APPLICATIONAL PROBLEMS (VOLUME II)

1. **A fruit shop keeper prepares 3 different varieties of gift packages. Pack I contains 6 apples.3 orange and 3 pomegranates. Pack II contains 5 apples ,4 oranges and 4 pomegranates and Pack III contains 6 apples, 6 oranges and 6 pomegranates. The cost of an apple, an orange and a pomegranate respectively are Rs. 30 ,Rs.15 and Rs.45.What is the cost of preparing each package of fruits?**

Let A be the cost matrix i.e. A = [30 15 45]

Let B be the Fruit matrix B =

Solution

Cost matrix $A = [30 \;\; 15 \;\; 45]$, Fruit matrix

$$B = \begin{array}{c} \\ \end{array}\begin{matrix} \text{P-I} & \text{P-II} & \text{P-III} \\ \end{matrix}$$
$$B = \begin{bmatrix} 6 & 5 & 6 \\ 3 & 4 & 6 \\ 3 & 4 & 6 \end{bmatrix} \begin{matrix} \text{Apples} \\ \text{Oranges} \\ \text{Pomegranates} \end{matrix}$$

Cost of packages are obtained by computing AB. That is, by multiplying cost of each item in A (cost matrix A) with number of items in B (Fruit matrix B).

$$AB = [30 \;\; 15 \;\; 45] \begin{bmatrix} 6 & 5 & 6 \\ 3 & 4 & 6 \\ 3 & 4 & 6 \end{bmatrix} = \begin{bmatrix} 360 \\ 390 \\ 540 \end{bmatrix}$$

Pack-I cost ₹ 360, Pack-II cost ₹ 390, Pack-III costs ₹ 540.

2.

Alcohol is removed from the body by the lungs, the kidneys, and by chemical processes in liver. At moderate concentration levels, the majority work of removing the alcohol is done by the liver; less than 5% of the alcohol is eliminated by the lungs and kidneys. The rate r at which the liver processes alcohol from the bloodstream is related to the blood alcohol concentration x by a rational function of the form $r(x) = \dfrac{\alpha x}{x+\beta}$ for some positive constants α and β. Find the maximum possible rate of removal.

As the alcohol concentration x increases the rate of removal increases.

Therefore, the maximum possible rate of removal $= \lim_{x\to\infty} r(x)$

$$= \lim_{x\to\infty} \frac{\alpha x}{x+\beta} = \lim_{x\to\infty} \frac{\alpha}{\left(1+\frac{\beta}{x}\right)} = \alpha.$$

3.

According to Einstein's theory of relativity, the mass m of a body moving with velocity v is $m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$, where m_0 is the initial mass and c is the speed of light. What happens to m as $v \to c^-$. Why is a left hand limit necessary?

$$\lim_{v\to c^-}(m) = \lim_{v\to c^-}\frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{m_0}{\sqrt{\lim_{v\to c^-}\left(1-\frac{v^2}{c^2}\right)}}$$

For $h > 0,\ c-h < v < c$. This implies, $(c-h)^2 < v^2 < c^2$.

That is, $\frac{(c-h)^2}{c^2} < \frac{v^2}{c^2} < 1$. That is, $\lim_{h\to 0}\frac{(c-h)^2}{c^2} < \lim_{h\to 0}\frac{v^2}{c^2} < \lim_{h\to 0} 1$.

That is, $1 < \lim_{h\to 0}\frac{v^2}{c^2} < 1$. That is, $1 < \lim_{v\to c^-}\frac{v^2}{c^2} < 1$. By Sandwich theorem, $\lim_{v\to c^-} = 1$.

Therefore, $\lim_{v\to c^-}(m) \to \infty$.

That is, the mass becomes very very large (infinite) as $v \to c^-$.

The left hand limit is necessary. Otherwise as $v \to c^+$ makes $1-\frac{v^2}{c^2} < 0$ and consequently we cannot find the mass.

4.

