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MODEL QUESTION PAPER

HIGHER SECONDARY MATHEMATICS VOLUME II

CLASS: 12^{TH} STANDARD

Time: 3 Hrs **Marks: 90**

PART – A (20x1=20)

Choose the correct answer

1. The position of a particle moving along a horizontal line at any time t is given by $s(t) = 3t^2 - 2t - 8$**. The time at which the particle is at rest is**

(1) $t = 0$ (2) $t = \frac{1}{3}$ (3) $t = 1$ (4) $t = 3$

2. The number given by the Rolle's theorem for the function $x^3 - 3x^2$, $x \in [0, 3]$ **is**

(1) 1 (2) $\sqrt{2}$ (3) $\frac{3}{2}$ (4) 2

3. The maximum value of the product of two positive numbers when their sum of the squares is 200 is

(1) 100 (2) $25\sqrt{7}$ (3) 28 (4) $24\sqrt{14}$

4. A circular template has a radius of 10 cm . The measurement of radius has an approximate error of 0.02 cm. Then the percentage error in calculating area of this template is

(1) 0.2% (2) 0.4% (3) 0.04% (4) 0.08%

5. If $g(x, y) = 3x^2 - 5y + 2y^2$, $x(t) = e^t$ **and** $y(t) = \cos t$, **then** $\frac{dg}{dt}$ **is equal to**

(1) $6e^{2t} + 5\sin t - 4\cos t \sin t$ (2) $6e^{2t} - 5\sin t + 4\cos t \sin t$

(3) $3e^{2t} + 5\sin t + 4\cos t \sin t$ (4) $3e^{2t} - 5\sin t + 4\cos t \sin t$

6. Linear approximation for $g(x) = \cos x$ **at** $x = \frac{\pi}{2}$ **is**

(1) $x + \frac{\pi}{2}$ (2) $-x + \frac{\pi}{2}$ (3) $x - \frac{\pi}{2}$ (4) $-x - \frac{\pi}{2}$

7. For any value of $n \in Z$, $\int_0^{\pi} e^{\cos^2 x} \cos^3[(2n+1)x]dx$ **is**

(1) $\frac{\pi}{2}$ (2) π (3) 0 (4) 2

8. If $f(x) = \int_0^x t\cos t\, dt$, **then** $\frac{df}{dx}$ **is**

(1) $\cos x - x\sin x$ (2) $\sin x + x\cos x$ (3) $x\cos x$ (4) $x\sin x$

9. The volume of solid of revolution of the region bounded by $y^2 = x(a - x)$ **about** x **axis is**

(1) πa^3 (2) $\frac{\pi a^3}{4}$ (3) $\frac{\pi a^3}{5}$ (4) $\frac{\pi a^3}{6}$

10. The order and degree of the differential equation $\sqrt{\sin x}(dx + dy) = \sqrt{\cos x}(dx - dy)$ **is**

(1) 1, 2 (2) 2, 2 (3) 1, 1 (4) 2, 1

11. The solution of the differential equation $\frac{dy}{dx}=\frac{y}{x}+\frac{\alpha\left(\frac{y}{x}\right)}{\alpha'\left(\frac{y}{x}\right)}$ **is**

(1) $x\alpha\left(\frac{y}{x}\right)=k$ **(2)** $\alpha\left(\frac{y}{x}\right)=kx$ **(3)** $y\alpha\left(\frac{y}{x}\right)=k$ **(4)** $\alpha\left(\frac{y}{x}\right)=ky$

12. P is the amount of certain substance left in after time t. If the rate of evaporation of the substance is proportional to the amount remaining, then

(1) $P=Ce^{kt}$ **(2)** $P=Ce^{-kt}$ **(3)** $P=Ckt$ **(4)** $Pt=C$

13. If $x=r\cos\theta,\ y=r\sin\theta$ **then** $\frac{\partial r}{\partial x}$ **is**

(1) $\sec\theta$ **(2)** $\sin\theta$ **(3)** $\cos\theta$ **(4)** $\operatorname{cosec}\theta$

