11th MATHS - APPLICATION PROBLEMS (VOLUME I)

1.

A man standing directly opposite to one side of a road of width x meter views a circular shaped traffic green signal of diameter a meter on the other side of the road. The bottom of the green signal is b meter height from the horizontal level of viewer's eye. If α denotes the angle subtended by the diameter of the green signal at the viewer's eye, then prove that

$$\alpha = \tan^{-1}\left(\frac{a+b}{x}\right) - \tan^{-1}\left(\frac{b}{x}\right).$$

Diameter of the signal = a

DA = b $\angle CBD = \alpha$

AB = x mts $\angle DBA = \beta$

From triangle ABD $\tan\beta = \frac{b}{x} = \frac{opp}{adj}$

$$\Rightarrow \beta = \tan^{-1}\left(\frac{b}{x}\right) \text{ ----------------- } (1)$$

From triangle ABC $\tan(\alpha+\beta) = \frac{a+b}{x}$

$$\Rightarrow \alpha+\beta = \tan^{-1}\left(\frac{a+b}{x}\right) \text{ ------------------------ } (2)$$

(2) - (1) $\Rightarrow \alpha = \tan^{-1}\left(\frac{a+b}{x}\right) - \tan^{-1}\left(\frac{b}{x}\right)$

2.

Suppose that a satellite in space, an earth station and the centre of earth all lie in the same plane. Let r be the radius of earth and R be the distance from the centre of earth to the satellite. Let d be the distance from the earth station to the satellite. Let 30° be the angle of elevation from the earth station to the satellite. If the line segment connecting earth station and satellite subtends angle α at the centre of earth, then prove that $d = R\sqrt{1+\left(\frac{r}{R}\right)^2 - 2\frac{r}{R}\cos\alpha}$.

Let S be the satellite CE = r SE = d SC = R

By cosine formula

$$D^2 = r^2 + R^2 - 2rR\cos\alpha$$

Divide by R^2 $\frac{d^2}{R^2} = \frac{r^2}{R^2} + 1 - \frac{2rR\cos\alpha}{R^2}$

$$d^2 = R^2\left[\frac{r^2}{R^2} + 1 - \frac{2rR\cos\alpha}{R^2}\right]$$

d = $R\sqrt{1 + \left(\frac{r}{R}\right)^2 - \frac{2rR\cos\alpha}{R^2}}$

Two vehicles leave the same place P at the same time moving along two different roads. One vehicle moves at an average speed of $60km/hr$ and the other vehicle moves at an average speed of 80 km/hr. After half an hour the vehicle reach the destinations A and B. If AB subtends $60°$ at the initial point P, then find AB.

AP = 3 PB =40 $\underline{|P} = 60^o$

By cosine formula

$AB^2 = AP^2+PB^2-2AP . PB \ \text{Cos } 60^o$

= $30^2 + 40^2 - 2 . 30 . 40$ ½

= 900+600-1200=1300=1 00 x 13

AB = 10 $\sqrt{13}$

3.

A man starts his morning walk at a point A reaches two points B and C and finally back to A such that $\angle A = 60°$ and $\angle B = 45°$, $AC = 4km$ in the $\triangle ABC$. Find the total distance he covered during his morning walk.

b= 4 km $\underline{|A} = 60^o$ $\underline{|B} = 45^o$

By using sine formula

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{\sin 60^o} = \frac{4}{\sin 45^o}$$

$$\Rightarrow \frac{2a}{\sqrt{3}} = 4\sqrt{2}$$

$\Rightarrow$ **a = 2**$\sqrt{6}$

By using Projection formula

C= a CosB + b CosA

$= 2\sqrt{6}Cos45^o + 4Cos60^o = 2\sqrt{6}Cos45^o + 4Cos60^o$

On simplification we ger **c =** $2\sqrt{3}+2 = 2\left(\sqrt{3}+1\right)$

Total distance = 4 +2 $\sqrt{6}$ **+** $2\sqrt{3}+2$

4.

A plane is 1 km from one landmark and 2 km from another. From the planes point of view the land between them subtends an angle of 60°. How far apart are the landmarks?

