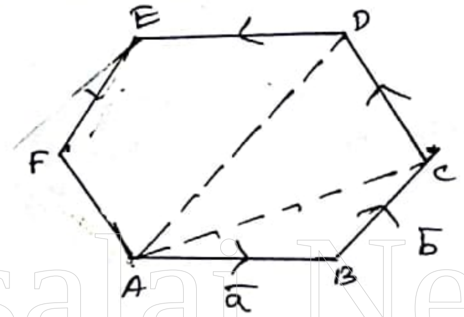
Vector Algebra - EXERCISE 8.1.

EX: 8.1. Represent Graphically the displacement of
i) 30 KM 60° west of north 2) 60KM 50° South of east.

EX: 8.2 If $\bar{a}, \bar{b}$ are vectors represented by two adjacent sides of regular hexagon, then find the vectors represented by other sides.



Sol: Let $\overline{AB} = \bar{a}$, $\overline{BC} = \bar{b}$

By Triangle law of addition

$$\overline{AC} = \bar{a} + \bar{b}$$

But $\overline{AD} = 2\,\overline{BC}$
$= 2\bar{b}$

Again by Δle law of addition.

$$\overline{AC} + \overline{CD} = \overline{AD}$$
$$\overline{CD} = \overline{AD} - \overline{AC}$$
$$= 2\bar{b} - (\bar{a} + \bar{b})$$
$$= \bar{b} - \bar{a}$$

$\overline{AB} = -\overline{DE} = -\bar{a}$

$\overline{EF} = -\overline{BC} = -\bar{b}$

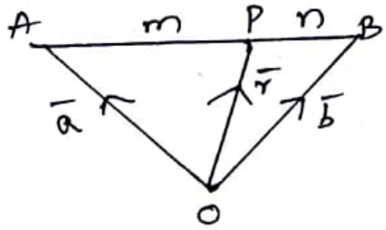
$\overline{FA} = -\overline{CD} = \bar{a} - \bar{b}$.

∴ $\overline{AB} = \bar{a}$
$\overline{BC} = \bar{b}$
$\overline{CD} = \bar{b} - \bar{a}$
$\overline{DE} = -\bar{a}$
$\overline{EF} = -\bar{b}$
$\overline{FA} = \bar{a} - \bar{b}$.

Theorem: 8.1. Section Formula (Internal Division)

Let O be the origin. Let A, B be two points let P be the point which divides the line segment AB internally in the ratio $m:n$. If $\bar{a}, \bar{b}$ are the p.v of the point A and B then the p.v of the point P is given by $\overline{OP} = \frac{n\bar{a} + m\bar{b}}{m+n}$.

Proof. Let $\overline{OA} = \bar{a}$, $\overline{OB} = \bar{b}$ and $\overline{OP} = \bar{r}$.



Let P divides AB in the ratio $m:n$ internally

$$\frac{AP}{PB} = \frac{m}{n}$$

$$\frac{|\overline{AP}|}{|\overline{PB}|} = \frac{m}{n}.$$

(ie) $n|\overline{AP}| = m|\overline{PB}|$

But $\overline{AP} = \overline{OP} - \overline{OA}$
$= \bar{r} - \bar{a}$

$\overline{PB} = \overline{OB} - \overline{OP} = \bar{b} - \bar{r}$

$\therefore n(\bar{r} - \bar{a}) = m(\bar{b} - \bar{r})$

$n\bar{r} - n\bar{a} = m\bar{b} - m\bar{r}$

$(n+m)\bar{r} = m\bar{b} + n\bar{a}$

$$\bar{r} = \frac{m\bar{b} + n\bar{a}}{m+n}$$

Note: 1) If P divides AB in the ratio $m:n$ externally then

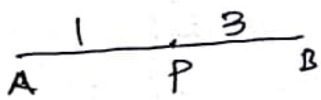
$$\bar{r} = \frac{m\bar{b} - n\bar{a}}{m-n}.$$

2) If P is the mid point of AB $m:n = 1:1$ then $\overline{OP}$

$$\bar{r} = \frac{\bar{a} + \bar{b}}{2}$$

Ex 8.3 Let A and B be two points with P.V. $2\bar{a} + 4\bar{b}$, $2\bar{a} - 8\bar{b}$. Find the P.V of the points which divides the line segment joining A and B in the ratio $1:3$ internally and externally.

Internally $\overline{OA} = 2\bar{a} + 4\bar{b}$
$\overline{OB} = 2\bar{a} - 8\bar{b}$



ratio $1:3$ internally.

$$\overline{OP} = \frac{1\overline{OB} + 3\overline{OA}}{1+3} = \frac{(2\bar{a} - 8\bar{b}) + 3(2\bar{a} + 4\bar{b})}{1+3}$$

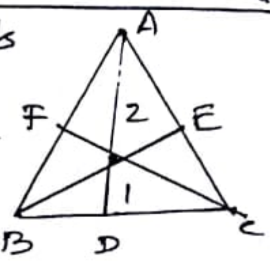
$$= \frac{8\bar{a} + 4\bar{b}}{4} = \frac{\cancel{4}(2\bar{a} + \bar{b})}{\cancel{4}}$$

Externally:
$$\overline{OP} = \frac{1\cdot\overline{OB} - 3\cdot\overline{OA}}{1-3} = \frac{(2\bar{a} - 8\bar{b}) - 3(2\bar{a} + 4\bar{b})}{-2}$$

$$= \frac{-4\bar{a} - 20\bar{b}}{-2} = \frac{\cancel{-2}(2\bar{a} + 10\bar{b})}{\cancel{-2}}$$

Theorem 8:3) The medians of a $\triangle^{le}$ are concurrent

In a $\triangle^{le}$ ABC D, E, F are the mid points of BC, CA, AB. ∴ AD, BE, CF are the medians.