14. A random variable X **has binomial distribution with** $n=25$ **and** $p=0.8$ **then standard deviation of** X **is**

(1) 6 **(2)** 4 **(3)** 3 **(4)** 2

15. Which of the following is a discrete random variable?

I. The number cars crossing a particular signal in a day

II. The number of customers in a queue to buy train tickets at a moment

III. The time taken to complete a telephone call

(1) I and II **(2) II only** **(3) III only** **(4) II and III**

16. If in 6 trials X **is a binomial variable which follows the relation** $9P(X=4)=P(X=2)$ **then the probability of success is**

(1) 0.125 **(2) 0.25** **(3) 0.375** **(4) 0.75**

17. The area of the region bounded by $y=x$, x **axis between** $x=1$ **and** $x=2$ **is**

(1) $\frac{3}{2}$ **(2)** $\frac{5}{2}$ **(3)** $\frac{1}{2}$ **(4)** $\frac{7}{2}$

18. The operation * defined by $a*b=\frac{ab}{7}$ **is not a binary operation on**

(1) Q^{+} **(2)** Z **(3)** R **(4)** C

19. Which of the following statements has the truth value T**?**

(1) $\sin x$ **is an even function**

(2) Every square matrix is non – singular

(3) The product of complex number and its conjugate is purely imaginary

(4) $\sqrt{5}$ **is an irrational number**

20. The proposition $p\wedge(\neg p\vee q)$ **is**

(1) a tatutology **(2) a contradiction**

(3) logically equivalent to $p\wedge q$ **(4) logically equivalent to** $p\vee q$

PART – B (7x2=14)

Answer any 7 of the following questions. Q.No 30 is compulsory

21. The temperature T **in Celsius in a long rod of length 10m, insulated at both ends, is a function of length** x **given by** $T=x(10-x)$**. Prove that the rate of change of temperature at the midpoint of the rod is zero.**

22. Write the Taylor series expansion of $\frac{1}{x}$ **about** $x=2$

23. The time T taken for a complete oscillation of a single pendulum with length l is given by the equation $T = 2\pi\sqrt{l/g}$ where g is a constant. Find the approximate percentage error in the calculated value of T corresponding to an error of 2 percent in the value of l

24. Evaluate $\int_2^3 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}$

25. Form the differential equation by eliminating the arbitrary constants A and B from $y = A\cos x + B\sin x$

26. Show that $y = ae^{-3x} + b$ where a and b are arbitrary constants is a solution of the differential equation $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} = 0$

27. If the probability mass function $f(x)$ of a random variable X is given by

x	**1**	**2**	**3**	**4**
$f(x)$	$\frac{1}{12}$	$\frac{5}{12}$	$\frac{5}{12}$	$\frac{1}{12}$

Find (i) $P(X \le 3)$ **(ii)** $P(X \ge 2)$

28. Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ **be any two boolean matrices of the same type. Find $A \vee B$ and $A \wedge B$**

29. Establish the equivalence property $p \rightarrow q \equiv \neg p \vee q$

30. If $U = (x-y)(y-z)(z-x)$ **then prove that** $U_x + U_y + U_z = 0$

PART – C (7x3=21)

Answer any 7 of the following questions. Q.No 40 is compulsory

31. Using Lagrange's mean value theorem determine the values of x for $f(x) = x^3 - 3x + 2,\ x \in [-2, 2]$

32. Find the linear approximation for $f(x) = \sqrt{1+x}$, $x \ge 1$ at $x_0 = 3, \Delta_x = 0.2$ **use the linear approximation to estimate** $f(3, 2)$

33. A coat of paint of thickness 0.2 cm is applied to the faces of a cube whose edge is 10cm. Use the differentials to find approximately how many cubic centimeters of paint is used to paint this cube. Also calculate the exact amount of paint used to paint this cube.

