Exercise: 9.2.

1. $\lim_{x \to 2} \frac{x^4 - 16}{x - 2}$.

Sol:

$$= \lim_{x \to 2} \frac{x^4 - 16}{x - 2}$$

$$= \lim_{x \to 2} \frac{x^4 - 2^4}{x - 2} = n a^{n-1}$$

$$= 4(2)^{4-1}$$

$$= 4(2)^3$$

$$= 4(8)$$

$$= 32.$$

$$\therefore \lim_{x \to a} \frac{x^n - a^n}{x - a} = n a^{n-1}$$

2. $\lim_{x \to 1} \frac{x^m - 1}{x^n - 1}$, m and n are integers.

Sol:

Given that $\lim_{x \to 1} \frac{x^m - 1}{x^n - 1}$

× by and ÷ by $x-1$

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$$= \lim_{x \to 1} \frac{x^m - 1}{x^n - 1} \times \frac{x-1}{x-1}$$

$$= \lim_{x \to 1} \frac{x^m - 1}{x-1} \times \frac{x-1}{x^n - 1}$$

$$= \lim_{x \to 1} \left[\frac{x^m - 1}{x-1} \times \frac{1}{\frac{x^n - 1}{x-1}} \right.$$

$$= \frac{\lim_{x \to 1} \frac{x^m - 1}{x-1}}{\lim_{x \to 1} \frac{x^n - 1}{x-1}} = \frac{m(1)^{m-1}}{n(1)^{n-1}}$$

$$= \frac{m}{n}$$

3. $\lim_{\sqrt{x} \to 3} \frac{x^2 - 81}{\sqrt{x} - 3}$

Sol:

$$= \lim_{\sqrt{x} \to 3} \frac{(\sqrt{x})^4 - 3^4}{\sqrt{3} - 3}$$

$x = (\sqrt{x})^2$

$x^2 = (\sqrt{x})^4$

$$= 4(3)^{4-1}$$

$$= 4(3)^3$$

$$= 4(27)$$

$$= 108$$

4. $\lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$, $x > 0$.

5.

Sol: $\lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$

$\times$ by and $\div$ by $\sqrt{x+h} + \sqrt{x}$ $\qquad a^2 - b^2 = (a+b)(a-b)$

$$= \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \times \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$

$$= \lim_{h \to 0} \frac{(\sqrt{x+h})^2 - (\sqrt{x})^2}{h[\sqrt{x+h} + \sqrt{x}]}$$

$$= \lim_{h \to 0} \frac{x+h-x}{h[\sqrt{x+h} + \sqrt{x}]}$$

$$= \lim_{h \to 0} \frac{h}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

$$= \frac{1}{\sqrt{x+0} + \sqrt{x}}$$

$$= \frac{1}{\sqrt{x} + \sqrt{x}}$$

$$= \frac{1}{2\sqrt{x}}.$$

5. $\lim_{x \to 5} \frac{\sqrt{x+4} - 3}{x-5}.$

Sol:

Sol:

Given that

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$$\lim_{x \to 5} \frac{\sqrt{x+4}-3}{x-5}$$

$\times$ ly and $\div$ by $\sqrt{x+4}+3$

$$= \lim_{x \to 5} \frac{\sqrt{x+4}-3}{x-5} \times \frac{\sqrt{x+4}+3}{\sqrt{x+4}+3}$$

$$= \lim_{x \to 5} \frac{(\sqrt{x+4})^2 - 3^2}{(x-5)(\sqrt{x+4}+3)}$$

$$= \lim_{x \to 5} \frac{x+4-9}{(x-5)(\sqrt{x+4}+3)}$$

$$= \lim_{x \to 5} \frac{x-5}{(x-5)(\sqrt{x+4}+3)}$$

$$= \lim_{x \to 5} \frac{1}{(\sqrt{x+4}+3)}$$

$$= \frac{1}{\sqrt{5+4}+3}$$

$$= \frac{1}{\sqrt{9}+3}$$

$$= \frac{1}{3+3}$$

$$= \frac{1}{6}$$ //.

$\lim_{x \to 2} \dfrac{1/x - 1/2}{x-2}$.

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Sol:

$= \lim_{x \to 2} \dfrac{1/x - 1/2}{x-2}$

$= \lim_{x \to 2} \dfrac{\frac{2-x}{2x}}{x-2}$

$= \lim_{x \to 2} \dfrac{2-x}{2x(x-2)}$

$= \lim_{x \to 2} \dfrac{-(x-2)}{2x(x-2)}$

$= \lim_{x \to 2} \dfrac{-1}{2x}$

$= \dfrac{-1}{2 \times 2}$

$= \dfrac{-1}{4}$

$\lim_{x \to 1} \dfrac{\sqrt{x} - x^2}{1-\sqrt{x}}$

Sol:

$\lim_{x \to 1} \dfrac{\sqrt{x} - (\sqrt{x})^4}{1-\sqrt{x}}$

$x = (\sqrt{x})^2$

$x^2 = ((\sqrt{x}^2))^2$

$= (\sqrt{x})^4$.

$\boxed{\therefore x^2 = (\sqrt{x})^4}$

$\lim_{x \to 1} \dfrac{\sqrt{x}\,[1-(\sqrt{x})^3]}{1-\sqrt{x}}$

$a^3 - b^3 = (a-b)(a^2+ab+b^2)$

$1^3 - \sqrt{x}^3 = (1-\sqrt{x})(1+\sqrt{x}+(\sqrt{x})^2)$

$= (1-\sqrt{x})(1+\sqrt{x}+x)$

$= \lim_{x\to 1} \dfrac{\sqrt{x}\,[(1-\sqrt{x})(1+\sqrt{x}+x)}{1-\sqrt{x}}$

$= \lim_{x\to 1} \sqrt{x}\,(1+\sqrt{x}+x)$

$= \sqrt{1}\,(1+\sqrt{1}+1)$

$= \sqrt{1}\,(1+1+1)$

$= 1(3)$

$= 3$

8. $\lim_{x\to 0} \dfrac{\sqrt{x^2+1}-1}{\sqrt{x^2+16}-4}$

Sol:

