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RAVI MATHS TUITION (HOME) CENTER, CH-82 PH-8056206308

11th chapter 1 sets relation function

11th Standard

Date : 25-Feb-19

Maths

Reg.No. : | | | | | | |

Time : 03:00:00 Hrs

Total Marks : 100

10 x 1 = 10

1) (c) n
2) (b) on to
3) (d) not a function
4) (d) (-∞,1]
5) (c) 1
6) (c) only one element
7) (d) N
8) (b) 4
9) (c) 512
10) (c) [0,1)

10 x 2 = 20

11) {x ∈ N : 4x + 9 < 52}

Let C = {x∈N:4x + 9 < 52}

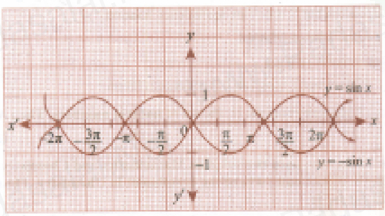
⇒ C = {x∈N:4x<52-9}

⇒ C={x∈N: 4x<43}

⇒ C= $\left\{x \in N : x < \frac{43}{4}\right\}$ ⇒ C={x∈N:x<10.75}

⇒ C={1,2,3,4,5,6,7,8,9,10}.

12) y = sin |x|



We know $|x| = \begin{cases} x \ if \ x \geq 0 \\ -x \ if \ x < 0 \end{cases}$

∴ sin Ixl = sin x if x ≥ 0

and sin Ixl = sin (-x)=-sin x if x < 0.

The graph of y = sin (-x) = - sinx is the reflection of the graph of sin x about Y-axis.

13) Given X = {a, b, c, d} and R = {(a, a)(b, b)(a, c)}.

To make the relation R reflexive we must have (c, c) and (d, d) ∈ R

∴ minimum number of ordered pairs to be included to R to make it reflexive is (c, c) and (d, d)

14) Given X = {a, b, c, d} and R = {(a, a)(b, b)(a, c)}.

To make R symmetric, we must have (c, a) ∈ R

∴ minimum number of ordered pairs to be included to R to make it symmetric is (c, a).

https://www.trbtnpsc.com/2018/06/latest-plus-one-11th-study-materials-tamil-medium-english-medium-new-syllabus-based.html

15) Given X = {a, b, c, d} and R = {(a, a)(b, b)(a, c)}.

To make R transitive, we must have (c, d) and (a, d) ∈R.

∴ (a, c) and (c, d) ∈ R ⇒ (a, d) ∈ R

∴ minimum number of ordered pairs to be included to make R transitive is (c, d) and (a, d)

16) Given relation is 2a + 3b = 30 for all a, b ∈ N.

2a + 3b = 30 ⇒ 2a =30 - 3b

$\Rightarrow \quad a = \frac{30-3b}{2}$

a	12	9	6	3
b	2	4	6	8

∴ The list of ordered pairs are (12, 2) (9, 4) (6, 6) (3, 8)

Reflexivity : (12, 12) ∉ R ⇒ R is not reflexive.

17) The relation is defined by aRb if a + b ≤ 6 for all a, b ∈N.

a+b ≤6 ⇒ a ≤ 6 - b

a	5	4	3	2	1
b	1	2	3	4	5

∴ The list of ordered pairs are (5,1) (4, 2) (3, 3) (2, 4) and (1, 5).

Hence, R is not an equivalence relation.

18) f(-3) = $(-3)^2$ - 3 $\quad \left[\therefore \quad f(x) = x^2 - 3 \quad when \quad x = -3\right]$

= 9 - 3 =6

f(5)=5^2+3(5)-2 $\quad \left[\therefore f(x) = x^2 + 3x - 2 \quad when \quad x = 5\right]$

= 25+ 15 - 2

= 38

f(2) = 2^2 - 3

= 4 - 3 = 1 $\quad \left[\therefore \quad f(x) = x^2 - 3 \quad when \quad x = 2\right]$

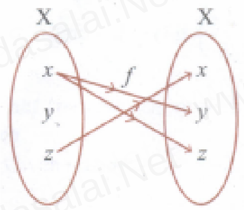
f(-1) = $(-1)^2$ + (-1) -5 $\left[\therefore \quad f(x) = x^2 + x - 5 \quad when \quad x = -1\right]$

= 1-1-5 = -5

f(0) = 0^2-3 = -3 $\quad \left[\therefore \quad f(x) = x^2 - 3 \quad when \quad x = 0\right]$

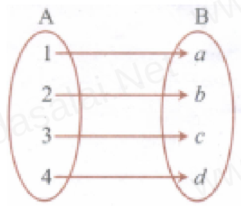
∴ f(-3)=6, f(5) = 38, f(2) = 1, f(-1) = -5, f(0) = -3

19) If X={x,y,z} and f={(x, y) (x, z) (z, x)} : (f : X → X).



f is not a function since the element x have two images namely y and z

20)



Let f: A → B defined by

f = {(1,a)(2,b)(3,c)(4,d)}

Here different elements have different images

∴ f is one-to-one.

Also Co-domain = {a,b,c,d} = Range.

∴ f is onto.

∴ f is one-to-one and onto.

10 x 3 = 30

21) Given cost of one meal = Rs.100

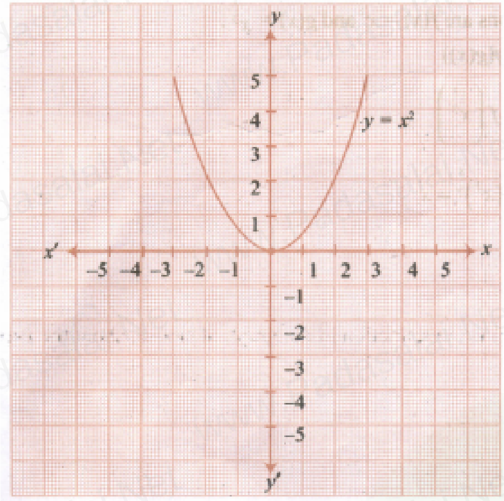
Number of customers is given by the function D(x)=200-x

∴ Day cost function = (200-x)(100)

=20000-100x

22) **Step 1 :**

Draw the Graph of $y = x^2$



Step 2 :

The graph of $y =(x -1)^2$, shifts to the right for one unit.

Step 3 :

The graph of $y = 3 (x - 1)^2$, compresses towards the Y-axis that is moves away from the X-axis since the multiplying factor is 3 which is greater than 1.

Step 4:

The graph of $y = 3 (x - 1)^2 + 5$, causes the shift to the upward for 5 units.

23) (B∪C) ={3, 4, 5, 6 ,7, 9}

Now, A×(B∪C) = {1, 2, 3} ×{3, 4, 5, 6, 7, 9}

={(1,3)(1,4)(1,5)(1,6)(1,7)(1,9)(2,3)(2,4)(2,5)(2,6)(2,7)(2,9)(3,3)(3,4)(3,5)(3,6)(3,7)(3,9)} .....(1)

Now A×B = {1,2,3} × {4,5,6,7}

={(1,4)(1,5)(1,6)(1,7)(2,4)(2,5)(2,6)(2,7)(3,4)(3,5)(3,6)(3,7)}

A×C = {1,2,3} × {3,4,5,9}

={(1,3)(1,4)(1,5)(1,9)(2,3)(2,4)(2,5)(2,9)(3,3)(3,4)(3,5)(3,9)}

RHS(A×B)∪(A×C)={(1,3)(1,4)(1,5)(1,6)(1,7)(1,9)(2,3)(2,4)(2,5)(2,6)(2,7)(2,9)(3,3)(3,4)(3,5)(3,6)(3,7)(3,9)} .....(2)

From (1) & (2), LHS = RHS

Hence verified

24) B-A = {4, 5, 6, 7}

LHS = C-(B-A) = {3, 9} ....(1)

C∩A = {3}

B' = {1,2,3,8,9}

C∩B' = {3, 9}

RHS = (C∩A)∪ (C∩B') = {3, 9} ....(2)

From (1) and (2), LHS = RHS

25) Given n(p(A)) = 1024 = 2^{10}

$\Rightarrow$ n(A) = 10 [$\therefore$ if n(A)=n, then n(p(A))=2^n]

n(p(B)) = 32 = 2^5

$\Rightarrow$ n(B) = 5.

We know that,

n(A$\cup$ B) = n(A) + n(B) - n(A$\cap$B)

$\Rightarrow$ 15 = 10+5 - (A$\cap$B)

$\Rightarrow$ n(A$\cap$B) = 0.

26) We know that n(A$\cup$ B) = n(A-B)+n(B-A)+n(A$\cap$B) if A and B are not disjoint.

$\Rightarrow$ n(A-B)+n(B-A) = n(A$\cup$B) - n(A$\cap$B)

$\Rightarrow$ n(AΔB) = 10-3

$\Rightarrow$ $\therefore$ n(AΔB)=7

$\therefore$ n[P(AΔB)] = 2^7=128

27) n(A$\times$ A) = 16. $\Rightarrow$ n(A)=4

Given S = {a,b) $\in$ A$\times$A:a<b}

$\therefore$ A = {0, 1, 2, -1}

(A$\times$A)={(0,0)(0,1)(0,2)(0,-1),(1,1) (1,2)(1,-1)(2,0)(2,1)(2,2)(2,-1)(-1,0)(-1,1)(-1,2)(-1,-1)}

Now, S={(0,1)(0, 2)(-1,0)(-1,1)(-1,2)}

$\therefore$ The remaining elements of S are (0,2)(-1,0)(-1,1)(1,2)

28) Let f(x) = $\frac{1}{1-2sinx}$

When the denominator is 0,

1-2 sin x = 0

$\Rightarrow$ 1 = 2 sin x

$\Rightarrow \quad sin \quad x = \frac{1}{2}$

$\Rightarrow \quad sin \quad x = sin\frac{\pi}{6}$

$\Rightarrow \quad x = n\pi + (-1)^n \frac{\pi}{6} n \in Z \quad [\because sin \quad x = sin\alpha \Rightarrow x = n\pi + (-1)^n \alpha \quad n \in Z]$

Domain of f(x) is R - $\left(n\pi + (-1)^n \frac{\pi}{6}\right), n \in Z$

29) Given f(x) = $\frac{\sqrt{4-x^2}}{\sqrt{x^2-9}}$

When x =2, f(x) = 0

When x = -2, f(x) = 0

For all the other values, we get negative value in the square root which is not possible.

