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# **BUSINESS MATHS 12<sup>TH</sup> E/M**

## **QUESTION & ANSWER BOOKLET**

**Prepared under the guidance of our respectable CEO of thiruvallur district.**

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## STD XII - BUSINESS MATHS

### CHAPTER -1

#### 2 MARKS

1. Find the rank of the matrix  $\begin{pmatrix} 0 & -1 & 5 \\ 2 & 4 & -6 \\ 1 & 1 & 5 \end{pmatrix}$

$$\text{Let } A = \begin{pmatrix} 0 & -1 & 5 \\ 2 & 4 & -6 \\ 1 & 1 & 5 \end{pmatrix}$$

$$|A| = \begin{vmatrix} 0 & -1 & 5 \\ 2 & 4 & -6 \\ 1 & 1 & 5 \end{vmatrix} = 6 \neq 0$$

$$\rho(A) = 3$$

2. Find the rank of the matrix  $\begin{pmatrix} 5 & 3 & 0 \\ 1 & 2 & -4 \\ -2 & -4 & 8 \end{pmatrix}$

$$\text{Let } A = \begin{pmatrix} 5 & 3 & 0 \\ 1 & 2 & -4 \\ -2 & -4 & 8 \end{pmatrix}$$

$$|A| = \begin{vmatrix} 5 & 3 & 0 \\ 1 & 2 & -4 \\ -2 & -4 & 8 \end{vmatrix} = 0$$

$$(\because \begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix} = 10 - 3 = 7 \neq 0)$$

$$\rho(A) = 2.$$

3. Find the rank of:-

i)  $\begin{pmatrix} 1 & 2 & -1 \\ 2 & 4 & 1 \\ 3 & 6 & 3 \end{pmatrix}$

ii)  $\begin{pmatrix} 1 & -1 & 3 \\ 2 & 1 & -2 \\ 3 & 3 & -7 \end{pmatrix}$

Find the rank of  $\begin{pmatrix} 3 & 1 & -5 & -1 \\ 1 & -2 & 1 & -5 \\ 1 & 5 & -7 & 2 \end{pmatrix}$

$$\text{Let } A = \begin{pmatrix} 3 & 1 & -5 & -1 \\ 1 & -2 & 1 & -5 \\ 1 & 5 & -7 & 2 \end{pmatrix}$$

$$\begin{array}{l} N \\ \begin{pmatrix} 1 & 5 & -7 & 2 \\ 1 & -2 & 1 & -5 \\ 3 & 1 & -5 & -1 \end{pmatrix} \end{array}$$

$$\begin{array}{l} N \\ \begin{pmatrix} 1 & 5 & -7 & 2 \\ 0 & -7 & 8 & -7 \\ 0 & -14 & 16 & -7 \end{pmatrix} \begin{array}{l} R_1 \\ R_2 \rightarrow R_2 - R_1 \\ R_3 \rightarrow R_3 - 3R_1 \end{array} \end{array}$$

$$\begin{array}{l} N \\ \begin{pmatrix} 1 & 5 & -7 & 2 \\ 0 & -7 & 8 & -7 \\ 0 & 0 & 0 & 7 \end{pmatrix} \begin{array}{l} R_1 \\ R_2 \rightarrow R_2 - R_1 \\ R_3 \rightarrow R_3 - 3R_1 \end{array} \end{array}$$

$$\therefore \rho(A) = 3$$

Find the rank of ..

$$\text{i) } \begin{pmatrix} 1 & -2 & 3 & 4 \\ -2 & 4 & -1 & -3 \\ -1 & 2 & 7 & 6 \end{pmatrix}$$

$$\text{ii) } \begin{pmatrix} 1 & -2 & 3 & 0 \\ -3 & 2 & 5 & 0 \\ 3 & 4 & 6 & 0 \end{pmatrix}$$

**3 MARKS**

4. Find the rank of the matrix  $A = \begin{pmatrix} 0 & 1 & 2 & 1 \\ 1 & 2 & 3 & 2 \\ 3 & 1 & 1 & 3 \end{pmatrix}$

The order of A is 3 x 4.

$$\therefore \rho(A) \leq 3.$$

$$A = \begin{pmatrix} 0 & 1 & 2 & 1 \\ 1 & 2 & 3 & 2 \\ 3 & 1 & 1 & 3 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & 2 & 1 \\ 3 & 1 & 1 & 3 \end{pmatrix} R_1 \leftrightarrow R_2$$

$$\sim \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & 2 & 1 \\ 0 & -5 & -8 & -3 \end{pmatrix} R_3 \rightarrow R_3 - 3R_1$$

$$\sim \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 2 & 2 \end{pmatrix} R_3 \rightarrow R_3 + 5R_2$$

$$\therefore \rho(A) = 3.$$

5. Find the rank of the matrix  $A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 3 & 4 & 5 & 2 \\ 2 & 3 & 4 & 0 \end{pmatrix}$

The order of A is 3 x 4

$$\therefore \rho(A) \leq 3.$$

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 3 & 4 & 5 & 2 \\ 2 & 3 & 4 & 0 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & -1 \\ 0 & 1 & 2 & -2 \end{pmatrix} \begin{matrix} R_2 \rightarrow R_2 - 3R_1 \\ R_3 \rightarrow R_3 - 2R_1 \end{matrix}$$

$$\sim \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & -2 & 1 \\ 0 & 0 & 0 & -1 \end{pmatrix} R_3 \rightarrow R_3 - R_2$$

The number of non zero rows is 3.

$$\therefore \rho(A) = 3.$$

6. Find the rank of  $\begin{pmatrix} -1 & 2 & -2 \\ 4 & -3 & 4 \\ -2 & 4 & -4 \end{pmatrix}$

$$\text{Let } A = \begin{pmatrix} -1 & 2 & -2 \\ 4 & -3 & 4 \\ -2 & 4 & -4 \end{pmatrix}$$

The order of A is 3 x 3.

$$\therefore \rho(A) \leq 3 [\text{Since minimum of } (3, 3) \text{ is } 3]$$

$$A = \begin{pmatrix} -1 & 2 & -2 \\ 4 & -3 & 4 \\ -2 & 4 & -4 \end{pmatrix}$$

$$\sim \begin{pmatrix} -1 & -2 & -2 \\ 4 & -3 & 4 \\ -2 & 4 & -4 \end{pmatrix} R_1 \rightarrow R_1(-1)$$

$$\sim \begin{pmatrix} 1 & -2 & 2 \\ 0 & 5 & -4 \\ -2 & 4 & -4 \end{pmatrix} R_2 \rightarrow R_2 - 4R_1$$

$$\sim \begin{pmatrix} 1 & -2 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 0 \end{pmatrix} R_3 \rightarrow R_3 + 2R_1$$

$$\therefore \rho(A) = 2.$$

7. Find the rank of  $\begin{pmatrix} 3 & 1 & -5 & -1 \\ 1 & -2 & 1 & -5 \\ 1 & 5 & -7 & 2 \end{pmatrix}$

$$A = \begin{pmatrix} 3 & 1 & -5 & -1 \\ 1 & -2 & 1 & -5 \\ 1 & 5 & -7 & 2 \end{pmatrix}$$

The order of A is 3 x 4.

$$\therefore \rho(A) \leq 3 [\text{Since minimum of } (3, 4) \text{ is } 3]$$

$$A = \begin{pmatrix} 3 & 1 & -5 & -1 \\ 1 & -2 & 1 & -5 \\ 1 & 5 & -7 & 2 \end{pmatrix}$$

$$\sim = \begin{pmatrix} 1 & 5 & -7 & 2 \\ 1 & -2 & 1 & -5 \\ 3 & 1 & -5 & -1 \end{pmatrix} R_1 \leftrightarrow R_3$$

$$\sim = \begin{pmatrix} 1 & 5 & -7 & 2 \\ 0 & -7 & 8 & -7 \\ 3 & 1 & -5 & -1 \end{pmatrix} R_2 \rightarrow R_2 - R_1$$

$$\sim = \begin{pmatrix} 1 & 5 & -7 & 2 \\ 0 & -7 & 8 & -7 \\ 0 & -14 & 16 & -7 \end{pmatrix} R_3 \rightarrow R_3 - 3R_1$$

$$\sim = \begin{pmatrix} 1 & 5 & -7 & 2 \\ 0 & -7 & 8 & -7 \\ 0 & 0 & 0 & 7 \end{pmatrix} R_3 \rightarrow R_3 - 2R_1$$

$$\therefore \rho(A) = 3.$$

8. Find the rank of  $\begin{pmatrix} 1 & -2 & 3 & 4 \\ -2 & 4 & -1 & -3 \\ -1 & 2 & 7 & 6 \end{pmatrix}$

$$A = \begin{pmatrix} 1 & -2 & 3 & 4 \\ -2 & 4 & -1 & -2 \\ -1 & 2 & 7 & 6 \end{pmatrix}$$

The order of  $\varphi$  is  $3 \times 4$

$$\therefore \rho(A) \leq \text{minimum of } (3, 4)$$

$$\therefore \rho(A) \leq 3$$

$$A = \begin{pmatrix} 1 & -2 & 3 & 4 \\ -2 & 4 & -1 & -3 \\ -1 & 2 & 7 & 6 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & -2 & 3 & 4 \\ 0 & 0 & 5 & 5 \\ -1 & 2 & 7 & 6 \end{pmatrix} R_2 \rightarrow R_2 + 2R_1$$

$$\sim \begin{pmatrix} 1 & -2 & 3 & 4 \\ 0 & 0 & 5 & 5 \\ 0 & 0 & 10 & 10 \end{pmatrix} R_3 \rightarrow R_3 + R_1$$

$$\sim \begin{pmatrix} 1 & -2 & 3 & 4 \\ 0 & 0 & 5 & 5 \\ 0 & 0 & 10 & 10 \end{pmatrix} R_3 \rightarrow R_3 + 2R_1$$

$$\therefore \rho(A) = 2.$$

### 5 MARKS

9. Solve by Cramer's rule  $x + y + z = 4$ ,  $2x - y + 3z = 1$ ,  $3x + 2y - z = 1$ .

$$\text{Solve: } \Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & -1 & 3 \\ 3 & 2 & -1 \end{vmatrix} = 13 \neq 0$$

$$\Delta x = \begin{vmatrix} 4 & 1 & 1 \\ 1 & -1 & 3 \\ 1 & 2 & -1 \end{vmatrix} = -13$$

$$\Delta y = \begin{vmatrix} 1 & 4 & 1 \\ 2 & 1 & 3 \\ 3 & 1 & -1 \end{vmatrix} = 39$$

$$\Delta z = \begin{vmatrix} 1 & 1 & 4 \\ 2 & -1 & 1 \\ 3 & 2 & 1 \end{vmatrix} = 26$$

$\therefore$  By Cramer's rule

$$x = \frac{\Delta x}{\Delta} = \frac{-13}{13} = -1$$



$$y = \frac{\Delta y}{\Delta} = \frac{39}{13} = 3$$

$$z = \frac{\Delta z}{\Delta} = \frac{26}{13} = 2$$

∴ The solution is  $(x, y, z) = (-1, 3, 2)$

10. The price of 3 Business Mathematics books, 2 Accountancy books and one Commerce books is ₹840 . The priced of 2 Business Mathematics books, one Accountancy books and one Commerce book is ₹570. The price of one Business Mathematics book, one Accountancy book and 2 Commerce books is ₹630. Find the cost of each book by using Cramer's Rule.

**Solve:** Let 'x' be the cost of a Business Mathematics book

Let 'y' be the cost of a Accountancy book

Let 'z' be the cost of a Commerce book.

$$3x + 2y + z = 840$$

$$2x + y + z = 570$$

$$x + y + 2z = 630$$

$$\Delta = \begin{vmatrix} 3 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 2 \end{vmatrix} \neq 0$$

$$\Delta x = \begin{vmatrix} 840 & 2 & 1 \\ 570 & 1 & 1 \\ 630 & 1 & 2 \end{vmatrix} = -240$$

$$\Delta y = \begin{vmatrix} 3 & 840 & 1 \\ 2 & 570 & 1 \\ 1 & 630 & 2 \end{vmatrix} = -306$$

$$\Delta z = \begin{vmatrix} 3 & 2 & 840 \\ 2 & 1 & 570 \\ 1 & 1 & 630 \end{vmatrix} = -360$$

$$x = \frac{\Delta x}{\Delta} = \frac{240}{-2} = 120$$

$$z = \frac{\Delta z}{\Delta} = \frac{360}{-2} = 180$$

$$y = \frac{\Delta y}{\Delta} = \frac{1300}{-7} = 150$$

∴ The cost of Business Mathematics books  
 The cost of an Accountancy book is ₹150 and  
 The cost of a Commerce book is ₹180.

11. An automobile company uses three types of steel  $S_1$ ,  $S_2$  and  $S_3$  for providing three different types of cars  $C_1$ ,  $C_2$  and  $C_3$ . Steel requirement  $P_1$  (in tonnes) for each type of car and total available Steel of all the three types are summarized in the following table.

Types of Steel	Types of Car			Total steel available
	$C_1$	$C_2$	$C_3$	
$S_1$	3	2	4	28
$S_2$	1	1	2	13
$S_3$	2	2	1	14

Determine the number of cars of each type which can be produced by Cramer's rule.

**Solve:**

$$3x + 2y + 4z = 28$$

$$x + y + 2z = 13$$

$$2x + 2y + z = 14$$

$$\text{Here: } \Delta = \begin{vmatrix} 3 & 2 & 4 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix} = -3 \neq 0$$

$$\Delta_x = -6, \Delta_y = -9, \Delta_z = -12$$

∴ By Cramer's rule

$$x = \frac{\Delta_x}{\Delta} = \frac{-6}{-3} = 2$$

$$y = \frac{\Delta_y}{\Delta} = \frac{-9}{-3} = 3$$

$$z = \frac{\Delta_z}{\Delta} = \frac{-12}{-3} = 4$$

∴ The numbers of cars of each type which can be produced are 2, 3 and 4.

12. Solve:  $2x + y - z = 3$ ,  $x + y + z = 1$ ,  $x - 2y - 3z = 4$

$$\Delta = \begin{vmatrix} 2 & 1 & -1 \\ 1 & 1 & 1 \\ 1 & -2 & -3 \end{vmatrix} = 2$$

$$\begin{vmatrix} 1 & 1 \\ -2 & -3 \end{vmatrix} - 1 \begin{vmatrix} 1 & 1 \\ 1 & -3 \end{vmatrix} - 1 \begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix}$$

$$= 2(-3 + 2) - 1(-3 - 1) - 1(-2 - 1)$$

$$= 2(-1) - 1(-4) - 1(-3)$$

$$= -2 + 4 + 3$$

$$= 5.$$

$$\Delta_x = -3 + 7 + 6$$

$$= 10$$

$$\Delta_y = -14 + 12 - 3$$

$$= -5$$

$$\Delta_z = 12 - 3 - 9$$

$$= 0.$$

$$x = \frac{\Delta_x}{\Delta}$$

$$= \frac{10}{5}$$

$$y = \frac{\Delta_y}{\Delta}$$

$$= \frac{-5}{5}$$

$$= -1$$

$$z = \frac{\Delta_z}{\Delta}$$

$$= \frac{0}{5} = 0$$

$\therefore$  Solution set is  $\{2, -1, 0\}$

13.  $x + y + z = 6$ ,  $2x + 3y - z = 5$ ,  $6x - 2y - 3z = -7$ .

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & -1 \\ 6 & -2 & -3 \end{vmatrix}$$

$$= -11 - 22$$

$$= -33 \neq 0$$

$$\Delta_x = \begin{vmatrix} 0 & 1 & 1 \\ 5 & 3 & -1 \\ -7 & -2 & -3 \end{vmatrix}$$

$$= -33$$

$$\Delta_y = \begin{vmatrix} 1 & 6 & 1 \\ 2 & 5 & -1 \\ 6 & -7 & -3 \end{vmatrix}$$

$$= -66$$

$$\Delta_z = \begin{vmatrix} 1 & 1 & 6 \\ 2 & 3 & 5 \\ 6 & -2 & -7 \end{vmatrix}$$

$$= -99$$

$$x = \frac{\Delta_x}{\Delta} = \frac{-33}{-33} = 1$$

$$y = \frac{\Delta_y}{\Delta} = \frac{-66}{-33} = 2$$

$$z = \frac{\Delta_z}{\Delta} = \frac{-99}{-33} = 3$$

$\therefore$  Solution set is  $\{1, 2, 3\}$

14. In a market survey there commodities A, B and C were considered. In finding out the index number some fixed weights were assigned to the three varieties in each of the commodities. The table below provides the information regarding the consumption of three commodities according to the three. Varieties and also the total weight received by the commodity.

