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10. Ordinary Differential Equations:

Differential equation:-

A differential equation is any equation which contains atleast one derivative of an unknown function, either ordinary derivative or Partial derivative.

Types of differential equation:-

* Ordinary differential equation
* Partial differential equation.

Ordinary differential equation:-

If a differential equation contains only ordinary derivatives of one or more functions with respect to a single independent variable, it is said to be an ordinary differential equation.

Eg: $\frac{dy}{dx} + \sin x \cdot y = \cos x$.

Partial differential equation:-

An equation involving only partial derivatives of one or more functions of two or more independent variables is called a Partial differential equation.

Eg: $\frac{\partial u}{\partial y} = -\frac{\partial u}{\partial x}$ $\quad$ $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

Order and degree of a differential equation:-

* The order of a differential equation is the highest order derivative present in the differential equation.

* If a differential equation is expressible in a polynomial form, then the integral power of the highest order derivatives appears is called the degree of the differential equation.

Remarks:

The degree of a differential equation is the Power of the highest order derivative involved in the differential in the differential equation when the differential equation Satisfies the following conditions.

* All of the derivatives in the equation are free from fractional Powers, if any.
* Highest order derivative should not be an argument of a transcendental function, trigonometric or exponential. etc.
* The co.eff of any term containing the highest order derivative should just be a function of x, y or some lower order derivative but not as transcendental, trigonometric, exponential logarithmic function of derivatives.

10.1 / 147 Determine the order and degree of the following differential equations.

(i) $\frac{dy}{dx} = x + y + 5$

order = 1 degree = 1

(ii) $\left[\frac{d^4y}{dx^4}\right]^3 + 4\left(\frac{dy}{dx}\right)^7 + 6y = 5\cos 3x$

order = 4 degree = 3

(iii) $\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 = x^2 \log\left(\frac{d^2y}{dx^2}\right)$

order = 2 degree - not defined

(iv) $3\left(\frac{d^2y}{dx^2}\right) = \left[4 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}$

Squaring on both sides

$$9\left(\frac{d^2y}{dx^2}\right)^2 = \left[4 + \left(\frac{dy}{dx}\right)^2\right]^3$$

order = 2 degree = 2

(v) $dy + (xy - \cos x)dx = 0$

$$dy = (\cos x - xy)\,dx$$

$$\frac{dy}{dx} = \cos x - xy$$

$$\frac{dy}{dx} + xy - \cos x = 0$$

order = 1 degree = 1

Ex: 10.1

$\frac{1}{148}$ For each of the following differential equations determine its order, degree (if exists)

(i) $\frac{dy}{dx} + xy = \cot x$

order = 1 degree = 1

(ii) $\left(\frac{d^3y}{dx^3}\right)^{2/3} - 3\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 4 = 0$

$$\left(\frac{d^3y}{dx^3}\right)^{2/3} = 3\frac{d^2y}{dx^2} - 5\frac{dy}{dx} - 4$$

cubing on both sides,

$$\left(\frac{d^3y}{dx^3}\right)^2 = \left(3\frac{d^2y}{dx^2} - 5\frac{dy}{dx} - 4\right)^3$$

order = 3 degree = 2

(iii) $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x\sin\left(\frac{d^2y}{dx^2}\right)$

order = 2 degree – Not defined.

(iv) $\sqrt{\frac{dy}{dx}} - 4\frac{dy}{dx} - 7x = 0$

$$\sqrt{\frac{dy}{dx}} = 4\frac{dy}{dx} + 7x$$

Squaring on both sides

$$\frac{dy}{dx} = \left(4\frac{dy}{dx} + 7x\right)^2$$

order = 1 degree = 2

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(v) $y\frac{dy}{dx} = \frac{x}{\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^3}$

$y\frac{dy}{dx}\left[\frac{dy}{dx} + \left[\frac{dy}{dx}\right]^3\right] = x$

$y\left(\frac{dy}{dx}\right)^2 + y\left(\frac{dy}{dx}\right)^4 = x$

order = 1 degree = 4

(vi) $x^2\frac{d^2y}{dx^2} + \left(1+\left(\frac{dy}{dx}\right)^2\right)^{\frac{1}{2}} = 0$

$x^2\frac{d^2y}{dx^2} = -\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{1}{2}}$

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Squaring on both sides

$x^4\left(\frac{d^2y}{dx^2}\right)^2 = 1+\left(\frac{dy}{dx}\right)^2$

order = 2 degree = 2

(vii) $\left(\frac{d^2y}{dx^2}\right)^3 = \sqrt{1+\frac{dy}{dx}}$

Squaring on both sides

$\left(\frac{d^2y}{dx^2}\right)^6 = 1+\frac{dy}{dx}$

order = 2 degree = 6

(viii) $\frac{d^2y}{dx^2} = xy + \cos\left(\frac{dy}{dx}\right)$

order = 2 degree - Not defined

(ix) $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + \int y\,dx = x^3$

differentiate on both sides

$\frac{d^3y}{dx^3} + 5\frac{d^2y}{dx^2} + y = 3x^2$

order = 3 degree = 1

(x) $x = e^{xy\left(\frac{dy}{dx}\right)}$

Taking log on both sides

$\log x = xy\frac{dy}{dx}$

order = 1 degree = 1

The ordinary differential equations are classified into two different types

* Linear
* Non linear.

Linear :-

A general linear ordinary differential equation of order n is any differential equation that can be written in the following form.

$$a_n(x)y^{(n)} + a_{n-1}(x)y^{(n-1)} + a_{n-2}(x)y^{(n-2)} + \dots + a_1(x)y' + a_0(y) = g(x)$$

where the co.efficients $a_n(x) \neq 0$ $a_0(x), a_1(x), \dots a_{n-1}(x)$ and $g(x)$ are any function of independent variable x.

Note :-

* The important thing to note about linear differential equations is that there are no products of the function, $y(x)$ and its derivatives and neither the function nor its derivatives occur to any power other than the first order.
* No transcendental functions of y or any of its derivatives occur in differential equation.
* Also note that neither the function nor its derivatives are 'inside' another function for instance $\sqrt{y'}$ or $e^{y'}$
* The co.efficients $a_0(x), a_1(x) \dots a_{n-1}(x)$ and $g(x)$ can be zero or non-zero functions, or constant or non-constant functions, linear or non linear functions.

only the function $y(x)$, and its derivatives are used in determining whether a differential equation is linear.

Non Linear :

A non linear ordinary differential equation is simply one that is not linear.

Some Examples :

* $\frac{dy}{dx} = ax^3$ is a first order O.D.E
* $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$ is a second order linear ODE
* $y'' + 2x^3y' = 7xy + x^2$,,
* $y'' + y' = \sqrt{x}$ is a second order linear ODE
* $y^2 + y' = \sqrt{x}$ is a second order non-linear ODE
* $y' = x\sin y$ is a first order non-linear ODE.

Formation of differential equations :-

* Physical situations.
* Geometrical Problems.

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Physical situations :

Ex : 10.2

$\frac{1}{151.}$ Express each of the following physical statements in the form of differential equation.

(i) Radium decays at a rate proportional to the amount Q present.

Radium decays at a rate proportional to the amount Q present.

$$\frac{dQ}{dt} \propto Q$$

$$\frac{dQ}{dt} = kQ \quad [k \text{ is a constant}]$$

(ii) The Population P of a city increases at a rate Proportional to the Product of Population and to the difference between 5,00,000 and the Population.

Rate of change of P w.r. to time is $\frac{dP}{dt}$

Given that

$$\frac{dP}{dt} \propto P(500000 - P)$$

$$\therefore \frac{dP}{dt} = KP(500000 - P)$$

where K is constant.

(iii) For a certain subtance, the rate of change of vapor Pressure P w.r. to temperature T is Proportional to the vapor Pressure and inversely Proportional to the square of the temperature.

Rate of change of P w.r. to temperature is $\frac{dP}{dT}$

Given that

$$\frac{dP}{dT} \propto \frac{P}{T^2}$$

$$\frac{dP}{dT} = \frac{KP}{T^2} \quad [K \text{ is a constant}]$$

(iv) A saving amount pays 8% interest per year, compounded continuously. In addition, the income from another investment is credited to the amount continuously at the rate of ₹400 per year.

Let x be the amount invested.

$$I = 8\% \text{ of } x = \frac{8}{100}x = \frac{2}{25}x$$

Given that

$$\frac{dx}{dt} = \frac{2x}{25} + 400.$$

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$\frac{2}{152}$

Assume that a spherical rain drop evaporates at a rate proportional to its surface area. Form a differential equation involving the rate of change of the radius of the rain drop.

Volume of sphere $= \frac{4}{3}\pi r^3$

$$\frac{dv}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$$

Surface area $= 4\pi r^2$

Given that

$$\frac{dv}{dt} \propto (S.A)$$

$$\frac{dv}{dt} = -K\, S.A$$

$$4\pi r^2 \cdot \frac{dr}{dt} = -K(4\pi r^2)$$

$$\frac{dr}{dt} = -K \quad [K \text{ is constant.}$$

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Formation of differential Eqns from Geometrical Problems :-

Algorithm :

* write the given equation involving independent variable x (say), dependent variable (y) (say) and the arbitrary constants.
* obtain the number of arbitrary constants in step I. let there be 'n' arbitrary constants.
* Differentiate the relation in step I n times w.r. to 'x'
* Eliminate arbitrary constants with the help of n equations involving the differential co-efficients obtained in step III and an equation in step I. The equation so obtained is the desired differential equation.

10.2/152. Find the differential equation for the family of all straight lines passing through the origin

The family of straight lines passing through (0,0) is

$$y = mx \quad \text{———} \quad ①$$

diff w.r. to 'x'

$$\frac{dy}{dx} = m$$

① becomes

$$y = x\frac{dy}{dx}$$

$$y\,dx - x\,dy = 0$$

This is the required equation.

10.3/152 Form the differential equation by eliminating the arbitrary constants A and B from

$y = A\cos x + B\sin x$

$$y = A\cos x + B\sin x \quad \text{———} \quad ①$$

diff w.r. to 'x'

$$y' = -A\sin x + B\cos x$$

again diff w.r. to 'x'

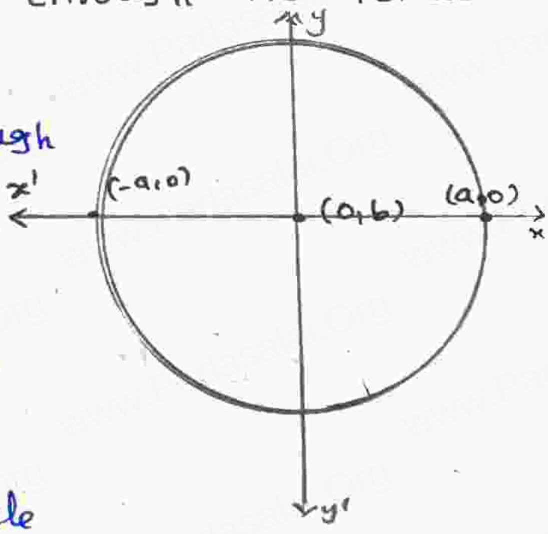
$$y'' = -A\cos x - B\sin x$$

$$y'' = -(A\cos x + B\sin x)$$

$$y'' = -y$$

$$\boxed{y'' + y = 0}$$

10.4/153. Find the differential equation of the family of circles passing through the points (a,0) and (-a,0)



A circle passes through the points (a,0) and (-a,0) and its centre is on y-axis

$$\text{radius} = \sqrt{(a-0)^2 + (0-b)^2}$$

$$= \sqrt{a^2 + b^2}$$

Equation of the circle

$$(x-0)^2 + (y-b)^2 = a^2 + b^2$$

$$x^2+(y-b)^2 = a^2+b^2 \quad \text{———} \quad (1)$$

diff w.r. to 'x'

$$2x+2(y-b)\frac{dy}{dx}=0$$

$$x+(y-b)\frac{dy}{dx}=0$$

$$x+(y-b)y'=0$$

$$(y-b)y'=-x$$

$$y-b=\frac{-x}{y'}$$

$$b=y+\frac{x}{y'}$$

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Sub $b=y+\frac{x}{y'}$ in (1)

$$x^2+\left(\cancel{y}-\cancel{y}-\frac{x}{y'}\right)^2 = a^2+\left(y+\frac{x}{y'}\right)^2$$

$$x^2+\cancel{\frac{x^2}{y'^2}} = a^2+y^2+\cancel{\frac{x^2}{y'^2}}+\frac{2xy}{y'}$$

$$x^2=a^2+y^2+\frac{2xy}{y'}$$

$$x^2-a^2-y^2=\frac{2xy}{y'}$$

$$(x^2-a^2-y^2)y'=2xy$$

$(x^2-a^2-y^2)y'-2xy=0$ which is the required differential equation.

10.5 / 153 Find the differential equation of the family of Parabolas $y^2=4ax$ where a is an arbitrary constant.

$$y^2=4ax \quad \text{———} \quad (1)$$

diff w.r. to 'x'

$$2yy'=4a$$

$$yy'=2a$$

$$a=\frac{yy'}{2} \quad \text{———} \quad (2)$$

Sub (2) in (1)

$$y^2=4\left(\frac{yy'}{2}\right)x$$

$$y^2=2yy'x$$

$$y=2y'x \Rightarrow y'=\frac{y}{2x}$$

$$\boxed{\frac{dy}{dx}=\frac{y}{2x}}$$

10.6 / 153. Find the differential equation of the family of all ellipses having foci on the x-axis and centre at the origin.

Eqn of the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad a>b \quad \text{——— (1)}$$

diff w.r. to 'x'

$$\frac{2x}{a^2}+\frac{2yy'}{b^2}=0$$

$$\frac{x}{a^2}+\frac{yy'}{b^2}=0 \quad \text{——— (2)}$$

Again diff w.r. to 'x'

$$\frac{1}{a^2}+\frac{yy''+y'^2}{b^2}=0 \quad \text{——— (3)}$$

from (1), (2) & (3)

$$\begin{vmatrix} x^2 & y^2 & 1 \\ x & yy' & 0 \\ 1 & yy''+y'^2 & 0 \end{vmatrix}=0$$

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$$x(yy''+y'^2)-yy'=0$$

∴ The Required equation is

$$x\left(y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2\right)-y\frac{dy}{dx}=0$$

Ex : 10.3

1 / 154. Find the differential equation of the family of (i) all non-vertical lines in a Plane. (ii) all non-horizontal lines in a plane.