$\therefore$ Domain = {2, -2}

30) As (a, a) $\in$ R for all a $\in$ S, R is reflexive.

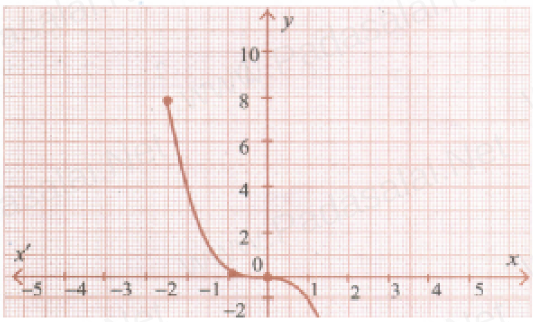
There is no pair (a, b) in R such that (b, a) $\notin$ R. In other words, for pair (a, b) $\in$ R, (b, a) is also in R. Thus R is symmetric.

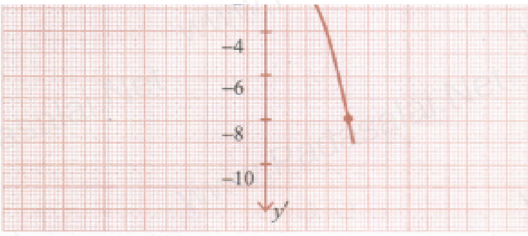
We cannot find two pairs (a, b) and (b, c) in R, such that (a, c) $\notin$ R. Thus the statement "R is not transitive" is not true; therefore, the statement "R is transitive" is true; hence R is transitive. Since R is reflexive, symmetric and transitive. Thus, this relation is an equivalence relation.

8 x 5 = 40

31) (i) y = $-x^3$

x	0	1	-1	2	-2
y	0	-1	1	-8	8



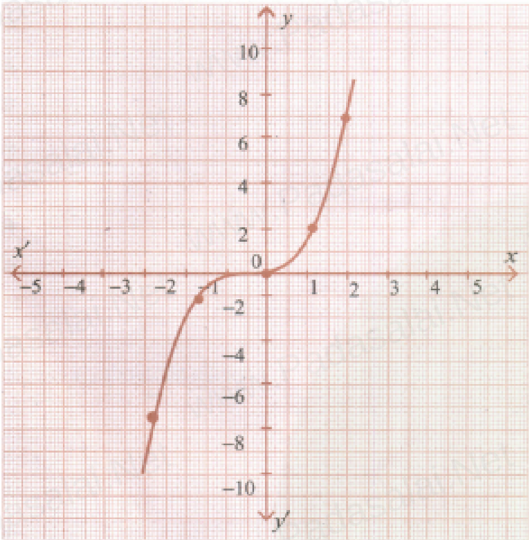


Let $f(x) = x^3$

Since $y = -f(x)$, this is the reflection of the graph off about the x-axis.

(ii) $y = x^3+1$

x	0	1	-1	2	-2
y	1	2	9	9	-7

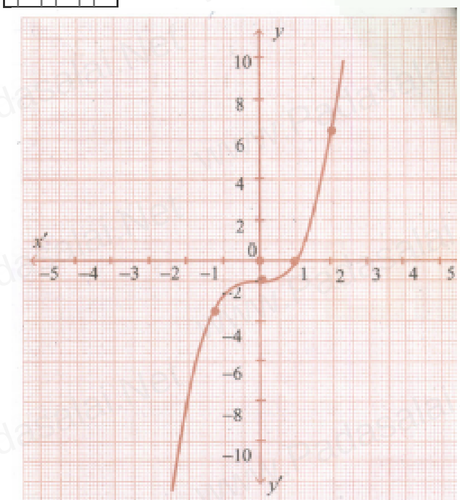


Let $f(x) = x^3$

Since $y = f(x) + 1$, this is the graph of f(x) shifts to the upward for one unit.

(iii) $y = x^3-1$

x	0	1	-1	2	-2
y	-1	0	-2	7	-9

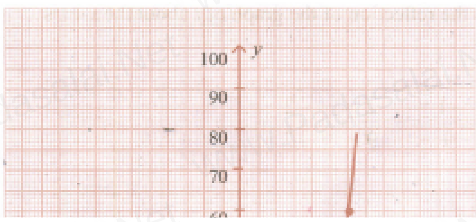


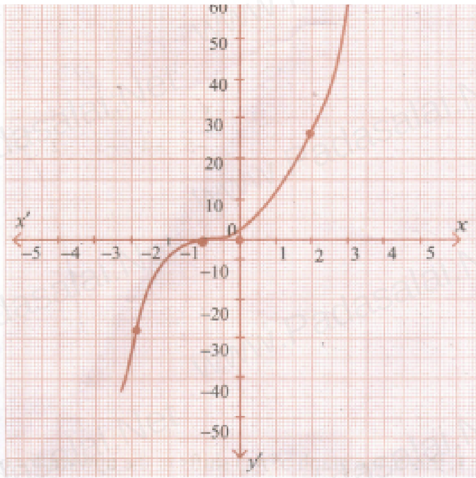
Let $f(x) = x^3$

Since $y = f(x) - 1$, this is the graph of (x) shifts to the downward for one unit.

(iv)

$y = (x+1)^3$

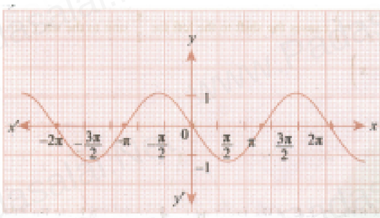




Let $f(x) = x^3$

$y = (x + 1)^3$, causes the graph of/ex) shifts to the left for one unit.

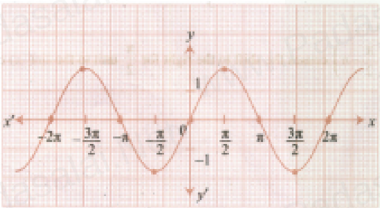
32) (i) y = sin (-x)



Let y = sin x.

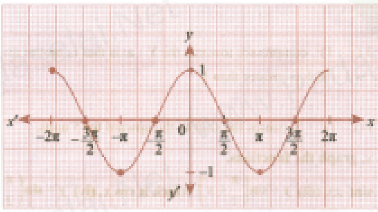
Then sin(-x) is the reflection of the graph of sin x, about y-axis.

(ii) y= -sin(-x)



-sin(-x) is the reflection of the graph of sin(-x) about the x-axis.

(iii) $y = sin\left(\frac{\pi}{2} + x\right)$

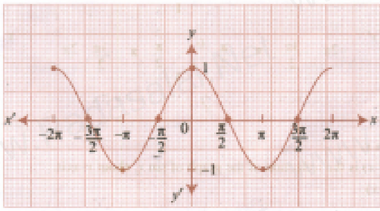


Lety= sinx.

Then $sin\left(\frac{\pi}{2} + n\right)$ causes the shift to the left for $\frac{\pi}{2}$ unit to the sin x curve.

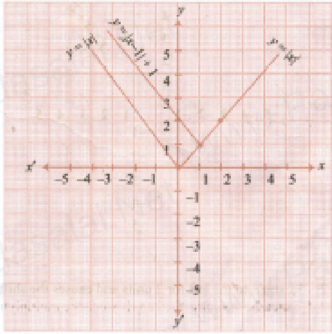
(iv)

$y = sin\left(\frac{\pi}{2} - x\right)$



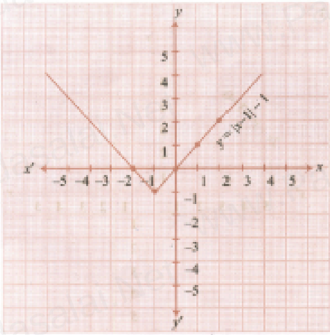
Let y = sinx.Then $sin\left(\frac{\pi}{2} - n\right)$ causes the shift to the left for $\frac{\pi}{2}$ unit to the sin (-x) curve.

33) (i) y = |x+1| + 1



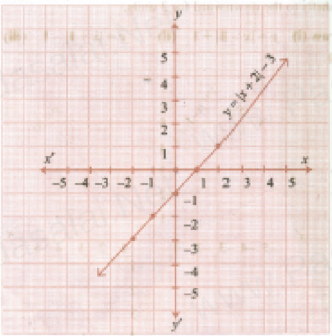
The graph of y =|x + 1|+1, shifts to the right for one unit and causes the shift to the upward for one unit.

(ii) y= |x+ 1|-1



The graph of y =|x - 1|-1, shifts to the left for one unit and causes the shift to the downward for one unit.

(iii) y = Ix+ 2|-3



The graph of y =|x + 2|-3, shifts to the left for 2 units and causes the shift to the downward for 3 units.

34) $-1 \leq \cos x \leq 1 = -3 \geq -3\cos x \geq -3$

$= -3 \leq -3\cos x \leq 3 \Rightarrow 1-3 \leq 1-3\ \cos x \leq 1+3$

$= -2 \leq 1-3\ \cos\ x$ and $1-3\cos\ x \leq 4$.

By taking reciprocals, we get $\Rightarrow \frac{1}{1-3\cos x} \leq -\frac{1}{2}$ and $\frac{1}{1-3\cos\ x} \geq \frac{1}{4}$.

Hence the range of f is $\left(-\infty, \frac{1}{2}\right] \cup \left[\frac{1}{4}, \infty\right)$.

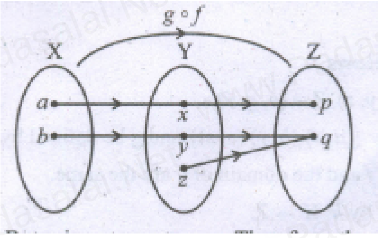
35) If x < -3 or x > 3, then x^2 will be greater than 9 and hence 9 - x^2 will become negative which has no square root in R. So x must lie on the interval [- 3, 3].

Also if $x \geq -1$ or $x \leq 1$, then x^2-1 will become negative or zero. If it is negative, x^2 - 1 has no square root in R. If it is zero.f is not defined. So, x must lie outside [- 1, 1]. That is x must lie on $(-\infty, -1] \cup [1, \infty)$, Combining these two conditions, the largest domain for f is $[-3, 3] \cap ((-\infty, -1) \cup (1, \infty))$. That is $[-3, -1) \cup (1, 3]$.

