1. NATURE OF PHYSICAL WORLD AND MESUREMENT.

3 AND 5 MARKS

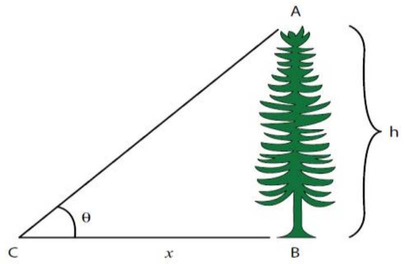
1. Write the advantages of SI system.
   - This system makes use of only one unit for one physical quantity, which means a rational system of units.
   - In this system, all the derived units can be easily obtained from basic and supplementary units, which means it is a coherent system of units.
   - It is a metric system which means that multiples and submultiples can be expressed as powers of 10.

2. Explain the use of screw gauge and vernier caliper in measuring smaller distance.

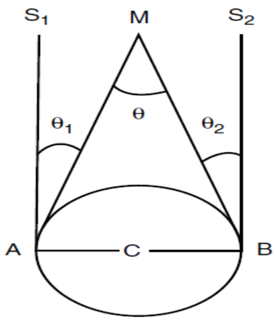
   Screw gauge:
   - The screw gauge is an instrument used for measuring accurately the dimensions of objects up to a maximum of about 50 mm.
   - The principle of the instrument is the magnification of linear motion using the circular motion of a screw.
   - The least count of screw gauge is 0.01 mm.

   Vernier caliper:
   - A vernier caliper is a versatile instrument for measuring the dimensions of an object namely diameter of a hole, or a depth of a hole.
   - The least count the vernier caliper is 0.1 mm.

3. How will you measure the height of an accessible object by triangulation method?
   - Let AB = h be the height of the tree or tower to be measured.
   - Let C be the point of observation at distance x from B.
   - $\angle ACB = \theta$
   - $\tan\theta = \frac{h}{x}$
   - $h = x\tan\theta$



4. Determine the distance of the Moon from the Earth by parallax method.
   - C is the centre of the Earth.
   - A and B are two diametrically opposite places on the surface of the Earth.
   - From A and B, the parallaxes θ_1 and θ_2 respectivily of Moon M with respect to some distant star are determined with the help of and astronomical telescope.
   - The total parallax of the Moon subtended on Earth
   - $\angle AMB = \theta_1 + \theta_2 = \theta$
   - $\theta = \frac{AB}{MC}$
   - $MC = \frac{AB}{\theta}$
   - Knowing the values of AB and θ we can calculate the distance MC of the Moon from the Earth.



5. Write a note on RADAR method to measure a larger distance.
   - The word RADAR stands for radio detection and ranging.
   - A radar can be used to measure accurately the distance of a nearby planet such as Mars.
   - In this method, radio waves are sent from transmitters which, after reflection from the planet, are detected by the receiver.
   - By measuring, the time interval (t) between the instants the radio waves are sent and received, the distance of the planet can be determined.
   - $Speed = \frac{distance\ \ travelled}{time\ \ taken}$
   - $d = \frac{vt}{2}$
   - v is the speed of the radio wave.
   - t is the total time taken by the radio wave.
   - This method can also be used to determine the height, at which an aeroplane flies from the ground.

6. What are gross errors? How they can be minimised?

The error caused due to the shear carelessness of an observer is called gross error.

- Reading an instruement without setting it properly.
- Taking observations in a wrong manner without bothering about the sources of errors and the precaustions.
- Recording wrong observations.
- Using wrong values of the observations in calculations.
- These errors can be minimized only when an observer is careful and mentally alert.

7. Explain the propagation of errors in the sum of two quantities.

- $A = A \pm \Delta A$
- $B = B \pm \Delta B$
- $Z = A + B$
- $Z = Z \pm \Delta Z$
- $Z \pm \Delta Z = (A \pm \Delta A) + (B \pm \Delta B)$
- $= (A + B) \pm (\Delta A + \Delta B)$
- $= Z \pm (\Delta A + \Delta B)$
- $\Delta Z = \Delta A + \Delta B$

8. Explain the propagation of errors in the difference of two quantities.

- $A = A \pm \Delta A$
- $B = B \pm \Delta B$
- $Z = A - B$
- $Z = Z \pm \Delta Z$
- $Z \pm \Delta Z = (A \pm \Delta A) - (B \pm \Delta B)$
- $= (A - B) \pm (\Delta A + \Delta B)$
- $= Z \pm (\Delta A + \Delta B)$
- $\Delta Z = \Delta A + \Delta B$

9. Explain the propagation of errors in the product of two quantities.

- $A = A \pm \Delta A$
- $B = B \pm \Delta B$
- $Z = AB$
- $Z = Z \pm \Delta Z$
- $Z \pm \Delta Z = (A \pm \Delta A)(B \pm \Delta B)$
- $= (AB) \pm (A\Delta B) \pm (B\Delta A) \pm (\Delta A.\Delta B)$
- Divide by Z on left side and AB on right side
- $1 \pm \frac{\Delta Z}{Z} = 1 \pm \frac{\Delta A}{A} \pm \frac{\Delta B}{B} \pm (\frac{\Delta A}{A}.\frac{\Delta B}{B})$
- $\frac{\Delta Z}{Z} = \left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right)$

10. Explain the propagation of errors in the division of two quantities.

- $A = A \pm \Delta A$
- $B = B \pm \Delta B$
- $Z = \frac{A}{B}$
- $Z = Z \pm \Delta Z$
- $Z \pm \Delta Z = \frac{A \pm \Delta A}{B \pm \Delta B} = \frac{A\left(1 \pm \frac{\Delta A}{A}\right)}{B\left(1 \pm \frac{\Delta B}{B}\right)}$
- $Z \pm \Delta Z = Z\left(1 \pm \frac{\Delta A}{A}\right)\left(1 \mp \frac{\Delta B}{B}\right)$
- $1 \pm \frac{\Delta Z}{Z} = 1 \pm \frac{\Delta A}{A} \mp \frac{\Delta B}{B} \mp (\frac{\Delta A}{A}.\frac{\Delta B}{B})$
- $\frac{\Delta Z}{Z} = \left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right)$

11. Explain the propagation of errors in the power of a quantity.

- $A = A \pm \Delta A$
- $Z = A^n$
- $Z = Z \pm \Delta Z$
- $Z \pm \Delta Z = (A \pm \Delta A)^n = A^n \left(1 \pm \frac{\Delta A}{A}\right)^n$
- $Z \pm \Delta Z = Z\left(1 \pm n\frac{\Delta A}{A}\right)$
- $1 \pm \frac{\Delta Z}{Z} = 1 \pm n\frac{\Delta A}{A}$
- $\frac{\Delta Z}{Z} = n\frac{\Delta A}{A}$

12. Write the rules for finding significant figures.

Rule	Example
All non-zero digits are significant	1342 has four significant figure
All zeros between two non zero digits are significant	2008 has four significant figures
All zeros to the right of a non-zero digit but to the left o a decimel point are significant.	30700. has five significant figures
The number without a decimal point, the terminal or trailin zeros are not significant.	30700 has three significant figures
All zeros are significant if they come from a measurement	30700 m has five significant figures
If the number is less than 1, the zeros on the right of the decimal point by to left of ther first non-zero digit are not significant.	0.00345 has three significant figures
All zeros to the right of a decimal point and to the right of non-zero digit are significant.	40.00 has four significant figures
The number of significant figures does not depend on the system of units used.	1.53 cm, 0.0153 m, 0.0000153 km all have three significant figures.

13. Write the rules for roundi off.

Rule	Example
If the digit to be dropped is smaller than 5, then the preceding digit should be left unchanged.	7.32 is rounded off to 7.3
If the digit to be dropped is greater thatn 5, then the preceding digt shoyld be increased by 1	17.26 is rounded off to 17.3
If the digit to be dropped is 5 followed by digits other than zero, then the preceding digit should be raised by 1	7.352 is rounded to 7.4
If the digit to be dropped is 5 or 5 followed by zeros, then the preceding digit is not changed if it is even	3.45 is rounded ot 3.4
If the digit to be dropped is 5 or 5 followd by zeros, then digit is raised by 1 if it is odd.	3.35 is rounded to 3.4

14. What are the uses of dimensional analysis?
- This method is used to convert a physical quantity from one system of units to another.
- To check the dimensional correctness of a given physical equation.
- To establish relations among various physical quantities.

15. What are the limitations of dimensional analysis?
- This method give no information about the dimensionless constants in the formula like 1, 2 π, e, etc.
- This method cannot decide whether the given quantity is a vector or a scalar.
- This method is not suitable to derive relations involving trigonometric, exponential and logarithmic functions.
- It cannot be applied to an equation involving more than three physical quantities.
- It can only check on whether a physical relation is dimensionally correct but not the correctness of the relation.

2. KINEMATICS

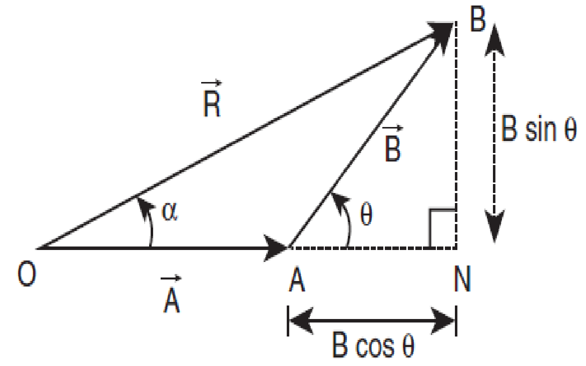
1. Explain in detain the triangle law of vector addition.

