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12th BM matrix 3 marks test 3

12th Standard 2019 EM

Date : 10-Jun-19

Business Maths

Reg.No. :

Time : 00:45:00 Hrs

Total Marks : 30

10 x 3 = 30

1) Find the rank of the matrix $\begin{pmatrix} 1 & 2 & -1 & 3 \\ 2 & 4 & 1 & -2 \\ 3 & 6 & 3 & -7 \end{pmatrix}$

2) Show that the equationsx+y=5, 2x+y=8 are consistent and solve them.

3) Show that the equations 2x+y=5,4x+2y=10 are consistent and solve them.

4) Show that the equations 3x−2y=6, 6x−4y=10 are inconsistent

5) Find k, if the equations x+2y−3z= −2,3x−y−2z=1,2x+3y−5z=k are consistent.

6) Find k, if the equations x+y+z=7,x+2y+3z=18,y+kz=6 are inconsistent

7) Investigate for what values of ‘a’ and ‘b’ the following system of equations x+y+z=6,x+2y+3z=10, x+2y+az = b have

(i) no solution

(ii) a unique solution

(iii) an infinite number of solutions.

8) Consider the matrix of transition probabilities of a product available in the market in two brands A and B.

$$\begin{matrix} A \\ B \end{matrix}\overset{\begin{matrix} A & \quad B \end{matrix}}{\begin{pmatrix} 0.9 & 0.1 \\ 0.3 & 0.7 \end{pmatrix}}$$

Determine the market share of each brand in equilibrium position.

9) Akash bats according to the following traits. If he makes a hit (S), there is a 25% chance that he will make a hit his next time at bat. If he fails to hit (F), there is a 35% chance that he will make a hit his next time at bat. Find the transition probability matrix for the data and determine Akash’s long- range batting average.

10) 80% of students who do maths work during one study period, will do the maths work at the next study period. 30% of students who do english work during one study period, will do the english work at the next study period.

Initially there were 60 students do maths work and 40 students do english work.

Calculate,

(i) The transition probability matrix

(ii) The number of students who do maths work, english work for the next subsequent 2 study periods.

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10 x 3 = 30

1) Let A = $\begin{pmatrix} 1 & 2 & -1 & 3 \\ 2 & 4 & 1 & -2 \\ 3 & 6 & 3 & -7 \end{pmatrix}$

Order of A is3×4

∴ρ(A) ≤ 3

Consider the third order minors

$\begin{vmatrix} 1 & 2 & -1 \\ 2 & 4 & 1 \\ 3 & 6 & 3 \end{vmatrix} = 0, \quad \begin{vmatrix} 1 & -1 & 3 \\ 2 & 1 & -2 \\ 3 & 3 & -7 \end{vmatrix} = 0$

$\begin{vmatrix} 1 & 2 & 3 \\ 2 & 4 & -2 \\ 3 & 6 & -7 \end{vmatrix} = 0, \quad \begin{vmatrix} 2 & -1 & 3 \\ 4 & 1 & -2 \\ 6 & 3 & -7 \end{vmatrix} = 0$

Since all third order minors vanishes, ∴ρ(A) ≤ 3

Now, let us consider the second order minors,

Consider one of the second order minors $\begin{vmatrix} 2 & -1 \\ 4 & 1 \end{vmatrix} = 6 \neq 0$

There is a minor of order 2 which is not zero.

$\therefore \rho(A) = 2$

2) The matrix equation corresponding to the given system is

$\begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 8 \end{pmatrix}$

A X=B

Matrix A	Augmented matrix [A,B]	Elementary Transformation
$\begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}$ $\sim \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 1 & 1 & 5 \\ 2 & 1 & 8 \end{pmatrix}$ $\sim \begin{pmatrix} 1 & 1 & 5 \\ 0 & -1 & -2 \end{pmatrix}$	$R_2 \rightarrow R_2 - 2R_1$
$\rho(A) = 2$	$\rho([A, B]) = 2$	

Number of non-zero rows is 2.

$\rho(A) = \rho([A, B]) = 2 =$ Number of unknowns.

The given system is consistent and has unique solution.

