UNIT – 06 GRAVITATION

TWO MARKS AND THREE MARKS:

01. State Kepler's three laws.

1. Law of orbits: Each planet moves around the Sun in an elliptical orbit with the Sun at one of the foci.
2. Law of area:
   The radial vector (line joining the Sun to a planet) sweeps equal areas in equal intervals of time
3. Law of period:
   The square of the time period of revolution of a planet around the Sun in its Elliptical orbit is directly proportional to the cube of the semi-major axis of The ellipse.

02. State Newton's Universal law of gravitation.

Newton's law of gravitation states that a particle of mass M_1 attracts any other particle of mass M_2 in the universe with an attractive force. The strength of this force of attraction was found to be directly proportional to the product of their masses and is inversely proportional to the square of the distance between them.

03. Will the angular momentum of a planet be conserved? Justify your answer.

Yes, Because $\vec{\tau} = \vec{r} \times \vec{F}$; $\vec{r} \times \left(\frac{GM_S M_E}{r^2}\hat{r}\right) = 0$

Since $\vec{r} = r\,\hat{r}$, $(\hat{r} \times \hat{r}) = 0$ So, $\vec{\tau} = \frac{d\vec{L}}{dt} = 0$

It implies that angular momentum is a constant vector. The angular momentum of the Earth about the Sun is constant throughout the motion.

04. Define the gravitational field. Give its unit.

The gravitational force experienced by unit mass placed at that point.

Unit $\vec{E}_1 = \frac{\vec{F}21}{m_2}$ in equation we get, $\vec{E} = -\frac{Gm_1}{r^2}\vec{r}$. its unit is N kg^{-1} (or) m s^{-2} .

05. What is meant by superposition of gravitational field?

Consider 'n' particles of masses, m_1 , m_2 ...m_n distributed in space at positions $\hat{r}_1, \hat{r}_2, \hat{r}_3$ ....etc, with respect to point P. The total gravitational field at a point P due to all the masses is given by the vector sum of the gravitational field due to the individual masses. This principle is known as superposition of gravitational fields.

$$\vec{E}_{total} = \vec{E}_1 + \vec{E}_2 + ...\vec{E}_n$$

$$= -\frac{Gm_1}{r_1^2}\vec{r}_1 - \frac{Gm_2}{r_2^2}\vec{r}_2 - .... \frac{Gm_n}{r_n^2}\vec{r}_n \ ; = -\sum_{i=1}^{n}\frac{Gm_i}{r_i^2}\vec{r}_i$$

06. Define gravitational potential energy.

Gravitational potential energy associated with this conservative force field. The gravitational potential energy is defined as the work done to bring the mass m_2 from infinity to a distance 'r' in the gravitational field of mass m_1. Its unit is joule.

07. Is potential energy the property of a single object? Justify.

Potential energy is a property of a system rational than of a single object due to its physical position. Because gravitational potential energy depends on relative position. So, a reference level at which to set the potential energy equal to zero.

08. Define gravitational potential.

The gravitational potential at a distance r due to a mass is defined as the amount of work required to bring unit mass from infinity to the distance r.

09. What is the difference between gravitational potential and gravitational potential energy?

Gravitational potential:

The amount of work done in bringing a body of unit mass from infinity to that point without acceleration. $V = -\frac{GM}{R}$

Gravitational potential Energy:

The energy stored in the body at that point. If the position of the body changes due to force acting on it, then change in its potential energy is equal to the amount of work done on the body by the forces acting on it. $U = -\frac{GMm}{R}$

10. What is meant by escape speed in the case of the Earth?

The minimum speed required by an object to escape from Earth's gravitational field.

ie. $Ve = \sqrt{2gR_E}$; $Ve = 11.2$ kms^{-1}

11. Why is the energy of a satellite (or any other planet) negative?

The negative sign in the total energy implies that the satellite is bound to the Earth and it cannot escape from the Earth.

As h approaches, ∞ the total energy tends to zero. Its physical meaning is that the satellite is completely free from the influence of Earth's gravity and is not bound to Earth at large distances.

12. What are geostationary and polar satellites?

Geostationary satellites:

The satellites revolving the Earth at the height of 36000 km above the equator, are appear to be stationary when seen from Earth is called geo-stationary satellites.

Polar satellites:

The satellites which revolve from north to south of the Earth at the height of 500 to 800 km from the Earth surface are called Polar satellites.

13. Define weight

The weight of an object is defined as the downward force whose magnitude W is equal to that of upward force that must be applied to the object to hold it at rest or at constant velocity relative to the earth. The magnitude of weight of an object is denoted as, W=N=mg.

14. Why is there no lunar eclipse and solar eclipse every month?

Moon's orbit is tilted 5^0 with respect to Earth's orbit, only during certain periods of the year; the Sun, Earth and Moon align in straight line leading to either lunar eclipse or solar eclipse depending on the alignment.

15. How will you prove that Earth itself is spinning?

The Earth's spinning motion can be proved by observing star's position over a night. Due to Earth's spinning motion, the stars in sky appear to move in circular motion about the pole star.

16. What is meant by state of weightlessness?

When downward acceleration of the object is equal to the acceleration due to the gravity of the Earth, the object appears to be weightless

17. Why do we have seasons on Earth?

The seasons in the Earth arise due to the rotation of Earth around the Sun with 23.5° tilt. Due to this 23.5° tilt, when the northern part of Earth is farther to the Sun, the southern part is nearer to the Sun. So when it is summer in the northern hemisphere, the southern hemisphere experience winter.

18. Water falls from the top of a hill to the ground. Why?

This is because the top of the hill is a point of higher gravitational potential than the surface of the Earth. i.e. Vhill > Vground.

19. What is the effect of rotation of the earth on the acceleration due to gravity?

The acceleration due to gravity decreases due to rotation of the earth. This effect is zero at poles and maximum at the equator.

20. A satellite does not need any fuel to aide around the earth. Why?

The gravitational force between satellite and earth provides the centripetal force required by the satellite to move in a circular orbit.

21. Why does a tide arise in the ocean?

Tides arise in the ocean due to the force of attraction between the moon and sea water.

22. Water falls from the top of a hill to the ground. Why?

This is because the top of the hill is a point of higher gravitational potential than the surface of the Earth. i.e. $V_{hill} > V_{ground}$.

23. When a man is standing in the elevator, what are forces acting on him.

1. Gravitational force which acts downward. If we take the vertical direction as positive y direction, the gravitational force acting on the man is $\vec{F}_G = -mg\hat{j}$
2. The normal force exerted by floor on the man which acts vertically upward, $\vec{N} = N\hat{j}$

24. Find the distance between Venus and Sun.

1) The distance between Venus and Sun. The distance between Earth and Sun is taken as one Astronomical unit (1 AU).
2) The trigonometric relation satisfied by this right angled triangle is Sin $\theta = \frac{r}{R}$
3) Where R = 1 AU. r = R Sin θ = (1 AU) (Sin 46^0). Here sin.46^0 = 0.77. Hence Venus is at a distance of 0.77 AU from Sun.

25. Find the expression of the orbital speed of satellite revolving around the earth.

Satellite of mass M to move in a circular orbit, centripetal force must be acting on the satellite. This centripetal force is provided by the Earth's gravitational force.

$$\frac{MV^2}{(R_E+h)} = \frac{GMM_E}{(R_E+h)^2}$$

$$V^2 = \frac{GM_E}{(R_E+h)} \;;$$

$$V = \sqrt{\frac{GM_E}{(R_E+h)}}$$

As h increases, the speed of the satellite decreases.

26. What are the points to be noted to study about gravitational field?

Case 1: If $r < r'$

Since gravitational force is attractive, m_2 is attracted by m_1.Then m_2 can move from r' to r without any external work. Here work is done by the system spending its internal energy and hence the work done is said to be negative.

Case 2: If $r > r'$

Work has to be done against gravity to move the object from r' to r. Therefore work is done on the body by external force and hence work done is positive.

27. What is meant by retrograde motion of planet?

1) The planets move eastwards and reverse their motion for a while and return to eastward motion again. This is called **"retrograde motion"** of planets.

2) To explain this retrograde motion, Ptolemy introduced the concept of "epicycle" in his geocentric model. According to this theory, while the planet orbited the Earth, it also underwent another circular motion termed as "epicycle". A combination of epicycle and circular motion around the Earth gave rise to retrograde motion of the planets with respect to Earth.

CONCEPTUAL QUESTIONS

01. In the following, what are the quantities which that are conserved?

a) Linear momentum of planet b) Angular momentum of planet
c) Total energy of planet d) Potential energy of a planet

Ans. (b & d) Angular momentum of planet, Potential energy of a Planet.

02. The work done by Sun on Earth in one year will be

a) Zero b) Non zero c) positive d) negative Ans . : Zero

03. The work done by Sun on Earth at any finite interval of time is

a) positive, negative or zero b) Strictly positive
c) Strictly negative d) It is always zero

Ans. d) it is always zero

04. If a comet suddenly hits the Moon and imparts energy which is more than the total energy of the Moon, what will happen?

If a comet hits the moon with large mass and with large velocity may destroy the moon completely or its impact makes the moon, go out of the orbit.

05. If the Earth's pull on the Moon suddenly disappears, what will happen to the Moon?

If the gravitational force suddenly disappears, moon will stop revolving around the earth and it will move in a direction tangential to its original orbit with a speed with which it was revolving around the earth.

06. If the Earth has no tilt, what happens to the seasons of the Earth?

If the Earth has no tilt, there will be no season as like now and the duration of day and night will be equal throughout the year.

07. A student was asked a question 'why are there summer and winter for us? He replied as 'since Earth is orbiting in an elliptical orbit, when the Earth is very far away from the Sun(aphelion) there will be winter, when the Earth is nearer to the Sun(perihelion) there will be winter'. Is this answer correct? If not, what is the correct explanation for the occurrence of summer and winter?

No, The seasons in the Earth arise due to the rotation of Earth around the Sun with 23.5° tilt. Due to this 23.5° tilt, when the northern part of Earth is farther to the Sun, the southern part is nearer to the Sun. So when it is summer in the northern hemisphere, the southern hemisphere experience winter.

08. The following photographs are taken from the recent lunar eclipse which occurred on January 31, 2018. Is it possible to prove that Earth is a sphere from these photographs?

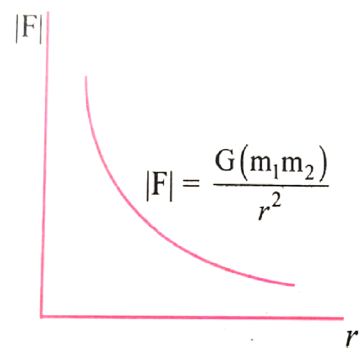
No. the moon goes around the earth in an elliptical orbit. This means its distance from us varies periodically as it goes around us.

FIVE MARKS:

01. Discuss the important features of the law of gravitation.

Important features of gravitational force:

1) As the distance between two masses increases, the strength of the force tends to decrease because of inverse dependence on r^2. Physically it implies that Ex. The planet Uranus experiences less gravitational force from the Sun than the Earth since Uranus is at larger distance from the Sun compared to the Earth.



2) The gravitational forces between two particles always constitute an action-reaction pair. It implies that the gravitational force exerted by the Sun on the Earth is always towards the Sun. The reaction-force is exerted by the Earth on the Sun. The direction of this reaction force is towards Earth.

3) The torque experienced by the Earth due to the gravitational force of the Sun is given by

$$\vec{\tau} = \vec{r} \times \vec{F} \; ; \; \vec{r} \times \left(\frac{GM_S M_E}{r^2}\hat{r}\right) = 0$$

Since $\vec{r} = r\,\hat{r}$, $(\hat{r} \times \hat{r}) = 0$ So, $\vec{\tau} = \frac{d\vec{L}}{dt} = 0$

It implies that angular momentum is a constant vector. The angular momentum of the Earth about the Sun is constant throughout the motion

4) Earth orbits around the Sun due to Sun's gravitational force, we assumed Earth and Sun to be point masses. This assumption is a good approximation because the distance between the two bodies is very much larger than their diameters.

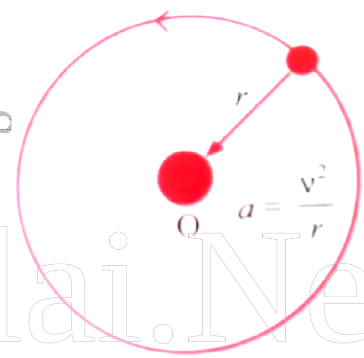
5) To calculate force of attraction between a hollow sphere of mass M with uniform density and point mass m kept outside the hollow sphere, we can replace the hollow sphere of mass M as equivalent to a point mass M located at the center of the hollow sphere.

6) If we place another object of mass 'm' inside this hollow sphere, the force experienced by this mass 'm' will be zero.

02. Explain how Newton arrived at his law of gravitation from Kepler's third law.

Newton's inverse square Law:

Newton considered the orbits of the planets as circular. For circular orbit of radius r, the centripetal acceleration towards the center is



$$a = \frac{v^2}{r} \quad \text{----------------------- 1}$$

Here v is the velocity and r, the distance of the planet from the center of the orbit. The velocity in terms of known quantities r and T, is

$$V = \frac{2\pi r}{T} \quad \text{--------------------- 2}$$

Here T is the time period of revolution of the planet. Substituting this value of v in equation (1) we get,

$$a = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = -\frac{4\pi^2 r}{T^2} \quad \text{------- 3}$$

Substituting the value of 'a' from (3) in Newton's second law, F = ma, where 'm' is the mass of the planet.

$$F = \frac{4\pi mr}{T^2} \quad \text{--------------------- 4}$$

From Kepler's third law,

$$\frac{r^3}{T^2} = k \text{ (Constant)} \quad \text{----------- 5}$$

$$\frac{r}{T^2} = \frac{k}{r^2} \quad \text{------------------ 6}$$

By substituting equation 6 in the force expression, we can arrive at the law of gravitation.

$$F = \frac{4\pi^2 mk}{r^2} \quad \text{----------7}$$

Here negative sign implies that the force is attractive and it acts towards the center. In equation (7), mass of the planet 'm' comes explicitly. But Newton strongly felt that according to his third law, if Earth is attracted by the Sun, then the Sun must also be attracted by the Earth with the same magnitude of force. So he felt that the Sun's mass (M) should also occur explicitly in the expression for force. From this insight, he equated the constant $4\pi^2 k$ to GM which turned out to be the law of gravitation.

$$F = \frac{GMm}{r^2}$$

Again the negative sign in the above equation implies that the gravitational force is attractive.

03. Explain how Newton verified his law of gravitation.

1) Newton verified his law of universal gravitation by comparing the acceleration of a terrestrial object to the acceleration of the moon.

2) He knew that the distance from the center of earth to the center of two spheres of known mass at either end of a light rod suspended by a then fiber from the center of the rod.

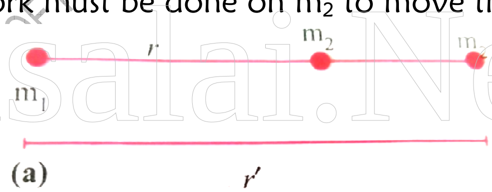
3) He had earlier found the small force that was needed to twist the fiber.

4) By bringing a third sphere close to one of the suspended spheres.

5) He was able to measure the force of gravity between the spheres and hence gravitation.

04. Derive the expression for gravitational potential energy.

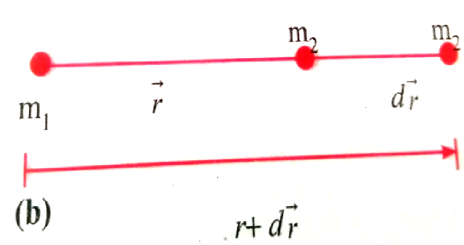
1) Two masses m_1 and m_2 are initially separated by a distance r'. Assuming m_1 to be fixed in its position, work must be done on m_2 to move the distance from r' to r as shown in Figure (a)



2) To move the mass m_2 through an infinitesimal displacement $d\vec{r}$ from r to $\vec{r} + d\vec{r}$ (shown in the Figure (b), work has to be done externally.

This infinitesimal work is given by

$dW = \vec{F}_{ext}.\, d\vec{r}$ --------------- 1



3) The work is done against the gravitational force, therefore,

$$\left|\vec{F}_{ext}\right| = \left|\vec{F}_G\right| = \frac{Gm_1m_2}{r^2} \quad \text{----------- 2}$$

Substituting equation (2) in (1), we get

$$dW = \frac{Gm_1m_2}{r^2}\hat{r}\,.\,d\vec{r} \quad ; \quad d\vec{r} = dr\,\hat{r}$$

$$\frac{Gm_1m_2}{r^2}\hat{r}\,.\,d\vec{r} \quad ; \quad \hat{r}.\hat{r} = 1 \text{ (Since both are unit vectors)}$$

$$dW = \frac{Gm_1m_2}{r^2}\,dr$$

4) Thus the total work done for displacing the particle from r' to r is $W = \int_{r'}^{r} dW = \int_{r'}^{r} \frac{Gm_1m_2}{r^2}\,dr$

$$W = -\left(\frac{Gm_1m_2}{r^2}\right)_{r'}^{r}$$

$$W = -\frac{Gm_1m_2}{r} + \frac{Gm_1m_2}{r'}$$

$W = U(r) - U(r')$

Where $U(r) = \frac{Gm_1m_2}{r}$

5) This work done W gives the gravitational potential energy difference of the system of masses m_1 and m_2 when the separation between them are r and r′ respectively.

05. Prove that at points near the surface of the Earth, the gravitational potential energy of the object is U = mgh.

1) Consider the Earth and mass system, with *r*, the distance between the mass m and the Earth's centre. Then the gravitational potential energy,

$U = -\frac{GM_em}{r}$ ----------------- 1

2) Here r = Re+h, where Re is the radius of the Earth. h is the height above the Earth's surface, $U = -G\frac{M_em}{(R_e+h)}$ ------------- 2

If h << Re, equation (2) can be modified as

$U = -G\frac{M_em}{R_e\left(1+\frac{h}{R_e}\right)}$; $U = -G\frac{M_em}{R_e}\left(1+\frac{h}{R_e}\right)^{-1}$ ---------- 3

3) By using Binomial expansion and neglecting the higher order terms, we get $U = -G\frac{M_em}{R_e}\left(1-\frac{h}{R_e}\right)$ ---------- 4

We know that, for a mass m on the Earth's surface,

$G\frac{M_em}{R_e} = mgR_e$ --------------- 5

Substituting equation (5) in (4) we get, $U = -mgR_e + mgh$

It is clear that the first term in the above expression is independent of the height h. For example, if the object is taken from h and it can be omitted.