Triangle law of vector addition:

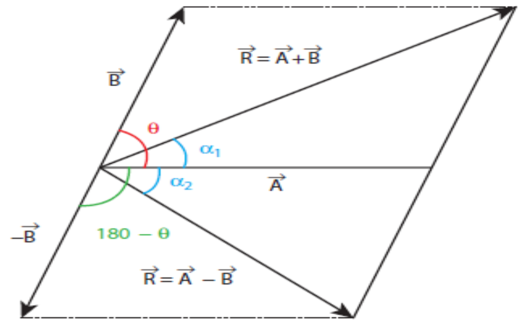
Represent the vectors $\vec{A}$ and $\vec{B}$ by the two adjacent sides of a triangle taken in the same order, then the resultant is given by the third side of the triangle.

Magnitude of resultant vector:
- $AN = B\cos\theta$
- $BN = B\sin\theta$
- **For** $\Delta\, OBN$, $OB^2 = ON^2 + BN^2$
- $R^2 = (A + B\cos\theta)^2 + (B\sin\theta)^2$
- $R^2 = A^2 + B^2\cos^2\theta + 2AB\cos\theta + B^2\sin^2\theta$
- $R^2 = A^2 + B^2(\cos^2\theta + \sin^2\theta) + 2AB\cos\theta$
- $R^2 = A^2 + B^2 + 2AB\cos\theta$
- $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

Direction of resultant vector:
- $\tan\alpha = \frac{B\sin\theta}{A+B\cos\theta}$
- $\alpha = \tan^{-1}\left(\frac{B\sin\theta}{A+B\cos\theta}\right)$



2. Explain the subtracton of vectors.
- $R = \sqrt{A^2 + B^2 + 2AB\cos(180° - \theta)}$
- $\cos(180° - \theta) = -\cos\theta$
- $R = \sqrt{A^2 + B^2 - 2AB\cos\theta}$
- $\tan\alpha_2 = \frac{B\sin(180° - \theta)}{A + B\cos(180° - \theta)}$
- $\sin(180° - \theta) = \sin\theta$
- $\tan\alpha_2 = \frac{B\sin\theta}{A - B\cos\theta}$
- $\alpha_2 = \tan^{-1}\left(\frac{B\sin\theta}{A - B\cos\theta}\right)$



3. Write the properties of scalar product or dot product.
   - The product quantity is always a scalar.
   - $\vec{A}.\vec{B}$ is positive if $\theta = 90°$
   - $\vec{A}.\vec{B}$ is positive if $90° < \theta < 180°$
   - $\vec{A}.\vec{B} = \vec{B}.\vec{A}$ (Commutative law)
   - $\vec{A}.(\vec{B}+\vec{C}) = \vec{A}.\vec{B} + \vec{A}.\vec{C}$ (Distributive law)
   - $\theta = \cos^{-1}\left[\frac{\vec{A}.\vec{B}}{A.B}\right]$
   - If $\theta = 0°\ then\ (\vec{A}.\vec{B})_{max} = AB$
   - If $\theta = 180°\ then\ (\vec{A}.\vec{B})_{min} = -AB$
   - If $\vec{A}.\vec{B} = 0$ then $\vec{A} \perp \vec{B}$
   - $(\vec{A})^2 = \vec{A}.\vec{A} = AA\cos\theta = A^2$
   - $\hat{n}.\hat{n} = 1$
   - $\hat{\imath}.\hat{\imath} = \hat{\jmath}.\hat{\jmath} = \hat{k}.\hat{k} = 1$
   - $\hat{\imath}.\hat{\jmath} = \hat{\jmath}.\hat{k} = \hat{k}.\hat{\imath} = 0$
   - $\vec{A}.\vec{B} = A_xB_x + A_yB_y + A_zB_z$
   - $A = \sqrt{A_x^2 + A_y^2 + A_z^2}$

4. Write the properties of vector product ot cross product.
   - The vector product of any two vectors is always another vector.
   - Its direction is perpendicular to the plane containing these two vectors.
   - $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$ but $\vec{A} \times \vec{B} = -[\vec{B} \times \vec{A}]$
   - If $\theta = 90°\ then\ (\vec{A} \times \vec{B})_{max} = AB\ \hat{n}$
   - If $\theta = 0°\ and\ \theta = 18\,0° then\ (\vec{A} \times \vec{B})_{min} = 0$
   - $\vec{A} \times \vec{A} = AA\sin\theta\,\hat{n} = \vec{0}$
   - $\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = \vec{0}$
   - $\hat{\imath} \times \hat{\jmath} = \hat{k},\ \hat{\jmath} \times \hat{k} = \hat{\imath},\ \hat{k} \times \hat{\imath} = \hat{\jmath}$
   - $\hat{\jmath} \times \hat{\imath} = -\hat{k},\ \hat{k} \times \hat{\jmath} = -\hat{\imath},\ \hat{\imath} \times \hat{k} = -\hat{\jmath}$
   - $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$
   - $|\vec{A} \times \vec{B}|$ gives the area of a parallelogram.
   - $\frac{1}{2}|\vec{A} \times \vec{B}|$ gives the area of a triangle.

5. Derive the kinematic equations of motion for constant acceleration.

velocity - time	displacement - time	velocity - acceleration
$a = \frac{dv}{dt}$	$v = \frac{ds}{dt}$	$a = \frac{dv}{ds}v$
$dv = a\,dt$	$ds = (u + at)dt$	$ds = \frac{1}{2a}d(v^2)$
$\int_u^v dv = \int_0^t adt$	$\int_0^s ds = u\int_0^t dt + a\int_0^t dt$	$\int_0^s ds = \frac{1}{2a}\int_u^v d(v^2)$
$v = u + at$	$s = ut + \frac{1}{2}at^2$	$v^2 = u^2 + 2as$
	$s = \frac{(u+v)t}{2}$	

6. Derive the kinematic equations of motion for a particle falling vertically.
   - Consider an object of mass m falling from a height h.
   - Let us choose the downward direction as positive y-axis.
   - $\vec{a} = g\hat{\jmath}\ ;\ a_y = g$

If the initia velocity of the object is u	If the particle starts from rest $(u = 0)$
$v = u + gt$	$v = gt$
$y = ut + \frac{1}{2}gt^2$	$y = \frac{1}{2}gt^2$
$v^2 = u^2 + 2gy$	$v^2 = 2gy$

7. Derive the kinematic equations of motion for a particle projected vertically.
   - Consider an object of mass m thrown vertically upwards with an initial velocity $\vec{u}$.
   - Vertical upward direction is taken as positive y axis.
   - $a = -g$
   - $v = u - gt$
   - $y = ut - \frac{1}{2}gt^2$
   - $v^2 = u^2 - 2gy$

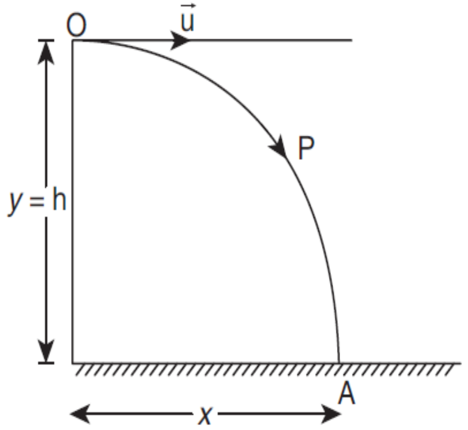
8. Derive the equations of motion for a particle **projected horizontally**.
   - Consider a projectile, say a ball, thrown horizontally with an initial velocity $\vec{u}$ from the top of a tower of height h.
   - Horizontal distance travelled by the ball $x(t) = x$
   - Veritical distance travelled by the ball $y(t) = y$

Motion along horizontal direction:

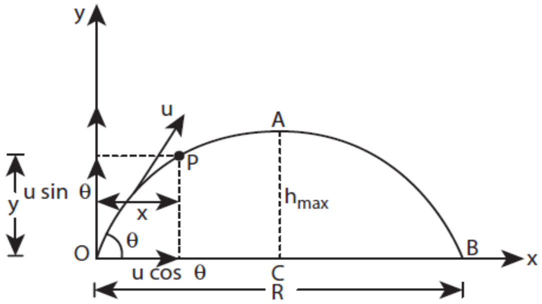
- $x = u_xt + \frac{1}{2}a_xt^2$
- $a_x = 0$
- $x = u_xt$

Motion along downward direction:

- $u_y = 0$
- $a_y = g$
- $y = u_y t + \frac{1}{2}gt^2$
- $y = \frac{1}{2}gt^2$
- $t = \frac{x}{u_x}$
- $y = \left(\frac{g}{2{u_x}^2}\right)x^2$
- $y = Kx^2$, ($K = \frac{g}{2{u_x}^2}$ *is a constant*)
- The above equation is the parapola.
- Thus the path followed by the projectile is a parapola.
- **Time of flight**: $T = \sqrt{\frac{2h}{g}}$
- **Horizontal range:** $R = uT = u\sqrt{\frac{2h}{g}}$
- **Resultant velocity:** $v = \sqrt{u^2 + g^2t^2}$
- Speed of the projectile when it hits the groud $v = \sqrt{u^2 + 2gh}$



9. Derive the equations of motion, range and maximum height reched by the particle **thrown at an oblique angle θ with respect to the horizontal direction**.