Now, the given system is transformed into

$\begin{pmatrix} 0 & 1 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$

x +y=5

y=2

$\therefore (1) \Rightarrow x + 2 = 5$

x=3

Solution is x=3,y=2

3) The matrix equation corresponding to the system is

$$\begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 10 \end{pmatrix}$$

A X=B

Matrix A	Augmented matrix [A,B]	Elementary Transformation
$\begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix}$ $\sim \begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix}$	$\begin{pmatrix} 2 & 1 & 5 \\ 4 & 2 & 10 \end{pmatrix}$ $\sim \begin{pmatrix} 2 & 1 & 5 \\ 4 & 2 & 10 \end{pmatrix}$	$R_2 \to R_2 - 2R_1$
$\rho(A) = 1$	$\rho([A, B]) = 1$	

$\rho(A) = \rho([A, B]) = 1 <$ number of unknowns

∴ The given system is consistent and has infinitely many solutions.

Now, the given system is transformed into the matrix equation.

$$\begin{pmatrix} 2 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$$

$\Rightarrow 2x + y = 5$

Let us take y=k,k ∈ R

$\Rightarrow 2x + k = 5$

$x = \frac{1}{2}(5 - k)$

$x = \frac{1}{2}(5 - k), y = k \quad for \quad all \quad k \in R$

Thus by giving different values for k, we get different solution. Hence the system has infinite number of solutions.

4) The matrix equation corresponding to the given system is

$$\begin{pmatrix} 3 & -2 \\ 6 & -4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ 10 \end{pmatrix}$$

AX=B

Matrix A	Augmented matrix [A,B]	Elementary Transformation
$\begin{pmatrix} 3 & -2 \\ 6 & -4 \end{pmatrix}$ $\sim \begin{pmatrix} 3 & -2 \\ 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 3 & -2 & 6 \\ 6 & -4 & 10 \end{pmatrix}$ $\sim \begin{pmatrix} 3 & -2 & 6 \\ 0 & 0 & -2 \end{pmatrix}$	$R_2 \to R_2 - 2R_1$
$\rho(A) = 1$	$\rho([A, B]) = 2$	

$\therefore \rho([A, B]) = 2, \rho(A) = 1$

$\rho(A) \neq \rho([A, B])$

∴ The given system is inconsistent and has no solution.

5) The matrix equation corresponding to the given system is

$$\begin{pmatrix} 1 & 2 & -3 \\ 3 & -1 & -2 \\ 2 & 3 & -5 \end{pmatrix}\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ k \end{pmatrix}$$

AX=B

Augmented matrix [A,B]	Elementary Transformation
$\begin{pmatrix} 1 & 2 & -3 & -2 \\ 3 & -1 & -2 & 1 \\ 2 & 3 & -5 & k \end{pmatrix}$	
$\sim \begin{pmatrix} 1 & 2 & -3 & -2 \\ 0 & -7 & 7 & 7 \\ 0 & -1 & 1 & 4+k \end{pmatrix}$	$R_2 \to R_2 - 3R_1$ $R_3 \to R_3 - 2R_1$
$\sim \begin{pmatrix} 1 & 2 & -3 & -2 \\ 0 & -7 & 7 & 7 \\ 0 & 0 & 0 & 21+7k \end{pmatrix}$	$R_3 \to 7R_3 - R_2$
$\rho(A) = 2, \rho([A,B]) = 2 \quad or \quad 3$	

For the equations to be consistent, $\rho([A,B]) = \rho(A) = 2$

∴ 21+7k=0

7k=-21

k=-3

6) The matrix equation corresponding to the given system is

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 0 & 1 & k \end{pmatrix}\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} 7 \\ 18 \\ 6 \end{pmatrix}$$

AX=B

Augmented matrix [A,B]	Elementary Transformation
$\begin{pmatrix} 1 & 1 & 1 & 7 \\ 1 & 2 & 3 & 18 \\ 0 & 1 & k & 6 \end{pmatrix}$	
$\sim \begin{pmatrix} 1 & 1 & 1 & 7 \\ 0 & 1 & 2 & 11 \\ 0 & 1 & k & 6 \end{pmatrix}$	$R_2 \to R_2 - R_1$
$\sim \begin{pmatrix} 1 & 1 & 1 & 7 \\ 0 & 1 & 2 & 11 \\ 0 & 0 & k-2 & -5 \end{pmatrix}$	$R_3 \to R_3 - R_2$

For the equations to be inconsistent

$\rho([A,B]) \neq \rho(A)$

It is possible if k−2=0.

