XI Std Maths EM.

Ch. VII MATRICES AND DETERMINANTS EXERCISE 7.1.

1. Construct an $m\times n$ Matrix $A=[a_{ij}]$ where a_{ij} is given by

1) $a_{ij}=\frac{(i-2j)^2}{2}$ with $m=2, n=3$.

2) $a_{ij}=\frac{|3i-4j|}{4}$ with $m=3, n=4$.

1) $m\times n=2\times 3$

$A=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23}\end{bmatrix}$

$a_{11}=\frac{1}{2}$

$a_{12}=\frac{(1-4)^2}{2}=\frac{9}{2}$

$a_{13}=\frac{(1-6)^2}{2}=\frac{25}{2}$

$a_{21}=\frac{(2-2)^2}{2}=0$

$a_{22}=\frac{(2-4)^2}{2}=\frac{4}{2}$

$a_{23}=\frac{(2-6)^2}{2}=\frac{16}{2}$

$=\begin{bmatrix} \frac{1}{2} & \frac{9}{2} & \frac{25}{2} \\ 0 & \frac{4}{2} & \frac{16}{2}\end{bmatrix}=\frac{1}{2}\begin{bmatrix} 1 & 9 & 25 \\ 0 & 4 & 16\end{bmatrix}$.

2) $m\times n=3\times 4$ $a_{ij}=\frac{|3i-4j|}{4}$

$A=\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34}\end{bmatrix}$

$a_{11}=\frac{1}{4}$, $a_{12}=\frac{5}{4}$, $a_{13}=\frac{9}{4}$, $a_{14}=\frac{13}{4}$

$a_{21}=\frac{2}{4}$, $a_{22}=\frac{2}{4}$, $a_{23}=\frac{6}{4}$, $a_{24}=\frac{10}{4}$

$a_{31}=\frac{5}{4}$, $a_{32}=\frac{1}{4}$, $a_{33}=\frac{3}{4}$, $a_{34}=\frac{7}{4}$

$=\frac{1}{4}\begin{bmatrix} 1 & 5 & 9 & 13 \\ 2 & 2 & 6 & 10 \\ 5 & 1 & 3 & 7\end{bmatrix}$

2) Find the values of p, q, r, s if

$\begin{bmatrix} p^2-1 & 0 & -31-q^3 \\ 7 & r+1 & 9 \\ -2 & 8 & s-1\end{bmatrix}=\begin{bmatrix} 1 & 0 & -4 \\ 7 & \frac{3}{2} & 9 \\ -2 & 8 & -\pi\end{bmatrix}$

$p^2-1=1$

$p^2=2$ $p=\pm\sqrt{2}$

$r+1=\frac{3}{2}$

$r=\frac{3}{2}-1=\frac{1}{2}$

$s-1=-\pi$

$s=1-\pi$

$-31-q^3=-4$

$q^3=-31+7$

$=-27$

$q=-3$.

$\therefore p=\pm\sqrt{2}, q=-3, r=\frac{1}{2}, s=1-\pi$

3) Determine the value of $x+y$ if $\begin{bmatrix} 2x+y & 4x \\ 5x-7 & 4x\end{bmatrix}=\begin{bmatrix} 7 & 7y-13 \\ y & x+6\end{bmatrix}$

$2x+y=7$ $2x+y=7$ $7x=14$ $y=7-4=3$

$5x-7=y \Rightarrow 5x-y=7$ $x=2$ $\therefore x+y=2+3=5$

4) Determine the matrices A and B if they satisfy

$2A - B = \begin{bmatrix} -6 & 6 & 0 \\ 4 & -2 & -1 \end{bmatrix}$ —① $A - 2B = \begin{bmatrix} 3 & 2 & 8 \\ -2 & 1 & -7 \end{bmatrix}$ —②

$4A - 2B = \begin{bmatrix} -12 & 12 & 0 \\ 8 & -4 & -2 \end{bmatrix}$

$A - 2B = \begin{bmatrix} 3 & 2 & 8 \\ -2 & 1 & -7 \end{bmatrix}$

(−) $3A = \begin{bmatrix} -15 & 10 & -8 \\ 10 & -5 & 5 \end{bmatrix}$

$A = \frac{1}{3}\begin{bmatrix} -15 & 10 & -8 \\ 10 & -5 & 5 \end{bmatrix}$

$2B = A - \begin{bmatrix} 3 & 2 & 8 \\ -2 & 1 & -7 \end{bmatrix}$

$2B = \frac{1}{3}\begin{bmatrix} -15 & 10 & -8 \\ 10 & -5 & 5 \end{bmatrix} - \frac{1}{3}\begin{bmatrix} 9 & 6 & 24 \\ -6 & 3 & -21 \end{bmatrix}$

$2B = \frac{1}{3}\begin{bmatrix} -24 & 4 & -24 \\ 16 & -8 & 26 \end{bmatrix} \Rightarrow B = \frac{1}{3}\begin{bmatrix} -12 & 2 & -12 \\ 8 & -4 & 13 \end{bmatrix}$

5) If $A = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$ compute A^4.

$A^2 = A \cdot A = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1+0 & a+a \\ 0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 1 & 2a \\ 0 & 1 \end{bmatrix}$

$A^3 = \begin{bmatrix} 1 & 3a \\ 0 & 1 \end{bmatrix}$

$A^4 = \begin{bmatrix} 1 & 4a \\ 0 & 1 \end{bmatrix}$

6) Consider the matrix. $A_\alpha = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ S.T $A_\alpha A_\beta = A_{\alpha+\beta}$.

$A_\alpha \cdot A_\beta = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix}$

$= \begin{bmatrix} \cos\alpha\cos\beta - \sin\alpha\sin\beta & -(\sin\alpha\cos\beta + \cos\alpha\sin\beta) \\ \sin\alpha\cos\beta + \cos\alpha\sin\beta & \cos\alpha\cos\beta - \sin\alpha\sin\beta \end{bmatrix}$

$A_{\alpha+\beta} = \begin{bmatrix} \cos(\alpha+\beta) & -\sin(\alpha+\beta) \\ \sin(\alpha+\beta) & \cos(\alpha+\beta) \end{bmatrix}$

7) If $A = \begin{bmatrix} 4 & 2 \\ -1 & x \end{bmatrix}$ and s.t $(A-2I)(A-3I)=0$ find x.

$$A-2I = \begin{bmatrix} 4 & 2 \\ -1 & x \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ -1 & x-2 \end{bmatrix}$$

$$A-3I = \begin{bmatrix} 4 & 2 \\ -1 & x \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -1 & x-3 \end{bmatrix}$$

$$(A-2I)(A-3I) = \begin{bmatrix} 2 & 2 \\ -1 & x-2 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & x-3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 2-2 & 4+2x-6 \\ -1-x+2 & -2+(x-2)(x-3) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$4+2x-6=0$$
$$2x=2$$
$$x=1$$

8) $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}$ S.T A^2 is unit matrix.

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a-a & b-b & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ unit matrix.

9) If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ and 1) $A^3-6A^2+7A+kI=0$ find k.

2) $A^3-6A^2+7A+2I=0$

$$A^2 = A\cdot A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1+0+4 & 0+0+0 & 2+0+6 \\ 0+0+2 & 0+4+0 & 0+2+3 \\ 2+0+6 & 0+0+0 & 4+0+9 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}$$

$$A^3 = A^2\cdot A = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 5+0+16 & 0+0+0 & 10+0+24 \\ 2+0+10 & 0+8+0 & 4+4+15 \\ 8+0+26 & 0+0+0 & 16+0+39 \end{bmatrix} = \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix}$$

$$A^3-6A^2+7A+kI = \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - \begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix} + \begin{bmatrix} 7 & 0 & 14 \\ 0 & 14 & 7 \\ 14 & 0 & 21 \end{bmatrix} + k\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix} + kI = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow kI = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} = 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow k=2$$

11) S.T $f(x)\cdot f(y)=f(x+y)$ if $f(x)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$ 6th problem model

$$f(x)\,f(y)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos y & -\sin y & 0\\ \sin y & \cos y & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}\cos x\cos y-\sin x\sin y & -(\sin x\cos y+\cos x\sin y) & 0\\ \sin x\cos y+\cos x\sin y & \cos x\cos y-\sin x\sin y & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$f(x+y)=\begin{bmatrix}\cos(x+y) & -(\sin(x+y)) & 0\\ \sin(x+y) & \cos(x+y) & 0\\ 0 & 0 & 1\end{bmatrix}\quad \therefore f(x)f(y)=f(x+y)$$

13) Verify the property $A(B+C)=AB+AC$ when the matrices A,B,C are given by $A=\begin{bmatrix}2 & 0 & -3\\ 1 & 4 & 5\end{bmatrix}$ $B=\begin{bmatrix}3 & 1\\ -1 & 0\\ 4 & 2\end{bmatrix}$ $C=\begin{bmatrix}4 & 7\\ 2 & 1\\ 1 & -1\end{bmatrix}$

$$B+C=\begin{bmatrix}3 & 1\\ -1 & 0\\ 4 & 2\end{bmatrix}+\begin{bmatrix}4 & 7\\ 2 & 1\\ 1 & -1\end{bmatrix}=\begin{bmatrix}7 & 8\\ 1 & 1\\ 5 & 1\end{bmatrix}$$

$$A(B+C)=\begin{bmatrix}2 & 0 & -3\\ 1 & 4 & 5\end{bmatrix}\begin{bmatrix}7 & 8\\ 1 & 1\\ 5 & 1\end{bmatrix}=\begin{bmatrix}14+0-15 & 16+0-3\\ 7+4+25 & 8+4+5\end{bmatrix}$$

$$=\begin{bmatrix}-1 & 13\\ 36 & 17\end{bmatrix}\text{ — LHS.}$$

$$AB=\begin{bmatrix}2 & 0 & -3\\ 1 & 4 & 5\end{bmatrix}\begin{bmatrix}3 & 1\\ -1 & 0\\ 4 & 2\end{bmatrix}=\begin{bmatrix}6+0-12 & 2+0-6\\ 3-4+20 & 1+0+10\end{bmatrix}=\begin{bmatrix}-6 & -4\\ 19 & 11\end{bmatrix}$$

$$AC=\begin{bmatrix}2 & 0 & -3\\ 1 & 4 & 5\end{bmatrix}\begin{bmatrix}4 & 7\\ 2 & 1\\ 1 & -1\end{bmatrix}=\begin{bmatrix}8+0-3 & 14+0+3\\ 4+8+5 & 7+4-5\end{bmatrix}=\begin{bmatrix}5 & 17\\ 17 & 6\end{bmatrix}$$

$$AB+AC=\begin{bmatrix}-1 & 13\\ 36 & 17\end{bmatrix}\text{ — RHS.}\quad \therefore LHS=RHS.$$

14) Find a Matrix A which satisfies the matrix relation

$$A\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix} = \begin{bmatrix}-7 & -8 & -9\\2 & 4 & 6\end{bmatrix}$$

To find the order of A: $(2\times 2)(2\times 3) = (2\times 3)$

Now Consider the A matrix of 2×2.

