Padasalai

RAVI MATHS TUITION CENTER, GKM COLONY, CH-82. PH- 8056206308

12th MATRIX - TEST 1

12th Standard 2019 EM

Date : 07-Jun-19

Business Maths

Reg.No. : | | | | | | |

Time : 01:30:00 Hrs

Total Marks : 50

10 x 1 = 10

1) If A=(1 2 3), then the rank of AA^T is

(a) 0 (b) 2 (c) 3 (d) 1

2) The rank of m×n matrix whose elements are unity is

(a) 0 (b) 1 (c) m (d) n

3) if $T = \begin{matrix} & \begin{matrix} A & B \end{matrix} \\ \begin{matrix} A \\ B \end{matrix} & \begin{pmatrix} 0.4 & 0.6 \\ 0.2 & 0.8 \end{pmatrix} \end{matrix}$ is a transition probability matrix, then at equilibrium A is equal to

(a) $\frac{1}{4}$ (b) $\frac{1}{5}$ (c) $\frac{1}{6}$ (d) $\frac{1}{8}$

4) If A= $\begin{pmatrix} 2 & 0 \\ 0 & 8 \end{pmatrix}$,then $\rho(A)$ is

(a) 0 (b) 1 (c) 2 (d) n

5) The rank of the matrix $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{pmatrix}$ is

(a) 0 (b) 1 (c) 2 (d) 3

6) The rank of the unit matrix of order n is

(a) n −1 (b) n (c) n +1 (d) n^2

7) If $\rho(A)$ =r then which of the following is correct?

(a) all the minors of order r which does not vanish (b) A has at least one minor of order r which does not vanish

(c) A has at least one (r+1) order minor which vanishes (d) all (r+1) and higher order minors should not vanish

8) IfA = $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ then the rank of AA^T is

(a) 0 (b) 1 (c) 2 (d) 3

9) If the rank of the matrix $\begin{pmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ -1 & 0 & \lambda \end{pmatrix}$ is 2. Then λ is

(a) 1 (b) 2 (c) 3 (d) only real number

10) The rank of the diagonal matrix $\begin{pmatrix} 1 & & & 0 \\ & 2 & & 0 \\ & & -3 & 0 \end{pmatrix}$

(a) 0 (b) 2 (c) 3 (d) 5

5 x 2 = 10

11) Find the rank of the following matrices. $\begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$

12) Solve the following system of equations by rank method
x+y+z=9,2x+5y+7z=52,2x−y−z =0

13) For what values of the parameterl , will the following equations fail to have unique solution: 3x−y+lz=1,2x+y+z=2,x+2y−lz = −1 by rank method.

14) Solve the following equations by using Cramer's rule
2x + 3y = 7; 3x + 5y = 9

15) Two types of soaps A and B are in the market. Their present market shares are 15% for A and 85% for B. Of those who bought A the previous year, 65% continue to buy it again while 35% switch over to B. Of those who bought B the previous year, 55% buy it again and 45% switch over to A. Find their market shares after one year and when is the equilibrium reached?

5 x 3 = 15

16) Find the rank of the matrix $\begin{pmatrix} 1 & 2 & -1 & 3 \\ 2 & 4 & 1 & -2 \\ 3 & 6 & 3 & -7 \end{pmatrix}$

17) Show that the equationsx+y=5, 2x+y=8 are consistent and solve them.

18) Show that the equations 2x+y+z=5,x+y+z=4,x−y+2z = 1 are consistent and hence solve them.

19) Find k, if the equations x+y+z=7,x+2y+3z=18,y+kz=6 are inconsistent

20) Consider the matrix of transition probabilities of a product available in the market in two brands A and B.

$$\begin{matrix} \\ A \\ B \end{matrix}\begin{pmatrix} A & B \\ 0.9 & 0.1 \\ 0.3 & 0.7 \end{pmatrix}$$

Determine the market share of each brand in equilibrium position.

3 x 5 = 15

21) Solve by Cramer's rule x+y+z=4,2x−y+3z=1,3x+2y−z = 1

22) The price of 3 Business Mathematics books, 2 Accountancy books and one Commerce book is Rs840. The price of 2 Business Mathematics books, one Accountancy book and one Commerce book is Rs 570. The price of one Business Mathematics book, one Accountancy book and 2 Commerce books is Rs 630. Find the cost of each book by using Cramer's rule.

23) An automobile company uses three types of Steel S_1, S_2 and S_3 for providing three different types of Cars C_1, C_2 and C_3. Steel requirement R (in tonnes) for each type of car and total available steel of all the three types are summarized in the following table.