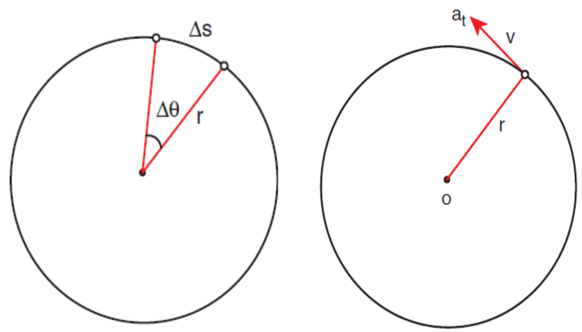


- Consider an object thrown with initial velocity $\vec{u}$ at an angle θ with the horizontal..
- $\vec{u} = u_x\hat{\imath} + u_y\hat{\jmath}$

Motion along horizontan direction	Motion along vertical direction
$a_x = 0$,	$a_y = -g$,
$u_x = u\cos\theta$	$u_y = u\sin\theta$
$v_x = u_x + a_x t$	$v_y = u_y + a_y t$
$v_x = u_x = u\cos\theta$	$v_y = u\sin\theta - gt$
$s_x = u_x t + \frac{1}{2}a_x t^2$	$s_y = u_y t + \frac{1}{2}a_y t^2$
$s_x = x$	$s_y = y$
$x = u\cos\theta\,.\,t$	$y = u\sin\theta\, t - \frac{1}{2}gt^2$
$t = \frac{x}{u\cos\theta}$	$y = x\tan\theta - \frac{1}{2}g\frac{x^2}{u^2\cos^2\theta}$

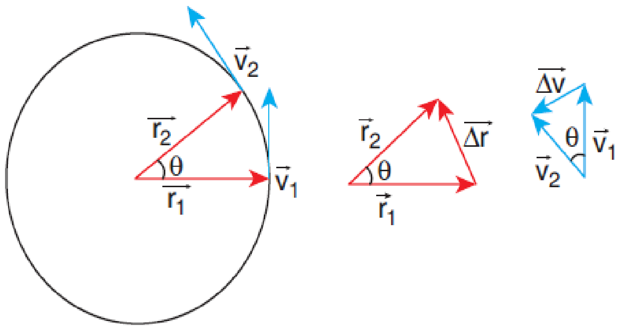
- The equation y represents the path followed by the projectile and it is an inverted parapola.
- **Maximum height:** $h_{max} = \frac{u^2\sin^2\theta}{2g}$
- **Time of flight:** $T_f = 2u\frac{\sin\theta}{g}$
- **Horizontal range:** $R_{max} = \frac{u^2\sin 2\theta}{g}$
- If $\theta = 45°$ then $R_{max} = \frac{u^2}{g}$

10. Derve the expression for tangential acceleration.
    - Consider an object moving along a circle of radius r.
    - In a time Δt, the object travels an arc distance Δs
    - $\Delta s = r\Delta\theta$
    - $\frac{\Delta s}{\Delta t} = r\frac{\Delta\theta}{\Delta t}$
    - If $\Delta t \to 0$ then $\frac{ds}{dt} = r\omega$
    - $\frac{ds}{dt} = v$
    - $v = r\omega$
    - $\vec{v} = r\vec{\omega}$
    - $\frac{dv}{dt} = r\frac{d\omega}{dt} = r\alpha$
    - $\frac{dv}{dt} = a_t$
    - $a_t = r\alpha$



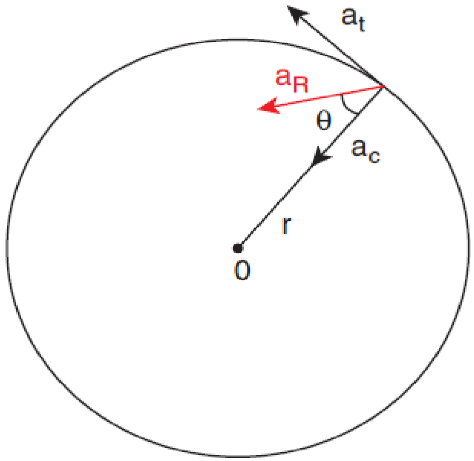
11. Derive the expression for centripetal acceleration.

- The centripetal acceleration is derived from a simple geometrical relationship between position and velocity vectors.
- Let the directions of position and velocity vectors shift through the same angle θ in small interval of time Δt.
- For uniform circular motion
- $r = r_1 = r_2$
- $v = v_1 = v_2$
- $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$
- $\Delta\vec{v} = \vec{v}_2 - v_1$
- $\frac{\Delta r}{r} = -\frac{\Delta v}{v} = \theta$
- $\Delta v = -v\left(\frac{\Delta r}{r}\right)$
- $\frac{\Delta v}{\Delta t} = -\frac{v}{r}\left(\frac{\Delta r}{\Delta t}\right)$
- $a = -\frac{v^2}{r}$
- $a = -\omega^2 r$



12. Derive the expression for total acceleration in the non – uniform circular motion.

- If the speed of the object in circular motion is not constant , then we have non-uniform circular motion.
- For example, when the bob attached to a string moves in vertical circle, the speed of the bob is not the same at all time.
- Whenever the speed is not same in circular motion, the particle will have both centripetal and tangential acceleration..
- $a_t = r\alpha$
- $a = \frac{v^2}{r}$
- $a_R = \sqrt{a_t^2 + \left(\frac{v}{r}\right)^2}$
- $\tan\theta = \frac{a_t}{\left(\frac{v^2}{r}\right)}$



3.LAWS OF MOTION

3 MARK QUESTIONS AND ANSWERS

1. What is inertia? Explain inertia of motion, inertia of rest and inertia of direction.

Inertia:

The inability of objects to move on its own or change its state of motion is called inertia.

(1) Inertia of rest:
The inability of an object to change its state of rest is called inertia of rest.

(2) Inertia of motion:
The inability of an object to change its state of motion is called inertia of motion.

(3) Inertia of direction:
The inability of an object to change its state of direction is called inertia of direction.

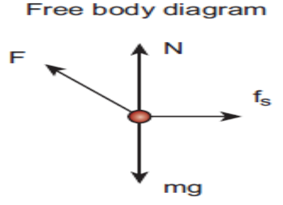
2. Prove that the impulse is the change in momentum.

- If a very large force acts on an object for a very short duration, then the force is called impulsive force or impulse.
- $Fdt = dp$
- $\int_{t_i}^{t_f} F dt = \int_{p_i}^{p_f} dp$
- $p_f - p_i = \Delta p = J$
- $t_f - t_i = \Delta t$
- $F\Delta t = \Delta p$
- $F\Delta t = J$
- J is impulse.
- This is equal to the change in momentum of the object.

3. Which one is easier to move an object? whether push or pull?

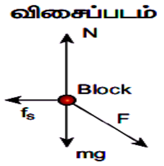
An object pushed at an angle θ :

- $N_{push} = mg + F\ \cos\theta$ ....(1)
- $f_s^{max} = \mu_s(mg + F\ \cos\theta)$



An object pulled at an angle θ :

- $N_{pull} = mg - F\ \cos\theta$ ....(2)
- $N_{pull} < N_{push}$
- It is easier to pull an object than to push to make it move



4. What are the steps to be followed for developing the free body diagram?
   - Identify the forces acting on the object.
   - Represent the object as a point.
   - Draw the vectors representing the forces acting on the object.

5. What are concurrent and coplanar forces?

Concurrent forces:

A collection of forces is said to be concurrent, if the lines of forces act at a common point.

Coplanar forces:

If the concurrent forces lie on the same plane, they are called as coplanar forces.

6. What is the meaning of law of conservation of momentum?
   - Law of conservation of momentum is a vector law.
   - It implies that the magnitude and dirction of total linear momentum is constant.
   - In some cases, the total linear momentum may acquire zero magnitude.

7. When a cricket player catches the ball, he pulls his hands gradually in the direction of the ball's motion. Why?
   * If he stops his hands soon after catchin the ball, the ball comes to rest very quickly.
   * It means that momentum of the ball is brought to rest very quickly.
   * So, the average force acting on the body will be very large.
   * Due to this large average force, the hands will get hurt.
   * To avoid getting hurt, the player brings the ball to rest slowly.

8. Cars are designed with air bags. Why?
   * When a car meets with an accident, its momentum reduces drastically in a very short time.
   * This is very dangerous for the passengers inside the car since they will experience a large force.
   * To prevent this fatal shock, cars are designed with air bags in such a way that when car meets with an accident, the momentum of the passengers will reduce slowly so that the average force acting on them will be smaller.

9. Shock absorbers are used in two wheelers. Why?
   * When there is a bump on the road, a sudden force is transferred to the vehicle.
   * The shock absorber prolongs the period of transfer of force on to the body of the rider.
   * Vehicles without shock absorbers will harm the body due to this reason.

10. Jumping on a concrete cemented floor is more dangerous than on the sand. Why?
    * Jumping on a concrete cemented floor is more dangerous than on the sand. Because,
    * Sand brings the body to rest slowly than the concrete floor.
    * So that the average force experienced by the body will be lesser.

11. Compare static friction and kinetic friction.

Static friction	Kinetic friction
It opposes the starting of motion	It opposes the relative motion of the object with respect to the surface.
Independent of surface of contact	Independent of surface of contact
μ_s depends on the nature of materials in mutual contact	μ_k depends on nature of materials and temperature of the surface
Depends on the magnitude of the applied force	Independent of magnitude of applied force.
It can take values from zero to $\mu_s N$	It can never be zero and always equals to $\mu_k N$
$f_s^{max} > f_k$	$f_k < f_s^{max}$
$\mu_s > \mu_k$	$\mu_k < \mu_s$

12. Obtain an expression for angle of friction.
   - The angle between the normal force and the resultant of normal force and friction force is called angle of friction.
   - $\vec{R} = \vec{N} + \vec{f}_s^{max}$
   - $R = \sqrt{(f_s^{max})^2 + N^2}$
   - $\tan\theta = \frac{f_s^{max}}{N}$
   - $\frac{f_s^{max}}{N} = \mu_s$
   - $\mu_s = \tan\theta$
   - The coefficient of static friction is equal to the tangent of the angle of friction.