36) Clearly, gof = {(1, 1), (2, 2), (3, 2)}. But fog is not defined because the range of g = {1, 2, 4} is not contained in the domain of f= {1,2, 3}.

37) To claim a statement is not true we have to prove by giving an example. Such examples are called counter examples).

Consider the diagram given below.



Clearly, f and gof are one-to-one. But g is not one to one. Thus from the above diagram, it shows that the statement is not true.

38) We know $|x| = \begin{cases} -x & if\ x \le 0 \\ x & if\ x > 0 \end{cases}$

So, $f(x) = \begin{cases} 2x - (-x) & if x \le 0 \\ 2x - x & if\ x > 0 \end{cases}$

Thus, $f(x) = \begin{cases} 3x & if x \le 0 \\ x & if\ x > 0 \end{cases}$

Also, $g(x) = \begin{cases} 2x + (-x) & if\ x \le 0 \\ 2x + x & if\ x > 0 \end{cases}$

Thus, $g(x) = \begin{cases} x & if x \le 0 \\ 3x & if x > 0 \end{cases}$

Let $x \le 0$. Then (gof) (x) = g(f{x)) = g(3x) = 3x.,

The last equality is taken because $3x \le 0$ whenever $x \le 0$.

Let x > 0. Then (gof)(x) =g(f(x)) = g(x) = 3x.

Thus (gof)(x)=3x for all x.

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10 x 1 = 10

1) The number of constant functions from a set containing m elements to a set containing n elements is

(a) mn (b) m (c) n (d) m+n

2) The function f:[0,2π]→[-1,1] defined by f(x)=sin x is

(a) one-to-one (b) on to (c) bijection (d) cannot be defined

3) Let X={1,2,3,4}, Y={a,b,c,d} and f={f(1,a),(4,b);(2,c);(3,d);(2,d)}. Then f is

(a) an one-to-one function (b) an onto function (c) a function which is not one-to-one (d) not a function

4) Let f:R→R be defined by f(x)=1-|x|. Then the range of f is

(a) R (b) (1,∞) (c) (-1,∞) (d) (-∞,1]

5) If A = {(x,y) : y = e^x, x∈R} and B = {(x,y) : y=e^{-x}, x ∈ R} then n(A∩B) is

(a) Infinity (b) 0 (c) 1 (d) 2

6) If A = {(x,y) : y = sin x, x ∈ R} and B = {(x,y) : y = cos x, X ∈ R} then A∩B contains

(a) no element (b) infinitely many elements (c) only one element (d) cannot be determined

7) Let A and B be subsets of the universal set N, the set of natural numbers. Then A'∪[[(A∩B)∪B'] is

(a) A (b) A' (c) B (d) N

8) If n((A x B) ∩(A x C)) = 8 and n(B ∩ C) = 2, then n(A) is

(a) 6 (b) 4 (c) 8 (d) 16

9) The number of relations on a set containing 3 elements is

(a) 9 (b) 81 (c) 512 (d) 1024

10) The range of the function $f(x) = |\lfloor x \rfloor - x|, x \in R$ is

(a) [0, 1] (b) [0,∞) (c) [0,1) (d) (0,1)

10 x 2 = 20

11) Write the following in roster form.{x ∈N:4x+9< 52}

12) From the curve y=sin x, draw y=sin|x|. (Hint: sin(-x)=-sin x)

13) Let X = {a, b, c, d}, and R = {(a, a) (b, b) (a, c)}. Write down the minimum number of ordered pairs to be included to R to make it

Reflexive

14) Let X = {a, b, c, d}, and R = {(a, a) (b, b) (a, c)}. Write down the minimum number of ordered pairs to be included to R to make it

Symmetric

15) Let X = {a, b, c, d}, and R = {(a, a) (b, b) (a, c)}. Write down the minimum number of ordered pairs to be included to R to make it

Transitive

16) On the set of natural number let R be the relation defined by aRb if 2a + 3b = 30. Write down the relation by listing all the pairs. Check whether it is reflexive

17) On the set of natural number let R be the relation defined by aRb if a + b $\leq$ 6. Write down the relation by listing all the pairs. Check whether it is equivalence.

18) Write the values of f at -3,5,2,-1,0 if

$$f(x)=\begin{cases} x^2+x-5 & if\ x\in(-\infty,0) \\ x^2+3x-2 & if\ x\in(3,\infty) \\ x^2 & if\ x\ \in(0,2) \\ x^2-3 & otherwise \end{cases}$$

19) State whether the following relations are functions or not. If it is a function check for one-to-oneness and ontoness. If it is not a function state why?

If X={x,y,z} and f={(x, y) (x, z) (z, x)} : (f : X $\rightarrow$ X).

20) Let A={1,2,3,4} and B = {a,b,c,d}. Give a function from A$\rightarrow$B for each of the following: one-to-one and onto.

10 x 3 = 30

21) The owner of small restaurant can prepare a particular meal at a cost of Rupee 100. He estimate that if the menu price of the meal is x rupees, then the number of customers who will order that meal at that price in an evening is given by the function D(x)=200-x. Express his day revenue total cost and profit on this meal as a function of x.

22) Write the steps to obtain the graph of the function $y=3(x-1)^2+5$ from the graph $y=x^2$

23) By taking suitable sets A, B, C, verify the following results:

$A\times(B\cup C)=(A\times B)\cup(A\times C)$

24) By taking suitable sets A, B, C, verify the following results:

$C-(B-A)=(C\cap A)\cup(C\cap B')$

25) If n (p(A)) = 1024, n(A∪B)=15 and n(p(B))=32, then find n(A∩B).

26) If n(A∩B)=3 and n(A∪B) = 10 then find n(P(AΔB))

27) If A$\times$ A has 16 elements, S={(a,b)∈A$\times$A:a< b};a< b};(-1, 2) and (0, 1) are two elements of S, then find the remaining elements of S.

28) Find the domain of $\frac{1}{1-2sinx}$

29) Find the largest possible domain of the real valued function f(x)=$\frac{\sqrt{4-x^2}}{\sqrt{x^2-9}}$

30) Check the relation R = {(1, 1) (2, 2) (3, 3) ... (n, n)} defined on the set S = {1, 2,3, .. n} for the three basic relations.

8 x 5 = 40

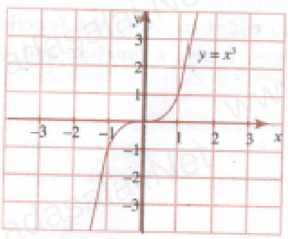
31) For the given curve $y=x^3$ given in figure draw, try to draw with the same scale

(i) $y=-x^3$

(ii) $y=x^3+1$

(iii) $y=x^3-1$

(iv) $y=(x+1)^3$



32) From the curve y=sin x, graph the functions.

(i) y=sin(-x)

(ii) y=-sin(-x)

(iii) $y=sin\left(\frac{\pi}{2}+x\right)$

(iv) $y=sin\left(\frac{\pi}{2}-x\right)$

33) From the curve y=|x|, draw

(i) y=|x+1|+1

(ii) y=|x-1|-1

(iii) y=|x+2|-3

34) Find the range of the function $f(x)=\frac{1}{1-3\cos x}$.

35) Find the largest possible domain for the real valued function given by $f(x)=\frac{\sqrt{9-x^2}}{x^2-1}$.

36) Let f = {(1, 4), (2, 5), (3, 5)} and g = {(4, 1), (5, 2), (6, 4)}. Find gof. Can you find fog?

37) Show that the statement, "if f and gof are one-to-one, then g is one-to-one" is not true.

38) Let f, g: $R \rightarrow R$ be defined as f (x) = 2x -|x| and g(x) = 2x + |x|. find fog.

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RAVI MATHS TUITION (HOME) CENTER, CH-82 PH-8056206308

11th chapter 2 Algebra

11th Standard

Date : 25-Feb-19

Maths

Reg.No. :

Time : 03:00:00 Hrs

Total Marks : 100

10 x 1 = 10

1) (b) [-11,7]

2) (b) (2,∞)

3) (b) [2,∞]

4) (c) -4

5) (b) 1

6) (b) 7

7) (b) -8

8) (c) 2

9) (c) 9,1

10) (a) $\frac{-1}{2}$

10 x 2 = 20

11) $\left(\left[(256)^{\frac{-1}{2}}\right]^{\frac{-1}{4}}\right)^3$ = $(256)^{\frac{-1}{2}\times\frac{-1}{4}\times 3}$ $\left[\because \frac{a^m}{a^n} = a^{m-n}\right]$

= $(256)^{\frac{3}{8}} = (2^8)^{\frac{3}{8}} = 2^{8\times\frac{3}{8}} = 2^3 = 8$

12) Given $\frac{3^{2n}9^2 3^{-n}}{3^{3n}}$ = 27

⇒ $\frac{3^{2n-n}.9^2}{3^{3n}}$ =27 [∵ $a^m.a^n=a^{m+n}$]

⇒ $\frac{3^n.(3^2)^2}{3^{3n}}$ =27

⇒ 3^{n-3n} (3^4)=27 $\left[\because \frac{a^m}{a^n} = a^{m-n} \& (a^m)^n = a^{mn}\right]$

⇒ $3^{-2n}.3^4$ =27

⇒ 3^{-2n+4} =3^3

Equating the powers both sides we get

-2n+4 = 3

⇒ -2n = 3-4 =-1

⇒ 2n = 1

⇒ n = $\frac{1}{2}$

13) $\frac{7+\sqrt{6}}{3-\sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}}$

⇒ $\frac{(7+\sqrt{6})(3+\sqrt{2})}{3^2-(\sqrt{2})^2} \Rightarrow \frac{21+7\sqrt{2}+3\sqrt{6}+\sqrt{129}-2}{9-2} \Rightarrow \frac{21+7\sqrt{2}+3\sqrt{6}+\sqrt{3}}{7}$

14) The means $-\frac{1}{4} - 3 \leq \frac{3}{4}x \leq \frac{1}{4}$.

$\Rightarrow -\frac{1}{4} - 3 \leq -\frac{3}{4}x \leq \frac{1}{4} - 3$

$\Rightarrow -\frac{13}{4} \leq -\frac{3}{4}x \leq -\frac{11}{4}$

Multiplying by 4 throughout we get,

$-13 \leq -3x \leq -11$

$\frac{-13}{-3} \geq x \geq \frac{-11}{3}$.

