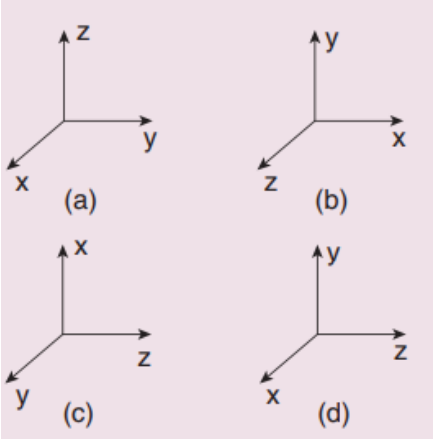
11th Physics - Unit 2 Kinematics

Solutions for the Textbook Problems

One Marks

1. Which one of the following Cartesian coordinate system is not followed in physics?



2. Identify the unit vector in the following.

(a) $\hat{i}+\hat{j}$ (b) $\frac{\hat{i}}{\sqrt{2}}$

(c) $\hat{k}-\frac{\hat{j}}{\sqrt{2}}$ (d) $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$

3. Which one of the following physical quantities cannot be represented by a scalar?

a) Mass b) Length

c) Momentum d) Magnitude of acceleration

4. Two objects of masses m_1 and m_2 fall from the heights h_1 and h_2 respectively. The ratio of the magnitude of their momenta when they hit the ground is

(a) $\sqrt{\frac{h_1}{h_2}}$ (b) $\sqrt{\frac{m_1h_1}{m_2h_2}}$

(c) $\frac{m_1}{m_2}\sqrt{\frac{h_1}{h_2}}$ (d) $\frac{m_1}{m_2}$

5. If a particle has negative velocity and negative acceleration, its speed

(a) increases (b) decreases

(c) remains same (d) zero

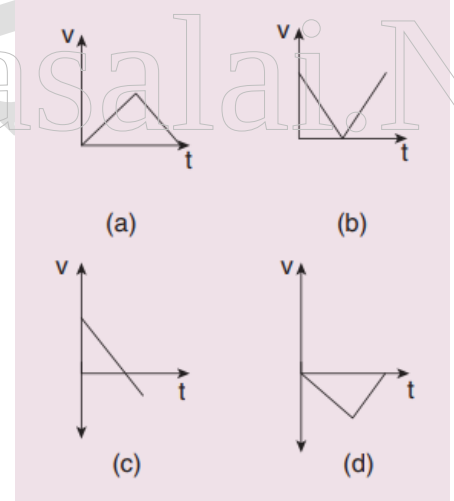
6. If the velocity is $\hat{v}=2\hat{\imath}+t^2\hat{\jmath}-9\hat{k}$, then the magnitude of acceleration at t = 0.5 s is

(a) 1m s^{-2} (b) 2 m s^{-2} (c) zero (d) -1 m s^{-2}

7. If an object is dropped from the top of a building and it reaches the ground at t = 4 s , then the height of the building is (ignoring air resistance) (g = 9.8 ms^{-2})

(a) 77.3 m (b) 78.4 m (c) 80.5 m (d) 79.2 m

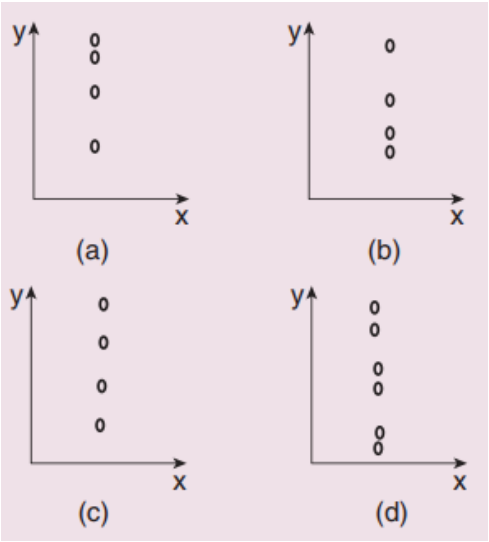
8. A ball is projected vertically upwards with a velocity v. It comes back to ground in time t. Which v-t graph shows the motion correctly?



9. If one object is dropped vertically downward and another object is thrown horizontally from the same height, then the ratio of vertical distance covered by both objects at any instant t is

(a) 1 (b) 2 (c) 4 (d) 0.5

10. A ball is dropped from some height towards the ground. Which one of the following represents the correct motion of the ball?