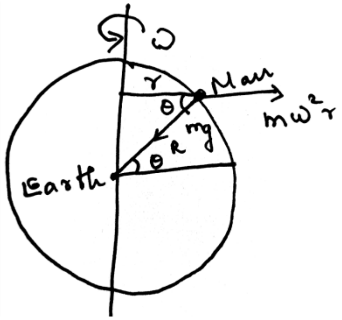
13. What are the methods of reducing friction?
   - In big machines used in industries, relative motion between different parts of the machine produces unwanted heat which reduces its efficiency.
   - Lubricants are used to reduce this kinetic friction.
   - Ball bearings provide another effective way to reduce kinetic friction in machines.
   - If ball bearings are fixed between two surfaces, during the relative motion only friction comes to effect and not kinetic friction.
   - Rolling friction is much smaller than the kinetic friction.
   - Hence machines are protected from the wear and tear over the years.

14. What are the ways of changing the velocity of the body by the force acting on it?
   1. Magnitude of the velocity can be changed without changing its direction.
   2. The direction of motion alone can be changed without changing the magnitude of velocity.
   3. Both magnitude and direction of the velocity can be changed.
      Example: simple pendulum.

15. Obtain an expression for centripetal acceleration.
   - If a particle is in uniform circular motion, there must be a centripetal acceleration towards the centre of the circle.
   - If there is acceleration then there must some force acting on it with respect to an inertial frame.
   - This force is called cenritpetal force.
   - $a = \frac{v^2}{r}$
   - $F_{cp} = \frac{mv^2}{r}$
   - $F_{cp} = m\omega^2 r$

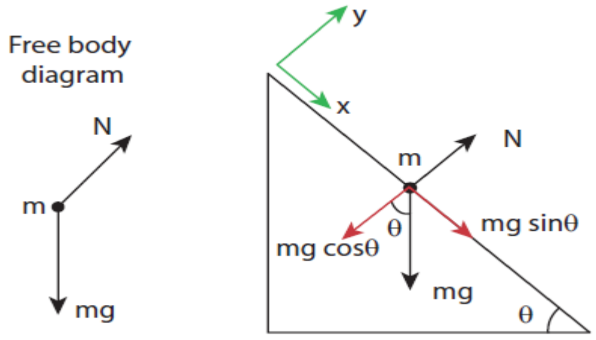
16. Give three examples of origin of the centripetal force.
   * In the case of a whirling motion a stone tied to a string, the centripetal force on the particle is provided by the tensional force on the string.
   * In motion of satellites around the Earth, the centripetal force is given by the Earth's gravitational force on the satellites.
   * When a car is moving on a circular track, the centripetal force is given by the frictional force between the road and the tyres.

17. Centrifugal force due to the rotation of the Earth.
   * Earth spins about its own axis with angular velocity ω.
   * Any object on the surgace of the Earth experience the centrifugal force.
   * Centrifugal force appears to act exactly in opposite direction from the axis of rotation.
   * Centrifugal force on the man standing on the Earth is $F_{cf} = m\omega^2 r$
   * $r = R\cos\theta$
   * R radius of the Earth.



5 MARK QUESTIONS AND ANSWERS

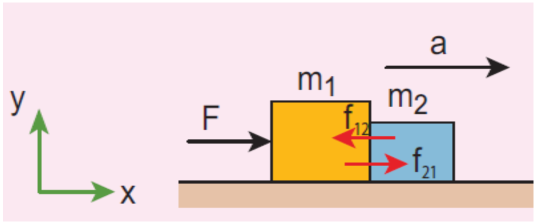
1. Obtain the expressions for acceleration of the object on the inclined plane and the speed of the object when it reaches the ground.



- When an object of mass m slides on a frictionless surface inclined at angle θ, the forces acting on it decides the
  - acceleration of the object
  - speed of the object when it reaches the bottom
- The force acting on the object is
  - Downward gravitational force (mg)
  - Normal force perpendicular to the inclined surface (N)

Motion along y direction	Motion along x direction
$N\hat{j} - mg\cos\theta\,\hat{j} = 0$	$mg\sin\theta\,\hat{\imath} = ma\hat{\imath}$
$N = mg\cos\theta$	$mg\sin\theta = ma$
	$a = g\sin\theta$
	$v = \sqrt{2sg\sin\theta}$

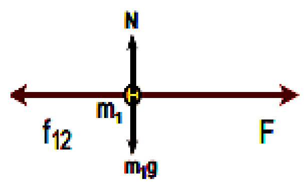
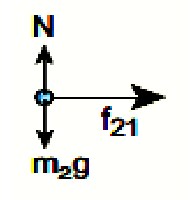
2. Explain the motion of two bodies in contact on a horizontal surface.



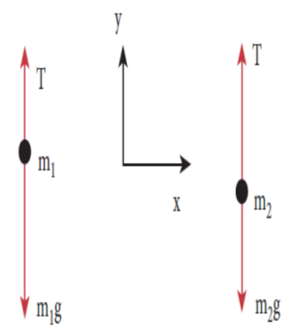
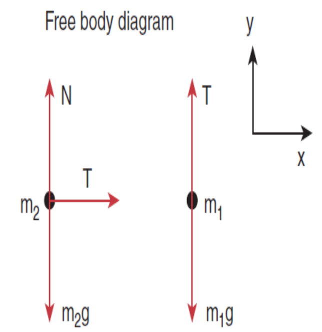
$$m = m_1 + m_2$$

$$F = ma$$

$$a = \frac{F}{m_1 + m_2}$$

Force on m_1	Force on m_2
	
$F\hat{\imath} - f_{12}\hat{\imath} = m_1 a\hat{\imath}$	$f_{21}\hat{\imath} = m_2 a\hat{\imath}$
$f_{12} = F - m_1 a$	$f_{21} = m_2 a$
$f_{12} = \frac{Fm_2}{m_1 + m_2}$	$f_{21} = \frac{Fm_2}{m_1 + m_2}$
$\vec{f}_{12} = -\frac{Fm_2}{m_1 + m_2}\hat{\imath}$	$\vec{f}_{21} = \frac{Fm_2}{m_1 + m_2}\hat{\imath}$
$\vec{f}_{12} = -\vec{f}_{21}$	

3. Explain the motion of blocks connected by a string in vertical motion and horizontal motion

Vertical motion	Horizontal motion
	
For mass m_1: $T\hat{j} - m_1 g\hat{j} = -m_1 a\hat{j}$ $T = m_1 g - m_1 a$ ………(1)	For mass m_1: $T\hat{j} - m_1 g\hat{j} = -m_1 a\hat{j}$ $T = m_1 g - m_1 a$ ………(1)
For mass m_2: $T\hat{j} - m_2 g\hat{j} = m_2 a\hat{j}$ $T = m_2 g + m_2 a$ ………(2)	For mass m_2: $T\hat{\imath} = m_2 a\hat{\jmath}$ $T = m_2 a$ ………(2)
From (1)and (2), $(m_1 - m_2)g = (m_1 + m_2)a$ $a = \left[\frac{m_1 - m_2}{m_1 + m_2}\right]g$	From (1) and (2) $m_1 g = (m_1 + m_2)a$ $a = \left(\frac{m_1}{m_1 + m_2}\right)g$
$T = \left(\frac{2m_1 m_2}{m_1 + m_2}\right)g$	$T = \left(\frac{2m_1 m_2}{m_1 + m_2}\right)g$

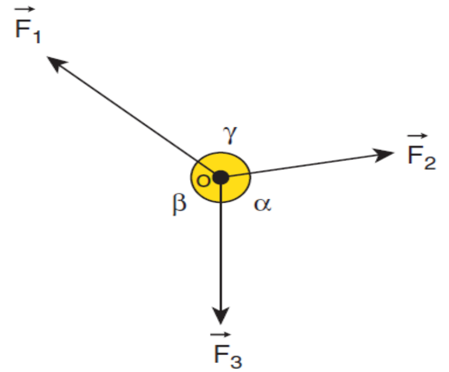
4. What are concurrent forces? State Lami's theorem.

Concurrent forces:

A collection of forces is said to be concurrent, if the lines of forces act at a common point.

Lami's theorem:

If a system of three concurrent and coplanar forces is in equilibrium, then Lami's theorem states that the magnitude of each force of the system is proportional to sine of the angle between the other two forces.



- Let us consider three coplanar and concurrent forces $\vec{F}_1, \vec{F}_2$ and $\vec{F}_3$ which act at a common point O as shown in figure.
- $\frac{|\vec{F}_1|}{\sin\alpha} = \frac{|\vec{F}_2|}{\sin\beta} = \frac{|\vec{F}_3|}{\sin\gamma}$
- Lami's theorem is useful to analyse the forces acting on objects which are in static equilibrium.

5. Prove the law of conservation of total linear momentum. Use it to find the recoil of a gun when a bullet is fired from it.
   - $\vec{F}_{12} = -\vec{F}_{21}$
   - $\vec{F}_{12} = \frac{d\vec{p}_1}{dt}, \vec{F}_{21} = \frac{d\vec{p}_2}{dt}$
   - $\frac{d\vec{p}_1}{dt} = -\frac{d\vec{p}_2}{dt}$
   - $\frac{d\vec{p}_1}{dt} + \frac{d\vec{p}_2}{dt} = 0$
   - $\frac{d}{dt}(\vec{p}_1 + \vec{p}_2) = 0$
   - $\vec{p}_{tot} = \vec{p}_1 + \vec{p}_2$

Law of conservation of total linear momentum:

If there are no external forces acting on the system, then the total linear momentum of the system is always a constant vector.

