RAVI MATHS TUITION (HOME) CENTER, CH-82 PH-8056206308

11th chapter 9 limits and derivative

11th Standard

Date : 01-Mar-19

Maths

Reg.No. : | | | | | | |

Time : 02:00:00 Hrs

Total Marks : 60

11 x 1 = 11

1) $lim_{x \to \infty} \frac{sin\, x}{x}$

(a) 1 (b) 0 (c) ∞ (d) -∞

2) $lim_{x \to 0} \frac{\sqrt{1-cos2x}}{x}$

(a) 0 (b) 1 (c) $\sqrt{2}$ (d) does not exist

3) Let the function f be defined by $f(x)=\begin{cases} 3x & 0 \le x \le 1 \\ -3x+5 & 1 < x \le 2 \end{cases}$,then

(a) $lim_{x \to 1} f(x) = 1$ (b) $lim_{x \to 1} f(x) = 3$ (c) $lim_{x \to 1} f(x) = 2$ (d) $lim_{x \to 1} f(x)$ does not exist

4) If $lim_{x \to 0} \frac{sin\, px}{tan\, 3x} = 4$, then the value of p is

(a) 6 (b) 9 (c) 12 (d) 4

5) $lim_{\alpha \to \pi/4} \frac{sin\alpha - cos\alpha}{\alpha - \frac{\pi}{4}}$ is

(a) $\sqrt{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) 1 (d) 2

6) At x= $\frac{3}{2}$ the function $f(x) = \frac{|2x-3|}{2x-3}$ is

(a) continuous (b) discontinuous (c) differentiable (d) non-zero

7) Let a function f be defined by $f(x) = \frac{x-|x|}{x}$ for x $\neq$0 and f(0)=2. Then f is

(a) continuous nowhere (b) continuous everywhere (c) continuous for all x except x = 1 (d) continuous for all x except x = 0

8) $\lim_{x \to 1} \frac{x^m-1}{x^n-1} is$

(a) mn (b) m+n (c) m-n (d) $\frac{m}{n}$

9) $\lim_{x \to \infty} \left(\frac{1}{x}+2\right)$ is equal to

(a) ∞ (b) 0 (c) 1 (d) 2

10) The function y = $\frac{|3x-4|}{3x-4}$ is discontinuous at x =

(a) 0 (b) $\frac{3}{4}$ (c) $\frac{4}{3}$ (d) 1

11) The rate of change of area A of a circle of radius r is

(a) $2\pi r$ (b) $2\pi r \frac{dr}{dt}$ (c) $\pi r^2 \frac{dr}{dt}$ (d) $\pi \frac{dr}{dt}$

7 x 2 = 14

12) Compute $lim_{x \to 1} \frac{x^3-1}{x-1}$

13) Find the positive integer n so that $lim_{x \to 3} \frac{x^n-3^n}{x-3} = 27$

14) Find the relation between a and b if $lim_{x \to 3} f(x)$ exists where $f(x) = \begin{cases} ax+b & \text{if } x>3 \\ 3ax-4b+1 & \text{if } x<3 \end{cases}$

15) Evaluate the following limits $lim_{x \to \infty} \frac{x^3+x}{x^4-3x^2+1}$

16) Examine the continuity of the following : $\frac{sinx}{x^2}$

17) Find the points of discontinuity of the function f, where

f(x)={ $x+2, \quad if \quad x \geq 2$
$x^2, \quad if \quad x < 2$

18) Evaluate $\lim_{x\to 2} \frac{x^2-3x+2}{x^2-x-2}$

5 x 3 = 15

19) Find $lim_{t\to 0} \frac{\sqrt{t^2+9}-3}{t^2}$.

20) Evaluate the following limits :

$lim_{x\to 0} \frac{\sqrt{x^2+1}-1}{\sqrt{x^2+16}-4}$

21) Evaluate the following limits :$lim_{x\to 0} \frac{1-cosx}{x^2}$

22) Evaluate the following limits :$lim_{x\to 0} \frac{1-cos^2x}{xsin2x}$

23) Find the points at which f is discontinuous. At which of these points f is continuous from the right, from the left, or neither? Sketch the graph of f.

$$f(x) = \begin{cases} 2x+1, & \text{if } x \leq -1 \\ 3x & \text{if } -1 < x < 1 \\ 2x-1, & \text{if } x \geq 1 \end{cases}$$

