Padasalai's
Telegram Groups!

(தலைப்பிற்கு கீழே உள்ள லிங்கை கிளிக் செய்து குழுவில் இணையவும்!)

- Padasalai's NEWS - Group
https://t.me/joinchat/NIfCqVRBNj9hhV4wu6_NqA

- Padasalai's Channel - Group
https://t.me/padasalaichannel

- Lesson Plan - Group
https://t.me/joinchat/NIfCqVWwo5iL-21gpzrXLw

- 12th Standard - Group
https://t.me/Padasalai_12th

- 11th Standard - Group
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- 10th Standard - Group
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- 9th Standard - Group
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- 6th to 8th Standard - Group
https://t.me/Padasalai_6to8

- 1st to 5th Standard - Group
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- TET - Group
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- PGTRB - Group
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- TNPSC - Group
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HIGHER SECONDARY – SECOND YEAR

BUSINESS MATHEMATICS
&
STATISTICS

QUESTION BANK

BASED ON NEW SYLLABUS

VOLUME – I & II

Contains

Points To Remember

Classification Of Text Book Problems

Answer to theory questions

Creative Questions

Answer to creative questions

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1 Applications of Matrices and Determinants

POINTS TO REMEMBER

: **Relation between adjoint and determinant of a matrix :**

- $|adjA| = |A|^{n-1}$

: CRAMER'S RULE :

- Cramer's rule is applicable only when $\Delta \neq 0$.

RANK OF A MATRIX

- Rank of a matrix

  The rank of a matrix A is the order of the largest non-zero minor of A
- The rank of a matrix A is the order of the largest non-zero minor of A
- $\rho(A) \geq 0$
- If A is a matrix of order $m \times n$, then $\rho(A) \leq$ minimum of $\{m, n\}$
- The rank of a zero matrix is '0'
- The rank of a non- singular matrix of order $n \times n$ is 'n'

CONDITION FOR EQUIVALENT MATRICES

- **Equivalent Matrices**

  Two Matrices A and B are said to be equivalent if one can be obtained from another by a finite number of elementary transformations and we write it as $A \sim B$.

ECHELON FORM OF A MATRIX

- **Echelon form**

  A matrix of order $m \times n$ is said to be in echelon form if the row having all its entries zero will lie below the row having non-zero entry.

RANK METHOD TO CHECK THE CONSISTENCY

- A system of equations is said to be consistent if it has at least one set of solution. Otherwise, it is said to be inconsistent
  - If $\rho([A, B]) = \rho(A)$, then the equations are consistent.
  - If $\rho([A, B]) = \rho(A) = n$, then the equations are consistent and have unique solution.
  - If $\rho([A, B]) = \rho(A) < n$, then the equations are consistent and have infinitely many solutions.
  - If $\rho([A, B]) \neq \rho(A)$ then the equations are inconsistent and has no solution.

SINGULAR & NON SINGULAR MATRIX

- If $|A| = 0$ then A is a singular matrix. Otherwise, A is a non singular matrix.
- In $AX = B$ if $|A| \neq 0$ then the system is consistent and it has unique solution.

CLASSIFICATION OF TEXT BOOK PROBLEMS

2 Marks

EXERCISE		EXAMPLE	Miscellaneous
Ex. No.,	Question Number	Question Number	Question Number
1.1	1. (i), (ii), (iii)	1.1, 1.2	1

3 Marks

EXERCISE		EXAMPLE	Miscellaneous
Ex. No.,	Question Number	Question Number	Question Number
1.1	1. (iv), (v), (vi), (vii), (viii), 2 (AB & BA Separately), 5	1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 1.10, 1.11	2, 3, 4, 10
1.2	1 (i), (ii), 2. 3, 4	1.19, 1.20, 1.21	
1.3	1	1.25, 1.26, 1.27	

5 Marks

EXERCISE		EXAMPLE	Miscellaneous
Ex. No.,	Question Number	Question Number	Question Number
1.1	3, 4, 6, 7, 8	1.12, 1.13, 1.14, 1.15, 1.16, 1.17, 1.18	5, 6, 7, 8, 9
1.2	1 (iii), (iv), (v) 5, 6	1.22, 1.23, 1.24	
1.3	2,3,4	1.28	

: CREATIVE QUESTIONS :

2 MARKS

1. Using Cramer's Rule, can you solve the system of linear equations, $2x + 3y = 7$, $4x + 6y = 14$? Justify your answer.
2. If $A = \begin{pmatrix} -1 & 2 \\ 4 & -4 \end{pmatrix}$ and $B^T = \begin{pmatrix} 1 & -4 \\ -2 & 4 \end{pmatrix}$ then find the rank of A+B
3. Find the value of 'λ' if the equations $2x + 3y + 4z = 9$, $2y + 3z = 5$, $\lambda z = 1$, are inconsistent.
4. $A = \begin{pmatrix} x & x & x \\ 4 & -2 & 1 \\ 2 & 3 & 4 \end{pmatrix}$ find x if $\rho(A) = 3$
5. Find the rank of the matrix $\begin{pmatrix} 2 & -4 & x \\ 4 & -8 & 2x \\ 5 & 7 & 1 \end{pmatrix}$
6. Write the Augmented matrix for the following system of equations,
$$x - 3y + 4z = 9, \quad 2x + 3z = 10, \quad 3x + y - 2z + 5 = 0$$
7. Show that the equations $4x - 3y = 8$, $2x + y = 5$ have unique solution.
8. For the matrices, $A = \begin{pmatrix} 1 & 8 \\ 1 & 12 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 5 \\ 0 & 7 \end{pmatrix}$ Prove that $\rho(A) + \rho(B) = |A|$
9. Solve : $2x + y = 4$, $x + 2y = 3$
10. Person X invested a sum of rupees ten thousands partly in two bonds yielding 5% and 6% interest respectively, and earns rupees 800 per year. Represent this given data as a system of linear equations.
11. 7 men and 4 women can jointly finish a piece of work in 10 days, whereas 4 men and 7 women can jointly finish the same work in 15 days, represent this data as a system of linear equations.
12. Two Internet service providers P and Q shares the market by having 40% and 60% customers. 60% Of those who are having P switch over Q and 50% of those who are having Q switch over to P.
Represent transition probability matrix for this data.
13. In a cricket match the winning chances for the two teams switches every hour. Based on that 12% supporters of team A switch over to B and 14% supporters of team B switch over to A.
Represent the transition probability matrix for this data.
14. Find the rank of the augmented matrix of the equations $\frac{1}{x} + \frac{2}{y} = 5$, $\frac{1}{x} + \frac{3}{y} = 8$
15. The pattern of sunny and rainy days on the planet Rainbow is a such that every sunny day is followed by another sunny day with probability 0.8, and every rainy day is followed by another rainy day with probability 0.6.
Represent this data as a transition probability matrix

3 MARKS

1. Solve by Cramer's rule : $11x - 13y = 7, \quad 3x + 2y = 13$

2. Using Cramer's rule, solve : $3e^x + 2e^y = 7, \; 4e^x + 7e^y = 18$

3. If $A = \begin{pmatrix} 0 & 3 & 2 & 4 \\ -2 & 5 & 8 & 0 \\ 2 & 1 & 4 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 & 5 & 0 \\ -3 & 4 & 6 & 5 \\ -1 & 2 & 3 & 4 \end{pmatrix}$ find the rank of A + B

4. If $A = \begin{pmatrix} 3 & 1 \\ 4 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} 5 & -2 \\ 3 & -3 \end{pmatrix}$ find the rank of AB

5. Examine the consistency of the following equations,
$$x - 4y + 7z = 14, \quad 3x + 8y - 2z = 13, \quad 7x - 8y + 26z = 5$$

6. Show that the equations $2x - y + z = 7, \; 3x + y - 5z = 13, \; x + y + z = 5$ are consistent and have unique solution.

7. Find 'k' if the equations $3x + 2y + 4z = 11, \; 2x + 3y = 6, \; x + 4y + kz = 2$ are inconsistent.

8. Construct a 3 x 4 matrix whose elements are same, then find its rank.

9. For the transition probability matrix $\begin{pmatrix} .6 & .4 \\ .45 & .55 \end{pmatrix}$ Find the new shares of X and Y after one period if their present share of 50% each.

10. Sum of the average marks of 2 students in an examination is 90, if the average marks of one student is double the average marks of the another. Then find the average marks of each student individually, using Cramer's rule.

11. Find the rank of the matrix $\begin{pmatrix} 4 & 2 & 1 & 3 \\ 6 & 3 & 4 & 7 \\ 2 & 1 & 0 & 1 \end{pmatrix}$

12. Find the rank of the matrix $\begin{pmatrix} 1 & 3 & -1 & 4 \\ 3 & 4 & 2 & 1 \\ -2 & 3 & 1 & 0 \end{pmatrix}$

13. If $A = \begin{pmatrix} 1 & 2 & -1 \\ 3 & 2 & 4 \\ 2 & 1 & 1 \end{pmatrix}$ then find $|\operatorname{adj}(A)|$

14. Find the value of x and y using Cramer's rule, $3x - 2y = 5, \; 5x - 3y = 9$

15. If $\operatorname{adj} A = \begin{pmatrix} 1 & 2 & -1 \\ 3 & 2 & 4 \\ 2 & 1 & 1 \end{pmatrix}$ then find $|A|$.

5 MARKS

1. Find x, y, z for the following system of equations
$$\frac{2}{x}+\frac{3}{y}+\frac{4}{z}=14 \quad , \quad \frac{3}{x}-\frac{2}{y}+\frac{1}{z}=3 \quad , \quad \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=5$$

2. Five years ago, the Father's age was six times the Son's age. The difference in their present ages is 25. Find their present ages using Cramer's rule.

3. ind x, y, z for the following system of equations
$$\frac{1}{x}+\frac{2}{y}+\frac{3}{z}=\frac{9}{4} \quad , \quad \frac{4}{x}-\frac{2}{y}+\frac{4}{z}=\frac{4}{3} \quad , \quad \frac{2}{x}+\frac{1}{y}+\frac{3}{z}=2$$

4. Find the solution of the system of equations using Cramner's rule
$7x - y - z = 0, \quad 10x - 2y + z = 8, \quad 6x + 3y - 2z = 7$

5. Find the solution of the system of equations using Rank method
$7x - y - z = 0, \quad 10x - 2y + z = 8, \quad 6x + 3y - 2z = 7$

6. A bag contains 3 types of coins namely ₹ 1, ₹2 and ₹ 5. There are 60 coins amounting to ₹ 200 in total. If the sum of the number of coins of ₹ 1 and ₹2 is equal to the number of ₹5 coins, find the number of coins in each category.

7. Solve by using Rank method : $x + y + 2z = 4, \; 2x + 2y + 4z = 8, \; 3x + 3y + 6z = 12$

8. In a stadium there were three types of chairs namely A, B, C. Totally there were 500 chairs. The prices for sitting on type A, Type B and Type C chairs are ₹ 500, ₹ 250, ₹ 100 respectively and when all the chairs are occupied the total collection will be ₹ 1,05,000. If the number chairs in Type A and Type B are equal, find the number of chairs in each Type.

9. Solve by Cramer's rule : $2x - z = 0, \; 5x + y = 4, \; y + 3z = 5$

10. Solve by rank method : $x - y + 2z = 3, \; 2x + z = 1, \; 3x + 2y + z = 4$

11. In a restaurant, the prices of the items Idly, Vada, Dosai are respectively x, y, z rupees. The following table shows the qty and the total bill amount of three customers,

Item → Bill No., ↓	Idly	Vada	Doasi	Bill Amount ₹
101	4	2	2	110
102	5	2	3	140
103	2	4	5	180

Using the above data, find the prices of each Idly, Vada and Dosai.

12. Metro Rail transit system has just gone into operation in a city. Of those who use the transit system this year, 15% will switch over to using their own car next year and 85% will continue to use the transit system. Of those who use their cars this year, 70% will continue to use their cars next year and 30% will switch over to the transit system. Suppose the population of the city remains constant and that 60% of the commuters use the transit system and 40% of the commuters use their own car this year,
(i) What will be the change in commuter's usage after one year.
(ii) What percent of commuters will be using the Metro train system in the long run?

13. The subscription department of a magazine sends out a letter to a large mailing list inviting subscriptions for the magazine. Some of the people receiving this letter already subscribe to the magazine while others do not. From this mailing list, 60% of those who already subscribe will subscribe again while 25% of those who do not now subscribe will subscribe. On the last letter it was found that 40% of those receiving it ordered a subscription. What percent of those receiving the current letter can be expected to order a subscription?

14. Two newspapers A and B are published in a city. Their present market shares are 25% for A and 75% for B. Of those who bought A the previous year, 55% continue to buy it again while 45% switch over to B. Of those who bought B the previous year, 75% buy it again and 25% switch over to A. Find their market shares after two years.

15. In a perfect competition between the two insurance companies shares with market shares 40% for company-A and 60% for company-B. Each year some dissatisfied clients switches over from one company to the other. In the last year 30% of the clients of company-A switched over to company-B and 40% of the clients from company-B switched over to company-A. Determine transition probability matrix for the given data, Calculate their market shares after one year. Also calculate when will the equilibrium level be reached?

ANSWERS TO CREATIVE QUESTIONS

2 Marks :

1. Can not use Cramer's rule, because $\Delta = 0$ 2. $\rho(A+B) = 0$

3. $\lambda = 0$ 4. $x \neq 0$ 5. Rank = 2   6. $\begin{pmatrix} 1 & -3 & 4 & 9 \\ 2 & 0 & 3 & 10 \\ 3 & 1 & -2 & -5 \end{pmatrix}$

9. x = 5/3 , y = 2/3   10. x+y=10000, 5x+6y=80000

11. 7/x + 4/y = 1/10, 4/x + 7/y = 1/15   12. $\begin{pmatrix} .4 & .6 \\ .5 & .5 \end{pmatrix}$ 13. $\begin{pmatrix} .88 & .12 \\ .14 & .36 \end{pmatrix}$

14. $\rho([A,B]) = 2$ 15. $\begin{pmatrix} .8 & .2 \\ .4 & .6 \end{pmatrix}$

3 Marks :

1. x=3, y=2   2. x=0, y=log 2   3. $\rho([A+B]) = 2$ 4. $\rho([AB]) = 2$

5. Consistent and have unique solution   7. k = -4

8. $\begin{pmatrix} x & x & x & x \\ x & x & x & x \\ x & x & -x & x \end{pmatrix}$, rank=1   9. 52.5% for X and 47.5% for Y

10. 60, 30   11. Rank = 2   12. Rank=3   13. 81

14. x=3, y=2   15. ± 3

5 Marks :

1. x= ½ , y= ½ . z=1   2. father's age 35, son's age 10

3. x=4, y=2, z=3   4. x=1, y=3 , z=4   5. x=1, y=3 , z=4

6. 10, 20, 30   7. x =4-t-2s, y=t, z=s, t,s € R   8. 100, 100, 300

9. x=1, y=-1, z=2   10. x=-1, y=2, z=3   11. 10, 15, 20

12. (i) x=63% y=37%   (ii) x=66.67%, y=33.33%   13. 39%

14. 34.75%, 65.25%   15. 52%, 48% and 57.14%, 43.86%

2 Integral Calculus – I

POINTS TO REMEMBER

Difference between F(x) and f(x)

- **The relation between the Primitive function and the derived function:**

  A function $F(x)$ is said to be a primitive function of the derived function $f(x)$, if $\frac{d}{dx}\left[F(x)\right]=f(x)$

MEANING OF INTEGRATION

- **Integration of a function:**

  The process of determining an integral of a given function is defined as integration of a function

PROPERTIES OF INDEFINITE INTEGRAL

$$\int a\, f(x)\ dx = a\int f(x)\ dx$$

$$\int\left[f(x)\pm g(x)\right]dx=\int f(x)\ dx \pm \int g(x)\ dx$$

STANDARD RESULTS OF INTEGRATION

1. $\int x^n\,dx = \frac{x^{n+1}}{n+1}+c$
2. $\int \frac{1}{x}dx = \log|x|+c$
3. $\int e^x\,dx = e^x+c$
4. $\int a^x\ dx = \frac{1}{\log a}a^x+c, a>0$ and $a\neq 1$
5. $\int \sin x\,dx = -\cos x+c$
6. $\int \cos x\,dx = \sin x+c$

SOME MORE RESULTS ON INTEGRATION

7. $\int \sec^2 x\,dx = \tan x + c$ 8. $\int \operatorname{cosec}^2 x\,dx = -\cot x + c$

9. $\int [f(x)]^n f'(x)\,dx = \dfrac{[f(x)]^{n+1}}{n+1} + c, n \neq -1$

10. $\int \dfrac{f'(x)}{f(x)}dx = \log|f(x)| + c$ 11. $\int \dfrac{f'(x)}{\sqrt{f(x)}}dx = 2\sqrt{f(x)} + c$

INTEGRATION BY PARTS

12. $\int u\,dv = uv - \int v\,du$

BERNOULLI'S FORMULA

13. $\int u\,dv = uv - u'v_1 + u''v_2 - u'''v_3 + \ldots$

SOME MORE RESULTS ON INTEGRATION

14. $\int \dfrac{dx}{a^2 - x^2} = \dfrac{1}{2a}\log\left|\dfrac{a+x}{a-x}\right| + c$ 15. $\int \dfrac{dx}{x^2 - a^2} = \dfrac{1}{2a}\log\left|\dfrac{x-a}{x+a}\right| + c$

16. $\int \dfrac{dx}{\sqrt{x^2 - a^2}} = \log\left|x + \sqrt{x^2 - a^2}\right| + c$ 17. $\int \dfrac{dx}{\sqrt{x^2 + a^2}} = \log\left|x + \sqrt{x^2 + a^2}\right| + c$

IMPORTANT PROPERTY ON INDEFINITE INTEGRAL

18. $\int e^x [f(x) + f'(x)]\,dx = e^x f(x) + c$ 19. $\int e^{ax} [a f(x) + f'(x)]\,dx = e^{ax} f(x) + c$

20. $\int \sqrt{x^2 - a^2}\,dx = \dfrac{x}{2}\sqrt{x^2 - a^2} - \dfrac{a^2}{2}\log\left|x + \sqrt{x^2 - a^2}\right| + c$

21. $\int \sqrt{x^2 + a^2}\,dx = \dfrac{x}{2}\sqrt{x^2 + a^2} + \dfrac{a^2}{2}\log\left|x + \sqrt{x^2 + a^2}\right| + c$

DEFINITE INTEGRAL

- **Definite integral:**

Let $f(x)$ be a continuous function on [a,b] and if $F(x)$ is anti derivative of $f(x)$, then $\int_a^b f(x)dx = F(b) - F(a)$.

PROPERTIES OF DEFINITE INTEGRAL

(i) $\int_a^b f(x)dx = \int_a^b f(t)dt$

(ii) $\int_a^b f(x)dx = -\int_b^a f(x)dx$

(iii) $\int_a^b [f(x) \pm g(x)]dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$

(iv) $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$

(v) $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

(vi) $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

(vii) a) If $f(x)$ is an even function, then $\int_{-a}^a f(x)dx = 2\int_0^a f(x)dx$

b) If $f(x)$ is an odd function, then $\int_{-a}^a f(x)dx = 0$

- **Particular case of Gamma Integral:**

If n is a positive integer, then $\int_0^\infty x^n e^{-ax}\, dx = \frac{n!}{a^{n+1}}$

- **Properties of gamma function:**

(i) $\Gamma(n) = (n-1)\Gamma(n-1), n>1$

(ii) $\Gamma(n+1) = n\Gamma(n), n>0$

(iii) $\Gamma(n+1) = n!$, n is a positive integer

(iv) $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

LIMIT OF A SUM

- **Definite integral as the limit of a sum:**

  Let $f(x)$ be a continuous real valued function in $[a, b]$, which is divided into n equal parts each of width h, then

$$\int_a^b f(x)dx = \lim_{\substack{h \to 0 \\ n \to \infty}} \sum_{r=1}^{n} h\, f(a + rh) \quad where\ h = \frac{b-a}{n}$$

RESULTS ON SUMMATION OF SERIES NEEDED IN LIMIT OF SUM

(i) $1 + 2 + 3 + \ldots\ldots n = \frac{n(n+1)}{2} = \sum_{r=1}^{n} r$

(ii) $1^2 + 2^2 + 3^2 + \ldots\ldots n^2 = \frac{n(n+1)(2n+1)}{6} = \sum_{r=1}^{r=n} r^2$

(iii) $1^3 + 2^3 + 3^3 + \ldots\ldots n^3 = \left[\frac{n(n+1)}{2}\right]^2 = \sum_{r=1}^{r=n} r^3$

CLASSIFICATION OF TEXT BOOK PROBLEMS

2 Marks

EXERCISE		EXAMPLE	Miscellaneous
Ex. No.,	Question Number	Question Number	Question Number
2.1	1, 2, 3	2.2, 2.3, 2.4, 2.5, 2.9, 2.10, 2.16	4
2.2	1	2.17, 2.19, 2.20, 2.21,2.22, 2.23	
2.4	1, 5	2.30, 2.31, 2.32, 2.40,2.41, 2.43	
2.6	1, 3, 5, 6, 7, 9	2.46, 2.48, 2.51, 2.52,2.53, 2.59	
2.7	1, 3, 8, 13, 14	2.66, 2.71, 2.74 → (i),(ii),(iii)	
2.8	1, 2	2.75, 2.76, 2.77, 2.80(i),(ii),(iii)	
2.9	1		
2.10,	1 → (i) , (ii), (iii), (iv)		

3 Marks

EXERCISE		EXAMPLE	Miscellaneous
Ex. No.,	Question Number	Question Number	Question Number
2.1	4, 5, 6, 7, 8	2.1,2.6,2.7,2.8,2.11,2.12,2.13	1, 2, 3, 5, 6 7, 8, 9
2.2	2, 3, 4, 5, 8	2.18,2.24,2.25,2.26,2.272.29	
2.3	1, 2, 3, 4, 5, 6, 7, 8	2.33,2.34,2.35,2.36,2.38	
2.4	2. 3, 4	2.42,2.44,2.45,2.47,2.49	
2.5	1, 2, 3, 4, 5, 6	2.50,2.54,2.55,2.56,2.57	
2.6	2, 4, 8, 13, 15	2.58,2.60,2.61,2.62,2.63	
2.7	2,4,5,6,7,9,10,11,12, 15, 16	2.64,2.65,2.67,2.68,2.69,2.70, 2.72,2.73,2.74 → (iv),(v)	
2.8	I → 3,4.5.6,7.8,9 II → 1,2,3,4	2.80 (iv)	
2.9	2, 3		
2.10,	I (v) , 2		

5 Marks

EXERCISE		EXAMPLE	Miscellaneous
Ex. No.,	Question Number	Question Number	Question Number
2.2	6, 7	2.14, 2.15	
2.5		2.28	
2.6	10, 11, 12, 14	2.37, 2.39	10
2.9	4, 5, 6	2.78, 2.79	
2.11	1, 2, 3, 4	2.81, 2.82, 2.83	

: CREATIVE QUESTIONS :

2 MARKS

Evaluate the following integrals.

