int_0^{\pi/3} \frac{\sin x}{\cos x} dx$

Put $\cos x = t \Rightarrow -\sin x\, dx = dt$

$\Rightarrow \sin x\, dx = -dt$

$\therefore \quad I = -\int_1^{1/2} \frac{dt}{t}$ [∵ when $x = 0$, $t = \cos 0 = 1$ when $x = \frac{\pi}{3}$, $t = \cos\frac{\pi}{3} = \frac{1}{2}$]

$= \int_{1/2}^{1} \frac{dt}{t} = [\log(t)]_{1/2}^{1} = \log 1 - \log\frac{1}{2}$

$= \log 1 - (\log 1 - \log 2)$

$= 0 - 0 + \log 2 = \log 2$

**25.** **Using the factorial representation of the gamma function, which of the following is the solution for the gamma function $\Gamma(n)$ when $n = 8$.** *[QY-2019]*

(a) 5040 (b) 5400
(c) 4500 (d) 5540

**[Ans: (a) 5040]**

*Hint:* $\Gamma(n)$ when $n = 8$ is 7!

$= 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 5040$

**26.** $\Gamma(n)$ **is**

(a) $(n-1)!$ (b) $n!$
(c) $n\,\Gamma(n)$ (d) $(n-1)\,\Gamma(n)$

**[Ans: (b) $(n-1)!$]**

**27.** $\Gamma(1)$ **is**

(a) 0 (b) 1 (c) $n$ (d) $n!$

*Hint:* $\Gamma(n) = (n-1)!$

$\Rightarrow \Gamma(1) = (1-1)! = 0! = 1$ **[Ans: (b) 1]**

**28.** **If $n > 0$, then $\Gamma(n)$ is** *[PTA-5]*

(a) $\int_0^1 e^{-x}x^{n-1}dx$ (b) $\int_0^1 e^{-x}x^{n}dx$
(c) $\int_0^\infty e^{-x}x^{-n}dx$ (d) $\int_0^\infty e^{-x}x^{n-1}dx$

**[Ans: (c) $\int_0^1 e^{-x}x^{n-1}dx$ ]**

**29.** $\Gamma\left(\frac{3}{2}\right)$ *[PTA-2;Govt.MQP-2019]*

(a) $\sqrt{\pi}$ (b) $\frac{\sqrt{\pi}}{2}$ (c) $2\sqrt{\pi}$ (d) $\frac{3}{2}$

*Hint:* $= \frac{1}{2}\times\sqrt{\pi} = \frac{\sqrt{\pi}}{2}$ **[Ans: (c) $\frac{\sqrt{\pi}}{2}$]**

**30.** $\int_0^\infty x^4 e^{-x}\, dx$ **is**

(a) 12 (b) 4 (c) 4! (d) 64

**[Ans: (c) 4!]**

*Hint:* $\int_0^\infty x^4 e^{-x}\, dx$

Here $a = 1, n = 4$

[∵ $\int_0^\infty x^n e^{-ax}\, dx = \frac{n!}{a^{n+1}}$]

$= \frac{4!}{(1)^5} = 4!$

## Miscellaneous problems

### Evaluate the following integrals

**1.** $\int \frac{1}{\sqrt{x+2}-\sqrt{x+3}} dx$

***Sol.*** $I = \int \frac{1}{\sqrt{x+2}-\sqrt{x+3}} dx$

Multiply and divide with the conjugate of the denominator,

We get

$I = \int \frac{\sqrt{x+2}+\sqrt{x+3}}{\left(\sqrt{x+2}-\sqrt{x+3}\right)\left(\sqrt{x+2}+\sqrt{x+3}\right)}$

$= \int \frac{\sqrt{x+2}+\sqrt{x+3}}{(x+2)-(x+3)} dx$

[∵ $(a+b)(a-b) = a^2 - b^2$]

$= \int \frac{\sqrt{x+2}+\sqrt{x+3}}{\cancel{x}+2-\cancel{x}-3} dx = -\int\left(\sqrt{x+2}+\sqrt{x+3}\right)dx$

$= \left[\frac{(x+2)^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \frac{(x+3)^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right] + c$

$= -\frac{2}{3}\left[(x+2)^{\frac{3}{2}} + (x+3)^{\frac{3}{2}}\right] + c$

## III. Match the following:

| | | | |
|---|---|---|---|
| 1. | $\int e^{-t}\, dt$ | a. | Improper definite integral |
| 2. | $\int_0^1 e^{-t}\, dt$ | b. | $\Gamma(n)$ |
| 3. | $\int_0^\infty e^{-t}\, dt$ | c. | $\frac{n!}{a^{n+1}}$ |
| 4. | For $n > 0, \int_0^\infty x^{n-1} e^{-x}\, dx$ | d. | Indefinite integral |
| 5. | $\int_0^\infty x^n e^{-ax}\, dx$ where $n$ is a positive integer. | e. | $n\,\Gamma(n), n>0$ |
| 6. | $\Gamma(n)$ | f. | $\sqrt{\pi}$ |
| 7. | $\Gamma(1)$ | g. | $n!$ where $n$ is a positive integer |
| 8. | $\Gamma(n+1)$ | h. | proper definite integer |
| 9. | $\Gamma\left(\frac{1}{2}\right)$ | i. | $(n-1)\,\Gamma(n-1), n > 1$ |
| 10. | $\Gamma(n+1)$ | j. | 1 |

[Ans: 1 - d, 2 - h, 3 - a, 4 - b, 5 - c, 6 - i, 7 - j, 8 - g, 9 - f, 10 - e]

## IV. Identify the Incorrect statement:

**1.**

(i) $\Gamma(7) = 6!$ (ii) $\Gamma(6) = 5!$

(iii) $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ (iv) $\Gamma(7) = 7!$

[Ans: (iv) $\Gamma(7) = 7!$]

*Hint:*

$\Gamma(n) = (n-1)\,\Gamma(n-1)$

**2.**

(i) $\sum_{r=1}^{n} r = n\frac{(n+1)}{2}$

(ii) $\sum_{r=1}^{n} r^3 = n\frac{(n+1)(2n+1)}{6}$

(iii) $\sum_{r=1}^{n} r^3 = \left[n\frac{(n+1)}{2}\right]^3$

(iv) $\sum_{r=1}^{n} r^3 = \left[n\frac{(n+1)}{2}\right]^2$

[Ans: (iii) $\sum_{r=1}^{n} r^3 = \left[n\frac{(n+1)}{2}\right]^3$ ]

**3.** (i) $\int \frac{1}{x} dx = \log|x| + c$

(ii) $\int \frac{1}{x} dx = \log x$

(iii) $\int x\, dx = \frac{x^2}{2} + c$

(iv) $\int e^x\, dx = e^x + c$ [Ans: (ii) $\int \frac{1}{x} dx = \log x$]

**4.**

(i) $\int u dv = uv - \int v du$ (ii) $\int u dv = uv - u^1 v_1$

(iii) $\int u dv = uv - u^1 v_1 + u^{11} v_2 - \ldots$

(iv) $\int u dv = uv - u^1 v_1 + u^{11} v_2 - u^{111} v_3 + \ldots\ldots$

Where $u$ and $v$ are functions of $x$

[Ans: (ii) $\int u dv = uv - u^1 v_1$]

**5.**

(i) $\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$

(ii) $\int_{-a}^{a} f(x)dx = 0$ if $f(x)$ is an odd function

(iii) $\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$ if $f(x)$ is an even function

(iv) $\int_a^b f(x)dx = \int_a^b f(t)dt$

[Ans: (i) $\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$]

**6.** **If $f(x)$ is an anti-derivative of $f(x)$, then**

(i) $\int_a^b f(x)dx = F(b) - F(a)$

(ii) $\int_a^b f(x)dx = [(F(a) - F(b))]$

(iii) $\int_a^b f(x)dx = -F(a) - F(b)$

(iv) $\int_a^b f(x)dx = [F(x)]_a^b$

[Ans: (iii) $\int_a^b f(x)dx = -F(a) - F(b)$]

$$f(a+rh) = f\left(2+r\cdot\frac{2}{n}\right)$$
$$= 2\left(2+\frac{2r}{n}\right)-1$$
$$= 4+\frac{4r}{n}-1 = 3+\frac{4r}{n}$$
$$\therefore \quad f(a+rh) = 3+\frac{4r}{n}$$
$$\therefore \int_2^4 (2x-1)\,dx = \lim_{n\to\infty}\sum_{r=1}^{n}\frac{2}{n}\left(3+\frac{4r}{n}\right)$$
$$= \lim_{n\to\infty}\sum_{r=1}^{n}\frac{2}{n}\left(3+\frac{4r}{n}\right) = \lim_{n\to\infty}\sum_{r=1}^{n}\left(\frac{6}{n}+\frac{8r}{n^2}\right)$$
$$= \lim_{n\to\infty}\left(\sum_{r=1}^{n}\frac{6}{n}+\sum_{r=1}^{n}\frac{8r}{n^2}\right)$$
$$= \lim_{n\to\infty}\left(\frac{6}{n}\cdot\sum_{r=1}^{n}1+\frac{8}{n^2}\cdot\sum_{r=1}^{n}r\right)$$
$$= \lim_{n\to\infty}\left(\frac{6}{n}\cdot n+\frac{8}{n^2}\cdot\frac{n(n+1)}{2}\right)$$

$$\left[\because \sum_{r=1}^{n}1 = n \;\&\; \sum_{r=1}^{n}r = \frac{n(n+1)}{2}\right]$$
$$= \lim_{n\to\infty}\left(6+\frac{4}{\cancel{n^2}}\cdot\cancel{n^2}\left(1+\frac{1}{n}\right)\right)$$
$$= 6+4\cdot\lim_{n\to\infty}\left(1+\frac{1}{n}\right)$$
$$= 6+4(1+0)$$
$$\left(\because \text{When } n\to n\to\infty, \frac{1}{n}\to 0\right)$$
$$= 6+4 = 10$$
$$\therefore \int_2^4 (2x-1)\,dx = 10$$
$$\left[\because \sum_{r=1}^{n} r = n \;\&\; \sum_{r=1}^{n} r\,n = \frac{n(n+1)}{2}\right]$$

***

Chapter 3

# INTEGRAL CALCULUS- II

## SNAPSHOT

Geometrical interpretation of definite integral is the area under a curve between the given limits.

Integration helps us to find out the total cost function and total revenue function from the marginal cost.

Consumer's surplus & producer's surplus theory was developed by the economist Marshal.

## FORMULAE TO REMEMBER

1. Area of the region bounded by $y = f(x)$, with X-axis and the ordinates at $x = a$ and $x = b$ is

$$A = \int_a^b y dx = \int_a^b f(x)dx$$

2. Area of the region bounded by $y = f(x)$, between the limits $x = a$, $x = b$ and lies below X - axis is

$$A = \int_a^b -y dx = -\int_a^b f(x)dx$$

3. Area of the region bounded by $x = f(y)$, between the limits $y = c$, $y = d$ with Y - axis and the area lies the right of Y-axis is

$$A = \int_c^d x dy = \int_c^d f(y)dy$$

4. Area bounded by $x = f(y)$ between the limits $y = c$, $y = d$ with Y-axis and the area lies to the left of Y-axis is

$$A = \int_c^d -x dy = -\int_c^d f(y)dy$$

5. Area between two curves from $x = a$ to $x = b$ is

$$A = \int_a^b [f(x) - g(x)]dx$$

**Sol:**

Given $MC = \frac{dC}{dx} \propto x$

$\Rightarrow \frac{dC}{dx} = k_1 x$

$\Rightarrow dC = k_1 x\, dx$

$\Rightarrow \int dC = k_1 \int x\, dx$

$\Rightarrow C = k_1 \frac{x^2}{2} + k_2$ ----- (1)

Given fixed cost is ₹ 5000

$\therefore$ When $x = 0$, $C = 5000$

$\Rightarrow 5000 = k_1(0) + k_2 \Rightarrow k_2 = 5000$

$\therefore$ (1) becomes $C = k_1 \frac{x^2}{2} + 5000$ ----- (2)

Also it is given that when $x = 50$, C = ₹ 5625

$\therefore$ (2) $5625 = k_1 \frac{x^2}{2} + 5000$

$\Rightarrow 5625 - 5000 = k_1 \times \frac{(50)^2}{2}$

$\Rightarrow 625 = k_1 \times \frac{(50)\times(50)}{2}$

$\Rightarrow \frac{625 \times 2}{50 \times 50} = k_1 \Rightarrow k_1 = \frac{1}{2}$

(2) becomes

$C = \frac{1}{2}\left(\frac{x^2}{2}\right) + 5000$

$\Rightarrow C = \frac{x^2}{4} + 5000$

**19. If MR = 20 – 5x + 3x², find total revenue function.** *[PTA-1;Govt.MQP-2019]*

**Sol:** Given $MR = 20 - 5x + 3x^2$

$\Rightarrow \frac{dR}{dx} = 20 - 5x + 3x^2$

$\Rightarrow dR = (20 - 5x + 3x^2)\, dx$

$\Rightarrow \int dR = \int (20 - 5x + 3x^2)\, dx$

$\Rightarrow R = 20x - \frac{5x^2}{2} + \frac{3x^3}{3} + k$

When $x = 0$, $R = 0 \Rightarrow k = 0$

$\therefore R = 20x - \frac{5x^2}{2} + x^3$

**20. If MR = 14 – 6x + 9x², find the demand function.**

**Sol:** Given $MR = 14 - 6x + 9x^2$

$\Rightarrow \frac{dR}{dx} = 14 - 6x + 9x^2$

$\Rightarrow dR = (14 - 6x + 9x^2)\, dx$

$\Rightarrow \int dR = \int (14 - 6x + 9x^2)\, dx$

$\Rightarrow R = 14x - \frac{6x^2}{2} + \frac{9x^3}{3} + k$

When $x = 0$, $R = 0 \Rightarrow k = 0$

$\therefore R = 14x - 3x^2 + 3x^3$

Demand function $P = \frac{R}{x} = \frac{14x - 3x^2 + 3x^3}{x}$

$\Rightarrow P = 14 - 3x + 3x^2$

## EXERCISE 3.3

**1. Calculate consumer's surplus if the demand function p = 50 – 2x and x = 20.**

**Sol:** Given demand function $P = 50 - 2x$

and $x = 20$

when $x_0 = 20$,

$p_0 = 50 - 2(20) = 50 - 40$

$= 10$

$p_0 x_0 = 20\,(10) = 200$

Consumer's Surplus $CS = \int_0^{x_0} f(x)\,dx - p_0 x_0$

$CS = \int_0^{20} (50 - 2x)\,dx - 200$

$= \left[50x - \frac{2x^2}{2}\right]_0^{20} - 200$

$= 50(20) - (20)^2 - 200$

$= 1000 - 400 - 200$

$= 1000 - 600$

$CS = 400$ units

**2. Calculate consumer's surplus if the demand function p = 122 – 5x – 2x² and x = 6**

**Sol:** Given demand function $p = 122 - 5x - 2x^2$

and $x = 6$

When $x_0 = 6$, $p_0 = 122 - 5(6) - 2(6)^2$

$= 122 - 30 - 72$

$= 122 - 102$

$p_0 = 20$

$p_0x_0 = 20 \times 6 = 120.$

Consumer's Surplus

$$CS = \int_0^x f(x)\,dx - p_0x_0$$

$$= \int_0^6 (122 - 5x - 2x^2)\,dx - 120$$

$$= \left[122x - \frac{5x^2}{2} - \frac{2x^3}{3}\right]_0^6 - 120$$

$$= 122(6) - 5\frac{(6^2)}{2} - 2\frac{(6^3)}{3} - 120$$

$$= 732 - \frac{180}{2} - \frac{432}{3} - 120$$

$= 732 - 90 - 144 - 120$

$= 732 - 354$

CS = 378 units.

