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# 1. Natural of Physical World and Measurement

**1. What is meant by Scientific method?**

The scientific method is a step-by-step approach in studying natural phenomena and establishing laws which govern these phenomena.

**2. What are the general features of scientific method?**

- Systematic Observation
- Controlled experimentation
- Qualitative and quantitative reasoning
- Mathematical modeling
- Prediction and verification or falsification of theories.

**3. What type of approaches are followed in studying physics?**

- Unification
- Reductionism

**4. What is Unification? Give the example.**

An attempt to explain various physical phenomena with a few concepts and laws is Unification.

Ex: Newton's universal law of gravitation explains various events like motion of freely falling body, motion of the planets around the sun, motion of the moon around the earth.

**5. What is reductionism? Give the example.**

An attempt to explain a macroscopic sysytem in terms of its microscopic constituents is reductionism.

Ex: Macroscopic properties like temperature, entropy, etc., of bulk systems can be easily interpreted in terms of the molecular motion(microscopic constituents).

**6. What is technology?**

Technology is the application of principles of physics for practical purposes.

**7. Describe the relation of Physics with Chemistry.**

- Studies of structure of atom, radioactivity, X-ray diffraction, etc., in physics have been used in chemistry to arrange elements in periodic table on the basis of atomic numbers.
- It is further helped to know the nature of valence and chemical bonding and to understand the complex chemical structures.
- Inter-disciplinary branches like Physical chemistry and Quantum chemistry plays vital role here.

**8. Describe the relation of Physics with Biology.**

- It is impossible to study biology without microscope designed using physical principles.
- Invention of electron microscope has made possible to see even the structure of a cell.
- X-ray diffraction and neutron diffraction techniques are helped to understand the structure of nucleic acids, which helps to control vital life processes.
- X-rays are used for diagnostic purposes.
- Radio-isotopes are used in radiotherapy for the treatment of cancer and other diseases.
- Now-a-days biological processes are being studied from the physical point of view.

**9. Describe the relation of Physics with mathematics.**

- Physics is a quantitative science.
- Physics is closely related to mathematics as a tool for its developement.

**10. Describe the relation of Physics with Astronomy.**

- Astronomical telescopes are used to study the motion of the planets and other celestial bodies in the sky.
- Radio telescopes are used to observe distant points of the universe.
- Studies of the universe are done using physical principles.

**11. Describe the relation of Physics with Geology.**

- Diffraction techniques helps to study the crystal structure of various rocks.
- Radioactivity is used to estimate the age of rocks, fossils and the age of the Earth.

**12. Describe the relation of Physics with Oceanography.**

- Oceanographers seek to understand the physical and chemical processes of the oceans.
- For that, they measure parameters such as temperature, salinity, current speed, gas fluxes and chemical components of the ocean.

**13. Describe the relation of Physics with Psychology.**

- All the psychological interactions can be derived from a physical process.
- The movements of neurotransmitters are governed by the physical properties of diffusion and molecular motion.
- The function of our brain is related to our underlying dualism (wave -particle nature).

**14. What is measurement?**

The comparison of any physical quantity with its standard unit is known as measurement.

**15. What is physical quantity? Give the examples.**

Quantities that can be measured and in terms of which laws of physics are described are called physical quantities. **Ex :** length, mass, time, force, energy, etc.,

**16. What is fundamental or base quantities? Give the examples.**

The quantities, which cannot be expressed in terms of any other physical quantities, are called fundamental or base quantities.

**Ex:** length, mass, time, electric current, temperature, luminous intensity and amount of substance.

**17. What is derived quantities? Give the examples.**

Quantities that can be expressed in term of fundamental quantities are called derived quantities.

**Ex:** area, volume, velocity, acceleration, force.

**18. What is an unit?**

An arbitrarily chosen standard of measurement of a quantity, which is accepted internationally is called unit of the quantity.

**19. What is System of Units?**

A complete set of units which is used to measure all kinds of fundamental and derived quantities is called a system of units.

**20. What is the f.p.s system?**

The f.p.s system is the British Engineering system in which length, mass and time are measured in foot, pound and second respectively.

**21. What is the c.g.s system?**

The c.g.s system is the Gaussian system in which length, mass and time are measured in centimeter, gram and second respectively.

**22. What is the m.k.s system?**

In the m.k.s system, length, mass and time are measured in meter, kilogram and second respectively.

**23. What are the advantages of SI unit system?**

- It is a rational system, in which only one unit is used for one physical quantity.
- It is a coherent system, which means all the derived units can be easily obtained form basic and supplementary units.
- It is a metric system, which means multiples and submultiples can be expressed as powers of 10.

**24. What is the SI standard of length? (or) What is one meter in SI system of units?**

One meter is the length of the path travelled by light in vacuum in $\frac{1}{299,792,458}$ of a second.

**25. What is the SI standard of mass? (or) What is one kilogram in SI system of units?**

One kilogram is the mass of the prototype cylinder of platinum iridium alloy (whose height is equal to its diameter), preserved at the International Bureau of Weights and Measures at Serves, near Paris, France.

**26. What is the SI standard of time? (or) What is one second in SI system of units?**

One second is the duration of 9,192,631,770 periods of radiation corresponding to the transition between the two hyperfine levels of the ground state of Cesium-133 atom.

**27. What is the SI standard of electric current? (or) What is one ampere in SI system of units?**

One ampere is the constant current, which flows through the two parallel straight conductors of infinite length with negligible cross-section, held one meter apart in vacuum, shall produce a force per unit length of $2 \times 10^{-7}$ $Nm^{-1}$ between them.

**28. What is the SI standard of temperature? (or) What is one kelvin in SI system of units?**

One kelvin is the fraction of $\frac{1}{273.16}$ of the thermodynamic temperature of the triple point of the water.

**29. What is the SI standard of amount of substance ? (or) What is one mole in SI system of units?**

One mole is the amount of substance which contains as many elementary entities as there are atoms in 0.012 kg of pure carbon-12.

**30. What is the SI standard of luminous intensity? (or) What is one candela in SI system of units?**

One candela is the luminous intensity of a source in a given direction that emits monochromatic radiation of frequency $5.4 \times 10^{-14}$ Hz and that has a radiant intensity of $\frac{1}{683}$ watt / steradian in that direction.

**31. what is length? Give its SI unit.**

Length is defined as the distance between any two points in space. Its SI unit is metre.

**32. what is one radian?**

One radian is the plane angle subtended by an arc whose arc length is equal to its radius.

**33. What is one steradian?**

One steradian is the solid angle subtended by the partial surface of a sphere whose suface area is equal to the square of its radius.

**34. Explain the use of Screw gauge in measuring smaller distances.**

- It is used to measure accurately the dimension of objects upto the maximum of 50 mm.
- The principle of the instrument is the magnification of linear motion using circular motion of a screw.
- The least count of the screw gauge is 0.01 mm.

**35. Explain the use of vernier caliper in measuring smaller distances.**

- It is a versatile instrument for measuring the dimensions of an object like diameter and depth of a hole.
- The least count of the vernier caliper is 0.1 mm.

**36. What are the methods adopted in measuring larger distances?**

- Triangulation method
- Parallax method
- Radar method

**37. What is Parallax?**

The shift in the position of an object (say a pen) when viewed with left and right eye alternatively is known as Parallax.

**(or)**

The apparent chage in position of an object with respect to its background, when viewed from two different locations is called Parallax.

**38. What is the abbreviation for RADAR?**

The word RADAR stands for RAdio Detection And Ranging.

**39. What is 1 light year ? Give its value.**

1 light year is the distance travelled by light in vacuum in one year.

1 light year = $9.467 \times 10^{15}$ m.

**40. What is 1 astronomical unit(AU)? Give its value.**

1 astronomical unit is the mean distance between earth and the sun.

1 AU = $1.496 \times 10^{11}$ m.

**41. What is 1 parsec (Parallactic second)? Give its value.**

1 parsec is the radial distance of an arc of arc length 1 AU subtends an angle of 1 second.

1 parsec = $3.08 \times 10^{16}$ m = 3.26 light year.

**42. Define mass?**

Mass of a body is defined as the quantity of matter contained in a body. The SI unit of mass is kilogram.

**43. What is the difference between Accuracy and Precision?**

| S.No. | Accuracy | Precision |
|---|---|---|
| 1 | Measurements close to true value. | Measurements close to each other. |
| 2 | All the accuracy values are precised. | All the precised values are not accurate. |

**44. What is meant by an error? Name its types.**

The uncertainity in a measurement is called an error.

**Types:** Systematic errors, Random errors & Gross errors

**45. What is meant by Systematic error?**

- Systematic errors are reproducible inaccuracies that are consistently in the same direction.
- These occur offen due to a problem that persists throughout the experiment.

**46. What are the Classifications of Systematic errors?**

- Instrumental errors
- Imperfections in experimental techniques or procedure.
- Personal errors
- Errors due to external causes.
- Least count error

**47. Describe Instrumental errors. How is it minimised?**

- It is happened when an instrument is not calibrated properly at the time of manufacture.
- For example, If a measurement is made with a meter scale whose end is worn out, result obtains error.
- These errors can be rectified by using the good quality instruments.

**48. Describe Imperfections in experimental technique or procedure. How can it be overcomed?**

- These errors arise due to the limitations in the experimental arrangements.
- For example, Calorimeter experiment is done without insulation makes radiation loss. This results errors.
- It can be overcomed by applying necessary correction.

**49. Describe the Personal errors.**

These errors occur due to individual performing experiment without initial setting up or careless observation without precautions.

**50. Describe the errors due to external causes.**

These errors are due to external conditions like change in temperature, humidity or pressure during an experiment.

**51. Describe the least count error. How can it be minimised?**

- Least count is the smallest value that can be measured by an instrument.
- The error due to the measurement in least count is called least count error.
- It can be minimised by using high precision instrument.

**52. Describe Random errors. How can it be minimised?**

- Random errors may arise due to random and unpredictable variation in experimental conditions like pressure,temperature, voltage supply,etc.,
- It is also due to personal errors.
- These errors are happened by chance, so it is called "Chance error".

- It can be minimised by calculating arithmatic mean of measurements taken. i.e. If 'n' number of readings $a_1, a_2, a_3, ...., a_n$ are done, the arithmatic mean is given by,

$$a_m = \frac{a_1 + a_2 + a_3 + \cdots + a_n}{n}$$

$$a_m = \frac{1}{n}\sum_{i=1}^{n} a_i$$

**53. Describe Gross error. How can it be minimised?**

The error caused due to the complete carelessness of an observer is called gross error.

**Example :**

- Reading an instrument without setting properly.
- Taking observations in a wrong manner without considering source of errors and the precautions.
- Recording wrong observations.
- Using wrong values of the observations in calculations.

These errors can be minimised only when an observer is careful and mentally alert.

**54. What is meant by Absolute error? Explain.**

- The magnitude of difference between true value and measured value of a quantity is called absolute error.
- If $a_1, a_2, a_3, ...., a_n$ are the measured values of any quantity, then the arithmatic mean is the true value of the measurements.

$$a_m = \frac{a_1 + a_2 + a_3 + \cdots + a_n}{n}$$

$$a_m = \frac{1}{n}\sum_{i=1}^{n} a_i$$

- The absolute error is given by,

$$\Delta a_1 = |a_m - a_1|$$
$$\Delta a_2 = |a_m - a_2|$$
.........................
.........................
$$\Delta a_n = |a_m - a_n|$$

**55. What is meant by mean Absolute error? Explain.**

The arithmatic mean of the magnitude of absolute errors in all the measurements is called the mean absolute error.

$$\Delta a_m = \frac{|\Delta a_1| + |\Delta a_2| + |\Delta a_3| + \cdots + |\Delta a_n|}{n}$$

$$\Delta a_m = \frac{1}{n}\sum_{i=1}^{n} |\Delta a_i|$$

**56. What is meant by Relative error? Explain.**

The ratio between mean absolute error to the mean value is called relative error. This is also called fractional error.

$$Relative\ error = \frac{Mean\ absolute\ error}{Mean\ value}$$
$$= \frac{\Delta a_m}{a_m}$$

**57. What is meant by Percentage error? Explain.**

The relative error expressed in percentage is called percentage error.

Percentage error = $\frac{\Delta a_m}{a_m}$ x 100 %

**58. What are the factors affecting error in final result?**

- The errors in the individual measurements.
- Nature of mathematical operations.

**59. What are significant figures?**

The number of digits which are counted reasonably sure in making a measurement are called significant figures.

**60. Explain the significant figures in addition and subtraction with the examples.**

In addtion and subtraction, the final result should retain as many decimal places as there are in the original number with the smallest number of decimal places.

**Example:**

**(i) Addition:**

3.1 + 1.780 + 2.046 = 6.926 is rounded off to **6.9** as the number 3.1 has least one decimal place.

**(ii) Subtraction:**

12.637 – 2.42 = 10.217 is rounded off to **10.22** as the number 2.42 has least two decimal places.

**61. Explain the significant figures in multipliication and division with the examples.**

In multiplication and division, the final result should retain as many significant figures as there are in the original number with the smallest number of significant figures.

**Examples:**

**(i) Multiplication:**

1.21 x 36.72 = 44.4312 is rounded off to **44.4** as the number 1.21 has least 3 significant figures.

**(ii) Division:**

36.72 ÷ 1.2 = 30.6 is rounded off to **31** as the number 1.2 has least 2 significant figures.

**62. What is dimension?**

The dimensions of a physical quantity are the powers to which the unit of base quantities are raised to represent a derived unit of that quantity.

**63. What is dimensional formula? Give an example.**

Dimensional formula is an expression which shows how and which of the fundamental units are required to represent the unit of a physical quantity.

**Ex:** $[M^0LT^{-2}]$ is the dimensional formula of acceleration.

**64. What is dimensional equation? Give an example.**

When the dimensional formula of a physical quantity is expressed in the form of an equation, such equation is known as the dimensional equation.

**Ex:** acceleration = $[M^0LT^{-2}]$

**65. What is dimensional variables?**

Physical quantities, which have dimension and have variable values are called dimensional variables.

**Examples:** length, velocity, acceleration, etc.,

**66. What is dimensionless variables?**

Physical quantities,which have no dimension and have variable values are called dimensional variables.

**Examples:** specific gravity, strain, refractive index,etc.,

**67. What is dimensional constants?**

Physical quantities, which have dimension and have constant values are called dimensional constants.

**Examples:** gravitational constant, planck's constant, etc.,

**68. What is dimensionless constants?**

Physical quantities,which have no dimension and have constant values are called dimensionless constants.

**Examples:** $\pi$, e, numbers, etc.,

**69. What is principle of homogeneity of dimensions?**

The principle of homogeneity of dimension states that the dimensions of all the terms in a physical expression should be the same.

**70. What are the applications of dimensional analysis method?**

- Convert a physical quantity from one system of units to another.
- Check the dimensional correctness of a given physical equation.
- Establish relations among various physical quantities.

**71. What are the limitations of dimensional analysis method?**

- It gives no information about the dimensionless constants like numbers, $\pi$, e, etc., in the formula.
- It cannot decide whether the given quantity is a scalar or vector.
- It is not suitable to derive relations involving trignometry, exponential and logarithmic functions.
- It cannot be applied to an equation involving more than three physical quantities.
- It can only check dimensional correctness of an equation but not the correctness of the equation.

## 5 Marks Q & A:

**1. Explain the propagation of error in the sum of two quantities?**

- Let $\Delta A$ and $\Delta B$ be the absolute errors in the two quantities A and B respectively.
- Then,
  Measured value of A = $A \pm \Delta A$
  Measured value of B = $B \pm \Delta B$
  Consider the sum, Z = A + B
- The error $\Delta Z$ in Z is given by,
  $Z \pm \Delta Z = (A \pm \Delta A) + (B \pm \Delta B)$
  $Z \pm \Delta Z = (A + B) \pm (\Delta A + \Delta B)$
  $Z \pm \Delta Z = Z \pm (\Delta A + \Delta B)$ $[\because Z = A + B]$
  $\Delta Z = \Delta A + \Delta B$
- The maximum possible error in the sum of two quantities is equal to the sum of the absolute errors in the individual quantities.

**2. Explain the propagation of error in the difference of two quantities?**

- Let $\Delta A$ and $\Delta B$ be the absolute errors in the two quantities A and B respectively.
- Then,
  Measured value of A = $A \pm \Delta A$
  Measured value of B = $B \pm \Delta B$
  Consider the difference, Z = A - B
- The error $\Delta Z$ in Z is given by,
  $Z \pm \Delta Z = (A \pm \Delta A) - (B \pm \Delta B)$
  $Z \pm \Delta Z = (A - B) \pm (\Delta A + \Delta B)$
  $Z \pm \Delta Z = Z \pm (\Delta A + \Delta B)$ $[\because Z = A - B]$
  $\Delta Z = \Delta A + \Delta B$
- The maximum possible error in the difference of two quantities is equal to the sum of the absolute errors in the individual quantities.

**3. Explain the propagation of error in the product of two quantities?**

- Let $\Delta A$ and $\Delta B$ be the absolute errors in the two quantities A and B respectively.
- Then,
  Measured value of A = $A \pm \Delta A$
  Measured value of B = $B \pm \Delta B$
  Consider the product, Z = A . B -----> (1)
- The error $\Delta Z$ in Z is given by,
  $Z \pm \Delta Z = (A \pm \Delta A) . (B \pm \Delta B)$
  $Z \pm \Delta Z = AB \pm A.\Delta B \pm B.\Delta A \pm \Delta A.\Delta B$ -----> (2)
  Dividing equation (2) by (1) we get,
  $$1 \pm \frac{\Delta Z}{Z} = 1 \pm \frac{\Delta B}{B} \pm \frac{\Delta A}{A} \pm \frac{\Delta A}{A}.\frac{\Delta B}{B}$$
  As $\frac{\Delta A}{A}$ and $\frac{\Delta B}{B}$ are both smaller values, their products $\frac{\Delta A}{A}.\frac{\Delta B}{B}$ can now be neglected. The maximum fractional error in Z is,
  $$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$
- The maximum fractional error in the product of two quantities is equal to the sum of the fractional errors in the individual quantities.

**4. Explain the propagation of error in the division or quotient of two quantities?**

- Let $\Delta A$ and $\Delta B$ be the absolute errors in the two quantities A and B respectively.
- Then,
  Measured value of A = $A \pm \Delta A$
  Measured value of B = $B \pm \Delta B$
  Consider the division, $Z = \frac{A}{B}$
- The error $\Delta Z$ in Z is given by,

$$Z \pm \Delta Z = \frac{A \pm \Delta A}{B \pm \Delta B} = \frac{A\left(1 \pm \frac{\Delta A}{A}\right)}{B\left(1 \pm \frac{\Delta B}{B}\right)}$$

$$Z \pm \Delta Z = \frac{A}{B}\left(1 \pm \frac{\Delta A}{A}\right)\left(1 \pm \frac{\Delta B}{B}\right)^{-1}$$

- By using binomial theorem, $(1+x)^n = 1 + nx$ , when $x<<1$, we get,

$$1 \pm \frac{\Delta Z}{Z} = \left(1 \pm \frac{\Delta A}{A}\right)\left(1 \mp \frac{\Delta B}{B}\right)$$

$$1 \pm \frac{\Delta Z}{Z} = 1 \pm \frac{\Delta A}{A} \mp \frac{\Delta B}{B} \pm \frac{\Delta A}{A}.\frac{\Delta B}{B}$$

As $\frac{\Delta A}{A}$ and $\frac{\Delta B}{B}$ are both smaller values, their products $\frac{\Delta A}{A}.\frac{\Delta B}{B}$ can now be neglected. The maximum fractional error in Z is,

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

- The maximum fractional error in the product of two quantities is equal to the sum of the fractional errors in the individual quantities.

**5. Explain the propagation of error in the power or a quantity?**

- Let $\Delta A$ and $\Delta B$ be the absolute errors in the two quantities A and B respectively.
- Then,
  Measured value of A = $A \pm \Delta A$
  Measured value of B = $B \pm \Delta B$
  Consider the $n^{th}$ power of A, $Z = A^n$
- The error $\Delta Z$ in Z is given by,

$$Z \pm \Delta Z = (A \pm \Delta A)^n = A^n\left(1 \pm \frac{\Delta A}{A}\right)^n$$

$$Z \pm \Delta Z = A^n\left(1 \pm \frac{\Delta A}{A}\right)^n$$

By using binomial theorem, we solve and get,

$$1 \pm \frac{\Delta Z}{Z} = 1 \pm n\frac{\Delta A}{A}$$

$$\frac{\Delta Z}{Z} = n\frac{\Delta A}{A}$$

The fractional error in the $n^{th}$ power of a quantity is n times the fractional error in that quantity.

**6. Explain the rules framed to count significant figures with the examples.**

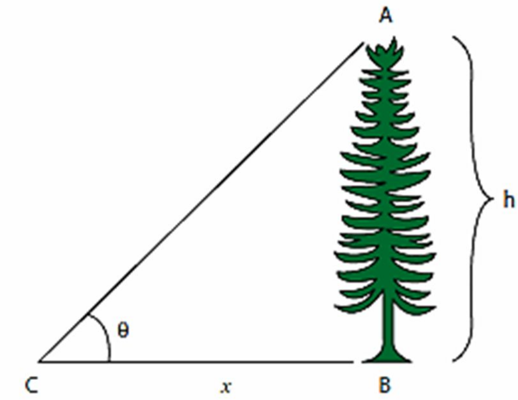
| S. No. | Rule | Example |
|---|---|---|
| 1. | All non-zero digits are significant | 1342 has **four** significant figures |
| 2. | All zeros between two non-zero digits are significant | 2008 has **four** significant figures |
| 3. | All zeros right to non-zero digit but left to decimal point are significant. | 30700. has **five** significant figures |
| 4. | The terminal or trailing zeros in the number without decimal point are not significant. | 30700 has **three** significant figures. |
| 5. | All zeros are significant if the number given with measurement unit. | 30700 m has **five** significant figures. |
| 6. | If a number is less than1, the zeros between decimal point and first non-zero digit are not significant but the zeros right to last non-zero digit are significant. | (i) 0.00345 has **three** significant figures.<br>(ii) 0.030400 has **five** significant figures.<br>(iii) 40.00 has **four** significant figures. |
| 7. | The number of significant figures doesn't depend on the system of units used | 1.53 cm, 0.0153 m, 0.0000153 km all have **three** significant figures. |

**7. Explain the rules framed for rounding off the numbers with the examples.**

| S. No. | Rule | Example |
|---|---|---|
| 1. | If the dropping digit is less than 5, then preceding digit kept unchanged. | 7.3<u>2</u> is rounded off to 7.3 |
| 2. | If the dropping digit is greater than 5, then preceding digit must be raised by 1. | 17.2<u>6</u> is rounded off to 17.3 |
| 3. | If the dropping digit is 5 followed by non-zero digits then preceding digit must be raised by1 | 7.3<u>5</u>2 is rounded off to 7.4 |
| 4. | If the dropping digit is 5 or 5 followed by zero, then preceding digit must be raised by 1 if it is odd. | 3.3<u>5</u> & 3.3<u>5</u>0 are rounded off to 3.4 |
| 5. | If the dropping digit is 5 or 5 followed by zero, then preceding digit is not changed if it is even. | 3.4<u>5</u> & 3.4<u>5</u>0 are rounded off to 3.4 |

**8. Explain the Triangulation method to find the height of the tree or the tower.**

- Let AB = h be the height of the tree or tower.
- Let C be the the point of observation at distance x from B.
- Place a range finder at C and measure the angle of elevation, ∠ACB = θ as shown in figure.

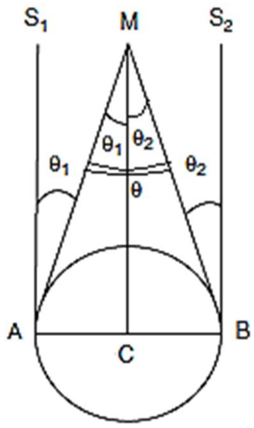



- From $\triangle ABC$, $tan\theta = \frac{AB}{BC} = \frac{h}{x}$

  or $\boxed{h = x \tan\theta}$
- Knowing the distance $x$, the height h can be determined.

**9. Explain the Parallax method to find the distance of the Moon from Earth.**

- Let C be the centre of the Earth.
- A & B are two diametrically opposite places on the surface of the Earth.
- AB is the diameter of the Earth and MC is the distance of the Moon from Earth.
- $\theta_1$ & $\theta_2$ are the parallactic angles of the Moon with respect to some distant stars $S_1$ & $S_2$ respectively.
- $\theta_1$ and $\theta_2$ are determined by using an astronomical telescope.

