Padasalai's Telegram Groups!

(தலைப்பிற்கு கீழே உள்ள லிங்கை கிளிக் செய்து குழுவில் இணையவும்!)

- Padasalai's NEWS - Group
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- Lesson Plan - Group
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- 12th Standard - Group
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- 11th Standard - Group
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- 10th Standard - Group
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- 9th Standard - Group
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- 6th to 8th Standard - Group
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K.K NAGAR, SALEM.

I – 75% EXAM – 2019

MATHS

Std : XII

Marks : 90

Date: 19.10.2019

Time : 2½ hrs

I. Choose the correct answer: (20 x 1 = 20)

1. If $A=\begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}$ and $AB = I_2$, then B =

   1) $\left(Cos^2\frac{\theta}{2}\right)A$ 2) $\left(Cos^2\frac{\theta}{2}\right)A^T$ 3) $(\cos^2\theta)\,I$ 4) $\left(\sin^2\frac{\theta}{2}\right)A^T$

2. The augmented matrix of a system of linear equation is $\begin{bmatrix} 1 & 2 & 7 & 3 \\ 0 & 1 & 4 & 6 \\ 0 & 0 & \lambda-7 & \mu+5 \end{bmatrix}$ the system has infinitely many solution if

   1) $\lambda = 7, \mu \neq -5$ 2)) $\lambda = -7, \mu = 5$ 3)) $\lambda \neq 7, \mu \neq -5$ 4) $\lambda = 7, \mu = -5$

3. if $|z-2+i| \leq 2$, then the greatest value of $|z|$ is

   1) $\sqrt{3}-2$ 2) $\sqrt{3}+2$ 3) $\sqrt{5}-2$ 4) $\sqrt{5}+2$

4. If $\omega \neq 1$ is a cubic root of unity and $(1+\omega)^7 = A + B\,\omega$, then (A, B) equals

   1) (1,0) 2) (−1,1) 3) (0,1) 4) (1,1)

5. According to the rational root theorem, which number is not possible rational zero of $4x^7+2x^4-10x^3-5$?

   1) −1 2) $\frac{5}{4}$ 3) $\frac{4}{5}$ 4) 5

6. $\tan^{-1}\left(\frac{1}{4}\right)+\tan^{-1}\left(\frac{2}{9}\right)$ is equal to

   1) $\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$ 2) $\frac{1}{2}\sin^{-1}\left(\frac{3}{5}\right)$ 3) $\frac{1}{2}\tan^{-1}\left(\frac{3}{5}\right)$ 4) $\tan^{-1}\left(\frac{1}{2}\right)$

7. If $\sin^{-1}x + \cot^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$, then x is equal to

   1) $\frac{1}{2}$ 2) $\frac{1}{\sqrt{5}}$ 3) $\frac{2}{\sqrt{5}}$ 4) $\frac{\sqrt{3}}{2}$

8. The circle $x^2+y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points if

   1) $15 < m < 65$ 2) $35 < m < 85$ 3) $-85 < m < -35$ 4) $-35 < m < 15$

9. If the two tangents drawn from a point P to the parabola $y^2 = 4x$ are at right angles then the locus of P is

   1) 2x+1=0 2) x= −1 3) 2x − 1 = 0 4) x=1

10. Consider the vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ such that $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = \vec{0}$. Let P_1 and P_2 be the planes determined by the pairs of vectors $\vec{a}, \vec{b}$ and $\vec{c}, \vec{d}$ respectively. Then the angle between P_1 and P_2 is

    1) 0° 2) 45° 3) 60° 4) 90°

11. If the length of the perpendicular from the origin to the plane $2x+3y+\lambda z=1$, $\lambda > 0$ is $\frac{1}{5}$, then the value of λ is