∴ The Solution set is $\left[\frac{-11}{3}, \frac{13}{3}\right]$

15) Given $log_9^{27} - log_{27}^{9}$

= $log_9^{3^3} - log_{27}^{3^2}$

= 3 $\log_9 3$-2 $\log_{27} 3$ [By power rule]

= $\frac{3}{log_3 9} - \frac{2}{log_3 27}$ [By change of the box rule]

= $\frac{3}{log_3 3^2} - \frac{2}{log_3 3^3}$

= $\frac{3}{2 \quad log_3 3} - \frac{2}{3 \quad log_3 3}$

= $\frac{3}{2} - \frac{2}{3}$ [∵ $\log_3{}^3$=1]

= $\frac{9-4}{6}$

16) LHS = $log\frac{a^2}{bc} + log\frac{b^2}{ca} + log\frac{c^2}{ab}$

= $log\left(\frac{a^2}{bc}.\frac{b^2}{ca}.\frac{c^2}{ab}\right)$

= $log\left(\frac{a^2b^2c^2}{a^2b^2c^2}\right)$

= log 1 = 0 =RHS

Hence proved

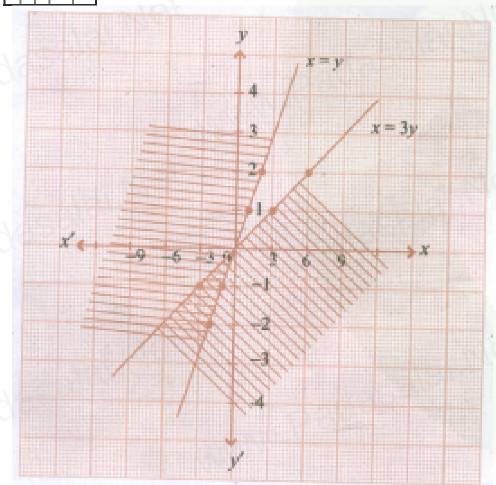
17) Given in equalities are $x \leq 3y,\ x \geq y$

Suppose $x = 3y \Rightarrow \frac{x}{3} = y$

x	0	3	6	-3
y	0	1	2	-1

If x=y

x	1	2	-1	-2
y	1	2	-1	-2



18) Given $\log_{5-x}$ $(x^2$-6x+65)=2

$(5-x)^2 = x^2$-6x+65 [Converting into expotential form]

⇒ 25 +x^2-10x =x^2-6x+65

⇒ -6x+65-25+10x = 0

⇒ 4x+40 = 0

⇒ 4x = -40

⇒ = $\frac{-40}{4}$ =-10

∴ x = -10

19) Given equation is $(2x+1)^2$-$(3x+2)^2$ = 0

⇒ (2x+1+3x+2)(2x+1-3x-2) = 0 [∵ a^2-b^2=(a+b)(a-b)]

⇒ (5x+3)(-x-1) = 0

⇒ (5x+3)(x+1) = 0

⇒ x = $\frac{-3}{5}$ or -1

∴ Solution set is $\left\{-1, \frac{-3}{5}\right\}$

20) Here a = 4, b = -1, c = -2

$\therefore$ D = $b^2 - 4ac = (-1)^2 - 4(4)(-2)$

= 1 + 32 = 33

Since D > 0, the two roots are real and distinct.

10 x 3 = 30

21) Given $\frac{1}{3-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2}$ ..(1)

Multiplying each term by the conjugate of the denominator we get

$\frac{1}{3-\sqrt{8}} = \frac{1}{3-\sqrt{8}} \times \frac{3+\sqrt{8}}{3+\sqrt{8}} = \frac{3+\sqrt{8}}{3^2-\sqrt{8}^2} = \frac{3+\sqrt{8}}{9-8} = 3+\sqrt{8}$

$\frac{1}{\sqrt{8}-\sqrt{7}} = \frac{1}{\sqrt{8}-\sqrt{7}} \times \frac{\sqrt{8}+\sqrt{7}}{\sqrt{8}+\sqrt{7}} = \frac{\sqrt{8}+\sqrt{7}}{8-7} = \frac{\sqrt{8}+\sqrt{7}}{1} = \sqrt{8}+\sqrt{7}$

$\frac{1}{\sqrt{7}-\sqrt{6}} = \frac{1}{\sqrt{7}-\sqrt{6}} \times \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}} = \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}^2+\sqrt{6}^2} = \frac{\sqrt{7}+\sqrt{6}}{7-6} = \sqrt{7}+\sqrt{6}$

$\frac{1}{\sqrt{6}-\sqrt{5}} = \frac{1}{\sqrt{6}-\sqrt{5}} \times \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}} = \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}^2+\sqrt{5}^2} = \frac{\sqrt{6}+\sqrt{5}}{6-5}$

$\frac{1}{\sqrt{5}-2} = \frac{1}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2} = \frac{\sqrt{5}+2}{\sqrt{5}^2+2^2} = \frac{\sqrt{5}+2}{5-4} = \sqrt{5}+2$

Substituting all these values in (1)we get

$\frac{1}{3-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2}$ =5

22) Given x = $\sqrt{2}+\sqrt{3}$

$\Rightarrow x^3 = \left(\sqrt{2}+\sqrt{3}\right)^2 = 2+3+2\sqrt{6} = 5+2\sqrt{6}$

$\therefore \frac{x^2+1}{x^2-1} = \frac{5+2\sqrt{6}+1}{5+2\sqrt{6}-2} = \frac{6+2\sqrt{6}}{3+2\sqrt{6}}$

$\Rightarrow \frac{6+2\sqrt{6}}{3+2\sqrt{6}} \times \frac{3-2\sqrt{6}}{3-2\sqrt{6}} = \frac{(6+2\sqrt{6})(3-2\sqrt{6})}{9-(2\sqrt{6})^2}$

$\Rightarrow \frac{18-12\sqrt{6}+6\sqrt{6}-4(\sqrt{6})^2}{9-24} = \frac{18-12\sqrt{6}-24}{-15}$

$\Rightarrow \frac{-6-6\sqrt{3}}{-15} = \frac{-3(2+2\sqrt{3})}{15}\ \frac{2+2\sqrt{3}}{5}$

23) LHS = $log2 + 16log\frac{16}{15} + 12log\frac{25}{24} + 7log\frac{81}{80}$

$= log2 + log\left(\frac{16}{15}\right)^{16} + log\left(\frac{25}{24}\right)^{12} + log\left(\frac{81}{80}\right)^{7}$

$= log2 \times \frac{(2^4)^{16}}{(3\times5)^{16}} \times \frac{(5^2)^{12}}{(2^2\times3)^{12}} \times \frac{(3^4)^7}{2^{28}\times5^7}$

$= log2^1 \times \frac{2^{64}}{3^{16}} \times \frac{5^{24}}{2^{36}\times3^{12}} \times \frac{3^{28}}{2^{28}\times5^7}$

$= log\frac{2^{1+64}.5^{24}.3^{28}}{3^{16+12}.5^{16+7}.2^{36+28}}\ \left[\because\ \frac{a^m}{a^n} = a^{m-n}\right]$

$= \log\frac{2^{65}\cdot5^{24}\cdot3^{28}}{3^{28}\cdot5^{23}\cdot2^{64}}$

=log 2^{65-64} x 5^{24-23} = log 2^1 x 5^1 = $\log_{10}{}^{10}$ = 1 = RHS

24) Given in equality is $-x^2 + 3x - 2 \geq 0$.

$\Rightarrow x^2 - 3x + 2 < 0$ [$\because$ a $\geq$ b $\Rightarrow$ -a < -b].

$\Rightarrow$ (x -1) (x -2) < 0.

The critical numbers are 1 and 2 and the possible intervals are (-∞ ,1) (1, 2) and (2, ∞)

Internals	Sign of (x -1)	Sign of (x - 2)	Sign of x^2 - 3x + 2
(- ∞, 1) (say x = 0)	-	-	+
(1, 2)(say x = 1.5)	+	-	-
(2,∞) (say x = 3)	+	+	+

The in equality x^2 - 3 x + 2 < 0 is satisfied only in the interval (1, 2).

$\therefore$ Solution set is (1, 2).

25) Let $\frac{log \quad x}{y-z}$ =

$\Rightarrow$ log x = k(y-z)=ky-kz ...(1)

log y = k(z-x)=kz-kx ...(2)

log z = k(x-y)=kx-ky ....(3)

Adding (1),(2) and (3)

log x +log y+ log z = ky-kz+kz-kx+kx-ky =0

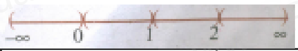
$\Rightarrow$ log xyz = 0 = log^1

$\Rightarrow$ xyz =1 Hence proved

26) Given inequality is $\frac{x^3(x-1)}{x-2} > 0$

The critical numbers are 0, 1, 2

The possible intervals are (-∞, 0) (0, 1) (1, 2) (2, ∞)



Intervals	Sign of x^3	Sign of (x - 1)	Sign of (x - 2)	Sign of $\frac{x^3(x-1)}{x-2}$
(- ∞, 0) Say x=-1	-	-	-	-
(0, 1) Say x=$\frac{1}{2}$	+	-	-	+
$(1,2) = \frac{1}{2}$ Say x = 1	+	+	-	-
(2,∞) Say x= 3	+	+	+	+

The given inequality $\frac{x^2(x-1)}{x-2} > 0$ is satisfied by the intervals (0,1) and (2, ∞)

∴ Solution set is (0, 1)∪(2, ∞)

∴ Solution set is $(0,1)\bigcup(2,\infty)$

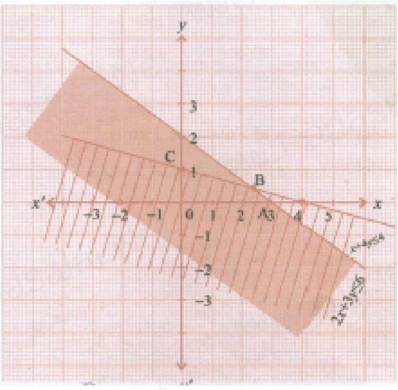
27) If 2x+3y=6

x	0	3
y	2	0

x+4y=4

x	0	4
y	1	0

x ≥ y ≥ 0 represents the area in the 1 quadrant.



