

DIFFERENTIAL EQUATIONS

Exercise 4.1

i) $\frac{dy}{dx} + 2y = x^3$

Order = 1 Degree = 1

ii) $\frac{d^3y}{dx^3} + 3\left(\frac{dy}{dx}\right)^3 + 2\frac{dy}{dx} = 0$

Order = 3 Degree = 1

iii) $\frac{d^2y}{dx^2} = \sqrt{y - \frac{dy}{dx}}$

Square On Both Sides

$$\left(\frac{d^2y}{dx^2}\right)^2 = y - \frac{dy}{dx}$$

$$\left(\frac{d^2y}{dx^2}\right)^2 = y - \frac{dy}{dx}$$

Order = 2 Degree = 2

iv) $\frac{d^3y}{dx^3} = 0$

Order = 3 Degree = 1

v) $\frac{d^2y}{dx^2} + y + \left[\frac{dy}{dx} - \frac{d^3y}{dx^3}\right]^{\frac{3}{2}} = 0$

$$\frac{d^2y}{dx^2} + y = -\left[\frac{dy}{dx} - \frac{d^3y}{dx^3}\right]^{\frac{3}{2}}$$

Square On both Sides

$$\left(\frac{d^2y}{dx^2}\right)^2 + y^2 = -\left[\frac{dy}{dx} - \frac{d^3y}{dx^3}\right]^{\frac{3}{2} \times 2}$$

$$\left(\frac{d^2y}{dx^2}\right)^2 + y^2 = -\left(\frac{dy}{dx} - \frac{d^3y}{dx^3}\right)^3$$

Order = 3 Degree = 3

vi) $(2 - y'')^2 = y''^2 + 2y'$

$$2^2 - 2(2)y'' + (y'')^2 = y''^2 + 2y'$$

$$4 - 4y'' = 2y'$$

$$4 - 4\frac{d^2y}{dx^2} = 2\frac{dy}{dx}$$

Order = 2 Degree = 1

vii) $\left(\frac{dy}{dx}\right)^5 + y = x - \frac{dx}{dy}$

Multiply $\frac{dy}{dx}$ on both Sides

$$\left(\frac{dy}{dx}\right)^3 \frac{dy}{dx} + y \frac{dy}{dx} = x \frac{dy}{dx} - \frac{dy}{dx} \left(\frac{dx}{dy}\right)$$

$$\left(\frac{dy}{dx}\right)^4 + y \frac{dy}{dx} = x \left(\frac{dy}{dx}\right)$$

Order = 1 Degree = 4

2i) $y = cx + c - c^3$

Sol: $y = cx + c - c^3 \rightarrow \textcircled{1}$

Differentiate w.r. to 'x'

$$\frac{dy}{dx} = c$$

Sub c in $\textcircled{1}$.

$$y = \frac{dy}{dx}x + \frac{dy}{dx} - \frac{dy}{dx}^3 //$$

ii) $y = c(x-c)^2$

Sol: $y = c(x-c)^2 \rightarrow \textcircled{1}$

Differentiate w.r. to 'x'

$$\frac{dy}{dx} = 2c(x-c) \rightarrow \textcircled{2}$$

$\textcircled{1} \div \textcircled{2}$

$$\frac{y}{\frac{dy}{dx}} = \frac{c(x-c)^2}{2c(x-c)}$$

$$\frac{y}{\frac{dy}{dx}} = \frac{x-c}{2}$$

$$\frac{2y}{\frac{dy}{dx}} = x-c$$

$$c = x - \frac{2y}{\frac{dy}{dx}}$$

$$y = x - \frac{2y}{\frac{dy}{dx}} \left(x - x - \frac{2y}{\frac{dy}{dx}} \right)^2$$

$$y = x - \frac{2y}{\frac{dy}{dx}} \left(-\frac{2y}{\frac{dy}{dx}} \right)^2$$

$$y = x - \frac{2y}{\frac{dy}{dx}} \left(-\frac{(2)^2 y^2}{\left(\frac{dy}{dx}\right)^2} \right)$$

$$y = x - \frac{2y}{\frac{dy}{dx}} \left(\frac{4y^2}{\left(\frac{dy}{dx}\right)^2} \right)$$

$$y = x - \frac{2y}{\frac{dy}{dx}} \left(\frac{4y^2}{\left(\frac{dy}{dx}\right)^3} \right)$$

$$\left(\frac{dy}{dx}\right)^3 y = 4xy^2 - 8y^3$$

$$\left(\frac{dy}{dx}\right)^3 y - 4xy^2 + 8y^3 = 0$$

$\div y$ on both sides.