1. $\int (x+1)^7 dx$ **2.** $\int (3x-4)^{(7/2)} dx$ **3.** $\int (5-6x)^2 dx$

4. $\int ½(3+\frac{x}{2})^5 dx$ **5.** $\int (\frac{1}{(x-1)^8}) dx$ **6.** $\int (\frac{1}{(5x+4)^{(7/2)}}) dx$

7. $\int (\frac{10}{(5x-2)}) dx$ **8.** $\int e^{(5x-1)} dx$ **9.** $\int 5^{(1-4x)} dx$

10. $\int ((2x+3) e^{(x^2+3x+5)}) dx$ **11.** $\int ((2^x).(2^{-x})) dx$ **12.** $\int (ae)^x dx$

13. $\int \sqrt{(1+\cos 2x)} dx$ **14.** $\int \sqrt{(1-\cos 2x)} dx$ **15.** $\int (x^2 e^{(3x)}) dx$

16. $\int (\frac{(4x+5)}{(2x^2+5x+4)}) dx$ **17.** $\int (\frac{-4}{\sqrt{(5-4x)}}) dx$ **18.** $\int (\frac{1}{(x^2+25)}) dx$

19. $\int (\frac{1}{(16-(3+x)^2)}) dx$ **20.** $\int (\sqrt{((x+1)^2-16)}) dx$

21. $\int (\frac{1}{(5-(2x+3)^2)}) dx$ **22.** $\int (\sqrt{(9-(2x+1)^2)}) dx$

23. $\int_{-\pi/2}^{\pi/2} \sin^3 x \, dx$ **24.** Find $\Gamma \frac{11}{2}$

25. $\int_{-2}^{2} (\frac{1}{(9+x^2)}) dx$ **26.** $\int_{-1}^{1} (5x^7-6x^5+3x^3-x+7) dx$

27. $\int_{0}^{\infty} (x^5)(e^{(-7x)}) dx$ **28.** Define Gamma function.

29. Write the formula used in definite integral as the limit of a sum.

3 MARKS

Evaluate the following integrals.

1. $\int (\frac{(9x^4-3x^3+3x-1)}{(3x-1)})dx$

2. $\int (\frac{(x+4)}{\sqrt{(5x-1)}})dx$

3. $\int (\frac{1}{(\sqrt{(5x+1)}-\sqrt{(5x-1)})})dx$

4. $\int (\frac{3}{(x(x+1))})dx$

5. If $f'(x) = \frac{1}{(1+x^2)}$ and $f(1) = \pi/4$, then find $f(x)$

6. $\int \sqrt{(1-\sin x)}dx$

7. $\int (\frac{1}{(x(\log x)^3)})dx$

8. $\int (\frac{1}{(x^2+6x+10)})dx$

9. $\int (\frac{1}{\sqrt{(2x^2+4x+7)}})dx$

10. $\int (\frac{1}{\sqrt{(1-x-x^2)}})dx$

11. $\int_1^2 (\frac{9}{((x+2)(x+1))})dx$

12. $\int_1^e (\frac{(\log x)^2}{x})dx$

13. $\int_{-1}^1 \log\frac{(1-x)}{(1+x)}dx$

14. State any three properties of definite integral.

15. If f(x) = { 2x – 4 , 1 ≤ x < 5
{ 0 , x ≥ 5

then find $\int_1^{10} f(x)dx$

16. $\int_{-\pi/2}^{\pi/2} \cos^2\Theta\, dx$

17. $\int_2^a 9x^3 dx=540$ then find the value of 'a'

18. State any three properties of Gamma Integrals

Evaluate :

19. $\int_0^1 x(1-x)^8 dx$

20. $\int_1^2 x(3-x)^6 dx$

5 MARKS

Evaluate the following integrals.

1. $\int_{0}^{\pi/2} \left(\frac{\sqrt{(\sin^3 x)}}{(\sqrt{(\sin^3 x)}+\sqrt{(\cos^3 x)})}\right) dx$

2. $\int_{0}^{3} \left(\frac{\sqrt{(x)}}{(\sqrt{(x)}+\sqrt{(3-x)})}\right) dx$

3. $\int_{0}^{\pi/2} \left(f \frac{(\sin x)}{(f(\sin x)+f(\cos x))}\right) dx$

4. $\int_{\pi/6}^{\pi/3} \left(\frac{1}{(1+\sqrt{(\cot x)})}\right) dx$

5. $\int_{\pi/6}^{\pi/3} \left(\frac{1}{(1+\sqrt{(\tan x)})}\right) dx$

6. $\int \left(\frac{(2x+1)}{((x-1)(x+2)^2)}\right) dx$

7. Evaluate the integral as the limit of a sum $\int_{2}^{3} (3x+4)\, dx$

8. $\int_{0}^{\infty} \left(\frac{(e^{(\arctan(x))})}{(1+x^2)}\right) dx$

9. $\int_{0}^{\pi/4} x \sec^2 x\, dx$

10. $\int_{0}^{2} \left(\frac{x}{(\sqrt{(3x+1)}+\sqrt{(2x+1)})}\right) dx$

11. $\int_{3}^{8} \left(\frac{\sqrt{(x)}}{(\sqrt{(x)}+\sqrt{(11-x)})}\right) dx$

12. $\int \left(\frac{(x+5)}{(x^2-3x+2)}\right) dx$

13. $\int \sin^{-1} x\, dx$

14. $\int_{0}^{\pi/2} \log \cot x\, dx$

15. $\int \left(e^x \frac{(\cos 2x)}{(\cos x+\sin x)}\right) dx$

16. $\int_{0}^{\pi/2} \log \tan x\, dx$

17. State any five properties of definite integral.

ANSWERS TO CREATIVE QUESTIONS

2 Marks :

1. $\frac{(x+1)^5}{5}+c$
2. $\frac{2}{27}(3X-4)^{(\frac{9}{2})}+c$
3. $\frac{(5-6X)^3}{-18}+c$
4. $\frac{(3+\frac{x}{2})^6}{3}+c$
5. $\frac{-1}{(7(x-1)^2)}+c$
6. $\frac{-2}{(25(5x+4)^{(5/2)})}+c$
7. $\log(5x-2)+c$
8. $\frac{e^{(5x-1)}}{5}+c$
9. $\frac{5^{(1-4X)}}{(-4\,log5)}+c$
10. $e^{(x^2+5x+5)}+c$
11. $x+c$
12. $\frac{(ae)^x}{(loga+1)}+c$
13. $\sqrt{(2)}\sin x+c$
14. $-\sqrt{(2)}\cos x+c$
15. $\frac{e^{3x}}{3}(x^2-\frac{2x}{3}+\frac{2}{5})+c$
16. $\log(2x^2+5x+4)+c$
17. $2\sqrt{(5-4x)}+c$
18. $\frac{1}{10}\ \tan^{-1}\ \frac{x}{5}+c$
19. $\frac{1}{18}\log(\frac{(7+x)}{(1-x)})+c$
20. $\frac{(x+1)}{2}\sqrt{((x+1)^2-16)}-8\log((x+1)+\sqrt{((x+1)^2-16)})+c$
21. $\frac{1}{(2\sqrt{5})}\ \log\ \frac{(\sqrt{(5)}=2x+3)}{(\sqrt{(5)}-(2x+3))}+c$
22. $\frac{(2x+1)}{4}\sqrt{(9-(2x+1)^2)}+\frac{9}{4}\ \sin^{-1}\ \frac{(2x+1)}{3}\ +c$
23. 0
24. $\frac{945}{32}\sqrt{(n)}$
25. 0
26. 0
27. $\frac{(5!)}{7^6}$

3 Marks :

1. $\frac{3}{4}x^4+x+c$ 2. $\frac{1}{5}[\frac{2}{15}(5x-1)^{(\frac{3}{2})}+42(5x-1)^{(\frac{1}{2})}]+c$

3. $\frac{1}{15}[(5x-1)^{(\frac{3}{2})}+(5x-1)^{(\frac{2}{2})}]+c$ 4. $\log(\frac{x}{(x+3)})+c$

5. $f(x) = \tan^{-1} x$ 6. $2[\sin\frac{x}{2}+\cos\frac{x}{2}]+c$ 7. $\frac{-1}{(2(\log x)^2)}+c$

8. $\tan^{-1}(x+3) + c$ 9. $\frac{1}{12}\log((x+1)+\sqrt{(2x^2+4x+7)})+c$

10. $\sin^{-1}(\frac{(2x+1)}{\sqrt{(3)}}) + c$ 11. $\log(\frac{16}{13})+c$ 12. $\frac{1}{3}$ 13. 0

15. 8 16. π/2 17. a = 4 19. $\frac{1}{90}$ 20. $\frac{1263}{56}$

5 Marks :

1. π/4 2. 3/2 3. π/4 4. π/12 or 15°

5. π/12 or 15° 6. $\frac{1}{3}\log(\frac{(x-1)}{(x+2)}-\frac{2}{(x+2)^3})+c$ 7. $\frac{23}{2}$

8. $e^{(\pi/2-1)}$ 9. π/4 – log √2 10. $\frac{2}{9}(7^{(\frac{3}{2})}-1)-\frac{1}{3}(5^{(\frac{3}{2})}-1)$

11. $\frac{5}{2}$ 12. 7 log(x-2) – 6 log (x-1) + c 13. $x \sin^{-1} x + \sqrt{(1-x^2)}$

14. 0 15. $e^x(\cos x)$ + c 16. 0

3 Integral Calculus – II

POINTS TO REMEMBER

DETERMINATION OF AREA USING INTEGRATION

WITH RESPECT TO X-AXIS

- The area of the region bounded by the curve $y = f(x)$ between limits $x = a$ *and* $x = b$ with x –axis if area lies above x-axis is $= \int_a^b y\,dx$.
- The area of the region bounded by the curve $y = f(x)$ between the limits $x = a$ and $x = b$ with $x - axis$ if area lies below $x - axis$ is $= \int_a -y\,dx$

WITH RESPECT TO Y-AXIS

- The area of the region bounded by the curve $x = g(y)$ between te limits $y = c$ and $y = d$ with y - axis if the area lies to the right of y- axis is $= \int_c^d x\ dy$
- The area of the region bounded by the curve $x = g(y)$ between the limits $y=d$ and $y=e$ with y-axis if the area lies to the left of y- axis is $= \int_d -x\,dy$

AREA BETWEEN TWO CURVES

- The area between the two given curves $y=f(x)$ and $y=g(x)$ from $x = a$ *to* $x = b$, is $\int_a^b \left(f(x) - g(x)\right) dx$.

USE OF INTEGRATION IN ECONOMICS

- Elasticity of demand is $\eta_d = \frac{-p}{x}\frac{dx}{dp}$

- Total inventory carrying cost $= c_1 \int_0^T I(x)\,dx$
- Amount of annuity after N Payment is $A = \int_0^N pe^{rt}\,dt$

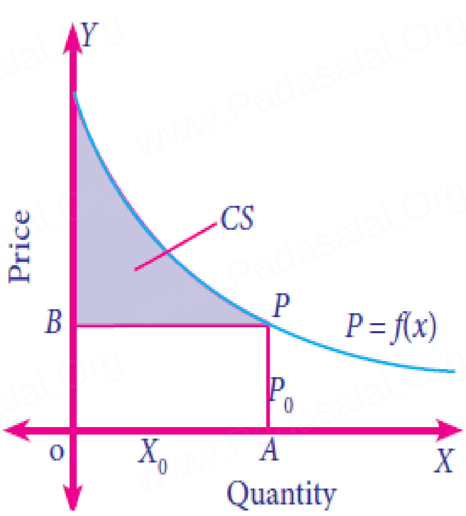
- Cost function is $C = \int (MC)\,dx + k.$
- Average cost function is $AC = \frac{C}{x}, x \neq 0$

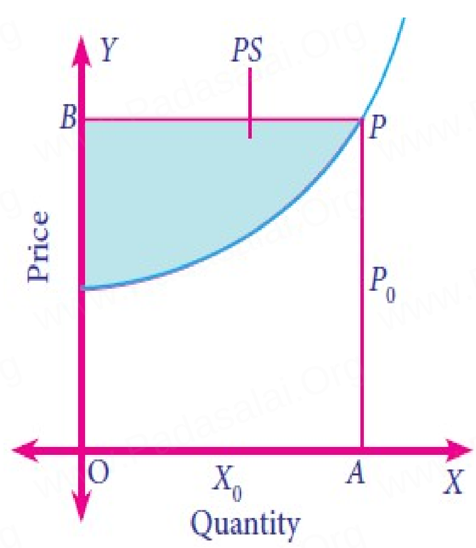
- Revenue function is $R = \int (MR)\,dx + k.$
- Demand function is $P = \frac{R}{x}$

- Profit function is =MR−MC $= R'(x) - C'(x)$

CONSUMER'S & PRODUCVER'S SURPLUS

- Consumer's surplus $= \int_0^{x_0} f(x)\,dx - x_0 p_0$
- Producer's surplus $= x_0 p_0 - \int_0^{x_0} p(x)\,dx$





CLASSIFICATION OF TEXT BOOK PROBLEMS

2 Marks

EXERCISE		EXAMPLE	Miscellaneous
Ex. No.,	Question Number	Question Number	Question Number
3.1	1, 2	3.1, 3.2, 3.3	
3.2	19		

3 Marks

EXERCISE		EXAMPLE	Miscellaneous
Ex. No.,	Question Number	Question Number	Question Number
3.1	4, 6	3.4	1, 2, 3, 4, 9, 10
3.2	1, 4, 5, 6, 7, 8, 9, 10, 11, 12, 15, 16, 17, 20	3.9, 3.10, 3.11, 3.12, 3.13, 3.15 , 3.16, 3.22, 3.23, 3.24	
3.3	1, 2, 3, 4, 5, 6, 7, 8	3.27, 3.28	

5 Marks

EXERCISE		EXAMPLE	Miscellaneous
Ex. No.,	Question Number	Question Number	Question Number
3.1	3, 5, 7	3.1, 3.5, 3.6, 3.7, 3.8	5, 6, 7, 8
3.2	2, 3, 13, 14, 18	3.14, 3.17, 3.18, 3.19, 3.20, 3.21, 3.25, 3.26	
3.3	9, 10, 11	3.29	

: CREATIVE QUESTIONS :

2 MARKS

1. Find the area of the region bounded by the parabola $x^2 = 4y$, y=2, y=4 and the y-axis.

2. Find the area of the region bounded by the parabola $y^2 = 4x$, x=1, x=4 and the x-axis.

3. Find the area under curve $y = 4x^2 - 8x + 4$ bounded by the y-axis, x-axis and the ordinate at x=2.

4. Find the area bounded by the semi cubical parabola $y^2 = x^3$ and the lines x=0, y=1 and y=2.

5. Find the demand function for which the elasticity of demand is 1.

6. Find the demand function for which the elasticity of demand is 2.

7. If marginal revenue is (100 + x) find the revenue function.

8. Find the demand function for which the elasticity of demand is 'n'.

9. If marginal cost is $MC = x^2 - 2$ when x=2, c = 4. Find cost function.

10. If marginal revenue is $MR = 3 - 2x - x^2$. Find revenue function.

11. Find area bounded by the line y=x+4, x-axis and the line x=1, x=2.

3 MARKS

1. Find the area under the curve $y = \frac{1}{(1+x^2)}$, x-axis, x=-1 and x=1

2. Find the area contained between the x-axis and one arch of the curve y=sin x.

3. Find the area contained between the x-axis and one arch of the curve y=cos x.

4. The marginal cost function of manufacturing x units of a commodity is $6+10x-6x^2$, find the total cost and average cost, given that, total cost of producing 1 unit is 20.

5. The marginal cost function of manufacturing x units of a commodity is $3x^2+3x+8$, if there is no fixed cost find the total cost and average cost.

6. The marginal cost function of manufacturing x units of a commodity is x^2-3x+2, if the fixed cost is 200, find the total cost and average cost.

7. If the marginal revenue for a commodity is MR= $\frac{e^x}{100}$ + x + x², find revenue function.

8. The marginal revenue function (in thousands of rupees) of a commodity is $7 + e^{-0.05x}$ where x is the number of units sold. Find the total revenue from the sale of 100 units (e^{-5} =0.0067)

9. Elasticity of function is given by $\frac{x}{(x-2)}$ find function when x=6, y=16.

10. Determine the cost of producing 3000 units of commodities of the marginal cost in rupees per unit is c'(x) = $\frac{x}{3000}$ + 2.50

11. The marginal cost at a production level of x units is given by $c'(x)=85 + 375/x^3$, find the cost of producing 5 incremental units after 10 units have been produced.

12. The demand function for a commodity p = $\frac{12}{(x+3)}$ find the consumer's surplus when the prevailing market price is 2.

13. The supply function for a commodity is $p = x^2 + x + 4$, where x denotes supply, find producer's surplus when price is 10.

14. The marginal cost function is given by MC = 3 - e^x find (i) C if C(0) = 10 (ii) AC

15. The marginal cost function is MC = 100/x find cost function if C(16) = 100, also find the average cost function.

5 MARKS

1. Find the area of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

2. Find the area of one loop of the curve $y^2 = x^2 (4 - x^2)$ between x=0 and x=2.

3. The demand and supply function under pure competition are $p_d=16 - x^2$ and $p_s = 2x^2+4$ find consumer's surplus and producer's surplus at the market equilibrium price.

4. The demand and supply curve are given by $p_d= \frac{16}{(x+4)}$ and $p_s = \frac{x}{2}$ find consumer's surplus and producer's surplus at the market equilibrium price.

5. The demand and supply function for a commodity are given by $p_d= 15 - x$ and $p_s = 0.3x + 2$ find consumer's surplus and producer's surplus at the market equilibrium price.

6. The demand and supply law under a pure competition are given by $p_d=23 - x^2$ and $p_s = 2x^2-4$, find consumer's surplus and producer's surplus at the market equilibrium price.

7. Under pure competition the demand and supply laws for a commodity are $p_d=56 - x^2$ and $p_s = 8+ \frac{x^2}{3}$ find consumer's surplus and producer's surplus at the market equilibrium price.

8. Find the consumer's and producer's surplus under market equilibrium of the demand function $p_d=20 - 3x - x^2$ and supply function $p_s = x-1$

9. The marginal cost of production of a firm is given by $C'(x)=5+0.13x$, The marginal revenue is given by $R'(x)=18$. The fixed cost is ₹120. Find the profit function.

10. The marginal cost $C'(x)$ and marginal revenue $R'(x)$ are given by $C'(x) =20 + \frac{x}{20}$ and $R'(x)=30$. The fixed cost is ₹ 200. Determine the maximum profit.

11. The marginal cost and marginal revenue with respect to a commodity of a firm are given by $C'(x) =4 + 0.08x$ and $R'(x)=42$. Find the total profit, given that the total cost at zero output is zero.