The velocity in ft/sec of a falling object is modeled by $r(t) = -\sqrt{\frac{32}{k}}\frac{1-e^{-2t\sqrt{32k}}}{1+e^{-2t\sqrt{32k}}}$, where k is a constant that depends upon the size and shape of the object and the density of the air. Find the limiting velocity of the object, that is, find $\lim_{t\to\infty} r(t)$.

$$\lim_{t\to\infty} r(t) = \lim_{t\to\infty} -\sqrt{\frac{32}{k}}\frac{1-e^{-2t\sqrt{32k}}}{1+e^{-2t\sqrt{32k}}}$$

$$= -\sqrt{\frac{32}{k}}\lim_{t\to\infty}\frac{1-e^{-2t\sqrt{32k}}}{1+e^{-2t\sqrt{32k}}}$$

$$= -\sqrt{\frac{32}{k}}\frac{(1-0)}{(1+0)} = -\sqrt{\frac{32}{k}} \text{ ft/sec.}$$

5.

Suppose that the diameter of an animal's pupils is given by $f(x)=\dfrac{160x^{-0.4}+90}{4x^{-0.4}+15}$, where x is the intensity of light and $f(x)$ is in *mm*. Find the diameter of the pupils with (a) minimum light (b) maximum light.

(a) For minimum light it is enough to find the limit of the function when $x \to 0^+$.

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{160x^{-0.4}+90}{4x^{-0.4}+15} = \lim_{x\to 0^+} \frac{160+90x^{0.4}}{4+15x^{0.4}}$$

$$= \frac{160}{4} = 40\text{mm} .$$

(b) For maximum light, it is enough to find the limit of the function when $x \to \infty$

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{160x^{-0.4}+90}{4x^{-0.4}+15} = \frac{90}{15} = 6\text{mm}$$

That is, the pupils have a limiting size of 6mm, as the intensity of light is very large.

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6.

An important problem in fishery science is to estimate the number of fish presently spawning in streams and use this information to predict the number of mature fish or "recruits" that will return to the rivers during the reproductive period. If S is the number of spawners and R the number of recruits, "Beverton-Holt spawner recruit function" is $R(S)=\frac{S}{(\alpha S+\beta)}$ where α and β are positive constants. Show that this function predicts approximately constant recruitment when the number of spawners is sufficiently large.

$$\lim_{s\to\infty} R(s)=\lim_{s\to\infty}\frac{s}{\alpha s+\beta}=\lim_{s\to\infty}\frac{\frac{s}{s}}{\frac{\alpha s+\beta}{s}}=\lim_{s\to\infty}\frac{1}{\alpha+\frac{\beta}{s}}=\frac{1}{\alpha}$$

7.

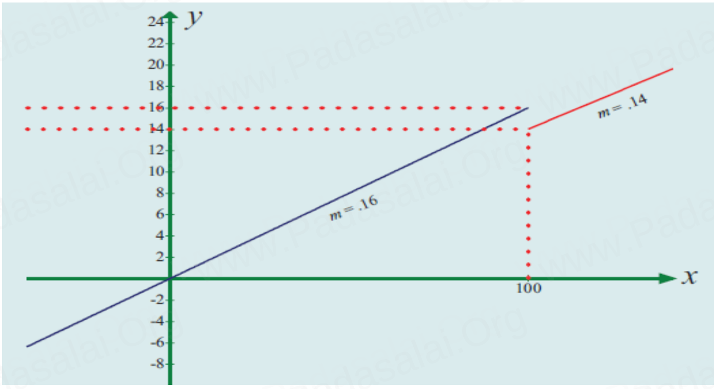
A tank contains 5000 litres of pure water. Brine (very salty water) that contains 30 grams of salt per litre of water is pumped into the tank at a rate of 25 litres per minute. The concentration of salt water after t minutes (in grams per litre) is $C(t)=\frac{30t}{200+t}$.

What happens to the concentration as $t\to\infty$?

$$\lim_{t\to\infty}\frac{30t}{20+t}=\lim_{t\to\infty}\frac{\frac{30t}{t}}{\frac{20}{t}+\frac{t}{t}}=\lim_{t\to\infty}\frac{30}{\frac{20}{t}+1}=\lim_{t\to\infty}\frac{30}{1}=30$$

8.