Let $A = \begin{bmatrix}x & y\\a & b\end{bmatrix}$

$$\begin{bmatrix}x & y\\a & b\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix} = \begin{bmatrix}-7 & -8 & -9\\2 & 4 & 6\end{bmatrix}$$

$$\begin{bmatrix}x+4y & 2x+5y & 3x+6y\\a+4b & 2a+5b & 3a+6b\end{bmatrix} = \begin{bmatrix}-7 & -8 & -9\\2 & 4 & 6\end{bmatrix}$$

$\Rightarrow$ $x+4y = -7$
$2x+5y = -8$

① $\times 2$ $2x+8y = -14$
$2x+5y = -8$
$(-)$ $3y = -6$
$y = -2$

$x - 8 = -7$
$x = -7+8 = 1$

$a+4b = 2$ — ①
$2a+5b = 4$ — ②

$2a+8b = 4$
$2a+5b = 4$
$3b = 0$
$b = 0$
$\therefore a = 2$.

$$\therefore A = \begin{bmatrix}1 & -2\\2 & 0\end{bmatrix}$$

18) Find the matrix A s.t $\begin{bmatrix}2 & -1\\1 & 0\\-3 & 4\end{bmatrix}A^T = \begin{bmatrix}-1 & -8 & -10\\1 & 2 & -5\\9 & 22 & 15\end{bmatrix}$

To find the order of A^T. $(3\times 2)(2\times 3) = (3\times 3)$

order of A^T is (2×3)

Consider $A^T = \begin{bmatrix}x & y & z\\a & b & c\end{bmatrix}$ Now $\begin{bmatrix}2 & -1\\1 & 0\\-3 & 4\end{bmatrix}\begin{bmatrix}x & y & z\\a & b & c\end{bmatrix} = \begin{bmatrix}-1 & -8 & -10\\1 & 2 & -5\\9 & 22 & 15\end{bmatrix}$

$$\begin{bmatrix} 2x-a & 2y-b & 2z-c \\ x & y & z \\ 3x+4a & 3y+4b & 3z+4c \end{bmatrix} = \begin{bmatrix} -1 & -8 & -10 \\ 1 & 2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$$

$x=1,\ y=2,\ z=-5.$

$\therefore$

$2-a=-1$, $3=a$ | $4-b=-8$, $b=12$ | $-10-c=-10$, $10-10=c$, $0=c.$

$\therefore A^T = \begin{bmatrix} 1 & 2 & -5 \\ 3 & 12 & 0 \end{bmatrix} \therefore A = \begin{bmatrix} 1 & 3 \\ 2 & 12 \\ -5 & 0 \end{bmatrix}$

15) $A^T = \begin{bmatrix} 4 & 5 \\ -1 & 0 \\ 2 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 2 & -1 & 1 \\ 7 & 5 & -2 \end{bmatrix}$ Verify that

1) $(A+B)^T = A^T+B^T = B^T+A^T$

2) $(A-B)^T = A^T - B^T$

3) $(B^T)^T = B.$

1) $A = \begin{bmatrix} 4 & -1 & 2 \\ 5 & 0 & 3 \end{bmatrix}$ $A^T = \begin{bmatrix} 4 & 5 \\ -1 & 0 \\ 2 & 3 \end{bmatrix}$

$B = \begin{bmatrix} 2 & -1 & 1 \\ 7 & 5 & -2 \end{bmatrix}$ $B^T = \begin{bmatrix} 2 & 7 \\ -1 & 5 \\ 1 & -2 \end{bmatrix}$

$A+B = \begin{bmatrix} 6 & -2 & 3 \\ 12 & 5 & 1 \end{bmatrix}$ $(A+B)^T = \begin{bmatrix} 6 & 12 \\ -2 & 5 \\ 3 & 1 \end{bmatrix}$

$A^T+B^T = \begin{bmatrix} 6 & 12 \\ -2 & 5 \\ 3 & 1 \end{bmatrix} \therefore (A+B)^T = A^T+B^T.$

$B = \begin{bmatrix} 2 & -1 & 1 \\ 7 & 5 & -2 \end{bmatrix}$

$B^T = \begin{bmatrix} 2 & 7 \\ -1 & 5 \\ 1 & -2 \end{bmatrix}$

$(B^T)^T = \begin{bmatrix} 2 & -1 & 1 \\ 7 & 5 & -2 \end{bmatrix} = B.$

$A-B = \begin{bmatrix} 2 & 0 & 1 \\ -2 & -5 & 5 \end{bmatrix}$ $A^T - B^T = \begin{bmatrix} 2 & -2 \\ 0 & -5 \\ 1 & 5 \end{bmatrix} \therefore (A-B)^T = B^T - A^T$

$(A-B)^T = \begin{bmatrix} 2 & -2 \\ 0 & -5 \\ 1 & 5 \end{bmatrix}$

19) If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ x & 2 & y \end{bmatrix}$ is a matrix s.t $AA^T = 9I$. find the value of x and y.

$$A^T = \begin{bmatrix} 1 & 2 & x \\ 2 & 1 & 2 \\ 2 & -2 & y \end{bmatrix}$$

$$A \cdot A^T = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ x & 2 & y \end{bmatrix} \begin{bmatrix} 1 & 2 & x \\ 2 & 1 & 2 \\ 2 & -2 & y \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$\begin{bmatrix} 1+4+4 & 2+2-4 & x+4+2y \\ 2+2-4 & 4+1+4 & 2x+2-2y \\ x+4+2y & 2x+2-2y & x^2+4+y^2 \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$x + 2y = -4$$
$$2x - 2y = -2$$
$$(+) \quad 3x = -6$$
$$x = -2$$

$$2y = -4 + 2$$
$$= -2$$
$$y = -1.$$

$\therefore x = -2 \quad y = -1.$

20) For what value of x the matrix $A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & x^3 \\ 2 & -3 & 0 \end{bmatrix}$ is ske-symmetric

i)

If A is skew symmetric then $A^T = -A$.

$$A^T = \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & -3 \\ 2 & x^3 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 4 \\ 1 & 0 & -x^3 \\ -2 & 3 & 0 \end{bmatrix}$$

$$\Rightarrow x^3 = 3$$
$$x = 3^{1/3}.$$

ii) If $\begin{bmatrix} 0 & p & 3 \\ 2 & q^2 & -1 \\ r & 1 & 0 \end{bmatrix}$ is a skew-symmetric find p, q, r.

Let $A = \begin{bmatrix} 0 & p & 3 \\ 2 & q^2 & -1 \\ r & 1 & 0 \end{bmatrix}$ If A is skew symmetric $A^T = -A$.

$$A^T = \begin{bmatrix} 0 & 2 & r \\ p & q^2 & 1 \\ 3 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -p & -3 \\ -2 & -q^2 & 1 \\ -r & -1 & 0 \end{bmatrix} \Rightarrow p = -2$$

$r = -3$

$q^2 = -q^2$

$2q^2 = 0$

$q = 0$

21) Construct the matrix $A = [a_{ij}]_{3\times3}$ where $a_{ij} = i-j$. State whether A is symmetric (or) skew symmetric.

$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$:

$a_{11} = 0$ $a_{21} = 1$ $a_{31} = 2$

$a_{12} = -1$ $a_{22} = 0$ $a_{32} = 1$

$a_{13} = -2$ $a_{23} = -1$ $a_{33} = 0$.

$$A = \begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix} \quad A^T = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & 1 \\ -2 & -1 & 0 \end{bmatrix}$$

$$= -\begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$$

$\therefore$ A is skew symmetric. $A^T = -A$

Ex 7.13. Express the matrix $A = \begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix}$ as a sum of symmetric and skew symmetric matrices.

$$A = \begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & -6 & -4 \\ 3 & 8 & 6 \\ 5 & 3 & 5 \end{bmatrix}$$

Let $P = \frac{1}{2}[A + A^T] = \frac{1}{2}\begin{bmatrix} 2 & -3 & 1 \\ -3 & 16 & 9 \\ 1 & 9 & 10 \end{bmatrix}$

$P^T = \frac{1}{2}\begin{bmatrix} 2 & -3 & 1 \\ -3 & 16 & 9 \\ 1 & 9 & 10 \end{bmatrix} = P$ $\therefore$ P^T is symmetric

Let $Q = \frac{1}{2}[A - A^T] = \frac{1}{2}\begin{bmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{bmatrix}$

$$Q^T = \frac{1}{2}\begin{bmatrix} 0 & -5 & -3 \\ 5 & 0 & -6 \\ 3 & 6 & 0 \end{bmatrix}$$

$$= -\frac{1}{2}\begin{bmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{bmatrix} = -Q$$

$\therefore$ Q is skew symmetric matrix.

$\therefore A = P + Q$

$$= \frac{1}{2}\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & -5 & -3 \\ 5 & 0 & -6 \\ 3 & 6 & 0 \end{bmatrix}$$

$\frac{16}{7.1}$) If A is a 3×4 matrix and B is a matrix s.t A^TB and BA^T are defined what is the order of the matrix B.

order of A is 3×4

$\therefore$ order of $A^T = 4\times3$

let order of B is $m\times n$.

order of $A^TB = (4\times3)(m\times n) \Rightarrow m = 3$, $n = 4$

order of $BA^T = (m\times n)(4\times3) \Rightarrow m = 3$, $n = 4$

$\therefore$ order of $B = m\times n = 3\times4$.

EX 7.12) If $A = \begin{bmatrix} 4 & 6 & 2 \\ 0 & 1 & 5 \\ 0 & 3 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 1 & -1 \\ 3 & -1 & 4 \\ -1 & 2 & 1 \end{bmatrix}$

Verify 1) $(AB)^T = B^TA^T$ 2) $(A+B)^T = A^T + B^T$ 3) $(A-B)^T = A^T - B^T$ 4) $(3A)^T = 3A^T$

$A = \begin{bmatrix} 4 & 6 & 2 \\ 0 & 1 & 5 \\ 0 & 3 & 2 \end{bmatrix}$ $A^T = \begin{bmatrix} 4 & 0 & 0 \\ 6 & 1 & 3 \\ 2 & 5 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 1 & -1 \\ 3 & -1 & 4 \\ -1 & 2 & 1 \end{bmatrix}$ $B^T = \begin{bmatrix} 0 & 3 & -1 \\ 1 & -1 & 2 \\ -1 & 4 & 1 \end{bmatrix}$

$$AB = \begin{bmatrix} 4 & 6 & 2 \\ 0 & 1 & 5 \\ 0 & 3 & 2 \end{bmatrix}\begin{bmatrix} 0 & +1 & -1 \\ 3 & -1 & 4 \\ -1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 16 & 2 & 22 \\ -2 & 9 & 9 \\ 7 & 1 & 14 \end{bmatrix}$$

$$(AB)^T = \begin{bmatrix} 16 & -2 & 7 \\ 2 & 9 & 1 \\ 22 & 9 & 14 \end{bmatrix}$$

Let $Q = \frac{1}{2}[A - A^T]$

$= \frac{1}{2}\begin{bmatrix} 0 & 9 & 9 \\ -9 & 0 & -3 \\ -9 & 3 & 0 \end{bmatrix}$ $Q^T = \frac{1}{2}\begin{bmatrix} 0 & -9 & -9 \\ 9 & 0 & 3 \\ 9 & -3 & 0 \end{bmatrix}$

$= -\frac{1}{2}\begin{bmatrix} 0 & 9 & 9 \\ -9 & 0 & -3 \\ -9 & 3 & 0 \end{bmatrix} = -Q$

$\therefore$ Q is skew symmetric.