∴ k=2

7) The matrix equation corresponding to the given system is

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & a \end{pmatrix}\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} 6 \\ 10 \\ b \end{pmatrix}$$

AX=B

Augmented matrix [A,B]	Elementary Transformation
$\begin{pmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & a & b \end{pmatrix}$	
$\sim \begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & a-1 & b-6 \end{pmatrix}$	$R_2 \to R_2 - R_1$ $R_3 \to R_3 - R_1$
$\sim \begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & a-3 & b-10 \end{pmatrix}$	

Case (i) For no solution:

The system possesses no solution only when \rho (A)\neq ([A,B]) which is possible only whena−3=0 andb−10\neq 0.

Hence for a=3,b\neq 10 , the system possesses no solution.

Case (ii) For a unique solution:

The system possesses a unique solution only when \rho (A)= ([A,B])=number of unknowns.

i.e when\rho (A)=\rho ([A,B])=3

Which is possible only when a−3\neq 0 and b may be any real number as we can observe .

Hence for a\neq and b \in R, the system possesses a unique solution.

Case (iii) For an infinite number of solutions:

The system possesses an infinite number of solutions only when

\rho (A)=\rho ([A,B])<number of unknowns

i,e when \rho (A)=\rho ([A,B])=2<3(number of unknowns) which is possible only when a−3=0,b−10=0

Hence for a = 3, b =10, the system possesses infinite number of solutions.

8) Transition probability matrix

T=_{ B }^{ A }\left(\begin{matrix} \overset { A }{ 0.9 } & \overset { B }{ 0.1 } \\ 0.3 & 0.7 \end{matrix} \right)

At equilibrium, (A B) T=(AB) where A+B=1

(A B) \left(\begin{matrix} 0.9 & 0.1 \\ 0.3 & 0.7 \end{matrix} \right) =(A B)

0.9A+ 0.3B = A

0.9A+0.3(1−A) = A

0.9A−0.3A+0.3 = A

0.6A + 0.3 = A

0.4A = 0.3

A=\frac { 0.3 }{ 0.4 } =\frac { 3 }{ 4 }

B=1-\frac { 3 }{ 4 } =\frac { 1 }{ 4 }

Hence the market share of brand A is 75% and the market share of brand B is 25%

9) The Transition probability matrix is T = \left(\begin{matrix} 0.25 & 0.75 \\ 0.35 & 0.65 \end{matrix} \right)

At equilibrium, (S F) \left(\begin{matrix} 0.25 & 0.75 \\ 0.35 & 0.65 \end{matrix} \right) =(S F) where S + F = 1

0.25 S + 0.35 F = S

0.25 S + 0.35 (1 – S) = S

On solving this, we get S=\frac { 0.35 }{ 1.10 }

\therefore Akash's batting average is 31.8%

10) (i) Transition probability matrix T= _{ E }^{ M }\left(\begin{matrix} \overset { M }{ 0.8 } & \overset { E }{ 0.2 } \\ 0.7 & 0.3 \end{matrix} \right)

After one study period, \left(\overset { M }{ 60\quad } \overset { E }{ 40 } \right) _{ E }^{ M }\left(\begin{matrix} \overset { M }{ 0.8 } & \overset { E }{ 0.2 } \\ 0.7 & 0.3 \end{matrix} \right) =\left(\overset { M }{ 76\quad } \overset { E }{ 24 } \right)

So in the very next study period, there will be 76 students do maths work and 24 students do the English work.

After two study periods, _{ E }^{ M }\left(\begin{matrix} \overset { M }{ 0.8 } & \overset { E }{ 0.2 } \\ 0.7 & 0.3 \end{matrix} \right) = (60.8+16.8 15.2+7.2)

=(77.6 22.4)

After two study periods there will be 78 (approx) students do maths work and 22 (approx) students do English work.