$U = mgh$

06. Explain in detail the idea of weightlessness using lift as an example.

i) When the lift falls (when the lift wire cuts) with downward acceleration a = g, the person inside the elevator is in the state of weightlessness or free fall.

ii) As they fall freely, they are not in contact with any surface (by neglecting air friction). The normal force acting on the object is zero. The downward acceleration is equal to the acceleration due to the gravity of the Earth. i.e (a = g). From equation N = m (g − a) we get, a = g ∴ N = m (g – g) = 0. This is called the state of weightlessness.

07. Derive an expression for escape speed.

1) Consider an object of mass M on the surface of the Earth. When it is thrown up with an initial speed v_i , the initial total energy of the object is

$$E_i = \frac{1}{2} MV_i^2 - \frac{GMM_E}{R_E} \quad \text{----------------} \quad 1$$

Where M_E, is the mass of the Earth and R_E - the radius of the Earth.

The term $-\frac{GMM_E}{R_E}$ is the potential energy of the mass M.

2) When the object reaches a height far away from Earth and hence treated as approaching infinity, the gravitational potential energy becomes zero [U (∞) = 0] and the kinetic energy becomes zero as well. Therefore the final total energy of the object becomes zero. This is for minimum energy and for minimum speed to escape. Otherwise Kinetic energy can be non-zero.

$E_f = 0$, According to the law of energy conservation, $E_i = E_f$ -------- 2

Substituting (1) in (2) we get,

$$\frac{1}{2} MV_i^2 - \frac{GMM_E}{R_E} = 0$$

$$\frac{1}{2} MV_i^2 = \frac{GMM_E}{R_E} \quad \text{-----------------} \quad 3$$

3) The escape speed, the minimum speed required by an object to escape Earth's gravitational field, hence replace, V_i with V_e. i.e,

$$\frac{1}{2} MV_e^2 = \frac{GMM_E}{R_E}$$

$$V_e^2 = \frac{GMM_E}{R_E} \cdot \frac{2}{M} \; ; \; V_e^2 = \frac{2GM_E}{R_E} \quad \text{-------------} \quad 4$$

$$\text{Using } g = \frac{GM_E}{R_e} \quad \text{– – – – – – – – – – – – – – – –} \quad 5$$

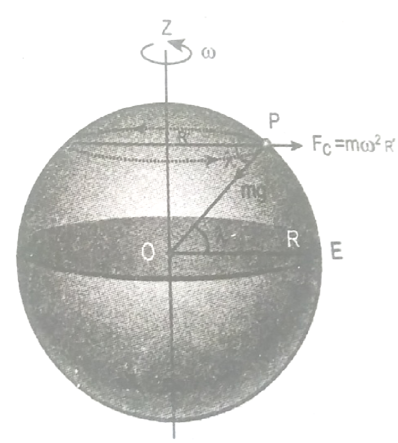
$$V_e^2 = 2gR_E \; ; \; V_e = \sqrt{2gRE} \quad \text{--------------} \quad 6$$

From equation (6) the escape speed depends on two factors: acceleration due to gravity and radius of the Earth. It is completely independent of the mass of the object.

08. Explain the variation of g with latitude.

Variation of g with latitude:

Whenever we analyze the motion of objects in rotating frames, we must take into account the centrifugal force. Even though we treat the Earth as an inertial frame, it is not exactly correct because the Earth spins about its own axis. So when an object is on the surface of the Earth, it experiences a centrifugal force that depends on the latitude of the object on Earth. If the Earth were not spinning, the force on the object would have been *mg*. However, the object experiences an additional centrifugal force due to spinning of the Earth.



This centrifugal force is given by $m\omega^2 R'$

$R' = R\cos\lambda$ ------------ 1

Where λ is the latitude. The component of centrifugal acceleration experienced by the object in the direction opposite to g is $a_c = \omega^2 R' \cos\lambda$

$= \omega^2 R\cos^2\lambda$ since $R' = R\cos\lambda$ Therefore,

$g' = g - \omega^2 R\cos^2\lambda$ ----------- 2

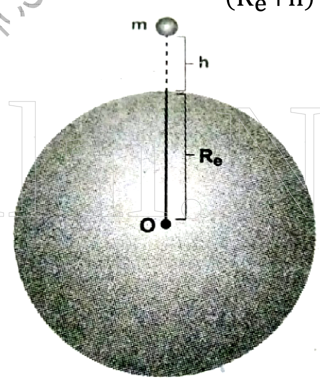
From the expression (2), we can infer that at equator, $\lambda = 0$; $g' = g - \omega^2 R$. The acceleration due to gravity is minimum. At poles λ = 90; $g' = g$, it is maximum. At the equator, g' is minimum.

09. Explain the variation of g with altitude.

Variation of g with altitude:

Consider an object of mass m at a height h from the surface of the Earth. Acceleration experienced by the object due to Earth is $g' = \frac{GM}{(R_e+h)^2}$

$$g' = \frac{GM}{R_e^2\left(1+\frac{h}{R_e}\right)^2} \; ; \; g' = \frac{GM}{R_e^2}\left(1+\frac{h}{R_e}\right)^{-2}$$



If h << Rₑ. We can use Binomial expansion.
Taking the terms upto first order

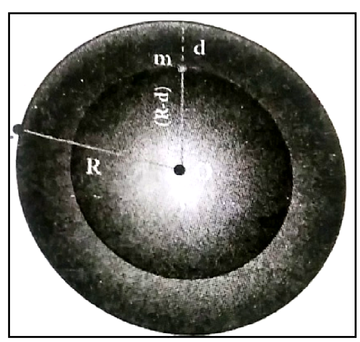
$$g' = \frac{GM}{R_e^2}\left(1 - 2\frac{h}{R_e}\right);$$

$$g' = g\left(1 - 2\frac{h}{R_e}\right)$$

We find that $g' < g$. This means that as altitude h increases the acceleration due to gravity g decreases.

10. Explain the variation of g with depth from the Earth's surface.

Variation of g with depth:



Consider a particle of mass m which is in a deep mine on the earth. Ex. Coal mines – in Neyveli). Assume the depth of the mine as d. To Calculate g at a depth d, consider the following points. The part of the Earth which is above the radius ($R_e - d$) do not contribute to the acceleration. The result is proved earlier and is given as $g' = \frac{GM'}{(R_e-d)^2}$ Here M is the mass of the Earth of radius (R_e – d). Assuming the density of earth ρ to be constant,

$$\rho = \frac{M'}{V'} \; ; \; \frac{M'}{V'} = \frac{M}{V} \text{ and } M' = \frac{M}{V}V'$$

$M' = \left(\frac{M}{\frac{4}{3}\pi R_e^3}\right)\left(\frac{4}{3}\pi(R_e - d)^3\right)$;

$M' = \frac{M}{R_e^3}(R_e - d)^3$

$g' = G\frac{M}{R_e^3}(R_e - d)^3 \cdot \frac{1}{(R_e-d)^2}$;

$g' = GM\,\frac{R_e\left(1-\frac{d}{R_e}\right)}{R_e^3}$

$g' = GM\,\frac{\left(1-\frac{d}{R_e}\right)}{R_e^2}$ thus $g' = g\left(1-\frac{d}{R_e}\right)$. Here also $g' < g$.

As depth increases, g′ decreases.

11. Derive the time period of satellite orbiting the Earth.

Time period of the satellite:

The distance covered by the satellite during one rotation in its orbit is equal to 2π (R_E +h) and time taken for it is the time period, T. Then

$$\frac{\text{Distance travelled}}{\text{Time taken}} = \frac{2\pi\,(\text{RE} + \text{h})}{T}$$

From equation, $\sqrt{\frac{GM_E}{(R_E+h)}} = \frac{2\pi\,(\text{RE}+\text{h})}{T}$ --------------- 1

$T = \frac{2\pi}{\sqrt{GM_E}}(R_E + h)^{\frac{3}{2}}$ ------------------- 2

Squaring both sides of the equation (2), we get $T^2 = \frac{4\pi^2}{GM_E}(R_E + h)^3$

$\frac{4\pi^2}{GM_E}$ = Constant say c, $T^2 = c\,(R_E + h)^3$ ---------------- 3

Equation (3) implies that a satellite orbiting the Earth has the same relation between time and distance as that of Kepler's law of planetary motion. For a satellite orbiting near the surface of the Earth, h is negligible compared to the radius of the Earth R_E . Then, $T^2 = \frac{4\pi^2}{GM_E}R_E^3$; $T^2 = \frac{4\pi^2}{\frac{GM_E}{R_E^2}}$

$T^2 = \frac{4\pi^2}{g}R_E$ Since $\frac{GM_E}{R_E^2} = g$; $T = 2\pi\sqrt{\frac{R_E}{g}}$

12. Derive an expression for energy of satellite.

Energy of an Orbiting Satellite

The total energy of a satellite orbiting the Earth at a distance h from the surface of Earth is calculated as follows; The total energy of the satellite is the sum of its kinetic energy and the gravitational potential energy. The potential energy of the satellite is, $U = \frac{GM_SM_E}{(R_E+h)}$

Here M_S - mass of the satellite, M_E- mass of the Earth, R_E - radius of the Earth.

The Kinetic energy of the satellite is $KE = ½\, M_SV^2$ ------------1

Here v is the orbital speed of the satellite and is equal to $v = \frac{GM_E}{(R_E+h)}$

Substituting the value of v in (1), the kinetic energy of the satellite becomes,

$$KE = \frac{1}{2}\frac{GM_S M_E}{2(R_E+h)}$$

Therefore the total energy of the satellite is $E = \frac{1}{2}\frac{GM_S M_E}{(R_E+h)} - \frac{GM_S M_E}{(R_E+h)}$

$$E = -\frac{GM_S M_E}{2(R_E+h)}$$

The negative sign in the total energy implies that the satellite is bound to the Earth and it cannot escape from the Earth.

13. Explain in detail the geostationary and polar satellites.

Geo-stationary and polar satellite

1) The satellites orbiting the Earth have different time periods corresponding to different orbital radii. Can we calculate the orbital radius of a satellite if its time period is 24 hours is calculated below. Kepler's third law is used to find the radius of the orbit.

$$T^2 = \frac{4\pi^2}{GM_E}(R_E + h)^3 \ ; \ (R_E + h)^3 = \frac{GM_E T^2}{4\pi^2}$$

$$(R_E + h) = \left(\frac{GM_E T^2}{4\pi^2}\right)^{\frac{1}{3}}$$

2) Substituting for the time period (24 hrs = 86400 seconds), mass, and radius of the Earth, h turns out to be 36,000 km. Such satellites are called **"geo-stationary satellites"**, since they appear to be stationary when seen from Earth.

3) geo-stationary satellites for the purpose of telecommunication. Another type of satellite which is placed at a distance of 500 to 800 km from the surface of the Earth orbits the Earth from north to south direction.

4) This type of satellite that orbits Earth from North Pole to South Pole is called a polar satellite. The time period of a polar satellite is nearly 100 minutes and the satellite completes many revolutions in a day.

5) A Polar satellite covers a small strip of area from pole to pole during one revolution. In the next revolution it covers a different strip of area since the Earth would have moved by a small angle. In this way polar satellites cover the entire surface area of the Earth.

14. Explain how geocentric theory is replaced by heliocentric theory using the idea of retrograde motion of planets.

1) To explain this retrograde motion, Ptolemy introduced the concept of "epicycle" in his geocentric model. According to this theory, while the planet orbited the Earth, it also underwent another circular motion termed as "epicycle".

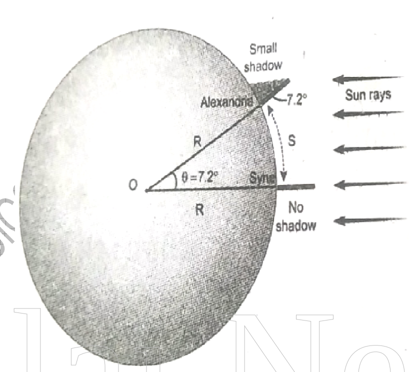
2) A combination of epicycle and circular motion around the Earth gave rise to retrograde motion of the planets with respect to Earth.

3) But Ptolemy's model became more and more complex as every planet was found to undergo retrograde motion. In the 15th century, the Polish astronomer Copernicus proposed.

4) The heliocentric model to explain this problem in a simpler manner. According to this model, the Sun is at the center of the solar system and all planets orbited the Sun.

5) The retrograde motion of planets with respect to Earth is because of the relative motion of the planet with respect to Earth.

15. **Explain in detail the Eratosthenes method of finding the radius of Earth.**



During noon time of summer solstice the Sun's rays cast no shadow in the city Syne which was located 500 miles away from Alexandria. At the same day and same time he found that in Alexandria the Sun's rays made 7.2 degree with local vertical as shown in the Figure. This difference of 7.2 degree was due to the curvature of the Earth.

The angle 7.2 degree is equivalent to $\frac{1}{8}$ radian. So, $\theta = \frac{1}{8}$ rad.

If S is the length of the arc between the cities of Syne and Alexandria, and if R is radius of Earth, then $S = R\,\theta = 500$ miles, so radius of the Earth

$R = \frac{500}{\theta}$ miles , $R = 500 \frac{miles}{\frac{1}{8}}$ $R = 4000$ miles.

1 mile is equal to 1.609 km. So, he measured the radius of the Earth to be equal to R = 6436 km, which is amazingly close to the correct value of 6378 km.

16. **Describe the measurement of Earth's shadow (umbra) radius during total lunar eclipse**

1) It is possible to measure the radius of shadow of the Earth at the point where the Moon crosses.

2) When the Moon is inside the umbra shadow, it appears red in color. As soon as the Moon exits from the umbra shadow, it appears in crescent shape.

3) By finding the apparent radii of the Earth's umbra shadow and the Moon, the ratio of the these radii can be calculated.

4) The apparent radius of Earth's umbra shadow = R_s = 13.2 cm

The apparent radius of the Moon = R_m = 5.15 cm

The ratio $\frac{R_S}{R_m} \approx 2.56$.

The radius of the Earth's umbra shadow is $R_s = 2.56 \times R_m$.

UNIT – 07 PROPERTIES OF MATTER

TWO MARKS AND THREE MARKS:

01. Define stress and strain.

The force per unit area is called as stress. Stress, $\sigma = \frac{Force}{Area} = \frac{F}{A}$

The SI unit of stress is N m^{-2} or Pascal (Pa) and its dimension is $[ML^{-1}T^{-2}]$.

The fractional change in the size of the object, in other words, strain measures the degree of deformation. Strain, $e = \frac{Change\ in\ Size}{Original\ size} = \frac{\Delta l}{l}$

02. State Hooke's law of elasticity.

Hooke's law is for a small deformation, when the stress and strain are proportional to each other.

03. Define Poisson's ratio.

The ratio of relative contraction (lateral strain) to relative expansion (longitudinal strain). It is denoted by the symbol μ.

Poisson s ratio, μ = Lateral strain / Longitudinal strain

04. Explain elasticity using intermolecular forces.

In a solid, inter-atomic forces bind two or more atoms together and the atoms occupy the positions of stable equilibrium. When a deforming force is applied on a body, its atoms are pulled apart or pushed closer. When the deforming force is removed, inter-atomic forces of attraction or repulsion restore the atoms to their equilibrium positions. If a body regains its original shape and size after the removal of deforming force, it is said to be elastic and the property is called elasticity.

05. Which one of these is more elastic, steel or rubber? Why?

Steel is more elastic than rubber because the steel has higher young's modulus than rubber. That's why, if equal stress is applied on both steel and rubber, the steel produces less strain.

06. A spring balance shows wrong readings after using for a long time. Why?

When the spring balances have been used for a long time they develop elastic fatigue in them and therefore the reading shown by such balances will be wrong.

07. What is the effect of temperature on elasticity?

If the temperature of the substance increases, its elasticity decreases.

08. Write down the expression for the elastic potential energy of a stretched wire.

Consider a wire whose un-stretch length is L and area of cross section is A. Let a force produce an extension l and further assume that the elastic limit of the wire has not been exceeded and there is no loss in energy. Then, the work done by the force F is equal to the energy gained by the wire.

The work done in stretching the wire by dl, dW = F dl

The total work done in stretching the wire from 0 to l is

$W = \int_0^l F\,dl$ ------------------1

From Young's modulus of elasticity, $Y = \frac{F}{A} \times \frac{L}{l} \Rightarrow F = \frac{YAl}{L}$ --------- 2

Substituting equation (2) in equation (1), we get

$W = \int_0^l \frac{YAl}{L} dl = \frac{YAl^2}{L.2} = ½ .Fl$

$W = \int \frac{YAl'}{L} dl' = \frac{YA}{L} \frac{l'^2}{2}\Big|_0^l = \frac{YA}{L}\frac{l^2}{2} = \frac{1}{2}\left(\frac{YAl}{L}\right) l = \frac{1}{2} Fl$

$W = \frac{1}{2} Fl$ = Elastic potential energy.

09. State Pascal's law in fluids.

If the pressure in a liquid is changed at a particular point, the change is transmitted to the entire liquid without being diminished in magnitude.

10. State Archimedes principle.

It states that when a body is partially or wholly immersed in a fluid, it experiences an upward thrust equal to the weight of the fluid displaced by it and its up-thrust acts through the centre of gravity of the liquid displaced.

11. What do you mean by up-thrust or buoyancy?

The upward force exerted by a fluid that opposes the weight of an immersed object in a fluid is called up-thrust or buoyant force and the phenomenon is called buoyancy.

12. State the law of floatation.

The law of floatation states that a body will float in a liquid if the weight of the liquid displaced by the immersed part of the body equals the weight of the body.

13. Define coefficient of viscosity of a liquid.

The coefficient of viscosity is defined as the force of viscosity acting between two layers per unit area and unit velocity gradient of the liquid. Its unit is Nsm^{-2} and dimension is $[ML^{-1}T^{-1}]$.

14. Distinguish between streamlined flow and turbulent flow.

Streamlined flow: When a liquid flows such that each particle of the liquid passing through a point moves along the same path with the same velocity as its predecessor then the flow of liquid is said to be a streamlined flow.

The velocity of the particle at any point is constant. It is also referred to as steady or laminar flow.

The actual path taken by the particle of the moving fluid is called a streamline, which is a curve, the tangent to which at any point gives the direction of the flow of the fluid at that point.