**3. The demand function $p = 85 - 5x$ and supply function $p = 3x - 35$. Calculate the equilibrium price and quantity demanded. Also calculate consumer's surplus.**

**Sol:** Given demand function $p_d = 85 - 5x$ and

Supply function $p_s = 3x - 35$

At equilibrium prices, $p_d = p_s$

$\Rightarrow 85 - 5x = 3x - 35$

$\Rightarrow 85 + 35 = 3x + 5x$

$\Rightarrow 120 = 8x$

$\Rightarrow x = \frac{120}{8} = 15$

When $x_0 = 15$, $p_0 = 85 - 5(15)$

$= 85 - 75$

$p_0 = 10$

$\therefore p_0x_0 = 15 \times 10 = 150$

Consumer's Surplus

$$CS = \int_0^x f(x)\,dx - p_0x_0$$

$$= \int_0^{15} (85 - 5x)\,dx - 150$$

$$= \left[85x - \frac{5x^2}{2}\right]_0^{15} - 150$$

$$= 85(15) - 5\frac{(15)^2}{2} - 150$$

$$= 1275 - \frac{1125}{2} - 150$$

$= 1275 - 562.5 - 150$

$= 1275 - 712.5$

CS $= 562.5$

**4. The demand function for a commodity is $p = e^{-x}$. Find the consumer's surplus when $p = 0.5$.** ***[PTA-6]***

**Sol:** Given demand function $p = e^{-x}$

and $p = 0.5$

When $p_0 = 0.5$, $0.5 = e^{-x}$

$$\Rightarrow \frac{1}{2} = e^{-x}$$

$$\Rightarrow \frac{1}{2} = \frac{1}{e^x} \quad \Rightarrow ex = 2$$

$\Rightarrow x = \log 2$

$$\therefore p_0x_0 = 0.5 \log 2 = \frac{1}{2}\log 2$$

Consumer's Surplus

$$= \int_0^x f(x)\,dx - p_0x_0$$

$$= \int_0^{\log 2} e^{-x}\,dx - \frac{1}{2}\log 2$$

$$= \left[\frac{e^{-x}}{-1}\right]_0^{\log 2} - \frac{1}{2}\log 2$$

$$= -\left[e^{-x}\right]_0^{\log 2} - \frac{1}{2}\log 2$$

$$= -\left(e^{-\log 2} - e^0\right) - \frac{1}{2}\log 2$$

$$= -\left(e^{\log\frac{1}{2}} - 1\right) - \frac{1}{2}\log 2$$

$$= -\left(\frac{1}{2} - 1\right) - \frac{1}{2}\log 2$$

$$= -\left(-\frac{1}{2}\right) - \frac{1}{2}\log 2$$

$$= \frac{1}{2} - \frac{1}{2}\log 2 = \frac{1}{2}(1 - \log_e 2)$$

$$\therefore CS = \frac{1}{2}(1 - \log_e 2)$$

## Miscellaneous problems

**1. A manufacture's marginal revenue function is given by MR = 275 − x − 0.3 $x^2$. Find the increase in the manufactures total revenue if the production is increased from 10 to 20 units.**

*Sol:* Given MR = $275 - x - 0.3x^2$

$\int MR = \int (275 - x - 0.3x^2)\, dx$

To find the total revenue, when it is increased from 10 to 20 units

$$R = \int_{10}^{20} (275 - x - 0.3x^2)\, dx$$

$$= \left(275x - \frac{x^2}{2} - 0.3\frac{x^3}{3}\right)_{10}^{20}$$

$$= \left(275x - \frac{x^2}{2} - 0.1x^3\right)_{10}^{20}$$

$$= \left[275(20) - \frac{20^2}{2} - 0.1(20)^3\right] - \left[275(10) - \frac{10^2}{2} - 0.1(10^3)\right]$$

= [5500 − 200 − 800] − [2750 − 50 − 100]

= [5500 − 1000] − [2750 − 150]

= 4500 − 2600

R = ₹ 1900

**2. A company has determined that marginal cost function for $x$ product of a particular commodity is given by MC = 125 + 10$x$ − $\frac{x^2}{9}$. Where C is the cost of producing $x$ units of the commodity. If the fixed cost is ₹ 250 what is cost of producing 15 units.**

*Sol:* Given MC = $125 + 10x - \frac{x^2}{9}$

$\Rightarrow \int MC = \int \left(125 + 10x - \frac{x^2}{9}\right) dx$

$\Rightarrow C = 125x + \frac{10x^2}{2} - \frac{x^3}{9 \times 3} + k$

$\Rightarrow C = 125x + 5x^2 - \frac{x^3}{27} + k$

Given fixed cost is ₹ 250,

When $x = 0$, C = 250 $\Rightarrow k = 250$

$\therefore C = 125x + 5x^2 - \frac{x^3}{27} + 250$

When $x$ = 15, C = ?

$\therefore C = 125(15) + 5(15)^2 - \frac{15^3}{27} + 250$

$= 1875 + 1125 - \frac{3375}{27} + 250$

= 3000 − 125 + 250

= ₹ 3125

∴ C = ₹ 3125

**3. The marginal revenue function for a firm is given by MR = $\frac{2}{x+3} - \frac{2x}{(x+3)^2} + 5$. Show that the demand function is P = $\frac{2}{x+3} + 5$.**

*Sol:*

Given MR = $\frac{2}{x+3} - \frac{2x}{(x+3)^2} + 5$

$\Rightarrow \int MR = \int \left(\frac{2}{x+3} - \frac{2x}{(x+3)^2} + 5\right) dx$

$\Rightarrow R = 2\log(x+3) - 2\int \frac{x}{(x+3)^2} dx + 5x + k$

$\Rightarrow R = 2\log(x+3) - 2\left[\int \frac{x+3-3}{(x+3)^2} dx\right] + 5x + k$

$\Rightarrow R = 2\log(x+3) - 2\left[\int \frac{1}{x+3} dx - 3\int \frac{dx}{(x+3)^2}\right] + 5x + k$

$\Rightarrow R = 2\log(x+3) - 2\log(x+3) + 6\frac{(x+3)^{-1}}{-1} + 5x + k$

$\Rightarrow R = \frac{-6}{x+3} + 5x + k$

when $x = 0$, R = 0 $\Rightarrow k = 0$

$0 = \frac{-6}{3} + 0 + k \Rightarrow k = +2$

$\therefore R = \frac{-6}{x+3} + 5x - 2$

Demand function P = $\frac{R}{x}$

$\Rightarrow P = \frac{-6}{x(x+3)} + 5 + \frac{2}{x}$

$\Rightarrow P = \frac{-6 + 2(x+3)}{x(x+3)} + 5$

$\Rightarrow P = \frac{-6 + 2x + 6}{x(x+3)} + 5$

$= \frac{2x}{x(x+3)} + 5$

$\Rightarrow P = \frac{2}{x+3} + 5$

## Additional Question

### I. Choose the correct answer

1. **The area bounded by $y = 2x - x^2$ and X-axis is ______ sq. units**

(a) $\frac{2}{3}$ (b) $\frac{4}{3}$ (c) 2 (d) 4

**[Ans: (b) $\frac{4}{3}$]**

*Hint:*

$$A = \int_0^2 y\,dx = \int_0^2 (2x - x^2)\,dx$$

$$= \left[2\frac{x^2}{2} - \frac{x^3}{3}\right]_0^2$$

$$= 4 - \frac{8}{3} = \frac{4}{3}$$

2. **The area of the region bounded by the ellipse**

(a) $\pi ab$ sq.units (b) $\frac{\pi a}{b}$ sq.units

(c) $2\pi ab$ sq.units (d) $\frac{\pi}{2}ab$ sq.units.

**[Ans: (a) $\pi ab$ sq.units]**

*Hint:* Area $= 4\int_0^a y\,dx$

3. **The area bounded by the curves $y = 2^x$, $x = 0$ and $x = 2$ is ______ sq. units.**

(a) $\log_e 2$ (b) $3\log_e 2$

(c) $\frac{3}{\log_e 2}$ (d) $2\log_e 3$

**[Ans: (c) $\frac{3}{\log_e 2}$]**

*Hint:*

$$\text{Area} = \int_0^2 y\,dx = \int_0^2 2^x\,dx = \left[\frac{2^x}{\log 2}\right]_0^2$$

$$= \frac{2^2}{\log 2} - \frac{2^0}{\log 2}$$

$$= \frac{4}{\log 2} - \frac{1}{\log 2} = \frac{3}{\log_e 2}$$

4. **The area of the region bounded by the line $2y = -x + 8$, X - axis and the lines $x = 2$ and $x = 4$ is ______ sq.units.**

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$ (c) 5 (d) $\frac{5}{2}$

**[Ans: (c) 5]**

*Hint:* $A = \int_2^4 y\,dx = \int_2^4 \left(\frac{-x}{2} + 4\right)dx = \left[\frac{-x^2}{4} + 4x\right]_2^4$

$= 12 - 7 = 5$

5. **The area enclosed by the curve $y = \cos^2 x$ in $[0, \pi]$ the lines $x = 0$, $x = \pi$ and the X-axis is ______ sq.units.**

(a) $\pi$ (b) $2\pi$ (c) $\frac{2}{\pi}$ (d) $\frac{\pi}{2}$

**[Ans: (d) $\frac{\pi}{2}$]**

*Hint:*

$$A = \int_0^\pi y\,dx = \int_0^\pi \cos^2 x\,dx$$

$$= \frac{1}{2}\int_0^\pi [1 + \cos 2x]\,dx$$

$$= \frac{1}{2}\left[x + \frac{\sin 2x}{2}\right]_0^\pi$$

$$= \frac{1}{2}[\pi + 0 - 0] = \frac{\pi}{2}$$

6. **The area of the region bounded by the line $y = 3x + 2$, the X-axis and the ordinates $x = -1$ and $x = 1$ is ______ sq. units.**

(a) $\frac{13}{3}$ (b) 13 (c) $\frac{26}{3}$ (d) $\frac{3}{13}$

**[Ans: (a) $\frac{13}{3}$]**

*Hint:* $A = \int_{-1}^{-\frac{2}{3}} -y\,dx + \int_{-\frac{2}{3}}^{1} y\,dx$

7. **The value of $\int_{-3}^{1} |x + 1|\,dx$ is ______.**

(a) 4 (b) $\frac{1}{4}$ (c) 8 (d) 2

**[Ans: (a) 4]**

*Hint:* $\int_{-3}^{1} |x+1|\,dx = \int_{-3}^{-1} -(x+1)\,dx + \int_{-1}^{3} (x+1)\,dx$

8. **The area lying above the X–axis and under the parabola $y = 4x - x^2$ is ______ sq. units**

(a) $\frac{16}{3}$ (b) $\frac{8}{3}$ (c) $\frac{32}{3}$ (d) $\frac{64}{3}$

**[Ans: (c) $\frac{32}{3}$]**

*Hint:* $A = \int_0^4 y\,dx$

$$\text{Required area} = \int_0^{2\pi} y\,dx$$

$$= \int_0^{\pi} y\,dx + \int_{\pi}^{2\pi} -y\,dx$$

[∵ the second area lies below the X-axis]

$$= \int_0^{\pi} \sin x\,dx + \int_{\pi}^{2\pi} -\sin x\,dx$$

$$= \left[-\cos x\right]_0^{\pi} - \left[-\cos x\right]_{\pi}^{2\pi}$$

$$= -\left[\cos x\right]_0^{\pi} + \left[\cos x\right]_{\pi}^{2\pi}$$

$$= -\left[\cos\pi - \cos 0\right] + \left[\cos 2\pi - \cos\pi\right]$$

$$= -\left[-1-1\right] + \left[+1-(-1)\right]$$

$[\therefore \cos\pi = -1 \; \cos 0 = 1 \;\; \& \cos 2\pi = 1]$

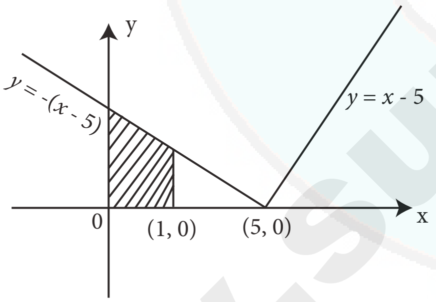
$$= -(-2) + (2)$$

$$= 2 + 2$$

$$A = 4 \text{ sq. units.}$$

**9. Sketch the graph of $y = |x - 5|$. Evaluate $\int_0^1 |x-5|\,dx$**

*Sol:*




$$y = |x-5|$$

$$= \begin{cases} -(x-5) & \text{if } \; x < 0 \\ +(x-5) & \text{if } \; x > 0 \end{cases}$$

$$\text{Required area} = \int_0^1 |x-5|\,dx$$

$$= \int_0^1 -(x-5)\,dx$$

$$= \int_0^1 (5-x)\,dx$$

$$= \left[5x - \frac{x^2}{2}\right]_0^1 = \left[5(1) - \frac{1}{2}\right] - 0$$

$$= 5 - \frac{1}{2} = \frac{10-1}{2}$$

$$= \frac{9}{2} \text{ sq. units.}$$

***

# Chapter 4

# DIFFERENTIAL EQUATIONS

## SNAPSHOT

* An ordinary differential equation is an equation that involves some ordinary derivatives $\left(\frac{dy}{dx}, \frac{d^2y}{dx^2}, \ldots\right)$ of a function $y = f(x)$. Here we have one independent variable.
* Sometimes a family of curves can be represented by a single equation with one or move arbitrary constants. By assigning different values for constants, we get a family of curves. The arbitrary constants are called the parameters of the family.
* Solution of the differential equation must contain the same number of arbitrary constants as the order of the equation. Such a solution is called General (complete) solution of the differential equation.
* The highest order derivative present in the differential equation is the order of the differential equation.
* Degree is the highest power of the highest order derivative in the differential equation.
* Variable separable method
  If in an equation, it is possible to collect all the terms of $x$ and $dx$ on one side and all the terms of $y$ and $dy$ on the other side, then the variable are said to be separable.
* Homogeneous differential equations:
  $\frac{dy}{dx} = \frac{f(x,y)}{g(x,y)}$ is homogeneous differential equation if $f(x, y)$ and $g(x, y)$ are homogenous functions of the same degree in $x$ and $y$.
* Linear differential equations of first order:
  General form of linear equation of first order is $\frac{dy}{dx} + Py = Q$.
* Second order first degree differential equations
  $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = f(x)$ is the general form of second order first degree differential equations.