Commodity variety	Variety			Total Weight
	I	II	III	
A	1	2	3	11
B	2	4	5	21

C	2	5	6	27
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Find the weight assigned to the three varieties by using Cramer's Rule.

$$x + 2y + 3z = 11$$

$$2x + 4y + 5z = 21$$

$$3x + 5y + 4z = 27$$

$$\Delta = -1 \neq 0$$

$$\Delta_x = -2$$

$$\Delta_y = -3$$

$$\Delta_z = -1$$

$$x = \frac{\Delta_x}{\Delta} = \frac{-2}{-1} = 2$$

$$y = \frac{\Delta_y}{\Delta} = \frac{-3}{-1} = 3$$

$$z = \frac{\Delta_z}{\Delta} = \frac{-1}{-1} = 1$$

Hence, the weight assigned to the varieties are 2, 3 and 1 respectively.

15. A total of ₹8,500 was invested in three interest earning accounts. The interest rates were 2%, 3%, 6% if the total simple interest for one year was ₹ 380 and the amount invested at 6% was equal to the sum of the amounts in the other two accounts, how much was invested in each account? (use Cramer's rule)

Solve:  $x + y + z = 8500 \rightarrow \textcircled{1}$

$$\frac{2x}{100} + \frac{3y}{100} + \frac{6z}{100} = 380$$

$$\begin{aligned} I &= \frac{Pnr}{100} \\ &= \frac{2x \times 1 \times 2}{100} \\ &= \frac{2x}{100} \end{aligned}$$

$$2x + 3y + 6z = 38000 \rightarrow \textcircled{2}$$

$$x + y - z = 0 \rightarrow \textcircled{3}$$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 6 \\ 1 & 1 & -1 \end{vmatrix}$$

$$= -2 \neq 0$$

$$\Delta_x = -500$$

$$\Delta_y = -8000$$

$$\Delta_z = -8500$$

$$x = \frac{\Delta_x}{\Delta} = \frac{500}{2} = 250$$

$$y = \frac{\Delta_y}{\Delta} = \frac{-8000}{-2} = 4000$$

$$z = \frac{\Delta_z}{\Delta} = \frac{-8500}{-2} = 4250$$

16. Consider the matrix of transition probabilities of a product available in the market in two brands A and B

$$\begin{matrix} & \begin{matrix} A & B \end{matrix} \\ \begin{matrix} A \\ B \end{matrix} & \begin{pmatrix} 0.9 & 0.1 \\ 0.3 & 0.7 \end{pmatrix} \end{matrix}$$

Determine the market share of each brand in equilibrium position.

Solve:

$$T = \begin{matrix} & \begin{matrix} A & B \end{matrix} \\ \begin{matrix} A \\ B \end{matrix} & \begin{pmatrix} 0.9 & 0.1 \\ 0.3 & 0.7 \end{pmatrix} \end{matrix}$$

$(AB)T = (AB)$  Where  $A+B=1$

$$0.9A + 0.3B = A$$

$$0.9A + 0.3(1 - A) = A$$

$$0.9A - 0.3A + 0.3 = A$$

$$A = \frac{0.3}{0.4}$$

$$A = \frac{3}{4}$$

$$B = 1 - \frac{3}{4}$$

$$B = \frac{1}{4}$$

17. 80% of students who do maths work during one study period, will do the maths work at the next study period 30% of students who do English work during one study period will do the English work at the next study period. Initially there were 60 students do maths work and 40 students do English work calculate.

(i) The transition probability matrix

(ii) The number of students who do maths work, English work for the next subsequent 2 study periods.

Solve:

$$\begin{matrix} & \begin{matrix} M & E \end{matrix} \\ \begin{matrix} M \\ E \end{matrix} & \begin{pmatrix} 0.78 & 0.22 \\ 0.77 & 0.23 \end{pmatrix}^2 \end{matrix}$$

$$= (46.8 + 30.8 \quad 13.2 + 9.2)$$

$$= (77.6 \quad 22.4)$$

18. A new transit system has just gone into operation in Chennai of those who use the transit system this year, 30% will switch over to using metro train next year and 70% will continue to use the transit system of those who use metro train this year, 70% will continue to use metro train next year and 30% will switch over to the transit system. Suppose the population of Chennai city remains constant and that 60% of the commuters use the transit system and 40% of the commuters use metro train this year.

(i) What percent of commuters will be using the transit system after one year?

(ii) What percent of commuters will be using the transit system in the long run?

Solve:

$$T = \begin{matrix} & \begin{matrix} A & B \end{matrix} \\ \begin{matrix} A \\ B \end{matrix} & \begin{pmatrix} .70 & .30 \\ .30 & .70 \end{pmatrix} \end{matrix}$$

$$A=60\% = .60$$

$$B=40\% = .4$$



$$\begin{pmatrix} .60 & .40 \end{pmatrix} \begin{pmatrix} .70 & .30 \\ .30 & .70 \end{pmatrix}$$

$$A=54\% \text{ and } B=46\%$$

$$A + B = 1$$

$$A \ B \begin{pmatrix} .7 & .3 \\ .3 & .7 \end{pmatrix} = (A \ B)$$

$$(.7A + .3B \quad .3A + .7B) = (A \ B)$$

$$A = \frac{.3}{.6} = \frac{1}{2} = 0.50$$

∴ Two types of soaps A and B are in the market. Their present market shares are 15% for A and 85% for B of those who bought A the previous year, 65% continue to buy it again while 35% switch over to B of those who bought B the previous year, 55% buy it again and 45% switch over to A. Find their market shares after one year and when is the equilibrium reached?

Solve:

$$T = \begin{matrix} A & B \\ A & \begin{pmatrix} .65 & .35 \\ .45 & .55 \end{pmatrix} \\ B & \end{matrix}$$

$$A = 15\% = .15$$

$$B = 85\% = .85$$

Percentage after one year is

$$= (.48 \quad .52)$$

At equilibrium,

$$(A \ B)T = (A \ B)$$

$$(A \ B) \begin{pmatrix} .65 & .35 \\ .45 & .55 \end{pmatrix} = (A \ B)$$

$$.65A + .45B = A$$

$$.65A + .45(1-A) = A$$

$$A = \frac{.45}{.8}$$

$$= .5625$$

$$= 56.25\%$$

$$B = 1 - A$$



$$= 1 - .5625$$

$$= .4375$$

$$= 43.75\%$$

∴ Equilibrium is reached then A=56.25% and B=43.75%

Transition probability Matrix

$$T = \begin{matrix} & \begin{matrix} A & B \end{matrix} \\ \begin{matrix} A \\ B \end{matrix} & \begin{pmatrix} .65 & .35 \\ .45 & .55 \end{pmatrix} \end{matrix}$$

Y after one year is

$$(.15, .85) \begin{pmatrix} .65 & .35 \\ .45 & .55 \end{pmatrix} = (.48, .52)$$

At equilibrium

$$(A \ B) T = (A \ B)$$

$$(A \ B) \begin{pmatrix} .65 & .35 \\ .45 & .55 \end{pmatrix} = (A \ B)$$

$$.65A + .45B = A$$

$$.65A + .45(1 - A) = A$$

$$A = \frac{.45}{.8} = .5625 = 56.25\%$$

$$B = 1 - A \quad (\because A + B = 1)$$

$$= 1 - .5625$$

$$= .4375$$

$$= 43.75$$

$$A^1 S = 56.25 \quad B^1 S = 43.75\%$$

Two products A and B currently share the market with shares 50% and 50% each respectively. Each week some brand switching takes place of those who

bought A the previous week, 60% buy it again whereas 40% switch over to B of those who bought B the previous week, 80% buy it again where as 20% switch over to A. Find their shares after one week and after two weeks. Of the price was continues, when is the equilibrium reached.

Transition probability matrix.

$$T = \begin{matrix} A & B \\ \begin{pmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{pmatrix} \end{matrix}$$

Shares after one week

$$= (0.5, 0.5) \begin{pmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{pmatrix} = (0.40, 0.60)$$

Shares after one week

$$A = 40\% \quad B = 60\%$$

Shares after two weeks

$$(0.4 \quad 0.6) \begin{pmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{pmatrix} = (0.36, 0.64)$$

$$A = 36\% \quad B = 64\%$$

At equilibrium

$$(A \ B)T = (A \ B)$$

$$A + B = 1$$

$$(A \ B) \begin{pmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{pmatrix} = (A, B)$$

$$0.6A + 0.2B = A$$

$$0.6A + 0.2(1 - A) = A$$

$$A = \frac{0.2}{0.6} = 0.33$$

$$B = 1 - A = 1 - 0.33 = 0.67$$

Equilibrium reach A=33%      B=67%

## CHAPTER -2

### 2 MARKS

1. Evaluate :  $\int \left(x + \frac{1}{x}\right)^2 dx$

$$\int \left(x^2 + 2 + \frac{1}{x^2}\right) dx$$

$$= \int x^2 dx + 2 \int dx + \int \frac{1}{x^2} dx$$

$$= \frac{x^3}{3} + 2x - \frac{1}{x} + C$$

**For Practice:**

2.  $\int \frac{8x+13}{\sqrt{4x+7}} dx$     3.  $\int \frac{e^{3x}+e^{5x}}{e^x+e^{-x}} dx$     4.  $\int \left(1 - \frac{1}{x^2}\right) e^{(x-1)} dx$     5.  $\int \log x dx$
6.  $\int x^5 e^{x^2} dx$     7.  $\int_{-1}^1 \log \left(\frac{2-x}{2+x}\right) dx$

### 3 MARKS

8. Evaluate:  $\int \frac{2x^2-14x+24}{x-3} dx$

Solve:  $\int \left(\frac{2x^2-14x+24}{x-3}\right) dx = \int \frac{(x-3)(2x-8)}{x-3} dx$

$= \int (2x - 8) dx$

$x^2 - 8x + C$

### **For Practice:**

9.  $\int \frac{1}{\sqrt{x+2}-\sqrt{x-2}} dx$

10.  $\int \sqrt{1 - \sin 2x} dx$

11.  $\int x^3 \log x dx$

12.  $\int e^x \left[ \frac{1}{x^2} - \frac{2}{x^3} \right] dx$

13.  $\int e^{3x} \left[ \frac{3x-1}{9x^2} \right] dx$

14. Find the integration for  $\frac{dy}{dx} = \frac{2x}{5x^2+1}$  limiting values as 0 and 1.

15. Evaluate:  $\int_1^T F(x) dx$ , where  $F(x) = \begin{cases} 7x + 3, & \text{if } 1 \leq x \leq 3 \\ 8x, & \text{if } 3 \leq x \leq 4 \end{cases}$

16. Evaluate:  $\int_{-1}^1 \frac{x^5}{a^2-x^2} dx$

17.  $\int_0^{72} \frac{\sin x}{\sin x + \cos x} dx$

18. Evaluate:  $\int_2^5 \frac{\sqrt{x}}{\sqrt{x} + \sqrt{7-x}} dx$

19.  $\int_0^{72} \frac{\sin^7 x}{\sin^7 x + \cos^7 x} dx$

### 5 MARKS

20. Evaluate the integral as the limit of a sum:  $\int_1^2 x^2 dx$

Solve:  $a = 1$ ,  $b = 2$ ,  $f(x) = x^2$

$$h = \frac{b-a}{h} = \frac{2-1}{h} = \frac{1}{h}$$

$$f(a+rh) = f\left(1 + r\frac{1}{h}\right) = f\left(1 + \frac{r}{h}\right) = \left(1 + \frac{r}{h}\right)^2$$

$$f(a+rh) = 1 + \frac{2r}{h} + \frac{r^2}{h^2}$$

$$\therefore \int_a^b f(x)dx = \lim_{h \rightarrow \infty} \sum_{r=1}^h hf(a+rh)$$

$$= \lim_{h \rightarrow \infty} \sum_{r=1}^n \left( \frac{1}{h} + \frac{2r}{h^2} + \frac{r^2}{h^3} \right)$$

$$\left[ 1 + 1 + \frac{2}{6} \right] = \frac{7}{3}$$

$$\therefore \int_1^2 x^2 dx = \frac{7}{3}$$

**For Practice:** Evaluate the following as the limit of sum:

21.  $\int_0^1 (x+4)dx$

22.  $\int_1^3 xdx$

23.  $\int_1^3 (2x+3)dx$

24.

$\int_0^1 x^2 dx$

25.  $\int_1^2 (2x+1)dx$

26.  $\int_0^1 x dx$

27. Evaluate:  $\int \frac{4x^2+2x+6}{(x+1)^2 (x-3)} dx$

28. Evaluate:  $\int \frac{3x^2-2x+5}{(x-1)(x^2+5)} dx$

### **CHAPTER -3 – Integral Calculus - II**

#### **2 MARKS**

1. Find the area bounded by  $y = 4x + 3$  with  $x$ -axis between the lines  $x=1$  and  $x=4$ .

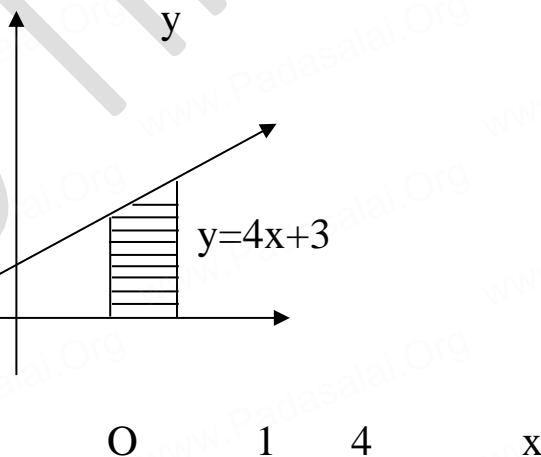
Solution:

$$\text{Area} = \int_a^b y dx$$

$$= \int_1^4 (4x + 3) dx$$

$$= [2x^2 + 3x]^4$$

$$= 39 \text{ sq. unit}$$



2. Find the area of the region bounded by the line  $x - 2y - 12 = 0$ , the  $y$ -axis and the lines  $y = 2$  and  $y = 5$ .

