EXERCISE PROBLEMS

XII L-3 Magnetism and Magnetic effects of electric current

1.) A bar magnet having a magnetic moment $\vec{M}$ is cut into four pieces i.e., first cut into two pieces along the axis of the magnet and each piece is further cut into two pieces. compute the magnetic moment of each piece.

when Bar magnet cut into two pieces, the magnetic strength is reduced to half and also magnetic moments reduces to half but length remains same $(2l)$.

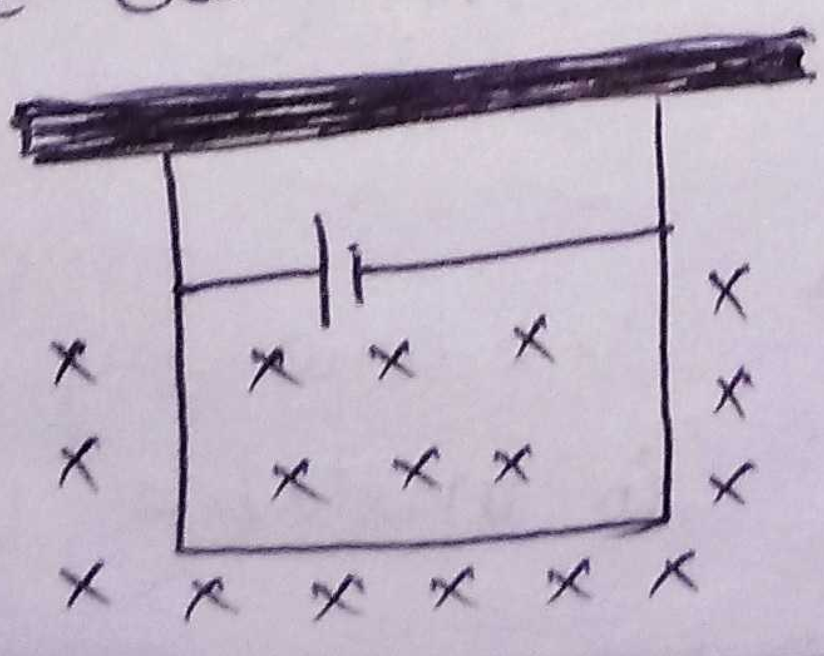
Magnetic moment $= \frac{M}{2}$

Again each piece is cut into two, so magnetic moment again becomes half i.e., $\frac{M/2}{2} = \frac{M}{4}$

$\therefore$ Magnetic moment of each piece

$\vec{M}_{new} = \frac{1}{4}\vec{M}$

2.) A conductor of linear mass density $0.2 g m^{-1}$ suspended by two flexible wire as shown in figure. Suppose the tension in the supporting wires is zero when it is kept inside the magnetic field of 1T whose direction is into the page. Compute the current inside and also the direction for the current. $(g = 10 ms^{-2})$



Linear mass density $\left(\frac{m}{l}\right) = 0.2 g m^{-1}$

$\frac{m}{l} = 0.2 \times 10^{-3} kg m^{-1}$

$= 2 \times 10^{-4} kg m^{-1}$

Magnetic field $(B) = 1T$

$g = 10 ms^{-2}$

when the conductor is placed in magnetic field of length 'l' and I is the current passing through the conductor experiences a Magnetic force of $F = BIl \sin\theta$. In the diagram $\theta = 90°$

$\therefore F = BIl \sin 90°$

$= BIl(1) = BIl$

The weight of the conductor having mass (m) $= mg$

The tension of the conducting wire 'T' means the tension in the wire becomes zero.

$T = mg - BIl = 0$

$BIl = mg$

$I = \frac{m}{l} g / B$

$\therefore I = \frac{2 \times 10^{-4} \times 10}{1} = 2 \times 10^{-3} A$

$\boxed{\therefore I = 2 mA}$

3.) A circular coil with cross-sectional area $0.1 cm^2$ is kept in a uniform magnetic field of strength 0.2T. If the current passing in the coil is 3A and plane of the loop is perpendicular to the direction of magnetic field calculate (a) total torque on the coil b) total force on the coil

c) Average force on each electron in the coil due to the magnetic field of the free electron density for the material of the wire is $10^{28} m^{-3}$.