Turbulent flow: When the speed of the moving fluid exceeds the critical speed, v_c the motion becomes turbulent.

The velocity changes both in magnitude and direction from particle to particle.

The path taken by the particles in turbulent flow becomes erratic and whirlpool-like circles called eddy current or eddies.

15. What is Reynold's number? Give its significance.

Reynold's number(R_C) is a dimensionless number, which is used to find out the nature of flow of the liquid. $R_C = \frac{\rho v D}{\eta}$

Where, ρ- density of the liquid, v –The velocity of flow of liquid.

D- Diameter of the pipe, η - The coefficient of viscosity of the fluid.

16. Define terminal velocity.

The maximum constant velocity acquired by a body while falling freely through a viscous medium is called the terminal velocity.

17. Write down the expression for the Stoke's force and explain the symbols involved in it.

Viscous force F acting on a spherical body of radius r depends directly on

i) radius (r) of the sphere

ii) velocity (v) of the sphere and

iii) coefficient of viscosity η of the liquid $F = 6\pi\eta r v$

18. State Bernoulli's theorem.

According to Bernoulli's theorem, the sum of pressure energy, kinetic energy, and potential energy per unit mass of an incompressible, non-viscous fluid in a streamlined flow remains a constant.

19. What are the energies possessed by a liquid? Write down their equations.

A liquid in a steady flow can possess three kinds of energy. They are (1) Kinetic energy, (2) Potential energy, and (3) Pressure energy, respectively.

KE = ½ mv^2 ------------ 1

PE = mgh --------------- 2 F x d = w = PV = pressure energy --------------3

20. Two streamlines cannot cross each other. Why?

No two streamlines can cross each other. If they do so, the particles of the liquid at the point of intersection will have two different directions for their flow, which will destroy the steady nature of the liquid flow.

21. Define surface tension of a liquid. Mention its S.I unit and dimension.

The surface tension of a liquid is defined as the energy per unit area of the surface of a liquid. (or) The surface tension of a liquid is defined as the force of tension acting perpendicularly on both sides of an imaginary line of unit length drawn on the free surface of the liquid.

Its unit is N m^{-1} and dimension is $[MT^{-2}]$.

22. How is surface tension related to surface energy?

Consider a rectangular frame of wire ABCD in a soap solution. Let AB be the movable wire. Suppose the frame is dipped in soap solution, soap film is formed which pulls the wire AB inwards due to surface tension. Let F be the force due to surface tension, then F = $(2T)l$

Here, 2 is introduced because it has two free surfaces. Suppose AB is moved by a small distance Δx to new a position A'B'. Since the area increases, some work has to be done against the inward force due to surface tension.

Work Done = Force x distance = $(2T)l\,(\Delta x)$

Increases in area of the film $\Delta A = (2l)\,(\Delta x) = 2l\Delta x$

Therefore, Surface energy = $\frac{\text{Work Done}}{\text{Increase in Surface area}}$

$= \frac{2Tl\Delta x}{2l\Delta x} = T$

Hence, the surface energy per unit area of a surface is numerically equal to the surface tension.

23. Define angle of contact for a given pair of solid and liquid.

The angle between the tangent to the liquid surface at the point of contact and the solid surface is known as the angle of contact.

24. Distinguish between cohesive and adhesive forces.

The force between the like molecules which holds the liquid together is called '*cohesive force*'. When the liquid is in contact with a solid, the molecules of the these solid and liquid will experience an attractive force which is called '*adhesive force*'.

25. What are the factors affecting the surface tension of a liquid?

(1) *The presence of any contamination or impurities* considerably affects the force of surface tension depending upon the degree of contamination.

(2) *The presence of dissolved substances* can also affect the value of surface tension. For example, a highly soluble substance like sodium chloride (NaCl) when dissolved in water (H_20) increases the surface tension of water. But the sparingly soluble substance like phenol or soap solution when mixed in water decreases the surface tension of water.

(3) *Electrification* affects the surface tension. When a liquid is electrified, surface tension decreases. Since external force acts on the liquid surface due to electrification, area of the liquid surface increases which acts against the contraction phenomenon of the surface tension. Hence, it decreases.

(4) *Temperature* plays a very crucial role in altering the surface tension of a liquid. Obviously, the surface tension decreases linearly with the rise of temperature.

26. What happens to the pressure inside a soap bubble when air is blown into it?

Pressure is greater inside the small build.

27. What do you mean by capillarity or capillary action?

The rise or fall of a liquid in a narrow tube is called capillarity or capillary action.

28. A drop of oil placed on the surface of water spreads out. But a drop of water place on oil contracts to a spherical shape. Why?

A drop of oil placed on the surface of water spreads because the force of adhesion between water and oil molecules dominates the cohesive force of oil molecules.

On the other hand, cohesive force of water molecules dominates the adhesive force between water and oil molecules. So drop of water on oil contracts to a spherical shape.

29. State the principle and usage of Venturimeter.

Bernoulli's theorem is the principle of Venturimeter.

Venturimeter is used to measure the rate of flow or flow speed of the incompressible fluid flowing through a pipe.

30. What are the applications of surface tension?

1) Oil pouring on the water reduces surface tension. So that the floating mosquitoes eggs drown and killed.
2) Finely adjusted surface tension of the liquid makes droplets of desired size, which helps in desktop printing, automobile painting and decorative items.
3) Specks of dirt are removed from the cloth when it is washed in detergents added hot water, which has low surface tension.
4) A fabric can be made waterproof, by adding suitable waterproof material (wax) to the fabric. This increases the angle of contact due to surface tension.

31. What physical quantity actually do we check by pressing the tyre after pumping?

After pumping the tyre, we actually check the compressibility of air by pressing the tyre. For smooth riding, rear tyre should have less compressibility than the front.

32. Give some examples for surface tension.

Clinging of painting brush hairs, when taken out of water.
Needle float on the water, Camphor boat.

33. How do water bugs and water striders walk on the surface of water?

When the water bugs or water striders are on the surface of the water, its weight is balanced by the surface tension of the water. Hence, they can easily walk on it.

34. What are the applications of viscosity?

1) Viscosity of liquids helps in choosing the lubricants for various machinery parts. Low viscous lubricants are used in light machinery parts and high viscous lubricants are used in heavy machinery parts.
2) As high viscous liquids damp the motion, they are used in hydraulic brakes as brake oil.
3) Blood circulation through arteries and veins depends upon the viscosity of fluids.
4) Viscosity is used in Millikan's oil-drop method to find the charge of an electron.

35. Explain the Stoke's law application in raindrop falling.

According to Stoke's law, terminal velocity is directly proportional to square of radius of the spherical body. So that smaller raindrops having less terminal velocity float as cloud in air. When they gather as bigger drops get higher terminal velocity and start falling.

36. Define Young's modulus. Give its unit.

Young's modulus is defined as the ratio of tensile or compressive stress to the tensile or compressive strain. Its unit is N m^{-2} or pascal.

37. What are the applications of elasticity?

Elasticity is used in structural engineering in which bridges and buildings are designed such a way that it can withstand load of flowing traffic, the force of winds and even its own weight.

The material of high Young's modulus is used in constructing beams.

38. Define Pressure. Give its unit and dimension.

The pressure is defined as the force acting per unit area. Its unit is N m^{-2} or pascal and dimension is $[ML^{-1}T^{-2}]$.

39. What is elasticity? Give examples.

Elasticity is the property of a body in which it regains its original shape and size after the removal of deforming force. **Ex:** Rubber, metals, steel ropes.

40. What is plasticity? Give an example.

Plasticity is the property of a body in which it does not regains its original shape and size after the removal of deforming force.
Ex: Glass.

CONCEPTUAL QUESTIONS

01. Why coffee runs up into a sugar lump (a small cube of sugar) when one corner of the sugar lump is held in the liquid?

The coffee runs up into the pores of sugar lump due to capillary action of the liquid.

02. Why two holes are made to empty an oil tin?

When oil comes out from a hole of an oil tin, pressure inside it decreased than the atmosphere. Therefore, the surrounding air rush up into the same hole prevents the oil to come out. Hence two holes are made to empty the oil tin.

03. We can cut vegetables easily with a sharp knife as compared to a blunt knife. Why?

Since the stress produced on the vegetables by the sharp knife is higher than the blunt knife, vegetables can be cut easily with the sharp knife.

04. Why the passengers are advised to remove the ink from their pens while going up in an aero-plane?

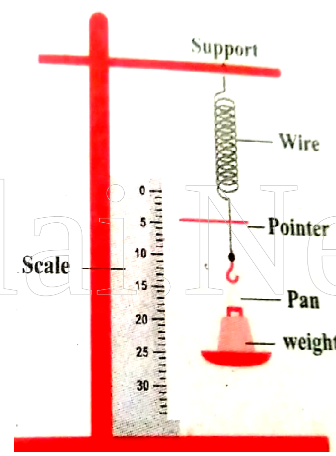
When an aero-plane ascends, the atmospheric pressure is decreased. Hence, the ink from the pen will leak out. So that, the passengers are advised to remove the ink from their pens while going up in the aero-plane.

05. We use straw to suck soft drinks, why?

When we suck the soft drinks through the straw, the pressure inside the straw becomes less than the atmospheric pressure. Due to the difference in pressure, the soft drink rises in the straw and we are able to enjoy it conveniently.

FIVE MARKS

01. State Hooke's law and verify it with the help of an experiment.



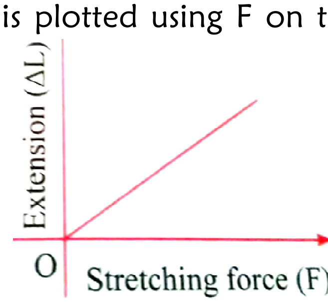
1) Hooke's law is for a small deformation, when the stress and strain are proportional to each other.

2) It can be verified in a simple way by stretching a thin straight wire (stretches like spring) of length L and uniform cross-sectional area A suspended from a fixed point O.

3) A pan and a pointer are attached at the free end of the wire as shown in Figure (a).

4) The extension produced on the wire is measured using a vernier scale arrangement. The experiment shows that for a given load, the corresponding stretching force is F and the elongation produced on the wire is ΔL.

5) It is directly proportional to the original length L and inversely proportional to the area of cross section A. A graph is plotted using F on the X- axis and ΔL on the Y- axis.



6) This graph is a straight line passing through the origin as shown in Figure (b).

Therefore, ΔL = (slope)F

Multiplying and dividing by volume,

V = A L,

F (slope) $= \frac{AL}{AL}\Delta L$

Rearranging, we get, $\frac{F}{A} = \left[\frac{L}{A(slope)}\right]\frac{\Delta L}{L}$ Therefore, , $\frac{F}{A}\ \alpha \left[\frac{\Delta L}{L}\right]$

Comparing with stress equation and strain equation, we get $\sigma\ \alpha\ \varepsilon$

i.e., the stress is proportional to the strain in the elastic limit.

02. Explain the different types of modulus of elasticity.

There are three types of elastic modulus.

(a) Young's modulus, (b) Rigidity modulus (or Shear modulus)

(c) Bulk modulus

Young's modulus:

When a wire is stretched or compressed, then the ratio between tensile stress (or compressive stress) and tensile strain (or compressive strain) is defined as Young's modulus.

$$= \frac{\text{Tensile stress or compressive stress}}{\text{Tensile starin or compressive strain}} \quad Y = \frac{\sigma_t}{\varepsilon_t} \text{ or } Y = \frac{\sigma_c}{\varepsilon_c}$$

The unit for Young modulus has the same unit of stress because, strain has no unit. So, S.I. unit of Young modulus is N m^{-2} or pascal.

Bulk modulus:

Bulk modulus is defined as the ratio of volume stress to the volume strain.

$$\text{Bulk modulus, } K = \frac{\text{Normal (Perpendicular)stress or pressure}}{\text{Volume strain}}$$

The normal stress or pressure is $\sigma_n = \frac{F_n}{\Delta A} = \Delta p$

The volume strain is $\varepsilon_v = \frac{\Delta V}{V}$

Therefore, Bulk modulus is $K = -\frac{\sigma_n}{\varepsilon_v} = -\frac{\Delta p}{\frac{\Delta V}{V}}$

The negative sign in the equation means that when pressure is applied on the body, its volume decreases. Further, the equation implies that a material can be easily compressed if it has a small value of bulk modulus.

The rigidity modulus or shear modulus:

The rigidity modulus is defined as Rigidity modulus or Shear modulus,

$$\eta_R = \frac{\text{Shearing stress}}{\text{Angle of shear or shearing strain}}$$

The shearing stress is $\sigma_s = \frac{\text{Trangential force}}{\text{Area over which it is applied}} = \frac{F_t}{\Delta A}$

The angle of shear or shearing strain $\varepsilon_s = \frac{x}{h} = \theta$

Therefore, Rigidity modulus is $\eta = \frac{\sigma_s}{\varepsilon_s} = \frac{\frac{F_t}{\Delta A}}{\frac{x}{h}} = \frac{\frac{F_t}{\Delta A}}{\theta}$

Further, the equation (7.9) implies, that a material can be easily twisted if it has small value of rigidity modulus. For example, consider a wire, when it is twisted through an angle θ, a restoring torque is developed, that is

$$\tau \propto \theta$$

This means that for a larger torque, wire will twist by a larger amount (angle of shear θ is large). Since, rigidity modulus is inversely proportional to angle of shear, the modulus of rigidity is small.

03. Derive an expression for the elastic energy stored per unit volume of a wire.

When a body is stretched, work is done against the restoring force (internal force). This work done is stored in the body in the form of elastic energy. Consider a wire whose un-stretch length is L and area of cross section is A. Let a force produce an extension *l* and further assume that the elastic limit of the wire has not been exceeded and there is no loss in energy. Then, the work done by the force F is equal to the energy gained by the wire.

The work done in stretching the wire by dl, dW = F dl

The total work done in stretching the wire from 0 to l is

$W = \int_0^l F\, dl$ ------------------1

From Young's modulus of elasticity, $Y = \frac{F}{A} \times \frac{L}{l} \Rightarrow F = \frac{YAl}{L}$ --------- 2

Substituting equation (2) in equation (1), we get

$W = \int_0^l \frac{YAl}{L} dl = \frac{YAl^2}{L.2} = ½ .Fl$

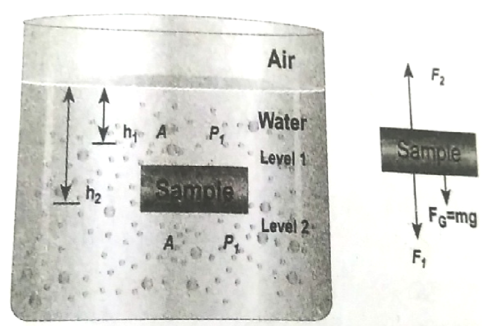
$W = \int \frac{YAl'}{L} dl' = \frac{YA}{L}\frac{l'^2}{2}\Big|_0^l = \frac{YA}{L}\frac{l^2}{2} = \frac{1}{2}\left(\frac{YAl}{L}\right) l = \frac{1}{2} Fl$

$W = \frac{1}{2} Fl$ = Elastic potential energy.

Energy per unit volume is called energy density,

$u = \frac{\text{Elastic potential energy}}{\text{Volume}} = \frac{\frac{1}{2}Fl}{AL} \quad \frac{1}{2}\frac{F}{A}\frac{l}{L} = \frac{1}{2}$ (Stress x Strain)

04. Derive an equation for the total pressure at a depth 'h' below the liquid surface.



Consider a water sample of cross sectional area in the form of a cylinder. Let h_1 and h_2 be the depths from the air-water interface to level 1 and level 2 of the cylinder, respectively as shown in Figure (a). Let F1 be the force acting downwards on level 1 and F_2 be the force acting upwards on level 2, such that, $F_1 = P_1 A$ and $F_2 = P_2 A$ Let us assume the mass of the sample to be *m* and under equilibrium condition, the total upward force (F_2) is balanced by the total downward force (F_1 + mg), in other words, the gravitational force will act downward which is being exactly balanced by the difference between the force. $F_2 - F_1$

$F_2 - F_1 = mg = F_G$

Where *m* is the mass of the water available in the sample element. Let ρ be the density of the water then, the mass of water available in the sample element is $m = \rho V = \rho A (h_2 - h_1)$ $V = A (h_2 - h_1)$

Hence, gravitational force,

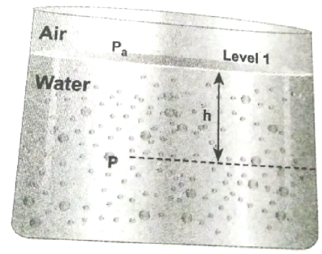
$F_G = \rho A (h_2 - h_1) g$

On substituting the value of W in equation

$$F_2=F_1+m\,g \Rightarrow P_2A = P_1A+ \rho A(h_2 - h_1)g$$

Cancelling out A on both sides, $P_2 = P_1 + \rho(h_2 - h_1)g$

If we choose the level 1 at the surface of the liquid (i.e., air-water interface) and the level 2 at a depth 'h' below the surface (as shown in Figure (b), then the value of h1 becomes zero ($h_1 = 0$) and in turn P1 assumes the value of atmospheric pressure (say Pa). In addition, the pressure (P_2) at a depth becomes P. Substituting these values in equation, we get



$$P = P_a + \rho gh$$

Which means, the pressure at a depth h is greater than the pressure on the surface of the liquid, where Pa is the atmospheric pressure which is equal to 1.013×10^5 Pa. If the atmospheric pressure is neglected or ignored then

$$P = \rho gh$$

05. State and prove Pascal's law in fluids.

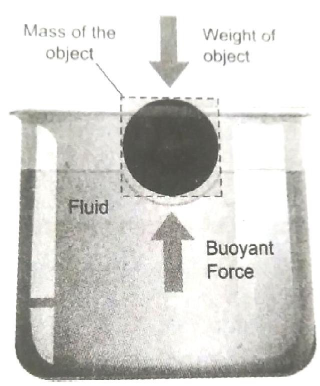
Hydraulic lift which is used to lift a heavy load with a small force. It is a force multiplier. It consists of two cylinders A and B connected to each other by a horizontal pipe, filled with a liquid (Figure). They are fitted with frictionless pistons of cross sectional areas A_1 and A_2 ($A_2 > A_1$). Suppose a downward force F is applied on the smaller piston, the pressure of the liquid under this piston increases to $P\left(\text{where}, P = \frac{F_1}{A_1}\right)$. But according to Pascal's law, this increased pressure P is transmitted undiminished in all directions. So a pressure is exerted on piston B. Upward force on piston B is

$$F_2 = P \times A_2 = \frac{F_1}{A_1} \times A_2 \Rightarrow F_2 = \frac{A_2}{A_1} \times F_1$$

Therefore by changing the force on the smaller piston A, the force on the piston B has been increased by the factor $\frac{A_2}{A_1}$ and this factor is called the mechanical advantage of the lift.