4 x 5 = 20

24)

Check if $lim_{x\to -58} f(x)$ exists or not, where $f(x) = \begin{cases} \frac{|x+5|}{x+5}, & \text{for } x \neq -5 \\ 0, & \text{for } x = -5 \end{cases}$

25) Evaluate the following limits :

$lim_{x\to a} \frac{\sqrt{x-b}-\sqrt{a-b}}{x^2-a^2} (a > b)$

26) Evaluate the following limits :$lim_{x\to 0} \frac{sin\ x(1-cos\ x)}{x^3}$

27) A function f is defined as follows :

f(x)={
$0, \quad for\ x < 0;$
$x, \quad for\ 0 \leq x < 1;$
$-x^2+4x-2, \quad for\ 1 \leq x < 3;$
$4-x, \quad for\ x \geq 3$

Is the function continuous?

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1) (b) 0

2) (d) does not exist

3) (d) $lim_{x\to 1} f(x)$ does not exist

4) (c) 12

5) (a) $\sqrt{2}$

6) (b) discontinuous

7) (d) continuous for all x except x = 0

8) (d) $\frac{m}{n}$

9) (d) 2

10) (c) $\frac{4}{3}$

11) (b) $2\pi r\frac{dr}{dt}$

7 x 2 = 14

12) $lim_{x\to 1}\frac{x^3-1}{x-1} = lim_{x\to 1}\frac{x^3-1}{x-1} = 3(1)^{3-1} = 3.$

13) $lim_{x\to 3}\frac{x^n-3^n}{x-3} = n.3^{n-1} = 27$

That is $n.3^{n-1} = 3\times 3^2 = 3\times 3^{3-1} \Rightarrow n = 3$.

14) $lim_{x\to 3^-} f(x) = 9a - 4b + 1$

$lim_{x\to 3^+} f(x) = 3a + b.$ Now the existence of limit forces us to have

$lim_{x\to 3^-} f(x) = lim_{x\to 3^+} f(x).$

$\Rightarrow$ 9a - 4b + 1 = 3a + b

$\Rightarrow$6a - 5b + 1 = 0.

15) $lim_{x\to\infty}\frac{x^3+x}{x^4-3x^2+1}$

Taking x^3 and x^4 common from numerator and denominator respectively, we get,

$lim_{x\to\infty}\frac{x^3(1+\frac{1}{x^2})}{x^4(1-\frac{3}{x^2}+\frac{1}{x^4})}$

$lim_{x\to\infty}\frac{1+\frac{1}{x^2}}{x(1-\frac{3}{x^2}+\frac{1}{x^4})} = \frac{1}{\infty} = 0\,[\because \frac{1}{x}\to 0\ as\ x\to\infty]$

$\therefore lim_{x\to\infty}\frac{x^3+x}{x^4-3x^2+1} = 0$

16) Let f(x)= $\frac{sinx}{x^2}$

f(x) does not exist for x=0. But sin x is continuous for all $x\in R$.

$\therefore$ f(x) is continuous only in **R** -{0}

17) Given f(x)={$x+2, \quad if \quad x \geq 2$
$x^2, \quad if \quad x < 2$

$lim_{x\to 2^-} f(x) = lim_{x\to 2^-} x^2 = 2^2 = 4$

$lim_{x\to 2^+} f(x) = lim_{x\to 2^+} x+2 = 2+2 = 4$

Also f(2)= x+2=2+2=4

$\therefore lim_{x\to 2^-} f(x) = lim_{x\to 2^+} f(x) = f(2) = 4$

$\therefore$ f(x)is continuous in R.

18) $$\lim_{x\to 2} \frac{x^2-3x+2}{x^2-x-2} = \lim_{x\to 2} \frac{(x-2)(x-1)}{(x-2)(x+1)} = \lim_{x\to 2} \frac{(x-1)}{(x+1)} = \frac{2-1}{2+1} = \frac{1}{3}$$

5 x 3 = 15

19) We can't apply the quotient theorem immediately. Use the algebra technique of rationalising the numerator

$$\frac{\sqrt{t^2+9}-3}{t^2} = \frac{(\sqrt{t^2+9}-3)(\sqrt{t^2+9}-3)}{t^2(\sqrt{t^2+9}-3)} = \frac{t^2+9-9}{t^2(\sqrt{t^2+9}-3)}$$

$$lim_{t\to 0} \frac{\sqrt{t^2+9}-3}{t^2} = lim_{t\to 0} \frac{t^2}{t^2\sqrt{t^2+9}+3} = lim_{t\to o} \frac{1}{\sqrt{t^2+9}+3} = \frac{1}{\sqrt{9}+3} = \frac{1}{6}.$$