Solve:

Given

$$x - 2y - 12 = 0$$

$$x = 2y + 12$$

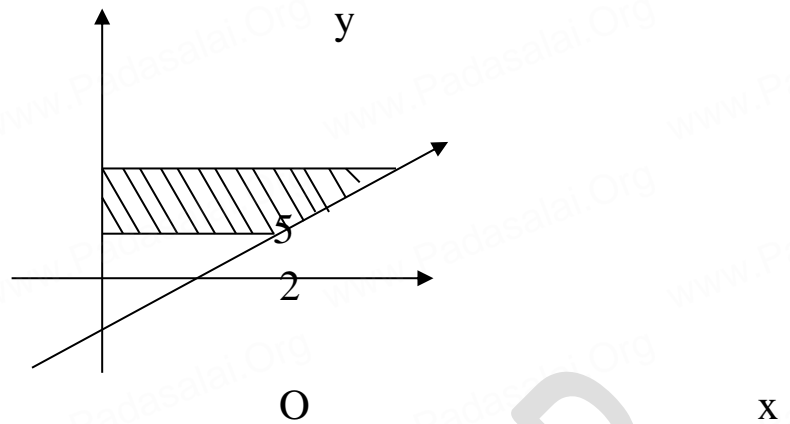
$$\text{Area} = \int_2^5 x dy$$

$$= \int_2^5 (2y + 12) dy$$

$$= [y^2 + 12y]_2^5 = 57 \text{ sq. units}$$

$$= \int_2^5 (2y + 12) dy$$

$$= [y^2 + 12y]_2^5 = 57 \text{ sq. units}$$



3. Find the area bounded by  $y=x$  between the line  $x=-1$  and  $x=2$  with  $x$ -axis.

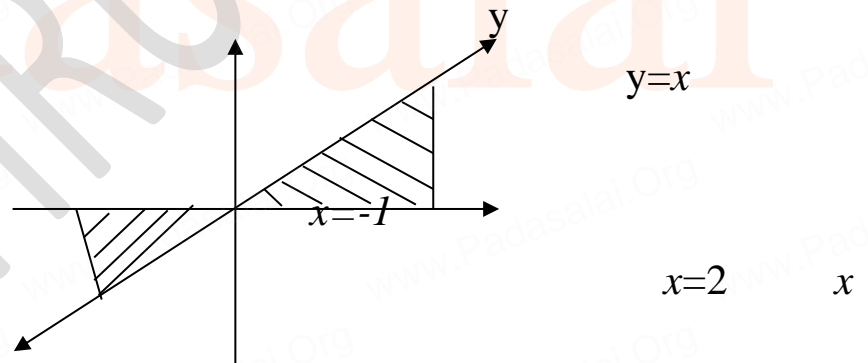
Solution:

Required Area:

$$= \int_{-1}^0 -x dx + \int_0^2 x dx$$

$$= -\left[\frac{x^2}{2}\right]_{-1}^0 + \left[\frac{x^2}{2}\right]_0^2$$

$$= \frac{5}{2} \text{ sq. units}$$



4. The marginal cost function  $MC=2+5e^x$

(i) Find  $C$  if  $C(0)=100$

(ii) Find  $AC$

Solution:  $MC=2+5e^x$

$$C = \int MC dx + k$$



$$= \int (2 + 5e^x) dx + k$$

$$C = 2x + 5e^x + k$$

$$x = 0 \Rightarrow c = 100$$

K=95
------

$$C = 2x + 5e^x + 95$$

$$AC = 2 + \frac{5e^x}{x} + \frac{95}{x}$$

5. Mr.Arul invests Rs.10,000 in ABC Bank each year, which pays an interest of 10% per annum compounded continuously for 5 year, How much amount will there be after 5 years.

$$e^{0.5} = 1.6487$$

Solution:

$$P = 10,000, \quad r = 0.1, \quad N = 5$$

$$\text{Annuity} = \int_0^5 10,000e^{0.1t} dt$$

$$= \frac{10000}{0.1} [e^{0.1t}]_0^5 = 100000(e^{0.5} - 1)$$

$$100000(0.6487) = \text{Rs. } 64870$$

For Practice:

- The marginal cost function is  $MC=300x^{215}$  and fixed cost is zero. Find out the total cost and average cost functions.
- If the marginal revenue function for a commodity is  $MR=9-4x^2$ . Find the demand function.
- If  $MR=20-5x+3x^2$ , Find total revenue function.
- If  $MR=14-6x+9x^2$ , find the demand function.



**3 MARKS**

10. Calculate the area bounded by the parabola  $y^2 = 4ax$  and its latus rectum.

Solution:

$y^2 = 4ax$  is the right open

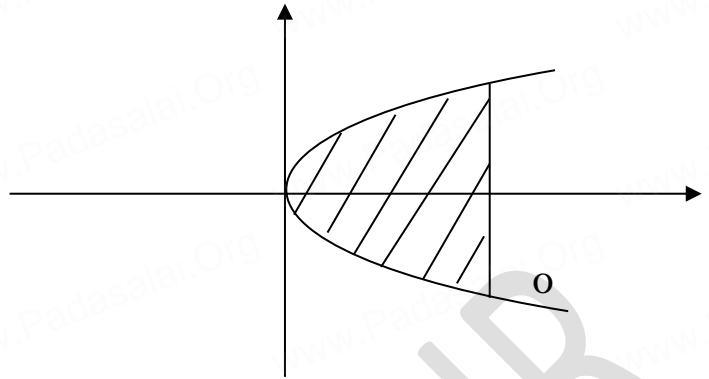
$$\Rightarrow y = \sqrt{4ax}$$

$$\text{Area} = 2 \int_0^a y \, dx$$

$$= 2 \int_0^a 2\sqrt{a} \sqrt{x} \, dx = 4\sqrt{a} \int_0^a x^{1/2} \, dx$$

$$= 4\sqrt{a} \left[ \frac{x^{3/2}}{3/2} \right]_0^a = \frac{8}{3} \sqrt{a} [a\sqrt{a} - 0]$$

$$= \frac{8}{3} a^2 \text{ sq. units.}$$



11. Given the marginal Revenue function  $\frac{4}{(2x+3)^2} - 1$  Show that the average revenue function is

$$P = \frac{4}{(6x+9)} - 1$$

Solution:

Given  $MR = \frac{4}{(2x+3)^2} - 1$

$$R = \int MR \, dx + k$$

$$R = \int (4(2x+3)^{-2} - 1) \, dx + k$$

$$R = \frac{-2}{(2x+3)} - x + k \rightarrow \textcircled{1}$$

When  $x = 0$ ,  $R = 0$   $k = \frac{2}{3}$

$$\therefore R = \frac{-2}{2x+3} - x + \frac{2}{3}$$

Average R- function:  $P = \frac{R}{x}$

$$P = \frac{-2}{(2x+3)x} - \frac{x}{x} + \frac{2}{3x}$$

$$P = \frac{4}{(6x+9)} - 1$$

12. The demand function for a commodity is  $P = e^{-x}$ . Find the consumer's surplus when  $P = 0.5$

Solve: Given:  $P = e^{-x}$  and  $P = 0.5$

$$0.5 = e^{-x}$$

$$\frac{1}{2} = e^{-x}$$

$$\frac{1}{2} = \frac{1}{e^x} \Rightarrow e^x = 2 \quad \boxed{x = \log 2}$$

$$P_o x_o = \frac{1}{2} \log_2$$

$$\text{Consumer's surplus} = \int_0^{x_o} f(x) dx - p_o x_o$$

$$= \int_0^{\log 2} e^{-x} dx - \frac{1}{2} \log 2$$

$$= -[e^{-x}]_0^{\log 2} - \frac{1}{2} \log 2$$

$$= -[e^{-\log 2} - 1] - \frac{1}{2} \log 2$$

$$= -\left[e^{\log \frac{1}{2}} - 1\right] - \frac{1}{2} \log 2$$

$$= -\left[\frac{1}{2} - 1\right] - \frac{1}{2}\log 2$$

$$= \frac{1}{2} - \frac{1}{2}\log 2 = \frac{1}{2}(1 - \log 2)$$

$$cs = \frac{1}{2}(1 - \log 2)$$

13. Calculate the producer's surplus at  $x=5$  for the supply function  $p=7+x$ .

Solution: Given,  $P = 7 + x$  and  $x = 5$

$$\Rightarrow P = 12$$

$$p_o x_o = (12)(5) = 60$$

$$p_o x_o = 60$$

Producer's surplus:

$$ps = p_o x_o - \int_0^{x_o} g(x) dx$$

$$= 60 - \int_0^5 (7 + x) dx$$

$$= 60 - \left[7x + \frac{x^2}{2}\right]_0^5$$

$$= 60 - \left[\left(25 + \frac{25}{2}\right) - (0)\right]$$

$$60 - 47.5 = 12.5$$

$$p5 = 12.5$$

14. The demand function for a commodity is the prevailing market price is Rs.6

Solution: Given  $p = \frac{36}{x+4}$  and  $p = 6$

$$p_o = 6 \Rightarrow x = 12$$

$$p_o x_o = (6)(2) = 12$$

$$p_o x_o = 12$$

Consumer's surplus:  $cs = \int_0^{x_o} f(x)dx - p_o x_o$

$$cs = \int_0^2 \frac{36}{x+4} dx - 12$$

$$cs = 36[\log(x+4)]_0^2 - 12$$

$$= 36 \left[ \log\left(\frac{6}{4}\right) - 12 \right]$$

$$cs = 36 \left[ \log\left(\frac{3}{2}\right) \right] - 12 \text{ units.}$$

For practice:

15. Sketch the graph  $y = 1x + 31$  and evaluate  $\int_{-6}^0 1 + 31dx$ .

16. Using integration find the area of the region bounded between the line  $x=4$  and the parabola :  $y^2=16x$ .

17. The rate of new product is given by  $f(x) = 100 - 90e^{-x}$ . Where  $x$  is the number of days the product is on the market. Find the total sale during the first four days ( $e^{-4} = 0.018$ ).

18. A company produces 50,000 unit per week with 200 workers. The rate of charge of productions with respect to the change in the number of additional labour  $x$  is represented as  $300 - 5x^{2/3}$ . If 64 additiona labours are employed, find not 4—

19. When the elasticity function is  $\frac{x}{x-2}$ . Find the function when  $x=6$  and  $y=16$ .

**5 MARKS**

20. Using integration find the area of the circle whose centre is at the origin and the radius is 'a' unit.

Solution:

Equation of the Circle is  $x^2 + y^2 = a^2$

Area = 4x[Area of the first Quadrant]

$$4x \int_0^a y \, dx$$

$$= 4x \int_0^a \sqrt{a^2 - x^2} \, dx$$

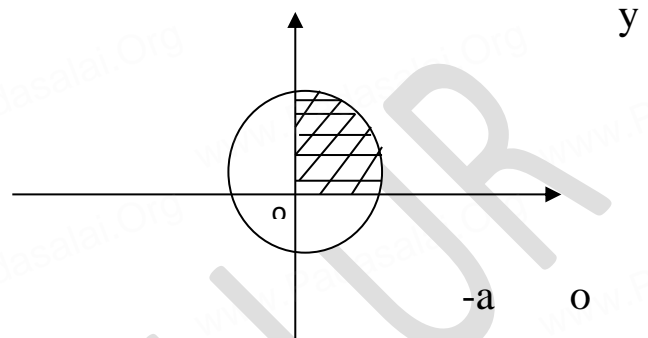
$$= 4 \left[ \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} \right]_0^a$$

$$= xa^2 \text{ sq units}$$

$$x^2 + y^2 = a^2$$

$$y^2 = a^2 - x^2$$

$$y = \sqrt{a^2 - x^2}$$



21. The marginal cost of production of a firm is given by  $c^1(x) = 5 + 0.13x$ , the marginal revenue is given by  $R^1(x) = 18$  and the fixed cost is Rs.120. Find the profit function.

Solution: Given  $c^1(x) = 5 + 0.13x$

$$R^1(x) = 18$$

Fixed cost is 120.

$$c(x) = \int c^1(x) \, dx$$

$$c(x) = 5x + \frac{0.13x^2}{2} + K_1$$

$$K_1 = 120$$

$$c = 5x + \frac{0.13x^2}{2} + 120 \rightarrow \textcircled{1}$$

$$R(x) = \int R^1(x) \, dx$$

$$R(x) = 18x + K_2$$

$$\Rightarrow K_2 = 0$$

$$R = 18x$$

Profit function:

$$P(x) = R(x) - c(x)$$

$$= 18x - \left( 5x + \frac{0.13x^2}{2} + 20 \right)$$

$$P = 13x - 0.065x^2 - 120$$

22. The demand equation for a products is

$x = \sqrt{100 - p}$  and the supply equation is and producer's surplus, under market equilibrium.

Solution:

Demand equation:

$$x = \sqrt{100 - p}$$

Squaring on bls

$$x^2 = 100 - p$$

$$p_d = 100 - x^2$$

Supply equation:

$$x = \frac{p}{2} - 10$$

$$x = \frac{p - 20}{2}$$

$$2x = p - 20$$

$$p_s = 2x + 20$$

At equilibrium, p

$$p_d = p_s$$

$$100 - x^2 = 2x + 20$$

$$x^2 + 2x + 20 - 100 = 0$$

$$x^2 + 2x - 80 = 0$$

$$(x + 10)(x - 8) = 0$$

$$x = -10, \quad x = 8$$

$$x_o = 8 \quad p_o = 36$$

$$p_o x_o = (36)(8) = 288$$

(i) Consumer's Surplus:

$$\begin{aligned}
 cs &= \int_0^{x_o} f(x)dx - p_o x_o \\
 &= \int_0^8 (100 - x^2)dx - 288 \\
 &= \left[ 100x - \frac{x^3}{3} \right]_0^8 - 288 \\
 cs &= \frac{1024}{3} \text{ units}
 \end{aligned}$$

(ii) Producer's Surplus:

$$\begin{aligned}
 ps &= p_o x_o - \int_0^{x_o} g(x)dx \\
 &= 288 - \int_0^8 (2x + 20)dx \\
 &= 288 - [x^2 + 20x]_0^8 \\
 ps &= 64 \text{ units}
 \end{aligned}$$

### **For Practice:**

23. The marginal cost and marginal revenue with respect to commodity of a time all given by  $c^1(x) = 8 + 6x$  and  $R^1(x) = 24$ . Find the total profit given that the total cost at zero output is zero.
24. The demand and supply function of a commodity are  $p_d = 18 - 2x - x^2$  and  $p_x = 2x - 3$ . Find the consumer's surplus and producer's surplus at equilibrium price.
25. Under perfect competition for a commodity the demand and supply laws are  $p_a = \frac{8}{x-1} - 2$  and  $p_s = \frac{x+3}{2}$  respectively. Find the consumer's and producer's surplus.



26. If the MC of a production of the company is directly proportional to the number of units( $x$ ) produced, then find the total cost function, when the fixed cost is Rs.5000 and the cost of producing 50 units is Rs.5625.
27. A firm has the marginal revenue function given by  $MR = \frac{a}{(x+b)^2} - c$  where  $x$  is the output and  $a, b, c$  are constants. Show that the demand function is given by  $x \frac{a}{b(p+c)-b}$ .



## **CHAPTER -4**

### **Examples 4.1**

$$v) y^1 + (y'')^2 = (x + y'')^2$$

$$y' + (y'')^2 = x^2 + 2xy'' + (y'')^2 \text{ using } [(a + b)^2 = a^2 + 2ab + b^2]$$

$$y' = x^2 + 2xy'' + (y'')^2 - (y'')^2$$

$$\frac{dy}{dx} = x^2 + 2x \frac{d^2y}{dx^2}$$

Order = 2, degree = 1

$$\left[1 + \frac{d^2y}{dx^2}\right]^{\frac{3}{2}} = a \frac{d^2y}{dx^2}$$

Squaring on both the sides

$$iv) \left[1 + \frac{d^2y}{dx^2}\right]^{\frac{3}{2} \times 2} = a^2 \left(\frac{d^2y}{dx^2}\right)^2$$

$$\left[1 + \frac{d^2y}{dx^2}\right]^3 = a^2 \left(\frac{d^2y}{dx^2}\right)^2$$

Order = 2, degree = 3

$$iii) \frac{d^2y}{dx^2} = \sqrt{y - \frac{dy}{dx}}$$

**2 MARKS**

Squaring on both side

$$\left(\frac{d^2y}{dx^2}\right)^2 = y - \frac{dy}{dx}$$

Order = 2, degree=2

### 3 MARKS

$$v) \frac{d^2y}{dx^2} + y + \left(\frac{dy}{dx} - \frac{d^3y}{dx^3}\right)^{3/2} = 0$$

$$\frac{d^2y}{dx^2} + y = -\left(\frac{dy}{dx} - \frac{d^3y}{dx^3}\right)^{3/2}$$

Squaring on both sides

$$\left(\frac{d^2y}{dx^2}\right)^2 + y^2 = \left(\frac{-dy}{dx}\right)^3 + \left(\frac{d^3y}{dx^3}\right)^3$$