A tomato wholesaler finds that the price of a newly harvested tomatoes is ₹0.16 per kg if he purchases fewer than 100 kgs each day. However, if he purchases at least 100 kgs daily, the price drops to ₹0.14 per kg. Find the total cost function and discuss the cost when the purchase is 100 kgs.



Let x denote the number of kilograms bought per day and C denote the cost. Then,

$$C(x) = \begin{cases} 0.16x, & \text{if } 0 \le x < 100 \\ 0.14x, & \text{if } x \ge 100 \end{cases}.$$

The sketch of this function is shown in Fig. 9.37.

It is discontinuous at $x = 100$ since $\lim_{x \to 100^-} C(x) = 16$ and $\lim_{x \to 100^+} C(x) = 14$.

Note that $C(100) = 14$. Thus, $\lim_{x \to 100^-} C(x) = 16 \ne 14 = \lim_{x \to 100^+} C(x) = C(100)$.

Note also that the function jumps from one finite value 14 to another finite value 16.

9.

A train started from Madurai Junction towards Coimbatore at 3pm (time $t = 0$) with velocity $v(t) = 20t + 50$ kilometre per hour, where t is measured in hours. Find the distance covered by the train at 5pm.

In calculus terminology, velocity $v = \frac{ds}{dt}$ is rate of change of position with time, where s is the distance. The velocity of the train is given by

$$v(t) = 20t + 50$$

Therefore, $\frac{ds}{dt} = 20t + 50$

To find the distance function s one has to integrate the derivative function.

That is, $s = \int (20t + 50)\, dt$

$$s = 10t^2 + 50t + c$$

The distance covered by the train is zero when time is zero. Let us use this initial condition $s = 0$ at $t = 0$ to determine the value c of the constant of integration.

$$\Rightarrow s = 10t^2 + 50t + c \quad \Rightarrow c = 0$$

Therefore, $s = 10t^2 + 50t$

The distance covered by the train in 2 hours (5pm-3pm) is given by substituting $t = 2$ in the above equation, we get

$$s = 10(2)^2 + 50(2) = 140 \text{ km.}$$

The rate of change of weight of person w in kg with respect to their height h in centimetres is given approximately by $\frac{dw}{dh} = 4.364 \times 10^{-5} h^2$. Find weight as a function of height. Also find the weight of a person whose height is 150 cm.

The rate of change of weight with respect to height is

$$\frac{dw}{dh} = 4.364\times10^{-5}h^2$$

$$w = \int 4.364\times10^{-5}h^2\ dh$$

$$w = 4.364\times10^{-5}\left(\frac{h^3}{3}\right) + c$$

One can obviously understand that the weight of a person is zero when height is zero.

Let us find the value c of the constant of integration by substituting the initial condition $w = 0$, at $h = 0$, in the above equation

$$w = 4.364\times10^{-5}\left(\frac{h^3}{3}\right) + c \Rightarrow c = 0$$

The required relation between weight and height of a person is

$$w = 4.364\times10^{-5}\left(\frac{h^3}{3}\right)$$

When the height h =150cm,

$$w = 4.364\times10^{-5}\left(\frac{150^3}{3}\right)$$

When the height $h = 150$cm, the weight is $w = 49kg$ (approximately)
Therefore, the weight of the person whose height 150cm is 49 kg.

10.

A tree is growing so that, after t - years its height is increasing at a rate of $\frac{18}{\sqrt{t}}$ cm per year. Assume that when $t = 0$, the height is 5 cm.

(i) Find the height of the tree after 4 years.

(ii) After how many years will the height be 149 cm?

The rate of change of height h with respect to time t is the derivative of h with respect to t.

$$\text{Therefore, } \frac{dh}{dt} = \frac{18}{\sqrt{t}} = 18t^{-\frac{1}{2}}$$

So, to get a general expression for the height, integrating the above equation with respect to t.

$$h = \int 18t^{-\frac{1}{2}}dt = 18(2t^{\frac{1}{2}}) + c = 36\sqrt{t} + c$$

Given that when $t = 0$, the height $h = 5$ cm.

$$5 = 0 + c \Rightarrow c = 5$$

$$h = 36\sqrt{t} + 5.$$

(i) To find the height of the tree after 4 years.

