Std : XI | *Mathematics* | Marks : 90

Date : 09.08.2019 | Reg. No: | | | | | | | | Time : 2.30 hrs.

I. Choose the best answers: **(20 x 1 = 20)**

1. The function $f: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = \sin x + \cos x$ is
   1) an odd function
   2) neither an odd function nor an even function
   3) an even function
   4) both odd function and even function
2. The range of the function $f(x) = |\lfloor x \rfloor - x|, x \in \mathbb{R}$ is
   1) [0, 1] 2) [0, ∞) 3) [0, 1) 4) (0, 1)
3. If $n(A) = 2$ and $n(B \cup C) = 3$, then $n[(A \times B) \cup (A \times C)]$ is
   1) 2^3 2) 3^2 3) 6 4) 5
4. The relation R defined on a set $A = \{0, -1, 1, 2\}$ by xRy if $|x^2 + y^2| \leq 2$, then which one of the following is true?
   1) $R = \{(0, 0), (0, -1), (0, 1), (-1, 0), (-1, 1), (1, 2), (1, 0)\}$
   2) $R^{-1} = \{(0, 0), (0, -1), (0, 1), (-1, 0), (1, 0)\}$
   3) Domain of R is $\{0, -1, 1, 2\}$
   4) Range of R is $\{0, -1, 1\}$
5. Domain and range of the equation $x - y^2 = 0$ is
   1) R,R 2) [0, ∞), R 3) R, (-∞, 0] 4) [0,∞), [0,∞)
6. If a relation contains a single element, then the relation is
   1) symmetric
   2) symmetric and transitive
   3) transitive
   4) reflexive
7. Let R be the universal relation on a set X with more than one element. Then R is
   1) not reflexive 2) not symmetric 3) transitive 4) none of the above
8. $[-2.9] =$ _____
   1) -2 2) -3 3) $2\frac{1}{9}$ 4) -2.5
9. The solution of $5x - 1 < 24$ and $5x + 1 > -24$ is
   1) (4, 5) 2) (–5, –4) 3) (–5, 5) 4) (–5, 4)
10. The solution set of the following inequality $|x - 1| \geq |x - 3|$ is
    1) [0, 2] 2) [2, ∞) 3) (0, 2) 4) (–∞, 2)
11. $|x - a| > r$ implies $x < a - r$ and $x > a + r$ (or)
    1) $x \in (-\infty, a - r) \cup (a - r, \infty)$
    2) $x \in (-\infty, a + r) \cup (a + r, \infty)$
    3) $x \in (-\infty, a - r) \cup (a + r, \infty)$
    4) $x \in (-\infty, r + a) \cup (r - a, \infty)$
12. The equation whose roots are numerically equal but opposite in sign to the roots of $3x^2 - 5x - 7 = 0$ is
    1) $3x^2 - 5x - 7 = 0$
    2) $3x^2 + 5x - 7 = 0$
    3) $3x^2 - 5x + 7 = 0$
    4) $3x^2 + x - 7 = 0$
13. If a and b are the real roots of equation $x^2 - kx + c = 0$, then the distance between the points $(a, 0)$ and $(b, 0)$ is
    1) $\sqrt{k^2 - 4c}$ 2) $\sqrt{4k^2 - c}$ 3) $\sqrt{4c - k^2}$ 4) $\sqrt{k - 8c}$
14. The number of roots of $(x + 3)^4 + (x + 5)^4 = 16$ is
    1) 4 2) 2 3) 3 4) 0
15. $7^\circ\, 30^1 =$ _____
    1) $\frac{\pi}{36}$ 2) $\frac{\pi}{12}$ 3) $\frac{\pi}{24}$ 4) $\frac{\pi}{13}$
16. $\cos 1^\circ + \cos 2^\circ + \cos 3^\circ + \ldots + \cos 179^\circ =$
    1) 0 2) 1 3) –1 4) 89
17. If $\cos 28^\circ + \sin 28^\circ = k^3$, then $\cos 17^\circ$ is equal to
    1) $\frac{k^3}{\sqrt{2}}$ 2) $-\frac{k^3}{\sqrt{2}}$ 3) $\pm\frac{k^3}{\sqrt{2}}$ 4) $-\frac{k^3}{\sqrt{3}}$
18. -3 radian
    1) -171° 41′ 2) -170° 26′ 3) -166° 25′ 4) -137° 67′
19. $\cos\theta = 0$ then θ *is*
    1) $(2n - 1)\frac{\pi}{2}, n \in z$
    2) $2n\pi, n \in z$
    3) $(2n + 1)\pi, n \in z$
    4) $(2n + 1)\frac{\pi}{2}, n \in z$
20. $\left(1 + \cos\frac{\pi}{8}\right)\left(1 + \cos\frac{3\pi}{8}\right)\left(1 + \cos\frac{5\pi}{8}\right)\left(1 + \cos\frac{7\pi}{8}\right) =$
    1) $\frac{1}{8}$ 2) $\frac{1}{2}$ 3) $\frac{1}{\sqrt{3}}$ 4) $\frac{1}{\sqrt{2}}$

II. Answer the following any 7 questions: (Q.No.30 is compulsory) **(7 x 2 = 14)**

21. If $n(p(A)) = 1024$, $n = (A \cup B) = 15$ and $n = (p(B)) = 32$, then find $n(A \cap B)$.
22. Let f and g be the two functions from $\mathbb{R}$ to $\mathbb{R}$ defined by $f(x) = 3x - 4$ and $g(x) = x^2 + 3$. Find $g \circ f$ and $f \circ g$.