(i) all non-vertical lines in a plane :

$$y=mx+c \quad m\neq 0$$

diff w.r. to 'x'

$$\frac{dy}{dx}=m$$

Again diff w.r. to 'x'

$$\boxed{\frac{d^2y}{dx^2}=0}$$

(ii) all non-horizontal lines in a plane.

$$y=mx+c \quad m\neq\infty$$

$$\frac{dy}{dx}=m \Rightarrow \frac{dx}{dy}=\frac{1}{m}$$

Again diff w.r. to 'x'

$$\boxed{\frac{d^2x}{dy^2}=0}$$

2/154 Form the differential equation of all straight lines touching the circle $x^2+y^2=r^2$

Eqn of the circle : $x^2+y^2=r^2$

let $y = mx \pm r\sqrt{1+m^2}$ be a tangent to the circle. —— (1)

$$\frac{dy}{dx} = m.$$

(1) becomes

$$y = xy' \pm r\sqrt{1+y'^2}$$

$$y - xy' = \pm r\sqrt{1+y'^2}$$

Squaring on both sides

$$(y-xy')^2 = r^2(1+y'^2)$$

$$\left[y - x\frac{dy}{dx}\right]^2 = r^2\left[1+\left(\frac{dy}{dx}\right)^2\right]$$ which is the required differential equation.

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3/154. Find the differential equation of the family of circles passing through the origin and having their centres on the x-axis.

All circles passing through origin and having their centre on the x-axis,

$$(x-a)^2+y^2 = a^2 \quad \text{—— (1)}$$

diff w.r. to 'x'

$$2(x-a)+2yy' = 0$$

$$x-a+yy' = 0$$

$$a = x+yy' \quad \text{—— (2)}$$

sub (2) in (1)

$$(\cancel{x}-\cancel{x}-yy')^2+y^2 = (x+yy')^2$$

$$(yy')^2+y^2 = (x+yy')^2$$

$$\cancel{(yy')^2}+y^2 = x^2+\cancel{(yy')^2}+2xyy'$$

$$y^2 = x^2+2xyy'$$

$$\boxed{x^2-y^2+2xyy' = 0}$$

which is the required differential equation.

4/154. Find the differential equation of the family of all the Parabolas with latus rectum 4a and whose axes are parallel to the x-axis.

Eqn of the Parabola

$$(y-k)^2 = 4a(x-h)$$

diff w.r. to 'x'

$$2(y-k)y' = 4a$$

$$(y-k)y' = 2a$$

$$y-k = \frac{2a}{y'}$$

Again diff w.r. to 'x'

$$y' = \frac{-2a}{y'^2} \cdot y''$$

$$y'^3 = -2ay''$$

$$\left(\frac{dy}{dx}\right)^3 = -2a\left(\frac{d^2y}{dx^2}\right)$$

$$2a\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = 0$$ which is the required differential equation.

5. Find the differential equation of the family of Parabolas with vertex at (0, −1) and having axis along the y-axis.

Eqn of the Parabola

$$(x-h)^2 = 4a(y-k)$$

Here, V(0, −1) $x^2 = 4a(y+1)$ —— (1)

diff w.r. to 'x'

$$2x = 4ay'$$

$$x = 2ay' \Rightarrow a = \frac{x}{2y'}$$ —— (2)

Sub (2) in (1)

$$x^2 = 4\left(\frac{x}{2y'}\right)(y+1)$$

$$y'x^2 = 2x(y+1)$$

$$x^2y' = 2x(y+1)$$

$$xy' = 2(y+1)$$

$$xy' - 2y - 2 = 0$$ which is the required differential equation.

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$\frac{6}{154}$. Find the differential equations of the family of all the ellipses having foci on the y-axis and the centre at the origin.

Eqn of the ellipse

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \qquad a > b \quad \text{---} (1)$$

diff w.r. to 'x'

$$\frac{2x}{b^2} + \frac{2yy'}{a^2} = 0$$

$$\frac{x}{b^2} + \frac{yy'}{a^2} = 0 \quad \text{---} (2)$$

again diff w.r. to 'x'

$$\frac{1}{b^2} + \frac{yy'' + y'^2}{a^2} = 0 \quad \text{---} (3)$$

from (1), (2) & (3)

$$\begin{vmatrix} x^2 & y^2 & 1 \\ x & yy' & 0 \\ 1 & yy''+y'^2 & 0 \end{vmatrix} = 0$$

$x(yy''+y'^2) - yy' = 0$ which is the required differential equation.

$\frac{7}{154}$. Find the differential equation corresponding to the family of curves represented by the equation $y = Ae^{8x} + Be^{-8x}$, where A and B are arbitrary constants.

$$y = Ae^{8x} + Be^{-8x} \quad \text{---} (1)$$

diff w.r. to 'x'

$$y' = 8Ae^{8x} - 8Be^{-8x} \quad \text{---} (2)$$

Again diff w.r. to 'x'

$$y'' = 64Ae^{8x} + 64Be^{-8x}$$

$$= 64[Ae^{8x} + Be^{-8x}]$$

$$y'' = 64y.$$

$$\boxed{y'' - 64y = 0}$$

which is the required differential equation.

8. Find the differential equation of the curve represented by $xy = ae^{x} + be^{-x} + x^{2}$

$$xy = ae^{x} + be^{-x} + x^{2}$$

$$xy - x^{2} = ae^{x} + be^{-x}$$

diff w.r.to 'x'

$$xy' + y - 2x = ae^{x} - be^{-x}$$

again diff w.r.to 'x'

$$xy'' + y'.1 + y' - 2 = ae^{x} + be^{-x}$$

$$xy'' + y'(1+1) - 2 = ae^{x} + be^{-x}$$

$$xy'' + 2y' - 2 = xy - x^{2}$$

$xy'' + 2y' + x^{2} - xy - 2 = 0$ which is the required differential equation.

Solutions of Ordinary differential equations:

A solution of a differential equation is an expression for the dependent variable in terms of the independent variable(s) which satisfies the diff. Eqn.

General Solution :-

The Solution which contains as many arbitrary constants as the order of the differential equation is called the general Solution.

Particular Solution :-

If we give particular values to the arbitrary constants in the general solution of differential equation, the resulting solution is called a Particular solution.

10.7 / 156 Show that $x^2+y^2=r^2$ where r is a Constant, is a solution of the differential equation $\frac{dy}{dx} = -\frac{x}{y}$

$$x^2+y^2=r^2$$

It has one arbitrary Constant.

diff w.r. to 'x'

$$2x+2y\frac{dy}{dx}=0$$

$$x+y\frac{dy}{dx}=0$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

$x^2+y^2=r^2$ Satisfies the differential equation $\frac{dy}{dx} = -\frac{x}{y}$

$\therefore$ $x^2+y^2=r^2$ is a Solution of $\frac{dy}{dx} = -\frac{x}{y}$.

10.8 / 156. Show that $y = mx + \frac{7}{m}$ $m \neq 0$ is a Solution of the differential equation $xy' + 7\frac{1}{y'} - y = 0$

$$y = mx + \frac{7}{m} \qquad (1)$$

'm' is an arbitrary constant.

diff w.r. to 'x'

$$\frac{dy}{dx} = m$$

$$y' = m$$

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Sub $y' = m$ in (1)

$$y = xy' + \frac{7}{y'}$$

$$xy' + \frac{7}{y'} - y = 0$$

$\therefore$ The given equation is a Solution of the diff. eqn $xy' + \frac{7}{y'} - y = 0$.

10.9 / 156 Show that $y = 2(x^2-1) + ce^{-x^2}$ is a solution of the differential equation $\frac{dy}{dx} + 2xy - 4x^3 = 0$

$$y = 2(x^2-1) + ce^{-x^2}$$

c is an arbitrary constant

$$y' = 2(2x) + c(e^{-x^2})(-2x)$$

$$y' = 4x - 2xce^{-x^2} \quad \text{— (1)}$$

Sub (1) in $\frac{dy}{dx} + 2xy - 4x^3 = 0$

$$L.H.S = 4x - 2xce^{-x^2} + 2x(2(x^2-1) + ce^{-x^2}) - 4x^3$$

$$= \cancel{4x} - \cancel{2xce^{-x^2}} + \cancel{4x^3} - \cancel{4x} + \cancel{2xce^{-x^2}} - \cancel{4x^3}$$

$$= 0 = R.H.S.$$

∴ The given function is a solution of the differential equation

$$\frac{dy}{dx} + 2xy - 4x^3 = 0.$$

10.10 / 157. Show that $y = a\cos(\log x) + b\sin(\log x)$, $x > 0$ is a solution of the differential eqn

$$x^2y'' + xy' + y = 0$$

$$y = a\cos(\log x) + b\sin(\log x)$$

where a and b are two arbitrary

we have differentiate two times.

$$y' = -\frac{a\sin\log x}{x} + \frac{b\cos(\log x)}{x}$$

$$xy' = -a\sin\log x + b\cos\log x.$$

Again diff w.r. to 'x'

$$xy'' + y' = -\frac{a\cos(\log x)}{x} - \frac{b\sin\log x}{x}$$

$$x(xy'' + y') = -(a\cos\log x + b\sin\log x)$$

$$x^2y'' + xy' = -y$$

$$x^2y'' + xy' + y = 0$$

∴ $y = a\cos\log x + b\sin\log x$ is a solution of given differential equation.

Ex : 10.4

$\frac{1}{156}$ Show that each of the following expressions is a solution of the corresponding given differential equation.

(i) $y = 2x^2$ $xy' = 2y$ —— ①

diff w.r. to 'x'

$y' = 4x$

Sub $y' = 4x$ in ①

L.H.S $= x(4x) = 4x^2 = 2(2x^2)$

$= 2y =$ R.H.S.

$\therefore$ $y = 2x^2$ is a solution of $xy' = 2y$

(ii) $y = ae^{x} + be^{-x}$: $y'' - y = 0$

where a, b are arbitrary constants.

we have to diff. the function 2 times.

$y = ae^{x} + be^{-x}$

diff w.r. to 'x'

$y' = ae^{x} - be^{-x}$

again diff w.r. to 'x'

$y'' = ae^{x} + be^{-x}$

$y'' = y$

$y'' - y = 0$

$\therefore$ $y = ae^{x} + be^{-x}$ is a solution of $y'' - y = 0$.

$\frac{2}{157.}$ Find the value of 'm' so that the function $y = e^{mx}$ is a solution of the given differential equation.

(i) $y' + 2y = 0$

$y' = me^{mx}$

$y' + 2y = 0$

$me^{mx} + 2y = 0$

$my + 2y = 0$

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$(m+2)y = 0$

$m+2 = 0$

$\boxed{m = -2}$

(ii) $y'' - 5y' + 6y = 0$

$y = e^{mx}$

$y' = me^{mx}$ $\quad y'' = m^2 e^{mx}$

$m^2 e^{mx} - 5me^{mx} + 6e^{mx} = 0$

$e^{mx}(m^2 - 5m + 6) = 0$

$(m^2 - 5m + 6)y = 0$

$m^2 - 5m + 6 = 0$

$(m-2)(m-3) = 0$

$m = 2, 3.$

3/157. The slope of the tangent to the curve at any point is the reciprocal of four times the ordinate at that point. The curve passes through (2,5). Find the eqn of the curve.

Given that the slope of the tangent is the reciprocal of four times the ordinate.

$\frac{dy}{dx} = \frac{1}{4y}$

$4y\,dy = dx$

$\int 4y\,dy = \int dx$

$4\frac{y^2}{2} = x + C$

$2y^2 = x + C$

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It passes through (2,5)

$2(25) = 2 + C$

$C = 50 - 2$

$\boxed{C = 48}$

$\therefore$ Required equation is

$2y^2 = x + 48.$

$\frac{4}{157}$. Show that $y = e^{-x} + mx + n$ is a solution of the differential eqn $e^{x}\left(\frac{d^2y}{dx^2}\right) - 1 = 0$

$$y = e^{-x} + mx + n$$

m, n are arbitrary constants

$$y' = -e^{-x} + m$$

$$y'' = e^{-x}$$

$$\frac{y''}{e^{-x}} = 1 \Rightarrow e^{x} y'' = 1$$

$$e^{x}\left(\frac{d^2y}{dx^2}\right) - 1 = 0$$

$\therefore$ $y = e^{-x} + mx + n$ is a solution of $e^{x}\left(\frac{d^2y}{dx^2}\right) - 1 = 0$.

$\frac{5}{157}$. Show that $y = ax + \frac{b}{x}$, $x \neq 0$ is a solution of the differential equation $x^2 y'' + xy' - y = 0$

$$y = ax + \frac{b}{x} \quad \text{—— (1)}$$

a and b are two arbitrary constants.

We have to differentiate two times.

diff w.r. to 'x'

$$y' = a - \frac{b}{x^2} \quad \text{—— (2)}$$

again diff w.r. to x

$$y'' = \frac{2b}{x^3} \quad \text{—— (3)}$$

Sub (1), (2) & (3) in $x^2 y'' + xy' - y = 0$

$$L.H.S = x^2 y'' + xy' - y$$

$$= x^2\left(\frac{2b}{x^3}\right) + x\left(a - \frac{b}{x^2}\right) - \left(ax + \frac{b}{x}\right)$$

$$= \frac{2b}{x} + \cancel{ax} - \frac{b}{x} - \cancel{ax} - \frac{b}{x}$$

$$= \frac{2b}{x} - \frac{2b}{x} = 0 = R.H.S$$

$\therefore$ $y = ax + \frac{b}{x}$ is a solution of $x^2 y'' + xy' - y = 0$

6/157 Show that $y = ae^{-3x} + b$, where a and b are arbitrary constants, is a solution of the differential equation $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} = 0$.

$$y = ae^{-3x} + b$$

diff w.r. to 'x'

$$y' = -3ae^{-3x} \quad \text{---} (1)$$

again diff w.r. to 'x'

$$y'' = 9ae^{-3x} \quad \text{---} (2)$$

Sub (1) & (2) in $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} = 0$

$$L.H.S = \frac{d^2y}{dx^2} + 3\frac{dy}{dx} = 9ae^{-3x} + 3(-3e^{-3x})$$

$$= 9ae^{-3x} - 9ae^{-3x} = 0 = RHS$$

$\therefore$ $y = ae^{-3x} + b$ is a solution of $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} = 0$.