Area of cross section $A = 0.1\ cm^2$

$= 0.1 \times (10^{-2})^2 = 1 \times 10^{-5} m$

Magnetic strength $B = 0.2T$

Current $I = 3A$

a) Total Torque on the coil $N = 1$

$\tau = NBIA \sin\theta$

$= 1 \times 0.2 \times 3 \times 1 \times 10^{-5} \times \sin 0°$

$= 0.6 \times 1 \times 10^{-5} (0)$

θ is the angle between normal drawn on the plane of the coil and the direction of magnetic field $(\theta = 0°)$

$\therefore \tau = 0$

(b) Total force on the coil

$F = BIl \sin\theta$

$\therefore F = BIl(0)$ $\quad \theta = 0$

$\boxed{F = 0}$

c) $n = 10^{28} m^{-3}$

Average force on each electron in the coil due to magnetic field

$\vec{F} = q(\vec{v} \times \vec{B}) = e(\vec{V_d} \times \vec{B})$

$\vec{F} = eV_d B \sin\theta$

$\vec{F} = eV_d B$ $\quad \theta = 90°$

But $I = neV_d A$

$V_d = \frac{I}{neA}$

$\therefore F = \cancel{e}\left(\frac{I}{n\cancel{e}A}\right)B$

$F = \frac{BI}{nA}$

$F = \frac{0.2 \times 3}{10^{28} \times 1 \times 10^{-5}} = \frac{0.6}{10^{23}}$

$\boxed{F = 0.6 \times 10^{-23} N}$

4.) A bar magnet is placed in a uniform magnetic field whose length is 0.8T. Suppose the bar magnet orient an angle 30° with the external field experiences a torque of 0.2 Nm. Calculate (i) the magnetic moment of the bar magnet ii) the work done by an applied force in moving it from most stable configuration to the most unstable configuration and also compute the work done by the applied magnetic field in this case.

