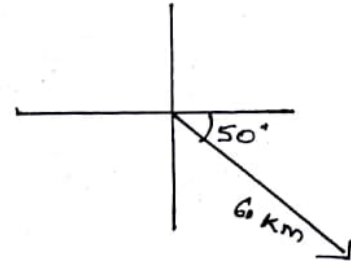
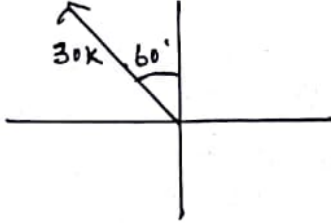


XI Std Maths - Unit - VIII 2018-2019.

Vector Algebra - EXERCISE 8.1.

EX: 8.1. Represent Graphically the displacement of
 i) 30 KM 60° west of north ii) 60 KM 50° South of east.



EX: 8.2 If $\vec{a}$, $\vec{b}$ are vectors represented by two adjacent sides of regular hexagon, then find the vectors represented by other sides.

Sol: Let $\vec{AB} = \vec{a}$, $\vec{BC} = \vec{b}$

By Triangle law of addition
 $\vec{AC} = \vec{a} + \vec{b}$

But $\vec{AD} = 2\vec{BC}$
 $= 2\vec{b}$

Again by triangle law of addition.

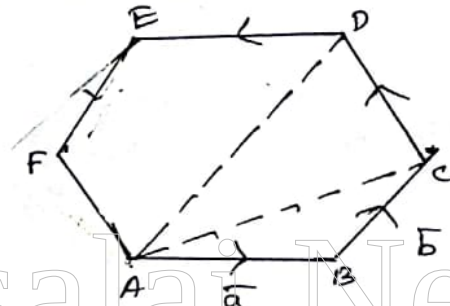
$$\begin{aligned}\vec{AC} + \vec{CD} &= \vec{AD} \\ \vec{CD} &= \vec{AD} - \vec{AC} \\ &= 2\vec{b} - (\vec{a} + \vec{b}) \\ &= \vec{b} - \vec{a}\end{aligned}$$

$$\vec{AB} = -\vec{DE} = -\vec{a}$$

$$\vec{EF} = -\vec{BC} = -\vec{b}$$

$$\vec{FA} = -\vec{CD} = \vec{a} - \vec{b}$$

$$\begin{aligned}\therefore \vec{AB} &= \vec{a} \\ \vec{BC} &= \vec{b} \\ \vec{CD} &= \vec{b} - \vec{a} \\ \vec{DE} &= -\vec{a} \\ \vec{EF} &= -\vec{b} \\ \vec{FA} &= \vec{a} - \vec{b}\end{aligned}$$



Theorem: 8.1. Section Formula (Internal Division)

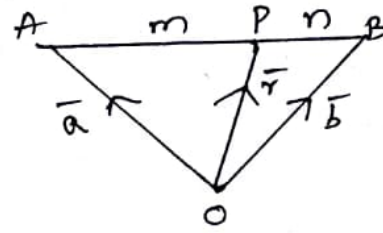
Let O be the origin. Let A, B be two points let P be the point which divides the line segment AB internally in the ratio m:n. If $\vec{a}$, $\vec{b}$ are the p.v of the points A and B then the p.v of the point P is given by $\vec{OP} = \frac{n\vec{a} + m\vec{b}}{m+n}$.

● Proof. Let $\vec{OA} = \vec{a}$, $\vec{OB} = \vec{b}$ and $\vec{OP} = \vec{r}$.

Let P divides AB in the ratio m:n internally

$$\frac{AP}{PB} = \frac{m}{n}$$

$$\frac{|\vec{AP}|}{|\vec{PB}|} = \frac{m}{n}$$



$$(n) |\vec{AP}| = m |\vec{PB}| \quad \text{But } \vec{AP} = \vec{OP} - \vec{OA} = \vec{r} - \vec{a}$$

$$\vec{PB} = \vec{OB} - \vec{OP} = \vec{b} - \vec{r}$$

$$\therefore n(\vec{r} - \vec{a}) = m(\vec{b} - \vec{r})$$

$$n\vec{r} - n\vec{a} = m\vec{b} - m\vec{r}$$

$$(n+m)\vec{r} = m\vec{b} + n\vec{a}$$

$$\vec{r} = \frac{m\vec{b} + n\vec{a}}{m+n}$$

Note: If P divides AB in the ratio m:n externally then

$$\vec{r} = \frac{m\vec{b} - n\vec{a}}{m-n}$$

2) If P is the mid point of AB m:n = 1:1 then $\vec{OP}$

$$\vec{r} = \frac{\vec{a} + \vec{b}}{2}$$

Ex 8.3 Let A and B be two points with p.v. $2\vec{a} + 4\vec{b}$, $2\vec{a} - 8\vec{b}$
Find the p.v of the points which divides the line segment joining A and B in the ratio 1:3 internally and externally.

Internally $\vec{OA} = 2\vec{a} + 4\vec{b}$
 $\vec{OB} = 2\vec{a} - 8\vec{b}$

ratio 1:3 internally.

$$\vec{OP} = \frac{1\vec{OB} + 3\vec{OA}}{1+3} = \frac{(2\vec{a} - 8\vec{b}) + 3(2\vec{a} + 4\vec{b})}{1+3}$$

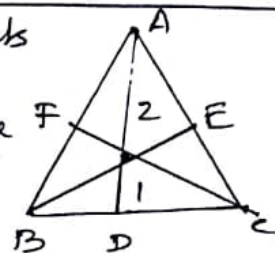
$$= \frac{2\vec{a} - 8\vec{b} + 6\vec{a} + 12\vec{b}}{4} = \frac{8\vec{a} + 4\vec{b}}{4} = 2\vec{a} + \vec{b}$$

Externally: $\vec{OP} = \frac{1\vec{OB} - 3\vec{OA}}{1-3} = \frac{(2\vec{a} - 8\vec{b}) - 3(2\vec{a} + 4\vec{b})}{-2}$

$$= \frac{2\vec{a} - 8\vec{b} - 6\vec{a} - 12\vec{b}}{-2} = \frac{-4\vec{a} - 20\vec{b}}{-2} = 2\vec{a} + 10\vec{b}$$

Theorem 8.3) The medians of a Δ are concurrent

In a Δ ABC, D, E, F are the mid points of BC, CA, AB. $\therefore$ AD, BE, CF are the medians.



$$\text{Let } \vec{OA} = \vec{a}, \vec{OB} = \vec{b}, \vec{OC} = \vec{c}$$

$$\therefore \vec{OD} = \frac{\vec{b} + \vec{c}}{2}, \vec{OE} = \frac{\vec{c} + \vec{a}}{2}, \vec{OF} = \frac{\vec{a} + \vec{b}}{2}$$

Let G_1 be the point on AD which divides AD in the ratio 2:1

$$\therefore \vec{OG}_1 = \frac{2 \cdot \vec{OD} + 1 \cdot \vec{OA}}{2 + 1} = \frac{2 \left(\frac{\vec{b} + \vec{c}}{2} \right) + 1 \cdot \vec{a}}{3} = \frac{\vec{a} + \vec{b} + \vec{c}}{3}$$

Let G_2 be the point on BE which divides with ratio 2:1,

and G_3 be the point on CF which divides in the ratio 2:1

$$\text{Hence we can find that } \vec{OG}_2 = \frac{\vec{a} + \vec{b} + \vec{c}}{2}$$

$$\vec{OG}_3 = \frac{\vec{a} + \vec{b} + \vec{c}}{2}$$

$\therefore G_1, G_2, G_3$ are the same point. Hence medians of a Δ are concurrent.

G is the centroid of the Δ . $\therefore \vec{OG} = \frac{\vec{a} + \vec{b} + \vec{c}}{3}$

Theorem 8.4) A quadrilateral is a parallelogram iff its diagonals bisect each other.

Proof: Let ABCD be a quadrilateral with diagonals AC and BD.

$$\vec{OA} = \vec{a}, \vec{OB} = \vec{b}, \vec{OC} = \vec{c}, \vec{OD} = \vec{d}$$

If ABCD is a parallelogram

$$\vec{AB} = \vec{DC}$$

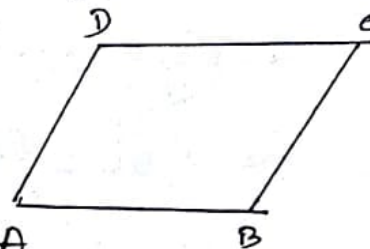
$$\vec{OB} - \vec{OA} = \vec{OC} - \vec{OD}$$

$$\vec{b} - \vec{a} = \vec{c} - \vec{d}$$

$$\vec{b} + \vec{d} = \vec{a} + \vec{c}$$

$$\frac{\vec{a} + \vec{c}}{2} = \frac{\vec{b} + \vec{d}}{2}$$

$\Rightarrow$ Mid points of AC and BD are same.



$$\text{If } \frac{\vec{a} + \vec{c}}{2} = \frac{\vec{b} + \vec{d}}{2} \Rightarrow \vec{a} + \vec{c} = \vec{b} + \vec{d}$$

$$\vec{c} - \vec{d} = \vec{b} - \vec{a}$$

$$\Rightarrow \vec{OC} - \vec{OD} = \vec{OB} - \vec{OA}$$

$$\Rightarrow \vec{DC} = \vec{AB}$$

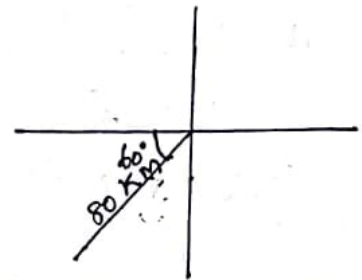
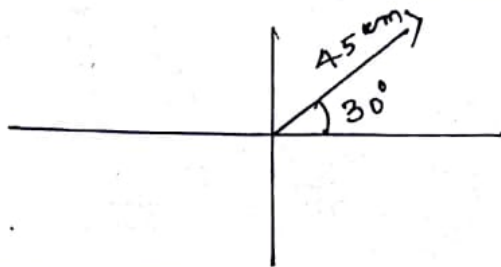
This shows that AB, CD are equal and parallel. Similarly we can prove BC, AD are parallel and equal.

$\therefore$ ABCD is Parallelogram.

EXERCISE - 8.1

1) Represent graphically the displacement of

- 1) 45 cm 30° north of east 2) 80 km, 60° south of west.



3) Let $\vec{a}$ and $\vec{b}$ be the p.o.v of the points A and B. P.T the p.v of the points which bisect the line segment AB are

$$\frac{\vec{a} + 2\vec{b}}{3}, \frac{\vec{b} + 2\vec{a}}{3}.$$

Let $\vec{OA} = \vec{a}$, $\vec{OB} = \vec{b}$

Let P be a point which divides AB in the ratio 1:2 internally.



$$\vec{OP} = \frac{1 \cdot \vec{b} + 2 \cdot \vec{a}}{1+2} = \frac{\vec{b} + 2\vec{a}}{3}$$

Let Q be the point on AB which divides AB in the ratio 2:1

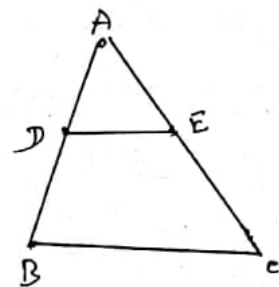
$$\frac{2 \cdot \vec{b} + 1 \cdot \vec{a}}{2+1} = \frac{2\vec{b} + \vec{a}}{3}$$

4) If D, E are the mid points of the sides AB and AC of a ΔABC

P.T $\vec{BE} + \vec{DE} = \frac{3}{2} \vec{BC}$

$$\begin{aligned} \vec{BE} &= \vec{BC} + \vec{CE} & \vec{DE} &= \vec{DB} + \vec{BE} \\ &= \vec{BC} + \frac{1}{2} \vec{CA} & &= \frac{1}{2} \vec{AB} + \vec{BC} \end{aligned}$$

$$\begin{aligned} \therefore \vec{BE} + \vec{DE} &= \vec{BC} + \frac{1}{2} \vec{CA} + \frac{1}{2} \vec{AB} + \vec{BC} \\ &= 2\vec{BC} + \frac{1}{2} (\vec{CA} + \vec{AB}) \\ &= 2\vec{BC} + \frac{1}{2} (\vec{CB}) \\ &= 2\vec{BC} - \frac{1}{2} \vec{BC} \\ &= \frac{3}{2} \vec{BC}. \end{aligned}$$



- 5) P.T the line segment joining the mid points of two sides of a triangle is parallel to the third side whose length is half of the length of the third side.