7/157. Show that the differential equation representing the family of curves $y^2 = 2a(x + a^{2/3})$ where a is a positive parameter is

$$\left(y^2 - 2xy\frac{dy}{dx}\right)^3 = 8\left(y\frac{dy}{dx}\right)^5$$

$$y^2 = 2a(x + a^{2/3}) \qquad \text{'a' is a Parameter.}$$

$$y^2 = 2ax + 2a^{5/3} \quad \text{---} (1)$$

diff w.r. to 'x'

$$2yy' = 2a$$

$$yy' = a \quad \text{---} (2)$$

Sub (2) in (1)

$$y^2 = 2x(yy') + 2(yy')^{5/3}$$

$$y^2 - 2xyy' = 2(yy')^{5/3}$$

cubing on both sides

$$(y^2 - 2xyy')^3 = 8(yy')^5$$

$\therefore$ $y^2 = 2a(x + a^{2/3})$ is a solution of given differential equation.

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$\frac{8}{158}$ Show that $y = a\cos bx$ is a solution of the differential equation $\frac{d^2y}{dx^2} + b^2y = 0$.

$$y = a\cos bx \quad \text{——— (1)}$$

diff w.r.to 'x'

$$y' = -ab\sin bx$$

again diff w.r.to 'x'

$$y'' = -ab^2\cos bx$$

$$y'' = -b^2(a\cos bx)$$

$$y'' = -b^2y$$

$$y'' + b^2y = 0$$

$\therefore$ $y = a\cos bx$ is a solution of $\frac{d^2y}{dx^2} + b^2y = 0$

Solutions of first order, first degree differential equations:

Methods:

* variable separable
* Homogeneous
* Linear.

* Variable separable method:

$$f_1(x)g_1(y)\,dx + f_2(x)g_2(y)\,dy = 0$$

$$f_1(x)g_1(y)\,dx = -f_2(x)g_2(y)\,dy$$

$$\frac{f_1(x)}{f_2(x)}\,dx = -\frac{g_2(y)}{g_1(y)}\,dy$$

$$\int\frac{f_1(x)}{f_2(x)}\,dx = -\int\frac{g_2(y)}{g_1(y)}\,dy + c$$

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10.11 / 159. Solve : $(1+x^2)\frac{dy}{dx} = 1+y^2$

$$\frac{dy}{1+y^2} = \frac{dx}{1+x^2}$$

$$\int \frac{dy}{1+y^2} = \int \frac{dx}{1+x^2}$$

$$\tan^{-1} y = \tan^{-1} x \pm c$$

$$\tan^{-1} x - \tan^{-1} y = c$$

$$\tan^{-1}\left(\frac{x-y}{1+xy}\right) = c$$

$$\frac{x-y}{1+xy} = \tan c = a$$

$$x - y = a(1+xy)$$

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10.12 / 159 Find the Particular Solution of $(1+x^3)dy - x^2y\,dx = 0$ Satisfying the condition $y(1)=2$

$$(1+x^3)dy - x^2y\,dx = 0$$

$$(1+x^3)dy = x^2y\,dx$$

$$\frac{dy}{y} = \frac{x^2}{1+x^3}dx$$

$$\int \frac{dy}{y} = \int \frac{x^2}{1+x^3}dx$$

$$\int \frac{dy}{y} = \frac{1}{3}\int \frac{3x^2}{1+x^3}dx \qquad \int \frac{f'(x)}{f(x)}dx = \log|f(x)|+c$$

$$\log y = \frac{1}{3}\log(1+x^3) + \log c$$

$$\log y = \log(1+x^3)^{1/3}\cdot c$$

$$y = (1+x^3)^{1/3}c$$

when $x = 1$, $y = 2$

$$2 = (1+1)^{1/3}\cdot c \Rightarrow c\,2^{1/3} = 2$$

$$c^3 \cdot 2 = 8$$

$$c^3 = 4$$

$$c = 4^{1/3}$$

$$\boxed{c = 4^{1/3}}$$

∴ The Required Solution is

$$y = 4^{1/3}(1+x^3)^{1/3}$$

cubing on both sides

$$y^3 = 4(1+x^3)$$

Ex : 10.5

4/161. Solve the following differential equations:-

(i) $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$

$\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$

$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + c$

$\int \frac{dy}{\sqrt{1-y^2}} = \int \frac{dx}{\sqrt{1-x^2}}$

$\sin^{-1} y = \sin^{-1} x + c$

$\sin^{-1} x - \sin^{-1} y = c$

$\sin^{-1}(x\sqrt{1-y^2} - y\sqrt{1-x^2}) = c$

$x\sqrt{1-y^2} - y\sqrt{1-x^2} = \sin C$

$x\sqrt{1-y^2} - y\sqrt{1-x^2} = K.$

(ii) $y\,dx + (1+x^2)\tan^{-1} x\, dy = 0$

$y\,dx = (1+x^2)\tan^{-1} x\, dy$

$-\frac{dy}{y} = \frac{dx}{(1+x^2)\tan^{-1} x}$

$\int \frac{f'(x)}{f(x)} dx = \log f(x) + c$

$-\int \frac{dy}{y} = \int \frac{dx}{(1+x^2)\tan^{-1} x}$

$-\log y = \log \tan^{-1} x + \log c$

$\log \tan^{-1} x + \log y = \log c$

$\log y \tan^{-1} x = \log c$

$\boxed{y \tan^{-1} x = c}$

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(iii) $\sin \frac{dy}{dx} = a \quad y(0) = 1$

$\frac{dy}{dx} = \sin^{-1} a$

$\int dy = \int \sin^{-1} a\, dx$

$y = x \sin^{-1} a + c$

when $x = 0, \quad y = 1$

$1 = 0 + c \Rightarrow c = 1$

$y = x \sin^{-1} a + 1$

$y-1 = x\sin^{-1}a$

(or)

$\frac{y-1}{x} = \sin^{-1}a$ (or) $\sin\left(\frac{y-1}{x}\right) = a$

(iv) $\frac{dy}{dx} = e^{x+y} + x^3 e^y$

$\frac{dy}{dx} = e^y(e^x + x^3)$

$\frac{dy}{e^y} = (e^x + x^3)\,dx$

$\int e^{-y}\,dy = \int (e^x + x^3)\,dx$

$-e^{-y} = e^x + \frac{x^4}{4} + c$

$e^x + e^{-y} + \frac{x^4}{4} = c.$

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(v) $(e^y+1)\cos x\,dx + e^y \sin x\,dy = 0$

$(e^y+1)\cos x\,dx = -e^y \sin x\,dy$

$\int \frac{\cos x}{\sin x}\,dx = \int -\frac{e^y}{e^y+1}\,dy$

$\int \frac{f'(x)}{f(x)}\,dx = \log f(x)$

$\log \sin x = -\log(e^y+1) + \log c$

$\log \sin x + \log(e^y+1) = \log c$

$\log \sin x\,(e^y+1) = \log c$

$\sin x\,(e^y+1) = c.$

(vi) $(y\,dx - x\,dy)\cot\left(\frac{x}{y}\right) = ny^2\,dx$

$\frac{y\,dx - x\,dy}{y^2}\cdot\cot\left(\frac{x}{y}\right) = n\,dx$

$\int \cot\left(\frac{x}{y}\right)\cdot d\left(\frac{x}{y}\right) = \int n\,dx$

$\log \sin\left(\frac{x}{y}\right) = nx + c$

$\sin\left(\frac{x}{y}\right) = e^{nx+c}$

$d\left(\frac{x}{y}\right) = \frac{y\,dx - x\,dy}{y^2}$

$d\left(\frac{y}{x}\right) = \frac{x\,dy - y\,dx}{x^2}$

$d(xy) = x\,dy + y\,dx$

(vii) $\frac{dy}{dx} - x\sqrt{25-x^2}\ \cancel{dx} = 0$

$$\frac{dy}{dx} = x\sqrt{25-x^2}$$

$$\int dy = \int x\sqrt{25-x^2}\,dx$$

$u = 25-x^2$

$du = -2x\,dx$

$\frac{-du}{2} = x\,dx$

$$y = \int \sqrt{u}\left(-\frac{du}{2}\right)$$

$$y = -\frac{1}{2}\left[\frac{2}{3}u^{3/2}\right] + c$$

$$y = -\frac{1}{3}u^{3/2} + c$$

$$y = -\frac{1}{3}(25-x^2)^{3/2} + c$$

$$3y + (25-x^2)^{3/2} = 3c$$

$$3y + (25-x^2)^{3/2} = C.$$

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(viii) $x\cos y\,dy = e^x(x\log x + 1)\,dx$

$$\cos y\,dy = \frac{e^x(x\log x+1)}{x}\,dx$$

$$\int \cos y\,dy = \int e^x\left(\log x + \frac{1}{x}\right)dx$$

$$\sin y = e^x \cdot \log x + c$$

$\int e^x(f(x) + f'(x))\,dx = e^x f(x) + c$

$f(x) = \log x$

$f'(x) = \frac{1}{x}$

(ix) $\tan y \frac{dy}{dx} = \cos(x+y) + \cos(x-y)$

$$\cos(A+B) + \cos(A-B) = 2\cos A\cos B$$

$$\tan y\frac{dy}{dx} = 2\cos x\cos y$$

$$\int \frac{\tan y}{\cos y}\,dy = \int 2\cos x\,dx$$

$$\int \sec y\tan y\,dy = 2\int \cos x\,dx$$

$$\sec y = 2\sin x + c.$$

Substitution method :-

(x) $\frac{dy}{dx} = \tan^2(x+y)$

$u = x+y$

$\frac{du}{dx} = 1 + \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{du}{dx} - 1$

$$\frac{du}{dx} - 1 = \tan^2 u$$

$$\frac{du}{dx} = 1 + \tan^2 u$$

$$\frac{du}{dx} = \sec^2 u$$

$$\frac{du}{\sec^2 u} = dx$$

$$\int \cos^2 u \, du = \int dx$$

$$\int \frac{1+\cos 2u}{2} du = \int dx$$

$$\frac{1}{2}\left[u + \frac{\sin 2u}{2}\right] = x + c$$

$$\frac{1}{2}\left[(x+y) + \frac{\sin 2(x+y)}{2}\right] = x + c$$

$$\frac{1}{2}\left[(x+y) + \frac{2\sin(x+y)\cos(x+y)}{2}\right] = x + c$$

$$\frac{1}{2}\left[(x+y) + \sin(x+y)\cos(x+y)\right] = x + c$$

$\frac{10.13}{159}$ Solve : $\frac{dy}{dx} = \sin^2(x-y+1)$

$u = x-y+1$

$\frac{du}{dx} = 1 - \frac{dy}{dx}$

$\frac{dy}{dx} = 1 - \frac{du}{dx}$

$$1 - \frac{du}{dx} = \sin^2 u$$

$$\frac{du}{dx} = 1 - \sin^2 u$$

$$\frac{du}{dx} = \cos^2 u$$

$$\frac{du}{\cos^2 u} = dx$$

$$\int \sec^2 u \, du = \int dx$$

$$\tan u = x + c$$

$$\tan(x-y+1) = x + c$$

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$\frac{10.14}{160}$ Solve : $\frac{dy}{dx} = \sqrt{4x+2y-1}$

$u = 4x+2y-1$

$\frac{du}{dx} = 4 + 2\frac{dy}{dx}$

$2\frac{dy}{dx} = \frac{du}{dx} - 4$

$\frac{dy}{dx} = \frac{1}{2}\frac{du}{dx} - 2$

$$\frac{1}{2}\cdot\frac{du}{dx} - 2 = \sqrt{u}$$

$$\frac{du}{dx} = 2(\sqrt{u}+2)$$

$$\frac{1}{2}\int \frac{du}{\sqrt{u}+2} = \int dx$$

Let $u = z^2$

$du = 2z.dz$

$$\frac{1}{2}\int \frac{2z.dz}{z+2} = x + c$$

$$\frac{1}{\cancel{2}}\cdot\cancel{2}\int \frac{z+2-2}{z+2} dz = x + c$$

$$\int\left(1 - \frac{2}{z+2}\right) dz = x + c$$

$$z - 2\log|z+2| = x + c$$

$$\sqrt{u} - 2\log|\sqrt{u}+2| = x + c$$

$$\sqrt{4x+2y-1} - 2\log\left|\sqrt{4x+2y-1}+2\right| = x + c$$

10.15
160

Solve : $\frac{dy}{dx} = \frac{x-y+5}{2(x-y)+7}$

$u = x - y$

$\frac{du}{dx} = 1 - \frac{dy}{dx}$

$\frac{dy}{dx} = 1 - \frac{du}{dx}$

$$1 - \frac{du}{dx} = \frac{u+5}{2u+7}$$

$$\frac{du}{dx} = 1 - \frac{u+5}{2u+7}$$

$$\frac{du}{dx} = \frac{2u+7-u-5}{2u+7}$$

$$\frac{du}{dx} = \frac{u+2}{2u+7}$$

$$\int \frac{2u+7}{u+2}du = \int dx$$

$$\int \frac{2u}{u+2}du + 7\int \frac{du}{u+2} = \int dx$$

$$2\int \frac{u+2-2}{u+2}du + 7\int \frac{du}{u+2} = \int dx$$

$$2\int 1 - \frac{2}{u+2}du + 7\int \frac{du}{u+2} = \int dx$$

$$\int 2du - \int \frac{4du}{u+2} + \int \frac{7du}{u+2} = \int dx$$

$$\Rightarrow \int 2du + \frac{3du}{u+2} = \int dx$$

$$2u + 3\log|u+2| = x + c$$

$$2(x-y) + 3\log|x-y+2| = x + c.$$

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10.16
161

Solve : $\frac{dy}{dx} = (3x+y+4)^2$

$u = 3x + y + 4$

$\frac{du}{dx} = 3 + \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{du}{dx} - 3$

$$\frac{du}{dx} - 3 = u^2$$

$$\frac{du}{dx} = u^2 + 3$$

$$\frac{du}{u^2+3} = dx$$

$$\int \frac{du}{u^2+\sqrt{3}^2} = \int dx \qquad \int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$$

$$\frac{1}{\sqrt{3}}\tan^{-1}\frac{u}{\sqrt{3}} = x + c$$

$$\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{3x+y+4}{\sqrt{3}}\right) = x + c.$$