Magnetic field strength $(B) = 0.8T$

The angle ~~inclined with a~~ magnetic field $\theta = 30°$

Torque $(\tau) = 0.2\ Nm$

Magnetic moment of the magnet $\boxed{\tau = MB \sin\theta}$

$M = \frac{\tau}{B \sin\theta}$

$= \frac{0.2}{0.8 \times \sin 30°}$

$= \frac{0.2 \times 2}{0.8 \times 1} = \frac{0.4}{0.8}$

$= \frac{4}{8} = 0.5 Am^2$

ii) Work done by an applied force in stable $U_i = -MB \cos\theta$

$\theta = 0°$

Workdone by an applied force in unstable $U_f = -MB \cos\theta$

Work done by the applied magnetic field $W = U_f - U_i$

$= -\mu B \cos 180° - (-\mu B \cos 0°)$

$= -\mu B(-1) + \mu B(1)$

$= \mu B + \mu B = 2\mu B$

$\therefore W = 2 \times 0.5 \times 0.8 = 0.8 J$

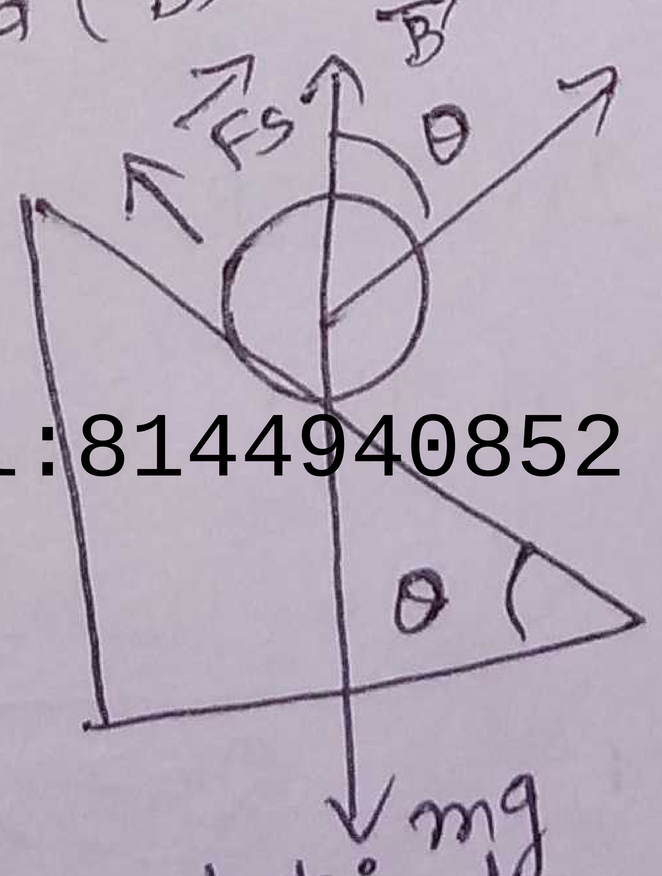
5.) A non conducting sphere has mass of 100g and radius 20cm. A flat compact coil of wire with turns 5 is wrapped tightly around it with each turns concentric with the sphere. This sphere is placed on an inclined plane such that plane of coil is parallel to the inclined plane. A uniform magnetic field of 0.5T exists in the region in vertically upwards direction. Compute the current I required to rest the sphere in equillibrium.

Mass of the non-conducting sphere $= 100g = 100 \times 10^{-3} kg = 0.1 kg$

Radius $R = 20cm = 20 \times 10^{-2} m$

Number of turns $(n) = 5$

Magnetic field $(B) = 0.5 T$



The sphere is in translational equillibrium thus $F_s - mg \sin\theta = 0$

$F_s = mg \sin\theta \longrightarrow (1)$

The frictional force (f_s) produces a anticlockwise torque of magnitude $\tau = f_s R$

$\therefore F_s R - \mu B \sin\theta = 0 \longrightarrow (2)$

$R \rightarrow$ Radius of sphere

Sub (1) in 2nd equation we get

$(mg \cancel{\sin\theta}) R = \mu B \cancel{\sin\theta}$

$mgR = \mu B$

Sub $\mu = NIA$, $A = \pi R^2$

$\therefore mgR = NI(\pi R^2) B$

$mgR = NI\pi R^2 B$

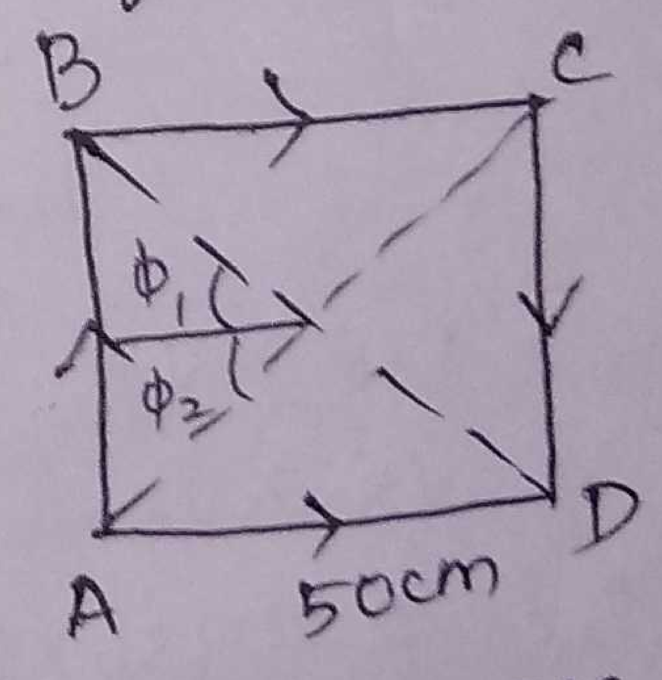
$I = \frac{mgR}{\pi N B R^2} = \frac{mg}{\pi NBR}$

$I = \frac{0.1 \times 10}{\pi \times 5 \times 0.5 \times 0.2}$

$= \frac{1}{\pi \times 0.5} = \frac{1}{\pi \times 1/2} = \frac{2}{\pi} A$

$I = 2/\pi \, A$

6.) Calculate the magnetic field at the center of a square loop which carries a current of 1.5A, length of each loop is 50cm.