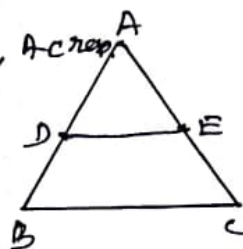
In a $\triangle ABC$ D, E are the mid points of AB, AC resp.

$$\therefore \vec{OD} = \frac{\vec{OA} + \vec{OB}}{2} \quad \vec{OE} = \frac{\vec{OA} + \vec{OC}}{2}$$

$$\begin{aligned} \vec{DE} &= \vec{OE} - \vec{OD} = \frac{\vec{OA} + \vec{OC}}{2} - \frac{\vec{OA} + \vec{OB}}{2} \\ &= \frac{\vec{OC} - \vec{OB}}{2} = \frac{1}{2} (\vec{BC}) \end{aligned}$$

$$|\vec{DE}| = \frac{1}{2} |\vec{BC}|$$

which implies that line joining of the mid points of the two sides is parallel to third side and length is equal to $\frac{1}{2}$ of the length of the third side.



- 6) P.T the line segments joining the mid points of the adjacent sides of a quadrilateral form a parallelogram.

ABCD is a quadrilateral in which E, F, G, H are the mid points of AB, BC, CD, DA resp.

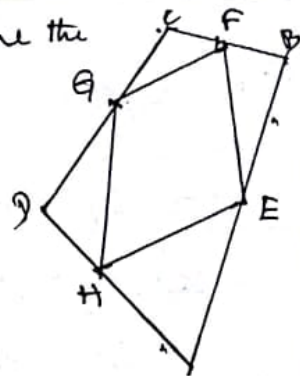
$$\vec{OE} = \frac{\vec{OA} + \vec{OB}}{2} \quad \vec{OG} = \frac{\vec{OD} + \vec{OC}}{2}$$

$$\vec{OF} = \frac{\vec{OB} + \vec{OC}}{2} \quad \vec{OH} = \frac{\vec{OD} + \vec{OA}}{2}$$

$$\text{Mid point of } \vec{EG} = \frac{\frac{\vec{OA} + \vec{OB}}{2} + \frac{\vec{OD} + \vec{OC}}{2}}{2} = \frac{\vec{OA} + \vec{OB} + \vec{OC} + \vec{OD}}{4}$$

$$\text{Mid point of } \vec{FH} = \frac{\frac{\vec{OB} + \vec{OC}}{2} + \frac{\vec{OD} + \vec{OA}}{2}}{2} = \frac{\vec{OA} + \vec{OB} + \vec{OC} + \vec{OD}}{4}$$

$\therefore$ The mid points of the diagonals EG and FH are same. EFGH is a parallelogram.



9) If D is the mid point of the side BC of a ΔABC P.T $\vec{AB} + \vec{AC} = 2\vec{AD}$.

Let D be the mid point of BC of the ΔABC

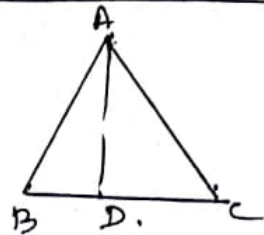
$$\vec{AB} = \vec{AD} + \vec{DB} \quad \text{--- ①}$$

$$\vec{AC} = \vec{AD} + \vec{DC}$$

$$= \vec{AD} - \vec{DB} \quad \text{--- ②} \because D \text{ is the midpoint of BC}$$

$$\vec{AB} + \vec{AC} = 2\vec{AD}$$

and DC and DB are opposite



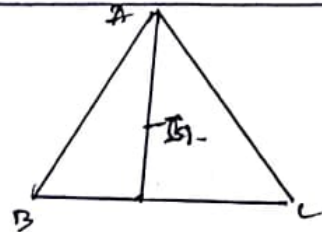
10) If G is the centroid of the ΔABC P.T $\vec{GA} + \vec{GB} + \vec{GC} = \vec{0}$

Let G be the centroid of the ΔABC

$$\vec{OG} = \frac{\vec{OA} + \vec{OB} + \vec{OC}}{3}$$

$$3\vec{OG} = \vec{OA} + \vec{OB} + \vec{OC} \quad \text{--- ①}$$

$$\begin{aligned} \text{Given } \vec{GA} + \vec{GB} + \vec{GC} &= \vec{OA} - \vec{OG} + \vec{OB} - \vec{OG} + \vec{OC} - \vec{OG} \\ &= (\vec{OA} + \vec{OB} + \vec{OC}) - 3\vec{OG} \\ &= 3\vec{OG} - 3\vec{OG} \\ &= \vec{0} \end{aligned}$$



11) Let A, B, C are the vertices of a Δ . Let D, E and F are the mid points of the sides BC, CA, AB resp. S.T $\vec{AD} + \vec{BE} + \vec{CF} = \vec{0}$.

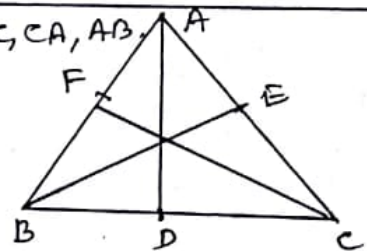
Let D, E, F are the mid points of BC, CA, AB.

$$\vec{AD} = \vec{AB} + \vec{BD} = \vec{AB} + \frac{1}{2}\vec{BC}$$

$$\vec{BE} = \vec{BC} + \vec{CE} = \vec{BC} + \frac{1}{2}\vec{CA}$$

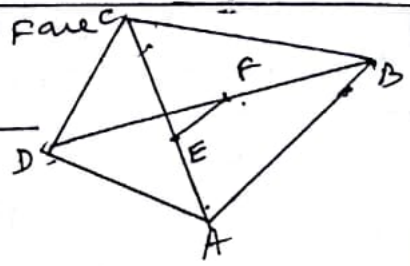
$$\vec{CF} = \vec{CA} + \vec{AF} = \vec{CA} + \frac{1}{2}\vec{AB}$$

$$\begin{aligned} \vec{AD} + \vec{BE} + \vec{CF} &= \frac{3}{2}\vec{AB} + \frac{3}{2}\vec{BC} + \frac{3}{2}\vec{CA} \\ &= \frac{3}{2}(\vec{AB} + \vec{BC} + \vec{CA}) \\ &= \frac{3}{2}(\vec{0}) = \vec{0} \end{aligned}$$



12) If ABCD is a quadrilateral and E, F are the mid points of AC and BD resp. then P.T $\vec{AB} + \vec{AD} + \vec{CB} + \vec{CD} = 4\vec{EF}$

ABCD is a quadrilateral in which, E, F are the mid points of AC, BD



$$\vec{AB} = \vec{AE} + \vec{EF} + \vec{FB}$$

$$\vec{AD} = \vec{AE} + \vec{EF} + \vec{FD}$$

$$\vec{CB} = \vec{CE} + \vec{EF} + \vec{FB}$$

$$\vec{CD} = \vec{CE} + \vec{EF} + \vec{FD}$$

$$\vec{AB} + \vec{AD} + \vec{CB} + \vec{CD} = 2(\vec{AE} + \vec{CE}) + 4\vec{EF} + 2(\vec{FB} + \vec{FD})$$

$$= 2(0) + 4\vec{EF} + 2(0)$$

$$= 4\vec{EF}$$

$\therefore \vec{AE}, \vec{CE}$ are equal but opposite
 $\vec{FB}, \vec{FD}$ are equal but opposite.

7) If $\vec{a}, \vec{b}$ represent a side and a diagonal of a parallelogram find the other sides and the other diagonal.

In a parallelogram ABCD

$$\vec{AB} = \vec{a}, \vec{AC} = \vec{b}$$

By T.L.A.

$$\vec{AB} + \vec{BC} = \vec{AC}$$

$$\vec{BC} = \vec{AC} - \vec{AB}$$

$$= \vec{b} - \vec{a}$$

$\vec{DA}$ is equal and opposite to $\vec{BC}$

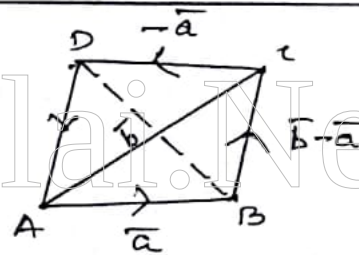
$$\therefore \vec{DA} = -(\vec{b} - \vec{a})$$

$\vec{CD}$ is equal and opposite to $\vec{AB}$

$$\vec{CD} = -\vec{a}$$

$$\vec{BD} = \vec{BC} + \vec{CD}$$

$$= (\vec{b} - \vec{a}) + (-\vec{a}) = \vec{b} - 2\vec{a}$$



8) If $\vec{PA} + \vec{OA} = \vec{QA} + \vec{OR}$ P, Q, R are collinear.

$$\text{Given } \vec{PA} + \vec{OA} = \vec{QA} + \vec{OR}$$

$$\vec{PA} = \vec{QR} \quad \text{--- ①}$$



$$\text{Again } \vec{PA} + \vec{AR} = \vec{QA} + \vec{OR} - \vec{QA}$$

$$= \vec{OR} - \vec{OP}$$

$$= \vec{PR} \quad \text{--- ②}$$

From ① and ②: P, Q, R are collinear.

EXERCISE - 8.2.

EX 8.4. Find the unit vector along the direction of $5\hat{i} - 3\hat{j} + 4\hat{k}$

$$\text{Let } \vec{a} = 5\hat{i} - 3\hat{j} + 4\hat{k}$$

$$|\vec{a}| = \sqrt{25+9+16} = \sqrt{50}$$

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{5\hat{i} - 3\hat{j} + 4\hat{k}}{\sqrt{50}}$$

Note! unit vector parallel to $\vec{a}$ but in the opposite direction
 $\hat{a} = -\frac{\vec{a}}{|\vec{a}|}$

EX: 8.5) Find the direction ratios and direction cosines of

- 1) $3\hat{i} + 4\hat{j} - 6\hat{k}$ 2) $3\hat{i} - 4\hat{k}$

$$\text{Let } \vec{a} = 3\hat{i} + 4\hat{j} - 6\hat{k}$$

$$\text{Dr's of } \vec{a} = 3, 4, -6. \quad |\vec{a}| = \sqrt{9+16+36} = \sqrt{61}$$

$$\text{DC's } \left(\frac{3}{\sqrt{61}}, \frac{4}{\sqrt{61}}, \frac{-6}{\sqrt{61}} \right)$$

$$\text{Let } \vec{b} = 3\hat{i} - 4\hat{k}$$

$$\text{Dr's of } \vec{b} = 3, 0, -4. \quad |\vec{b}| = \sqrt{9+16} = 5$$

$$\text{DC's } \left(\frac{3}{5}, 0, \frac{-4}{5} \right)$$

EX 8.6) Find the direction cosines of a vector whose dr's are 2, 3, -6.

2) Can a vector have direction angles $30^\circ, 45^\circ, 60^\circ$

3) Find the dc's of $\vec{AB}$ where $A(2, 3, 1)$ $B(3, -1, 2)$

4) Find the dc's of the line joining $(2, 3, 1)$ $(3, -1, 2)$

5) The dr's of a vector are 2, 3, 6 and its magnitude is 5

Find the vector.

1) dc's are $\frac{2}{\sqrt{4+9+36}}, \frac{3}{\sqrt{49}}, \frac{-6}{\sqrt{49}}$

$$(i) \left(\frac{2}{7}, \frac{3}{7}, \frac{-6}{7} \right)$$

2) If α, β, γ are the angles with Ox, Oy, Oz then

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

$$(ii) \cos^2 30^\circ + \cos^2 45^\circ + \cos^2 60^\circ = \left(\frac{\sqrt{3}}{2} \right)^2 + \left(\frac{1}{\sqrt{2}} \right)^2 + \left(\frac{1}{2} \right)^2$$

$$= \frac{3}{4} + \frac{1}{2} + \frac{1}{4} \neq 1.$$

$\therefore$ They are not the direction angles of a vector.

$$3) \vec{AB} = \vec{OB} - \vec{OA} = \hat{i} - 4\hat{j} + \hat{k}$$

$$|\vec{AB}| = \sqrt{1+16+1} = \sqrt{18}$$

$$\therefore \text{D.C's} \left(\frac{1}{\sqrt{18}}, -\frac{4}{\sqrt{18}}, \frac{1}{\sqrt{18}} \right)$$

$$4) \vec{AB} = \vec{OB} - \vec{OA} = \hat{i} - 4\hat{j} + \hat{k}$$

$$\text{The d.c's of AB are } \left(\frac{1}{\sqrt{18}}, -\frac{4}{\sqrt{18}}, \frac{1}{\sqrt{18}} \right)$$

Suppose if we take second point as first point

$$\text{d.c's are } \left(-\frac{1}{\sqrt{18}}, \frac{4}{\sqrt{18}}, \frac{1}{\sqrt{18}} \right)$$

$$5) \text{ d.c's are } \left(\frac{2}{7}, \frac{3}{7}, \frac{6}{7} \right)$$

$$\therefore \text{Unit vector is } \frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}$$

$$\text{The required vector is } \frac{5}{7}(2\hat{i} + 3\hat{j} + 6\hat{k})$$

EX 8.7) S.T the points whose P.V are $2\hat{i} + 3\hat{j} - 5\hat{k}$, $3\hat{i} + \hat{j} - 2\hat{k}$ and $6\hat{i} - 5\hat{j} + 7\hat{k}$ are collinear.

$$\text{Let } \vec{OA} = 2\hat{i} + 3\hat{j} - 5\hat{k}$$

$$\vec{OB} = 3\hat{i} + \hat{j} - 2\hat{k}$$

$$\vec{OC} = 6\hat{i} - 5\hat{j} + 7\hat{k}$$

$$\vec{AB} = \vec{OB} - \vec{OA} = \hat{i} - 2\hat{j} + 3\hat{k}$$

$$\vec{AC} = \vec{OC} - \vec{OA} = 4\hat{i} - 8\hat{j} + 12\hat{k}$$

$$= 4(\hat{i} - 2\hat{j} + 3\hat{k})$$

$\therefore$ The three points are collinear.