$= \lim_{x\to 0} \dfrac{\sqrt{x^2+1}-1}{\sqrt{x^2+16}-4}$

$= \lim_{x\to 0} \dfrac{(x^2+1)^{1/2}-1}{(x^2+16)^{1/2}-(16)^{1/2}}$

$\times$ by and $\div$ by x^2

$$= \lim_{x \to 0} \frac{(x^2+1)^{1/2} - 1^{1/2}}{(x^2+16)^{1/2} - (16)^{1/2}} \times \frac{x^2}{x^2}$$

$$= \lim_{x \to 0} \frac{(x^2+1)^{1/2} - (1)^{1/2}}{x^2} \times \frac{x^2}{(x^2+16)^{1/2} - (16)^{1/2}}$$

$$= \lim_{x \to 0} \frac{(x^2+1)^{1/2} - (1)^{1/2}}{(x^2+1) - 1} \times \frac{(x^2+16) - 16}{(x^2+16)^{1/2} - (16)^{1/2}}$$

$$= \lim_{x \to 0} \frac{(x^2+1)^{1/2} - (1)^{1/2}}{(x^2+1) - 1} \times \lim_{x \to 0} \frac{1}{\frac{(x^2+16)^{1/2} - (16)^{1/2}}{(x^2+16) - 16}}$$

$$= \frac{1}{2}(1)^{1/2 - 1} \times \frac{1}{\frac{1}{2}(16)^{1/2 - 1}}$$

$$= \frac{1}{2}(1) \times \frac{1}{\frac{1}{2}(16)^{-1/2}}$$

$$= (16)^{1/2}$$

$$= \sqrt{16}$$

$$= 4$$

4) $\lim_{x \to 0} \frac{\sqrt{1+x} - 1}{x}$

Sol:

Given that

$$\lim_{x \to 0} \frac{\sqrt{1+x} - 1}{x}$$

$\times$ by and $\div$ by $\sqrt{1+x} + 1$

$$\lim \frac{\sqrt{1+x} - 1}{x} \times \frac{\sqrt{1+x} + 1}{\sqrt{1+x} + 1}$$

$$= \lim_{x \to 0} \frac{(x^2+1)^{1/2} - 1^{1/2}}{(x^2+16)^{1/2} - (16)^{1/2}} \times \frac{x^2}{x^2}$$

$$= \lim_{x \to 0} \frac{(x^2+1)^{1/2} - (1)^{1/2}}{x^2} \times \frac{x^2}{(x^2+16)^{1/2} - (16)^{1/2}}$$

$$= \lim_{x \to 0} \frac{(x^2+1)^{1/2} - (1)^{1/2}}{(x^2+1) - 1} \times \frac{(x^2+16) - 16}{(x^2+16)^{1/2} - (16)^{1/2}}$$

$$= \lim_{x \to 0} \frac{(x^2+1)^{1/2} - (1)^{1/2}}{(x^2+1) - 1} \times \lim_{x \to 0} \frac{1}{\frac{(x^2+16)^{1/2} - (16)^{1/2}}{(x^2+16) - 16}}$$

$$= \frac{1}{2}(1)^{1/2 - 1} \times \frac{1}{\frac{1}{2}(16)^{1/2 - 1}}$$

$$= \frac{1}{2} \times \frac{1}{\frac{1}{2}(16)^{-1/2}}$$

$$= (16)^{1/2}$$

$$= \sqrt{16}$$

$$= 4.$$

9) $\lim_{x \to 0} \frac{\sqrt{1+x} - 1}{x}$.

Sol:

Given that

$$\lim_{x \to 0} \frac{\sqrt{1+x} - 1}{x}$$

$\times$ ly and $\div$ by $\sqrt{1+x} + 1$

$$\lim_{x \to 0} \frac{\sqrt{1+x} - 1}{x} \times \frac{\sqrt{1+x} + 1}{\sqrt{1+x} + 1}$$

$$= \lim_{x \to 0} \frac{(\sqrt{1+x})^2 - 1^2}{x(\sqrt{1+x}+1)}$$

$$= \lim_{x \to 0} \frac{1+x-1}{x[\sqrt{1+x}+1]}$$

$$= \lim_{x \to 0} \frac{x}{x[\sqrt{1+x}+1]}$$

$$= \lim_{x \to 0} \frac{1}{(\sqrt{1+x}+1}$$

$$= \frac{1}{(\sqrt{1+0}+1)}$$

$$= \frac{1}{1+1}$$

$$= \frac{1}{2}$$

10. $\lim_{x \to 1} \frac{\sqrt[3]{7+x^3} - \sqrt{3+x^2}}{x-1}$.

Sol:

Given that

$$= \lim_{x \to 1} \frac{\sqrt[3]{7+x^3} - \sqrt{3+x^2}}{x-1}$$

$$= \lim_{x \to 1} \frac{(7+x^3)^{1/3} - 8^{1/3}}{(7+x^3)-8} \times \frac{x^3-1}{x-1} -$$

$$\frac{(3+x^2)^{1/2} - 4^{1/2}}{(3+x^2)-4} \times \frac{x^2-1}{x-1}$$

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$$= \lim_{7+x^3 \to 8} \frac{(7+x^3)^{1/3} - 8^{1/3}}{(7+x^3)-8} \times \frac{(x-1)(x^2+x+1)}{(x-1)}$$

$$\lim_{3+x^2 \to 4} \frac{(3+x^2)^{1/2} - 4^{1/2}}{3+x^2-4} \times \frac{(x+1)(x-1)}{(x-1)}$$

[$\because x \to 1,\ x^3 - 1$ and $7+x^3 \to 7+1 = 8$ and

$x \to 1,\ x^2 - 1$ and $3+x^2 = 3+1 = 4$] .

$$= \lim_{7+x^3 \to 8} \frac{(7+x^3)^{1/3} - 8^{1/3}}{(7+x^3)^{1/3} - 8} (x^2+x+1) -$$

$$\lim_{3+x^2} \frac{(3+x^2)^{1/2} - 4^{1/2}}{(3+x^2)-4} \times (x+1)$$

$$= \frac{1}{3}(8)^{1/3 - 1} \cdot (1+1+1) - \frac{1}{2}(4)^{1/2 - 1} (1+1)$$

$$= \frac{1}{3}(8)^{-2/3} \times 3 \left(-\tfrac{1}{2}\right) 4^{-1/2} (2)$$

$$= \frac{1}{8^{2/3}} - \frac{1}{4^{1/2}}$$

$$= \frac{1}{(8)^{1/3}} - \frac{1}{\sqrt{4}}$$

$$= \frac{1}{\cancel{\sqrt[3]{64}}\ (64)^{1/3}} - \frac{1}{2}$$

$$= \frac{1}{4} - \frac{1}{2}$$

$$= \frac{1-2}{4}$$

$$= \frac{-1}{4}$$

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11. $\lim_{x \to 2} \frac{2 - \sqrt{x+2}}{\sqrt[3]{2} - \sqrt[3]{4-x}}$