Types of Steel	Types of Car			Total Steel available
	C_1	C_2	C_3	
S_1	3	2	1	28

S_2	1	1	2	13
S_3	2	2	2	14

Determine the number of Cars of each type which can be produced by Cramer's rule.

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1) (d) 1

2) (b) 1

3) (a) $\frac{1}{4}$

4) (c) 2

5) (d) 3

6) (b) n

7) (b) A has at least one minor of order r which does not vanish

8) (d) 3

9) (a) 1

10) (c) 3

5 x 2 = 10

11) Let $A=\begin{pmatrix}5&6\\7&8\end{pmatrix}$

Order of A is 2 x 2

$\therefore \rho(A)\le 2$ [Since minimum of (2,2) is 2]

Consider the second order minor

$\begin{vmatrix}5&6\\7&8\end{vmatrix}=40-42$

= $-2\neq 0$

There is a minor of order 2, which is not zero

$\therefore \rho(A)=2$

12) Given non-homogeneous equations are

x+yz=9

2x+5y+7z = 52

2x-y-z = -6

The matrix equation corresponding to the given system is

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & 5 & 7 \\ 2 & -1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 9 \\ 52 \\ -6 \end{pmatrix}$$

Augmented matrix	Elementary Transformation
$\begin{pmatrix} 1 & 1 & 1 & 9 \\ 2 & 5 & 7 & 52 \\ 2 & -1 & -1 & -6 \end{pmatrix}$	
$\sim \begin{pmatrix} 1 & 1 & 1 & 9 \\ 0 & 3 & 5 & 34 \\ 2 & -1 & -1 & -6 \end{pmatrix}$	$R_2 \to R_2 - 2R_1$
$\sim \begin{pmatrix} 1 & 1 & 1 & 9 \\ 0 & 3 & 5 & 34 \\ 0 & -3 & -3 & -24 \end{pmatrix}$	$R_3 \to R_3 - 2R_1$
$\sim \begin{pmatrix} 1 & 1 & 1 & 9 \\ 0 & 3 & 5 & 34 \\ 0 & 0 & 2 & 10 \end{pmatrix}$	$R_3 \to R_3 + R_2$

The last equivalent matrix is in echelon form andit has three non-zero rows.

$\therefore \rho(A) = 3$

$\rho([A, B]) = 3$ Number of unknowns

∴ The given system is consistent and has unique solution.

To find the solution, let us rewrite the above echelon from into the matrix form

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 3 & 5 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 9 \\ 34 \\ 10 \end{pmatrix}$$

⇒ x+y+z=9 .. (1)

3y+5z = 34 ..(2)

2z = 10 ..(3)

From (3), $2z = 10 \Rightarrow z = \frac{10}{2} = 5$

Substituting Z = 5 in (2) we get,

3y + 5 (5) = 34

⇒ 3y+25=34

⇒ 3y = 34 - 25

⇒ 3y = 9

Substituting y = 3 and z = 5 in (1) we

⇒ x+3+5 = 9

⇒ x+8 = 9

⇒x = 9-8

⇒x = 1

∴ Solution set is {I, 3, 5}

13) Given non-homogeneous equations are

$3x - y + \lambda z = 1$

$2x + y + z = 2$

$x + 2y - \lambda z = -1$

The matrix equation corresponding to the given system is

Augmented matrix [A,B]	Elementary Transformation
$\begin{pmatrix} 3 & -1 & \lambda & 1 \\ 2 & 1 & 1 & 2 \\ 1 & 2 & - & -1 \end{pmatrix}$	
$\sim \begin{pmatrix} 1 & 2 & -\lambda & -1 \\ 2 & 1 & 1 & 2 \\ 3 & -1 & \lambda & 1 \end{pmatrix}$	$R_1 \leftrightarrow R_3$
$\sim \begin{pmatrix} 1 & 2 & -\lambda & -1 \\ 0 & -3 & 1+2\lambda & 4 \\ 3 & -1 & \lambda & 1 \end{pmatrix}$	$R_2 \rightarrow R_2 - 2R_1$
$\begin{pmatrix} 1 & 2 & -\lambda & -1 \\ 0 & -3 & 1+2\lambda & 4 \\ 0 & -7 & 4\lambda & 4 \end{pmatrix}$	$R_3 \rightarrow R_3 - 3R_1$
$\sim \begin{pmatrix} 1 & 2 & -\lambda & -1 \\ 0 & -1 & \frac{1+2\lambda}{3} & \frac{4}{3} \\ 0 & - & \frac{4\lambda}{7} & \frac{4}{7} \end{pmatrix}$	$R_2 \rightarrow R_2 \div 3$ $R_3 \rightarrow R_3 \div 7$
$\begin{pmatrix} 1 & 2 & -\lambda & -1 \\ 0 & -1 & \frac{1+2}{3} & \frac{4}{3} \\ 0 & 0 & \frac{-7-2\lambda}{21} & \frac{-16}{21} \end{pmatrix}$	$R_3 \rightarrow R_3 - R_2$