Recoil momentum of a gun:

- Consider the firing of a gun.
- $\vec{p}_1$ – momentum of the bullet
- $\vec{p}_2$ - momentum of the gun
- Initially the gun and the bullet are at rest.
- Total momentum before firing the gun is $\vec{p}_1 + \vec{p}_2 = 0$
- According to the law of conservation of linear momentum, total linear momentum has to be zero after the firing also.
- When the gun is fired, the momentum of the bullet changes to $\vec{p}_1{}'$
- Momentum of the gun changes to $\vec{p}_2{}'$
- $\vec{p}_1{}' + \vec{p}_2{}' = 0$
- $\vec{p}_1{}' = -\vec{p}_2{}'$
- The momentum of the gun is exactly equal and opposite direction to the momentum of the bullet.
- This is the reason after firing, the gun suddenly moves backward with the momentum $-\vec{p}_2{}'$
- It is called 'recoil momentum'.

6. What is friction? What are its types? Suggest a few methods to reduce friction.

Friction:

If a very gentle force in the horizontal direction is given to an object at rest on the table, an opposing force called the frictional force which always opposes the relative motion between an object and the surface where it is placed.

Types of friction:

1. Static friction:

- Static friction is the force which opposes the initiation of motion of an object on the surface.
- When the object is at rest on the surface, only two forces act on it.
- They are the downward gravitational force and upward normal force.
- The resultant of these two forces on the object is zero.
- As a result the object is at rest.
- If some external force is applied on the object parallel to the surface on which the object is at rest, the surface exerts exactly an equal and opposite force on the object to resists its motion and tries to keep the object at rest.
- It implies that the external force and frictional force are equal and opposite.
- If the external force is increased, after a particular limit, the surface cannot provide sufficient opposing frictional force to balance the external force on the object.
- Then the object starts to slide.
- This is the maximum static friction that can be exerted by the surface.
- The empirical formula for static friction: $0 \leq f_s \leq \mu_s N$
- μ_s is the coefficient of static friction.
- It depends on the nature of the surfaces in contact.

- If the object is at rest and no external force is applied on the object, the static friction acting on the object is zero.
- If the object is at rest and there is an external force applied parallel to the surface, then the force of static friction acting on the object is exactly equal to the external force applied on the object.

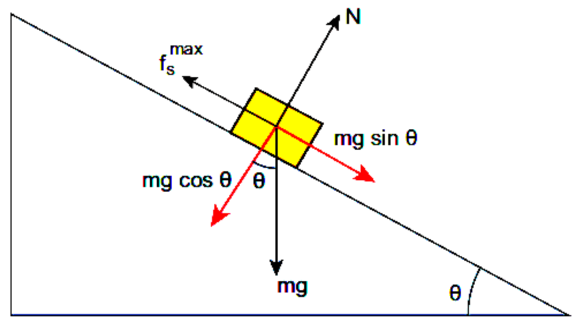
2. Kinetic friction

- If the external force acting on the object is greater than the maximum static friction, the object begins to slide.
- When an object slides, the surface exerts a frictional force called kinetic friction.
- It is also called sliding friction.
- $f_k = \mu_k N$
- μ_k is the coefficient of kinetic friction.
- $\mu_k < \mu_s$

Methods of reducing friction:

- In big machines used in industries, relative motion between different parts of the machine produces unwanted heat which reduces its efficiency.
- Lubricants are used to reduce this kinetic friction.
- Ball bearings provide another effective way to reduce kinetic friction in machines.
- If ball bearings are fixed between two surfaces, during the relative motion only friction comes to effect and not kinetic friction.
- Rolling friction is much smaller than the kinetic friction.
- Hence machines are protected from the wear and tear over the years.

7. Prove that the angle of repose is equal to the angle of friction.

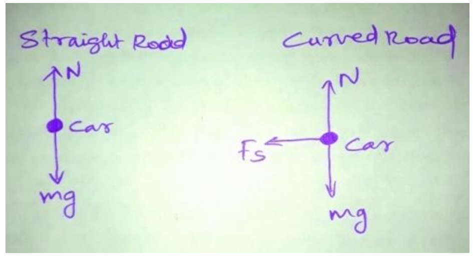


- Angle of repose is the angle of inclined plane with the horizontal such that an object placed on it begins to slide.
- The gravitational force mg is resolved into two components.
- $mg \sin\theta$ Parallel to the inclined plane.
- $mg \cos\theta$ Perpendicular to the inclined plane.
- $mg \sin\theta$ Component parallel to the inclined plane tries to move the object down.
- Maximum static friction is $f_s^{max} = \mu_s N$
- $\frac{f_s^{max}}{N} = \mu_s$ ---------(1)
- $f_s^{max} = mg \sin\theta$ ---------(2)
- $N = mg \cos\theta$ ---------------(3)
- (2)/(3)→ $\frac{f_s^{max}}{N} = \frac{\sin\theta}{\cos\theta}$
- $\mu_s = \tan\theta$
- The above equation is also suitable for angle of friction.
- Hence, angle of repose is same as the angle of friction.

8. Explain about rolling friction.

- When an object moves on a surface, essentially it is sliding on it with the help of wheels.
- But wheels move on the surfacr through rolling motion.
- In rolling motion when wheels move on a surface, the point of contact with the surface is always at rest.
- Hence the frictional force is very less.
- At the same time if the object moves without wheels, frictional force is larger.
- This makes it difficult to move the object.
- Ideally in pure rolling, motion of the point of contact with the surface should be at rest, but in practice it is not so.
- Due to the elastic nature of the surface at the point of contact there will be some deformation on the object at this point on the wheel.
- Due to this deformation, there will be minimal friction between wheel and surface.
- It is called rolling friction.
- Rolling friction is much smaller than kinetic friction.

9. Obtain the conditions for safe turn and skidding of a vehicle on a leveled circular road.



- When a vehicle travels in a curved path, there must be centripetal force acting on it.
- This centripetal force is provided by the frictional force between the tyre and surface of the road.
- Consider a vehicle of mass ‘m’ moving at a speed ‘v’ in the circular track of radius ‘r’.

- There are three forces acting on the vehicle when it moves.
- Gravitational force acting downwards
- Normal force acting upwards
- Frictional force acting horizontally inwards along the road.

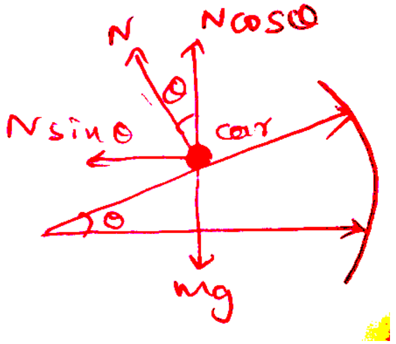
Codintion for safe turn:

- $\frac{mv^2}{r} \leq \mu_s mg$
- $\mu_s \geq \frac{v^2}{rg}$
- $\sqrt{\mu_s rg} \geq v$

Condition for skidding:

- $\frac{mv^2}{r} > \mu_s mg$
- $\mu_s < \frac{v^2}{rg}$
- $\sqrt{\mu_s rg} < v$

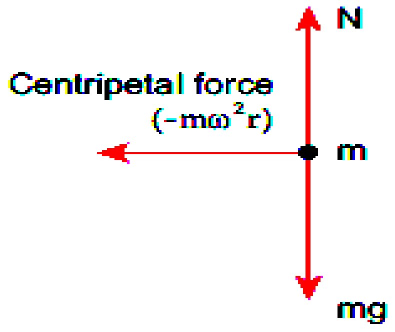
10. Explain the need for banking of trakcs.
    - In a leveled circular road, skidding mainly depends on the coefficient of static friction.
    - The coefficient of static friction depends on the nature of the surface which has a maximum limiting value.
    - To avoid this problem, usually the outer edge of the road is slightly raised compared to the inner edge.
    - This is called banking of tracks.
    - This introduces an inclination and the angle is called banking angle.
    - When the car takes a turn, there are two forces acting on the car
    - Gravitational force mg acting downwards
    - Normal force N acting perpendicular to surface
    - $N\cos\theta = mg$
    - $N\sin\theta = \frac{mv^2}{r}$
    - $\tan\theta = \frac{v^2}{rg}$
    - $v = \sqrt{rg\tan\theta}$
    - The banking angle and the radius of curvature of the road determines the safe speed of the car at the turning.
    - If the speed of the car exceeds this safe speed, then it starts to skid outward but frictional force comes into effect and provides an additional centripetal force to prevent the outward skidding.
    - At the same time, if the speed of the car is little lesser than safe speed, it starts to skid inward and frictional force comes into effect, which



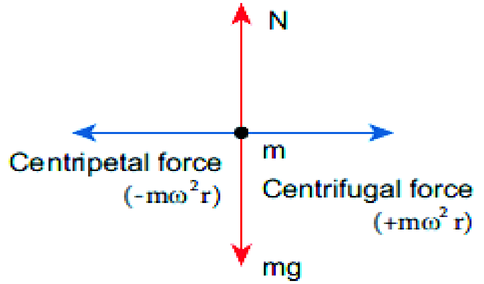
reduces centripetal force to prevent inward skidding.