When $t = 4$ years,

$$h = 36\sqrt{t} + 5 \Rightarrow h = 36\sqrt{4} + 5 = 77$$

The height of the tree after 4 years is 77 cm

(ii) When $h = 149$cm

$$h = 36\sqrt{t} + 5 \Rightarrow 149 = 36\sqrt{t} + 5$$

$$\sqrt{t} = \frac{149-5}{36} = 4 \Rightarrow t = 16$$

Thus after 16 years the height of the tree will be 149 cm.

11.

At a particular moment, a student needs to stop his speedy bike to avoid a collision with the barrier ahead at a distance 40 metres away from him. Immediately he slows (retardation) the bike under braking at a rate of 8 metre/second^2. If the bike is moving at a speed of 24m/s, when the brakes are applied, would it stop before collision?

Let a be the acceleration, v be the velocity of the car, and s be the distance.

Stated in calculus terminology, velocity, $v = \frac{ds}{dt}$, is the rate of change of position with time, and acceleration, $a = \frac{dv}{dt}$, is rate of change of velocity with time.

The acceleration to be negative because if you take the direction of movement to be positive, then for a bike that is slowing down, its acceleration vector will be oriented in the opposite direction of its motion (retardation).

Given that the retardation of the car is 8 meter/second^2.

Therefore, $a = \frac{dv}{dt} = -8$ meter/second^2.

Therefore, $v = \int a\,dt = \int -8dt = -8t + c_1$

$v = -8t + c_1$.

When the brakes are applied,

$t = 0$, and $v = 24$m/s.

So, $24 = -8(0) + c_1 \Rightarrow c_1 = 24$

Therefore, $v = -8t + 24$.

That is, $\frac{ds}{dt} = -8t + 24$.

It is required to find the distance, not the velocity, so need more integration in order.

$$s = \int v\,dt = \int (-8t + 24)dt$$

To determine c_2, the stopping distance s is measured from where, and when, the brakes are applied so that at $t = 0$, $s = 0$.

$$s = -4t^2 + 24t + c_2 \Rightarrow 0 = -4(0)^2 + 24(0) + c_2 \Rightarrow c_2 = 0$$

$$s = -4t^2 + 24t$$

The stopping distance s could be evaluated if we knew the braking time. The time can be determined from the speed statement.

The bike stops when $v = 0$, $\Rightarrow v = -8t + 24 \Rightarrow 0 = -8t + 24 \Rightarrow t = 3$.

When $t = 3$, we get

$$s = -4t^2 + 24t \Rightarrow s = -4(3)^2 + 24(3)$$

$$s = 36 \text{ metres}$$

The bike stops at a distance 4 metres to the barrier.

12.

A ball is thrown vertically upward from the ground with an initial velocity of 39.2 m/sec. If the only force considered is that attributed to the acceleration due to gravity, find

(i) how long will it take for the ball to strike the ground?

(ii) the speed with which will it strike the ground? and

(iii) how high the ball will rise?

a = -9.8

$$\frac{dv}{dt} = -9.8 \Rightarrow \int dv = -9.8dt \Rightarrow v = -9.8t + C$$

t= 0 v=39.2 hence c_1 = 39.2

therefore v = -9.8+39.2

$$v = \frac{ds}{dt} = (-9.8t + 39.2) \text{ ------------- (i)}$$

$$\int ds = \int \ ds = (-9.8t + 39.2)\,dt$$

$$s = -9.8\frac{t^2}{2} + 39.2 + C_2$$

When t = 0 s=0

Hence C_2=

S= $-4.9t^2$+39.2t

The ball rise high level v=0 substitute in (I)

0 = -9.8t+39.2 $\Rightarrow$ t=4

13.