OABC is the required shaded region

28) Let $\frac{x}{(x+3)(x-4)} = \frac{A}{x+3} + \frac{B}{x-4}$ where A and B are constants.

Then, $\frac{x}{(x+3)(x-4)} = \frac{A(x-4)+B(x+3)}{(x+3)(x-4)}$ which gives x=A(x-4)+B(x+3)

When x=-4, we have B=$\frac{4}{7}$

When x=-3 we have A=$\frac{3}{7}$

Hence, $\frac{x}{(x+3)(x-4)} = \frac{3}{7(x+3)} + \frac{4}{7(x-4)}$

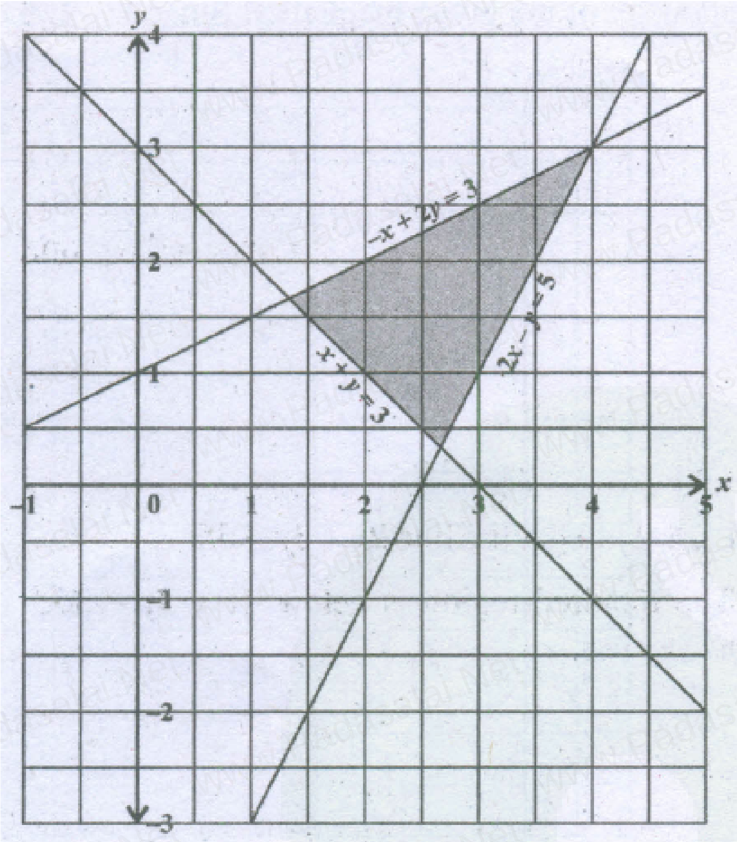
https://www.trbtnpsc.com/2018/06/latest-plus-one-11th-study-materials-tamil-medium-english-medium-new-syllabus-based.html

29) Observe that a straight line can be drawn if we identify any two points on it. For example, (3, 0) and (0, 3) can be easily identified as two .points on the straight line x +y = 3.

Draw the three straight lines x +y = 3, 2x - y = 5 and -x + 2y = 3.

Now (0, 0) does not satisfy x +y ≥ 3. Thus, the half plane bounded by x +y = 3, not containing the origin, is the solution set of x +y ≥ 3.

Similarly, the half-plane bounded by 2x - y ≤ 5 containing the origin represents the solution set of the 2x - y ≤ 5



The region represented by -x + 2y ≤ 3 is the half space bounded by the straight line the line -x + 2y = 3 that contains the origin.

The region common to the above three half planes represents the solution set of the given linear inequalities.

30) $\sqrt{98} = \sqrt{49 \times 2} = 7\sqrt{2}; \sqrt{18} = \sqrt{9 \times 2}3\sqrt{2}$

$\sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2}; \sqrt{75} = \sqrt{25 \times 3}5\sqrt{3}$

$\sqrt{27} = \sqrt{9 \times 3}3\sqrt{3}$

$\therefore \sqrt{98} + \sqrt{50} - \sqrt{18} + \sqrt{75} - \sqrt{27} = 7\sqrt{2} + 5\sqrt{2} - 3\sqrt{2} + 5\sqrt{3} - 3\sqrt{3}$

= $9\sqrt{2} + 2\sqrt{3}$

8 x 5 = 40

31) Given $a^2+b^2 = 7ab$

Adding 2ab both sides we get,

$a^2+b^2+2ab = 7ab + 2ab$

⇒ $(a+b)^2 = 9ab$

⇒ $\frac{(a+b)^2}{9}$ =ab

⇒ $\left(\frac{a+b^2}{9}\right)^2$ =ab

Taking square root,we get

$\frac{a+b}{3}$ =ab

$log\left(\frac{a+b}{3}\right) = log(ab)^{\frac{1}{2}}$

=$\frac{1}{2}$ log (ab)

log $\left(\frac{a+b}{3}\right)$ = $\frac{1}{2}$ [log a+log b]

Hence proved.

32) Let $\frac{log_e^x}{b-c} = \frac{log_e^y}{c-a} = \frac{log_e^z}{a-b} = k$

$log_e^x = k(b-c), log_e^y = k(c-a)$ and

$log_e^z = k(a-b)$ ...(1)

$\Rightarrow$ $x = e^{k(b-c)}, y = e^{k(c-a)}$ and $z = e^{k(a-b)}$ ....(2)

$xyz = e^{k(b-c)}.e^{k(c-a)}.e^{k(a-b)}$

$= e^{k(b-c+c-a+a-b)=ek(0)} = e^0 = 1$

$\Rightarrow$ xyz = 1

33) Let x be the number of litres of 30% acid solution

∴ Total mixture = (600+x)litres

30% of x + 12% of 600 > 5% of (600+x)

$\Rightarrow \frac{30x}{100} + \frac{12}{100} \times 600 > \frac{15}{100}(600+x)$

⇒ 30x + 7200 > 9000 + 15x [Multiplying by 100]

⇒ 30x + 7200 - 15x > 9000 [Subtracting 15x]

⇒ 15x + 7200 > 900

⇒ 15x > 1800

⇒ x > 120 ....(1)

Also, 30% of x + 12% of 600 < $\frac{18}{100}$(600 + x)

$\frac{30x}{100} + \frac{12}{100} \times 600 < \frac{18}{100}(600+x)$

⇒ 30x + 7200 < 18 (600 + x) [Multiplying by 100]

⇒ 30x + 7200 < 10, 800 + 18x

⇒ 12x + 7200 < 10, 800 [Subtracting 18x]

⇒ 12x < 10, 800 - 7200 [Subtracting 7200]

⇒ 2x < 3600

⇒ $x < \frac{3600}{12}$

⇒ x < 300 ....(2)

From (1) and (2), 1200 < x < 300

The number of litres of the 30% acid solution will have to be greater than 120 litres and less than 300 litres.

34) Let x be the smaller of two positive odd integers so that other one is x + 2

Given x > 10, ...(1) and x + 2 > 10.

⇒ x > 10-2

⇒ x > 8 ....(2)

And (x) + (x + 2) < 40 ....(3)

From (1) and (2) we get, x > 10 ....(4)

From (3) we get

2x+2 <40

⇒ 2x<40-2

⇒ 2x< 38

⇒ x< $\frac{38}{2}$

⇒ x = 19 ....(5)

From (4) and (5) we get,

10 < x < 19.

Since x is an odd natural number, x can take the values 11, 13, 15, 17.

Hence the required possible consecutive pairs will be (11, 13), (13, 15), (15, 17)

35) Since the degree of the numerator is equal to the degree of the denominator, let us divide the numerator by the denominator

$$\begin{array}{r|l} & x-5 \\ \therefore x^2+5x+6 & x^3+2x+1 \\ & x^3+5x^2+6x \\ & -5x^2-4x+1 \\ & -5x^2-25x-30 \\ & 21x+31 \end{array}$$

$\therefore \frac{x^2+x+1}{x^2-5x+6} = 1 + \frac{6x-5}{x^2-5x+6}$

Consider $\frac{6x-5}{x^2-5x+6} = \frac{6x-5}{(x-3)(x-2)} = \frac{A}{(x-3)} + \frac{B}{(x-2)}$

$\Rightarrow \frac{6x-5}{x^2-5x+6} = \frac{A(x-2)+B(x-3)}{(x-3)(x-2}$

$\Rightarrow$ 6x-5 = A(x-2)+B(x-3)

Putting x=2 in (2) we get

7 = B(-1) $\Rightarrow$ B=-7

Putting x=3 in (2) we get

13 = A(1) $\Rightarrow$ A=13

$\therefore \frac{6x-5}{x^2-5x+6} = \frac{13}{x-3} - \frac{7}{x-2}$

Substituting in (1) we get

$\frac{x^2+x+1}{x^2-5x+6} = 1 + \frac{13}{x-3} - \frac{7}{x-2}$

36) $\frac{x+12}{(x+1)^2(x-2)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x-2}$

$\Rightarrow$ x + 12 = A(x+1)(x-2)+B(x-2)+C(x+1)^2 (1)

Putting x = - 1 in (1) we get,

11=B(-3) $\Rightarrow \boxed{B = \frac{-11}{3}}$

Putting x = 2 in (1) we get,

14C=C(9) $\Rightarrow \boxed{C = \frac{14}{9}}$

Equating the Co-efficient of x^2 in (1) we get,

0 = A + C $\Rightarrow$ A = - C $\Rightarrow \boxed{A = \frac{-14}{9}}$

$\therefore \frac{x+12}{(x+1)^2(x-2)} = \frac{\frac{-14}{9}}{x+1} - \frac{\frac{11}{3}}{(x+1)^2} + \frac{\frac{14}{9}}{x-2} = -\frac{14}{9(x+1)} - \frac{11}{3(x+1)^2} + \frac{14}{9(x-2)}$

37) Let $\sqrt{7-4\sqrt{3}} = a + b\sqrt{3}$ with a,b rationals.

