



# Padalsalai's Telegram Groups!

( தலைப்பிற்கு கீழே உள்ள லிங்கை கிளிக் செய்து குழுவில் இணையவும்! )

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**SAIVEERA ACADEMY**  
**REVOLUTION FOR LEARNING , COIMBATORE 8098850809**  
**12<sup>TH</sup> CHAPTER – 7 FULL TEST**

**Marks: 80****Time : 1 hr 30 min****I.Choose the best answers****20 × 1 = 20**

1. The angular displacement of a fly wheel in radius is given by  $\theta = 9t^2 - 2t^3$ . The time when the angular acceleration zero is  
 1) 2.5 s                                      2) 3.5 s                                      3) 1.5 s                                      4) 4.5 s
2. A missile fired from ground level rises x metres vertically upwards in “t “ seconds and  $x = t(100 - 12.5t)$  Then the maximum height reached by the missiles is  
 1) 100m                                      2) 150 m                                      3) 250 m                                      4) 200m
3. The curve  $y = f(x)$  and  $y = g(x)$  cut orthogonally if at the point of intersection  
 1) slope of  $f(x)$  = slope of  $g(x)$                                       2) slope of  $f(x)$  + slope of  $g(x)$  = 0  
 3) slope of  $f(x)$  / slope of  $g(x)$  = -1                                      4) [ slope of  $f(x)$  ] [ slope of  $g(x)$  ] = -1
4.  $\lim_{x \rightarrow 0} \frac{x}{\tan x}$  is  
 1) 1                                      2) - 1                                      3) 0                                      4)  $\infty$
5. f is a differentiable function defined on an interval I with positive derivative. Then f is  
 1) increasing on I                                      2) decreasing on I  
 3) strictly increasing on I                                      4) strictly decreasing on I
6. If the gradient of a curve changes from positive just before P to negative just after then “P “ is a  
 1) minimum point                                      2) maximum point                                      3) inflection point                                      4) discontinuous point
7. The function  $f(x) = x^3$  has  
 1) absolute maximum                                      2) absolute minimum                                      3) local maximum                                      4) no extrema
8. The statement “ If f is continuous on a closed interval [ a, b ] then f attains an absolute maximum value f ( c ) and an absolute minimum value f(d) at some number c and d in [ a , b ] “ is  
 1) The extreme value theorem                                      2) Fermat’s theorem  
 3) Law of Mean                                      4) Rolle’s theorem
9. Identify the correct statement:  
 i) a continuous function has local maximum then it has absolute maximum  
 ii) a continuous function has local minimum then it has absolute minimum  
 iii) a continuous function has absolute maximum then it has local maximum  
 iv) a continuous function has absolute minimum then it has local minimum  
 1)(i) and (ii)                                      2) (i) and (iii)                                      3) (iii) and (iv)                                      4) (i) , (iii) and (iv)
10. Which of the following statements are correct?  
 i) Rolle’s theorem is a particular case of Lagranges law of mean  
 ii) Lagranges law of mean is a particular case of generalized law of mean ( Cauchy)  
 iii) Lagranges law of mean is a particular case of Rolle’s theorem.  
 iv) Generalized law of mean is a particular case of Lagranges law of mean.  
 1)(ii) , (iii)                                      2) (iii) , (iv)                                      3) (i) , (ii)                                      4) (i) , (iv)
11. Find the point on the curve  $6y = x^3 + 2$  at which y-coordinate changes 8 times as fast as x-coordinate is  
 (1) (4,11)                                      (2) (4,-11)                                      (3) (-4,11)                                      (4) (-4,-11)
12. The minimum value of the function  $|3 - x| + 9$  is  
 (1) 0                                      (2) 3                                      (3) 6                                      (4) 9

13. The point of inflection of the curve  $y = (x - 1)^3$  is

- (1) (0,0) (2) (0,1) (3) (1,0) (4) (1,1)

14. The maximum product of two positive numbers, when their sum of the squares is 200, is

- (1) 100 (2)  $25\sqrt{7}$  (3) 28 (4)  $24\sqrt{14}$

15. The curve  $y = ax^4 + bx^2$  with  $ab > 0$

- (1) has no horizontal tangent (2) is concave up  
(3) is concave down (4) has no points of inflection