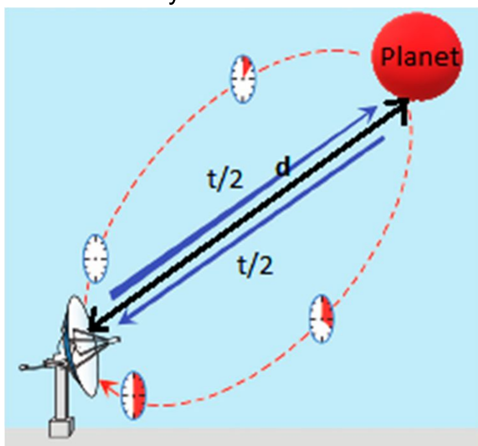



- Thus, the total parallactic angle(parallax) of the Moon subtended on Earth, ∠AMB = $\theta_1 + \theta_2 = \theta$.
- From the figure, $\theta = \frac{AB}{AM} = \frac{AB}{MC}$ $[\because AM = MC]$

  $$\boxed{MC = \frac{AB}{\theta}}$$
- Knowing the values of AB and θ, We can calculate distance of the Moon from Earth (MC).

**10. Explain the RADAR method to find the distance of a nearby Planet.**

- The word RADAR stands for RAdio Detection And Ranging.
- In this method, radio waves are sent from transmitters which after reflected from the planet are detected by the receiver.




- By measuring the time interval between the instants the radio waves are sent and received, the distance of the planet(d) can be detemined as,

  Distance = Speed of radio waves X time taken,

  $$\boxed{d = \frac{v \times t}{2}}$$
- Where v is the speed of radio waves and t is the time taken by radio waves to travel forward and backward hence t/2 is the time taken to cover the distance d.

**11. Explain the conversion of physical quantity from one system of units to another with the examples.**

- The product of the numerical value (n) and its corresponding unit [u] is constant.

  n [u] = constant
- Consider Physical quantity which has dimension 'a' in mass, 'b' in length and 'c' in time.
- If the fundamental unit in one system are $M_1$ , $L_1$ & $T_1$ and in other system are $M_2$ , $L_2$ & $T_2$, then we can write,

  $$n_1[M_1^a\,L_1^b\,T_1^c] = n_2[M_2^a\,L_2^b\,T_2^c]$$

**Example: 1**

**Convert 76 cm of mercury into Nm$^{-2}$ using the method of dimension.**

Data => h = 76 cm ; ρ = 13.6 g cm$^{-3}$ ; g = 980 cm s$^{-2}$.

**Solution:**

$P_1$ = hρg =76 x 13.6 x 980 = 1.01 x 10$^6$ dyne cm$^{-2}$.

- The dimension formula of pressure P is [M L$^{-1}$ T$^{-2}$]

  So, **a = 1 , b = -1 , c = -2**

  $$P_1[M_1^a\,L_1^b\,T_1^c] = P_2[M_2^a\,L_2^b\,T_2^c]$$

  $$P_2 = P_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

| | | |
|---|---|---|
| $M_1$ = 1 g | $L_1$ = 1 cm | $T_1$ = 1 s |
| $M_2$ = 1 kg | $L_2$ = 1 m | $T_2$ = 1 s |

$$P_2 = 1.01 \times 10^6 \left[\frac{1\ g}{1\ kg}\right]^1 \left[\frac{1\ cm}{1\ m}\right]^{-1} \left[\frac{1\ s}{1\ s}\right]^{-2}$$

$$P_2 = 1.01 \times 10^6 \left[\frac{10^{-3}\ kg}{1\ kg}\right]^1 \left[\frac{10^{-2}\ m}{1\ m}\right]^{-1} \left[\frac{1\ s}{1\ s}\right]^{-2}$$

$= 1.01 \times 10^6 \times 10^{-3} \times 10^2 \times 1$

$P_2 = 1.01 \times 10^5\ Nm^{-2}$.

**Example 2:**

**If the value of universal gravitational constant in SI is $6.6 \times 10^{-11}\ Nm^2\ kg^{-2}$ , then find its value in CGS System?**

**Solution:**

Let $G_{SI}$ be the gravitational constant in SI system and $G_{cgs}$ be in cgs system.

$G_{SI} = 6.6 \times 10^{-11}\ Nm^2\ kg^{-2}$

❖ The dimensional formula for G is $M^{-1} L^3 T^{-2}$ .

**a = -1, b = 3, c = -2**

$$G_{cgs} = G_{SI} \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

| | | |
|---|---|---|
| $M_1$ = 1 kg | $L_1$ = 1 m | $T_1$ = 1 s |
| $M_2$ = 1 g | $L_2$ = 1 cm | $T_2$ = 1 s |

$$G_{cgs} = 6.6 \times 10^{-11} \left[\frac{1\ kg}{1\ g}\right]^{-1} \left[\frac{1\ m}{1\ cm}\right]^3 \left[\frac{1\ s}{1 s}\right]^{-2}$$

$$G_{cgs} = 6.6 \times 10^{-11} \left[\frac{1\ kg}{10^{-3}\ kg}\right]^{-1} \left[\frac{1\ m}{10^{-2}\ m}\right]^3 \left[\frac{1\ s}{1 s}\right]^{-2}$$

$= 6.6 \times 10^{-11} \times 10^{-3} \times 10^6 \times 1$

$G_{cgs} = 6.6 \times 10^{-8}$ dyne $cm^2\ g^{-2}$

**12.Check the dimensional correctness of a given equation with the examples.**

**Example 1:**

Consider an equation, **v = u + at**

Apply dimensions on both sides, we get,

$[LT^{-1}] = [LT^{-1}] + [LT^{-2}]\ [T^{-1}]$

$[LT^{-1}] = [LT^{-1}] + [LT^{-1}]$

Since dimensions on both sides are same, the given equation is dimensionally correct.

**Example 2:**

Consider an equation, $\frac{1}{2}\boldsymbol{mv^2} = \boldsymbol{mgh}$

Apply dimensions on both sides, we get,

$[M]\ [LT^{-1}]^2 = [M]\ [LT^{-2}]\ [L]$

$[ML^2T^{-2}] = [ML^2T^{-2}]$

Since dimensions on both sides are same, the given equation is dimensionally correct.

**13.Explain how to establish the relation among various physical quantities with the examples.**

❖ If physical quantity Q depends on $Q_1$, $Q_2$ and $Q_3$, then we write, $Q \propto Q_1^a\ Q_2^b\ Q_3^c$

$$Q = k\ Q_1^a\ Q_2^b\ Q_3^c$$

❖ Where k is dimensionless constant. Applying the dimensions of $Q_1$, $Q_2$ and $Q_3$ and using principle of homogeneity, the powers of M, L and T are made equal on both sides. Then we get values of a, b and c to form relation.

**Example 1:**

**Obtain an expression for the time period of T of a simple pendulam. The time period T depends upon (i) mass 'm' of the bob (ii) length 'l' of the pendulam and (iii) acceleration due to gravity 'g' at the place where the pendulam is suspended. (Constant k = 2π)**

**Solution:**

$T\ \alpha\ m^a\ l^b\ g^c$

$T = k\ m^a\ l^b\ g^c$ -----> (1)

❖ Here k is dimensional constant. Applying dimensions on both sides, we get,

$[T] = [M^a]\ [L^b]\ [LT^{-2}]^c$

$[M^0\ L^0\ T^1] = [M^a\ L^{b+c}\ T^{-2c}]$

❖ Comparing the powers of M, L, T on both sides, a = 0, b + c = 0, -2c = 1 .

❖ Solving it, we get, a = 0, b =1/2, c= -1/2

❖ From equation (1),

$T = 2\pi\ m^0\ l^{1/2}\ g^{-1/2}$

$$\boxed{T = 2\pi\sqrt{\frac{l}{g}}}$$

**Example 2:**

**The force F acting on a body moving in a circular path depends on mass of the body (m), velocity and radius (r) of the circular path.Obtain the expression for the force by dimensional analysis method. (Take the value of k = 1)**

**Solution:**

$F\ \alpha\ m^a\ v^b\ r^c$

$F = k\ m^a\ v^b\ r^c$ -----> (1)

❖ Here k is dimensional constant. Applying dimensions on both sides, we get,

$[M\ L\ T^{-2}] = [M^a]\ [LT^{-1}]^b\ [L]^c$

$[M\ L\ T^{-2}] = [M^a\ L^{b+c}\ T^{-b}]$

❖ Comparing the powers of M, L, T on both sides, a = 1, b + c = 1, -b = -2 .

❖ Solving it, we get, a = 1, b = 2, c= -1

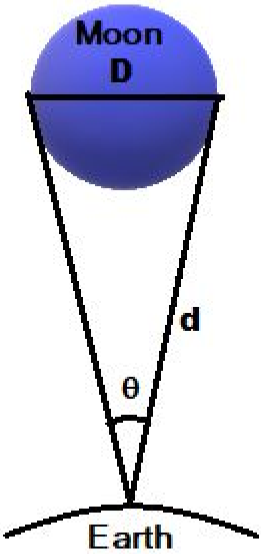
❖ From equation (1),

$F = m^1\ v^2\ r^{-1}$

$$\boxed{F = \frac{mv^2}{r}}$$

## Book back Q & A:

1. **How will you measure the diameter of the Moon using parallax method?**
   - Let θ be the angle subtended by the Moon at the surface of the Earth.
   - d is the distance of the Moon from Earth.
   - D is the diameter of the Moon.




   - From figure,

     Arc angle, $\theta = \frac{D}{d}$
   - From this, diameter of the Moon, D = d . θ
   - Knowing the values of **d** and θ, we can calculate the diameter of the Moon.

## Mark distribution

| Exam | Total marks | Pass mark |
|---|---|---|
| Theory | 70 | 15 |
| Practical | 20 | 20 |
| Internal Assessment | 10 | (or) Exam attended |
| Total | 100 | 35 |

## Internal Assessment:

1. **Attendance:** 2
   Above 80% - 2 Marks
   75-80 % - 1 Mark
2. **Internal class test:** 4
   (Calculated to 4 marks from best three test)
3. **Assignment :** 2
4. **Co-curricular activities** 2
   (Any 3 activities out of 33 given)

Total : 10

## External Exam:

1. **Record Note** 3
2. **Expt. Skill** 2
3. **Practical Exam** 15

Total : 20

## Question Pattern :

| Part/ Question type | Marks | No. of questions asked | No. of questions to be answered | Total marks |
|---|---|---|---|---|
| I 1 marks | 1 | 15 | 15 | 15 |
| II Short Q | 2 | 8+1(9) | 5+1(6) | 12 |
| III Brief Q | 3 | 8+1(9) | 5+1(6) | 18 |
| IV Long Q | 5 | 5 (with internal choice) | 5 | 25 |
| | | | மொத்தம் | 70 |

Note : Part II and Part III have one compulsory question respectively.

## 2. Kinematics

**1. What is kinematics?**

Kinematics is the branch of mechanics which deals with the motion of objects without taking force into account.

**2. What is meant by Frame of reference?**

Frame of reference is a coordinate system with respect to which position of an object is described.

**3. What is meant by Cartesian coordinate system?**

Cartesian coordinate system is the frame of reference with respect to which the position of the object is described in terms of position coordinates(x,y,z).

**4. What is the point mass? Give the examples.**

The mass of an object, which is concentrated at a point is called "point mass". It has no internal structures like shape and size.

Example: (i) In the event of motion of Earth around the Sun, Earth can be treated as point mass.
(ii) When stone is thrown in space, stone is considered as point mass.

**5. What are the types of motion?**

- Linear motion
- Circular motion
- Rotational motion
- Vibratory (or) Oscillatory motion.

**6. What is linear motion? Give the examples.**

When an object is moving in a straight line, it is called linear motion.

Example: (i) An athlete running on a straight track.
(ii) A particle falling vertically downwards.

**7. What is circular motion? Give the examples.**

When an object is moving in a circular path, it is called circular motion.

Example: (i) The whirling motion of a stone attached to a string.
(ii) The motion of a satellite around the Earth.

**8. What is Rotational motion? Give the examples.**

If any object is revolving about an axis, the motion is called Rotational motion.

Example: (i) Rotation of a disc about its central axis.
(ii) Spinning of the Earth about its own axis.

**9. What is vibratory motion? Give the examples.**

If an object executes to and fro motion about a fixed point, it is called vibratory or oscillatory motion.

Example: (i) Vibration of a string on a guitar.
(ii) movement of a swing.

**10. What is one dimensional motion? Give the examples.**

Motion of a particle along a straight line is called one dimensional motion.

Example: (i) Motion of a train along a straight track.
(ii) An object falling freely down under gravity.

**11. What is two dimensional motion? Give the examples**

Curved motion of a particle in a plane is called two dimensional motion.

Example: (i) Motion of a coin on a carrom board.
(ii) An insect crawling over the floor.

**12. What is three dimensional motion?Give the examples**

If a particle moving in a three dimensional space, then it is called three dimensional motion.

Example: (i) A bird flying in the sky.
(ii) Random motion of molecules.
(iii) Flying kite on a windy day.

**13. What is Scalar? Give examples**

A physical quantity which can be described only by magnitude is called Scalar.

Ex: Distance, mass, temperature, speed, energy, etc.,

**14. What is Vector? Give examples**

A physical quantity which can be described by both magnitude and direction is called Vector.

Ex: Force, velocity, displacement, acceleration, etc.,

**15. How to denote a vector quantity?**

A vector quantity can be geometrically represented by line arrow, in which lengh of the line denotes magnitude and arrow denotes its direction.

**16. What are the types of vectors?**

- Equal vectors
- Collinear vectors
- Parallel vectors
- Anti-parallel vectors
- Unit vectors
- Orthogonal unit vectors

**17. What is equal vectors?**

Two vectors of same physical quantity having same magnitude and direction are called equal vectors.

**18. What is collinear vectors?**

Two vectors acting along the same line act either both in same direction or opposite to each other are called collinear vectors.

**19. What is parallel vectors?**

Two vectors act in the parallel lines along the same direction are called parallel vectors.

**20. What is anti-parallel vectors?**

Two vectors act in the parallel lines along the opposite directions are called anti-parallel vectors.

**21. What is unit vector?**

A vector with unit magnitude is called unit vector. It is equal to the ratio of a vector and its magnitude.

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|}$$

**22. What is orthogonal unit vector?**

If unit vectors are mutually perpendicular to each other, then they are called orthogonal unit vectors.

**23. State triangle law of addition of two inclined vectors.**

It is stated that if two vectors are represented by the two adjacent sides of a triangle in same order, then the resultant is given by the third side of the triangle in opposite order.

**24. Define Scalar or Dot product of two vectors.**

The scalar or dot product of two vectors is defined as the product of the magnitude of the both vectors and the cosine of the angle between them.

$$\vec{A} \cdot \vec{B} = ABCos\theta = C$$

**25. Define Vector or Cross product of two vectors.**

The Vector or Cross product of two vectors is defined as the product of the magnitude of the both vectors and the sine of the angle between them.

$$\vec{A} \times \vec{B} = ABSin\theta\ \hat{n} = \vec{C}$$

**26. State right hand thumb rule in vector product.**

According to this law, if the curvature of the right hand fingers represents rotating direction of a vector $\vec{A}$ towards another vector $\vec{B}$, then the stretched thumb points out the direction of resultant vector $\vec{C}$.

**27. What is distance? Give its unit.**

Distance is the actual path length travelled by an object in the given interval of time during the motion. Its unit is metre.

**28. What is displacement? Give its unit.**

Displacement is the shortest distance between initial and final position of the object in the given interval of time during the motion. Its unit is metre.

.

**29. What are the differences between distance and displacement?**

| S. No. | Distance | Displacement |
|---|---|---|
| 1 | It is total length of path travelled. | It is shortest distance between initial and final position of an object. |
| 2 | It is a scalar quantity. | It is a vector quantity. |
| 3 | It can be zero or positive but not negative | It can be zero, positive and negative. |
| 4 | It may be equal to or greater than the displacement. | It may be equal to or less than the distance. |
| 5 | It has many values between two positions of an object. | It has only one value between two positions of an object. |

**30. Define average velocity.**

The average velocity is defined as the ratio of change in displacement vector to the corresponding time interval. $\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t}$

**31. Define average speed.**

The average speed is defined as the ratio of total path length travelled by the particle to a given interval of time.

**32. Define instantaneous velocity. Give its unit.**

The velocity at an instant is defined as the change in position vector with respect to time. Its unit is $ms^{-1}$.

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

**33. What are the differences between velocity and average velocity?**

| S.No | Velocity (or) Instantaneous velocity | Average velocity |
|---|---|---|
| 1. | Velocity at an instant of time (or) Rate of change of displacement vector | Ratio of change in displacement vector to the time interval. |
| 2. | It is measured at particular instant of time in motion. | It is measured for a given interval of time in motion. |
| 3. | $\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$ | $\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t}$ |

**34. What is momentum (or) linear momentum? Give its unit.**

The momentum or linear momentum is the product of mass of a particle and its velocity. Its unit is kg $ms^{-1}$ . i.e. $\vec{p} = m\vec{v}$

**35. What is relative velocity?**

The velocity of one object with respect to another object is called relative velocity.

**36. What is uniform motion?**

If an object is moving with constant velocity, then the motion is called uniform motion.

**37. What is non-uniform or accelerated motion?**

If an object is moving with various velocity with time, then the motion is called non-uniform or accelerated motion.

**38. What is uniform accelerated motion?**

If change in velocity of an object in given interval of time is constant, then the motion is called uniform accelerated motion.

**39. What is non-uniform accelerated motion?**

If change in velocity of an object in given interval of time is not constant, then the motion is called non-uniform accelerated motion.

**40. Define average acceleration.**

Average acceleration is defined as the ratio of change in velocity over the given time interval.

$$\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t}$$

**41. Define instantaneous acceleration. Give its unit.**

The acceleration at an instant is defined as the change in velocity with respect to time. Its unit is $ms^{-2}$.

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

**42. What is free fall of a body?**

The motion of a body falling towards the Earth from a small altitude, purely under gravitational force is called free fall of a body.

**43. What is meant by a projectile? Give the examples.**

An object is thrown in the air with some initial velocity and allowed to move under gravity is called a projectile.

<u>Ex:</u>

- An object dropped from window of a moving train
- A bullet fired from a rifle.
- A ball thrown in any direction.

**44. What are the types of projectile motion?**

- Projectile given initial velocity in the horizontal direction.
- Projectile given initial velocity at an angle to the horizontal.

**45. What are the assumptions made in projectile motion?**

- Air resistance is neglected.
- The effect due to rotation of Earth and curvature of earth is negligible.
- The acceleration due to gravity is constant throughout the motion of the projectile.

**46. What is trajectory?**

The path followed by the projectile is called trajectory.

**47. What is time of flight?**

The time interval between the instant of projection and the instant when the projectile hits the ground.

**48. What is horizontal range?**

The maximum horizontal distance between the point of projection and the point where the projectile hits the ground.

**49. What is maximum height?**

The maximum vertical distance travelled by the projectile during its journey is called maximum height.

**50. What is angular displacement? Give its unit.**

The angle described by the particle about the axis of rotation in a given time is called angular displacement. Its unit is radian.

$$\theta = \frac{S}{r}$$

**51. What is angular velocity? Give its unit.**

The rate of change of angular displacement is called angular velocity. Its unit is rad $s^{-1}$ .

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$$

**52. What is angular acceleration? Give its unit.**

The rate of change of angular velocity is called angular acceleration. Its unit is rad $s^{-2}$ .

$$\vec{\alpha} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\omega}}{\Delta t}$$

**53. What is tangential acceleration?**

The acceleration which is acting along the direction of linear velocity and tangent to the circular motion is called tangential acceleration.

**54. What is uniform circular motion?**

When an object is moving on a circular path with constant speed, it is called uniform circular motion.

**55. What is non-uniform circular motion?**

When an object is moving on a circular path with change in speed and direction, it is called non-uniform circular motion.

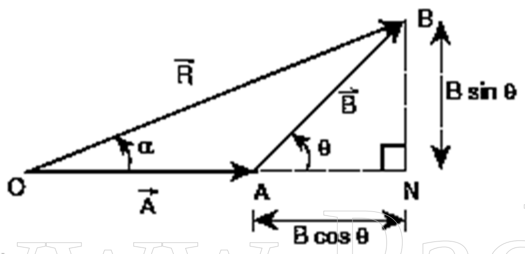
**56. What is centripetal acceleration or radial acceleration or normal acceleration?**

The acceleration which is acting towards the center along the radial direction and perpendicular to linear velocity of circular motion is called centripetal acceleration.

## 5 Marks Q & A:

**1. Find the magnitude and direction of resultant of the two vectors by using triangle law of vector addition.**

- Let $\vec{A}$ and $\vec{B}$ are two vectors they are inclined at angle $\theta$ between them.
- According to triangle law of vector addition, head of the vector $\vec{A}$ is connected to tail of the vector $\vec{B}$ and both are represented in adjescent side of a triangle in some order.
- Let $\vec{R}$ be the resultant vector, which is represented in third closing side of the triangle in opposite order.
- Let $\alpha$ be the angle made by the resultant vector $\vec{R}$ with vector $\vec{A}$.
- Thus we can write, $\vec{R} = \vec{A} + \vec{B}$




**(a) Magnitude of resultant vector :**

- From ⊿ABN,

$$\cos\theta = \frac{AN}{B} \quad ; \quad AN = B\cos\theta$$
$$\sin\theta = \frac{BN}{B} \quad ; \quad BN = B\sin\theta$$

- From ⊿OBN,

$OB^2 = ON^2 + BN^2$

$$R^2 = (A + B\cos\theta)^2 + (B\sin\theta)^2$$

$$R^2 = A^2 + B^2\cos^2\theta + 2AB\cos\theta + B^2\sin^2\theta$$

$$\boxed{R = |\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}}$$

**(b) Direction of resultant vector :**

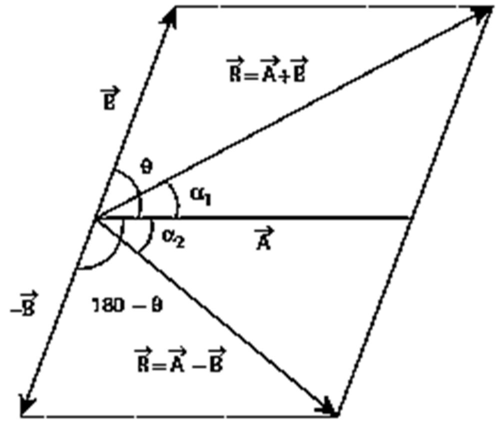
- From ⊿OBN,

$$\tan\alpha = \frac{BN}{ON} = \frac{BN}{OA + AN}$$

$$\boxed{\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}}$$

**2. Discuss Subtraction of two vectors geometrically and write the equations for magnitude and direction of resultant vector.**

- Let $\vec{A}$ and $\vec{B}$ are two vectors they are inclined at angle $\theta$ between them.
- Obtain $-\vec{B}$ as in figure and now angle between $\vec{A}$ and $\vec{B}$ is $180^0 - \theta$.