1/161. If F is the constant force generated by the motor of an automobile of mass M, its velocity v is given by $M.\frac{dv}{dt} = F - kv$. where k is a constant. Express v in terms of 't' given that $v=0$ when $t=0$.

$$M\frac{dv}{dt} = F - kv$$

$$\frac{dv}{F-kv} = \frac{1}{M}.dt$$

$$\int \frac{dv}{F-kv} = \frac{1}{M}\int dt$$

$$-\frac{1}{k}\log|F-kv| = \frac{1}{M}t + c$$

[k is a constant

multiply by $(-k)$

$$\log|F-kv| = -\frac{k}{M}t + c$$

when $v=0$ $t=0$

$$\log F = 0 + c$$

$$\boxed{c = \log F}$$

∴ The Required Solution is

$$\log|F-kv| = \frac{-kt}{M} + \log F$$

$$\log|F-kv| - \log F = -\frac{kt}{M}$$

$$\log\left|\frac{F-kv}{F}\right| = -\frac{kt}{M}$$

$$\frac{F-kv}{F} = e^{-kt/M}$$

$$F - kv = F(e^{-kt/M})$$

$$F = e^{kt/M}(F-kv)$$

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2/161. The velocity v of a parachute falling vertically satisfies the equation $v.\frac{dv}{dx} = g\left(1-\frac{v^2}{k^2}\right)$ where g and k are constants. If v and x are both initially zero, find v in terms of 'x'

$$v.\frac{dv}{dx} = g\left(1-\frac{v^2}{k^2}\right)$$

$$v\frac{dv}{dx} = \frac{g}{k^2}(k^2-v^2)$$

$$\int \frac{v\,dv}{k^2-v^2} = \int \frac{g}{k^2}\,dx$$

$$-\frac{1}{2}\int \frac{-2v}{k^2-v^2}\,dv = \frac{g}{k^2}\int dx$$

$$-\frac{1}{2}\log|k^2-v^2| + \log c = \frac{g}{k^2}x.$$

$$\log \frac{c}{\sqrt{k^2-v^2}} = \frac{gx}{k^2}$$

$$\frac{c}{\sqrt{k^2-v^2}} = e^{\frac{gx}{k^2}}$$

when $v = x = 0$

$$\frac{c}{\sqrt{k^2}} = e^0 \Rightarrow \boxed{c = k}$$

$$\frac{k}{\sqrt{k^2-v^2}} = e^{\frac{gx}{k^2}}$$

Squaring on both sides

$$\frac{k^2}{k^2-v^2} = e^{\frac{2gx}{k^2}}$$

$$k^2 \cdot e^{-\frac{2gx}{k^2}} = k^2 - v^2$$

$$v^2 = k^2 - k^2 e^{-2gx/k^2}$$

$$v^2 = k^2(1 - e^{-2gx/k^2})$$

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3/161 Find the equation of the curve whose slope is $\frac{y-1}{x^2+x}$ and which passes through the point (1, 0)

Given that $\frac{dy}{dx} = \frac{y-1}{x^2+x}$

$$\frac{dy}{y-1} = \frac{dx}{x^2+x}$$

$$\int \frac{dy}{y-1} = \int \frac{dx}{(x+\frac{1}{2})^2 - \frac{1}{4}}$$

$$\log(y-1) = \frac{1}{2(\frac{1}{2})}\log\left(\frac{x+\frac{1}{2}-\frac{1}{2}}{x+\frac{1}{2}+\frac{1}{2}}\right) + \log c$$

$$\log(y-1) = \log\left(\frac{x}{x+1}\right) + \log c$$

$$y-1 = \frac{xc}{x+1}$$

It passes through $(1,0)$

$$-1 = \frac{c}{1+1} \Rightarrow \boxed{c=-2}$$

$\therefore$ The Required Solution is

$$y-1 = \frac{-2x}{x+1}$$

$$y = \frac{-2x}{x+1}+1$$

$$y = \frac{-2x+x+1}{x+1}$$

$$\boxed{y = \frac{1-x}{1+x}}$$

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Homogeneous differential Equations:

Homogeneous function of degree n:

A function $f(x,y)$ is said to be a homogeneous function of degree n in the variables x and y if $f(tx,ty) = t^n f(x,y)$ for some $n \in R$ for all suitably restricted x, y and t. This is known as Euler's homogeneity.

Homogeneous Differential equation:-

An ordinary differential equation is said to be in homogeneous form, if the equation is written as $\frac{dy}{dx} = f\left(\frac{y}{x}\right)$

Solution of homogeneous differential eqn:

* $\frac{dy}{dx} = f\left(\frac{y}{x}\right)$

  (or)

  $\frac{dy}{dx} = \frac{f_1(x,y)}{f_2(x,y)}$

* Put $y = vx$, $\frac{dy}{dx} = v + x\frac{dv}{dx}$
* Thus the eqn becomes, $x\frac{dv}{dx} = g(v)$ -
* Solve, using variable separable method.
* Finally put $v = y/x$.
* This leads to the following result.

Another form of homogeneous differential equation :-

$$\frac{dx}{dy} = \frac{f_1(x,y)}{f_2(x,y)}$$

Put $x = vy$ $\quad \frac{dx}{dy} = v + y \cdot \frac{dv}{dy}$

Ex : 10.6

Solve the following differential equations :-

$\frac{1}{165}$ $\left[x + y\cos\left(\frac{y}{x}\right)\right]dx = x\cos\left(\frac{y}{x}\right) \cdot dy$

$$\frac{dy}{dx} = \frac{x + y\cos\left(\frac{y}{x}\right)}{x\cos\left(\frac{y}{x}\right)}$$

This is the homogeneous equation.

Put $y = vx$ $\quad \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$v + x\frac{dv}{dx} = \frac{x + vx\cos v}{x\cos v}$$

$$= \frac{\cancel{x}(1 + v\cos v)}{\cancel{x}\cos v}$$

$$x\frac{dv}{dx} = \frac{1 + v\cos v}{\cos v} - v$$

$$= \frac{1 + \cancel{v\cos v} - \cancel{v\cos v}}{\cos v}$$

$$x\frac{dv}{dx} = \frac{1}{\cos v}$$

$$\int \cos v \, dv = \int \frac{dx}{x}$$

$$\sin v = \log x + \log c$$

$$\sin\frac{y}{x} = \log xc$$

$$xc = e^{\sin y/x}$$

$$x = ce^{\sin y/x}.$$

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$\frac{2}{165}$ $(x^3 + y^3)dy - x^2y\,dx = 0$

$$(x^3 + y^3)dy = x^2y\,dx$$

$$\frac{dy}{dx} = \frac{x^2y}{x^3 + y^3}$$

It is a homogeneous diff. eqn.

Put $y = vx$ $\quad \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$v+x\frac{dv}{dx} = \frac{x^2 vx}{x^3+v^3x^3} = \frac{x^3 v}{x^3(1+v^3)}$$

$$v+x\frac{dv}{dx} = \frac{v}{1+v^3}$$

$$x\frac{dv}{dx} = \frac{v}{1+v^3} - v$$

$$= \frac{v-v-v^4}{1+v^3}$$

$$x\frac{dv}{dx} = \frac{-v^4}{1+v^3}$$

$$\int \frac{1+v^3}{v^4}dv = -\int \frac{dx}{x}$$

$$\int\left(\frac{1}{v^4}+\frac{1}{v}\right)dv = -\int\frac{dx}{x}$$

$$-\frac{1}{3v^3}+\log v = -\log x + \log c$$

$$\log v + \log x + \log c = \frac{1}{3v^3}$$

$$\log vxc = \frac{1}{3v^3}$$

$$\log\left(\frac{y}{x}\cdot x\cdot c\right) = \frac{x^3}{3y^3}$$

$$\log cy = \frac{x^3}{3y^3}$$

$$cy = e^{x^3/3y^3}$$

$$y = c\cdot e^{x^3/3y^3}$$

3/166 $$y e^{x/y}dx = (xe^{x/y}+y)dy$$

$$\frac{dx}{dy} = \frac{xe^{x/y}+y}{ye^{x/y}}$$

It is a homogeneous diff. eqn.

Put $x = vy$ $\quad \frac{dx}{dy} = v + y\cdot\frac{dv}{dy}$

$$v+y\frac{dv}{dy} = \frac{vye^v+y}{ye^v} = \frac{ve^v+1}{e^v}$$

$$y\frac{dv}{dy} = \frac{ve^v+1}{e^v} - v$$

$$= \frac{\cancel{ve^v}+1-\cancel{ve^v}}{e^v}$$

$$y\frac{dv}{dy} = \frac{1}{e^v}$$

$$\int e^v dv = \int\frac{dy}{y}$$

$$e^v = \log y + \log c$$

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$$e^{v} = \log yc$$

$$e^{x/y} = \log yc$$

4/166. Solve : $2xy\,dx + (x^2+2y^2)\,dy = 0$

$$(x^2+2y^2)\,dy = -2xy\,dx$$

$$\frac{dy}{dx} = \frac{-2xy}{x^2+2y^2}$$

It is a homogeneous diff. eqn

Put $y = vx \quad \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$v + x\frac{dv}{dx} = \frac{-2xvx}{x^2+2v^2x^2}$$

$$v + x\frac{dv}{dx} = \frac{-2\cancel{x^2}v}{\cancel{x^2}(1+2v^2)} = \frac{-2v}{1+2v^2}$$

$$x\frac{dv}{dx} = \frac{-2v}{1+2v^2} - v$$

$$= \frac{-2v - v - 2v^3}{1+2v^2}$$

$$= \frac{-3v - 2v^3}{1+2v^2}$$

$$\int \frac{1+2v^2}{2v^3+3v}\,dv = -\int \frac{dx}{x}$$

$$\frac{1}{3}\int \frac{3+6v^2}{2v^3+3v}\,dv = -\int \frac{dx}{x}$$

$$\frac{1}{3}\log|2v^3+3v| = -\log x + \log c$$

$$\log|2v^3+3v|^{1/3} = \log\frac{c}{x}.$$

$$(2v^3+3v)^{1/3} = \frac{c}{x}$$

$$2v^3+3v = \frac{c}{x^3}$$

$$\frac{2y^3}{x^3} + 3\frac{y}{x} = \frac{c}{x^3}$$

$$\frac{2y^3+3x^2y}{x^3} = \frac{c}{x^3}$$

$$\boxed{2y^3+3x^2y = c}$$

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$\frac{5}{166}$ $(y^2-2xy)\,dx = (x^2-2xy)\,dy$

$$\frac{dy}{dx} = \frac{y^2-2xy}{x^2-2xy}$$

It is a homogeneous differential equation

Put $y = vx \quad \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$v + x\frac{dv}{dx} = \frac{v^2x^2 - 2xvx}{x^2-2xvx}$$

$$= \frac{\cancel{x^2}(v^2-2v)}{\cancel{x^2}(1-2v)}$$

$$x\frac{dv}{dx} = \frac{v^2-2v}{1-2v} - v$$

$$= \frac{v^2-2v-v+2v^2}{1-2v}$$

$$x\frac{dv}{dx} = \frac{3v^2-3v}{1-2v} = \frac{3(v^2-v)}{1-2v}$$

$$\int \frac{1-2v}{v^2-v} = \int 3\frac{dx}{x}$$

$$-\log|v^2-v| = 3\log x + \log c$$

$$\log\frac{1}{v^2-v} = \log x^3c$$

$$\frac{1}{v^2-v} = x^3c$$

$$v^2 - v = \frac{c}{x^3}$$

$$\frac{y^2}{x^2} - \frac{y}{x} = \frac{c}{x^3}$$

$$y^2 - xy = \frac{c}{x}$$

$$xy^2 - x^2y = c$$

$$\boxed{xy(y-x) = c}$$

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$\frac{6}{166}$ $x\frac{dy}{dx} = y - x\cos^2\left(\frac{y}{x}\right)$

$$\frac{dy}{dx} = \frac{y - x\cos^2\left(\frac{y}{x}\right)}{x}$$

Put $y = vx \quad \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$v + x\frac{dv}{dx} = \frac{vx - x\cos^2 v}{x}$$

$$v + x\frac{dv}{dx} = \frac{\cancel{x}(v - \cos^2 v)}{\cancel{x}}$$

$$\cancel{v} + x\frac{dv}{dx} = \cancel{v} - \cos^2 v$$

$$\frac{dv}{\cos^2 v} = -\frac{dx}{x}$$

$$\int \sec^2 v\, dv = -\int \frac{dx}{x}$$

$$\tan v = -\log x + \log c$$

$$\tan\frac{y}{x} = \log\frac{c}{x}$$

$$\frac{c}{x} = e^{\tan y/x}$$

$$\boxed{xe^{\tan y/x} = c}$$

7/166. $(1+3e^{y/x})dy + 3e^{y/x}(1-\frac{y}{x})dx = 0$ given that $y=0$ when $x=1$.

$$(1+3e^{y/x})dy = -3e^{y/x}(1-\frac{y}{x})dx$$

$$\frac{dy}{dx} = \frac{3e^{y/x}(\frac{y}{x}-1)}{1+3e^{y/x}}$$

It is a homogeneous differential equation

Put $y = vx$ $\quad \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$v + x\frac{dv}{dx} = \frac{3e^v(v-1)}{1+3e^v}$$

$$x\frac{dv}{dx} = \frac{3e^v(v-1)}{1+3e^v} - v$$

$$= \frac{\cancel{3ve^v} - 3e^v - v - \cancel{3ve^v}}{1+3e^v}$$

$$= \frac{-(v+3e^v)}{1+3e^v}$$

$$\int \frac{1+3e^v}{v+3e^v}dv = \int -\frac{dx}{x}$$

$$\log|v+3e^v| = -\log x + \log c$$

$$v + 3e^v = \frac{c}{x}$$

$$\frac{y}{x} + 3e^{y/x} = \frac{c}{x}$$

$$y + 3xe^{y/x} = c$$

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Given that $y=0$ when $x=1$

$$0+3(1)e^{0} = c$$

$$\boxed{c=3}$$

$\therefore$ The Required Solution is

$$y+3xe^{y/x} = 3.$$

8/166. $(x^2+y^2)dy = xy\,dx$. It is given that $y(1)=1$ and $y(x_0)=e$. Find the value of x_0

$$(x^2+y^2)dy = xy\,dx$$

$$\frac{dy}{dx} = \frac{xy}{x^2+y^2}$$

This is a differential equation.