12. The elasticity of demand with respect to price for a commodity is $\frac{(x-5)}{x}$, $x\neq5$ when the demand is x. Find the demand function if the price is 2 when demand is 7. Also find the revenue function.

13. The elasticity of demand with respect to price for a commodity is a constant and is equal to 3. Find the demand function and hence the total revenue function, given that when the price is 1 the demand is 4.

14. Find the area of the region enclosed by $y^2=x$ and $y=x-2$

15. The price of a machine is ₹ 6,00,000 with an estimated life of 12 years. The estimated salvage value is ₹ 40,000. The machine can be rented at ₹ 72,000 per year. The present value of the rental payment is calculated at 9% interest rat. Find out whether it is advisable to rent the machine. ($e^{-1.08}$ =0.3396)

ANSWERS TO CREATIVE QUESTIONS

2 Marks :

1. $\frac{8}{3}(4-\sqrt{2})$ sq. units
2. $\frac{28}{3}(4-\sqrt{2})$ sq. units
3. $\frac{20}{3}$ sq. units
4. $\frac{3}{5}(2^{(\frac{5}{3})}-1)$ sq. units
5. xp = k
6. x p^2 = k
7. R = $100x+\frac{x^2}{2}$
8. $x\,p^n=k$
9. $C=\frac{x^3}{3}-2x+\frac{16}{3}$
10. $R=3x+x^2-\frac{x^3}{3}$
11. $\frac{11}{3}$ sq. units.

3 Marks :

1. π/2 sq. units
2. 2 sq. units
3. 2 sq. units
4. $C=6x+5x^2-2x^3+11$, $AC=6+5x2-2x^2+\frac{11}{x}$
5. $C=x^3+\frac{3}{2}x^2+8$, $AC=x^2+\frac{3}{2}x+\frac{8}{x}$
6. $C=x^3-\frac{3}{2}x^2+2x+200$, $AC=x^2-\frac{3}{2}x+2+\frac{200}{x}$
7. $R=\frac{e^x}{100}+\frac{x^2}{2}+\frac{x^3}{3}-\frac{1}{100}$
8. 719.866 ≈ 720
9. $\frac{-1}{32}$
10. ₹ 9000
11. $C=85x-\frac{375}{2x^2}-843$
12. $C.S=12\log 2-6$ units
13. $\frac{88}{3}$ units
14. $C=3x-e^x+11$, $AC=3-\frac{e^x}{x}+\frac{11}{x}$
15. $C=100(1+\log(\frac{x}{16}))$ $AC=\frac{100}{x}(1+\log(\frac{x}{16}))$

5 Marks :

1. π a b sq. units   2. $\frac{16}{3}$ sq. units

3. C.S = $\frac{16}{3}$ units. P.S = $\frac{32}{3}$ units   4. C.S = $16\log 2 - 8$ units P.S= 4 units

5. C.S = 50 units, P.S = 15 units   6. C.S = 18 units, P.S = 36 units

7. C.S = 144 units, P.S = 48 units   8. C.S = $\frac{63}{2}$ units, P.S = $\frac{9}{2}$ units

9. Profit = 13 x – 0.065 x^2 – 120   10. Max. Profit = 800

11. Profit = 38x – 0.04 x^2 12. P = $\frac{4}{(x-5)}$, R = $\frac{4x}{(x-5)}$

13. P = $\left(\frac{4}{x}\right)^{\left(\frac{1}{3}\right)}$ 14. $\frac{9}{2}$ sq. units

15. Present value of 12 years is ₹ 5,28,320
    Cost of Machine is ₹ 5,60,000, It is better to rent the machine.

4 Differential Equations

POINTS TO REMEMBER

ORDER & DEGREE OF A DIFFERENTIAL EQUATION

- Order of the highest order derivative present in the differential equation is the order of the differential equation.

- A function which satisfies the given differential equation is called its solution.

TO FORM A DIFFERENTIAL EQUATION

- To form a differential equation from a given function we differentiate the function successively as many times as the number of arbitrary constants in the given function and then eliminate the arbitrary constants.

TO SOLVE HOMOGENEOUS DIFFERENTIAL EQUATION

$f(x,y)$ and $g(x,y)$ are homogeneous functions of same degree

Put $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

TO SOLVE FIRST ORDER LINEAR DIFFERENTIAL EQUATION

- A differential equation of the form $\frac{dy}{dx}+Py=Q$ where P and Q are constants or functions of x only is called a first order linear differential equation.

The solution is $ye^{\int pdx}=\int Qe^{\int pdx}dx+c$

For the differential equation $\frac{dx}{dy}+Px=Q$ (linear in x) where P and Q are functions of y alone, the solution is $xe^{\int pdy}=\int Qe^{\int pdy}dy+c$

SECOND ORDER LINEAR DIFFERENTIAL EQUATION

- A general second order linear differential equation with constant coefficients is of the form $a\frac{d^2y}{dx^2}+b\frac{dy}{dx}+cy=f(x)$

TO SOLVE SECOND ORDER LINEAR DIFFERENTIAL EQUATION

*** Form the Characteristic/Auxillary Equation**

***Write the Complementary function as follows**

	Nature of roots	Complementary function
1	Real and different $(m_1 \neq m_2)$	$Ae^{m_1x}+Be^{m_2x}$
2	Real and equal $m_1=m_2=m(say)$	$(Ax+B)e^{mx}$
3	Complex roots $(\alpha \pm i\beta)$	$e^{\alpha x}(A\cos\beta x+B\sin\beta x)$

*** Determine the particular integral (P.I)**

*** Write the General equation as**

General solution is $y=C.F+P.I$

CLASSIFICATION OF TEXT BOOK PROBLEMS

2 Marks

EXERCISE		EXAMPLE	Miscellaneous
Ex. No.,	Question Number	Question Number	Question Number
4.1	1 → (i) to (vii) 2 → (i), (iii), (iv)	4.1 → (i) to (vii) 4.2 → (ii) ,(iii)	
4.2	3 (i)	4.6	
4.5	1, 2, 3, 4, 7	4.25, 4.26, 4.27	

3 Marks

EXERCISE		EXAMPLE	Miscellaneous
Ex. No.,	Question Number	Question Number	Question Number
4.1	2 → (ii), 3, 4, 5, 6, 7	4.2 → (i), 4.3, 4.4, 4.5	2, 3, 5, 10
4.2	1 → (i),(ii), 2, 3→(ii), 4, 5, 6 → (i),(ii), 7	4.7, 4.9, 4.10, 4.12	
4.4	1, 2	4.20,	
4.5	5, 6	4.28, 4.29	

5 Marks

EXERCISE		EXAMPLE	Miscellaneous
Ex. No.,	Question Number	Question Number	Question Number
4.2	---	4.2(Full), 4.8, 4.11 ,4.13, 4.14	1, 4, 6, 7, 8, 9
4.3	FULL EXERCISE	4.15,4.16, 4.17, 4.18, 4.19	
4.4	3, 4, 5, 6, 7, 8(repetition of 5), 9	4.21, 4.22, 4.23, 4.24	
4.5	8, 9, 10, 11, 12, 13	4.30, 4.31, 4.32	

: CREATIVE QUESTIONS :

2 MARKS

1. Define differential equation. What are the types of the differential equations?

2. Define Order and Degree of differential equations.

3. Form the differential equation of y=A sin3x+B cos3x

4. Form the differential equation of $y=Ae^{3x}+Be^{-3x}$

5. Solve : $\frac{dy}{dx}=\frac{\sqrt{(1-y^2)}}{\sqrt{(1-x^2)}}$

6. Solve : $\frac{dy}{dx}=\frac{(1+y^2)}{\sqrt{(1-x^2)}}$

7. Solve : $\frac{dy}{dx}=\frac{(y+2)}{(x-1)}$

8. Write the general form of first order linear differential equation and its solution.

9. Find the integrating factor (I.F) of $\frac{dy}{dx}+2y\tan x=\sin x$

10. Find the particular integral (P.I) of $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=10e^x$

11. Solve : $(D^2-9)y=0$

12. Solve : $\frac{d^2y}{dx^2}+\frac{dy}{dx}=0$

13. Solve : $\frac{dy}{dx}=\frac{(x+1)}{(2-y)}$

14. Solve : $\frac{dy}{dx}=\frac{y}{x}$

15. Form the differential equation representing the family of curves y = A sin(B + x)

3 MARKS

1. Form the differential equation of $y = mx + \frac{a}{m}$, where *m* is arbitrary constant.

2. Solve: $\frac{dy}{dx} = 2xy + 2ax$

3. Solve: $\frac{dy}{dx} = 1 + x + y + xy$

4. What is the general form of the demand function which has a constant elasticity of -1?

5. Solve : $\frac{dy}{dx} + y = x$

6. Solve : $\frac{dy}{dx} + \frac{y}{x} = \sin(x^2)$

7. Solve : $(3D^2 + 7D - 6)y = 0$

8. Solve : $\frac{dy}{dx} = \frac{(2x^2 + 1)}{x}$

9. Solve : $\frac{dy}{dx} = e^{(x+y)}$

10. Solve : y log x dx – x dy = 0

11. Solve : $\frac{dy}{dx} = y \tan x$, given that y = 1 when x = 0

12. Find the equation of curve passing through the point (-2,3). Given that the slope of the tangent to the curve at any point (x,y) is $\frac{2x}{y^2}$

13. Solve : $(D^2 + 2kD + k^2)y = 10e^{(-kx)}$

14. Solve : $(D^2 - 2D + 4)y = 10$

15. Solve : $\frac{dy}{dx} + ay = e^x (where\ a \neq -1)$

5 MARKS

1. Find the equation of the curve passing through (1,0) and which has slope $1+\frac{y}{x}$ at (x, y)

2. Solve : $(x+1)\frac{dy}{dx}-y=e^{x}(x+1)^{2}$

3. A man plans to invest some amount in a small saving scheme with a guaranteed compound interest at the rate of 12% for 5 years. How much should be invested if he wants the amount ₹ 25000 at the end of 5 year period. ($e^{-0.6}=0.5488$)

4. Solve : $(D^2 + 1)y = 0$ when x=0, y=2 and when x = π/2, y =-2

5. The sum of ₹ 1000 is compounded continuously, the nominal rate of interest being 4% per annum. In how many years will the amount be twice the original principle? $(\log_e 2 = 0.6931)$

6. A bank pays interest by continuous compounding. That is by treating the interest rate as the instantaneous rate of change of principal. Suppose in an account interest accrues at 8% per year compounded continuously. Calculate the percentage increase in such an account over a year. (Take $e^{.08} \approx 1.0833$)

7. Radium disappears at a rate proportional to the amount present. If 5% of the original amount disappears in 50 years, how much will remain at the end of 100 years. (Take A_0 as the initial amount)

8. The rate at which the population of a city increases at any time is proportional to the population at that time. If there were 1,30,000 people in the city in 1960 and 1,60,000 in 1990, what population may be anticipated in 2020.
($\log_e \frac{16}{13} = .2070$; $e^{.42} = 1.52$)

9. In a bank, principal increases continuously at the rate of 5% per year. An amount of ₹ 1000 is deposited with this bank, how much will it be worth after 10 years? ($e^{0.5} = 1.658$)

10. In a culture, the bacteria count is 1,00,000. The number is increased by 10% per hour. In how many hours will the count reach 2,00,000, if the rate of growth of bacteria is proportional to the number present? $(\log_e 2 = 0.6931)$

11. In a bank, principal increases continuously at the rate of r% per year. Find the value of r if ₹ 100 doubles itself in 10 years. $(\log_e 2 = 0.6931)$

12. At present, a firm is manufacturing 2000 items. It is estimated that the rate of change of production P w.r.t. additional number of workers x is given by $\frac{dp}{dx} = 100 - 12\sqrt{(x)}$. If the firm employs 25 more workers, then what is the level of production.

13. A curve passes through the point (1, π/6). Let the slope of the curve at each point (x,y) be $\frac{y}{x} + sec\left(\frac{y}{x}\right), x > 0.$ Then find the equation of the curve.

14. Let the population of rabbits surviving at a time t be governed by the differential equation $\frac{dP}{dt} = \frac{1}{2}P - 200$. If P(0) = 100, then what is the population at the time 't'?

15. The function y = g(x) is the solution of the differential equation $\frac{dy}{dx} + \frac{xy}{(x^2-1)} = \frac{(x^4+2x)}{\sqrt{(x^2-1)}}$ in (-1,1). Then find f(2).

16. Solve : $\frac{dy}{dx} = \frac{(x+y)}{x}$

17. Solve : (x – y) dy – (x + y) dx = 0

ANSWERS TO CREATIVE QUESTIONS

2 Marks :

1. A differential equation is an equation with a function and one or more of its derivatives. The types are ordinary differential equations and partial differential equations.
2. The highest order derivative present in the differential equation is the Order of the differential equation. Degree is the highest power of the highest order derivative in the differential equation.

3. y''+9y=0 4. y''=9y 5. $\sin^{-1} y = \sin^{-1} x + c$

6. $\tan^{-1} y = \sin^{-1} x + c$ 7. y+2=(x-1) c

8. $\frac{dy}{dx}+Py=Q$, $y e^{(\int P dx)}=\int (Q e^{(\int P dx)})+c$ 9. $\sec^2 x$ 10. $5 x^2 \; e^x$

11. $y = A e^{-3x} + B e^{3x}$ 12. $y = A + B \; e^{-x}$ 13. $4y - y^2 = x^2 + 2x + c$

14. y = cx 15. y'' + y = 0

3 Marks :

1. $y y' = x(y')^2 + a$ 2. $\log(y + a) = x^2 + c$ 3. $\log(1+y) = x + x^2/2 + c$

4. x = p c 5. $y e^x = e^x (x - 1) + c$ 6. $2 xy + \cos(x^2) = c$

7. $y = \; A e^{-3x} + B e^{(\frac{2}{3}x)}$ 8. $y = x + \log x + c$ 9. $e^x + e^{-y} = c$

10. $\log y = \frac{((\log x)^2)}{2} + c$ 11. y = sec x 12. $y^3 = 3x^2 + 15$

13. $y = (Ax+B)e^{-kx} + 5x^2 e^{-kx}$ 14. $y = e^x(A\cos\sqrt{3}x + B\sin\sqrt{3}x) + \frac{5}{2}$

15. $(1+a)\, y e^{ax} = e^{((1+a)x)} + c$

5 Marks :

1. $x = e^{(\frac{x}{y})}$ or y = x log x 2. $\frac{y}{(x+1)} = e^x + c$ or $y = (e^x + c)(x+1)$

3. ₹ 13,720 4. y = 2(cos x – sin x) 5. 17 years(app)

6. 8.33% 7. A = 0.9025Ao or 90.25% of Ao 8. 197600

9. ₹ 1648 10. 13.862 hours 11. 6.931%

12. 3500 13. sin(y/x) = log x + 1/2 14. $P=400-300 \; e^{(\frac{t}{2})}$

15. $f(2) = 16\sqrt{\frac{3}{(5)}}$ 16. $x = c\, e^{(\frac{y}{x})}$ 17. $\tan^{-1} (\frac{y}{x}) = \log c\sqrt{(x^2+y^2)}$

5 Numerical Methods

POINTS TO REMEMBER

RELATION ON OPERATORS,

FORWARD DIFFERENCE - Δ ,

SHIFT – E ,

BACKWARD DIFFERENCE - ∇

- $\Delta f(x) = f(x+h) - f(x)$
- $\nabla f(x) = f(x) - f(x-h)$
- $\nabla f(x+h) = \Delta f(x)$

$\Delta \equiv E - 1$

$\nabla \equiv \frac{E-1}{E}$

$E\Delta \equiv \Delta E$

- $Ef(x) = f(x+h)$
- $E^n f(x) = f(x+nh)$

NEWTON'S FORWARD & BACKWARD INTERPOLATION FORMULA

- Newton's forward interpolation formula:

$$y_{(x=x_0+nh)} = y_0 + \frac{n}{1!}\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!}\Delta^3 y_0 + \ldots$$

- Newton's backward interpolation formula:

$$y_{(x=x_n+nh)} = y_n + \frac{n}{1!}\nabla y_n + \frac{n(n+1)}{2!}\nabla^2 y_n + \frac{n(n+1)(n+2)}{3!}\nabla^3 y_n + \ldots$$

Lagrange's Formula

- Lagrange's interpolation formula:

$$y = f(x) = \frac{(x-x_1)(x-x_2)...(x-x_n)}{(x_0-x_1)(x_0-x_2)...(x_0-x_n)}y_0 + \frac{(x-x_0)(x-x_2)...(x-x_n)}{(x_1-x_0)(x_1-x_2)...(x_1-x_n)}y_1 + ...$$

$$+ \frac{(x-x_0)(x-x_1)...(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)...(x_n-x_{n-1})}y_n$$

EXTRAPOLATION

If y is to be estimated for the values of x which lies outside the given set of the values of it, is called extrapolation.

$$E\left[f(x_0)\right] = f(x_0+h)$$

$$\Delta^n\left(\frac{1}{x}\right) = \frac{(-1)^n n!}{x(x+1)(x+2)......(x+n)}$$

(i) $(1+\Delta)(1-\nabla) = 1$

(ii) $\Delta\nabla \equiv \Delta - \nabla$

(iii) $\nabla \equiv E^{-1}\Delta$

CLASSIFICATION OF TEXT BOOK PROBLEMS

2 Marks

EXERCISE		EXAMPLE	Miscellaneous
Ex. No.,	Question Number	Question Number	Question Number
		5.4 → (i),(ii),(iii)	1 f(0) 2 → (i), (ii), (iii)
		5.6	

3 Marks

EXERCISE		EXAMPLE	Miscellaneous
Ex. No.,	Question Number	Question Number	Question Number
5.1	1, 2, 3, 4, 6, 7	5.1, 5.2, 5.3,5.7, 5.8, 5.9, 5.10	1 $\Delta f(0)$, $\Delta^2 f(0)$ 3
5.2	1, 2	5.12	

5 Marks

EXERCISE		EXAMPLE	Miscellaneous
Ex. No.,	Question Number	Question Number	Question Number
5.1	5, 8	5.5, 5.11	1 (full) 4, 5, 6, 7, 8, 9, 10
5.2	3, 4, 5, 6, 7, 8, 9, 10, 11, 12	5.13, 5.14, 5.15, 5.16, 5.17, 5.18, 5.19, 5.20, 5.21, 5.22	

: CREATIVE QUESTIONS :

2 MARKS

1. Construct a forward difference table for the following data f(0) = 1, f(1) = 3, f(3) = 55

2. Find $\Delta^3(1/x)$ by taking 1 as the interval of differencing.

3. What is the relation between the operator Δ, and ∇ ?

4. Show that $E\Delta \equiv \Delta E$

5. Show that $\nabla \equiv \frac{(E-1)}{E}$

6. Write the properties of the shifting operator E.

7. Define: $E^n f(x)$

8. Show that $\Delta = E - 1$

9. Explain Forward Difference Operator.

10. Explain Backward Difference Operator.

11. Write any two properties of the operator Δ.

12. Find $\Delta^2(x^2)$

13. If $f(x) = x^2 + 2x + 3$ and the interval of differencing is unity then find $\Delta f(x)$.

14. Given f(0) = 5, f(1) = 6, f(5) = 50, f(4) = 105 find $\Delta^3 y_0$

15. For the given tabulated value of y = f(x) find $\Delta^3 y_2$, Δy_3

x :	1941	1951	1961	1971	1981	1991
y :	20	24	29	36	46	51

3 MARKS

1. Write the properties of the operator Δ.

2. Construct a forward difference table for $y = f(x) = x^3 + 3x + 3$ for x = 1,2,3,4,5

3. By constructing a difference table and using the third order differences as constant, find the 5[th] term of the series 4, 13, 34, 73, ______

4. From the following data, estimate the population for the year 1986, graphically,

year :	1960	1970	1980	1990	2000
Population : (in thousands)	12	15	20	26	33

5. Using graphic method, find the value of y when x = 27, From the following data,

x	:	10	15	20	25	30
y	:	35	32	29	26	23

6. The population of town is as follows,

year	:	1940	1950	1960	1970	1980	1990
Population (in lakhs)	:	20	24	29	36	46	50

Estimate the population for the year 1976 graphically.

7. Using Graphic method, find the value of y when x = 42, from the following data

x	:	20	30	40	50
y	:	51	43	34	24

8. From the following data find f(3)

x	:	1	2	3	4	5
y	:	2	5	--	14	32

9. Find the missing entry from the following,

x	:	0	5	10	15	20	25
y	:	7	11	14	--	24	32

10. From the following data, estimate the export for the year 2015

year	:	2014	2015	2016	2017	2018
Export (in tons)	:	443	--	369	397	467

11. Find the missing term from the following data

x	:	1	2	3	4
y	:	100	–	126	157

12. Find F(2) for the following data, F(0) = 1, F(1) = 3, F(3) = 55.

13. A second degree polynomial passes through (1,5), (2,0), (3,-3), (4,-4). Find the polynomial.

14. Construct a forward difference table for the following data,

x	:	0 – 40	40 - 60	60 – 80	80 – 100	100 – 120
y	:	250	120	100	70	50

15. A second degree polynomial passes through (1,5), (2,10), (3,17), (4,26). Find the polynomial and find the value of x = 5.