06. State and prove Archimedes principle.

It states that when a body is partially or wholly immersed in a fluid, it experiences an upward thrust equal to the weight of the fluid displaced by it and its up-thrust acts through the centre of gravity of the liquid displaced.



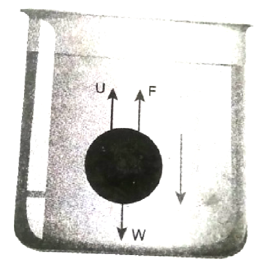
Up-thrust or buoyant force = weight of liquid displaced.

07. Derive the expression for the terminal velocity of a sphere moving in a high viscous fluid using stokes force.

Expression for terminal velocity:

Consider a sphere of radius *r* which falls freely through a highly viscous liquid of coefficient of viscosity η. Let the density of the material of the sphere be ρ and the density of the fluid be σ.

Gravitational force acting on the sphere, F_G = mg = $\frac{4}{3}\pi r^3 \rho g$ (downward force)



Up thrust, U = $\frac{4}{3}\pi r^3 \sigma g$ (upward force)

Viscous force F = $6\pi\eta r v_t$

At terminal velocity v_t, downward force = upward force

$F_G - U = F \Rightarrow \frac{4}{3}\pi r^3 \rho g - = \frac{4}{3}\pi r^3 \sigma g = 6\pi\eta r v_t$

$V_t = \frac{2}{9} \times \frac{r^2(\rho-\sigma)}{\eta} g \Rightarrow V_t \infty r^2$

Here, it should be noted that the terminal speed of the sphere is directly proportional to the square of its radius. If σ is greater than ρ, then the term (ρ - σ) becomes negative leading to a negative terminal velocity.

08. Derive Poiseuille's formula for the volume of a liquid flowing per second through a pipe under streamlined flow.

Consider a liquid flowing steadily through a horizontal capillary tube. Let $v = \left(\frac{V}{t}\right)$ be the volume of the liquid flowing out per second through a capillary tube. It depends on (1) coefficient of viscosity (η) of the liquid, (2) radius of the tube (r), and (3) the pressure gradient$\left(\frac{P}{l}\right)$. Then, $v \alpha \eta^a r^b \left(\frac{P}{l}\right)^c$;

$v = k\, \eta^a r^b \left(\frac{P}{l}\right)^c$ where, k is a dimensionless constant. Therefore,

$[v] = \frac{\text{Volume}}{\text{time}} = [L^3T^{-1}]$, $\left[\frac{dP}{dx}\right] = \frac{\text{Pressure}}{\text{distance}} = [ML^{-2}T^{-2}]$,

$[\eta] = [ML^{-1}T^{-1}]$ and $[r] = [L]$

Substituting in equation, So, equating the powers of M, L, and T on both sides, we get a + c = 0, −a + b −2c =3, and −a −2c = −1

We have three unknowns *a*, *b*, and *c.* We have three equations, on solving, we get a = −1, b = 4, and c = 1

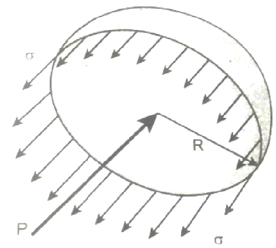
Therefore, equation becomes, $v = k\eta^{-1}r^4\left(\frac{P}{l}\right)^1$

Experimentally, the value of *k* is shown to be $\frac{\pi}{8}$, we have $v = \frac{\pi r^4 P}{8\eta l}$

09. Obtain an expression for the excess of pressure inside a i) liquid drop ii) liquid bubble iii) air bubble.

i) Excess of pressure inside air bubble in a liquid.

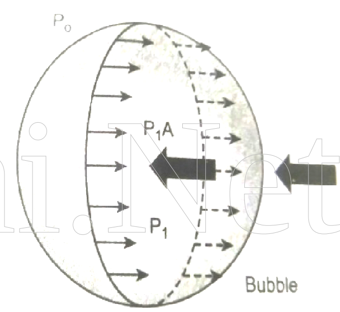
Consider an air bubble of radius R inside a liquid having surface tension T as shown in Figure (a). Let P_1 and P_2 be the pressures outside and inside the air bubble, respectively. Now, the excess pressure inside the air bubble is



$\Delta P = P_1 - P_2$. To find the excess pressure inside the air bubble, let us consider the forces acting on the air bubble.

ii) Excess pressure inside a soap bubble.

Consider a soap bubble of radius R and the surface tension of the soap bubble be T as shown in Figure (b). A soap bubble has two liquid surfaces in contact with air, one inside the bubble and other outside the bubble. Therefore, the force on the soap bubble due to surface tension is $2\times2\pi RT$. The various forces acting on the soap bubble are,



i) Force due to surface tension $F_T = 4\pi RT$ towards right

ii) Force due to outside pressure $F_{P1} = P_1\pi R^2$ towards right

iii) Force due to inside pressure $F_{P2} = P_2\pi R^2$ towards left

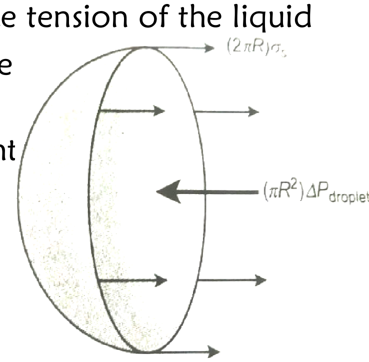
As the bubble is in equilibrium, $F_{P2} = F_T + F_{P1}$

$P_2\pi R^2 = 4\pi RT + P_1\pi R^2 \Rightarrow (P_2 - P_1)\pi R^2 = 4\pi RT$

Excess pressure is $\Delta P = P_2 - P_1 = \frac{4T}{R}$

iii) Excess pressure inside the liquid drop

Consider a liquid drop of radius R and the surface tension of the liquid is T as shown in Figure. The various forces acting on the



i) Force due to surface tension $F_T = 2\pi RT$ towards right

ii) Force due to outside pressure $F_{P1} = P_1\pi R^2$ towards right

iii) Force due to inside pressure $F_{P2} = P_2\pi R^2$ towards left

As the liquid drop is in equilibrium, $F_{P2} = F_T + F_{P1}$

$P_2\pi R^2 = 2\pi RT + P_1\pi R^2 \Rightarrow (P_2 - P_1)\pi R^2 = 2\pi RT$

Excess pressure is $\Delta P = P_2 - P_1 = \frac{2T}{R}$

10. What is capillarity? Obtain an expression for the surface tension of a liquid by capillary rise method.

Consider a capillary tube which is held vertically in a beaker containing water; the water rises in the capillary tube to a height h due to surface tension.

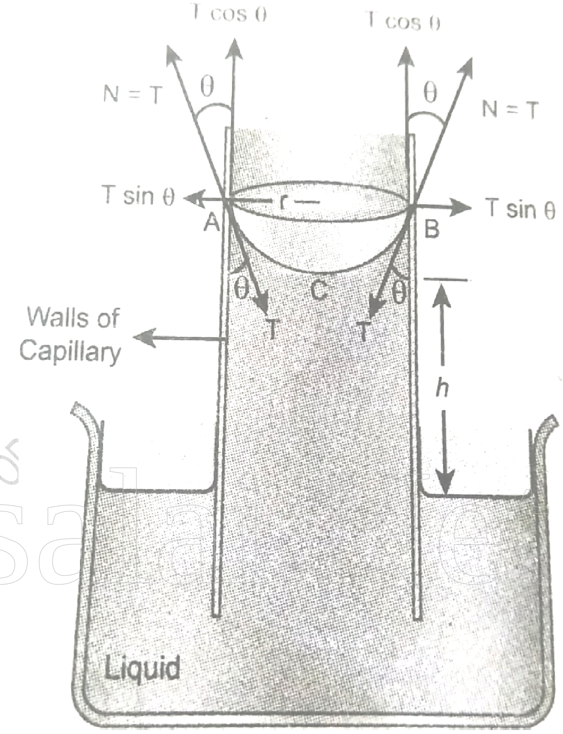
The surface tension force F_T, acts along the tangent at the point of contact downwards and its reaction force upwards. Surface tension T, is resolved into two components i) Horizontal component T sinθ and ii) Vertical component T cosθ acting upwards, all along the whole circumference of the meniscus.

Total upward force = $(T\cos\theta)(2\pi r) = 2\pi r T\cos\theta$

Where θ is the angle of contact, r is the radius of the tube. Let ρ be the density of water and h be the height to which the liquid rises inside the tube.

Then, $\begin{pmatrix}\text{the volume of}\\ \text{liquid column in}\\ \text{the tube, V}\end{pmatrix} = \begin{pmatrix}\text{Volume of the liquid}\\ \text{column of radius r}\\ \text{height h}\end{pmatrix} + \begin{pmatrix}\text{Volume of liquid of}\\ \text{radius r and height}\\ \text{r} - \text{Volume of the}\\ \text{hemisphere of radius r}\end{pmatrix}$

$V = \pi r^2 h + \left(\pi r^2 \text{x}\, r - \frac{2}{3}\pi r^3\right) \Rightarrow \pi r^2 h + \frac{1}{3}\pi r^3$



The upward force supports the weight of the liquid column above the free surface, therefore, $2\pi r T\cos\theta = \pi r^2\left(h + \frac{1}{3}r\right)\rho g \Rightarrow$

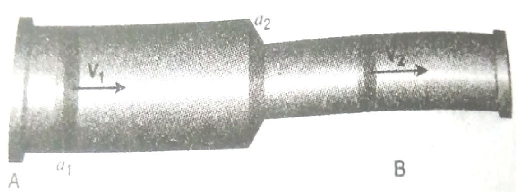
$T = \frac{r\left(h+\frac{1}{3}r\right)\rho g}{2\cos\theta}$

If the capillary is a very fine tube of radius (i.e., radius is very small) then $\frac{r}{3}$ can be neglected when it is compared to the height h. Therefore,

$T = \frac{r\rho g h}{2\cos\theta}$

11. Obtain an equation of continuity for a flow of fluid on the basis of conservation of mass.

Consider a pipe AB of varying cross sectional area a_1 and a_2 such that $a_1 > a_2$. A non-viscous and incompressible liquid flows steadily through the pipe, with velocities v_1 and v_2 in area a_1 and a_2, respectively as shown in Figure.



Let m_1 be the mass of fluid flowing through section A in time Δt, $m_1 = (a_1 v_1 \Delta t)\,\rho$

Let m_2 be the mass of fluid flowing through section B in time Δt, $m_2 = (a_2 v_2 \Delta t)\,\rho$

For an incompressible liquid, mass is conserved $m_1 = m_2$

$a_1 v_1 \Delta t\,\rho = a_2 v_2 \Delta t\,\rho$

$a_1 v_1 = a_2 v_2 \Rightarrow a\,v = \text{constant}$

which is called the equation of continuity and it is a statement of conservation of mass in the flow of fluids.

In general, a v = constant, which means that the volume flux or flow rate remains constant throughout the pipe. In other words, the smaller the cross section, greater will be the velocity of the fluid.

12. State and prove Bernoulli's theorem for a flow of incompressible, non-viscous, and streamlined flow of fluid.

Bernoulli's theorem :

According to Bernoulli's theorem, the sum of pressure energy, kinetic energy, and potential energy per unit mass of an incompressible, non-viscous fluid in a streamlined flow remains a constant.

$\frac{P}{\rho}+\frac{1}{2}v^2+gh=$ Constant, this is known as Bernoulli's equation.

Proof:

Let us consider a flow of liquid through a pipe AB as shown in Figure. Let V be the volume of the liquid when it enters *A* in a time *t* which is equal to the volume of the liquid leaving *B* in the same time. Let a_A, v_A and P_A be the area of cross section of the tube, velocity of the liquid and pressure exerted by the liquid at *A* respectively.

Let the force exerted by the liquid at *A* is $F_A = P_A a_A$

Distance travelled by the liquid in time t is $d = v_A t$

Therefore, the work done is $W = F_A d = P_A a_A v_A t$

But $a_A v_A t = a_A d = V$, volume of the liquid entering at A.

Thus, the work done is the pressure energy (at A), $W = F_A d = P_A V$

Pressure energy per unit volume at A $= \frac{\text{Pressure energy}}{\text{Volume}} = \frac{P_A V}{V} = P_A$

Pressure energy per unit mass at A $= \frac{\text{Pressure energy}}{\text{Mass}} = \frac{P_A V}{m} = \frac{P_A}{\frac{m}{V}} = \frac{P_A}{\rho}$

Since m is the mass of the liquid entering at A in a given time, therefore, pressure energy of the liquid at A is $E_{PA} = P_A V = P_A V \times \left(\frac{m}{m}\right) = m\frac{P_A}{\rho}$

Potential energy of the liquid at A, $P_{EA} = mg\,h_A$,

Due to the flow of liquid, the kinetic energy of the liquid at A,

$KE_A = \frac{1}{2} mV_A^2$

Therefore, the total energy due to the flow of liquid at A,

$E_A = EP_A + KE_A + PE_A$

$E_A = m\frac{P_A}{\rho} + \frac{1}{2} mV_A^2 + mgh_A$

Similarly, let a_B, v_B, and P_B be the area of cross section of the tube, velocity of the liquid, and pressure exerted by the liquid at B. Calculating the total energy at E_B, we get $E_B = m\frac{P_B}{\rho} + ½\, mV_B^2 + mgh_B$

From the law of conservation of energy, $E_A = E_B$

$E_A = m\frac{P_A}{\rho} + ½\, mV_A^2 + mgh_A = E_B = m\frac{P_B}{\rho} + ½\, mV_B^2 + mgh_B$

$\frac{P_A}{\rho} + ½\, V_A^2 + gh_A = \frac{P_B}{\rho} + ½\, V_B^2 + gh_B$ = constant

Thus, the above equation can be written as $\frac{P}{\rho g} + \frac{1}{2}\frac{v^2}{g} + h$ = constant

13. Describe the construction and working of venturimeter and obtain an equation for the volume of liquid flowing per second through a wider entry of the tube.

Venturimeter

This device is used to measure the rate of flow (or say flow speed) of the incompressible fluid flowing through a pipe. It works on the principle of Bernoulli's theorem.

Let P_1 be the pressure of the fluid at the wider region of the tube A. Let us assume that the fluid of density 'ρ' flows from the pipe with speed 'v_1' and into the narrow region, its speed increases to 'v_2'. According to the Bernoulli's equation, this increase in speed is accompanied by a decrease in the fluid pressure P_2 at the narrow region of the tube B. Therefore, the pressure difference between the tubes A and B is noted by measuring the height difference ($\Delta P = P_1 - P_2$) between the surfaces of the manometer liquid.

From the equation of continuity, we can say that

$Av_1 = a\, v_2$ which means that $V_2 = \frac{A}{a} v_1$

Using Bernoulli's equation, $P_1 + \rho\frac{V1^2}{2} = P_2 + \rho\frac{V2^2}{2} = P_2 + \rho\frac{1}{2}\left(\frac{A}{a}v_1\right)^2$

From the above equation, the pressure difference,

$\Delta P = P_1 - P_2 = \rho\frac{V1^2}{2}\left(\frac{A^2-a^2}{a^2}\right)$

Thus, the speed of flow of fluid at the wide end of the tube A

$v_1^2 = \frac{2(\Delta P)a^2}{\rho(A^2-a^2)} \Rightarrow V_1 = \sqrt{\frac{2(\Delta P)a^2}{\rho(A^2-a^2)}}$

The volume of the liquid fl owing out per second is

$AV_1 = \sqrt{\frac{2(\Delta P)a^2}{\rho(A^2-a^2)}}$

$= aA\sqrt{\frac{2(\Delta P)}{\rho(A^2-a^2)}}$

14. Write any two applications of Bernoulli's theorem.

(a) Blowing off roofs during wind storm

1) In olden days, the roofs of the huts or houses were designed with a slope. One important scientific reason is that as per the Bernoulli's principle, it will be safeguarded except roof during storm or cyclone.

2) During cyclonic condition, the roof is blown off without damaging the other parts of the house.

3) In accordance with the Bernoulli's principle, the high wind blowing over the roof creates a low-pressure P_1.

4) The pressure under the roof P_2 is greater. Therefore, this pressure difference $(P_2 – P_1)$ creates an up thrust and the roof is blown off.

(b) Aerofoil lift

1) The wings of an airplane (aerofoil) are so designed that its upper surface is more curved than the lower surface and the front edge is broader than the real edge.

2) As the aircraft moves, the air moves faster above the aerofoil than at the bottom.

3) According to Bernoulli's Principle, the pressure of air below is greater than above, which creates an up-thrust called the dynamic lift to the aircraft.

15. Write the applications of elasticity.

1) The elastic behavior is one such property which especially decides the structural design of the columns and beams of a building.

2) As far as the structural engineering is concerned, the amount of stress that the design could withstand is a primary safety factor.

3) A bridge has to be designed in such a way that it should have the capacity to withstand the load of the flowing traffic, the force of winds, and even its own weight.

4) The elastic behavior or in other words the bending of beams is a major concern over the stability of the buildings or bridges.

5) To reduce the bending of a beam for a given load, one should use the material with a higher Young's modulus of elasticity (Y).

6) The Young's modulus of steel is greater than aluminium or copper. Iron comes next to steel.

7)This is the reason why steel is mostly preferred in the design of heavy duty machines and iron rods in the construction of buildings.

UNIT – 08 HEAT AND THERMODYNAMICS

TWO MARKS AND THREE MARKS:

01. 'An object contains more heat'- is it a right statement? If not why?

Heat is not a quantity. Heat is energy in transit which flows from higher temperature object to lower temperature object. Once the heating process is stopped we cannot use the word heat. When we use the word 'heat', it is the energy in transit but not energy stored in the body. An Object has more heat is wrong, instead object is hot will be appropriate.

02. Obtain an ideal gas law from Boyle's and Charles' law.

1) Acceleration to Boyle's law P $\alpha \frac{1}{V}$

2) Acceleration to Charle's law V α T. By combining these two equations we have PV = CT. Here C is a positive constant.