Also $\frac{2y}{4}\left(\frac{dy}{dx}\right)=a$

$\Rightarrow \frac{y}{2}\left(\frac{dy}{dx}\right)=a$ (3)

Substituting (2) and (3) in (1) we get,

$$y^2 = 2y\left(\frac{dy}{dx}\right)\left[x+\frac{y}{2}\left(\frac{dy}{dx}\right)\right]$$

$$\Rightarrow y^2 = 2xy\left(\frac{dy}{dx}\right)+y^2\left(\frac{dy}{dx}\right)^2$$

Dividing by $y$ we get,

$$y = 2x\left(\frac{dy}{dx}\right)+y\left(\frac{dy}{dx}\right)^2$$

## EXERCISE 4.2

**1. Solve (i) $\frac{dy}{dx} = ae^y$**

**Sol.** Given $\frac{dy}{dx} = ae^y$

Separating the variables we get,

$\frac{dy}{e^y} = adx \Rightarrow e^{-y}\,dy = adx$

Integrating both sides we get,

$$\int e^{-y}dy = a\int dx$$

$$-e^{-y} = ax + c$$

$$ax + e^{-y} + c = 0$$

**(ii)** $\frac{1+x^2}{1+y} = xy\frac{dy}{dx}$

**Sol.** Given $\frac{1+x^2}{1+y} = xy\frac{dy}{dx}$

Separating the variables we get,

$$\frac{(1+x^2)dx}{x} = y(1+y)dy$$

$$\Rightarrow \left(\frac{1}{x}+\frac{x^2}{x}\right)dx = (y+y^2)dy$$

$$\Rightarrow \left(\frac{1}{x}+x\right)dx = (y+y^2)dy$$

Integrating both sides we get,

$$\int\left(\frac{1}{x}+x\right)dx = \int(y+y^2)dy$$

$$\Rightarrow \log x+\frac{x^2}{2} = \frac{y^2}{2}+\frac{y^3}{3}+c$$

**2. Solve: $y(1-x) - x\frac{dy}{dx} = 0$** [PTA-6]

**Sol.** Given $y(1-x) - x\frac{dy}{dx} = 0$

$$\Rightarrow y(1-x) = x\frac{dy}{dx}$$

Separating the variables we get,

$$\Rightarrow \frac{(1-x)}{x}dx = \frac{dy}{y}$$

$$\Rightarrow \left(\frac{1}{x}-1\right)dx = \frac{dy}{y}$$

Integrating both sides we get,

$$\int\left(\frac{1}{x}-1\right)dx = \int\frac{dy}{y}$$

$$\Rightarrow \log x - x = \log y + c$$

**3. Solve (i) $ydx - xdy = 0$**

**Sol.** Given $ydx - xdy = 0$

$$\Rightarrow ydx = xdy$$

Separating the variables we get,

$$\frac{dx}{x} = \frac{dy}{y}$$

Integrating both sides we get,

$$\int\frac{dx}{x} = \int\frac{dy}{y}$$

$$\log x = \log y + \log c$$

$$\log x = \log cy$$

$[\because \log m + \log n = \log mn]$

$$x = cy$$

**(ii)** $\frac{dy}{dx}+e^x+ye^x = 0$ [PTA-1]

$$\Rightarrow \frac{dy}{dx} = -e^x(1+y)$$

Separating the variables we get,

$$\Rightarrow \frac{dy}{1+y} = -e^x\,dx$$

$$\Rightarrow \log(1+y) = -e^x + c$$

$\Rightarrow \quad \log p = .08t + c$

$\Rightarrow \quad p = e^{.08t}\, e^{c}$

$\Rightarrow \quad p = c_1 e^{.08t}$ where $c_1 = e^c$

when $t = 0, p = ₹1,00,000$

$\therefore$ (1) becomes $1,00,000 = c_1 e^{.08(0)}$

$\Rightarrow \quad 1,00,000 = c_1$

$\therefore (1) \quad p = 1,00,000\, e^{0.08t}$

when $t = 10$,

$p = 1,00,000\, e^{0.08(10)}$

$= 1,00,000\, (e^{0.8})$

$= 1,00,000\, (2.2255)$

$[\because e^{0.8} = 2.2255]$

$= ₹\ 2,22,550$

## EXERCISE 4.5

**Solve the following differential equations**

**1.** $\dfrac{d^2y}{dx^2} - 6\dfrac{dy}{dx} + 8y = 0$ *[PTA-4]*

**Sol.**

The auxiliary equation is

$m^2 - 6m + 8 = 0$

$\Rightarrow (m-4)(m-2) = 0$

$\Rightarrow \quad m = 2, 4$

(8 = (–4)(–2), –6 = –4 + (–2))

The roots are real and different

$\therefore$ Complementary function CF is $Ae^{2x} + Be^{4x}$

$\therefore$ The general solution is $y = Ae^{2x} + Be^{4x}$

**2.** $\dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} + 4y = 0$

**Sol.**

The auxiliary equation is

$m^2 - 4m + 4 = 0$

$\Rightarrow \quad (m-2)^2 = 0$

$\Rightarrow \quad m = 2, 2$

The roots are real and equal

$\therefore$ Complementary function CF is $(Ax + B)e^{2x}$

$\therefore$ The general solution is $y = (Ax + B)e^{2x}$

**3.** $(D^2 + 2D + 3)y = 0$

**Sol.**

The auxiliary equation is

$m^2 + 2m + 3 = 0$

Here $a = 1, b = 2, c = 3$

$$\therefore m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{4 - 4(1)(3)}}{2(1)}$$

$$= \frac{-2 \pm \sqrt{4 - 12}}{+2}$$

$$= \frac{-2 \pm \sqrt{-8}}{+2}$$

$$= \frac{-2 \pm i\sqrt{4 \times 2}}{2}$$

$$= \frac{-2 \pm i2\sqrt{2}}{2} = \not{2}\left(\frac{-1 \pm i\sqrt{2}}{\not{2}}\right)$$

$= -1 \pm i\sqrt{2}$

$\therefore \alpha = -1, \beta = \sqrt{2}$

The Complementary function

$CF = e^{\alpha x}[A \cos \beta x + B \sin \beta x]$

$= e^{-x}[A \cos \sqrt{2}\, x + B \sin \sqrt{2}\, x]$

$\therefore$ The general solution is

$y = e^{-x}[A \cos \sqrt{2}\, x + B \sin \sqrt{2}\, x]$

**4.** $\dfrac{d^2y}{dx^2} - 2k\dfrac{dy}{dx} + k^2y = 0$

**Sol.**

The auxiliary equation is

$m^2 - 2km + k^2 = 0$

$\Rightarrow \quad (m - k)^2 = 0$

$\Rightarrow \quad m = k, k$

The roots are real and equal

$\therefore$ Complimentary function CF is $(Ax + B)e^{kx}$

The general solution is

$y = (Ax + B)e^{kx}$

put $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$ is

$$v + x\frac{dv}{dx} = \frac{x.vx - v^2x^2}{x^2 + x - vx} = \frac{x^2(v - v^2)}{x^2(1-v)} = \frac{v - v^2}{1+v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{v-v^2}{1+v} - v = \frac{v - v^2 - v(1+v)}{1+v}$$

$$= \frac{v - v^2 - v - v^2}{1+v}$$

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{-2v^2}{1+v}$$

$$\Rightarrow \quad \frac{(1+v)dv}{2v^2} = -\frac{dx}{x}$$

**24. Which of the following is the homogeneous differential equation?** *[QY-2019]*

(a) $(3x - 5)dx = (4y - 1)dy$

(b) $xy\,dx - (x^3 + y^3)dy = 0$

(c) $y^2dx + (x^2 - xy - y^2)dy = 0$

(d) $(x^2 + y)dx = (y^2 + x)dy$

**[Ans: (c) $y^2dx + (x^2 - xy - y^2)dy = 0$]**

**Hint:** Only in (*c*) each and every term is of degree 2

**25. The solution of the differential equation**

$$\frac{dy}{dx} = \frac{y}{x} + \frac{f\left(\frac{y}{x}\right)}{f'\left(\frac{y}{x}\right)}$$ *[PTA-2]*

(a) $f\left(\frac{y}{x}\right) = kx$ (b) $xf\left(\frac{y}{x}\right) = k$

(c) $f\left(\frac{y}{x}\right) = ky$ (d) $yf\left(\frac{y}{x}\right) = k$

**[Ans: (a) $f\left(\frac{y}{x}\right) = kx$]**

**Hint:** $$\frac{dy}{dx} = \frac{y}{x} = \frac{f\left(\frac{y}{x}\right)}{f^1\left(\frac{y}{x}\right)}$$

$$\Rightarrow \frac{dy}{dx} - \frac{y}{x} = \frac{f\left(\frac{y}{x}\right)}{f^1\left(\frac{y}{x}\right)}$$

$$P = -Yx; \quad Q = \frac{f\left(\frac{y}{x}\right)}{f^1\left(\frac{y}{x}\right)}$$

only when $f\left(\frac{y}{x}\right) = kx$, Q will be a function of $x$

## MISCELLANEOUS PROBLEMS

**1. Suppose that $Q_d = 30 - 5p + 2\frac{dp}{dt} + \frac{d^2P}{dt^2}$ and $Q_s = 6 + 3P$. Find the equilibrium price for market clearance.**

*Sol.* Given $Q_d = 30 - 5p + 2\frac{dp}{dt} + \frac{d^2p}{dt^2}$ and

$Q_3 = 6 + 3p$

At equilibrium, $Q_d = Q_s$

$$\Rightarrow \quad 30 - 5p + 2\frac{dp}{dt} + \frac{d^2p}{dt^2} = 6 + 3p$$

$$\Rightarrow \frac{d^2p}{dt^2} + 2\frac{dp}{dt} - 5p + 30 - 6 - 3p = 0$$

$$\Rightarrow \quad \frac{d^2p}{dt^2} + 2\frac{dp}{dt} - 8p = -24$$

Auxiliary equation is $m^2 + 2m - 8 = 0$

$\Rightarrow (m + 4)(m - 2) = 0$

$\Rightarrow \quad m = -4, 2$

−8, 2; 4 −2

$\therefore$ C.F. is $Ae^{-4t} + Be^{2t}$

Particular Integral (P.I.) $= \frac{-24}{(D+4)(D-x)}e^{0x}$

$$= \frac{-24}{(0+4)(0-2)} = \frac{-24}{-8} = 3$$

$y = CF + PI$

$\therefore$ The general solution is

$y = Ae^{-4t} + Be^{2t} + 3$

**2. Form the differential equation having for its general solution $y = ax^2 + bx$** *[PTA-5]*

*Sol.* Given equation is $y = ax^2 + bx$ (1)

Differentiating w. r. t. '$x$', we get,

$$\frac{dy}{dx} = 2ax + b \quad (2)$$

$$m^2 - 5m - 6 = 0$$
$$(m-6)(m+1) = 0$$
$$\Rightarrow m = 6, -1$$
$$\text{C.F} = Ae^{6t} + Be^{-t}$$
$$\text{P.I} = \frac{1}{\phi(D)} f(x)$$
$$= \frac{1}{D^2 - 5D - 6}(-24)e^{0t} = \frac{-24}{-6} \text{ (Replace D by 0)}$$
$$= 4$$

The general solution is P = C.F + P.I

$$= Ae^{6t} + Be^{-t} + 4$$

**4. Suppose that the quantity needed $Q_d = 42 - 4p - 4\frac{dp}{dt} + \frac{d^2p}{dt^2}$ and quantity supplied $Q_s = -6 + 8p$ where $p$ is the price. Find the equilibrium price for market clearance.** *[PTA-2]*

**Sol.**

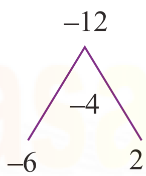
For market clearance, $Q_d = Q_s$

$$\Rightarrow 42 - 4p - 4\frac{dp}{dt} + \frac{d^2p}{dt^2} = -6 + 8p$$
$$\Rightarrow 48 - 12p - 4\frac{dp}{dt} + \frac{d^2p}{dt^2} = 0$$
$$\Rightarrow \frac{d^2p}{dt^2} - 4\frac{dp}{dt} - 12p = -48$$

The auxiliary equation is $m^2 - 4m - 12 = 0$

$\Rightarrow (m - 6)(m + 2) = 0$

$\Rightarrow m = -2, 6$




The roots are real and different

$\therefore$ C.F. is $Ae^{-2t} + Be^{6t}$

$$\text{P.I.} = \frac{48}{(D-6)(D+2)} e^{0t} = \frac{-48}{(0-6)(0+2)}$$
$$= \frac{-48}{-12} = 4$$

$\therefore$ The general solution is

P = C.F. + P.I.

$\Rightarrow P = Ae^{-2t} + Be^{6t} + 4$

**5. A man plans to invest some amount in a small saving scheme with a guaranteed compound interest compounded continuously at the ratio of 12 percent for 5 years. How much should he invest if he wants an amount of Rs. 25000 at the end of 5 year period? ($e^{-0.6} = 0.5488$)** *[PTA-5]*

**Sol.** Let P($t$) denotes the amount of money in the account at time $t$. Then the differential equation governing the growth of money is

$$\frac{dp}{dt} = \frac{12}{100}p \Rightarrow \frac{dp}{dt} = 0.12p$$

Separating the variables,

$$\frac{dp}{P} = 0.12\, dt$$

Integrating, $\int \frac{dp}{P} = \int 0.12\, dt + c$

$\Rightarrow \log p = 0.12t + c$

$\Rightarrow P = e^{0.12t + c} \Rightarrow P = e^{0.12t} . e^c$

$\Rightarrow P = e^{0.12t} . c1$ (1)

When $t = 0, p = 0 \Rightarrow p = e^0 (c_1) \Rightarrow c_1 = p$

When $t = 5$, and $p = 25000$

$$25000 = e^{.12(5)} . P \quad [\because c_1 = p]$$
$$\Rightarrow 25000 = e^{0.6}P$$
$$\Rightarrow \frac{25000}{e^{0.6}} = p$$
$$\Rightarrow 25000\,(e^{-0.6}) = p$$
$$\Rightarrow 25000\,(0.5488) = p$$
$$\Rightarrow ₹\ 13720$$

Hence, to get an amount of ₹ 25000 at the end of 5 years, ₹ 13720 must be invested.