Put $y=vx$ $\quad \frac{dy}{dx} = v+x\frac{dv}{dx}$

$$v+x\frac{dv}{dx} = \frac{x\,vx}{x^2+v^2x^2} = \frac{v}{1+v^2}$$

$$x\frac{dv}{dx} = \frac{v}{1+v^2} - v$$

$$= \frac{v-v-v^3}{1+v^2} = \frac{-v^3}{1+v^2}$$

$$\int \frac{1+v^2}{v^3}dv = -\int\frac{dx}{x}$$

$$\int \frac{1}{v^3}dv + \int\frac{1}{v}dv = -\int\frac{1}{x}dx$$

$$\frac{-1}{2v^2} + \log v = -\log x + \log c$$

$$\frac{1}{2v^2} = \log vxc$$

$$vxc = e^{\frac{1}{2v^2}}$$

$$\frac{y}{\cancel{x}}\cdot\cancel{x}\cdot c = e^{\frac{x^2}{2y^2}}$$

$$y = ce^{x^2/2y^2}$$

when $x=1$, $y=1$

$$1 = c\cdot e^{1/2}$$

$$c = e^{-1/2} \qquad \boxed{c = 1/\sqrt{e}}$$

$\therefore$ The Required Solution is

$$y = \frac{1}{\sqrt{e}}e^{x^2/2y^2}$$

when $x=x_0$ $\quad y=e$

$$e = \frac{1}{\sqrt{e}}e^{\frac{x_0^2}{2e^2}}$$

$$e\sqrt{e} = e^{x_0^2/2e^2}$$

$$e^{3/2} = e^{\frac{1}{2}\left(\frac{x_0^2}{e^2}\right)} \Rightarrow \frac{3}{\cancel{2}} = \frac{1}{\cancel{2}}\left(\frac{x_0^2}{e^2}\right)$$

$$x_0^2 = 3e^2 \qquad \boxed{x_0 = \sqrt{3}e}$$

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Solve : $(x^2 - 3y^2)dx + 2xy\,dy = 0$

$$2xy\,dy = -(x^2 - 3y^2)\,dx$$

$$\frac{dy}{dx} = \frac{3y^2 - x^2}{2xy}$$

It is a homogeneous diff. equation.

Put $y = vx$ $\quad \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$v + x\frac{dv}{dx} = \frac{3v^2x^2 - x^2}{2xvx}$$

$$= \frac{3v^2 - 1}{2v}$$

$$x\frac{dv}{dx} = \frac{3v^2-1}{2v} - v$$

$$= \frac{3v^2 - 1 - 2v^2}{2v}$$

$$= \frac{v^2-1}{2v}$$

$$\int \frac{2v}{v^2-1}\,dv = \int \frac{dx}{x}$$

$$\log|v^2-1| = \log x + \log c$$

$$\log|v^2-1| = \log xc$$

$$\log\left|\frac{y^2}{x^2} - 1\right| = \log xc$$

$$\frac{y^2 - x^2}{x^2} = xc$$

$$y^2 - x^2 = x^3 c.$$

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Solve : $(y + \sqrt{x^2+y^2})dx - x\,dy = 0 \qquad y(1) = 0$

$$(y + \sqrt{x^2+y^2})\,dx = x\,dy$$

$$\frac{dy}{dx} = \frac{y + \sqrt{x^2+y^2}}{x}$$

It is a homogeneous diff. eqn.

Put $y = vx$ $\quad \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$v + x\frac{dv}{dx} = \frac{vx + \sqrt{x^2 + v^2x^2}}{x}$$

$$v + x\frac{dv}{dx} = v + \sqrt{1+v^2}$$

$$x\frac{dv}{dx} = \sqrt{1+v^2}$$

$$\frac{dv}{\sqrt{1+v^2}} = \frac{dx}{x}$$

$$\int \frac{dv}{\sqrt{1+v^2}} = \int \frac{dx}{x} \qquad \int \frac{dx}{\sqrt{x^2+a^2}} = \log|x+\sqrt{x^2+a^2}|+c$$

$$\log|v+\sqrt{1+v^2}| = \log x + \log c$$

$$v+\sqrt{1+v^2} = xc$$

$$\frac{y}{x} + \sqrt{1+\frac{y^2}{x^2}} = xc$$

$$\frac{y}{x} + \frac{\sqrt{x^2+y^2}}{x} = xc$$

$$y+\sqrt{x^2+y^2} = x^2 c$$

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when $x = 1, \quad y = 0$

$$0+\sqrt{0+1} = c$$

$$\boxed{c=1}$$

$\therefore$ The Required Solution is

$$y+\sqrt{x^2+y^2} = x^2$$

10.20 / 165 Solve : $y^2+x^2\frac{dy}{dx} = xy\frac{dy}{dx}$

$$\frac{dy}{dx}(x^2-xy) = -y^2$$

$$\frac{dy}{dx} = \frac{y^2}{xy-x^2}$$

It is a homogeneous differential equation.

Put $y = vx \qquad \frac{dy}{dx} = v+x\frac{dv}{dx}$

$$v+x\frac{dv}{dx} = \frac{v^2x^2}{x^2v-x^2} = \frac{v^2\cancel{x^2}}{\cancel{x^2}(v-1)}$$

$$x\frac{dv}{dx} = \frac{v^2}{v-1} - v$$

$$= \frac{\cancel{v^2}-\cancel{v^2}+v}{v-1}$$

$$\int \frac{v-1}{v}\,dv = \int \frac{dx}{x}$$

$$\int\left(1-\frac{1}{v}\right)dv = \int\frac{dx}{x}$$

$$v - \log v = \log x + \log c$$

$$\log vxc = v$$

$$\log \frac{y}{\cancel{x}}\cdot\cancel{x}\cdot c = \frac{y}{x}$$

$$yc = e^{y/x}$$

$$y = ce^{y/x}$$

10.21 / 165 Solve : $(1+2e^{x/y})dx + 2e^{x/y}(1-\frac{x}{y})dy = 0$

$$2e^{x/y}(1-\frac{x}{y})dy = -(1+2e^{x/y})dx.$$

$$\frac{dx}{dy} = \frac{2e^{x/y}(\frac{x}{y}-1)}{1+2e^{x/y}}$$

It is a homogeneous differential equation.

Put $x = vy$ $\frac{dx}{dy} = v + y \cdot \frac{dv}{dy}$

$$v + y\frac{dv}{dy} = \frac{2e^{v}(v-1)}{1+2e^{v}}$$

$$y\frac{dv}{dy} = \frac{2ve^{v}-2e^{v}}{1+2e^{v}} - v$$

$$= \frac{\cancel{2ve^{v}} - 2e^{v} - v - \cancel{2ve^{v}}}{1+2e^{v}}$$

$$y\frac{dv}{dy} = \frac{-(v+2e^{v})}{1+2e^{v}}$$

$$\int \frac{1+2e^{v}}{v+2e^{v}}dv = -\int \frac{dy}{y}$$

$$\log v+2e^{v} = -\log y + \log c$$

$$v+2e^{v} = \frac{c}{y}$$

$$\frac{x}{y} + 2e^{x/y} = \frac{c}{y}$$

$$\boxed{x + 2ye^{x/y} = c}$$

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10.19 / 164 Solve : $(2x+3y)dx + (y-x)dy = 0$

$$(y-x)dy = -(2x+3y)dx$$

$$\frac{dy}{dx} = \frac{2x+3y}{x-y}$$

Put $y = vx$ $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$$v + x\frac{dv}{dx} = \frac{2x+3vx}{x-vx}$$

$$= \frac{\cancel{x}(2+3v)}{\cancel{x}(1-v)}$$

$$x\frac{dv}{dx} = \frac{2+3v}{1-v} - v$$

$$= \frac{2+3v-v+v^2}{1-v}$$

$$x\frac{dv}{dx} = \frac{2+2v+v^2}{1-v}$$

$$\int \frac{1-v}{v^2+2v+2}\,dv = \int \frac{dx}{x} \quad \text{——— } (1)$$

consider $\int \frac{1-v}{v^2+2v+2}\,dv$

$$1-v = A\cdot\frac{d}{dx}(v^2+2v+2) + B$$

$$1-v = A(2v+2) + B$$

Equating the co.eff of v and constant.

$2A = -1$ $\qquad$ $2A + B = 1$

$\boxed{A = -\frac{1}{2}}$ $\qquad$ $-1 + B = 1$

$\boxed{B = 2}$

$$\int \frac{1-v}{v^2+2v+2}\,dv = \int -\frac{1}{2}\frac{(2v+2)}{v^2+2v+2}\,dv + \int \frac{2\,dv}{v^2+2v+2}$$

$$= -\frac{1}{2}\log|v^2+2v+2| + \int \frac{2\,dv}{(v+1)^2 - 1 + 2}$$

$$= -\frac{1}{2}\log|v^2+2v+2| + 2\int \frac{dv}{(v+1)^2+1}$$

$$= -\frac{1}{2}\log|v^2+2v+2| + 2\cdot\frac{1}{1}\tan^{-1}\left(\frac{v+1}{1}\right)$$

$$= -\frac{1}{2}\log|v^2+2v+2| + 2\tan^{-1}(v+1) \quad \text{——— } (2)$$

Sub (2) in (1)

$$-\frac{1}{2}\log|v^2+2v+2| + 2\tan^{-1}(v+1) = \log x + \log c$$

$$2\tan^{-1}(v+1) = \log xc + \log\sqrt{v^2+2v+2}$$

$$2\tan^{-1}(v+1) = \log xc\sqrt{v^2+2v+2}$$

$$2\tan^{-1}\left(\frac{y}{x}+1\right) = \log\left[xc\sqrt{\frac{y^2}{x^2}+2\frac{y}{x}+2}\right]$$

$$2\tan^{-1}\left(\frac{x+y}{x}\right) = \log\left[\cancel{x}c\,\frac{\sqrt{y^2+2xy+2x^2}}{\cancel{x}}\right]$$

$$2\tan^{-1}\left(\frac{x+y}{x}\right) = \log c\sqrt{2x^2+2xy+y^2}$$

[OR]

$$2\tan^{-1}\left(\frac{x+y}{x}\right) = \frac{1}{2}\log(2x^2+2xy+y^2) + \log c$$

$$4\tan^{-1}\left(\frac{x+y}{x}\right) = \log|2x^2+2xy+y^2| + K$$

First order linear differential equations:

A first order linear differential equation of the form

$$\frac{dy}{dx} + Py = Q$$

where P and Q are the functions of x only. Here no product of y and its derivative $\frac{dy}{dx}$ occur and the dependent variable y and its derivative w.r.to independent variable x occurs only in the first degree.

Integrating factor $= e^{\int P dx}$

General Solution:

$$y e^{\int P dx} = \int Q e^{\int P dx} \cdot dx + c$$

Another form:-

$$\frac{dx}{dy} + Px = Q$$

Integrating factor $= e^{\int P dy}$

General solution:-

$$x e^{\int P dy} = \int Q e^{\int P dy} dy + c.$$

10.22 / 167 Solve : $\frac{dy}{dx} + 2y = e^{-x}$

$P = 2 \qquad Q = e^{-x}$

$\int P dx = \int 2 dx = 2x$

$I.F = e^{\int P dx} = e^{2x}$

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General Solution:-

$$y e^{\int P dx} = \int Q e^{\int P dx} dx + c$$

$$y e^{2x} = \int e^{-x} \cdot e^{2x} dx + c$$

$$y e^{2x} = \int e^{x} dx + c$$

$$y e^{2x} = e^{x} + c.$$

10.23 / 167 Solve : $[y(1-x\tan x)+x^2\cos x]dx - x\,dy = 0$

$$x\,dy = [y(1-x\tan x)+x^2\cos x]dx$$

$$x\frac{dy}{dx} = y(1-x\tan x)+x^2\cos x.$$

$$x\frac{dy}{dx} + (x\tan x-1)y = x^2\cos x$$

$\div\ x$

$$\frac{dy}{dx} + \left(\tan x-\frac{1}{x}\right)y = x\cos x$$

$$P = \tan x - \frac{1}{x} \qquad Q = x\cos x$$

$$\int P\,dx = \int\left(\tan x - \frac{1}{x}\right)dx = \log\sec x - \log x$$

$$= \log\frac{\sec x}{x} = \log\frac{1}{x\cos x}$$

$$e^{\int P\,dx} = e^{\log\frac{1}{x\cos x}} = \frac{1}{x\cos x}$$

General Solution :

$$y\,e^{\int P\,dx} = \int Q\,e^{\int P\,dx}\,dx + c$$

$$\frac{y}{x\cos x} = \int x\cos x\cdot\frac{1}{x\cos x}\cdot dx + c$$

$$\frac{y}{x\cos x} = \int dx + c$$

$$\frac{y}{x\cos x} = x + c \qquad [\text{OR}]$$

$$y = (x+c)\,x\cos x.$$

10.24 / 168 Solve : $\frac{dy}{dx} + 2y\cot x = 3x^2\operatorname{cosec}^2 x$

$$P = 2\cot x \qquad Q = 3x^2\operatorname{cosec}^2 x$$

$$\int P\,dx = \int 2\cot x\,dx = 2\log\sin x = \log\sin^2 x.$$

$$I.F = e^{\int P\,dx} = e^{\log\sin^2 x} = \sin^2 x.$$

General Solution :