Note: If $\vec{AC} = 4\vec{AB}$
 $\vec{AC}$ and $\vec{AB}$ are $\parallel$

A — B — C

When the three points are collinear then only AB and AC are $\parallel$.

EX 8.8) Find the point whose P.V has magnitude 5 and $\parallel$ to $4\hat{i} - 3\hat{j} + 10\hat{k}$

$$\text{Let } \vec{a} = 4\hat{i} - 3\hat{j} + 10\hat{k}$$

$$|\vec{a}| = \sqrt{16+9+100} = \sqrt{125} = 5\sqrt{5}$$

$$\hat{a} = \frac{4\hat{i} - 3\hat{j} + 10\hat{k}}{5\sqrt{5}}$$

$$5\hat{a} = \frac{4\hat{i} - 3\hat{j} + 10\hat{k}}{\sqrt{5}} \therefore \text{Required points are } \left(\frac{4}{\sqrt{5}}, -\frac{3}{\sqrt{5}}, \frac{10}{\sqrt{5}} \right) \frac{10}{\sqrt{5}} = 2\sqrt{5}$$

EX 8.9) P.T the points whose pv's $2\hat{i} + 4\hat{j} + 3\hat{k}$, $4\hat{i} + \hat{j} + 9\hat{k}$, $10\hat{i} - \hat{j} + 6\hat{k}$ forms a rt triangle

$$\begin{aligned} \text{let } \vec{OA} &= 2\hat{i} + 4\hat{j} + 3\hat{k} & \vec{AB} &= \vec{OB} - \vec{OA} = 2\hat{i} - 3\hat{j} + 6\hat{k} \\ \vec{OB} &= 4\hat{i} + \hat{j} + 9\hat{k} & \vec{BC} &= \vec{OC} - \vec{OB} = 6\hat{i} - 2\hat{j} - 3\hat{k} \\ \vec{OC} &= 10\hat{i} - \hat{j} + 6\hat{k} & \vec{CA} &= \vec{OA} - \vec{OC} = -8\hat{i} + 5\hat{j} - 3\hat{k} \end{aligned}$$

$\vec{AB} + \vec{BC} + \vec{CA} = 0$ $\therefore$ First it is a triangle.

$$|\vec{AB}| = \sqrt{4+9+36} = 7$$

$$|\vec{BC}| = \sqrt{36+4+9} = 7$$

$$|\vec{CA}| = \sqrt{64+25+9} = \sqrt{98}$$

$$|\vec{AB}|^2 + |\vec{BC}|^2 = |\vec{CA}|^2$$

$$49 + 49 = 98. \therefore \text{given points forms a rt } \Delta.$$

EX 8.10) S.T the vectors $5\hat{i} + 6\hat{j} + 7\hat{k}$, $7\hat{i} - 8\hat{j} + 9\hat{k}$, $3\hat{i} + 20\hat{j} + 5\hat{k}$ are coplanar.

$$\text{let } 5\hat{i} + 6\hat{j} + 7\hat{k} = 5(7\hat{i} - 8\hat{j} + 9\hat{k}) + t(3\hat{i} + 20\hat{j} + 5\hat{k})$$

$$\begin{array}{rcl} 7s + 3t = 5 & \text{--- (1)} & \textcircled{1} \times 5 \quad 35s + 15t = 25 \\ -8s + 20t = 6 & \text{--- (2)} & \textcircled{2} \times 3 \quad 27s + 15t = 21 \\ 9s + 5t = 7 & \text{--- (3)} & \hline 8s = 4 \end{array}$$

$$\therefore s = \frac{1}{2} \quad t = \frac{1}{2}$$

$\therefore$ we can write one vector is a linear combination of other two vectors. Hence given vectors are coplanar.

7/8.2) S.T the vectors $2\hat{i} - \hat{j} + \hat{k}$, $3\hat{i} - 4\hat{j} - 4\hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ form a right angled Δ .

$$\vec{AB} = 2\hat{i} - \hat{j} + \hat{k} \quad |\vec{AB}| = \sqrt{4+1+1} = \sqrt{6}$$

$$\vec{BC} = 3\hat{i} - 4\hat{j} - 4\hat{k} \quad |\vec{BC}| = \sqrt{9+16+16} = \sqrt{41}$$

$$\vec{CA} = \hat{i} - 3\hat{j} - 5\hat{k} \quad |\vec{CA}| = \sqrt{1+9+25} = \sqrt{35}$$

$$\vec{CA} + \vec{AB} = \vec{BC} \therefore \text{First it is a } \Delta.$$

$$|\vec{CA}|^2 + |\vec{AB}|^2 = |\vec{BC}|^2$$

$$35 + 6 = 41. \therefore \text{it is right angled } \Delta.$$

8.2) Find the value of λ for which the vectors $\vec{a} = 3\hat{i} + 2\hat{j} + 9\hat{k}$
 $\vec{b} = \hat{i} + \lambda\hat{j} + 3\hat{k}$ are parallel.

If $\vec{a}$ and $\vec{b}$ are $\parallel$ $\vec{b} = t\vec{a}$

$$\hat{i} + \lambda\hat{j} + 3\hat{k} = 3\left(\hat{i} + \frac{2}{3}\hat{j} + 3\hat{k}\right)$$

$$\Rightarrow \lambda = 2/3.$$

9) S.T the following vectors are coplanar.

1) $\hat{i} - 2\hat{j} + 3\hat{k}$ $-2\hat{i} + 3\hat{j} - 4\hat{k}$, $-\hat{j} + 2\hat{k}$

2) $5\hat{i} + 6\hat{j} + 7\hat{k}$, $7\hat{i} - 8\hat{j} + 9\hat{k}$, $3\hat{i} + 20\hat{j} + 5\hat{k}$.

1) $\hat{i} - 2\hat{j} + 3\hat{k} = s(-2\hat{i} + 3\hat{j} - 4\hat{k}) + t(-\hat{j} + 2\hat{k})$

$$1 = -2s \Rightarrow s = -1/2$$

$$-2 = 3s - t \Rightarrow -2 = 3(-1/2) - t$$

$$t = -3/2 + 2 = 1/2.$$

$$\therefore \hat{i} - 2\hat{j} + 3\hat{k} = -\frac{1}{2}(-2\hat{i} + 3\hat{j} - 4\hat{k}) + \frac{1}{2}(-\hat{j} + 2\hat{k})$$

$\therefore$ we can write one vector as a linear combination of other two vectors. Hence they are coplanar.

2) $5\hat{i} + 6\hat{j} + 7\hat{k} = s(7\hat{i} - 8\hat{j} + 9\hat{k}) + t(3\hat{i} + 20\hat{j} + 5\hat{k})$

$$5 = 7s + 3t \quad \text{--- (1)}$$

$$6 = -8s + 20t \quad \text{--- (2) This is Example 8.10.}$$

$$7 = 9s + 5t \quad \text{--- (3)}$$

10) S.T the points whose P.V $4\hat{i} + 5\hat{j} + \hat{k}$, $-\hat{j} - \hat{k}$, $3\hat{i} + 9\hat{j} + 4\hat{k}$,
 $-4\hat{i} + 4\hat{j} + 4\hat{k}$ are coplanar.

$$\vec{OA} = 4\hat{i} + 5\hat{j} + \hat{k}$$

$$\vec{OB} = -\hat{j} - \hat{k}$$

$$\vec{OC} = 3\hat{i} + 9\hat{j} + 4\hat{k}$$

$$\vec{OD} = -4\hat{i} + 4\hat{j} + 4\hat{k}$$

$$\vec{AB} = -4\hat{i} - 6\hat{j} - 2\hat{k} \quad (\vec{OB} - \vec{OA})$$

$$\vec{BC} = 3\hat{i} - 6\hat{j} - 2\hat{k}$$

$$\vec{CD} = 7\hat{i} - 5\hat{j}$$

$$\vec{CD} = 7\hat{i} - 5\hat{j}$$

$$-4\hat{i} - 6\hat{j} - 2\hat{k} = s(3\hat{i} - 6\hat{j} - 2\hat{k}) + t(7\hat{i} - 5\hat{j})$$

$$-4 = 3s + 7t$$

$$-6 = 10s - 5t$$

$$-2 = 5s \Rightarrow s = -2/5$$

$$-4 = 3(-\frac{7}{5}) - 7\lambda$$

$$-7\lambda = -\frac{6}{5} + 4 = \frac{-6+20}{5}$$

$$= \frac{14}{5} \Rightarrow \lambda = \frac{14}{5 \times 7}$$

$$\therefore -4\hat{i} - 6\hat{j} - 2\hat{k} = -\frac{2}{5}(3\hat{i} - 6\hat{j} - 2\hat{k}) + \frac{2}{5}(-7\hat{i} - 5\hat{j})$$

$\therefore$ one vector can be written as sum of two linear vectors. Hence the given vectors are coplanar.

11) If $\vec{a} = 2\hat{i} + 3\hat{j} - 4\hat{k}$, $\vec{b} = 3\hat{i} - 4\hat{j} - 5\hat{k}$ c) $-3\hat{i} + 2\hat{j} + 3\hat{k}$.
Find the magnitude of 1) $\vec{a} + \vec{b} + \vec{c}$ 2) $3\vec{a} - 2\vec{b} + 5\vec{c}$.

$$\begin{aligned}\vec{a} &= 2\hat{i} + 3\hat{j} - 4\hat{k} \\ \vec{b} &= 3\hat{i} - 4\hat{j} - 5\hat{k} \\ \vec{c} &= -3\hat{i} + 2\hat{j} + 3\hat{k}\end{aligned}$$

$$\textcircled{1} \vec{a} + \vec{b} + \vec{c} = 2\hat{i} + 3\hat{j} - 4\hat{k}$$

$$|\vec{a} + \vec{b} + \vec{c}| = \sqrt{4+9+16} = \sqrt{29}$$

$$\text{d.c.s} = \frac{2}{\sqrt{29}}, \frac{3}{\sqrt{29}}, \frac{-4}{\sqrt{29}}$$

$$\begin{aligned}3\vec{a} &= 6\hat{i} + 9\hat{j} - 12\hat{k} \\ -2\vec{b} &= -6\hat{i} + 8\hat{j} + 10\hat{k} \\ 5\vec{c} &= -15\hat{i} + 10\hat{j} + 15\hat{k}\end{aligned}$$

$$3\vec{a} - 2\vec{b} + 5\vec{c} = -15\hat{i} + 27\hat{j} + 13\hat{k}$$

$$|3\vec{a} - 2\vec{b} + 5\vec{c}| = \sqrt{1123}$$

$$\text{d.c.s} = \frac{-15}{\sqrt{1123}}, \frac{27}{\sqrt{1123}}, \frac{13}{\sqrt{1123}}$$

12) The P.V of the vertices of a triangle are $\hat{i} + 2\hat{j} + 3\hat{k}$, $3\hat{i} - 4\hat{j} + 5\hat{k}$ and $-2\hat{i} + 3\hat{j} - 7\hat{k}$. Find the perimeter of the Δ -

$$\begin{aligned}\vec{OA} &= \hat{i} + 2\hat{j} + 3\hat{k} \\ \vec{OB} &= 3\hat{i} - 4\hat{j} + 5\hat{k} \\ \vec{OC} &= -2\hat{i} + 3\hat{j} - 7\hat{k}\end{aligned}$$

$$\vec{AB} = \vec{OB} - \vec{OA} = 2\hat{i} - 6\hat{j} + 2\hat{k}$$

$$\vec{BC} = \vec{OC} - \vec{OB} = -5\hat{i} + 7\hat{j} - 12\hat{k}$$

$$\vec{CA} = \vec{OA} - \vec{OC} = 3\hat{i} - \hat{j} + 10\hat{k}$$

$$|\vec{AB}| = \sqrt{4+36+4} = \sqrt{44}$$

$$|\vec{BC}| = \sqrt{25+49+144} = \sqrt{218}$$

$$|\vec{CA}| = \sqrt{9+1+100} = \sqrt{110}$$

$$\therefore \text{Perimeter of the } \Delta = \sqrt{44} + \sqrt{218} + \sqrt{110}$$