- Thus, Resultant $\vec{R} = \vec{A} + (-\vec{B}) = \vec{A} - \vec{B}$
- According to triangle law of vectors,

**(a) Magnitude of difference :**

$$R = |\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos(180^0 - \theta)}$$

Since, $\cos(180^0 - \theta) = -\cos\theta$

$$\boxed{|\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 - 2AB\cos\theta}}$$

**(b) Direction of difference :**

$$\tan\alpha = \frac{B\sin(180^0 - \theta)}{A + B\cos(180^0 - \theta)}$$

But $\sin(180^0 - \theta) = \sin\theta$

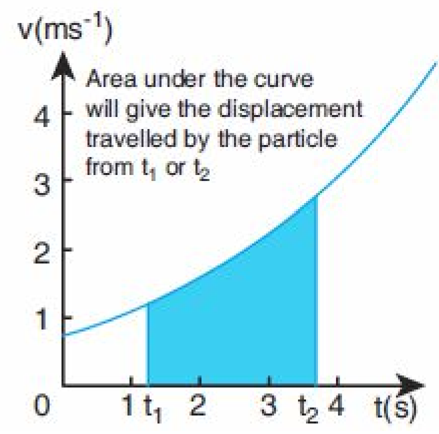
$$\boxed{\tan\alpha = \frac{B\sin\theta}{A - B\cos\theta}}$$

**3. Give the Comparison of the properties of Scalar and Vector product.**

| S. No | Scalar / Dot product | Vector / Cross product |
|---|---|---|
| 1 | Product quantity C=$\vec{A}\cdot\vec{B}$ is always a scalar. $\vec{A}\cdot\vec{B} = +ve$ if θ is acute (θ<90$^0$) $\vec{A}\cdot\vec{B} = -ve$ if θ is obtuse (90$^0$ > θ < 180$^0$) | Product quantity $\vec{C} = \vec{A}\times\vec{B}$ is always a Vector. $\vec{C}$ is always orthogonal to $\vec{A}$ & $\vec{B}$ but $\vec{A}$ and $\vec{B}$ may or may not be mutually orthogonal. |
| 2 | It obeys Commutative law. $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$ | It doesn't obey Commutative law. $\vec{A}\times\vec{B} \neq \vec{B}\times\vec{A}$. But, $\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$ and $\lvert\vec{A}\times\vec{B}\rvert = \lvert\vec{B}\times\vec{A}\rvert$. |
| 3 | It obeys Distributive law. $\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B}+\vec{B}\cdot\vec{A}$ | It obeys Distributive law. $\vec{A}\times(\vec{B}+\vec{C}) = \vec{A}\times\vec{B}+\vec{B}\times\vec{A}$ |
| 4 | When $\vec{A}$ & $\vec{B}$ are parallel, θ = 0$^0$ , $(\vec{A}\cdot\vec{B})_{max} = AB$ | When $\vec{A}$ & $\vec{B}$ are parallel, θ = 0$^0$ , $(\vec{A}\times\vec{B})_{min} = 0$ |
| 5 | When $\vec{A}$ & $\vec{B}$ are anti-parallel, θ = 180$^0$ , $(\vec{A}\cdot\vec{B})_{min} = -AB$ | When $\vec{A}$ & $\vec{B}$ are anti-parallel, θ = 180$^0$ , $(\vec{A}\times\vec{B})_{min} = 0$ |
| 6 | When $\vec{A}$ & $\vec{B}$ are perpendicular, θ = 90$^0$, $\vec{A}\cdot\vec{B} = 0$ | When $\vec{A}$ & $\vec{B}$ are perpendicular, θ = 90$^0$ , $(\vec{A}\times\vec{B})_{max} = AB\,\hat{n}$ |
| 7 | Self-dot product of a vector, $\vec{A}\cdot\vec{A} = AAcos0^0 = A^2$ | Self-cross product of a vector, $\vec{A}\times\vec{A} = AAsin0^0\hat{n} = \vec{0}$ |
| 8 | Self-dot product of a unit vector, $\hat{n}\cdot\hat{n} = 1\times 1\ cos0^0 = 1$ $\hat{\imath}\cdot\hat{\imath} = \hat{\jmath}\cdot\hat{\jmath} = \hat{k}\cdot\hat{k} = 1$ | Self-cross product of a unit vector, $\vec{A}\times\vec{A} = AAsin0^0\hat{n} = \vec{0}$ $\hat{\imath}\times\hat{\imath} = \hat{\jmath}\times\hat{\jmath} = \hat{k}\times\hat{k} = \vec{0}$ |
| 9 | Dot product of orthogonal unit vectors, $\hat{\imath}\cdot\hat{\jmath} = \hat{\jmath}\cdot\hat{k} = \hat{k}\cdot\hat{\imath} = 0$ | Cross product of orthogonal unit vectors, $\hat{\imath}\times\hat{\jmath} = \hat{k}$ ; $\hat{\jmath}\times\hat{\imath} = -\hat{k}$ $\hat{\jmath}\times\hat{k} = \hat{\imath}$ ; $k\times\hat{\jmath} = -\hat{\imath}$ $\hat{k}\times\hat{\imath} = \hat{\jmath}$ ; $\hat{\imath}\times\hat{k} = -\hat{\jmath}$ |
| 10 | Scalar product of vector components, $\vec{A}\cdot\vec{B} = (A_x\hat{\imath}+A_y\hat{\jmath}+A_z\hat{k})\cdot(B_x\hat{\imath}+B_y\hat{\jmath}+B_z\hat{k})$ $\vec{A}\cdot\vec{B} = A_xB_x+A_yB_y+A_zB_z$ | Vector product of vector components, $\vec{A}\cdot\vec{B} = \begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix}$ $= \hat{\imath}(A_yB_z - A_zB_y) + \hat{\jmath}(A_zB_x - A_xB_z) + \hat{k}(A_xB_y - A_yB_x)$ |

**4. Elucidate the significance of velocity – time graph.**

❖ By using velocity – time graph we can find out the distance and displacement by calculating the area under the curve.




❖ We know, velocity $v = \frac{dx}{dt}$

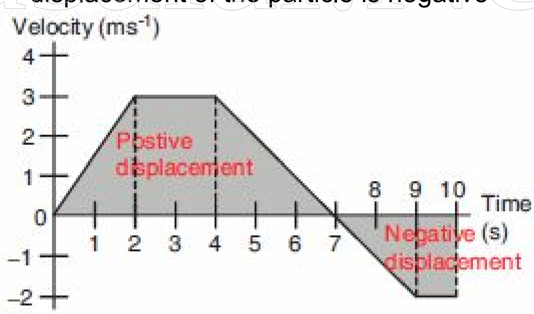
or dx = v dt

By integrating both sides, we get,

$$\int_{x_1}^{x_2} dx = \int_{t_1}^{t_2} v\,dt$$

Displacement, $x_2 - x_1$ = Area under the curve

❖ If area under the curve is negative, the displacement of the particle is negative




**5. Discuss about relative velocity.**

❖ When two objects A and B moving with different velocities, then the velocity of one object with respect to another is called relative velocity.

**(a) Case 1 : A and B moving in same direction.**

❖ Let $V_A$ and $V_B$ are the uniform velocities of A and B respectively.

❖ If A and B are moving in same direction,

The relative velocity of A with respect to B is,

$$\overrightarrow{V_{AB}} = \overrightarrow{V_A} - \overrightarrow{V_B}$$

The relative velocity of B with respect to A is,

$$\overrightarrow{V_{BA}} = \overrightarrow{V_B} - \overrightarrow{V_A}$$

❖ Thus, magnitude of relative velocity of one object with respect to other is equal to difference in magnitude of two velocities.

**(b) Case 2 : A and B moving in opposite direction.**

- Let $V_A$ and $V_B$ are the uniform velocities of A and B respectively.
- If A and B are moving in opposite direction,

The relative velocity of A with respect to B is,

$$\overrightarrow{V_{AB}} = \overrightarrow{V_A} - (-\overrightarrow{V_B}) = \overrightarrow{V_A} + \overrightarrow{V_B}$$

The relative velocity of B with respect to A is,

$$\overrightarrow{V_{BA}} = -\overrightarrow{V_B} - \overrightarrow{V_A} = -(\overrightarrow{V_A} + \overrightarrow{V_B})$$

- Thus, magnitude of relative velocity of one object with respect to other is equal to sum of magnitude of two velocities.

**(c) Case 3 : A and B moving with an angle θ.**

- Let $V_A$ and $V_B$ are the uniform velocities of A and B respectively inclined at an angle θ between them.
- The relative velocity of A with respect to B is

$$\overrightarrow{V_{AB}} = \overrightarrow{V_A} - \overrightarrow{V_B}$$

Then, the magnitude of $\overrightarrow{V_{AB}}$ is given by,

$$V_{AB} = \sqrt{V_A^2 + V_B^2 - 2V_AV_B\cos\theta}$$

The direction of $\overrightarrow{V_{AB}}$ is given by,

$$\tan\beta = \frac{V_B\sin\theta}{V_A - V_B\cos\theta}$$

- **(1) When θ = 0°** ( $V_A$ & $V_B$ in same direction)

  $V_{AB} = V_A - V_B$ along the direction of $\overrightarrow{V_A}$.

  $V_{BA} = V_B - V_A$ along the direction of $\overrightarrow{V_B}$.

- **(2) When θ = 180°** ($V_A$ & $V_B$ in opposite direction)

  $V_{AB} = V_A + V_B$ along the direction of $\overrightarrow{V_A}$.

  $V_{BA} = V_B + V_A$ along the direction of $\overrightarrow{V_B}$.

- **(3) When θ = 90°**

  ($V_A$ & $V_B$ in perpendicualr direction)

  $V_{AB} = \sqrt{V_A^2 + V_B^2}$ along the direction of $\overrightarrow{V_A}$.

  $V_{BA} = \sqrt{V_B^2 + V_A^2}$ along the direction of $\overrightarrow{V_B}$.

**6. Write the expression for magnitude and direction of relative velocity of rain with respect to man.**

- If velocity of man $\overrightarrow{V_M}$ is right angle to velocity of rain $\overrightarrow{V_R}$ falling veritically downwards, Then the relative velocity of the rain with respect to the man is,

$$\overrightarrow{V_{RM}} = \overrightarrow{V_R} - \overrightarrow{V_M}$$

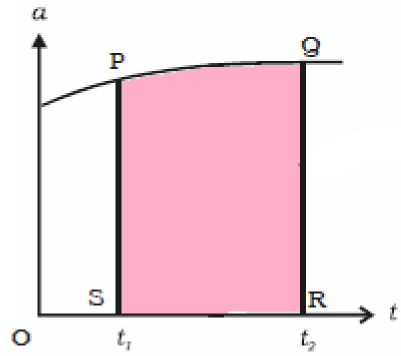
- Magnitude $V_{RM} = \sqrt{V_R^2 + V_M^2}$

  Direction $\theta = \tan^{-1}\frac{V_M}{V_R}$

- In order to save himself from rain he should hold the umbrella at an angle θ with the verticle.

**7. Elucidate the significance of acceleration–time graph.**

- By using acceleration– time graph we can find out the speed and velocity by calculating the area under the curve.




- We know, acceleration $a = \frac{dv}{dt}$

  or dv = a dt

By integrating both sides, we get,

$$\int_{v_1}^{v_2} dv = \int_{t_1}^{t_2} a\,dt$$

Velocity, $V_2 - V_1$ = Area under the curve

**8. Obtain kinematic equations for uniformally accelerated motion.**

- Consider an object moving along a straight line with uniform or constant acceleration 'a'.
- Let 'u' be the initial velocity at time t=0 and 'v' be the final velocity at time t.
- Let 'S' be the displacement.

**(a) Velocity – time relation :**

- Acceleration, $a = \frac{dv}{dt}$

  or $dv = a\,dt$

- By integrating both sides, we get,

$$\int_u^v dv = \int_0^t a\,dt = a\int_0^t dt = a[t]_0^t$$

$$v - u = at$$

$$\boxed{v = u + at}$$

**(b) Displacement – time relation :**

- Velocity, $v = \frac{dS}{dt}$

  or $dS = v\,dt = (u + at)dt$

  $[\because v = u + at]$

- By integrating both sides, we get,

$$\int_0^S dS = \int_0^t (u + at)\,dt$$

$$\int_0^S dS = u\int_0^t dt + a\int_0^t t dt$$

$$\boxed{S = ut + \frac{1}{2}at^2}$$

**(c) Velocity - displacement relation :**

- Acceleration, $a = \frac{dv}{dt} = \frac{dv}{dS}\frac{dS}{dt} = \frac{dv}{dS}v$

$$ds = \frac{1}{a} v\, dv$$

- By integrating both sides, we get,

$$\int_0^S dS = \frac{1}{a}\int_u^v v\, dv = \frac{1}{a}\left[\frac{v^2}{2}\right]_u^v$$

$$S = \frac{1}{2a}(v^2 - u^2)$$

$$v^2 - u^2 = 2aS$$

$$\boxed{v^2 = u^2 + 2aS}$$

**(d) Displacement – average velocity relation :**

- Final Velocity, $v = u + at$

$$at = v - u \quad \text{-----> (1)}$$

- We know displacement,

$$S = ut + \frac{1}{2}at^2$$

- Substituting equation(1) , we get,

$$S = ut + \frac{1}{2}(v-u)t$$

$$S = ut + \frac{1}{2}vt - \frac{1}{2}ut$$

$$\boxed{S = \frac{(u+v)\,t}{2}}$$

**9. Derive the equations of motion for a particle falling vertically.**

- Consider an object of mass 'm' falling from a height h.
- Assume that there is no air resistance and acceleration due to gravity is constant near the surface of the Earth.
- If the object is thrown with an initial velocity u along the Y-axis, then its final velocity and displacement at any time 't' is v and y respectively. Further acceleration a is equal to g.

- Therefore equations of motion are,

$$v = u + gt$$

$$y = ut + \frac{1}{2}gt^2$$

$$v^2 = u^2 + 2gy$$

- Suppose initial velocity u = 0, then

$$v = gt$$

$$y = \frac{1}{2}gt^2$$

$$v^2 = 2gy$$

- Time taken by the object to reach the ground(T), If t= T and y = h, then

$$h = \frac{1}{2}gT^2$$

$$T = \sqrt{\frac{2h}{g}}$$

- The Speed of the object when it reaches the ground,

$$v_{ground}^2 = 2gh$$

$$v_{ground} = \sqrt{2gh}$$

**10. Derive the equations of motion for a particle projected vertically upward.**

- Consider an object of mass 'm' thrown vertically upward with an initial velocity u.
- Assume that there is no air resistance and acceleration due to gravity is constant near surface of the Earth.
- The final velocity and displacement at any time 't' is v and y respectively. Further acceleration a is equal to -g.

- Therefore equations of motion are,

$$v = u - gt$$

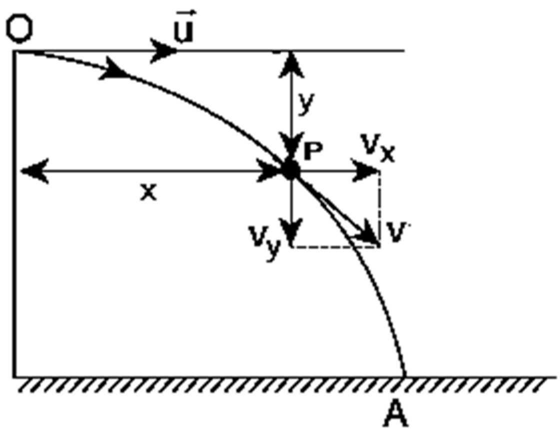
$$y = ut - \frac{1}{2}gt^2$$

$$v^2 = u^2 - 2gy$$

**11. Obtain the following expressions in the event of horizontal projection of a projectile from the top of a tower of height 'h' (a) the path of the projectile (b) time of flight (c) horizontal range (d) resultant velocity and (e) speed of the projectile when hits the ground.**

- Consider an object is thrown horizontally with initial velocity u along x-direction.

- Since acceleration due to gravity acts vertically downwards, velocity along the horizontal x-direction $u_x$ doesn't change throught the motion. Whereas velocity along the y-direction $u_y$ is changed.




**(a) The path of the projectile :**

**(i) Motion along horizontal direction:**

- The horizontal distance travelled by the projectile at a point P after a time t can be written as,

$$S_x = u_x t + \frac{1}{2}a_x t^2$$

- Here, $S_x = x$ , $u_x = u$ and $a_x = 0$, Therefore,

$$x = ut$$

$$t = \frac{x}{u} \text{ ---------> (1)}$$

**(ii) Motion along downward direction:**

- The downward distance travelled by the projectile at a point P after a time t can be written as,

$$S_y = u_y t + \frac{1}{2} a_y t^2$$

- Here, $S_y = y$ , $u_y = 0$ and $a_y = g$ , Therefore,

$$y = \frac{1}{2} g t^2$$

- Substituting equation (1), we get,

$$y = \frac{1}{2} g \left(\frac{x}{u}\right)^2 = \left(\frac{g}{2u^2}\right) x^2$$

$$y = Kx^2 \text{ --------> (2)}$$

Where $K = \frac{g}{2u^2}$ is a constant.

- The equation(2) represents the equation of a parabola. Thus, the path travelled by the projectile is a parabola.

**(b) <u>Time of flight :</u> ($T_f$)**

- The time of flight($T_f$) is the time taken by the projectile to hit the ground after thrown.

- The downward distance travelled by the projectile at a time t can be written as,

$$S_y = u_y t + \frac{1}{2} a_y t^2$$

- Here substituting the values $S_y = h$, $t = T_f$ , $u_y = 0$, and $a_y = g$ we get,

$$h = \frac{1}{2} g T_f^2$$

- Therefore, $\boxed{T_f = \sqrt{\frac{2h}{g}}}$

**(c) <u>Horizontal range :</u> (R)**

- The horizontal range(R) is the maximum horizontal distance covered by the projectile from the foot of the tower to the point where the projectile hits the ground.

- The horizontal distance travelled by the projectile at a time t can be written as,

$$S_x = u_x t + \frac{1}{2} a_x t^2$$

- Here, $S_x = R$ , $u_x = u$ , $a_x = 0$ and $t = T_f$

$$R = uT_f$$

- Therefore, $\boxed{R = u\sqrt{\frac{2h}{g}}}$ $\left[\because T_f = \sqrt{\frac{2h}{g}}\right]$

**(d) Resultant Velocity at any time : (v)**

- The velocity of the projectile at point p after the time t has two components $V_x$ and $V_y$.

- The velocity component along x-direction is,

$$v_x = u_x + a_x t$$

Since, $u_x = u$, $a_x = 0$, we get, $v_x = u$

- The velocity component along y-direction is,

$$v_y = u_y + a_y t$$

Since, $u_y = 0$, $a_y = g$, we get, $v_y = gt$

- Hence the resultant velocity at any time t is,

$$\vec{v} = v_x \hat{\imath} + v_y \hat{\jmath}$$

$$\vec{v} = u\,\hat{\imath} + gt\,\hat{\jmath}$$

- The magnitude of resultant velocity or speed is given by,

$$v = \sqrt{v_x^2 + v_y^2}$$

$$\boxed{v = \sqrt{u^2 + g^2 t^2}}$$

**(e) <u>Speed of the projectile when hits the ground :</u>**

- As the horizontal component of the velocity is same as initial velocity, $v_x = u$

- The vertical component of the velocity at a time t is,

$$v_y = u_y + a_y t$$

- Here $u_y = 0$ , $a_y = g$ and $t = T_f$ . Substituting this we get,

$$v_y = gT_f$$

or $$v_y = g\sqrt{\frac{2h}{g}}$$

$$v_y = \sqrt{2\text{gh}}$$

- The speed of the projectile when hits the ground,

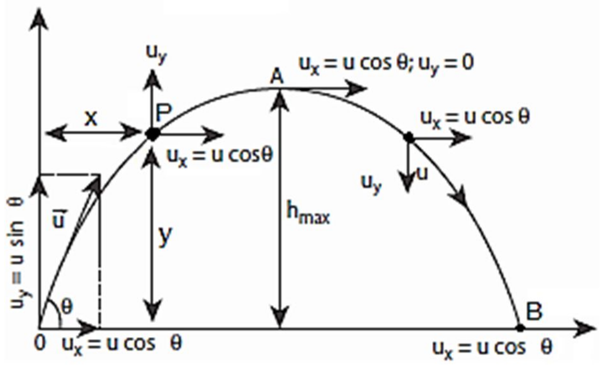
$$v = \sqrt{v_x^2 + v_y^2}$$

$$\boxed{v = \sqrt{u^2 + 2gh}}$$

**12. Obtain the following expressions in the event of angular projection of a projectile with the horizontal (a) the path of the projectile (b) maximum height (c) time of flight (d) horizontal range.**

- Consider an object is thrown with initial velocity $u$ at an angle θ with the horizontal.

- Since acceleration due to gravity acts vertically downwards, velocity along the horizontal x-direction $u_x$ doesn't change throught the motion. Whereas velocity along the y-direction $u_y$ is changed.




- **The path of the projectile :**

**(i) Motion along x-direction:**

- The horizontal distance travelled by the projectile at a point P after a time t can be written as,

$$S_x = u_x t + \frac{1}{2}a_x t^2$$

- Here, $S_x = x$ , $u_x = u\cos\theta$ and $a_x = 0$, Therefore,

$$x = u\cos\theta \cdot t$$

$$t = \frac{x}{u\cos\theta} \text{ ---------> (1)}$$

**(ii) Motion along y-direction:**

- The downward distance travelled by the projectile at a point P after a time t can be written as,

$$S_y = u_y t + \frac{1}{2}a_y t^2$$

- Here, $S_y = y$ , $u_y = u\sin\theta$ and $a_y = -g$ , Therefore,

$$y = u\sin\theta \cdot t - \frac{1}{2}gt^2$$

- Substituting equation (1), we get,

$$y = u\sin\theta \cdot \frac{x}{u\cos\theta} - \frac{1}{2}g\left(\frac{x}{u\cos\theta}\right)^2$$

$$y = x\tan\theta - \frac{1}{2}g\frac{x^2}{u^2\cos^2\theta}$$

- Thus, the path travelled by the projectile is an inverted parabola.

- **Maximum Height : ($h_{max}$)**

- The maximum vertical distance travelled by the projectile during its journey is called maximum height.

- For the vertical part of the motion,

$$v_y^2 = u_y^2 + 2a_yS_y$$

- Here, $V_y = 0$, $S_y = h_{max}$ , $u_y = u\sin\theta$ and $a_y = -g$ , Therefore,

$$0 = u^2\sin^2\theta - 2gh_{max}$$

$$\boxed{h_{max} = \frac{u^2\sin^2\theta}{2g}}$$

- **Time of flight : ($T_f$)**

- The time of flight($T_f$) is the time taken by the projectile to hit the ground after thrown.

- The downward distance travelled by the projectile at a time t can be written as,

$$S_y = u_y t + \frac{1}{2}a_y t^2$$

- Here substituting the values $S_y = 0$, $t = T_f$ , $u_y = u\sin\theta$ , and $a_y = -g$ we get,

$$0 = u\sin\theta - \frac{1}{2}gT_f^2$$

Therefore, $\boxed{T_f = \frac{2u\sin\theta}{g}}$

- **Horizontal range : (R)**

- The horizontal range(R) is the maximum horizontal distance distance between the point of projection and the point where the projectile hits the ground.

- The horizontal distance travelled by the projectile at a time t can be written as,

$$S_x = u_x t + \frac{1}{2}a_x t^2$$

- Here, $S_x = R$ , $u_x = u\cos\theta$ , $a_x = 0$ and $t = T_f$

$$R = u\cos\theta \cdot T_f$$

$$R = u\cos\theta \cdot \frac{2u\sin\theta}{g} = \frac{2u^2\sin\theta\cos\theta}{g}$$

$$\left[\because T_f = \frac{2u\sin\theta}{g}\right]$$

- Therefore, $\boxed{R = \frac{u^2\sin 2\theta}{g}}$

$$[\because \sin 2\theta = 2\sin\theta \cdot \cos\theta]$$

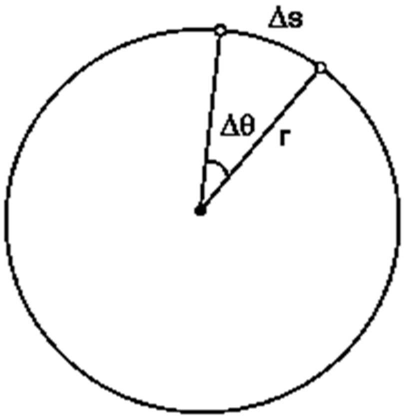
- For maximum range, $\sin 2\theta = 1$

$$2\theta = \frac{\pi}{2}$$

$$\theta = \frac{\pi}{4}$$

The maximum range is, $\boxed{R = \frac{u^2}{g}}$

**13. Obtain the relation between linear velocity and angular velocity.**



- Consider an object moving along a circle of radius r. In a time Δt, the object travels an arc distance ΔS as shown in figure. The corresponding angle subtended is Δθ.
- From figure, the ΔS can be written as,

$$\Delta S = r\Delta\theta$$

- Dividing both sides by Δt, we get,

$$\frac{\Delta S}{\Delta t} = r\frac{\Delta\theta}{\Delta t}$$

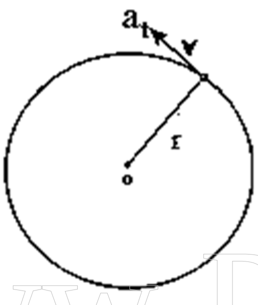
- In the limit $\Delta t \to 0$, the above equation becomes,

$$\frac{dS}{dt} = r\frac{d\theta}{dt}$$

- Here, $v = \frac{dS}{dt}$ and $\omega = \frac{d\theta}{dt}$ , therefore,

$$\boxed{v = r\omega}$$

**14. Derive an expression for tangential acceleration in the circular motion.**




- Consider the circular motion of radius r with linear velocity v and angular velocity ω. In this,

$$v = r\omega$$

- Differentiating the above equation with respect to time, we get,

$$\frac{dv}{dt} = r\frac{d\omega}{dt}$$

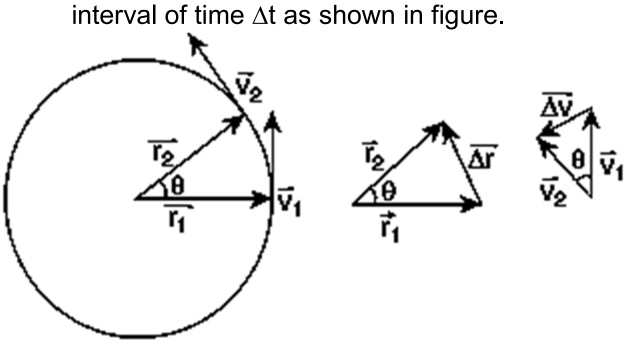
- Here, $a_t = \frac{dv}{dt}$ and $\alpha = \frac{d\omega}{dt}$ , therefore,

$$\boxed{a_t = r\alpha}$$

where $a_t$ is the tangential acceleration and α is the angular acceleration

**15. Derive an expression for centripetal acceleration in the uniform circular motion.**

- Consider the position vectors and velocity vectors shift through the some angle θ in a small interval of time Δt as shown in figure.