3) So we can write the constant C as k times the number of particles N. Here k is the Boltzmann constant (1.381×10^{-23} JK^{-1}) and it is found to be a universal constant. So the ideal gas law can be stated as follows PV = NkT

03. Define one mole.

One mole of any substance is the amount of that substance which contains Avogadro number (N_A) of particles (such as atoms or molecules).

04. Define specific heat capacity and give its unit.

Specific heat capacity of a substance is defined as the amount of heat energy required to raise the temperature of 1kg of a substance by 1 Kelvin or 1°C

ΔQ = ms ΔT

Therefore, $s = \frac{1}{m}\frac{\Delta Q}{\Delta T}$

Where s – Specific heat capacity of a substance and its value depends only on the nature of the substance not amount of substance.

ΔQ - Amount of heat energy ; ΔT - Change in temperature ;
m – Mass of the substance ; The SI unit for specific heat capacity is $Jkg^{-1}K^{-1}$

05. Define molar specific heat capacity.

Molar specific heat capacity is defined as heat energy required to increase the temperature of one mole of substance by 1K or 1°C. $C = \frac{1}{\mu}\frac{\Delta Q}{\Delta T}$

Here C is known as molar specific heat capacity of a substance and μ is number of moles in the substance.

The SI unit for molar specific heat capacity is J mol^{-1} K^{-1} .

06. What is a thermal expansion?

Thermal expansion is the tendency of matter to change in shape, area, and volume due to a change in temperature.
All three states of matter (solid, liquid and gas) expand when heated. When a solid is heated, its atoms vibrate with higher amplitude about their fixed points. The relative change in the size of solids is small.

07. Give the expressions for linear, area and volume thermal expansions.

Linear Expansion:

$\alpha_L = \frac{\Delta L}{L\Delta T}$; Where, αL = coefficient of linear expansion.

ΔL = Change in length; L = Original length ; ΔT = Change in temperature.

Area Expansion:

$\alpha_A = \frac{\Delta A}{A\Delta T}$; Where, αA = coefficient of area expansion.

ΔA = Change in area; A = Original area ; ΔT = Change in temperature

Volume Expansion:

$\alpha_V = \frac{\Delta V}{V\Delta T}$ Where, αV = coefficient of volume expansion;

ΔV = Change in volume; V = Original volume ; ΔT = Change in temperature. Unit of coefficient of linear, area and volumetric expansion of solids is $°C^{-1}$ or K^{-1}

08. Define latent heat capacity. Give its unit.

Latent heat capacity of a substance is defined as the amount of heat energy required to change the state of a unit mass of the material.

Q = m x L ; $L = \frac{Q}{m}$

Where L = Latent heat capacity of the substance; Q = Amount of heat; m = mass of the substance. The SI unit for Latent heat capacity is $J\ kg^{-1}$.

09. State Stefan-Boltzmann law.

Stefan Boltzmann law states that, the total amount of heat radiated per second per unit area of a black body is directly proportional to the fourth power of its absolute temperature.

$E \alpha T^4$ or $E = \sigma T^4$; Where, σ is known as Stefan's constant. Its value is $5.67 \times 10^{-8}\ W\ m^{-2}\ k^{-4}$

10. What is Wien's law?

Wien's law states that, the wavelength of maximum intensity of emission of a black body radiation is inversely proportional to the absolute temperature of the black body. $\lambda_m \,\alpha\, \frac{1}{T}$ or $\lambda_m = \frac{b}{T}$. Where, b is known as Wien's constant. Its value is 2.898×10^{-3} m K

11. Define thermal conductivity. Give its unit.

The quantity of heat transferred through a unit length of a material in a direction normal to unit surface area due to a unit temperature difference under steady state conditions is known as thermal conductivity of a material.

$\frac{Q}{L} = \frac{KA\Delta T}{L}$; Where, K is known as the coefficient of thermal conductivity.

The SI unit of thermal conductivity is J s^{-1} m^{-1} K^{-1} or W m^{-1} K^{-1}.

12. What is a black body?

A black body is an object that absorbs all electromagnetic radiations. It is a perfect absorber and radiator of energy with no reflecting power.

13. What is a thermodynamic system? Give examples.

Thermodynamic system:

A thermodynamic system is a finite part of the universe. It is a collection of large number of particles (atoms and molecules) specified by certain parameters called pressure (P), Volume (V) and Temperature (T). The remaining part of the universe is called surrounding. Both are separated by a boundary.

Examples: A thermodynamic system can be liquid, solid, gas and radiation. Bucket of water, Air molecules in the room, Human body, Fish in the sea.

14. What are the different types of thermodynamic systems?

Open system can exchange both matter and energy with the environment.
Closed system exchange energy, but not matter with the environment.
Isolated system can exchange neither energy nor matter with the environment.

15. What is meant by 'thermal equilibrium'?

Two systems are said to be in thermal equilibrium with each other if they are at the same temperature, which will not change with time.

16. What is mean by state variable? Give example.

In thermodynamics, the state of a thermodynamic system is represented by a set of variables called thermodynamic variables.

Examples: Pressure, temperature, volume and internal energy etc.

The values of these variables completely describe the equilibrium state of a thermodynamic system.

17. What are intensive and extensive variables? Give examples.

Extensive variable depends on the size or mass of the system.
Example: Volume, total mass, entropy, internal energy, heat capacity etc.
Intensive variables do not depend on the size or mass of the system.
Example: Temperature, pressure, specific heat capacity, density etc.

18. What is an equation of state? Give an example.

Equation of state:

The equation which connects the state variables in a specific manner is called equation of state. A thermodynamic equilibrium is completely specified by these state variables by the equation of state. If the system is not in thermodynamic equilibrium then these equations cannot specify the state of the system.

Example of equation of state called vander Waals equation. Real gases obey this equation at thermodynamic equilibrium. The air molecules in the room truly obey vander Waals equation of state. But at room temperature with low density we can approximate it into an ideal gas.

19. State Zeroth law of thermodynamics.

The zeroth law of thermodynamics states that if two systems, A and B, are in thermal equilibrium with a third system, C, then A and B are in thermal equilibrium with each other.

20. Define the internal energy of the system.

The internal energy of a thermodynamic system is the sum of kinetic and potential energies of all the molecules of the system with respect to the center of mass of the system.

The energy due to molecular motion including translational, rotational and vibrational motion is called internal kinetic energy (E_K) The energy due to molecular interaction is called internal potential energy (E_P).

Example: Bond energy. $U = E_K + E_P$

21. Are internal energy and heat energy the same? Explain.

No, but they are related. If heat energy is added to substance, its internal energy will increase. Internal energy is a means are of the amount of kinetic and potential energy possessed by particles in a substation.

Heat energy concerns only transfer of internal energy from the hotter to a colder body.

22. Define one calorie.

The amount of heat required at a pressure of standard atmosphere to raise the temperature of 1g of water 1^0C.

23. Did joule converted mechanical energy to heat energy? Explain.

1) Yes, In his experiment, two masses were attached with a rope and a paddle wheel. When these masses fall through a distance h due to gravity, both the masses lose potential energy equal to 2mgh.

2) When the masses fall, the paddle wheel turns. Due to the turning of wheel inside water, frictional force comes in between the water and the paddle wheel.

3) This causes a rise in temperature of the water. This implies that gravitational potential energy is converted to internal energy of water.

4) The temperature of water increases due to the work done by the masses. In fact, Joule was able to show that the mechanical work has the same effect as giving heat.

24. State the first law of thermodynamics.

Change in internal energy (ΔU) of the system is equal to heat supplied to the system (Q) minus the work done by the system (W) on the surroundings.

25. Can we measure the temperature of the object by touching it?

1) No, When you stand bare feet with one foot on the carpet and the other on the tiled floor, your foot on tiled floor feels cooler than the foot on the carpet even though both the tiled floor and carpet are at the same room temperature.

2) It is because the tiled floor transfers the heat energy to your skin at higher rate than the carpet. So the skin is not measuring the actual temperature of the object; instead it measures the rate of heat energy transfer.

3) But if we place a thermometer on the tiled floor or carpet it will show the same temperature.

26. Give the sign convention for Q and W.

System gains heat	-	Q is positive
System loses heat	-	Q is negative
Work done on the system	-	W is negative
Work done by the system	-	W is positive

27. Define the quasi-static process.

A quasi-static process is an infinitely slow process in which the system changes its variables (P,V,T) so slowly such that it remains in thermal, mechanical and chemical equilibrium with its surroundings throughout. By this infinite slow variation, the system is always almost close to equilibrium state.

28. Give the expression for work done by the gas.

In general the work done by the gas by increasing the volume from V_i to V_f is given by $W = \int_{V_f}^{V_i} PdV$

29. What is PV diagram?

PV diagram is a graph between pressure P and volume V of the system. The P-V diagram is used to calculate the amount of work done by the gas during expansion or on the gas during compression.

30. Explain why the specific heat capacity at constant pressure is greater than the specific heat capacity at constant volume.

Because when heat is added at constant pressure the substance, expands and work. i.e. more amount of energy has to be supplied to a constant pressure to increase the system's temperature by the same amount. Some of this energy is lost due to expansion work done by the system.

31. Give the equation of state for an isothermal process.

The equation of state for isothermal process is given by PV = Constant

32. Give an expression for work done in an isothermal process.

$$W = \mu RT \text{ in } \left(\frac{V_f}{V_i}\right)$$

33. Express the change in internal energy in terms of molar specific heat capacity.

If Q is the heat supplied to mole of a gas at constant volume and if the temperature changes by an amount ΔT , we have $Q = \mu C_v \Delta T$ -------------1

By applying the first law of thermodynamics for this constant volume process (W=0, since dV=0), we have $Q = \Delta U - 0$ ------------2

By comparing the equations (1) and (2), $\Delta U = \mu C_v \Delta T$ or $C_v = \frac{1}{\mu}\frac{\Delta U}{\Delta T}$

If the limit ΔT goes to zero, we can write $C_v = \frac{1}{\mu}\frac{dU}{dT}$

Since the temperature and internal energy are state variables, the above relation holds true for any process.

34. Apply first law for (a) an isothermal (b) adiabatic (c) isobaric processes.

Isothermal : Q = W ; Q – Heat ; W – Wire
Adiabatic : ΔU = W Change internal Energy; Isobaric: ΔU = Q – PΔU

35. Give the equation of state for an adiabatic process.

The equation of state for an adiabatic process is given by PV^{γ} = Constant. Here γ is called adiabatic exponent $\left(\gamma = \frac{C_p}{C_v}\right)$ which depends on the nature of the gas. The equation implies that if the gas goes from an equilibrium state (P_i,V_i) to another equilibrium state (P_f ,V_f) adiabatically then it satisfies the relation.

36. Give an equation state for an isochoric process.

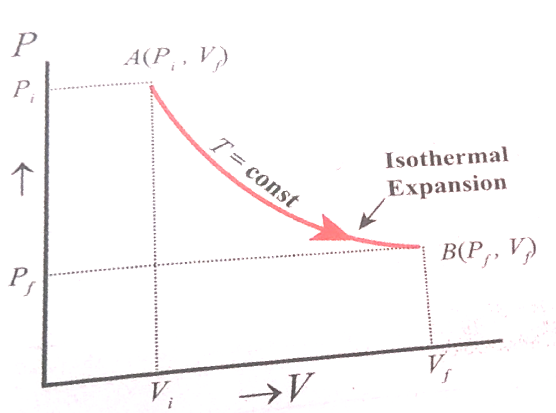
The equation of state for an isochoric process is given by P= $\left(\frac{\mu R}{V}\right)$T,
Where, $\left(\frac{\mu R}{V}\right)$ = Constant

37. If the piston of a container is pushed fast inward. Will the ideal gas equation be valid in the intermediate stage? If not, why?

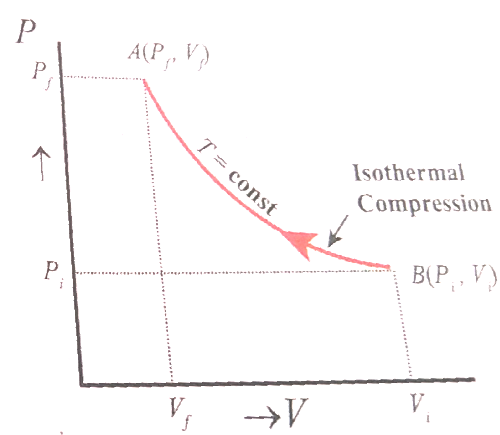
Decrease in volume leading to increase in temperature work is done on the gas. Ideal gas equation PV = RT. When piston be pushed further the parameters V and R are taken as constant. The equation becomes
P = kT. i.e P α T

38. Draw the PV diagram for ;

a. Isothermal process

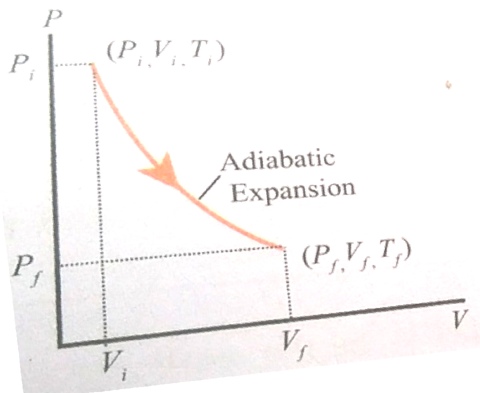


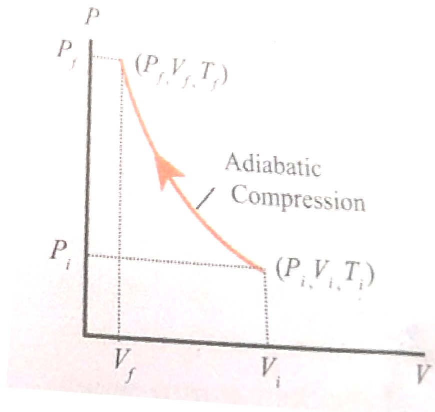
(a)



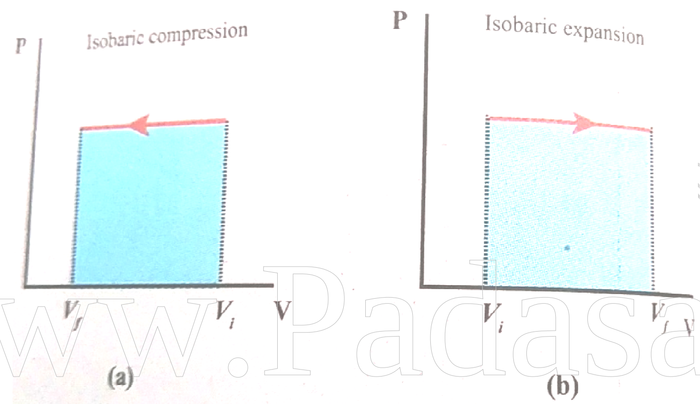
(b)

b. Adiabatic process

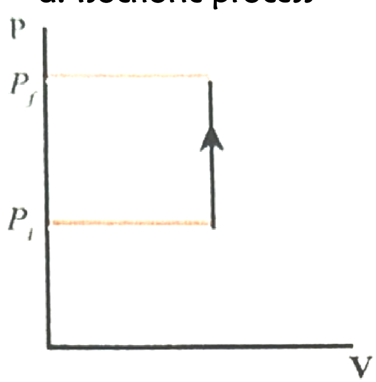


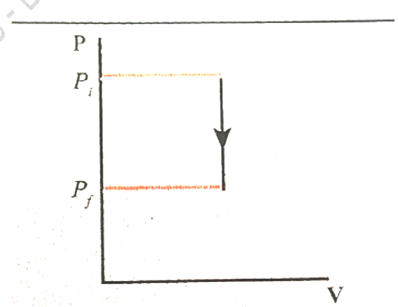


c. isobaric process



d. Isochoric process





39. What is a cyclic process?

This is a thermodynamic process in which the thermodynamic system returns to its initial state after undergoing a series of changes. Since the system comes back to the initial state, the change in the internal energy is zero. In cyclic process, heat can flow in to system and heat flow out of the system.

40. What is meant by reversible and irreversible processes?

Reversible process: A thermodynamic process can be considered reversible only if it possible to retrace the path in the opposite direction in such a way that the system and surroundings pass through the same states as in the initial, direct process. Example: A quasi–static isothermal expansion of gas, slow compression and expansion of a spring.

Irreversible process: All natural processes are irreversible. Irreversible process cannot be plotted in a PV diagram, because these processes cannot have unique values of pressure, temperature at every stage of the process.

41. State Clausius form of the second law of thermodynamics

"Heat always flows from hotter object to colder object spontaneously". This is known as the Clausius form of second law of thermodynamics.

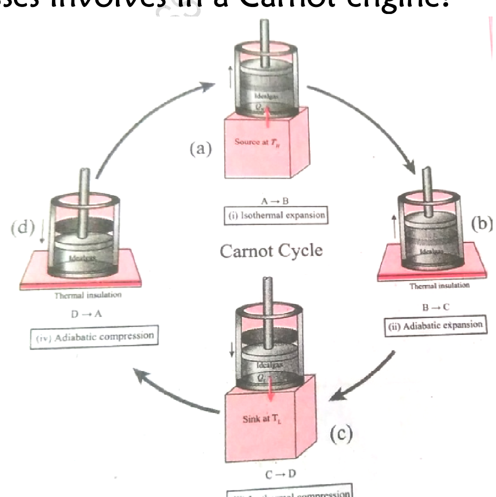
42. State Kelvin-Planck statement of second law of thermodynamics.

Kelvin-Planck statement: It is impossible to construct a heat engine that operates in a cycle, whose sole effect is to convert the heat completely into work. This implies that no heat engine in the universe can have 100% efficiency.

43. Define heat engine.

Heat engine is a device which takes heat as input and converts this heat in to work by undergoing a cyclic process.

44. What are processes involves in a Carnot engine?



45. Can the given heat energy be completely converted to work in a cyclic process? If not, when can, the heat can completely converted to work?

1) No, In a cyclic process, the complete heat energy is not completely converted to work. The whole heat cannot be converted into work, as it will violate second law of thermodynamics.

2) In an Isothermal process the whole heat can be converted into work. For an isothermal process dQ = dT, which shows that whole heat can be converted into work.

46. State the second law of thermodynamics in terms of entropy.

"For all the processes that occur in nature (irreversible process), the entropy always increases. For reversible process entropy will not change". Entropy determines the direction in which natural process should occur.

47. Why does heat flow from a hot object to a cold object?

Because entropy increases when heat flows from hot object to cold object.