    1) $2\sqrt{3}$ 2) $3\sqrt{2}$ 3) 0 4) 1

12. The slope of the line normal to the curve $f(x) = 2\cos 4x$ at $x = \frac{\pi}{12}$ is

    1) $-4\sqrt{3}$ 2) −4 3) $\frac{\sqrt{3}}{12}$ 4) $4\sqrt{3}$

13. The number given by the Mean value theorem for the function $\frac{1}{x}, x \in [1,9]$ is
1) 2 2) 2.5 3) 3 4) 3.5

14. If $v(x,y) = \log(e^x + e^y)$, then $\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y}$ is equal to
1) $e^x + e^y$ 2) $\frac{1}{e^x+e^y}$ 3) 2 4) 1

15. If $u(x,y) = x^2 + 3xy + y - 2019$, then $\frac{\partial u}{\partial x}|_{(4,-5)}$ is equal to
1) -4 2) -3 3) -7 4) 13

16. The inverse of $\begin{pmatrix} 2 & -1 \\ 5 & -2 \end{pmatrix}$ is
1) $\begin{pmatrix} 2 & 1 \\ 5 & 2 \end{pmatrix}$ 2) $\begin{pmatrix} 2 & -1 \\ 5 & -2 \end{pmatrix}$ 3) $\begin{pmatrix} -2 & 1 \\ -5 & 2 \end{pmatrix}$ 4) $\begin{pmatrix} -2 & 1 \\ -5 & 1 \end{pmatrix}$

17. The values of $\sqrt[4]{-1}$ are
1) $1, -1$ 2) $\pm\frac{1}{\sqrt{2}}(1 \pm i)$ 3) 0,1,2,4 4) $\pm 1, \pm i$

18. The number of real and imaginary roots of $x^9 + 9x^7 + 7x^5 + 5x^3 + 3x$ are
1) 0,9 2) 1,8 3) 2,7 4) 3,6

19. If $\cos^{-1} x + 3\sin^{-1} x = \pi$ then the value of x is
1) 0 2) $\frac{1}{2}$ 3) $\frac{1}{\sqrt{2}}$ 4) 1

20. The point of contact of the parabola with the tangent is
1) $\left(\frac{\pm am}{\sqrt{1+m^2}}, \frac{\pm 1}{\sqrt{1+m^2}}\right)$ 2) $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$ 3) $\left(\frac{-a^2m}{c}, \frac{b^2}{c}\right)$ 4) $\left(\frac{-a^2m}{c}, \frac{-b^2}{c}\right)$

II. Answer any seven questions: (Question No. 30 is compulsory) **(7 x 2 = 14)**

21. Solve the following system of homogenous equations.
$3x + 2y + 7z = 0,\ 4x - 3y - 2z = 0,\ 5x + 9y + 23z = 0$

22. Write in polar form of the following complex number $\frac{i-1}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}}$

23. Examine for the rational roots of $2x^3 - x^2 - 1 = 0$.

24. Find the domain of $\cos^{-1}\left(\frac{2+\sin x}{3}\right)$.

25. Find the equation of the tangent and normal to the circle $x^2 + y^2 = 25$ at P(-3,4).

26. Find the magnitude and the direction cosines of the torque about the point (2,0,-1) of a force $2\hat{i} + \hat{j} - \hat{k}$, whose line of action passes through the origin.

27. Evaluate the following limit, if necessary use l'Hôpital Rule:
$\lim_{x\to\infty} \frac{x}{\log x}$.

28. Assuming $\log_{10} e = 0.4343$, find an approximate value of $\log_{10} 1003$.

29. If the planes $2x - ay + z - 3 = 0$ and $4x + y - bz - 5 = 0$ are parallel, then find the value of a and b.

30. Discuss the concavity of $y = e^{-x}$.

III. Answer any seven questions: (Question No. 40 is compulsory) **(7 x 3 = 21)**

31. Verify $(AB)^{-1} = B^{-1}A^{-1}$ with $A = \begin{bmatrix} 0 & -3 \\ 1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} -2 & -3 \\ 0 & -1 \end{bmatrix}$.

32. If $|z| = 1$, show that $2 \le |z^2 - 3| \le 4$.

33. Find the sum of the squares of the roots of $ax^4 + bx^3 + cx^2 + dx + e = 0, a \neq 0$.

34. If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$ and $0 < x, y, z < 1$, show that $x^2 + y^2 + z^2 + 2xyz = 1$.