- In uniform circular motion,
  $r = |\vec{r_1}| = |\vec{r_2}|$ and $v = |\vec{v_1}| = |\vec{v_2}|$
- From figure, the geometrical relationship between the magnitude of position and velocity vectors is given by,

$$\frac{\Delta r}{r} = -\frac{\Delta v}{v} = \theta$$

- Here the negative sign implies that Δv points radially inward, towards the center of the circle.

$$\Delta v = -v\left(\frac{\Delta r}{r}\right)$$

- Dividing both sides by Δt, we get,

$$\frac{\Delta v}{\Delta t} = -\frac{v}{r}\left(\frac{\Delta r}{\Delta t}\right)$$

- Applying the limit $\Delta t \to 0$, We get,

$$\frac{dv}{dt} = -\frac{v}{r}\left(\frac{dr}{dt}\right)$$

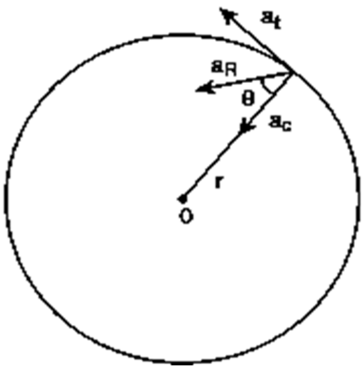
- Since $a_c = \frac{dv}{dt}$ and $v = \frac{dr}{dt}$ , We can write,

$$\boxed{a_c = -\frac{v^2}{r}}$$

Where $a_c$ is the centripetal acceleration.

**16. Derive an expression for total acceleration in the non-uniform circular motion.**

- Consider the non-uniform circular motion of an object. (**Ex:** vertical circular motion)
- In non-uniform circular motion both centripetal and tangential acceleration act on the object as shown in figure.




- The resultant acceleration is obtained by vector sum of centripetal and tangential acceleration.
- Hence, $\vec{a_R} = \vec{a_t} + \vec{a_c}$
- Therefore, the magnitude of resultant acceleration is given by,

$$a_R = \sqrt{a_t^2 + a_c^2}$$

$$\boxed{a_R = \sqrt{a_t^2 + \left(\frac{v^2}{r}\right)^2}} \qquad \left[\because a_c = \frac{v^2}{r}\right]$$

# 3. Laws of motion

**1. State the Newton's first law of motion.**

Every body continues its state of rest or in uniform motion until external force acting on it.

**2. What is inertia? What are its types?**

The inability of an object to change its state of rest or motion.

Types :
- Inertia of rest
- Inertia of motion
- Inertia of direction

**3. What is inertia of rest? Give an example.**

The inability of an object to change its state of rest is called inertia of rest.

Example:

When a bus start to move from rest position, all the passengers inside the bus suddenly will be pushed back. Here passengers cannot change their state of rest on its own that's why they pushed back.

**4. What is inertia of motion? Give an example.**

The inability of an object to change its state of motion on its own is called inertia of rest.

Example:

When a bus in motion suddenly braked, all the passengers inside the bus will move forward. Here passengers cannot change their state of motion on its own that's why they moved forward.

**5. What is inertia of direction? Give an example.**

The inability of an object to change its state of direction on its own is called inertia of rest.

Example:

When a stone attached to a string is in whirling motion suddenly cut out, the stone will move in the tangential direction of the circle. Here the whirling stone cannot change its state of direction on its own that's why it couldn't continue its circular motion.

**6. State Newton's second law of motion.**

The force acting on an object is equal to the rate of change of its momentum.

**7. Define one Newton.**

One Newton is defined as the force which acts on 1 kg of mass to give an acceleration 1 ms$^{-2}$ in the direction of the force.

**8. State Newton's third law.**

For every action there is an equal and opposite reaction.

**9. What is free body diagram?**

Free body diagram is a simple tool to analyse the motion of the object using Newton's laws.

**10. What are the steps followed in developing the free body diagram?**
- Identify the forces acting on the object
- Represent the object as a point.
- Draw the vectors representing the forces acting on the object.

**11. What is concurrent forces?**

The lines of forces acting at a common point are called concurrent forces.

**12. What is coplanar forces?**

The lines of forces they are in the same plane are called coplanar forces.

.

**13. State Lami's theorem.**

If a system of three concurrent and coplanar forces is in equilibrium, each force is directly proportional to sine of the angle between the other two forces.

**14. State law of conservation of total linear momentum.**

If there is no external force acting on the system, the total linear momentum of the system is always a constant vector.

**15. What is impulsive force or impulse? Give its unit.**

If a very large force acts on an object in a very short time, the force is called impulsive force.Its unit is Ns.

$$J = F \times \Delta t$$

**16. Illustrate the average force with the examples.**
- When a cricket player catches the ball, he pulls his hands gradually in the direction of the ball's motion because to reduce average large force which hurts his hands.
- When a car meets with an accident, the air bag system inside a car prevents the passengers by reducing average forces acting on them.
- When a two wheeler bumps on the road, the shock absorbers make comfort to rider by reducing average force.
- Jumping on a concrete cemented road is more dangerous than jumping on the sand since the sand reduces the average force on jumping.

**17. What is meant by static friction?**

Static friction is the force which opposes the initiation of motion of an object on the surface.

**18. What is meant by kinetic friction?**

Kinetic friction is the force which opposes the motion of an object during movement.

**19. Define angle of friction.**

The angle of friction is defined as the angle between the normal force(N) and resultant force(R) of normal force and maximum friction force($f_s^{max}$).

**20. Define angle of repose.**

The angle of repose is defined as the angle of the inclined plane at which the object starts to slide.

**21. Describe the applications of angle of repose.**

- Antilons make sand traps in such way that its angle of inclination is made equal to angle of repose. So that insects enter the edge of the trap start to slide towards the bottom where the antilons hide itself.
- Children sliding boards are always inclined just above the angle of repose. So that children playing on that slide smoothly. At the same time, much greater inclined angle may hurt the sliding children.

**22. Compare the static and kinetic friction.**

| S.No. | Static friction | Kinetic friction |
|---|---|---|
| 1. | It opposes initiation of motion. | It opposes relative motion of the object with respect to the surface. |
| 2. | Independent of surface contact | Independent of surface contact |
| 3. | $\mu_s$ depends on the nature of material in mutual contact. | $\mu_k$ depends on the nature of material and temperature of the surface. |
| 4. | Depends on the magnitude of applied force. | Independent of magnitude of applied force. |
| 5. | It takes values from 0 to $\mu_s$N. | It is always equal to $\mu_k$N. |
| 6. | $f_s^{max} > f_k$ | $f_k < f_s^{max}$ |
| 7. | $\mu_s > \mu_k$ | $\mu_k < \mu_s$ |

**23. State the empirical laws of static and kinetic friction.**

- The empirical law of static friction states that the static frictional force is directly proportional to the normal force. i.e. $f_s = \mu_s N$ where, $0 \le f_s \le \mu_s N$ .
- The empirical law of kinetic friction states that the kinetic frictional force is directly proportional to the normal force. i.e. $f_k = \mu_k N$ .

**24. What is rolling friction?**

The rolling friction is the minimal force, which opposes the rotational motion of the wheel on the surface.

**25. What is centripetal force?**

If a particle is in uniform circular motion with respect to an inertial frame, there is a force acting towards the center of the circle is called centripetal force.

**26. Suggest a few methods to reduce friction.**

- By using lubricants in machinary parts.
- By using ball bearings.

**27. What is meant by pseudo force?**

The pseudo force is a fictitious force. It is just an apparent but it makes real effect. It is represented only in non-inertial frames.

**Example :** centrifugal force.

**28. What are inertial frames?**

The frame of reference, which is not accelerated, is known as inertial frame. Newton's laws are applicable in these frames.

**29. What are non-inertial frames?**

The frame of reference, which is accelerated, is known as non-inertial frame. Newton's laws are not applicable in these frames.

**30. Illustrate the centripetal force with the examples.**

- In the whirling motion of a stone tied with a string, the centripetal force is given by tensional force through the string.
- In the motion of satellites around the Earth, the gravitational force gives the centripetal force.
- When a car is moving on a circular track, the frictional force between road and tyre gives the centripetal force.
- When the planets orbit around the Sun experience the centripetal force towards the sun is given by the gravitational force of the Sun.

**31. What is meant by banking of tracks?**

When the coefficient of static friction is not enough on the leveled circular road, the outer edge of the road is slightly raised compared to the inner egde to avoid skidding. It is called banking of tracks.

**32. What is centrifugal force?**

If a particle is in circular motion with respect to a non-inertial frame, there is a pseudo force acting away from the center of the circle is called centrifugal force.

**33. Compare the centripetal and centrifugal forces.**

| S.No. | Centripetal force | Centrifugal force |
|---|---|---|
| 1. | It is a real force given by external agencies like gravitational force, tensional force, normal force,etc. | It is a pseudo force or fictitious force cannot be derived from any external agencies. |
| 2. | Acts in both inertial and non-inertial frames | Acts only in non-inertial frames(rotating frames) |
| 3. | It acts towards the axis of rotation or center of the circular motion. | It acts away from the axis of rotation or center of the circular motion |
| 4. | Real force and has real effects. | Pseudo force but has real effects. |
| 5. | It orginates from interaction of two objects | It orginates from inertia of the object. |
| 6. | It is included in free body diagram for both inertial and non-inertial frames. | It is included in free body diagram for only non-inertial frames. |
| 7. | Magnitudely it is equal to centrifugal force. | Magnitudely it is equal to centripetal force. |

## Conceptual Questions:

**34. Why it is not possible to push a car from inside?**

It is not possible to push a car from inside because the pushing force is equalised by the reactional force of the car seat.

**35. There is a limit beyond which polishing of a surface increases frictional resistance rathar than decreasing it why?**

Polishing the surface beyond the certain limit induces the electrostatic addisive force on the surface, which will inturn developes the frictional resistance.

**36. Can a single isolated force exist in nature? Explain your answer.**

No. It cannot. According to Newton's third law "For every action there is an equal and opposite reaction". So the forces always exist in pairs.

**37. Why does a parachute descend slowly?**

The large area covered by the parachute experiences more air resistive force acting opposite to downward gravitational force. So that the parachute descends slowly.

**38. When we walk on ice one should take short steps. Why?**

As the surface of the ice is very smooth, in order to avoid skidding, short steps help us to make necessary static friction to walk.

**39. When a person walks on a surface, the frictional force exerted by the surface on the person is opposite to the direction of motion. True or false?**

False. When the person walks on the surface, he pushes the surface backward, whereas surface gives frictional force forward which is in the direction of motion.

**40. Can the coefficient of friction be more than one?**

Yes. The coefficient of friction can be more than one. It means friction is greater than normal force. For example, rubber has coefficient of friction 1.16.

**41. Can we predict the direction of motion of a body from the direction of force on it?**

No. It cannot. The direction of motion can be along the direction of force or opposite to force or perpendicular force or without the force.

**42. The momentum of a system of particle is always conserved. True or false?**

False. The momentum of a system of particle is conserved only when external force acting on it is zero.

**43. When you walk on the tiled floor where water is spilled, you are likely to slip. Why?**

Water on tiled floor reduces the coefficient of friction of the surface. So when we walk on wet tile, it drags our leg to slide. Now the friction becomes kinetic friction, which is much weaker than static friction. That's why we likely to slip.

**44. When a bicycle moves in the forward direction, what is the direction of frictional force in the rear and front wheels?**

- When a bicycle moves in the forward direction, static friction in the rear wheel acts forward.
- So that front wheel gets backward static friction. When wheels slip friction becomes kinetic friction.
- In addition to static friction, rolling friction also acts both wheels in the backward direction.

**45. Under What condition will a car skid on a leveled circular road?**

When a car moves on a leveled circular road with greater speed, static friction given by road not able provide enough centripetal force to turn. So that car will start to skid.

**46. It is dangerous to stand near the open door (or) steps while travelling in the bus. Why?**

When the bus takes sudden turn, the person standing near the open door or steps is pushed away from the bus due to centrifugal force.

## 5 Marks Q & A:

**1. Discuss the significance of Newton's laws.**

**(a) Newton's laws are vector laws.**

- From Newton's 2nd law, $\vec{F} = m\vec{a}$
- It can be written in the components as,

$$F_x\hat{\imath} + F_y\hat{\jmath} + F_z\hat{k} = ma_x\hat{\imath} + ma_y\hat{\jmath} + ma_z\hat{k}$$

- By comparing components on both sides,
- $F_x = ma_x$. The acceleration along x-direction depends on component of force along x direction.
- $F_y = ma_y$. The acceleration along y-direction depends on component of force along y direction.
- $F_z = ma_z$. The acceleration along z-direction depends on component of force along z direction.
- So that Force acting along one direction doesn't affect force acting along the other direction.

**(b) The acceleration experienced by the body at time depends only on the force at that instant.**

- Time dependent force can be written as,

$$\vec{F}(t) = m\vec{a}(t)$$

- So that acceleration of the object doesn't depend on the previous history of the force.
- For example, when a ball is bowled, the acceleration of the ball leaves the hand doesn't depend on the force in which it is bowled.

**(c) Direction of motion doesn't depend on the direction of force.**

**Case(i): Force and motion in the same direction.**

When an apple falls from a tree, direction of motion of the apple is along the gravitational force.

**Case(ii): Force and motion not in the same direction.**

The Moon experiences a force in different direction when it revolves elliptically around the Earth.

**Case(iii): Force and motion in the opposite direction.**

If an object is thrown vertically upwards, the direction of motion and gravitational force are opposite.

**Case(iv): Zero net force, but there is motion.**

When a raindrop gets detached from the cloud, downward gravitational force is equalised by the air drag (viscous) force in upward direction in certain time. Now raindrops moves with constant velocity without the net force till the surface of the Earth.

**(d) Net force of multiple forces provides acceleration.**

If multiple forces $\vec{F_1}, \vec{F_2}, \vec{F_3}, \ldots \vec{F_n}$ act on the same body, then the total force $(\vec{F}_{net})$ is equal to the vector sum of the individual forces. Their net force provides the acceleration.

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots + \vec{F}_n$$

**(e) Newton's 2nd law is second order differential equation**

- Since the acceleration is the second order derivative of position vector of the body, i.e. $\left[\vec{a} = \frac{d^2\vec{r}}{dt^2}\right]$ the force can be written as,

$$\vec{F} = m\vec{a} = m\frac{d^2\vec{r}}{dt^2}$$

- So that Newton's 2nd law is second order differential equation.

**(f) Newton's first and second laws are internally consistent.**

- If force acting on the body is zero, according to Newton's 2nd law,

$$m\frac{d\vec{v}}{dt} = 0$$

- It implies $\vec{v}$ = constant. It is essentially Newton's first law. Though Newton's 2nd law is internally consistent with first law, it cannot be derived from each other.

**(g) Newton's second law is cause and effect relation.**

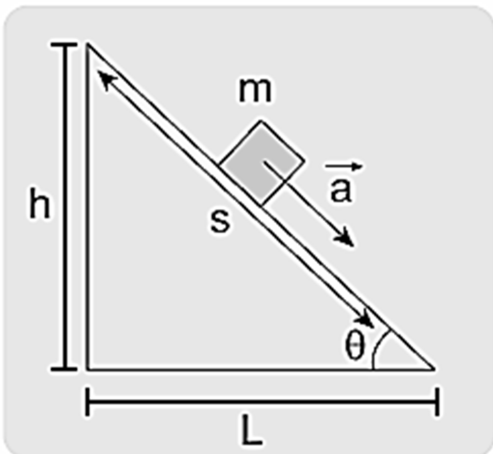
- Since Newton's 2nd law is cause and effect relation, conventionally cause (Force) should be written in right and effect $(m\vec{a})$ in the left of the equation.

$$m\vec{a} = \vec{F}$$

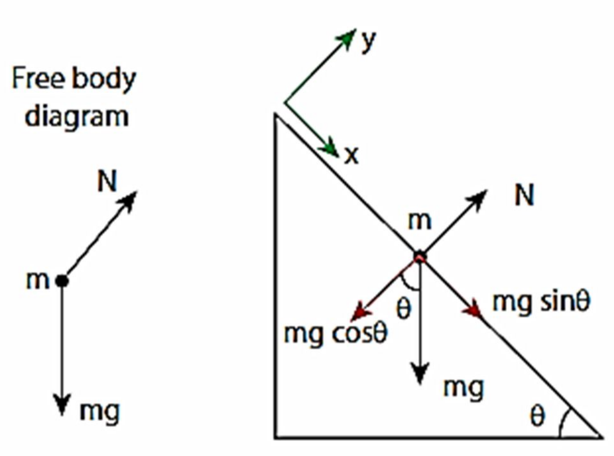
$$\frac{d\vec{p}}{dt} = \vec{F}$$

**2. Obtain the expressions for acceleration and speed of an object moving in an inclined plane.**

- When an object of mass m slide on a frictionless inclined surface at an angle θ.
- The forces acting on the object is (i) Downward gravitational force (ii) Normal force perpendicular to the inclined surface.



- To draw free body diagram the block is assumed to be point mass. Now the coordinate system is taken parallel to inclined surface.
- The gravitational force mg is resolved into mgsinθ and mgcosθ. They are parallel and perpendicular to inclined surface respectively.
- The angle made by the mg with mg cosθ is θ as shown in figure.




- As the normal force N is compensated by mgcosθ, there is no motion along y-axis.

$$-mgcos\theta\hat{j} + N\hat{j} = 0$$
$$N\hat{j} = mgcos\theta\hat{j}$$

- Comparing components on both sides,

$$N = mgcos\theta$$

- Since the component mgsinθ is not compensated by any force, the object starts to slide along x-direction. By using Newton's 2nd law, We write,

$$mgsin\theta\hat{\imath} = ma\hat{\imath}$$

- Comparing components on both sides,

$$mgsin\theta = ma$$

- The acceleration of the object is, $\boxed{a = gsin\theta}$
- If θ =90$^0$ , the object moves vertically downward with acceleration $a = g$ .
- Applying 3rd equation of motion along x-direction, we get,

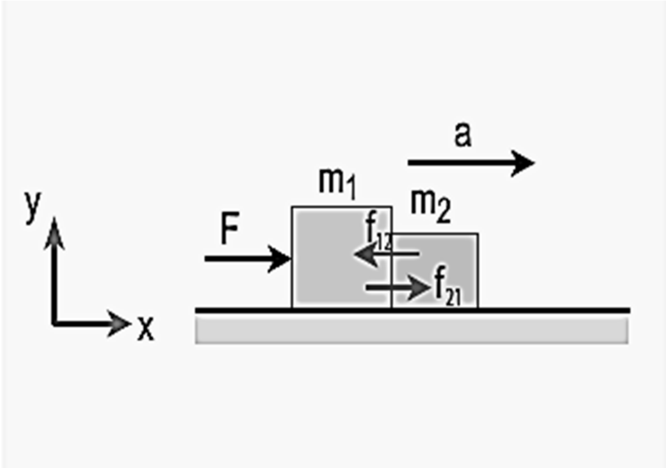
$$v^2 = u^2 + 2aS$$

- Here the initial speed u = 0 and a = gsinθ , the speed of the object sliding can written as,

$$\boxed{v = \sqrt{2Sg\ sin\theta}}$$

**3. Obtain the expressions for acceleration of two bodies of different masses and show that forces acting on each other is equal and opposite.**

- Consider two blocks of masses $m_1$ and $m_2$ ($m_1$>$m_2$) kept in contact with each other on horizontal frictionless surface as shown in figure.




- By the application of a horizontal force F, both the blocks move with acceleration a simultaneously along F.
- If m = $m_1$ + $m_2$ , according to Newton's 2nd law,

$$\vec{F} = m\vec{a}$$

- If motion is along x-direction,

$$F\hat{\imath} = ma\hat{\imath}$$

- Comparing the components on both sides,

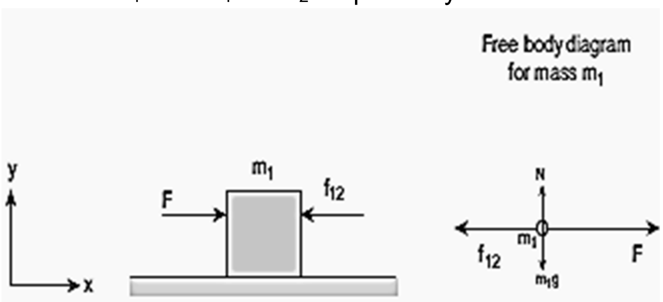
$$F = ma$$
$$F = (m_1 + m_2)a \quad [m = m_1 + m_2]$$

- The acceleration of the system is given by,

$$\boxed{a = \frac{F}{m_1 + m_2}}$$

<u>Proof :</u> **Forces acting on each other is equal and opposite.**

- Let $F_{12}$ and $F_{21}$ are forces of contact exerted by $m_2$ on $m_1$ and $m_1$ on $m_2$ respectively.




- According to the above free body diagram,

$$F\hat{\imath} - f_{12}\hat{\imath} = m_1a\hat{\imath}$$

- By comparing the components, we get,

$$F - f_{12} = m_1a$$
$$f_{12} = F - m_1a$$

- Substituting the value of 'a', we get,

$$f_{12} = F - m_1\left(\frac{F}{m_1 + m_2}\right)$$

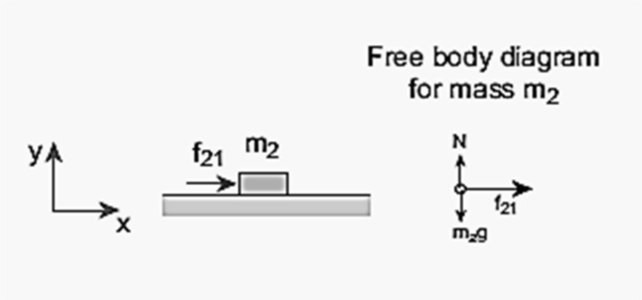
$$f_{12} = F\left[1 - \frac{m_1}{m_1 + m_2}\right]$$

$$f_{12} = \frac{Fm_2}{m_1 + m_2}$$

- In vector form, the contact force on $m_1$ by $m_2$ is given by,

$$\boxed{\vec{f}_{12} = -\frac{Fm_2}{m_1 + m_2}\hat{\imath}}$$

The –ve sign indicates that $\vec{f}_{12}$ is along negative x-direction.




- According to the above free body diagram,

$$f_{21}\hat{\imath} = m_2 a\hat{\imath}$$

- By comparing the components, we get,

$$f_{21} = m_2 a$$

- Substituting the value of 'a', we get,

$$f_{21} = \frac{Fm_2}{m_1 + m_2}$$

- In vector form, the contact force on $m_1$ by $m_2$ is given by,

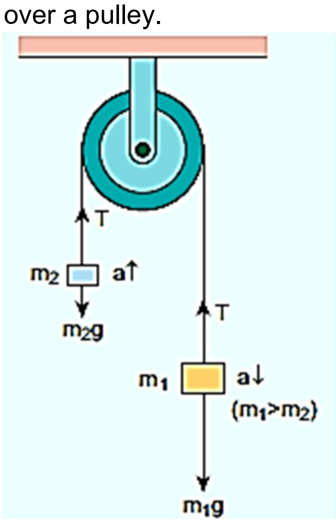
$$\boxed{\vec{f}_{21} = \frac{Fm_2}{m_1 + m_2}\hat{\imath}}$$

$\vec{f}_{21}$ is along positive x-direction.

- Therefore, $\vec{f}_{12} = -\vec{f}_{21}$, which confirm Newton's 3rd law.

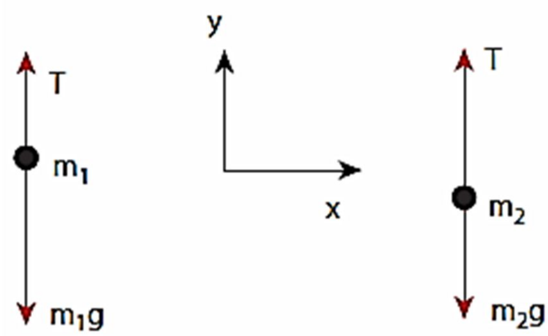
**4. Explain the motion of blocks connected by a string in verticle direction.**

- Consider two blocks of masses $m_1$ and $m_2$ ($m_1$>$m_2$) are connected by a light inextensible string that passes over a pulley.




- Let T be the tension in the string. When the system is released $m_1$ moves downward and $m_2$ moves upward with the same acceleration 'a'.

Free body diagram




**(a) To find acceleration:**

- Applying Newton's 2nd law for $m_2$, we get,

$$T\hat{\jmath} - m_2 g\hat{\jmath} = m_2 a\hat{\jmath}$$

- Comparing the components on both sides,

$$T - m_2 g = m_2 a \quad \text{------> (1)}$$

- Applying Newton's 2nd law for $m_1$, we get,

$$T\hat{\jmath} - m_2\hat{\jmath} = -m_1 a\hat{\jmath}$$

The –ve sign on R.H.S denotes $m_1$ moves along the negative y-direction.