20) $$lim_{x\to 0} \frac{\sqrt{x^2+1}-1}{\sqrt{x^2+16}-4} = lim_{x\to 0} \frac{(x^2+1)^{1/2}-1^{1/2}}{(x^2+16)^{1/2}-(16)^{1/2}}$$

Multiplying and dividing byx^2, we get

$$= lim_{x\to 0} \frac{(x^2+1)^{1/2}-1^{1/2}}{x^2} \times \frac{x^2}{(x^2+16)^{1/2}-(16)^{1/2}}$$

$$= lim_{x\to 0} \frac{(x^2+1)^{1/2}-1^{1/2}}{(x^2+1)-1} \times \frac{x^2+16-16}{(x^2+16)^{1/2}-(16)^{1/2}}$$

$$= [lim_{x^2+1\to 1} \frac{(x^2+1)^{1/2}-1^{1/2}}{(x^2+1)-1}] \times \frac{1}{[lim_{x^2+16\to 16} \frac{(x^2+16)^{1/2}-(16)^{1/2}}{(x^2+16)-16}}$$

$$= \frac{1}{2}(1)^{\frac{1}{2}-1} \times \frac{1}{\frac{1}{2}(16)^{\frac{1}{2}-1}} [\because lim_{x\to a} \frac{x^n-a^n}{x-a} = n.a^{n-1}]$$

$$= \frac{1}{2} \times \frac{1}{\frac{1}{2}(16)^{-1/2}} = (16)^{1/2} = \sqrt{16} = 4$$

21) $$lim_{x\to 0} \frac{1-cosx}{x^2} = lim_{x\to 0} \frac{2sin^2\frac{x}{2}}{x^2} [\because cos2x = 1-2sin^2x]$$

$$= 2.lim_{x\to 0} \frac{sin^2(\frac{x}{2})}{\frac{x^2}{4}\times 4} = \frac{2}{4}[lim \frac{x}{2} \to 0 \frac{sin(\frac{x}{2})}{(\frac{x}{2})}]^2$$

$$= \frac{2}{4} \times 1^2 = \frac{2}{4} = \frac{1}{2} \ [\because lim_{\theta\to o} \frac{sin\theta}{\theta} = 1]$$

$$\therefore lim_{x\to 0} \frac{1-cosx}{x^2} = \frac{1}{2}$$

22) $$lim_{x\to 0} \frac{1-cos^2x}{xsin2x} = lim_{x\to 0} \frac{sin^2x}{x(2sinxcosx)} = \frac{1}{2} lim_{x\to 0} \frac{sinx}{xcosx}$$

$$= \frac{1}{2}(lim_{x\to 0} \frac{sinx}{x}) \times lim_{x\to 0} \frac{1}{cos\ x} = \frac{1}{2} \times 1 \times \frac{1}{1} = \frac{1}{2} \quad [\because cos0 = 1]$$

$$\therefore lim_{x\to 0} \frac{1-cos^2x}{xsin2x} = \frac{1}{2}$$

23) $lim_{x \to 1^-} f(x) = lim_{x \to 1^-} 3x = 3$

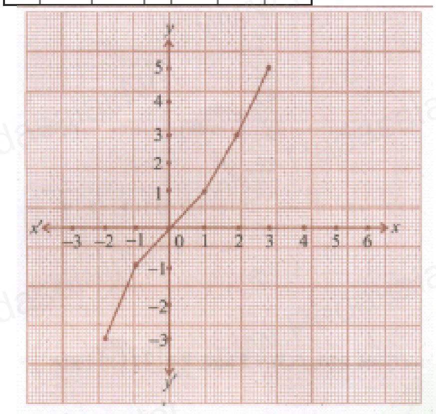
$lim_{x \to 1^+} f(x) = lim_{x \to 1^+} 2x - 1 = 2 - 1 = 1$

$lim_{x \to 1^-} f(x) \neq lim_{x \to 1^+} f(x)$

and $f(1) = 2x - 1 = 2(1) - 1 = 1$

$\therefore f(x)$ is not continuous at x=1

x	-2	-1	0	1	2	3
f(x)	2x+1 -3	2x+1 -1	3x 0	2x-1 1	2x-1 3	2x-1 5



4 x 5 = 20

24) (i) f(-5⁻)