13) Find the unit vector parallel to $3\vec{a} - 2\vec{b} + 4\vec{c}$
 8.2) $\vec{a} = 3\hat{i} - \hat{j} - 4\hat{k}$ $\vec{b} = -2\hat{i} + 4\hat{j} - 3\hat{k}$, $\vec{c} = \hat{i} + 2\hat{j} - \hat{k}$

$$\begin{aligned} 3\vec{a} &= 9\hat{i} - 3\hat{j} - 12\hat{k} \\ -2\vec{b} &= 4\hat{i} - 8\hat{j} + 6\hat{k} \\ 4\vec{c} &= 4\hat{i} + 8\hat{j} - 4\hat{k} \end{aligned}$$

$$\begin{aligned} 3\vec{a} - 2\vec{b} + 4\vec{c} &= 17\hat{i} + 3\hat{j} + 10\hat{k} \end{aligned}$$

$$\begin{aligned} \text{let } |17\hat{i} + 3\hat{j} + 10\hat{k}| &= \sqrt{289 + 9 + 100} \\ &= \sqrt{398} \end{aligned}$$

$$\text{unit vector || to } 3\vec{a} - 2\vec{b} + 4\vec{c} = \frac{17\hat{i} + 3\hat{j} + 10\hat{k}}{\sqrt{398}}$$

15) The PV of ~~2, 3, 4~~ are the points P, Q, R, S are $\hat{i} + \hat{j} + \hat{k}$
 8.2) $2\hat{i} + 5\hat{j}$, $3\hat{i} + 2\hat{j} - 3\hat{k}$ and $\hat{i} - 6\hat{j} - \hat{k}$ resp.
 P.T the lines PQ and RS are parallel.

$$\vec{OP} = \hat{i} + \hat{j} + \hat{k}$$

$$\vec{OQ} = 2\hat{i} + 5\hat{j}$$

$$\vec{OR} = 3\hat{i} + 2\hat{j} - 3\hat{k}$$

$$\vec{OS} = \hat{i} - 6\hat{j} - \hat{k}$$

$$\vec{PQ} = \vec{OQ} - \vec{OP} = \hat{i} + 4\hat{j} - \hat{k}$$

$$\vec{RS} = \vec{OS} - \vec{OR} = -2\hat{i} - 8\hat{j} + 2\hat{k}$$

$$\begin{aligned} \vec{RS} &= -2(\hat{i} + 4\hat{j} - \hat{k}) \\ &= -2\vec{PQ} \end{aligned}$$

∴ PQ, RS are parallel.

16) Find the value or values of ^{for} m which $m(\hat{i} + \hat{j} + \hat{k})$ is a unit vector

$$\begin{aligned} \hat{a} &= \frac{\vec{a}}{|\vec{a}|} = \frac{m(\hat{i} + \hat{j} + \hat{k})}{\frac{1}{m}} \\ &= \pm \frac{(\hat{i} + \hat{j} + \hat{k})}{\frac{1}{m}} \end{aligned}$$

$$\begin{aligned} \Rightarrow |\vec{a}| &= \sqrt{1+1+1} = \sqrt{3} \\ |\vec{a}| &= \pm \frac{1}{m} \\ \therefore m &= \pm \frac{1}{\sqrt{3}} \end{aligned}$$

17) S.T the points A(1,1,1) B(1,2,3) C(2,-1,1) are the vertices of isosceles triangle

$$\vec{OA} = \hat{i} + \hat{j} + \hat{k}$$

$$\vec{OB} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\vec{OC} = 2\hat{i} - \hat{j} + \hat{k}$$

$$\vec{AB} = \hat{j} + 2\hat{k}$$

$$\vec{BC} = \hat{i} - 3\hat{j} - 2\hat{k}$$

$$\vec{CA} = -\hat{i} + 2\hat{j}$$

$$|\vec{AB}| = \sqrt{1+4} = \sqrt{5}$$

$$|\vec{BC}| = \sqrt{1+9+4} = \sqrt{14}$$

$$|\vec{CA}| = \sqrt{1+4} = \sqrt{5}$$

$$|\vec{AB}| = |\vec{CA}| \therefore \text{it is isosceles } \Delta.$$

$\frac{1}{8.2}$) Verify whether the following ratios are d.c's of some vector or not.

1) $\frac{1}{5}, \frac{3}{5}, \frac{4}{5}$ 2) $\frac{1}{\sqrt{2}}, \frac{1}{2}, \frac{1}{2}$ 3) $\frac{4}{3}, 0, \frac{3}{4}$.

W.K.T $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$

1) $\frac{1}{25} + \frac{9}{25} + \frac{16}{25} \neq 1 \therefore$ not d.c's

2) $\frac{1}{2} + \frac{1}{4} + \frac{1}{4} = 1 \therefore$ ② is d.c's

3) $\frac{16}{9} + 0 + \frac{9}{16} \neq 1 \therefore$ not d.c's.

$\frac{2}{8.2}$) Find the direction cosines of a vector whose d.r's are

1) 1, 2, 3 2) 3, -1, 3 3) 0, 0, 7.

1) Let $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k}$

$|\vec{r}| = \sqrt{1+4+9} = \sqrt{14} \therefore$ d.c's $\left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)$

2) Let $\vec{r} = 3\hat{i} - \hat{j} + 3\hat{k}$

$|\vec{r}| = \sqrt{9+1+9} = \sqrt{19}$ d.c's $\left(\frac{3}{\sqrt{19}}, \frac{-1}{\sqrt{19}}, \frac{3}{\sqrt{19}}\right)$

3) Let $\vec{r} = 7\hat{k}$

$|\vec{r}| = \sqrt{49} = 7$ d.c's $\left(0, 0, \frac{7}{7}\right)$

$\frac{3}{8.2}$) Find the d.c's and d.r's of the following vectors.

1) $3\hat{i} - 4\hat{j} + 8\hat{k}$ 2) $3\hat{i} + \hat{j} + \hat{k}$ 3) $\hat{j}$ 4) $5\hat{i} - 3\hat{j} - 4\hat{k}$

5) $3\hat{i} - 3\hat{k} + 4\hat{j}$ 6) $\hat{i} - \hat{k}$

Let $\vec{x} = 3\hat{i} - 4\hat{j} + 8\hat{k}$

$|\vec{x}| = \sqrt{9+16+64} = \sqrt{89}$

d.r's : (3, -4, 8)

d.c's $\left(\frac{3}{\sqrt{89}}, \frac{-4}{\sqrt{89}}, \frac{8}{\sqrt{89}}\right)$

2) $\vec{x} = 3\hat{i} + \hat{j} + \hat{k}$

$|\vec{x}| = \sqrt{9+1+1} = \sqrt{11}$

d.r's (3, 1, 1)

d.c's $\left(\frac{3}{\sqrt{11}}, \frac{1}{\sqrt{11}}, \frac{1}{\sqrt{11}}\right)$

3) $\vec{x} = \hat{j}$

$|\vec{x}| = \sqrt{1} = 1$

d.r's (0, 1, 0)

d.c's (0, 1, 0)

4) $\vec{x} = 5\hat{i} - 3\hat{j} + 48\hat{k}$ $ \vec{x} = \sqrt{2338}$	d.r's $(5, -3, -48)$ d.c's $\left(\frac{5}{\sqrt{2338}}, \frac{-3}{\sqrt{2338}}, \frac{-48}{\sqrt{2338}}\right)$
5) $\vec{x} = 3\hat{i} + 4\hat{j} - 3\hat{k}$ $ \vec{x} = \sqrt{9+16+9} = \sqrt{34}$	d.r's $(3, 4, -3)$ d.c's $\left(\frac{3}{\sqrt{34}}, \frac{4}{\sqrt{34}}, \frac{-3}{\sqrt{34}}\right)$
6) $\vec{x} = \hat{i} - \hat{k}$ $ \vec{x} = \sqrt{1+1} = \sqrt{2}$	d.r's $(1, 0, -1)$ d.c's $\left(\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}\right)$

5) $\frac{1}{2}, \frac{1}{2\sqrt{2}}, a$ are the direction cosines of some vector find a .

$$\cos \alpha = \frac{1}{2}, \cos \beta = \frac{1}{2\sqrt{2}}, \cos \gamma = a$$

$$\text{w.k.T } \cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

$$\frac{1}{4} + \frac{1}{2} + a^2 = 1$$

$$a^2 = \frac{1}{4}$$

$$a = \pm \frac{1}{2}$$

4) A triangle is formed by joining the points $(1, 0, 0)$ $(0, 1, 0)$ $(0, 0, 1)$. Find the d.c's of the medians.

$$A(1, 0, 0) B(0, 1, 0) C(0, 0, 1)$$

Let D, E, F are the mid points of BC, CA, AB resp.

AD, BE, CF are the medians.

$$\text{Mid point of BC is } D = (0, \frac{1}{2}, \frac{1}{2})$$

$$A = (1, 0, 0)$$

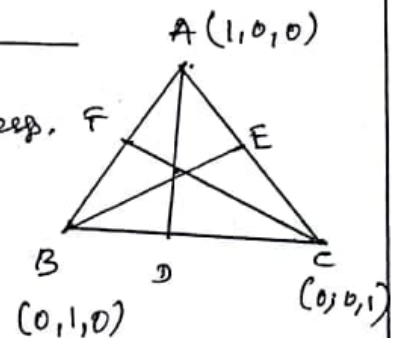
$$\text{D.r's of AD} = (-1, \frac{1}{2}, \frac{1}{2})$$

$$\sqrt{1 + \frac{1}{4} + \frac{1}{4}} = \sqrt{\frac{6}{4}}$$

$$\text{D.c's of AD} = \left(-\frac{1 \cdot \sqrt{2}}{\sqrt{3}}, \frac{\frac{1}{2} \cdot \sqrt{2}}{\sqrt{3}}, \frac{\frac{1}{2} \cdot \sqrt{2}}{\sqrt{3}}\right)$$

$$= \left(-\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$$

∴ the d.c's of other medians are $\left(\frac{1}{\sqrt{6}}, -\frac{\sqrt{2}}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right), \left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}}\right)$



4/8/2)

EXERCISE - 8.3.

EX. 8.11) Find $\vec{a} \cdot \vec{b}$ when 1) $\vec{a} = \hat{i} - \hat{j} + 5\hat{k}$ $\vec{b} = 3\hat{i} - 2\hat{k}$
 2) $\vec{a}, \vec{b}$ represents the points $(2, 3, -1)$
 $(-1, 2, 3)$

$$1) \vec{a} = \hat{i} - \hat{j} + 5\hat{k} \quad \vec{b} = 3\hat{i} - 2\hat{k} \quad \vec{a} \cdot \vec{b} = 3 + 0 - 10 = -7$$

$$2) \vec{a} = 2\hat{i} + 3\hat{j} - \hat{k} \quad \vec{b} = -\hat{i} + 2\hat{j} + 3\hat{k} \quad \vec{a} \cdot \vec{b} = -2 + 6 - 3 = 1$$

EX 8.12) Find $(\vec{a} + 3\vec{b}) \cdot (2\vec{a} - \vec{b})$ if $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} + 2\hat{j} - \hat{k}$

$$\vec{a} = \hat{i} + \hat{j} + 2\hat{k} \quad (\vec{a} + 3\vec{b}) = 10\hat{i} + 7\hat{j} - \hat{k}$$

$$3\vec{b} = 9\hat{i} + 6\hat{j} - 3\hat{k}$$

$$2\vec{a} = 2\hat{i} + 2\hat{j} + 4\hat{k} \quad (2\vec{a} - \vec{b}) = -\hat{i} + 5\hat{k}$$

$$\vec{b} = 3\hat{i} + 2\hat{j} - \hat{k}$$

$$(\vec{a} + 3\vec{b}) \cdot (2\vec{a} - \vec{b}) = -10 + 0 + 5 = -5$$

EX 8.13) If $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$
 s.t. $\vec{a} + \lambda\vec{b}$ is $\perp$ to $\vec{c}$ then find λ .

$$\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$$

$$\lambda\vec{b} = -\lambda\hat{i} + 2\lambda\hat{j} + \lambda\hat{k}$$

$$\vec{a} + \lambda\vec{b} = (2-\lambda)\hat{i} + (2+2\lambda)\hat{j} + (3+\lambda)\hat{k}$$

$$\vec{c} = 3\hat{i} + \hat{j}$$

$$\vec{a} + \lambda\vec{b} \text{ and } \vec{c} \text{ are } \perp \therefore (\vec{a} + \lambda\vec{b}) \cdot \vec{c} = 0$$

$$(\vec{a} + \lambda\vec{b}) \cdot \vec{c} = (2-\lambda)3 + (2+2\lambda)1 = 0$$

$$6 - 3\lambda + 2 + 2\lambda = 0$$

$$8 = \lambda$$

EX 8.14) If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ p.t. $\vec{a}, \vec{b}$ are $\perp$.