23. The weight of the muscles of a man is a function of his body weight x and can be expressed as $w(x) = 0.35x$. Determine the domain of this function.
24. Solve $\frac{1}{|2x-1|} < 6$ and express the solution using the interval notation.
25. Find the values of p for which the difference between the roots of the equation $x^2 + px + 8 = 0$ *is 2.*
26. Compute $\log_3 5 \log_{25} 27$.
27. Prove that $\frac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1} = \frac{1+\sin\theta}{\cos\theta}$.
28. In a circle of diameter 40 *cm*, a chord is of length 20*cm*. Find the length of the minor arc of the chord.
29. Prove that $\cos(\frac{3\pi}{4}+x) - \cos(\frac{3\pi}{4}-x) = -\sqrt{2}\sin x$
30. Draw the rough diagram $y = e^x$ and $y = \log_e x$.

III. Answer the following any 7 questions: (Q.No.40 is compulsory) **(7 x 3 = 21)**

31. In a survey of 5000 persons in a town, it was found that 45% of the persons know language *A*, 25% know language *B*,10% know language *C*,5% know languages *A* and *B*, 4% know languages*B* and *C*, and 4% know languages *A* and *C*. If 3% of the persons know all the three languages, find the number of persons who knows only language *A*.
32. In the set *Z* of integers, define *mRn* if $m - n$ is divisible by 7. Prove that *R* is equivalence relation.
33. From the curve $y = \sin x$, draw $y = \sin|x|$.(Hint: $\sin(-x) = -\sin x$)
34. Our monthly electricity bill contains a basic charge, which does not change with number of units used, and a charge that depends only on how many units we use. Let us say electricity board charges Rs.110 as basic charge and charges Rs.4 for each unit we use. If a person wants to keep his electricity bill below Rs.250, then what should be his electricity usage?
35. Find the number of solutions of $x^2 + |x - 1| = 1$.
36. If $x^2 + x + 1$ is a factor of the polynomial $3x^3 + 8x^2 + 8x + a$, then find the value of a.
37. Solve $\frac{x^2-4}{x^2-2x-15} \leq 0$.
38. Eliminate θ from the equations $a\sec\theta - c\tan\theta = b$ and $b\sec\theta + d\tan\theta = c$.
39. A train is moving on a circular track of 1500 *m* radius at the rate of 66*km/ hr*. What angle will it turn in 20 seconds?
40. Find the value of $\frac{\sin 300°.\tan 330°.\sec 420°}{\cot 135°.\cos 210°.\mathit{cosec}\ 315°}$.

IV. Answer all the questions: **(7 x 5 = 35)**

41.a) Let $X = \{a,b,c,d\}$ and $R = \{(a,a)\ (b,b),\ (a,c)\}$. Write down the minimum number of ordered pairs to be included to *R* to make it.
(i) reflexive (ii) symmetric (iii) transitive (iv) equivalence

(OR)

b) On the set of natural numbers let *R* be the relation defined by *aRb* if $a + b \leq 6$. Write down the relation by listing all the pairs. Check whether it is
(i) reflexive (ii) symmetric (iii) transitive (iv) equivalence

42.a) i) Find the domain of $f(x) = \frac{1}{1-2\cos x}$. (ii) Find the range of the function $f(x) = \frac{1}{1-3\cos x}$.(2+3)

(OR)

b) Find the largest possible domain of the real valued function $f(x) = \frac{\sqrt{4-x^2}}{\sqrt{x^2-9}}$.

43.a) From the curve y = x, draw
(i) $y = -x$ (ii) $y = 2x$ (iii) $y = x + 1$ (iv) $y = \frac{1}{2}x + 1$ (v) $2x + y + 3 = 0$.

(OR)

b) A manufacture has 600 litres of a 12 percent solution of acid. How many litres of a 30 percent acid solution must be added to it so that the acid content in the resulting mixture will be more than 15 percent but less than 18 percent?

44.a) Without sketching the graphs, find whether the graphs of the following functions will intersect the x –axis and if so in how many points.
(i) $y = x^2 + x + 2$, (ii) $y = x^2 - 3x - 7$, (iii) $y = x^2 + 6x + 9$

(OR)

b) Use the method of undetermined coefficients to find the sum of $1 + 2 + 3 + \ldots\ldots + (n-1) + n,\ n \in \mathbb{N}$

45.a) Resolve into partial fractions. $\frac{2x^2+5x-11}{x^2+2x-3}$ **(OR)**

b) Solve the linear inequalities and exhibit the solution set graphically:
$x + y \geq 3, 2x - y \leq 5, -x + 2y \leq 3.$

46.a) If $\tan^2\theta = 1 - k^2$, show that $\sec\theta + \tan^3\theta\ \mathrm{cosec}\ \theta = (2 - k^2)^{3/2}$. Also, find the values of k for which this result holds.

(OR)

b) If $\sin x = \frac{4}{5}$ (in I quadrant) and $\cos y = \frac{-12}{13}$ (in II quadrant) , then find (i) $\sin(x-y)$, (ii) $\cos(x-y)$.

47.a) If $x\cos\theta = y\cos(\theta + \frac{2\pi}{3}) = z\cos(\theta + \frac{4\pi}{3})$, find the value of $xy + yz + zx$.

(OR)

b) Prove that : $\{1 + \cot\alpha - \sec(\alpha + \frac{\pi}{2})\}\{1 + \cot\alpha + \sec(\alpha + \frac{\pi}{2})\} = 2\cot\alpha$