11. If a particle executes uniform circular motion in the xy plane in clock wise direction, then the angular velocity is in

(a) +y direction (b) +z direction

(c) -z direction (d) -x direction

12. If a particle executes uniform circular motion, choose the correct statement

(a) The velocity and speed are constant.

(b) The acceleration and speed are constant.

(c) The velocity and acceleration are constant.

(d) The speed and magnitude of acceleration are constant.

13. If an object is thrown vertically up with the initial speed u from the ground, then the time taken by the object to return back to ground is

(a) $\frac{u^2}{2g}$ (b) $\frac{u^2}{g}$

(c) $\frac{u}{2g}$ (d) $\frac{2u}{g}$

14. Two objects are projected at angles 30° and 60° respectively with respect to the horizontal direction. The range of two objects are denoted as R_{30° and R_{60° . Choose the correct relation from the following

(a) $R_{30^0} = R_{60^0}$

(b) $R_{30^0} = 4R_{60^0}$

(c) $R_{30^0} = \frac{R_{60^0}}{2}$

(d) $R_{30^0} = 2\,R_{60^0}$

Solution

1.Option D

In Physics we prefer right handed coordinate system. The condition to check it is as follows.

$\hat{x} \times \hat{y} = \hat{z}$ using (cross product rule)

2. Option D

Magnitude of unit vector = 1

a) $|\hat{\imath} \times \hat{\jmath}| = \sqrt{2} \neq 1$

b) $\left|\frac{\hat{\imath}}{\sqrt{2}}\right| = \sqrt{\frac{1}{2}} \neq 1$

c) $\hat{k} - \frac{\hat{\jmath}}{\sqrt{2}} = \sqrt{1 + \frac{1}{2}} \neq 1$

d) $\frac{\hat{\imath}+\hat{\jmath}}{\sqrt{2}} = \sqrt{\frac{1}{2} + \frac{1}{2}} = 1$

3. Option C

Momentum is a vector hence it cannot be represented by a scalar.

4. Option C

Since they are falling, initial velocity is zero.

$u_1 = u_2 = 0$

$v^2 = u^2 + 2as$

$v_1^2 = 2gh_1$

$v_2^2=2gh_2$

$$\frac{m_1\ v_1}{m_2\ v_2} = \frac{m_1\sqrt{2gh_1}}{m_2\sqrt{2gh_2}}$$

5. Option A

Negative acceleration increases the negative velocity whereas it decreases positive velocity.

6. Option A

$$\frac{d\vec{v}}{dt} = \vec{a}; \quad \vec{a} = \frac{d(2\hat{\imath}+t^2\hat{\jmath}-9\hat{k})}{dt} = 2t\,\hat{\jmath}\,;$$
$$|\hat{a}| = 2t; \quad |\hat{a}|_{t=0.5} = 1ms^{-2}$$

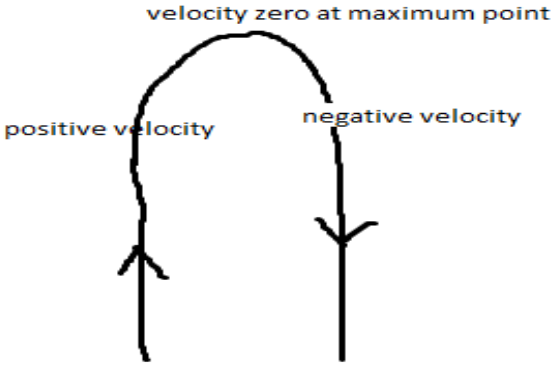
7. Option B

$s = ut + \frac{1}{2}\,at^2$; since the object is dropped $u = 0,\ a = 9.8ms^{-2}$.

$$s = \frac{1}{2}(9.8)(4) = 78.4m$$

8. Option C

Graph C alone matches our scenario.



9. Option A

Since horizontal velocity has no effect on vertical direction and distance covered by both the objects are same (action of gravity only).

10. Option A

Due to gravity the velocity increases with time hence distance travelled by ball in successive equal time interval increases.

11. Option C

Clockwise direction is conventionally taken to be negative and perpendicular to the plane in which circular motion takes place (Check Cross Product Rule).

12. Option D

A circular motion always posses centripetal acceleration whose direction changes continuously with time though it's radial, therefore only possible choice is D.