Sol:

$= \lim_{x \to 2} \frac{2 - \sqrt{x+2}}{\sqrt[3]{2} - \sqrt[3]{4-x}}$

$= \lim_{x \to 2} \frac{-[\sqrt{x+2} - 2]}{-[\sqrt[3]{4-x} - \sqrt[3]{2}]}$

$= \lim_{x \to 2} \frac{\sqrt{x+2} - 2}{\sqrt[3]{4-x} - \sqrt[3]{2}}$

$x = \sqrt{x}$

$2 = \sqrt{4}$

$= \lim_{x \to 2} \frac{(x+2)^{1/2} - (2^2)^{1/2}}{(4-x)^{1/3} - (2)^{1/3}}$

$\times$ by and $\div$ by $x - 2$

$= \lim_{x \to 2} \frac{(x+2)^{1/2} - (4)^{1/2}}{x-2} \times \frac{x-2}{(4-x)^{1/3} - 2^{1/3}}$

$= \lim_{x \to 2} \frac{(x+2)^{1/2} - 4^{1/2}}{(x+2) - 4} \times \lim_{x \to 2} \frac{-(2-x)}{(4-x)^{1/3} - 2^{1/3}}$

$= \left(\frac{1}{2}\right) (4)^{1/2 - 1} \lim_{x \to 2} \frac{-1}{\frac{(4-x)^{1/3} - 2^{1/3}}{2-x}}$

$= \frac{1}{2} (4)^{-1/2} \lim_{x \to 2} \frac{-1}{\frac{(4-x)^{1/3} - 2^{1/3}}{(4-x) - 2}}$

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$$= \frac{1}{2} \left(\frac{1}{4}\right)^{1/2} \left[\frac{-1}{1/3\ (2)^{1/3 - 1}} \right]$$

$$= \frac{1}{2} \times \frac{1}{\sqrt{4}} \left[\frac{-1}{1/3\ 2^{2/3}} \right]$$

$$= \frac{1}{2} \times \frac{1}{2} \left[-3 \times 2^{2/3} \right]$$

$$= \frac{-3}{4} (2^2)^{1/3}$$

$$= -\frac{3}{4} (4)^{1/3}$$

$$= \frac{-3}{4} \sqrt[3]{4}$$

12. $\lim_{x \to 0} \frac{\sqrt{1+x^2} - 1}{x}$.

Sol:

$$= \lim_{x \to 0} \frac{\sqrt{1+x^2} - 1}{x}$$

$\times$ by and $\div$ by $\sqrt{1+x^2} + 1$

$$= \lim_{x \to 0} \frac{\sqrt{1+x^2} - 1}{x} \times \frac{\sqrt{1+x^2} + 1}{\sqrt{1+x^2} + 1}$$

$$= \lim_{x \to 0} \frac{(\sqrt{1+x^2})^2 - 1^2}{x\left[\sqrt{1+x^2} + 1\right]}$$

$$= \lim_{x \to 0} \frac{1+x^2 - 1}{x\left[\sqrt{1+x^2} + 1\right]}$$

$$= \lim_{x \to 0} \frac{x^2}{x\sqrt{1+x^2}+1}$$

$$= \lim_{x \to 0} \frac{x}{\sqrt{1+x^2}+1}$$

$$= \frac{0}{\sqrt{1+0}+1}$$

$$= \frac{0}{1+1}$$

$$= \frac{0}{2}$$

$$= 0$$

15. $\lim_{x \to 0} \frac{\sqrt{1-x}-1}{x^2}$

Sol:

$$= \lim_{x \to 0} \frac{\sqrt{1-x}-1}{x^2}$$

$\times$ ly and $\div$ by $\sqrt{1-x}+1$

$$= \lim_{x \to 0} \frac{\sqrt{1-x}-1}{x^2} \times \frac{\sqrt{1-x}+1}{\sqrt{1-x}+1}$$

$$= \lim_{x \to 0} \frac{(\sqrt{1-x})^2 - 1^2}{x^2[\sqrt{1-x}+1]}$$

$$= \lim_{x \to 0} \frac{1-x-1}{x^2(\sqrt{1-x}+1)}$$

$$= \lim_{x \to 0} \frac{-x}{x^2(\sqrt{1-x^2}+1)}$$

$$= \lim_{x \to 0} \frac{-1}{x(\sqrt{1-x^2}+1)}$$

$$= \frac{-1}{0}$$

$$= -\infty$$

limit does not exist.

14. $\lim_{x \to 5} \frac{\sqrt{x-1}-2}{x-5}$

Sol:

$$= \lim_{x \to 5} \frac{\sqrt{x-1}-2}{x-5}$$

$\times$ ly and $\div$ by $\sqrt{x-1}+2$

$$= \lim_{x \to 5} \frac{\sqrt{x-1}-2}{x-5} \times \frac{\sqrt{x-1}+2}{\sqrt{x-1}+2}$$

$$= \lim_{x \to 5} \frac{(\sqrt{x-1})^2-(2)^2}{(x-5)(\sqrt{x-1}+2)}$$

$$= \lim_{x \to 5} \frac{x-1-4}{x-5(\sqrt{x-1}+2)}$$

$$= \lim_{x \to 5} \frac{x-5}{x-5(\sqrt{x-1}+2)}$$

$$= \lim_{x \to 5} \frac{1}{(\sqrt{x-1}+2)}$$

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$$= \lim_{x \to 5} \frac{1}{\sqrt{x-1}+2}$$

$$= \frac{1}{\sqrt{5-1}+2}$$

$$= \frac{1}{\sqrt{4}+2}$$

$$= \frac{1}{2+2}$$

$$= \frac{1}{4}$$ //.

15. $\lim_{x \to a} \frac{\sqrt{x-b}-\sqrt{a-b}}{x^2-a^2}$ $(a>b)$.