## Govt. Exam Question & Answers

### 1 Marks

**1. The order and degree of the differential equation of $y = mx + c$ is _______** *[HY-2019]*

(a) 2, 2 (b) 2, 1 (c) 1, 2 (d) 1, 1

[*Ans.* (b) 2, 1]

**Hint:**

*a*, *b*, *d* are homogeneous differential equations but *c* is a differential equation.

**3. The integrating factor of the differential equation $x\frac{dy}{dx} - y = 2x^2$ is**

(a) $e^{\int \frac{1}{x}dx}$ (b) $e^{-\log x}$

(c) $\frac{1}{x}$ (d) $\int 2x\, dx$

**[Ans: (d) $\int 2x\, dx$]**

**Hint:**

Hint: *a*, *b*, *c* are integrating factors but not *d*.

**4. The general solution of the differential equation $\frac{dy}{dx} = e^{x+y}$ is**

(a) $e^x + e^{-y} = C$ (b) $e^x + e^y = C$

(c) $e^x + \frac{1}{e^y} = C$ (d) $e^{-y} + e^x = C$

**[Ans: (b) $e^x + e^y = C$]**

**Hint:**

*a*, *c*, *d* are solutions in different forms but not *b*.

**5. The equation of the curve whose slope is given by $\frac{dy}{dx} = \frac{2y}{x}$, $x > 0$, $y > 0$ and which passes through the point (1, 1) is**

(a) $x^2 = y$ (b) $x = \sqrt{y}$

(c) $x^4 = y^3$ (d) $x^4 = y^2$

**[Ans: (c) $x^4 = y^3$]**

**Hint:** *a*, *b*, *d* represents the curve but not *c*.

## CHOOSE THE INCORRECT STATEMENT

**1.** (a) $\frac{dy}{dx} = 2xy$ is a differential equation

(b) The order of a differential equation is the order of highest order derivative in the equation

(c) The order of a differential equation is a positive integer.

(d) The order of a differential equation need not be positive always

**[Ans: (d) The order of a differential equation need not be positive always]**

**2. In the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \sin\left(\frac{dy}{dx}\right) = 0$**

(a) order 2 degree 2

(b) order 2 degree not defined

(c) $\left(\frac{d^2y}{dx^2}\right)^2 = -\sin\left(\frac{dy}{dx}\right)$

(d) $\frac{d^2y}{dx^2} = \sqrt{-\sin\left(\frac{dy}{dx}\right)}$

**[Ans: (a) order 2 degree 2]**

**Hint:** Since the differential equation cannot be expressed as a polynomial in differential co-efficients, its degree is not defined.

**3. In $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = \sin x$**

(a) order 2, degree 1

(b) order 2, degree not defined

(c) It is a linear differential equation

(d) Auxiliary equation is $m^2 - 5m + 6 = 0$

**[Ans: (b) order 2, degree not defined]**

**4. In the family of curves given by $y = Ae^x$ where A is the parameter**

(a) $\frac{d^2y}{dx^2} = 0$ (b) $\frac{dy}{dx} = Ae^x$

(c) $\frac{dy}{dx} = y$

(d) For different values of A we get different members of the family

**[Ans: (a) $\frac{d^2y}{dx^2} = 0$]**

**5.** a) The solution of a differential equation is a relation between the variables involved which satisfies the differential equation

b) The solution which contains as many as arbitrary constants as the order of the differential equation is the general solution

c) Solutions obtained by giving particular values to the arbitrary constants in the general solution is the particular solution

d) $y = 3\cos x + 2\sin x$ is the general solution of $\frac{d^2y}{dx^2} + y = 0$

**[Ans: (d) $y = 3\cos x + 2\sin x$ is the general solution of $\frac{d^2y}{dx^2} + y = 0$]**

Chapter 5

# NUMERICAL METHODS

## SNAPSHOT

* Forward difference operator ($\Delta$) = (Delta)

  $\Delta yn = y_{n+1} - y_n$, $n = 0, 1, 2, \ldots$
* $\Delta f(x) = f(x + h) - f(x)$, $h$ is the equal interval of spacing

### PROPERTIES OF OPERATOR $\Delta$:

* If $c$ is a constant then $\Delta c = 0$

  $\Delta$ is distributive $\Rightarrow \Delta\,(f(x) + g(x)) = \Delta f(x) + \Delta g(x)$
* If $c$ is a constant, then $\Delta c \cdot f(x) = c \cdot \Delta f(x)$.
* If $m$ and $n$ are positive integers then

  $\Delta^m \cdot \Delta^n f(x) = \Delta^{m+n} \cdot f(x)$
* $\Delta\,[f(x) \cdot g(x)] = f(x) \cdot \Delta g(x) + g(x) \cdot \Delta f(x)$
* $\Delta\left[\dfrac{f(x)}{g(x)}\right] = \dfrac{g(x)\,\Delta f(x) - f(x)\cdot\Delta g(x)}{g(x)\cdot g(x+h)}$
* $\Delta^2 y_n = \Delta y_{n+1} - \Delta y_n$
* $\Delta^3 y_n = \Delta^2 y_{n+1} - \Delta^2 y_{n,n} = 0, 1, 2 \ldots$
* $\Delta^k y_n = \Delta^{k-1} y_{n+1} - \Delta^{k-1} y_{n,n} = n = 0, 1, 2, \ldots\ldots$

### BACKWARD DIFFERENCE OPERATOR $\nabla$ (NEPLA)

* $\nabla y_1 = y_1 - y_0$.
* $\nabla^k y_n = \nabla^{k-1} y_n - \nabla^{k-1} y_{n-1}$, $n = 1, 2, 3 \ldots$
* $\nabla f(x + 2h) = f(x + 2h) - f(x + h)$, $h$ is the interval of spacing.
* $\nabla^n f(x + nh) = \nabla^n f(x)$.

### SHIFTING OPERATOR (E)

* $E\,[f(x_0)] = f(x_0 + h)$
* $E^2 f(x) = E[f(x + h)] = f(x + 2h)$
* $E^n f(x) = f(x + nh)$ &

  $E^{-n} f(x) = f(x - nh)$.

**12. Using interpolation, find the value of $f(x)$ when $x = 15$** *[Govt.MQP-2019]*

| $x$ | 3 | 7 | 11 | 19 |
|---|---|---|---|---|
| $f(x)$ | 42 | 43 | 47 | 60 |

**Sol.**

Using interpolation, find the value of $f(x)$ when $x = 1$

Here the intervals are unequal. By Lagrange's interpolation formula, we have

$x_0 = 3,\quad x_1 = 7,\quad x_2 = 11,\quad x_3 = 19$

$y_0 = 42,\quad y_1 = 43,\quad y_2 = 47,\quad y_3 = 60$ and $x = 15$

$$\therefore y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \times y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \times y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} \times y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} \times y_3$$

$$= \frac{(15-7)(15-11)(15-19)}{(3-7)(3-11)(3-19)} \times 42 + \frac{(15-3)(15-11)(15-19)}{(7-3)(7-11)(7-19)} \times 43 + \frac{(15-3)(15-7)(15-19)}{(11-3)(11-7)(11-19)} \times 47 + \frac{(15-3)(15-7)(15-11)}{(19-3)(19-7)(19-11)} \times 60$$

$$= \frac{(8)(4)(-4)}{(-4)(-8)(-16)} \times 42 + \frac{(12)(4)(-4)}{(4)(-4)(-12)} \times 43 + \frac{(12)(8)(-4)}{(8)(4)(-8)} \times 47 + \frac{(12)(8)(4)}{(16)(12)(8)} \times 60$$

$$= \frac{42}{4} - 43 + \frac{12 \times 47}{8} + \frac{60 \times 4}{16}$$

$$= \frac{21}{2} - 43 + \frac{3 \times 47}{2} + 15$$

$= 10.5 - 43 + 70.5 + 15$

$y = 53$

Hence when $x = 15$, $f(x) = 53$.

## EXERCISE 5.3

### Choose the correct answer

**1.** $\Delta^2(y_0) =$ *[PTA-4; QY-2019]*

(a) $y_2 - 2y_1 + y_0$ (b) $y_2 + 2y_1 - y_0$
(c) $y_2 + 2y_1 + y_0$ (d) $y_2 + y_1 + 2y_0$

**[Ans: (a) $y_2 = 2y_1 + y_0$]**

**Hint:**

$\Delta^2(y_0) = \Delta y_1 - \Delta y_0$ [$\because \Delta y_1 = y_2 - y_1$ & $\Delta y_0 = y_1 - y_0$]

$= (y_2 - y_1) - (y_1 - y_0)$

$= y_2 - 2y_1 + y_0$

**2.** $\Delta f(x)$ *[PTA-3]*

(a) $f(x + h)$ (b) $f(x) - f(x + h)$
(c) $f(x + h) - f(x)$ (d) $f(x) - f(x - h)$

**[Ans: (c) $f(x + h) - f(x)$]**

**Hint:** $\Delta f(x) = f(x + h) - f(x)$

**3.** $E \equiv$ *[Govt.MQP-2019]*

(a) $1 + \Delta$ (b) $1 - \Delta$
(c) $1 + \Delta$ (d) $1 - \Delta$

**[Ans: (a) $\Delta + 1$]**

**Hint:** $\Delta = E - 1 \Rightarrow E = \Delta + 1$

**4.** **If $h = 1$, then $\Delta(x^2) =$** *[PTA-6]*

(a) $2x$ (b) $2x - 1$ (c) $2x + 1$ (d) 1

**[Ans: (c) $2x + 1$]**

**Hint:**

$\Delta f(x) = f(x + h) - f(x)$

$\Rightarrow \Delta(x^2) = (x + h)^2 - x^2$

$\Rightarrow \Delta(x^2) = (x + 1)^2 - x^2$ [$\because h = 1$]

$\Rightarrow \Delta(x^2) = x^2 + 2x + 1 - x^2 = 2x + 1.$

**5.** **If $c$ is a constant, then $\Delta c =$**

(a) c (b) $\Delta$ (c) $\Delta^2$ (d) 0

**[Ans: (d) 0]**

**6.** **If $m$ and $n$ are positive integers then $\Delta^m \Delta^n f(x) =$** *[QY-2019]*

(a) $\Delta^{m+n} f(x)$ (b) $\Delta^m f(x)$
(c) $\Delta^n f(x)$ (d) $\Delta^{m-n} f(x)$

**[Ans: (a) $\Delta^{m+n} f(x)$]**

**7.** **If $n$ is a positive integer then $\Delta^n [\Delta^{-n} f(x)]$**

(a) $f(2x)$ (b) $f(x + h)$ *[PTA-5]*
(c) $f(x)$ (d) $\Delta f(x)$

**[Ans: (c) $f(x)$]**

**Hint:** $\Delta n[\Delta{-n} f(x)] = \Delta n - n f(x) = f(x)$

## 5 Marks

**1. Calculate the value of $y$ when $x = 7.5$ from the table given below** *[PTA-6]*

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| $y$ | 1 | 8 | 27 | 64 | 125 | 216 | 343 | 512 |

**Sol.** Since the required value is at the end of the table, apply backward interpolation formula.

| x | y | Δy | $\Delta^2$y | $\Delta^3$y | $\Delta^4$y |
|---|---|---|---|---|---|
| 1 | 1 | | | | |
| | | 7 | | | |
| 2 | 8 | | 12 | | |
| | | 19 | | 6 | |
| 3 | 27 | | 18 | | 0 |
| | | 37 | | 6 | |
| 4 | 64 | | 24 | | 0 |
| | | 61 | | 6 | |
| 5 | 125 | | 30 | | |
| | | 91 | | 6 | |
| 6 | 216 | | 36 | | 0 |
| | | 127 | | 6 | |
| 7 | 343 | | 42 | | |
| | | 169 | | | |
| 8 | 512 | | | | |

$y_{(x = x_n + nh)}$

$= y_n + \frac{n}{1!}\nabla y_n + \frac{n(n+1)}{2!}\nabla^2 y_n + \frac{n(n+1)(n+2)}{3!}\nabla^3 y_n + ....$

to find $y$ at $x = 7.5$ $\therefore$ $x_n + nh = 7.5$, $x_n = 8$, $h = 1$

$\Rightarrow n = -0.5$

$$y_{(x=7.5)} = 512 + \frac{-0.5}{1!}169 + \frac{-0.5(-0.5+1)}{2!}42 + \frac{-0.5(-0.5+1)(-0.5+2)}{3!}6 + ....$$

$= 421.87$

## Govt. Exam Question & Answers

## 1 Marks

1. If $m$ and $n$ are positive integers then $\Delta^m \Delta^n f(x) =$
   (a) $\Delta^{m+n} f(x)$ (b) $\Delta^m f(x)$ *[HY-2019]*
   (c) $\Delta^n f(x)$ (d) $\Delta^{m-n} f(x)$
   [***Ans.*** (a) $\Delta^{m+n} f(x)$]

## 2 Marks

**2. Prove that $\nabla \equiv \frac{E-1}{E}$** *[Govt.MQP-2019]*

**Sol.**
$$\nabla \equiv \frac{E-1}{E}$$
$$\nabla x = f(x) - f(x-h)$$
$$= f(x) - E^{-1} f(x)$$
$$= (1 - E^{-1}) f(x)$$
$\Rightarrow$ $\nabla \equiv 1 - E^{-1}$

i.e., $\nabla \equiv 1 - \frac{1}{E}$

Hence $\nabla \equiv \frac{E-1}{E}$

**2.** Find $\Delta^2 e^x$. *[QY-2019]*

**Sol.**
$$\Delta^2 e^x = \Delta.[\Delta e^x]$$
$$= \Delta.[e^{x+h} - e^x]$$
$$= \Delta[e^x e^h - e^x]$$
$$= \Delta e^x [e^h - 1]$$
$$= (e^h - 1)\,\Delta e^x$$
$$= (e^h - 1).(e^h - 1).e^x$$
$$= (e^h - 1)^2.e^x$$

**3.** Given $U_0 = 1$ ; $U_1 = 11$, $U_2 = 21$, $U_3 = 28$ and $U_4 = 29$ Find $\Delta^2 U_0$? *[QY-2019]*

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

**5. Using Lagrange's formula find the value of $y$ when $x = 4$ from the following table.**

| $x$ | 0 | 3 | 5 | 6 | 8 |
|---|---|---|---|---|---|
| $y$ | 276 | 460 | 414 | 343 | 110 |