$$|\vec{a} + \vec{b}|^2 = |\vec{a} - \vec{b}|^2$$

$$|\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b} = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a} \cdot \vec{b}$$

$$2\vec{a} \cdot \vec{b} + 2\vec{a} \cdot \vec{b} = 0$$

$$4\vec{a} \cdot \vec{b} = 0$$

$$\vec{a} \cdot \vec{b} = 0$$

$$\Rightarrow \vec{a} \perp \vec{b}$$

EX 8.15) For any vector $\vec{r}$ P.T $\vec{r} = (\vec{r} \cdot \hat{i})\hat{i} + (\vec{r} \cdot \hat{j})\hat{j} + (\vec{r} \cdot \hat{k})\hat{k}$

$$\text{Let } \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{r} \cdot \hat{i} = (x\hat{i} + y\hat{j} + z\hat{k}) \cdot \hat{i}$$

$$= x$$

$$(\vec{r} \cdot \hat{i})\hat{i} = x\hat{i}$$

$$\text{IIIy } (\vec{r} \cdot \hat{j})\hat{j} = y\hat{j}$$

$$(\vec{r} \cdot \hat{k})\hat{k} = z\hat{k}$$

$$\therefore (\vec{r} \cdot \hat{i})\hat{i} + (\vec{r} \cdot \hat{j})\hat{j} + (\vec{r} \cdot \hat{k})\hat{k} \\ = x\hat{i} + y\hat{j} + z\hat{k} \\ = \vec{r}$$

EX 8.16) Find the angle between the vectors $5\hat{i} + 3\hat{j} + 4\hat{k}$ and $6\hat{i} - 8\hat{j} - \hat{k}$.

$$\vec{a} = 5\hat{i} + 3\hat{j} + 4\hat{k} \Rightarrow |\vec{a}| = \sqrt{25 + 9 + 16} = \sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2}$$

$$\vec{b} = 6\hat{i} - 8\hat{j} - \hat{k} \Rightarrow |\vec{b}| = \sqrt{36 + 64 + 1} = \sqrt{101}$$

$$\vec{a} \cdot \vec{b} = 30 - 24 - 4 = 2$$

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} = \frac{2}{5\sqrt{2}\sqrt{101}}$$

$$\theta = \cos^{-1}\left(\frac{\sqrt{2}}{5\sqrt{101}}\right)$$

EX 8.17) Find the projection of $\vec{AB}$ on $\vec{CD}$ where A, B, C, D are the points $(4, -3, 0)$ $(7, -5, -1)$ $(-2, 1, 3)$ $(0, 2, 5)$

$$\vec{AB} = \vec{OB} - \vec{OA} = 3\hat{i} - 2\hat{j} - \hat{k}$$

$$\vec{CD} = \vec{OD} - \vec{OC} = 2\hat{i} + \hat{j} + 2\hat{k}$$

$$\vec{AB} \cdot \vec{CD} = 6 - 2 - 2 = 2$$

$$|\vec{CD}| = \sqrt{4 + 1 + 4} = 3$$

$$\text{Projection of } \vec{AB} \text{ on } \vec{CD} = \frac{\vec{AB} \cdot \vec{CD}}{|\vec{CD}|} = \frac{2}{3}$$

EX 8.18) If $\vec{a}$, $\vec{b}$, $\vec{c}$ are the unit vectors satisfying $\vec{a} - \sqrt{3}\vec{b} + \vec{c} = 0$ then find the angle between $\vec{a}$ and $\vec{c}$.

$$\text{Given } \vec{a} - \sqrt{3}\vec{b} + \vec{c} = 0$$

$$\vec{a} + \vec{c} = \sqrt{3}\vec{b}$$

$$|\vec{a} + \vec{c}| = \sqrt{3}|\vec{b}|$$

$$|\vec{a} + \vec{c}|^2 = 3|\vec{b}|^2$$

$$|\vec{a}|^2 + |\vec{c}|^2 + 2\vec{a} \cdot \vec{c} = 3|\vec{b}|^2$$

$$1 + 1 + 2|\vec{a}||\vec{c}|\cos \theta = 3$$

$$2\cos \theta = 3 - 2 = 1$$

$$\cos \theta = \frac{1}{2}$$

$$\theta = \pi/3$$

EX 8.19) S.T the points $(4, -3, 1)$ $(2, -4, 5)$ and $(1, -1, 0)$ form a rt angled Δ .

$$\vec{OA} = 4\hat{i} - 3\hat{j} + \hat{k}$$

$$\vec{OB} = 2\hat{i} - 4\hat{j} + 5\hat{k}$$

$$\vec{OC} = \hat{i} - \hat{j}$$

$$\vec{AB} = \vec{OB} - \vec{OA} = -2\hat{i} - \hat{j} + 4\hat{k}$$

$$\vec{BC} = \vec{OC} - \vec{OB} = -\hat{i} + 3\hat{j} - 5\hat{k}$$

$$\vec{CA} = \vec{OA} - \vec{OC} = 3\hat{i} - 2\hat{j} + \hat{k}$$

Now $\vec{AB} + \vec{BC} + \vec{CA} = 0 \therefore$ the three points forms a Δ .

$$\vec{AB} \cdot \vec{CA} = -6 + 2 + 4 = 0$$

$$\angle A = \pi/2.$$

$\therefore$ it is a rt angled Δ .

1/8.3) 1) Find $\vec{a} \cdot \vec{b}$ when 1) $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$ $\vec{b} = \hat{i} - 4\hat{j} - 2\hat{k}$
2) $\vec{a} = 2\hat{i} + 2\hat{j} - \hat{k}$ $\vec{b} = 6\hat{i} - 3\hat{j} + 2\hat{k}$

$$1) \vec{a} = \hat{i} - 2\hat{j} + \hat{k}$$

$$\vec{b} = 3\hat{i} - 4\hat{j} - 2\hat{k}$$

$$\vec{a} \cdot \vec{b} = 3 + 8 - 2 = 9$$

$$2) \vec{a} = 2\hat{i} + 2\hat{j} - \hat{k}$$

$$\vec{b} = 6\hat{i} - 3\hat{j} + 2\hat{k}$$

$$\vec{a} \cdot \vec{b} = 12 - 6 - 2 = 4.$$

2/8.3) Find the value of λ for which the vectors $\vec{a}$ and $\vec{b}$ are $\perp$.
1) $\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$ $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$, 2) $\vec{a} = 2\hat{i} + 4\hat{j} - \hat{k}$
 $\vec{b} = 3\hat{i} - 2\hat{j} + \lambda\hat{k}$.

If $\vec{a}, \vec{b}$ are $\perp$ $\vec{a} \cdot \vec{b} = 0$

$$1) \vec{a} \cdot \vec{b} = 2 - 2\lambda + 3 = 0$$

$$5 = 2\lambda \Rightarrow \lambda = 5/2$$

$$2) \vec{a} \cdot \vec{b} = 6 - 8 - \lambda = 0$$

$$-\lambda = 2$$

$$\lambda = -2$$

3/8.3) If $\vec{a}$ and $\vec{b}$ are two vectors s.t $|\vec{a}| = 10$, $|\vec{b}| = 15$
 $\vec{a} \cdot \vec{b} = 75\sqrt{2}$ find the angle between $\vec{a}$ and $\vec{b}$.

$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$$

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} = \frac{75\sqrt{2}}{10 \times 15} = \frac{1}{\sqrt{2}}$$

$$\cos \theta = \frac{1}{\sqrt{2}} \quad \theta = \pi/4.$$

4/8.3) Find the angle between the vectors

$$1) 2\hat{i} + 3\hat{j} - 6\hat{k}, 6\hat{i} - 3\hat{j} + 2\hat{k}$$

$$2) \hat{i} - \hat{j}, \hat{j} - \hat{k}$$

1) $\vec{a} = 2\hat{i} + 3\hat{j} - 6\hat{k}$ $|\vec{a}| = \sqrt{4+9+36} = 7$,
 $\vec{b} = 6\hat{i} - 3\hat{j} + 2\hat{k}$ $|\vec{b}| = \sqrt{36+9+4} = 7$.
 $\vec{a} \cdot \vec{b} = 12 - 9 - 12 = -9$ $\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} = \frac{-9}{7 \cdot 7}$

2) $\vec{a} = \hat{i} - \hat{j} \Rightarrow |\vec{a}| = \sqrt{1+1} = \sqrt{2}$ $\theta = \cos^{-1} \left(-\frac{9}{49} \right)$
 $\vec{b} = \hat{j} - \hat{k} \Rightarrow |\vec{b}| = \sqrt{1+1} = \sqrt{2}$

$\vec{a} \cdot \vec{b} = -1$
 $\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} = \frac{-1}{\sqrt{2} \cdot \sqrt{2}} = -\frac{1}{2}$ θ lies in II quadrant
 $\theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

5/8.3 If $\vec{a}, \vec{b}, \vec{c}$ are the three vectors s.t. $\vec{a} + 2\vec{b} + \vec{c} = 0$ and $|\vec{a}| = 3$ $|\vec{b}| = 4$ $|\vec{c}| = 7$ find the angle between $\vec{a}$ and $\vec{b}$.

$\vec{a} + 2\vec{b} + \vec{c} = 0$
 $|\vec{a} + 2\vec{b}| = |-\vec{c}|$
 $|\vec{a} + 2\vec{b}|^2 = |\vec{c}|^2$
 $|\vec{a}|^2 + 4|\vec{b}|^2 + 4\vec{a} \cdot \vec{b} = |\vec{c}|^2$
 $9 + 64 + 4\vec{a} \cdot \vec{b} = 49$
 $4\vec{a} \cdot \vec{b} = -24$
 $\vec{a} \cdot \vec{b} = -6$
 $|\vec{a}| |\vec{b}| \cos \theta = -6$
 $3 \times 4 \times \cos \theta = -6$

$\Rightarrow \cos \theta = -\frac{6}{12} = -\frac{1}{2}$

θ lies in II quadrant

$\theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

6/8.3 S.T the vectors $\vec{a} = 2\hat{i} + 3\hat{j} + 6\hat{k}$, $\vec{b} = 6\hat{i} + 2\hat{j} - 3\hat{k}$, $\vec{c} = 3\hat{i} - 6\hat{j} + 2\hat{k}$ are mutually orthogonal

$\vec{a} = 2\hat{i} + 3\hat{j} + 6\hat{k}$ $\vec{a} \cdot \vec{b} = 12 + 6 - 18 = 0$
 $\vec{b} = 6\hat{i} + 2\hat{j} - 3\hat{k}$ $\vec{b} \cdot \vec{c} = 18 - 12 - 6 = 0$
 $\vec{c} = 3\hat{i} - 6\hat{j} + 2\hat{k}$ $\vec{c} \cdot \vec{a} = 6 - 18 + 12 = 0$

$\therefore \vec{a}, \vec{b}, \vec{c}$ are mutually orthogonal.

7/8.3 S.T the vectors $-\hat{i} - 2\hat{j} - 6\hat{k}$, $2\hat{i} - \hat{j} + \hat{k}$, $-\hat{i} + 3\hat{j} + 5\hat{k}$ forms a right angled triangle.

$$\vec{a} = -\hat{i} - 2\hat{j} - 6\hat{k}$$

$$\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$$

$$\vec{c} = -\hat{i} + 3\hat{j} + 5\hat{k} \quad \text{form}$$

$$\vec{a} + \vec{c} = \vec{b} \quad \therefore \text{Form it is a line.}$$

$$\vec{a} \cdot \vec{b} = -2 + 2 - 6 \neq 0$$

$$\vec{b} \cdot \vec{c} = -2 - 3 + 5 = 0 \quad \therefore \vec{b} \cdot \vec{c} = 0$$

$\therefore$ it is a right angled line.

8.3) If $|\vec{a}| = 5$, $|\vec{b}| = 6$, $|\vec{c}| = 7$ and $\vec{a} + \vec{b} + \vec{c} = 0$ find $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$.

$$|\vec{a} + \vec{b} + \vec{c}| = 0$$

$$|\vec{a} + \vec{b} + \vec{c}|^2 = 0$$

$$|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$

$$25 + 36 + 49 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0,$$

$$2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = -110$$

$$\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = -55.$$

9.3) S.T the points $(2, -1, 3)$ $(4, 3, 1)$ $(3, 1, 2)$ are collinear.

$$\vec{OA} = 2\hat{i} - \hat{j} + 3\hat{k}$$

$$\vec{OB} = 4\hat{i} + 3\hat{j} + \hat{k}$$

$$\vec{OC} = 3\hat{i} + \hat{j} + 2\hat{k}$$

$$\vec{AB} = \vec{OB} - \vec{OA} = 2\hat{i} + 4\hat{j} - 2\hat{k}$$

$$\vec{BC} = \vec{OC} - \vec{OB} = -\hat{i} - 2\hat{j} + \hat{k}$$

$$\vec{AB} = -2(-\hat{i} - 2\hat{j} + \hat{k})$$

$$= -2\vec{BC}$$

$\therefore A, B, C$ are collinear.

