SRI VIJAY VIDYALAYA MAT. HR. SEC. SCHOOL, KRISHNAGIRI.
III-Cyclic Test 2019-20
Maths

Class: XI
Date : 22.08.2019

Time : 1.30 Hours
Marks : 50

I. Choose the best answer (10x1=10)

1. What must be the matrix X, if $2x + \begin{bmatrix}1 & 2\\3 & 4\end{bmatrix} = \begin{bmatrix}3 & 8\\7 & 2\end{bmatrix}$?

(1) $\begin{bmatrix}1 & 3\\2 & -1\end{bmatrix}$ (2) $\begin{bmatrix}1 & -3\\2 & -1\end{bmatrix}$ (3) $\begin{bmatrix}2 & 6\\4 & -2\end{bmatrix}$ (4) $\begin{bmatrix}2 & -6\\4 & -2\end{bmatrix}$

2. If $A = \begin{bmatrix}a & x\\y & a\end{bmatrix}$ and if xy = 1, then det (AA^T) is equal to

(1) $(a-1)^2$ (2) $(a^2+1)^2$ (3) a^2-1 (4) $(a^2-1)^2$

3. If $a \neq b$, b, c satisfy $\begin{vmatrix}a & 2b & 2c\\3 & b & c\\4 & a & b\end{vmatrix} = 0$, then abc =

(1) a + b + c (2) 0 (3) b^3 (4) ab + bc.

4. If $\Delta = \begin{vmatrix}a_{11} & a_{12} & a_{13}\\a_{21} & a_{22} & a_{23}\\a_{31} & a_{32} & a_{33}\end{vmatrix}$ and A_{ij} is cofactor of a_{ij}, then value of Δ is given by

(1) $a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33}$ (2) $a_{11}A_{11} + a_{12}A_{21} + a_{13}A_{31}$

(3) $a_{21}A_{11} + a_{22}A_{12} + a_{23}A_{13}$ (4) $a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$

5. If $\vec{a} + \vec{2b}$ and $3\vec{a} + m\vec{b}$ are parallel, then the value of m is

(1) 3 (2) 1/3 (3) 6 (4) 1/6

6. If $\lambda\hat{\imath} + 2\lambda\hat{\jmath} + 2\lambda\hat{k}$ is a unit vector, then the value of λ is

(1) $\frac{1}{3}$ (2) $\frac{1}{4}$ (3) $\frac{1}{9}$ (4) $\frac{1}{2}$

7. If $|\vec{a} + \vec{b}| = 60$, $|\vec{a} - \vec{b}| = 40$ and $|\vec{b}| = 46$, then $|\vec{a}|$ is

(1) 42 (2) 12 (3) 22 (4) 32

8. If $\vec{a}$ and $\vec{b}$ are two vectors of magnitude 2 and inclined at an angle 60°, then the angle between $\vec{a}$ and $\vec{a} + \vec{b}$ is

(1) 30° (2) 60° (3) 45° (4) 90°

9. If $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$ and $\vec{b} = 2\hat{\imath} + x\hat{\jmath} + \hat{k}$, $\vec{c} = \hat{\imath} - \hat{\jmath} + 4\hat{k}$ and $\vec{a}.(\vec{b} \times \vec{c}) = 70$, then x is equal to

(1) 5 (2) 7 (3) 26 (4) 10

10. The value of λ when the vectors $\vec{a} = 2\hat{\imath} + \lambda\hat{\jmath} + \hat{k}$, and $\vec{b} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ are orthogonal is

(1) 0 (2) 1 (3) $\frac{3}{2}$ (4) $\frac{-5}{2}$

PART - B

II. Answer any 4 questions (Q.No.16 is compulsory) 4X2=8

11. If $A = \begin{bmatrix}1 & a\\0 & 1\end{bmatrix}$ then compute A^4.

12. Prove that $\begin{vmatrix}Sec^2\theta & tan^2\theta & 1\\tan^2\theta & sec^2\theta & -1\\38 & 36 & 2\end{vmatrix} = 0$