Since

$\frac{4\lambda}{7} - \frac{1+2\lambda}{3}$

= $\frac{12\lambda - 7 - 14\lambda}{21} = \frac{-7-2\lambda}{21}$

and $\frac{4}{7} - \frac{4}{3} = \frac{12-28}{21}$

= $\frac{-16}{21}$

∴Sincethe systemisfailtohave unique solution either it can have infinitely many solution or it may be inconsistent.

This can happen only when $\frac{-7-2\lambda}{21} = 0$

$\Rightarrow -7 - 2\lambda = 0$

$\Rightarrow -7 = 2\lambda$

$\Rightarrow \lambda = \frac{-7}{2}$

14) $\Delta = \begin{vmatrix} 2 & 3 \\ 3 & 5 \end{vmatrix} = 10 - 9 = 1 \neq 0$

Since $\Delta \neq 0$ we can apply Cramer's rule and the system is consistent with unique solution.

$\Delta x = \begin{vmatrix} 7 & 3 \\ 9 & 5 \end{vmatrix} = 7(5) - 9(3)$

= 35 - 27 = 8

$\Delta y = \begin{vmatrix} 2 & 7 \\ 3 & 9 \end{vmatrix} = 2(9) - 3(7)$

= 18 - 21 =-3

$\therefore x = \frac{\Delta x}{\Delta} = \frac{8}{1} = 8$

$y = \frac{\Delta y}{\Delta} = \frac{-3}{1} = 3$

$\therefore$ Solution set is {8, -3)

15) Transition probability matrix

(A B) T = (A B)

$$T = \begin{matrix} & \begin{matrix} A & B \end{matrix} \\ \begin{matrix} A \\ B \end{matrix} & \begin{pmatrix} \cdot 65 & \cdot 35 \\ \cdot 45 & \cdot 55 \end{pmatrix} \end{matrix}$$

Where A represents the percent of people those who bought soap A and B represents the percent of people those who bought soap B.

By the given data

A = 15%=·15

and B = 85% = ·85

Percentage after one year is

$(\cdot 15 \quad \cdot 85) \begin{pmatrix} \cdot 65 & \cdot 35 \\ \cdot 45 & \cdot 55 \end{pmatrix}$

= ((.15)(·65) + (·85)(-45) ·15(-35)+ ·85(-55))

= (-0975 + ·3825 ·0525 + -4675)

= (-48 ·52)

Hence, market share after one year is 48% and 52%

At equilibrium,

$(A \quad B) \begin{pmatrix} \cdot 65 & \cdot 35 \\ \cdot 45 & \cdot 55 \end{pmatrix} = (A \quad B)$

(-65A+A5B ·35A+·55B) = (A B)

Equating the corresponding entries on both sides

we get

$\Rightarrow \cdot 65A + \cdot 45B = A$

$\Rightarrow \cdot 65A + \cdot 45(1 - A) = A$

[Since A+B = 1 =} B= 1-A]

$\Rightarrow \cdot 65A + \cdot 45 - \cdot 45A = A$

$\Rightarrow \cdot 45 = A - \cdot 65A + \cdot 45A$

$\Rightarrow \cdot 45 = A(\cdot 35 + 45)$

$\Rightarrow \cdot 45 = A(\cdot 35 + 45)$

$\Rightarrow \cdot 45 = A(-8)$

$\Rightarrow A = \frac{\cdot 45}{\cdot 8} = \cdot 5625 = 56.25$

$\therefore B = 1 - A = 1 - \cdot 5625 = \cdot 4375$

= 43.75%

$\therefore$ Equilibrium is reached when A = 56.25% and B = 43.75%

5 x 3 = 15

16) Let A = $\begin{pmatrix} 1 & 2 & -1 & 3 \\ 2 & 4 & 1 & -2 \\ 3 & 6 & 3 & -7 \end{pmatrix}$