A wound is healing in such a way that t days since Sunday the area of the wound has been decreasing at a rate of $-\frac{3}{(t+2)^2}$ cm^2 per day. If on Monday the area of the wound was $2\,cm^2$

(i) What was the area of the wound on Sunday?

(ii) What is the anticipated area of the wound on Thursday if it continues to heal at the same rate?

Given $\frac{dA}{dt} = -\frac{3}{(t+2)^2}$

$$\int dA = -3\int (t+2)^{-2} dt$$

$$A = -3\frac{(t+2)^{-1}}{-1} + C \quad \Rightarrow \quad A = \frac{3}{t+2} + c$$

For Monday t = 1 When A =2 $2 = \frac{3}{3} + c \quad \Rightarrow$ c = 2-1 =1

Therefore $A = \frac{3}{t+2} + 1$

On Sunday t =o A = ? $A = \frac{3}{2} + 1 = 1.5 + 2 = 2.5$

On thrursday t= 4 $A = \frac{3}{4+2} + 1 = \frac{3}{6} + 1 = \frac{1}{2} + 1 = 1.5$

14.

Three candidates X, Y, and Z are going to play in a chess competition to win FIDE (World Chess Federation) cup this year. X is thrice as likely to win as Y and Y is twice as likely as to win Z. Find the respective probability of X, Y and Z to win the cup.

Let A, B, C be the event of winning FIDE cup respectively by X, Y, and Z this year.

Given that X is thrice as likely to win as Y.

$$A : B \; :: \; 3 : 1. \qquad (1)$$

Y is twice as likely as to win Z

$$B : C \; :: \; 2 : 1 \qquad (2)$$

From (1) and (2)

$$A : B : C \; :: \; 6 : 2 : 1$$

$A = 6k$, $B = 2k$, $C = k$, where k is proportional constant.

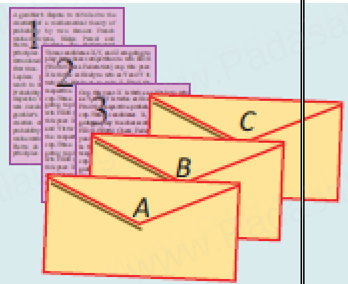
Probability to win the cup by X is $P(A) = \frac{6k}{9k} = \frac{2}{3}$

Probability to win the cup by Y is $P(B) = \frac{2k}{9k} = \frac{2}{9}$ and

Probability to win the cup by Z is $P(C) = \frac{k}{9k} = \frac{1}{9}$.

15.

Three letters are written to three different persons and addresses on three envelopes are also written. Without looking at the addresses, what is the probability that (i) exactly one letter goes to the right envelopes (ii) none of the letters go into the right envelopes?



Let A, B, and C denote the envelopes and 1, 2, and 3 denote the corresponding letters.

The different combination of letters put into the envelopes are shown in the table.

Let c_i denote the outcomes of the events.

Let X be the event of putting the letters into the exactly only one right envelopes.

Let Y be the event of putting none of the letters into the right envelope.

Outcomes

Envelope	c_1	c_2	c_3	c_4	c_5	c_6
A	1	1	2	2	3	3
B	2	3	1	3	1	2
C	3	2	3	1	2	1

$S = \{c_1, c_2, c_3, c_4, c_5, c_6\}$, $n(S) = 6$

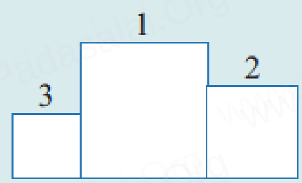
$$X = \{c_2, c_3, c_6\}, \quad n(X) = 3$$

$$Y = \{c_4, c_5\} \quad n(Y) = 2$$

$$P(X) = \frac{3}{6} = \frac{1}{2} \quad P(Y) = \frac{2}{6} = \frac{1}{3}.$$

16.

For a sports meet, a winners' stand comprising of three wooden blocks is in the form as shown in figure. There are six different colours available to choose from and three of the wooden blocks is to be painted such that no two of them has the same colour. Find the probability that the smallest block is to be painted in red, where red is one of the six colours.