$$\left(\frac{dy}{dx}\right)^3 - 4xy + 8y^2 = 0$$

iii) $x^2 + y^2 = a^2$

Differentiate w.r. to 'x'

$$2x + 2y \frac{dy}{dx} = 0$$

$\div 2$

$$x + y \left(\frac{dy}{dx}\right) = 0$$

$$3) (x-\alpha)^2 + (y-\beta)^2 = r^2$$

$$\text{Sol: } (x-\alpha)^2 + (y-\beta)^2 = r^2 \rightarrow (1)$$

Differentiate w.r.to 'x'

$$2(x-\alpha) + 2(y-\beta) = 0$$

$$\div 2 \quad (x-\alpha) + (y-\beta) \frac{dy}{dx} = 0 \rightarrow (2)$$

Again Differentiate w.r.to 'x'

$$(1-0) + (y-\beta) \frac{d^2y}{dx^2} + \frac{dy}{dx} \left(\frac{dy}{dx} \right) = 0$$

$$1 + (y-\beta) \frac{d^2y}{dx^2} + \left(\frac{dy}{dx} \right)^2 = 0$$

$$y-\beta = \frac{-1 - \left(\frac{dy}{dx} \right)^2}{\frac{d^2y}{dx^2}}$$

$$y-\beta = - \frac{1 + \left(\frac{dy}{dx} \right)^2}{\frac{d^2y}{dx^2}}$$

Substitute in (2)

$$(x-\alpha) + \left[\frac{-1 + \left(\frac{dy}{dx} \right)^2}{\frac{d^2y}{dx^2}} \right] \frac{dy}{dx} = 0$$

$$(x-\alpha) = \left[\frac{1 + \left(\frac{dy}{dx} \right)^2}{\frac{d^2y}{dx^2}} \right] \frac{dy}{dx}$$

Sub in eqn (1).

$$\left[\frac{1 + \left(\frac{dy}{dx} \right)^2}{\frac{d^2y}{dx^2}} \right]^2 + \left[\frac{1 + \left(\frac{dy}{dx} \right)^2}{\frac{d^2y}{dx^2}} \right]^2 = r^2$$

$$\frac{\left[1 + \left(\frac{dy}{dx} \right)^2 \right]^2 \frac{dy^2}{dx^2}}{\left[\frac{d^2y}{dx^2} \right]^2} + \frac{\left[1 + \left(\frac{dy}{dx} \right)^2 \right]^2}{\left[\frac{d^2y}{dx^2} \right]^2} = r^2$$

$$\left[1 + \left(\frac{dy}{dx} \right)^2 \right]^2 \frac{dy^2}{dx^2} + 1 = r^2 \left[\frac{d^2y}{dx^2} \right]^2$$

$$\left[1 + \left(\frac{dy}{dx} \right)^2 \right]^2 \left[1 + \frac{dy^2}{dx^2} \right] = r^2 \left[\frac{d^2y}{dx^2} \right]^2$$

$$1 + \left(\frac{dy}{dx} \right)^3 = r^2 \left[\frac{d^2y}{dx^2} \right]^2$$

4)

Sol:

$y = mx$
Differentiate w.r.to 'x'

$$\frac{dy}{dx} = m$$

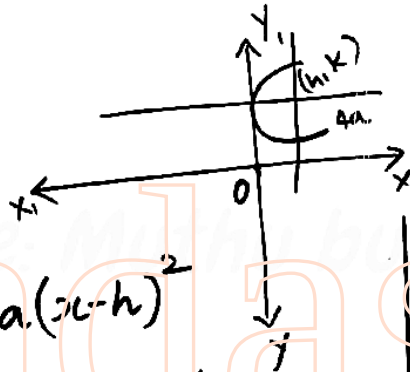
$$\therefore y = \frac{dy}{dx} x$$

$$2 \cdot \frac{-(dy/dx)^2 \frac{dy}{dx}}{\frac{d^2y}{dx^2}} = 4a$$

$$-\left(\frac{dy}{dx}\right)^3 = \frac{2a}{1} \left(\frac{d^2y}{dx^2}\right)$$

$$-\left(\frac{dy}{dx}\right)^3 = 2a \left(\frac{d^2y}{dx^2}\right)$$

5)



$$(y-k)^2 = 4a(x-h)$$

Differentiate w.r.to 'x'

$$2(y-k) \frac{dy}{dx} = 4a(1-0)$$

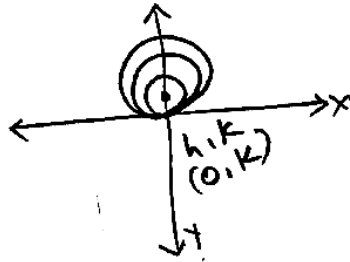
$$2(y-k) \frac{d^2y}{dx^2} + \frac{dy}{dx} \left(\frac{dy}{dx} - 0\right) = 0$$

$$y-k \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$$

$$y-k = \frac{-(dy/dx)^2}{\frac{d^2y}{dx^2}}$$

$$2a \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = 0$$

6)



$$(x-h)^2 + (y-k)^2 = k^2$$

$$(x-0)^2 + (y-k)^2 = k^2$$

$$x^2 + y^2 - 2yk + k^2 = k^2$$

$$x^2 + y^2 = 2yk \rightarrow \textcircled{1}$$

Differentiate w.r. to x

$$2x + 2y \left(\frac{dy}{dx} \right) = 2k \left(\frac{dy}{dx} \right)$$

$\div 2$

$$x + y \left(\frac{dy}{dx} \right) = k \frac{dy}{dx} \rightarrow \textcircled{2}$$

$\textcircled{1} \div \textcircled{2}$

$$\frac{x^2 + y^2}{x + y \left(\frac{dy}{dx} \right)} = \frac{2yk}{\frac{dy}{dx} k}$$

$$\frac{x^2 + y^2}{x + y \left(\frac{dy}{dx} \right)} = \frac{2y}{\frac{dy}{dx}}$$

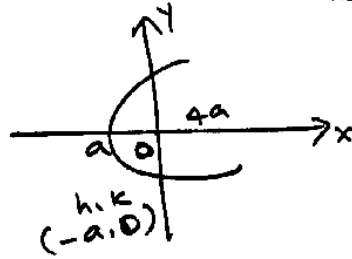
$$x^2 \frac{dy}{dx} + y^2 \frac{dy}{dx} = 2y \left[x + y \left(\frac{dy}{dx} \right) \right]$$

$$x^2 \frac{dy}{dx} + y^2 \frac{dy}{dx} = 2xy + 2y \frac{dy}{dx}$$

$$x^2 \frac{dy}{dx} + y^2 \frac{dy}{dx} - 2y \frac{dy}{dx} = 2xy$$

$$x^2 \frac{dy}{dx} - y^2 \frac{dy}{dx} = 2xy$$

7)



$$(y-k)^2 = 4a(x-h)$$

$$(y-1)^2 = 4a(x+a)$$

$$y^2 = 4a(x+a)$$

Differentiate w.r.to x

$$2y \frac{dy}{dx} = 4a \rightarrow (1)$$

$$\frac{d}{dx} \left(\frac{y^2}{2} \right) = a$$

$$\frac{y}{2} \left(\frac{dy}{dx} \right) = a \rightarrow (2)$$

$$y^2 = 2y \frac{dy}{dx} \left(x + \frac{y}{2} \left(\frac{dy}{dx} \right) \right)$$

$$y^2 = 2xy \frac{dy}{dx} + \frac{2y^2}{2} \left(\frac{dy}{dx} \right)^2$$

$$y^2 = 2xy \frac{dy}{dx} + y^2 \left(\frac{dy}{dx} \right)^2$$

$\div y$

$$y = 2x \frac{dy}{dx} + y \left(\frac{dy}{dx} \right)^2 //$$

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