5 MARKS

1. Estimate the production for the year 1962 and 1965, from the following data,

year	:	1961	1962	1963	1964	1965	1966	1967
Production (in tons)	:	200	–	260	306	–	390	430

2. Using Lagranges's formula find y(11) from the following data

x	:	6	7	10	12
y	:	13	14	15	17

3. Apply Lagrange's formula to find the value of y when x = 5, given that

x	:	1	2	3	4	7
y	:	2	4	8	16	128

4. Using Lagrange's formula find the value of y when x = 42 from the following data,

x	:	40	50	60	70
y	:	31	73	124	159

5. Using Lagrange's formula find f(4) when f(0) =276, f(3) = 460, f(5) = 414, f(6) = 343, f(8) = 110

6. Find the number of men getting wages between ₹ 30 and ₹ 35 from the following

wages	x:	20 – 30	30 – 40	40 – 50	50 – 60
No.,of Men	y :	9	30	35	42

7. Find y when x = 0.2 given that

x	:	0	1	2	3	4
y	:	176	185	194	202	212

8. If $y_{75} = 2459$, $y_{80} = 2018$, $y_{85} = 1180$, $y_{90} = 402$, find y_{82}.

9. From the following data calculate the value of $e^{1.75}$

x	:	1.7	1.8	1.9	2.0	2.1
e^x	:	5.474	6.050	6.686	7.389	8.166

10. From the data find the number of students whose height is between 80cm and 70cm

Hieght in cms :	40-60	60-80	80-100	100-120	120-140
No.,of students :	250	120	100	70	50

11. Using Gregory Newton's formula, find y when x = 85

x	:	50	60	70	80	90	100
y	:	184	204	226	250	276	304

12. From the following data find the area of a circle of diameter 96 units by using Gregory Newton's formula,

Diameter	x:	80	85	90	95	100
Area	y:	5026	5674	6362	7088	7854

13. From the following data, What might be the population in 1925

x	:	1891	1901	1911	1921	1931
y (in thousands)	:	46	66	81	93	101

14. Find the value of sin 52°

Θ° :	45°	50°	55°	60°
sin Θ :	0.7071	0.7660	0.8192	0.8660

15. Using Newton's backward interpolation formula, find the cubic polynomial.

x	:	0	1	2	3
f(x)	:	1	3	16	49

ANSWERS TO CREATIVE QUESTIONS

2 Marks :

1.

x	y	Δy	$\Delta^2 y$
0	1		
		2	
1	3		50
		52	
3	55		

2. $\dfrac{-6}{(x(x+1)(x+2)(x+3))}$

3. $\nabla f(x+h) = \Delta f(x)$, $\nabla f(x+2h) = \Delta f(x+h)$

6. (i) $E(f_1(x) + E(f_2(x) + \ldots\ldots\ldots + E(f_n(x)$ (ii) $c\,E(f(x))$ (iii) $E^{m+n} f(x)$
(iv) $E^n(E^{-n}(f(x)) = f(x)$

7. $E^n(f(x) = f(x+nh)$

11. (i) $\Delta c = 0$ (ii) $\Delta(f(x)+g(x)) = \Delta f(x) + \Delta g(x)$ (iii) $\Delta[c\, f(x)]=c\, \Delta f(x)$

12. 2 13. 2x + 3 14. -32 15. -8, 10

3 Marks :

1. (i) $\Delta c = 0$ (ii) Δ is distributive (iii) $c\, \Delta f(x)$ 2. Table upto $\Delta^4 y = 0$

3. k= 136 4. 24 5. x = 27, y = 24.8 6. 41 Lakhs

7. 33 8. 7 9. 18 10. 384 11. 107

12. $x^2 - 8x + 12$ 13. Table upto $\Delta^4 y = 20$ 14. f(2) = 21

15. x^2+2x+2, 37

5 Marks :

1. 220 tons, 350 tons 2. 15.6666 3. 32.93 4. 37.48

5. 453.311 6. 15 7. 177.6176 8. 1704.408

9. 5.7549375 10. 54 11. 262.75 12. 7237.87

13.0.7880032 14. 96.8368 15. $2x^3 - 2x^2 + x + 1$

6 Random Variable and Mathematical Expectation

POINTS TO REMEMBER

Random variable

A variable which can assume finite number of possible values or an infinite sequence of countable real numbers is called a discrete random variable.

A random variable X which can take on any value (integral as well as fraction) in the interval is called continuous random variable.

Probability Mass function

(i) $p(x_i) \geq 0 \; \forall i$, (ii) $\sum_{i=1}^{\infty} p(x_i) = 1$

Probability density function

(i) $f(x) \geq 0 \; \forall x$ and

(ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$.

Discrete distribution function

$$F_X(x) = P(X \leq x), \text{ for all } x \in R$$

$$i.e., F_X(x) = \sum_{x_i \leq x} p(x_i)$$

Continuous distribution function

cumulative distribution function (c.d.f)

$$F_X(x) = P[X \leq x] = \int_{-\infty}^{x} f(t)\,dt, -\infty < x < \infty$$

Properties of cumulative distribution function

(i) $0 \leq F(x) \leq 1, \; -\infty < x < \infty$

(ii) $F(-\infty) = \lim_{x \to -\infty} F(x) = 0$ and $F(+\infty) = \lim_{x \to \infty} F(x) = 1$.

(iii) $F(\cdot)$ is a monotone, non-decreasing function; that is, $F(a) \leq F(b)$ for $a < b$.

(iv) $F(\cdot)$ is continuous from the right; that is, $\lim_{h \to 0} F(x+h) = F(x)$.

(v) $F'(x) = \frac{d}{dx}F(x) = f(x) \geq 0$

(vi) $F'(x) = \frac{d}{dx}F(x) = f(x) \Rightarrow dF(x) = f(x)dx$

$dF(x)$ is known as probability differential of X.

(vii) $P(a \le x \le b) = \int_a^b f(x)dx = \int_{-\infty}^{b} f(x)dx - \int_{-\infty}^{a} f(x)dx$

$= P(X \le b) - P(X \le a)$

$= F(b) - F(a)$

Expected value

$E(X) = \sum_x x\, p(x)$

$E(X) = \int_{-\infty}^{\infty} x\, f(x)\, dx$

The mean of X, denoted by μ_x or $E(X)$.

Variance

$Var(X) = \sum \left[x - E(X)\right]^2 p(x)$

$Var(X) = \int_{-\infty}^{\infty} \left[x - E(X)\right]^2 f_X(x)\, dx$

$Var(X) = E\left[X - E(X)\right]^2 = E\left(X^2\right) - \left[E(X)\right]^2$

represents the moment of inertia of the same density with respect to a perpendicular axis through the center of gravity.

Properties of Mathematical expectation

(i) $E(a) = a$, where 'a' is a constant

(ii) $E(aX) = aE(X)$

(iii) $E(aX+b) = aE(X)+b$, where 'a' and 'b' are constants.

(iv) If $X \ge 0$, then $E(X) \ge 0$

(v) $V(a) = 0$

(vi) If X is random variable, then $V(aX+b) = a^2 V(X)$

r^{th} moment of X, usually denoted by μ'_r

$$\mu'_r = E\left(X^r\right) = \begin{cases} \sum_x x^r p(x), & \text{for discrete random variable} \\ \int_{-\infty}^{\infty} x^r f(x)dx, & \text{for continuous random variable} \end{cases}$$

r^{th} central moment of X about a $\qquad \mu_r = E[(X - \mu_X)^r]$

standard deviation of X, denoted by σ_X $\qquad +\sqrt{Var[X]}$

CLASSIFICATION OF TEXT BOOK PROBLEMS

2 Marks

EXERCISE		EXAMPLE	Miscellaneous
Ex. No.,	Question Number	Question Number	Question Number
6.1	1, 4, 5, 11, 12, 13, 14, 15, 16, 17,	6.1, 6.10	6, 7, 8, 10
6.2	1, 2, 4, 6, 7, 8, 9, 10, 11, 15	6.13, 6.15	

3 Marks

EXERCISE		EXAMPLE	Miscellaneous
Ex. No.,	Question Number	Question Number	Question Number
6.1	2, 3, 18, 19, 20	6.3, 6.4, 6.5, 6.6, 6.8	3, 4, 9
6.2	3, 5, 12,13, 14,	6.16, 6.17, 6.18, 6.19, 6.20, 6.21, 6.22	

5 Marks

EXERCISE		EXAMPLE	Miscellaneous
Ex. No.,	Question Number	Question Number	Question Number
6.1	6, 7, 8, 9, 10	6.2, 6.7, 6.9, 6.11	1, 2, 5
6.2	---	6.12, 6.14, 6.23, 6.24	

: ANSWER TO THEORY QUESTIONS :

EXERCISE : 6.1

11. Define random variable.

A random variable (r.v.) is a real valued function defined on a sample space S and taking values in $(-\infty, \infty)$ or whose possible values are numerical outcomes of a random experiment.

12. Explain what are the types of random variable.

1. **Discrete random variable**

A variable which can assume finite number of possible values or an infinite sequence of countable real numbers is called a discrete random variable.

2. **Continuos random variable**

A random variable X which can take on any value (integral as well as fraction) in the interval is called continuous random variable.

13. Define discrete random variable

A variable which can assume finite number of possible values or an infinite sequence of countable real numbers is called a discrete random variable.

14. What do you understand by continuous random variable?

A random variable X which can take on any value (integral as well as fraction) in the interval is called continuous random variable.

15. Describe what is meant by a random variable.

In a random experiment when the experiment is performed more than once then we associate a real number with each outcome. The function or variable that associate to these real numbers is called a random variable.

For example, when a coin is tossed three times, the number of times we get HEAD will be 0 or 1 or 2 or 3, so the variable representing the number of heads is a random variable.

There are two types of random variable, (i) discrete --> takes contable numbers (ii) continuous --> takes any value in an interval.

16. Distinguish between discrete and continuous random variable.

Discrete random variable takes countable number of possible values whereas Continuous random variable takes any value in an interval.

Example, Number of telephone calls at a particular time is discrete and Electricity consumption in kilowatt hours.

17. Explain the distribution function of a random variable.

For a discrete random variable, the distribution function is

$$F_X(x) = P(X \le x), \text{ for all } x \in R \qquad i.e., F_X(x) = \sum_{x_i \le x} p(x_i)$$

For a continuous random variable, the distribution function or cumulative distribution function is

$$F_X(x) = P[X \le x] = \int_{-\infty}^{x} f(t)\,dt, -\infty < x < \infty$$

18. Explain the terms (i) probability mass function (ii) probability density function and (iii) probability distribution function.

(i) Probability mass function :

If X is a discrete random variable with distinct values $x_1, x_2, \ldots, x_n, \ldots$, then the function, denoted by $P_X(x)$ and defined by

$$P_X(x) = p(x) = \begin{cases} P(X = x_i) = p_i = p(x_i) & \text{if } x = x_i, i = 1, 2, \ldots, n, \ldots \\ 0 & \text{if } x \neq x_i \end{cases}$$

This is defined to be the probability mass function or discrete probability function of X.

The probability mass function $p(x)$ must satisfy the following conditions

(i) $p(x_i) \geq 0 \; \forall i$, (ii) $\sum_{i=1}^{\infty} p(x_i) = 1$

(ii) Probability density function :

The probability that a random variable X takes a value in the interval $[t_1, t_2]$ is given by $f_X(x)$ called the probability density function

The probability density functions $f_X(x)$ or simply by $f(x)$ must satisfy the following conditions. (i) $f(x) \geq 0 \; \forall \; x$ and (ii) $\int_{-\infty}^{\infty} f(x)dx = 1$.

(iii) Probability distribution function :

The discrete cumulative distribution function or distribution function of a real valued discrete random variable X takes the countable number of points $x_1, x_2, \ldots$ with corresponding probabilities $p(x_1), p(x_2), \ldots$ and then the cumulative distribution function is defined by

$$F_X(x) = P(X \leq x), \text{ for all } x \in R$$

(or)

If X is a continuous random variable with the probability density function $f_X(x)$, then the function $F_X(x)$ is defined by $F_X(x) = P[X \leq x]$

20. State the properties of distribution function.

Properties of cumulative distribution function

(i) $0 \leq F(x) \leq 1, \; -\infty < x < \infty$

(ii) $F(-\infty) = \lim_{x \to -\infty} F(x) = 0$ and $F(+\infty) = \lim_{x \to \infty} F(x) = 1$.

(iii) $F(\cdot)$ is a monotone, non-decreasing function; that is, $F(a) \leq F(b)$ for $a < b$.

(iv) $F(\cdot)$ is continuous from the right; that is, $\lim_{h \to 0} F(x+h) = F(x)$.

(v) $F'(x) = \frac{d}{dx}F(x) = f(x) \geq 0$

(vi) $F'(x) = \frac{d}{dx}F(x) = f(x) \Rightarrow dF(x) = f(x)dx$

$dF(x)$ is known as probability differential of X.

(vii) $P(a \leq x \leq b) = \int_a^b f(x)dx = \int_{-\infty}^{b} f(x)dx - \int_{-\infty}^{a} f(x)dx$

$= P(X \leq b) - P(X \leq a)$

$= F(b) - F(a)$

EXERCISE : 6.2

7. What are the properties of Mathematical expectation?

Properties of Mathematical expectation

(i) $E(a)=a$, where 'a' is a constant (ii) $E(aX)=aE(X)$

(iii) $E(aX+b)=aE(X)+b$, where '*a*' and '*b*' are constants.

(iv) If $X\geq 0, then\, E(X)\geq 0$ (v) $V(a)=0$

(vi) If X is random variable, then $V(aX+b)=a^2V(X)$

8. What do you understand by Mathematical expectation?
Mathematical expectation refers to total of the product of each value of a random variable and it's corresponding probability,
For a discrete random variable $E(X)=\sum_x x\, p(x)$
For a continuous random variable $E(X)=\int_{-\infty}^{\infty} x\, f(x)\, dx$

9. How do you define variance in terms of Mathematical Expectation?
$Var(X)=E[X-E(X)]^2=E(X^2)-[E(X)]^2$

10. Define Mathematical expectation in termes of discrete random variable.
Mathematical expectation refers to total of the product of each value of a random variable and it's corresponding probability,
For a discrete random variable $E(X)=\sum_x x\, p(x)$

11. State the definition of Mathematical expectation using continuous random variable.
Mathematical expectation refers to total of the product of each value of a random variable and it's corresponding probability,
For a continuous random variable $E(X)=\int_{-\infty}^{\infty} x\, f(x)\, dx$

: CREATIVE QUESTIONS :

2 MARKS

1. Consider the experiment of tossing 2 coins. Let X be a random variable denoting the number of heads obtained as follows

X :	0	1	2
P(X=x) :	1/4	1/2	1/4

Is P(X) is a p.m.f

2. If a discrete random variable has the p.m.f

X :	0	1	2	3
P(X=x) :	k	2k	3k	5k

find k.

3. For the following probability distribution, construct c.d.f

X :	-2	-1	0	1	2	3
P(X=x) :	0.1	0.1	0.2	0.2	0.3	0.1

4. For the following discrete random variable find (i) $P(X \le 1)$ (ii) $P(0 < X < 2)$

X :	0	1	2	3
P(X = x) :	1/6	1/2	3/10	1/30

5. A random variable has the following probability distribution

X :	-2	0	5
P(X = x) :	1/4	1/4	1/2

Evaluate : (i) $P(X < 0)$ (ii) $P(0 \le X \le 10)$

6. Find 'k' if the following function is a p.m.f $P(x) = \begin{cases} k/6, & x = 0 \\ k/3, & x = 2 \\ k/2, & x = 4 \\ 0, & \text{otherwise} \end{cases}$

7. If the function f(x) is given by $f(x) = c\, e^{-x}$, $0 < x < \infty$ is a p.d.f. Find 'c'

8. Let X be the life expectancy of a certain type of bulb in hours. Determine 'a' so that the function may be a p.d.f $f(x) = \begin{cases} a/x^2, & 1000 \le X \le 2000 \\ 0, & \text{otherwise} \end{cases}$

9. The probability that a man fishing at a pond will catch 1, 2 , 3, 4 fishes are 0.4, 0.3, 0.2, 0.1 respectively. What is the expected number of fish caught?

10. A person receives a sum of rupees equal to the square of the number that appears on the face when a balanced die is tossed. How much money can he expect to receive?

11. A player tosses two fair coins. He wins ₹ 5 if 2 heads appears and ₹ 2 if 1 head appears and ₹ 1 if no head appears, find his expected amount of gain.

12. Find E(X) and $E(X^2)$ for the following probability distribution,

X :	1	2	3	4
P(X=x) :	0.1	0.3	0.4	0.2

13. Let X be a continuous random variable follows the probability distribution

$$f(x) = \begin{cases} 1/2, & -1 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

Find E(X), $E(X^2)$

14. Verify that, $f(x) = \begin{cases} 3x^2, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$ is a p.d.f. Also find $P(X < 1/3)$

15. Find E(2X-7) for the following,

X :	-3	-2	-1	0	1	2	3
P(X=x) :	0.05	0.1	0.3	0	0.3	0.15	0.1

16. A player tossed two fair coins. If 2 heads shows he wins ₹ 4 and if one head shows he wins ₹ 2 but if two tails shows he must pay ₹ 3 as penalty. Calculate the expected value of the amount he won by him.

17. A balanced die is rolled. A person receive ₹ 10 when the number 1 or 3 or 5 occurs and loss ₹ 5 when 2 or 4 or 6 occurs. How much money can he expect and the average per roll in the long run.

18. Write any two examples of discrete random variable.

19. How many types of random variable? What are they?

20. Write any two examples of continuous random variable.

21. Write any two examples of a random variable.

22. Find (i) var(5) (ii) var(5X) if var(X) = 3

23. Prove that (i) var(4X+2) = 16 var(X) (ii) E(4) = 4

24. Find 'a' if $f(x) = \begin{cases} a(x-1)^2, & 1 < x < 2 \\ 0, & \text{otherwise} \end{cases}$ is a p.d.f

25. The following represents the probability of D, the daily demand of a certain product,

D :	1	2	3	4	5
P(D) :	0.1	0.1	0.3	0.3	0.2

3 MARKS

1. Find the mean, variance and standard deviation of the following probability distribution,

X :	-3	-2	-1	0	1	2	3
P(X=x) :	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$

2. A random variable X has the probability function as follows,

X :	-1	0	1
P(X=x) :	0.2	0.3	0.5

Evaluate : (i) E(3X+1) (ii) $E(X^2)$ (iii) var(X)

3. Find the mean, variance and standard deviation for the following probability function

X :	1	2	3	4
P(X=x) :	0.1	0.3	0.4	0.2

4. A multinational bank is concerned about the time(in minutes) of its customer's before they would use ATM for their transaction. A study of a random sample of 500 customers results the following probability distribution,

X :	0	1	2	3	4	5	6	7	8
P(X=x):	0.2	0.18	0.16	0.12	0.10	0.09	0.08	0.04	0.03

Find Mean and Variance.

5. A player tosses two fair coins. He wins ₹ 5 if 2 heads appears and ₹ 2 if 1 head appears and ₹ 1 if no head appears, find the amount he gains and also find variance.

6. A person receives a sum of rupees equal to the square of the number that appears on the face when a balanced die is tossed. How much money can he expect to receive? Also find variance.

7. A random variable has the probability distribution,

X :	0	1	2	3
P(X=x) :	$\frac{1}{16}$	$\frac{3}{8}$	k	$\frac{5}{16}$

find k and c.d.f of X

8. For the following probability density function $f(x) = \begin{cases} \frac{1}{3}, & -1 < x < 1 \\ 0, & \text{otherwise} \end{cases}$

Find (i) E(X) (ii) $E(X^2)$ and (iii) var(X)

9. A continuous random variable has the p.d.f $f(x) = \begin{cases} k(2-x), & 0 < x < 2 \\ 0, & \text{otherwise} \end{cases}$

find the value of 'k'.

10. A random variable has the following probability distribution,

X :	-2	-1	0	1	2	3
P(X=x) :	0.1	k	0.2	2k	0.3	k

Find k. And construct c.d.f of X.

11, Three fair coins are tossed simultaneously. Find the probability mass function of the number of heads.