35. Find the centre, foci, and eccentricity of the hyperbola $11x^2 - 25y^2 - 44x + 50y - 256 = 0$.

36. If a plane meets the coordinate axes at A,B,C such that the centriod of the triangle ABC is the point (u, v, w), find the equation of the plane.

37. Find the asymptote of the following curve:
$f(x) = \frac{3x}{\sqrt{x^2+2}}$

38. Show that the percentage error in the n^{th} root of a number is approximately $\frac{1}{n}$ times the percentage error in the number.

39. Find the equation of ellipse, if $e = \frac{2}{3}$ length of latus rectum is 5 centre is (0,0).

40. Find the approximate value of $\sqrt[4]{80}$.

IV. Answer all the questions: **(7 x 5 = 35)**

41. a) The upward speed $v(t)$ of a rocket at time t is approximated by $v(t) = at^2 + bt + c,\ 0 \le t \le 100$ where a,b, and c are constants. It has been found that the Speed at times $t = 3, t = 6, and\ t = 9$ seconds are respectively, 64,133, and 208 miles per Second respectively. Find the speed at time $t = 15$ seconds. (Use Gaussian elimination method.)

(OR)

b) Find the real values of α for which the system of equations

$x + 3y + 5z = \alpha x$
$5x + y + 3z = \alpha y$
$3x + 5y + z = \alpha z$ has infinitely number of solutions.

42. a) If $z = x + iy$ and $\arg\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$, show that $x^2 + y^2 + 3x - 3y + 2 = 0$.

(OR)

b) If $2\cos\alpha = x + \frac{1}{x}$ and $2\cos\beta = y + \frac{1}{y}$, show that

(i) $\frac{x^m}{y^n} - \frac{y^n}{x^m} = 2i \sin(m\alpha - n\beta)$ (ii) $x^m y^n + \frac{1}{x^m y^n} = 2\cos(m\alpha + n\beta)$.

43. a) Find all zeros of the polynomial $x^6 - 3x^5 - 5x^4 + 22x^3 - 39x^2 - 39x + 135$, if it is known that $1 + 2i$ and $\sqrt{3}$ are two of its zeros.

(OR)

b) Solve for x: $tan^{-1}\left(\frac{x-1}{x+1}\right) + tan^{-1}\left(\frac{2x-1}{2x+1}\right) = tan^{-1}\left(\frac{23}{36}\right)$.

44. a) Find the equation of the circle passing through the points (1,1), (2,-1), and (3,2).

(OR)

b) Two coast guard stations are located 600 km apart at points A(0,0) and B(0,600). A distress signal from a ship at P is received at slightly different times by two stations. It is determined that the ship is 200 km farther from station A than it is from station B. Determine the equation of hyperbola that passes through the location of the ship.

45. a) Find the non-parametric form of vector equation and Cartesian equation of the plane passing Through the point (1, -2, 4) and perpendicular to the plane $x + 2y - 3z = 11$ and parallel to the line $\frac{x+7}{3} = \frac{y+3}{-1} = \frac{z}{1}$.

(OR)

b) Find the coordinates of the foot of the perpendicular and length of the perpendicular from the point (4,3,2) to the plane $x + 2y + 3z = 2$.

46. a) A particle moves along a horizontal line such that its position at any time $t \ge 0$ is given by $s(t) = t^3 - 6t^2 + 9t + 1$, where s is measured in metres and t in seconds?

(i) At what time the particle is at rest?
(ii) At what time the particle changes its direction?
(iii) Find the total distance travelled by the particle in the first 2 seconds.

(OR)

b) For the function $f(x) = 4x^3 + 3x^2 - 6x + 1$ find the intervals of monotonicity, local extrema, intervals of concavity and points of inflection.

47. a) Let $f(x,y) = \sin(xy^2) + e^{x^3+5y}$ for all $(x,y) \in R^2$. Calculate $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial^2 f}{\partial y \partial x}$ and $\frac{\partial^2 f}{\partial x \partial y}$.

(OR)

b) Find the approximate value of $\sqrt[3]{1.02} + \sqrt[4]{1.02}$.