Let S be the sample space and A be the event that the smallest block is to be painted in red.

$$n(S) = 6P_3 = 6 \times 5 \times 4 = 120$$

$$n(A) = 5 \times 4 = 20$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{20}{120} = \frac{1}{6}.$$

	6	6	3
$n(S)$	6	5	4
$n(A)$	5	4	Red

17.

A man has 2 ten rupee notes, 4 hundred rupee notes and 6 five hundred rupee notes in his pocket. If 2 notes are taken at random, what are the odds in favour of both notes being of hundred rupee denomination and also its probability?

Let S be the sample space and A be the event of taking 2 hundred rupee note.

Therefore, $n(S) = 12c_2 = 66$, $n(A) = 4c_2 = 6$ and $n(\bar{A}) = 66 - 6 = 60$

Therefore, odds in favour of A is 6: 60

That is, odds in favour of A is 1: 10, and $P(A) = \frac{1}{11}$.

18.

The probability that a girl, preparing for competitive examination will get a State Government service is 0.12, the probability that she will get a Central Government job is 0.25, and the probability that she will get both is 0.07. Find the probability that (*i*) she will get atleast one of the two jobs (*ii*) she will get only one of the two jobs.

Let I be the event of getting State Government service and C be the event of getting Central Government job.

Given that $P(I) = 0.12$, $P(C) = 0.25$, and $P(I \cap C) = 0.07$

(*i*) P (at least one of the two jobs) $= P(I \text{ or } C) = P(I \cup C)$

$= P(I) + P(C) - P(I \cap C)$

$= 0.12 + 0.25 - 0.07 = 0.30$

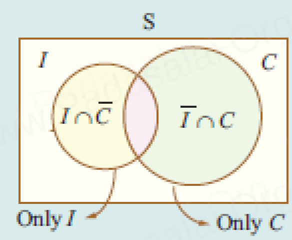
(*ii*) P(only one of the two jobs) = P[only I or only C].

$= P(I \cap \bar{C}) + P(\bar{I} \cap C)$

$= \{P(I) - P(I \cap C)\} + \{P(C) - P(I \cap C)\}$

$= \{0.12 - 0.07\} + \{0.25 - 0.07\}$

$= 0.23.$



19.

The probability that a new railway bridge will get an award for its design is 0.48, the probability that it will get an award for the efficient use of materials is 0.36, and that it will get both awards is 0.2. What is the probability, that (i) it will get at least one of the two awards (ii) it will get only one of the awards.

1.

Let S be the will get an award for its design P(S) =0.48

Let M will get an award for the efficiency of use of materials P(M)=0.36

$S \cap M$ = that it will get both awards

$P(S \cap M) = 0.2$

Probability that it will get atleast one of the two awards

P(SUM)=P(S)+P(M)-P(S∩M)

= 0.48+0.36-0.2=0.64

2.Probability that it will get only one of the awards

$= P\left(S \cap \overline{M}\right) + P\left(\overline{S} \cap M\right)$

$= P(S) - P(S \cap M) + P(M) - P(S \cap M)$

=0.48-0.2 + 0.36 -0.2=0.44

20.

An anti-aircraft gun can take a maximum of four shots at an enemy plane moving away from it. The probability of hitting the plane in the first, second, third, and fourth shot are respectively 0.2, 0.4, 0.2 and 0.1. Find the probability that the gun hits the plane.

Let H_1, H_2, H_3 and H_4 be the events of hitting the plane by the anti-aircraft gun in the first second, third and fourth shot respectively.