12. Find the p.m.f of number of girl child in families with 4 children assuming equal probability for boys and girls.

13. Find μ and σ^2 when E(X+3) = 10 and $E(X+3)^2 = 116$

14. Four fair coins are tossed once. Find the p.m.f , mean & variance for the number of heads.

15. Find mean and $E(X^2)$ if $f(x) = \begin{cases} \lambda \exp^{(-\lambda x)}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}$

5 MARKS

1. Verify that $f(x) = \begin{cases} 3x^2, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$ is a p.d.f and
   Evaluate : (i) $P(X \leq 1/3)$ (ii) $P(1/3 \leq X \leq 1/2)$

2. Let X be acontinuous random variable with p.d.f $f(x) = \begin{cases} ax, & 0 < x \leq 1 \\ a, & 1 < x < 2 \\ ax+3a, & 2 \leq x \leq 3 \\ 0, & \text{otherwise} \end{cases}$
   Find 'a'. Compute : (i) $P(X \leq 1.5)$, (ii) $P(2 \leq x \leq 3)$

3. A random variable has the following probability distribution,

X :	0	1	2	3	4	5	6	7	8
P(X=x) :	k	3k	5k	7k	9k	11k	13k	15k	17k

   (i) Determine tha valueof k
   (ii) Find P(X < 3), P(X > 3) and P(0 < X < 5)

4. Given the p.d.f of a continuos R.V X as $f(x) = \begin{cases} c\,x(1-x), & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$
   Find 'c' and also c.d.f.

5. The kms X in thousands which car owners get with a certain kind of tyre is a R.V having p.d.f

$$f(x) = \begin{cases} \frac{1}{20} e^{-(\frac{x}{20})}, & x > 0 \\ 0, & x < 0 \end{cases}$$

Find the probability that one of these tyres will last (i) atmost 10000 kms (ii) between 16000 and 24000 kms (iii) atleast 30000 kms

6. Find the mean and variance of the probability density function $f(x) = \begin{cases} k e^{(-kx)}, & k > 0 \\ 0, & \text{otherwise} \end{cases}$

7. A random variable has the following probability distribution,

X :	1	2	3	4	5
P(X = x) :	a^2	$2a^2$	$3a^2$	2a	3a

If X is a p.m.f find 'a'. also find $P(2 \le X \le 5)$ and $P(3 < X)$.

8. Suppose that the life in hours of a certain part of radio tube is a continuous random variable with a p.d.f given by

$$f(x) = \begin{cases} \frac{100}{x^2}, & x \ge 0 \\ 0, & \text{otherwise} \end{cases}$$

(i) What is the probability that all of the three tube in a given radio set will have to be replaced during the first of 150 hours of operation.

(ii) What is the probability that none of three of the original tubes will have to be replaced during the first 150 hours of operation.

9. Find the mean and variance of the following probability distribution,

$$f(x) = \begin{cases} 2 \ \exp^{(-2x)}, & x \ge 0 \\ 0, & x < 0 \end{cases}$$

10. In a pack of 52 cards, two cards are drawn at random simultaneously. If the number of queen cards drawn is a random variable, find the mean and variance of the distribution.

11. An urn contains 3 white balls and 2 red balls. A sample of 2 balls is chosen at random from the urn. Let X denotes the number of red balls chosen. Find mean and variance of the distribution.

12. Give the p.d.f of a continuous random variable X as, $f(x)=\begin{cases} kx(1-x), & 0<x<1 \\ 0, & \text{otherwise} \end{cases}$ find k, mean and variance.

ANSWERS TO CREATIVE QUESTIONS

2 Marks :

1. Yes 2. $k = \frac{1}{11}$ 3. $F(x) = \begin{cases} 0 & x < -2 \\ 0.1, & -2 \le x < -1 \\ 0.2, & -1 \le x < 0 \\ 0.4, & 0 \le x < 1 \\ 0.6, & 1 \le x < 2 \\ 0.9, & 2 \le x < 3 \\ 1, & x \ge 3 \end{cases}$

4. (1) $P(x \le 1) = \frac{2}{3}$ (2) $P(0 < x < 2) = \frac{1}{2}$

5. $P(x < 0) = \frac{1}{4}$, $P(0 < x < 10) = \frac{3}{4}$ 6. k = 1 7. c = 1 8. $a = \frac{1}{2000}$

9. E(X) = 2 10. $E(X) = \frac{91}{6}$ 11. $E(X) = \frac{5}{2}$ 12. E(X) = 2.7, $E(X^2) = 8.1$

13. E(X) = 0, $E(X^2) = \frac{1}{3}$ 14. $P(x \le \frac{1}{3}) = \frac{1}{27}$ 15. E(2X-7) = -6.5

16. $E(X) = \frac{5}{4}$ 17. $E(X) = \frac{5}{2}$

18. (1) Consider the experiment of tossing a coin twice. (2) consider the experiment of rolling a dice once.

19. Random variables are two types,
1. Discrete random variable, 2. Continuous random variable.

20. (1) The amount of rain fall on a rainy day (2) The heights of Individuals (3) The weight of Individuals

21. (1) The amount of a sugar in an orange (2) The time required to run a mile

22. var(5) = 0, var(5X) = 75 24. a = 3 25. E(D) = 3.4

3 Marks :

1. Mean = 0, var(X) = 4, S.D = 2 2. E(X) = 0.3, E(3X+1)=1.9, $E(X^2)$=0.7, var(X) = 0.61

3. Mean = 2.7, var(X) = 0.81 4. Mean = 2.71, var(X) = 5.1659

5. Mean = 5/2, var(X) = 1.25 6. $E(X) = \frac{91}{6}$ 7. k=1/4

$F(x) = \begin{cases} 0, & x < 0 \\ 1/16, & 0 \le x < 1 \\ 7/16, & 1 \le x < 2 \\ 11/16, & 2 \le x < 3 \\ 1 & x \ge 3 \end{cases}$

8. E(X) = 0, $E(X^2)$ = 1/9, var(X) = 1/9 9. k = 1/2

k=0.1

10. $F(x) = \begin{cases} 0 & x < -2 \\ 0.1, & -2 \le x < -1 \\ 0.2, & -1 \le x < 0 \\ 0.4, & 0 \le x < 1 \\ 0.6, & 1 \le x < 2 \\ 0.9, & 2 \le x < 3 \\ 1, & x \ge 3 \end{cases}$

11.

X :	0	1	2	3
P(X=x):	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Mean = 2, variance = 1

12.

X :	0	1	2	3	4
P(X=x):	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{16}$

13. 7, 16

14.

X :	0	1	2	3	4
P(X=x):	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{16}$

Mean : 2, Variance : 1

15. Mean = $\frac{1}{\lambda}$ $E(X^2) = \frac{2}{\lambda^2}$

5 Marks :

1. (i) 1/27 (ii) 19/216

2. a=1/2, $P(x \le 1.5) = 1/2$, $P(2 \le x \le 3) =$

3. k = 1/81 $P(x < 3) = 1/9$, $P(x > 3) = 65/81$, $P(0 < x < 5) = 24/81 = 8/27$

4. c = 6, $F(x) = \begin{cases} 0, & x \le 0 \\ 3x^2 - 2x^3, & 0 < x < 1 \\ 1, & x \ge 1 \end{cases}$

5. (i) 0.3935, (ii) 0.1481, (iii) 0.2231

6. (i) 1/k (ii) $1/k^2$

7. (i) a = 1/6, (ii) 17/36, (iii) 5/6

8. (i) 1/27 (ii) 8/27

9. Mean = 1/2, var(X) = 1/4

10. Mean=2/13, Variance=8/13

11. Mean = 4/5, Variance = 12/25

12. k = 6, Mean=1/2, Variance =1/20

7 Probability Distributions

POINTS TO REMEMBER

Bernoulli trial

Random experiment whose outcome is either Success or Failure

Binomial distribution

$$P\,(x = x) = p(x) = \begin{cases} {}^{n}C_{x}\,p^{x}q^{n-x} & x = 0,1,2,\ldots.n; q = 1-p \\ 0, & otherwise \end{cases}$$

$X \sim B(n,p)$ is a binomial variate.

Properties of Binomial distribution

Mean = np, Variance = npq, Variance < Mean

symmetrical if $p = q = 0.5$

skew symmetric if $p \neq q$

positively skewed if $p < 0.5$ negatively skewed if $p > 0.5$

POISSON DISTRIBUTION

Poisson distribution is a limiting case of binomial distribution when $n \to \infty$ $p \to 0$ $np = \lambda$ is finite

$$P(x,\lambda) = P(X=x) = \begin{cases} \dfrac{e^{-\lambda}\lambda^{x}}{x!}, & x = 0,1,2\ldots\ldots;\lambda > 0 \\ 0 & otherwise \end{cases}$$

Mean = Variance = λ

Poisson distribution can never be symmetrical

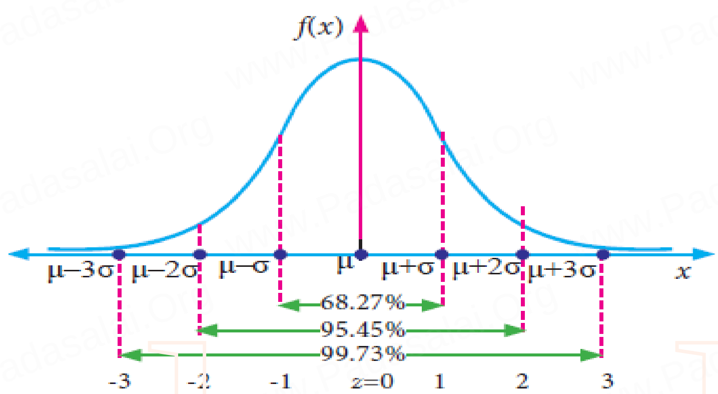
Poisson distribution is for rare events

NORMAL DITRIBUTION

$$f(x:\mu,\sigma)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\}\quad \begin{array}{c}-\infty<x<\infty,\\ -\infty<\mu<\infty,\\ \sigma>0\end{array}$$

mean μ and variance σ^2

Normal distribution is a limiting case of Binomial distribution when $n\to\infty$ neither p (or q) is very small



STANDARD NORMAL DISTRIBUTION $Z \sim N(0,1)$

$$\varphi(z)=\frac{1}{\sqrt{2\pi}}e^{-z^2/2}\qquad -\infty<z<\infty$$

CLASSIFICATION OF TEXT BOOK PROBLEMS

2 Marks

EXERCISE		EXAMPLE	Miscellaneous
Ex. No.,	Question Number	Question Number	Question Number
7.1	1, 2, 4, 8, 11, 15, 16	7.2, 7.3, 7.5	---
7.2	1, 2, 5	7.14, 7.15	
7.3	1, 2, 3	---	

3 Marks

EXERCISE		EXAMPLE	Miscellaneous
Ex. No.,	Question Number	Question Number	Question Number
7.1	5, 9, 12, 17, 18, 19	7.1, 7.4, 7.6, 7.7, 7.9, 7.10, 7.12, 7.13,	2, 3, 5, 9, 10
7.2	6, 7, 11	7.16, 7.19, 7.20	
7.3	---	7.24, 7.27, 7.30, 7.31	

5 Marks

EXERCISE		EXAMPLE	Miscellaneous
Ex. No.,	Question Number	Question Number	Question Number
7.1	3, 6, 7, 10, 13, 14, 20	7.8, 7.11	1, 4, 6, 7, 8
7.2	4, 8, 9, 10, 12	7.17, 7.18	
7.3	4, 5, 6, 7, 8, 9, 10	7.21, 7.22, 7.23, 7.25, 7.26,7.28, 7.29	

: ANSWER TO THEORY QUESTIONS :

EXERCISE : 7.1

1. Define Binomial distribution.

A random variable X is said to follow binomial distribution with parameter n and p, if it assumes only non- negative value and its probability mass function in given by

$$P(x = x) = p(x) = \begin{cases} {}^nC_x p^x q^{n-x} \; x = 0,1,2,\ldots..n; q = 1-p \\ 0, otherwise \end{cases}$$

2. Define Bernoulli trials.

A random experiment whose outcomes are of two types namely success S and failure F, occurring with probabilities p and q respectively, is called a Bernoulli trial.

Some examples of Bernoulli trials are :

(i) Tossing of a coin (Head or tail)

(ii) Throwing of a die (getting even or odd number)

4. Write down the conditions for which the binomial distribution can be used.

Conditions for the binomial probability distribution are

(i) the trials are independent

(ii) the number of trials is finite

(iii)each trial has only two possible outcomes called success and failure.

(iv) the probability of success in each trial is a constant.

EXERCISE : 7.2

1. Define Poisson distribution.

A random variable X is said to follow a Poission distribution with parameter λ if it assumes only non-negative values and its probability mass function is given by

$$P(x, \lambda) = P(X=x) = \begin{cases} \dfrac{e^{-\lambda}\lambda^x}{x!}, & x = 0,1,2\ldots\ldots; \lambda > 0 \\ 0 & otherwise \end{cases}$$

2. Write any two examples for Poisson distribution.

(Any two of the following)

(i) Number of bacteria in one cubic centimeter.

(ii) Number of printing mistakes per page in a text book

(iii) the number of alpha particles emitted by a radioactive substance in a fraction of a second.

(iv) Number of road accidents occurring at a particular interval of time per day.

(v) Number of lightnings per second.

3. Write the conditions for which the Poisson distribution is a limiting case of binomial distribution.

Poisson distribution is a limiting case of binomial distribution under the following conditions :

(i) n, the number of trials is indefinitely large i.e $n \to \infty$.

(ii) p, the constant probability of success in each trial is very small, i.e. $p \to 0$

(iii) $np = \lambda$ is finite. Thus $p = \frac{\lambda}{n}$ and $q = 1 - \left(\frac{\lambda}{n}\right)$ where λ is a positive real number.

5. Mention the properties of Poisson distribution.

(i) Mean = Variance = λ , it is the only distribution with mean = variance

(ii) It is not skewed

(iii) It is used for rare events.

EXERCISE : 7.3

1. Define Normal distribution

A random variable X is said to follow a normal distribution with parameters mean μ and variance σ^2, if its probability density function is given by

$$f(x : \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\} \quad \begin{matrix} -\infty < x < \infty, \\ -\infty < \mu < \infty, \\ \sigma > 0 \end{matrix}$$

2. Define standard normal variate.

A random variable Z = (X–μ)/σ follows the standard normal distribution. Z is called the standard normal variate with mean 0 and standard deviation 1 i.e Z ~ N(0,1). Its Probability density function is given by :

$$\varphi(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2} / 2 \quad -\infty < z < \infty$$

3. Write down the conditions in which the Normal distribution is a limiting case of binomial distribution.

Normal distribution is a limiting case of Binomial distribution under the following conditions:

(i) n, the number of trials is infinitely large, i.e. $n \to \infty$

(ii) neither p (or q) is very small,

4. Write down any five chief characteristics of Normal probability curve.

(i) the curve is bell- shaped and symmetrical about the line $x=u$

(ii) Mean, median and mode of the distribution coincide.

(iii) x – axis is an asymptote to the curve. (tails of the cuve never touches the horizontal (x) axis)

(iv) No portion of the curve lies below the x-axis as $f(x)$ being the probability function can never be negative.

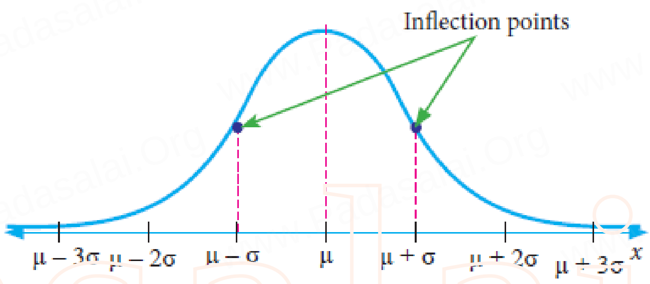


Fig 7.3

(v) The Points of inflexion of the curve are $x = \mu \pm \sigma$

(vi) The curve of a normal distribution has a single peak i.e it is a unimodal.

(vii) As x increases numerically, $f(x)$ decreases rapidly, the maximum probability occurring at the point $x = \mu$ and is given by $[p(x)]\max = 1/\sigma\sqrt{2\pi}$

(viii) The total area under the normal curve is equal to unity and the percentage distribution of area under the normal curve is given below

(a) About 68.27% of the area falls between $\mu - \sigma$ and $\mu + \sigma$

$P(\mu - \sigma < X < \mu + \sigma) = 0.6826$

(b) About 95.5% of the area falls between $\mu - 2\sigma$ and $\mu + 2\sigma$

$P(\mu - 2\sigma < X < \mu + 2\sigma) = 0.9544$

(c) About 99.7% of the area falls between $\mu - 3\sigma$ and $\mu + 3\sigma$

$P(\mu - 3\sigma < X < \mu + 3\sigma) = 0.9973$

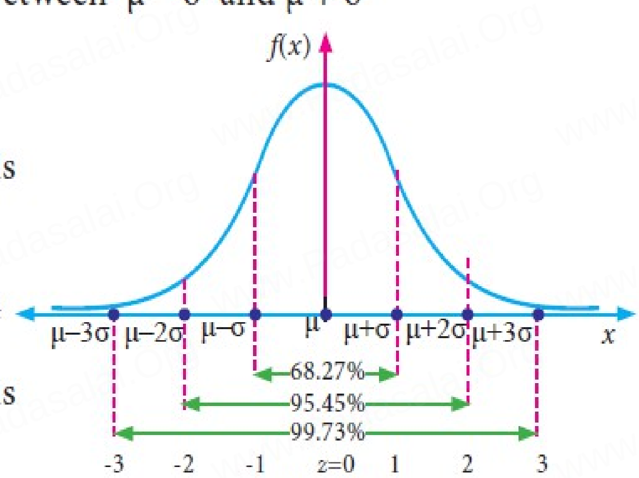


Fig 7.4

: CREATIVE QUESTIONS :

2 MARKS

1. The mean of a binomial distribution is 6 and its S.D is 3, Is this statement true or false? Give your comment.

2. A die is thrown 120 times and getting 1 or 5 is considered a success. Find the mean and variance of the number of successes.

3. In a binomial distribution consisting of 5 independent trials, probabilities of 1 and 2 successes are 0.4096 and 0.2048 respectively. Find the parameter 'p' of the distribution.

4. For a binomial distribution, the mean is 6 and the S.D is √2. Write down all the terms of the distribution.

5. If X ~ B(8,1/3), find E(2X + 3)

6. Let X have a Poisson distribution with P(X=0) = 0.0183 and P(X=1) = 0.0652, Find mean

7. Prove that in a Poisson distribution mean = λ

8. Let X have a Poisson distribution with mean 4. Find var(3X + 5)

9. In a Poisson distribution if P(X=2) = P(X=1) , find λ

10. The number of accidents in a year involving taxi drivers in a city follows a Poisson distribution with mean 3. Out of 1000 taxi drivers, find the number of taxi drivers with no accidents in a year. $[e^{-3}=0.0498]$

11. X is normally distributed with mean 12 and S.D 4. Find the maximum value of the probability function.

12. If the parameter of the normal distribution is
$$f(x) = \frac{1}{\sqrt{(72\pi)}} \exp^{-\left(\frac{((x-10)^2)}{72}\right)}, \quad -\infty < x < \infty$$
. Find its point of inflection.

13. The random variable X is normally distributed with mean 70 and S.D 10, what is the probability that X is between 50 and 90.

14. If X~N(100,36), then find mean,S.D and median of the Normal distribution.

15. If the weight of the students are normally distributed with mean 68 kg and S.D 5 kg. How much percentage of students have weight between 63 and 73 kg.

3 MARKS

1. In tossing of a coin 10 times, find the chance of getting atleast six heads.

2. The sum and product of the mean and variance of a binomial distribution are 10 abd 24, Find the distribution.

3. On an average if one vessel in every ten is wrecked. Find the probability that out of five vessels expected to arrive, atleast four will arrive safely.

4. If X ~ B(n,p) such that 4 P(x=4) = P(x = 2) and n=4. Find mean and S.D

5. If the sum of mean and variance of a binomial distribution is 4.8 for 5 trials, find the distribution.

6. In a poisson distribution if 13 P(x=0)=10 P(x=1) find P(x=3) $[e^{-1.3}=0.3543]$

7. If the number of incoming calls per minute in a telephone is a random variable having a Poisson distribution with λ = 0.2, find the probability that there will be exactly 2 calls during a period of 4 minutes. $e^{-0.2}=0.9802$ ¿

8. Find the probability that atmost 5 defective bolts will be found in a box of 200 bolts, if it is known that 2% of such bolts are expected to be defective. $[e^{-4}=0.01832]$

9. An insurance company insure 4000 people against loss of both eyes in car accidents. Based on previous data, the rates were computed on the assumption that on the average 10 persons in 1,00,000 will have car accidents each year that results in this type of injury. What is the probability that more than 3 will claim the insurance in a year. $[e^{-0.4}=0.6703]$

10. The random variable X is normally distributed with mean 80 and S.D 5. What is the probability that X is between 70 and 85. Find without using area table.

11. If X is nortmal variate with mean 80 and S.D 10, compute
    (i) $P(X \leq 80)$ (ii) $P(X \geq 90)$ (iii) $P(X \leq 60)$

12. If the height of 300 students are normally distributed with mean 64.5 inches and S.D 3.3 inches. Find the height below which 99% of student lies.

13. Marks in an aptitude test given to 500 students of a school was found to be normally distributed. 10% of the students scored below 40 marks and 10% of the students scored above 90 marks. Find the number of students scored between 40 and 90.