10.3) If $\vec{a}, \vec{b}$ are the unit vectors and θ is the angle between $\vec{a}$ and $\vec{b}$
S.T 1) $\sin \theta/2 = \frac{1}{2} |\vec{a} - \vec{b}|$ 2) $\cos \theta/2 = \frac{1}{2} |\vec{a} + \vec{b}|$ 3) $\tan \theta/2 = \frac{|\vec{a} - \vec{b}|}{|\vec{a} + \vec{b}|}$

$$|\vec{a} - \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a} \cdot \vec{b}$$

$$= 1 + 1 - 2|\vec{a}||\vec{b}|\cos \theta$$

$$= 2(1 - \cos \theta)$$

$$= 2 \cdot 2 \sin^2 \theta/2$$

$$|\vec{a} - \vec{b}| = 2 \sin \theta/2 \Rightarrow \frac{1}{2} |\vec{a} - \vec{b}| = \sin \theta/2 \quad \text{--- ①}$$

$$\begin{aligned}
 |\vec{a} + \vec{b}|^2 &= |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b} \\
 &= 1 + 1 + 2|\vec{a}| \cdot |\vec{b}| \cos \theta \\
 &= 2 + 2 \cos \theta \\
 &= 2(1 + \cos \theta) \\
 &= 2 \cdot 2 \cos^2 \frac{\theta}{2} \\
 |\vec{a} + \vec{b}| &= 2 \cos \frac{\theta}{2} \Rightarrow \frac{1}{2} |\vec{a} + \vec{b}| = \cos \frac{\theta}{2} \quad \text{--- (2)}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{1} \quad \frac{\sin \frac{\theta}{2}}{\cos \frac{\theta}{2}} &= \frac{\frac{1}{2} |\vec{a} - \vec{b}|}{\frac{1}{2} |\vec{a} + \vec{b}|} = \tan \frac{\theta}{2}
 \end{aligned}$$

12.3) Find the projection of the vector $\vec{i} + 3\vec{j} + 7\vec{k}$ on the vector $2\vec{i} + 6\vec{j} + 3\vec{k}$.

$$\begin{aligned}
 \text{Let } \vec{a} &= \vec{i} + 3\vec{j} + 7\vec{k} \\
 \vec{b} &= 2\vec{i} + 6\vec{j} + 3\vec{k}
 \end{aligned}$$

$$\vec{a} \cdot \vec{b} = 2 + 18 + 21 = 41$$

$$|\vec{b}| = \sqrt{4 + 36 + 9} = 7$$

$$\text{Projection of } \vec{a} \text{ on } \vec{b} \text{ is } \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = \frac{41}{7}$$

13.3) Find λ when the projection of $\vec{a} = \lambda\vec{i} + \vec{j} + 4\vec{k}$ on $\vec{b} = 2\vec{i} + 6\vec{j} + 3\vec{k}$ is 4 units.

$$\vec{a} = \lambda\vec{i} + \vec{j} + 4\vec{k}$$

$$\vec{b} = 2\vec{i} + 6\vec{j} + 3\vec{k}$$

$$\begin{aligned}
 \vec{a} \cdot \vec{b} &= 2\lambda + 6 + 12 \\
 &= 2\lambda + 18
 \end{aligned}$$

$$|\vec{b}| = \sqrt{4 + 36 + 9} = 7$$

$$\text{Projection of } \vec{a} \text{ on } \vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = 4 \Rightarrow \frac{2\lambda + 18}{7} = 4$$

$$2\lambda + 18 = 28$$

$$2\lambda = 10$$

$$\lambda = 5$$

14.3) Three vectors $\vec{a}, \vec{b}, \vec{c}$ are s.t. $|\vec{a}| = 2$, $|\vec{b}| = 3$, $|\vec{c}| = 4$ and $\vec{a} + \vec{b} + \vec{c} = 0$. Then find $4\vec{a} \cdot \vec{b} + 3\vec{b} \cdot \vec{c} + 3\vec{c} \cdot \vec{a}$.

$$\vec{a} + \vec{b} + \vec{c} = 0$$

$$|\vec{a} + \vec{b}| = |-\vec{c}|$$

$$|\vec{a} + \vec{b}|^2 = |\vec{c}|^2$$

$$|\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b} = |\vec{c}|^2$$

$$4 + 9 + 2\vec{a} \cdot \vec{b} = 16$$

$$\vec{a} \cdot \vec{b} = \frac{3}{2}$$

$$|\vec{a} + \vec{c}|^2 = |\vec{b}|^2$$

$$|\vec{a}|^2 + |\vec{c}|^2 + 2\vec{a} \cdot \vec{c} = |\vec{b}|^2$$

$$4 + 16 + 2\vec{a} \cdot \vec{c} = 9$$

$$\vec{a} \cdot \vec{c} = -\frac{11}{2}$$

$$|\vec{b} + \vec{c}|^2 = |\vec{a}|^2$$

$$|\vec{b}|^2 + |\vec{c}|^2 + 2\vec{b} \cdot \vec{c} = |\vec{a}|^2$$

$$9 + 16 + 2\vec{b} \cdot \vec{c} = 4$$

$$\vec{b} \cdot \vec{c} = -\frac{21}{2}$$

$$\therefore 4\vec{a} \cdot \vec{b} + 3\vec{b} \cdot \vec{c} + 3\vec{c} \cdot \vec{a} = 4 \cdot \frac{3}{2} - \frac{3 \cdot 21}{2} - \frac{3 \cdot 11}{2} = \frac{12 - 63 - 33}{2}$$

$$= -42$$

EXERCISE - 8.4.

EX 8.20) Find $|\vec{a} \times \vec{b}|$ if $\vec{a} = 3\hat{i} + 4\hat{j}$ and $\vec{b} = \hat{i} + \hat{j} + \hat{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 0 \\ 1 & 1 & 1 \end{vmatrix} = \hat{i}(4-0) - \hat{j}(3-0) + \hat{k}(3-4)$$

$$= 4\hat{i} - 3\hat{j} - \hat{k}$$

$$|\vec{a} \times \vec{b}| = \sqrt{16+9+1} = \sqrt{26}$$

EX 8.21) If $\vec{a} = -3\hat{i} + 4\hat{j} - 7\hat{k}$ and $\vec{b} = 6\hat{i} + 2\hat{j} - 3\hat{k}$

Verify 1) $\vec{a}$ and $\vec{a} \times \vec{b}$ are $\perp$
2) $\vec{b}$ and $\vec{a} \times \vec{b}$ are $\perp$.

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 4 & -7 \\ 6 & 2 & -3 \end{vmatrix} = \hat{i}(-12+14) - \hat{j}(9+42) + \hat{k}(-6-24)$$

$$= 2\hat{i} - 51\hat{j} - 30\hat{k}$$

$$\vec{a} \cdot (\vec{a} \times \vec{b}) = (-3\hat{i} + 4\hat{j} - 7\hat{k}) \cdot (2\hat{i} - 51\hat{j} - 30\hat{k})$$

$$= -6 + 204 + 210 = 0$$

$\therefore \vec{a}$ and $\vec{a} \times \vec{b}$ are $\perp$.

$$\vec{b} \cdot (\vec{a} \times \vec{b}) = (6\hat{i} + 2\hat{j} - 3\hat{k}) \cdot (2\hat{i} - 51\hat{j} - 30\hat{k})$$

$$= 12 - 102 + 90 = 0$$

$\therefore \vec{b}$, $\vec{a} \times \vec{b}$ are $\perp$.

EX 8.22) Find the vectors of magnitude 6 which are $\perp$ to both vectors $\vec{a} = 4\hat{i} - \hat{j} + 3\hat{k}$, $\vec{b} = -2\hat{i} + \hat{j} - 2\hat{k}$.

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 3 \\ -2 & 1 & -2 \end{vmatrix} = \hat{i}(2-3) - \hat{j}(-8+6) + \hat{k}(4-2)$$

$$= -\hat{i} + 2\hat{j} + 2\hat{k}$$

$$|\vec{a} \times \vec{b}| = \sqrt{1+4+4} = 3$$

$$\hat{n} = \pm \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|} = \pm \frac{(-\hat{i} + 2\hat{j} + 2\hat{k})}{3}$$

$$b\hat{n} = \pm \frac{2}{3}(-\hat{i} + 2\hat{j} + 2\hat{k})$$

EX 8.23) Find the cosine and sine angle between the vectors

$$\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k} \quad \vec{b} = 4\hat{i} - 2\hat{j} + 2\hat{k}$$

$$\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k} \Rightarrow |\vec{a}| = \sqrt{4+1+9} = \sqrt{14}$$

$$\vec{b} = 4\hat{i} - 2\hat{j} + 2\hat{k} \Rightarrow |\vec{b}| = \sqrt{16+4+4} = \sqrt{24}$$

$$\vec{a} \cdot \vec{b} = 8 - 2 + 6 = 12.$$

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} = \frac{12}{\sqrt{14} \sqrt{24}}$$

$$\begin{aligned} \vec{a} \times \vec{b} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 3 \\ 4 & -2 & 2 \end{vmatrix} = \hat{i}(2+6) - \hat{j}(4-12) + \hat{k}(-4-4) \\ &= 8\hat{i} + 8\hat{j} - 8\hat{k} \\ &= 8(\hat{i} + \hat{j} - \hat{k}) \end{aligned}$$

$$|\vec{a} \times \vec{b}| = 8\sqrt{1+1+1} = 8\sqrt{3}.$$

$$\begin{aligned} \sin \theta &= \frac{|\vec{a} \times \vec{b}|}{|\vec{a}| |\vec{b}|} = \frac{8\sqrt{3}}{\sqrt{14} \sqrt{24}} = \frac{8 \times \sqrt{3}}{\sqrt{14} \times \sqrt{2} \times \sqrt{12}} = \frac{\sqrt{4} \times \sqrt{3}}{\sqrt{7} \times \sqrt{4} \times \sqrt{3}} \\ &= \frac{2}{\sqrt{7}} \end{aligned}$$

Ex 8.24) Find the area of the parallelogram whose adjacent sides are $\vec{a} = 3\hat{i} + \hat{j} + 4\hat{k}$ $\vec{b} = \hat{i} - \hat{j} + \hat{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 4 \\ 1 & -1 & 1 \end{vmatrix} = \hat{i}(1+4) - \hat{j}(3-4) + \hat{k}(-3-1) \\ = 5\hat{i} + \hat{j} - 4\hat{k}$$

$$|\vec{a} \times \vec{b}| = \sqrt{25+1+16} = \sqrt{42}.$$

Area of the Parallelogram $|\vec{a} \times \vec{b}| = \sqrt{42}$ units.

Ex 8.25) For any two vectors $\vec{a}$ and $\vec{b}$ P.T $|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = |\vec{a}|^2 |\vec{b}|^2$

$$\begin{aligned} |\vec{a} \times \vec{b}| &= |\vec{a}| |\vec{b}| \sin \theta & \vec{a} \cdot \vec{b} &= |\vec{a}| |\vec{b}| \cos \theta \\ |\vec{a} \times \vec{b}|^2 &= |\vec{a}|^2 |\vec{b}|^2 \sin^2 \theta & (\vec{a} \cdot \vec{b})^2 &= |\vec{a}|^2 |\vec{b}|^2 \cos^2 \theta \quad \text{--- ②} \end{aligned}$$

$$\begin{aligned} \text{①} + \text{②} \quad |\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2 &= |\vec{a}|^2 |\vec{b}|^2 (\sin^2 \theta + \cos^2 \theta) \\ &= |\vec{a}|^2 |\vec{b}|^2 \end{aligned}$$

Ex 8.26) Find the area of the Δ having the points A (1, 0, 0) B (0, 1, 0) C (0, 0, 1)

$$\begin{aligned} \vec{OA} &= \hat{i} & \vec{AB} &= \vec{OB} - \vec{OA} = -\hat{i} + \hat{j} \\ \vec{OB} &= \hat{j} & \vec{AC} &= \vec{OC} - \vec{OA} = -\hat{i} + \hat{k} \\ \vec{OC} &= \hat{k} \end{aligned}$$

$$\text{Area of the } \Delta = \frac{1}{2} |\vec{AB} \times \vec{AC}|$$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} = \hat{i}(1-0) - \hat{j}(-1-0) + \hat{k}(0+1) \\ = \hat{i} + \hat{j} + \hat{k}$$

$$|\vec{AB} \times \vec{AC}| = \sqrt{1+1+1} = \sqrt{3}$$

$$\text{Area of the } \Delta ABC = \frac{1}{2} |\vec{AB} \times \vec{AC}| = \frac{1}{2} \cdot \sqrt{3} = \frac{\sqrt{3}}{2}$$

1/8.4) Find the magnitude of $\vec{a} \times \vec{b}$ if $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$ $\vec{b} = 3\hat{i} + 5\hat{j} - 2\hat{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 3 \\ 3 & 5 & -2 \end{vmatrix} = \hat{i}(-2-15) - \hat{j}(-4-9) + \hat{k}(10-3) \\ = -17\hat{i} + 13\hat{j} + 7\hat{k}$$

$$|\vec{a} \times \vec{b}| = \sqrt{289 + 169 + 49} = \sqrt{507}$$

2/8.4) p.T $\vec{a} \times (\vec{b} + \vec{c}) + \vec{b} \times (\vec{c} + \vec{a}) + \vec{c} \times (\vec{a} + \vec{b}) = \vec{0}$

$$\text{LHS: } \vec{a} \times (\vec{b} + \vec{c}) + \vec{b} \times (\vec{c} + \vec{a}) + \vec{c} \times (\vec{a} + \vec{b}) \\ = \vec{a} \times \vec{b} + \vec{a} \times \vec{c} + \vec{b} \times \vec{c} + \vec{b} \times \vec{a} + \vec{c} \times \vec{a} + \vec{c} \times \vec{b} \\ = \vec{a} \times \vec{b} + \vec{a} \times \vec{c} + \vec{b} \times \vec{c} - \vec{a} \times \vec{b} - \vec{a} \times \vec{c} - \vec{b} \times \vec{c} \\ = \vec{0}$$

3/8.4) Find the vectors of magnitude $10\sqrt{3}$ that are $\perp$ to the plane which contains $\hat{i} + 2\hat{j} + \hat{k}$ and $\hat{i} + 3\hat{j} + 4\hat{k}$.