Order=3, degree=3

$$(2 - y'')^2 = y''^2 + 2y'$$

$$4 - 4y'' + (y'')^2 = (y'')^2 + 2y' \quad \text{By using } [(a - b)^2 = a^2 - 2ab + b^2]$$

$$4 - 4y'' = (y'')^2 + 2y' - (y'')^2$$

$$4 - 4y'' = 2y'$$

$$4 - 4\frac{d^2y}{dx^2} = 2\frac{dy}{dx}$$

Order = 2, degree=1

$$1. \text{ Solve: } 9y'' - 12y' + 4y = 0$$

$$(9D^2 - 12D + 4)y = 0$$

**Auxiliary Equation**

$$(9m^2 - 12m + 4) = 0$$

$$(3m - 2)^2 = 0$$

$$\Rightarrow (3m - 2)(3m - 2)$$

$$3m = 2, \quad 3m = 2$$

$$m = \frac{2}{3}, m = \frac{2}{3}$$

$$(a + b)^2 = (a^2 + 2ab + b^2)$$

$$(3m - 2)^2 = 9m^2 - 12m + 4$$

Roots are real and equal

$$C.I = y = (Ax+B)^{emx}$$

$$Y = (Ax+B)^{e1/3x}$$

2. Solve  $\frac{d^2y}{dx^2} - \frac{4dy}{dx} + 5y = 0$ .

$$\frac{d^2}{dx^2} = D^2, \frac{d}{dx} = D$$

$$(D^2 - 4D + 5)y = 0$$

Auxiliary Equation is

$$(m^2 - 4m + 5) = 0$$

$$(m - 2)^2 - 4 + 5$$

$$(m - 2)^2 = 1$$

$$m - 2 = \pm \sqrt{1}$$

$$m = 2 \pm \sqrt{1}$$

$$\alpha + 1B \quad \alpha = 2, B = 1$$

The roots are Imaginary

$$C.I = e^{2x}[A \cos Bx + B \sin Bx]$$

General Equation :  $\Rightarrow y = e^{2x}[A \cos x + B \sin x]$

3. Solve  $\frac{d^2x}{dt^2} - \frac{3dx}{dt} + 2x = 0$  given that when  $t = 0$

$$x = 0, \frac{dx}{dt} = 1$$

$$\frac{d}{dt} = D$$

$$\Rightarrow (D^2 - 3D + 2)x = 0$$

Auxiliary Equation:

$$(m^2 - 3m + 2) = 0$$

$$\begin{array}{ccc} & & 2 \\ & \diagdown & / \\ -1 & & -2 \\ & \diagup & \diagdown \\ & & -3 \end{array}$$

$$(m - 1)(m - 2)$$

$$m - 1 = 0, m - 2 = 0$$

$$m_1 = 1, m_2 = 2$$

The roots are real and unequal

$$\text{So C.F} = Ae^{m_1x} + Be^{m_2x}$$

$$x = Ae^x + Be^{2t} \rightarrow \textcircled{1}$$

Substitute  $t=0, x=0$  (given) in (1) equation.

$$0 = A + B \rightarrow \textcircled{2} \quad e^0 = 1$$

Differentiate  $\textcircled{1}$  with resp't

$$\frac{dx}{dt} = Ae^t + 2Be^{2t}$$

$$\frac{dx}{dt} = 1, t = 0 \text{ (given)}$$

$$\text{when } \frac{dx}{dt} = 1 \Rightarrow$$

$$1 = Ae^0 + 2Be^{2(0)}$$

$$1 = A + 2B$$

$$1 = A + 2B \rightarrow \textcircled{3}$$

Solve  $\textcircled{2}$  &  $\textcircled{3}$  equation

$$2 \times 1 A + B = 0$$

$$3 \times 1 \frac{A}{(-)} + \frac{+2B}{(-)} + \frac{1}{(-)} \frac{1}{-B = -1} B = 1$$

Substitute B=1 in ② Equation

$$A + 1 = 0$$

$$A = -1$$

Substitute A & B in ① Equation

$$x = 1e^t + 1(B)e^{2t}$$

$$x = l^{2t} - l^t$$

### 5 MARKS

1. Solve:  $(3D^2 + D - 14)y = 4 - 13e^{-7/3x}$

$$(3D^2 + D - 14)y = 4 - 13e^{-7/3x}$$

The auxiliary equation is

$$3m^2 + m - 14 = 0$$

$$(3m + 7)(3m - 2) = 0$$

$$m = -7/3, 2$$

$$C.F = Ae^{-7/3x} + Be^{2x}$$

$$PI = \frac{1}{0D} f(x) = \frac{1}{3D + D - 14} \left[ 4 - 13e^{-7/3x} \right]$$

$$= \frac{1}{3D + D - 14} (4) + \frac{1}{3D + D - 14} \left[ -13e^{-7/3x} \right]$$

$$PI_1 + PI_2$$

$$P.I_1 = \frac{1}{3D + D - 14} 4e^{0x}$$

$$= \frac{1}{0 + 0 - 14} 4e^{0x}$$

$$P.I_1 = -4/14$$

$$P.I_2 = \frac{1}{3D + D - 14} x(-13)e^{-7/3x}$$

Replace D by  $-7/3$  Here  $3D^2 + D - 14 = 0$  when  $D = 7/3$   $PI_2 =$   
 $x \frac{1}{6D+1} (-13e^{-7/3x})$

Replace D by  $-7/3$

$$P.I_2 = x \frac{1}{6(-7/3) + 1} (-13e^{-7/3x})$$

$$= x \frac{1}{-13} (-13e^{-7/3x})$$

$$= xe^{-7/3x}$$

The general solution is

$$y = C.F + P.I_1 + P.I_2$$

$$y = Ae^{-7/3x} + Be^{2x} - \frac{2}{7} + xe^{-7/3x}$$

$$2. (D^2 + D - 6)y = e^{3x} + e^{-3x}$$

$$A.E = m^2 + m - 6$$

$$(m + 3)(m - 2) = 0$$

$$m = -3 \text{ or } m = 2$$

$$C.F = Ae^{-3x} + Be^{2x}$$

$$P.I = \frac{e^{-3x}}{(D + 3)(D - 2)} = \frac{e^{-3x}}{(-3 + 3)(3 - 2)}$$

$$= \frac{e^{3x}}{6} \cdot x(+1) = \frac{+e^{3x}x}{6}$$

$$P.I_2 = \frac{e^{3x}}{(D + 3)(D - 2)} = \frac{e^{3x}}{(2 + 3)(2 - 3)}$$

$$= \frac{e^{2x}}{(-3 + 3)(-3 - 2)} = \frac{xe^{-3x}}{-5}$$

$$y = Ae^{-3x} + Be^{2x} + \frac{e^{3x}}{6} - \frac{xe^{3x}}{5}$$

$$3. (D^2 - 10D + 25)y = 4e^{5x} + 5$$

$$A.E = m^2 - 10m + 25 = 0$$

$$(m - 5)(m - 5) = 0$$

$$C.F = (Ax + B)e^{5x}$$

$$P.I_1 = \frac{4e^{5x}}{(D - 5)(D - 5)} = \frac{4e^{5x}}{(5 - 5)(5 - 5)}$$

$$= \frac{x^2}{2} 4e^{5x} = 2x^2 e^{5x}$$

$$P.I_2 = \frac{5e^{0x}}{(0-5)(0-5)} = \frac{5e^{0x}}{(-5)(-5)}$$

$$= \frac{5}{25} = 1/5$$

$$y = C.F \neq PI_1 + PI_2$$

$$= (Ax + B)e^{5x} + 2x^2 e^{5x} + \frac{1}{5}$$

4.  $(D^2 - 2D - 15)y = 0$  given that  $\frac{dy}{dx} = D$  and  $\frac{d^2y}{dx^2} = 2$  when  $x=D$

Given:

$$(D^2 - 2D - 15)y = 0$$

Auxiliary Equation:

$$m^2 - 2m - 15 = 0$$

$$m^2 + 3m - 5m(m + 3) = 0$$

$$(m - 5)(m + 3) = 0$$

$$m_2 = 5 \quad m_1 = 3$$

$$\begin{aligned} C.F: Ae^{m_1x} + Be^{m_2x} \\ = Ae^{-3x} + Be^{5x} \end{aligned}$$

$$\text{General Solution} = \boxed{Ae^{-3x} + Be^{5x}} \rightarrow \textcircled{1}$$

$$\frac{dy}{dx} = \boxed{Ae^{-3x}(-3) + Be^{5x}(5)} \rightarrow \textcircled{2}$$



$$\frac{d^2y}{dx^2} = Ae^{-3x}(9) + Be^{5x}(25) \rightarrow \textcircled{3}$$

$$\frac{dy}{dx} = 0 \text{ when } x = 0 \text{ sub: in } \textcircled{2}$$

$$0 = Ae^{-3(0)}(-3) + Be^{5(0)}(5)$$

$$0 = -3A + 5B \rightarrow \textcircled{4}$$

$$\frac{d^2y}{dx^2} = 2 \text{ when } x = 0 \text{ sub: } \textcircled{3}$$

$$2 = Ae^{-3(0)}9 + Be^{5(0)}(25)$$

$$2 = 9A + 25B \rightarrow \textcircled{5}$$

$$\textcircled{4} \times 3 \Rightarrow 0 = -9A + 15B$$

$$\textcircled{5} \Rightarrow 2 = 9A + 25B$$

$$2 = 40B$$

$$\frac{2}{40} = B$$

$$\therefore B = \frac{1}{20}$$

Sub:  $B = \frac{1}{20}$  in  $\textcircled{4}$ :

$$0 = -3A + 5\left(\frac{1}{20}\right)$$

$$0 = -3A + \frac{1}{4}$$

$$3A = \frac{1}{4}$$

$$\therefore A = \frac{1}{12}$$

$$y = Ae^{-3x} + Be^{5x}$$

Sub:  $A = \frac{1}{12}$  and  $B = \frac{1}{20}$  in ①

$$\therefore y = \frac{e^{-3x}}{12} + \frac{e^{5x}}{20}$$

5.  $(D^2 - 3D + 2)y = e^{3x}$  which shall vanish for  $x = 0$  and for  $x = \log 2$

Given:  $(D^2 - 3D + 2)y = e^{3x}$

Auxiliary Equation:  $m^2 - 3m + 2 = 0$

$$m^2 - m - 2m + 2 = 0$$

$$m(m - 1) - 2(m - 1) = 0$$

$$\therefore m_2 = 2, m_1 = 1$$

$$\frac{C.F: Ae^{m_1x} + Be^{m_2x}}{Ae^x + Be^{2x}}$$

$$P.I = \frac{e^{3x}}{D^2 - 3D + 2} = \frac{e^{3x}}{9 - 9 + 2} = \frac{e^{3x}}{2}$$

General Solution:  $y = CF + PT$

$$y = Ae^x + Be^{2x} + e^{3x/2}$$

when  $x=0, y=0$

$$0 = Ae^0 + Be^{2(0)} + e^{3(0)/2}$$

$$0 = A + B + \frac{1}{2}$$

$$A + B = -\frac{1}{2} \rightarrow \textcircled{1}$$

**When  $x=\log 2, y=0$**

$$0 = Ae^{\log 2} + Be^{2\log 2} + \frac{e^{3\log 2}}{2}$$

$$0 = Ae^{\log 2} + Be^{\log 4} + \frac{e^{\log 8}}{2}$$

$$0 = A(2) + B(4) + \frac{8}{2}$$

$$0 = 2A + 4B + 4$$

$$2A + 4B = -4 \rightarrow \textcircled{2}$$

$$\textcircled{2} \div 2 \Rightarrow A + 2B = -2$$

$$\textcircled{1} \Rightarrow A + B = -\frac{1}{2}$$

$$\textcircled{2} - \textcircled{1} \Rightarrow$$

$$\therefore B = -\frac{3}{2}$$

Sub:  $B = -\frac{3}{2}$  in  $\textcircled{1}$

$$A \frac{-3}{2} = \frac{-1}{2}$$

$$A = \frac{-1}{2} + \frac{3}{2}$$

$$A = \frac{2}{2}$$

$$\therefore A = 1$$

Sub:  $A=1$ ,  $B=-\frac{3}{2}$  in general solution.

$$y = e^x \frac{-3e^{2x}}{2} + \frac{e^{3x}}{2}$$

1. Suppose that the quantity demanded

$$Q_d = 29 - 2P - \frac{5dp}{dt} + \frac{d^2p}{dt^2} \text{ and quantity supplied}$$

$Q_s = 5 + 4P$  where  $p$  is the price. Find the equilibrium price for market clearance.

At equilibrium,

$$Q_d = Q_s$$

$$29 - 2P - 5 \frac{dp}{dt} + \frac{d^2p}{dt^2} = 5 + 4P$$

$$\frac{d^2p}{dt^2} - 5 \frac{dp}{dt} + 29 - 5 - 2p - 4p = 0$$

$$\frac{d^2p}{dt^2} - 5 \frac{dp}{dt} + 24 - 6p = 0$$

$$\frac{d^2p}{dt^2} - \frac{5dp}{dt} - 6p = -24$$

$$(D^2 - 5D - 6)P = -24$$

The auxiliary equation is

$$m^2 - 5m - 6 = 0$$

$$(m - 6)(m + 1) = 0$$

$$m = 6, -1$$

The roots are real and different

$$C.F = Ae^{m_1x} + Be^{m_2x}$$

$$C.F = Ae^{6t} + Be^{-t}$$

$$P.I = \frac{1}{D^2 - 5D - 6}(-16) = \frac{-24e^{0t}}{(D - 6)(D + 1)}$$

$$= \frac{-24}{(0 - 6)(0 + 1)} = \frac{-24}{-6} = 4$$

$$P = P.I + C.F$$

$$P = Ae^{6t} + Be^{-t} + 4$$

2. Suppose that the quantity demanded

$$Q_d = 13 - 6P + 2 \frac{dp}{dt} + \frac{d^2p}{dt^2} \text{ and quantity supplied}$$

$Q_s = -3 + 2p$  where P is the price. Find the equilibrium price for market clearance

**At Equilibrium**

$$Q_d = Q_s$$

$$13 - 6P + \frac{2dp}{dt} + \frac{d^2p}{dt^2} = -3 + 2P$$

$$\frac{d^2P}{dt^2} + \frac{2dp}{dt} + 13 + 13 - 6p - 2p = 0$$

$$\frac{d^2p}{dt^2} + \frac{2dp}{dt} + 16 - 8P = 0$$

$$\frac{d^2p}{dt^2} + \frac{2dp}{dt} - 8P = -16$$

$$(D^2 + 2D - 8)P = -16$$

The auxiliary equation is

$$m^2 + 2m - 8 = 0$$

$$(m + 4)(m - 2) = 0$$

$$m = -4, 2$$

The roots are real and different.

$$C.F = Ae^{m_1x} + Be^{m_2x}$$

$$C.F = Ae^{-4t} + Be^{2t}$$

$$P.I. = \frac{1}{D^2 + 2D - 8}(-16) = \frac{-16e^{0t}}{(D + 4)(D - 2)}$$

$$= \frac{-16}{(0 + 4)(0 - 2)} = \frac{-16}{-8} = 2$$

$$P = C.F + P.I$$

$$P = Ae^{-4t} + Be^{2t} + 2$$

## CHAPTER -5

### NUMERICAL METHODS

#### Missing Items:

Given:  $U_0 = 1, U_1 = 11, U_2 = 21, U_3 = 28$  and  $U_4 = 29$  find  $\Delta^4 y_0$

$$\begin{aligned}
 \Delta^4 U_0 &= (E - 1)^4 U_0 \\
 &= (E^4 - 4E^3 + 6E^2 - 4E + 1)U_0 \\
 &= E^4 U_0 - 4E^3 U_0 + 6E^2 U_0 - 4E U_0 + U_0 \\
 &= U_4 - 4U_3 + 6U_2 - 4U_1 + U_0 \\
 &= 29 - 4(28) + 6(21) - 4(11) + 1 \\
 &= 156 - 156 = 0
 \end{aligned}$$

Given :  $y_3 = 2, y_4 = -6, y_5 = 8, y_6 = 9$  and  $y_7 = 17$  calculate  $\Delta^4 y_3$

$$\begin{aligned}
 \Delta^4 y_3 &= (E - 1)^4 y_3 \\
 &= (E^4 - 4E^3 + 6E^2 - 4E + 1)y_3 \\
 &= (E^4 y_3 - 4E^3 y_3 + 6E^2 y_3 - 4E y_3 + y_3) \\
 &= y_7 - 4y_6 + 6y_5 - 4y_4 + y_3 \\
 &= 17 - 4(9) + 6(8) - 4(-6) + 2 \\
 &= 17 - 36 + 48 + 24 + 2 \\
 &= 55
 \end{aligned}$$