13. If G is the centroid of a triangle ABC, prove that $\overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC} = \vec{0}$

14. If $\frac{1}{2}, \frac{1}{\sqrt{2}}$, a are the direction cosines of some vector, then find a.

15. For any vector $\vec{r}$ prove that $\vec{r} = (\vec{r}.\hat{\imath})\hat{\imath} + (\vec{r}.\hat{\jmath})\hat{\jmath} + (\vec{r}.\hat{k})\hat{k}$

16. For any two vectors $\vec{a}$ and $\vec{b}$, prove that $|\vec{a} \times \vec{b}|^2 + (\vec{a}.\vec{b})^2 = |\vec{a}|^2 |\vec{b}|^2$

PART - C

III. Answer any 4 questions (Q.No.22 is compulsory) **4X3=12**

17. Solve for x if $[x\ 2\ -1]\begin{bmatrix}1 & 1 & 2\\ -1 & -4 & 1\\ -1 & -1 & -2\end{bmatrix}\begin{bmatrix}x\\ 2\\ 1\end{bmatrix} = 0$

18. Show that $\begin{vmatrix}o & c & b\\ c & 0 & a\\ b & a & 0\end{vmatrix}^2 = \begin{vmatrix}b^2+c^2 & ab & ac\\ ab & c^2+a^2 & bc\\ ac & bc & b^2+a^2\end{vmatrix}$

19. If D and E are the mid points of the sides AB and AC of a triangle ABC, Prove that $\overrightarrow{BE} + \overrightarrow{DC} = \frac{3}{2}\overrightarrow{BC}$

20. Show that the points whose position vectors are $2\hat{i}+3\hat{j}-5\hat{k}$, $3\hat{i}+\hat{j}-2\hat{k}$ and $6\hat{i}-5\hat{j}+7\hat{k}$ collinear

21. Show that the points (4, -3, 1), (2, -4, 5) and (1, -1, 0) from a right angled triangle.

22. Let $\vec{a}, \vec{b}, \vec{c}$ be unit vectors such that $\vec{a}.\vec{b} = \vec{a}.\vec{c} = 0$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$. prove that $\vec{a} = \pm\frac{2}{\sqrt{3}}(\vec{b} \times \vec{c})$.

PART - D

IV.ANSWER ALL THE FOLLOWING QUESTION:- **4X5=20**

23. a) Prove that $\begin{vmatrix}1 & x^2 & x^3\\ 1 & y^2 & y^3\\ 1 & z^2 & z^3\end{vmatrix}$ = (x-y) (y-z) (z-x) (xy +yz+zx) Using factor theorem.

(OR)

b) Express the matrix $A = \begin{bmatrix}1 & 3 & 5\\ -6 & 8 & 3\\ -4 & 6 & 5\end{bmatrix}$ as the sum of a symmetric and a skew - symmetric matrices.

24. a) Prove that $\begin{vmatrix}1+a & 1 & 1\\ 1 & 1+b & 1\\ 1 & 1 & 1+c\end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$

(OR)

b) Prove that medians of a triangle are concurrent by vector method.

25. a) Show that the points whose position vectors $4\hat{i}+5\hat{j}+\hat{k}$, $-\hat{j}-\hat{k}$, $3\hat{i}+9\hat{j}+4\hat{k}$ and $-4\hat{i}+4\hat{j}+4\hat{k}$ are coplanar

(OR)

b)If $\vec{a}$, $\vec{b}$ are unit vectors and θ is the angle between them, show that

i) $\sin\frac{\theta}{2} = \frac{1}{2}|\vec{a} - \vec{b}|$ ii) $\cos\frac{\theta}{2} = \frac{1}{2}|\vec{a} + \vec{b}|$ iii) $\tan\frac{\theta}{2} = \frac{|\vec{a}-\vec{b}|}{|\vec{a}+\vec{b}|}$