16. The angle made by any tangent to the curve  $y = x^5 + 8x + 1$  with x axis is a

- 1) obtuse 2) right angle 3) acute angle 4) none of these

17. The equation of the tangent to the curve  $x = t \cos t$ ,  $y = t \sin t$  at the origin

- 1)  $x = 0$  2)  $y = 0$  3)  $x + y = 7$  4)  $x + y = 0$

18. The function  $f(x) = x^9 + 3x^7 + 64$  is increasing on

- 1)  $(-\infty, 0)$  2)  $(0, \infty)$  3)  $\mathbb{R}$  4) none of these

19. If the curves  $y = 2e^x$  and  $y = ae^{-x}$  intersect orthogonally, then  $a =$

- 1)  $\frac{1}{2}$  2)  $-\frac{1}{2}$  3) 2 4)  $2e^2$

20. The tangent to the curve  $y^2 - xy + 9 = 0$  is vertical when

- (1)  $y = 0$  (2)  $y = \pm\sqrt{3}$  (3)  $y = 1$  (4)  $y = \pm 3$

## II. Answer the following questions

**5 × 2 = 10**

1. Using the Rolle's theorem, determine the values of  $x$  at which the tangent is parallel to the  $x$ -axis for the following function  $f(x) = \sqrt{x} - \frac{x}{3}$ ,  $x \in [0, 9]$

2. Using the Lagrange's mean value theorem determine the values of  $x$  at which the tangent is parallel to the secant line at the end points of the given interval:  $f(x) = (x - 2)(x - 7)$ ,  $x \in [3, 11]$

3. Write the Taylor series expansion of  $\frac{1}{x}$  about  $x = 2$  by finding the first three non-zero terms

4. Evaluate :  $\lim_{x \rightarrow 1^-} \left( \frac{\log(1-x)}{\cot(\pi x)} \right)$

5. Find the intervals of monotonicity and hence find the local extrema for the function

$$f(x) = x^{\frac{2}{3}}$$

## III. Answer the following questions

**5 × 3 = 15**

1. Salt is poured from a conveyer belt at a rate of 30 cubic metre per minute forming a conical pile with a circular base whose height and diameter of base are always equal. How fast is the height of the pile increasing when the pile is 10 metre high?

2. Find the equations of the tangents to the curve  $y = \frac{x+1}{x-1}$  which are parallel to the line  $x + 2y = 6$

3. Find the acute angle between  $y = x^2$  and  $y = (x - 3)^2$ .

4. Evaluate :  $\lim_{x \rightarrow 0^+} (\cos x)^{\frac{1}{x^2}}$

5. Discuss the monotonicity and local extrema of the function  $f(x) = \log(1 + x) - \frac{x}{1+x}$ ,  $x > -1$  hence find the domain where,  $\log(1 + x) > \frac{x}{1+x}$

**IV. Answer the following questions** **$7 \times 5 = 35$** 

1. A particle moves along a line according to the law  $s(t) = 2t^3 - 9t^2 + 12t - 4$ , where  $t \geq 0$ .
  - (i) At what times the particle changes direction?
  - (ii) Find the total distance travelled by the particle in the first 4 seconds.
  - (iii) Find the particle's acceleration each time the velocity is zero.
2. A police jeep, approaching an orthogonal intersection from the northern direction, is chasing a speeding car that has turned and moving straight east. When the jeep is 0.6 km north of the intersection and the car is 0.8 km to the east. The police determine with a radar that the distance between them and the car is increasing at 20 km/hr. If the jeep is moving at 60 km/hr at the instant of measurement, what is the speed of the car?
3. Write the Maclaurin series expansion of the following function  $\tan^{-1} x$ ;  $-1 \leq x \leq 1$
4. Find the intervals of concavity, point of inflection and local extrema of the function  $f(x) = 4x^6 - 6x^4$
- 5.i). We have a 12 square unit piece of thin material and want to make an open box by cutting small squares from the corners of our material and folding the sides up. The question is, which cut produces the box of maximum volume?
- ii) Prove that among all the rectangles of the given area square has the least perimeter
5. A rectangular page is to contain  $24 \text{ cm}^2$  of print. The margins at the top and bottom of the page are 1.5 cm and the margins at other sides of the page is 1 cm. What should be the dimensions of the page so that the area of the paper used is minimum.
6. A farmer plans to fence a rectangular pasture adjacent to a river. The pasture must contain 1,80,000 sq.mtrs in order to provide enough grass for herds. No fencing is needed along the river. What is the length of the minimum needed fencing material?
7. A hollow cone with base radius  $a$  cm and height  $b$  cm is placed on a table. Show that the volume of the largest cylinder that can be hidden underneath is  $\frac{4}{9}$  times volume of the cone