#### **3. From the following table find the missing value:**

$x$	2	3	4	5	6
-----	---	---	---	---	---



$f(x)$	45.0	49.2	54.1	-	67.4
--------	------	------	------	---	------

$$\Delta^4 y_0 = 0$$

$$(E - 1)^4 y_0 = 0$$

$$(E^4 - 4E^3 + 6E^2 - 4E + 1)y_0 = 0$$

$$E^4 y_0 - 4E^3 y_0 + 6E^2 y_0 - 4E y_0 + y_0 = 0$$

$$y_4 - 4y_3 + 6y_2 - 4y_1 + y_0 = 0$$

$$67.4 - 4y_3 + 6(54.1) - 4(4.2) + 45 = 0$$

$$240.2 = 4y_3$$

$$\therefore y_3 = 60.05$$

1. If  $y = x^3 - x^2 + x - 1$  calculate the values of  $y$  for  $x = 0, 1, 2, 3, 4, 5$  and form the forward differences table.

$$y = x^3 - x^2 + x - 1$$

$$x = 0, 1, 2, 3, 4, 5$$

$$y = f(x)$$

$x$	$y$	$\Delta y_0$	$\Delta^2 y_0$	$\Delta^3 y_0$	$\Delta^4 y_0$	$\Delta^5 y_0$
0	-1	1				
1	0	5	4			
2	5	15	10	6	0	
3	20	31	16	6		0
4	51	33	22	6	0	
5	104					

**3 MARKS (or) 5 MARKS**

2. Evaluate  $\Delta \left[ \frac{1}{(x+1)(x+2)} \right]$  by taking '1' as the integral of differencing.

$$\Delta \left[ \frac{1}{(x+1)(x+2)} \right]$$

$$\frac{1}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$$

$$\frac{1}{(x+1)(x+2)} = \frac{A(x+2) + B(x+1)}{(x+1)(x+2)}$$

$$1 = A(x+2) + B(x+1)$$

$$\text{Take } x = -2$$

$$1 = A(-2+2) + B(-2+1)$$

$$1 = 0 + -B$$

$$B = -1$$

where  $x = -1$

$$1 = A(-1+2) + B(-1+1)$$

$$1 = A + 0$$

$$A = 1$$

$$\frac{1}{(x+1)(x+2)} = \frac{1}{x+1} - \frac{1}{x+2}$$

$$\Delta \left[ \frac{1}{(x+1)(x+2)} \right] = \Delta \left( \frac{1}{x+1} \right) - \Delta \left( \frac{1}{x+2} \right)$$

$$= \left[ \frac{1}{(x+h+1)} - \frac{1}{x+1} \right] - \left[ \frac{1}{x+h+2} - \frac{1}{x+2} \right]$$

$$= \frac{1}{x+1+1} - \frac{1}{x+1} - \frac{1}{x+1+2} + \frac{1}{x+2}$$

$$\begin{aligned}
 & \frac{1}{x+2} - \frac{1}{x+1} - \frac{1}{x+3} + \frac{1}{x+2} \\
 &= \frac{2}{(x+2)} - \frac{1}{x+1} - \frac{1}{x+3} \\
 &= \frac{2(x+1)(x+3) - (x+2)(x-3) - (x+2)(x+1)}{(x+1)(x+2)(x+3)} \\
 &= \frac{2(x^2 + 4x + 3) - (x^2 - 5x + 6) - (x^2 + 3x + 2)}{(x+1)(x+2)(x+3)} \\
 &= \frac{2x^2 + 3x + 6 - x^2 - 5x - 6 - x^2 - 3x - 2}{(x+1)(x+2)(x+3)} \\
 &= \frac{2x^2 - 2x^2 - 8x - 8x + 6 - 6 - 2}{(x+1)(x+2)(x+3)} \\
 &= \frac{-2}{(x+1)(x+2)(x+3)}
 \end{aligned}$$

1. Estimate the production for 1964 and 1966 from the following data

Year	1961	1962	1963	1964	1965	1966	1967
Production	200	220	260	-	350	-	430

Solution :

$$\Delta^5 y_k = 0$$

$$(E-1)^5 y_k = 0$$

$$(E^5 - 5E^4 + 10E^3 - 10E^2 + 5E - 1) y_k = 0$$

$$E^5 y_k - 5E^4 y_k + 10E^3 y_k - 10E^2 y_k + 5E y_k - y_k = 0 \quad (1)$$

Put  $K = 0$  in (1)

$$E^5 y_0 - 5E^4 y_k + 10E^3 y_0 - 10E^2 y_0 + 5E y_0 - y_0 = 0$$

$$y_5 - 5y_4 + 10y_3 - 10y_2 + 5y_1 - y_0 = 0$$

$$y_5 - 5(350) + 10y_3 - 10(260) + 5(220) - 200 = 0$$

$$y_5 + 10y_3 = 3450 \quad (2)$$

Put  $k=1$  is (1)

$$E^5 y_1 - 5E^4 y_1 + 10E^3 y_1 - 10E^3 y_1 - y_1 = 0$$

$$y_6 - 5y_5 + 10y_4 - 10y_3 + 5y_2 - g_1 = 0$$

$$430 - 5y_5 + 10(350) - 10y_3 + 5(260) - 220 = 0$$

$$5y_5 + 10y_3 = 5010$$

Solve (2) and (3)

(2)  $\rightarrow \times 5$

$$5y_5 + 50y_3 = 17250$$

$$\underline{5y_5 + 10y_3 = 5010}$$

$$40y_3 = 12240$$

$$y_3 = 306$$

Sub  $y_3 = 306$  in (1)

$$y_5 + 3060 = 3450$$

$$y_5 = 390$$

$$\therefore y_3 = 306$$

$$y_5 = 390$$

2. Find the missing entries from the following:

$x$	0	1	2	3	4	5
$y$	0	-	8	15	-	35
$= f(x)$						

Solution:-

No. of values=4

$$\Delta^4 f(x) = 0$$

$$\Delta^4 y_k = 0$$

$$(E - 1)^4 y_k = 0$$

$$(E^4 - 4E^3 + 6E^2 - 4E + 1)y_k = 0$$

$$E^4 u_k - 4E^3 y_k + 6E^2 y_k - 4E y_k + y_k = 0 \quad (1)$$

Put  $k=0$  in (1)

$$E^4 y_0 - 4E^3 y_0 + 6E^2 y_0 - 4E y_0 + y_0 = 0$$

$$y_4 - 4y_3 + 6y_2 - 4y_1 + y_0 = 0$$

$$y_4 - 4(15) + 6(8) - 4y_1 + 0 + 0$$

$$y_4 - 4y_1 = 12$$

2

Put  $k=1$  in (1)

$$E^4 y_1 - 4E^3 y_1 + 6E^2 y_1 - 4E y_1 + y_1 = 0$$

$$y_5 - 4y_4 + 6y_3 - 4y_2 + y_1 = 0$$

$$35 - 4y_4 + 60 - 32 + y_1 = 0$$

$$-4y_4 + y_1 = 63 \quad (3)$$

Solve (2) and (3)

$$4 \times 1 \quad 4y_4 - 16y_1 = 48$$

$$-4y_4 + y_1 = -63$$

$$\begin{array}{rcl} -15y_1 & = & -15 \\ y_1 & = & 1 \end{array}$$

Sub:  $y_1 = 1$  in (1)

$$y_4 - 4(1) = 12$$

$$y_4 = 8$$

$$y_1 = 1 \quad y_4 = 8$$

**GREGORY FORWARD FORMULA****Example: 5:17**

The Population of a certain town is as follow:

**5 MARKS**

Year: $x$	1941	1957	1961	1971	1981	1991
Population in lakhs: $y$	20	24	29	36	46	57

Using appropriate interpolation formula, estimate the population during the period 1946.

Solution:

$$y = y_0 + \frac{n}{1!} \Delta y_0 + \frac{n(n-1)}{2!} \Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!} \Delta^3 y_0 + \dots$$

$x = 1946$ $x_0 + nh = 1946$ $x_0 = 1941$ $h = 10$ $1941 + n(10) = 1946$ $n = 0.5$	$n = \frac{x - x_0}{h}$ $\Rightarrow \frac{x - x_0}{h}$ $\Rightarrow \frac{5}{10}$ $\Rightarrow 0.5$
---	---

$x$	$y$	$\Delta y$	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
-----	-----	------------	--------------	--------------	--------------	--------------

1941	20					
		4				
1957	24		1			
		5		1		
1961	29		2		0	
		7		1		-9
1971	36		3		-9	
		10		-8		
1981	46		-5			
		5				
1991	57					

$$\begin{aligned}
 y &= 20 + \frac{0.5}{1!}(4) + \frac{0.5(0.5-1)}{2!}(1) + \frac{0.5(0.5-1)(0.5-2)}{3!}(1) \\
 &\quad + \frac{0.5(0.5-1)(0.5-2)(0.5-3)}{4!}(0) \\
 &\quad + \frac{0.5(0.5-1)(0.5-2)(0.5-3)(0.5-4)}{5!} \\
 &\Rightarrow 20 + 2 - 0.125 + 0.0625 - 0.24609 \\
 &\Rightarrow 21.69 \text{ lakhs}
 \end{aligned}$$

### **Example: 5:2**

(5) In an examination the Number of candidates who secured marks between certain interval were as follows:

Marks	0-19	20-39	40-59	60-79	80-99
No. of candidates	41	62	65	50	17

Estimate the Numbers of candidates whose marks are less than 70.

Solution:

$$y = y_n + \frac{n}{1!} \nabla y_n + \frac{n(n+1)}{2!} \nabla^2 y_n + \frac{n(n+1)(n+2)}{3!} \nabla^3 y_n + \dots$$

$$x = 70$$



$$x_n = 99$$

$$h = 20$$

$$n = \frac{x - x_n}{h}$$

$$\Rightarrow \frac{70 - 99}{20}$$

$$\Rightarrow -1.45$$

x	y	$\Delta y$	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0>20	41	62			
0>40	103		3		
0>60	168	65	-15	-18	
0>80	218	50	-33	-18	0
0>100	235	17			

$$\Rightarrow 235 + \frac{(-1.45)}{1} (17) + \frac{(-1.45)(-1.45 + 1)2 \times 1}{2 \times 1}$$

$$\frac{(-1.45)(-1.45 + 1)(-1.45 + 2)}{3 \times 2 \times 1} (-18) + \frac{(-1.45)(-1.45 + 1)(-1.45 + 2)(-1.45)}{4 \times 3 \times 2 \times 1}$$

$$\Rightarrow 235 - 24.65 - 10.766 - \frac{6.459}{6} + 0$$

$$\Rightarrow 235 - 24.65 - 10.766 - 1.0765$$

$\Rightarrow 198.50$

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## **CHAPTER -5**

### **GREGORY FORWARD FORMULA**

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**Example: 5:20****5 MARKS**

1. Estimate the premium for policies maturing at the age of 63.

Age	45	50	55	60	65
Premium	114.84	96.16	83.32	74.48	68.48

‘Let age=x, Premium=y

$$y_{(x=x_n+nh)} = y_n + \frac{n}{1!} \nabla y_n + \frac{n(n+1)}{2!} \nabla^2 y_n + \frac{n(n+1)(n+2)}{3!}$$

$$x \nabla^3 y_n + \frac{n(n-1)(n+2)(n+3)}{4!} \nabla + \nabla y_n^+$$

x	y	$\nabla y$	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
45	114.84				
		-18.68	5.84		
50	96.16			-1.84	
		-12.84	4		0.68
55	83.32			-1.16	
		-8.84	2.84		
60	74.48				
		-6			
65	68.48				

$$x_n + nh = 63, \quad x_n = 65.48, \quad h = 5$$

$$65.48 + n(5) = 63, \quad n = \frac{-1.096}{5}$$

$$n = \frac{-2}{5}$$

$$y_{(x=63)} = 68.48 + \frac{-2}{5} (-6) + \frac{-2}{5} \left( \frac{-2}{5} + 1 \right) (2.84) +$$

$$\frac{-2}{5} \left( \frac{-2}{5} + 1 \right) \left( \frac{-2}{5} + 2 \right) (-1.16)$$

$$+ \frac{-2}{5} \left( \frac{-2}{5} + 1 \right) \left( \frac{-2}{5} + 2 \right) \left( \frac{-2}{5} + 3 \right) (0.6)$$

$$= 68.48 + 2.4 - 0.3408 + 0.07424 - 0.28288$$

$$= 70.585152$$

$$y(63) = 70.59$$

### **Example: 5:2**

6) Find the value of  $f(x)$  and  $x=32$

$x$	30	35	40	45	50
$f(x)$	15.9	14.9	14.1	13.3	12.5

$$y_{(x=x_n+nh)} = y_n + \frac{n}{1!} \Delta y_n + \frac{n(n-1)}{2!} \Delta^2 y_n +$$

$$\frac{n(n-1)(n-2)}{3!} \Delta^3 y_n + \frac{n(n-1)(n-2)(n-3)}{4!} \Delta^4 y_n + \dots$$

$X$	$y$	$\Delta y$	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
30	15.9	-1			
35	14.9	-0.8	0.2		
40	14.1	-0.8	0	-0.2	
45	13.3	-0.8	2		
50	12.5				

$$x_n + nh = 32, x_n = 30.0, h = 5$$

$$15.9 + n(5) = 32, \quad n = \frac{32 - 30.0}{5}$$

$$\begin{aligned}
 n &= \frac{2}{5} = 0.4 \\
 &= 15.9 + \frac{0.4}{1}(-1) + \frac{0.4(0.4-1)}{2 \times 1}(0.2) + \frac{0.4(0.4-1)(0.4-2)}{3 \times 2}(-0.2) \\
 &\quad + \frac{0.4(0.4-1)(0.4-2)(0.4-3)}{4 \times 3 \times 2 \times 1}(0.2) \\
 &= 15.9 - 0.4 + \frac{0.4(-0.6)(0.2)}{2} + \frac{0.4(-0.6)(-1.6)(-0.2)}{6} \\
 &\quad + \frac{0.4(-0.6)(-1.6)(-2.6)}{24}(0.2) \\
 &= 15.9 - 0.4 + \frac{(-0.048)}{2} - \frac{0.0768}{6} - \frac{0.19968}{24} \\
 &= 15.9 - 0.4 + (-0.024) - 0.0128 - 0.00832 \\
 &= 15.45488 \\
 f(32) &= 15.45
 \end{aligned}$$

### Lagrange's

Using Lagrange's interpolation formula find  $y(10)$  from the following table.