26. a) i) If $\vec{a}, \vec{b}, \vec{c}$ are position vectors of the vertices A, B, C of a triangle ABC, show that the area of the triangle ABC is $\frac{1}{2}|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$. Also deduce the condition for collinearity of the points A, B and C.

ii) Show that $\vec{a} \times (\vec{b} + \vec{c}) + \vec{b} \times (\vec{c} + \vec{a}) + \vec{c} \times (\vec{a} + \vec{b}) = \vec{0}$

(OR)

b) If ABCD is quadrilateral and E and F are the midpoints of AC and BD respectively, then prove that $\overrightarrow{AB} + \overrightarrow{AD} + \overrightarrow{CB} + \overrightarrow{CD} = 4\,\overrightarrow{EF}$

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Sri vijay vidyalaya mat. hr. Sec. School - Krishnagiri

XI III - cyclic Test

Sub: maths [Answer key]

I PART-A

1. (1) $\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$
2. (4) $(a^2-1)^2$
3. (3) b^3
4. (4) $a_{11}A_{11}+a_{21}A_{21}+a_{31}A_{31}$
5. (3) 6
6. (1) $1/3$
7. (3) 22
8. (1) $30°$
9. (3) 26
10. (4) $-5/2$

PART-B

11) Sol:

$A^2 = \begin{bmatrix} 1 & 2a \\ 0 & 1 \end{bmatrix}$

$A^4 = \begin{bmatrix} 1 & 4a \\ 0 & 1 \end{bmatrix}$

12) $L.H.S = \begin{vmatrix} 1 & \tan^2\theta & 1 \\ -1 & \sec^2\theta & -1 \\ 2 & 36 & 2 \end{vmatrix}$ $C_1 \to C_1 - C_3$

$= 0 = R.H.S$

13) $3\vec{OG} = \vec{OA}+\vec{OB}+\vec{OC}$

$\therefore L.H.S = \vec{OA}-\vec{OG}+\vec{OB}-\vec{OG}+\vec{OC}-\vec{OG}$

$= \vec{0}$

14) $(\frac{1}{2})^2+(\frac{1}{\sqrt{2}})^2+a^2=1$

$\frac{1}{4}+\frac{1}{2}+a^2=1$

$a^2=\frac{1}{4} \Rightarrow \boxed{a=\pm\frac{1}{2}}$

15) Let $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$

$\vec{r}.\hat{i}=x$, $\vec{r}.\hat{j}=y$, $\vec{r}.\hat{k}=z$

$\therefore R.H.S = L.H.S$

16) $L.H.S = |\vec{a}|^2|\vec{b}|^2(\sin^2\theta+\cos^2\theta)$

$=|\vec{a}|^2|\vec{b}|^2 = R.H.S.$

PART-C

17) $[x-2+1 \quad x-8+1 \quad 2x+2+2]\begin{bmatrix} x \\ 2 \\ 1 \end{bmatrix}=0$

$(x-1 \quad x-7 \quad 2x+4)\begin{bmatrix} x \\ 2 \\ 1 \end{bmatrix}=0$

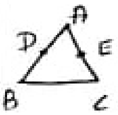
$x^2+3x-10=0$

$x=-5, 2.$

18) $L.H.S = \begin{vmatrix} 0 & c & b \\ c & 0 & b \\ b & a & 0 \end{vmatrix}\begin{vmatrix} 0 & c & b \\ c & 0 & a \\ b & a & 0 \end{vmatrix}$

$= \begin{vmatrix} c^2+b^2 & ab & ac \\ ab & c^2+a^2 & bc \\ ac & bc & b^2+a^2 \end{vmatrix}$