$\lfloor A = 60^o$ **b = 2 km** **c = 1 km**

By using cosine formula

$a^2=b^2+c^2-2bcCos60^o$

= 4 + 1 - 2 . 2 . 1 . ½ = 5-2 =3

$a^2 = 3$

a = $\sqrt{3}$ **if θ =45° the book answer is correct**

5.

A fighter jet has to hit a small target by flying a horizontal distance. When the target is sighted, the pilot measures the angle of depression to be 30°. If after 100 km, the target has an angle of depression of 45°, how far is the target from the fighter jet at that instant?

Let C be the position of the target

A, B are the position of the fighter Jet

$\lfloor A = 30^o$ $\lfloor B = 45^o$ **therefore** $\lfloor C = 105^o$

AB = c = 100 KM

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{\sin 30^o} = \frac{b}{\sin 45^o} = \frac{c}{\sin 105^o}$$

We know that sin 105° = $\dfrac{\sqrt{3}+1}{2\sqrt{2}}$

Therefore $\frac{a}{\frac{1}{2}} = \frac{1002\sqrt{2}}{\sqrt{3}+1} = \frac{100\sqrt{2}}{\sqrt{3}+1}\frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{100\left(\sqrt{6}-\sqrt{2}\right)}{3-1}$

$a = 50\left(\sqrt{6}-\sqrt{2}\right)$ KM

6.

A farmer wants to purchase a triangular shaped land with sides $120 feet$ and $60 feet$ and the angle included between these two sides is $60°$. If the land costs ₹500 per sq.ft, find the amount he needed to purchase the land. Also find the perimeter of the land.

AB = 120º AC =60ft $\underline{|A} = 60^o$

Area of triangle = Δ = $\frac{1}{2}(AB)(AC)\sin 60^o$

$= \frac{1}{2} \times 120 \times 60 \times \frac{\sqrt{3}}{2}$ **=1800 x 1.732 sq feet**

Total cost = 500 x 1800 x 1.732 = Rs. 1558800

$a^2=b^2+c^2-2bc\cos 60^o$

= 14400+3600-2 x 120 x 60 x $\frac{1}{2}$ = 18000-7200

a^2 = 10800 = 400 x 27

a = 20$\sqrt{27}$

therefore perimeter of the triangle = 120 +60+20$\sqrt{27}$ feet = 180+20$\sqrt{27}$ feet

7.

A straight tunnel is to be made through a mountain. A surveyor observes the two extremities A and B of the tunnel to be built from a point P in front of the mountain. If $AP = 3km$, $BP = 5\ km$ and $\angle APB = 120°$, then find the length of the tunnel to be built.

AP = 3 km BP = 5 km $\underline{|APB} = 120^o$

$AB^2 = AP^2+BP^2$-2 AP BP Cos $\underline{|P}$

= 9 + 25 -2 .3.5. Cos $\left(\pi - \frac{\pi}{3}\right)$

= 9 + 25 +15=49

AB^2= 7 KM

8.

Two Navy helicopters A and B are flying over the Bay of Bengal at same altitude from the sea level to search a missing boat. Pilots of both the helicopters sight the boat at the same time while they are apart $10\ km$ from each other. If the distance of the boat from A is $6\ km$ and if the line segment AB subtends $60°$ at the boat, find the distance of the boat from B.

$\angle C = 60^o$ **b = 6 c = 10**

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{6}{\sin B} = \frac{10}{\sin 60^o}$$

Therefore Sin B = $\frac{6\times\sqrt{3}}{10} = \frac{3\sqrt{3}}{10}$

By projection formula

a= b CosC + c CosB

a=6cos60+ c $\sqrt{1-\sin^2 B}$

= 6 . ½ + 10 $\sqrt{\frac{100-27}{100}}$ **=**

= $3+\frac{10\sqrt{73}}{10} = 3+\sqrt{73}$ **KM**

9.

A researcher wants to determine the width of a pond from east to west, which cannot be done by actual measurement. From a point P, he finds the distance to the eastern-most point of the pond to be $8\ km$, while the distance to the western most point from P to be $6\ km$. If the angle between the two lines of sight is $60°$, find the width of the pond.