$$y\,e^{\int P\,dx} = \int Q\,e^{\int P\,dx}\,dx + c$$

$$y\sin^2 x = \int 3x^2\cancel{\operatorname{cosec}^2 x}\cdot\cancel{\sin^2 x}\,.dx$$

$$y\sin^2 x = \frac{3x^3}{3} + c$$

$$y\sin^2 x = x^3 + c$$

10.25 / 168 Solve : $(1+x^3)\frac{dy}{dx}+6x^2 \cdot y = 1+x^2$

$\div (1+x^3)$

$$\frac{dy}{dx}+\frac{6x^2}{1+x^3}y=\frac{1+x^2}{1+x^3}$$

$$P=\frac{6x^2}{1+x^3} \qquad Q=\frac{1+x^2}{1+x^3}$$

$$\int P\,dx = 2\int\frac{3x^2}{1+x^3}dx = 2\log(1+x^3) = \log(1+x^3)^2$$

$$I.F = e^{\int P\,dx} = e^{\log(1+x^3)^2} = (1+x^3)^2$$

General Solution :

$$y e^{\int P\,dx} = \int Q e^{\int P\,dx}.dx+c$$

$$y(1+x^3)^2 = \int\frac{1+x^2}{1+x^3}.(1+x^3)^2.dx+c$$

$$y(1+x^3)^2 = \int(1+x^2)(1+x^3)\,dx+c$$

$$= \int(1+x^2+x^3+x^5)dx+c$$

$$y(1+x^3)^2 = x+\frac{x^3}{3}+\frac{x^4}{4}+\frac{x^6}{6}+c.$$

10.26 / 169 Solve : $ye^y dx = (y^3+2xe^y)dy$

$$\frac{dx}{dy}=\frac{y^3+2xe^y}{ye^y}$$

$$\frac{dx}{dy}=\frac{y^2}{e^y}+\frac{2}{y}x$$

$$\frac{dx}{dy}-\frac{2}{y}x = y^2e^{-y}$$

$$P=-\frac{2}{y} \qquad Q=y^2.e^{-y}$$

$$\int P\,dy = \int -\frac{2}{y}dy = -2\log y = \log\frac{1}{y^2}$$

$$I.F = e^{\int P\,dy} = e^{\log\frac{1}{y^2}} = \frac{1}{y^2}$$

General Solution :

$$\cancel{x}e^{\int P\,dy} = \int Q e^{\int P\,dy}dy+c$$

$$\frac{x}{y^2}=\int y^2e^{-y}.\frac{1}{y^2}dy+c$$

$$\frac{x}{y^2} = -e^{-y}+c$$

$$\frac{x}{y^2}+e^{-y}=c$$

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Ex: 10.7

Solve the following differential equations:

1. $\cos x \frac{dy}{dx} + y\sin x = 1$

$\div \cos x \qquad \frac{dy}{dx} + y\tan x = \sec x$

$P = \tan x \qquad Q = \sec x$

$\int P dx = \int \tan x . dx = \log \sec x$

$I.F = e^{\int P dx} = \sec x$

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General Solution :-

$$y e^{\int P dx} = \int Q e^{\int P dx} . dx + c$$

$$y \sec x = \int \sec x . \sec x \, dx + c$$

$$y \sec x = \int \sec^2 x \, dx + c$$

$$y \sec x = \tan x + c.$$

[OR]

$$y = \sin x + c \cos x.$$

2. $(1-x^2)\frac{dy}{dx} - xy = 1$

$\div (1-x^2) \qquad \frac{dy}{dx} - \frac{x}{1-x^2} y = \frac{1}{1-x^2}$

$P = \frac{-x}{1-x^2} \qquad Q = \frac{1}{1-x^2}$

$\int P dx = \frac{1}{2}\int \frac{-2x}{1-x^2} dx = \frac{1}{2}\log(1-x^2) = \log\sqrt{1-x^2}$

$I.F = e^{\int P dx} = \sqrt{1-x^2}$

General Solution :-

$$y e^{\int P dx} = \int Q e^{\int P dx} dx + c$$

$$y\sqrt{1-x^2} = \int \frac{1}{1-x^2}\sqrt{1-x^2}\, dx$$

$$y\sqrt{1-x^2} = \int \frac{1}{\sqrt{1-x^2}} dx$$

$$y\sqrt{1-x^2} = \sin^{-1} x + c$$

[OR]

$$y = \frac{\sin^{-1} x}{\sqrt{1-x^2}} + \frac{c}{\sqrt{1-x^2}}$$

3. / 169

Solve : $\frac{dy}{dx} + \frac{y}{x} = \sin x$

$P = \frac{1}{x}$ $\quad Q = \sin x$

$\int P dx = \int \frac{1}{x} dx = \log x$

$I.F = e^{\int P dx} = e^{\log x} = x$

General solution :-

$y e^{\int Pdx} = \int Q e^{\int Pdx} dx + c$

$xy = \int \sin x \cdot x \, dx$

$xy = \int x \sin x \, dx$

$xy = [x(-\cos x) - 1(-\sin x)] + c$

$xy = -x\cos x + \sin x + c$

$xy + x\cos x - \sin x = c$

[OR]

$x(y + \cos x) = \sin x + c.$

4 / 169

$(x^2+1)\frac{dy}{dx} + 2xy = \sqrt{x^2+4}$

$\div (x^2+1)$

$\frac{dy}{dx} + \frac{2x}{x^2+1} y = \frac{\sqrt{x^2+4}}{x^2+1}$

$P = \frac{2x}{x^2+1}$ $\quad Q = \frac{\sqrt{x^2+4}}{x^2+1}$

$\int P dx = \int \frac{2x}{x^2+1} dx = \log(x^2+1)$

$e^{\int Pdx} = e^{\log(x^2+1)} = x^2+1$

General solution :

$y e^{\int Pdx} = \int Q e^{\int Pdx} dx + c$

$y(x^2+1) = \int \frac{\sqrt{x^2+4}}{x^2+1} \cdot (x^2+1) \, dx$

$y(x^2+1) = \int \sqrt{x^2+4} \, dx$

$\int \sqrt{x^2+a^2} dx = \left[\frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log|x+\sqrt{x^2+a^2}| + c\right]$

$y(x^2+1) = \frac{x}{2}\sqrt{x^2+4} + \frac{4}{2}\log|x+\sqrt{x^2+4}| + c$

$y(x^2+1) = \frac{x}{2}\sqrt{x^2+4} + 2\log|x+\sqrt{x^2+4}| + c$

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5/169

$$(2x - 10y^3)dy + y\,dx = 0$$

$$y\,dx = (10y^3 - 2x)dy$$

$$y\frac{dx}{dy} = 10y^3 - 2x$$

$$\frac{dx}{dy} = 10y^2 - \frac{2}{y}x$$

$$\frac{dx}{dy} + \frac{2}{y}x = 10y^2$$

$$P = \frac{2}{y} \qquad Q = 10y^2$$

$$\int P\,dy = \int \frac{2}{y}dy = 2\log y = \log y^2$$

$$I.F = e^{\int P\,dy} = e^{\log y^2} = y^2$$

General Solution:

$$x \cdot e^{\int P\,dy} = \int Q e^{\int P\,dy}\,dy + c$$

$$xy^2 = \int 10y^2 \cdot y^2\,dy + c$$

$$xy^2 = 10\int y^4\,dy + c$$

$$xy^2 = 10\frac{y^5}{5} + c$$

$$xy^2 = 2y^5 + c.$$

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6/169

$$x\sin x\frac{dy}{dx} + (x\cos x + \sin x)y = \sin x$$

$\div (x\sin x)$

$$\frac{dy}{dx} + \left(\frac{x\cos x + \sin x}{x\sin x}\right)y = \frac{1}{x}$$

$$\frac{dy}{dx} + \left(\cot x + \frac{1}{x}\right)y = \frac{1}{x}$$

$$P = \cot x + \frac{1}{x} \qquad Q = \frac{1}{x}$$

$$\int P\,dx = \int\left(\cot x + \frac{1}{x}\right)dx = \log\sin x + \log x$$

$$= \log x\sin x$$

$$I.F = e^{\int P\,dx} = x\sin x$$

General Solution :-

$$ye^{\int P\,dx} = \int Q e^{\int P\,dx}\,dx + c$$

$$xy\sin x = \int \frac{1}{\cancel{x}} \cdot \cancel{x}\sin x\,dx + c$$

$$xy\sin x = -\cos x + c$$

$$xy\sin x + \cos x = c$$

7/169. $\left(y - e^{\sin^{-1} x}\right)\frac{dx}{dy} + \sqrt{1-x^2} = 0$

$\left(y - e^{\sin^{-1} x}\right)\frac{dx}{dy} = -\sqrt{1-x^2}$

$-\sqrt{1-x^2}\cdot\frac{dy}{dx} = y - e^{\sin^{-1}x}$

$\sqrt{1-x^2}\,\frac{dy}{dx} = -y + e^{\sin^{-1}x}$

$\div\left(\sqrt{1-x^2}\right)$

$\frac{dy}{dx} + \frac{1}{\sqrt{1-x^2}}\,y = \frac{e^{\sin^{-1}x}}{\sqrt{1-x^2}}$

$P = \frac{1}{\sqrt{1-x^2}}$ $\quad Q = \frac{e^{\sin^{-1}x}}{\sqrt{1-x^2}}$

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$\int P\,dx = \int \frac{1}{\sqrt{1-x^2}}dx = \sin^{-1}x$

$e^{\int P dx} = e^{\sin^{-1}x}$

General solution :

$y e^{\int P dx} = \int Q e^{\int P dx} dx + c$

$y e^{\sin^{-1}x} = \int \frac{e^{\sin^{-1}x}}{\sqrt{1-x^2}}\cdot e^{\sin^{-1}x}\cdot dx + c$

$u = \sin^{-1}x$

$du = \frac{1}{\sqrt{1-x^2}}dx$

$y e^{\sin^{-1}x} = \int (e^u)^2\cdot du + c$

$y e^{\sin^{-1}x} = \int e^{2u} du + c$

$y e^{\sin^{-1}x} = \frac{e^{2u}}{2} + c$

$y e^{\sin^{-1}x} = \frac{e^{2\sin^{-1}x}}{2} + c.$

8/169 $\frac{dy}{dx} + \frac{y}{(1-x)\sqrt{x}} = 1-\sqrt{x}$

$P = \frac{1}{\sqrt{x}(1-x)}$ $\quad Q = 1-\sqrt{x}$

$\int P dx = \int \frac{1}{\sqrt{x}(1-x)}dx$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}dx$

$2du = \frac{dx}{\sqrt{x}}$

$= \int \frac{2\,du}{1-u^2}$

$= \cancel{2}\left[\frac{1}{\cancel{2}(1)}\log\left|\frac{1+u}{1-u}\right|\right]$

$= \log\left(\frac{u+1}{1-u}\right)$

$e^{\int P dx} = \frac{u+1}{1-u} = \frac{1+\sqrt{x}}{1-\sqrt{x}}$

General Solution :-

$y e^{\int P dx} = \int Q\cdot e^{\int P dx} dx + c$

$$y\left(\frac{1+\sqrt{x}}{1-\sqrt{x}}\right) = \int \cancel{1-\sqrt{x}}\cdot\left(\frac{1+\sqrt{x}}{\cancel{1-\sqrt{x}}}\right)dx$$

$$y\left(\frac{1+\sqrt{x}}{1-\sqrt{x}}\right) = x + \frac{2}{3}x^{3/2} + c$$

9/169 $(1+x+xy^2)\frac{dy}{dx} + (y+y^3) = 0$

$$(1+x+xy^2)\frac{dy}{dx} = -(y+y^3)$$

$$\frac{dx}{dy} = -\frac{(1+x+xy^2)}{y+y^3}$$

$$= \frac{-1}{y(1+y^2)} - \frac{x\cancel{(1+y^2)}}{y\cancel{(1+y^2)}}$$

$$\frac{dx}{dy} + \frac{x}{y} = \frac{-1}{y(1+y^2)}$$

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$$P = \frac{1}{y} \qquad Q = \frac{-1}{y(1+y^2)}$$

$$\int P\,dy = \int \frac{1}{y}dy = \log y$$

$$e^{\int P dy} = e^{\log y} = y$$

General Solution :-

$$x\cdot e^{\int P dy} = \int Q e^{\int P dy} dy + c$$

$$xy = \int \frac{-1}{y(1+y^2)}\cdot y\,dy$$

$$xy = \int \frac{-1}{1+y^2}dy$$

$$xy = -\tan^{-1}y + c$$

$$xy + \tan^{-1}y = c$$

10/169. $\frac{dy}{dx} + \frac{y}{x\log x} = \frac{\sin 2x}{\log x}$

$$P = \frac{1}{x\log x} \qquad Q = \frac{\sin 2x}{\log x}$$

$$\int P\,dx = \int \frac{1}{x\log x}dx = \log\log x$$

$$I.F = e^{\int P dx} = e^{\log\log x} = \log x.$$

General Solution :

$$y\,e^{\int P dx} = \int Q e^{\int P dx}dx + c$$

$$y\log x = \int \frac{\sin 2x}{\cancel{\log x}}\cdot \cancel{\log x}\,dx + c$$

$$y\log x = -\frac{\cos 2x}{2} + c$$

$$2y\log x + \cos 2x = c$$

11/169. $(x+a)\frac{dy}{dx} - 2y = (x+a)^4$

$\div (x+a)$

$$\frac{dy}{dx} - \frac{2}{x+a}y = (x+a)^3$$

$P = -\frac{2}{x+a}$ $\quad Q = (x+a)^3$

$$\int P\,dx = -2\int\frac{dx}{x+a} = -2\log(x+a) = \log\frac{1}{(x+a)^2}$$

$$I.F = e^{\int P dx} = \frac{1}{(x+a)^2}$$

General Solution :

$$y\,e^{\int P dx} = \int Q\,e^{\int P dx}dx + c$$

$$\frac{y}{(x+a)^2} = \int (x+a)^3 \cdot \frac{1}{(x+a)^2}dx + c$$

$$\frac{y}{(x+a)^2} = \int (x+a)\,dx$$

$$\frac{y}{(x+a)^2} = \frac{x^2}{2} + ax + c.$$

12/169 $\frac{dy}{dx} = \frac{\sin^2 x}{1+x^3} - \frac{3x^2}{1+x^3}y$

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$$\frac{dy}{dx} + \frac{3x^2}{1+x^3}y = \frac{\sin^2 x}{1+x^3}$$