Squaring on both sides, we get 7-4$\sqrt{3}$ = $a^2+3b^2+2ab\sqrt{3}$

So, $a^2+3b^2=7$ and 2ab=-4

Therefore a=$\frac{-2}{b}$

From $a^2+3b^2=7$ we get $(\frac{-2}{b})^2 + 3b^2 = 7$

which gives $\frac{4}{b^2} + 3b^2 = 7$ or $3b^4-7b^2+4=0$

Solving for b^2 we get $b^2=\frac{(7\pm\sqrt{49-48})}{8}$, which gives $b^2=1$ or $b^2=\frac{4}{3}$.

Thus, b=$\pm$1 or b=$\pm \frac{2}{\sqrt{3}}$

Since b is rational, we have b=$\pm$1 and hence the corresponding values for a are ∓ 2.

Since $\sqrt{7-4\sqrt{3}} > 0$ we have $\sqrt{7-4\sqrt{3}} = 2 - \sqrt{3}$.

38) $\frac{3(x-2)}{5} \leq \frac{5(2-x)}{3}$

$9(x-2) \leq 25(2-x)$

$9x - 18 \leq 50 - 25x \Rightarrow 9x + 25x \leq 50 + 18$

$34x \leq 68 \Rightarrow \frac{34x}{34} \leq \frac{68}{64}$

$x \leq 2$

$x \in (-\infty, 2]$

(ii) Multiplying by 3, throughout,

$5 - x < \frac{3x}{2} - 12$

Multiplying by 2, we get,

10-2x < 3x-24

$\Rightarrow$ 10+24< 3x+2x

$\Rightarrow = \frac{24}{5}$

$\Rightarrow x \geq \frac{24}{5}$

$\therefore$ Solution set is $\left[\frac{24}{5}, \infty\right)$

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RAVI MATHS TUITION (HOME) CENTER, CH-82 PH-8056206308

11th chapter 2 Algebra

11th Standard

Date : 25-Feb-19

Maths

Reg.No. :

Time : 03:00:00 Hrs

Total Marks : 100

10 x 1 = 10

1) If |x+2|≤9,then x belongs to

(a) (-∞,-7) (b) [-11,7] (c) (-∞,-7)∪[11,∞) (d) (-11,7)

2) If $\frac{|x-2|}{x-2} \geq 0$,then x belongs to

(a) [2,∞] (b) (2,∞) (c) (-∞,2) (d) (-2,∞)

3) The solution set of the following inequality |x-1| ≥ |x-2| is

(a) [0,2] (b) [2,∞] (c) (0,2) (d) (-∞,2)

4) The value of $log_3 \frac{1}{81}$ is

(a) -2 (b) -8 (c) -4 (d) -9

5) The value of $\log_a b \log_b c \log_c a$ is

(a) 2 (b) 1 (c) 3 (d) 4

6) If 3 is the logarithm of 343 then the base is

(a) 5 (b) 7 (c) 6 (d) 9

7) If a and b are the roots of the equation x^2-kx+16=0 and a^2+b^2=32 then the value of k is

(a) 10 (b) -8 (c) -8,8 (d) 6

8) The number of solution of x^2+|x-1|=1 is

(a) 1 (b) 0 (c) 2 (d) 3

9) If 8 and 2 are the roots of x^2+ax+c=0 and 3,3 are the roots of x^2+dx+b=0;then the roots of the equation x^2+ax+b = 0 are

(a) 1,2 (b) -1,1 (c) 9,1 (d) -1,2

10) If $\frac{1-2x}{3+2x-x^2} = \frac{A}{3-x} + \frac{B}{x+1}$,then the value of A+B is

(a) $\frac{-1}{2}$ (b) $\frac{-2}{3}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$

10 x 2 = 20

11) Evaluate $\left(\left[(256)^{\frac{-1}{4}}\right]^{\frac{-1}{4}}\right)^3$

12) Simplify and hence the value of n: $\frac{3^{2n}9^2 3^{-n}}{3^{3n}} = 27$

13) Simplify by rationalising the denominator $\frac{7+\sqrt{6}}{3-\sqrt{2}}$

14) Solve for x $\left|3 - \frac{3}{4}x\right| \leq \frac{1}{4}$

15) Compute $log_9^{27} - log_{27}^{9}$

16) Prove $log\frac{a^2}{bc} + log\frac{b^2}{ca} + log\frac{c^2}{ab} = 0$

17) Determine the region in the Plane determined by the inequalities. $x \leq 3y, x \geq y$

18) Solve $\log_{5-x}$ (x^2-6x+65)=2

19) Solve $(2x+1)^2-(3x+2)^2=0$

https://www.trbtnpsc.com/2018/06/latest-plus-one-11th-study-materials-tamil-medium-english-medium-new-syllabus-based.html

20) Discuss the nature of roots of $4x^2 - x - 2 = 0$

10 x 3 = 30

21) Simplify $\frac{1}{3-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2}$

22) If $x=\sqrt{2} + \sqrt{3}$ find $\frac{x^2+1}{x^2-1}$

23) Prove that $log2 + 16log\frac{16}{15} + 12log\frac{25}{24} + 7log\frac{81}{80} = 1$

24) Solve $-x^2 + 3x - 2 \geq 0$

25) If $\frac{log\, x}{y-z} = \frac{log\, y}{z-x} = \frac{log\, z}{x-y}$, then prove that xyz= 1

26) Find all values of x for which $\frac{x^3(x-1)}{x-2} > 0$.

27) Determine the region in the plane determined by the inequalities.

$2x + 3y \leq 6, \quad x + 4y \leq 4, \quad x \geq 0, \quad y \geq 0.$

28) Resolve into partial fractions: $\frac{x}{(x+3)(x-4)}$

29) Solve the linear inequalities and exhibit the solution set graphically: $x+y \geq 3$, $2x-y \leq 5$, $-x+2y \leq 3$.

30) Simplify: $\sqrt{98} + \sqrt{50} - \sqrt{18} + \sqrt{75} - \sqrt{27}$

8 x 5 = 40

31) If $a^2+b^2 = 7ab$.Show that $\log\left(\frac{a+b}{3}\right) = \frac{1}{2}$ (log a +log b)

32) If $\frac{log_e^x}{b-c} = \frac{log_e^y}{c-a} = \frac{log_e^z}{a-b}$, show that xyz = 1

33) A manufacturer has 600 litres of a 12 percent solution of acid. How many litres of a 30 percent acid solution must be added to it so that the acid content in the resulting mixture will be more than 15 percent but less than 18 percent?

34) Find all pairs of consecutive odd natural numbers both of which are larger than 10 and their sum is less than 40.

35) Resolve the following rational expressions into partial fractions.

$\frac{x^2+x+1}{x^2-5x+6}$

36) Resolve the following rational expressions into partial fractions.

$\frac{x+12}{(x+1)^2(x-2)}$

37) Find the square root of $7-4\sqrt{3}$

38) **Solve:**

(i) $\frac{3(x-2)}{5} \leq \frac{5(2-x)}{3}$

(ii) $\frac{5-x}{3} < \frac{x}{2} - 4$.

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RAVI MATHS TUITION (HOME) CENTER, CH-82 PH-8056206308

Chapter 10 diff. Calculus

11th Standard

Date : 25-Feb-19

Maths

Reg.No. :

Time : 03:00:00 Hrs

Total Marks : 100

10 x 1 = 10

1) $\frac{d}{dx}(\frac{2}{\pi}\sin x^o)$is

(a) $\frac{\pi}{180}\cos x^o$ (b) $\frac{1}{90}\cos x^o$ (c) $\frac{\pi}{90}\cos x^o$ (d) $\frac{2}{\pi}\cos x^o$

2) If y=cos (sin x^2),then $\frac{dy}{dx}$ at x= $\sqrt{\frac{\pi}{2}}$ is

(a) -2 (b) 2 (c) $-2\sqrt{\frac{\pi}{2}}$ (d) 0

3) If f(x)=x $\tan^{-1}$ x, then f'(1) is

(a) $1+\frac{\pi}{4}$ (b) $\frac{1}{2}+\frac{\pi}{4}$ (c) $\frac{1}{2}-\frac{\pi}{4}$ (d) 2

4) If f(x)=x+2, then f'(f(x)) at x= 4 is

(a) 8 (b) 1 (c) 4 (d) 5

5) If $y=\frac{(1-x)^2}{x^2}$,then $\frac{dy}{dx}$ is

(a) 8 (b) 1 (c) 4 (d) 5

6) It is given that f '(a) exists, then $\lim_{x\to a}\frac{xf(a)-af(x)}{x-a}$ is

(a) f(a)-af'(a) (b) f '(a) (c) -f '(a) (d) f(a)+af '(a)

7) If $f(x)=\begin{cases}x+2, & -1<x<3\\ 5 & x=3\\ 8-x & x>3\end{cases}$,then at x=3, f '(x) is

(a) 1 (b) -1 (c) 0 (d) does not exist

8) The derivative of f(x)=x|x| at x =−3 is

(a) 6 (b) -6 (c) does not exist (d) 0

9) If $f(x)=\begin{cases}2a-x, & \text{for } -a<x<a\\ 3x-2a & \text{for } x\geq a\end{cases}$,then which one of the following is true?

(a) f(x)is not differentiable at x = a (b) f(x) is discontinuous at x = a (c) f(x) is continuous for all x in **R**

(d) f(x) is differentiable for all x ≥ a

10) The number of points in **R** in which the function $f(x) = |x-1| + |x-3| + \sin x$ is not differentiable, is

(a) 3 (b) 2 (c) 1 (d) 4

10 x 2 = 20

11) Differentiate the following with respect to x: y=xe^x log x

12) Differentiate the following with respect to x :$y=\frac{\log x}{e^x}$

13) Differentiate : y = $e^{\sin x}$.

14) Find $\frac{dy}{dx}$ if x=at^2 ; y=2at, t ≠ 0.

15) Find y',y''and y''' if y=x^3-6x^2-5x+3.

16) Find the derivatives of the following $(\cos x)^{\log x}$

17) Find the derivatives of the following $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

18) Find $\frac{dy}{dx}$ if $x^4 + x^2y^2 + y^4 = 50.$

19) Find $\frac{dy}{dx}$ if $x = a\log t, \quad y = b\sin t.$

20) If $y = 500e^{7x} + 600e^{-7x}$ show that $\frac{d^2y}{dx^2} = 49y.$

10 x 3 = 30

21) Find the derivatives of the following functions with respect to corresponding independent variables: $y = \frac{x}{\sin x + \cos x}$

22) Find f '(x) if f(x)= $\frac{1}{3\sqrt{x^2+x+1}}$

23) Differentiate the following: $y=(2x-5)^4(8x^2-5)^{-3}$

24) Differentiate: $y = \frac{x^{\frac{3}{4}}\sqrt{x^2+1}}{(3x+2)^5}$

25) If $y=\tan^{-1}\left(\frac{1+x}{1-x}\right)$, find y'.