### 5 MARKS

$x$	5	6	9	11
$y$	12	13	14	16

$$x_0 = 5, x_1 = 6, x_2 = 9, x_3 = 11$$

$$y_0 = 12, y_1 = 13, y_2 = 14, y_3 = 16$$

$$\begin{aligned}
 y = f(x) &= \frac{(x - x_1)(x - x_2)(x - x_3)}{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)} y_0 \\
 &\quad + \frac{(x - x_0)(x - x_2)(x - x_3)}{(x_1 - x_0)(x_1 - x_2)(x_1 - x_3)} y_1 \\
 &\quad + \frac{(x - x_0)(x - x_1)(x - x_3)}{(x_2 - x_0)(x_2 - x_1)(x_2 - x_3)} y_2 \\
 &\quad + \frac{(x - x_0)(x - x_1)(x - x_2)}{(x_3 - x_0)(x_3 - x_1)(x_3 - x_2)} y_3
 \end{aligned}$$

$$\begin{aligned}
 & x y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} x y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} x y_3 \\
 &= \frac{(x-6)(x-9)(x-11)}{(5-6)(5-9)(5-11)} x(12) + \frac{(x-5)(x-9)(x-11)}{(6-5)(6-9)(6-11)} x(13) + \\
 & \quad \frac{(x-5)(x-6)(x-11)}{(9-5)(9-6)(9-11)} x(14) + \frac{(x-5)(x-6)(x-9)}{(11-5)(11-6)(11-9)} x(16)
 \end{aligned}$$

Put  $x=10$ ,

$$\begin{aligned}
 y = 10 &= f(10) \\
 &= \frac{(10-6)(10-9)(10-11)}{(-1)(-4)(-6)} (12) \\
 &+ \frac{(10-5)(10-9)(10-11)}{(1)(-3)(-5)}
 \end{aligned}$$

$$\begin{aligned}
 & (13) + \frac{(10-5)(10-6)(10-11)}{(4)(3)(-2)} + \frac{(10-5)(10-6)(10-9)}{(6)(5)(2)} (16) \\
 &= \frac{4(1)(-1)}{(-1)(-4)(-6)} (12) + \frac{5(1)(-1)}{1(-3)(-5)} (13) + \frac{5(4)(-1)}{4(3)(2)} (14) +
 \end{aligned}$$

$$\frac{5(4)(1)}{(6)(5)(2)} (16)$$

$$= \frac{1}{6} (12) - \frac{13}{3} + \frac{5(4)}{6} + \frac{4 \times 16}{12}$$

$$= \frac{12}{6} - \frac{13}{3} + \frac{70}{6} + \frac{64}{12}$$

$$\frac{432 - 936 + 2520 + 1152}{216}$$

$$\frac{4104 - 936}{216} = \frac{3168}{216}$$

$$= 14.66$$

1. Using interpolation estimate the business done in 1985 from the following data

**5 MARKS**

Year	1982	1983	1984	1986
Business done (in Lakhs)	150	235	365	525

$$x_0 = 1982, x_1 = 1983, x_2 = 1984, x_3 = 1986$$

$$y_0 = 150, y_1 = 235, y_2 = 365, y_3 = 525$$

$$y = f(x) = \frac{(x - x_1)(x - x_2)(x - x_3)}{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)} x y_0 +$$

$$\frac{(x - x_0)(x - x_2)(x - x_3)}{(x_1 - x_0)(x_1 - x_2)(x_1 - x_3)} x y_1 + \frac{(x - x_0)(x - x_1)(x - x_3)}{(x_2 - x_0)(x_2 - x_1)(x_2 - x_3)}$$

$$+ \frac{(x - x_0)(x - x_1)(x - x_2)}{(x_3 - x_0)(x_3 - x_1)(x_3 - x_2)} x y_3$$

$$= \frac{(x - 1983)(x - 1984)(x - 1986)}{(1982 - 1983)(1982 - 1984)(1982 - 1986)} x 150$$

$$+ \frac{(x - 1982)(x - 1984)(x - 1986)}{(1983 - 1982)(1983 - 1984)(1983 - 1986)}$$

$$x 235 + \frac{(x - 1982)(x - 1983)(x - 1986)}{(1984 - 1982)(1984 - 1983)(1984 - 1986)} x 365 +$$

$$\frac{(x - 1982)(x - 1983)(x - 1984)}{(1986 - 1982)(1986 - 1983)(1986 - 1984)} x 525$$

Put  $x=1985$

$$f(1985) = \frac{(1985 - 1983)(1985 - 1984)(1985 - 1986)}{(-1)(-2)(-3)} x 1150 +$$



$$\begin{aligned}
& \frac{(1985 - 1982)(1985 - 1984)(1985 - 1986)}{(1)(-1)(-3)} \times 235 + \\
& \frac{(1985 - 1982)(1985 - 1983)(1985 - 1986)}{(2)(1)(2)} \times 365 + \\
& \frac{(1985 - 1982)(1985 - 1983)(1985 - 1986)}{(4)(3)(2)} \times 525 \\
& = \frac{(2)(1)(-1)}{-8} \times 150 + \frac{(3)(1)(-1)}{3} \times 235 + \frac{(3)(2)(-1)}{-4} \times 365 + \frac{(3)(2)(1)}{24} \times 525 \\
& = \left(\frac{1}{4} \times 150\right) + (-235) + \left(\frac{3}{2} \times 365\right) + \left(\frac{1}{4} \times 525\right) \\
& \frac{150}{4} - 235 + \frac{1095}{2} + \frac{525}{4} = \frac{300 - 1880 + 4380 + 1050}{8} \\
& = \frac{5730 - 1880}{8} = \frac{3850}{8} = 481.25
\end{aligned}$$

## **CHAPTER -6**

Random variable and Mathematical Expectation

### **2 MARKS**

1. Number of cars of some of the households are given below.

No. of cars	0	1	2	3	4
No. of households	30	320	380	190	80

Compute probability mass function and also verify  $P(x_i)$  is a probability mass function?

Soln:

Let  $x$  be the no. of cars.

$X=x_i$	No. of households	$P(x_i)$
0	30	0.03
1	320	0.32
2	380	0.38
3	190	0.19
4	80	0.08
	<b>1000</b>	<b>1.00</b>

here (i)  $p(x_i) \geq 0, \forall i$  and

(ii)  $\sum_{i=1}^n p(x_i) = 1$

$\therefore p(x)$  is a probability mass function.

2. Find the expected value for the random variable of an unbiased die.

3. Let  $X$  be a random variable and  $y = 2x+1$ . What is the variance of  $Y$  if variance of  $X$  is 5?

### 3 MARKS

4. A coin is tossed thrice. Let  $X$  be the number of observed heads. Find the cumulative distribution function of  $X$ .

Soln: The sample space ( $S$ )

$$S = \{C(HHH), (HHT), (HTH), (HTT), (THH), (THT), (TTH), (TTT)\}$$

$X$  takes the values: 3, 2, 2, 1, 2, 1, 1 and 0.

Range of $X$	0	1	2	3
$P(x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$
$F(x)$	$\frac{1}{8}$	$\frac{4}{8}$	$\frac{7}{8}$	1

Thus we have,

$$F_x(x) = \begin{cases} 0, & \text{For } x < 0 \\ \frac{1}{8}, & \text{for } 0 \leq x < 1 \\ \frac{4}{8}, & \text{for } 1 \leq x < 2 \\ \frac{7}{8}, & \text{for } 2 \leq x < 3 \\ 1, & \text{for } x \leq 3 \end{cases}$$

**For practice:**

5) Suppose, the life in hours of a radio tube has the following p,d,f:

$$f(x) = \begin{cases} \frac{100}{x^2}, & \text{when } x \geq 100 \\ 0, & \text{when } x < 100 \end{cases}$$

**Find the distribution function.**

6. Suppose the p.m.f of the discrete random variable of

X=x	0	1	2	3
P(x)	0.2	0.1	0.4	0.3

What is the value of  $E(3x+2x^2)$  ?

7. If  $F(x)$  is defined by  $F(x) = Ke^{-2x}$ ;  $0 \leq x < \infty$  is a density function.

Determine the constant K and also the mean.

8. A continuous random variable  $x$  has the following distribution function.

$$F(x) = \begin{cases} \theta, & \text{of } x \leq 1 \\ K(x-1)^4 & \text{of } 1 < x \leq 3 \\ 1, & \text{of } x > 3 \end{cases}$$

**5 MARKS**

9) a continuous random variable  $x$  has the following probability function.

x=x	0	1	2	3	4	5	6	7
p(x)	0	k	2k	2k	3k	k <sup>2</sup>	2k <sup>2</sup>	7k <sup>2</sup> +k

(i) Find K (ii) Evaluate  $p(x < 6)$ ,  $p(x \geq 6)$  and  $p(0 < x < 5)$  (iii) If  $p(x \leq x) > \frac{1}{2}$ , then find the minimum value of  $x$ .

Solution:

(i) Find k:  $\sum_{x=0}^7 p(x) = 1$

$$p(0) + p(1) + p(2) + p(3) + p(4) + p(5) + p(6) + p(7) = 1$$

$$0 + k + 2k + 2k + 3K + k^2 + 2k^2 + 7k^2 + k = 1$$

$$10k^2 + 9k - 1 = 0$$

$$(k + 1) \left( k - \frac{1}{10} \right) = 0$$

$$k = -1, \quad k = \frac{1}{10}$$

K must not a negative value.  $\therefore K = \frac{1}{10}$

(ii)  $p(x < 6) = p(0) + p(1) + p(2) + p(3) + p(4) + p(5)$

$$= 0 + K + 2K + 2K + 3K + K^2$$

$$= K^2 + 8K$$

$$R(x < 6) = \frac{1}{100} + \frac{8}{10} = 0.01 + 0.8 = 0.81$$

$$p(x \geq 6) = p(6) + p(7)$$

$$= 2k^2 + 7k^2 + K$$

$$= 9k^2 = 9 \left( \frac{1}{100} \right) = 0.09$$

$$p(0 < x < 5) = p(1) + p(2) + p(3) + p(4)$$

$$= K + 2K + 2K + 3K$$

$$= 8k = 0.8$$

(iii)  $p(x \leq x) > \frac{1}{2}$

$$(a) p(x \leq 0) = 0 < \frac{1}{2}$$

$$(b) p(x \leq 1) = K = 0.1 < \frac{1}{2}$$

$$(c) p(x \leq 2) = 0.3 < \frac{1}{2}$$

$$(d) p(x \leq 3) = 0.5 < \frac{1}{2}$$

$$(e) p(x \leq 4) = 0.8 > \frac{1}{2}$$

$\therefore$  The minimum value of  $x$  is 4.

### **For Practice:**

10) the length of time (in minutes) that a certain person speaks on the telephone is found to be random phenomenon, with a probability function specified by the probability density function For  $x$  as,

$$F(x) = \begin{cases} Ae^{-x/5}, & \text{For } x \geq 0 \\ 0, & \text{other wise} \end{cases}$$

(i) Find the value of  $A$  that makes for  $x$  a p.d.f.

(ii) What is the probability that the number of minutes that person will take over the phone is

(i) More than 10 minutes      (ii) less than 5 minutes

(iii) between 5 and 10 minutes.

11) Determine the mean and variance of a discrete random variable, given its distribution as follows,

$x=x$	1	2	3	4	5	6
$f(x)$	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$	1

12) The number of miles an automobile lasts before it reaches a critical point in tread wear can be represented by a p.d.f.

$$f(x) = \begin{cases} \frac{1}{30} e^{-\frac{x}{30}} & ; \text{for } x > 0 \\ 0 & ; \text{for } x \leq 0 \end{cases}$$

Find the expected number of miles (in thousands) a tire would last until it reaches the critical tread wear point.

13) The probability density function of a random variable  $x$  is  $F(x) = ke^{-1x}$ ,  $-\infty < x < \infty$  Find the value of  $k$  and also find mean and variance for the random variable.

## **CHAPTER -6**

### **Probability distributions:**

### **2 MARKS**

1. The mean of a binomial distribution is 5 and standard deviation is 2, Determine the distribution.

Soln: Given mean =  $np = 5$

$$SD = \sqrt{npq} = 2$$

$$var = npq = 4$$



$$\frac{npq}{np} = \frac{4}{5} = 1 \quad q = 4/5 \therefore p = 1 - q = 1/5$$

$$\therefore np = 5 \Rightarrow n = 25$$

$$p(x = r) = nCr p^r q^{n-r} = 25Cr \left(\frac{1}{5}\right)^r \left(4/5\right)^{25-r}$$

2. The mortality rate for a certain disease is 7 in 1000. What is the probability for just 2 deaths on account of this disease in a group of 400?

Soln: Given:  $p = 7/1000; n = 400$

$$\lambda = np = \frac{7 \times 400}{1000} = 2.8$$

$$p(x = x) = \frac{e^{-\lambda} \lambda^x}{x!} = \frac{e^{-2.8} (2.8)^x}{x!}$$

$$p(x = 2) = \frac{e^{-2.8} (2.8)^2}{2!} = \frac{0.06 \times 7.84}{2} = 0.2354$$

### Sums for Practice:

#### 2 MARKS

3. Mention the properties of poisson distribution.
4. In a family of 3 children, what is the probability that there will be exactly 2 girls.
5. The mean of Binomial distribution is 20 and standard deviation is 4. Find the parameters of the distribution.
6. Verify the following statement:  
The mean of a B.D is 12 and its S.D is 4. (S.D – Standard deviation)
7. In tossing of a 5 fair coins, find the chance of getting exactly 3 head.
8. If the probability of success is 0.09, how many trials are needed to have a probability of atleast one success as  $1/3$  or more?
9. Write down any 3 characteristics of normal probability curve.
10. The average daily procurement of milk by village society is 800 litres with a standard deviation of 100 litres. Find out proportion of societies procuring milk between 800 litres to 1000 litres per day.

#### 3 MARKS:



11. X is normally distributed with mean 12 and SD 4. Find  $(X \leq 20)$  and  $P(0 \leq x \leq 12)$
12. In a P.D the first probability term is 0.2725. Find the next probability term:
13. Assume that a drug causes a serious side effect at a rate of 3 patients per one hundred. What is the probability that atleast one person will have side effects in a random sample of ten patients taking the drug?
14. If the chance of running a bus service according to schedule is 0.8. Calculate the prob on a day schedule with 10 services atleast one is late.
15. When counting red blood cells, a square grid is used, over which a drop of blood is evenly distributed. Under the microscope an average of 8 erythrocytes are observed per single square. What is the probability that exactly 5 erthrocytes are found in one square?
- 16.3 marks (model sum)

The average number of customers, who appear in a counter of a certain bank per minute is two. Find the probability the during a given minute (i) No customer appears

(ii) 3 or more customers appear. ( $e^{-2} = 0.1353$ )

Soln:  $\lambda = 2$

$$(i) P(x = x) = \frac{e^{-\lambda} \lambda^x}{x!} = \frac{e^{-2} (2^x)}{x!}$$

$$\begin{aligned} (ii) p(x \geq 3) &= 1 - p(x < 3) = 1 - [p(x = 0) + p(x = 1) + p(x = 2)] \\ &= 1 - e^{-2} \left[ \frac{2^0}{0!} + \frac{2^1}{1!} + \frac{2^2}{2!} \right] = 1 - 0.1353[5] \\ &= 1 - 0.6765 = 0.3235 \end{aligned}$$

### 5 MARKS

17. In a distribution 30% of the items are under 50 and 10% are over 86. Find the mean and S.D of the distribution.

Soln: By data

$$p(-\infty < \leq \leq) = 0.3$$

$$p(0 < \leq, z_1) = 0.2$$

$$\Rightarrow Z_1 = 0.53 (\because \text{it lies on left side})$$

$$\text{Also: } Z_1 = \frac{x_1 - \mu}{\sigma} \Rightarrow \frac{50 - \mu}{\sigma} = -0.53$$

$$50 - \mu = -0.53\sigma \quad \text{①}$$

$$p(Z_2 < z < \infty) = 0.1$$

$$p(0 < z < z_2) = 0.4 \Rightarrow Z_2 = 1.28 \text{ (from the normal table)}$$

$$z_2 = \frac{x_2 - \mu}{\sigma} \Rightarrow \frac{86 - \mu}{\sigma} = 1.28$$

$$86 - \mu = 1.28\sigma \quad \text{②}$$

$$\text{②} - \text{①} \Rightarrow$$

$$86 - \mu = 1.28\sigma$$

$$50 - \mu = -0.53\sigma$$

$$36 = 1.81\sigma$$

$$8 = \frac{36}{1.81} = 19.89$$

$$\sigma \approx 20$$

$$\sigma = 20 \text{ in } \text{②} \Rightarrow$$

$$86 - \mu = 1.28(20)$$

$$\mu = 86 - 25.6 = 60.4$$

$\therefore$  mean = 60.4 and S.D =  $\sigma$  = 20

### **Sums for Practice:**

18. The sum and product of the mean and variance of a B.D are 24 and 128. Find the distribution.

19. Forty percent of business travellers carry a laptop. In a sample of 15 business travelers.

- (i) What is the probability that 3 will have a laptop?
- (ii) What is the probability that 12 of the travellers will not have a laptop.
- (iii) What is the probability that atleast 3 of the travellers have a laptop.

20. The distribution of the number of road accidents per day in a city is poisson with mean 4. Find the number of days out of 100 days when there will be (i) no accident.

- (ii) atleast 2 accidents and (iii) at most 3 accidents.

21. A sample of 125 dry battery cells tested to find the length of life produced the following result with mean 12 and SD 3 hours. Assuming that the data to be normally distributed, what percentage of battery cells are expected to have life.

- (i) more than 13 hours
- (ii) less than 5 hours
- (iii) between 9 and 14 hours.

22. Time taken by a construction company to construct a flyover is a normal variate with mean 400 labour days and standard deviation of 100 labour days. If the company promises to construct the flyover in 450 days or less and agree to pay a penalty of ₹10,000 for each labour day spent in excess of 450 days. What is the probability that

- (i) the company pays a penalty of atleast ₹ 2,00,000?
- (ii) the company takes atmost 500 days to complete the flyover?

## **CHAPTER -8**

### **2 MARKS**

1. A server channel monitored for an hour was found to have an estimated mean of 20 transactions transmitted per minute. The variance = 4. Find the standard error.