19) $\vec{BE}+\vec{DC} = \vec{OE}-\vec{OB}+\vec{OC}-\vec{OD}$



$= \frac{\vec{OA}+\vec{OC}}{2}-\vec{OB}+\vec{OC}-(\frac{\vec{OB}+\vec{OA}}{2})$

$= 3\vec{OC}-3\vec{OB}$

$= 3\vec{BC}$

20) $\vec{OA} = 2\hat{i}+3\hat{j}-5\hat{k}$

$\vec{OB} = 3\hat{i}+\hat{j}-2\hat{k}$, $\vec{OC}=6\hat{i}-5\hat{j}+7\hat{k}$

$\vec{AB} = \hat{i}-2\hat{j}+3\hat{k}$

$\vec{AC} = 4\hat{i}-8\hat{j}+12\hat{k})$

$\vec{AC} = 4\vec{AB} \Rightarrow \vec{AC} \parallel \vec{AB}$ (B is common)

$\therefore$ Three pts are collinear.

21) Let $\vec{OA} = 4\hat{i}+3\hat{j}+\hat{k}$

$\vec{OB} = 2\hat{i}-4\hat{j}+5\hat{k}$, $\vec{OC}=\hat{i}-\hat{j}$

$\vec{AB} = -2\hat{i}-\hat{j}+4\hat{k}$

$\vec{BC} = -\hat{i}+3\hat{j}-5\hat{k}$, $\vec{CA}=3\hat{i}-2\hat{j}+\hat{k}$

$\therefore \vec{AB}+\vec{BC}+\vec{CA}=\vec{0}$ form a Δ.

$\therefore \vec{AB}.\vec{CA}=0 \Rightarrow \vec{AB}\perp\vec{CA}$

$\therefore$ Hence they form a right angled Δ

22) $\vec{a}.\vec{b}=\vec{a}.\vec{c}=0$

$\vec{a}\perp\vec{b}$ and $\vec{a}\perp\vec{c}$

$\vec{a} \parallel (\vec{b}\times\vec{c})$

$\Rightarrow \vec{a}=\pm\lambda(\vec{b}\times\vec{c})$

$|\vec{a}| = \lambda|\vec{b}\times\vec{c}|$

$1 = \lambda|\vec{b}||\vec{c}|\sin\frac{\pi}{3}$

$1 = \lambda(\frac{\sqrt{3}}{2}) \Rightarrow \lambda = \frac{2}{\sqrt{3}}$

$\therefore \vec{a} = \pm\frac{2}{\sqrt{3}}(\vec{b}\times\vec{c}).$

PART - D

23) a) $\Delta = \begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix}$

Put $x = y \Rightarrow \Delta = 0$

$\therefore (x-y)$ is a factor of Δ

|||ly $(y-z)$ and $(z-x)$ also a factor of Δ.

$m = 5 - 3 = 2$

other factor is $k(x^2+y^2+z^2) + l(xy+yz+zx)$

Put $x=0, y=1, z=2$

$5k + 2l = 2$

Put $x=0, y=-1, z=1$

$2k - l = -1$

$\therefore k = 0, \ l = 1$

$\therefore \Delta = (x-y)(y-z)(z-x)(xy+yz+zx)$

(OR)

b) $A = \begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix}, \ A^T = \begin{bmatrix} 1 & -6 & -4 \\ 3 & 8 & 6 \\ 5 & 3 & 5 \end{bmatrix}$

Let $P = \frac{1}{2}(A+A^T) = \frac{1}{2}\begin{bmatrix} 2 & -3 & 1 \\ -3 & 16 & 9 \\ 1 & 9 & 10 \end{bmatrix}$

$P^T = P \Rightarrow P$ is symmetric.

Let $Q = \frac{1}{2}(A-A^T) = \frac{1}{2}\begin{bmatrix} 0 & 9 & 9 \\ -9 & 0 & -3 \\ -9 & 3 & 0 \end{bmatrix}$

$\Rightarrow Q^T = -Q \Rightarrow Q$ is skew symmetric

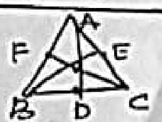
$\therefore A = P + Q$

24) Soln:

a) $L.H.S = \begin{vmatrix} a & 0 & 1 \\ -b & b & 1 \\ 0 & -c & 1+c \end{vmatrix}$

$= abc + bc + ac + ab$

$= abc\left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$

b) 

$\overrightarrow{OG_1} = \overrightarrow{OG_2} = \overrightarrow{OG_3}$

$= \dfrac{\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC}}{3}$

$\therefore$ Hence G_1, G_2, G_3 are same point.