26) Find the derivative of $\tan^{-1}(1+x^2)$ with respect to x^2+x+1.

27) Find the derivatives of the following $\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}$

28) Find the derivatives of the following x = a (cos t + t sin t) ; y = a (sin t - t cos t)

29) Find the derivatives of the following $\sin^{-1}(3x-4x^3)$

30) Show that the function $f(x) = \begin{cases} x-1, & x < 2 \\ 2x-3, & x \geq 2 \end{cases}$ is not differentiable at x = 2.

8 x 5 = 40

31) Find the second order derivative if x and y are given by
x = a cos t
y = a sin t.

32) Find the derivative of $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ with respect to $\tan^{-1}x$.

33) If u= $\tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$ and v= $\tan^{-1}x$, find $\frac{du}{dv}$

34) If $y=e^{\tan^{-1}x}$, Show that $(1+x^2)y''+(2x-1)y'=0$.

35) If $y = \frac{\sin^{-1}x}{\sqrt{1-x^2}}$, show that $(1-x^2)y_2-3xy_1-y=0$.

36) If x = a (θ + sin θ), y = a (1 - cos θ) then prove that at $\theta = \frac{\pi}{2}, y'' = \frac{1}{a}$

37) If sin y=x sin (a+y),then prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}, a \neq n\pi.$

38) If $y = (\cos^{-1}x)^2$, prove that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0.$ Hence find y_2 when x=0

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10 x 1 = 10

1) (b) $\frac{1}{90}\cos x^o$

2) (d) 0

3) (b) $\frac{1}{2}+\frac{\pi}{4}$

4) (b) 1

5) (d) 5

6) (a) f(a)-af'(a)

7) (d) does not exist

8) (a) 6

9) (a) f(x)is not differentiable at x = a

10) (b) 2

10 x 2 = 20

11) $\frac{dy}{dx} = xe^x(\frac{1}{x}) + e^x.\log x(1) + x\log x(e^x)$

$= e^x + e^x\log x + xe^x\log x = e^x(1+\log x + x\log x).$

12) $y = \frac{\log x}{e^x} = e^{-x}.\log x$

$\frac{dy}{dx} = e^{-x}(\frac{1}{x}) + \log x(e^{-x})(-1)$

$= e^{-x}[\frac{1}{x} - \log x].$

13) Take u = sin x so that

y = e^u

$\frac{dy}{dx} = \frac{d(e^u)}{du}\times\frac{du}{dx} = e^x \times \cos x = \cos x e^{\sin x}$

14) We have x = at^2 ; y=2at

$\frac{dy}{dx} = \frac{y'(t)}{x'(t)} = \frac{2a}{2at} = \frac{1}{t}.$

15) We have, y =x^3-6x^2-5x+3 and

y' = 3x^2-12x-5

y'' = 6x - 12

y'''= 6.

16) Let y=(cosx) logx

Taking logarithm,

log y=logx.log(cosx)

$\Rightarrow \frac{1}{y}\frac{dy}{dx}$=logx.$\frac{d}{dx}$log(cosx)+log(cosx)$\frac{d}{dx}$(logx)

$\Rightarrow \frac{1}{y}\frac{dy}{dx}$=logx x $\frac{1}{\cos x}$(-sinx)+log(cosx) x $\frac{1}{x}$

$\Rightarrow \frac{1}{y}\frac{dy}{dx}$=-logx tanx +$\frac{\log(\cos x)}{x}$

$\Rightarrow \frac{dy}{dx}$=y[$\frac{\log(\cos x)}{x}$ -logx.tanx]

$\Rightarrow \frac{dy}{dx}$=(cosx)logx[$\frac{\log(\cos x)}{x}$ -logx.tanx]

17) $\frac{1}{a^2}(2x) + \frac{1}{b^2}(2y)\frac{dy}{dx} = 0$

$\Rightarrow \frac{2x}{a^2} + \frac{2y}{b^2}\left(\frac{dx}{dy}\right) = 0$

$\Rightarrow \frac{2y}{b^2}\left(\frac{dx}{dy}\right) = \frac{-2x}{a^2}$

$\Rightarrow \frac{dy}{dx} = \frac{-2x}{a^2}\times\frac{b^2}{2y} = \frac{-b^2x}{a^2x}$

18) Given $x^4 + x^2y^2 + y^4 = 50$ Differentiating both sides with respect to 'x' we have,

$4x^3 + x^2.2y\frac{dy}{dx} + y^2.2x + 4y^3\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx}(2x^2y + 4y^3) = -4x^3 - 2xy^2 \Rightarrow \frac{dy}{dx} = -\frac{4x^3-2xy^2}{2x^2y+4y^3} = -\frac{2x(2x^2+y^2)}{2y(x^2+2y^2)} \therefore \frac{dy}{dx} = -\frac{x(2x^2+y^2)}{y(x^2+2y^2)}$

19) $Given \quad x = a\log t \qquad y = b\sin t. \Rightarrow \frac{dx}{dt} = \frac{a}{t} \qquad \Rightarrow \frac{dy}{dt} = b\cos t \therefore \frac{dy}{dx} = \frac{\frac{dx}{dt}}{\frac{dy}{dt}} = \frac{b\cos t}{\frac{a}{t}} = \frac{bt\cos t}{a}$

20) Given y=$y = 500e^{7x} + 600e^{-7x}$ ....(1)

Differentiating both sides with respect to 'x' we have

$\frac{dy}{dx} = 500e^{7x}(7) + 600e^{-7x}(-7)\frac{dy}{dx} = 7(500e^{7x} - 600e^{-7x})$

Differentiating again with respect to 'x' we have

$\frac{d^2y}{dx^2} = 7(500(e^{7x})(7) - 600(e^{-7x})(-7)) = 49(500 \quad e^{7x} + 600 \quad e^{-7x}) = \quad 49 \quad y \quad (from \quad (1))$

$\therefore \frac{d^2y}{dx^2} = 49y$ Hence proved.

10 x 3 = 30

21) Given $y = \frac{x}{\sin x + \cos x}$

Using quotient rule,

$\frac{dy}{dx} = \frac{(sinx + cosx)\frac{d}{dx}(x) - x.\frac{d}{dx}(sinx + cosx)}{(sinx + cosx)^2} = \frac{(sinx + cosx)(1) - x(cosx - sinx)}{(sinx + cosx)^2}$

$= \frac{sinx + cosx - xcosx + xsinx}{(sinx + cosx)^2} = \frac{(1-x)cosx + (1+x)sinx}{(sinx + cosx)^2}.$

22) First we write : f(x) = $(x^2+x+1)^{\frac{-1}{3}}$

Then, f '(x) $= -\frac{1}{3}(x^2 + x + 1)^{\frac{-1}{3}-1}\frac{d}{dx}(x^2 + x + 1)$

$= -\frac{1}{3}(x^2 + x + 1)^{\frac{-4}{3}} \times (2x + 1)$

$= -\frac{1}{3}(2x + 1)(x^2 + x + 1)^{\frac{-4}{3}}.$

23) Given $y=(2x-5)^4 (8x^2-5)^{-3}$

Let u=2x-5, $v=8x^2-5$

$\Rightarrow \frac{du}{dx} = 2$ and $\frac{dv}{dx} = 16x$

$\Rightarrow y=u^4.v^{-3}$

$\therefore \frac{dy}{dx} = u^4.\frac{d}{dx}(v^{-3}) + v^{-3}.\frac{d}{dx}(u^4)$ [Product rule]

$=u^4.(-3)v^{-3-1}\frac{dv}{dx} + v^{-3}.4u^3.\frac{du}{dx} = -3u^4v^{-4}.\frac{dv}{dx} + 4u^3v^{-3}\frac{du}{dx} = -3u^4v^{-4}(16x) + 4u^3v^{-3}(2)$

$=-48xu^4v^{-4} + 8u^3v^{-3} = -48x\frac{(2x-5)^4}{(8x^2-5)^4} + 8\frac{(2x-5)^3}{(8x^2-5)^3} = \frac{8(2x-5)^3}{(8x^2-5)^3}\left[\frac{-6x(2x-5)}{8x^2-5} + 1\right]$

$= \frac{8(2x-5)^3}{(8x^2-5)^3}\left[\frac{-12x^2 + 30x + 8x^2 - 5}{8x^2 - 5}\right] = \frac{8(2x-5)^3}{(8x^2-5)^4}.$

24) Taking logarithm on both sides of the equation and using the rules of logarithm we have,

$\log y = \frac{3}{4}\log x + \frac{1}{2}\log(x^2 + 1) - 5\log(3x + 2)$

Differentiating implicitly

$\frac{y'}{y} = \frac{3}{4x} + \frac{1}{2}\frac{2x}{(x^2+1)} - \frac{5 \times 3}{3x+2}$

$= \frac{3}{4x} + \frac{x}{(x^2+1)} - \frac{15}{3x+2}$

Therefore, $\frac{dy}{dx} = y' = \frac{x^{\frac{3}{4}}\sqrt{x^2+1}}{(3x+2)^5}\left[\frac{3}{4x} + \frac{x}{x^2+1} - \frac{15}{3x+2}\right]$

25) Let x = tan θ

Then $\frac{1+x}{1-x} = \frac{1 + tan\theta}{1 - tan\theta} = tan(\frac{\pi}{4} + \theta).$

$tan^{-1}(\frac{1+x}{1-x}) = tan^{-1}[tan(\frac{\pi}{4} + \theta)] = \frac{\pi}{4} + \theta = \frac{\pi}{4} + tan^{-1}x$