- Comparing the components on both sides,

$$T - m_2 g = -m_1 a$$

$$m_2 g - T = m_1 a \quad \text{-------> (2)}$$

- Adding equations (1) & (2), we get,

$$m_1 g - m_2 g = m_1 a + m_2 a$$

$$(m_1 - m_2)g = (m_1 + m_2)a$$

$$\boxed{a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)g} \text{ ---→ (3)}$$

- If $m_1 = m_2$, then a = 0. It shows if masses are equal whole system will be at rest.

- In vector form, $\vec{a} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g\,\hat{\jmath}$ for $m_1$

$\vec{a} = -\left(\frac{m_1 - m_2}{m_1 + m_2}\right) g\,\hat{\jmath}$ for $m_2$

**(b) To find the tension in the string :**

- Substituting equation (3) in (1), we get,

$$T - m_2 g = m_2\left(\frac{m_1 - m_2}{m_1 + m_2}\right)g$$

$$T = m_2 g + m_2\left(\frac{m_1 - m_2}{m_1 + m_2}\right)g$$

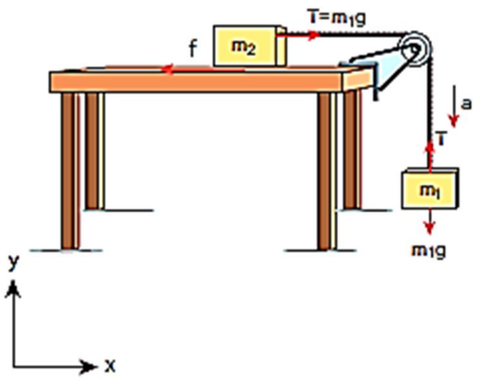
$$T = m_2 g\left(1 + \frac{m_1 - m_2}{m_1 + m_2}\right)$$

$$T = m_2 g\left(\frac{m_1 + m_2 + m_1 - m_2}{m_1 + m_2}\right)$$

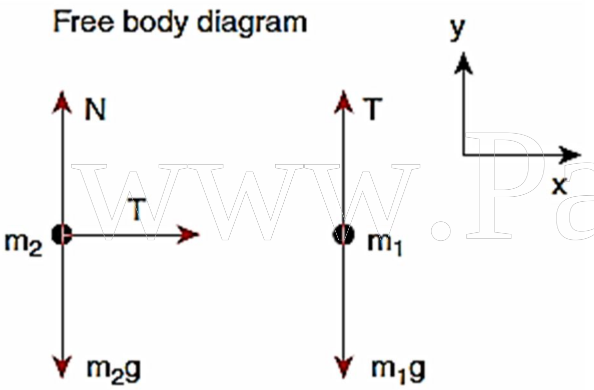
$$\boxed{T = \left(\frac{2m_1 m_2}{m_1 + m_2}\right)g}$$

**5. Explain the motion of blocks connected by a string in horizontal direction.**

- Consider the $m_2$ is kept on a frictionless horizontal table.The mass $m_2$ is connected with hanging mass $m_1$ by a string which pass through a small pulley as shown in figure.




- As both the blocks are connected to the unstretchable string, $m_1$ moves downward and $m_2$ moves horizontal with same acceleration.
- Forces acting on $m_1$ and $m_2$ are as shown in free body diagram.




- Applying Newton's 2nd law for $m_1$, we get,

$$T\hat{j} - m_1 g\hat{j} = m_1 a\hat{j}$$

- Comparing the components on both sides,

$$T - m_1 g = m_1 a \text{ ------> (1)}$$

- Applying Newton's 2nd law for $m_2$, we get,

$$T\hat{i} = m_2 a\hat{i}$$

- Comparing the components on both sides,

$$T = m_2 a \text{ ------> (2)}$$

- As gravitation force on $m_2$ and Normal force are balanced, there is no vertical acceleration in $m_2$.

$$N\hat{j} - m_2 g\hat{j} = 0$$

- Comparing the components on both sides,

$$N - m_2 g = 0$$
$$N = m_2 g$$

- Substituting equation (2) in (1), we get,

$$m_2 a - m_1 g = m_1 a$$
$$m_2 a + m_1 a = m_1 g$$

$$\boxed{a = \left(\frac{m_1}{m_1 + m_2}\right) g} \text{ ---→ (3)}$$

- Substituting equation (3) in (2), we get,

$$\boxed{T = \left(\frac{m_1 m_2}{m_1 + m_2}\right) g}$$

- It is shown that tension in the string for horizontal motion is half that in vertical motion for same $m_1$ and $m_2$.

**6. State and Prove the law of conservation of linear momentum.**

- **Statement :** If there is no external force acting on the system, the total linear momentum of the system is always a constant vector.
- When two particles interact with each other, $F_{12}$ and $F_{21}$ are the forces exerted by the particle 2 on 1 and by the particle 1 on 2 respectively.
- According to Newton's 3rd law,

$$\vec{F}_{12} = -\vec{F}_{21} \text{-------->(1)}$$

- According to Newton's 2nd law,

$$\vec{F}_{12} = \frac{d\vec{p}_1}{dt} \quad and \quad \vec{F}_{21} = \frac{d\vec{p}_2}{dt} \text{ ---→ (2)}$$

Here $p_1$ and $p_2$ are the linear momentum of particle 1 and 2.

- Substituting equation (2) in (1), we get,

$$\frac{d\vec{p}_1}{dt} = -\frac{d\vec{p}_2}{dt}$$
$$\frac{d\vec{p}_1}{dt} + \frac{d\vec{p}_2}{dt} = 0$$
$$\frac{d}{dt}(\vec{p}_1 + \vec{p}_2) = 0$$

$$\boxed{\vec{p}_1 + \vec{p}_2 = constant}$$

- Hence the the total linear momentum ( $\vec{p}_1 + \vec{p}_2$) of the system is a constant vector.

**7. Using conservation of linear momentum, find the recoil velocity of a gun when a bullet is fired from it.**

- Consider $\vec{p}_1$ and $\vec{p}_2$ are the initial momentum of a bullet and a gun before firing.
- Since initially both the gun and bullet are at rest,

$$\vec{p}_1 = 0 \;\; and \;\; \vec{p}_2 = 0$$

- Hence, total linear momentum before firing can be written as,

$$\vec{p}_1 + \vec{p}_2 = 0$$

- When the gun is fired, the momentum of the bullet changes from $\vec{p}_1$ to $\vec{p}_1'$ and the momentum of the gun changes from $\vec{p}_2$ to $\vec{p}_2'$.
- According to conservation of linear momentum, total linear momentum after firing must be equal to total linear momentum before firing. So that,

$$\vec{p}_1' + \vec{p}_2' = 0 \text{ ------> (1)}$$

- Let $m_b$ & $m_g$ are the mass of the bullet and the gun and $v_b$ & $v_g$ are the velocity and recoil velocity of the bullet and the gun respectively.
- Hence $\vec{p}_1'$ and $\vec{p}_2'$ can be written as,

$$\vec{p}_1' = m_b\vec{v}_b \quad \text{and} \quad \vec{p}_2' = m_g\vec{v}_g$$

- Substituting the valus of $\vec{p}_1'$ and $\vec{p}_2'$ in equation (1), we get,

$$m_b\vec{v}_b + m_g\vec{v}_g = 0$$

- Hence, the recoil velocity of the gun is given by,

$$\boxed{\vec{v}_g = -\frac{m_b}{m_g} \times \vec{v}_b}$$

**8. Obtain the impulse – momentum equation.**

- If a large force F acts on a object in a very short time dt, Newton's 2nd law can be written as,

$$F = \frac{dp}{dt}$$
$$dp = F\,dt$$

- Integrating over the time from an initial time $t_i$ to a final time $t_f$, we get

$$\int_{p_i}^{p_f} dp = \int_{t_i}^{t_f} F\,dt$$

Here $p_i$ and $p_f$ are the initial and final momentum at time $t_i$ and $t_f$ .

- If force F is constant over the time interval, then

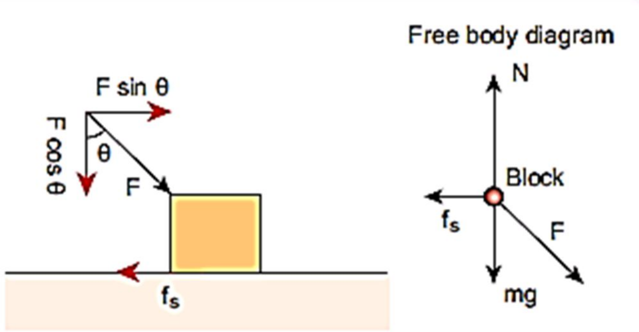
$$\int_{p_i}^{p_f} dp = F\int_{t_i}^{t_f} dt$$
$$p_f - p_i = F(t_f - t_i)$$
$$\boxed{\Delta p = F\Delta t} \text{------>(1)}$$
$$\Delta p = J \qquad [\because J = F\,\Delta t]$$

- Here $\Delta p = p_f - p_i$ , change in momentum and $\Delta t = t_f - t_i$ , time interval.
- The equation (1) is called momentum – impulse equation.

**9. Using free body diagram, show that it is easy to pull an object than push it.**

<u>(a) Pushing an object :</u>

- When an object is pushed at an arbitrary angle θ, the applied force F can be resolved into two components as shown in figure.




- From the diagram the normal force N is balance by the total downward force mg + Fcosθ. Thus,

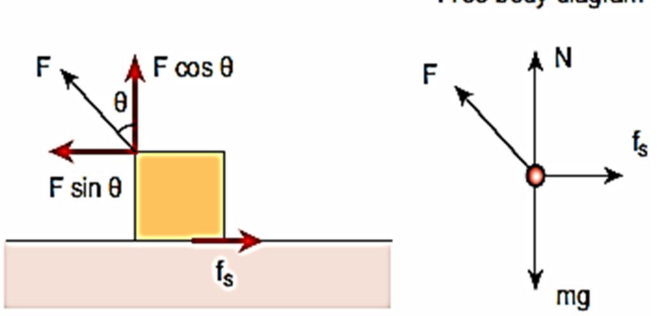
$$N_{push} = mg + F\,cos\theta$$

- In this case, maximum static friction $f_s^{max}$ can be written as,

$$f_s^{max} = \mu_s\,N_{push}$$
$$f_s^{max} = \mu_s\,(mg + F\,cos\theta) \text{ --->(1)}$$

<u>(b) Pulling an object :</u>

- When an object is pulled at an arbitrary angle θ, the applied force F can be resolved into two components as shown in figure.




- From the diagram the normal force N is balance by the total downward force mg - Fcosθ. Thus,

$$N_{pull} = mg - F\,cos\theta$$

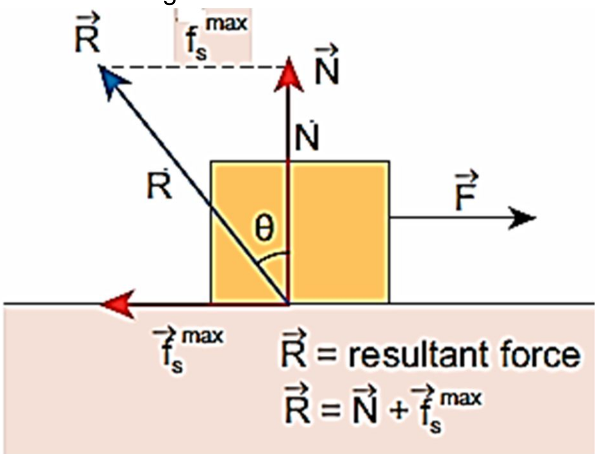
- In this case, maximum static friction $f_s^{max}$ can be written as,

$$f_s^{max} = \mu_s\,N_{pull}$$
$$f_s^{max} = \mu_s\,(mg - F\,cos\theta) \text{--->(2)}$$

- From equation (1) and (2), to overcome $f_s^{max}$ and to move the object, it is easier to pull an object than to push it.

**10. Prove that the coefficient of static friction is equal to tangent of the angle of friction.**

- Let N be the normal force, $f_s^{max}$ be the maximum friction force, R be the resultant force of N and $f_s^{max}$ and θ be the angle between R and $f_s^{max}$ as shown in figure.




- From the figure, the resultant force is given by,

$$R = \sqrt{(f_s^{max})^2 + N^2}$$

- And from the figure,

$$\tan\theta = \frac{f_s^{max}}{N}$$

- Since $f_s^{max} = \mu_s N$, $\quad \mu_s = \frac{f_s^{max}}{N}$

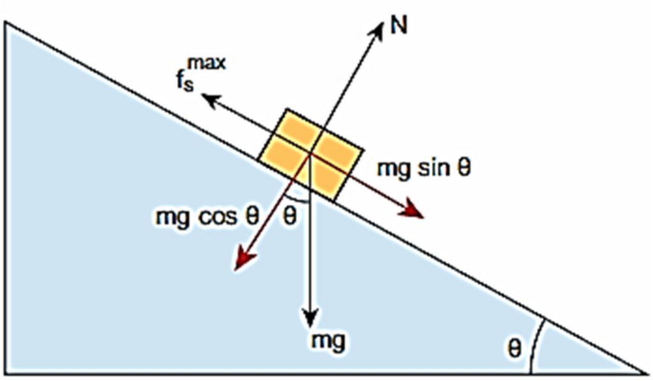
- Hence, $\tan\theta = \mu_s$

or $\boxed{\mu_s = \tan\theta}$

- Therefore, the coefficient of static friction is equal to tangent of the angle of friction.

**11. Show that in an inclined plane, the angle of friction is equal to angle of repose.**

- Consider an inclined plane on which an object is placed as shown in figure.




- Let θ be the angle of inclined plane with the horizontal, which is made equal to the angle of repose. So that the object placed on it start to slide.

- From the figure, the component mg cosθ is balanced by Normal force N can be written as,

$$N = mg\cos\theta$$

- When the object start to slide, the maximum static friction is given by,

$$f_s^{max} = \mu_s N$$
$$f_s^{max} = \mu_s\, mg\cos\theta \text{ ----->(1)}$$

- From the figure, $f_s^{max}$ can also be wrtten as,

$$f_s^{max} = mg\sin\theta \text{ ----->(2)}$$

- Equating equation (1) and (2), we get

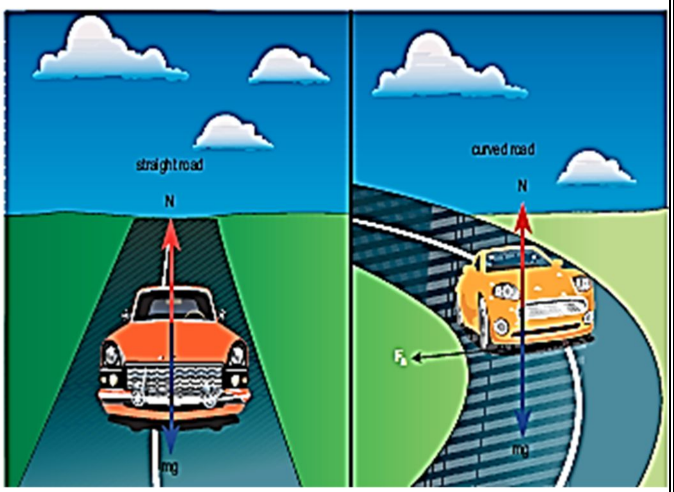
$$\mu_s\, mg\cos\theta = mg\sin\theta$$
$$\mu_s = \frac{\sin\theta}{\cos\theta}$$
$$\boxed{\mu_s = \tan\theta} \text{ ------> (3)}$$

- It shows that equation (3) is like the definition of angle of friction $\mu_s = \tan\theta$ where θ is the angle of friction.

- Thus, the angle of repose θ in equation (3) is same as the angle of friction.

**12. Obtain the conditions for safe and unsafe(skid) turn of a car on a leveled circular road.**

- Consider a car of mass 'm' moving at a speed 'v' in the circular track of radius 'r'.




- When the car is on the road, the normal force N is balanced by gravitational force mg is given by,

$$N = mg$$

- When the car turns on the circular track, the static friction provides the centripetal force can be expressed as,

$$\frac{mv^2}{r} = F_s$$

- As we know, $F_s \le \mu_s mg$ , there are two conditions possible.

**(a) For safe turn :**

$$\boxed{\frac{mv^2}{r} \leq \mu_s mg \quad (or) \quad \mu_s \geq \frac{v^2}{rg} \quad (or) \quad \sqrt{\mu_s rg} \geq v}$$

- In this case static friction gives necessary centripetal force to bend the car on the road.
- Here the co-efficient of friction between tyre and the surface of the road determines what maximum speed the car can have for safe turn.

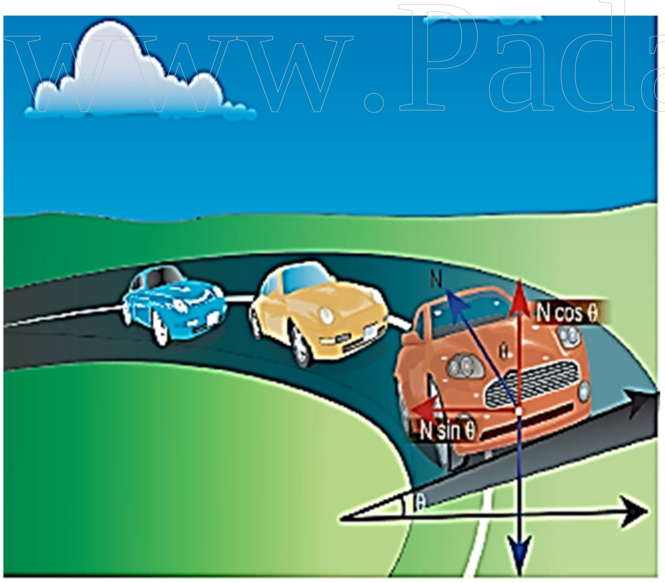
**(b) For unsafe(skid) turn :**

$$\boxed{\frac{mv^2}{r} > \mu_s mg \quad (or) \quad \mu_s < \frac{v^2}{rg}}$$

- In this case, static friction is not able to provide enough centripetal force to turn, the car will start to skid.

**13. Obtain the expression for safe speed of a car when it turns on a banking of tracks and discuss how it prevents from skidding.**

- Consider a banking of track, whose outer edge is raised at an angle θ with the horizontal as shown in figure.




- So that the normal force makes same angle θ with the vertical, can be resolved into N cosθ and N sinθ
- From the diagram, the component N cosθ is balanced by mg is written as,

$$N\,cos\theta = mg \quad \text{----->} (1)$$

- From the diagram, the centripetel force is given by N sinθ can be written as,

$$N\,sin\theta = \frac{mv^2}{r} \dashrightarrow (2)$$

- Dividing equation (2) by (1), we get

$$tan\theta = \frac{v^2}{rg}$$

$$\boxed{v = \sqrt{rg\,tan\theta}}$$

- The banking angle θ and radius of curvature of the road or track(r) determines the safe speed of the car at the turning.
- When the car just exceeds the safe speed, it will start to skid outward but the frictional force will provide additional centripetal force to prevent the outward skidding.
- When the car little slows the safe speed, it will start to skid inward but frictional force will reduce centripetal force to prevent the inward skidding.
- However, frictional force cannot prevent the car from skidding when the car speed is much greater than the safe speed.

# 4. Work, Enegy and Power

**1. What is work? Give its SI unit and dimension.**

- Work is said to be done by the force when the force applied on a body displaces it. Its SI unit is joule.

$$W = \vec{F}.\vec{dr} = F\ dr\ cos\theta$$

- Work is a scalar quantity. Its dimentional formula is $[ML^2T^{-2}]$.

**2. Explain how the definition of work in physics is different from general perception.**

- In general, any activity refers to work. It may be physical or mental work.
- But in Physics, work is treated as a physical quantity with a precise definition.

**3. Define Energy. Give its SI unit and dimension.**

- Energy is defined as the capacity to do work. Its SI unit is joule.
- Energy magnitudely equal to work. It is also a scalar. Its dimension is $[ML^2T^{-2}]$.

**4. Write some other units used in energy and equate them to joule.**

- 1 erg (CGS unit) = $10^7$ J
- 1 electron volt (1 eV) = $1.6 \times 10^{-19}$ J
- 1 calorie (1 cal) = 4.186 J
- 1 kilowatt hour (1kWh) = $3.6 \times 10^6$ J =1 unit.

**5. What are the types of mechanical energy?**

- Kinetic energy.
- Potential energy.

**6. What is Kinetic energy?**

The energy possessed by a body by virtue of its motion is called Kinetic energy.

**7. State Work – Kinetic energy theorem.**

The work done by the force on the body changes the kinetic energy of the body. This is called Work – Kinetic energy theorem.

**8. What is Potential energy?**

The energy possessed by a body by virtue of its its position is called Potential energy.

**9. What are the types of Potential energy?**

- Gravitational potential energy.
- Elastic potential energy.
- Electrostatic potential energy.

**10. What is elastic potential energy?**

The potential energy possessed by a spring due to a deforming force which stretches or compress the spring is termed as elastic potential energy.

**11. What is conservative force? Give examples.**

- If the work done by or against the force in moving body doesn't depend the nature of the path between initial and final position of the body, the force is called conservative force.
- **Example:** Elastic spring force, electrostatic force, magnetic force, gravitational force, etc.

**12. What is non-conservative force? Give examples.**

- If the work done by or against the force in moving body depends on the path between initial and final position of the body, the force is called non-conservative force.
- **Examples:** Frictional forces, viscous force

**13. Write difference between conservative and non-conservative forces.**

| S. No. | Conservative force | Non-conservative force |
|---|---|---|
| 1. | It is independent of path. | It depends on the path. |
| 2. | Work done in a round trip is zero. | Work done in a round trip is not zero. |
| 3. | Work done is completely recoverable. | Work done is not completely recoverable. |
| 4. | Total energy remains constant. | Energy dissipated as heat energy. |
| 5. | Force is the negative gradient of potential energy. | No such relation exist. |



**14. State law of conservation of energy.**

The law of conservation of energy states that energy can neither be created nor be destroyed. One form of energy can be transformed to another form but total energy of an isolated system remains constant.

**15. Define power. Give its unit.**

- Power is defined as the rate of work done or energy delivered. Its unit is watt.
- $Power(P) = \frac{Work\ done(W)}{Time\ taken(t)}$

**16. Define average power.**

- The average power is defined as the ratio of the total work done to the total time taken.
- $Average\ Power(P_{avg}) = \frac{Total\ Work\ done}{Total\ Time\ taken}$

**17. Define instantaneous power.**

- The instantaneous power is defined as the power delivered at an instant. (i.e $\Delta t \to 0$)
- $P_{avg} = \frac{dw}{dt}$

**18. Define one watt.**

One watt is defined as the power when one joule of work is done in one second.

**19. Write some other units used in power and equate them to watt.**
- 1 kW = $10^3$ W
- 1 MW = $10^6$ W
- 1 GW = $10^9$ W
- 1 hp (horse power) = 746 W

**20. What is meant by collision? Give the examples.**
- The interaction of two bodies with or without physical contacts, is known as collision.
- **Examples:** Carom, billiards, marbles, etc.

**21. What are the types of collisions?**
- Elastic collision.
- Inelastic collision.

**22. What is elastic collision? Give an example.**
- The collision in which total kinetic energy before collision is equal to the total kinetic energy after collision, is known as elastic collision.
- **Example:** electron-electron collision.

**23. What is inelastic (or) plastic collision? Give an example.**
- The collision in which total kinetic energy before collision is not equal to the total kinetic energy after collision, is known as inelastic collision.
- **Example:** Clay putty or bubblegum is thrown on a moving vehicle.

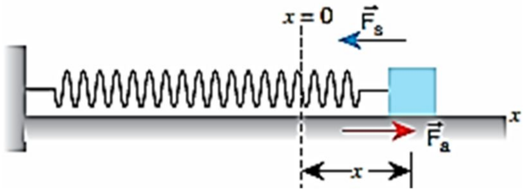
**24. Compare between elastic and inelastic collisions.**

| S. No. | Elastic collision | Inelastic collision |
|---|---|---|
| 1. | Total momentum is conserved. | Total momentum is conserved. |
| 2. | Total kinetic energy is conserved. | Total kinetic energy is not conserved. |
| 3. | Forces involved are conservative forces. | Forces involved are non-conservative forces. |
| 4. | Mechanical energy is not dissipated. | Mechanical energy is dissipated into heat, light, sound, etc. |

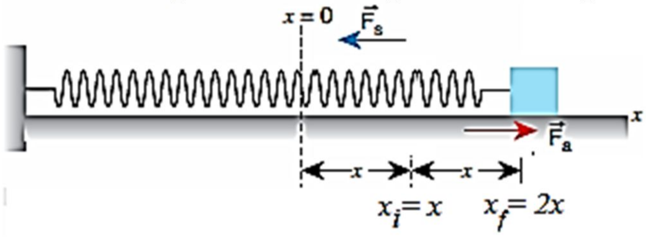
**25. Define COR (or) coefficient of restitution(e).**
- It is defined as the ratio of velocity of seperation after collision to the velocity of approach before collision.
- In general, values of COR(**e**) lies beween 0<e<1.
- For perfect elastic collision, e = 1.
- For perfect inelastic collision, e = 0.

# Conceptual Questions:

**26. A spring which initially in unstretched condition, is first stretched by a length x and again by a further length x. The work done in the first case $W_1$ is one third of the work done in second case $W_2$ . True or false?**
- True.