$\therefore A = P + Q = \frac{1}{2}\begin{bmatrix} 2 & -3 & 1 \\ -3 & 16 & 9 \\ 1 & 9 & 10 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & 9 & 9 \\ -9 & 0 & -3 \\ -9 & 3 & 0 \end{bmatrix}$

17) 7.1) Express the following matrices as the sum of symmetric and skew symmetric matrices.

1) $\begin{bmatrix} 4 & -2 \\ 3 & -5 \end{bmatrix}$ 2) $\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$

1) Let $A = \begin{bmatrix} 4 & -2 \\ 3 & -5 \end{bmatrix}$ $A^T = \begin{bmatrix} 4 & 3 \\ -2 & -5 \end{bmatrix}$

Let $P = \frac{1}{2}[A + A^T] = \frac{1}{2}\begin{bmatrix} 8 & 1 \\ 1 & -10 \end{bmatrix}$

$P^T = \frac{1}{2}\begin{bmatrix} 8 & 1 \\ 1 & -10 \end{bmatrix} = P$ $\therefore$ P is symmetric.

Let $Q = \frac{1}{2}[A - A^T] = \frac{1}{2}\begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$

$Q^T = \frac{1}{2}\begin{bmatrix} 0 & 5 \\ -5 & 0 \end{bmatrix} = -\frac{1}{2}\begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$

$\therefore$ Q is skew symmetric.

$A = P + Q = \frac{1}{2}\begin{bmatrix} 8 & 1 \\ 1 & -10 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$.

2) Let $A = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$ $A^T = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$

Let $P = \frac{1}{2}[A + A^T] = \frac{1}{2}\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix}$ $P^T = \frac{1}{2}\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix}$

$= P$

$\therefore$ P is symmetric matrix.

$$B^T A^T = \begin{bmatrix} 0 & 3 & -1 \\ 1 & -1 & 2 \\ -1 & 4 & 1 \end{bmatrix} \begin{bmatrix} 4 & 0 & 0 \\ 6 & 1 & 3 \\ 2 & 5 & 2 \end{bmatrix} = \begin{bmatrix} 16 & -2 & 7 \\ 2 & 9 & 1 \\ 22 & 9 & 14 \end{bmatrix}$$

$\therefore (AB)^T = B^T A^T$.

2) $$A+B = \begin{bmatrix} 4 & 6 & 2 \\ 0 & 1 & 5 \\ 0 & 3 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 1 & -1 \\ 3 & -1 & 4 \\ -1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 7 & 1 \\ 3 & 0 & 9 \\ -1 & 5 & 3 \end{bmatrix}$$

$$(A+B)^T = \begin{bmatrix} 4 & 3 & -1 \\ 7 & 0 & 5 \\ 1 & 9 & 3 \end{bmatrix}$$

$$A^T + B^T = \begin{bmatrix} 4 & 0 & 0 \\ 6 & 1 & 3 \\ 2 & 5 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 3 & -1 \\ 1 & -1 & 2 \\ -1 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 3 & -1 \\ 7 & 0 & 5 \\ 1 & 9 & 3 \end{bmatrix}$$

$\therefore (A+B)^T = A^T + B^T$.

3) $$A - B = \begin{bmatrix} 4 & 6 & 2 \\ 0 & 1 & 5 \\ 0 & 3 & 2 \end{bmatrix} - \begin{bmatrix} 0 & 1 & -1 \\ 3 & -1 & 4 \\ -1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 5 & 3 \\ -3 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$(A-B)^T = \begin{bmatrix} 4 & -3 & 1 \\ 5 & 2 & 1 \\ 3 & 1 & 1 \end{bmatrix}$$

$$A^T - B^T = \begin{bmatrix} 4 & 0 & 0 \\ 6 & 1 & 3 \\ 2 & 5 & 2 \end{bmatrix} - \begin{bmatrix} 0 & 3 & -1 \\ 1 & -1 & 2 \\ -1 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -3 & 1 \\ 5 & 2 & 1 \\ 3 & 1 & 1 \end{bmatrix}$$

$\therefore (A-B)^T = A^T - B^T$

4) $$3A = \begin{bmatrix} 12 & 18 & 6 \\ 0 & 3 & 15 \\ 0 & 9 & 6 \end{bmatrix} \quad (3A)^T = \begin{bmatrix} 12 & 0 & 0 \\ 18 & 3 & 9 \\ 6 & 15 & 6 \end{bmatrix} = 3 \begin{bmatrix} 4 & 0 & 0 \\ 6 & 1 & 3 \\ 2 & 5 & 3 \end{bmatrix}$$

$= 3A^T$.

Ex 7.11 A fruit shopkeeper prepares 3 different varities of gift packages Pack I contains 6 apples 3 oranges 3 pomegranates, Pack II contains 5 apples, 4 oranges 4 pomegranates. Pack III contains 6 apples 6 oranges 6 pomegranates. The cost of an apple an orange and a pomegranate are Rs30 Rs15 Rs 45. What is the cost of preparing each package of fruits.

Cost Matrix $A = \begin{bmatrix} 30 & 15 & 45 \end{bmatrix}$

Fruit Matrix $B = \begin{bmatrix} 6 & 5 & 6 \\ 3 & 4 & 6 \\ 3 & 4 & 6 \end{bmatrix}$

$$AB = \begin{bmatrix} 30 & 15 & 45 \end{bmatrix} \begin{bmatrix} 6 & 5 & 6 \\ 3 & 4 & 6 \\ 3 & 4 & 6 \end{bmatrix} = \begin{bmatrix} 360 \\ 390 \\ 540 \end{bmatrix}$$

Cost Pack I = 360

Pack II = 390

Pack III = 540.

24) 7.1) A shopkeeper in a Nuts and Spices shop makes gifts packs of cashew nuts, raisins, almonds.

Pack I contains 100 gm of cashew nuts 100 gms raisins 50 gm almonds

II contains 200 gm of cashew nuts 100 gms raisins 100 "

III contains 250 " 250 " 150 "

The cost of 50 gm of cashew nuts Rs 50

" 50 raisins Rs 10

" 50 " almonds Rs 60.

Cost Matrix $A = \begin{bmatrix} 50 & 10 & 60 \end{bmatrix}$

Nuts Matrix $B = \begin{bmatrix} 2 & 4 & 5 \\ 2 & 2 & 5 \\ 1 & 2 & 3 \end{bmatrix}$

$$AB = \begin{bmatrix} 50 & 10 & 60 \end{bmatrix} \begin{bmatrix} 2 & 4 & 5 \\ 2 & 2 & 5 \\ 1 & 2 & 3 \end{bmatrix} =$$

$$= \begin{bmatrix} 100 + 20 + 60 \\ 200 + 20 + 120 \\ 250 + 50 + 180 \end{bmatrix} = \begin{bmatrix} 180 \\ 340 \\ 480 \end{bmatrix}$$

∴ Cost of pack I = 180

II = 340

III = 480.

$\frac{12}{7.1}$) If A is a square matrix s.t $A^2=A$, then find $7A-(I+A)^3$

$$(I+A)^2 = (I+A)(I+A)$$
$$= I(I+A)+A(I+A)$$
$$= I^2+IA+AI+A^2$$
$$= I+2A+A$$
$$=(I+3A)$$
$$(I+A)^2(I+A) = (I+3A)(I+A)$$
$$= I^2+IA+3IA+3A^2$$
$$= I+4A+3A$$
$$= I+7A$$
$$7A-(I+A)^3 = -I$$

$\frac{10}{7.1}$) Give your own examples of matrices satisfying the following conditions in each case.

1) A and B s.t $AB \neq BA$

2) A and B s.t $AB=0=BA$ $A\neq 0, B\neq 0$.

3) A and B s.t $AB=0$ and $BA\neq 0$.

3) $A=\begin{bmatrix}1&0\\0&0\end{bmatrix}$ $B=\begin{bmatrix}0&0\\3&0\end{bmatrix}$

$$AB=\begin{bmatrix}1&0\\0&0\end{bmatrix}\begin{bmatrix}0&0\\3&0\end{bmatrix}=\begin{bmatrix}0&0\\0&0\end{bmatrix}$$

$AB=0$
But $BA\neq 0$.

$$BA=\begin{bmatrix}0&0\\3&0\end{bmatrix}\begin{bmatrix}1&0\\0&0\end{bmatrix}=\begin{bmatrix}0&0\\3&0\end{bmatrix}\neq 0$$

2) $A=\begin{bmatrix}1&1\\1&1\end{bmatrix}$ $B=\begin{bmatrix}1&-1\\-1&1\end{bmatrix}$

$$AB=\begin{bmatrix}1-1&-1+1\\1-1&-1+1\end{bmatrix}=\begin{bmatrix}0&0\\0&0\end{bmatrix}$$

$$BA=\begin{bmatrix}1&-1\\-1&1\end{bmatrix}\begin{bmatrix}1&1\\1&1\end{bmatrix}=\begin{bmatrix}1-1&1-1\\-1+1&-1+1\end{bmatrix}=\begin{bmatrix}0&0\\0&0\end{bmatrix}$$ $\therefore AB=BA$.

1) $A=\begin{bmatrix}1&0\\-1&7\end{bmatrix}$ $B=\begin{bmatrix}0&1\\7&-1\end{bmatrix}$

$$AB=\begin{bmatrix}1&0\\-1&7\end{bmatrix}\begin{bmatrix}0&1\\7&-1\end{bmatrix}=\begin{bmatrix}0&1\\49&-8\end{bmatrix}$$

$$BA=\begin{bmatrix}0&1\\7&-1\end{bmatrix}\begin{bmatrix}1&0\\-1&7\end{bmatrix}=\begin{bmatrix}-1&7\\8&-7\end{bmatrix}$$ $\therefore AB\neq BA$.

Matrix and determinants — Tips and properties.

1) A matrix is a rectangular arrangement of entries of elements displayed in rows and columns put with in a square bracket [ ].