Sn:

$$= \lim_{x \to a} \frac{\sqrt{x-b}-\sqrt{a-b}}{x^2-a^2}$$

$\times$ ly and $\div$ by $\sqrt{x-b}+\sqrt{a-b}$

$$= \lim_{x \to a} \frac{\sqrt{x-a}-\sqrt{a-b}}{x^2-a^2} \times \frac{\sqrt{x-b}+\sqrt{a-b}}{\sqrt{x-b}+\sqrt{a-b}}$$

$$= \lim_{x \to a} \frac{(\sqrt{x-b})^2-(\sqrt{a-b})^2}{(x^2-a^2)(\sqrt{x-b}+\sqrt{a-b})}$$

$$= \lim_{x \to a} \frac{(x-b)-(a-b)}{(x-a)(x+a)(\sqrt{x-b}+\sqrt{a-b})}$$

$$= \lim_{x \to a} \frac{(x-b-a+b)}{(x-a)(x+a)(\sqrt{x-b}+\sqrt{a-b})}$$

$$= \lim_{x \to a} \frac{x-a}{(x-a)(x+a)\left(\sqrt{x-b}+\sqrt{a-b}\right)}$$

$$= \lim_{x \to a} \frac{1}{(x+a)\left(\sqrt{x-b}+\sqrt{a-b}\right)}$$

$$= \frac{1}{(a+a)\left(\sqrt{a-b}+\sqrt{a-b}\right)}$$

$$= \frac{1}{2a \; 2\sqrt{a-b}}$$

$$= \frac{1}{4a\sqrt{a-b}}$$

Exercise : 9.3.

1. a) Find the left and right limits of $f(x) = \dfrac{x^2-4}{(x^2+4x+4)(x+3)}$ at $x = -2$.

Sol:

$$\frac{\text{anything}}{0} = \infty$$

$$\frac{0}{\text{anything}} = 0$$

$$f(x) = \frac{x^2-4}{(x^2+4x+4)(x+3)}$$

$$f(-2^-) = \lim_{x \to 2^-} \frac{(x)^2-4}{(x^2+4x+4)(x+3)}$$

$$= \lim_{x \to 2^-} \frac{(x+2)(x-2)}{(x+2)(x+2)(x+3)}$$

$$= \lim_{x \to 2^-} \frac{x-2}{(x+2)(x+3)}$$

$$= \lim_{x \to 5} \frac{1}{\sqrt{x-1} + 2}$$

$$= \frac{1}{\sqrt{5-1} + 2}$$

$$= \frac{1}{\sqrt{4} + 2}$$

$$= \frac{1}{2+2}$$

$$= \frac{1}{4} \,//.$$

15. $\lim_{x \to a} \frac{\sqrt{x-b} - \sqrt{a-b}}{x^2 - a^2}$ $(a > b)$.

Sn:

$$= \lim_{x \to a} \frac{\sqrt{x-b} - \sqrt{a-b}}{x^2 - a^2}$$

$\times$ ly and $\div$ by $\sqrt{x-b} + \sqrt{a-b}$

$$= \lim_{x \to a} \frac{\sqrt{x-a} - \sqrt{a-b}}{x^2 - a^2} \times \frac{\sqrt{x-b} + \sqrt{a-b}}{\sqrt{x-b} + \sqrt{a-b}}$$

$$= \lim_{x \to a} \frac{(\sqrt{x-b})^2 - (\sqrt{a-b})^2}{(x^2 - a^2)(\sqrt{x-b} + \sqrt{a-b})}$$

$$= \lim_{x \to a} \frac{(x-b) - (a-b)}{(x-a)(x+a)(\sqrt{x-b} + \sqrt{a-b})}$$

$$= \lim_{x \to a} \frac{(x-b-a+b)}{(x-a)(x+a)(\sqrt{x-b} + \sqrt{a-b})}$$

$$= \lim_{x \to a} \frac{x-a}{(x-a)(x+a)(\sqrt{x-b}+\sqrt{a-b})}$$

$$= \lim_{x \to a} \frac{1}{(x+a)(\sqrt{x-b}+\sqrt{a-b})}$$

$$= \frac{1}{(a+a)(\sqrt{a-b}+\sqrt{a-b})}$$

$$= \frac{1}{2a\ 2\sqrt{a-b}}$$

$$= \frac{1}{4a\sqrt{a-b}}$$

Exercise: 9.3.

1. a) Find the left and right limits of $f(x) = \frac{x^2-4}{(x^2+4x+4)(x+3)}$ at $x = -2$.

Sol:

$\frac{\text{anything}}{0} = \infty$

$\frac{0}{\text{anything}} = 0$

$$f(x) = \frac{x^2-4}{(x^2+4x+4)(x+3)}$$

$$f(-2^-) = \lim_{x \to 2^-} \frac{(x)^2-4}{(x^2+4x+4)(x+3)}$$

$$= \lim_{x \to 2^-} \frac{(x+2)(x-2)}{(x+2)(x+2)(x+3)}$$

$$= \lim_{x \to 2^-} \frac{x-2}{(x+2)(x+3)}$$

$$= \lim_{x \to 2} \frac{x-2}{(x+2)(x+3)}$$

$\to (-4-2) = -6$

$(x-2) \Rightarrow (-4+2) = -2$

$(x+2) \Rightarrow (-4+3) = -1$

$(x+3) \Rightarrow$

$$= \frac{\text{Negative}}{0(-ve)}$$

$$= +\infty$$

$\Rightarrow f(-2^-) \to \infty$ as $x \to 2^-$

$$f(-2^-) = \lim_{x \to -2^+} \frac{x-2}{(x+2)(x+3)}$$

nearest value should take

$-2 \quad -1 \quad 0$

$(x-2) \Rightarrow (-1-2)$

$= -3 \; (-ve)$

$(x+2) = (-2+2)$

$= 0 = -ve$

$$= \frac{(-ve)}{0(+ve)}$$

$$= -\infty$$

$(x+3) = (1+3)$

$= 4 \; (+ve)$

$f(-2^+) \to -\infty$ as $x \to 2^+$.

b) $f(x) = \tan x$ at $x = \pi/2$.

Sol:

$$\lim_{x \to \pi/2} f(x) = \lim_{x \to \pi/2^-} \tan x = \infty$$

$\therefore \lim_{x \to \pi/2^-} f(x) \to \infty \quad x \to \frac{\pi}{2}^-$

$$\lim_{x \to \frac{\pi}{2}^+} f(x) = \lim_{x \to \frac{\pi}{2}^+} \tan x$$

$= -\infty$

$\therefore \lim_{x \to \pi/2^+} f(x) \to -\infty$ as $x \to \pi^+/2$

Note:

$x \to \pm\infty$ of rational expression can be found by first dividing the numerator and denomiator by the highest power of x that appears in the denominator and then calculating the limit of $x \to \infty$ of both numerator and denominator.