- In 1st case, $x_i = 0$ and $x_f = x$
- $W_1 = \frac{1}{2}k\left(x_f^2 - x_i^2\right) = \frac{1}{2}k(x^2 - 0) = \frac{1}{2}kx^2$




- In 2nd case, $x_i = x$ and $x_f = 2x$
- $W_2 = \frac{1}{2}k\left(x_f^2 - x_i^2\right)$
- $W_2 = \frac{1}{2}k(4x^2 - x^2) = \frac{1}{2}3kx^2 = 3W_1$
- Therefore, $W_1 = \frac{W_2}{3}$

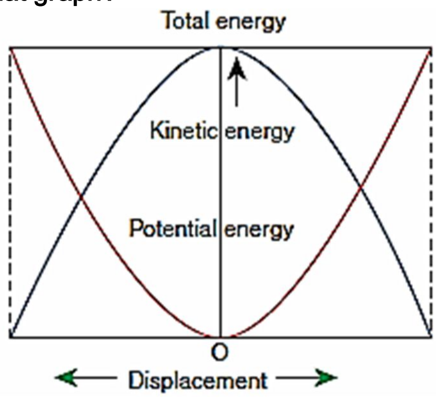
**27. Which is conserved in inelastic collision? Total energy (or ) Kinetic energy?**

Total energy, because in the inelastic collison total kinetic energy after the collision is changed.

**28. Is there any net work done by external forces on a car moving with a constant speed along a straight road?**

No. When the car is moving with constant speed in a straight road, according to Newton's law there will be no acceleration and external force. Hence, there is no external work done on a car.

**29. A car starts from rest and moves on a surface with uniform acceleration. Draw the graph of kinetic energy versus displacement. What information do you can get from that graph?**




In a frictionless environment, the energy gets transferred from kinetic to potential and potential kinetic repeatedly such that the total energy of the car remains constant.

**30. A charged particle moves towards another charged particle. Under what conditions the total momentum and the total energy of the system conserved?**

When they undergo collision process, the total momentum and the total energy of the system are conserved.

## 5 Marks Q & A:

**1. Write a brief note on work done.**

- Work is said to be done by the force when the force applied on a body displaces it. Its SI unit is joule.

$$W = \vec{F}.\overrightarrow{dr} = F\ dr\ cos\theta$$

- Work is a scalar quantity. Its dimentional formula is $[ML^2T^{-2}]$.
- The work done by the force depends on the force (F), displacement(dr) and the angle(θ) between them.
- Work done is zero in the following cases.
  (i) **When the force is zero (F =0)**
  Ex: A body is moving on a horizontal frictionless surface without force.

  (ii) **When the displacement is zero (dr =0)**
  Ex: When force is applied on a regid wall, there is no dispalcement.

  (iii) **When the force and displacement are perpendicular (θ = 90$^0$)**
  Ex: When a body moves in a horizontal direction, the gravitational force and displacement are perpendicular to each other.
- Work done is negative, when the force and displacement are opposite to each other.

  Ex: When the goal keeper catches the ball in foot ball game, the applied force and displacement of the ball are opposite to each other till the ball comes to rest.

**2. Describe the work done by a constant force.**

- When a constant force F acts on a body, the small work done(dw) by the force for the smaller displacement(dr) is given by,

$$dw = (Fcos\theta)dr$$

- The total work done from initial ($r_i$) and final ($r_f$) dispalcement is given by,

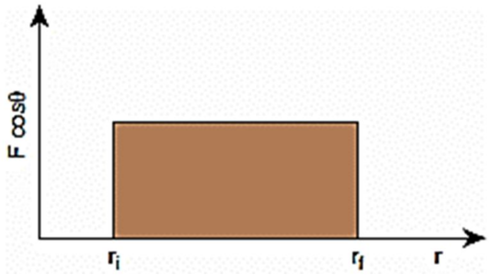
$$W = \int_{r_i}^{r_f} dW$$

$$W = \int_{r_i}^{r_f} (Fcos\theta)\, dr$$

$$W = (Fcos\theta)\int_{r_i}^{r_f} dr$$

$$W = (Fcos\theta)(r_f - r_i)$$

- The graphical representation of the work done by a constant force is shown in figure.




**3. Describe the work done by a variable force.**

- When a variable force F acts on a body, the small work done(dw) by the force for the smaller displacement is given by,

$$dw = (Fcos\theta)dr$$

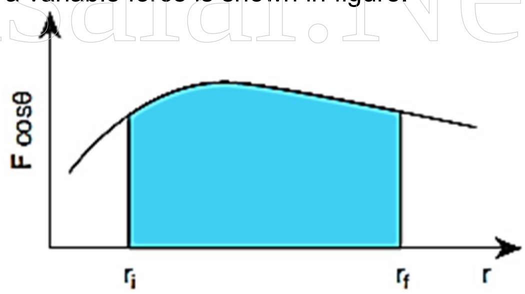
Where F and θ are variables.

- The total work done from initial ($r_i$) and final ($r_f$) dispalcement is given by,

$$W = \int_{r_i}^{r_f} dW$$

$$W = \int_{r_i}^{r_f} (Fcos\theta)\, dr$$

- The graphical representation of the work done by a variable force is shown in figure.




**4. State and prove Work – Energy theorem.**

- Work-Energy theorem: The work done by the force on the body changes the kinetic energy of the body.
- Cosider a body of mass m at rest on a frictionless horizontal surface.
- The work(W) done by the constant force (F) for a displacement (s) in the same direction is,

$$W = Fs \text{ --------> (1)}$$

- The constant force is given by,

$$F = ma \text{ -------> (2)}$$

- The 3$^{rd}$ equation of motion can be written as,

$$v^2 = u^2 + 2as$$

$$a = \frac{v^2 - u^2}{2s}$$

❖ Substituting the value of 'a' in equation(2), we get,

$$F = m\left(\frac{v^2 - u^2}{2s}\right) ---\to (3)$$

❖ Substituting equation(3) in (1), We have,

$$W = m\left(\frac{v^2 - u^2}{2s}\right) s$$

$$W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

❖ As the right hand side of the equation represents change in kinetic energy(ΔKE) of the body, then we can write,

$$\boxed{W = \Delta KE}$$

❖ This is the equation of Work-Energy theorem.

**5. Derive the relation between momentum and kinetic energy.**

❖ Consider an object of mass m moving with a velocity $\vec{v}$.

❖ The linear momentum is, $\vec{p} = m\vec{v}$ ------->(1)

❖ KInetic energy is, $KE = \frac{1}{2}mv^2$ ---------->(2)

❖ Multiply and divide the equation(2) by 'm',we get,

$$KE = \frac{1}{2}\frac{m^2(\vec{v}.\vec{v})}{m}$$

$$KE = \frac{1}{2}\frac{(m\vec{v}).(m\vec{v})}{m}$$

$$KE = \frac{1}{2}\frac{(\vec{p}.\vec{p})}{m} \qquad [\because p = mv]$$

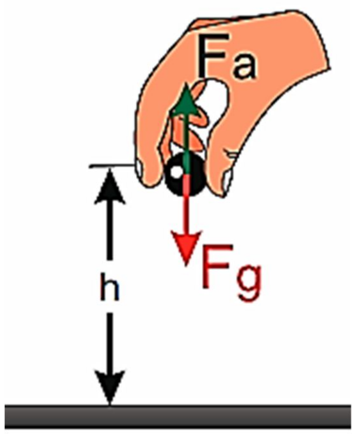
$$KE = \frac{p^2}{2m} \qquad [\because \vec{p}.\vec{p} = p^2]$$

❖ The magnitude of linear momentum can be written as,

$$\boxed{p = \sqrt{2m(KE)}}$$

**6. Derive an expression for the potential energy near the surface of the Earth.**

❖ Consider a body of mass m being moved from ground to the height h against the gravitational force as shown in figure.




❖ The gravitational force force acting on the body is,

$$\vec{F}_g = -mg\,\hat{j}$$

❖ If the body is lifted with constant velocity, the applied force ($F_a$) is equal and opposite to the gravitational force ($F_g$). So that,

$$\vec{F}_a = -\vec{F}_g = mg\,\hat{j}$$

❖ The gravitational potential energy(U) can be written as,

$$U = \int \vec{F}_a \cdot \vec{dr} = |\vec{F}_a||\vec{dr}|cos\theta$$

❖ Since the force and displacement are in the same direction, $\theta = 0^0$ . Hence, $\cos 0^0 = 1$ , $|\vec{F}_a| = mg$ and $|\vec{dr}|$= dr.

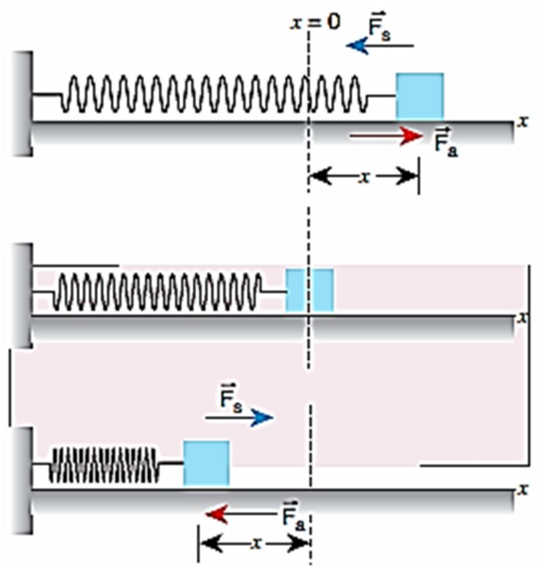
$$U = mg\int_0^h dr$$

$$U = mg[r]_0^h$$

$$\boxed{U = mgh}$$

**7. Obtain an expression for elastic potential energy of a spring stretched along horizontal direction.**

❖ Consider a spring-mass system. One end of the spring is fixed to a rigid wall and the other end is attached to the mass 'm', which is placed on a smooth horizontal table as shown in figure.




❖ Here x=0 is the equilibrium position of the mass. In this position potential energy is zero.

❖ Now the spring is stretched by a distance 'x' along the direction of applied force $\vec{F}_a$ .

❖ So that a restoring force($\vec{F}_s$) is set in the spring, which is equal and opposite to the applied force.

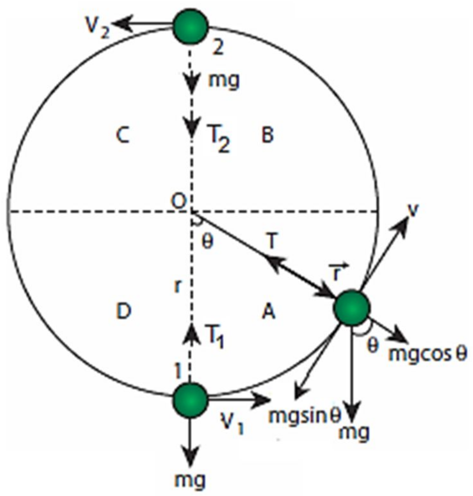
$$\vec{F}_a = -\vec{F}_s$$

- According to Hooke's law,
$$\vec{F_s} = -k\vec{x}$$
The –ve sign indicates that the spring force is always opposite to the displacement $\vec{x}$. Here k is the force constant.

- Now the applied force can be written as,
$$\vec{F_a} = k\vec{x}$$
- The work done by the applied force on the spring stretched to a smaller displacement $\overrightarrow{dx}$ , is stored as elastic potential energy(dU).
$$dU = \vec{F_a} \cdot \overrightarrow{dx} = |\vec{F_a}||\overrightarrow{dx}|cos\theta$$
- For the displacement $\vec{x}$,
$$U = \int dU = \int_0^x F_a\, dx\, cos\theta$$
- Since $F_a = kx$ and θ = 0, the potential energy can be written as,
$$U = \int_0^x kx\, dx$$
$$U = k\left[\frac{x^2}{2}\right]_0^x$$
$$U = \frac{1}{2}kx^2$$
- If the position of the mass changed from $x_i$ to $x_f$ , the potential energy can be written as,
$$U = \frac{1}{2}k(x_f^2 - x_i^2)$$
- Thus, we observe that the elastic potential energy depends on force constant k and elongation or compression x.

**8. Obtain an expression for difference in tension of a string at lowest and highest points of a vertical circular motion of a body. Also find minimum speed of the body at lowest and highest points.**

- Consider a body mass 'm' attached to one end of a massless inextensible string, which executes vertical circular motion as shown in figure.




- Let $\vec{r}$ be the radius of the circle, which is equal to length of the string, and θ be the angle made by the radial vector $\vec{r}$ with the vertical downward direction.

- In the tangential direction, applying Newton's 2nd law,
$$mgsin\theta = ma_t$$
$$mgsin\theta = -m\left(\frac{dv}{dt}\right) ---\rightarrow (1)$$
Where $a_t = -\frac{dv}{dt}$ is tangential retardation.

- In the radial direction,
$$T - mgcos\theta = ma_r$$
$$T - mgcos\theta = \frac{mv^2}{r} ---\rightarrow (2)$$
Where $a_r = \frac{v^2}{r}$ is the centripetal acceleration.

- Consider $v_1$ , $T_1$ and $v_2$ , $T_2$ are the velocities and tensions at lowest point 1 and highest point 2 respectively.

- <u>**Tension at lowest point (1) :**</u>
Here θ = $0^0$ , T= $T_1$ and v = $v_1$ . Substituting these values in equation (2), we get,
$$T_1 - mg = \frac{mv_1^2}{r}$$
$$T_1 = \frac{mv_1^2}{r} + mg ---\rightarrow (3)$$

- <u>**Tension at highest point (2) :**</u>
Here θ = $180^0$ , T= $T_2$ and v = $v_2$ . Substituting these values in equation (2), we get,
$$T_2 + mg = \frac{mv_2^2}{r}$$
$$T_2 = \frac{mv_2^2}{r} - mg ---\rightarrow (4)$$

- From equations (3) & (4), it is seen that $T_1 > T_2$.

- <u>**Difference in tension :**</u>
Subtracting equation(4) from (3), we get,
$$T_1 - T_2 = \frac{mv_1^2}{r} + mg - \frac{mv_2^2}{r} + mg$$
$$T_1 - T_2 = \frac{m}{r}[v_1^2 - v_2^2] + 2mg ---\rightarrow (5)$$

- Applying law of conservation of energy at point 1 and 2, we have,
Total energy at point 1 = Total energy at point 2
$$E_1 = E_2$$
$$U_1 + KE_1 = U_2 + KE_2 ---\rightarrow (6)$$
Where $U_1$ , $U_2$ and $KE_1$ , $KE_2$ are the potential and kinetic energies at points 1 and 2.

- Here $U_1 = 0$ (since point 1 is base point) , $U_2 = mg\,(2r)$ , $KE_1 = \frac{1}{2}mv_1^2$ and $KE_2 = \frac{1}{2}mv_2^2$ , So that from equation(6),

$$0 + \frac{1}{2}mv_1^2 = mg(2r) + \frac{1}{2}mv_2^2$$

- Rearranging we get,

$$\frac{1}{2}m(v_1^2 - v_2^2) = 2mgr$$
$$v_1^2 - v_2^2 = 4gr \longrightarrow (7)$$

- Substituting equation(7) in (5), we have,

$$T_1 - T_2 = \frac{m}{r}[4gr] + 2mg$$

- Therefore, the difference in tension is,

$$\boxed{T_1 - T_2 = 6mg}$$

- <u>**Minimum speed at the highest point (2) :**</u>
If the tension $T_2 = 0$, the body will get minimum speed to move on vertical circle. Therefore, from equation (4), we get,

$$0 = \frac{mv_2^2}{r} - mg$$
$$\frac{mv_2^2}{r} = mg$$
$$v_2^2 = rg$$
$$\boxed{v_2 = \sqrt{gr}} \longrightarrow (8)$$

- Hence, the body must have a speed $v_2 \geq \sqrt{gr}$ at point 2 to stay in the circular path.

- <u>**Minimum speed at the lowest point (1) :**</u>
To have minimum speed at point 2, the body must have minimum speed at point 1.

- From equation(7),

$$v_1^2 - v_2^2 = 4gr$$

- Substituting $v_2 = \sqrt{gr}$ , we get,

$$v_1^2 - gr = 4gr$$
$$v_1^2 = 5gr$$
$$\boxed{v_1 = \sqrt{5gr}} \longrightarrow (9)$$

- Hence, the body must have a speed $v_1 \geq \sqrt{5gr}$ at point1 to stay in the circular path.

- From equations (8) and (9), it is clear that, the minimum speed $v_1$ at point 1 should be $\sqrt{5}$ times greater than the minimum speed $v_2$ at point 2 in order to loop the circle.

**9. Derive the relation between power and velocity.**

- The work done by a force $\vec{F}$ for a displacement $\overrightarrow{dr}$ can be written as,

$$dw = \vec{F} \cdot \overrightarrow{dr}$$

- Dividing by 'dt' on both sides,

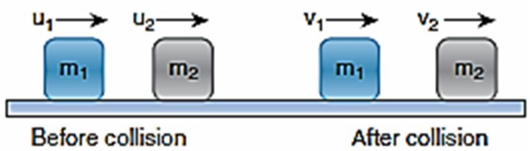
$$\frac{dw}{dt} = \vec{F} \cdot \frac{\overrightarrow{dr}}{dt}$$

- But, the power $P = \frac{dw}{dt}$ and the velocity $\vec{v} = \frac{\overrightarrow{dr}}{dt}$, Therefore,

$$\boxed{P = \vec{F} \cdot \vec{v}}$$

**10. Arrive at an expression for elastic collision in one dimension and discuss various cases.**

- Consider two elastic bodies of masses $m_1$ and $m_2$ moving in a straight line along +ve x-direction on a frictionless horizontal surface as shown in figure.




- Let $u_1$ & $v_1$ and $u_2$ & $v_2$ be the initial and final velocities of masses $m_1$ & $m_2$ respectively.

- When $u_1 > u_2$ , collision happends. For elastic collision, the total linear momentum and kinetic energies of two masses before and after collision must remain same.

- According to law of conservation of linear momentum,

$$\begin{Bmatrix}\text{Total linear momentum} \\ \text{before collision } (p_1)\end{Bmatrix} = \begin{Bmatrix}\text{Total linear momentum} \\ \text{after collision } (p_2)\end{Bmatrix}$$

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
$$m_1(u_1 - v_1) = m_2(v_2 - u_2) \longrightarrow (1)$$

- For elastic collision,

$$\begin{Bmatrix}\text{Total kinetic energy} \\ \text{before collision } (KE_1)\end{Bmatrix} = \begin{Bmatrix}\text{Total kinetic energy} \\ \text{after collision } (KE_2)\end{Bmatrix}$$

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$
$$m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2)$$
$$m_1(u_1 + v_1)(u_1 - v_1) = m_2(v_2 + u_2)(v_2 - u_2)$$
----->(2)

- Dividing equation (2) by (1), we get,

$$\frac{m_1(u_1 + v_1)(u_1 - v_1)}{m_1(u_1 - v_1)} = \frac{m_2(v_2 + u_2)(v_2 - u_2)}{m_2(v_2 - u_2)}$$
$$u_1 + v_1 = v_2 + u_2$$
$$u_1 - u_2 = v_2 - v_1$$
$$u_1 - u_2 = -(v_1 - v_2) \longrightarrow (3)$$

- It shows that for any elastic head on collision, relative speed before collision is equal and opposite to relative speed after collision.
- From equation (3),

$$v_1 = v_2 + u_1 - u_2 \longrightarrow (4)$$

and

$$v_2 = u_1 + v_1 - u_2 \longrightarrow (5)$$

- **To find final velocities $v_1$ and $v_2$ :**
  Substituting equation(5) in (1), we get,

$$m_1(u_1 - v_1) = m_2(u_1 + v_1 - u_2 - u_2)$$
$$m_1(u_1 - v_1) = m_2(u_1 + v_1 - 2u_2)$$
$$m_1u_1 - m_1v_1 = m_2u_1 + m_2v_1 - 2m_2u_2$$
$$m_1u_1 - m_2u_1 + 2m_2u_2 = m_1v_1 + m_2v_1$$
$$(m_1 - m_2)u_1 + 2m_2u_2 = (m_1 + m_2)v_1$$
$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2 \longrightarrow (6)$$

- Simillarly, substituting equation(5) in (1), we get,

$$v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 \longrightarrow (7)$$

**Case 1: When two bodies have same mass. i.e. $m_1 = m_2$**

- From equations (6) & (7),
  $v_1 = u_2$ and $v_2 = u_1$
- It shows that after collision their velocities are exchanged.

**Case 2 : When two bodies have same mass and second mass is at rest. i.e. $m_1 = m_2$ & $u_2 = 0$.**

- From equations (6) & (7),
  $v_1 = 0$ and $v_2 = u_1$
- It shows that after collision when 1st body comes to rest, the 2nd body moves with the initial velocity of 1st body.

**Case 3 : The 1st body is very much lighter than 2nd body and second mass is at rest.. i.e. $m_1 << m_2$ & $u_2 = 0$.**

- In this case, $m_1 + m_2 \approx m_2$ and $\frac{m_1}{m_2} \approx 0$
- From equation (6),

$$v_1 = \left(\frac{m_1}{m_2} - 1\right)u_1 + 2\,u_2$$
$$v_1 = (0 - 1)u_1 + 2\,(0)$$
$$v_1 = -u_1$$

- From equation (7),

$$v_2 = \left(2 \times \frac{m_1}{m_2}\right)u_1 + \left(1 - \frac{m_1}{m_2}\right)u_2$$
$$v_2 = (0)u_1 + 1\,(0)$$
$$v_2 = 0$$

- It shows that after collision 1st body returns back with its initial velocity and the 2nd body remains at rest.

**Case 4 : The 2nd body is very much lighter than 1st body and second mass is at rest.. i.e. $m_2 << m_1$ & $u_2 = 0$.**

- In this case, $m_1 + m_2 \approx m_1$ and $\frac{m_2}{m_1} \approx 0$
- From equation (6),

$$v_1 = \left(1 - \frac{m_2}{m_1}\right)u_1 + \left(2 \times \frac{m_2}{m_1}\right)u_2$$
$$v_1 = (1 - 0)u_1 + 0$$
$$v_1 = u_1$$

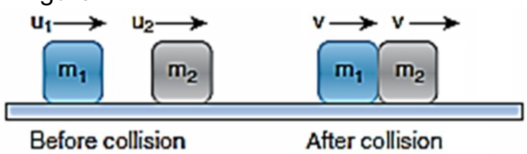
- From equation (7),

$$v_2 = 2\,u_1 + \left(\frac{m_2}{m_1} - 1\right)u_2$$
$$v_2 = 2\,u_1 + (0 - 1)\,(0)$$
$$v_2 = 2\,u_1$$

- It shows that after collision 1st body moves with its initial velocity and the 2nd body moves with two times the initial velocity of 1st body.

**11. Arrive an expression for common velocity after collision in one-dimensional perfect inelastic collision of two bodies.**

- Consider a perfect inelastic collision of two masses $m_1$ and $m_2$ moving in a straight line along +ve x-direction on a frictionless horizontal surface.
- After the collision, the objects stick together and move with common velocity 'v' as shown in figure.




- .According to law of conservation of linear momentum,

$$\begin{Bmatrix}\text{Total linear momentum}\\ \text{before collision } (p_1)\end{Bmatrix} = \begin{Bmatrix}\text{Total linear momentum}\\ \text{after collision } (p_2)\end{Bmatrix}$$

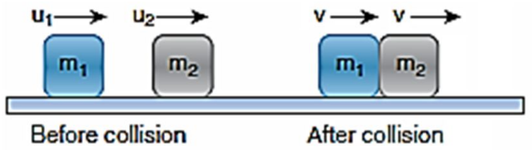
$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$

- Hence, the common velocity after collision is,

$$\boxed{v = \frac{m_1u_1 + m_2u_2}{(m_1 + m_2)}}$$

**12. Find the loss of kinetic energy in one-dimensional perfect inelastic collision of two bodies.**

- Consider a perfect inelastic collision of two masses $m_1$ and $m_2$ moving in a straight line along +ve x-direction on a frictionless horizontal surface.
- After the collision, the objects stick together and move with common velocity 'v' as shown in figure.




- Total kinetic energy before collision,

$$KE_i = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$$

- Total kinetic energy after collision,

$$KE_f = \frac{1}{2}(m_1+m_2)v^2$$

- Then the loss of kinetic energy is,

$$\Delta Q = KE_f - KE_i$$

$$\Delta Q = \frac{1}{2}(m_1+m_2)v^2 - \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$$

- But $v = \frac{m_1u_1+m_2u_2}{(m_1+m_2)}$, therefore loss of kinetic energy,

$$\boxed{\Delta Q = \frac{1}{2}\left(\frac{m_1m_2}{m_1+m_2}\right)(u_1-u_2)^2}$$

# 5. Motion of System of Particles and Rigid bodies

**1. What is a rigid body?**

A rigid body is the one, which maintains its dimension and fixed shape even when an external force acts on it.