48. Define the coefficient of performance.

COP is a measure of the efficiency of a refrigerator. It is defined as the ratio of heat extracted from the cold body (sink) to the external work done by the compressor W. COP = $\beta = \frac{Q_L}{W}$

49. Can water be boiled without heating?

Yes, at low pressure, the water boils fast at low temperature below the room temperature, when the pressure is made low, the water starts boiling without supplying any heat.

50. As air is a bad conductor of heat, why do we not feel warm without clothes?

This is conductor when we are without clothes air carries away heat from our body due to convection and hence we feel cold.

51. Why is it hotter at the same distance over the top of a fire than in front of it?

At a point in front of fire, heat is received due to the process of radiation only, while at a point above the fire, heat reaches both due to radiation and convection.

52. Define Triple point.

Triple point the triple point of a substance is the temperature and pressure at which the three phases (gas, liquid and solid) of that substance coexist in thermodynamic equilibrium. The triple point of water is at 273.1 K

53. Write the applications of thermal conversion.

1) Boiling water in a cooking pot is an example of convection. Water at the bottom of the pot receives more heat. Due to heating, the water expands and the density of water decreases at the bottom.

2) Due to this decrease in density, molecules rise to the top. At the same time the molecules at the top receive less heat and become denser and come to the bottom of the pot.

3) This process goes on continuously. The back and forth movement of molecules is called convection current.

4) To keep the room warm, we use room heater. The air molecules near the heater will heat up and expand.

5) As they expand, the density of air molecules will decrease and rise up while the higher density cold air will come down. This circulation of air molecules is called convection current.

54. Write the main features of prevost theory?

1) Every object emits heat radiations at all finite temperatures (except 0 K) as well as it absorbs radiations from the surroundings. For example, if you touch someone, they might feel your skin as either hot or cold.

2) A body at high temperature radiates more heat to the surroundings than it receives from it. Similarly, a body at a lower temperature receives more heat from the surroundings than it loses to it.

3) Prevost applied the idea of 'thermal equilibrium' to radiation. He suggested that all bodies radiate energy but hot bodies radiate more heat than the cooler bodies. At one point of time the rate of exchange of heat from both the bodies will become the same. Now the bodies are said to be in 'thermal equilibrium'. Only at absolute zero temperature a body will stop emitting.

55. Draw and explain the distribution of radiation intensity.

1) It implies that if temperature of the body increases, maximal intensity wavelength (λm) shift s towards lower wavelength (higher frequency) of electromagnetic spectrum.

2) From the graph it is clear that the peak of the wavelengths is inversely proportional to temperature. The curve is known as 'black body radiation curve'.

FIVE MARKS:

01. Explain the meaning of heat and work with suitable examples.

Meaning of work:

1) When you rub your hands against each other the temperature of the hands increases. You have done some work on your hands by rubbing. The temperature of the hands increases due to this work. Now if you place your hands on the chin, the temperature of the chin increases.

2) This is because the hands are at higher temperature than the chin. In the above example, the temperature of hands is increased due to work and temperature of the chin is increased due to heat transfer from the hands to the chin.

3) By doing work on the system, the temperature in the system will increase and sometimes may not. Like heat, work is also not a quantity and

through the work energy is transferred to the system . So we cannot use the word 'the object contains more work' or 'less work'.

4) Either the system can transfer energy to the surrounding by doing work on surrounding or the surrounding may transfer energy to the system by doing work on the system. For the transfer of energy from one body to another body through the process of work, they need not be at different temperatures.

02. Discuss the ideal gas laws.

Boyle's law, Charles' law and ideal gas law:

1) For a given gas at low pressure (density) kept in a container of volume V, experiments revealed the following information.
When the gas is kept at constant temperature, the pressure of the gas is inversely proportional to the volume.

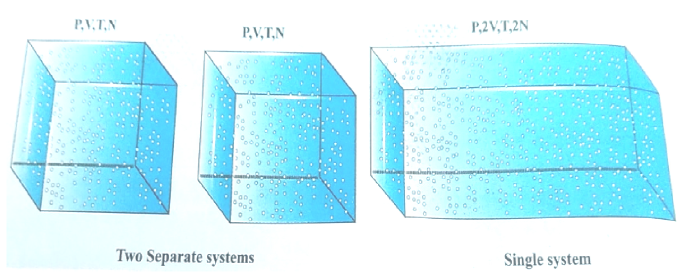
P $\alpha \frac{1}{V}$ It was discovered by Robert Boyle (1627-1691) and is known as Boyle's law.

2) When the gas is kept at constant pressure, the volume of the gas is directly proportional to absolute temperature. V α T . It was discovered by Jacques Charles (1743-1823) and is known as Charles' law.
By combining these two equations we have PV = CT. Here C is a positive constant.

3) C is proportional to the number of particles in the gas container by considering the following argument.

4) If we take two containers of same type of gas with same volume V, same pressure P and same temperature T, then the gas in each container obeys the above equation. PV = CT.

5) If the two containers of gas is considered as a single system, then the pressure and temperature of this combined system will be same but volume will be twice and number of particles will also be double as shown in figure.



For this combined system, V becomes 2V, so C should also double to match with the ideal gas equation $\frac{P(2V)}{T}$ = 2C.

6) It implies that C must depend on the number of particles in the gas and also should have the dimension of $\left[\frac{PV}{T}\right]$ = JK^{-1}.

7) we can write the constant C as k times the number of particles N.

Here k is the Boltzmann constant (1.381×10^{-23} JK^{-1}) and it is found to be a universal constant. So the ideal gas law can be stated as follows PV = NkT

03. Explain in detail the thermal expansion.

1) Thermal expansion is the tendency of matter to change in shape, area, and volume due to a change in temperature.

2) All three states of matter (solid, liquid and gas) expand when heated. When a solid is heated, its atoms vibrate with higher amplitude about their fixed points. The relative change in the size of solids is small. Railway tracks are given small gaps so that in the summer, the tracks expand and do not buckle. Railroad tracks and bridges have expansion joints to allow them to expand and contract freely with temperature changes.

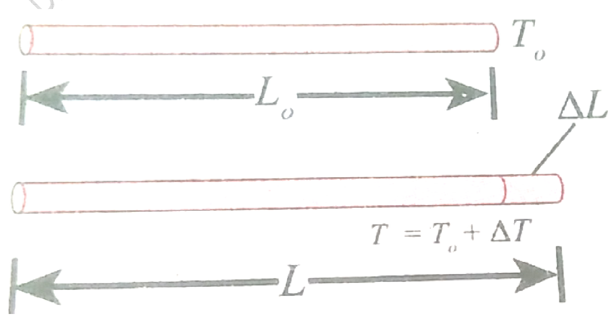
3) Liquids, have less intermolecular forces than solids and hence they expand more than solids. This is the principle behind the mercury thermometers.

4) In the case of **gas** molecules, the intermolecular forces are almost negligible and hence they expand much more than solids. For example in hot air balloons when gas particles get heated, they expand and take up more space.

5) The increase in dimension of a body due to the increase in its temperature is called thermal expansion.

6) The expansion in length is called **linear expansion.** Similarly the expansion in area is termed as **area expansion** and the expansion in volume is termed as **volume expansion.**

Linear Expansion:

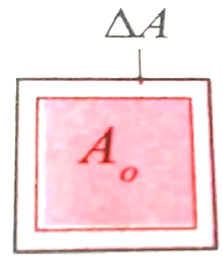


In solids, for a small change in temperature ΔT, the fractional change in length $\left(\frac{\Delta L}{L}\right)$ is directly proportional to ΔT. $\frac{\Delta L}{L} = a_L \Delta T$

Therefore, $\alpha_L = \frac{\Delta L}{L\Delta T}$; Where, αL = coefficient of linear expansion.

ΔL = Change in length; L = Original length ; ΔT = Change in temperature.

Area Expansion:



For a small change in temperature ΔT the fractional change in area $\left(\frac{\Delta A}{A}\right)$ of a substance is directly proportional to ΔT and it can be written as

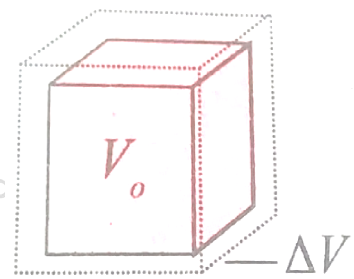
$$\frac{\Delta A}{A} = \alpha_A \Delta T$$

Therefore,

$\alpha_A = \frac{\Delta A}{A \Delta T}$; Where, αA = coefficient of area expansion.

ΔA = Change in area; A = Original area ; ΔT = Change in temperature

Volume Expansion:



For a small change in temperature ΔT the fractional change in volume$\left(\frac{\Delta V}{V}\right)$ of a substance is directly proportional to ΔT.

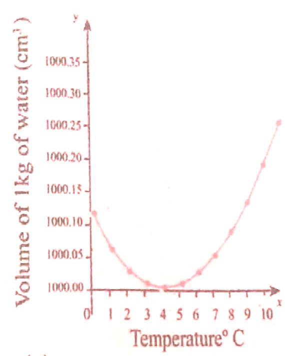
$\frac{\Delta V}{V} = \alpha_V \Delta T$, Therefore, $\alpha_V = \frac{\Delta V}{V \Delta T}$

Where, αV = coefficient of volume expansion;

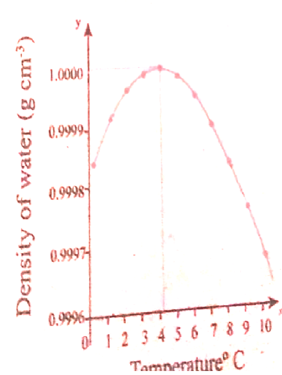
ΔV = Change in volume; V = Original volume ; ΔT = Change in temperature. Unit of coefficient of linear, area and volumetric expansion of solids is $°C^{-1}$ or K^{-1}

04. Describe the anomalous expansion of water. How is it helpful in our lives?

Anomalous expansion of water:



(a)



1) Liquids expand on heating and contract on cooling at moderate temperatures. But water exhibits an anomalous behavior. It contracts on heating between 0°C and 4°C.

2) The volume of the given amount of water decreases as it is cooled from room temperature, until it reach 4°C .

3) Below 4°C the volume increases and so the density decreases. This means that the water has a maximum density at 4°C . This behavior of water is called anomalous expansion of water.

4) In cold countries during the winter season, the surface of the lakes will be at lower temperature than the bottom.

5) Since the solid water (ice) has lower density than its liquid

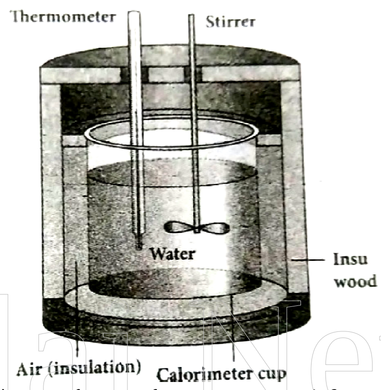
form, below 4°C, the frozen water will be on the top surface above the liquid water (ice fl oats).

6) This is due to the anomalous expansion of water. As the water in lakes and ponds freeze only at the top the species living in the lakes will be safe at the bottom.

05. Explain Calorimetry and derive an expression for final temperature when two thermodynamic systems are mixed.

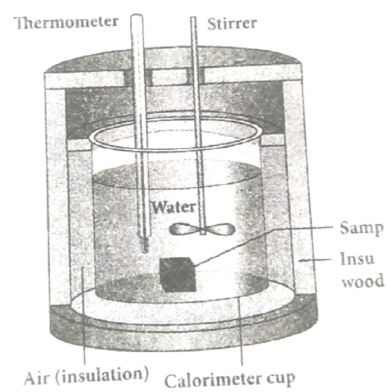
Calorimetry :

1) Calorimetry means the measurement of the amount of heat released or absorbed by thermodynamic system during the heating process. When a body at higher temperature is brought in contact with another body at lower temperature, the heat lost by the hot body is equal to the heat gained by the cold body. No heat is allowed to escape to the surroundings. It can be mathematically expressed as $Q_{gain} = -Q_{lost}$; $Q_{gain} + Q_{lost} = 0$



2) Heat gained or lost is measured with a calorimeter. Usually the calorimeter is an insulated container of water as shown in Figure.

3) A sample is heated at high temperature (T_1) and immersed into water at room temperature (T_2) in the calorimeter. After some time both sample and water reach a final equilibrium temperature T_f . Since the calorimeter is insulated, heat given by the hot sample is equal to heat gained by the water. It is shown in the Figure.



$Q_{gain} = -Q_{lost}$

Note the sign convention. The heat lost is denoted by negative sign and heat gained is denoted as positive.

From the definition of specific heat capacity

$$Q_{gain} = m_2 s_2 (T_f - T_2)$$
$$Q_{lost} = m_1 s_1 (T_f - T_1)$$

Here s_1 and s_2 specific heat capacity of hot sample and water respectively. So we can write

$$m_2 s_2 (T_f - T_2) = - m_1 s_1 (T_f - T_1)$$
$$m_2 s_2 T_f - m_2 s_2 T_2 = - m_1 s_1 T_f + m_1 s_1 T_1$$
$$m_2 s_2 T_f + m_1 s_1 T_f = m_2 s_2 T_2 + m_1 s_1 T_1$$

The final temperature $T_f = \frac{m_1 S_1 T_1 + m_2 S_2 T_2}{m_1 S_1 + m_2 S_2}$

06. Discuss various modes of heat transfer.

Conduction:

Conduction is the process of direct transfer of heat through matter due to temperature difference. When two objects are in direct contact with one another, heat will be transferred from the hotter object to the colder one. Thermal conductivity depends on the nature of the material.

Convection:

Convection is the process in which heat transfer is by actual movement of molecules in fluids such as liquids and gases. In convection, molecules move freely from one place to another.

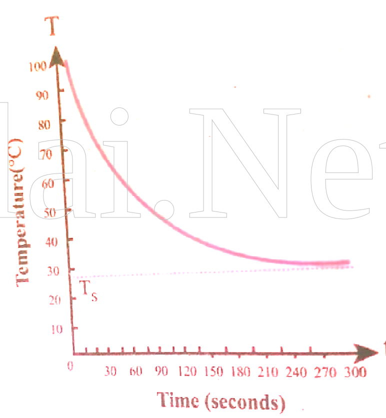
Radiation:

Radiation is a form of energy transfer from one body to another by electromagnetic waves. Radiation which requires no medium to transfer energy from one object to another.

Example: 1. Solar energy from the Sun. 2. Radiation from room heater.

07. Explain in detail Newton's law of cooling.

Newton's law of cooling:



1) Newton's law of cooling states that the rate of loss of heat of a body is directly proportional to the difference in the temperature between that body and its surroundings .

$\frac{dQ}{dt} \alpha$ – (T – T) ------------- 1

2) The negative sign indicates that the quantity of heat lost by liquid goes on decreasing with time. Where, T = Temperature of the object T_s = Temperature of the surrounding.

From the graph in Figure , it is clear that the rate of cooling is high initially and decreases with falling temperature.

3) Let us consider an object of mass m and specific heat capacity s at temperature T. Let Ts be the temperature of the surroundings. If the temperature falls by a small amount dT in time dt, then the amount of heat lost is, dQ = msdT ---------------- 2

4) Dividing both sides of equation (2) by $\frac{dQ}{dt} = \frac{\text{msdT}}{dt}$ -------------3

From Newton's law of cooling $\frac{dQ}{dt} \alpha$ – (T – T_S)

$\frac{dQ}{dt} = -$ a (T – T_S) ---------- 4

Where a is some positive constant. From equation (2) and (4)

– a (T – T_S) = ms $\frac{dT}{dt}$

$\frac{dT}{(T - TS)} = -\frac{a}{ms}dt$ --------------------- 5

Integrating equation (5) on both sides,

$\int_0^{\infty} \frac{dT}{(T - TS)} = - \int_0^{t} \frac{a}{ms} dt$

$\ln (T - T_S) = \frac{a}{ms} t + b_1$

Where b_1 is the constant of integration. taking exponential both sides, we get,

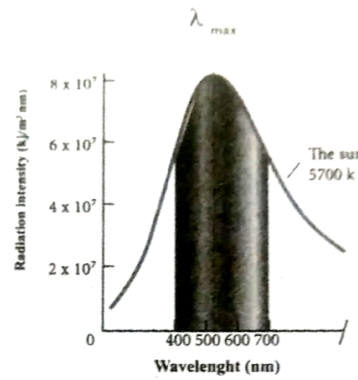
$T = T_S + b_{2\,e} \frac{a}{ms} t$. Here $b_2 = eb_1$ = Constant

08. Explain Wien's law and why our eyes are sensitive only to visible rays?

1) Wien's law states that, the wavelength of maximum intensity of emission of a black body radiation is inversely proportional to the absolute temperature of the black body.

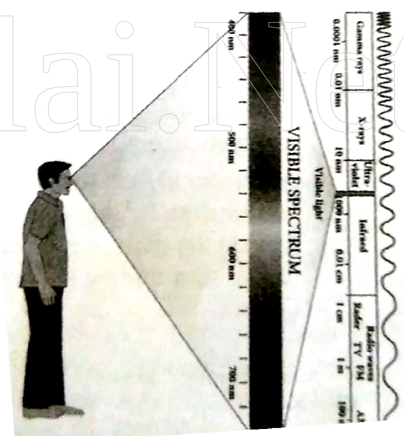
$\lambda_m \, \alpha \, \frac{1}{T}$ or $\lambda_m = \frac{b}{T}$ ---------1

Where, b is known as Wien's constant.
Its value is 2.898×10^{-3} m K



2) The Sun is approximately taken as a black body. Since any object above 0 K will emit radiation, Sun also emits radiation. Its surface temperature is about 5700 K. By substituting this value in the equation (1).

$\lambda_m = \frac{b}{T} = \frac{2.898 \times 10^{-8}}{5700} \approx 508\text{nm}$



3) It is the wavelength at which maximum intensity is 508nm. Since the Sun's temperature is around 5700K, the spectrum of radiations emitted by Sun lie between 400 nm to 700 nm which is the visible part of the spectrum.

4) The humans evolved under the Sun by receiving its radiations. The human eye is sensitive only in the visible not in infrared or X-ray ranges in the spectrum.

5) Suppose if humans had evolved in a planet near the star Sirius (9940K), then they would have had the ability to see the Ultraviolet rays!