*Setting Goals is the first step  
in turning the  
invisible into the visible*

**Key**

1. 3) 1.5 s
2. 4) 200m
3. 4) [ slope of  $f(x)$  ] [ slope of  $g(x)$  ] = -1
4. 1)1
5. 3) strictly increasing on  $I$
6. 2) maximum point
7. 4) no extrema
8. 1) The extreme value theorem
9. 1)(i) and (ii)
10. 3) (i) , (ii)
11. (1) (4,11)
12. (4) 9
13. (3) (1,0)
14. (1) 100
15. (4) has no points of inflection
16. 3) acute angle
17. 2)  $y = 0$
18. 3) R
19. 1)  $\frac{1}{2}$
20. (4)  $y = \pm 3$

# Padasalai



**SAIVEERA ACADEMY**  
**REVOLUTION FOR LEARNING COIMBATORE - 8098850809**  
**12TH APPLICATION OF DIFFERENTIAL CALCULUS**  
**EX : 7.3 – 7.8 TEST**

**Marks: 70****Time: 1 hr 15 min****I. Answer all the question****10 × 2 = 20**

1. Compute the value of 'c' satisfied by Rolle's theorem for the function

$$f(x) = \log\left(\frac{x^2+6}{5x}\right) \text{ in the interval } [2,3].$$

2. A truck travels on a toll road with a speed limit of 80 km/hr. The truck completes a 164 km journey in 2 hours. At the end of the toll road the trucker is issued with a speed violation ticket. Justify this using the Mean Value Theorem.

3. Prove, using mean value theorem, that  $|\sin \alpha - \sin \beta| \leq |\alpha - \beta|$ ,  $\alpha, \beta \in \mathbb{R}$

4. Explain why Rolle's theorem is not applicable to the following functions in the respective intervals.

i)  $f(x) = \left|\frac{1}{x}\right|$ ,  $x \in [-1, 1]$

ii)  $f(x) = x - 2 \log x$ ,  $x \in [2, 7]$

5. Explain why Lagrange's mean value theorem is not applicable to the following functions in the respective intervals

i)  $f(x) = \frac{x+1}{x}$ ,  $x \in [-1, 2]$

ii)  $f(x) = |3x + 1|$ ,  $x \in [-1, 3]$

6. Expand  $\log(1+x)$  as a Maclaurin's series upto 4 non-zero terms for  $-1 < x \leq 1$ .

7. If  $\lim_{\theta \rightarrow 0} \left(\frac{1 - \cos m\theta}{1 - \cos n\theta}\right) = 1$  then prove that  $m = \pm n$ .

8. Find the absolute extrema of the function  $f(x) = 3\cos x$  on the closed interval  $[0, 2\pi]$ .

9. Prove that the function  $f(x) = x - \sin x$  is increasing on the real line. Also discuss for the existence of local extrema.

10. Find the local maximum and minimum of the function  $x^2 y^2$  on the line  $x + y = 10$

**II. Answer all the question****5 × 3 = 15**

1. Show that the value in the conclusion of the mean value theorem for  $f(x) = Ax^2 + Bx + C$  on any interval  $[a, b]$  is  $\frac{a+b}{2}$

2. Expand the polynomial  $f(x) = x^2 - 3x + 2$  in powers of  $x - 1$ .

3. Evaluate :  $\lim_{x \rightarrow \alpha} (1 + 2x)^{\frac{1}{2 \log x}}$

4. Find intervals of concavity and points of inflexion for the following functions

$$f(x) = x(x - 4)^3$$

5. Find the intervals of monotonicities and hence find the local extremum for the following functions:  $f(x) = 2x^3 + 3x^2 - 12x$

**III. Answer all the question****7 × 5 = 35**

1. Expand  $\tan x$  in ascending powers of  $x$  upto 5th power for  $-\frac{\pi}{2} < x < \frac{\pi}{2}$

2. Evaluate :  $\lim_{x \rightarrow \frac{\pi}{2}} (\sin x)^{\tan x}$

3. Find the intervals of monotonicities and hence find the local extremum for the following functions:  $f(x) = \sin x \cos x + 5$ ,  $x \in (0, 2\pi)$

4. Find the points on the unit circle  $x^2 + y^2 = 1$  nearest and farthest from  $(1, 1)$ .

5. Find the dimensions of the rectangle with maximum area that can be inscribed in a circle of radius 10 cm.
6. A manufacturer wants to design an open box having a square base and a surface area of 108 sq.cm. Determine the dimensions of the box for the maximum volume.
7. A hollow cone with base radius  $a$  cm and height  $b$  cm is placed on a table. Show that the volume of the largest cylinder that can be hidden underneath is  $\frac{4}{9}$  times volume of the cone.

*"The future depends on what you do today"*

# Padasalai