$\angle A = 60^o$ **b = 6 c = 8**

By cosine formula

$a^2=b^2+c^2-2bcCosA$

= 36+64-2 x 6 x cos 60°

= 100 – 96x1/2 =52

a^2 = 52 ⇒ a = $\sqrt{52} = \sqrt{4\times13} = 2\sqrt{13}$ **KM**

10.

Two soldiers A and B in two different underground bunkers on a straight road, spot an intruder at the top of a hill. The angle of elevation of the intruder from A and B to the ground level in the eastern direction are 30° and 45° respectively. If A and B stand $5km$ apart, find the distance of the intruder from B.

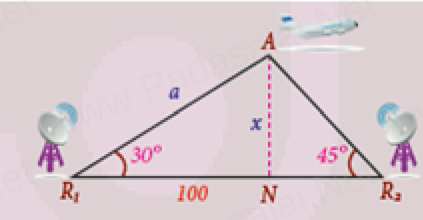
From the diagram $\lfloor P = 180^o - (135^o - 15^o) = 15^o$

$\frac{5}{\sin\theta} = \frac{x}{\sin 30^o}$ $\Rightarrow$ **2x =** $\frac{5}{\sin 15^o}$

$\Rightarrow 2x = \frac{5}{\frac{\sqrt{3}-1}{2}} \Rightarrow$ **x=** $\frac{5\sqrt{2}}{\sqrt{3}-1}$

11.

Suppose two radar stations located 100 km apart, each detect a fighter aircraft between them. The angle of elevation measured by the first satiation is 30^0, whereas the angle of elevation measured by the second station is 45^0. Find the altitude of the air craft at that instant



Let R_1 and R_2 be two radar stations and A be the position of fighter aircraft at the time of detection.
Let x be the required altitude of the aircraft.
Draw $\perp AN$ from A to R_1R_2 meeting at N.

$\lfloor A = 180^o - (30^o + 45^o) = 105^o$

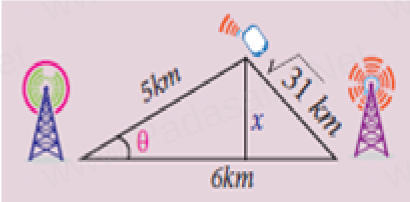
$\frac{a}{\sin A} = \frac{100}{\sin 105^o}$

$\Rightarrow \frac{100}{\frac{\sqrt{3}+1}{2\sqrt{2}}} \times \frac{1}{\sqrt{2}}$

= $\frac{200(\sqrt{3}-1)}{2} = 100(\sqrt{3}-1)$ **=a**

Sin30° = $\frac{x}{a}$ $\Rightarrow \frac{1}{2} = \frac{x}{a}$ $\Rightarrow$ **x = 100** $(\sqrt{3}-1)$ **½ =50**$(\sqrt{3}-1)$ **KM**

12. Suppose that there are two cell phone towers within range of a cell phone. The two towers are located at 6 km apart along a straight highway, running east to west t and the cell phone is north of the highway. The signal is 5 km from the first tower and √31 km from the second tower. Determine the position of the cell phone north and east of the first tower and how far it is from the highway



Let θ be the position of the cell phone from north to east of the first tower

By Cosine formula

$$\left(\sqrt{31}\right)^2 = 25+36-25\times 6 Cos\theta$$

$$60\cos\theta = 61-31=30$$

$$Cos\theta = \frac{30}{60} = \frac{1}{2} \quad \Rightarrow \theta = 60^\circ = \frac{\pi}{3}$$

13.

An Engineer has to develop a triangular shaped park with a perimeter 120 m in a village. The park to be developed must be of maximum area. Find out the dimensions of the park.

Perimeter of the triangle = 120 m

For a fixed perimeter 2s the area of the triangle is maximum when a = b =c

Therefore the side of the triangle = $\frac{120}{3} = 40$

Hence a = b= c = 40 m

14.

A rope of length 12 m is given. Find the largest area of the triangle formed by this rope and find the dimensions of the triangle so formed.