Degree of $q(x)$ is greater than the degree of $p(x)$. $\lim_{n \to \infty} \frac{p(x)}{q(x)} = 0$.

Degree of $p(x)$ = Degree of $q(x)$

$$\lim_{n \to \infty} \frac{p(x)}{q(x)} = \frac{\text{highest degree of } x \text{ in } p(x)}{\text{highest degree of } x \text{ in } q(x)}$$

Exercise: 9.3.

1. Evaluate the following limit.

$$\lim_{x \to 3^-} \frac{x^2-9}{x^2(x^2-6x+9)}.$$

Sol:

$$\lim_{x \to 3^-} \frac{x^2-9}{x^2(x^2-6x+9)}$$

9
-3 -3
-6

$$= \lim_{x \to 3^-} \frac{(x-3)(x+3)}{x^2(x-3)(x-3)}$$

$$= \lim_{x \to 3^-} \frac{(x+3)}{x^2(x-3)}$$

$$= \frac{(-)}{(+)0}$$

$$= -\infty$$

$$\lim_{x \to 3^+} \frac{x^2-9}{x^2(x^2-6x+9)} = \lim_{x \to 3^+} \frac{x+3}{x^2(x-3)}$$

$$= \frac{(+)}{(+)0}$$

$$= \infty.$$

3. $\lim_{x \to \infty} \frac{3}{x-2} - \frac{2x+11}{x^2+x-6}.$

Sol:

$$= \lim_{x \to \infty} \frac{3}{x-2} - \frac{2x+11}{x^2+x-6}$$

$$= \lim_{x \to \infty} \frac{3}{x-2} - \frac{2x+11}{(x-2)(x+3)}$$

$$= \lim_{x \to \infty} \frac{3(x+3)-(2x+11)}{(x-2)(x+3)}$$

$$= \lim_{x \to \infty} \frac{3x+9-2x-11}{(x-2)(x+3)}$$

$$= \lim_{x \to \infty} \frac{x-2}{(x-2)(x+3)}$$

$$= \lim_{x \to \infty} \frac{1}{x+3}$$

$\frac{1}{x} = \frac{1}{\infty} = \frac{1}{1/0} = \frac{1}{1} \times 0 = 0.$

$$= \lim_{x \to \infty} \frac{1}{x(1+3/x)}$$

$$= \frac{1}{\infty}$$

$$= 0$$

4. $\lim_{x \to \infty} \frac{x^3+x}{x^4-3x^2+1}$

Sol:

$$= \lim_{x \to \infty} \frac{x^3+x}{x^4-3x^2+1}$$

$\frac{1}{\infty} = 0$

$$= \lim_{x \to \infty} \frac{\cancel{x^3}\left(1+\frac{1}{x^2}\right)}{\cancel{x^4}\left(1-\frac{3}{x^2}+\frac{1}{x^4}\right)}$$

$$= \frac{1+0}{\infty(1-0+0)}$$

$$= \frac{1}{\infty}$$

$$= 0$$

5. $\lim_{x \to \infty} \frac{x^4-5x}{x^2-3x+1}$

Sol:

$$= \lim_{x \to \infty} \frac{x^4-5x}{x^2-3x+1}$$

$$= \lim_{x\to\infty} \frac{x^4\left(1-\frac{5}{x^3}\right)}{x^2\left(1-\frac{3}{x}+\frac{1}{x^2}\right)}$$

$$= \frac{\infty(1-0)}{(1-0+0)}$$

$$= \infty.$$

6. $\lim_{x\to\infty} \frac{1+x-3x^3}{1+x^2+3x^3}$.

Sol:

$$= \lim_{x\to\infty} \frac{1+x-3x^3}{1+x^2+3x^3}$$

$$= \lim_{x\to\infty} \frac{x^3\left(\frac{1}{x^3}+\frac{1}{x^2}-3\right)}{x^3\left(\frac{1}{x^3}+\frac{1}{x}+3\right)}$$

$$= \frac{0+0-3}{0+0+3}$$

$$= \frac{-3}{3}$$

$$= -1.$$

7. $\lim_{x\to\infty} \left(\frac{x^2}{2x^2-1} - \frac{x^2}{2x+1}\right)$

Sol:

$$= \lim_{x\to\infty} \left(\frac{x^2}{2x^2-1} - \frac{x^2}{2x+1}\right)$$

$$= \lim_{x\to\infty} \frac{x^3(2x+1) - x^2(2x^2-1)}{(2x^2-1)(2x+1)}$$

$$= \lim_{x\to\infty} \frac{\cancel{2x^4} + x^3 - \cancel{2x^4} + x^2}{(2x^2-1)(2x+1)}$$

$$= \lim_{x\to\infty} \frac{x^3 + x^2}{(2x^2-1)(2x+1)}$$

$$= \lim_{x\to\infty} \frac{x^3\left(1+\frac{1}{x}\right)}{x^2\left(2-\frac{1}{x^2}\right)x\left(2+\frac{1}{x}\right)}$$

$$= \lim_{x\to\infty} \frac{1+\frac{1}{x}}{\left(2-\frac{1}{x^2}\right)\left(2+\frac{1}{x}\right)}$$

$$= \frac{1+0}{(2-0)(2+0)}$$

$$= \frac{1}{2\times 2}$$

$$= \frac{1}{4}$$

Show that

i) $\lim_{n\to\infty} \frac{1+2+3+\cdots+n}{3n^2+7n+2} = \frac{1}{6}$

Sol:

L.H.S:

$$\lim_{n\to\infty} \frac{1+2+3+\cdots+n}{3n^2+7n+2}$$

$$= \lim_{n\to\infty} \frac{\frac{n(n+1)}{2}}{n^2\left(3+\frac{7}{n}+\frac{2}{n^2}\right)}$$

$$= \lim_{n\to\infty} \frac{n\times n\left(1+\frac{1}{n}\right)}{2n^2\left(3+\frac{7}{n}+\frac{2}{n^2}\right)}$$

$$= \frac{1+0}{2(3+0+0)}$$

$$= \frac{1}{2(3)}$$

$$R.H.S. = \frac{1}{6}$$

ii) $\lim_{n\to\infty} \frac{1^2+2^2+\cdots+(3n)^2}{1+2+\cdots+5n(2n+3)} = \frac{9}{25}$