General Form of matrix: If a matrix A has m rows and n columns then it is written as $A = [a_{ij}]$ $1 \le i \le m$, $1 \le j \le n$

2) If a matrix A has m rows and n columns then the order of the matrix is $m \times n$ (read as m by n)

3) Suppose a matrix is of order $m \times n$ then it has mn elements.

4) A matrix having only one row is called row matrix

5) A matrix having only one column it is column matrix

6) A matrix A is said to be zero matrix (or) Null matrix denoted by 0 if $a_{ij} = 0$ $1 \le i \le m$ $1 \le j \le n$

(ie) All elements in the matrix are zero

7) Square matrix: A matrix in which number of rows is equal to number of columns is called square matrix.

8) Diagonal Matrix A square matrix is called diagonal matrix if $a_{ij} = 0$ when $i \ne j$.

9) A diagonal matrix whose entries along the principal diagonal are equal is called scalar matrix.

10) A square matrix in which all the diagonal entries are 1 and the rest are all zero is called unit matrix.

11) A square matrix is said to be upper triangular matrix if all the elements below the main diagonal are zero.

12) A square matrix is said to be a lower triangular matrix if all the elements above the main diagonal are zero.

13) A square matrix which is either upper triangular or lower triangular is called triangular matrix.

14) Two matrices $A = [a_{ij}]$ and $B = [b_{ij}]$ are equal (where A = B) iff 1) both A and B are same order.
2) corresponding elements of A and B are equal.

15) A matrix is said to be conformable for multiplication with a matrix B if the number of column of A is equal to number of rows of B.

suppose A is of order (2×3)

B is of order (3×2) then we can multiply AB.

$\therefore$ The order of AB is $(2\times 3)(3\times 2) = 2\times2$.

16) It is possible ~~AB≠0~~ $AB=0$ $A\neq0$ $B\neq0$.

If $AB=0$ not necessarily $A=0$ or $B=0$.

17) In the matrix $(A+B)^2 \neq A^2 \pm 2AB + B^2$

$(A+B)(A-B) \neq A^2-B^2$

20) Transpose of a matrix: The transpose of a matrix is obtained by interchanging rows and columns of A and is denoted by A^T.

21) If A is of order $m\times n$ and A^T is of order $n\times m$.

22) 1) $(A^T)^T = A$ 2) $(KA)^T = KA^T$

3) $(A+B)^T = A^T+B^T$ 4) $(AB)^T = B^TA^T$. (reversal law)

23) A matrix A is said to be symmetric if $A^T = A$.

24) A square matrix A is said to be skew symmetric if $A^T = -A$

25) For any matrix A, A^T+A is symmetric

$A-A^T$ is skew symmetric.

26) A square matrix can be expressed as the sum of a symmetric matrix and a skew symmatric matrix.

$$A = \frac{1}{2}(A+A^T) + \frac{1}{2}(A-A^T)$$

Properties of determinants

Property 1) The determinant of a matrix remains unaltered if its rows are changed into column and columns into rows (ie) $|A| = |A^T|$.

Proof: $|A| = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

$$= a_1(b_2c_3 - b_3c_2) - b_1(a_2c_3 - a_3c_2) + c_1(a_2b_3 - a_3b_2)$$

Expansion through row wise.

Change the rows into columns and columns into rows.

$|A^T| = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$ Expansion through column wise.

$$= a_1(b_2c_3 - c_2b_3) - b_1(a_2c_3 - c_2a_3) + c_1(a_2b_3 - b_2a_3)$$

$\therefore |A^T| = |A|$.

2) If any two rows/columns of a determinant are interchange then the determinant changes in sign, but its absolute value remains unaltered.

$|A| = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

$$= a_1(b_2c_3 - b_3c_2) - b_1(a_2c_3 - a_3c_2) + c_1(a_2b_3 - a_3b_2)$$

Interchange second row and third row

$|A'| = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \\ a_2 & b_2 & c_2 \end{vmatrix} = a_1(b_3c_2 - b_2c_3) - b_1(a_3c_2 - a_2c_3) + c_1(a_3b_2 - a_2b_3)$

$$= -a_1(b_2c_3 - b_3c_2) + b_1(a_2c_3 - a_3c_2) - c_1(a_2b_3 - a_3b_2)$$

$$= -[a_1(b_2c_3 - b_3c_2) - b_1(a_2c_3 - a_3c_2) + c_1(a_2b_3 - a_3b_2)]$$

$|A'| = -|A|$.

3) If there are n interchanges of rows (columns) of a matrix A then the determinant of the resulting matrix is $(-1)^n |A|$.

4) If two rows (or) two columns of matrix A are identical then its determinants is zero.

Proof. $|A| = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_2 & b_2 & c_2 \end{vmatrix}$ $2^{nd}, 3^{rd}$ rows are identical

Now interchange 2^{nd} and 3^{rd} rows.

$$-|A| = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$-|A| = |A| \Rightarrow |A| + |A| = 0$$
$$2|A| = 0$$
$$|A| = 0.$$

5) If each element in a row (or column) of a matrix is multiplied by a scalar k, then the determinant is multiplied by the same scalar k.

$$|A| = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1(b_2c_3 - b_3c_2) - b_1(a_2c_3 - a_3c_2) + c_1(a_2b_3 - a_3b_2)$$

$$|A'| = \begin{vmatrix} ka_1 & kb_1 & kc_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$= ka_1(b_2c_3 - b_3c_2) - kb_1(a_2c_3 - a_3c_2) + kc_1(a_2b_3 - a_3b_2)$$

$$= k[a_1(b_2c_3 - b_3c_2) - b_1(a_2c_3 - a_3c_2) + c_1(a_2b_3 - a_3b_2)]$$

$$\underline{|A'| = k|A|}.$$

Note: If A is of square matrix of order n

$|AB| = |A||B|$

$AB = 0$ either $A = 0$, or $B = 0$

$|A^n| = |A|^n$.

6) If each element of a row (or) column of a determinant is expressed as sum of two (or) more terms then the whole determinant is expressed as sum of two (or) more determinants

Proof:
$$\begin{vmatrix} a_1+m_1 & b_1 & c_1 \\ a_2+m_2 & b_2 & c_2 \\ a_3+m_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} m_1 & b_1 & c_1 \\ m_2 & b_2 & c_2 \\ m_2 & b_3 & c_3 \end{vmatrix}.$$

LHS. $(a_1+m_1)(b_2c_3-b_3c_2)-(a_2+m_2)(b_1c_3-b_3c_1)+(a_3+m_3)(b_1c_2-b_2c_1)$

$= a_1(b_2c_3-b_3c_2)-a_2(b_1c_3-b_3c_1)+a_3(b_1c_2-b_2c_1)$
$+m_1(b_2c_3-b_3c_2)-m_2(b_1c_3-b_3c_1)+m_3(b_1c_2-b_2c_1)$

$$= \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} m_1 & b_1 & c_1 \\ m_2 & b_2 & c_2 \\ m_2 & b_3 & c_3 \end{vmatrix}$$

7) If, to each element of any row or column of a determinant the equi-multiples of the corresponding entries of one (or) more rows (columns) are added (or) subtracted then the value of the determinant remains unchanged.

$$|A| = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$R_1+PR_2+qR_3$

$$|A'| = \begin{vmatrix} a_1+Pa_2+qa_3 & b_1+Pb_2+qb_3 & c_1+Pc_2+qc_3 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} Pa_2 & Pb_2 & Pc_2 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} qa_3 & qb_3 & qc_3 \\ a_2 & b_2 & c_3 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + P\begin{vmatrix} a_2 & b_2 & c_2 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + q\begin{vmatrix} a_3 & b_3 & c_3 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$|A'| = |A| + 0 + 0$

Factor Theorem: If each element of a matrix A is a polynomial in x and if $|A|$ vanishes for $x = a$ then $x - a$ is a factor of $|A|$

1. If r rows (columns) are identical in a determinant of order n $(n \geq r)$ when we put $x = a$ then $(x-a)^{r-1}$ is a factor of $|A|$.

2. If the degree of the leading diagonal terms are equal to the degree of the factors then the remaining factor be K.

3. If the degree of the leading diagonal terms is greater than the degree of the factors by 1 then the remaining factor be $k(a+b+c)$.

4) If the degree of the leading diagonal terms is greater than the degree of the factors by 2 then the remaining factor be $k(a^2+b^2+c^2)+l(ab+bc+ca)$

By giving suitable values for a, b, c on both sides we can get the value of k. suitable value means, For not getting 0 on both sides we have to substitute the value for a, b, c. suppose if there are $a-b$, $b-c$, $c-a$ are the factors give $a=0$ $b=1$, $c=2$, and if a, b, c are the factors Put $a=1$, $b=1$, $c=1$ any ~~...~~ how we may get $\neq 0$ on both sides.

Relation between a Determinant and its cofactors:

Let $|A| = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ Let $A_1, B_1, C_1 \ldots$ are the Cofactors of $a_1, b_1, c_1, \ldots$ of $|A|$

then $a_1A_1 + b_1B_1 + c_1C_1 = |A|$ But $a_1A_2 + b_1B_2 + c_1C_2 = 0$ and so on.

(ie) Note that the sum of the Product of elements of any row (or) Column with their corresponding cofactors is value of the determinant

2) If elements of any row or column are multiplied with with the corresponding cofactors of any other row or column Then the sum is zero.