Let H be the event that anti-aircraft gun hits the plane. Therefore $\bar{H}$ is the event that the plane is not shot down. Given that

$P(H_1) = 0.2 \quad \Rightarrow P(\bar{H}_1) = 1 - P(H_1) = 0.8$

$P(H_2) = 0.4 \quad \Rightarrow P(\bar{H}_2) = 1 - P(H_2) = 0.6$

$P(H_3) = 0.2 \quad \Rightarrow P(\bar{H}_3) = 1 - P(H_3) = 0.8$

$P(H_4) = 0.1 \quad \Rightarrow P(\bar{H}_4) = 1 - P(H_4) = 0.9$

The probability that the gun hits the plane is

$P(H) = 1 - P(\bar{H}) = 1 - P(\overline{H_1 \cup H_2 \cup H_3 \cup H_4})$

$= 1 - P\left(\bar{H}_1 \cap \bar{H}_2 \cap \bar{H}_3 \cap \bar{H}_4\right)$

$= 1 - P(\bar{H}_1)P(\bar{H}_2)P(\bar{H}_3)P(\bar{H}_4)$

$= 1 - (0.8)(0.6)(0.8)(0.9) = 1 - 0.3456$

$P(H) = 0.6544$

21.

X speaks truth in 70 percent of cases, and Y in 90 percent of cases. What is the probability that they likely to contradict each other in stating the same fact?

X speaks truth in 70 percent of cases, and Y in 90 percent of cases. What is the probability that they likely to contradict each other in stating the same fact?

Solution

Let A be the event of X speaks the truth, B be the event of Y speaks the truth

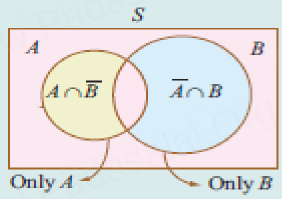
$\therefore \overline{A}$ is the event of X not speaking the truth and $\overline{B}$ is the event of Y not speaking he truth.

Let C be the event that they will contradict each other.

Given that

$P(A) = 0.70 \quad \Rightarrow P(\overline{A}) = 1 - P(A) = 0.30$

$P(B) = 0.90 \quad \Rightarrow P(\overline{B}) = 1 - P(B) = 0.10$



C = (A speaks truth and B does not speak truth or B speaks truth and A does not speak truth)

$C = [(A \cap \overline{B}) \cup (\overline{A} \cap B)]$ (see figure)

since $(A \cap \overline{B})$ and $(\overline{A} \cap B)$ are mutually exclusively,

$$P(C) = P(A \cap \overline{B}) + P(\overline{A} \cap B)$$

$$= P(A)\,P(\overline{B}) + P(\overline{A})\,P(B)$$

(Since A, B are independent event, A, $\overline{B}$ are also independent events)

$$= (0.70)\ (0.10) + (0.30)\ (0.90)$$

$$= 0.070 + 0.270 = 0.34$$

$$P(C) = 0.34.$$

22.

A main road in a City has 4 crossroads with traffic lights. Each traffic light opens or closes the traffic with the probability of 0.4 and 0.6 respectively. Determine the probability of

(i) a car crossing the first crossroad without stopping

(ii) a car crossing first two crossroads without stopping

(iii) a car crossing all the crossroads, stopping at third cross.

(iv) a car crossing all the crossroads, stopping at exactly one cross.

Let A_i be the event that the traffic light opens at i th cross, for i= 1, 2, 3, 4.

Let B_i be the event that the traffic light closes at i th cross, for i= 1, 2, 3, 4.

The traffic lights are all independent.

Therefore A_i and B_i are all independent events, for i= 1, 2, 3, 4.

Given that

$P(A_i) = 0.4, \ i = 1, 2, 3, 4$

$P(B_i) = 0.6, \ i = 1, 2, 3, 4$

(i) Probability of car crossing the first crossroad without stopping,

$P(A_1) = 0.4.$

(ii) Probability of car crossing first two crossroads without stopping,

$$P(A_1 \cap A_2) = P(A_1 A_2) = (0.4)(0.4) = 0.16$$

(iii) Probability of car crossing all the crossroads, stopping at third cross

$$P(A_1 \cap A_2 \cap B_3 \cap A_4) = P(A_1 A_2 B_3 A_4) = (0.4)(0.4)(0.6)(0.4) = 0.0384$$

(iv) Probability of car crossing all the crossroads, stopping at exactly one of the crossroads is

$$P(B_1 A_2 A_3 A_4 \cup A_1 B_2 A_3 A_4 \cup A_1 A_2 B_3 A_4 \cup A_1 A_2 A_3 B_4)$$

$$= P(B_1 A_2 A_3 A_4) + P(A_1 B_2 A_3 A_4) + P(A_1 A_2 B_3 A_4) + P(A_1 A_2 A_3 B_4)$$

$$= 4(0.4)(0.4)(0.6)(0.4) = 4(0.0384) = 0.1536$$

23.