$P = \frac{3x^2}{1+x^3}$ $\quad Q = \frac{\sin^2 x}{1+x^3}$

$$\int P\,dx = \int\frac{3x^2}{1+x^3}dx = \log(1+x^3)$$

$$I.F = e^{\int P dx} = (1+x^3)$$

General Solution :

$$y\,e^{\int P dx} = \int Q\,e^{\int P dx}dx + c$$

$$y(1+x^3) = \int\frac{\sin^2 x}{\cancel{1+x^3}}(\cancel{1+x^3})\,dx + c$$

$$y(1+x^3) = \int\frac{1-\cos 2x}{2}dx$$

$$y(1+x^3) = \frac{1}{2}\left[x - \frac{\sin 2x}{2}\right] + c$$

$$y(1+x^3) - \frac{1}{2}\left[x - \frac{\sin 2x}{2}\right] = c$$

13/169

$x\frac{dy}{dx}+y=x\log x$

$\div x \quad \frac{dy}{dx}+\frac{1}{x}y=\log x$

$P=\frac{1}{x} \qquad Q=\log x$

$\int P\,dx=\int\frac{1}{x}dx=\log x$

$I.F=e^{\int P\,dx}=e^{\log x}=x$

General Solution :-

$y e^{\int P\,dx}=\int Q e^{\int P\,dx}dx+c$

$xy=\int \log x\cdot x\,dx+C$

$xy=\int x\cdot\log x+C$

$\int u\,dv=uv-\int v\,du$

$u=\log x \qquad \int dv=\int x\,dx$

$du=\frac{1}{x}dx \qquad v=\frac{x^2}{2}$

$xy=\frac{x^2}{2}\log x-\int\frac{x^2}{2}\cdot\frac{1}{x}\cdot dx+c$

$xy=\frac{x^2}{2}\log x-\int\frac{1}{2}x\,dx+C$

$xy=\frac{x^2}{2}\log x-\frac{1}{2}\frac{x^2}{2}+C$

$xy=\frac{x^2}{2}\log x-\frac{1}{4}x^2+C.$

$xy=\frac{x^2}{2}\left[\log x-\frac{1}{2}\right]+C.$

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14/169.

$x\frac{dy}{dx}+2y-x^2\log x=0$

$\div x \quad \frac{dy}{dx}+\frac{2}{x}y=x\log x$

$P=\frac{2}{x} \qquad Q=x\log x.$

$\int P\,dx=\int\frac{2}{x}dx=2\log x=\log x^2$

$I.F=e^{\int P\,dx}=x^2$

General Solution :

$y e^{\int P\,dx}=\int Q e^{\int P\,dx}dx+c$

$\int u\,dv=uv-\int v\,du$

$u=\log x \qquad \int dv=x^3dx$

$du=\frac{1}{x}dx \qquad v=\frac{x^4}{4}$

$x^2y=\int x\log x\cdot x^2\cdot dx+c$

$x^3y=\int x^3\log x\cdot dx$

$=\frac{x^4}{4}\cdot\log x-\int\frac{x^4}{4}\cdot\frac{1}{x}dx$

$=\frac{x^4}{4}\log x-\frac{1}{4}\int x^3dx$

$=\frac{x^4}{4}\log x-\frac{1}{4}\cdot\frac{x^4}{4}+C.$

$x^3y=\frac{x^4}{4}\left[\log x-\frac{1}{4}\right]+C.$

15/169 $\frac{dy}{dx} + \frac{3y}{x} = \frac{1}{x^2}$ given that $y=2$ and $x=1$

$P = \frac{3}{x}$ $Q = \frac{1}{x^2}$

$\int P dx = \int \frac{3}{x} dx = 3\log x = \log x^3$

$I.F = e^{\int P dx} = e^{\log x^3} = x^3$

General Solution :

$y e^{\int P dx} = \int Q e^{\int P dx} dx + c$

$y x^3 = \int \frac{1}{x^2} \cdot x^3 \cdot dx + c$

$x^3 y = \int x \, dx + c$

$x^3 y = \frac{x^2}{2} + c.$

when $x = 1$, $y = 2$

$(1)(2) = \frac{1}{2} + c.$

$c = 2 - \frac{1}{2} = \frac{3}{2}$ $\boxed{c = 3/2}$

$\therefore$ The Required Solution is

$x^3 y = \frac{x^2}{2} + \frac{3}{2}$

$2x^3 y = x^2 + 3.$

Applications of first Order Ordinary differential equations :

10.27/170 The growth of a Population is Proportional to the number Present. If the Population of a Colony doubles in 50 years, in how many years will the Population become triple ?

Population (A)	Time (t)
A_0	0
$2A_0$	50
$3A_0$	?

Let A be the Population at any time 't'

$\frac{dA}{dt} \propto A$

$\frac{dA}{dt} = kA$

$$A = ce^{kt} \quad \text{——— ①}$$

when $t = 0 \quad A = A_0$

$$A_0 = ce^0 \Rightarrow \boxed{c = A_0}$$

① becomes,

$$A = A_0 e^{kt}$$

when $t = 50 \quad A = 2A_0$

$$2A_0 = A_0 e^{50k}$$

$$e^{50k} = 2$$

$$50k = \log 2$$

$$k = \frac{1}{50}\log 2$$

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when $t = t_1 \quad A = 3A_0$

$$3\cancel{A_0} = \cancel{A_0} e^{kt_1}$$

$$e^{kt_1} = 3$$

$$kt_1 = \log 3$$

$$t_1 = \frac{1}{k}\log 3$$

$$t_1 = \frac{50}{\log 2}.\log 3 = 50\left(\frac{\log 3}{\log 2}\right)$$

∴ The population is tripled in $50\left(\frac{\log 3}{\log 2}\right)$ yrs.

10.28 / 171 A radioactive isotope has an initial mass 200 mg, which two years later is 150 mg. Find the expression for the amount of the isotope remaining at any time. What is its half-life?

[Half life means the time taken for the radioactivity of a specified isotope to fall to half its original value.]

Mass of isotope (A)	Time (t)
200mg	0
150mg	2
?	t_1
100mg	?

let A be the mass isotope at any time 't'

$$\frac{dA}{dt} \propto A$$

$$\therefore \frac{dA}{dt} = kA$$

$$A = ce^{kt} \quad \text{—— (1)}$$

when $A = 200$ mg $\quad t = 0$

$$200 = ce^{0}$$

$$\boxed{c = 200}$$

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(1) becomes

$$A = 200\,e^{kt}$$

when $A = 150$ mg $\quad t = 2$

$$150 = 200\,e^{2k}$$

$$e^{2k} = \frac{3}{4}$$

$$2k = \log\frac{3}{4} \Rightarrow \boxed{k = \frac{1}{2}\log\left(\frac{3}{4}\right)}$$

when $t = t_1$, $A = ?$

$$A = 200\,e^{kt_1}$$

$$= 200\,e^{\frac{1}{2}\log\left(\frac{3}{4}\right)\cdot t_1}$$

(i) $\therefore$ The mass of isotope remaining after t years is

$$A = 200\,e^{\frac{t_1}{2}\cdot\log\left(\frac{3}{4}\right)}$$

(ii) when $A = 100$ mg $\quad t_h = ?$

$$100 = 200\,e^{kt_2}$$

$$e^{kt_2} = \frac{1}{2}$$

$$kt_2 = \log\frac{1}{2}$$

$$t_2 = \frac{1}{k}\cdot\log\frac{1}{2}$$

$$t_2 = \frac{2\log\left(\frac{1}{2}\right)}{\log(3/4)} \text{ years.}$$

Ex : 10.8

$\frac{1}{174}$ The rate of increase in the number of bacteria in a certain bacteria culture is proportional to the number present. Given that the number triples in 5 hrs. find how many bacteria will be present after 10 hrs?

Amount of bacteria (A)	Time (t)
A_0	0
$3A_0$	5
?	10

let A be the amount of bacteria at any time 't'

$$\frac{dA}{dt} \propto A \Rightarrow \frac{dA}{dt} = kA$$

$$A = ce^{kt} \text{ ——— } (1)$$

when $t = 0$, $A = A_0$

$$A_0 = ce^0 \Rightarrow \boxed{c = A_0}$$

(1) becomes

$$A = A_0 e^{kt}$$

when $t = 5$ $A = 3A_0$

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$$3\cancel{A_0} = \cancel{A_0} e^{5k}$$

$$\boxed{e^{5k} = 3}$$

when $t = 10$ $A = ?$

$$A = A_0 \cdot e^{10k}$$

$$= A_0 (e^{5k})^2 = A_0 (3)^2$$

$$= 9A_0$$

$\therefore$ At the end of 10hrs the number of bacteria as 9 times the original number of bacteria.

2
174 Find the Population of a city at any time t. given that the rate of increase of Population is Proportional to the Population at that instant and that in a Period of 40 yrs the Population increased from 3,00,000 to 4,00,000

Population (A)	Time (t)
3,00,000	0
4,00,000	40
?	t_1

Let A be the Population at any time 't'

$$\frac{dA}{dt} \propto A \Rightarrow \frac{dA}{dt} = KA$$

$$A = Ce^{Kt} \quad \text{——} \quad (1)$$

when $t=0$, $A = 3,00,000$

$$3,00,000 = Ce^{0}$$

$$\boxed{C = 3,00,000}$$

(1) becomes,

$$A = 3,00,000\, e^{Kt}$$

when $t = 40$ $A = 4,00,000$

$$400000 = 300000\, e^{40K}$$

$$4 = 3e^{40K}$$

$$e^{40K} = \frac{4}{3}$$

$$40K = \log \frac{4}{3}$$

$$\boxed{K = \frac{1}{40} \log \frac{4}{3}}$$

when $t = t_1$ $A = ?$

$$A = 300000\, e^{Kt_1}$$

$$= 300000\, e^{\frac{1}{40}\log(4/3)\, t_1}$$

$$= 300000\, e^{t_1 \log(4/3)^{1/40}}$$

$$= 300000\, e^{\log(4/3)^{t_1/40}}$$

$$= 300000 \left(\frac{4}{3}\right)^{t_1/40}$$

$\therefore$ At any time 't_1' the Population will be $3,00,000 \left(\frac{4}{3}\right)^{t_1/40}$.

3/174. The equation of electromotive force for an electric circuit containing resistance and self-inductance is $E = Ri + L\frac{di}{dt}$, where E is the electromotive force is given to the circuit, R the resitance and L, the co.efficient of induction Find the current i at time 't' when E = 0.

Given that

$$L\frac{di}{dt} + Ri = E$$

$$\div L \qquad \frac{di}{dt} + \frac{R}{L}i = \frac{E}{L}$$

It is a linear differential equation.

$$P = \frac{R}{L} \qquad Q = \frac{E}{L}$$

$$\int P\,dt = \int \frac{R}{L}.dt = \frac{R}{L}t$$

$$I.F = e^{\int P dt} = e^{\frac{R}{L}t}$$

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General Solution:

$$y e^{\int P dx} = \int Q e^{\int P dx}.dx + c$$

$$i e^{\frac{R}{L}t} = \int \frac{E}{L}.e^{\frac{R}{L}t}.dt + c$$

$$i e^{\frac{R}{L}t} = \frac{E}{L}.\frac{e^{\frac{R}{L}t}}{\frac{R}{L}} + c$$

$$i e^{\frac{R}{L}t} = \frac{E}{R}e^{R/L\,t} + c$$

$$\div e^{\frac{R}{L}t}$$

$$i = \frac{E}{R} + ce^{-\frac{R}{L}t}$$

when $E = 0$ $\quad i = \frac{0}{R} + ce^{-Rt/L}$

$$\boxed{i = ce^{-Rt/L}}$$

4/174 The engine of a moter boat moving at 10m/s is shut off. Given that the retardation at any time (after shutting off the engine] equal to the velocity at that time. Find the velocity after 2 sec of switching off the engine.

Given that the retardation at any Sub sequent time is equal to the velocity at that time,

$$-\frac{dv}{dt} = v$$

[retardation = - Accelaration]

$$\int \frac{dv}{v} = \int -dt$$

$$\log v = -t + c$$

when $v = 10, \quad t = 0$

$$\log 10 = -0 + c$$

$$\boxed{c = \log 10}$$

$$\log v = -t + \log 10$$

$$\log v - \log 10 = -t$$

$$\log \frac{v}{10} = -t$$

$$\frac{v}{10} = e^{-t}$$

$$v = 10e^{-t}$$

when $t = 2 \quad v = ?$

$$v = 10e^{-2}$$

$$\boxed{v = \frac{10}{e^2}}$$

5/174 Suppose a Person deposits 10000 Indian rupees in a bank account at the rate of 5% Per annum Compounded Continuously. How much money will be in his bank account 18 months later?

Amount	Time (yrs)
10000	0
?	$1\frac{1}{2}$ yrs

Let A be the amount at any time 't'

$$\frac{dA}{dt} \propto A$$

$$\boxed{k = 5\%}$$

$$\frac{dA}{dt} = kA$$

$$\frac{dA}{dt} = \frac{5}{100} A$$

$$A = Ce^{0.05t} \quad \text{———} \quad (1)$$

when $t = 0$ $A = 10000$

$$10000 = Ce^{0}$$

$$\boxed{c = 10000}$$

(1) becomes,

$$A = 10000\, e^{0.05t}$$

when $t = \frac{3}{2}$ $A = ?$

$$A = 10000\, e^{0.05(1.5)}$$

$$= 10000\, e^{0.075}$$

$\therefore$ At the end of $1\frac{1}{2}$ years the amount will be $10000\, e^{0.075}$.