- However if the speed of the vehicle is sufficiently greater than the correct speed, then firctional force cannot sotp the car from skidding.

11. Explain centrifugal force with example.
    - Centrifugal force is a psudo force.
    - Its magnitude is equal and opposite to the centripetal force.
    - It exists in rotating frames only.
    - Consider the case of a whirling motion of a stone tied to string.
    - Assume that the stone hs angular velocity ω in the inertial frame.
    - If the motion of the stone is observed from frame which is also rotating along with the stone with same angular velocity ω then, the stone appears to be at rest.
    - This implies that in addition to the inward centripetal force $-m\omega^2$ there must be an equal and opposite force that acts on the stone outward with value $+m\omega^2 r$.
    - So the total force acting on the stone in a rotating frame is equal to zero.
    - The outward force $+m\omega^2 r$ is called the centrifugal force.



In inertial frame



In rotating frame

12. Explain the effects of centrifugal force.
- When a car takes a turn in curved road, person inside the car feels an outward force which pushes the person away.
- This outward force is also called centrifugal force.
- If there is sufficient friction between the person and the seat, it will prevent the person from moving outwards.
- When a car moving in a straight line suddenly takes a turn, the objects not fixed to the car try to continue in linear motion due to their inertia of direction.
- While observing this motion from an inertial frame, it appears as a straight line.
- But, when it is observed from the rotating frame it appears to move outwards.
- A person standing on a rotating platform feels an outward centrifugal force and is likely to be pushed away from the platform.
- Many a time the frictional force between the platform and the person is not sufficient to overcome outward push.
- To avoid this, usually the outer edge of the platform is little inclined upwards which exerts a normal force on the person which prevents the person from falling.

13. Calculate the centripetal acceleration of the Moon towards the Earth.
- The Moon orbits the Earth once in 27.3 days in an almost circular orbit.
- Centripetal acceleration of the Moon due to Earth's gravity is $a_m = \omega^2 R_m$
- R_m is the distance between Earth and the Moon.
- It is 60 times the radius of the Earth.
- $R_m = 60R$
- $R = 6.4 \text{ x} 10^6 m$
- $R_m = 384 \text{ x} 10^6 m$
- $\omega = \frac{2\pi}{T}$
- $T = 2.358 \text{ x} 10^6 \ s$
- $a_m = \omega^2 R_m = \frac{4\pi^2}{T^2} R_m$
- $a_m = 0.00272 \ m \ s^{-2}$

14. Compare centripetal force and centrifugal force.

Centripetal force	**Centrifugal force**
It is a real force	It is a pseudo force
It arises from gravitational force, tension in the string, normal force	It is not arises from gravitational force, tension in the string, normal force
Acts in both inertial and non – inertial frames	Acts only in rotating frames
It acts towards the axis of rotation	It acts outwards from the axis of rotation
Real force and has real effects	Pseudo force but has real effects
Origin of centripetal force is interaction between two objects	Origin of centrifugal force is inertia.

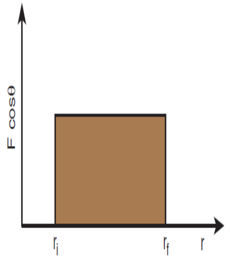
4. WORK, ENERGY AND POWER

3 MARK QUESTIONS

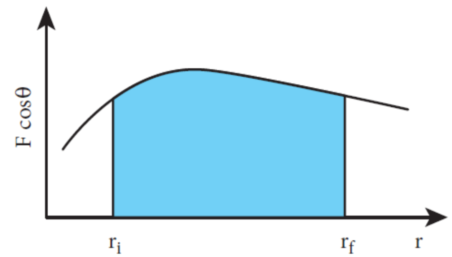
1. Explain how the definition of work in physics is different from general perception.
- The term work is used in diverse contexts in daily life.
- It refers to both physical as well as mental work.
- In fact, any activity can generally be called as work.
- But in physics, the term work is treated as a physical quantity with a precise definition.
- Work is said to be done by the force when the force applied on a body displaces it.
- To do work, energy is required.
- In simple words, energy is defined as the ability to do work.

2. Write the mathematical expression for workdone and also mention the cases when workdone becomes zero.
- $W = F\, dr \cos\theta$
- Work done is zero in the following cases
  - When the force is zero
  - When the displacement is zero
  - When the force and displacement are perpendicular to each other.

3. Obtain the expression for work done by the constant force.
- $dW = (F\cos\theta) dr$
- $W = \int_{r_i}^{r_f} dW = (F\cos\theta)\int_{r_i}^{r_f} dr$
- $W = (F\cos\theta)(r_f - r_i)$



4. Obtain the expression for work done by the variable force.
- $dW = (F\cos\theta) dr$
- $W = \int_{r_i}^{r_f} dW$
- $W = \int_{r_i}^{r_f} F\cos\theta \, dr$

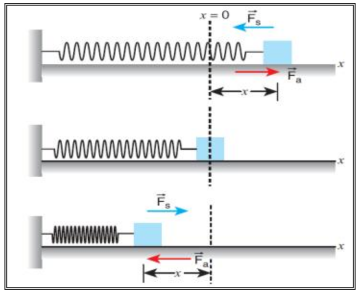


5. Derive a relation between kinetic energy and momentum.
- $KE = \frac{1}{2}mv^2$
- $KE = \frac{1}{2}m(\vec{V}.\vec{V})$
- $= \frac{1}{2}\frac{(m\,\vec{V}.m\vec{V})}{m}$
- $= \frac{1}{2}m(\vec{V}.\vec{V})$
- $= \frac{1}{2}\frac{(\vec{p}.\vec{p})}{m}$
- $KE = \frac{p^2}{2m}$
- $p = \sqrt{2m\,(KE)}$

6. What is potential energy? Mention its types.
- The energy possessed by the body by virtue of its position is called poetential energy.
- Types of potential energy
  - Gravitational potential energy
  - Elastic potential energy
  - Electrostatic potential energy.

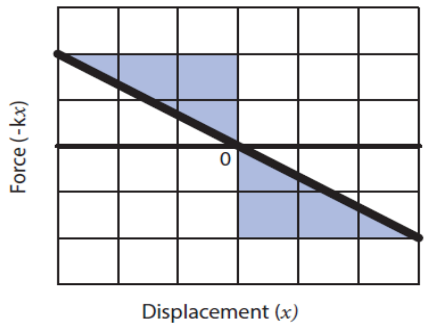
7. Obtain the expression for the potential energy near the surface of the Earth.
- Let us consider a body of mass m being moved from ground to the height h against the gravitational force.
- $\vec{F_g} = -mg\,\hat{j}$
- $\vec{F_a} = -\vec{F_g}$
- $\vec{F_a} = +mg\,\hat{j}$
- $U = \int \vec{F_a}\,.d\vec{r}$
- $U = \int_o^h F_a\; dr \cos\theta$
- $F_a = mg, \cos 0° = 1$
- $U = mg\int_o^h dr$
- $U = mgh$

8. Derive an expression for the elastic potential energy stored in a spring.
- When a spring is elongated, it develops a restoring force.
- The potential energy possessed by a spring due to a deforming force which stretches the spring is termed as elastic potential energy.

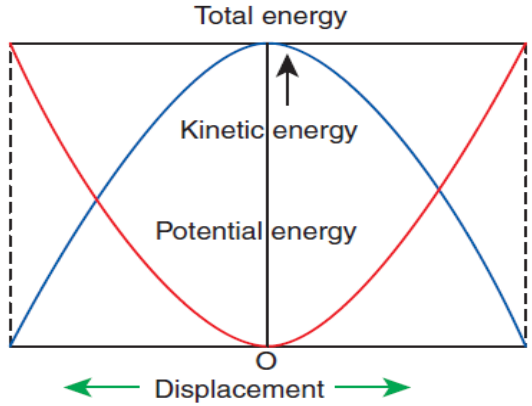


- $\vec{F_s} = -k\,\vec{x}$
- $\vec{F_a} = -\vec{F_s}$
- $\vec{F_a} = +k\,\vec{x}$
- $U = \int \vec{F_a}\,.d\vec{x}$
- $U = \int_o^x F_a\; dx \cos\theta$
- $F_a = kx, \cos 0° = 1$
- $U = k\int_o^x x\; dx$
- $U = \frac{1}{2}kx^2$
- *If the initial position is not zero,* $U = \frac{1}{2}k(x_f{}^2 - x_i{}^2)$

9. Draw the force – displacement graph for a sprig.
- Area $= \frac{1}{2}kx^2$



10. Draw and explain the graph between potential energy and displacement for a spring mass system.
- A compressed or extended spring will transfer its stored potential energy into kinetic energy of the mass attached to the spring.
- In frictionless environment, the energy gets transferred from kinetic energy to potential energy and vise versa such that the total energy of the systan reamis constant.
- At the mean position, $\Delta KE = \Delta U$



11. Compare conservative and non conservative forces.

Conservative forces	Non conservative forces
Work done is independent of path	Work done depends upon the path
Work done in round trip is zero	Work done in a round trip is not zero
Total energy remains constant	Energy is dissipated as heat energy
Work done is completely recoverable	Work done is not recoverable
$F = -\frac{dU}{dx}$	No such relation exists
Examples: Electrostatic force, Gravitational force	Frictional forces, viscous forces