09. Discuss the

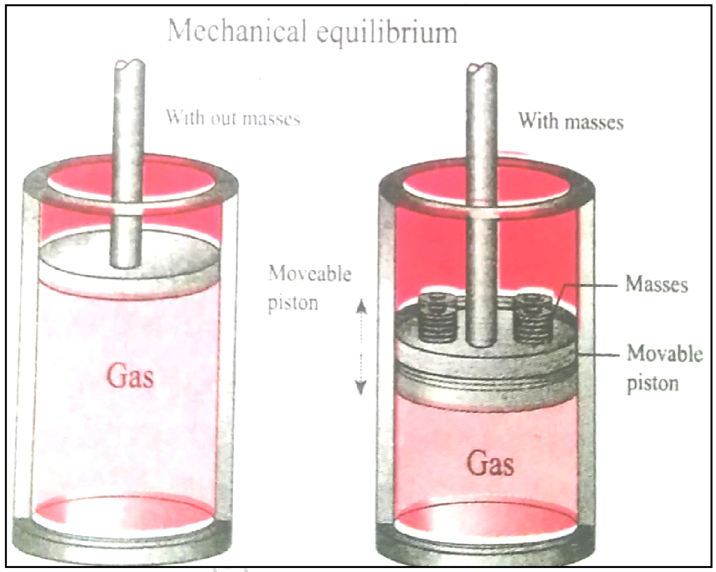
a. Thermal equilibrium
b. Mechanical equilibrium
c. Chemical equilibrium
d. Thermodynamic equilibrium.

a. Thermal equilibrium:

Two systems are said to be in thermal equilibrium with each other if they are at the same temperature, which will not change with time.

b. Mechanical equilibrium:

Consider a gas container with piston as shown in Figure. When some mass is placed on the piston, it will move downward due to downward gravitational force and after certain humps and jumps the piston will come to rest at a new position. When the downward gravitational force given by the piston is balanced by the upward force exerted by the gas, the system is said to be in mechanical equilibrium. A system is said to be in mechanical equilibrium if no unbalanced force acts on the thermo dynamic system or on the surrounding by thermodynamic system.



c. Chemical equilibrium:

If there is no net chemical reaction between two thermodynamic systems in contact with each other then it is said to be in chemical equilibrium.

d. Thermodynamic equilibrium:

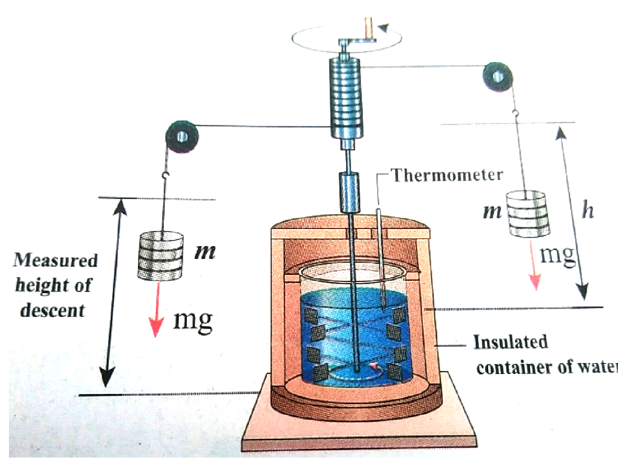
If two systems are set to be in thermodynamic equilibrium, then the systems are at thermal, mechanical and chemical equilibrium with each other. In a state of thermodynamic equilibrium the macroscopic variables such as pressure, volume and temperature will have fixed values and do not change with time.

10. Explain Joule's Experiment of the mechanical equivalent of heat.

1) Joule showed that mechanical energy can be converted into internal energy and vice versa. In his experiment, two masses were attached with a rope and a paddle wheel.

2) When these masses fall through a distance h due to gravity, both the masses lose potential energy equal to 2mgh.

3) When the masses fall, the paddle wheel turns. Due to the



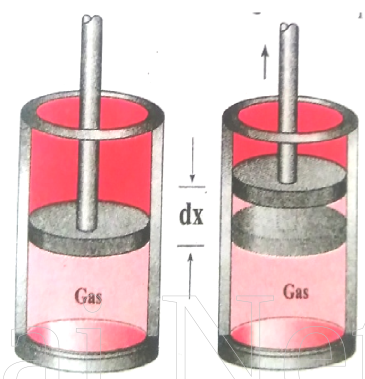
turning of wheel inside water, frictional force comes in between the water and the paddle wheel.

4) This causes a rise in temperature of the water. This implies that gravitational potential energy is converted to internal energy of water.

5) The temperature of water increases due to the work done by the masses. In fact, Joule was able to show that the mechanical work has the same effect as giving heat.

6) He found that to raise 1 g of an object by 1°C , 4.186 J of energy is required. In earlier days the heat was measured in calorie. 1 cal = 4.186 J This is called Joule's mechanical equivalent of heat.

11. Derive the expression for the work done in a volume change in a thermodynamic system.



Work done in volume changes

1) Consider a gas contained in the cylinder fitted with a movable piston. Suppose the gas is expanded quasi-statically by pushing the piston by a small distance dx.

2) Since the expansion occurs quasi-statically the pressure, temperature and internal energy will have unique values at every instant. The small work done by the gas on the piston. $dW = Fdx$ ------------- 1

3) The force exerted by the gas on the piston $F = PA$. Here A is area of the piston and P is pressure exerted by the gas on the piston.

Equation (1) can be rewritten as $dW = PA\,dx$ ------------ 2

4) But $Adx = dV$= change in volume during this expansion process. So the small work done by the gas during the expansion is given by
$dW = PdV$

5) Note here that is positive since the volume is increased. Here, is positive. In general the work done by the gas by increasing the volume from V_i to V_f is given by $W = \int_{V_i}^{V_f} PdV$ ------------- 4

Suppose if the work is done on the system, then $V_i > V_f$. Then, W is negative.

6) Note here the pressure P is inside the integral in equation (4). It implies that while the system is doing work, the pressure need not be constant.

7) To evaluate the integration we need to first express the pressure as a function of volume and temperature using the equation of state.

12. Derive Mayer's relation for an ideal gas.

Meyer's relation

1) Consider μ mole of an ideal gas in a container with volume V, pressure P and temperature T.

2) When the gas is heated at constant volume the temperature increases by dT. As no work is done by the gas, the heat that flows into the system will increase only the internal energy. Let the change in internal energy be dU.

If Cv is the molar specific heat capacity at constant volume,

$$dU = \mu C_v dT \text{ -------------- } 1$$

3) Suppose the gas is heated at constant pressure so that the temperature increases by dT. If 'Q' is the heat supplied in this process and 'dV' the change in volume of the gas. $Q = \mu C_p dT$ ------------ 2

4) If W is the work done by the gas in this process, then

$W = PdV$ -------------3

But from the first law of thermodynamics, $Q = dU + W$ ---------4

Substituting equations (1), (2) and (3) in (4), we get,

$\mu C_p dT = \mu C_v\, dT + PdV$ ------------5

5) For mole of ideal gas, the equation of state is given by

$PV = \mu RT \Rightarrow PdV + VdP = \mu RdT$ ----------- 6

Since the pressure is constant, $dP=0$

$\therefore C_p dT = C_v dT + RdT$

$\therefore C_P = C_v + R$ (or) $C_p - C_v = R$ ------------ 7

This relation is called Meyer's relation

13. Explain in detail the isothermal process.

Isothermal process

1) It is a process in which the temperature remains constant but the pressure and volume of a thermodynamic system will change. The ideal gas equation is

$PV = \mu RT$, Here, T is constant for this process

So the equation of state for isothermal process is given by $PV = constant$ ------ 1

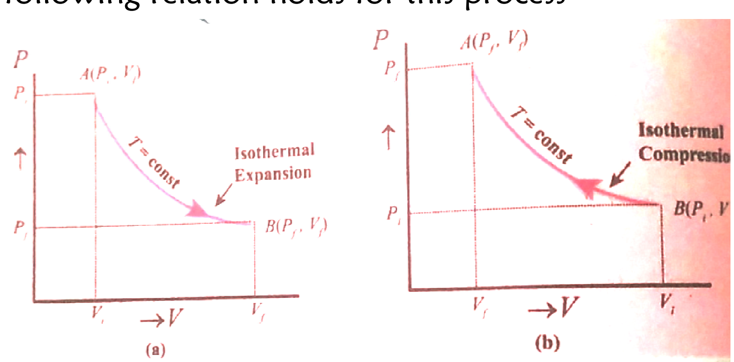
2) This implies that if the gas goes from one equilibrium state (P_1,V_1) to another equilibrium state (P_2,V_2) the following relation holds for this process

$P_1V_1 = P_2V_2$ --------------2

3)Since

PV = constant, P is inversely proportional to $P \propto \frac{1}{V}$.

This implies that PV graph is a hyperbola. The pressure-volume graph for constant temperature is also



called isotherm. PV diagram for quasi-static isothermal expansion and quasi-static isothermal compression.

4) We know that for an ideal gas the internal energy is a function of temperature only. For an isothermal process since temperature is constant, the internal energy is also constant. This implies that dU or ΔU = 0.

For an isothermal process, the first law of thermodynamics can be written as follows, Q = W ------------ 3

5) From equation (3), we infer that the heat supplied to a gas is used to do only external work.

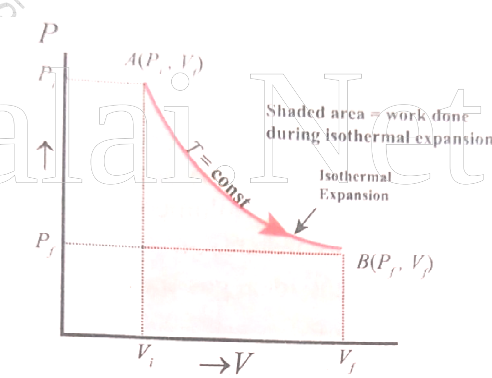
6) The isothermal compression takes place when the piston of the cylinder is pushed. This will increase the internal energy which will flow out of the system through thermal contact.

14. Derive the work done in an isothermal process

Work done in an isothermal process:

1) Consider an ideal gas which is allowed to expand quasi-statically at constant temperature from initial state (P_i,V_i) to the final state (P_f , V_f). We can calculate the work done by the gas during this process. From equation the work done by the gas,

$$W = \int_{V_i}^{V_f} P dV \text{ -------------- } 1$$



(a)

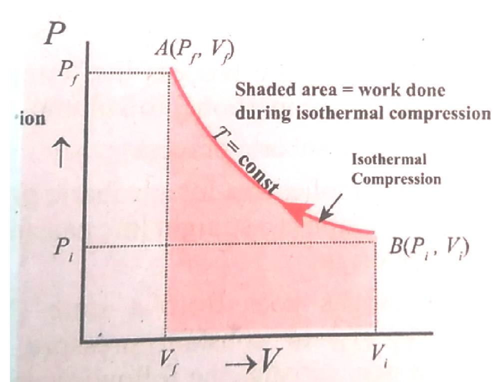
2) As the process occurs quasi-statically, at every stage the gas is at equilibrium with the surroundings. Since it is in equilibrium at every stage the ideal gas law is valid. Writing pressure in terms of volume and temperature,

$$P = \frac{\mu RT}{V} \text{ ---------------------} 2$$

Substituting equation (2) in (1) we get,

$$W = \int_{V_i}^{V_f} \frac{\mu RT}{V} dV$$

$$W = \mu RT \int_{V_i}^{V_f} \frac{dV}{V} \text{ -------------- } 3$$



(b)

In equation (3), we take μRT out of the integral, since it is constant throughout the isothermal process.

By performing the integration in equation (3),

we get $W = \mu RT \ln\left(\frac{V_f}{V_i}\right)$ ---------------- 4

3) Since we have an isothermal expansion, $\frac{V_f}{V_i} > 1$, So $\ln\left(\frac{V_f}{V_i}\right) > 0$.

As a result the work done by the gas during an isothermal expansion is positive.

The above result in equation (4) is true for isothermal compression also. But in an isothermal compression $\frac{V_f}{V_i}$ <1, So In $\left(\frac{V_f}{V_i}\right)$ <0. As a result the work done on the gas in an isothermal compression is negative.

4) In the PV diagram the work done during the isothermal expansion is equal to the area under the graph. Similarly for an isothermal compression, the area under the PV graph is equal to the work done on the gas which turns out to be the area with a negative sign.

15. Explain in detail an adiabatic process.

Adiabatic process

1) This is a process in which no heat flows into or out of the system (Q=0). But the gas can expand by spending its internal energy or gas can be compressed through some external work. So the pressure, volume and temperature of the system may change in an adiabatic process.

2) The equation of state for an adiabatic process is given by

PV^γ = Constant--------------1

Here γ is called adiabatic exponent $\left(\gamma = \frac{C_p}{C_v}\right)$ which depends on the nature of the gas.

3) The equation (8.35) implies that if the gas goes from an equilibrium state (P_i,V_i) to another equilibrium state (P_f ,V_f) adiabatically then it satisfies the relation

$P_iV_i^\gamma = P_fV_f^\gamma$ ------------- 2

4) The PV diagram of an adiabatic expansion and adiabatic compression process. The PV diagram for an adiabatic process is also called adiabat.

5) Note that the PV diagram for isothermal and adiabatic processes look similar. But actually the adiabatic curve is steeper than isothermal curve.

6) To rewrite the equation (1) in terms of T and V. From ideal gas equation, the pressure P = $\frac{\mu RT}{V}$. Substituting this equation in the equation (1), we have $\frac{\mu RT}{V}V^\gamma$ = Constant or $\frac{T}{V}$ V^γ = $\frac{Constant}{\mu R}$

7) Note here that is another constant. So it can be written as

$T V^{\gamma-1}$ = Constant ------------3

The equation (3) implies that if the gas goes from an initial equilibrium state (T_i, V_i) to final equilibrium state (T_f, V_f) adiabatically then it satisfies the relation $T_iV_i^{\gamma-1} = T_fV_f^{\gamma-1}$ --------------- 4

The equation of state for adiabatic process can also be written in terms of T and P as $T^\gamma P^{1-\gamma}$ = constant.

16. Derive the work done in an adiabatic process

Work done in an adiabatic process:

1) Consider μ moles of an ideal gas enclosed in a cylinder having perfectly non conducting walls and base. A frictionless and insulating piston of cross sectional area A is fitted in the cylinder. Let W be the work done when the system goes from the initial state (P_i,V_i,T_i) to the final state (P_f,V_f,T_f) adiabatically. $W = \int_{V_i}^{V_f} PdV$ ---------------- 1

2) By assuming that the adiabatic process occurs quasi-statically, at every stage the ideal gas law is valid. Under this condition, the adiabatic equation of state is PV^γ = constant (or) $P = \frac{Constant}{V^\gamma}$ can be substituted in the equation (1), we get $W_{adia} = \int_{V_i}^{V_f} \frac{Constant}{V^\gamma} dV$

$$= \text{Constant} \int_{V_i}^{V_f} V^\gamma dV$$

$$= \text{Constant} \left[\frac{V^{-\gamma+1}}{-\gamma+1}\right]_{V_i}^{V_f}$$

$$= \frac{\text{Constant}}{1-\gamma}\left[\frac{\text{Constant}}{V_f^{\gamma-1}} - \frac{\text{Constant}}{V_i^{\gamma-1}}\right]$$

But, $P_iV_i^\gamma = P_fV_f^\gamma$ = constant.

$$W_{adia} = \frac{1}{1-\gamma}\left[\frac{P_fV_f^\gamma}{V_f^{\gamma-1}} - \frac{P_iV_i^\gamma}{V_i^{\gamma-1}}\right]$$

$$W_{adia} = \frac{1}{1-\gamma}[P_fV_f - P_iV_i] \text{ -------- } 2$$

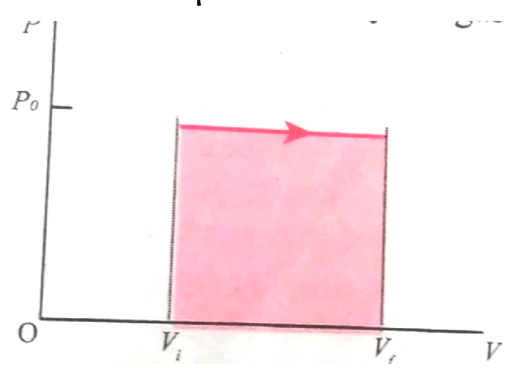
From ideal gas law, $P_fV_f = \mu RT_f$ and $P_iV_i = \mu RT_i$

Substituting in equation (2), we get, $W_{adia} = \frac{\mu R}{\gamma-1}[T_i - T_f]$

3) In adiabatic expansion, work is done by the gas. i.e., W_{adia} is positive. As $T_i > T_f$, the gas cools during adiabatic expansion. In adiabatic compression, work is done on the gas. i.e., W_{adia} is negative. As $T_i < T_f$, the temperature of the gas increases during adiabatic compression.

17. Explain the isobaric process and derive the work done in this process

Isobaric process



1) This is a thermodynamic process that occurs at constant pressure. Even though pressure is constant in this process, temperature, volume and internal energy are not constant. From the ideal gas equation, we have

$V = \left(\frac{\mu R}{P}\right)T$ ----------1 Here $\frac{\mu R}{P}$ = Constant

2) In an isobaric process the temperature is directly proportional to volume. $V \propto T$ (Isobaric process) ---- (2)

This implies that for a isobaric process, the V-T graph is a straight line passing through the origin.

3) If a gas goes from a state (V_i ,T_i) to (V_f ,T_f) at constant pressure, then the system satisfies the following equation $\frac{T_f}{V_f} = \frac{T_i}{V_i}$

The work done in an isobaric process: Work done by the gas $W = \int_{V_i}^{V_f} PdV$

In an isobaric process, the pressure is constant, so P comes out of the integral,

$W = P\int_{V_i}^{V_f} dV$ $W = P\,[V_f - V_i] = P\Delta V$ -----------3

4) Where ΔV denotes change in the volume. If ΔV is negative, W is also negative. This implies that the work is done on the gas. If ΔV is positive, W is also positive, implying that work is done by the gas.

5) The equation (3) can also be rewritten using the ideal gas equation.

From ideal gas equation PV = μRT and $V = \frac{\mu RT}{P}$

Substituting this in equation (3) we get, $W = \mu RT_f\left(1 - \frac{T_i}{T_f}\right)$

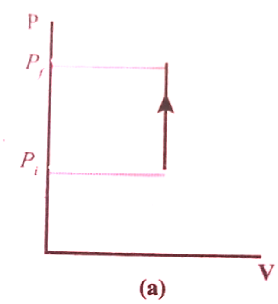
6) In the PV diagram, area under the isobaric curve is equal to the work done in isobaric process. The shaded area in the following Figure is equal to the work done by the gas.