$\frac{6}{174}$ Assume that the rate at which radioactive nuclei decay is proportional to the number of such nuclei that are present in a given sample. In a certain sample 10% of the original number of radioactive nuclei have undergone disintegration in a period of 100 yrs. what percentage of the original radioactive nuclei will remain after 1000 years.?

Radio active nuclei (A)%	Time (t)
A_0	0
$90\ A_0$	100
?	1000

let A be the amount of radioactive nuclei at any time 't'

$$\frac{dA}{dt} \propto A$$

$$\frac{dA}{dt} = KA$$

$$A = ce^{kt} \quad \text{———} \quad (1)$$

when $t=0$ $A = A_0$

$A_0 = ce^0$

$\boxed{c = A_0}$

(1) becomes,

$A = A_0 e^{kt}$

when $t = 100$ $A = 90\% A_0$

$= \frac{9}{10} A_0$

$\frac{9}{10} \cancel{A_0} = \cancel{A_0} e^{kt}$

$\boxed{e^{100k} = \frac{9}{10}}$

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when $t = 1000$ $A = ?$

$A = A_0 e^{1000k}$

$= A_0 (e^{100k})^{10}$

$= A_0 \left(\frac{9}{10}\right)^{10}$

$\therefore$ Required Percentage after 1000 yrs

$= \frac{\cancel{A_0} \left(\frac{9}{10}\right)^{10}}{\cancel{A_0}} \times 100$

$= \frac{9^{10}}{10^8} \%$

$\therefore \frac{9^{10}}{10^8} \%$ of radioactive element will remain after 1000 yrs.

$\frac{10.29}{172}$ In a murder investigation, a corpse was found by a detective at exactly 8 P.m. Being alert, the detective also measured the body temperature and found it to be 70°F. Two hrs later, the detective measured the body temperature again and found it to be 60°F. If the room temperature is 50°F and assuming that the body temperature of the person before death was 98.6°F at what time did the murder occur?

[$\log 2.43 = 0.88789$ $\log(0.5) = -0.69315$]

Time (hrs)	Temperature (°F)
0	70°F
2	60°F
?	98.6°F

let T be the temperature of the body at any time 't'

$\boxed{S = 50°F}$

Given that

$$\frac{dT}{dt} \propto (T-S) \quad \text{[Newton's law of cooling]}$$

$$\frac{dT}{dt} = K(T-S)$$

$$T-S = ce^{Kt}$$

$$T = S + ce^{Kt}$$

$$T = 50 + ce^{Kt} \quad \text{——— (1)}$$

when $t = 0$ $T = 70$

$$70 = 50 + ce^{0}$$

$$\boxed{c = 20}$$

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(1) becomes

$$T = 50 + 20e^{Kt}$$

when $t = 2$ $T = 60$

$$60 = 50 + 20e^{2K}$$

$$20e^{2K} = 10$$

$$e^{2K} = \frac{10}{20} = \frac{1}{2}$$

$$2K = \log(0.5)$$

$$K = \frac{1}{2}(\log 0.5)$$

when $t = ?$ $T = 98.6$

$$98.6 = 50 + 20e^{Kt}$$

$$20e^{Kt} = 48.6$$

$$e^{Kt} = \frac{48.6}{20} = 2.43$$

$$Kt = \log(2.43)$$

$$t = \frac{1}{K}\log(2.43)$$

$$\therefore t = \frac{2\log(2.43)}{\log 0.5} = \frac{2(0.88789)}{-0.69315} = -2.56$$

It appears that the person was murdered at $(8.00 - 2.56) \simeq 5.00$ P.m.

7/174 Water at temperature $100^\circ c$ cools in 10 min to $80^\circ c$ in a room temperature of $25^\circ c$ find (i) The temperature of water after 20 minutes (ii) The time when the temperature is $40^\circ c$.

$\left[\log_e \frac{11}{15} = -0.3101 \quad \log_e 5 = 1.6094\right]$

Time (min)	Temperature ($^\circ C$)
0	100
10	80
20	?
?	40

Let T be the temperature of water at any time 't'

$\boxed{S = 25^\circ c}$

By Newton's law of Cooling

$$\frac{dT}{dt} \propto T-S$$

$$\frac{dT}{dt} = k(T-S)$$

$$T-S = ce^{kt}$$

$$T = S + ce^{kt}$$

$$T = 25 + ce^{kt} \quad \text{——— (1)}$$

when $t=0$ $T=100$

$$100 = 25 + ce^{0}$$

$$\boxed{c = 75}$$

(1) becomes

$$T = 25 + 75e^{kt}$$

when $t = 10$ $T = 80$

$$80 = 25 + 75e^{10k}$$

$$75e^{10k} = 55$$

$$e^{10k} = \frac{55}{75} = \frac{11}{15}$$

$$\boxed{e^{10k} = \frac{11}{15}}$$

$$10k = \log \frac{11}{15}$$

$$k = \frac{1}{10}(-0.3101)$$

$$\boxed{k = -0.03101}$$

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when $t = 20$ $T = ?$

(i) $T = 25 + 75 e^{20k}$

$= 25 + 75 (e^{10k})^2$

$= 25 + \cancel{75}^{5} \times \frac{11}{\cancel{15}} \times \frac{11}{\cancel{15}_3}$

$= 25 + \frac{121}{3} = 25 + 40.33$

$= 65.33°C$

∴ The temperature of water after 20min is $65.33°C$

(ii) $T = 40$ $t = ?$

$T = 25 + 75 \cdot e^{kt}$

$40 = 25 + 75 e^{kt}$

$75 e^{kt} = 40 - 25 = 15$

$e^{kt} = \frac{15}{75} = \frac{1}{5}$

$e^{kt} = \frac{1}{5}$

$e^{-kt} = 5$

$-kt = \log 5$

$t = \frac{-1}{k} \log 5$

$= \frac{-1}{-3.101} \times 1.6094$

$= \frac{1.6094}{0.03101} = 51.89$

$\simeq 51.9$ min.

when the temperature is $40°c$, the time will be 51.9 min (approx)

$\frac{8}{174}$ At 10.00 A.M. a woman took a cup of hot instant coffee from her microwave oven and placed it on a nearby kitchen counter to cool. At this instant the temperature of the coffee was $180°F$, and 10 min later it was $160°F$. Assume that constant temperature of the kitchen was $70°F$.

(i) what was the temperature of the coffee at 10.15 A.M ?

(ii) The woman likes to drink coffee when its temperature is between $130^\circ F$ and $140^\circ F$ between what time should she have drunk the coffee?

Time (min)	Temperature ($^\circ F$)
0	180
10	160
15	?

$\boxed{S = 70}$

Let T be the temperature at any time 't'

By Newton's law of Cooling

$$\frac{dT}{dt} \propto T - S$$

$$\frac{dT}{dt} = k(T-S)$$

$$T - S = ce^{kt}$$

$$T = S + ce^{kt}$$

$$T = 70 + ce^{kt} \quad \text{--- (1)}$$

when $t = 0$, $T = 180^\circ$

$$180 = 70 + ce^{0}$$

$$\boxed{c = 110^\circ}$$

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(1) becomes

$$T = 70 + 110e^{kt} \quad \text{--- (2)}$$

when $t = 10$ $T = 160^\circ$

$$160 = 70 + 110e^{k(10)}$$

$$110e^{10k} = 90$$

$$\boxed{e^{10k} = \frac{9}{11}}$$

$\log_e \frac{9}{11} = -0.2006$

$$10k = \log\frac{9}{11}$$

$$\boxed{k = \frac{1}{10}\log\frac{9}{11}}$$

when $t = 15\text{min}$ $T = ?$

$$T = 70 + 110e^{k(15)}$$

$$= 70 + 110\, e^{\frac{30k}{2}}$$

$= 70+110(e^{10k})^{\frac{3}{2}}$

$= 70+110\left(\frac{9}{11}\right)^{3/2}$

$= 70+110\left(\frac{27}{11\sqrt{11}}\right)$ $\left[\because \sqrt{11} = 3.3166\right]$

$= 70+\frac{270}{\sqrt{11}}$

$= 70+\frac{270}{3.3166} = 70+81.4086$

$= 151.40\,^\circ F$

After 15 min the temperature will be $151.4^\circ F$ (Approx)

(ii) when $T = 130^\circ$ $t = ?$

from ②

$130 = 70+110e^{kt}$

$110e^{kt} = 60$

$e^{kt} = \frac{6}{11}$ $\left[\log_e \frac{6}{11} = -0.6061\right]$

$kt = \log\frac{6}{11}$

$t = \frac{1}{k}\log\frac{6}{11}$

$= \frac{10.\log\frac{6}{11}}{\log\frac{9}{11}} = \frac{10(-0.6061)}{-0.2006}$

$= 10(3.0216) = 30.216$ min.

when $T = 140^\circ$ $t = ?$

$140 = 70+110e^{kt}$

$110e^{kt} = 70$ $\left[\log_e \frac{7}{11} = -0.4519\right]$

$e^{kt} = \frac{7}{11}$

$kt = \log\frac{7}{11}$

$t = \frac{1}{k}\log\frac{7}{11} = \frac{10\log\frac{7}{11}}{\log\frac{9}{11}}$

$= \frac{10(-0.4519)}{-0.2006} = 10(2.2531)$

$= 22.531$ min.

$\therefore$ The time, when the temperature is between $130^\circ F$ and $140^\circ F$ $\approx$ 10.22 AM – 10.30 A.M (Approximately)

9/IT4 A pot of boiling water at 100°c is removed from a stove at time t=0 and left to cool in the kitchen. After 5 min the water temperature has decreased to 80°c and another 5min later it has dropped to 65°c. Determine the temperature of the kitchen.

Time (min)	Temperature (°c)
0	100
5	80
10	65

$S = ?$

Let T be the temperature of boiling water at any time 't'

By Newton's law of cooling,

$$\frac{dT}{dt} \propto T-S$$

$$\frac{dT}{dt} = k(T-S)$$

$$T-S = ce^{kt}$$

$$T = S+ce^{kt} \quad \text{——— (1)}$$

when $t=0$ $T=100$

$$100 = S+ce^{0}$$

$$\boxed{C = 100-S}$$

(1) becomes,

$$T = S+(100-S)e^{kt}$$

when $t=5$ $T=80$

$$80 = S+(100-S)e^{5k}$$

$$80-S = (100-S)e^{5k}$$

$$e^{5k} = \frac{80-S}{100-S} \quad \text{——— (2)}$$

when $t=10$, $T=65$

$$65 = S+(100-S)e^{10k}$$

$$65-S = (100-S)e^{10k}$$

$$e^{10k} = \frac{65-s}{100-s}$$

$$(e^{5k})^2 = \frac{65-s}{100-s}$$

$$\left(\frac{80-s}{100-s}\right)^2 = \frac{65-s}{100-s}$$

$$\frac{(80-s)^2}{100-s} = 65-s$$

$$(80-s)^2 = (65-s)(100-s)$$

$$6400+s^2-160s = 6500-65s-100s+s^2$$

$$6400+\cancel{s^2}-160s = 6500-165s+\cancel{s^2}$$

$$165s-160s = 6500-6400$$

$$5s = 100$$

$$s = 20^\circ$$

$\therefore$ The Temperature of the Kitchen is $20^\circ c$.

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A tank initially contains 50 lts of pure water. Starting at time $t=0$ a brine containing with 2gms of dissolved salt per litre flows into the tank at the rate of 3lts per minite. The mixture is kept uniform by stirring and the well-stirred mixture simultaneously flows out of the tank at the same rate. Find the amount of salt present in the tank at any time 't' $t>0$.

Let $x(t)$ denote the amount of salt in the tank at any time 't'

$$\frac{dx}{dt} = \text{Input} - \text{output}$$

Inflow rate $= 2(3) = 6$

outflow rate $= \frac{3}{50}x$

$$\frac{dx}{dt} = 6 - \frac{3}{50}x$$

$$= -\frac{3}{50}(x-100)$$

$$\frac{dx}{x-100} = -\frac{3}{50}dt$$

$$\log|x-100| = -\frac{3t}{50} + \log c$$

$$x-100 = ce^{-3t/50}$$

$$x = 100 + ce^{-3t/50}$$

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when $t=0$, $x=0$

$$100+ce^{0}=0$$

$$\boxed{c=-100}$$

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$$\therefore x = 100-100e^{-\frac{3t}{50}}$$

when $t=t_1$ $x=?$

$$x = 100-100e^{-3t_1/50}$$

$$= 100(1-e^{3t_1/50})$$

$\therefore$ The amount of salt present in the tank at any time t is

$$x = 100(1-e^{3t/50})$$

10.30 / 173 A tank contains 1000 litres of water in which 100 grams of salt is dissolved. Brine runs in a rate of 10 litres per minute. and each litre contains 5gms of dissolved salt. The mixture of the tank is kept uniform by stirring. Brine runs ~~out~~ at 10 lts per minute. Find the amount of salt at any time 't'

Let $x(t)$ denote the amount of salt in the tank at any time 't'

$$\frac{dx}{dt} = \text{in flow rate} - \text{out flow rate}$$

$$\text{Inflow} = 5(10) = 50\,gm$$

$$\text{out flow} = \frac{10}{1000}x = 0.01x$$

$$\frac{dx}{dt} = 50-0.01x$$

$$= -0.01(x-5000)$$

$$\int\frac{dx}{x-5000} = \int -0.01\,dt$$

$$\log|x-5000| = -0.01t+\log c$$

$$x-5000 = ce^{-0.01t}$$

$$x = 5000+ce^{-0.01t}$$

when $t=0$ $x=100$

$$100 = 5000+ce^{0}$$

$$\boxed{c=-4900}$$

$\therefore$ The amount of the salt in the tank at time 't' is $x = 5000-4900e^{-0.01t}$.