*Sol.*

Given $x_0 = 0,\quad x_1 = 3,\quad x_2 = 5,\quad x_3 = 6,\ x_4 = 8$

$y_0 = 276, y_1 = 460, y_2 = 414, y_3 = 343, y_4 = 110$

Lagrange's formula is

$$y = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)} y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)} y_3$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_0)(x_4-x_1)(x_4-x_2)(x_4-x_3)} y_4$$

$$\Rightarrow y = 276\frac{(1)(-1)(-2)(-4)}{(-3)(-5)(-6)(-8)} + 460\frac{(4)(-1)(-2)(-4)}{(3)(-2)(-3)(-5)} + 414\frac{(4)(1)(-2)(-4)}{(5)(2)(-1)(-3)}$$

$$+ 343\frac{(4)(1)(-1)(-4)}{(6)(3)(1)(-2)} + 110\frac{(4)(1)(-1)(-2)}{(8)(5)(3)(2)}$$

$$\Rightarrow y = -3.066 + 163.555 + 441.6 - 152.44 + 3.666$$

$$\Rightarrow y = 453.311.$$

***

Chapter 6

# RANDOM VARIABLE AND MATHEMATICAL EXPECTATION

## MUST KNOW DEFINITIONS

- A random variable is a real valued function defined on a sample space S and taking values in $(-\infty,\infty)$.
- A variable which can assume finite number of possible values or an infinite sequence of countable real numbers is **discrete random variable.**
- Probability mass function $P(x) = \begin{cases} P(X = x_i) = p_i = p(x_i) & \text{if } x = x_i,\ i = 1, 2, \ldots\ldots n \\ 0, & \text{if } x \neq x_i \end{cases}$
- Cummulative distribution function for discrete r. v. is defined by

$$F_X(x) = P(X \le x), \text{ for all } x \in \mathbb{R}.$$
$$= \sum_{x_i \le x}' P(x_i)$$

- Cummulative distribution function for continuous random variable. is

$$F_X(x) = P[X \le x] = \int_{-\infty}^{x} f(t)\, dt,\ -\infty < x < \infty$$

- $$\mu'_r = E(X^r) = \sum_x x^r\ p(x) \text{ for discrete random variable.}$$

$$= \int_{-\infty}^{x} x^r . f(x) dx \text{ for continuous random variable}$$

$$\mu_r = E[(X - \mu_x)^r].$$

**29.** **The height of persons in a country is a random variable of the type**

(a) discrete random variable

(b) continuous random variable

(c) both (a) and (b)

(d) neither (a) nor (b)

**[Ans: (b) continuous random variable]**

**30.** **The distribution function $F(x)$ is equal to**

(a) $P(X = x)$ (b) $P(X \le x)$

(c) $P(X \ge x)$ (d) all of these

**[Ans: (b) $P(X \le x)$]**

## Miscellaneous problems

**1.** **The probability function of a random variable X is given by** *[PTA-5;PTA-6]*

$$p(x) = \begin{cases} \frac{1}{4}, & \text{for } x = -2 \\ \frac{1}{4}, & \text{for } x = 0 \\ \frac{1}{2}, & \text{for } x = 10 \\ 0, & \text{elsewhere} \end{cases}$$

**Evaluate the following probabilities.**

(i) $P(X \le 0)$, (ii) $P(X < 0)$,

(iii) $P(|X| \le 2)$ and (iv) $P(0 \le X \le 10)$

*Sol.* Given probability function is

$$p(x) = \begin{cases} \frac{1}{4}, & \text{for } x = -2 \\ \frac{1}{4}, & \text{for } x = 0 \\ \frac{1}{2}, & \text{for } x = 10 \\ 0, & \text{elsewhere} \end{cases}$$

| $X = x$ | $-2$ | $0$ | $10$ |
|---|---|---|---|
| $P(X = x)$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{2}$ |

(i) $P(X \le 0) = P(X = -2) + P(X = 0)$

$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

(ii) $P(X < 0) = P(X = -2)$

$= \frac{1}{4}$

(iii) $P(|X| \le 2) = P(-2 < X < 2)$

$= P(X = -2) + P(X = 0) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

(iv) $P(0 \le X \le 10) = P(X = 0) + P(X = 10)$

$= \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$

**2.** **Let X be a random variable with cumulative distribution function**

$$F(x) = \begin{cases} 0, & \text{if } x < 0 \\ \frac{x}{8}, & \text{if } 0 \le x < 1 \\ \frac{1}{4} + \frac{x}{8}, & \text{if } 1 \le x < 2 \\ \frac{3}{4} + \frac{x}{12}, & \text{if } 2 \le x < 3 \\ 1, & \text{for } 3 \le x \end{cases}$$

**(a) Compute (i) $P(1 \le X \le 2)$ and**

**(ii) $P(X = 3)$.**

**(b) Is X a discrete random variable? Justify your answer.**

*Sol.* Given probability distribution function is

$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{x}{8} & \text{if } 0 \le x < 1 \\ \frac{1}{4} + \frac{x}{8} & \text{if } 1 \le x < 2 \\ \frac{3}{4} + \frac{x}{12} & \text{if } 2 \le x < 3 \\ 1 & \text{if } 3 \le x \end{cases}$$

we know $F'(x) = f(x)$

Since $E(X^2) = E(X)$ we get,

$$4 - 7p = 2 - 3p \Rightarrow 4 - 2 = -3p + 7p$$

$$\Rightarrow 2 = 4p \Rightarrow p = \frac{2}{4} = \frac{1}{2}$$

$$\therefore p = \frac{1}{2}$$

**5. The probability distribution of a random variable X is**

| X | 1 | 2 | 4 | 2A | 3A | 5A |
|---|---|---|---|---|---|---|
| P(X) | $\frac{1}{2}$ | $\frac{1}{5}$ | $\frac{3}{25}$ | $\frac{1}{10}$ | $\frac{1}{25}$ | $\frac{1}{25}$ |

**Calculate (i) A if E(X) = 2.94 (ii) V(X)**

*Sol.* $E(X) = \sum x_i^2 p_i$

$$\Rightarrow E(X) = 1(\frac{1}{2}) + 2(\frac{1}{5}) + 4(\frac{3}{25}) + 2A(\frac{1}{10}) + 3A(\frac{1}{25}) + 5A(\frac{1}{25})$$

$$\Rightarrow \frac{1}{2} + \frac{2}{5} + \frac{12}{25} + \frac{A}{5} + \frac{3A}{25} + \frac{A}{5}$$

$$= \frac{69}{50} + \frac{13A}{25}$$

Since $E(X) = 2.94$

$$\frac{69}{50} + \frac{13A}{25} = 2.94 \Rightarrow \frac{13A}{25} = 2.94 - \frac{69}{50}$$

$$= 2.94 - 1.38 = 1.56$$

$$\Rightarrow A = \frac{1.56 \times 25}{13} = \frac{39}{13} = 3.$$

$$E(X^2) = 1(\frac{1}{2}) + 4(\frac{1}{5}) + 16(\frac{3}{25}) + (2A)^2 (\frac{1}{10}) + (3A)^2(\frac{1}{25}) + (5A)^2 (\frac{1}{25}) \quad [\because A = 3]$$

$$= \frac{1}{2} + \frac{4}{5} + \frac{48}{25} + 36(\frac{1}{10}) + 81(\frac{1}{25}) + 225 (\frac{1}{25})$$

$$E(X^2) = \frac{25+40+96+180+162+450}{50}$$

$$= \frac{953}{50} = 19.06$$

$$V(X) = E(X^2) - [E(X)]^2$$

$$= 19.06 - (2.94)^2 \quad [\because E(X) = 2.94]$$

$$= 19.06 - 8.6436$$

$$V(X) = 10.4164$$

***

Chapter 7

# PROBABILITY DISTRIBUTIONS

## MUST KNOW DEFINITIONS

- A random variable X is said to follow Binomial distribution if its probability mass function is given by

$$P(X = x) = P(x) = \begin{cases} {}^nC_x p^x q^{n-x}, x = 0,1,2,.....n;\ q = 1-p \\ 0, \text{ otherwise} \end{cases}$$

- A random variable X is said to follow Poission distribution if its probability mass function is given by

$$P(X = x) = \begin{cases} \dfrac{e^{-\lambda}.\lambda^x}{x!}, x = 0, 1, 2, ......\ \lambda > 0 \\ 0, \text{ otherwise} \end{cases}$$

- A random variable X is said to follow normal distribution if its probability density function is given by

$$f(x\,;\mu\,,5) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\} \quad \begin{matrix} -\infty < x < \infty, \\ -\infty < \mu < \infty, \\ \sigma > 0 \end{matrix}$$

## FORMULAE TO REMEMBER

**Properties of Binomial Distribution :**

- Mean = $np$ and variance = $npq$
- For binomial distribution, variance < mean
- Binomial distribution is symmetrical if $p = q = 0.5$.
- It is skew symmetric if $p \neq q$.
- It is positively skewed if $p < 0.5$.
- It is negatively skewed if $p > 0.5$.

### Properties of Normal Distribution :

- The curve is bell shaped and symmetrical about the line $x = \mu$.
- Mean, median and mode of the distribution coincide.
- X - axis is an asymptote to the curve.
- No portion of the curve lies below the X - axis.
- The points of inflexion of the curve are $x = \mu \pm \sigma$.
- The curve is unimodal.
- The max probability occurs at $x = \mu$ and it is $\frac{1}{\sigma\sqrt{2\pi}}$.
- $p(\mu-\sigma < X < \mu+\sigma) = 0.6826$
- $p(\mu-2\sigma < X < \mu+2\sigma) = 0.9544$
- $p(\mu-3\sigma < X < \mu+3\sigma) = 0.9973$

### Properties of Standard Normal Distribution :

- Its probability density function is $\varphi(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}, -\infty < z < \infty$
- The area under the standard normal curve is 1.
- 68.26% of area lies between $z = -1$ to $z = 1$.
- 95.44% of area lies between $z = -2$ and $z = 2$.
- 99.74% of area lies between $z = -3$ and $z = 3$.

## TEXTUAL QUESTIONS

### EXERCISE 7.1

**1. Define Binomial distribution.**

**Sol:** A random variable X is said to follow binomial distribution with parameter $n$ and $p$, if it assumes only non-negative value and its probability mass function is given by

$$P(X = x) = p(x) = \begin{cases} {}^nC_x p^x q^{n-x}, & x = 0,1,2,.....n; \\ 0, & \text{otherwise} \end{cases}$$
$q = 1 - p$

**2. Define Bernoulli trials.**

**Sol:** A random experiment whose outcomes are of two types namely success S and failure F, occurring with probabilities $p$ and $q$ is called a Bernoulli trial.

**3. Derive the mean and variance of binomial distribution.**

**Sol:** The mean of the binomial distribution

$$E(X) = \sum_{x=0}^{n} x.p(x) = \sum_{x=0}^{n} x.\binom{n}{x} p^x q^{n-x}$$

$$= p.\sum_{x=0}^{n} x.\binom{n}{x} p^{x-1} q^{n-x}$$ [Take $p$ common]

$$= p.\sum_{x=1}^{n} \cancel{x}.\left(\frac{n}{\cancel{x}}\right).\binom{n-1}{x-1} p^{x-1} q^{n-x}$$

$$= np.\sum_{x=1}^{n} \binom{n-1}{x-1} p^{x-1} q^{n-x}$$

$$= np(q+p)^{n-1}$$ [using binomial theorem]

$(x+a)^n = x^n + {}^nC_1 x^{n-1} a^1 + .... + a^n$

$$= np(1)^{n-1}$$ [$\because p + q = 1$]

$$= np$$

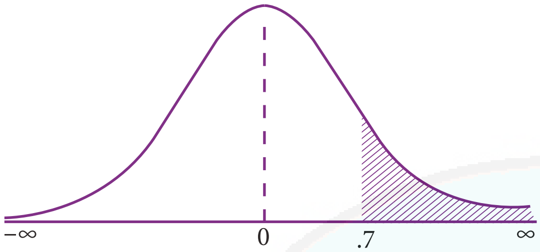
$\therefore$ Mean $= E(X) = np$ ... (1)

$\therefore$ P(X ≥ 470) = P(Z ≥ 0.7) = P(0.7 < Z < ∞)

= P(0 < Z < ∞) – P(0 < Z < 0.7)

= 0.5 – 0.2580

P(X ≥ 470) = 0.2420.




**(ii) P (atmost 500 days) = P(X ≤ 500)**

When X = 500, $Z = \frac{X-\mu}{\sigma}$

$= \frac{500-400}{100} = \frac{100}{100} = 1$

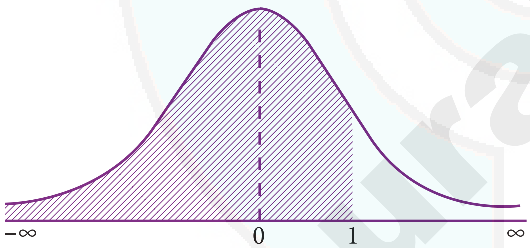
$\therefore$ P(X ≤ 500) = P(Z ≤ 1)

= P(–∞ < Z < 1)

= P(–∞ < Z < 0) + P(0 < Z < 1)

= 0.5 + 0.3413 = 0.8413

P(X ≤ 500) = 0.8413.




## EXERCISE 7.4

### Choose the correct answer :

**1. Normal distribution was invented by** *[HY-2019]*

(a) Laplace (b) De-Moivre

(c) Gauss (d) all the above

**[Ans: (b) De-Moivre]**

**2. If X ~ N(9, 81) the standard normal variate Z will be** *[PTA-3]*

(a) $Z = \frac{X-81}{9}$ (b) $Z = \frac{X-9}{81}$

(c) $Z = \frac{X-9}{9}$ (d) $Z = \frac{9-X}{9}$

**[Ans: (c) $Z = \frac{X-9}{9}$]**

Hint: $\mu = 9, \sigma^2 = 81 \Rightarrow \sigma = 9$

$\therefore Z = \frac{X-\mu}{\sigma} = \frac{X-9}{9}$

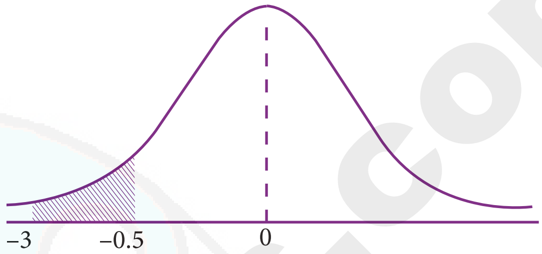
**3. If Z is a standard normal variate, the proportion of items lying between Z = –0.5 and Z = –3.0 is**

(a) 0.4987 (b) 0.1915

(c) 0.3072 (d) 0.3098

**[Ans: (c) 0.3072]**

Hint: P(–0.5 < Z < –3)




= P(–3 < Z < 0) – P(–0.5 < Z < 0)

= P(0 < Z < 3) – P(0 < Z < 0.5)

= 0.4987 – 0.1915 = 0.3072

**4. If X ~ N(μ, σ²), the maximum probability at the point of inflexion of normal distribution is**

(a) $\left(\frac{1}{\sqrt{2\pi}}\right)e^{\frac{1}{2}}$ (b) $\left(\frac{1}{\sqrt{2\pi}}\right)e^{\left(-\frac{1}{2}\right)}$

(c) $\left(\frac{1}{\sigma\sqrt{2\pi}}\right)e^{\left(-\frac{1}{2}\right)}$ (d) $\left(\frac{1}{\sqrt{2\pi}}\right)$

**[Ans: (c) $\left(\frac{1}{\sigma\sqrt{2\pi}}\right)e^{\left(-\frac{1}{2}\right)}$]**

**5. In a parametric distribution the mean is equal to variance is :**

(a) binomial (b) normal

(c) poisson (d) all the above

**[Ans: (c) poisson]**

**6. In turning out certain toys in a manufacturing company, the average number of defectives is 1%. The probability that the sample of 100 toys there will be 3 defectives is**

(a) 0.0613 (b) 0.613

(c) 0.00613 (d) 0.3913

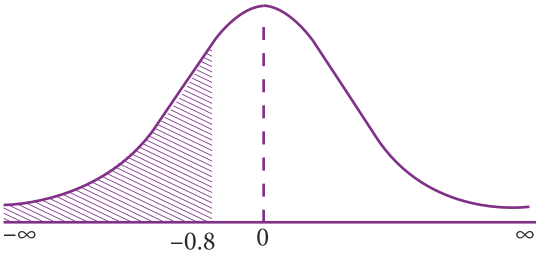
**[Ans: (a) 0.0613]**

Hint: $\phi = 1\% = \frac{1}{100}, n = 100$

$\Rightarrow \lambda = np = \frac{1}{100} \times 100 = 1$

$P(X=3) = \frac{e^{-\lambda}\cdot\lambda^x}{x!} = \frac{e^{-1}\cdot 1^3}{3!}$

$= \frac{0.3679}{6} = 0.0613$




$= P(-\infty < Z < 0)$

$- P(-0.8 < Z < 0)$

$= 0.5 - P(0 < Z < 0.8)$ [By symmetry]

$= 0.5 - 0.2881 = 0.2119$

Hence, the probability that a baby is born with weight less than 3100g is 0.2119.