$y = \frac{\pi}{4} + tan^{-1}x$

$y' = \frac{1}{1+x^2}.$

26) Let f(x) = $tan^{-1}(1+x^2)$

g(x) = x^2+x+1

$\frac{df}{dg} = \frac{f'(x)}{g'(x)}$

$f'(x) = \frac{2x}{1+x^2}$

$g'(x) = 2x + 1$

$\frac{df}{dg} = \frac{\frac{2x}{1+x^2}}{2x+1} = \frac{2x}{(2x+1)(1+x^2)}$

27) Let y=$tan^{-1}\sqrt{\frac{1-cosx}{1+cosx}}$ [$\therefore 2sin^2\frac{x}{2} = 1 - cosx$ and $2\ cos^2\frac{x}{2} = 1 + cosx$]

$\Rightarrow$ y=$tan^{-1}\sqrt{\frac{2sin^2\frac{x}{2}}{2cos^2\frac{x}{2}}} = tan^{-1}\sqrt{tan^2\frac{x}{2}} = tan^{-1}\left(tan\frac{x}{2}\right)$

$\Rightarrow y = \frac{x}{2}$

$\therefore \frac{dy}{dx} = \frac{1}{2}(1) = \frac{1}{2}.$

28) x=a(cost+t sint); y=a (sint-cost)

$\frac{dx}{dt}$=a(-sint+t cost+sint)=a.(t cost)

y=a(sint-t cost)

$\frac{dy}{dt}$=a(cost-t(-sint)-cost)=a(cost+t sint-cost)=a t sint

$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{atsint}{atcost}$=tan t

29) Let y=$\sin^{-1}(3x-4x^3)$

Let x=sinθ

$\Rightarrow$ θ=$\sin^{-1}$x

$\therefore$ y=$\sin^{-1}$(3 sinθ-4 $\sin^3$θ)

=$\sin^{-1}$(sin3θ) [∵ sin3θ=3 sinθ-4 $\sin^3$θ]

y=3θ=3 $\sin^{-1}$x

$\therefore \frac{dy}{dx} = 3 \times \frac{1}{\sqrt{1-x^2}} = \frac{3}{\sqrt{1-x^2}}$.

30) $f'\left(2^-\right) = \lim_{x\to 2^-}\frac{f(x)-f(2)}{x-2} = \lim_{x\to 2^-}\frac{(x-1)-(1)}{x-2}$ $[\because f(x) = 2x-3; f(2) = 4-3 = 1] = \lim_{x\to 2^-}\frac{x-2}{x-2} = 1$ $\ldots(1) f'\left(2^+\right) = \lim_{x\to 2^+}\frac{f(x)-f(2)}{x-2} =$

8 x 5 = 40

31) Differentiating the function implicitly with respect to x, we get

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{a\cos t}{-a\sin t} = -\frac{\cos t}{\sin t}$

$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$

$= \frac{d}{dx}\left(\frac{-\cos t}{\sin t}\right)$

$= \frac{d}{dx}\left(\frac{-\cos t}{\sin t}\right)\frac{dt}{dx} = -[-cosec^2 t] \times \frac{1}{x'(t)}$

$= cosec^2 t \times \frac{1}{-a\sin t}$

$= -\frac{cosec^2 t}{a}$

32) Let u=$\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ and v=$\tan^{-1}$(x)

Now u=$\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

Put x=tanθ

$\Rightarrow$ θ=$\tan^{-1}$x

u=$\sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$

=$\sin^{-1}$(sin2θ)=2θ=2$\tan^{-1}$x

$\therefore \frac{du}{dx} = \frac{1}{1+x^2}$ and $\frac{dv}{dx} = \frac{1}{1+x^2}$

$\therefore \frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{\frac{2}{1+x^2}}{\frac{1}{1+x^2}}$

$\frac{du}{dv} = \frac{2}{1+x^2} \times \frac{1+x^2}{1}$=2

$\therefore \frac{du}{dv}$=2

33) Let u=$\tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$ and =$\tan^{-1}$x

Put x=tanθ in u

$\Rightarrow$ θ=$\tan^{-1}$x

$\therefore$ u=tan-1

$$\left(\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right) = \tan^{-1}\left(\frac{\sec\theta-1}{\tan\theta}\right) = \tan^{-1}\left(\frac{\frac{1}{\cos\theta}-1}{\frac{\sin\theta}{\cos\theta}}\right) = \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$$

= tan x

$$\tan^{-1}\left(\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right) = \tan^{-1}\left(\frac{\tan\frac{\theta}{2}}{\cos\frac{\theta}{2}}\right) = \tan^{-1}\left(\tan\frac{\vartheta}{2}\right) = \frac{\vartheta}{2} = \frac{1}{2}$$ -1

$\therefore \frac{du}{dx} = \frac{1}{2(1+x^2)}$ $\frac{dv}{dx} = \frac{1}{1+x^2}$

$\therefore \frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{\frac{1}{2(1+x^2)}}{\frac{1}{1+x^2}} = \frac{1}{2(1+x^2)} + \frac{1+x^2}{1} = \frac{1}{2}$

$\therefore \frac{du}{dv} = \frac{1}{2}$.

34) Given $y=e^{\tan^{-1}x}$

$y'=e^{\tan^{-1}x}.\frac{d}{dx}(\tan^{-1}x)$

$\Rightarrow y'=e^{\tan^{-1}x}.\frac{1}{1+x^2}$

$\Rightarrow (1+x^2)y'=y$ [$\because y=e^{\tan^{-1}x}$]

Differentiating again with respect to 'x', we get

$(1+x^2)y''+y'(2x)=y'$

$\Rightarrow (1+x^2)y''+2x\,y'-y'=0$

$\Rightarrow (1+x^2)y''+(2x-1)y'=0$

35) Given $y=\frac{\sin^{-1}x}{\sqrt{1-x^2}}$

$\Rightarrow y\sqrt{1-x^2}=\sin^{-1}x$

Squaring, $y^2(1-x^2)=(\sin^{-1}x)^2$ [$\because y=e^{\tan^{-1}x}$]

Differentiating with respect to 'x', we get

$y^2(-2x)+(1-x^2)2yy'=2.\sin^{-1}x.\frac{d}{dx}(\sin^{-1}x)$

$\Rightarrow -2xy^2+(1-x^2)2yy'=2.\sin^{-1}x.\frac{1}{\sqrt{1-x^2}}$ $\left[\because y=\frac{\sin^{-1}x}{\sqrt{1-x^2}}\right]$

$\Rightarrow -2xy^2+(1-x^2)2yy'=2y$

Dividing by 2y throughout we get,

$-xy+(1-x^2)y'=1$

Differentiating again with respect to 'x', we get

$\Rightarrow [xy'+y(1)]+(1-x^2)y''+y'(-2x)=0$

$\Rightarrow xy'-y+(1-x^2)y''-2xy'=0$

$\Rightarrow (1-x^2)y''-3xy'-y=0$

Also, $(1-x^2)y_2-3xy_1-y=0$

36) Given $x=a(\theta+\sin\theta)$

$\Rightarrow \frac{dx}{d\theta}=a(1+\cos\theta)$

$\Rightarrow \frac{dx}{d\theta}=a\left(2\cos^2\frac{\theta}{2}\right)$ [$\because y=e^{\tan^{-1}x}$]

Also, $y=a(1-\cos\theta)$

$\Rightarrow \frac{dy}{d\theta}=a(\sin\theta)=2a\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

$\therefore y'=\frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=\frac{2a\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2a\cos^2\frac{\theta}{3}}=\tan\frac{\theta}{2}$

Differentiating again with respect to 'x' we get

$y''=$

$$\sec^2\frac{\theta}{2}.\frac{1}{2}.\frac{d\theta}{dx}=\sec^2\frac{\theta}{2}\left(\frac{1}{2}\right)\left(\frac{1}{2a\cos^2\frac{\theta}{2}}\right)=\frac{1}{4a}\sec^4\frac{\theta}{2}$$

$\therefore \theta=\frac{\pi}{2}$

$y''=\frac{1}{4a}\left[\sec\left(\frac{\pi}{4}\right)\right]^4=\frac{1}{4a}.(\sqrt{2})^4=\frac{4}{4a}=\frac{1}{a}.$

37) Given $\sin y=x\sin(a+y)$ ....(1)

Differentiating with respect to 'x' we get,

$\cos y\frac{dy}{dx}=x\cos(a+y)\left(\frac{dy}{dx}\right)+\sin(a+y)(1)$ [product rule]

$\Rightarrow \frac{dy}{dx}(\cos y-x\cos(a+y))=\sin(a+y)$

$\Rightarrow \frac{dy}{dx}=\frac{\sin(a+y)}{\cos y-x\cos(a+y)}=\frac{\sin(a+y)}{\cos y-\frac{\sin y}{\sin(a+y)}.\cos(a+y)}$ [from(1)]

$\Rightarrow \frac{dy}{dx}=\frac{\sin^2(a+y)}{\sin(a+y)\cos y-\sin y\cos(a+y)}=\frac{\sin^2(a+y)}{\sin(a+y-y)}$ [$\because \sin(A+B)=\sin A\cos B-\cos A\sin B$]

$\frac{dy}{dx}=\frac{\sin^2(a+y)}{\sin x}.$

38) Given $y=(\cos^{-1}x)^2$

Differentiating with respect to 'x' we get

$y'=2.\cos^{-1}x.\frac{d}{dx}(\cos^{-1}x)$

$\Rightarrow$ y;$=2.\cos^{-1}x\frac{-1}{\sqrt{1-x^2}}$

$\Rightarrow y'\sqrt{1-x^2}=-2\cos^{-1}x$

Squaring both sides we get,

$(y')^2(1-x^2)=4(\cos^{-1}x)^2$

Differentiating again with respect to 'x' we get

$(y')^2(-2x)+(1+x^2)2y'.y''=4(2)\cos^{-1}x.\frac{d}{dx}(\cos^{-1}x)$

$\Rightarrow -2x(y')^2+2(1-x^2)y'.y''=8\cos^{-1}x.\frac{-1}{\sqrt{1-x^2}}$

$\Rightarrow -2x(y')^2+2(1-x^2)y'.y''=4\left(\frac{-2\cos^{-1}x}{\sqrt{1-x^2}}\right)$

$\Rightarrow -2x(y')^2+2(1-x^2)y'.y''=4y'$ [From (1)]

Dividing throughout by 2 y' we get

$-xy'+(1-x^2)y''=2$

$\Rightarrow (1-x^2)y''-xy'-2=0$

$\Rightarrow$ i.e., $(1-x_2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}-2=0$