13. Option C

(Time of Flight) $T_f = \frac{2u\sin\theta}{g}$; Here $\theta = 90°$

14. Option A

(Range) $R_f = \frac{u^2\sin 2\theta}{g}$; $\sin 60° = \sin 120°$

15. Option B

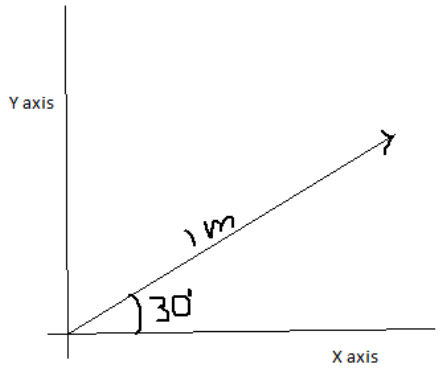
$s = ut + \frac{1}{2}\,at^2$; u=0

$$a = \frac{2s}{t^2} = \frac{2(50)}{4} = 25ms^{-2}$$

Exercise Problems

1. The position vectors particle has length 1m and makes 30° with the x-axis. What are the lengths of the x and y components of the position vector?

Solution:

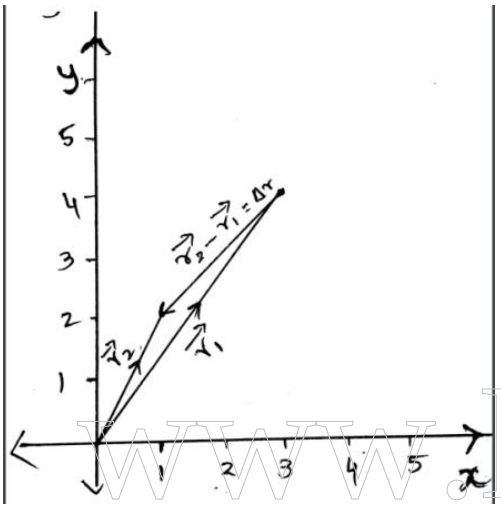


$x = r\cos\theta = r\cos 30° = (1)\frac{\sqrt{3}}{2}$

$y = r\sin\theta = (1)sin30° = \frac{1}{2}$

2. A particle has its position moved from $\vec{r_1} = 3\hat{\imath} + 4\hat{\jmath}$ *to* $\vec{r_2} = \hat{\imath} + 2\hat{\jmath}$. Calculate the displacement vector $\Delta\vec{r}$ and draw the $\vec{r_1}, \vec{r_2}$ *and* $\Delta\vec{r}$ in a two dimensional Cartesian coordinate system.

Solution:



$\Delta\vec{r} = \vec{r_2} - \vec{r_1} = \hat{\imath} + 2\hat{\jmath} - (3\hat{\imath} + 4\hat{\jmath})$

$= \hat{\imath} + 2\hat{\jmath} - 3\hat{\imath} - 4\hat{\jmath} = -2\hat{\imath} - 2\hat{\jmath}.$

3. Calculate the average velocity of the particle whose position vector changes from $\vec{r_1} = 5\hat{\imath} + 6\hat{\jmath}$ *to* $\vec{r_2} = 2\hat{\imath} + 3\hat{\jmath}$ in a time 5 second.

Solution:

$$\Delta\vec{v} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r_2} - \vec{r_1}}{\Delta t} = \frac{2\hat{\imath} + 3\hat{\jmath} - 5\hat{\imath} - 6\hat{\jmath}}{5}$$

$$= -\frac{3}{5}(\hat{\imath} + \hat{\jmath})$$

4. Convert the vector $\vec{r} = 3\hat{\imath} + 2\hat{\jmath}$ into a unit vector.

Solution:

$$\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{3\hat{\imath} + 2\hat{\jmath}}{\sqrt{3^2 + 2^2}} = \frac{3\hat{\imath} + 2\hat{\jmath}}{\sqrt{13}}$$

5. What are the resultants of the vector product of two given vectors given by $\vec{A} = 4\hat{\imath} - 2\hat{\jmath} + \hat{k}$ *and* $\vec{B} = 5\hat{\imath} + 3\hat{\jmath} - 4\hat{k}$?

Solution:

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 4 & -2 & 1 \\ 5 & 3 & -4 \end{vmatrix}$$

$$= \hat{\imath}(8 - 3) - \hat{\jmath}(-16 - 5) + \hat{k}(12 + 10)$$

$$= 5\hat{\imath} + 20\hat{\jmath} + 22\hat{k}$$

6. An object is projected at an angle such that the horizontal range is 4 times of the maximum height. What is the angle of projection of the object?