**10. People's monthly electric bills in chennai are normally distributed with a mean of ₹225 and a standard deviation of ₹55. Those people spend a lot of time online. In a group of 500 customers, how many would we expect to have a bill that is ₹100 or less?.**

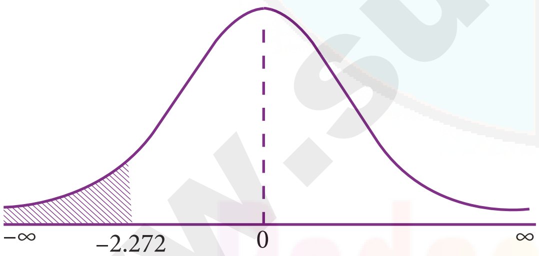
**Sol:** Given $\mu = 225, \sigma = 55$

$P(X \leq 100)$

When $X = 100, Z = \dfrac{X-\mu}{\sigma}$

$= \dfrac{100-225}{55} = -2.272$

$\therefore P(X \leq 100) = P(Z \leq -2.272)$




$P(-\infty < Z < -2.272) = P(-\infty < Z < 0)$

$- P(-2.272 < Z < 0)$

$= 0.5 - P(0 < Z < 2.272)$ [By symmetry]

$= 0.5 - 0.4884 = 0.0116$

$\therefore$ Probability of one customer to have a bill for ₹ 100 or less is 0.0116.

$\therefore$ Out of 500 customers, the number of persons to have a bill for ₹ 100 or less is

$0.0116 \times 500 = 5.8 \cong 6.$

## PTA Question & Answers

### 1 Marks

1. **For Binomial Distribution** *[PTA-1]*

(a) Mean = Median (b) Mean > Variance

(c) Mean < Variance (d) Mean = S.D.

**[Ans: (b)** Mean > Variance **]**

### 2 Marks

1. **The mean of Binomials distribution is 20 and standard deviation is 4. Find the parameters of the distribution.** *[PTA-3]*

**Sol:** The parameters of Binomial distribution are n and p

For Binomial distribution Mean = $np$ =20

Standard deviation = $\sqrt{npq} = 4$

$\therefore npq = 6 \Rightarrow npq/np = \dfrac{16}{20} = \dfrac{4}{5}$

$q = \dfrac{4}{5} \Rightarrow p = 1-q = 1-(\dfrac{4}{5}) = \dfrac{1}{5}$

since $np = 20$

$n = \dfrac{20}{p}$

$n = 100$

### 3 Marks

1. **The average daily procurement of milk by village society in 800 litres with a standard deviation of 100 litres. Find out proportion of societies procuring milk between 800 litres to 1000 litres per day.** *[PTA-1]*

**Sol:** We are given mean $\mu = 800$ and standard deviation $\sigma = 100$ . Probability that the procurement of milk between 800 litres to 1000 litres per day is

$P(800 < X < 1000)$

$P(\dfrac{800-800}{100} < z < \dfrac{1000-800}{100}$

$P(0 < Z < 2) = 0.4772$ (table value)

Therefore 47.75 percent of societies procure milk between 800 litres to 1000 litres per day.

**Sol:** Let $p$ be the probability of happening of an event.

$$\text{Given } p = 0.002 = \frac{2}{1000}$$

$$\text{Also } n = 1000$$

$$\therefore \text{Mean} = np = 1000 \times \frac{2}{1000} = 2$$

$$\therefore \lambda = 2$$

Hence, X follows Poisson distribution with

$$P(x, \lambda) = \frac{e^{-\lambda}.\lambda^0}{x!}$$

$\therefore$ P(event happens exactly twice)

$$= P(X = 2) = \frac{e^{-\lambda}.\lambda^2}{2!} = \frac{e^{-2}.(2^2)}{2}$$

$$= e^{-2}(2) = 2(0.1353) = 0.2706$$

$$\therefore P(X = 2) = 0.2706$$

## Practice 3 mark questions

**1. The probability that an event A happens in one treat of an experiment is 0.4. Three independent treats of the experiment are performed. Find the probability that the event A happens at least once.**

**Sol:** Given

$$p = 0.4, n = 3$$

$$q = 1 - p = 1 - 0.4 = 0.6$$

$$P(X = x) = nC_x\, p^x q^{n-x}$$

$$P(X \geq 1) = P(X = 1) + P(X = 2) + P(X = 3)$$

$$= 3C_1 (0.4)^1 (0.6)^2 + 3C_2 (0.4)^2 (0.6) + 3C_3 (0.4)^3 (0.6)^0$$

$$= 3 \times \left(\frac{4}{10}\right)\left(\frac{36}{100}\right) + 3\left(\frac{16}{100}\right)\left(\frac{6}{100}\right) + \frac{64}{1000}$$

$$= \frac{1}{1000}(432 + 288 + 64) = \frac{784}{1000}$$

$$P(X \geq 1) = 0.784$$

**2. The standard deviation of a binomial distribution $(q + p)^{16}$ is 2. Find its mean.**

**Sol:** Given $n = 16$, S.D $= 2 \Rightarrow \sqrt{npq} = 2$

$$\Rightarrow npq = 4$$

$$\therefore 16(pq) = 4 \Rightarrow pq = \frac{4}{16} = \frac{1}{4}$$

$$\Rightarrow q = \frac{1}{4p} \quad ...(1)$$

$$\text{Since } p + q = 1 \Rightarrow p + \frac{1}{4p} = 1$$

$$\Rightarrow \frac{4p^2 + 1}{4p} = 1$$

$$\Rightarrow 4p^2 + 1 = 4p \Rightarrow 4p^2 - 4p + 1 = 0$$

$$\Rightarrow (2p - 1)^2 = 0 \Rightarrow 2p - 1 = 0$$

$$\Rightarrow 2p = 1 \Rightarrow p = \frac{1}{2}$$

$$\therefore q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\text{Mean} = np = 16 \times \frac{1}{2} = 8$$

**3. If a random variable X follows Poisson distribution such that P(X = 2) = 9. P(X = 4) + 90 P(X = 6) then find the mean and variance.**

**Sol:** Given P(X = 2) = 9 P(X = 4) + 90 P(X = 6)

Since X follows Poisson distribution with

$$P(X, \lambda) = \frac{e^{-\lambda}.\lambda^0}{x!}$$

$$\frac{e^{-\lambda}.\lambda^2}{2!} = \frac{e^{-\lambda}.\lambda^4}{4!} + \frac{90e^{-\lambda}.\lambda^6}{6!}$$

$$\frac{\cancel{e^{-\lambda}}.\lambda^2}{2} = \cancel{e^{-\lambda}}\left(\frac{9\lambda^4}{24} + \frac{90\lambda^6}{720}\right)$$

$$\Rightarrow \frac{\cancel{\lambda^2}}{2} = \lambda^2\left(\frac{3\lambda^2}{8} + \frac{\lambda^4}{8}\right)$$

$$\Rightarrow \frac{1}{2} = \frac{3\lambda^2 + \lambda^4}{8}$$

$$\Rightarrow \lambda^4 + 3\lambda^2 = 4$$

$$\Rightarrow \lambda^4 + 3\lambda^2 - 4 = 0$$

$$\text{Put } \lambda^2 = t$$

$$\Rightarrow t^2 + 3t - 4 = 0 \Rightarrow (t - 1)(t + 4) = 0$$

$$\Rightarrow t = 1 \text{ or } t = -4$$

$$\therefore t = 1 \; [t = -4 \text{ is not possible}]$$

$$\therefore \lambda^2 = 1 \Rightarrow \lambda = 1$$

$$\therefore \text{Mean} = 1$$

For Poisson distribution, mean = variance = 1

**4. Find the value of K if X is a normal variate whose $p.d.f$ is given by $f(x) = \frac{1}{K}e^{8x-4x^2}$, $-\infty < X < \infty$**

**Sol:** The p.d.f for the normal distribution is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, \quad -\infty < x < \infty \quad ...(1)$$

$$\text{Given } f(x) = \frac{1}{K}e^{8x-4x^2}$$

Chapter

8

# SAMPLING TECHNIQUES AND STATISTICAL INFERENCE

## MUST KNOW DEFINITIONS

| | |
|---|---|
| **Sampling** | ➢ **Sampling** is the process of selecting a sample from a population.<br>➢ **Types of sampling :**<br>**1.** Non-Random sampling or Non-probability sampling.<br>**2.** Random Sampling or Probability sampling. |
| **Stratified Random Sampling** | ➢ First divide the population into sub-populations, which are called strata. The collection of all the samples from all strata gives the stratified random samples. |
| **Systematic Sampling** | ➢ Randomly select the first sample from the first $k$ units. Then every $k^{th}$ member, starting with the first selected sample, is included in the sample. |
| **Sampling distribution** | ➢ It is a frequency distribution which is formed with various values of a statistic computed from different samples of the same size drawn from the same population. |
| **Estimation** | ➢ The method of obtaining the most likely value of the population parameter using statistic is called estimation. |
| **Estimator** | ➢ Sample statistic used to estimate a population parameter. |
| **Estimate** | ➢ An estimate is a specific observed value of a statistic. |
| **Point Estimation** | ➢ When a single value is used as an estimate. |
| **Interval Estimation** | ➢ Finding limits within which the parameter would be expected to lie. |
| **Null Hypothesis** | ➢ A hypothesis which is tested for possible rejection under the assumption that it is true. |
| **Alternative Hypothesis** | ➢ Any hypothesis which is complementary to the null hypothesis. |
| **Critical value** | ➢ The value of test statistic which separates the critical region and the acceptance region.<br>**Type I Error :** The error of rejecting $H_0$ when it is true.<br>**Type II Error :** The error of accepting $H_0$ when it is false. |

## FORMULAE TO REMEMBER

**Merits and Demerits of simple random sampling :**

**Merits** ➢ **1.** Personal bias is completely eliminated.
**2.** It is economical as it saves time, money and labour.
**3.** It requires minimum knowledge about the population in advance.

**20. In a sample of 400 population from a village 230 are found to be eaters of vegetarian items and the rest non-vegetarian items. Compute the standard error assuming that both vegetarian and non-vegetarian foods are equally popular in that village?**

*Solution:* Sample size $n = 400$

Probability of vegetarian $= \frac{230}{400} = 0.575$

$\therefore$ P = 0.575

Probability of non-vegetarian $q = 1 - p = 1 - 0.575$

$\Rightarrow q = 0.425$

Standard error for sample proportion $= \sqrt{\frac{pq}{n}}$

$= \sqrt{\frac{(0.575)(0.425)}{400}} = \sqrt{0.000610} = 0.0246$

S.E = 0.025 (app)

## EXERCISE 8.2

**1. Mention two branches of statistical inference?**

*Ans.* The two branches of statistical inference are Estimation and testing of hypotheses.

**2. What is an estimator?**

*Ans.* An estimator is a sample statistic used to estimate a population parameter.

**3. What is an estimate?**

*Ans.* An estimate is a specific observed value of a statistic.

**4. What is point estimation?** *[Govt.MQP-2019]*

*Ans.* An estimate of a population parameter given by a single number is called as point estimation.

**5. What is interval estimation?**

*Ans.* Generally, there are situations where point estimation is not desirable and we are interested in finding limits within which the parameter would be expected to lie is called an interval estimation.

**6. What is confidence interval?**

*Ans.* The interval $[c_1, c_2]$ within which the unknown value of the population parameter is expected to lie is known as Confidence Interval.

**7. What is null hypothesis? Give an example.**

*Ans.* Null hypothesis is the hypothesis which is tested for possible rejection under the assumption that it is true and it is denoted by $H_0$.

**Example :** If we want to find the population mean has a specific value $\mu_0$, then the null hypothesis $H_0$ is $H_0 : \mu = \mu_0$

**8. Define alternative hypothesis.** *[PTA-2]*

*Ans.* Any hypothesis which is complementary to the null hypothesis is called as the alternative hypothesis and it is usually denoted by $H_1$.

**9. Define critical region.**

*Ans.* A region corresponding to a test statistic in the sample space which tends to rejection of $H_0$ is called critical region.

**10. Define critical value.**

*Ans.* The value of test statistic which separates the critical region and the acceptance region is called the critical value.

**11. Define level of significance.**

*Ans.* The probability of type I error is known as level of significance.

**12. What is type I error?**

*Ans.* The error of rejecting $H_0$ when it is true is type I error.

**13. What is single tailed test?**

*Ans.* When the hypothesis about the population parameter is rejected only for the value of sample statistic falling into one of the tails of the sampling distribution, then it is known as one-tailed test.