Current through a square loop is $I = 1.5 A$

Length of each loop $l = 50cm = 50 \times 10^{-2} m$

Magnetic field at the centre of a square loop = ?

According to Biot Savart law

$$B = \frac{\mu_0 I}{4\pi a}(\sin\phi_1 + \sin\phi_2)$$

$$\phi_1 = \phi_2 = 45^\circ$$

$$\sin 45^\circ = \frac{1}{\sqrt{2}}$$

The magnetic field at one side $(B) = \frac{\mu_0 I}{4\pi(l/2)}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)$

$a = \frac{l}{2}$

$$B = \frac{\mu_0 I \times 2 \times 2}{4\pi L \times \sqrt{2}} = \frac{\mu_0 I (\cancel{4})}{\cancel{4}\pi L\sqrt{2}}$$

$$B = \frac{\mu_0 I}{\sqrt{2} L \pi}$$

But the square has 4 sides

$$\therefore B = \frac{\mu_0 I}{\sqrt{2} L \pi} \times 4$$

$$= \frac{4\pi \times 10^{-7} \times 4 \times 1.5}{\sqrt{2} \times 50 \times 10^{-2} \times 3.14}$$

$$= \frac{16\cancel{\pi} \times \overset{0.3}{\cancel{1.5}} \times 10^{-7}}{1.414 \times \cancel{5} \times 10^{-1} \times \cancel{\pi}}$$

$$= \frac{4.8 \times 10^{-6}}{1.414}$$

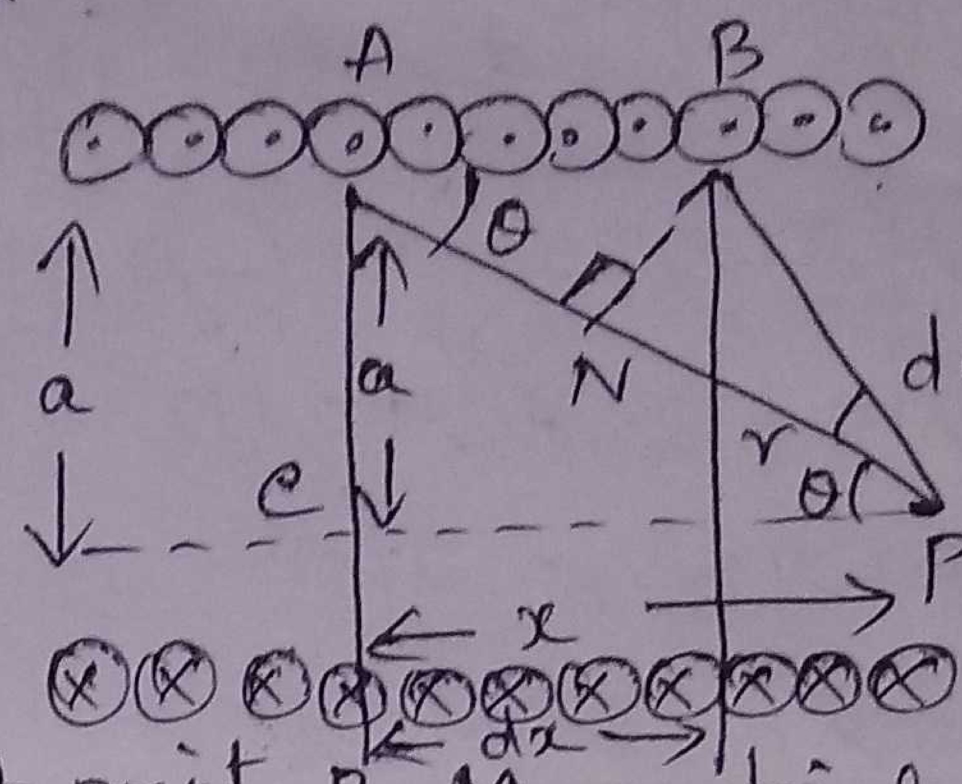
$$\frac{4.8 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{\overset{2.4}{\cancel{4.8}} \times 1.414 \times 10^{-6}}{\cancel{2}}$$