14. If X is normally distributed with mean 6 and S.D 5. Find P(X-6 < 10)

15. A large number of measurements is normally distributed with mean 65.5" and S.D of 6.2". Find the percentage of measurements that fall between 54.8" and 68.8"

5 MARKS

1. On the average 20% of the products manufactured by ABC company are found to be defective. If we select 6 of these products at random and X denotes the number of defective products. Find the probability of
(i) two products are defective (ii) atmost two are defective
(iii) atleast two are defective.

2. The overall percentage of passes in an examination is 80. If 6 candidates appear in the examination. What is the probability that
(i) atleast 5 pass the examination (ii) atmost 1 fail in the examination

3. If the probability of success is 40 percentage in a game. How many games should be played to have a probability of atleast two success as 1/3 or more.

4. A hotel has 5 rooms. The demand of rooms on each day is distributed as a Poisson variate, with mean 2. Calculate the number of days in the month of June,
(i) No room is hired (ii) some demand is refused.

5. On an average 1 in 1000 houses in a certain district has a fire accident during a year. If there are 2000 houses in that district. What is the probability that exactly 5 houses will have fire accident during the year and how many houses will escape from the fire accidents during the same year.

6. In a normal distribution 20% of the items are less than 100 and 30% are over 200. Find the mean and S.D of the distribution.

7. The life of army shoes is normally distributed with mean 8 months and S.D 2 months, If 5000 pairs are issued, how many pairs would be expected to need replacement with in a year.

8. The I.Q of group of 1000 school children has a mean 96 and S.D 12. Assuming that the distribution of I.Q among the school children is normally, find the number of children having I.Q (i) less than 72 (ii) between 80 and 120.

9. In a sample of 1000 candidates the mean of certain test is 45 and S.D is 15. Assuming normality of the distribution. Find the range of marks which have the central 40% candidates scored.

10. If the heigth of 500 studentsare normally distributed with mean 60 inches. If 10% of students have height more than 86, find the S.D and also find the number of students who have the height less than or equal to 65 inches.

ANSWERS TO CREATIVE QUESTIONS

2 Marks :

1. False. Since q is greater than 1 2. Mean = 80, varriance = 80/3 3. p=1/5

4. $P(X=x) = {}_9C_x\,(1/3)^x\,(2/3)^{9-x}$, x=0,1,2,....9 5. 25/3 6. 4 8.36

9. 3 10. 50 11. $\frac{1}{(4\sqrt{(2\pi)})}$ 12. x = 4 and x = 16

13. 0.9544 14. μ = 100 , σ = 6, Median = 100 15. 341

3 Marks :

1. 193/512 2. $P(X=x) = {}_{18}C_x\,(1/3)^x\,(2/3)^{18-x}$, x=0,1,2,....18 3. 0.91854

4. 2, $2/\sqrt{3}$ 5. $P(X=x) = {}_5C_x\,(4/5)^x\,(1/5)^{5-x}$, x=0,1,2,....5 6. 0.1297

7. 0.2954 8. 0.9685 9. 0.0008 10. 0.8185

11. 0.5, 0.1587 12. 72.189 13. 400 14. 0.0228 15. 66.61

5. Marks :

1. $15\left(\frac{4^4}{5^6}\right)$, $11\left(\frac{4^4}{5^5}\right)$, $1-2\left(\frac{4}{5}\right)^5$ 2. $2\left(\frac{4}{5}\right)^5$, $\left(\frac{1}{5}\right)^4$ 3. 3 or more

4. 4 days (app), one day (app)

5. 36 houses, 135 housss 6. μ=161.53, σ =73.26

7. 0.4886 8. 23, 885 9. approximately between 37 and 53

10. 20.3

8 Sampling Techniques and Statistical Inference

POINTS TO REMEMBER

Sample group of individuals from a population

Sampling
procedure or process of selecting a sample from a population

Parameter: population mean (μ), variance(σ^2)

Statistic : Any statistical measure computed from sample is known as statistic.
Sample Mean, Saample Variance, Sample S.D, etc.,

Types of sampling

1. Non-Random sampling or Non-probability sampling.
2. Random Sampling or Probability sampling.
   - (i) Simple random sampling
   - (ii) Stratified random sampling
   - (iii) Systematic sampling

(i) Simple random sampling

(A) Lottery method
(B) Table of Random number

Merits

1. Personal bias is completely eliminated.
2. This method is economical as it saves time, money and labour.
3. The method requires minimum knowledge about the population in advance.

Demerits

1. This requires a complete list of the population but such up-to-date lists are not available in many enquiries.
2. If the size of the sample is small, then it will not be a representative of the population.

(ii) Stratified Random Sampling

divide the population into sub-populations, called strata
A sample is drawn from each stratum at random.

Merits

(a) A random stratified sample is superior to a simple random sample because it ensures representation of all groups and thus it is more representative of the population which is being sampled.

(b) A stratified random sample can be kept small in size without losing its accuracy.

(c) It is easy to administer,if the population under study is sub-divided.

(d) It reduces the time and expenses in dividing the strata into geographical divisions, since the government itself had divided the geographical areas.

Demerits

(a) To divide the population into homogeneous strata (if not divided), it requires more money, time and statistical experience which is a difficult one.

(b) If proper stratification is not done, the sample will have an effect of bias.

(c) There is always a possibility of faulty classification of strata and hence increases variability.

(iii) Systematic Sampling

randomly select the first sample from the first k units Then every k^{th} member, starting with the first selected sample, is included in the sample.

Merits

1. This is simple and convenient method.
2. This method distributes the sample more evenly over the entire listed population.
3. The time and work is reduced much.

Demerits

1. Systematic samples are not random samples.
2. If N is not a multiple of n, then the sampling interval (k) cannot be an integer, thus sample selection becomes difficult.

Errors, which arise in the normal course of investigation or enumeration

(i) Sampling Errors

The errors involved in the collection, processing and analysis of the data may be broadly classified into two categories

(ii) Non-Sampling Errors

The errors that arise due to human factors

Standard Error

S.No	Statistic	Standard Error
1.	Sample mean	$\sigma/\sqrt{n}$
2.	Observed sample proportion	$\sqrt{PQ/n}$
3.	Sample standard deviation	$\sqrt{\sigma^2/2n}$
4.	Sample variance	$\sigma^2\sqrt{2/n}$
5.	Sample quartiles	$1.36263\,\sigma/\sqrt{n}$
6.	Sample median	$1.25331\,\sigma/\sqrt{n}$
7.	Sample correlation coefficient	$(1-\rho^2)/\sqrt{n}$

Statistical Inference

(i) Estimation (ii) Testing of Hypothesis

(i) Estimation

The method of obtaining the most likely value of the population parameter using statistic is called estimation.

1. **Point Estimation** When a single value is used as an estimate
2. **Interval Estimation** finding limits within which the parameter would be expected to lie

Estimator:

Any sample statistic which is used to estimate an unknown population parameter is called an estimator ie., an estimator is a sample statistic used to estimate a population parameter.

Estimate:

When we observe a specific numerical value of our estimator, we call that value is an estimate. In other words, an estimate is a specific observed value of a statistic.

Characteristic of a good estimator

(i) Unbiasedness (ii) Consistency(iii) Efficiency (iv) Sufficiency.

Confidence interval $\bar{x} - Z_{\alpha/2} SE < \mu < \bar{x} + Z_{\alpha/2} SE$

Normal Probability Table

Critical Values Z_α	Level of significance (α)			
	1%	2%	5%	10%
Two-tailed test	$\|Z_\alpha\|=2.58$	$\|Z_\alpha\|=2.33$	$\|Z_\alpha\|=1.96$	$\|Z_\alpha\|=1.645$
Right tailed test	$Z_\alpha=2.33$	$Z_\alpha=2.055$	$Z_\alpha=1.645$	$Z_\alpha=1.28$
Left tailed test	$Z_\alpha=-2.33$	$Z_\alpha=-2.055$	$Z_\alpha=-1.645$	$Z_\alpha=-1.28$

(ii) Testing of Hypothesis

Hypothesis testing is also referred to as "Statistical Decision Making".

There are two types of statistical hypothesis

(i) Null hypothesis (ii) Alternative hypothesis

"Null hypothesis is the hypothesis which is tested for possible rejection under the assumption that it is true", and it is denoted by H_0 .

Any hypothesis which is complementary to the null hypothesis is called as the alternative hypothesis and is usually denoted by H_1.

Types of Errors in Hypothesis testing

Type I error: The error of rejecting H_0 when it is true.

Type II error: The error of accepting when H_0 it is false.

Critical region or Rejection region

A region corresponding to a test statistic in the sample space which tends to rejection of H_0 is called critical region or region of rejection.

The region complementary to the critical region is called the region of acceptance.

Level of significance

CLASSIFICATION OF TEXT BOOK PROBLEMS

2 Marks

EXERCISE		EXAMPLE	Miscellaneous
Ex. No.,	Question Number	Question Number	Question Number
8.1	1,2,3,4,5,6,12,14,15	8.1, 8.2, 8.6, 8.10	---
8.2	1,2,3,4,5,6,7,8,9,10,11,12,13		

3 Marks

EXERCISE		EXAMPLE	Miscellaneous
Ex. No.,	Question Number	Question Number	Question Number
8.1	10, 11, 13, 16, 17, 18	8.4, 8.5, 8.7, 8.8, 8.9	2, 3, 5
8.2	14	---	

5 Marks

EXERCISE		EXAMPLE	Miscellaneous
Ex. No.,	Question Number	Question Number	Question Number
8.1	7,8,9, 19, 20	8.3	1, 4, 6, 7
8.2	15, 16, 17	8.11, 8.12, 8.13, 8.14, 8.15, 8.16, 8.17, 8.18, 8.19	

: ANSWER TO THEORY QUESTIONS :

EXERCISE : 8.1

1.What is a population ?

Population refers to all individuals under the study is called as population.

2. What is a sample?

A group of individuals selected from the population to make representation to the entire population under study is called a sample.

3. What is statistic?

Any statistical measure such as mean, variance, standard deviation, etc., computed from the sample is known as statistic.

4. Define Parameter.

The statistical constants of the population like mean (μ), variance(σ^2) is referred as parameter.

5. What is sampling distribution of a statistic?

Sampling distribution of a statistic is the frequency distribution which is formed with various values of a statistic computed from different samples of the same size drawn from the same population.

6. What is standard error?

The standard deviation of the sampling distribution of a statistic is known as its standard error (S.E). Some of the S.E are sample mean = $\sigma/\sqrt{n}$, sample proportion = $\sqrt{(PQ/n)}$

EXERCISE : 8.2

1. Mention two branches of statistical inference.

1. Estimation 2. Testing of Hypothesis are the two branches of statistical inference.

2. What is an estimator?

Any sample statistic which is used to estimate an unknown population parameter is called an estimator ie., an estimator is a sample statistic used to estimate a population parameter.

3. What is an estimate?

When we observe a specific numerical value of our estimator, we call that value is an estimate. In other words, an estimate is a specific observed value of a statistic.

4. What is a point estimation?

When a single value is used as an estimate, the estimate is called a point estimate of the population parameter. In other words, an estimate of a population parameter given by a single number is called as point estimation.

5. What is interval estimation?

Generally, there are situations where point estimation is not desirable and we are interested in finding limits within which the parameter would be expected to lie is called an interval estimation.

6. What is confidence interval?

choose a small value of α which is known as level of significance(1% or 5%) and determine two constants say, c_1 and c_2 such that $P(c_1 < \theta < c_2 | t) = 1 - \alpha$.

The quantities c_1 and c_2, so determined are known as the Confidence Limits and the interval $[c_1, c_2]$ within which the unknown value of the population parameter is expected to lie is known as Confidence Interval. $(1-\alpha)$ is called as confidence coefficient.

7. What is null hypothesis? Give example.

According to Prof. R.A.Fisher, "Null hypothesis is the hypothesis which is tested for possible rejection under the assumption that it is true", and it is denoted by H_0.

For example: If we want to find the population mean has a specified value μ_0, then the null hypothesis H_0 is set as follows $H_0 : \mu = \mu_0$

8. Define alternate hypothesis.

Any hypothesis which is complementary to the null hypothesis is called as the alternative hypothesis and is usually denoted by H_1.

For example: If we want to test the null hypothesis that the population has specified mean μ i.e., $H_0 : \mu = \mu_0$ then the alternative hypothesis could be any one among the following:

(i) $H_1 : \mu \neq \mu_0$ ($\mu >$ or $\mu < \mu$ (ii) $H_1 : \mu > \mu_0$ (iii) $H_1 : \mu < \mu_0$

9. Define critical region.

A region corresponding to a test statistic in the sample space which tends to rejection of H_0 is called critical region or region of rejection.

10. Define critical value.

The value of test statistic which separates the critical (or rejection) region and the acceptance region is called the critical value or significant value. It depend upon

(i) The level of significance

(ii) The alternative hypothesis whether it is two-tailed or single tailed.

: CREATIVE QUESTIONS :

2 MARKS

1. Using the nfollowing random number table (Kendall – Babington Smith)

23	15	75	48	59	01	83	72	59	93	76	24	97	08	86	95	23	03	67	44
05	54	55	50	43	10	53	74	35	08	90	61	18	37	44	10	96	22	13	43
14	87	16	03	50	32	40	43	62	23	50	05	10	03	22	11	54	36	08	34
38	97	67	49	51	94	05	17	58	53	78	80	59	01	94	32	42	87	16	95
97	31	26	17	18	99	75	53	08	70	94	25	12	58	41	54	88	21	05	13

Draw a random sample of 10 four digit numbers starting from 2000 – 8050

2. A server channel monitored for an hour was found to have an estimated mean of 30 transactions per minute. The variance is known to be 6. Find the standard error.

3. Find the sample size for the given standard deviation 12 and the standard error with respect of sample mean 3.

4. A sample of 100 students is chosen from a large group of students. The average height of these students is 160 cm and standard deviation (S.D) is 7 cm. Obtain the standard error for the average height of large group of students of 158 cm.

5. Faulty selection of the sample instead of correct selection by defective sampling technique. The investigator substitutes a convinient sample if the original sample is not available while investigation.

6. Give two examples for non-sampling errors.

7. Find the standard error with standard deviation 20 and sample of 1000.

8. Using the following Tippet's random number table

2952	6641	3992	9792	7969	5911	3170	5624
4167	9524	1545	1396	7203	5356	1300	2693
2670	7483	3408	2762	3563	1089	6913	7991
0560	5246	1112	6107	6008	8125	4233	8776
2754	9143	1405	9025	7002	6111	8816	6446

Draw a sample of 10 three digit numbers which are odd numbers.

9. Determine the standard error of population for a random sample of 500 apples was taken from a large consignment and 50 were found to be bad.

10. Define : Type II error

11. Define : Unbiasedness

12. Define: Consistency

13. What is the number of ways in which one can select 2 customers out of 10 customers?

14. Out of 500 students, 300 students did the assignment given by the teacher. Find the standard error of the sample proportion.

15. A sample of 50 students is chosen from XI standard students. The average weight of the students is 50 kg and standard deviation is 2 Kg. Obtain the standard error for the average weight of large group of students of 55 Kg.

3 MARKS

1. A die is thrown 9000 times and a throw of 5 or 6 is observed 3000 times. Find the standard error of the proportion for an unbiased die.

2. The standard deviation of a sample of size 50 is 6.3. Determine the standard error when population standard deviation is 6.5?

3. A whole seller of pineapple claims that only 3% of the apples supplied by him are defective. A random sample of 500 apples contained 36 defective apples. Calculate the standard error concerning good apples.

4. A sample of 1000 students whose mean weight is 120 lbs from a school in Tamilnadu state was taken and their average weight was found to be 124 lbs with a S.D of 32 lbs. Calculate the standard error of mean.

5. A random sample of 50 observations was drawn from a large population and its standard deviation was found to be 2.5. Calculate the suitable standard error that this sample is taken from a population with S.D 3.

6. In a sample of 500 population from a village, 200 were found to be eaters of vegetarian items and the rest non-vegetarian items. Compute the standard error assuming that both vegetarian and non-vegetarian foods are equally popular in that village.

7. A sample of 200 items drawn from the Universe with mean value 81 and S.D 6 has a mean value 80. Is the difference in the mean significant?

8. A random sample of size 50 with mean 67.9 is drawn from a normal population. If it is known that the standard error of the sample mean, is $\sqrt{(0.7)}$ find 95% confidence interval for the population mean.

9. A random sample of 500 apples was taken from a large consignment and 45 of them were found to be bad. Find the limits at which the bad apples lie at 99% confidence level.

10. Out of 1000 T.V viewers 320 watched a particular programme. Find 95% confidence limits for T.V viewers who watched this programme.

11. From a school students, sample of 150 selected at random to test the accuracy of solving a problem in business mathematics and of these 10 did a mistake. Find the limits within which the number of students who did the problem wrongly at 99% confidence level

12. Out of a large population of customer's ledge accounts, a sample of 200 accounts was taken to test accuracy of posting and balancing wherein 35 mistakes were found. Find 95% confidence limits which the number of defective cases can be expected to lie.

13. A random sample of 50 branches of state bank of India in a district showed a mean annual profit of Rs.75 Lakhs and a standard deviation of 10 Lakhs. Find the 95% confidence limits for the estimate of mean profit of population.

14. A random sample of 55 from a specific population yields the mean of 73.5 and standard deviation of 13.2. Construct 90% confidence interval for the population mean.

15. Find the sample size for the given standard deviation 6 and the standard error with respect of sample mean is 1.8

5 MARKS

1. Measurements of the weights of a random sample of 200 ball bearings made by a certain machine during one week showed mean of 0.824 newtons and a standard deviation of 0.042 newtons. Find (a) 95% and (b) 99% confidence limits for the mean weight of all the ball bearings.

2. Out of 1000 customers ledger accounts a sample of 200 accounts was taken to test the accuracy of posting and balancing wherein 35 mistakes were found. Find 95% confidence limits within which the number of defective cases can be expected to lie.

3. A sample poll of 100 voters choosen at random from all voters in a given district indicated that 55% of them were in favour of a particular candidate. Find (a) 95% and (b) 99% confidence limits for the proportion of all voters in favour of that candidate.

4. The mean life time of 50 Electric bulbs produced by a manufacturing company is estimated to be 825 hours with a standard deviation of 110 hours. If μ is the mean life time of all the bulbs produced by the company. Test the hypothesis that μ= 500 hours at 5% level of significance.

5. A company markets car tyres. Their lives are normally distributed with a mean of 50,000 kilometers and standard deviation of 2000 kilometers. A test sample of 64 tyres has a mean life of 51250 kilometers . Can you conclude that the sample mean differs significantly from the population mean? (test at 5% level of significance)

6. A sample of 400 students is found to have a mean height of 171.38 cm. Can it reasonably be regarded as sample from a population with mean height of 171.17 cm and standard deviaton of 3.3 cms (test at 5% level of significance)

7. The income distribution of the population of a village has a mean of Rs.6000 and a variance of Rs.32400. Could a sample of 64 persons with a mean income of Rs.5950 belong to this population? (test at both 5% and 1% level of significance)

8. To list the conjecture of the management that 60% employees favour a new bonus scheme, a sample of 150 employees was drawn and their opinion was taken whether they favoured it or not. Only 55 employees out of 150 favoured the new bonus scheme. Test the conjecture at 1% level of significance.

9. An inventor has developed a new, energy-efficient lawn mover engine. He claims that the engine will run continuously for 5 hours (300 minutes) on a single gallon of regular gasoline. From his stock of 2000 engines, the inventor selects a simple random sample of 50 engines for testing. The engines run for an average of 295 minutes, with a standard deviation of 20 minutes. Test the null hypothesis that the mean run time is 300 minutes against the alternative hypothesis that the mean run time is not 300 minutes. Use a 0.05 level of significance.

10. Bon Air Elementary school has 1000 students. The principal of the school thinks that the average IQ of the students at Bon Air is at least 110. To prove her point. She administers an IQ test to 20 randomly selected students. Among the sampled students, the average IQ is 108 with a standard deviation of 10. Based on these results, should the principal accept or reject her original hypothesis? Assume a sign level of 0.01 (Assume that test scores in the population of engines are normally distributed).

11. The average IQ for the adult population is 100 with a standard deviation of 15. A researcher believes this value has changed. The researcher decides to test the IQ of 75 random adults. The average IQ of the samples is 105. Is there enough evidence to suggest the average IQ has changed.

ANSWERS TO CREATIVE QUESTIONS

2 Marks :

1. 2315, 3897, 5550, 6749, 2617, 5901, 4310, 5032, 5194, 5374

2. 0.316 3. 16 4.0.7

5. (a) Faculty selection of the sample instead of correct sample by defective sampling technique.
   (b) The investigator substitutes a convenientsample if the original sample is not available while investigations.

6. (a) Due to lack of trained and qualified investigators
   (b) Due to framing of a wrong questions.

7. 2 8. 641, 423, 143, 545, 405, 107, 969, 203, 563, 911

9. 0.0134 10. The error of accepting H_o when it is false.