**2. Define center of mass.**

The center of mass of a body is defined as a point where the entire mass of the body appears to be concentrated.

**3. What is a point mass?**

A point mass is a hypothetical point which has non-zero mass and no size or shape.

**4. Define torque. Give its unit.**

Torque is defined as the moment of the external applied force about a point or axis of rotation. Its unit is N m.

i.e. $\vec{\tau} = \vec{r} \times \vec{F}$

**5. Define angular momentum. Give its unit.**

The angular momentum of a point mass is defined as the moment of its linear momentum. Its unit is kg $m^2$ $s^{-1}$ .

i.e. $\vec{L} = \vec{r} \times \vec{p}$

**6. What is meant by mechanical equilibrium of a rigid body?**

A rigid body is said to be in mechanical equilibrium when both its linear momentum and angular momentum remain constant.

**7. What are the types of equilibrium?**

- Translational equilibrium
- Rotational equilibrium
- Static equilibrium
- Dynamic equilibrium
- Stable equlibrium
- Unstable equlibrium
- Neutral equilibrium

**8. What are translational equilibrium conditions?**

- Linear momentum is constant
- Net force is zero

**9. What are rotational equilibrium conditions?**

- Angular momentum is constant
- Net torque is zero

**10. What are static equilibrium conditions?**

- Linear and angular momentum are zero
- Net force and net torque are zero

**11. What are dynamic equilibrium conditions?**

- Linear and angular momentum are constant
- Net force and net torque are zero

**12. What are the stable equilibrium conditions?**

- Linear and angular momentum are zero
- The body tries to come back to equilibrium if slightly disturbed and released.
- The center of mass of the body shifts slightly higher if disturbed from equilibrium
- Potential energy of the body is minimum and it increases if disturbed.

**13. What are the unstable equilibrium conditions?**

- Linear and angular momentum are zero
- The body cannot come back to equilibrium if slightly disturbed and released.
- The center of mass of the body shifts slightly lower if disturbed from equilibrium
- Potential energy of the body is not minimum and it decreases if disturbed.

**14. What are the neutral equilibrium conditions?**

- Linear and angular momentum are zero
- The body remains at the same equilibrium if slightly disturbed and released.
- The center of mass of the body does not shifts higher or lower if disturbed from equilibrium
- Potential energy remains same even if disturbed.

**15. Distinguish between stable and unstable equilibrium.**

| S.No. | Stable equilibrium | Unstable equilibrium |
|---|---|---|
| 1. | The body returns back to equilibrium after slight disturbance. | The body does not return back to equilibrium after slight disturbance. |
| 2. | The center of mass of the body shifts higher during disturbance. | The center of mass of the body shifts lower during disturbance. |
| 3. | Potential energy is minimum and increased during disturbance. | Potential energy is maximum and decreased during disturbance. |

**16. Define a couple.**

A couple is defined as a pair of forces which are equal and opposite and seperated by a perpendicular distance causes a turning effect.

**17. Give some examples for couple.**

- Opening a cap of a pen.
- Turning a steering wheel of a car.
- opening a water tap.

**18. State principle of moments.**

When an object is in equilibrium the sum of the anticlockwise moments about a turning point must be equal to the sum of the clockwise moments.

**19. What is center of gravity?**

The center of gravity of a body is the point at which the entire weight of the body acts irrespective of the position and orientation of the body.

**20. What do you mean by the moment of inertia? Give its unit and dimension.**

The moment of inertia is a measure of rotational inertia. The unit of moment of inertia is, kg $m^2$. Its dimension is M $L^2$.

**21. Define moment of inertia.**

Moment of inertia is defined as the sum of the products of the mass and the square of the perpendicular distance to the axis of rotation of each particle in a body rotating about an axis.

$$i.e.\ \ I = \sum_{i=0}^{n} m_i\, r_i^2$$

**22. Mention any two physical significance of moment of inertia.**

- ❖ Lesser the moment of inertia, greater the speed of rotation.
- ❖ Greater the mass concentrated away from the axis of rotaion, greater the moment of inertia.

**23. What is radius of gyration? Give its unit.**

The radius of gyration of an object is the perpendicular distance from the axis of rotation to an equivalent point mass, which has same mass and moment of inertia of the object. Its unit is metre.

**24. Define radius of gyration.**

Radius of gyration is defined as the root mean square (rms) distance of the particles of the body from the axis of rotation.

$$i.e.\ K = \sqrt{\frac{r_1^2 + r_2^2 + r_3^2 + \cdots + r_n^2}{n}}$$

**25. State parallel axis theorem.**

The moment of inertia of a body about any axis is equal to the sum of its moment of inertia about a parallel axis through its center of mass and the product of the mass of the body and the square of the perpendicular distance between the two axes.

i.e. $I = I_C + md^2$

**26. State perpendicular axis theorem.**

The moment of inertia of a plane laminar body about an axis perpendicular to its plane is equal to the sum of moments of inertia about two perpendicular axes lying in the plane of the body such that all the three axes are mutually perpendicular and concurrent.

i.e. $I_z = I_x + I_y$

**27. State law of conservation of angular momentum.**

When no external torque acts on the body, the net angular momentum of a rotating rigid body remains constant.

**28. Distinguish between sliding and slipping.**

| S.No. | Sliding | Slipping |
|---|---|---|
| 1. | The translation is more than rotation. | The rotation is more than translation. |
| 2. | Relative velocity between point of contact and the surface is non-zero. | Relative velocity between point of contact and the surface is zero. |
| 3. | It happens when the moving vehicle suddenly stopped on a slippery road. | It happens when the vehicle is start to move on a slippery road or in mud. |

**29. What is the condition for pure rolling?**

In pure rolling, the total kinetic energy must be equal to the sum of kinetic energies of translational and rotational motion.

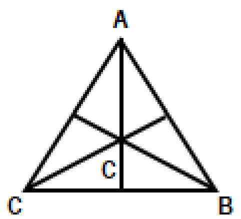
**30. Comparison of translational and rotational quantities.**

| S. No | Translational Motion | Rotational motion about a fixed axis |
|---|---|---|
| 1. | Displacement, x | Angular displacement, θ |
| 2. | Time, t | Time, t |
| 3. | Velocity, $v = \frac{dx}{dt}$ | Angular velocity, $\omega = \frac{d\theta}{dt}$ |
| 4. | Acceleration, a= $\frac{dv}{dt}$ | Angular acceleration, $\alpha = \frac{d\omega}{dt}$ |
| 5. | Mass, m | Moment of inertia, I |
| 6. | Force, $F = ma$ | Torque, $\tau = I\alpha$ |
| 7. | Linear momentum, $p = mv$ | Angular momentum, L = $I\omega$ |
| 8. | Impulse, $F\,\Delta t = \Delta p$ | Angular Impulse, $\tau\,\Delta t = \Delta L$ |
| 9. | Work done by force, w = $Fs$ | Work done by torque, $w = \tau\theta$ |
| 10. | Kinetic energy, $KE = \frac{1}{2}mv^2$ | Rotational Kinetic energy, $KE = \frac{1}{2}I\omega^2$ |
| 11. | Power, $P = Fv$ | Rotational Power, $P = \tau\omega$ |

**31. Find out the center of mass for the given geometrical structures. a) Equilateral triangle b) Cylinder c) Square.**

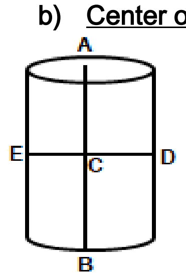
- ❖ For evenly distributed mass, center of mass will be at the geometrical center of the uniform shape.

a) <u>Center of mass for equilateral triangle :</u>



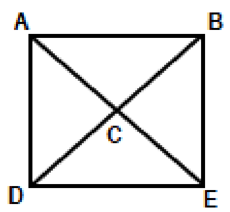

Draw the Perpendicular lines from vertices A, B and C to opposite sides. The lines meet at one point C, which is the center of mass.

b) <u>Center of mass for Cylinder :</u>




Draw the perpendicular cross line ED at the mid of the height of the cylinder. This intersect the axis of cylinder at C, Which is the center of mass.

c) <u>Center of mass for Square :</u>




Draw the diagonals AE and BD, which will intersect at C. The point C is the center of mass.

## Conceptual Questions:

**32. When a tree is cut, the cut is made on the side facing the direction in which the tree is required to fall. Why?**

The side on which the cut is made is no longer supported by the normal force from the bottom, therefore, the gravitational force acts on the tree, tries to rotate it. The torque provided by the gravity will rotate the tree such that the tree falls on the side where it was cut.

**33. Why does a porter bend forward while carrying a sack of rice on his back?**

A porter bends forward while carrying a sack of rice on his back because to change the position of centre of gravity such that he gets the stability.

**34. Why is it much easier to balance a meter scale on your fingertip than balancing on a match stick?**

The center of gravity of the meter scale is higher than the center of gravity of the matchstick. Higher the center of gravity makes lesser torque. So that it is easier to balance a meter scale on your fingertip than balancing on a match stick.

**35. Two identical water bottles one empty and the other filled with water are allowed to roll down an inclined plane. Which one of them reaches the bottom first? Explain your answer.**

Water filled bottle. Because the moment of inertia of the empty bottle is higher than the moment of inertia of the water filled bottle.

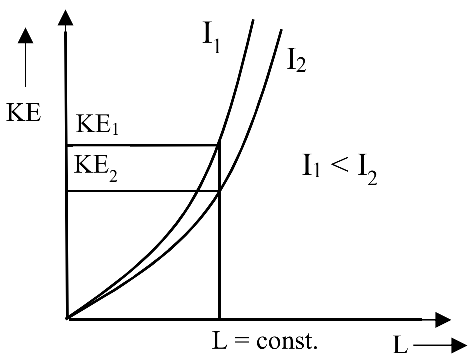
**36. Write the relation between angular momentum and rotational kinetic energy. Draw a graph for the same. For two objects of same angular momentum, compare the moment of inertia using the graph.**

<u>Relation:</u> Rotational kinetic energy,

$$KE = \frac{L^2}{2I}$$

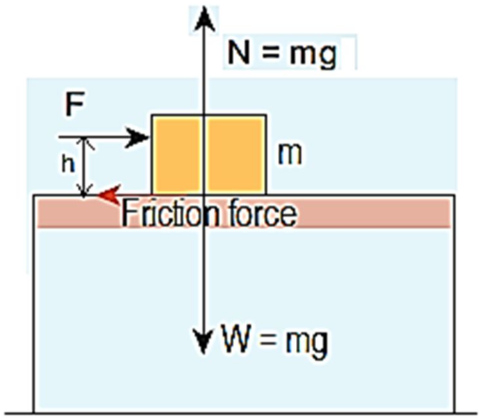
where L is angular momentum and I is moment of inertia.

<u>Graph between KE and L :</u>




The graph shows that of the two bodies of same angular momentum, those one have less rotaional kinetic energy has higher moment of inertia.

**37. A rectangle block rests on a horizontal table. A horizontal force is applied on the block at a height h above the table to move the block. Does the line of action of the normal force N exerted by the table on the block depend on h?**




- Yes. The line of action of the normal force N exerted by the table on the block depend on h.
- When height of the appiled force 'h' increases, a torque is produced by the applied force and frictional force such that block start to tilt.
- To balance this effect, line of action of normal force shift away from applied force and make a opposite torque, joining with gravitational force 'W'.

**38. Three identical solid spheres move down through three inclined planes A, B and C all same dimensions. A is without friction, B is undergoing pure rolling and C is rolling with slipping. Compare the kinetic energies $E_A$, $E_B$ and $E_C$ at the bottom.**

- In this case, when three identicle solid spheres starts to move on the inclined planes, they all have same potential energy.
- During the motion, the potential energy is converted into kinetic energy.
- According to law of conservation of energy, at the bottom all the potential energy is converted into kinetic energy.
- Such that all three spheres have same kinetic energy at the bottom whatever be the type of motion. i.e. $E_A = E_B = E_C$ .

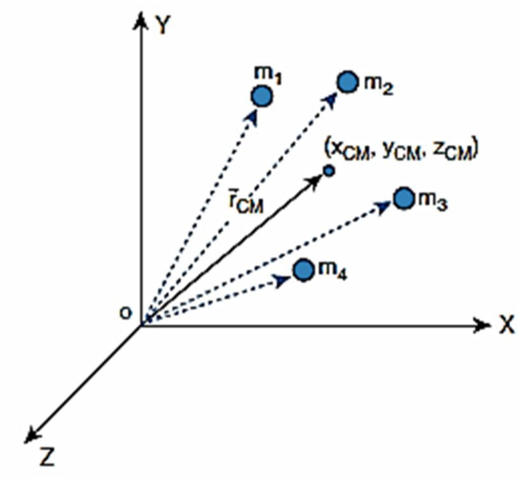
**39. Give an example to show that the following statement is false. 'Any two forces acting on a body can be combined into single force that would have same effect'.**

- Consider two equal and opposite forces acting on a wheel.
- If two foces combined and acting on single point on the wheel, there will be no effect. However, if they act seperatly on the edges of the wheel, there will be a rotating effect.
- This example falsifies the given statement.

## 5 Marks Q & A:

**1. Find the center of mass for a collection of 'n' point masses.**

- Consider the point masses $m_1, m_2, m_3, \ldots m_n$ whose position coordinates from orgin O along X-direction are $x_1, x_2, x_3, \ldots x_n$ as shown in figure.




- The equation for the X coordinate of the center of mass is,

$$X_{CM} = \frac{\sum m_i x_i}{\sum m_i}$$

Where, $\sum m_i = M$ , is the total mass of all the particles.

- Hence,

$$X_{CM} = \frac{\sum m_i x_i}{M}$$

- Similarly, the Y and Z coordinates of center of mass can be written as,

$$Y_{CM} = \frac{\sum m_i y_i}{M}$$

$$Z_{CM} = \frac{\sum m_i z_i}{M}$$

- The position of the center of mass of these masses is ( $X_{CM}$ , $Y_{CM}$ , $Z_{CM}$ ). In general, the position of center of mass in vector form can be expresssed as,

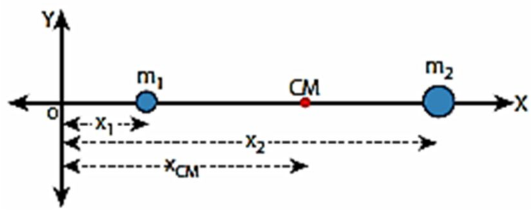
$$\vec{r}_{CM} = \frac{\sum m_i \vec{r}_i}{M}$$

Where, $\vec{r}_{CM} = X_{CM}\hat{\imath} + Y_{CM}\hat{\jmath} + Z_{CM}\hat{k}$ is the position vector of the center of mass and $\vec{r}_i = X_i\hat{\imath} + Y_i\hat{\jmath} + Z_i\hat{k}$ is the position vector of the distributed point mass.

**2. Find the center of mass of two point masses by shifting the origin.**

- Consider the point masses $m_1$ and $m_2$ which are positioned as $x_1$ and $x_2$ along X-axis. The center of mass can be found in this system in three ways as follows.

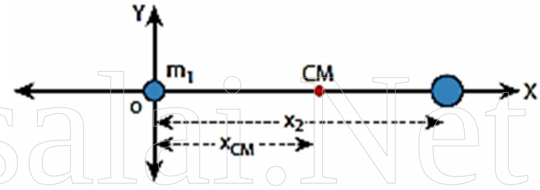
- <u>When the masses are on positive X-axis :</u>
  - The origin is taken arbitrarily as shown in figure.




  - The center of mass along X-axis is,

$$X_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

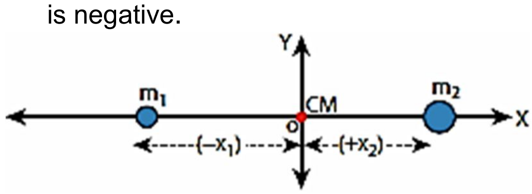
- <u>When the origin coincides with any one of the masses:</u>
  - If the orgin coincide with mass $m_1$ as shown in figure, its position $x_1 = 0$.




  - The center of mass along X-axis is,

$$X_{CM} = \frac{m_1(0) + m_2 x_2}{m_1 + m_2} = \frac{m_2 x_2}{m_1 + m_2}$$

- <u>When the origin coincides with center of the mass itself :</u>
  - If the origin coincide with center of mass as shown in figure, $X_{CM} = 0$. Hence, the position $x_1$ is negative.




  - The center of mass along X-axis is,

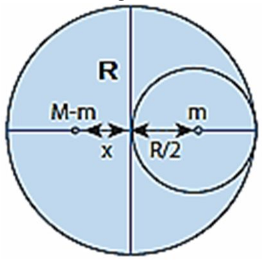
$$0 = \frac{m_1(-x_1) + m_2 x_2}{m_1 + m_2}$$

$$m_1 x_1 = m_2 x_2$$

  - The above equation is known as principle of moments.

**3. From a uniform disc of radius R, a small disc of radius $\frac{R}{2}$ is cut and removed as shown in the diagram. Find the center of mass of the remaining portion of the disc.**

- Let us consider the mass of the uncut full disc be M. Its center of mass would be at the geometric center of the disc on which the origin coincides.
- Now the small disc of the mass m is cut from the disc M. So that the center of mass the small disc is at R/2 as shown in figure.




- Hence, the center of mass of remaining disc is shifted to x distance left to the origin.
- Applying principle of moments,

$$(M-m)x = (m)\frac{R}{2}$$

$$x = \left[\frac{m}{(M-m)}\right]\frac{R}{2} \longrightarrow (1)$$

- If σ is the surface mass density (i.e. $\sigma = \frac{M}{\pi R^2}$), the mass m of small disc is,

$$m = surface\ mass\ density \times surface\ area$$

$$m = \sigma \times \pi\left(\frac{R}{2}\right)^2$$

$$m = \frac{M}{\pi R^2} \times \pi\left(\frac{R}{2}\right)^2 = \frac{M}{\pi R^2} \times \pi \times \frac{R^2}{4} = \frac{M}{4}$$

- Substituting the value of m in equation(1), we get,

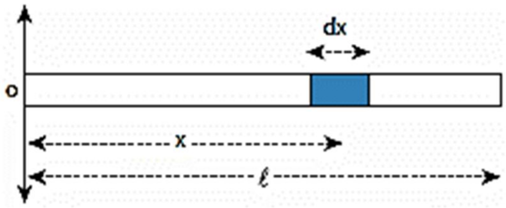
$$x = \left[\frac{\frac{M}{4}}{\left(M-\frac{M}{4}\right)}\right]\frac{R}{2} = \left[\frac{\frac{M}{4}}{\left(\frac{3M}{4}\right)}\right]\frac{R}{2}$$

$$x = \frac{R}{6}$$

- Therefore, the center of mass of the remaining portion is at a distance of R/6 left to the center of disc.

**4. Locate the center of mass of a uniform rod of mass M and length $\ell$.**

- Consider a uniform rod of mass M and length $\ell$ whose one end coincides with the origin as shown in figure.




- The rod is along x-axis. Let dm be the small mass of elemental length dx at a distance x from the origin.
- If λ is the linear mass density (i.e. $\lambda = \frac{M}{\ell}$), the mass of small element dm is,

$$dm = \frac{M}{\ell}dx$$

- Now the center of mass of the rod is,

$$X_{CM} = \frac{\int x dm}{\int dm}$$

$$X_{CM} = \frac{\int_0^\ell x\frac{M}{\ell}dx}{M} = \frac{1}{\ell}\int_0^\ell x\, dx$$

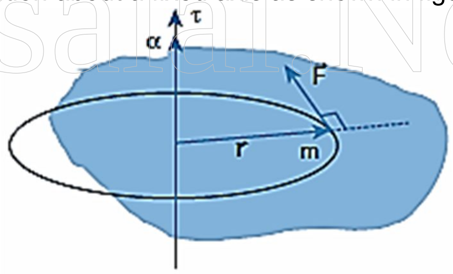
$$X_{CM} = \frac{1}{\ell}\left[\frac{x^2}{2}\right]_0^\ell = \frac{1}{\ell}\left[\frac{\ell^2}{2}\right]$$

$$X_{CM} = \frac{\ell}{2}$$

- Therefore, the center of mass of the uniform rod is at its geometrical center.

**5. Obtain the relation between torque and angular acceleration.**

- Consider a rigid body rotating about a fixed axis. A point mass m in the body will execute a circular motion about a fixed axis as shown in figure.




- Let $\vec{F}$ be a tangential force acting on the point mass produces the torque for rotation and $\vec{r}$ be the position vector of the point mass.
- The torque produced by the force on the point mass m about the axis of rotation is written as,

$$\tau = rF\sin 90^0 = rF \qquad [\because \sin 90^0 = 1]$$

$$\tau = r\,ma \qquad [\because F = ma]$$

$$\tau = r\,m\,r\alpha = mr^2\alpha \qquad [\because a = r\alpha]$$

$$\tau = mr^2\alpha \longrightarrow (1)$$

- For all particles of the body, $mr^2 = \sum m_i r_i^2$
- Therefore, $\tau = (\sum m_i r_i^2)\alpha$

$$\tau = I\alpha$$

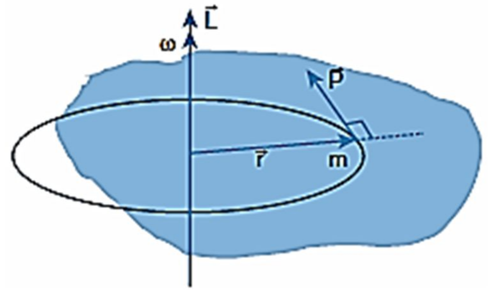
Where, $I = \sum m_i r_i^2$, moment of inertia of the rigid body.

- In vector form,

$$\vec{\tau} = I\vec{\alpha}$$

**6. Obtain the relation between angular momentum and angular velocity.**

- Consider a rigid body rotating about a fixed axis. A point mass m in the body will execute a circular motion about a fixed axis as shown in figure.




- let $\vec{L}$ be the angular momentum which is perpendicular to position vector $\vec{r}$ and linear momentum vector $\vec{p}$.

- As the angular momentum is the moment of linear momentum,

$$L = r\,p\sin 90^0 \qquad [\because \sin 90^0 = 1]$$

$$L = r\,mv = rm\,r\omega \qquad [\because p = mv]$$

$$L = mr^2\omega$$

- For all particles of the body, $mr^2 = \sum m_i r_i^2$

- Therefore, $L = (\sum m_i r_i^2)\omega$

$$L = I\omega$$

Where, $I = \sum m_i r_i^2$, moment of inertia of the rigid body.

- In vector form,

$$\vec{L} = I\vec{\omega}$$

**7. Obtain the relation between torque and angular momentum.**

- The angular momentum expression is,

$$L = I\omega ---\rightarrow (1)$$

- The torque expression is,

$$\tau = I\alpha = I\left(\frac{d\omega}{dt}\right) \qquad \left[\because \alpha = \frac{d\omega}{dt}\right]$$

$$\tau = \frac{d(I\omega)}{dt} ---\rightarrow (2)$$

- Substituting equation(1) in (2), we get,

$$\tau = \frac{dL}{dt}$$

- In vector form,

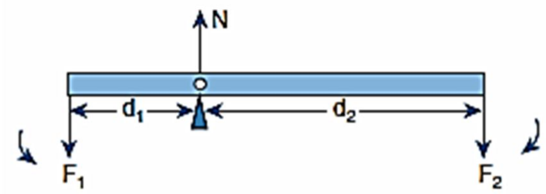
$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

**8. State and prove principle of moments. Also get the expression for mechanical advantage.**

- **Statement :** When an object is in equilibrium the sum of the anticlockwise moments about a turning point must be equal to the sum of the clockwise moments.

- **Proof :**
  Consider a light rod of negligible mass which is pivoted at a point along its length.

- Let two parallel forces $F_1$ and $F_2$ act at the two ends at distances $d_1$ and $d_2$ from the point of pivot and the normal reaction force N at the point of pivot as shown in figure.




- Since the rod has to remain stationary in horizontal position, it should be in transitional and rotational equilibrium. Then, both the net force and net torque must be zero.

- For net force to be zero,

$$-F_1 + N - F_2 = 0$$

- For net torque to be zero,

$$d_1F_1 - d_2F_2 = 0$$

$$d_1F_1 = d_2F_2 --\rightarrow (1)$$

- The above equation represents principle of moments.

- The beam balance used for weighing goods uses this princilple with $d_1 = d_2$ and $F_1 = F_2$ .

- **Mechanical Advantage (MA) :**
  From the equation(1),

$$\frac{F_1}{F_2} = \frac{d_2}{d_1} --\rightarrow (2)$$

- If $F_1$ is the load and $F_2$ is our effort, we get advantage when $d_1 < d_2$ .This implies that $F_1 > F_2$. Hence we can lift a large load with small effort.

- In equation(2), the term $\left(\frac{d_2}{d_1}\right)$ is called mechanical advantage of the simple lever. The pivot point is called fulcrum.