$$B = 3.3936 \times 10^{-6}$$

$$= 3.34 \times 10^{-6}\ T$$

7) S.T the magnetic field at any point on the axis of the solenoid having turns per unit length is

$$B = \frac{1}{2}\mu_0 n I(\cos\theta_1 - \cos\theta_2)$$



At point P Magnetic field

$$dB = \frac{\mu_0 N I a^2}{2(a^2+x^2)^{3/2}}$$

$N = n\,dx$

$$\therefore dB = \frac{\mu_0 (n\,dx) I a^2}{2(a^2+x^2)^{3/2}} \rightarrow (1)$$

From the diagram $\sin\theta = \frac{BN}{AB} = \frac{r\,d\theta}{dx}$

$$dx = \frac{r\,d\theta}{\sin\theta} \rightarrow (2)$$

In ΔACP $a^2+x^2 = r^2 \Rightarrow r^2 = a^2+x^2$

$r = (a^2+x^2)^{1/2}$

$$r^3 = r \cdot r^2 = (a^2+x^2)(a^2+x^2)^{1/2} = (a^2+x^2)^{3/2} \rightarrow (3)$$

Sub (2) & (3) in eq (1)

we get $dB = \frac{\mu_0 \left(n \frac{r\,d\theta}{\sin\theta}\right) I a^2}{2r^3}$

$$dB = \frac{\mu_0 n I a^2\,d\theta}{2r^2 \sin\theta}$$

In ΔACP $\sin\theta = \frac{a}{r} \Rightarrow \sin^2\theta = \frac{a^2}{r^2}$

Sub this in the above equation

$$dB = \frac{\mu_0 n I a^2\,d\theta}{2r^2 \sin\theta} = \frac{\mu_0 n I\,d\theta\,(\sin^2\theta)}{2\sin\theta}$$

$$dB = \frac{\mu_0 n I}{2}\sin\theta\,d\theta = \frac{1}{2}\mu_0 n I \sin\theta\,d\theta$$

∴ The magnetic field at any point on the axis of the solenoid

$$B = \int_{\theta_1}^{\theta_2} dB = \int_{\theta_1}^{\theta_2} \tfrac{1}{2}\mu_0 n I \sin\theta \, d\theta$$

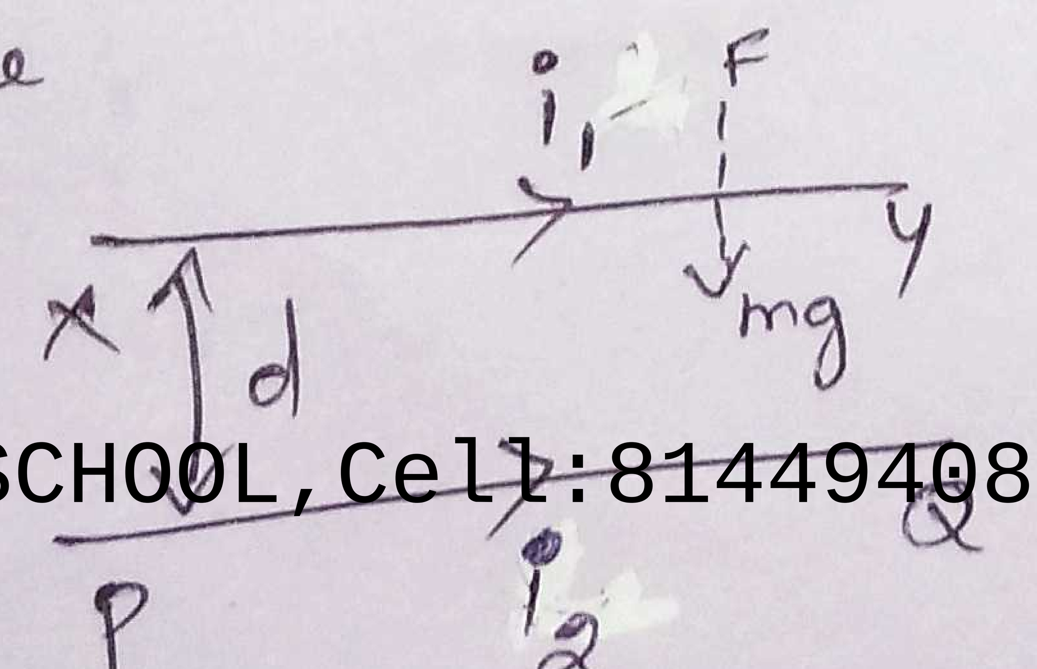
$$= \tfrac{1}{2}\mu_0 n I \int_{\theta_1}^{\theta_2} \sin\theta \, d\theta$$