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 1 & 3 & 4 \end{vmatrix} = \hat{i}(8-3) - \hat{j}(4-1) + \hat{k}(3-2) \\ = 5\hat{i} - 3\hat{j} + \hat{k}$$

$$|\vec{a} \times \vec{b}| = \sqrt{25 + 9 + 1} = \sqrt{35}$$

$$\hat{n} = \pm \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|} = \pm \frac{(5\hat{i} - 3\hat{j} + \hat{k})}{\sqrt{35}}$$

$$10\sqrt{3} \hat{n} = \pm 10\sqrt{3} \frac{(5\hat{i} - 3\hat{j} + \hat{k})}{\sqrt{35}}$$

4/8.4) Find the unit vectors $\perp$ to each of the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ where $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$.

$$\text{Let } \vec{c} = \vec{a} + \vec{b} = 2\hat{i} + 3\hat{j} + 4\hat{k} \\ \vec{d} = \vec{a} - \vec{b} = -\hat{j} - 2\hat{k}$$

$$\vec{c} \times \vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 0 & -1 & -2 \end{vmatrix} = \hat{i}(-6+4) - \hat{j}(-4-0) + \hat{k}(-2-0) \\ = -2\hat{i} + 4\hat{j} - 2\hat{k} = -2(\hat{i} + 2\hat{j} + \hat{k})$$

$$|\vec{c} \times \vec{a}| = \sqrt{4+16+4} = \sqrt{24}.$$

$$\hat{n} = \pm \frac{\vec{c} \times \vec{a}}{|\vec{c} \times \vec{a}|} = \pm \frac{(-2\hat{i} + 4\hat{j} - 2\hat{k})}{\sqrt{24}} \\ = \pm \frac{2(\hat{i} - 2\hat{j} + \hat{k})}{2\sqrt{6}}$$

5/8.4) Find the area of the parallelogram whose adjacent sides are determined by the vectors $\hat{i} + 2\hat{j} + 3\hat{k}$, $3\hat{i} - 2\hat{j} + \hat{k}$.

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 3 & -2 & 1 \end{vmatrix} = \hat{i}(2+6) - \hat{j}(1-9) + \hat{k}(-2-6) \\ = 8\hat{i} + 8\hat{j} - 8\hat{k} \\ = 8(\hat{i} + \hat{j} - \hat{k})$$

$$|\vec{a} \times \vec{b}| = 8\sqrt{1+1+1} = 8\sqrt{3}.$$

Area of the parallelogram $|\vec{a} \times \vec{b}| = 8\sqrt{3}$ sq. units.

6/8.4) Find the area of the Δ whose vertices are A (3, -1, 2) B (1, -1, -3) C (4, -3, 1)

$$\vec{AB} = \vec{OB} - \vec{OA} = -2\hat{i} - 5\hat{k}$$

$$\vec{AC} = \vec{OC} - \vec{OA} = \hat{i} - 2\hat{j} - \hat{k}$$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 0 & -5 \\ 1 & -2 & -1 \end{vmatrix} = \hat{i}(0-10) - \hat{j}(2+5) + \hat{k}(4-0) \\ = -10\hat{i} - 7\hat{j} + 4\hat{k}$$

$$|\vec{AB} \times \vec{AC}| = \sqrt{100+49+16} = \sqrt{165}.$$

$$\text{Area of the } \Delta = \frac{1}{2} |\vec{AB} \times \vec{AC}| = \frac{1}{2} \sqrt{165}.$$

7/8.4) If $\vec{a}, \vec{b}, \vec{c}$ are the P.V of A, B, C of a Δ ABC, s.t the area of the $\Delta BC = \frac{1}{2} |\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$ also deduce the condition for collinearity of the three points A, B, C.

$$\begin{aligned} \text{Let } \vec{OA} &= \vec{a} & \vec{AB} &= \vec{OB} - \vec{OA} = \vec{b} - \vec{a} \\ \vec{OB} &= \vec{b} & \vec{AC} &= \vec{OC} - \vec{OA} = \vec{c} - \vec{a} \\ \vec{OC} &= \vec{c} \end{aligned}$$

$$\begin{aligned} \vec{AB} \times \vec{AC} &= (\vec{b} - \vec{a}) \times (\vec{c} - \vec{a}) \\ &= \vec{b} \times \vec{c} - \vec{b} \times \vec{a} - \vec{a} \times \vec{c} + \vec{a} \times \vec{a} \\ &= \vec{b} \times \vec{c} + \vec{a} \times \vec{b} + \vec{c} \times \vec{a} \end{aligned}$$

$$|\vec{AB} \times \vec{AC}| = |\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$$

$$\text{Area of the } \triangle = \frac{1}{2} |\vec{AB} \times \vec{AC}|$$

$$= \frac{1}{2} |\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$$

If the three points are collinear Area of the $\triangle$ is 0.

$$\frac{1}{2} |\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}| = 0$$

$$|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}| = 0.$$

10.4) Find the angle between the vectors $2\hat{i} + \hat{j} - \hat{k}$ and $\hat{i} + 2\hat{j} + \hat{k}$ using vector product.

$$\begin{aligned} \vec{a} &= 2\hat{i} + \hat{j} - \hat{k} \Rightarrow |\vec{a}| = \sqrt{4+1+1} = \sqrt{6} \\ \vec{b} &= \hat{i} + 2\hat{j} + \hat{k} \quad |\vec{b}| = \sqrt{1+4+1} = \sqrt{6} \end{aligned}$$

$$\begin{aligned} \vec{a} \times \vec{b} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 1 & 2 & 1 \end{vmatrix} = \hat{i}(1+2) - \hat{j}(2+1) + \hat{k}(4-1) \\ &= 3\hat{i} - 3\hat{j} + 3\hat{k} \\ &= 3(\hat{i} - \hat{j} + \hat{k}) \end{aligned}$$

$$|\vec{a} \times \vec{b}| = 3\sqrt{1+1+1} = 3\sqrt{3}$$

$$\sin \theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}| |\vec{b}|} = \frac{3\sqrt{3}}{\sqrt{6}\sqrt{6}} = \frac{\sqrt{3}}{2}$$

$$\begin{aligned} \theta &= \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) \\ &= \frac{\pi}{3} \end{aligned}$$

EXERCISE - 8.5 (one mark)

1. The value of $\vec{AB} + \vec{BC} + \vec{CD} + \vec{DA}$ is

- 1) $\vec{AD}$ 2) $\vec{CA}$ 3) $\vec{0}$ 4) $-\vec{AD}$

By polygon law $\vec{AB} + \vec{BC} + \vec{CD} + \vec{DA} = \vec{0}$.

$\therefore$ The starting point and end point are same at the same time the ^{vectors} continuously going on.

2) If $\vec{a} + 2\vec{b}$ and $3\vec{a} + m\vec{b}$ are parallel then the value of m is

- 1) 3 2) $\frac{1}{3}$ 3) 6 4) $\frac{1}{6}$

$$\vec{a} + 2\vec{b} = 3(\vec{a} + \frac{m}{3}\vec{b})$$

$$\Rightarrow \frac{m}{3} = 2 \Rightarrow m = 6.$$

3) The unit vector parallel to the resultant of the vectors $\vec{i} + \vec{j} - \vec{k}$ and $\vec{i} - 2\vec{j} + \vec{k}$ is

- 1) $\frac{\vec{i} - \vec{j} + \vec{k}}{\sqrt{5}}$ 2) $\frac{2\vec{i} + \vec{j}}{\sqrt{5}}$ 3) $\frac{2\vec{i} - \vec{j} + \vec{k}}{\sqrt{5}}$ 4) $\frac{2\vec{i} - \vec{j}}{\sqrt{5}}$

Let Resultant vector $\vec{a} = 2\vec{i} - \vec{j}$

$$|\vec{a}| = \sqrt{4+1} = \sqrt{5}$$

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{2\vec{i} - \vec{j}}{\sqrt{5}}$$

4) A vector $\vec{OP}$ makes 60° and 45° with the positive direction of the x axis and y axis. Then the angle between $\vec{OP}$ and the z axis is

- 1) 45° 2) 60° 3) 90° 4) 30°

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

$$(\cos 60^\circ)^2 + (\cos 45^\circ)^2 + \cos^2 \gamma = 1$$

$$\frac{1}{4} + \frac{1}{2} + \cos^2 \gamma = 1$$

$$\cos^2 \gamma = \frac{1}{4}$$

$$\cos \gamma = \frac{1}{2}$$

$$\gamma = 60^\circ$$

5) If $\vec{BA} = 3\vec{i} + 2\vec{j} + \vec{k}$ and the P.V of B is $\vec{i} + 3\vec{j} - \vec{k}$ then the P.V of A is

- 1) $4\vec{i} + 2\vec{j} + \vec{k}$ 2) $4\vec{i} + 5\vec{j}$ 3) $4\vec{i}$ 4) $-4\vec{i}$

$$\vec{BA} = \vec{OA} - \vec{OB} = 3\vec{i} + 2\vec{j} + \vec{k}$$

$$\begin{aligned}\vec{OA} &= (3\vec{i} + 2\vec{j} + \vec{k}) + \vec{OB} \\ &= (3\vec{i} + 2\vec{j} + \vec{k}) + (\vec{i} + 3\vec{j} - \vec{k}) \\ &= 4\vec{i} + 5\vec{j}\end{aligned}$$

- 6) A vector makes equal angle with the positive direction of the co-ordinate axes. Then each angle is equal to
1) $\cos^{-1}(\frac{1}{3})$ 2) $\cos^{-1}(\frac{2}{3})$ 3) $\cos^{-1}(\frac{1}{\sqrt{3}})$ 4) $\cos^{-1}(\frac{2}{\sqrt{3}})$

Given $\alpha = \beta = \gamma$.

$$\cos^2 \alpha + \cos^2 \alpha + \cos^2 \alpha = 1$$

$$3\cos^2 \alpha = 1$$

$$\cos^2 \alpha = \frac{1}{3} \Rightarrow \cos \alpha = \frac{1}{\sqrt{3}}$$

$$\alpha = \cos^{-1}(\frac{1}{\sqrt{3}})$$

7) The vectors $\vec{a} - \vec{b}$, $\vec{b} - \vec{c}$, $\vec{c} - \vec{a}$ are

- 1) Parallel to each other 2) unit vector.
3) mutually $\perp$ vectors 4) coplanar vectors.