Perimeter of the triangle =12 m

For a fixed perimeter 2s the area of the triangle is maximum when a= b = c

Therefore the side of the triangle = $\frac{12}{3} = 4$

Hence a =b = c = 4

Area of the triangle = $\frac{s^2}{3\sqrt{3}}$ **since 2s = 12 ⇒ s= 6**

$$= \frac{6^2}{3\sqrt{3}} = \frac{12}{\sqrt{3}} = \frac{12}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{12\sqrt{3}}{3} = 4\sqrt{3} \text{ sq mts}$$

15. Using Heron's formula, show that the equilateral triangle has the maximum area for any fixed perimeter.

Hint ; In $xyz \le k$, maximum occurs when x= y= z

Let ABC be a triangle with constant perimeter 2s (since s is constant)

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

Δ is maximum when $s(s-a)(s-b)(s-c)$ is maximum

$$(s-a)(s-b)(s-c) \le \left(\frac{(s-a)(s-b)(s-c)}{3}\right)^3 = \frac{s^3}{27}$$

Since Geometric Mean $\le$ Arithmetic mean

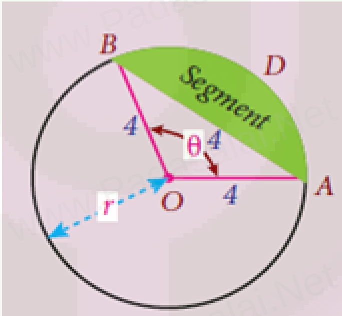
Equality occurs when s-a =s-b=s-c

Ie. When a = b =c maximum of $(s-a)(s-b)(s-c) = \dfrac{s^3}{27}$

Therefore for a fixed perimeter 2s the area of the triangle is maximum when a = b= c

Maximum Area = $\sqrt{\dfrac{ss^3}{2}} = \dfrac{s^2}{3\sqrt{3}}$ sq units

16. The Government plans to have a circular zoological park of diameter 8 km. A separate area in the form of a segment formed by a chord of length 4 km is to be allotted exclusively for a veterinary hospital in the park. Find the area of the segment to be allotted for the veterinary hospital.



Let AB be the chord and O be the centre of the circular park.
Let $\angle AOB = \theta$.

Area of the segment = Area of the sector − Area of $\triangle OAB$.

$$= \frac{1}{2}r^2\theta - \frac{1}{2}r^2 \sin\theta$$

$$= \left(\frac{1}{2} \times 4^2\right)[\theta - \sin\theta] = 8[\theta - \sin\theta] \qquad \text{...(i)}$$

But $\cos\theta = \dfrac{4^2 + 4^2 - 4^2}{2(4)(4)} = \dfrac{1}{2}$

Thus, $\theta = \dfrac{\pi}{3}$

From (i), area of the the segment to be allotted for the veterinary hospital

$$= 8\left[\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right] = \frac{4}{3}\left[2\pi - 3\sqrt{3}\right] m^2$$

17. A foot ball player can kick a foot ball from ground level with an initial velocity of 80ft/second. Find the maximum horizontal distance the foot ball travels and at what angle (Take g=32)

The formula for horizontal distance R is given by

$$R = \frac{u^2 \sin 2\alpha}{g} = \frac{(80 \times 80)\sin 2\alpha}{32} = 10 \times 20 \sin 2\alpha.$$

Thus, the maximum distance is 200 ft.
Hence, he has to kick the football at an angle of $\alpha = 45°$ to reach the maximum distance.

18. A ripple tank demonstrates the effect of two water waves being added together. The two waves are described by h= 8 cost and h = 6 sint where t ∈ [0,2π) is in seconds and h is the height in millimeters above still water. Find the maximum height of the resultant wave and the value of t at which it occurs.

Let H be the height of the resultant wave at time t. Then H is given by

$$H = 8\cos t + 6\sin t$$

Let $8\cos t + 6\sin t = k\cos(t-\alpha) = k(\cos t\cos\alpha + \sin t\sin\alpha)$

Hence, $k = 10$ and $\tan\alpha = \frac{3}{4}$, so that

$H = 10\cos(t-\alpha)$

Thus, the maximum of $H = 10$ mm. The maximum occurs when $t = \alpha$, where $\tan\alpha = \frac{3}{4}$.