$$B = \tfrac{1}{2}\mu_0 n I \left(-\cos\theta\right)_{\theta_1}^{\theta_2}$$

$$= \tfrac{1}{2}\mu_0 n I \left[-\cos\theta_2 - (-\cos\theta_1)\right]$$

$$\boxed{B = \tfrac{1}{2}\mu_0 n I \left(\cos\theta_1 - \cos\theta_2\right)}$$

8) Let I_1 and I_2 be the steady currents passing through a long horizontal wire XY and PQ respectively. Suppose the wire PQ is fixed in horizontal plane and the wire XY is allowed to move freely in a vertical plane. Let the wire XY is in equillibrium at a height d over the parallel wire PQ as shown on figure



The current i_2 flowing in the direction of PQ experience Magnetic force on XY

$$F = \frac{\mu_0}{4\pi}\frac{2 I_1 I_2}{d}$$

$$F_1 = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d} \text{ (Upward)}$$

The Downward force

$$F_2 = mg$$

At Equillibrium $F_1 = F_2$

$$\therefore mg = \frac{\mu_0 I_1 I_2}{2\pi d} \rightarrow (1)$$

If the wire XY is pulled the distance of y along PQ

$$F' = \frac{\mu_0 I_1 I_2}{2\pi(d-y)} \text{ (Upward force)} \rightarrow (2)$$

The resultant force of XY

$$F_R = F' - mg = \frac{\mu_0 I_1 I_2}{2\pi(d-y)} - mg$$

$$\therefore F_R = \frac{\mu_0 I_1 I_2}{2\pi(d-y)} - \frac{\mu_0 I_1 I_2}{2\pi d} \quad \left(mg = \frac{\mu_0 I_1 I_2}{2\pi d}\right)$$

$$\therefore F_R = \frac{\mu_0 I_1 I_2}{2\pi}\left(\frac{1}{d-y} - \frac{1}{d}\right)$$

$$= \frac{\mu_0 I_1 I_2}{2\pi}\left[\frac{d-(d-y)}{d(d-y)}\right]$$

$$= \frac{\mu_0 I_1 I_2}{2\pi}\left\{\frac{y}{d(d-y)}\right\} = \frac{\mu_0 I_1 I_2 y}{2\pi d^2}$$

$\because d >> x$ $\therefore dy$ neglected

$$F_R = \frac{\mu_0 I_1 I_2 y}{2\pi d^2} \rightarrow (3)$$

$$F_R \propto y \Rightarrow F_R = -m\omega^2 y$$

$$\left[\text{S.H.M } a_y = -\omega^2 y\right]$$

$$F_R = m a_y$$

XY comes to uniform Oscillation

Hence $F_R = -ky \rightarrow (4)$

$(3) \Rightarrow F_R = -\left(\dfrac{\mu_0 I_1 I_2}{2\pi d^2}\right) y$

Comparing (3) & (4)

$$k = \frac{\mu_0 I_1 I_2}{2\pi d^2} = \frac{mg}{d}$$

Time period $T = 2\pi\sqrt{m/k}$

$$= 2\pi\sqrt{\frac{\cancel{m}}{\cancel{m}g/d}}$$

$$\boxed{T = 2\pi\sqrt{d/g}}$$