EXERCISE- 7.2

EX 7.16) Find $|A|$ if $A = \begin{bmatrix} 0 & \sin\alpha & \cos\alpha \\ \sin\alpha & 0 & \sin\beta \\ \cos\alpha & -\sin\beta & 0 \end{bmatrix}$

$|A| = 0(\quad) - \sin\alpha(0 - \cos\alpha\sin\beta) + \cos\alpha(-\sin\alpha\sin\beta)$

$= +\sin\alpha\cos\alpha\sin\beta - \sin\alpha\cos\alpha\sin\beta$

$= 0$

EX 7.17) compute $|A|$ using Sarrus method if $A = \begin{bmatrix} 3 & 4 & 1 \\ 0 & -1 & 2 \\ 5 & -2 & 6 \end{bmatrix}$

$$|A| = \begin{matrix} 3 & 4 & 1 & 3 & 4 \\ 0 & -1 & 2 & 0 & -1 \\ 5 & -2 & 6 & 5 & -2 \end{matrix}$$

$|A| = (-18 + 40) - (-\cancel{15} - 12 - 0)$

$= -18 + 40 + \cancel{18}^{5} + 12$

$= 5\cancel{7} - \cancel{28}$

$= 39$

EX 7.18 If a, b, c and x are positive ~~real numbers~~ real number then S.T.

$$\begin{vmatrix} (a^x + a^{-x})^2 & (a^x - a^{-x})^2 & 1 \\ (b^x + b^{-x})^2 & (b^x - b^{-x})^2 & 1 \\ (c^x + c^{-x})^2 & (c^x - c^{-x})^2 & 1 \end{vmatrix} = 0$$

SOL: $\begin{vmatrix} a^{2x} + a^{-2x} + 2 & a^{2x} + a^{-2x} - 2 & 1 \\ b^{2x} + b^{-2x} + 2 & b^{2x} + b^{-2x} - 2 & 1 \\ c^{2x} + c^{-2x} + 2 & c^{2x} + c^{-2x} - 2 & 1 \end{vmatrix}$

$C_1 - C_2$

$= \begin{vmatrix} 4 & a^{2x} + a^{-2x} - 2 & 1 \\ 4 & b^{2x} + b^{-2x} - 2 & 1 \\ 4 & c^{2x} + c^{-2x} - 2 & 1 \end{vmatrix}$

$= 4 \begin{vmatrix} 1 & a^{2x} + a^{-2x} - 2 & 1 \\ 1 & b^{2x} + b^{-2x} - 2 & 1 \\ 1 & c^{2x} + c^{-2x} - 2 & 1 \end{vmatrix} = 0.$

EX 7.19) without expanding the determinants S.T $|B| = 2|A|$

where $B = \begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}$ and $A = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

$$|B| = \begin{vmatrix} \cancel{2(b+c)} & \cancel{2(c+a)} & \cancel{2(a+b)} \\ 2(a+b+c) & 2(a+b+c) & 2(a+b+c) \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} \quad R_1 \to R_1 + R_2 + R_3.$$

$$2\begin{vmatrix} a+b+c & a+b+c & a+b+c \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}$$

$$= 2\begin{vmatrix} a+b+c & a+b+c & a+b+c \\ -b & -c & -a \\ -c & -a & -b \end{vmatrix} \quad \begin{matrix} R_2 - R_1 \\ R_3 - R_1. \end{matrix}$$

$$= 2\begin{vmatrix} a & b & c \\ -b & -c & -a \\ -c & -a & -b \end{vmatrix} \quad R_1 + R_2 + R_3$$

$$= 2\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 2|A|$$

EX 7.20) Evaluate $\begin{vmatrix} 2014 & 2017 & 0 \\ 2020 & 2023 & 1 \\ 2023 & 2026 & 0 \end{vmatrix}$

Sol: $\begin{vmatrix} 2014 & 3 & 0 \\ 2020 & 3 & 1 \\ 2023 & 3 & 0 \end{vmatrix}$ $C_2 \to C_2 - C_1.$

$$= 3\begin{vmatrix} 2014 & 1 & 0 \\ 2020 & 1 & 1 \\ 2023 & 1 & 0 \end{vmatrix} =$$

$$= -3\begin{vmatrix} 0 & 1 & 2014 \\ 1 & 1 & 2020 \\ 0 & 1 & 2023 \end{vmatrix} = -3\left[-1(2023-2014)\right]$$

$$= -3(-9) = 27.$$

(EX.7.22) P.T $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix} = (x-y)(y-z)(z-x)$

sol:

$$\begin{array}{ccc} c_1 & c_1-c_2 & c_2-c_3 \end{array}$$

$$\begin{vmatrix} 1 & 0 & 0 \\ x & x-y & y-z \\ x^2 & x^2-y^2 & y^2-z^2 \end{vmatrix} = 1\left[(x-y)(y^2-z^2) - (x^2-y^2)(y-z)\right]$$

$$= (x-y)(y-z)(y+z) - (x+y)(x-y)(y-z)$$

$$= (x-y)(y-z)\left[(y+z) - (x+y)\right]$$

$$= (x-y)(y-z)(z-x)$$

$\frac{1}{7.2}$) Without expanding the determinant $\begin{vmatrix} s & a^2 & b^2+c^2 \\ s & b^2 & c^2+a^2 \\ s & c^2 & a^2+b^2 \end{vmatrix} = 0$

sol: $s\begin{vmatrix} 1 & a^2 & a^2+b^2+c^2 \\ 1 & b^2 & a^2+b^2+c^2 \\ 1 & c^2 & a^2+b^2+c^2 \end{vmatrix}$ (c_2+c_3)

$$= s(a^2+b^2+c^2)\begin{vmatrix} 1 & a^2 & 1 \\ 1 & b^2 & 1 \\ 1 & c^2 & 1 \end{vmatrix} = s(a^2+b^2+c^2)(0) = 0$$

$\frac{2}{7.2}$) S.T $\begin{vmatrix} b+c & bc & b^2c^2 \\ c+a & ca & c^2a^2 \\ a+b & ab & a^2b^2 \end{vmatrix} = 0.$

sol:

$$= (bc)(ca)(ab)\begin{vmatrix} \frac{1}{b}+\frac{1}{c} & 1 & bc \\ \frac{1}{c}+\frac{1}{a} & 1 & ca \\ \frac{1}{a}+\frac{1}{b} & 1 & ab \end{vmatrix} \begin{array}{l} \frac{R_1}{bc} \\ \frac{R_2}{ca} \\ \frac{R_3}{ab} \end{array}$$

$$= (a^2b^2c^2)(abc)\begin{vmatrix} \frac{1}{b}+\frac{1}{c} & 1 & \frac{1}{a} \\ \frac{1}{c}+\frac{1}{a} & 1 & \frac{1}{b} \\ \frac{1}{a}+\frac{1}{b} & 1 & \frac{1}{c} \end{vmatrix}$$

$$= a^3b^3c^3\begin{vmatrix} \frac{1}{a}+\frac{1}{b}+\frac{1}{c} & 1 & \frac{1}{a} \\ \frac{1}{a}+\frac{1}{b}+\frac{1}{c} & 1 & \frac{1}{b} \\ \frac{1}{a}+\frac{1}{b}+\frac{1}{c} & 1 & \frac{1}{c} \end{vmatrix} \quad (c_1+c_3)$$

$$a^3b^3c^3\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix} 1 & 1 & 1/a \\ 1 & 1 & 1/b \\ 1 & 1 & 1/c \end{vmatrix} = 0$$

$\frac{3}{7.2}$) P.T $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$.

Sol: $abc\begin{vmatrix} a & c & a+c \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$ $\frac{C_1}{a}, \frac{C_2}{b}, \frac{C_3}{c}$

$= abc\begin{vmatrix} -2b & -2b & 0 \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$ $R_1-R_2-R_3$

$= -2abc\begin{vmatrix} +b & +b & 0 \\ a+b & b & a \\ b & b+c & c \end{vmatrix} = -2abc\begin{vmatrix} b & b & 0 \\ a & 0 & a \\ 0 & c & c \end{vmatrix}$ R_2-R_1, R_3-R_1

$= -2abc\left[b(0-ca) - b(ac)\right]$

$= -2abc\left[-abc-abc\right]$

$= (-2abc)(-2abc)$

$= 4a^2b^2c^2$.

$\frac{4}{7.2}$) P.T $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$.

Sol: $abc\begin{vmatrix} \frac{1}{a}+1 & \frac{1}{b} & 1/c \\ 1/a & 1+1/b & 1/c \\ 1/a & 1/b & 1+1/c \end{vmatrix}$ $\frac{C_1}{a}, \frac{C_2}{b}, \frac{C_3}{c}$

$C_1+C_2+C_3$

$= abc\begin{vmatrix} 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & \frac{1}{b} & 1/c \\ 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & 1+\frac{1}{b} & 1/c \\ 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & 1/b & 1+\frac{1}{c} \end{vmatrix}$

$= abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix} 1 & \frac{1}{b} & 1/c \\ 1 & 1+\frac{1}{b} & 1/c \\ 1 & 1/b & 1+1/c \end{vmatrix}$.

$= abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix} 1 & 1/b & 1/c \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$ R_2-R_1, R_3-R_1

$= abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \cdot 1$.

5) P.T $\begin{vmatrix} \sec^2\theta & \tan^2\theta & 1 \\ \tan^2\theta & \sec^2\theta & -1 \\ 38 & 3b & 2 \end{vmatrix} = 0.$

7.2

Sol: $c_1 - c_2$

$\begin{vmatrix} \sec^2\theta - \tan^2\theta & \tan^2\theta & 1 \\ \tan^2\theta - \sec^2\theta & \sec^2\theta & -1 \\ 2. & 3b & 2 \end{vmatrix}$

$= \begin{vmatrix} 1 & \tan^2\theta & 1 \\ -1 & \sec^2\theta & -1 \\ 2 & 3b & 2 \end{vmatrix} = 0.$

6) S.T $\begin{vmatrix} x+2a & y+2b & z+2c \\ x & y & z \\ a & b & c \end{vmatrix} = 0$

7.2

Sol. $\begin{vmatrix} 2a & 2b & 2c \\ x & y & z \\ a & b & c \end{vmatrix}$ $R_1 - R_2$

$= 2\begin{vmatrix} a & b & c \\ x & y & z \\ a & b & c \end{vmatrix} = 0.$

7) Write the general form of 3×3 skew-symmetric matrix and

7.2 Prove that its determinents is 0.

Let $A = \begin{vmatrix} 0 & 2 & 3 \\ -2 & 0 & 4 \\ -3 & -4 & 0 \end{vmatrix}$ is a skew symmetric matrix.

(Skew symmetric matrix is a matrix in which $a_{ii} = 0$ $_{i=j}$ and $a_{ij} = -a_{ji}$.)

$= 0(\quad) - 2(0+12) + 3(8-0)$

$= -24 + 24 = 0.$

8) If $\begin{vmatrix} a & b & a\alpha+b \\ b & c & b\alpha+c \\ a\alpha+b & b\alpha+c & 0 \end{vmatrix} = 0$ P.T a, b, c are in G and α is root of $ax^2+2bx+c=0$.

7.2

$C_3 = C_3 - xC_1 - yC_2$

Sol: $\begin{vmatrix} a & b & ax+by \\ b & c & bx+cy \\ ax+by & bx+cy & 0 \end{vmatrix} = \begin{vmatrix} a & b & 0 \\ b & c & 0 \\ ax+by & bx+cy & -(a\alpha^2+2b\alpha+c) \end{vmatrix} = 0.$

$= -(a\alpha^2+2b\alpha+c)(ac-b^2) = 0$

$\Rightarrow ac - b^2 = 0 \Rightarrow$ a, b, c are in G.P.

$ac = b^2$

$a\alpha^2 + 2b\alpha + c = 0 \Rightarrow \alpha$, are the roots of $ax^2+2bx+c=0$

9) 7.2 P.T $\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ca \\ 1 & c & c^2-ab \end{vmatrix} = 0$

Sol: $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \begin{vmatrix} 1 & a & +bc \\ 1 & b & +ca \\ 1 & c & +ab \end{vmatrix}.$

$= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - abc \begin{vmatrix} 1 & a & 1/a \\ 1 & b & 1/b \\ 1 & c & 1/c \end{vmatrix}.$

$= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix}.$

$= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0.$

13) 7.2 Find the value of $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix} = ?$ if $x, y, z \neq 1$.