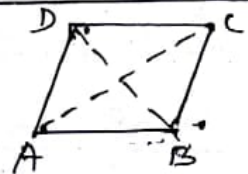
$$\begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{vmatrix} = 1(1) - 1(1) = 0 \therefore \text{They are coplanar.}$$

8) If ABCD is a parallelogram then $\vec{AB} + \vec{AD} + \vec{CB} + \vec{CD}$ is equal to

- 1) $2(\vec{AB} + \vec{AD})$ 2) $4\vec{AC}$ 3) $4\vec{BD}$ 4) $\vec{0}$

$$\begin{aligned}\vec{AB} + \vec{AD} &= \vec{AB} + \vec{BC} \\ \vec{CB} + \vec{CD} &= -\vec{BC} - \vec{AB}\end{aligned}$$

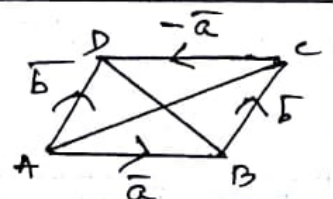
$$\therefore \vec{AB} + \vec{AD} + \vec{CB} + \vec{CD} = \vec{0}.$$



9) One of the diagonals of the parallelogram ABCD with adjacent sides $\vec{a}$ and $\vec{b}$ is $\vec{a} + \vec{b}$ then the other diagonal is

- 1) $\vec{a} - \vec{b}$ 2) $\vec{b} - \vec{a}$ 3) $\vec{a} + \vec{b}$ 4) $\frac{\vec{a} + \vec{b}}{2}$

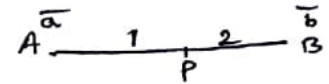
$$\begin{aligned}\vec{BD} &= \vec{BC} + \vec{CD} \\ &= \vec{b} - \vec{a}\end{aligned}$$



10) If $\vec{a}, \vec{b}$ are the P.V. of A and B. Then which of the following points whose P.V. lies on AB.

- 1) $\vec{a} + \vec{b}$ 2) $\frac{2\vec{a} - \vec{b}}{2}$, 3) $\frac{2\vec{a} + \vec{b}}{3}$, 4) $\frac{\vec{a} - \vec{b}}{3}$

$$\vec{OP} = \frac{1 \cdot \vec{b} + 2\vec{a}}{1+2} = \frac{2\vec{a} + \vec{b}}{3}$$



11) If $\vec{a}, \vec{b}, \vec{c}$ are the P.V. of three collinear points then which of the following is True.

- 1) $\vec{a} = \vec{b} + \vec{c}$ 2) $2\vec{a} = \vec{b} + \vec{c}$ 3) $\vec{b} = \vec{c} + \vec{a}$ 4) $4\vec{a} + \vec{b} + \vec{c} = 0$

X $\vec{OA} = \vec{a}, \vec{OB} = \vec{b}, \vec{OC} = \vec{c}$

If the three points are collinear. $\vec{AB} + \vec{BC} = \vec{AC}$

$$\vec{OB} - \vec{OA} + \vec{OC} - \vec{OB} = \vec{OC} - \vec{OA}$$

12) If $\vec{r} = \frac{9\vec{a} + 7\vec{b}}{16}$ then the point P whose P.V. $\vec{r}$ divides the line joining the points with P.V. $\vec{a}, \vec{b}$ in the ratio

- 1) 7:9 internally 2) 9:7 internally
3) 9:7 externally 4) 7:9 externally.

$$\vec{r} = \frac{7\vec{b} + 9\vec{a}}{7+9} = \frac{7\vec{b} + 9\vec{a}}{16}$$

13) If $\lambda\hat{i} + 2\lambda\hat{j} + 2\lambda\hat{k}$ is a unit vector then the value of λ is

- 1) $\frac{1}{3}$ 2) $\frac{1}{4}$ 3) $\frac{1}{9}$ 4) $\frac{1}{2}$

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\lambda\hat{i} + 2\lambda\hat{j} + 2\lambda\hat{k}}{1}$$

$$|\vec{a}| = \sqrt{\lambda^2 + 4\lambda^2 + 4\lambda^2} = 1 \Rightarrow 3\lambda = 1$$

$$\lambda = \frac{1}{3}$$

14) Two vertices of a triangle have P.V. $3\hat{i} + 4\hat{j} - 4\hat{k}$ and $2\hat{i} + 3\hat{j} + 4\hat{k}$. If the P.V. of centroid is $\hat{i} + 2\hat{j} + 3\hat{k}$ then the P.V. of the third vertex is

- 1) $-2\hat{i} - \hat{j} + 9\hat{k}$ 2) $-2\hat{i} - \hat{j} - 6\hat{k}$ 3) $2\hat{i} - \hat{j} + 6\hat{k}$ 4) $-2\hat{i} + \hat{j} + 6\hat{k}$

$$\vec{OA} = 3\hat{i} + 4\hat{j} - 4\hat{k}$$

$$\vec{OB} = 2\hat{i} + 3\hat{j} + 4\hat{k}$$

$$\vec{OA} + \vec{OB} = 5\hat{i} + 7\hat{j}$$

$$\vec{OA} + \vec{OB} + \vec{OC} = 3(\hat{i} + 2\hat{j} + 3\hat{k})$$

$$\vec{OC} = (3\hat{i} + 6\hat{j} + 9\hat{k}) - (5\hat{i} + 7\hat{j})$$

$$= -2\hat{i} - \hat{j} + 9\hat{k}$$

15) If $|\vec{a} + \vec{b}| = 60$, $|\vec{a} - \vec{b}| = 40$ $|\vec{b}| = 46$ then $|\vec{a}|$ is

- 1) 42 2) 12 3) 22 4) 32

$$|\vec{a} + \vec{b}|^2 + |\vec{a} - \vec{b}|^2 = 60^2 + 40^2$$

$$2[|\vec{a}|^2 + |\vec{b}|^2] = 3600 + 1600$$

$$= 5200$$

$$|\vec{a}|^2 + 46^2 = 2600$$

$$|\vec{a}|^2 = 2600 - 2116$$

$$= 484 \Rightarrow |\vec{a}| = 22$$

$$\begin{array}{r} 46 \times 46 \\ \underline{276} \quad 3 \\ 184 \quad 2 \\ \underline{2116} \\ 22 \times 22 \\ \underline{484} \end{array}$$

16) If $\vec{a}$ and $\vec{b}$ having same magnitude and angle between them is 60° and their scalar product is $\frac{1}{2}$ then $|\vec{a}|$ is

- 1) 2 2) 3 3) 7 4) 1

$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta = \frac{1}{2}$$

$$|\vec{a}| |\vec{a}| \cos 60^\circ = \frac{1}{2}$$

$$|\vec{a}|^2 = \frac{\frac{1}{2}}{\frac{1}{2}} \Rightarrow |\vec{a}| = 1$$

17) The value of $\theta \in (0, \pi/2)$ for which the vectors $\vec{a} = (\sin \theta)\hat{i} + (\cos \theta)\hat{j}$ and $\vec{b} = \hat{i} - \sqrt{3}\hat{j} + 2\hat{k}$ are $\perp$ is equal to 1) $\pi/3$ 2) $\pi/6$ 3) $\pi/4$ 4) $\pi/2$

$$\vec{a} \cdot \vec{b} = \sin \theta - \sqrt{3} \cos \theta = 0$$

$$\sin \theta = \sqrt{3} \cos \theta$$

$$\frac{\sin \theta}{\cos \theta} = \sqrt{3} \Rightarrow \tan \theta = \sqrt{3}$$

$$\theta = \pi/3$$

18) If $|\vec{a}| = 13$ $|\vec{b}| = 5$ and $\vec{a} \cdot \vec{b} = 60$ then $|\vec{a} \times \vec{b}|$ is

- 1) 15 2) 35 3) 45 4) 25

$$|\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2 = |\vec{a}|^2 |\vec{b}|^2$$

$$= 169 \times 25$$

$$|\vec{a} \times \vec{b}|^2 = 4225 - 3600$$

$$= 625$$

$$|\vec{a} \times \vec{b}| = 25$$

19) Vectors $\vec{a}$ and $\vec{b}$ inclined at 120° . If $|\vec{a}| = 1$, $|\vec{b}| = 2$ find $(\vec{a} + 3\vec{b}) \times (3\vec{a} - \vec{b})$ is

- 1) 225 2) 275 3) 325 4) 300

$$(\vec{a} + 3\vec{b}) \times (3\vec{a} - \vec{b}) = 3\vec{a} \times \vec{a} - \vec{a} \times \vec{b} + 9\vec{b} \times \vec{a} - 3\vec{b} \times \vec{b}$$

$$= -10 \vec{a} \times \vec{b}$$

$$= -10 \cdot |\vec{a}| |\vec{b}| \sin 120^\circ$$

$$\therefore [(\vec{a} + 3\vec{b}) \times (3\vec{a} - \vec{b})]^2 = 100 \times 3 = +10 \cdot 1 \cdot 2 \cdot \sin 60^\circ$$

$$= 300 \quad = 10 \times 1 \times 2 \times \frac{\sqrt{3}}{2} = 10\sqrt{3}$$

20) If $\vec{a}$ and $\vec{b}$ are two vectors of magnitude 2 and inclined at an angle 60° then angle between $\vec{a}$ and $\vec{a} + \vec{b}$

1) 30° 2) 60° 3) 45° 4) 90° .

$$\vec{a} \cdot (\vec{a} + \vec{b}) = |\vec{a}|^2 + \vec{a} \cdot \vec{b}$$

$$= |\vec{a}|^2 + |\vec{a}| |\vec{b}| \cos \theta$$

21) If the projection of $5\vec{i} - \vec{j} - 3\vec{k}$ on $\vec{i} + 3\vec{j} + \lambda\vec{k}$ is same as the projection of $\vec{i} + 3\vec{j} + \lambda\vec{k}$ on $5\vec{i} - \vec{j} - 3\vec{k}$ then λ is

1) ± 4 2) ± 3 3) ± 5 4) ± 1 .

$$\frac{(5\vec{i} - \vec{j} - 3\vec{k}) \cdot (\vec{i} + 3\vec{j} + \lambda\vec{k})}{\sqrt{1+9+\lambda^2}} = \frac{(\vec{i} + 3\vec{j} + \lambda\vec{k}) \cdot (5\vec{i} - \vec{j} - 3\vec{k})}{\sqrt{25+1+9}}$$

$$\frac{5-3-3\lambda}{\sqrt{10+\lambda^2}} = \frac{5-3-3\lambda}{\sqrt{35}} \Rightarrow \frac{2-3\lambda}{\sqrt{10+\lambda^2}} = \frac{2-3\lambda}{\sqrt{35}}$$

$$\sqrt{35} = \sqrt{10+\lambda^2}$$

Sq. on both sides.

$$35 = 10 + \lambda^2 \Rightarrow \lambda^2 = 25$$

$$\lambda = \pm 5$$

22) If $(1, 2, 4)$ and $(2, -3\lambda, -3)$ are the initial and terminal points of the vectors $\vec{i} + 5\vec{j} - 7\vec{k}$ then the value of λ is equal to

1) $7/3$ 2) $-7/3$ 3) $-5/3$ 4) $5/3$.

$$\vec{OB} - \vec{OA} = \vec{i} + (3\lambda - 2)\vec{j} - 7\vec{k} = \vec{i} + 5\vec{j} - 7\vec{k}$$

$$\Rightarrow -3\lambda - 2 = 5$$

$$-3\lambda = 7 \quad \lambda = -7/3$$

23) If the points whose P.V $10\vec{i} + 3\vec{j}$, $12\vec{i} - 5\vec{j}$, $a\vec{i} + 11\vec{j}$ are collinear then a is equal to 1) 6 2) 3 3) 5 4) 8

$$\vec{OA} = 10\vec{i} + 3\vec{j}$$

$$\vec{OB} = 12\vec{i} - 5\vec{j}$$

$$\vec{OC} = a\vec{i} + 11\vec{j}$$

$$\vec{AB} = \vec{OB} - \vec{OA} = 2\vec{i} - 8\vec{j}$$

$$\vec{AC} = \vec{OC} - \vec{OA} = (a-10)\vec{i} + 8\vec{j}$$

$$= -[10-a]\vec{i} - 8\vec{j}$$

$$\Rightarrow 2\vec{i} = 10 - a$$

$$a = 10 - 2$$

$$= 8.$$

24) If $\vec{a} = \vec{i} + \vec{j} + \vec{k}$, $\vec{b} = 2\vec{i} + x\vec{j} + \vec{k}$, $\vec{c} = \vec{i} - \vec{j} + 4\vec{k}$ and $\vec{a} \cdot (\vec{b} \times \vec{c}) = 70$ then x is equal to

1) 5 2) 7 3) 26 4) 10.

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & x & 1 \\ 1 & -1 & 4 \end{vmatrix} = 70$$

$$1(4x+1) - 1(8-1) + 1(-2-x) = 70$$

$$4x+1-7-2-x=70$$

$$3x = 78$$

$$x = 26.$$

25) $\vec{a} = \vec{i} + 2\vec{j} + 2\vec{k}$ $|\vec{b}| = 5$ and the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$. Then the area of the Δ formed by these two vectors as two sides is. 1) $7/4$ 2) $15/4$ 3) $3/4$

$$\text{Area of the } \Delta = \frac{1}{2} |\vec{a} \times \vec{b}|$$

$$|\vec{a}| = \sqrt{1+4+4} = 3$$

$$= \frac{1}{2} |\vec{a}| |\vec{b}| \sin \theta$$

$$= \frac{1}{2} 3 \times 5 \times \sin 30^\circ$$

$$= \frac{1}{2} \times 3 \times 5 \times \frac{1}{2} = \frac{15}{4}$$