**14. A sample of 100 items, draw from a universe with mean value 4 and S.D 3, has a mean value 63.5. Is the difference in the mean significant?**

*Solution:* Sample size $n = 100$, *[PTA-3]*

Sample mean $\overline{X} = 3.5$

Population mean $\mu = 4$

Population standard deviation $\sigma = 3$

**Null Hypotheses :** There is no significant difference in the mean. i.e., $H_0 : \mu = 4$

**Alternative Hypotheses :** There is significant difference in the mean.

i.e., $H_1 : \mu \neq H$

The level of significance $\alpha = 5\% = 0.05$

Applying the test statistic,

$$Z = \frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

## Govt. Exam Question & Answers

### 3 Marks

**1. A random sample of 500 apples was taken from large consignment and 45 of them were found to be bad. Find the limits at which the bad apples lie at 99% confidence level.** *[HY-2019]*

**Solution:** Given $n = 500$, $p = \frac{45}{500} = 0.9$,

$q = 1 - 0.09 = 0.91$

99% Confidence limits for the bad apples:

$$p \pm Z_{\frac{\alpha}{2}}\sqrt{\frac{pq}{n}} \Rightarrow 0.09 \pm 2.58\sqrt{\frac{(0.09)(0.91)}{500}}$$

$\Rightarrow (0.057, 0.123)$

∴ The bad apples in the consignment lie between 5.7% and 12.3%

### 5 Marks

**2. A sample of 100 measurements at breaking strength of cotton thread gave a mean of 7.4 gram and a standard deviation of 1.2 gram. Find 95% confidence limits for the mean breaking strength of cotton thread.** *[Govt.MQP-2019]*

**Sol.** Given, sample size = 100, $\bar{x} = 7.4$, since σ is unknown but s = 1.2 is known.

In this problem, we consider $\sigma = s$, $Z_{\alpha/2} = 1.96$

$$\text{S.E.} = \frac{\sigma}{\sqrt{n}} = \frac{s}{\sqrt{n}} = \frac{1.2}{\sqrt{100}} = 0.12$$

Hence 95% confidence limits for the population mean are

$$\bar{x} - Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

$7.4 - (1.96 \times 0.12) \leq \mu \leq 7.4 + (1.96 \times 0.12)$

$7.4 - 0.2352 \leq \mu \leq 7.4 + 0.2352$

$7.165 \leq \mu \leq 7.635$

This implies that the probability that the true value of the population mean breaking strength of the cotton threads will fall in this interval (7.165,7.635) at 95%

**3. An auto company decided to introduce a new six cylinder car whose mean petrol consumption is claimed to be lower than that of the existing auto engine. It was found that the mean petrol consumption for the 50 cars was 10 km per litre with a standard deviation of 3.5 km per litre. Test at 5% level of significance, whether the claim of the new car petrol consumption is 9.5 km per litre on the average is acceptable.** *[HY-2019]*

**Sol.** Sample size $n = 50$ Sample mean $x = 10$ km
Sample standard deviation $s = 3.5$ km

Population mean $\mu = 9.5$ km

Since population SD is unknown we consider $\sigma = s$

The sample is a large sample and so we apply Z-test

Null Hypothesis: There is no significant difference between the sample average and the company's claim, i.e., $H_0 : \mu = 9.5$

Alternative Hypothesis: There is significant difference between the sample average and the company's claim, i.e., $H_1 : \mu \neq 9.5$ (two tailed test)

The level of significance $\alpha = 5\% = 0.05$

Applying the test statistic

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1); Z = \frac{10 - 9.5}{\frac{3.5}{\sqrt{50}}} \sim N(0,1)$$

$$= \frac{0.5}{0.495} = 1.01$$

Thus the calculated value 1.01 and the significant value or table value $Z_{\frac{\alpha}{2}} = 1.96$

Comparing the calculated and table value ,Here

$Z < Z_{\frac{\alpha}{2}}$ i.e., $1.01 < 1.96$.

Inference: Since the calculated value is less than table value i.e., $Z < Z_{\frac{\alpha}{2}}$ at 5% level of significance, the null hypothesis $H_0$ is accepted. Hence we conclude that the company's claim that the new car petrol consumption is 9.5 km per litre is acceptable.

As the significance level is α = 0.005, $Z_{\frac{\alpha}{2}} = 1.96$

∴ 95% confidence limits for the population mean is $\bar{x} - z_{\frac{\alpha}{2}}(\text{S.E}) \leq \mu \leq \bar{x} + z_{\frac{\alpha}{2}}(\text{S.E})$

$\Rightarrow 75 - (1.96)(1.414) \leq \mu \leq 75 + (1.96)(1.414)$

$\Rightarrow 75 - 2.771 \leq \mu \leq 75 + 2.771$

$\Rightarrow 72.23 \leq \mu \leq 77.77$

Hence, the 95% confidence interval of the population mean is (72.23, 77.77)

**4. A company market car tyres. Their lives are normally distributed with a mean of 50,000 kms and standard derivation of 2000 kms. A test sample of 64 tyres has a mean life of 51250 km. Can you conclude that the sample mean differs significantly from the population mean? (Test at 5% level)**

***Sol.*** Given sample size $n$ = 64

Sample mean $\bar{x}$ = 51250

Null hypotheses : $H_0$ : Population mean μ = 50000

Alternative hypotheses: $H_1$ : μ ≠ 50,000

The test statistic, $z = \dfrac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \dfrac{51250 - 50000}{\frac{2000}{\sqrt{64}}} = 5$

∴ $z = 5$

As the significance level is α = 0.05, $Z_{\frac{\alpha}{2}} = 1.96$

Here $z > z_\alpha$ as 5 < 1.96

Inference : As $z > z_\alpha$, $H_0$ is rejected. Hence, we can conclude that the sample mean differs significantly from the population mean.

## PRACTICE 5 MARK QUESTIONS

**1. Measurements of the weights of a random sample of 200 ball bearings made by certain machine during one week showed a mean of 0.824 newtons and a S.D. of 0.042 newton's. Find a) 95% and b) 99% confidence limits for the mean weight of all the ball bearings.**

***Sol.*** Given sample size $n$ = 200

Sample mean $\bar{x}$ = 0.824

Sample S.D. $s$ = 0.042

Standard error = $\dfrac{s}{\sqrt{n}} = \dfrac{0.042}{\sqrt{200}}$

$= \dfrac{0.042}{14.14} = 0.00270$

(a) As the level of significance is α = 0.05, $Z_{\frac{\alpha}{2}} = 1.96$

∴ 95% confidence limits for μ are given by $\bar{x} - z_{\frac{\alpha}{2}}(\text{S.E}) \leq \mu \leq \bar{x} + z_{\frac{\alpha}{2}}(\text{S.E})$

$\Rightarrow 0.824 - (1.96)(0.00270) \leq \mu \leq 0.824 + (1.96)(0.00270)$

$\Rightarrow 0.824 - 0.00582 \leq \mu \leq 0.824 + 0.00582$

$\Rightarrow 0.818 \leq \mu \leq 0.832$

Hence, the 95% confidence limits for μ is (0.818, 0.832)

(b) As the level of significance is α = 0.001, $Z_{\frac{\alpha}{2}} = 2.58$

∴ 99% confidence limits for μ are given by $\bar{x} - z_{\frac{\alpha}{2}}(\text{S.E}) \leq \mu \leq \bar{x} + z_{\frac{\alpha}{2}}(\text{S.E})$

$\Rightarrow 0.824 - (2.58)(0.00270) \leq \mu \leq 0.824 + (2.58)(0.00270)$

$\Rightarrow 0.824 - 0.00582 \leq \mu \leq 0.824 + 0.005825$

$\Rightarrow 0.816 \leq \mu \leq 0.832$

Hence, the 99% confidence limits for μ is (0.816, 0.832)

Chapter 9

# APPLIED STATISTICS

## MUST KNOW DEFINITION

* When quantitative data are arranged in the order of their occurrence, the resulting series is called the time series.
* Components of time series are

  (a) Secular trend (b) Seasonal variations (c) Cyclic variations (d) irregular variations
* Measurements of trends are

  (a) Free hand or graphic method (b) Method of semi averages

  (c) Method of moving averages (d) Method of least squares
* Index numbers are statistical methods used to measure the relative change in the level of a variable w.r.t time, geographical location, income, profession etc.
* Cost of living index number is constructed to study the effect of changes in the price of goods for a current period as compared with base period.

  SQC (Statistical Quality Control) is a statistical technique which would help in elimination of assignable causes and bring the production process under control.

## IMPORTANT FORMULAE TO REMEMBER

* Mathematical model for a Time-series

  (a) Additive Model : Y = T + S + C + I

  Y $\rightarrow$ Original value
  T $\rightarrow$ Trend value, C $\rightarrow$ Cyclic component
  S $\rightarrow$ Seasonal component
  I $\rightarrow$ Irregular component

  (b) Multiplicative model : Y = T × S × C × I

  Weighted Index number

  (i) Price index $(P_{01}) = \dfrac{\sum p_1 w}{\sum p_0 w} \times 100$

  $p_1 \rightarrow$ current year price
  $p_0 \rightarrow$ base year price
  $q_0 \rightarrow$ base year quantity
  $q_1 \rightarrow$ current year quantity

## EXERCISE 9.1

**1. Define Time series.**

**Sol.** When quantitative data are arranged in the order of their occurrence, the resulting series is called the time series.

**2. What is the need for studying time series?**

**Sol.** Time series has an important objective to identify the variations and try to eliminate the variations and also helps us to estimate or predict the future values.

**3. State the uses of time series.**

**Sol.** (i) It helps in the analysis of the past behavior

(ii) It helps in forecasting and for future plans

(iii) It helps in the evaluation of current achievements

(iv) It helps in making comparative studies between one time period and others.

**4. Mention the components of the time series.**

**Sol.** The components of time series are

(i) Secular trend (ii) Seasonal variations

(iii) Cyclic variations (iv) Irregular variations

**5. Define secular trend.**

**Sol.** The tendency of time series to increase or decrease or stagnates during a long period of time is secular trend.

**6. Write a brief note on seasonal variations.**

**Sol.** Tendency movements are due to nature, which repeat themselves periodically in every seasons. These variations repeat themselves in less than one year time. It is measured in an interval of time.

Seasonal variations may be influenced by natural force, social customs and traditions.

**7. Explain cyclic variations.**

**Sol.** Cyclic variations are not necessarily uniformly periodic in nature. This may or may not follow exactly similar patterns after equal intervals of time.

Generally, one cyclic period ranges from 7 to 9 years and there is no hard and fast rule in the fixation of years for a cyclic period.

**8. Discuss about irregular variation.**

**Sol.** Irregular variations do not have particular pattern and there is no regular period of time of their occurrences. Normally they are short terms variations but its occurrence sometimes has its effect so intense that they may give rise to new cyclic or other movements of variations. For example floods, wars, earthquakes, Tsunami, strikes, lockouts etc.

**9. Define seasonal index.**

**Sol.** Seasonal index is a measure of how a particular season compares with the average season.

**10. Explain the method of fitting a straight line.**

**Sol.** The line of least fit is a line from which the sum of the deviations of various points is zero. This is the best method for obtaining the trend values.

(i) The straight line trend is represented by

$$Y = a + b\,X \quad \text{....(1)}$$

where Y is the actual value and X is time

(ii) The constants '$a$' and '$b$' are estimated by solving the following two normal Equations

$\Sigma Y = n\,a + b\Sigma X$

$\Sigma XY = a\,\Sigma X + b\,\Sigma X^2$ where $n$ is the number of years given in the data.

(iii) By substituting the values of '$a$' and '$b$' in the trend equation (1), we get the line of best fit.

**11. State the two normal equations used in fitting a straight line.** *[PTA-1]*

**Sol.** The two normal equations are

$\Sigma Y = n\,a + b\Sigma X$

$\Sigma XY = a\,\Sigma X + b\,\Sigma X^2$ where $n$ is the number of years given in the data.

**12. State the different methods of measuring trend.**

**Sol.** The different methods of measurements of trends are

(i) Free hand or graphic method

(ii) Method of semi averages

(iii) Method of moving averages

(iv) Method of least squares

## 5 Marks

**1. Construct the Laspeyre's Paasche's and Fisher's price index number for the following data and also comment on the result.** *[Govt.MQP-2019]*

| Commodities | Basic Year | | Current Year | |
|---|---|---|---|---|
| | Price | Quantity | Price | Quantity |
| Rice | 15 | 5 | 16 | 8 |
| Wheat | 10 | 6 | 18 | 9 |
| Rent | 8 | 7 | 15 | 8 |
| Fuel | 9 | 5 | 12 | 6 |
| Miscellaneous | 16 | 6 | 15 | 10 |

| Commodities | Basic Year | | Current Year | |
|---|---|---|---|---|
| | Price | Quantity | Price | Quantity |
| Rice | 15 | 5 | 16 | 8 |
| Wheat | 10 | 6 | 18 | 9 |
| Rent | 8 | 7 | 15 | 8 |
| Fuel | 9 | 5 | 12 | 6 |
| Misc. | 16 | 6 | 15 | 10 |
| | | | | Total |

| $p_0q_0$ | $p_0q_1$ | $p_1q_0$ | $p_1q_1$ |
|---|---|---|---|
| 75 | 120 | 80 | 128 |
| 60 | 90 | 108 | 162 |
| 56 | 64 | 105 | 120 |

Laspeyre's price index number

$$P_{01}^{L} = \frac{\Sigma p_1q_0}{\Sigma p_0q_0}\times 100 = \frac{397}{280}\times 100 = 141.7857$$

Paasche's price index number

$$P_{01}^{P} = \frac{\Sigma p_1q_1}{\Sigma p_0q_1}\times 100 = \frac{559}{405}\times 100 = 138.0246$$

Fisher's price index number

$$P_{01}^{F} = \left(\sqrt{\frac{\Sigma p_1q_0\times\Sigma p_1q_1}{\Sigma p_0q_0\times\Sigma p_0q_1}}\right)\times 100 = \left(\sqrt{\frac{397\times 559}{280\times 405}}\right)\times 100 = 139.8925$$

On an average, there is an increase of 41.78%, 38.02% and 39.89% in the price of the commodities by Laspeyre's, Paasche's, Fisher's price index number respectively, when the base year compared with the current year.

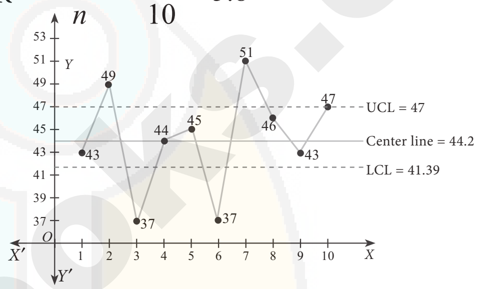
**2. Given the values of sample mean ($\overline{X}$) and the range (R) for ten samples of size 5 each. Draw mean chart and comment on the state of control of the process.** *[HY-2019]*

| Sample number | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $\overline{X}$ | 43 | 49 | 37 | 44 | 45 | 37 | 51 | 46 | 43 | 47 |
| R | 5 | 6 | 5 | 7 | 7 | 4 | 8 | 6 | 4 | 6 |

**Given the following control chart constraint for : $n = 5$, A2 = 0.58, D3 = 0 and D = 2.115.**

*Sol.* $\overline{X} = \frac{\sum \overline{X}}{10} = \frac{442}{10} = 44.2$

$\overline{R} = \frac{\sum R}{n} = \frac{58}{10} = 5.8$




UCL = $\overline{\overline{X}} + A2\overline{R}$ = 44.2 + 0.483(5.8) = 47.00

CL = $\overline{\overline{X}}$ = 44.2

LCL = $\overline{\overline{X}} - A_2\overline{R}$ = 44.2 − 0.483(5.8) = 41.39

The above diagram shows all the three control lines with the data points plotted, since four points falls out of the control limits, we can say that the process is out of control.