19.

What must be the radius of a circular running path, around which an athlete must run 5 times in order to describe 1 km?

For 5 rounds = 1 km = 1000 mts

For 1 round = $\frac{1000}{5} = 200$ **mts**

$2\pi r = 200$

$\pi r = 100$

$\frac{22}{7} r = 100$

$r = \frac{100 \times 7}{22} = \frac{700}{22} = 31^o 49'5'$

20.

In a circle of diameter 40 cm, a chord is of length 20 cm. Find the length of the minor arc of the chord.

Since the diameter = 40 cm r = 2 cm

Chord of length = l = 20 cm

The triangle is an equilateral triangle Hence θ=60°

The length of the minor arc = rθ=20 x $\frac{\pi}{3} = \frac{20\times 22}{7\times 3} = \frac{440}{21}$ cm

21.

Find the degree measure of the angle subtended at the centre of circle of radius 100 cm by an arc of length 22 cm.

Since $l = r\theta$

$$\theta = \frac{l}{r} = \frac{22}{100} = 0.22^{o} = 0.22\times\frac{180}{\pi}$$

$$\theta = \frac{0.22\times 180\times 7}{22} = 12.6 = 12\frac{6}{10}\times 60 = 12^{o}36'$$

22.

What is the length of the arc intercepted by a central angle of measure 41° in a circle of radius 10 ft?

θ = 41° = $\frac{\pi}{180}\times 41$

Radius = 10 ft

Hence length of the arc= $10\times 41^{o} = 10\times 41\times\frac{\pi}{180} = 7.16\ ft$

23.

If in two circles, arcs of the same length subtend angles 60° and 75° at the centre, find the ratio of their radii.

Since $60^{o} = \frac{\pi}{3}$ $\quad 75^{o} = \frac{5\pi}{12}$

$\theta_1 = \frac{l}{r_1}$ $\quad \Rightarrow l = r_1\theta_1 \quad l = r_2\theta_2$ Since same length

$r_1\theta_1 = r_2\theta_2$

$$\frac{r_1}{r_2} = \frac{\theta_1}{\theta_2} \qquad \frac{r_1}{r_2} = \frac{\frac{5\pi}{12}}{\frac{\pi}{3}} == \times\frac{5\pi}{12}\times\frac{3}{\pi} = \frac{5}{4}$$

Therefore the ratio of their radii is 5 : 4

24.

The perimeter of a certain sector of a circle is equal to the length of the arc of a semi-circle having the same radius. Express the angle of the sector in degrees, minutes and seconds.

Perimeter of a sector of a circle =length of the arc of a semi circle

$$2r + r\theta = \pi r$$

$$r(2+\theta) = \pi r$$

$$2+\theta = \pi$$

$$\theta = \pi - 2 \quad \text{radian}$$

$$= \frac{180}{\pi}(\pi - 2) = \frac{180\pi}{\pi} - \frac{180\times 2}{\pi}$$

$$= 180 - \frac{360}{\pi} = 180 - 114^\circ 32' 44'' = 65^\circ 27' 16''$$

25.

Find the length of an arc of a circle of radius 5 cm subtending a central angle of 15°

Let s be the length of the arc of a circle of radius r subtending a central angle θ. Then $s = r\theta$.

We have, $\theta = 15^\circ = 15 \times \frac{\pi}{180} = \frac{\pi}{12}$ radians

So that, $s = r\theta$ gives $s = 5 \times \frac{\pi}{12} = \frac{5\pi}{12}\ cm$

26.

If the arc of the same length in two circles subtended central angle of 30° and 80° find the ratio of their radii

Let r_1 and r_2 be the radii of the two given circles and l be the length of the arc.