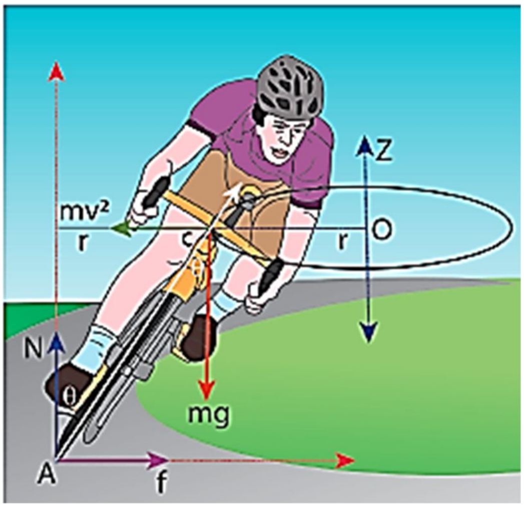
- Thus, the mechanical advantage(MA) is expressed as,

$$Mechanical\ Advantage(MA) = \frac{d_2}{d_1}$$

- There are many simple machines that work on the above principle.

**9. Explain why a cyclist bends while negotiating a curve road? Arrive at the expression for angle of bending for a given velocity.**

❖ Consider a cyclist negotiating a circular level road(not banked) of radius r with a speed v about the center O as shown in figure.

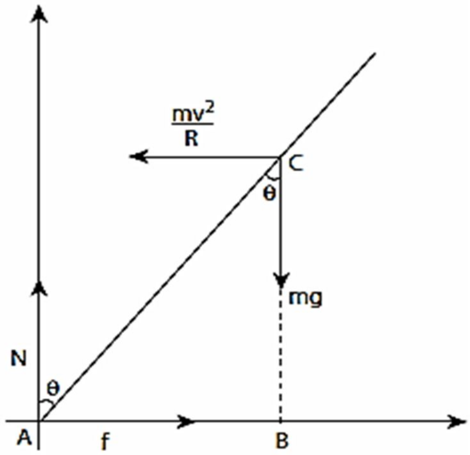



❖ Let m be the mass of the system, which includes cycle and cyclist and C be the center of gravity of this system.

❖ Let us consider horizontal is along x-axis and vertical is along Z-axis.

❖ The system as a frame rotating about Z-axis and the system is at rest in this rotating frame Z.

❖ In this rotating frame, the centrifugal force $\frac{mv^2}{r}$ acts away from center O and passing through the center of gravity C as shown below.




❖ As the system is in rotational equilibrium, the net torque must be zero. Thus,

$$\vec{\tau}_{net} = 0$$

$$-mg\ AB + \frac{mv^2}{r} BC = 0$$

Here, the clock wise moment (mg AB) is taken as negative and the anti clockwise moment $\left(\frac{mv^2}{r} BC\right)$ is taken as positive.

$$mg\ AB = \frac{mv^2}{r} BC$$

❖ But from ΔABC, AB = AC sinθ & BC = AC cosθ. Therefore, the above equation can be written as,

$$mg\ AC\ \sin\theta = \frac{mv^2}{r} AC\ \cos\theta$$

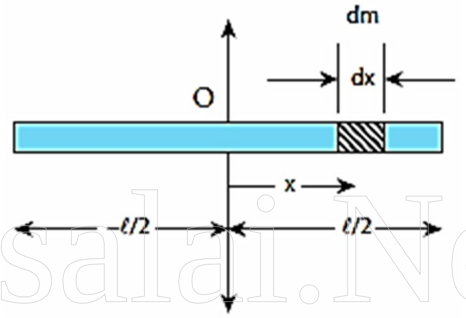
$$\tan\theta = \frac{v^2}{rg}$$

$$\theta = \tan^{-1}\left[\frac{v^2}{rg}\right]$$

❖ It shows that while negotiating a circular road of radius r at velocity v, a cyclist has to bend an angle θ from vertical, to avoid a fall.

**10. Obtain the expression for moment of inertia of a uniform rod.**

❖ Consider a uniform rod of mass M and length $\ell$ as shown in figure.






❖ Let us consider the rod is along the x-axis and the moment of inertia of the rod is found about the axis, which passes through center of mass (here the geometrical center) of the rod 'O'.

❖ Now the moment of inertia of an infinitesimal small mass 'dm' of length dx of the rod, which is at a distance 'x' from O can be expressed as,

$$dI = (dm)x^2 \longrightarrow (1)$$

❖ The moment of inertia(I) of the entire rod can be found by integrating the equation(1) as,

$$I = \int dI = \int_{-l/2}^{+l/2} (dm)x^2 \longrightarrow (2)$$

❖ If λ is linear mass density(i.e. $\lambda = \frac{M}{\ell}$), the small mass dm can be written as,

$$dm = \lambda\, dx = \frac{M}{\ell} dx$$

❖ Substituting the 'dm' value in equation(2), we get,

$$I = \int_{-l/2}^{+l/2} \left(\frac{M}{\ell} dx\right) x^2 = \frac{M}{\ell} \int_{-l/2}^{+l/2} x^2\, dx$$

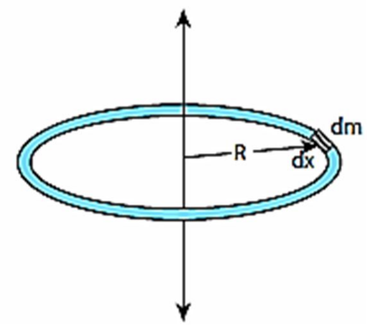
$$I = \frac{M}{\ell}\left[\frac{x^3}{3}\right]_{-l/2}^{+l/2}$$

$$I = \frac{M}{\ell}\left[\frac{\ell^3}{24} + \frac{\ell^3}{24}\right] = 2\frac{M}{\ell}\left[\frac{\ell^3}{24}\right]$$

$$I = \frac{1}{12}M\ell^2$$

**11. Obtain the expression for moment of inertia of a uniform ring.**

- Consider a uniform ring of mass M and radius R as shown in figure.




- The moment of inertia of the ring is found about the axis, which passes through its center and perpendicular to the plane.
- Now the moment of inertia of an infinitesimal small mass 'dm' of length dx of the ring, which is at a distance 'R' from the center can be expressed as,

$$dI = (dm)R^2 \longrightarrow (1)$$

- The moment of inertia(I) of the entire ring can be found by integrating the equation(1) as,

$$I = \int dI = \int_0^{2\pi R} (dm)R^2 \longrightarrow (2)$$

- If λ is linear mass density (i.e. $\lambda = \frac{M}{2\pi R}$), the small mass dm can be written as,

$$dm = \lambda\, dx = \frac{M}{2\pi R}\, dx$$

- Substituting the 'dm' value in equation(2), we get,

$$I = \int_0^{2\pi R}\left(\frac{M}{2\pi R}\, dx\right)R^2 = \frac{MR}{2\pi}\int_0^{2\pi R} dx$$

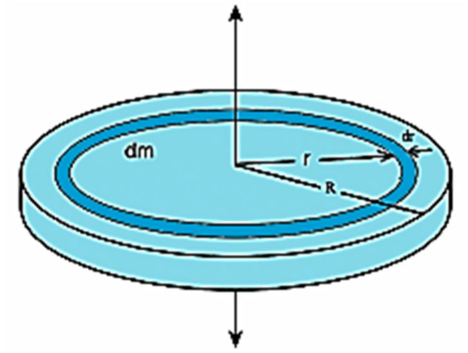
$$I = \frac{M}{2\pi}[x]_0^{2\pi R}$$

$$I = \frac{M}{2\pi}[2\pi R - 0]$$

$$I = MR^2$$

**12. Obtain the expression for moment of inertia of a uniform disc.**

- Consider a uniform disc of mass M and radius R as shown in figure.




- The moment of inertia of the disc is found about the axis, which passes through its center and perpendicular to the plane. This disc is made up of many infinitesimally small rings.
- Now consider the moment of inertia of an ring, which has infinitesimal small mass 'dm' , thickness dr and radius 'r', which can be expressed as,

$$dI = (dm)r^2 \longrightarrow (1)$$

- The moment of inertia(I) of the entire disc can be found by integrating the equation(1) as,

$$I = \int dI = \int_0^{2\pi R} (dm)r^2 \longrightarrow (2)$$

- If σ is surface mass density (i.e. $\sigma = \frac{M}{\pi R^2}$), the small mass dm can be written as,

$$dm = \sigma\, 2\pi r\, dr = \frac{M}{\pi R^2}\, 2\pi r\, dr = \frac{2M}{R^2}\, r\, dr$$

- Substituting the 'dm' value in equation(2), we get,

$$I = \int_0^R\left(\frac{2M}{R^2}\, r\, dr\right)r^2 = \frac{2M}{R^2}\int_0^R r^3 dr$$

$$I = \frac{M}{2\pi}\left[\frac{r^4}{4}\right]_0^R$$

$$I = \frac{M}{2\pi}\left[\frac{R^4}{4} - 0\right]$$

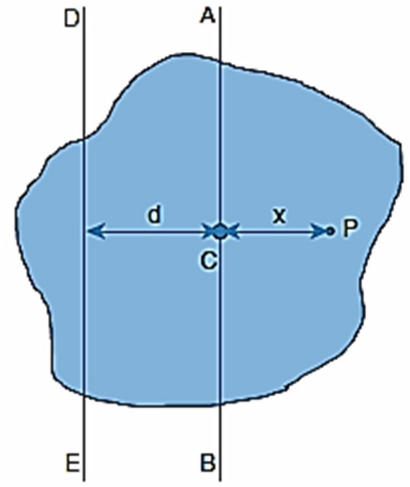
$$I = \frac{1}{2}MR^2$$

**13. State and prove parallel axis theorem.**

- **Statement :** The moment of inertia of a body about any axis is equal to the sum of its moment of inertia about a parallel axis through its center of mass and the product of the mass of the body and the square of the perpendicular distance between the two axes.

❖ Proof :

Let us consider a rigid body as shown in figure.




❖ Let $I_C$ be the moment of inertia of the body about an axis AB, which passes through center of mass.

❖ Consider I is the moment of inertia of the body to be found about an axis DE, which is parallel to AB. and d is the perpendicular distance between DE and AB.

❖ Let P be the point mass of mass m, which is located at a distance x from its center of mass.

❖ The moment of inertia of the point mass about the axis DE is,

$$dI = m(x+d)^2$$

❖ The moment of inertia of the whole body about the axis DE is,

$$I = \sum m(x+d)^2$$
$$I = \sum m(x^2+d^2+2xd)$$
$$I = \sum (mx^2+md^2+2dmx)$$

$$I = \sum mx^2 + \sum md^2 + 2d\sum mx$$

❖ Here, $\sum mx^2 = I_C$ , the moment of inertia of the body about the center of mass and $\sum mx = 0$ (since x has +ve and –ve values about the axis AB)

❖ Therefore, The moment of inertia of the whole body about the axis DE can be expressed as,

$$I = I_C + \sum m\, d^2$$

❖ But $\sum m = M$ , mass of the whole body. Thus,

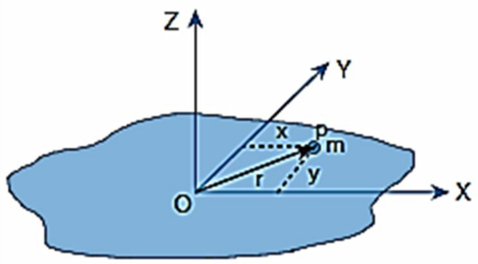
$$I = I_C + Md^2$$

❖ Hence, the parallel axis theorem is proved.

**14. State and prove perpendicular axis theorem.**

❖ Statement : The moment of inertia of a plane laminar body about an axis perpendicular to its plane is equal to the sum of moments of inertia about two perpendicular axes lying in the plane of the body such that all the three axes are mutually perpendicular and concurrent.

❖ Consider a plane laminar object of negligible thickness on which the origin O lies. The mutually perpendicular axes X and Y are lying on the the plane and z-axis is perpendicular to palne as shown in figure.




❖ Let us consider a point mass P of mass m, which is at a distance r from origin O.

❖ The moment of inertia of the point mass about the Z-axis is,

$$dI_Z = mr^2$$

❖ The moment of inertia of the whole body about the Z-axis is,

$$I_Z = \sum mr^2$$

❖ Here, $r^2 = x^2 + y^2$, So that,

$$I_Z = \sum m(x^2+y^2)$$
$$I_Z = \sum mx^2 + \sum my^2$$

❖ But $\sum mx^2 = I_Y$ , the moment of inertia of the body about the Y-axis and $\sum my^2 = I_X$, the moment of inertia of the body about the X-axis.

❖ Therefore, $I_Z = I_Y + I_X$

or $I_Z = I_X + I_Y$

❖ Hence, the perpendicular axis theorem is proved.

**15. Discuss the conservation of angular momentum with example.**

❖ According to law of conservation of angular momentum, when no external torque acts on the body, the net angular momentum of a rotating rigid body remains constant.

i.e. If $\tau = \frac{dL}{dt} = 0$ , L = Constant.

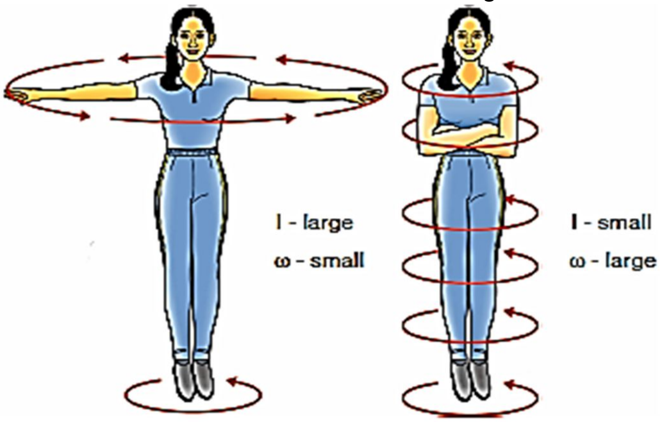
- But $L = I\omega$ , So that,

$$I\omega = Constant$$

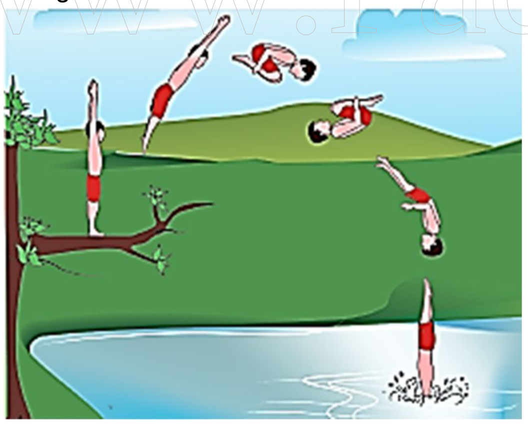
or
$$I \propto \frac{1}{\omega}$$

- It shows that if the moment of inertia I increases, the angular velocity ω decreases and vice versa.

- <u>Example:</u>
  - The ice dancer spins slowly when the hands are stretched out as this position increases moment of inertia and spis faster when the hands are brought close to the body as it decreases moment of inertia as shown in figure.



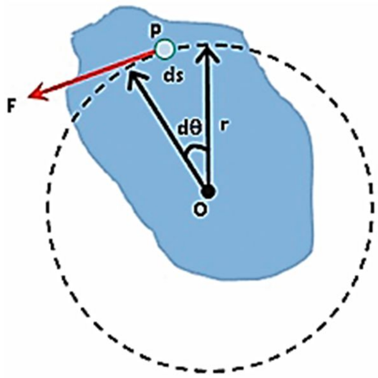

  - A diver while in air increases the number of somersaults by curling his body close, to decrease the moment of inertia as shown in figure.





**16. Find the work done by a torque.**

- Consider a rigid body rotating about a fixed axis. Let P be a point on the body which rotates about the axis perpendicular to the plane of the paper as shown in figure.




- When a tangential force applied on the body, it produces a small displacement ds on the point P.

- Now the work done by a force F for the displacement 'ds' is,

$$dw = F\ ds$$

- From the figure, ds can be expressed as,

$$ds = r\ d\theta$$

- Substituting ds value, dw becomes,

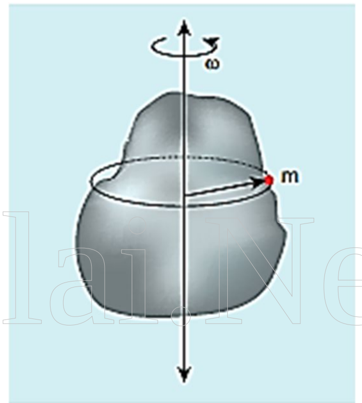
$$dw = F\ r\ d\theta$$

$$dw = \tau\ d\theta \qquad [\because\ Torque\ \tau = F\ r]$$

- This the work done by a torque.

**17. Obtain an expression for rotational kinetic energy.**

- Consider a rigid body with all particles rotating with angular velocity ω about an axis as shown in figure.




- The tangential velocity v is varied for every particle in rotation, based on its positions from the axis of rotation.

- If $m_i$ is the mass of a $i$th particle with tangential velocity $v_i$ , situated at a distance $r_i$ from axis of rotation, the kinetic energy of this particle is,

$$KE_i = \frac{1}{2}m_i v_i^2$$

- We know, $v_i = r_i\omega$ , so that,

$$KE_i = \frac{1}{2}m_i(r_i\omega)^2 = \frac{1}{2}(m_i r_i^2)\omega^2$$

- Now the kinetic energy for whole body is,

$$KE_i = \frac{1}{2}\left(\sum m_i r_i^2\right)\omega^2$$

- But $\sum m_i r_i^2 = I$, the moment of inertia of the whole body. Therefore, the rotational kinetic energy becomes,

$$KE_i = \frac{1}{2}I\omega^2$$

**18. Obtain the relation between rotational kinetic energy and angular momentum.**

- Consider a rigid body of moment of inertia 'I' rotate with angular velocity ω.
- The angular momentum of the rigid body is,

$$L = I\omega$$

- The rotational kinetic energy of the rigid body is,

$$KE = \frac{1}{2}I\omega^2$$

- Multiplying and dividing the R.H.S of the equation by L, we get,

$$KE = \frac{1}{2}\frac{I^2\omega^2}{I} = \frac{1}{2}\frac{(I\omega)^2}{I}$$

$$KE = \frac{L^2}{2I}$$

- This is the relation between rotational kinetic energy and angular momentum.

**19. Arrive at an expression for kinetic energy in pure rolling with center of mass as reference.**

- As the pure rolling consist of both translational and rotational motion, the total kinetic energy of pure rolling is the sum of kinetic energies of translational and rotational motion.

$$KE = KE_{TRANS} + KE_{ROT}$$

- If M be the mass of the rolling object, $V_{CM}$ be the velocity of center of mass, $I_{CM}$ be the moment of inertia about the center of mass and ω be the angular velocity, then,

$$KE = \frac{1}{2}Mv_{CM}^2 + \frac{1}{2}I_{CM}\omega^2$$

- But $I_{CM} = MK^2$ and $\omega = \frac{v_{CM}}{R}$ . Here K is the radius of gyration. So that,

$$KE = \frac{1}{2}Mv_{CM}^2 + \frac{1}{2}(MK^2)\frac{v_{CM}^2}{R^2}$$

$$KE = \frac{1}{2}Mv_{CM}^2 + \frac{1}{2}Mv_{CM}^2\left(\frac{K^2}{R^2}\right)$$

$$KE = \frac{1}{2}Mv_{CM}^2\left[1 + \frac{K^2}{R^2}\right]$$

**20. Arrive at an expression for kinetic energy in pure rolling with point of contact as reference.**

- If $I_O$ is the moment of inertia of the object about the point of contact, the rotational kinetic energy is,

$$KE = \frac{1}{2}I_O\omega^2$$

- By parallel axis theorem,

$$I_O = I_{CM} + MR^2$$

- But $I_{CM} = MK^2$ and $\omega = \frac{v_{CM}}{R}$ . Here K is the radius of gyration. So that,

$$I_O = MK^2 + MR^2$$

- Substituting the values of $I_O$ and ω in KE relation, we get,

$$KE = \frac{1}{2}(MK^2 + MR^2)\frac{v_{CM}^2}{R^2}$$

$$KE = \frac{1}{2}Mv_{CM}^2\left[1 + \frac{K^2}{R^2}\right]$$

**21. A solid sphere is undergoing pure rolling. What is the ratio of its translational and rotational kinetic energy?**

- The expression for total kinetic energy in pure rolling is,

$$KE = KE_{TRANS} + KE_{ROT} \longrightarrow (1)$$

- For any object the total kinetic energy can be arrived as,

$$KE = \frac{1}{2}Mv_{CM}^2 + \frac{1}{2}Mv_{CM}^2\left(\frac{K^2}{R}\right) \longrightarrow (2)$$

- Comparing the equations (1) & (2), we get,

$$KE_{TRANS} = \frac{1}{2}Mv_{CM}^2$$

$$KE_{ROT} = \frac{1}{2}Mv_{CM}^2\left(\frac{K^2}{R}\right)$$

- Now the ratio between $KE_{TRANS}$ and $KE_{ROT}$ is,

$$KE_{TRANS} : KE_{ROT} = 1 : \left(\frac{K^2}{R}\right)$$

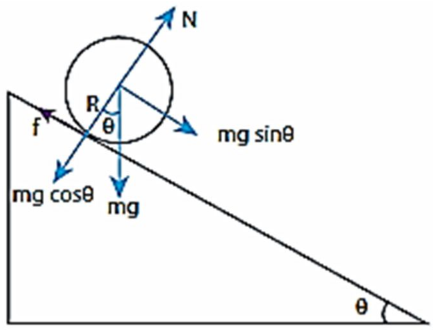
- For solid sphere, $\frac{K^2}{R} = \frac{2}{5}$ , Therefore,

$$KE_{TRANS} : KE_{ROT} = 1 : \frac{2}{5}$$

or $$KE_{TRANS} : KE_{ROT} = 5 : 2$$

**22. Discuss the rolling on inclined plane and arrive at the expressions for the acceleration, the final velocity and the time taken for rolling down the inclined plane.**

- Consider a round object of mass m and radius R is rolling down on an inclined plane without slipping as shown in figure.



**(a) Acceleration of the rolling object :**

- From the figure, it is seen that the component of gravitational force(mg cosθ) perpendicular to inclined plane is balanced by the normal force N.

- Therefore, the component of gravitational force (mg sinθ) parallel to inclined plane and the frictional force f, combinely causes the motion.

- For translational motion,

$$mg\ sin\theta - f = ma \dashrightarrow (1)$$

- For rotational motion, let us take a torque with respect to the center of the object. mg sinθ cannot make any torque as it passes through the center of the object, but the frictional force f can set a torque as,

$$\tau = Rf$$

- But we know, $\tau = I\alpha$ ,thus,

$$Rf = I\alpha$$

- Substituting, the angular acceleration $\alpha = \frac{a}{R}$ and the moment of inertia $I = mK^2$ , we get,

$$Rf = mK^2\left(\frac{a}{R}\right)$$

$$f = ma\left(\frac{K^2}{R^2}\right) \dashrightarrow (2)$$

- Substituting equation (2) in (1), we have,

$$mg\ sin\theta - ma\left(\frac{K^2}{R^2}\right) = ma$$

$$mg\ sin\theta = ma + ma\left(\frac{K^2}{R^2}\right)$$

$$a\left(1+\frac{K^2}{R^2}\right) = g\ sin\theta$$

$$a = \frac{g\ sin\theta}{\left(1+\frac{K^2}{R^2}\right)} \dashrightarrow (3)$$

**(b) Final velocity of the rolling object :**

- When the object starts rolling on the inclined plane at the height h from rest, initial velocity u = 0 and the length of the inclined plane $s = \frac{h}{sin\theta}$ .

- Substituting the values of u, s and 'a' in 3rd equation of motion $v^2 = u^2 + 2as$,

$$v^2 = 2 \times \frac{g\ sin\theta}{\left[1+\frac{K^2}{R^2}\right]}\left[\frac{h}{sin\theta}\right]$$

$$v^2 = \frac{2gh}{\left[1+\frac{K^2}{R^2}\right]}$$

$$v = \sqrt{\frac{2gh}{\left[1+\frac{K^2}{R^2}\right]}} \dashrightarrow (4)$$

**(c) Time taken for rolling down the inclined plane :**

- If the object starts rolling from rest, initial velocity u = 0. Therefore, from 1st equation of motion $v = u + at$,

$$v = at$$

or

$$t = \frac{v}{a}$$

- Substituting equations (3) & (4), we have,

$$t = \frac{v\sqrt{\frac{2gh}{\left[1+\frac{K^2}{R^2}\right]}}}{\frac{g\ sin\theta}{\left(1+\frac{K^2}{R^2}\right)}}$$

$$t = v\sqrt{\frac{2gh}{\left[1+\frac{K^2}{R^2}\right]}} \times \left[\frac{\left(1+\frac{K^2}{R^2}\right)}{g\ sin\theta}\right]$$

$$t = \sqrt{\frac{2h\left(1+\frac{K^2}{R^2}\right)}{g\ sin\theta}}$$

- It suggest that for a given inclined angle, the object with least value of radius of gyration K will reach the bottom of the inclined plane first.