12. Prove that the total linear momentum is conserved in all collision processes.
    - $\Delta\vec{p}_1 = \vec{F}_{12}\Delta t$
    - $\Delta\vec{p}_2 = \vec{F}_{21}\Delta t$
    - $\Delta(\vec{p}_1 + \vec{p}_2) = (\vec{F}_{12} + \vec{F}_{21})\Delta t$
    - $\vec{F}_{12} = -\vec{F}_{21}$
    - $\Delta(\vec{p}_1 + \vec{p}_2) = 0$
    - If $\Delta t \rightarrow 0$ then $\frac{d(\vec{p}_1+\vec{p}_2)}{dt} = 0$
    - Hence total linear momentum is conserved in all collision processes.
13. Obtain an expression for velocity in completely inelastic collision.
    - In a perfectly inelastic collision, the objects stick together permanently after collision such that they move with common velocity.
    - Momentum before collision $= m_1u_1 + m_2u_2$
    - Momentum after collision $= (m_1 + m_2)v$
    - Momentum before collision = Momentum after collision
    - $m_1u_1 + m_2u_2 = (m_1 + m_2)v$
    - $v = \frac{m_1u_1+m_2u_2}{(m_1+m_2)}$
14. Obtain an expression for loss of kinetic energy in perfect inelastic collision.
    - In perfect inelastic collision, the loss in kinetic energy is transformed into another form of energy like sound, heat, thermal, light, etc.
    - $KE_i = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$
    - $KE_f = \frac{1}{2}(m_1 + m_2)v^2$
    - $\Delta Q = KE_i - KE_f$
    - $\Delta Q = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 - \frac{1}{2}(m_1 + m_2)v^2$
    - $\Delta Q = \frac{1}{2}\left(\frac{m_1m_2}{m_1+m_2}\right)(u_1 - u_2)^2$
15. What is coefficient of restitution?
    - Coefficient of restitution is defined as the ratio of velocity of separation after collision and velocity of approach before collision.
    - $e = \frac{(v_2-v_1)}{u_1-u_2}$
    - For perfect elastic collision, $e = 1$
    - $0 < e < 1$
16. Compare elastic and inelastic collisons

Elastic collision	Inelastic collision
Total momentum is conserved	Total momentum is conserved
Total kinetic energy is conserved	Total kinetic energy is not conserved
Forces involved are conservative forces	Forces involved are non-conservative forces
Mechanical energy is not dissipated	Mechanical energy is dissipated into heat, sound, light etc.

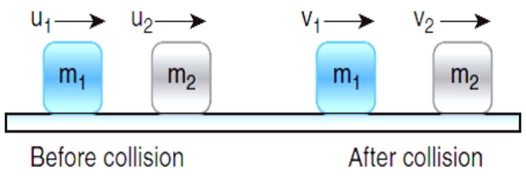
5 MARK QUESTIONS

1. State and prove work – energy theorem and give its importance.

   Work – Energy Theorem:

   Work done by the forces on a body is equal to the change in kinetic energy.
    - $W = Fs$
    - $F = ma$
    - $v^2 = u^2 + 2as$
    - $a = \frac{v^2-u^2}{2s}$
    - $F = m\left(\frac{v^2-u^2}{2s}\right)$
    - $W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$
    - $KE = \frac{1}{2}mv^2$
    - Kinetic energy of the body is always positive.
    - $\Delta KE = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$
   - $W = \Delta KE$
   - The work – kinetic energy theorem implies the following:
     - If the work done is positive, then the kinetic energy increases.
     - If the work done is negative, then the kinetic energy decreases.
     - If there is no work done on the body, the there is no change in kinetic energy.
2. Obtain a relation between power and velocity.
    - $W = \int \vec{F}.d\vec{r}$
    - $W = \int dW = \int \frac{dW}{dt}dt$
    - $\int \vec{F}.d\vec{r} = \int(\vec{F}.\vec{v})dt \quad (\vec{v} = \frac{d\vec{r}}{dt})$
    - $\int \frac{dW}{dt}dt = \int(\vec{F}.\vec{v})dt$
    - $\int\left(\frac{dW}{dt} - \vec{F}.\vec{v}\right)dt = 0$
    - $\frac{dW}{dt} = \vec{F}.\vec{v}$

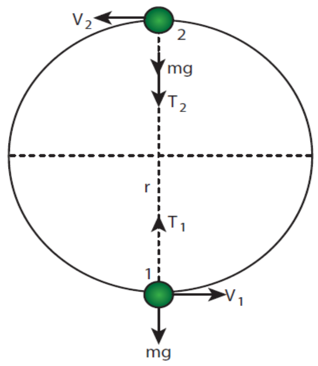
3. Obtain the expression for velocities of bodies in one dimensional collision and also explain the sepecial cases.



- Initial momentum before collison $= m_1u_1 + m_2u_2$
- Final momentum after collison $= (m_1 + m_2)v$
- Initial momentum before collison = Final momentum after collision
- $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$
- $m_1(u_1 - v_1) = m_2(v_2 - u_2)$
- Initial kinetic energy before collison $= \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$
- Final kinetic energy after collison $= \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$
- In an elastic collision,
- Initial kinetic energy before collison = Final kinetic energy after collison
- $\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$
- $m_1(u_1 + v_1)(u_1 - v_1) = m_2(v_2 + u_2)(v_2 - u_2)$
- $u_1 - u_2 = -(v_1 - v_2)$
- $v_1 = v_2 + u_2 - u_1$
- $v_2 = u_1 + v_1 - u_2$
- $v_1 = \left(\frac{m_1-m_2}{m_1+m_2}\right)u_1 + \left(\frac{2m_2}{m_1+m_2}\right)u_2$
- $v_2 = \left(\frac{2m_1}{m_1+m_2}\right)u_1 \left(\frac{m_2-m_1}{m_1+m_2}\right)u_2$
- Case 1: If $m_1 = m_2$ then $v_1 = u_2$, $v_2 = u_1$
- Case 2: If $m_1 = m_2$ and $u_2 = 0$ then $v_1 = 0$, $v_2 = u_1$
- Case 3: If $m_1 \ll m_2$ and $u_2 = 0$ then $v_1 = -u_1 > v_2 = 0$
- Case 4: If $m_2 \ll m_1$ and $u_2 = 0$ then $v_1 = u_1 > v_2 = 2u_1$

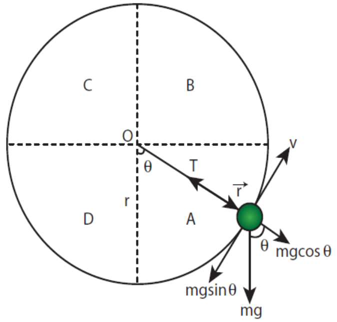
4. Explain in detail the motion along the vertical circle.

- Imagine that a body of mass m attached to one end of a massless and inextensible string.



- That executes circular motion in vertical plane.
- The other end of the string is fixed.
- There are two forces acting on the string
  - Gravitational force
  - Tension along the string
- $mg\sin\theta = -m\left(\frac{dv}{dx}\right)$
- $T - mg\cos\theta = \frac{mv^2}{r}$
- The four important facts to be understood from the above two equations as follows

1. The mass is having tangential acceleration for all values of θ. It is clear that this vertical circular motion is not a uniform circular motion.
2. The magnitude of velocity is not constant in the course of motion, the tension in the string is also not constant.
3. In sections A and D, ***tension cannot vanish even when the velocity vanishes.***
4. In sections B and C***, velocity cannot vanish even when the tension vanishes.***



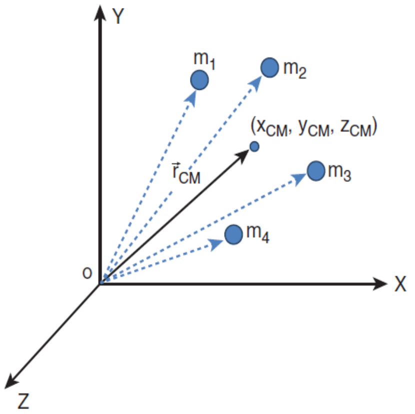
Motion in vertical circle for lowest and highest points:

- At the lowest point, $T_1 = \frac{mv_1^2}{r} + mg$
- At the highest point, $T_2 = \frac{mv_2^2}{r} - mg$
- $T_1 - T_2 = \frac{m}{r}(v_1^2 - v_2^2) + 2mg$
- At points 1 and 2, $E_1 = E_2$
- $E_1 = \frac{1}{2}mv_1^2$
- $E_2 = 2mgr + \frac{1}{2}mv_2^2$
- $v_1^2 - v_2^2 = 4gr$
- $T_1 - T_2 = 6\,mg$
- Minimum speed at the highest point (2), $v_2 = \sqrt{gr}$
- Minimum speed at the lowest point (1), $v_1 = \sqrt{5gr}$

5. MOTION OF SYSTEM OF PARTICLES AND RIGID BODIES

(3 and 5 mark question answers)

1. Obtain an expression for the center of mass for distributed point masses.



- A point mass is hypothetical point particle which has non zero mass and no size or no shape.
- $X_{CM} = \frac{\sum m_i x_i}{\sum x_i}$
- $\sum m_i = M$
- $X_{CM} = \frac{\sum m_i x_i}{M}$
- $Y_{CM} = \frac{\sum m_i y_i}{M}$
- $Z_{CM} = \frac{\sum m_i z_i}{M}$
- $\vec{r}_{CM} = \frac{\sum m_i \vec{r}_i}{M}$

2. Obtain the expression for the center of mass for the two point masses system.
   * When the masses are on positive x axis, $X_{CM} = \frac{m_1x_1+m_2x_2}{m_1+m_2}$
   * When the origin coincides with any one of the masses, $x_1 = 0, X_{CM} = \frac{m_2x_2}{m_1+m_2}$
   * When the origin coincides with the center of mass itself, $m_1x_1 = m_2x_2$