$\theta_1 = 30^\circ = \frac{\pi}{6}$ radians

$\theta_2 = 80^\circ = \frac{4\pi}{9}$ radians

Given that $l = r_1\theta = r_2\theta$

Thus, $\frac{\pi}{6} r_1 = \frac{4\pi}{9} r_2$

$\frac{r_1}{r_2} = \frac{8}{3}$ which implies $r_1 : r_2 = 8 : 3$.

27.

A train is moving on a circular track of 1500 m radius at the rate of 66 km/hr. What angle will it turn in 20 seconds?

r = 1500 mts

In 20 seconds the length is $= \frac{66\times1000}{3600}\times 20 = \frac{3300}{9}$ = length

$\theta = \frac{l}{r} = \frac{3300}{9\times1500} = \frac{11}{45}$

28.

A circular metallic plate of radius 8 cm and thickness 6 mm is melted and moulded into a pie (a sector of the circle with thickness) of radius 16 cm and thickness 4 mm. Find the angle of the sector.

r= 8 cm = 80 m.m

thickness = 6 mm

radius = 16 cm = 160 mm

θ is a central angle

The circular metallic plate melted = made in to a volume of a sector

$$(\pi \times 80 \times 80 \times 6 = \pi \times 160 \times 160 \times 4)\ \frac{\theta}{360}$$

$$\theta = \frac{80\times80\times6\times360}{160\times160\times4} = 135^o = \frac{3\pi}{4}$$

29.

A and B are working on similar jobs but their annual salaries differ by more than Rs 6000. If B earns rupees 27000 per month, then what are the possibilities of A's salary per month?

Let the salary of A = x

There are two possibilities (i) Salary of A < salary of B

(ii) Salary of A >salary of B

(i)

Therefore the salary A will be = 6000 +x (differ more than Rs.6000)

Salary of B is Rs.27,000

Salary of A < salary of B

6000+x < 27000

X < 27000-6000

X < 21000

The salary of A is less than Rs 21,000

(OR)

(ii)

The salary of A will be = x-6000 (differ less than Rs.6000)

Salary of A >salary of B

Salary of A > 27000

x-6,000 > 27000

x > 33000

The salary of B is greater than Rs 33,000

30.

A Plumber can be paid according to the following schemes: In the first scheme he will be paid rupees 500 plus rupees 70 per hour, and in the second scheme he will paid rupees 120 per hour. If he works x hours to complete the job, then for what value of x does the first scheme give better wages?

The plumber works x hours

Wages from the first scheme = 500+70x

Wages from the second scheme=120x

First scheme give better wages mean $\Rightarrow 500+70x > 120x$

$\Rightarrow 500 > 120x-70x$

$\Rightarrow 500 > 50x$

$\Rightarrow 10 > x$

Therefore the value of x is less than 10

X = 1,2,3,4,5,6,7,8,9

31.

A model rocket is launched from the ground. The height h reached by the rocket after t seconds from lift off is given by $h(t) = -5t^2 + 100t, 0 \leq t \leq 20$. At what times the rocket is 495 feet above the ground?

Given $h(t) = -5t^2+100t$ $0 \leq t \leq 20$

Height must be lies between 0 and 495

$0 \leq h(t) \leq 495$

$0 \leq -5t^2+100t \leq 495$

$0 \leq -5t^2+100t -495 \leq 0$

ie., $-5t^2+100t -495 = 0$

Divide by (-5) $t^2-20t+99=0$

$(t-11)(t-9) = 0$

$t = 11$ $t = 9$

When t = 11 sec (or) 9 sec the rocket is above the ground

32.

Find all pairs of consecutive odd natural numbers both of which are larger than 10 and their sum is less than 40.

Let x be the small odd number

Another number = x +2

Given x>10 and also x+2 >10

Their Sum = x+(x+2) <40

2x < 40-2=38

X < 19

⇒ 10 <x < 19

Moreover x is odd

Therefore the possible values of x are 11, 13,15, 17

All possible pair of consecutive odd natural numbers

(11,13) (13,15) ,(15,17), (17,19) \

33.

A manufacturer has 600 litres of a 12 percent solution of acid. How many litres of a 30 percent acid solution must be added to it so that the acid content in the resulting mixture will be more than 15 percent but less than 18 percent?