For x < - 5, |x + 5| = - (x + 5)

Thus f(-5⁻)=$lim_{x \to -5^-} \frac{-(x+5)}{(x+5)} = -1$

(ii) f(-5⁺)

For x > - 5, |x + 5| = (x + 5)

Thus f(-5⁺)=$lim_{x \to -5^+} \frac{(x+5)}{(x+5)} = 1$

Note that f(-5⁻) ≠ f(-5⁺) . Hence the limit does not exist.

25) $lim_{x \to a} \frac{\sqrt{x-b} - \sqrt{a-b}}{x^2 - a^2} (a > b)$

Multiplying and dividing by $(\sqrt{x-b} + \sqrt{a-b})$ we get,

$= lim_{x \to a} \frac{\sqrt{x-b} - \sqrt{a-b}}{x^2 - a^2} \frac{\sqrt{x-b} + \sqrt{a-b}}{\sqrt{x-b} + \sqrt{a-b}}$

$= lim_{x \to a} \frac{(x-b) - (a-b)}{(x^2 - a^2)[\sqrt{x-b} + \sqrt{a-b}]}$

$= \lim_{x \to a} \frac{x - b - a + b}{(x+a)(x-a)\left[\sqrt{x-b} + \sqrt{a-b}\right]}$

$= \lim_{x \to a} \frac{x-a}{(x+a)(x-a)\left[\sqrt{x-b} + \sqrt{a-b}\right]}$

$= \frac{1}{(a+a)[\sqrt{a-b} + \sqrt{a-b}]} = \frac{1}{2a[2\sqrt{a-b}]}$

$= \frac{1}{4a[\sqrt{a-b}]}$

26) $lim_{x \to 0} \frac{sin\ x(1 - cos\ x)}{x^3} = lim_{x \to 0} \frac{sin\ x \times (2sin^2 \frac{x}{2})}{x^3}$

$= 2[lim_{x \to 0} \frac{sin\ x}{x}] \times lim_{x \to 0} \frac{sin^2 \frac{x}{2}}{x^2} = 2(1) \times lim_{\frac{x}{2} \to 0} \frac{sin^2 \frac{x}{2}}{\frac{x^2}{4} \times 4}$

$2 \times [lim_{\frac{x}{2} \to 0} \frac{sin^2 \frac{x}{2}}{\frac{x^2}{4}}] \times \frac{1}{4} = 2 \times 1 \times \frac{1}{4} = \frac{1}{2}$

$\therefore lim_{x \to 0} \frac{sin\ x(1 - cos\ x)}{x^3} = \frac{1}{2}$

27) Given f(x)={ $\begin{cases} 0, & for\ x<0; \\ x, & for\ 0\le x<1; \\ -x^2+4x-2, & for\ 1\le x<3; \\ 4-x, & for\ x\ge 3 \end{cases}$

(i) At the point x=0

$lim_{x\to 0^-} f(x) = lim_{x\to 0^-} 0 = 0$

$lim_{x\to 0^+} f(x) = x = 0$

and f(0)=x=0

$lim_{x\to 0^-} f(x) = lim_{x\to 0^+} f(x) = f(0) = 0$

$\therefore f(x)$ is continuous at x=0

(ii) At the point x=1

$lim_{x\to 1^-} f(x) = x = 1$

$lim_{x\to 1^+} f(x) = -x^2+4x-2 = -1+4-2 = 4-3 = 1$

f(1)=$-x^2+4x-2 = -1+4-2 = 1$

$lim_{x\to 1^-} f(x) = lim_{x\to 1^+} f(x) = f(1) = 1$

$\therefore f(x)$ is continuous at x=1

(iii) At the point x=3

$lim_{x\to 3^-} f(x) = lim_{x\to 3^-} -x^2+4x-2 = -(3)^2+4(3)-2$

=-9+12-2=1

$and\ lim_{x\to 3^+} f(x) = lim_{x\to 3^+} 4-x = 4-3 = 1$

f(3)=4-3=1

$\therefore lim_{x\to 3^-} f(x) = lim_{x\to 3^+} f(x) = f(3) = 1$

$\therefore f(x)$ is continuous at x=3