Order of A is3×4

∴ρ(A)≤3

Consider the third order minors

$\begin{vmatrix} 1 & 2 & -1 \\ 2 & 4 & 1 \\ 3 & 6 & 3 \end{vmatrix} =$ 0, $\begin{vmatrix} 1 & -1 & 3 \\ 2 & 1 & -2 \\ 3 & 3 & -7 \end{vmatrix} = 0$

$\begin{vmatrix} 1 & 2 & 3 \\ 2 & 4 & -2 \\ 3 & 6 & -7 \end{vmatrix} = 0,$ $\begin{vmatrix} 2 & -1 & 3 \\ 4 & 1 & -2 \\ 6 & 3 & -7 \end{vmatrix} = 0$

Since all third order minors vanishes, ∴ρ(A)≤3

Now, let us consider the second order minors,

Consider one of the second order minors $\begin{vmatrix} 2 & -1 \\ 4 & 1 \end{vmatrix} = 6 \neq 0$

There is a minor of order 2 which is not zero.

$\therefore \rho(A) = 2$

17) The matrix equation corresponding to the given system is

$\begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 8 \end{pmatrix}$

A X=B

Matrix A	Augmented matrix [A,B]	Elementary Transformation
$\begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}$ $\sim \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 1 & 1 & 5 \\ 2 & 1 & 8 \end{pmatrix}$ $\sim \begin{pmatrix} 1 & 1 & 5 \\ 0 & -1 & -2 \end{pmatrix}$	$R_2 \to R_2 - 2R_1$
$\rho(A) = 2$	$\rho([A, B]) = 2$	

Number of non-zero rows is 2.

$\rho(A) = \rho([A, B]) = 2 =$ Number of unknowns.

The given system is consistent and has unique solution.

Now, the given system is transformed into

$\begin{pmatrix} 0 & 1 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$

x +y=5

y=2

$\therefore (1) \Rightarrow x + 2 = 5$

x=3

Solution is x=3,y=2

18) The matrix equation corresponding to the given system is

$$\begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & -1 & 2 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \\ 1 \end{pmatrix}$$

A X=B

Augmented matrix [A,B]	Elementary Transformation
$\begin{pmatrix} 2 & 1 & 1 & 5 \\ 1 & 1 & 1 & 4 \\ 1 & -1 & 2 & 1 \end{pmatrix}$	
$\sim \begin{pmatrix} 1 & 1 & 1 & 4 \\ 2 & 1 & 1 & 5 \\ 1 & -1 & 2 & 1 \end{pmatrix}$	$R_1 \leftrightarrow R_2$
$\begin{pmatrix} 1 & 1 & 1 & 4 \\ 0 & -1 & -1 & -3 \\ 1 & -2 & 1 & -3 \end{pmatrix}$	$R_2 \rightarrow R_2 - 2R_1$ $R_3 \rightarrow R_3 - R_1$
$\sim \begin{pmatrix} 1 & 1 & 1 & 4 \\ 2 & -1 & -1 & -3 \\ 0 & 0 & 3 & 3 \end{pmatrix}$	$R_3 \rightarrow R_3 - 2R_2$
$\rho(A) = 3, \rho([A, B]) = 3$	

Obviously the last equivalent matrix is in the echelon form. It has three non-zero rows.

$\rho(A) = 3$, $\rho([A, B]) = 3$= Number of unknowns .

The given system is consistent and has unique solution.

To find the solution, let us rewrite the above echelon form into the matrix form.

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & -1 & -1 \\ 0 & 0 & 3 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \\ 3 \end{pmatrix}$$

x+y+z=4 (1)

y+z=3 (2)

3z=3 (3)

$(3) \Rightarrow z = 1$

$(2) \Rightarrow y = 3 - z = 2$

$(1) \Rightarrow x = 4 - y - z$

x=1

∴ x=1, y=2,z=1

19) The matrix equation corresponding to the given system is

$$\begin{pmatrix}1&1&1\\1&2&3\\0&1&k\end{pmatrix}\begin{pmatrix}X\\Y\\Z\end{pmatrix}=\begin{pmatrix}7\\18\\6\end{pmatrix}$$

AX=B

Augmented matrix [A,B]	Elementary Transformation
$\begin{pmatrix}1&1&1&7\\1&2&3&18\\0&1&k&6\end{pmatrix}$	$R_2 \to R_2 - R_1$
$\sim\begin{pmatrix}1&1&1&7\\0&1&2&11\\0&1&k&6\end{pmatrix}$	$R_3 \to R_3 - R_2$
$\sim\begin{pmatrix}1&1&1&7\\0&1&2&11\\0&0&k-2&-5\end{pmatrix}$	

For the equations to be inconsistent

$\rho([A,B]) \neq \rho(A)$

It is possible if k−2=0.