Sol: $= \begin{vmatrix} \frac{\log x}{\log x} & \frac{\log y}{\log x} & \frac{\log z}{\log x} \\ \frac{\log x}{\log y} & \frac{\log y}{\log y} & \frac{\log z}{\log y} \\ \frac{\log x}{\log z} & \frac{\log y}{\log z} & \frac{\log z}{\log z} \end{vmatrix} = \frac{1}{\log x \log y \log z} \begin{vmatrix} \log x & \log y & \log z \\ \log x & \log y & \log z \\ \log x & \log y & \log z \end{vmatrix}$

$= 0.$

15) 7.2 Without expanding evaluate the following determinants.

1) $\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$ 2) $\begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$

1) $3x \begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 2 & 3 & 4 \end{vmatrix} = 0.$

2) $\begin{vmatrix} x+y+z & x+y+z & x+y+z \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$

$= x+y+z \begin{vmatrix} 1 & 1 & 1 \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix} = 0.$

16) 7.2 If A is a square matrix $|A| = 2$ and find $|AA^T|$.

Sol: $|A| = 2 \Rightarrow |A^T| = 2$

$|AA^T| = |A||A^T| = 2 \times 2 = 4.$

17) 7.2 If A and B are square matrix of order 3 s.t $|A| = -1$, $|B| = 3$ Find the value of $|3AB|$

Sol: $|3AB| = 27|A||B| = 27(-1)(3) = -81.$

$\frac{18}{7.2}$ If $\lambda = -2$ determine the value of $\begin{vmatrix} 0 & 2\lambda & 1 \\ \lambda^2 & 0 & 3\lambda^2+1 \\ -1 & 6\lambda-1 & 0 \end{vmatrix}$.

Sol: $\begin{vmatrix} 0 & -4 & 1 \\ 4 & 0 & 13 \\ -1 & -13 & 0 \end{vmatrix} = +4(13)+1(-4\times 13)$

$= \cancel{104}\ 0$

$\frac{19}{7.2}$) Determine the roots of the eqn. $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0.$

Sol: $\begin{vmatrix} 1 & 4 & 20 \\ 0 & 6 & 15 \\ 0 & -2-2x & 5-5x^2 \end{vmatrix} = 0$

$1(6(\cancel{10}+5x^2)-15(-2-2x)) = 0$

$30(2+x^2)+30(1+2x) = 0$

$30(2+x^2+1+2x) = 0$

$x^2+2x+3 = 0.$

$x^2+2x+x+3 = 0$

$6(5-5x^2)+15(2+2x)$

$30-30x^2+30+30x$

$30x^2-30x-60 = 0$

$x^2-x-2 = 0 \quad -2, 1$

$x^2-2x+x-2 = 0$

$x(x-2)+1(x-2) = 0$

$(x-2)(x+1) = 0$

$x = 2, -1.$

$\frac{20}{7.2}$) verify that $\det(AB) = (\det A)(\det B)$

(or) $|AB| = |A||B|$. $A = \begin{bmatrix} 4 & 3 & -2 \\ 1 & 0 & 7 \\ 2 & 3 & -5 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 3 & 3 \\ -2 & 4 & 0 \\ 9 & 7 & 5 \end{bmatrix}$

$AB = \begin{bmatrix} 4 & 3 & -2 \\ 1 & 0 & 7 \\ 2 & 3 & -5 \end{bmatrix}\begin{bmatrix} 1 & 3 & 3 \\ -2 & 4 & 0 \\ 9 & 7 & 5 \end{bmatrix} = \begin{bmatrix} 4-6-18 & 12+12-14 & 12+0-10 \\ 1+0+63 & 3+0+49 & 3+0+35 \\ 2-6-45 & 6+12-35 & 6+0-25 \end{bmatrix}$

$= \begin{bmatrix} -20 & 10 & 2 \\ 64 & 52 & 38 \\ -49 & -17 & -19 \end{bmatrix}$

$\frac{21}{7.2}$) using co factors of elements of second row, evaluate $|A|$ where $A = \begin{bmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{bmatrix}$

$$|A| = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix} = -2(9-16)+0(\quad)-1(10-3)$$

$$= 14-7=7.$$

$\frac{12}{7.2}$) If a, b, c are all positive and p^{th}, q^{th}, r^{th} terms of GP

S.T $\Delta = \begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} = 0$

Let the first term of GP be A and common ratio is R.

$a = AR^{p-1} \Rightarrow \log a = \log A + (p-1)\log R$

$b = AR^{q-1} \Rightarrow \log b = \log A + (q-1)\log R$

$c = AR^{r-1} \Rightarrow \log c = \log A + (r-1)\log R$.

$$\therefore \Delta = \begin{vmatrix} \log A + (p-1)\log R & p & 1 \\ \log A + (q-1)\log R & q & 1 \\ \log A + (r-1)\log R, & r & 1 \end{vmatrix}$$

$$= \begin{vmatrix} \log A & p & 1 \\ \log A & q & 1 \\ \log A & r & 1 \end{vmatrix} + \begin{vmatrix} p\log R & p & 1 \\ q\log R & q & 1 \\ r\log R & r & 1 \end{vmatrix} - \begin{vmatrix} \log R & p & 1 \\ \log R & q & 1 \\ \log R & r & 1 \end{vmatrix}.$$

$$= \log A \begin{vmatrix} 1 & p & 1 \\ 1 & q & 1 \\ 1 & r & 1 \end{vmatrix} + \log R \begin{vmatrix} p & p & 1 \\ q & q & 1 \\ r & r & 1 \end{vmatrix} - \log R \begin{vmatrix} 1 & p & 1 \\ 1 & q & 1 \\ 1 & r & 1 \end{vmatrix}$$

$$= \log A(0) + \log R(0) - \log R(0)$$

$$= 0$$

$\frac{23)}{7.1}$ If A and B are symmetric Matrices of same order, P.T

1) $AB+BA$ is a symmetric

2) $AB-BA$ is a skew symmetric.

Given A and B are symmetric Matrix.

$A^T = A$, $B^T = B$.

Consider $(AB+BA)^T = (AB)^T + (BA)^T$

$= B^T A^T + A^T B^T$

$= BA + AB$

$= AB + BA$. ∴ is symmetric.

If A and B any two matrices.

$A^T = A$, $B^T = B$.

$(AB-BA)^T = (AB)^T - (BA)^T$

$= B^T A^T - A^T B^T$.

$= BA - AB$

$= -(AB-BA)$

∴ $(AB-BA)^T = -(AB-BA)$ AB − BA is skew symmetric.

$\frac{10}{7.2}$ If a, b, c are p^{th}, q^{th}, r^{th} terms of an AP Find $\begin{vmatrix} a & b & c \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$

Let A be the first term and d be the common difference of AP.

$A+(p-1)d = a$

$A+(q-1)d = b$

$A+(r-1)d = c$.

$\begin{vmatrix} a & b & c \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$

$= \begin{vmatrix} A+(p-1)d & A+(q-1)d & A+(r-1)d \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$

$= \begin{vmatrix} A & A & A \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix} + \begin{vmatrix} pd & qd & rd \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix} - \begin{vmatrix} d & d & d \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$.

$= A \begin{vmatrix} 1 & 1 & 1 \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix} + d \begin{vmatrix} p & q & r \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix} - d \begin{vmatrix} 1 & 1 & 1 \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$

$= 0 + 0 + 0$

$= 0$

EXERCISE - 7.3.

EX 7.23) using factor theorem, P.T $\begin{vmatrix} x+1 & 3 & 5 \\ 2 & x+2 & 5 \\ 2 & 3 & x+4 \end{vmatrix} = (x-1)^2(x+9)$

Let $|A| = \begin{vmatrix} x+1 & 3 & 5 \\ 2 & x+2 & 5 \\ 2 & 3 & x+4 \end{vmatrix}$

Put $x=1$ $|A| = \begin{vmatrix} 2 & 3 & 5 \\ 2 & 3 & 5 \\ 2 & 3 & 5 \end{vmatrix}$ Since 3 rows are identical $(x-1)^2$ is a factor.

Put $x=-9$ $|A| = \begin{vmatrix} -8 & 3 & 5 \\ 2 & -7 & 5 \\ 2 & 3 & -5 \end{vmatrix}$

$\overset{C_1+C_2+C_3}{=} \begin{vmatrix} 0 & 3 & 5 \\ 0 & -7 & 5 \\ 0 & 3 & -5 \end{vmatrix} = 0$ $\therefore x+9$ is a factor

The degree of the leading terms is 3. The degree of the factors is also 3

$\therefore$ The remaining factor be k.

$\therefore \begin{vmatrix} x+1 & 3 & 5 \\ 2 & x+2 & 5 \\ 2 & 3 & x+4 \end{vmatrix} = k(x-1)^2(x+9)$

Put $x=2$

Put $x=2$ $\begin{vmatrix} 3 & 3 & 5 \\ 2 & 4 & 5 \\ 2 & 3 & 6 \end{vmatrix} = k \cdot 1 \cdot 11$

By giving suitable value to x we get k. Suitable value means not getting 0 on both sides.

$3(24-15)-2(18-15)+2(15-20) = 11k$

$27-6-10 = 11k \Rightarrow k=1$

$\therefore \begin{vmatrix} x+1 & 3 & 5 \\ 2 & x+2 & 5 \\ 2 & 3 & x+4 \end{vmatrix} = (x-1)^2(x+9)$

Ex T-24) P.T $\begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$

$$|A| = \begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix}$$

Put $x=y$ $\quad |A| = \begin{vmatrix} 1 & y^2 & y^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix} = 0$ $\quad R_1, R_2$ are identical.

Similarly we can prove that if $y=z$, $z=x$ then $\Delta = 0$

$\therefore (y-z), (z-x)$ are also the factor of Δ.

The degree of leading diagonal elements is 5.

The degree of the factors is 3.

$\therefore$ The remaining factor will be $k(x^2+y^2+z^2)+l(xy+yz+zx)$

$$\therefore \begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix} = [(x-y)(y-z)(z-x)][k(x^2+y^2+z^2)+l(xy+yz+zx)]$$

Put $x=0, y=-1, z=1$

$$\begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & -1 \\ 1 & 1 & 1 \end{vmatrix} = [(1)(-2)(1)][k(0+1+1)+l(-1)].$$

$$= -2[2k-l].$$

$$\cancel{2} = -\cancel{2}(2k-l] \Rightarrow 2k-l = -1 \quad \text{--- (1)}$$

Put $x=0, y=1, z=2$

$$\begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 2 & 8 \end{vmatrix} = (-1)(-1)(2)[k(0+1+4)+l(2)]$$

$$\cancel{4}^{2} = \cancel{2}[5k+2l]$$

$$5k+2l = 2$$

From (1) $\quad 4k-2l = -2$

(+) $\quad k = 0 \Rightarrow 2l = 2$

$$l = 1$$

$$\therefore \begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx).$$ 1

7.25) P.T $|A| = \begin{vmatrix} (q+r)^2 & p^2 & p^2 \\ q^2 & (r+p)^2 & q^2 \\ r^2 & r^2 & (p+q)^2 \end{vmatrix} = 2pqr(p+q+r)^3$

Put $p=0$ $\begin{vmatrix} (q+r)^2 & 0 & 0 \\ q^2 & r^2 & q^2 \\ r^2 & r^2 & q^2 \end{vmatrix} = 0$ c_2 is identical to c_3 $c_2 \equiv c_3$.