Sol: L.H.S:

$$\lim_{n\to\infty} \frac{1^2+2^2+\cdots+(3n)^2}{1+2+\cdots+5n(2n+3)}$$

Formula:

$$1^2+2^2+3^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$1+2+3+\cdots+n = \frac{n(n+1)}{2}$$

$$= \lim_{n\to\infty} \frac{\frac{(\cancel{3}n)(3n+1)(2\cdot 3n+1)}{\cancel{6}\,2}}{\frac{5n(5n+1)}{\cancel{2}\,1}(2n+3)}$$

$$= \lim_{n\to\infty} \frac{n(3n+1)(6n+1)}{5n(5n+1)(2n+3)}$$

$$= \lim_{n\to\infty} \frac{n\left(3+\frac{1}{n}\right) n\left(6+\frac{1}{n}\right)}{5n\left(5+\frac{1}{n}\right) n\left(2+\frac{3}{n}\right)}$$

$$= \frac{(3+0)(6+0)}{5(5+0)(2+0)}$$

$$\Rightarrow \frac{(3)(6)}{5(5)(2)} = \frac{\cancel{18}\,9}{25\times\cancel{2}\,1}$$

$$R.H.S = \frac{9}{25}.$$

iii) $\lim_{n\to\infty} \frac{1}{1\cdot2} + \frac{1}{2\cdot3} + \frac{1}{3\cdot4} + \dots + \frac{1}{n(n+1)} = 1.$

Sol:

$$= \lim_{n\to\infty} \frac{1}{1\cdot2} + \frac{1}{2\cdot3} + \frac{1}{3\cdot4} + \dots + \frac{1}{n(n+1)}$$

$$= \lim_{n\to\infty} \left(\frac{1}{1} - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{1}{n} - \frac{1}{n+1}\right)$$

$$= \lim_{n\to\infty} \left(1 - \frac{1}{n+1}\right)$$

$$= \lim_{n\to\infty} \left(\frac{n+\cancel{1}-\cancel{1}}{n+1}\right)$$

$1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{n}$

$$= \lim_{n\to\infty} \frac{n}{n+1}$$

$$= \lim_{n\to\infty} \frac{\cancel{n}}{\cancel{n}\left(1+\frac{1}{n}\right)}$$

$$= \lim_{n \to \infty} \frac{n}{n\left(1+\frac{1}{n}\right)}$$

$$= \lim_{n \to \infty} \frac{1}{1+\frac{1}{n}}$$

$$= \frac{1}{1+0}$$

$$= 1$$

9. An important problem in fishery science is to estimate the number of fish presently spawing in streams and use this information to predict the number of mature fish or "recruits" that will return to the rivers during the reproductive period. If S is the number of spawners and R the number of recruits. Beverton recruit function $R(S) = \frac{S}{(\alpha S+\beta)}$ where α and β are positive constant. Show that this function predicts approximately constant recruitment when the number of spawners is sufficiently large.

Sol:

$$R(S) = \frac{S}{2S+\beta}$$

$$\lim_{S \to \infty} R(S) = \lim_{S \to \infty} \frac{S}{2S+\beta}$$

$$= \lim_{s \to \infty} \frac{s}{s\left(\alpha + \frac{\beta}{s}\right)}$$

$$= \lim_{s \to \infty} \frac{1}{\alpha + \frac{\beta}{s}}$$

$$= \frac{1}{\alpha + 0}$$

$$= \frac{1}{\alpha} = \text{constant.}$$

10. A tank contains 5000 litres of pure water Brine (very salty water) that contains 30 grams of salt per litre of water is pumped into the tank at a rate of 25 litre per minute. The concentration after t minutes (in grams of salt water per litre) is $c(t) = \frac{30t}{200+t}$. What happens to the concentration as $t \to \infty$?

Sol:

Given that

$$= \lim_{n \to \alpha} \frac{30t}{200+t}$$

$$= \lim_{n \to \alpha} \frac{30t}{t\left(1+\frac{200}{t}\right)}$$

$$= \frac{30}{1+0}$$

$= 30.$

Example: 9.25.

Alcohol is removed from the body by the lungs, the kidneys and by chemical processes in liver. At moderate concentration levels the majority work of removing the alcohol is done by the liver; less than 5% of the alcohol is eliminated by the lungs and kidneys. The rate r at which the liver processes alcohol from the blood stream is related to the blood alcohol concentration x by a relation function of the form $r(x) = \frac{\alpha x}{x+\beta}$ for some positive constants α and β. Find the maximum possible rate of removal.

Sol:

$$\lim_{x \to \alpha} r(x) = \lim_{x \to \alpha} \frac{\alpha x}{x+\beta}$$

$$= \lim_{x \to \alpha} \frac{\alpha x}{x\left(1+\frac{\beta}{x}\right)}$$

$$= \lim_{x \to \alpha} \frac{\alpha}{1+\frac{\beta}{x}}$$

$$= \frac{\alpha}{1+0}$$

$$= \alpha.$$

Example : 9.26.

 According to Einstein's theory of relativity

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the mass m of a body moving with velocity v is $m = \dfrac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$ where m_0 is the initial mass and c is the speed of light. what happens to m as $v \to c^-$. why is a left hand limit necessary?

Sol:

$$\lim_{v\to c^-} = \lim_{v\to c^-} \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$= \frac{m_0}{\sqrt{\lim_{v\to c^-}\left(1-\frac{v^2}{v^2}\right)}}$$

$h > 0$

$$c-h < v^2 < c$$

$$(c-h)^2 < v^2 < c^2$$

$$\frac{(c-h)^2}{c^2} < \frac{v^2}{c^2} < \frac{c^2}{c^2}$$

$$\frac{(c-h)^2}{c^2} < \frac{v^2}{c^2} < 1$$

$$= \lim_{h\to 0} \frac{(c-h)^2}{c^2} < \lim_{h\to 0} \frac{v^2}{c^2} < \lim_{h\to 0} 1$$

$$\frac{c^2}{c^2} < \frac{v^2}{c^2} < 1$$

$$1 < \frac{v^2}{c^2} < 1$$

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$\therefore \lim_{v \to c^-} \frac{v^2}{c^2} = 1.$

$= \lim_{v \to c^-} (m) \to \infty$

That is the mass becomes very very large (infinite) as $v \to c^-$

The left hand limit is necessary. Otherwise as $v \to c^+$ makes $1 - \frac{v^2}{c^2} < 0$ and consequently.