11. Unbiasedness: An estimator $T_n = T(x_1, x_2, ..., x_n)$ is said to be an unbiased estimator of $\gamma(\theta)$ if $E(T_n) = \gamma(\theta)$, for all $\theta \varepsilon \theta$ (parameter space). (i.e)An estimator is said to be unbiased if its expected value is equal to the population parameter. Example: $E(\overline{x}) = \mu$

12. Consistency: An estimator $T_n = T(x_1, x_2, ..., x_n)$ is said to be consistent estimator of $\gamma(\theta)$, if T_n converges to $\gamma(\theta)$ in Probability, i.e., $T_n \xrightarrow{P} \gamma(\theta) \, as \, n \to \infty$, for all $\theta \varepsilon \Theta$.

13. 45 14. 0.022 15. 0.28

3 Marks :

1. 0.00496 2. 0.61 3. 0.0116 4. 1.01 5. 0.3

6. 0.022 7. 2.36 8. (66.2, 67.54) 9. (0.057, 0.123)

10. (29.2%, 34.8%) 11. (0.0184, 0.1216)

12. (0.122, 0.228) = (12.2%, 2.28%) 13. (72.23, 77.77)

14. (70.57, 76.43) 15. 11

5 Marks :

1. (i) (0.818, 0.829) (ii) (0.816,0.832)

2. No of defective cases lie between 1230 and 2270

3. (i) 45% and 65% (ii) 42% and 68%

4. 4.82, rejected 5. 5, rejected 6. 1.273, accepted

7. 2.2, at 5%, Ho, rejected, at 1% Ho accepted

8. 5.9, rejected 9. 1.76, accepted 10. 0.894, accepted

11. 3.33, rejected

9 Applied Statistics

POINTS TO REMEMBER

Time Series Analysis

Components of Time Series

(i) Secular Trend (ii) Seasonal variations (iii) Cyclic variations (iv) Irregular variations

Measurements of Trends

(i) Freehand or Graphic Method.
(ii) Method of Semi-Averages.
(iii) Method of Moving Averages.
(iv) Method of Least Squares.

The straight line trend is represented by the equation $Y = a + bX$ ...(1)

where Y is the actual value, X is time, a, b are constants

The constants 'a' and 'b' are estimated by solving the following two normal Equations

$$\Sigma Y = n\,a + b\,\Sigma X \quad ...(2)$$

$$\Sigma XY = a\,\Sigma X + b\,\Sigma X^2 \quad ...(3)$$

Where 'n' = number of years given in the data.

(i) Additive Model:

Y= T+S+C+I

(ii) Multiplicative Model:

Y = T × S × C × I

Y = Original value , T = Trend Value , S = Seasonal component

C = Cyclic component , I = Irregular component

Methods of measuring Seasonal Variations By Simple Averages :

$$\text{Seasonal Index (S.I)} = \frac{\textit{Seasonal Average}}{\textit{Grand average}} X\,100$$

INDEX NUMBER

Index Numbers are the indicators which reflect the changes over a specified period

Classification of Index Numbers:

(i) Price Index Number

(ii) Quantity Index Number

(iii) Cost of living Index Number

Weighted Index Number

Price Index (P_{01}) = $\frac{\sum p_1 w}{\sum p_0 w} \times 100$, Let us consider the following notations,

p_1 - current year price p_0 - base year price

q_1 - current year quantity q_0 - base year quantity

where suffix '0' represents base year and '1' represents current year.

Laspeyre's price index number

$$P_{01}^L = \frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100$$

quantity of the base year is used as weight.

Paasche's price index number

$$P_{01}^P = \frac{\sum p_1 q_1}{\sum p_0 q_1} X 100$$

quantity of the current year is used as weight

Fisher's price index number

$$P_{01}^F = \sqrt{P_{01}^L \times P_{01}^P} = \sqrt{\frac{\sum p_1 q_0 \times \sum p_1 q_1}{\sum p_0 q_0 \times \sum p_0 q_1}} \times 100$$

Fisher's price index number is the geometric mean between Laspeyre's and Paasche's price index number

Since Fisher's Price Index Number satisfies both TRT & FRT, it is termed as an Ideal Index Number

Test of adequacy for an Index Number

(i) Time Reversal Test (ii) Factor Reversal Test

$$P_{01}^F \times P_{10}^F = 1$$

$$P_{01} \times Q_{01} = \frac{\sum p_1 q_1}{\sum p_0 q_0}$$

Methods of constructing Cost of Living Index Number

(i) Aggregate Expenditure Method (or) Weighted Aggregative Method.

(ii) Family Budget Method.

Family Budget Method

$$\text{Cost of Living Index Number} = \frac{\sum PV}{\sum V}$$

where $P = \frac{p_1}{p_0} \times 100$ is the price relative.

$V = \sum p_0 q_0$ is the value relative.

Statistical Quality Control (SQC)

(i) Charts for Mean ($\overline{X}$) (ii) Charts for Range (R)

Case (i) when $\overline{X}$ and SD are given	Case (ii) when $\overline{X}$ and SD are not given	Case (i) when SD is given	Case (ii) when SD is not given
$UCL = \overline{\overline{X}} + 3\frac{\sigma}{\sqrt{n}}$	$UCL = \overline{\overline{X}} + A_2\overline{R}$	$UCL = \overline{R} + 3\sigma_R$	$UCL = D_4\overline{R}$
$CL = \overline{\overline{X}}$	$CL = \overline{\overline{X}}$	$CL = \overline{R}$	$CL = \overline{R}$
$LCL = \overline{\overline{X}} - 3\frac{\sigma}{\sqrt{n}}$	$LCL = \overline{\overline{X}} - A_2\overline{R}$	$LCL = \overline{R} - 3\sigma_R$	$LCL = D_3\overline{R}$

CLASSIFICATION OF TEXT BOOK PROBLEMS

2 Marks

EXERCISE		EXAMPLE	Miscellaneous
Ex. No.,	Question Number	Question Number	Question Number
9.1	1,2,3,4,5,7,8,9,11,16	9.1, 9.3	---
9.2	1,2,3,4,5,6,7,8,9,10, 11, 12, 13	---	
9.3	1,2,3,4,5,6,7,8,9,10, 11,12,13	---	

3 Marks

EXERCISE		EXAMPLE	Miscellaneous
Ex. No.,	Question Number	Question Number	Question Number
9.1	6,10, 12, 14, 15	9.2, 9.4, 9.5	1,2,6,7
9.2	14, 20, 21, 22	9.15, 9.16, 9.17, 9.18	
9.3	---	9.19, 9.20	

5 Marks

EXERCISE		EXAMPLE	Miscellaneous
Ex. No.,	Question Number	Question Number	Question Number
9.1	13, 17, 18, 19, 20, 21, 22	9.6, 9.7, 9.8, 9.9	3,4,5,8,10,11
9.2	15, 16, 17, 18, 19	9.10, 9.11, 9.12, 9.13, 9.14	
9.3	14, 15, 16,17, 18, 19, 20, 21	9.21, 9.22, 9.23	

: ANSWER TO THEORY QUESTIONS :

EXERCISE : 9.1

1. Define Time Series.

A Time Series consists of data arranged chronologically

When quantitative data are arranged in the order of their occurrence, the resulting series is called the Time Series

2. What is the need for studying time series?

time series helps us to study and analyze the time related data which involves in business fields, economics, industries, etc…

3. State the uses of time series

It helps in the analysis of the past behavior.

It helps in forecasting and for future plans.

It helps in the evaluation of current achievements.

It helps in making comparative studies between one time period and others.

4. Mention the components of the time series.

There are four types of components in a time series. They are

(i) Secular Trend (ii) Seasonal variations (iii) Cyclic variations (iv) Irregular variations

5. Define secular trend.

It is a general tendency of time series to increase or decrease or stagnates during a long period of time. An upward tendency is usually observed in population of a country, production, sales, prices in industries, income of individuals etc., A downward tendency is observed in deaths, epidemics, prices of electronic gadgets, water sources, mortality rate etc…. It is not necessarily that the increase or decrease should be in the same direction throughout the given period of time.

6. Write a brief note on seasonal variations.

As the name suggests, tendency movements are due to nature which repeat themselves periodically in every seasons. These variations repeat themselves in less than one year time. It is measured in an interval of time. Seasonal variations may be influenced by natural force, social customs and traditions. These variations are the results of such factors which uniformly and regularly rise and fall in the magnitude. For example, selling of umbrellas' and raincoat in the rainy season, sales of cool drinks in summer season, crackers in Deepawali season, purchase of dresses in a festival season, sugarcane in Pongal season.

7. Explain cyclic variations

These variations are not necessarily uniformly periodic in nature. That is, they may or may not follow exactly similar patterns after equal intervals of time. Generally one cyclic period ranges from 7 to 9 years and there is no hard and fast rule in the fixation of years for a cyclic period. For example, every business cycle has a Start- Boom- Depression- Recover, maintenance during booms and depressions, changes in government monetary policies, changes in interest rates.

8. Discuss about irregular variation.

These variations do not have particular pattern and there is no regular period of time of their occurrences. These are accidently changes which are purely random or unpredictable. Normally they are short-term variations, but its occurrence sometimes has its effect so intense that they may give rise to new cyclic or other movements of variations. For example floods, wars, earthquakes, Tsunami, strikes, lockouts etc…

9. Define seasonal index.

Seasonal variation in a time series refer to those variations which occur regularly and periodically within a period of less than one year.

Seasonal Index is a measure of how a particular season through some cycle compares with the average season of that cycle.

11. State the two normal equations used in fitting a straight line.

$\Sigma Y = n\,a + b\,\Sigma X$

$\Sigma XY = a\,\Sigma X + b\,\Sigma X^2$

Where 'n' = number of years given in the data.

The constants 'a' and 'b' are estimated

The constant 'a' takes only positive values, but constant 'b' takes both positive and negative values

12. State the different methods of measuring trend.

(i) Freehand or Graphic Method.

(ii) Method of Semi-Averages.

(iii) Method of Moving Averages.

(iv) Method of Least Squares.

EXERCISE : 9.2

1. Define Index Number.

Index Numbers are the indicators which reflect the changes over a specified period of time in price of different commodities, production, sales, cost of living etc... Index Numbers are statistical methods used to measure the relative change in the level of a variable or group of variables with respect to time, geographical location or other characteristics such as income, profession etc.

2. State the uses of Index Number

(i) It is an important tool for the formulating decision and management policies.

(ii) It helps in studying the trends and tendencies.

(iii) It determines the inflation and deflation in an economy.

3. Mention the classification of Index Number.
   (i) **Price Index Number**
   (ii) **Quantity Index Number**
   (iii) **Cost of living Index Number**

4. Define Laspeyre's price index number.
   Laspeyre's price index number

$$P_{01}^{L} = \frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100$$

   p_1 - current year price
   p_0 - base year price
   q_0 - base year quantity

   In Laspeyre's price index number, the quantity of the base year is used as weight.

5. Explain Paasche's price index number.
   In Paasche's price index number, the quantity of the current year is used as weight.

$$P_{01}^{P} = \frac{\sum p_1 q_1}{\sum p_0 q_1} \times 100$$

   p_1 - current year price
   p_0 - base year price
   q_1 - current year quantity

6. Write note on Fisher's price index number.
   Fisher's price index number is the geometric mean between Laspeyre's and Paasche's price index number

$$P_{01}^{F} = \sqrt{P_{01}^{L} \times P_{01}^{P}}$$

   Fisher's Index number is ideal index number because it satisfies both time reversal and factor reversal tests.

7. State the test of adequacy of index number.
   i. Time Reversal Test ii. Factor Reversal Test

8. Define Time Reversal Test
   Time Reversal Test

   It is an important test for testing the consistency of a good index number. This test maintains time consistency by working both forward and backward with respect to time (here time refers to base year and current year). Symbolically the following relationship should be satisfied, $P_{01} \times P_{10} = 1$

9. Explain Factor reversal test
   Factor Reversal Test

   This is another test for testing the consistency of a good index number. The product of price index number and quantity index number from the base year to the current year should be equal to the true value ratio. That is, the ratio between the total value of current period and total value of the base period is known as true value ratio.

   Factor Reversal Test is

$$P_{01} \times Q_{01} = \frac{\sum p_1 q_1}{\sum p_0 q_0}$$

10. Define true value ratio.
   The ration between the total value of current period and total value of the base period is known as true value ratio. ie., $\frac{\sum p_1 q_1}{\sum p_0 q_0}$ is the true value ratio.

11. Discuss about Cost of living index number.

Cost of Living Index Number is constructed to study the effect of changes in the price of goods and services of consumers for a current period as compared with base period.

The change in the cost of living index number between any two periods means the change in income which will be necessary to maintain the same standard of living in both the periods.
Therefore the cost of living index number measures the average increase in the cost to maintain the same standard of life.

12. Define Family budget method

Family Budget Method

In this method, the weights are calculated by multiplying prices and quantity of the base year. (i.e.) $V=\sum p_0q_0$. The formula is given by,

$$\text{Cost of Living Index Number} = \frac{\sum PV}{\sum V}$$

where $P=\frac{p_1}{p_0}\times 100$ is the price relative.

$V=\sum p_0q_0$ is the value relative.

13. State the uses of Cost of Living Index Number.

(i) It indicates whether the real wages of workers are rising or falling for a given time.

(ii) It is used by the administrators for regulating dearness allowance or grant of bonus to the workers.

EXERCISE : 9.3

1. Define Statistical Quality Control.

Quality Control is a powerful technique used to diagnose the lack of quality in any of the raw materials, processes, machines etc… It is essential that the end products should possess the qualities that the consumer expects from the manufacturer.

2. Mention the types of causes for variation in a production process.

There are two causes of variation which affects the quality of a product, namely

1. Chance Causes (or) Random causes

2. Assignable Causes

3. Define chance cause.

The minor causes which do not affect the quality of the products to an extent are called Chance Causes (or) Random Causes. For example, Rain, Floods, power cuts, etc.,

4. Define Assignable cause.
   The assignable causes may occur in at any stage of the process, right from the arrival of the raw materials to the final delivery of the product.
   Some of the important factors of assignable causes are defective raw materials, fault in machines, unskilled manpower, worn out tools, new operation, etc.,

5. What do you mean by product control?
   Product control means that controlling the quality of the product by critical examination through sampling inspection plans.
   Product control aims at a certain quality level to be gauranteed to the customers.

6. What do you mean by process control?
   The main objective in any production process is to control and maintain a satisfactory quality level of the manufactured product. This is done by 'Process Control'.
   In process control the proportion of defective items in the production process is to be minimized and it is achieved through the technique of control charts.

7. Define a control chart.
   Shewhart's control charts provide an answer to both. It is a simple technique used for detecting patterns of variations in the data. Control charts are simple to construct and easy to interpret. A typical control charts consists of the following three lines.
   (i) Centre Line (CL) indicates the desired standard level of the process.
   (ii) Upper Control Limit (UCL) indicates the upper limit of tolerance.
   (iii) Lower Control Limit (LCL) indicates the lower limit of tolerance.

8. Name the control charts for variables.
   (i) Charts for Mean ($\overline{X}$) (ii) Charts for Range (R)

9. Define mean chart.
   The mean or $\overline{X}$ chart is to show the **quality averages** of the samples taken from the given process. The control limits of the $\overline{X}$ shows the presence or absence of assignable causes in the production process. It is required for decision making to accept or reject the process.

10. Define R Chart
    The Range or R chart is to show the **variablity or dispersion** of the samples taken from the given process.
    The control limits of the R shows the presence or absence of assignable causes in the production process. It is required for decision making to accept or reject the process.

11. What are the uses of statistical quality control?
    The control limits of the R and $\overline{X}$ shows the presence or absence of assignable causes in the production process. It is required for decision making to accept or reject the process.

12. What are the control limits for the mean chart?

control limits for $\overline{X}$ chart in two different cases is

Case (i) when $\overline{X}$ and SD are given	Case (ii) when $\overline{X}$ and SD are not given
$UCL = \overline{\overline{X}} + 3\frac{\sigma}{\sqrt{n}}$	$UCL = \overline{\overline{X}} + A_2\overline{R}$
$CL = \overline{\overline{X}}$	$CL = \overline{\overline{X}}$
$LCL = \overline{\overline{X}} - 3\frac{\sigma}{\sqrt{n}}$	$LCL = \overline{\overline{X}} - A_2\overline{R}$

13. Write the control limits for the R chart.

control limits for R chart in two different cases are

Case (i) when SD is given	Case (ii) when SD is not given
$UCL = \overline{R} + 3\sigma_R$	$UCL = D_4\overline{R}$
$CL = \overline{R}$	$CL = \overline{R}$
$LCL = \overline{R} - 3\sigma_R$	$LCL = D_3\overline{R}$

: CREATIVE QUESTIONS :

2 MARKS

1. What are the two commonm models used for decomposition of time series?
2. What is additive model?
3. What is multiplicative model?
4. Who is the father of statistical quality control?
5. In the method of least squares 'a' and 'b' refers to what? Write the equation of the trend line.
6. Name the two types in construction of Index numbers.
7. Give any two examples for secular trend.
8. Give any two examples for cyclic variation.
9. Give any two examples for irregular variation.
10. Write the weight of Laspeyre's and Paasche's Price index number.
11. What is the relation between Fisher's price index number , Laspeyre's and Paasche's Price index numbers.

3 MARKS

1. Find the trend values to the following data by the method of semi-averages.

Year	2010	2011	2012	2013	2014	2015	2016
Sales	102	105	114	110	108	116	112

2. Find the trend values by the method of semi-averages.

Year	2005	2006	2007	2008	2009	2010	2011	2012
Sales	270	240	230	230	220	200	210	200

3. Draw a trend line by graphic method(free hand)

Year	2010	2011	2012	2013	2014	2015	2016
Production	20	22	25	26	25	27	30

4. Draw a trend line by graphic method (free hand)

Year	2011	2012	2013	2015	2016
Sales	20	24	25	38	60

5. Calculate the cost of living index by aggregate expenditure method.

Commodity	Quantity 2014	Price	
		2014	2017
A	100	8	12
B	25	6	7.5
C	10	5	5.25
D	20	48	52
E	65	15	16.5

F	30	19	27

6. Construct cost of living index for 2019 taking 2018 as the base year, using Aggregate expenditure method.

Commodity	Quantity 2018	Price	
		2018	2019
A	6	5.75	6
B	1	5	8
C	6	6	9
D	4	8	10
E	2	2	1.8
F	1	20	15

7. Below are given figures of production (in thousand tonnes) of a sugar factory. Obtain the trend values be 3-year moving average.

Year	1980	1981	1982	1983	1984	1985	1986
Production	80	90	92	83	94	99	92

8. Using four yearly moving average calculate the trend values

Year	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990
Production	464	515	518	467	502	540	557	571	586	612

9. **Construct the cost of living Index Number for 2003 on the basis of 2000 from the following data using family Budget method.**

Items	Price		Weights
	2000	2003	
Food	200	280	30
Rent	100	200	20
Clothing	150	120	20
Fuel & lighting	50	100	10
Miscellaneous	100	200	20

10. **Calculate the 3-yearly Moving Averages of the production figures (in mat. tonnes) given below**

Year	1973	1974	1975	1976	1977	1978	1979	1980	1981
Production	15	21	30	36	42	46	50	56	63

5 MARKS

1. From the data given under calculate seasonal indices

Quarter	Year				
	2004	2005	2006	2007	2008
I	40	42	41	45	44
II	35	37	35	36	38
III	38	39	38	36	38
IV	40	38	40	41	42

2. Calculate the seasonal indices by the method of simple average

Quarter	Year				
	2014	2015	2016	2017	2018
I	78	76	72	74	76
II	66	74	68	70	74
III	84	82	80	84	86
IV	80	78	70	74	82

3. Calculate the seasonal indices by the method of simple average

Year	I Quarter	II Quarter	III Quarter	IV Quarter
1985	68	62	61	63
1986	65	58	66	61
1987	68	63	63	67

4. Calculate the seasonal indices by the method of simple average

Year	Quarters			
	I	II	III	IV
2002	72	68	80	70
2003	76	70	82	74
2004	74	66	84	80
2005	76	74	84	78
2006	78	74	86	82

5. From the data given below, construct a cost living index number by family budget method for 2016 from 2006

Commodity	P	Q	R	S	T	U
Quantity in 2006 (Base Year)	50	25	10	20	30	40
Price per unit in 2006 (₹)	10	5	8	7	9	6
Price per unit in 2016 (₹)	6	4	3	8	10	12

6. Calculate the cost living index using Family budget method.

Commodity	A	B	C	D	E	F	G	H
Quantity in Base Year	20	50	50	20	40	50	60	40
Price in Base year	10	30	40	200	25	100	20	150
Price in Current yea	12	35	50	300	50	150	25	180

7. Compute (i) Laspeyre's (ii) Paasche's and (iii) Fisher's Index numbers

Commodity	PRICE		QUANTITY	
	BASE YEAR	CURRENT YEAR	BASE YEAR	CURRENT YEAR
A	6	10	50	50
B	2	2	100	120
C	4	6	60	60
D	10	12	30	25

8. Compute (i) Laspeyre's (ii) Paasche's and (iii) Fisher's Index numbers

Commodity	2018		2017	
	Price	Quanity	Price	Quantity
A	4	6	2	8
B	6	5	5	10
C	5	10	4	14
D	2	13	2	19

9. Construct Fisher'sindex number and show that it satisfies Factor reversal and Time reversal tests.

Commodity	Price		Quantity	
	Base Year	Current Year	Base Year	Current Year
A	6	10	50	56
B	2	2	100	120
C	4	6	60	60
D	10	12	30	24
E	8	12	40	36

10. Calculate the values for control line, control limits for mean chart and range chart and determine whether the process is in control.