$\therefore$ p is a factor similarly if $q=0, r=0$ then $|A|=0$

$\therefore q, r$ are also factors.

If $p+q+r=0 \Rightarrow q+r=-p$ $r+p=-q$ $p+q=-r$.

$\therefore |A| = \begin{vmatrix} p^2 & p^2 & p^2 \\ q^2 & q^2 & q^2 \\ r^2 & r^2 & r^2 \end{vmatrix} = 0$ c_1, c_2, c_3 are identical

$\therefore (p+q+r)^2 = 0$.

The degree of the leading term is 6. and the degree of the factors is 5. $\therefore$ The remaining factor be $K(a+b+c)$

$$\therefore \begin{vmatrix} (q+r)^2 & p^2 & p^2 \\ q^2 & (r+p)^2 & q^2 \\ r^2 & r^2 & (p+q)^2 \end{vmatrix} = pqr(p+q+r)^2 \cdot K(p+q+r)$$

$$= K\,pqr(p+q+r)^3.$$

Put $p=1, q=1, r=1$

$$\begin{vmatrix} 4 & 1 & 1 \\ 1 & 4 & 1 \\ 1 & 1 & 4 \end{vmatrix} = K(1+1+1)^3 = 27K.$$

$4(16-1)-1(4-1)+1(1-4)=27K$

$60-3-3=27K \Rightarrow 27K=54$

$K=2$

$\therefore \begin{vmatrix} (q+r)^2 & p^2 & p^2 \\ q^2 & (r+p)^2 & q^2 \\ r^2 & r^2 & (p+q)^2 \end{vmatrix} = 2pqr(p+q+r)^3.$

<u>EX: 7.26</u> In a $\triangle$le ABC if $\begin{vmatrix} 1 & 1 & 1 \\ 1+\sin A & 1+\sin B & 1+\sin C \\ \sin A(1+\sin A) & \sin B(1+\sin B) & \sin C(1+\sin C) \end{vmatrix}$

P.T $\triangle ABC$ is an isosceles $\triangle$le

Put $\sin B = \sin A$

$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 1+\sin A & 1+\sin A & 1+\sin C \\ \sin A(1+\sin A) & \sin A(1+\sin A) & \sin A(1+\sin C) \end{vmatrix} = 0.$

$|A|=0$.

Illy if $\sin B = \sin C$ and $\sin C = \sin A$

$B=C$ ~~BC~~ or $C=A$.

$\therefore A=B$ (or) The triangle

In all cases atleast two angles are equal $\therefore$ The triangle is isosceles.

$\frac{1}{7.3}$ S.T $\begin{vmatrix} x & a & a \\ a & x & a \\ a & a & x \end{vmatrix} = (x-a)^2(x+2a)$

Put $x=a$ $\begin{vmatrix} a & a & a \\ a & a & a \\ a & a & a \end{vmatrix} = 0$ c_1, c_2, c_3 are identical

$\therefore (x-a)^2$ is a factor.

Put $x=-2a$. $\begin{vmatrix} -2a & a & a \\ a & -2a & a \\ a & a & -2a \end{vmatrix} = \begin{vmatrix} 0 & 0 & 0 \\ a & -2a & a \\ a & a & -2a \end{vmatrix} = 0$

$\therefore x+2a$ is a factor.

The degree of the leading diagonal is 3. The degree of the factors are 3. $\therefore$ The remaining factor be K.

$\therefore \begin{vmatrix} x & a & a \\ a & x & a \\ a & a & x \end{vmatrix} = k(x-a)^2(x+2a)$

Put $x=0$

$\begin{vmatrix} 0 & a & a \\ a & 0 & a \\ a & a & 0 \end{vmatrix} = ka^2 \cdot 2a = k2a^3.$

$-a(-a^2)+a(a^2)$

$2a^3 = k2a^3 \therefore k=1.$

$\therefore \begin{vmatrix} x & a & a \\ a & x & a \\ a & a & x \end{vmatrix} = (x-a)^2(x+2a).$

$\frac{2}{7.3}$ S.T $\begin{vmatrix} b+c & a-c & a-b \\ b-c & c+a & b-a \\ c-b & c-a & a+b \end{vmatrix} = 8abc.$

Put $a=0$ $|A|$ $\begin{vmatrix} b+c & -c & -b \\ b-c & c & b \\ c-b & c & b \end{vmatrix} \overset{c_1+c_2+c_3}{=} \begin{vmatrix} 0 & -c & -b \\ 0 & c & b \\ 0 & c & b \end{vmatrix} = 0.$

III^{ly} if $b=0, c=0$ $|A|=0$ $\therefore$ a, b, c are the factors of $|A|$.

The degree of the leading diagonal is 3 and The degree of the factors are 3. $\therefore$ The remaining factor is K.

$\therefore \begin{vmatrix} b+c & a-c & a-b \\ b-c & c+a & b-a \\ c-b & c-a & a+b \end{vmatrix} = kabc$ put $a=1, b=1, c=1.$

$= \begin{vmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{vmatrix} = k\cancel{abc} \Rightarrow k=8$

$\therefore \begin{vmatrix} b+c & a-c & a-b \\ b-c & c+a & b-a \\ c-b & c-a & a+b \end{vmatrix} = 8abc.$

$\frac{3}{7.3}$ Solve $\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = 0$

$$\begin{vmatrix} x+a & b & c \\ x & -x & 0 \\ 0 & x & -x \end{vmatrix} \begin{matrix} \\ R_1 - R_2 \\ R_3 - R_2 \end{matrix} = 0$$

$$(x+a)(+x^2) - x(-xb - xc) = 0$$

$$+x^2(x+a) + x^2(b+c) = 0$$

$$x^2(b+c+x+a) = 0$$

$$\Rightarrow b+c+x+a = 0$$

$$-(b+c+a) = x.$$

$\frac{4}{7.3}$) S.T $|A| \begin{vmatrix} b+c & a & a^2 \\ c+a & b & b^2 \\ a+b & c & c^2 \end{vmatrix} = (a+b+c)(a-b)(b-c)(c-a)$

Put $a=b$ $\quad |A| = \begin{vmatrix} b+c & b & b^2 \\ c+b & b & b^2 \\ 2b & c & c^2 \end{vmatrix} = \begin{vmatrix} 0 & 0 & 0 \\ c+b & b & b^2 \\ 2b & c & c^2 \end{vmatrix} R_1 - R_2 = 0.$

$\therefore$ $a-b$ is a factor of $|A|$ Similarly if $b=c$, $c=a$ then $|A|=0$

$\therefore$ $b-c$, $c-a$ are the factors of $|A|$.

$\because$ The degree of the leading diagonal is 4.

The degree of the factors is 3.

$\therefore$ The remaining factor is $k(a+b+c)$

$$\therefore \begin{vmatrix} b+c & a & a^2 \\ c+a & b & b^2 \\ a+b & c & c^2 \end{vmatrix} = k(a+b+c)(a-b)(b-c)(c-a).$$

Put $a=0$, $b=1$, $c=2$ $\quad \begin{vmatrix} 3 & 0 & 0 \\ 2 & 1 & 1 \\ 1 & 2 & 4 \end{vmatrix} = k(0+1+2)(-1)(-1)(2)$

$3(4-2) = 6k.$

$6 = 6k \quad \therefore k = 1.$

$\therefore \begin{vmatrix} b+c & a & a^2 \\ c+a & b & b^2 \\ a+b & c & c^2 \end{vmatrix} = (a+b+c)(a-b)(b-c)(c-a)$

$\frac{6}{7.3}$) S.T $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix} = (x-y)(y-z)(z-x).$

Put $x = y$ $\quad |A| = \begin{vmatrix} 1 & 1 & 1 \\ y & y & z \\ y^2 & y^2 & z^2 \end{vmatrix} = 0$ $\quad c_1, c_2$ identical.

$\therefore x-y$ is a factor of $|A|$.

Similarly if $y = z$, $z = x$ then $|A| = 0 \therefore (y-z), (z-x)$ are the factors of $|A|$.

$\therefore$ The degree of the leading diagonal is 3 and the degree of the factors is 3 the remaining factor is k.

$\therefore \begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix} = k(x-y)(y-z)(z-x).$

Put $x = 0$, $y = 1$, $z = 2$

$\begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{vmatrix} = k(-1)(-1)(2)$

$1(4-2) = 2k$

$2 = 2k \Rightarrow k = 1.$

$\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix} = (x-y)(y-z)(z-x).$

$\frac{5}{7.3}$) Solve $\begin{vmatrix} 4-x & 4+x & 4+x \\ 4+x & 4-x & 4+x \\ 4+x & 4+x & 4-x \end{vmatrix} = 0.$

$$|A| = \begin{vmatrix} 8 & 8 & 8+2x \\ 8+2x & 8 & 8 \\ 4+x & 4+x & 4-x \end{vmatrix} \begin{matrix} R_1+R_2 \\ R_2+R_3 \\ \end{matrix}$$

$$= \begin{vmatrix} C_1-C_2 & C_2-C_3 & \\ 0 & -2x & 8+2x \\ 2x & 0 & 8 \\ 0 & 2x & 4-x \end{vmatrix} = 0$$

$$= 0(\quad) \mp 2x \left(-2x(4-x) - 2x(8+2x)\right)$$

$$+4x^2 (4-x+8+2x) = 0$$

$\cancel{4-x-8-2x=0}$

$\cancel{3x=12}$

$x=0$ $\cancel{x=4}$. $x=-12$

Product of determinants:

Ex-7.27) Verify that $|AB| = |A||B|$ if $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ $B = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$$AB = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta + \sin^2\theta & \sin\theta\cos\theta - \sin\theta\cos\theta \\ \sin\theta\cos\theta - \sin\theta\cos\theta & \sin^2\theta + \cos^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$|A| = \cos^2\theta + \sin^2\theta = 1$

$|B| = \cos^2\theta + \sin^2\theta = 1$

$|A|\cdot|B| = 1\cdot 1 = 1$

$|AB| = 1$

$\therefore |AB| = |A||B|$.