∴ k=2

20) Transition probability matrix

$T = \begin{matrix} A \\ B \end{matrix}\begin{pmatrix} \overset{A}{0.9} & \overset{B}{0.1} \\ 0.3 & 0.7 \end{pmatrix}$

At equilibrium, (A B) T=(AB) where A+B=1

(A B) $\begin{pmatrix}0.9&0.1\\0.3&0.7\end{pmatrix}$ =(A B)

0.9A+ 0.3B = A

0.9A+0.3(1−A) = A

0.9A−0.3A+0.3 = A

0.6A + 0.3 = A

0.4A = 0.3

A=$\frac{0.3}{0.4} = \frac{3}{4}$

B=1-$\frac{3}{4} = \frac{1}{4}$

Hence the market share of brand A is 75% and the market share of brand B is 25%

3 x 5 = 15

21) Here $\triangle = \begin{vmatrix}1&1&1\\2&-1&3\\3&2&-1\end{vmatrix} = 13 \neq 0$

∴ We can apply Cramer's Rule and the system is consistent and it has unique solution.

$\triangle_x = \begin{vmatrix}1&1&1\\2&-1&3\\3&2&-1\end{vmatrix} = -13 \quad \triangle_y = \begin{vmatrix}1&4&1\\2&1&3\\3&1&-1\end{vmatrix} = 39 \quad \triangle_z = \begin{vmatrix}1&1&4\\2&-1&1\\3&2&1\end{vmatrix} = 26$

∴ By Cramer's rule

$x = \frac{\triangle x}{\triangle} = \frac{-13}{13} = -1 \quad y = \frac{\triangle y}{\triangle} = \frac{39}{13} = 3 \quad z = \frac{\triangle z}{\triangle} = \frac{26}{13} = 2$

∴ The solution is (x,y,z) = (−1,3,2)

22) Let 'x' be the cost of a Business Mathematics book

Let 'y' be the cost of a Accountancy book.

Let 'z' be the cost of a Commerce book.

∴ 3x+2y+z=840

2x+y+z=570

x+y+2z=630

Here

$$\triangle = \begin{vmatrix} 3 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 2 \end{vmatrix} = -2 \neq 0 \quad \triangle_x = \begin{vmatrix} 840 & 2 & 1 \\ 570 & 1 & 1 \\ 630 & 1 & 2 \end{vmatrix} = -240$$

$$\triangle_y = \begin{vmatrix} 3 & 840 & 1 \\ 2 & 570 & 1 \\ 1 & 630 & 2 \end{vmatrix} = -300 \quad \triangle_z = \begin{vmatrix} 3 & 2 & 840 \\ 2 & 1 & 570 \\ 1 & 1 & 630 \end{vmatrix} = -360$$

∴ By Cramer's rule

$$x = \frac{\triangle x}{\triangle} = \frac{-240}{-2} = 120 \quad y = \frac{\triangle y}{\triangle} = \frac{300}{-2} = 150 \quad z = \frac{\triangle z}{\triangle} = \frac{360}{-2} = 180$$

∴The cost of a Business Mathematics book is Rs 120,

the cost of a Accountancy book is Rs 150 and

the cost of a Commerce book is Rs 180.

23) Let 'x' be the number of cars of type C1

Let 'y' be the number of cars of type C2

Let 'z' be the number of cars of type C3

3x+2y+4z=28

x+y2z=13

2x+2y+z=14

Here

$$\triangle = \begin{vmatrix} 3 & 2 & 4 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix} = -3 \neq 0 \quad \triangle_x = \begin{vmatrix} 28 & 2 & 4 \\ 13 & 1 & 2 \\ 14 & 2 & 1 \end{vmatrix} = -6$$

$$\triangle_y = \begin{vmatrix} 3 & 28 & 4 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix} = -9 \quad \triangle_z = \begin{vmatrix} 3 & 2 & 28 \\ 1 & 1 & 13 \\ 2 & 2 & 14 \end{vmatrix} = -12$$

∴By Cramer's rule

$$x = \frac{\triangle x}{\triangle} = \frac{-6}{-3} = 2 \quad y = \frac{\triangle y}{\triangle} = \frac{-9}{-3} = 3 \quad z = \frac{\triangle z}{\triangle} = \frac{-12}{-3} = 4$$

∴The number of cars of each type which can be produced are 2, 3 and 4.

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