7) The first law of thermodynamics for isobaric process is given by

ΔU = Q - PΔV

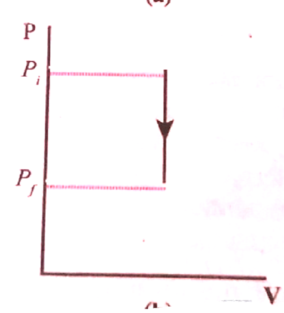
18. Explain in detail the isochoric process.

Isochoric process



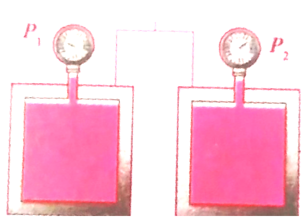
1) This is a thermodynamic process in which the volume of the system is kept constant. But pressure, temperature and internal energy continue to be variables. The pressure - volume graph for an isochoric process is a vertical line parallel to pressure axis.

2) The equation of state for an isochoric process is given by $P = \left(\frac{\mu R}{V}\right)$



Where, $\left(\frac{\mu R}{V}\right)$ = Constant

It that the pressure is directly proportional to temperature. This implies that the P-T graph for an isochoric process is a straight line passing through origin. If a gas goes from state (P_i,T_i) to (P_f,T_f) at constant volume, then the system satisfies the following equation $\frac{P_i}{T_i} = \frac{P_f}{T_f}$



For an isochoric processes, ΔV=0 and W=0. Then the first law becomes

ΔU = Q

3) Implying that the heat supplied is used to increase only the internal energy. As a result the temperature increases and pressure also increases.

4) Suppose a system loses heat to the surroundings through conducting walls by keeping the volume constant, then its internal energy decreases. As a result the temperature decreases; the pressure also decreases.

19. What are the limitations of the first law of thermodynamics?

Limitations of first law of thermodynamics

The first law of thermodynamics explains well the inter convertibility of heat and work. But it does not indicate the direction of change.

For example,

a. When a hot object is in contact with a cold object, heat always flows from the hot object to cold object but not in the reverse direction. According to first law, it is possible for the energy to flow from hot object to cold object or from cold object to hot object. But in nature the direction of heat flow is always from higher temperature to lower temperature.

b. When brakes are applied, a car stops due to friction and the work done against friction is converted into heat. But this heat is not reconverted to the kinetic energy of the car. So the first law is not sufficient to explain many of natural phenomena.

20. Explain the heat engine and obtain its efficiency.

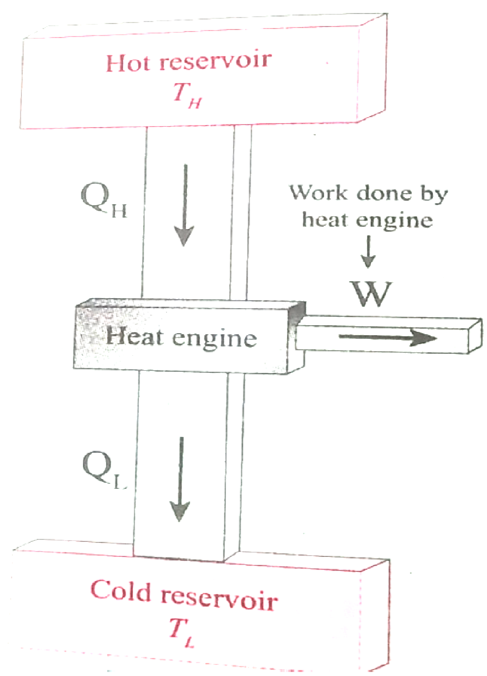
Heat engine is a device which takes heat as input and converts this heat in to work by undergoing a cyclic process.

A heat engine has three parts:

(a) Hot reservoir (b) Working substance

(c) Cold reservoir

A Schematic diagram for heat engine is given below in the figure



1) Hot reservoir (or) Source: It supplies heat to the engine. It is always maintained at a high temperature T_H

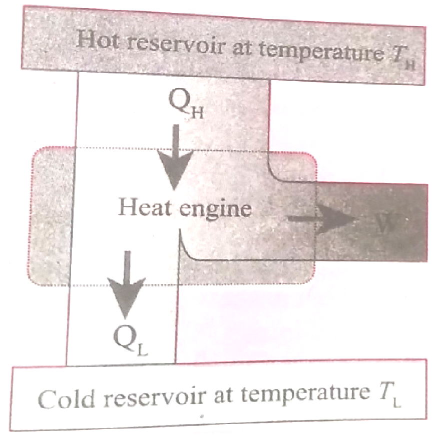
2) Working substance: It is a substance like gas or water, which converts the heat supplied into work.

i) A simple example of a heat engine is a steam engine. In olden days steam engines were used to drive trains. The working substance in these is water which absorbs heat from the burning of coal.

ii) The heat converts the water into steam. This steam is does work by rotating the wheels of the train, thus making the train move.

3) Cold reservoir (or) Sink: The heat engine ejects some amount of heat (Q_L) in to cold reservoir after it doing work. It is always maintained at a low temperature T_L.

For example, in the automobile engine, the cold reservoir is the surroundings at room temperature. The automobile ejects heat to these surroundings through a silencer.



4) The heat engine works in a cyclic process. After a cyclic process it returns to the same state. Since the heat engine returns to the same state after it ejects heat, the change in the internal energy of the heat engine is zero.

5) The efficiency of the heat engine is defined as the ratio of the work done (output) to the heat absorbed (input) in one cyclic process. Let the working substance absorb heat Q_H units from the source and reject Q_L units to the sink after doing work W units

We can write Input heat = Work done + ejected heat

$$Q_H = W + Q_L$$
$$W = Q_H - Q_L$$

Then the efficiency of heat engine $\eta = \frac{Output}{Input} = \frac{W}{Q_H} = \frac{Q_H - Q_L}{Q_H}$

$$\eta = \frac{Output}{Input} = \frac{W}{Q_H} = 1 - \frac{Q_L}{Q_H}$$

6) Note here that Q_H, Q_L and W all are taken as positive, a sign convention followed in this expression.

Since $Q_L < Q_H$, the efficiency (η) always less than 1. This implies that heat absorbed is not completely converted into work. The second law of thermodynamics placed fundamental restrictions on converting heat completely into work.

21. Explain in detail Carnot heat engine.

A reversible heat engine operating in a cycle between two temperatures in a particular way is called a Carnot Engine. The Carnot engine has four parts which are given below.

1) Source: It is the source of heat maintained at constant high temperature TH. Any amount of heat can be extracted from it, without changing its temperature.

2) Sink: It is a cold body maintained at a constant low temperature TL. It can absorb any amount of heat.

3) Insulating stand: It is made of perfectly non-conducting material. Heat is not conducted through this stand.

4) Working substance: It is an ideal gas enclosed in a cylinder with perfectly non-conducting walls and perfectly conducting bottom. A non-conducting and frictionless piston is fitted in it.

Carnot's cycle:

i) The working substance is subjected to four successive reversible processes forming what is called Carnot's cycle.

ii) Let the initial pressure, volume of the working substance be P_1, V_1.

Step A to B: Quasi-static isothermal expansion from (P_1, V_1, T_H) to (P_2, V_2, T_H):

5) The cylinder is placed on the source. The heat (Q_H) flows from source to the working substance (ideal gas) through the bottom of the cylinder. Since the process is isothermal, the internal energy of the working substance will not change. The input heat increases the volume of the gas. The piston is allowed to move out very slowly (quasi-statically).

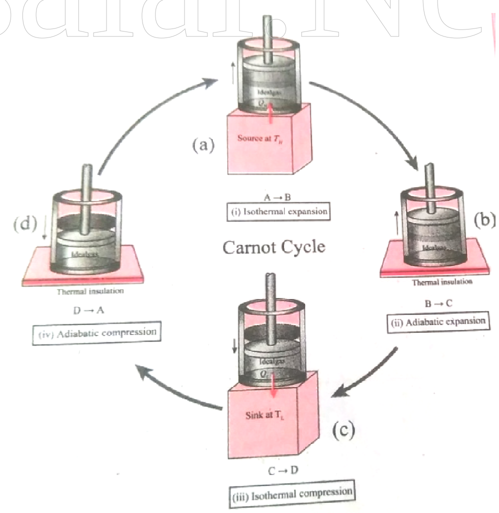
6) W_1 is the work done by the gas in expanding from volume V_1 to volume V_2 with a decrease of pressure from P_1 to P_2. This is represented by the P-V diagram along the path AB.

7) Then the work done by the gas (working substance) is given by

$$\therefore Q_H = W_{A \to B} = \int_{V_1}^{V_2} P dV$$

Since the process occurs quasi-statically, the gas is in equilibrium with the source till it reaches the final state. The work done in the isothermal expansion is given by the equation.

Step B to C: Quasi-static adiabatic expansion from (P_2, V_2, T_H) to (P_3, V_3, T_L)



1) The cylinder is placed on the insulating stand and the piston is allowed to move out. As the gas expands adiabatically from volume V_2 to volume V_3 the pressure falls from P_2 to P_3.
2) The temperature falls to T_L. This adiabatic expansion is represented by curve BC in the P-V diagram. This adiabatic process also occurs quasi-statically and implying that this process is reversible and the ideal gas is in equilibrium throughout the process. The work done by the gas in an adiabatic expansion is given by,

$$W_{B \to C} = \int_{V_2}^{V_3} P dV = \frac{\mu R}{\gamma - 1}[T_H - T_L] = \text{Area under the curve BC}$$

Step C → D: Quasi-static isothermal compression from (P_3, V_3, T_L) to (P_4, V_4, T_L):

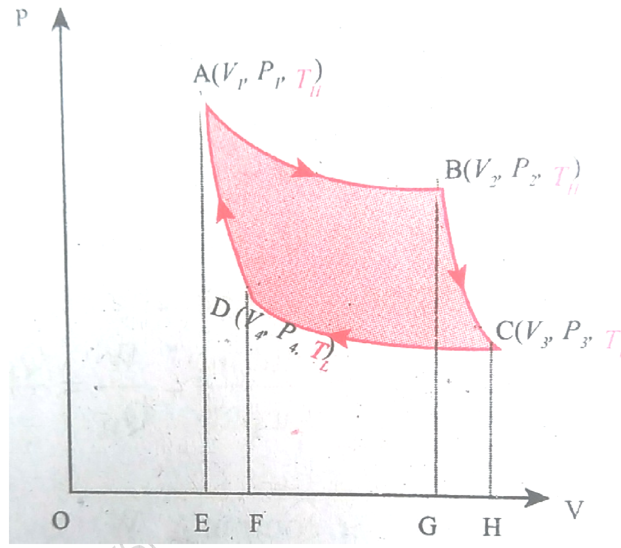
1) The cylinder is placed on the sink and the gas is isothermally compressed until the pressure and volume become P_4 and V_4 respectively. This is

represented by the curve CD in the PV diagram. Let $W_{C\to D}$ be the work done on the gas. According to first law of thermodynamics

$$W_{C\to D} = \int_{V_3}^{V_4} PdV = \mu RT_L \ln = \frac{V_4}{V_3} = -\mu RT_L \ln = \frac{V_3}{V_4}$$

= – Area under the curve CD

Here V_3 is greater than V_4. So the work done is negative, implying work is done on the gas.



Step D→A: Quasi-static adiabatic compression from (P_4, V_4, T_L) to (P_1, V_1, T_H):

1) The cylinder is placed on the insulating stand again and the gas is compressed adiabatically till it attains the initial pressure P_1, volume V_1 and temperature T_H. This is shown by the curve DA in the P-V diagram.

$$W_{D\to A} = \int_{V_4}^{V_1} PdV = \frac{\mu R}{\gamma - 1}[T_L - T_H] = \text{Area under the curve DA}$$

2) In the adiabatic compression also work is done on the gas so it is negative. Let 'W' be the net work done by the working substance in one cycle

∴W=Work done by the gas – work done on the gas

$= W_{A\to B} + W_{B\to C} - W_{C\to D} - W_{D\to A}$ since $W_{B\to C} = W_{D\to A}$

$= W_{A\to B} - W_{C\to D}$

The net work done by the Carnot engine in one cycle

$W = W_{A\to B} - W_{C\to D}$ ---------------1

Equation (1) shows that the net work done by the working substance in one cycle is equal to the area (enclosed by ABCD) of the P-V diagram.

3) It is very important to note that after one cycle the working substance returns to the initial temperature T_H. This implies that the change in internal energy of the working substance after one cycle is zero.

22. Derive the expression for Carnot engine efficiency.

Efficiency of a Carnot engine

1) Efficiency is defined as the ratio of work done by the working substance in one cycle to the amount of heat extracted from the source.

$$\eta = \frac{Work\ done}{Heat\ extracted} = \frac{W}{Q_H} \text{ ---------- 1}$$

From the first law of thermodynamics, $W = Q_H - Q_L$

$$\eta = \frac{QH - QL}{Q_H} = 1 - \frac{Q_L}{Q_H} \text{ -------------- 2}$$

Applying isothermal conditions, we get,

$Q_H = \mu RT_H \ln \frac{V_2}{V_4}$; $Q_L = \mu RT_L \ln \frac{V_3}{V_4}$ ------------ 3

Here we omit the negative sign. Since we are interested in only the amount of heat (Q_L) ejected into the sink, we have, $\frac{Q_L}{Q_H} = \frac{T_L \ln \frac{V_3}{V_4}}{T_H \ln \frac{V_2}{V_1}}$ ---------4

By applying adiabatic conditions, we get, $T_H V_2^{\gamma-1} = T_L V_3^{\gamma-1}$

By dividing the above two equations, we get, $T_H V_1^{\gamma-1} = T_L V_4^{\gamma-1}$

By dividing the above two equations, we get, $\left(\frac{V_2}{V_1}\right)^{\gamma-1} = \left(\frac{V_3}{V_4}\right)^{\gamma-1}$

Which implies that, $\frac{V_2}{V_1} = \frac{V_3}{V_4}$ ---------------5

Substituting equation (5) in (4), we get, $\frac{Q_L}{Q_H} = \frac{T_L}{T_H}$

The efficiency η - $1 - \frac{T_L}{T_H}$

Note : T_L and T_H should be expressed in Kelvin scale.

23. Explain the second law of thermodynamics in terms of entropy.

Entropy and second law of thermodynamics

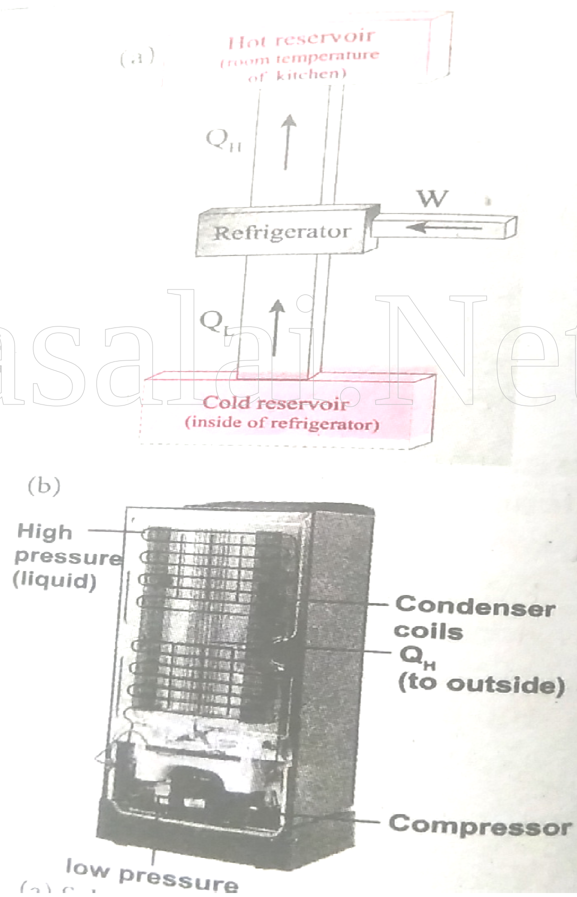
1) We have seen in the equation that the quantity $\frac{Q_H}{T_H}$. Is equal to $\frac{Q_L}{T_L}$ the quantity $\frac{Q}{T}$ is called entropy. It is a very important thermodynamic property of a system.
2) It is also a state variable. $\frac{Q_H}{T_H}$ is the entropy received by the Carnot engine from hot reservoir and $\frac{Q_L}{T_L}$ is entropy given out by the Carnot engine to the cold reservoir. For reversible engines (Carnot Engine) both entropies should be same, so that the change in entropy of the Carnot engine in one cycle is zero. But for all practical engines like diesel and petrol engines which are not reversible engines, they satisfy the relation $\frac{Q_L}{T_L} > \frac{Q_H}{T_H}$.
3) In fact we can reformulate the second law of thermodynamics as follows "For all the processes that occur in nature (irreversible process), the entropy always increases. For reversible process entropy will not change". Entropy determines the direction in which natural process should occur.
4) Because entropy increases when heat flows from hot object to cold object. If heat were to flow from a cold to a hot object, entropy will decrease leading to violation of second law thermodynamics.
5) Entropy is also called 'measure of disorder'. All natural process occur such that the disorder should always increases.
6) Consider a bottle with a gas inside. When the gas molecules are inside the bottle it has less disorder. Once it spreads into the entire room it leads to more disorder.

7) In other words when the gas is inside the bottle the entropy is less and once the gas spreads into entire room, the entropy increases. From the second law of thermodynamics, entropy always increases.
8) If the air molecules go back in to the bottle, the entropy should decrease, which is not allowed by the second law of thermodynamics.
9) The same explanation applies to a drop of ink diffusing into water. Once the drop of ink spreads, its entropy is increased. The diffused ink can never become a drop again. So the natural processes occur in such a way that entropy should increase for all irreversible process.

24. Explain in detail the working of a refrigerator.

REFRIGERATOR

A refrigerator is a Carnot's engine working in the reverse order.



Working Principle:

The working substance (gas) absorbs a quantity of heat Q_L from the cold body (sink) at a lower temperature T_L. A certain amount of work W is done on the working substance by the compressor and a quantity of heat Q_H is rejected to the hot body (source) ie, the atmosphere at T_H. When you stand beneath of refrigerator, you can feel warmth air.

From the first law of thermodynamics ,

we have $Q_L + W = Q_H$

As a result the cold reservoir (refrigerator) further cools down and the surroundings (kitchen or atmosphere) gets hotter.