Solution:

Given

$R = 4h_{max}$

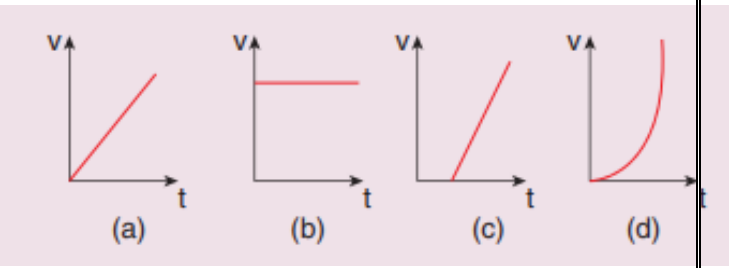
$$\frac{2u^2\sin\theta\cos\theta}{g} = 4\frac{u^2\sin^2\theta}{2g}$$

$1 = \tan\theta$

$\tan^{-1} 1 = \theta$

$\theta = 45°$

7. An object at an angle such that the horizontal range is 4 times of the Unit 2 Kinematics 101 maximum height. What is the angle of projection of the object?



Solution:

In a v-t graph, slope gives acceleration. Using the slope of each graph following conclusions have been made.

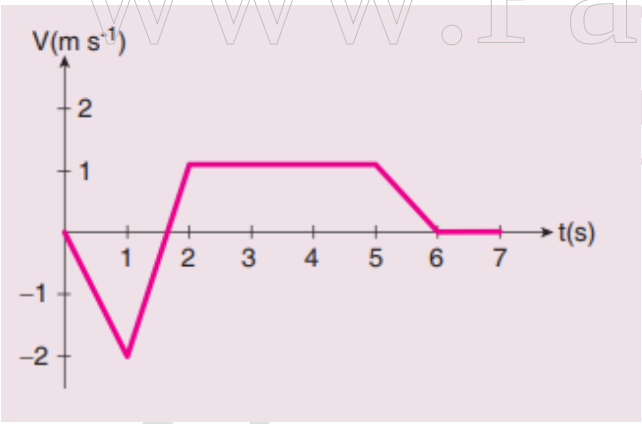
a) Slope is constant (a straight line) hence acceleration is constant.

b) Slope is zero, hence acceleration is zero.

c) Slope is constant since the line is more steeper than "a", acceleration is constant but a greater value than "a".

d) Slope increases with time hence acceleration is variable.

8. The following velocity–time graph represents a particle moving in the positive x–direction. Analyse its motion from 0 to 7 s. Calculate the displacement covered and distance travelled by the particle from 0 to 2 s.



Solution:

In a v-t graph area under curve gives the displacement.

Distance = 1.75m and Displacement = - 1.25m

9. A particle is projected at an angle of θ with respect to the horizontal direction. Match the following for the above motion.

(a) v_x – decreases and increases

(b) v_y – remains constant

(c) Acceleration – varies

(d) Position vector – remains downward

Solution:

v_x = remains constant

Explanation: For a particle being projected the acceleration is along vertical due to gravity and hence horizontal velocity remains constant

v_y = decreases and increases

Explanation: Due to negative acceleration the positive velocity decreases until it becomes zero at maximum height and from then on it increases in negative direction.

a= remains downward

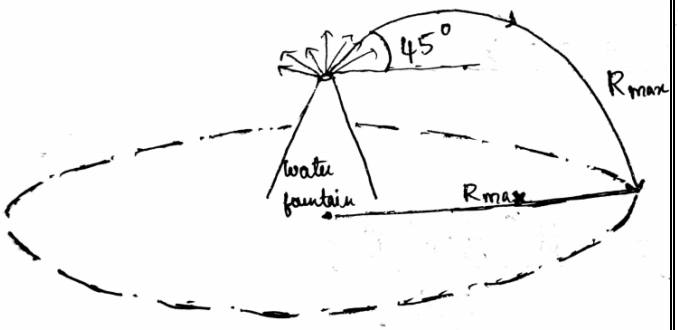
Explanation: Acceleration due to gravity.

r = varies

Explanation: It is based on the choice of origin.