Soln: Given  $Var = \sigma^2 = 4$

$$\begin{aligned}
 SD &= \sigma = 2 \\
 n &= 1 \text{ hr} = 60 \text{ min} \\
 \bar{x} &= 20 \text{ min} \\
 \therefore \text{std error} &= \frac{\sigma}{\sqrt{n}} = \frac{2}{\sqrt{60}} \\
 &= 0.2582
 \end{aligned}$$

2. The standard deviation of a sample of size 50 is 6.3. Determine the standard error whose population standard deviation is 6?

Soln:

$$\begin{aligned}
 n &= 50, s = 6.3 \text{ (sample)} \\
 (\text{populak}) \sigma &= 6
 \end{aligned}$$

$$\text{The std error for sample S.D } = \sqrt{\frac{\sigma^2}{2n}}$$

$$\begin{aligned}
 &= \sqrt{\frac{6 \times 6}{2 \times 50}} \\
 &= \frac{6}{10} = 0.6
 \end{aligned}$$

$\therefore$  Standard error for Sample S.D } = 0.6

3. A die is thrown 9000 times and a a throw of 3 or 4 is observed 3240 times. Find the standard error of the proportion for an unbiased die.

Soln: Sample size  $n=9000$

Population proportion  $p=p(\text{getting 3 ro 4})$

$$= \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$$

$$p = 0.333,$$

$$Q = 1 - P = 1 - 0.3333 = 0.6667$$

The S.E for sample proportion is given by  $SE = \sqrt{\frac{PQ}{n}} =$

$$\sqrt{\frac{(0.333)(0.6667)}{9000}} = 0.0049$$

4. In a sample of 400 population from a village 230 are found to be eaters of vegetarian items and the rest non-vegetarian items. Complete the S.E assuming that both veg and non-veg foods are equally popular in that village?

Soln:

$$n = 400$$

$$p = q = 1/2 (\because \text{both veg \& non veg are equal})$$

$$\begin{aligned} \text{S.E. for sample proportion } &= \sqrt{\frac{pq}{n}} = \sqrt{\frac{1/2 \times 1/2}{400}} \\ &= \frac{1/2}{20} = \frac{1}{40} = 0.025 \end{aligned}$$

### **Sums for practice:**

5. Find the sample size for the given std deviation 10 and the S.E with respect of sample mean is 3.

6. A wholesaler in apples claims that only 4% of the apples supplied by him 600 apples contained 36 defective apples. Calculate the standard error concerning of good apples.

7. A sample of 1000 students whose mean weight is 119 lbs (pounds) from a school in Tamil Nadu state was taken and their average weight was found to be 120 lbs with a standard deviation of 30 lbs. Calculate the standard error of mean.

8. A random sample of 60 observations was drawn from a large population and its standard deviation was found to be 2.5. Calculate the suitable standard error that this sample is taken from a population with standard deviation 3?

### **3 MARKS**

9. A sample of 100 measurements at breaking strength of cotton thread gave a mean of 7.4 and a standard deviation of 1.2 gms. Find 95% confidence limits for the mean breaking strength of cotton thread.

Soln: Given  $n = 100$ ,  $\bar{x} = 7.4$ ,  $s = 1.2$

Here  $\sigma = s = 1.2$ ,  $z_{\alpha/2} = 1.96$

$$\text{S.E} = \sigma / \sqrt{n} = \frac{1.2}{\sqrt{100}} = 0.12$$



95% confidence limits for the population mean area

$$\bar{x} - z_{\alpha/2} \left( \frac{\sigma}{\sqrt{n}} \right) \leq \mu \leq \bar{x} + z_{\alpha/2} \left( \frac{\sigma}{\sqrt{n}} \right)$$

$$7.4 - (1.96 \times 0.12) \leq \mu \leq 7.4 \pm (1.96 \times 0.12)$$

$$7.165 \leq \mu \leq 7.635$$

10. A machine produces a component of a product with a standard deviation of 1.6 cm in length. A random sample of 64 component was selected from the output and this sample has a mean length of 90 cm. The customer will reject the part if it is either less than 88 cm or more than 92cm. Does the 95% confidence interval for the true mean length of all the components produced ensure acceptance by the customer?

Soln:

Given:

Sample	Population
$\bar{x} = 90$	$\sigma = 1.6$
$N=64$	

$$Z_{\alpha/2} = 1.96 \text{ (5\% level of significance)}$$

$$S.E = \frac{\sigma}{\sqrt{n}} = \frac{1.6}{\sqrt{64}} = \frac{1.6}{8} = \frac{16}{80} = \frac{1}{5} = 0.2$$

$\mu$  = mean length (population) confidence interval is  $\bar{x} \pm Z_{\alpha/2} \left( \frac{\sigma}{\sqrt{n}} \right) \leq \mu \leq \bar{x} +$

$$Z_{\alpha/2} \left( \frac{\sigma}{\sqrt{n}} \right)$$

$$\text{ie, } 90 - 1.96(0.2) \leq \mu \leq 90 + 1.96(0.2)$$

$$89.61 \leq \mu \leq 90.39$$

$\Rightarrow$  that the probability of the population mean length of the component will fall in this interval (89.61, 90.39) at 95%.

11. The mean life time of a sample of 169 light bulbs manufactured by a company is found to be 1350 hours with a standard deviation of 100 hours. Establish 90% confidence limits within which the mean life time of light bulbs is expected to lie?

Soln:

Sample	Population
$\bar{x}$ $= 1350 \text{ hrs}$	$\sigma$ $= 100 \text{ hrs}$

n=169	
-------	--

### 5 MARKS

12. (i) A sample of 900 members has a mean 3.4cm and SD 2.61cm. Is the sample taken from a large population with mean 3.25cm and S.D 2.62cm?  
(ii) If the population is normal and its mean is unknown, find the 95% and 98% confidence limits of true mean.

Soln:

Sample	Population
$n = 900$	$\mu = 3.25 \text{ CM}$
$\bar{x}=3.4\text{cm}$	$\sigma = 2.61$
$S=2.61\text{cm}$	

Null hypothesis  $H_0: \mu = 3.25\text{cm}$  (ie the sample has been taken from the population with mean=3.25)

Alternative hypothesis:  $H_1: \mu \neq 3.25\text{cm}$

Level of significance  $\alpha = 5\%$

Significant or table value  $Z_{\alpha/2} = 1.96$

Test Statistic:

$$Z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{3.4 - 3.25}{\frac{2.61}{\sqrt{900}}}$$

$$Z = \frac{0.15}{0.087} = 1.724 \text{ (Calculated value)}$$

$$\therefore Z < Z_{\alpha/2} = 1.724 < 1.96$$

Calculated value < table value.

$\therefore$  The sample has been drawn from the population mean.

(iii) Confidence limit:

95% confidential limits are

$$\bar{x} - Z_{\alpha/2} \left( \frac{\sigma}{\sqrt{n}} \right) \leq \mu \leq \bar{x} + Z_{\alpha/2} \left( \frac{\sigma}{\sqrt{n}} \right)$$

$$3.4 - (1.96 \times 0.087) \leq \mu \leq 3.4 + (1.96 \times 0.087)$$

$$3.229 \leq \mu \leq 3.571$$

98% confidential limits are

$$3.4 - (2.33 \times 0.087) \leq \mu \leq 3.4 + (2.33 \times 0.087)$$

$$3.197 \leq \mu \leq 3.603.$$

$\therefore$  95% confidential limits is (3.229, 3.571) and 98% confidential limits is (3.197, 3.603).

Sums for practice:

13. The mean weekly sales of soap bars in departmental stores were 146.3 bars per store. After an advertising campaign the mean weekly sales in 400 stores for a typical week increased to 153.7 and showed a standard deviation of 17.2. Was the advertising campaign successful?

14. The wages of the factory workers are assumed to be normally distributed with mean and variance 25. A random sample of 50 workers gives the total wages equal to ₹ 2550. Test the hypothesis  $\mu = 52$ , against the alternative hypothesis  $\mu = 49$  at 1% level of significance.

15. An ambulance service claim that it takes on the average 8.9 minutes to reach its destination in emergency calls. To check on this claim, the agency which licenses ambulance services, has then timed on 50 emergency calls, getting a mean of 9.3 minutes with a standard deviation of 1.6 minutes. What can they conclude at the level of significance.

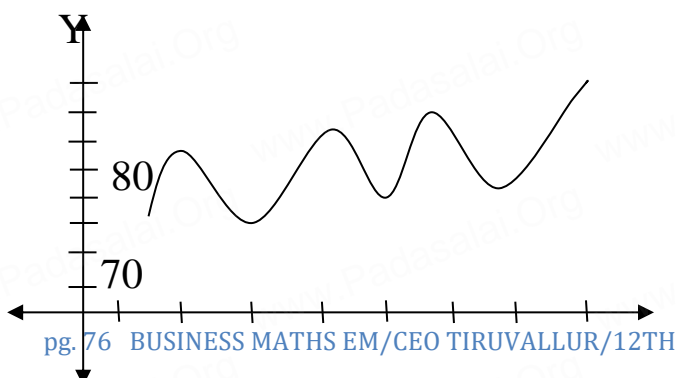


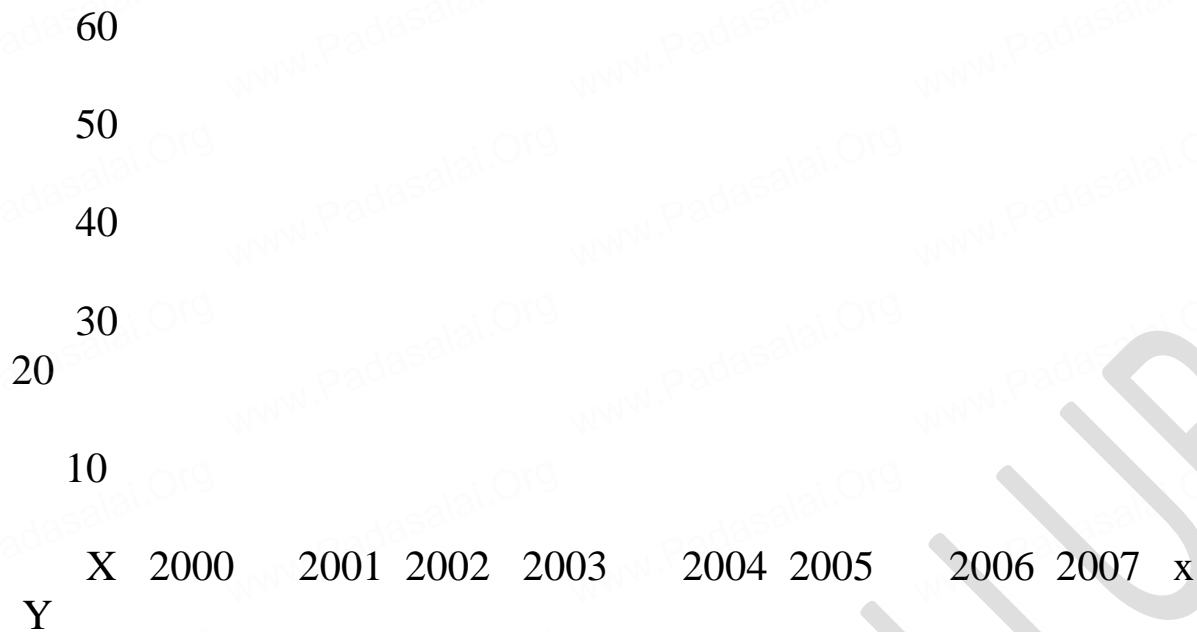
**BUSINESS MATHS**  
**IX. APPLIED STATISTICS**  
**2 MARKS**

1. Fit a st.line by the method of freehand method for the given data.

Year: 2000 2001 2002 2003 2004

Sales: 30 4625 5940





## 2. Uses of Index Number:

- (i) It is an important tool for the formulating decision and management policies.
- (ii) It helps in studying the trends and tendencies.

## 3 MARKS

## 3. Fit a trend line by the method of semi-averages for the given data.

Year:	2000	2001	2002	2003	2004	2005	2006
Sales:	105	115	120	100	110	125	135

Soln: The number of years is odd (seven)

We will leave the middle year's production value.

Year	Production	Average
2000	105	$\frac{105 + 115 + 120}{3}$ $= 113.33$
2001	115	
2002	120	
2003	100(left out)	$\frac{110 + 125 + 135}{3}$ $= 123.33$
2004	110	
2005	125	
2006	135	

4. Calculate three-yearly moving averages of number of students studying in a higher secondary school in a particular village from the following data.

Year	1995	1995	1997	1998	1999	2000
No. of Students:	332	317	357	392	402	405
Year	2001	2002	2003	2004		
No. of Students:	410	427	435	438		
<b>Year</b>	<b>No. of students</b>	<b>3-yearly moving total</b>	<b>3-yearly moving averages</b>			
1995	332	-	-			
1996	317	1006	335.33			
1997	357	1066	355.33			
1998	392	1151	383.67			
1999	402	1199	399.67			
2000	405	1217	405.67			
2001	410	1242	414.00			
2002	427	1272	424.00			
2003	435	1300	433.33			
2004	438	-	-			

5. Find the trend of production by the method of a five-yearly period of moving average for the following data.

Year	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990
Prod :	126	123	117	128	125	124	130	114	122	129	118	123

Soln:

Year	Production	Five-year moving total	Five-year moving average
1979	126	-	-

1980	123	-	-
1981	117	619	123.8
1982	128	617	123.4
1983	125	624	124.8
1984	124	621	124.2
1985	130	615	123
1986	114	619	123.8
1987	122	613	122.6
1988	129	606	121.2
1989	118	-	-
1990	123	-	-

6. Calculate the cost of living Index number for the year 2015 with respect to base year: 2010 of the following data.

Commodities	Number of units q <sub>0</sub> (2010)	Price (2010)	Price (2015)	From the table we have to find	
				P <sub>0</sub> q <sub>0</sub>	P <sub>1</sub> q <sub>0</sub>
Rice	5	1500	1750	7500	8750
Sugar	3.5	1100	1200	3850	4200
Pulses	3	800	950	2400	2850
Cloth	2	1200	1550	2400	3100
Ghee	0.75	550	700	412.5	525
Rent	12	2500	3000	30000	36000
Fuel	8	750	600	6000	4800
Misc	10	3200	3500	32000	35000
				84562.5	95225.

Hence, the cost of living Index number for a particular class of people for the year 2015 is increased by 12.61% as compared to the year 2010.

7. Construct the cost of living Index number for 2011 on the basis of 2007 from the given data using family budget method.

Commodities	Price		Weights	$P = \frac{p_1}{p_0} \times 100$	pv
	2007 P <sub>0</sub>	2011 P <sub>1</sub>			
A	350	400	40	114.285	4571.4

B	175	250	35	142.857	4999.99
C	100	115	15	115	1725
D	75	105	20	114	2280
E	60	80	25	133.33	3333.3
			135		16909.7315

$$\text{Cost of living Index number} = \frac{\sum PV}{\sum V} = \frac{16909.7315}{135} = 125.2572$$

Hence the cost of living Index number for a particular class of people for the year 2011 is increased by 25.25% as compared to the year 2007.

8. A machine drills hole in a pipe with a mean diameter of 0.532 cm and a standard deviation of 0.002 cm. Calculate the control limits for mean of samples 5.

Sol:

Given  $\bar{x} = 0.532, \sigma = 0.002, n = 5$

The control limits for  $\bar{x}$ - chart is

$$\text{UCL} = \bar{x} + 3 \frac{\sigma}{\sqrt{n}} = 0.532 + \frac{3(0.002)}{\sqrt{5}} = 0.5346$$

$$\text{CL} = \bar{x} = 0.532$$

$$\text{LCL} = \bar{x} - 3 \frac{\sigma}{\sqrt{n}} = 0.532 - \frac{3(0.0002)}{\sqrt{5}} = 0.5293$$

For practice

9. Using fisher's ideal formula. Compute price Index number for 1999 with 1996 as base year, given the following:

Year	Commodity: A		Commodity: B		Commodity: C	
	Price	Quantity	Price	Quantity	Price	Quantity
1996	5	10	8	6	6	3
1999	4	12	7	7	5	4

10. The following data gives the readings for 8 samples of size 6 each in the production of a certain product. Find the control limits using mean chart.

Sample	1	2	3	4	5	6
Mean	300	342	351	319	326	333
Range	25	37	20	28	30	22

**5 MARKS**

11. Given below are the data relating to the sales of a product in a district. Fit a straight line trend by the method of least squares and tabulate the trend values.