(i) Let x denotes the number of solution of acid More than 15%

Given that 12% of 600 +30%of x > 15% (600+x)

$$\left(\frac{12}{100}\times 600\right)+\frac{30x}{100}>(600+x)\frac{15}{100}$$

7200+30x > 9000+15x (multiply by 100 through out)

30x -15x > 9000-7200

15 x > 1800

X > 120

(ii) Less than 18%

30% of x + 12%(600) < (600+x) 18%

30x +7200 < 10800+18x

30x – 18x < 10800 -7200

12 x < 3600

X < 300

34.

To secure A grade one must obtain an average of 90 marks or more in 5 subjects each of maximum 100 marks. If one scored 84, 87, 95, 91 in first four subjects, what is the minimum mark one scored in the fifth subject to get A grade in the course?

To get A grade ⇒ Average ≥ 90

$$\frac{84+87+95+91+x}{5} \geq 90$$

357 +x ≥ 450 (cross multiply)

X ≥ 450 -357

X ≥ 93

Minimum marks one should scored 93 to get A grade in the course.

35. **A girl A is reading a book having 446 pages and she has already finished reading 271 pages . She wants to finish this book within a week. What is the minimum number of pages she should read per day to complete reading the book with in a week?**

Let the number of pages = x (per day)

Total number of pages 446

she can read 7x pages with in a week (7 days for a week)

(7 (number of pages)= 7x)

she already finished reading 271 pages

hence 7x+271 ≥ 446

hence 7x ≥ 446-271

7x ≥ 17 5

÷by 7 x ≥ 25

⇒ she should read at least 25 pages per day (minimum number of pages to be read)

36.

Our monthly electricity bill contains a basic charge, does not change with number of units used , and a charge that depends only on how many units we use. Let us say Electricity Board charges Rs.110 as basic charge and charges Rs. 4 for each unit we use. If a person wants to keep his electricity bill below Rs. 250. Then what should be his electricity usage?

X denotes number of units used (Note that x ≥ 0)

Electricity bill = electiricty Board chrges + 4 (number of units used)

= Rs 110+4x

If the person wants his bill to be below Rs 250

We have to solve the inequality 110+4x <250

4x < 250 -110

4 x < 140

÷ 4 $x < \frac{140}{4} = 35$

Therfore x < 35

The person should keep his usage below 35 units

(so that his bill below Rs 250)

37.

A simple cipher takes a number and codes it, using the function $f(x) = 3x - 4$. Find the inverse of this function, determine whether the inverse is also a function and verify the symmetrical property about the line $y = x$ (by drawing the lines).

f(x) = 3x-4

$y = 3x - 4 \Rightarrow y + 4 = 3x \Rightarrow x = \frac{y+4}{3}$

We have to draw the graph of $y = 3x-4$ and $y = \frac{x+4}{3}$

These two lines are symmetric with respect to the line $y = x$

38.

The formula for converting from Fahrenheit to Celsius temperatures is $y = \frac{5x}{9} - \frac{160}{9}$. Find the inverse of this function and determine whether the inverse is also a function.

$y = \frac{5x}{9} - \frac{160}{9}$

$y = \frac{5x-160}{9}$ = f(x)

$9y = 5x - 160$

$5x = 9y + 160$ = g(y)

We see that fog = y = I_y

gof = x = I_x

therefore the functions are inverse to each other

hence f and g are bijective functions

39.

The owner of a small restaurant can prepare a particular meal at a cost of Rupees 100. He estimates that if the menu price of the meal is x rupees, then the number of customers who will order that meal at that price in an evening is given by the function $D(x) = 200 - x$. Express his day revenue, total cost and profit on this meal as functions of x.

Number of customers = 200 –x

Cost of one meal = Rs.100

Total Cost = (Cost) x (number of customers)

= 100 (200-x)

Revenue on one meal = x

Total Revenue = (Revenue on one meal) x (number of customer)

= x (200-x)

Profit = (Revenue)- (Cost)

= x (200-x) - 100 (200-x)

Taking common term (200-x)

Profit = (200-x) (x-100)

40.