Example: 9.27.

The velocity in ft/sec of a falling object is modeled by $r(t) = -\sqrt{\frac{32}{k}} \frac{1-e^{-2t\sqrt{32k}}}{1+e^{-2t\sqrt{32k}}}$ where k is a constant that depends upon the size and shape of the object and the density of the air. Find the limiting velocity of the object that is find $\lim_{t \to \infty} r(t)$

Sol:

$= \lim_{t \to \infty} r(t) = \lim_{t \to \infty} -\sqrt{\frac{32}{k}} \frac{1-e^{-2t\sqrt{32k}}}{1-e^{-2t\sqrt{32k}}}$

$e^{-\infty} = 0$

$e^0 = 1$

$= \sqrt{\frac{32}{k}} \lim_{t \to \infty} \frac{1-e^{-2t\sqrt{32k}}}{1+e^{-2t\sqrt{32k}}}$

$e^{\infty} = \infty$

$$= -\sqrt{\frac{32}{k}}\ \frac{(1-e)^{-\infty}}{(1+e)^{-\infty}}$$

$$= -\sqrt{\frac{32}{k}}\ \frac{(1-e)}{1+e}$$

$$= -\sqrt{\frac{32}{k}}\ ft/sec.$$

Example : 9.28 :

Suppose that the diameter of an animal's pupils is given by $f(x) = \frac{160x^{-0.4}+90}{4x^{-0.4}+15}$ where x is the intensity of light and $f(x)$ is in mm. Find the diameter of the pupil with

a) minimum light.

Sol:

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{160x^{-0.4}+90}{4x^{-0.4}+15}$$

$$= \lim_{x\to 0} \frac{\frac{160}{x^{0.4}}+90}{\frac{4}{x^{0.4}}+15}$$

$$= \lim_{x\to 0} \frac{160+90x^{0.4}}{4+15x^{0.4}}$$

$$= \frac{160}{4}$$

$$= 40$$

b) maximum light.

Sol:

$\lim_{x \to \infty} y(x) = \lim_{x \to \infty} \frac{100x^{-0.4}+90}{4x^{0.4}+15}$

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$= \frac{90}{15}$

$= 6mm.$

Result:

* $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$
* $\lim_{\theta \to 0} \frac{1-\cos\theta}{\theta} = 0$
* $\lim_{x \to 0} \frac{e^x - 1}{x} = 1, \quad \lim_{x \to 0} \frac{e^{-x}-1}{x} = -1$
* $\lim_{x \to 0} \frac{\log(1+x)}{x} = 1$
* $\lim_{x \to 0} \frac{a^x - 1}{x} = \log a, \; a > 0.$
* $\lim_{x \to 0} \frac{\sin^{-1}x}{x} = 1$
* $\lim_{x \to 0} \frac{\tan^{-1}x}{x} = 1$
* $\lim_{x \to 0} \left(1+\frac{1}{x}\right)^x = e$
* $\lim_{x \to 0} (1+x)^{1/x} = e$
* $\lim_{x \to \alpha} \left(1+\frac{k}{x}\right)^x = e^k.$

Exercise: 9.4.

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Evaluate the following limits

1) $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{7x}$.

Sol:

$$=\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{7x}$$

$$=\left[\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}\right]^{7}$$

$$=e^{7}.$$

2. $\lim_{x\to 0}(1+x)^{1/3x}$.

Sol:

$$=\lim_{x\to 0}(1+x)^{1/3x}$$

$$=\left[\lim_{x\to 0}(1+x)^{1/x}\right]^{1/3}$$

$$=e^{1/3}$$

3. $\lim_{x\to\infty}\left(1+\frac{k}{x}\right)^{m/x}$

Sol:

$$=\lim_{x\to\infty}\left(1+\frac{k}{x}\right)^{m/x}$$

Put $\frac{1}{x}=t$

$x\to\infty$ $\frac{1}{x}\to 0$

$$= \lim_{t \to 0} (1+kt)^{mt}$$

$$= (1+0)^0$$

$$= 1.$$

4. $\lim_{x \to \infty} \left(\frac{2x^2+3}{2x^2+5} \right)^{8x^2+3}$.

Sol:

$$= \lim_{x \to \infty} \left(\frac{2x^2+3}{2x^2+5} \right)^{8x^2+3}$$

$$= \lim_{x \to \infty} \left[\frac{2x^2 \left(1 + \frac{3}{2x^2}\right)}{2x^2 \left(1 + \frac{5}{2x^2}\right)} \right]^{8x^2+3}$$

5.

$$= \lim_{x \to \infty} \left(\frac{1 + \frac{3/2}{x^2}}{1 + \frac{5/2}{x^2}} \right)^{8x^2+3}$$

$$= \lim_{x \to \infty} \left(\frac{1 + \frac{3/2}{x^2}}{1 + \frac{5/2}{x^2}} \right)^{8x^2} \lim_{x \to \infty} \left(\frac{1 + \frac{3/2}{x^2}}{1 + \frac{5/2}{x^2}} \right)^{3}$$

$$= \left[\frac{\lim_{x \to \infty} \left(1 + \frac{3/2}{x^2}\right)^{x^2}}{\lim_{x \to \infty} \left(1 + \frac{5/2}{x^2}\right)^{x^2}} \right]^8 \left[\frac{1+0}{1+0} \right]^3$$

6.

$$= \left[\frac{e^{3/2}}{e^{5/2}} \right]^8 \cdot 1$$

$$= \left(\frac{1}{e^{5/2} e^{-3/2}}\right)^8$$

$$= \left(\frac{1}{e^{5/2-3/2}}\right)^8$$

$$= \left(\frac{1}{e^{\frac{5-3}{2}}}\right)^8$$

$$= \left(\frac{1}{e^{2/2}}\right)^8$$

$$= \left(\frac{1}{e}\right)^8$$

$$= \frac{1}{e^8}$$

5. $\lim\limits_{x \to \infty} \left(1 + \frac{3}{x}\right)^{x+2}$.

Sol:

$$= \lim_{x \to \infty} \left(1 + \frac{3}{x}\right)^{x+2}$$

$$= \lim_{x \to \infty} \left(1 + \frac{3}{x}\right)^{x} \lim_{x \to \infty} \left(1 + \frac{3}{x}\right)^{2}$$

$$= e^3 \, (1+0)^2$$

$$= e^3 . 1$$

$$= e^3.$$

6. $\lim\limits_{x \to 0} \frac{\sin^3 (x/2)}{x^3}$.