Hence the medians of Δ are concurrent.

25) a) $\vec{a} = \overrightarrow{AB} = -4\hat{i} - 6\hat{j} - 2\hat{k}$

$\vec{b} = \overrightarrow{AC} = -4\hat{i} + 4\hat{j} + 3\hat{k}$

$\vec{c} = \overrightarrow{AD} = -8\hat{i} - \hat{j} + 3\hat{k}$

$\therefore \vec{a} = s\vec{b} + t\vec{c}$

$\therefore t = 2/3$ and $s = -4/3$

which are satisfies third eqn.

$\therefore$ Hence points are coplanar.

b) $|\vec{a} - \vec{b}|^2 = 2 - 2\cos\theta = 2(1-\cos\theta)$

$= 4\sin^2 \theta/2$

$|\vec{a} - \vec{b}| = 2\sin\theta/2 \Rightarrow \sin\frac{\theta}{2} = \frac{1}{2}|\vec{a} - \vec{b}|$

$|\vec{a} + \vec{b}|^2 = 2 + 2\cos\theta = 2(1+\cos\theta)$

$|\vec{a} + \vec{b}|^2 = 4\cos^2\theta/2 \Rightarrow \cos\frac{\theta}{2} = \frac{1}{2}|\vec{a} + \vec{b}|$

$\therefore \tan\frac{\theta}{2} = \dfrac{\sin\theta/2}{\cos\theta/2} = \dfrac{|\vec{a} - \vec{b}|}{|\vec{a} + \vec{b}|}$

26) a) (i) Area of $\Delta = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$

$= \frac{1}{2}|(\vec{b} - \vec{a}) \times (\vec{c} - \vec{a})|$

$= \frac{1}{2}|(\vec{a} \times \vec{b}) + (\vec{b} \times \vec{c}) + (\vec{c} \times \vec{a})|$

$\therefore A, B, C$ are collinear

then area of $\Delta = 0$

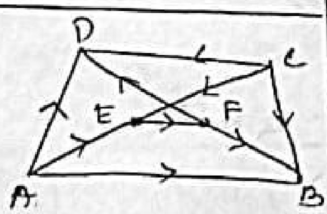
$\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = \vec{0}$

(ii) $L.H.S = (\vec{a} \times \vec{b}) + (\vec{a} \times \vec{c}) + (\vec{b} \times \vec{c}) + (\vec{b} \times \vec{a}) + (\vec{c} \times \vec{a}) + (\vec{c} \times \vec{b})$

$= (\vec{a} \times \vec{b}) + (\vec{a} \times \vec{c}) + (\vec{b} \times \vec{c}) - (\vec{a} \times \vec{b}) - (\vec{a} \times \vec{c}) - (\vec{b} \times \vec{c})$

$= \vec{0}$

[OR]

b) 

$\overrightarrow{AB} = \overrightarrow{AE} + \overrightarrow{EF} + \overrightarrow{FB}$

$\overrightarrow{AD} = \overrightarrow{AE} + \overrightarrow{EF} + \overrightarrow{FD}$

$\overrightarrow{CB} = \overrightarrow{CE} + \overrightarrow{EF} + \overrightarrow{FB}$

$\overrightarrow{CD} = \overrightarrow{CE} + \overrightarrow{EF} + \overrightarrow{FD}$

$\therefore \overrightarrow{AB} + \overrightarrow{AD} + \overrightarrow{CB} + \overrightarrow{CD} = 4\overrightarrow{EF}$