10. A water fountain on the ground sprinkles water all around it. If the speed of the water coming out of the fountain is v. Calculate the total area around the fountain that gets wet.

Solution:



In water fountain, the water is projected with initial velocity v and the maximum range occurs when $\theta = 45°$ and the total area around the fountain that gets wet is a circle whose radius is R_{max}.

$$R = \frac{v^2 \sin 45°}{g} = \frac{v^2}{g}$$

$$Area = \pi R^2 = \frac{\pi v^4}{g^2}$$

11. The following table gives the range of a particle when thrown on different planets. All the particles are thrown at the same angle with the horizontal and with the same initial speed. Arrange the planets in ascending order according to their acceleration due to gravity, (g value).

Planet	Range
Jupiter	50 m
Earth	75 m
Mars	90 m
Mercury	95 m

Solution:

$R = \frac{u^2 \sin 2\theta}{g}$; $R \propto \frac{1}{g}$

$g_{jupiter} > g_{earth} > g_{mars} > g_{mercury}$

12. The resultant of two vectors A and B is perpendicular to vector A and its magnitude is equal to half of the magnitude of vector B. Then the angle between A and B is

Solution:

Magnitude of resultant of A and B is given by,
R = √(A² + B² + 2ABcosθ)

According to the question ,
Magnitude of resultant = half of Magnitude of B
√(A² + B² + 2A.Bcosθ) = B/2

Taking square both sides
A² + B² + 2A.Bcosθ = B²/4
A² + 2ABcosθ + 3B²/4 = 0

Also, A and R is perpendicular with each other ,
A.(A + B) = 0
A.A + A.B = 0
A² + A.Bcosθ = 0
cosθ = - A/B

A² + 2A.B(-A/B) + 3B²/4 = 0

A² - 2A² + 3B²/4 = 0

A = √3B/2

Now, cosθ = -A/B = -√3B/2B = -√3/2

cosθ = cos150° ⇒Φ = 150°

Hence angle between A and B = 150°

13. Compare the components for the following vector equations

a) $T\hat{j} - mg\hat{j} = ma\hat{j}$ b) $\vec{T} + \vec{F} = \vec{A} + \vec{B}$
c) $\vec{T} - \vec{F} = \vec{A} - \vec{B}$ d) $T\hat{j} + mg\hat{j} = ma\hat{j}$

Solution:

a) T – mg = ma

b) $T_x + F_x = A_x + B_x$

14. Calculate the area of the triangle for which two of its sides are given by the vectors
$\vec{A} = 5\hat{\imath} - 3\hat{\jmath}, \vec{B} = 4\hat{\imath} + 6\hat{\jmath}$.

Solution:

Area of a triangle whose sides are vector A and B

$$\frac{1}{2}|\vec{A} \times \vec{B}|$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 5 & -3 & 0 \\ 4 & 6 & 0 \end{vmatrix}$$

$$= \hat{\imath}(0) - \hat{\jmath}(0) + \hat{k}(30 + 12) = 42\hat{k}$$

$$\frac{1}{2}\left|\vec{A}\times\vec{B}\right| = \frac{1}{2}\sqrt{(42)^2} = 21$$

15. If Earth completes one revolution in 24 hours, what is the angular displacement made by Earth in one hour. Express your answer in both radian and degree.

Solution:

Earth completes one revolution in 24 hours . Hence angular displacement in 24 hours = 360°
angular displacement in 1 hour =360/24=15°.

Hence, angular displacement made by earth in one hour = 15°

The relation between radian and degree,

360° = 2π radian

1° = 2π/360 radian = π/180°

15° = 15π/180 = π/12 radian.

16. A object is thrown with initial speed 5 ms^{-1} with an angle of projection 30° . What is the height and range reached by the particle?

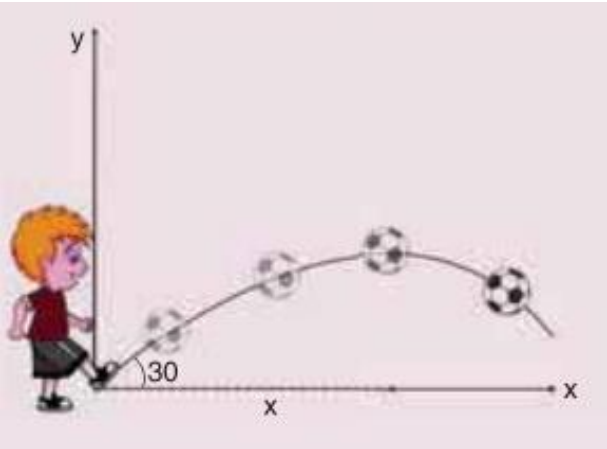
Solution:

$u = 5ms^{-1};\ \theta = 30°$

$$h_{max} = \frac{u^2\sin^2\theta}{2g} = \frac{25\times(\frac{1}{2})}{2\times 9.8} = 0.318m$$

$$R = \frac{u^2\sin 2\theta}{g} = \frac{25\times(\frac{\sqrt{3}}{2})}{9.8} = 2.21m$$

17. A foot-ball player hits the ball with speed 20 ms^{-1} with angle 30° with respect to horizontal direction as shown in the figure. The goal post is at distance of 40 m from him. Find out whether ball reaches the goal post?



Solution:

$u = 20ms^{-1}; \theta = 30°$

$$R = \frac{u^2\sin 2\theta}{g} = \frac{400\times\left(\frac{\sqrt{3}}{2}\right)}{9.8} = 35.3m$$

which is lesser than the distance at which post is located hence the ball will reach it.

18. If an object is thrown horizontally with an initial speed 10 m s^{-1} from the top of a building of height 100 m. what is the horizontal distance covered by the particle?

Solution:

Since the object is horizontally projected its vertical velocity is zero.

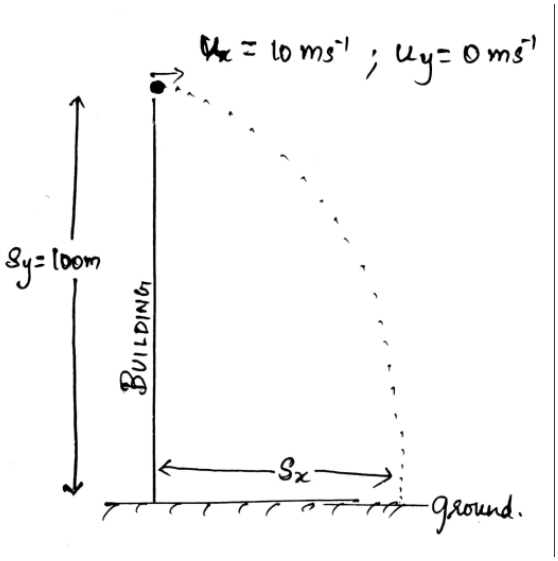
$u_x = 10ms^{-1};\ u_y = 0$

We resolve the motion into horizontal and vertical.

$$s_y = u_y t + \frac{1}{2}gt^2$$

$$\sqrt{\frac{2s_y}{g}} = t = \sqrt{\frac{200}{9.8}} = 4.5s$$

$$s_x = u_x \times t = 10\times 4.5 = 45m$$



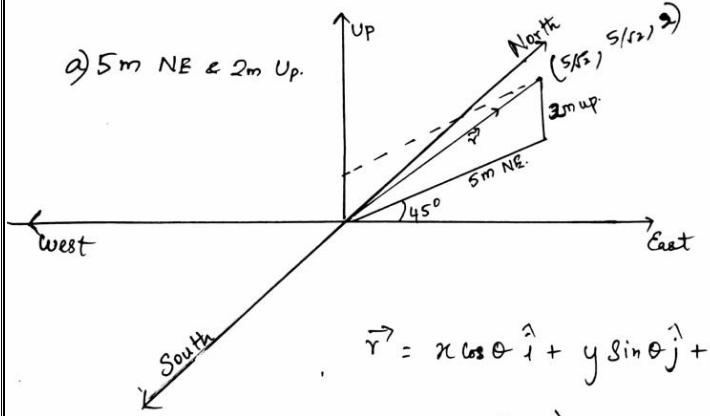
20. Consider the x–axis as representing east, the y–axis as north and z–axis as vertically upwards. Give the vector representing each of the following points.