## Additional Question

# I. Choose the correct answer:

**1. A time series is a set of data recorded**

(a) Periodically
(b) at equal time intervals
(c) at successive points of time
(d) all the above **[Ans: (d) all the above]**

**2. The component of a time series attached to long time variation is termed as**

(a) Cyclic variations (b) Secular trend
(c) Irregular variation (d) all the above

**[Ans: (b) Secular trend]**

**3. The component of a time series which is attached to short term fluctuations is**

(a) Seasonal variations (b) Cyclic variation
(c) Irregular variation (d) all the above

**[Ans: (d) all the above]**

Chapter 10

# OPERATIONS RESEARCH

## MUST KNOW DEFINITIONS

| | | |
|---|---|---|
| **Transportation problem** | : | The objective of transportation problem is to determine the amount to be transported from each origin to each destinations such that the total transportation cost is minimized. |
| **Feasible solution** | : | A feasible solution to a transportation problem is a set of non negative values $x_{ij}$ $(i = 1,2 \ldots m, j = 1, 2, \ldots n)$ that satisfies the constraints. |
| **Basic feasible solution** | : | If a solution contains not more than $m+n-1$ allocations where $m$ is the number of rows and $n$ is the number of columns then the solution is called basic feasible solutions. |
| **Optimal solution** | : | It is a feasible solution which optimizes (minimises) the total transportation cost. |
| **Non degenerate basis feasible solution** | : | It is a basic feasible solution contains exactly $m+n-1$ allocations in independent positions. |
| **Degeneracy** | : | If a solution contains less than $m+n-1$ allocations, it is called a degenerate basic feasible solution. |
| **Methods of finding basic feasible solutions** | : | 1) North West Corner rule - (NWC)<br>2) Least Cost Method - (LCM)<br>3) Vogel's Approximation Method (VAM) |
| **Assignment Problem** | : | To assign the different jobs to different machines (one job per machine) to minimize the overall cost is known as assignment problem. |

$$x_{ij} = \begin{cases} 1, \text{ if } i^{th} \text{ job is assigned to } j^{th} \text{ machine} \\ 0, \text{ if } i^{th} \text{ job is not assigned to } j^{th} \text{ machine} \end{cases}$$

Thus, the allocations are

| | D | E | F | G | Available |
|---|---|---|---|---|---|
| A | (200) 11 | (50) 13 | 17 | 14 | 250 |
| B | 16 | (175) 18 | (125) 14 | 10 | 300 |
| C | 21 | 24 | (150) 13 | (250) 10 | 400 |
| Required | 200 | 225 | 275 | 250 | |

∴ The transportation schedule is

A→D, A→E, B→E, B→F, C→F, C→G

Hence, the total transportation cost is

= 200 (11) + 50 (13) + 175 (18) + 125 (14) + 150 (13) + 250 (10)

= 2200 + 650 + 3150 + 1750 + 1950 + 2500

= ₹ 12,200

## EXERCISE 10.2

**1. What is the Assignment problem?** *[HY-2019]*

**Ans.** To assign the different jobs to the different machines (one job per machine) to minimize the overall cost is known as assignment problem.

**2. Give mathematical form of assignment problem.** *[PTA-1]*

**Ans.** The mathematical form of assignment problem is

Minimize Z $= \sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij}\, x_{ij}$ subject to the

Constraints $\sum_{i=1}^{n} x_{ij} = 1, j = 1,2,\ldots n$

$\sum_{j=1}^{n} x_{ij} = 1, i = 1, 2, \ldots n$ and

$x_{ij} = 0$ or $1$ for all $i, j$

**3. What is the difference between Assignment Problem and Transportation Problem?** *[PTA-3; PTA-5]*

**Ans.** The assignment problem is a special case of transportation problem where the number of sources and destinations are equal. Here, jobs represent sources and machines represent destinations.

**4. Three jobs A, B and C one to be assigned to three machines U, V and W. The processing cost for each job machine combination is shown in the matrix given below. Determine the allocation that minimizes the overall processing cost.**

Machine

| Job | U | V | W |
|---|---|---|---|
| A | 17 | 25 | 31 |
| B | 10 | 25 | 16 |
| C | 12 | 14 | 11 |

**(Cost is in ₹ per unit)**

***Solution:***

Here number of rows and columns are equal

∴ The given assignment problem is balanced.

**Step 1 :** Select a minimum element in each row and subtract this from all the elements in its row.

| | U | V | W | Row minimum |
|---|---|---|---|---|
| A | 0 | 8 | 14 | 17 |
| B | 0 | 15 | 6 | 10 |
| C | 1 | 3 | 0 | 11 |

Here column V has no zero. Go to Step 2.

**Step 2 :** Select the minimum element in each column and subtract this from all the elements in its column.

| | U | V | W |
|---|---|---|---|
| A | 0 | 5 | 14 |
| B | 0 | 12 | 6 |
| C | 1 | 0 | 0 |
| Column minimum | | 3 | |

New, each row and column contains atleast one zero. Hence, assignments can be made.

**Step 3 :** Examine the rows with exactly one zero. Mark it by ☐ and draw a vertical line. After examining the row, examine the column with one zero mark it by ☐ and draw a horizontal line.

Transportation schedule :

A→I, A→II, A→III, A→IV, B→I, C→IV, D→II

Total transportation cost:

= (6×5)+(6+1)+(17×3)+(5×3)+(15×3)+(12×3)+(19×1)

= 30 + 6 + 51 + 15 + 45 + 36 + 19 = ₹ 202

## Govt. Exam Question & Answers

### 2 Marks

**1. Define: Feasible solution to transportation process.** *[Govt.MQP-2019]*

*Solution:* A feasible solution to a transportation problem is a set of non-negative values xij(i = 1,2,..,m, j = 1,2,…n) that satisfies the constraints.

### 5 Marks

**1. Find the initial basic feasible solution to the following transportation problem by north west corner method.** *[Govt.MQP-2019]*

| | D | E | F | G | Available |
|---|---|---|---|---|---|
| A | 11 | 13 | 17 | 14 | 250 |
| B | 16 | 18 | 14 | 10 | 300 |
| C | 21 | 24 | 13 | 10 | 400 |
| Required | 200 | 225 | 275 | 250 | |

*Solution:*

Here total availability = 250 + 300 + 400 = 950
total requirement = 200 + 225 + 275 + 250 = 950

⇒ total availability = total requirement

∴ The given problem is a balanced transportation problem

Hence, there exists a feasible solution to the given problem.

**I. allocation**

| | D | E | F | G | $a_i$ |
|---|---|---|---|---|---|
| A | (200) 11 | 13 | 17 | 14 | 250/50 |
| B | 16 | 18 | 14 | 10 | 300 |
| C | 21 | 24 | 13 | 10 | 400 |
| $b_j$ | 200/0 | 225 | 275 | 250 | |

[∵ min (200, 250) = 200]

**II. allocation**

| | E | F | G | $a_i$ |
|---|---|---|---|---|
| A | (50) 13 | 17 | 14 | 50/0 |
| B | 18 | 14 | 10 | 300 |
| C | 24 | 13 | 10 | 400 |
| $b_j$ | 225/175 | 275 | 250 | |

[∵ min (225, 50) = 50]

**III. allocation**

| | E | F | G | $a_i$ |
|---|---|---|---|---|
| B | (175) 18 | 14 | 10 | 300/125 |
| C | 24 | 13 | 10 | 410 |
| $b_j$ | 175/0 | 275 | 250 | |

[∵ min (175, 300) = 175]

**IV. allocation**

| | F | G | $a_i$ |
|---|---|---|---|
| B | (125) 14 | 10 | 125/0 |
| C | 13 | 10 | 400 |
| $b_j$ | 275/150 | 250 | |

[∵ min (275, 125) = 125]

**V. allocation**

| | F | G | $a_i$ |
|---|---|---|---|
| C | (150) 13 | 10 | 400/250 |
| $b_j$ | 150/0 | 250 | |

[∵ min (150, 400) = 150]

**VI. allocation**

| | G | $a_i$ |
|---|---|---|
| C | (250) 10 | 250 |
| $b_j$ | 250 | |

[∵ min (250, 250) = 250]

Thus, the allocations are

| | D | E | F | G | Available |
|---|---|---|---|---|---|
| A | (200)11 | (50)13 | 17 | 14 | 250 |
| B | 16 | (175)18 | (125)14 | 10 | 300 |
| C | 21 | 24 | (150)13 | (250)10 | 400 |
| Required | 200 | 225 | 275 | 250 | |

∴ The transportation schedule is

A→D, A→E, B→E, B→F, C→F, C→G

12th STD

## Govt. Model Question Paper - 2019-20

REG. NO.

# Business Mathematics and Statistics

Time Allowed : 3.00 Hours Maximum Marks : 90

**Introductions :**

(a) Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.

(b) Use **Blue** or **Black** ink to write and underline and pencil to draw diagrams

### PART - I

**Note :** (i) Answer all questions are **compulsory.**

(ii) Choose the most suitable answer from the given **four** correct alternatives and write the option code and the corresponding answer. [20 × 1 = 20]

**1.** If the rank of the matrix $\begin{pmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ -1 & 0 & \lambda \end{pmatrix}$ is 2, then the value of λ is

(a) 1 (b) 2
(c) 3 (d) – 1

**2.** The system of equations $4x+6y = 5$, $10x + 15y = 13$ has

(a) an unique solution (b) no solution
(c) infinitely many solutions (d) none of these

**3.** $\int \frac{2x+3}{\sqrt{x^2+3x+2}}dx$ is

(a) $\sqrt{x^2+3x+2}+c$

(b) $2\sqrt{x^2+3x+2}+c$

(c) $\log(x^2 + 3x + 2) + c$

(d) $\frac{2}{3}(x^2+3x+2)^{\frac{3}{2}}+c$

**4.** $\Gamma\left(\frac{3}{2}\right)$ is

(a) $\sqrt{\pi}$ (b) $\frac{\sqrt{\pi}}{2}$ (c) $2\sqrt{\pi}$ (d) $\frac{3}{2}$

**5.** The marginal revenue and marginal cost functions of a company are MR = 10 – 5x and MC = – 24 + 2x where x is the product, then the profit function is

(a) $9x^2 + 54x$ (b) $9x^2 – 54x$
(c) $34x-\frac{7x^2}{2}$ (d) $34x+\frac{-7x^2}{2}+k$

**6.** Area bounded by $y = x$ between the limits 0 and 2 is

(a) 1 sq.unit (b) 3 sq.units
(c) 2 sq.units (d) 4 sq.units

**7.** The differential equation $\left(\frac{dx}{dy}\right)^3+2y^{\frac{1}{3}}=x$ is

(a) of order 2 and degree 1
(b) of order 1 and degree 3
(c) of order 1 and degree 6
(d) of order 1 and degree 2

**8.** The P.I of $(3D^2 + D – 14)y = 13e^{2x}$ **is**

(a) $\frac{x}{2}e^{2x}$ (b) $xe^{2x}$
(c) $\frac{x^2}{2}e^{2x}$ (d) $13xe^{2x}$

**9.** E ≡

(a) 1 + Δ (b) 1– Δ
(c) 1 + Δ (d) 1– Δ

**10.** For the given data, the value of $\Delta^3 y_0$ is

| *x* | 5 | 6 | 9 | 11 |
|---|---|---|---|---|
| *y* | 12 | 13 | 15 | 18 |

(a) 1 (b) 0
(c) 2 (d) –1

12th STD

## COMMON QUARTERLY EXAMINATION SEPTEMBER - 2019

Reg. No.

Time Allowed : 3.00 Hours **Business Mathematics & Statistics** Maximum Marks : 90

**Instructions :**

1. Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.
2. Use **Blue** or **Black** ink to write and underline and pencil to draw diagrams.

**PART - I [20 × 1 = 20]**

**Note :** (i) Answer all the questions.

(ii) Choose the most appropriate answer from the given four alternatives and write the option code and the corresponding answer.

**1.** The rank of the matrix of order '$n$' is

(a) $n-1$ (b) $n$ (c) $n+1$ (d) $n^2$

**2.** The system of equations $2x - y = 1$, $3x + 2y = 12$ has

(a) a unique solution

(b) no solution

(c) infinitely many solution

(d) none of these

**3.** $|A_{n\times n}| = 3$ $|\text{adj } A| = 243$ then the value $n$ is

(a) 4 (b) 5 (c) 6 (d) 7

**4.** If $\begin{matrix} & A & B \\ A & 0.7 & 0.3 \\ B & 0.6 & x \end{matrix}$ is transition Probability matrix, then the value of $x$ is

(a) 0.2 (b) 0.3 (c) 0.4 (d) 0.7

**5.** $\int \frac{e^x}{e^x+1}dx$ is

(a) $\log\left|\frac{e^x}{e^x-1}\right|+c$ (b) $\log\left|\frac{e^x+1}{e^x}\right|-c$

(c) $\log|e^x|+c$ (d) $\log|e^x+1|+c$

**6.** Using the factorial representation of the gamma function, which of the following is the solution for the gamma function $l(n)$ when $n = 8$

(a) 5040 (b) 5400 (c) 4500 (d) 5540

**7.** $\int_0^x e^{-5x}x^7\,dx$ is

(a) $\frac{5!}{7^6}$ (b) $\frac{7!}{5^8}$ (c) $\frac{5!}{6^7}$ (d) $\frac{7!}{(-5)^8}$

**8.** The value of $\int_2^3 f(5-x)\,dx - \int_2^3 f(x)dx$ is

(a) 1 (b) 0 (c) $-1$ (c) 5

**9.** The demand and supply functions are given by $D(x) = 16 - x^2$, $S(x) = 2x^2 + 4$ are under perfect competition, then the equilibrium price $x$ is

(a) 2 (b) 3 (c) 4 (d) 5

**10.** The producer's surplus when the supply function for a commodity is $p = 3 + x$ and $x_0 = 3$ is

(a) $\frac{5}{2}$ units (b) $\frac{9}{2}$ units

(c) $\frac{3}{2}$ units (d) $\frac{7}{2}$ units

**11.** The area bounded by the parabola $y^2 = 4x$ bounded by its latus rectum is

(a) $\frac{16}{3}$ Sq.units (b) $\frac{8}{3}$ Sq.units

(c) $\frac{72}{3}$ Sq.units (d) $\frac{1}{3}$ Sq.units

**12.** If the marginal revenue $MR = 35 + 7x - 3x^2$, then the average revenue AP is

(a) $35x + \frac{7x^2}{2} - x^3$ (b) $35x + \frac{7x}{2} - x^2$

(c) $35 + \frac{7x}{2} - x^2$ (d) $35 + 7x + x^2$