Sample Number	1	2	3	4	5	6	7	8	9	10
Mean	11.2	11.8	10.8	11.6	11	9.6	10.4	9.6	10.6	10
Range(R)	7	4	8	5	7	4	8	4	7	9

ANSWERS TO CREATIVE QUESTIONS

2 Marks :

1. Multicative model, additive model

2. $y_t = T_t + S_t + C_t + I_t$

3. $y_t = T_t \times S_t \times C_t \times I_t$

4. Walter Andrew Shewhart

5. In the method of least squares 'a' gives the mean of Y and 'b' gives rate of change (slope) Trend line equation is Y = a + b X

6. (1) Unweighted index number (2) weighted index number

7. Time series relating to population, price, production, literacy etc., may show increasing trend and time series relating to birth rate , death rate, poverty may show decreasing trend.

8. Changes in Government monetory policies, changes in interest rates, changes in business cycle.

9. Floods, wars, earth quakes, Tsunami, strikes, etc.,

10. In Laspeyre's price index number weight is q_0 – base year quantity
   In Paasche's price index number weight is q_1 – current year quantity

11. Fisher's index number is the geometric mean of Laspeyre's and Paasche's Index numbers.

   ie., $P_{01}^F = \sqrt{P_{01}^L \times P_{01}^P}$

,

3 Marks :

1.

Year :	2010	2011	2012	2013	2014	2015	2016
Trend:	105.75	107	108.25	109.50	110.75	112	113.25

2.

Year :	2005	2006	2007	2008	2009	2010	2011	2012
Trend:	255.625	246.875	238.125	229.375	220.625	211.875	208.125	194.375

5. Cost Living Index = 124.46

6. Cost living index = 119.09

7. – , 87.33, 88.33, 89.67, 92, 95, –.

8. –, –, 495.75, 503.63, 511.63, 529.50, 553, 572.50, –, –.

9. Cost of Living Index (C.L.I) $= \frac{\Sigma PV}{\Sigma V} = \frac{15800}{100} = 158$

10. --, 22, 29, 36, 41.33, 46, 50.67, 56.33, --

5 Marks :

1.

Quarter :	I	II	III	IV
S.I :	108.30	92.54	96.55	102.68

2.

Quarter :	I	II	III	IV
S.I :	98.4293	92.1465	108.9	100.52

3.

Quarter :	I	II	III	IV
S.I :	105.10	95.68	99.35	99.87

4.

Quarter :	I	II	III	IV
S.I :	98.4	92.14	108.9	100.52

5. CLI = 101.1 6. CLI = 124.34

7. P_{01}^{L} = 136.54, P_{01}^{p} = 135,92, P_{01}^{F} = 136.23

8. P_{01}^{L} = 125, P_{01}^{p} = 126.21, P_{01}^{F} = 125.6 9. Fisher Index Number=139.79

10. Process is in control

10 Operations Research

POINTS TO REMEMBER

Transportation Problem

The objective of transportation problem

the amount to be transported

from each origin

to each destinations

such that the total transportation cost is minimized.

Feasible Solution: A feasible solution to a transportation problem is a set of non-negative values x_{ij} (i=1,2,..,m, j=1,2,…n) that satisfies the constraints.

*** feasible solution that minimizes the total transportation cost ==> optimal solution**

*** not morethan m+n–1 allocations ==> basic feasible solution**

*** exactly m+n-1 allocations ==> Non-degenerate basic feasible solution**

*** less than m+n-1 allocations ==> degeneracy**

Methods of finding initial Basic Feasible Solutions

North-West Corner Rule (NWC)

Least Cost Method (LCM)

Vogel's Approximation Method(VAM)

Assignment Problem

The objective of assignment problem

to assign the

different jobs

to the different machines

to minimize the overall cost

Using minimum(-z) we can find maximum(z) in problems that need maximization

The assignment problem is a special case of transportation problem

The optimum assignment schedule remains unaltered if we add or subtract a constant from all the elements of the row or column of the assignment cost matrix.

Hungarian method provides optimum assignment schedule in an assignment problem.

In this method only when the minimum number of vertical and/or horizontal lines that covers all the cells containing zero = max(no., of rows , no., of columns), we gets the optimum solution.

If not then **add** the minimum of the values in the cells that are not having any line on them, to the cells that are at the intersection of the vertical and horizontal lines, and subtract the same from the remaining cells that has no lines on them.

DECISION THEORY

Decision making may be defined as - " a process of best selection from a set of alternative courses of action, that course of action which is supposed to meet objectives upto satisfaction of the decision maker."

Acts (or courses of action) => **each course of action is referred as an act**

Events (or States of nature) => **The occurrences which are outside of the decision maker's control and which determine the level of success for an act is referred as events or states of nature or outcome.**

Pay-off => **The results of combinations of an act with each of the event, is the outcome and monetary gain or loss of each such outcome is the pay-off.**

Types of decision making

Decision making under certainty

CLASSIFICATION OF TEXT BOOK PROBLEMS

2 Marks

EXERCISE		EXAMPLE	Miscellaneous
Ex. No.,	Question Number	Question Number	Question Number
10.1	1, 2, 3, 4	--	--
10.2	1, 2, 3		

3 Marks

EXERCISE		EXAMPLE	Miscellaneous
Ex. No.,	Question Number	Question Number	Question Number
10.1	5, 6, 7, 10, 11	10.1, 10.2, 10.3, 10.4	1,7
10.2	4	10.9	
10.3	1,2, 3, 4	10.10, 10.11, 10.12	

5 Marks

EXERCISE		EXAMPLE	Miscellaneous
Ex. No.,	Question Number	Question Number	Question Number
10.1	8, 9, 12	10.5, 10.6	2 , 3, 4, 5, 6
10.2	5, 6, 7, 8	10.7, 10.8,	

: ANSWER TO THEORY QUESTIONS :

EXERCISE : 10.1

1. What is transportation problem?

   The transportation problem is to identify the quantity of homogeneous items **to be transported** from each **origin (source)** to each **destination** with the objective of **minimising the total transportation cost**.

   Example : Managing water supply from water distribution points to various places in a city, so as to minimise the transportation cost.

2. Write mathematical form of transportation problem.

   The objective function is Minimize Z = $\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$ Subject to the constraints

   $\sum_{j=1}^{n} x_{ij} = a_i$, i=1,2,.......m (supply constraints)

   $\sum_{i=1}^{m} x_{ij} = b_j$, j=1,2,.......$n$ (demand constraints)

   $x_{ij} \geq 0$ for all i,j. (non-negative restrictions)

3. What is feasible solution and non-degenerate solution in transportation problem?

   In a transportation problem, the set of non-negative values x_{ij} (i=1,2,3,..m, j=1,2,...n) that satisfies the constraints is called **feasible solution**.

   If it contains **not more than** m+n-1 allocations then it is called **basic feasible solution**.

   In a basic feasible solution if there are **exactly** m+n-1 allocations in independent positions, then it is called non-degerate basic feasible solution.

4. What do you mean by balanced transportation problem?

   In a transportation problem, if the total supply is **equal to** the total demand, it is said to be balanced transportation problem.

EXERCISE : 10.2

1. What is the Assignment problem?

For 'm' jobs to be performed on 'n' machines (one job per machine).
The assignment of different jobs to the different machines to **minimize** the **overall cost** is known as Assignment problem.

2. Give mathematical form of Assignment problem.

$$\text{Minimize } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} x_{ij}$$

Subject to the constrains

$$\sum_{i=1}^{n} x_{ij} = 1, \ j = 1, 2, \ldots..n$$

$$\sum_{j=1}^{n} x_{ij} = 1, \ i = 1, 2, \ldots..n \quad \text{and} \quad x_{ij} = 0 \ \text{(or)} \ 1 \ \text{for all } i, j$$

3. What is the difference between Assignment problem and Transportation problem?

With the objective of minimising the total cost,
the assignment problem is to assign the job to the machine whereas
the transportation problem is to allocate the quantity to be taken from the source to the destination.

In assignment problem the supply and demand are equal to 1, whereas in transportation problem it varies at each of the source as well as at each of the destination.

: CREATIVE QUESTIONS :

2 MARKS

1. What is the objective of a transpotation problem?
2. Define Basic feasible solution in a transportation problem.
3. Define optimal solution.
4. What is meant by degeneracy?
5. Define : Decision making
6. What is meant by (i) Act (ii) Event?
7. What are the two types of decision making?
8. Define : Pay off
9. Check whether the given transportation problem is a balanced

	W1	W2	W3	W4	W5	SUPPLY (units)
P1	2	8	14	35	51	50
P2	24	12	16	20	1	57
P3	14	32	1	23	26	55
Demand (Units)	24	46	25	40	30	

10. Check whether the given transportation problem is a balanced
There are 3 plants P1, P2, P3 each producing 5000, 2050 and 1500 units of a similar product. There are 4 warehouses w1, W2, W3, W4 each having demand of 3900, 2000, 1100, 1550 units respectively.

11. From the following Initial Basic Solution table, write the Transportation schedule and calculate the total transportation.

	A	B	C	a_i
L	**(5)** 3	7	6	**5**
M	**(2)** 3	**(6)** 3	1	**8**
N	5	**(3)** 4	**(4)** 7	**7**
O	1	6	**(14)** 2	**14**
b_j	**7**	**9**	**18**	

12. Write the steps used in Minimax ctiteria in taking the best action.

13. Write the steps used in Maximin ctiteria in taking the best action.

14. In a conditional pay offs for each action – event, the Minimum pay off are -10, -4, 0 for the Alternative X, Y, Z respectively, Using Maximin crtiteria which alternative is the best to choose.

15. From the following Initial Basic Solution table, find the allocations indicated as (x), (y) and (z).

	A	B	C	a_i
L	**(5)** 3	7	6	**5**
M	**(x)** 3	**(y)** 3	1	**9**
N	5	**(z)** 4	**(4)** 7	**8**
O	1	6	**(14)** 2	**14**
b_j	**z+2**	**12**	**18**	

3 MARKS

1. Compare the three methods of solving a transportation problem. Which method is considered to be the best?

2. From the following Initial Basic Solution table, find the allocations indicated as (x), (y) and (z).

	A	B	C	a_i
L	**(5)** 13	17	1 6	**5**
M	**(x)** 13	**(y)** 1 3	1 1	**9**
N	1 5	**(z)** 1 4	**(2y)** 1 7	**14**
O	11	1 6	**(14)** 12	**14**
b_j	**z+2**	**12**	**18**	

3. Check whether the following assigment problem is balanced, if not state the reason and make the problem as balanced.

		Men			
		1	2	3	4
Task	X	12	29	18	20
	Y	16	30	9	16
	Z	35	22	18	12

4. Three projects X, Y, Z are to be assigned to three teams L, M, N of a software company. The processing time (in days) for each project – team combination is shown in the matrix given below. Determine the allocation that minmizes the overall project completion time.

		Team		
		L	M	N
	X	12	20	26
Project	Y	5	20	11
	Z	7	9	6

5. For an assignment problem, following is the table obtained after subtracting the minimum value from the rows and also from the columns, is not giving the optimum solution.

		Team		
		L	M	N
	X	0	0	0
Project	Y	0	1	5
	Z	0	2	2

Make necessary steps to get the optimum solution, and give the optimum assignment schedule.

6. For the given pay-off matrix, choose the best alternative for the given states of nature under (i) Maximin (ii) Minimax princple

Alternative	States of Nature		
	Good	Fair	Bad
A	100	60	+50
B	80	50	+10
C	40	20	+5

7. Determine the Initial Basic Feasible solution for the given transportation problem using North-west corner rule

	W1	W2	W3	W4	W5	SUPPLY (units)
P1	5	18	14	35	31	50
P2	14	22	10	20	6	75
P3	24	15	11	13	26	175
Demand (Units)	95	85	45	50	25	

8. Determine the Initial Basic Feasible solution for the given transportation problem using Least cost method

	W1	W2	W3	W4	SUPPLY (units)
A1	5	12	14	15	30
A2	3	9	6	6	40
A3	8	5	11	1	50
Demand (Units)	25	35	25	35	

9. Find the initial basic feasible solution for the following transportation problem by Vogel's approximation method.

	L	M	N	Supply(a_i)
X	5	1	8	12

Y	2	4	0	14
Z	3	6	7	4
Demand(b_j)	9	10	11	

10. From the following Initial Basic Solution table, find the allocations indicated as (x), (y) and (z).

	A	B	C	a_i
L	(x) 3	17	(z) 8	**10**
M	13	**(8)** 5	(2) 11	**10**
N	(y) 3	**(4)** 8	17	**15**
O	11	(5) 16	**(10)** 12	**15**
b_j	**15**	**17**	**18**	

Also find the total transportation cost.

5 MARKS

1. There are 3 plants P1, P2, P3 each producing 50, 100 and 150 units of a similar product. There are 5 warehouses w1, W2, W3, W4, W5 each having demand of 110, 75, 50, 45, 40 units respectively. The cost of sending one unit from various plants to the warehouse differ as given by the cost matrix below. Find a suitable distribution pattern so that transportation cost is minimum. (Using VAM)

	W1	W2	W3	W4	W5	SUPPLY (units)
P1	2	8	14	35	51	50
P2	24	12	16	20	1	100
P3	14	32	1	23	26	150
Demand (Units)	105	75	45	40	35	

2. To meet out the water scarcity in a city, the water supply board set up three water supply points S1, S2, S3 with the capacity of 20, 30, 25 (thousands) litres of water (per day) for supply to 5 areas A1, A2, A3, A4, A5 each having demand for 15, 20, 10, 16, 14 (thousands) litres of water (per day) respectively. The cost of sending one thousand litres of water from water supply points differ as given by the cost matrix below. Find a suitable distribution pattern so that transportation cost is minimum.

	A1	A2	A3	A4	A5	SUPPLY (units)
S1	2	8	14	20	15	20
S2	8	12	16	22	5	30
S3	14	32	3	15	12	25

Demand (Units)	15	20	10	16	14	

3. The Owner of a small factory has 4 machinist available to assign to jobs for the day. 5 jobs are offered with expected profit in rupees for each machine on each job as follows,

	A	**B**	**C**	**D**	**E**
1	62	78	50	101	82
2	71	84	61	73	59
3	87	92	111	71	81
4	87	64	87	77	80
5	42	67	69	54	73

Determine the assignment of machinists to jobs that will result in a maximum profit.

Hint : Maximum Z = Minimum(-Z)

4. A car hire company has one car at each of five depots a,b,c,d, and e. A customer in each of the 5 towns A,B,C,D and E require a car. The distance (in miles) between the depots (origins) and the towns (destinations) where the customers are given in the following distance matrix.

	a	**b**	**c**	**d**	**e**
A	260	230	275	290	300
B	235	230	220	260	375
C	210	285	225	270	240
D	150	210	180	180	150
E	155	135	205	135	205

How should the cars be assigned to the customers so as to minimize the distance travelled?

5. Solve the following assignment problem, Cell values represent cost of assigning job A,B,C and D to the operators I, II, III and IV

		Men			
		I	II	III	IV
Task	A	5	3	2	8
	B	7	9	2	6
	C	8	4	5	7
	D	5	7	7	8

ANSWERS TO CREATIVE QUESTIONS

2 Marks :

1. What is the objective of a transpotation problem?
   The objective of a transportation problem is to determine the quantity to be transported from each orgin to each destination such that the total transportation cost is minimized.

2. Define Basic feasible solution in a transportation problem.
   In a transportation problem, If it contains not more than m+n-1 allocations then its is called basic feasible solution, where m is the number of origins(rows) and n is the number of destinations(columns).

3. Define optimal solution.
   In a transportation problem, optimal solution is a basic feasible solution which minimizes the total transportation cost.

4. What is meant by degeneracy?
   In a transportation problem, if the basic feasible solution contains less than m+n-1 allocations, then it is said to be degeracy, where m is the number of origins(rows) and n is the number of destinations(columns).

5. Define : Decision making
   A process of best selection from a set of alternative courses of action, that course of action which is supposes to meet objectives upto the satisfaction of the decision maker.

6. What is meant by (i) Act (ii) Event?
   In decision making each course of action is referred as an act.

   The occurrances which are outside of the decision maker's control and which determine the level of success for an act is referred as events or states of nature or outcome.

7. What are the two types of decision making?
   1. Decision making under certainty - Only one possible state of nature exists
   2. Decision making under uncertainty - Only pay-offs are known

8. Define : Pay off
   The results of combinations of an act with each of the event, is the outcome and monetary gain or loss of each such outcome is the pay-off.
   It should be in quantitative form.

9. Total supply = 162
   Total demand = 165
   since they are not equal, it is not a balanced transportation problem.

10. Total supply = 8550 = Total demand
since they are equal, it is a balanced transportation problem.

11. $L \to A,\ M \to A,\ M \to B, N \to B, N \to C,\ N \to C$, Total transportation cost = ₹ 107

12. Steps for Minimax criterian
(i) Determine the highest/maximum outcome for each alternative.
(ii) Choose the alternative associated with the minimum of these

13. Steps for Maximin criterian
(i) Determine the lowest/minimum outcome for each alternative.
(ii) Choose the alternative associated with the maximum of these

14. Alternative Z is to be choosen.

15. (x) = (1), (y) = (8) , (z) = (4)

3 Marks :

1. There are three methods of solving a transportation problem namely,
   i. North-west corner method
   ii. Least cost method
   iii. Vogel's approximation method

   In North-west corner rule, we allocate from top-left cell without considering the cost factor. So, this method can be used to get an initial basic feasible solution that need not be an optimal solution.
   Whereas in Least cost and Vogel's methods we give priority to the least cost so they give better initial basic feasible solution. In particular, since vogel's approximation method takes care of both the least and the second least cost values, it can give the best initial basic feasible solution very close to the optimal solution.

2. (x) = (7), (y) = (2) , (z) = (10)

3. The number of rows is 3 whereas the number of columns is 4 hence the problem is not a balanced problem, to make it balanced include a dummy row with all entries as 0.

4. $Y \to L,\ X \to M,\ Z \to N$ Minimum Time = 31 days

5.

		L	M	N
Project	X	0	0	0
	Y	0	1	5
	Z	0	2	2

Add 1 at the intersection, subtract 1 from the uncovered data.

Now the table becomes as below, which gives optimum solution.

		L	M	N
Project	X	1	0	0
	Y	0	0	4
	Z	0	1	1

X -> N, Y -> M, Z -> L

6. (i) A (ii) C

7. P1 -> W1, P2->W1, P2->W2, P3->W3, P3->W4,P3->W5
Minimum cost = 5x50 + 14x45 + 22x30 + 11x45 + 13x50 + 26x25 = ₹ 3335

8. P1->W2, P1-W3, P2->W1, P2->W3, P3->W2, P3->W4
Minimum cost = 12x20 + 14x10 + 3x25 + 6x15 + 5x15 + 1x35 = ₹ 655

9. X -> L, X->M, Y->L, Y->N, Z->L, Minimum cost = ₹ 38

10. (x) =(4), (y) = (11), (z) = (6)

<u>5 Marks :</u>

1. Using VAM ==> P1-W2=50,P2-W2=25, P2-W4=40, P2-W5=35, P3-W1=105, P3-W3=45
   Minimum cost = 3050
   m+n-1=7, Number of allocations = 6 < 7 therefore it is deneracy.

2. Using North-west corner ==> S1-A1=15, S1-A2=5,S2-A2=15,S2-A3=10,S2-A4=5,S3-A4=11, S3-A5=14 and Minimum cost = 853 x 1000 = 8,53,000 per day (or)
   Using Least cost ==> S1-A1 = 15, S1-A2=5, S2-A2=15, S2-A4=1, S2-A5=14, S3-A2=10, S3-A4=15 and Minimum Cost = 447x1000 = 4,47,000 per day (or)
   Using VAM ==> S1-A1=15,S1-A2=5,S2-A2=15,S2-A4=1,S3-A3=10,S3-A4=15 and Minimum cost = 527 x 1000 = 5,27,000

3. 1 → D, 2 → B, 3 → C, 4 → A,, 5 → E Maximum profit = ₹ 456

4. A → b, B → c, C → a, D → e , E → d Minimum distance = 945 miles

5. X -> L, X->M, Y->L, Y->N, Z->L,