3. Write the center of mass of the following.
   1. Velocity 2. Acceleration 3. External force
   * Velocity, $\vec{v}_{CM} = \frac{\sum m_i \vec{v}_i}{M}$
   * Acceleration, $\vec{a}_{CM} = \frac{\sum m_i \vec{a}_i}{M}$
   * External force, $\vec{F}_{ext} = M\vec{a}_{CM}$

4. Obtain a relation between torque and angular acceleration.
   * $\tau = rF \sin 90°$
   * $\tau = rF$
   * $\tau = r\,ma$
   * $\tau = (mr^2)\alpha$
   * $\vec{\tau} = (\sum m_i r_i^2)\vec{\alpha}$
   * $\vec{\tau} = I\vec{\alpha}$

5. Obtain a relation between angular momentum and angular velocity.
   * $L = rmv \sin 90°$
   * $L = rmv$
   * $L = (mr^2)\omega$
   * $\vec{L} = (\sum m_i r_i^2)\vec{\omega}$
   * $\vec{L} = I\vec{\omega}$

6. Obtain a relation between torque and angular momentum.
   * $L = I\omega$
   * $\tau = I\alpha$
   * $\tau = I\frac{d\omega}{dt}$
   * $\tau = \frac{d(I\omega)}{dt}$
   * $\tau = \frac{dL}{dt}$

7. What are the types of equilibrium?
   * Translational equilibrium
   * Rotational equilibrium
   * Static equilibrium
   * Dynamic equilibrium
   * Stable equilibrium
   * Unstable equilibrium
   * Neutral equilibrium

8. Obtain an expression for radius of gyration.
   * $I = MK^2$
   * $I = m_1{r_1}^2 + m_2{r_2}^2 + \cdots + m_n{r_n}^2$
   * $m = m_1 = m_2 = \cdots = m_n$
   * $I = m({r_1}^2 + {r_2}^2 + \cdots + {r_n}^2)$
   * $I = \frac{nm\,({r_1}^2+{r_2}^2+\cdots+{r_n}^2)}{n}$
   * $I = MK^2$
   * $K = \sqrt{\frac{{r_1}^2+{r_2}^2+\cdots+{r_n}^2}{n}}$

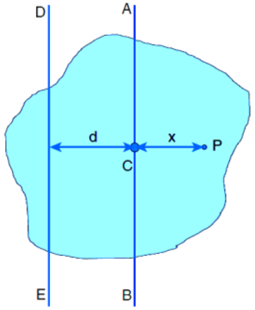
9. Obtain the expression for moment of inertia of uniform rod, uniform ring and uniform disc.

Uniform Rod	Uniform Ring	Uniform Disc
$dI = (dm)x^2$	$dI = (dm)R^2$	$dI = (dm)r^2$
$\lambda = \frac{M}{l}$	$\lambda = \frac{M}{2\pi R}$	$\sigma = \frac{M}{\pi R^2}$
$dm = \frac{M}{l}dx$	$dm = \frac{M}{2\pi R}dx$	$dm = \frac{M}{\pi R^2}dx$
$I = \int (dm)x^2$	$I = \int (dm)R^2$	$I = \int (dm)r^2$
$I = \frac{M}{l}\int_{-l/2}^{l/2} x^2\, dx$	$I = \frac{MR}{2\pi}\int_{0}^{2\pi R} dx$	$I = \frac{2M}{R^2}\int_{0}^{R} r^3 dr$
$I = \frac{Ml^2}{12}$	$I = MR^2$	$I = \frac{MR^2}{2}$

10. State and prove parallel axes theorem.

Parallel axes theorem:

The theorem states that the moment of inertia of a body about any axis is equal to the sum of its moment of inertia about a parallel axis through its center of mass and the product of the mass of the body and square of the perpendicular distance between the two axes.



* $I = \sum m(x+d)^2$
* $I = \sum(mx^2 + md^2 + 2dmx)$
* $I = \sum mx^2 + \sum m\, d^2 + 2d\sum mx$
* $\sum mx^2 = I_C, \sum mx = 0$
* $I = I_C + Md^2$

11. State and prove perpendicular axis theorem.

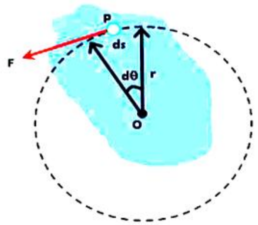
Perpendicular axis theorem:

The theorem states that the moment of inertia of a plane laminar body about an axis perpendicular to its plane is equal to the sum moments of inertia about two perpendicular axes lying in the plane of the body such that all the three axes are mutually perpendicular and have a common point.

* $I_Z = \sum mr^2$
* $r^2 = x^2 + y^2$
* $I_Z = \sum m(x^2 + y^2)$
* $I_Z = \sum mx^2 + \sum my^2$
* $I_Z = I_X + I_Y$

12. Obtain an expression for the work done by the torque.

* $dW = Fds$
* $ds = rd\theta$
* $dW = Frd\theta$
* $dW = \tau d\theta$



13. Obtain an expression for the kinetic energy of a rigid body in pure rolling motion.

* $KE_i = \frac{1}{2}m_i {v_i}^2$
* $KE_i = \frac{1}{2}(m_i {r_i}^2)\, \omega^2$
* $KE = \frac{1}{2}(m_i {r_i}^2)\, \omega^2$
* $KE = \frac{1}{2}I\, \omega^2$

14. Obtain a relation between kinetic energy and angular momentum.

* $L = I\omega$
* $KE = \frac{1}{2}I\, \omega^2$
* $\omega = \frac{L}{I}$
* $KE = \frac{L^2}{2I}$

15. Obtain an expression for the power delivered by torque.

* Power delivered is the work done in unit time.
* $P = \frac{dw}{dt}$
* $dw = \tau d\theta$
* $P = \tau\frac{d\theta}{dt}$
* $P = \tau\omega$

16. Compare the translational and rotational quantities.

Translational motion	Rotational motion
Displacement, x	Angular displacement, θ
Time, t	Time, t
Velocity, $v = \frac{dx}{dt}$	Angular velocity, $\omega = \frac{d\theta}{dt}$
Acceleration, $a = \frac{dv}{dt}$	Angular acceleration, $\alpha = \frac{d\omega}{dt}$
Mass, m	Moment of inertia, I
Force, $F = ma$	Torque, $\tau = I\alpha$
Momentum, $p = mv$	Angular momentum, $L = I\omega$
Impulse, $F\Delta t = \Delta p$	Impulse, $\tau\Delta t = \Delta L$
Work done, $W = Fs$	Work done, $W = \tau d\theta$
Kinetic energy, $KE = \frac{1}{2}m\, v^2$	Kinetic energy, $KE = \frac{1}{2}I\, \omega^2$
Power, $P = Fv$	Power, $P = \tau\omega$

17. Obtain an expression for the kinetic energy of a rigid body in pure rolling motion

- $KE = KE_{TRANS} + KE_{ROT}$
- $KE = \frac{1}{2}Mv_{CM}^2 + \frac{1}{2}I_{CM}\omega^2$

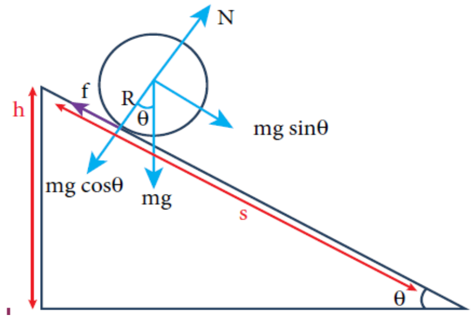
With center of mass as a reference:

* $KE = \frac{1}{2}Mv_{CM}^2 + \frac{1}{2}Mv_{CM}^2\left(\frac{K^2}{R^2}\right)$
* $KE = \frac{1}{2}Mv_{CM}^2\left(1 + \frac{K^2}{R^2}\right)$

With point of contact as reference:

* $KE = \frac{1}{2}I_0\,\omega^2$
* $I_0 = I_{CM} + MR^2$
* $I_0 = MK^2 + MR^2$
* $KE = \frac{1}{2}M(K^2 + R^2)\frac{v_{CM}^2}{R^2}$
* $KE = \frac{1}{2}Mv_{CM}^2\left(1 + \frac{K^2}{R^2}\right)$

18. Explain rolling on inclined plane and obtain an expression for its acceleration .



* $mg\sin\theta - f = ma$
* $Rf = I\alpha$
* $I = MK^2$
* $\alpha = \frac{a}{R}$
* $f = ma\left(\frac{K^2}{R^2}\right)$
* $a = \frac{g\sin\theta}{\left(1+\frac{K^2}{R^2}\right)}$
* $v = \sqrt{\frac{2gh}{\left(1+\frac{K^2}{R^2}\right)}}$
* $t = \sqrt{\frac{2h\left(1+\frac{K^2}{R^2}\right)}{g\sin^2\theta}}$

19. Explain the law of conservation of angular momentum with suitable examples.

Law of conservation of angular momentum

When no external torque act on the body, the net angular momentum of a rotating rigid body remains constant.

* $\tau = \frac{dL}{dt}$
* If $\tau = 0$ then $L = I\omega =$ constant.

Examples:

* An ice dancer spins slowly when the hands are stretched out and spins faster when the hands are brought close to the body.
* Stretching of hands away from the body increases moment of inertia, thus the angular velocity decreases resulting in slower spin.
* When the hands are brought close to the body, the moment of inertia decreases and thus angular velocity increases resulting in faster spin.
* A diver while in air curls the body close to decrease the moment of inertia, which in turn helps to increase the number of somersaults in the air.