Sol:

$$= \lim_{x \to 0} \frac{\sin^3 (x/2)}{x^3}$$

$$= \lim_{x \to 0} \frac{\sin^3 (x/2)}{8 \times \frac{x^3}{8}}$$

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$$= \frac{1}{8} \lim_{x \to 0} \frac{\sin^3(x/2)}{(x/2)^3}$$

$$= \frac{1}{8} \left[\lim_{x \to 0} \frac{\sin(x/2)}{x/2} \right]^3$$

$$= \frac{1}{8} (1)^3$$

$$= \frac{1}{8}$$

7. $\lim_{x \to 0} \frac{\sin \alpha x}{\sin \beta x}$.

Sol:

$$= \lim_{x \to 0} \frac{\sin \alpha x}{\sin \beta x}$$

$$= \lim_{x \to 0} \frac{\alpha \cancel{x} \frac{\sin \alpha x}{\alpha x}}{\beta \cancel{x} \frac{\sin \beta x}{\beta x}}$$

$$= \frac{\alpha}{\beta} \frac{\lim_{x \to 0} \frac{\sin \alpha x}{\alpha x}}{\lim_{x \to 0} \frac{\sin \beta x}{\beta x}}$$

$$= \frac{\alpha}{\beta} \cdot \frac{1}{1}$$

$$= \frac{\alpha}{\beta}$$

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9) $\lim_{\alpha \to 0} \frac{\sin(\alpha^n)}{(\sin\alpha)^m}$

Sol:

$$= \lim_{\alpha \to 0} \frac{\sin(\alpha^n)}{(\sin\alpha)^m}$$

$$= \lim_{\alpha \to 0} \frac{\frac{\alpha^n . \sin(\alpha^n)}{\alpha^n}}{\frac{\alpha^m (\sin\alpha)^m}{\alpha^m}}$$

$(\sin)^m = \sin^m \alpha.$

$$= \frac{\alpha^n}{\alpha^m} \frac{\lim_{\alpha \to 0} \frac{\sin(\alpha^n)}{\alpha n}}{\lim_{\alpha \to 0} \left(\frac{\sin\alpha}{\alpha}\right)^m}$$

$$= \frac{\alpha^n}{\alpha^m} \frac{1}{\left(\lim_{n \to \alpha} \frac{\sin\alpha}{\alpha}\right)^m}$$

$$= \frac{\alpha^n}{\alpha^m} \cdot \frac{1}{1^m}$$

$$= \frac{\alpha^n}{\alpha^m}$$

$$= \alpha^n . \alpha^{-m}$$

$$= \alpha^{n-m}$$

10) $\lim_{x \to 0} \frac{\sin(a+x) - \sin(a-x)}{x}$

Sol:

$$= \lim_{x \to 0} \frac{\sin(a+x) - \sin(a-x)}{x}$$

$$= \lim_{x \to 0} \frac{2\cos a \cdot \sin x}{x}$$

$$= 2\cos a \lim_{x \to 0} \frac{\sin x}{x}$$

$\sin(A+B) - \sin(A-B) = 2\cos A \sin B.$

$$= 2\cos a \cdot 1$$

$$= 2\cos a.$$

12. $\lim_{x \to 0} \frac{2 \text{ arc} \sin x}{3x}.$

Sol:

$$= \lim_{x \to 0} \frac{2 \text{ arc} \sin x}{3x}$$

$$= \lim_{x \to 0} \frac{2\sin^{-1} x}{3x}.$$

$\text{arc} \sin x = \sin^{-1} x$
$\text{arc} \tan x = \tan^{-1} x$

$$= \frac{2}{3} \lim_{x \to 0} \frac{\sin^{-1} x}{x}$$

$$= \frac{2}{3}$$

13. $\lim_{x \to 0} \frac{1 - \cos x}{x^2}$

Sol:

$$= \lim_{x \to 0} \frac{1 - \cos x}{x^2}$$

$$= \lim_{x \to 0} \frac{2\sin^2 x/2}{x^2}$$

$$= 2 \lim_{x \to 0} \frac{\sin^2 x/2}{4 \times x^2/4}$$

$$= \frac{2}{4} \lim_{x \to 0} \left[\frac{\sin(x/2)}{(x/2)}\right]^4$$

$$= \frac{1}{2} (1)^2$$

$$= \frac{1}{2}.$$

14. $\lim_{x \to 0} \frac{\tan 2x}{x}.$

Sol:

$$= \lim_{x \to 0} \frac{\tan 2x}{x}$$

$$= \lim_{x \to 0} \frac{\tan 2x}{2x \cdot 1/2}$$

$$= 2 \lim_{x \to 0} \frac{\tan 2x}{2x}$$

$$= 2(1)$$

$$= 2.$$

16. $\lim_{x \to 0} \frac{3^x - 1}{\sqrt{x+1} - 1}.$

Sol:

$$= \lim_{x \to 0} \frac{3^x - 1}{\sqrt{x+1} - 1}$$

$$= \lim_{x \to 0} \frac{3^x - 1}{\sqrt{x+1} - 1} \times \frac{\sqrt{x+1} + 1}{\sqrt{x+1} + 1}$$

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$$= \lim_{x\to 0} \frac{(3^x-1)(\sqrt{x-1}+1)}{x+1-1}$$

$$= \lim_{x\to 0} \frac{3^x-1}{x} \cdot \lim_{x\to 0} \sqrt{x+1}+1$$

$$\left[\because \lim_{x\to 0} \frac{a^x-1}{x} = \log a\right]$$

$$= \log 3 (2)$$

$$= 2 \log 3$$

7) $\lim_{x\to 0} \frac{1-\cos^2 x}{x \sin 2x}$.

Sol:

$$= \lim_{x\to 0} \frac{1-\cos^2 x}{x \sin 2x}$$

$$= \lim_{x\to 0} \frac{\sin^2 x}{x \cdot 2\sin x \cos x}$$

$$= \lim_{x\to 0} \frac{\tan x}{2x}$$

$$= \frac{1}{2} \lim_{x\to 0} \frac{\tan x}{0}$$

$$= \frac{1}{2}(1)$$

$$= \frac{1}{2}.$$

S. Arun

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