The function for exchanging American dollars for Singapore Dollar on a given day is $f(x) = 1.23x$, where x represents the number of American dollars. On the same day the function for exchanging Singapore Dollar to Indian Rupee is $g(y) = 50.50y$, where y represents the number of Singapore dollars. Write a function which will give the exchange rate of American dollars in terms of Indian rupee.

Given x is number of American dollar

y is number of Singapore dollar

f(x) = 1.23x = Singapore dollar

g(x) = 50.50y = Indian rupee

Now the function which will give the exchange rate of American dollars in terms of Indian rupee

gof(x) = g(f(x))

= 50.50 (1.23x) = 62.115 x

41.

A salesperson whose annual earnings can be represented by the function $A(x) = 30,000 + 0.04x$, where x is the rupee value of the merchandise he sells. His son is also in sales and his earnings are represented by the function $S(x) = 25,000 + 0.05x$. Find $(A + S)(x)$ and determine the total family income if they each sell Rupees $1,50,00,000$ worth of merchandise.

Annual earnings =A (x) = 30,000n+0.04x (x = rupee value)

Sales earnings = S(x) = 25,000 +0.05x

Total income = A + S

=30,000n+0.04x + 25,000 +0.05x

= 55,000 + 0.09x

Given x= 1,50,00,000

Total income = 55,000+0.09 (1,50,00,000)

= 55,000 + 13,50,000

= 14,05,000

42.

The total cost of airfare on a given route is comprised of the base cost C and the fuel surcharge S in rupee. Both C and S are functions of the mileage m; $C(m) = 0.4m + 50$ and $S(m) = 0.03m$. Determine a function for the total cost of a ticket in terms of the mileage and find the airfare for flying 1600 miles.

Cost function = C = 0.4m+50

Fuel surcharge function = S = 0.03

Total cost of the function = T = C+S

=(0.4m+50) + (0.03)

T = 0.43m + 50

Given m = 1600 miles

T = 0.43(1600) +50

= 688 +50 = Rs.738

43.

The distance of an object falling is a function of time t and can be expressed as $s(t) = -16t^2$. Graph the function and determine if it is one-to-one.

$s(t_1) = s(t_2)$

$-16\, t_1^2 = -16\, t_2^2$

$t_1^2 = t_2^2$

$t_1^2 - t_2^2 = 0$

$(t_1+t_2)\,(t_1-t_2) = 0$

$t_1 = -t_2 \quad t_1 = t_2$

Since time cannot be negative therefore $t_1=t_2$

Hence it is one to one function

44.

The weight of the muscles of a man is a function of his body weight x and can be expressed as $W(x) = 0.35x$. Determine the domain of this function.

X denotes the body weight of a man

Weight of a man is not zero

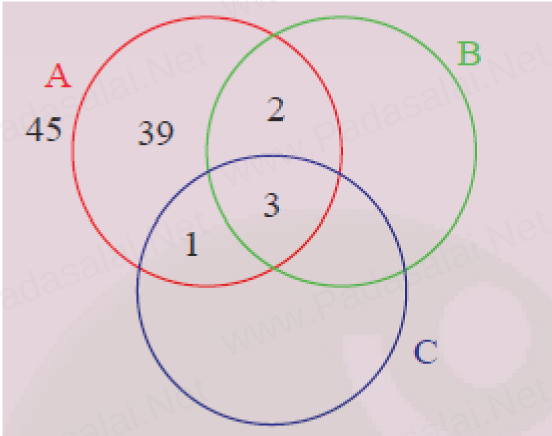
For all values of $x>0$ the weight can be defined

Hence the Domain of the function is $(0, \infty)$

45.

In a survey of 5000 persons in a town , it was found that 45% of the persons know Language A 25% know Language B , 10% know Language C , 5% know Language A and B, 45 know Languages B and C, and 4% know Languages A and C .If 3% of the persons know all the the three languages , find the number of persons who know only language A

From the Venn diagram we see that



The percentage of persons who know Language A only = 39 %

The number of persons who knows Only Language A out of 5000 = 5000 x 39/100 =50 x 39 =1950

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