a) 5 m north east and 2 m up

b) 4 m south east and 3 m up

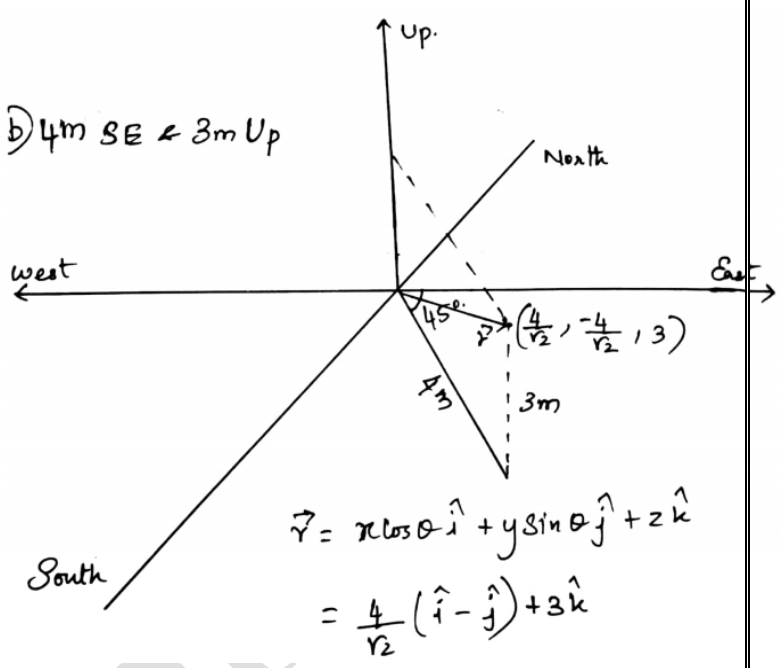
c) 2 m north west and 4 m up

Solution:



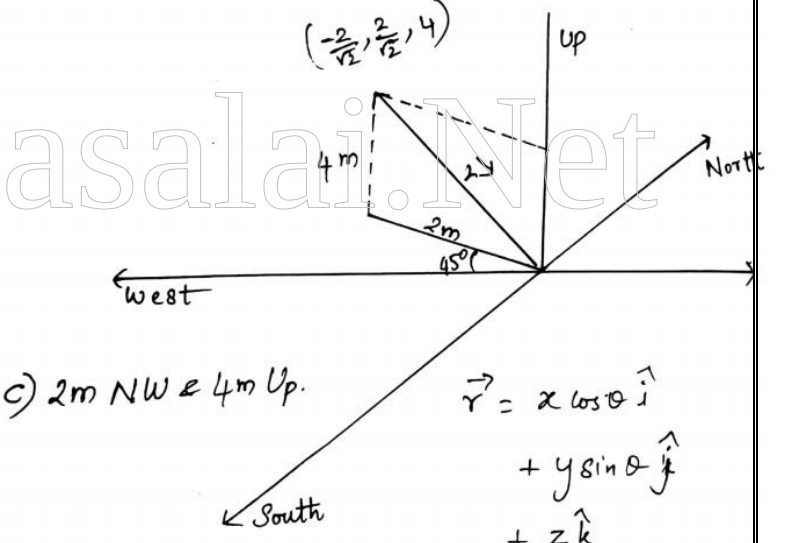
$$\vec{r} = x\cos\theta\,\hat{i} + y\sin\theta\,\hat{j} + z\hat{k}$$

$$= \frac{5}{\sqrt{2}}(\hat{i}+\hat{j}) + 2\hat{k}$$



$$\vec{r} = x\cos\theta\,\hat{i} + y\sin\theta\,\hat{j} + z\hat{k}$$

$$= \frac{4}{\sqrt{2}}(\hat{i}-\hat{j}) + 3\hat{k}$$



c) 2m NW & 4m Up.

$$\vec{r} = x\cos\theta\,\hat{i} + y\sin\theta\,\hat{j} + z\hat{k}$$

$$\vec{r} = \frac{-2}{\sqrt{2}}\hat{i} + \frac{2}{\sqrt{2}}\hat{j} + 4\hat{k}$$

$$= (-\hat{i}+\hat{j})\sqrt{2} + 4\hat{k}$$

21. The Moon is orbiting the Earth approximately once in 27 days, what is the angle transversed by the Moon per day?

Solution:

Angle transversed by Moon in 27 days = $360°$

Angle tranversed in one day = $\frac{360°}{27} = 13.33°$

22. An object of mass m has angular acceleration α $= 0.2rad\ s^{-2}$What is the angular displacement covered by the object aft er 3 second? (Assume that the object started with angle zero with zero angular velocity).

Solution:

$\omega_i = 0; t = 3s;$

$\theta = \omega_i t + \frac{1}{2}\alpha t^2 = \frac{1}{2}(0.2)9 = 0.9rad$

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