A factory has two machines I and II. Machine I produces 40% of items of the output and Machine II produces 60% of the items. Further 4% of items produced by Machine I are defective and 5% produced by Machine II are defective. An item is drawn at random. If the drawn item is defective, find the probability that it was produced by Machine II.

$$P(A_1) = 0.40, \quad P(B/A_1) = 0.04$$

$$P(A_2) = 0.60, \quad P(B/A_2) = 0.05$$

$$P(A_2/B) = \frac{P(A_2)\,P(B/A_2)}{P(A_1)\,P(B/A_1) + P(A_2)\,P(B/A_2)}$$

$$P(A_2/B) = \frac{(0.60)(0.05)}{(0.40)(0.04)+(0.60)(0.05)} = \frac{15}{23}.$$

24.

A construction company employs 2 executive engineers. Engineer-1 does the work for 60% of jobs of the company. Engineer-2 does the work for 40% of jobs of the company. It is known from the past experience that the probability of an error when engineer-1 does the work is 0.03, whereas the probability of an error in the work of engineer-2 is 0.04. Suppose a serious error occurs in the work, which engineer would you guess did the work?

$$P(A_1) = 0.60, \quad P(B/A_1) = 0.03$$
$$P(A_2) = 0.40, \quad P(B/A_2) = 0.04$$

$$P(A_1/B) = \frac{P(A_1)\,P(B/A_1)}{P(A_1)\,P(B/A_1) + P(A_2)\,P(B/A_2)}$$

$$= \frac{(0.60)(0.03)}{(0.60)(0.03)+(0.40)(0.04)}$$

$$P(A_1/B) = \frac{9}{17}.$$

$$P(A_2/B) = \frac{P(A_2)\,P(B/A_2)}{P(A_1)\,P(B/A_1) + P(A_2)\,P(B/A_2)}$$

$$P(A_2/B) = \frac{(0.40)(0.04)}{(0.60)(0.03)+(0.40)(0.04)}$$

$$P(A_2/B) = \frac{8}{17}.$$

Since $P(A_1/B) > P(A_2/B)$, the chance of error done by engineer-1 is greater than the chance of error done by engineer-2. Therefore one may guess that the serious error would have been be done by engineer-1.

25.

A consulting firm rents car from three agencies such that 50% from agency L, 30% from agency M and 20% from agency N. If 90% of the cars from L, 70% of cars from M and 60% of the cars from N are in good conditions (i) what is the probability that the firm will get a car in good condition? (ii) if a car is in good condition, what is probability that it has come from agency N?

$$P(A_1) = 0.50, \quad P(G/A_1) = 0.90$$

$$P(A_2) = 0.30, \quad P(G/A_2) = 0.70$$

$$P(A_3) = 0.20, \quad P(G/A_3) = 0.60.$$

(i) Since A_1, A_2 and A_3 are mutually exclusive and exhaustive events and G is an event in S, then the total probability of event G is P(G).

$$P(G) = P(A_1)\,P(G/A_1) + P(A_2)\,P(G/A_2) + P(A_3)\,P(G/A_3)$$

$$P(G) = (0.50)(0.90) + (0.30)(0.70) + (0.20)(0.60)$$

$$P(G) = 0.78.$$

(ii) The conditional probability A_3 given G is $P(A_3/G)$

By Bayes' theorem,

$$P(A_3/G) = \frac{P(A_3)\,P(G/A_3)}{P(A_1)\,P(G/A_1) + P(A_2)\,P(G/A_2) + P(A_3)\,P(G/A_3)}$$

$$P(A_3/G) = \frac{(0.20)(0.60)}{(0.50)(0.90) + (0.30)(0.70) + (0.20)(0.60)}$$

$$= \frac{2}{13}.$$

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