EX 7.28) S.T $\begin{vmatrix} 0 & c & b \\ c & 0 & a \\ b & a & 0 \end{vmatrix}^2 = \begin{vmatrix} b^2+c^2 & ab & ac \\ ab & c^2+a^2 & bc \\ ac & bc & a^2+b^2 \end{vmatrix}$.

LHS: $\begin{vmatrix} 0 & c & b \\ c & 0 & a \\ b & a & 0 \end{vmatrix}^2 = \begin{vmatrix} 0 & c & b \\ c & 0 & a \\ b & a & 0 \end{vmatrix} \begin{vmatrix} 0 & c & b \\ c & 0 & a \\ b & a & 0 \end{vmatrix}$ Row by Row.

$$= \begin{vmatrix} 0+c^2+b^2 & 0+0+ab & 0+ac+0 \\ 0+0+ab & c^2+0+a^2 & 0+bc+0 \\ 0+ac+0 & cb+0+0 & 0+a^2+b^2 \end{vmatrix}$$

$$= \begin{vmatrix} c^2+b^2 & ab & ac \\ ab & c^2+a^2 & bc \\ ac & bc & a^2+b^2 \end{vmatrix}$$

EX 7.29) S.T $\begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ca-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix} = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2$

Sol: $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2 = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

$$= \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} (-1) \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}.$$

interchange 2nd, 3rd rows in II det and taking − outside. Then multiply the first row with (−1)

$$= \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} \begin{vmatrix} -a & -b & -c \\ c & a & b \\ b & c & a \end{vmatrix}.$$

$$= \begin{vmatrix} -a^2+bc+bc & -ab+ab+c^2 & -ac+b^2+ac \\ -ab+c^2+ab & -b^2+ca+ca & -bc+bc+a^2 \\ -ac+ac+b^2 & -bc+a^2+bc & -c^2+ab+ab \end{vmatrix}$$

(Taking row by column method)

$$= \begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ca-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix}$$

$\therefore$ ~~AHS~~ = ~~RHS~~.

Ex 7.30) P.T $\begin{vmatrix} 1 & x & x \\ x & 1 & x \\ x & x & 1 \end{vmatrix}^2 = \begin{vmatrix} 1-2x^2 & -x^2 & -x^2 \\ -x^2 & -1 & x^2-2x \\ -x^2 & x^2-2x & -1 \end{vmatrix}$

LHS: $\begin{vmatrix} 1 & x & x \\ x & 1 & x \\ x & x & 1 \end{vmatrix}^2 = \begin{vmatrix} 1 & x & x \\ x & 1 & x \\ x & x & 1 \end{vmatrix} \begin{vmatrix} 1 & x & x \\ x & 1 & x \\ x & x & 1 \end{vmatrix}$.

$$= \begin{vmatrix} 1 & x & x \\ x & 1 & x \\ x & x & 1 \end{vmatrix} \times (-1)(-1) \begin{vmatrix} 1 & x & x \\ -x & -1 & -x \\ -x & -x & -1 \end{vmatrix}.$$

$$= \begin{vmatrix} 1 & x & x \\ x & 1 & x \\ x & x & 1 \end{vmatrix} \begin{vmatrix} 1 & x & x \\ -x & -1 & -x \\ -x & -x & -1 \end{vmatrix}$$ Row by Column.

$$= \begin{vmatrix} 1-x^2-x^2 & x-x-x^2 & x-x^2-x \\ x-x-x^2 & x^2-1-x^2 & x^2-x-x \\ x-x^2-x & x^2-x-x & x^2-x^2-1 \end{vmatrix}$$

$$= \begin{vmatrix} 1-2x^2 & -x^2 & -x^2 \\ -x^2 & -1 & x^2-2x \\ -x^2 & x^2-2x & -1 \end{vmatrix} = RHS.$$

Ex 7.31) If A_i, B_i, C_i are the cofactors of a_i, b_i, c_i resp $i = 1$ to 3.

$|A| = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ S.T $\begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix} = |A|^2$

12 consider $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix}$ Row by Row

$$= \begin{vmatrix} a_1A_1 + b_1B_1 + c_1C_1 & a_2A_1 + b_2B_1 + c_2C_1 \\ a_1A_2 + b_1B_2 + c_1C_2 & a_2A_2 + b_2B_2 + c_2C_2 \\ a_1A_3 + b_1B_3 + c_1C_3 & a_2A_3 + b_2B_3 + c_2C_3 \end{vmatrix}$$

$$= \begin{vmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{vmatrix} = |A|^3$$

$$|A| \begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_c & C_3 \end{vmatrix} = |A|^3 \Rightarrow \begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix} = \frac{|A|^3}{|A|} = |A|^2$$

Area of the $\triangle$le.

Ex 7.32) If the area of the $\triangle$le with vertices $(-3,0)$ $(3,0)$ $(0,k)$ is 9 square units find the value of k.

$$\text{Area of the } \triangle\text{le} = \frac{1}{2} \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = \frac{1}{2} \begin{vmatrix} -3 & 0 \\ 3 & 0 \\ 0 & k \\ -3 & 0 \end{vmatrix}.$$

$$= \frac{1}{2}\left[(0-0) + 3k + 3k\right] = 9$$

$$= \frac{\cancel{6}^{3}k}{\cancel{2}} = 9$$

$$k = \pm 3.$$

Ex 7.33) Find the area of the $\triangle$le whose vertices are $(-2,-3)$ $(3,2)$ and $(-1,-8)$

1

Area of the $\triangle le$ = $\frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} -2 & -3 \\ 3 & 2 \\ -1 & -8 \\ -2 & -3 \end{vmatrix}$

$= \frac{1}{2}\left[(\cancel{+}4+9)+(-24+2)+(3-16)\right]$

$= \frac{1}{2}\left|\left[5-22-13\right]\right|$

$= \frac{1}{2}\,\cancel{30}\ \underline{\underline{15}}$

EX 7.34) S.T the points $(a, b+c)$ $(b, c+a)$ $(c, a+b)$ are collinear.

Area of the $\triangle le$: $\frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} a & b+c \\ b & c+a \\ c & a+b \\ a & b+c \end{vmatrix}$

$= \frac{1}{2}\left[(a(c+a)-b(b+c))+(b(a+b)-c(c+a))+(c(b+c)-a(a+b))\right]$

$= \frac{1}{2}\left[\cancel{ac}+\cancel{a^2}-\cancel{b^2}\cancel{+bc}+\cancel{ab}+\cancel{b^2}-\cancel{c^2}-\cancel{ca}+\cancel{bc}+\cancel{c^2}-\cancel{a^2}-\cancel{ab}\right]$

$= \frac{1}{2}(0) = 0$

∴ The given points are collinear.

1) 7.4) Find the area of the triangle whose vertices are $(0,0)$ $(1,2)$ $(4,3)$

Area of the $\triangle le$ = $\frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 0 & 0 \\ 1 & 2 \\ 4 & 3 \\ 0 & 0 \end{vmatrix}$

$= \frac{1}{2}\left|\left[(0)+(3-8)+0\right]\right|$

$= \frac{5}{2}$ sq. units.

$\frac{2}{7.4}$ If (k, 2) (2, 4) (3, 2) are vertices of the $\triangle$le of area 4 sq. units find the value of k.

Area of the $\triangle$le $= \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} k & 2 \\ 2 & 4 \\ 3 & 2 \\ k & 2 \end{vmatrix} = 4.$

$= \frac{1}{2}\left[(4k-\cancel{4}) + (\cancel{4}-12) + (6-2k)\right] = 4.$

$= \frac{1}{2}[2k-6] = \pm 4$

$2k-6 = \pm 8$

$2k = \pm 14$

$k = 7.$

$\frac{3}{7.4}$) 1) $|A| = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{vmatrix} \begin{matrix} R_2 - R_1 \\ R_3 - R_2 \end{matrix}$

$= 0$

$\frac{46 \times 3}{138}$

$\therefore$ It is a singular matrix.

2) $|A| = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix} = 2(0-20) + 3(-42-4) + 5(30-0)$

$= -40 - 138 + 150 \neq 0.$

$\therefore$ A is not a singular matrix (ie) it is non singular.

3) $|A| = \begin{vmatrix} 0 & a-b & k \\ b-a & 0 & 5 \\ -k & -5 & 0 \end{vmatrix}$

$= 0(0+25) - (a-b)(0+5k) + k(-5(b-a) - 0)$

$= 0 - (a-b)5k - 5k(b-a)$

$-(a-b)5k + (a-b)5k = 0$

$\therefore$ it is singular matrix.

$\frac{4}{7.4}$) Determine the values of a and b so that the following matrices are singular.

i) $A = \begin{bmatrix} 7 & 3 \\ -2 & a \end{bmatrix}$ ii) $A = \begin{bmatrix} b-1 & 2 & 3 \\ 3 & 1 & 2 \\ 1 & -2 & 4 \end{bmatrix}$

$|A| = \begin{vmatrix} 7 & 3 \\ -2 & a \end{vmatrix} = 0$

$= 7a + 6 = 0$

$7a = -6$

$a = -6/7$

$|A| =$

$(b-1)(4+4) - 2(12-2) + 3(-6-1) = 0$

$8b - 8 - 20 - 21 = 0$

$8b = 49$

$b = \frac{49}{8}$

$\frac{5}{7.4}$) If $\cos 2\theta = 0$ determine $\begin{vmatrix} 0 & \cos\theta & \sin\theta \\ \cos\theta & \sin\theta & 0 \\ \sin\theta & 0 & \cos\theta \end{vmatrix}^2$

$\frac{6}{7.4}$ Find the value of $\begin{vmatrix} \log_3 64 & \log_4 3 \\ \log_3 8 & \log_4 9 \end{vmatrix} \begin{vmatrix} \log_2 3 & \log_8 3 \\ \log_3 4 & \log_3 4 \end{vmatrix}$

$= \begin{vmatrix} \log_3 64 \log_2 3 + \log_4 3 \log_3 4 & \log_3 64 \log_8 3 + \log_4 3 \cdot \log_3 4 \\ \log_3 8 \cdot \log_2 3 + \log_4 9 \log_3 4 & \log_3 8 \cdot \log_8 3 + \log_4 9 \cdot \log_3 4 \end{vmatrix}$

$= \begin{vmatrix} 6 \log_3 2 \log_2 3 + 1 & 6 \log_3 2 \log_8 3 + 1 \\ 3 \log_3 2 \cdot \log_2 3 + 2 \log_4 3 \log_3 4 & 1 + 2 \log_4 3 \log_3 4 \end{vmatrix}$

$= \begin{vmatrix} 7 & 3 \\ 5 & 3 \end{vmatrix} = 21 - 15 = 6.$

$\frac{6 \log_3 2}{\log_3 8} = \frac{6 \log_3 2}{3 \log_3 2}$

$= 6 \cdot \frac{1}{3} = 2$