Year:	1995	1996	1997	1998	1999	2000	2001	2002
Sales:	6.7	5.3	4.3	6.1	5.6	7.9	5.8	6.1

Soln:

Computation of trend values by the method of least squares.

In case of even number of years, let us consider

$$X = \frac{x - \text{Arithmetic mean of two middle years}}{0.5}$$

Year	Sales (Y)	$X = \frac{x - 1998.5}{0.5}$	XY	$X^2$	Trend values ( $Y_t$ )
1995	6.7	-7	-46.9	49	5.6166
1996	5.3	-5	-26.5	25	5.719
1997	4.3	-3	-12.9	9	5.8214
1998	6.1	-1	-6.1	1	5.9238
1999	5.6	1	5.6	1	6.0262
2000	7.9	3	23.7	9	6.1285
2001	5.8	5	29	25	6.2309
2002	6.1	7	42.7	49	6.3333
	47.8		8.6	168	

$$a = \frac{\sum y}{n} = \frac{47.8}{8} = 5.975$$

$$b = \frac{\sum xy}{\sum x^2} = \frac{8.6}{168} = 0.0512$$

$$y = a + bx = 5.975 + 0.0512x$$

$$5.975 + 0.0512\left(\frac{x - 1998.5}{0.5}\right)$$

12. Calculate the seasonal index for the quarterly production of a product. Using the method of simple averages.

Year	I quarter	II quarter	III quarter	IV quarter
2005	255	351	425	400
2006	269	310	396	410
2007	291	332	358	395
2008	198	289	310	357
2009	200	290	331	359



	2010	250	300	350	400
Quarterly Total	1463		1872	2170	2321
Quarterly Averages	243.83		312	361.67	386.83

$$\text{S.I for I quarter} = \frac{\text{Average of I quarter}}{\text{Grand average}} \times 100$$

$$\text{Grand Average} = \frac{1304.333}{4} = 326.0833$$

$$\text{S.I for I quarter} = \frac{243.8333}{326.0833} \times 100 = 74.77$$

$$\text{S.I for II quarter} = \frac{312}{326.0833} \times 100 = 95.68$$

$$\text{S.I for III quarter} = \frac{361.6667}{326.0833} \times 100 = 110.91$$

$$\text{S.I for IV quarter} = \frac{386.833}{326.0833} \times 100 = 118.63$$

13. Construct the Laspeyres's , Paache's and Fisher's price index number for the following data. Comment on the result.

Sol:

Commodities	Base price	Year quantity	Current price	Year quantity	$p_0q_0$	$p_0q_1$	$p_1q_0$	$p_1q_1$
	15							
Rice	10	5	16	8	75	120	80	128
Wheat	8	6	18	9	60	90	108	162
Rent	9	7	15	8	56	64	105	120
Fuel		5	12	6	45	54	60	72
Transport	11	4	11	7	44	77	44	77
Miscellaneous	16	6	15	10	96	160	90	150
					376	565	487	709

Laspeyres's Price Index number

$$P_{01}^L = \frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100 = \frac{487}{376} \times 100 = 129.5212$$

Paasche's Price Index number

$$P_{01}^P = \frac{\sum p_1 q_1}{\sum p_0 q_1} \times 100 = \frac{709}{565} \times 100 = 125.4867$$

Fisher's Price Index number.

$$P_{01}^P = \left( \sqrt{\frac{\sum p_1 q_0}{\sum p_0 q_0} \times \frac{\sum p_1 q_1}{\sum p_0 q_1}} \right) \times 100 = \sqrt{\frac{487 \times 709}{376 \times 565}} \times 100 = 127.4879$$

On an average, there is an increase of 29.52%, 25.48% and 27.48% in the price of the commodities by Laspeyre's Paasche's, Fisher's price index number respectively, when the base year compared with the current year.

14. Construct fisher's price index number and prove that it satisfies both time reversal test and factor reversal test for data.

Commodities	Base price	year quantity	current price	year quantity	$p_0 q_0$	$p_0 q_1$	$p_1 q_0$	$p_1 q_1$
Rice	40	5	48	4	200	160	240	192
Wheat	45	2	42	3	90	135	84	126
Rent	90	4	95	6	360	540	380	570
Fuel	85	3	80	2	255	170	240	160
Transport	50	5	65	8	250	400	325	520
Miscellaneous	65	1	72	3	65	195	72	216
					1220	1600	1341	1784

Fisher's price index number:

$$P_{01}^F = \sqrt{\frac{\sum p_1 q_0}{\sum p_0 q_0} \times \frac{\sum p_1 q_1}{\sum p_0 q_1}} \times 100 = \sqrt{\frac{1341}{1220} \times \frac{1784}{1600}} \times 100 = 110.706$$

Time Reversal Test:  $P_{01} \times P_{10} = 1$

$$P_{01} \times P_{10} = \sqrt{\frac{\sum P P L_o \times \sum P_1 q_1 \times \sum P_o q_1 \times \sum P_o q_o}{\sum P_o q_o \times \sum P_o q_1 \times \sum P_1 q_1 \times \sum P_1 q_o}} = \sqrt{\frac{1341 \times 1784 \times 1600 \times 1220}{1220 \times 1600 \times 1784 \times 1341}} = 1$$

Factor Reversal Test:

$$P_{01} \times q_{01} = \frac{\sum P_1 q_1}{\sum P_0 q_0}$$



$$\begin{aligned}
 P_{01} \times Q_{01} &= \sqrt{\frac{\sum P_1 q_0 \times \sum P_1 q_1 \times q_1 P_0 \times \sum q_1 P_1}{\sum P_0 q_0 \times \sum P_0 q_1 \times \sum q_0 P_0 \times \sum q_0 P_1}} \\
 &= \sqrt{\frac{1341 \times 178 \times 1600 \times 1784}{1220 \times 1600 \times 1220 \times 1341}} \\
 &= \sqrt{\frac{1784 \times 1784}{1220 \times 1220}} = \frac{1784}{1220} = \frac{\sum P_1 q_1}{\sum P_0 q_0}
 \end{aligned}$$

∴ Time Reversal and Factor Reversal Tests are Verified.

15. You are given below the values of sample mean ( $\bar{x}$ ) and the range (R) for ten samples of size 5 each. Draw mean chart and comment on the state of control of the process.

Sample Number	1	2	3	4	5	6	7	8
	9	10						
$\bar{x}$	43	49	37	44	45	37	51	46
R	43	47						
	5	6	5	7	7	4	8	6
	4	6						

Given the following control chart constraint for  $n=5$ ,  $A_2=0.58$ ,  $D_3=0$  and  $D_4=2.115$ .

Sol:

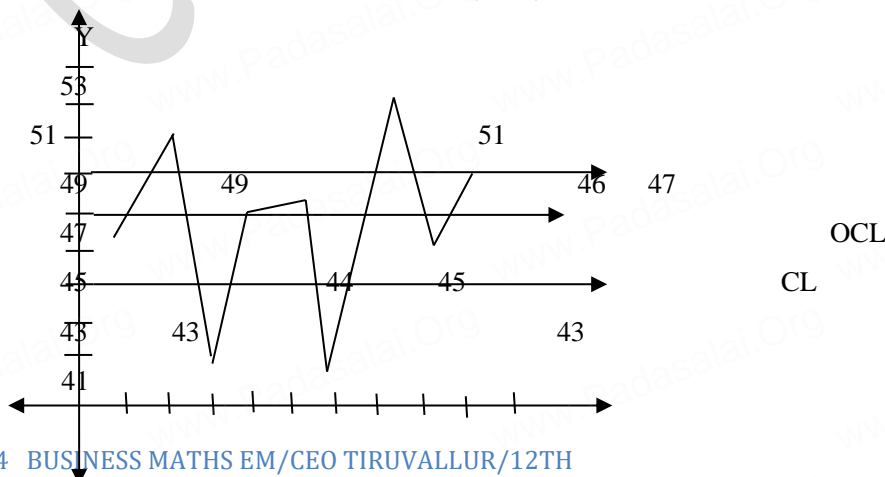
$$\bar{\bar{x}} = \frac{\sum \bar{x}}{10} = \frac{442}{10} = 44.2$$

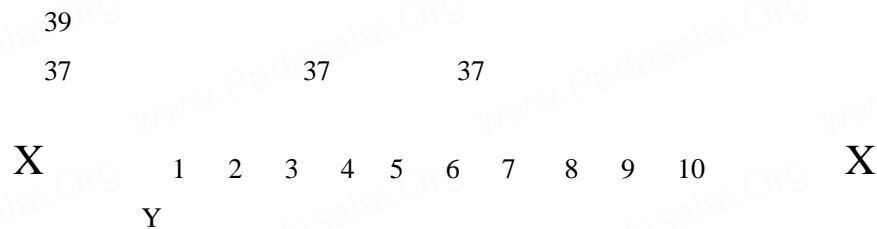
$$\bar{R} = \frac{\sum R}{10} = \frac{58}{10} = 5.8$$

$$UCL = \bar{\bar{x}} + A_2 \bar{R} = 44.2 + 0.483(5.8) = 47$$

$$CL = \bar{\bar{x}} = 44.2$$

$$LCL = \bar{\bar{x}} - A_2 \bar{R} = 44.2 - 0.483(5.8) = 41.39$$





The above diagram shows all the three control lines with the data points plotted. Since four points fall out of the control limits, we can say that the process is out of control.

### **For practice:**

16. In a production process, eight samples of size 4 are collected and their means and ranges are given below. Construct mean chart and range chart with control limits.

Sample Number	1	2	3	4	5	6
$\bar{x}$	12	13	11	12	14	13
R	16	15	2	5	4	3

17. The following data gives the readings for 8 samples of size 6 each in the production of a certain product. Find the control

Sample	1	2	3	4	5	6
Mean	300	342	351	319	326	333
Range	25	37	20	28	30	22

Given  $n=6$ ,  $A_2=0.483$

18. Use the method of monthly averages to find the monthly. Indices for the following data of production of a commodity for the years 2002, 2003, and 2004.

2002	15	18	17	19	16	20	21	18
17	15	14	18					
2003	20	18	16	13	12	15	22	16
18	20	17	15					
2004	18	25	21	11	14	16	19	20
17	16	18	20					

## OPERATION RESEARCH

### 2 MARKS

1. What is feasible solution and non degenerate solution in transportation problem
2. What do you mean by balanced transportation problem

### 3 MARKS

3. What is the difference between Assignment problem and transportation problem.
4. Obtain an initial basic feasible solution to the following transportation problem by using least – cost method.

			D1	D2	D3
9	8	5	01	25	
6	8	4	02	35	
7	6	9	03	40	

Demand    30            25            45

Sol:

$$\text{Total supply} = 25 + 35 + 40 = 100$$

$$\text{Total demand} = 30 + 25 + 45 = 100$$

$$\text{Total supply} = \text{Total demand.}$$

The given problem is balanced transportation problem. Hence there exists a feasible solution to the given problem.

I. allocation

	D1	D2	D3
9	8	5	01 25
6	8	4	02 35
7	6	9	03 40

Demand 30 25 45

[∴ The least cost is 4 and it is in ( $O_2D_3$ )  
 $\min(45, 35) = 35$ ]

II. allocation.

	D1	D2	D3
9	8	5	01
7	6	9	03

Demand 30 25 45

[∴ The least cost is 5 and it is in ( $O_1D_3$ )  
 $\min(10, 25) = 10$ ]

III. allocation

	D1	D2
9	8	01 15
7	6	03 40/15

Demand 30 25/0

[∴ The least cost is 6 and it is in ( $O_3D_2$ )  
 $\min(25, 40) = 25$ ]

IV. allocation D1

9

01 

7
---

 20

03 15/0

[∴ The least cost is 7 and it is in ( $O_3D_1$ )

$$\min(30, 15) = 15]$$

V. allocation

D1  

9
---

 20/5  
 15/0

$$\min(20, 15) = 15.$$

Thus the allocations are.

	D1	D2	D3
01	9	8	5
02	6	8	4
03	7	6	9
	40		

Demand 30 25 45

The transportation schedule is

$O_1 \rightarrow D_1, O_1 \rightarrow D_3, O_2 \rightarrow D_3, O_3 \rightarrow D_1, O_3 \rightarrow D_2, O_3 \rightarrow D_3$

$$\text{cost} = (9 \times 15) + (5 \times 10) + (35 \times 4) + (15 \times 7) + (25 \times 6)$$

$$= 135 + 50 + 140 + 105 + 150 = ₹580.$$

5. Three jobs A, B, and C one to be assigned to three machines U, V and W. The processing cost for each job machine combination is shown in the matrix given below. Determine the allocation that minimizes the overall processing cost.

Machine

Job	U	V	W
A	17	25	31
B	10	25	16
C	12	14	11

C

Sol:

Here the number of rows and columns are equal.

∴ The given assignment problem is balance.

Step 1: Select a minimum element in each row and subtract this from all the elements in its row.

	U	V	W	Row min
A	0	8	14	17
B	0	15	6	10
C	1	3	0	11

Here column V has no zero. Go to step 2.

Step 2 : Select the minimum element in each column and subtract this from all the elements in its column.

	U	V	W
A	0	5	14
B	0	12	6
C	1	0	0

Now, each row and column contains atleast one zero. Hence assignment can be made.

Step 3:

Examine the rows with exactly one zero mark it by  $\square$  and draw a vertical line. After examining the row, examine the column with one zero and mark it by  $\square$  and draw a horizontal line.

	U	V	W
A	$\square$ 0	5	14
B	0	12	6
C	1	$\square$ 0	0

Now the elements not lying on the line are  $\begin{matrix} 5 & 14 \\ 12 & 6 \end{matrix}$  and min is 5 subtract 5 from all these number and add 5 to 1 which lies in the intersecting lines. Other numbers remains the same.

A new cost matrix will be formed and repeat step 3.

	U	V	W
A	0	$\square$ 0	9
B	0	7	1
C	6	$\square$ 5	5

0	7	1	A
6	10	0	B

C

Thus all the 3 assignment schedule and total cost is

	Job	Machine	Cost
A	v		-25
B	u		-10
C	w		11

Total cost = ₹46

6. Given the following pay-off matrix (in rupees) for three strategies and two states of nature.

Strategy	States of nature	
	E <sub>1</sub>	E <sub>2</sub>
S <sub>1</sub>	40	60
S <sub>2</sub>	10	-20
S <sub>3</sub>	-40	150

Select a strategy using each of the following rule (i) maximum (ii) Minimax  
Sol:

Strategy	State of nature		Maximum Pay off	Minimum Pay off
	E <sub>1</sub>	E <sub>2</sub>		
S <sub>1</sub>	40	60	60	40
S <sub>2</sub>	10	-20	10	-20
S <sub>3</sub>	-40	150	150	-40

(i) Max (40, -20, -40) = 40

∴ strategy s<sub>1</sub> is the best according to maximum criteria.

(ii) Min (60, 10, 150) = 10



∴ strategy  $s_2$  is the best according to mini max principle.

### **For practice**

7. A business man has three alternatives open to him each of which can be followed by any of the four possible events. The conditional pay offs for each action – event combination are given below.

Alternative Pay-offs (conditional events)

	A	B	C	D
X	8	0	-10	6
Y	-4	12	18	-2
Z	14	6	0	8.

Determine which alternative should the business man choose, if he adopts the maximum principle.

### **5 MARKS**

8. Obtain an initial basic feasible solution to the following transportation problem by using (i) least cost method.

(ii) Northwest corner rule.

(iii) Vogel's approximation method.

	I	II	III
A	7		
B	12		
C	11		

10    10    10

9. Obtain the initial solution for the following problem

	A	B	C
Sources	1	5	
	2	8	
	3	7	

4    14

7    9    18



10. Obtain an initial basic feasible solution to the following transportation problem using yogel's approximation method.

	Ware houses				Stores			
		I	II	III	IV			
Availability(9i)								

5	1	3	3	34
3	3	5	4	15
6	4	4	3	12
4	1	4	5	10

11. Solve the following assignment problem. Cell values represent cost of assigning job A, B,C and D to the machines I,II,III and IV.

Machines.

Jobs

	I	II	III	IV
A	10	12	19	11
B	5	10	7	8
C	12	14	13	11
D	8	15	11	9