34. Evaluate $\int_0^1 \frac{\sin(3\tan^{-1}x)\tan^{-1}x}{1+x^2}\,dx$

35. Solve $\frac{dy}{dx} + 2y\cot x = 3x^2 \operatorname{cosec}^2 x$

36. The engine of a motor boat moving at 10 m/s is shut off. Given that the retardation at any subsequent time (after shutting the engine) equal to the velocity at that time. Find the velocity after 2 seconds of switching off the engine.

37. Suppose two coins are tossed once. If X denotes the number of tails(i) Write down the sample space. (ii) find the inverse image of 1 (iii) the values of the random variable and number of elements in its inverse images.

38. Find the mean and variance of a random variable X whose probability density function is $f(x)=\begin{cases}\lambda e^{-\lambda x} & for\ x\geq 0\\ 0 & otherwise\end{cases}$

39. Examine the closure, commutative and associative properties satisfied by $*$ on Q given by $a*b=(a+b)/2,\ a,b\in Q$

40. Find the volume of a solid formed by revolving the region bounded by $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,\ a>b$ **about the minor axis.**

PART – D (7x5=35)

Answer all the questions

41. a) A stone is dropped into a pond causing ripples in the form of concentric circles. The radius r of the outer ripple is increasing at a constant rate at 2 cm per second. When the radius is 5 cm find the rate of changing of the total area of the disturbed water.

OR

b) A steel plant is capable of producing x tones per day of a low – grade steel and y tones per day of a high – grade steel, where $y=40-5x/10-x$. If the fixed market price of low – grade steel is half that of high – grade steel, then what should be optimal productions in low – grade steel and high – grade steel in order to have maximum receipts.

42. a) $W(x,y,z)=xy+yz+zx,\ x=u-v,\ y=uv,\ z=u+v,\ u,v\in R$ **. Find** $\frac{\partial W}{\partial u},\frac{\partial W}{\partial v}$ **and evaluate them at** $\left(\frac{1}{2},1\right)$

OR

b) If $u=\tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$ **then prove that** $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\sin 2u$

43. a) Evaluate $\int_1^2 (4x^2-1)dx$ **as the limits of sums**

OR

b) The region enclosed by the circle $x^2+y^2=a^2$ is divided into two segments by the line $x=h$. Find the area of the smaller segment

44. a) Find the equation of the curve whose slope is $y-1/x^2+x$ and which passes through the point $(1,0)$

OR

b) $(x^2+y^2)dy=xydx$ **. It is given that $y(1)=1$ and $y(x_0)=e$. Find the value of x_0**

45. a) Find the constant C such that $f(x)=\begin{cases}Cx^2, & 1<x<4\\ 0, & otherwise\end{cases}$ **is a density function. Compute (i)** $P(1.5<X<3.5)$ **(ii)** $P(X\leq 2)$ **(iii)** $P(3<X)$

OR

b) On the average 20% of the products manufactured by ABC company are found to be defective. If we select 6 of these products at random and X denotes the number of defective products find probability that (i) two products are defective (ii) at most one product is defective (iii) at least two products are defective.

46. a) Verify (i) closure property, (ii) commutative property, (iii) associative property, (iv) existence of identity and (v) existence of inverse for the operation $\times_{11}$ on a subset $A=\{1, 3, 4, 5, 9\}$ of the set of remainders $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

OR

b) Let A be $Q\setminus\{1\}$ Define $*$ on A by $x*y=x+y-xy$. Is * binary on A? if so examine the commutative, associative, existence of identity, existence of inverse properties for the operation * on A

47. a) For the function $f(x)=4x^3+3x^2-6x+1$ find the intervals of monotonicity, local extrema, intervals of concavity and points of inflection.

OR

b) Assume that the rate at which radioactive nuclei decay is proportional to the number of such nuclei that are present in a given sample. In a certain sample 10% of the original number of radioactive nuclei have undergone disintegration in a period of 100 years. What percentage of the original radioactive nuclei will remain after 1000 years?

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