Let $\overline{OA} = \bar{a}$, $\overline{OB} = \bar{b}$, $\overline{OC} = \bar{c}$

$$\therefore \quad \overline{OD} = \frac{\bar{b}+\bar{c}}{2}, \quad \overline{OE} = \frac{\bar{c}+\bar{a}}{2} \quad \overline{OF} = \frac{\bar{a}+\bar{b}}{2}$$

Let G_1 be the point on AD which divides AD in the ratio 2:1

$$\therefore \overline{OG_1} = \frac{2 \cdot \overline{OD} + 1 \cdot \overline{OA}}{2+1} = \frac{\cancel{2}\left(\frac{\bar{b}+\bar{c}}{\cancel{2}}\right)+1 \cdot \bar{a}}{3} = \frac{\bar{a}+\bar{b}+\bar{c}}{3}$$

Let G_2 be the point on BE which divides in the ratio 2:1.

and G_3 be the point on CF which divides in the ratio 2:1

$|||^{ly}$ we can find that $\overline{OG_2} = \frac{\bar{a}+\bar{b}+\bar{c}}{2}$

$$\overline{OG_3} = \frac{\bar{a}+\bar{b}+\bar{c}}{2}$$

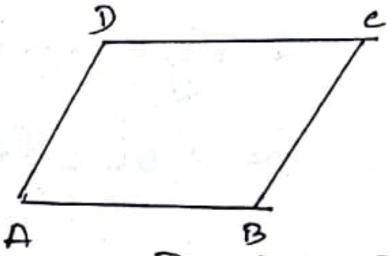
∴ G_1, G_2, G_3 are the same point. Hence medians of a $\triangle^{le}$ are concurrent.

G is the centroid of the $\triangle^{le}$. ∴ $\overline{OG} = \frac{\bar{a}+\bar{b}+\bar{c}}{3}$

Theorem 8.4) A quadrilateral is a parallelogram iff its diagonals bisect each other.

Proof: Let ABCD be a quadrilateral with diagonals AC and BD.

$\overline{OA} = \bar{a}$, $\overline{OB} = \bar{b}$, $\overline{OC} = \bar{c}$ $\overline{OD} = \bar{d}$



I If ABCD is a parallelogram

$$\overline{AB} = \overline{DC}$$
$$\overline{OB} - \overline{OA} = \overline{OC} - \overline{OD}$$
$$\bar{b} - \bar{a} = \bar{c} - \bar{d}$$
$$\bar{b} + \bar{d} = \bar{a} + \bar{c}$$
$$\frac{\bar{a}+\bar{c}}{2} = \frac{\bar{b}+\bar{d}}{2}$$

⇒ Mid points of AC and BD are same.

II If $\frac{\bar{a}+\bar{c}}{2} = \frac{\bar{b}+\bar{d}}{2} \Rightarrow \bar{a}+\bar{c} = \bar{b}+\bar{d}$

$$\bar{c} - \bar{d} = \bar{b} - \bar{a}$$
$$\Rightarrow \overline{OC} - \overline{OD} = \overline{OB} - \overline{OA}$$
$$\Rightarrow \overline{DC} = \overline{AB}$$

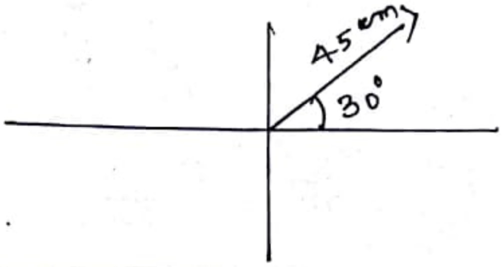
This shows that AB, CD are equal and parallel $|||^{ly}$ we can prove BC, AD are parallel and equal

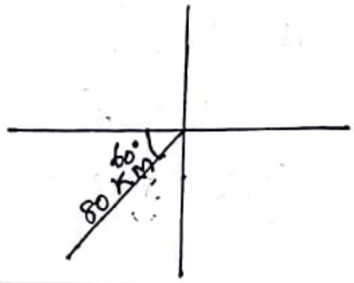
∴ ABCD is parallelogram.

EXERCISE - 8.1

1) Represent graphically the displacement of

1) 45 cm 30° north of east 2) 80 KM, 60° south of west.





3) Let $\bar{a}$ and $\bar{b}$ be the po.v of the points A and B. P.T the P.V of the points which trisects the line segment AB are $\frac{\bar{a}+2\bar{b}}{3}$, $\frac{\bar{b}+2\bar{a}}{3}$.

Let $\overline{OA} = \bar{a}$, $\overline{OB} = \bar{b}$

Let P be a point which divides AB in the ratio 1 : 2 internally.

$$\overline{OP} = \frac{1\cdot\bar{b} + 2\cdot\bar{a}}{1+2} = \frac{\bar{b}+2\bar{a}}{3}$$

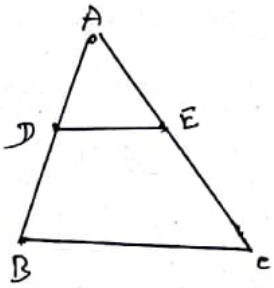
Let Q be the point on AB which divides AB in the ratio 2 : 1

$$\frac{2\cdot\bar{b}+\bar{a}}{2+1} = \frac{2\bar{b}+\bar{a}}{3}.$$

4) If D, E are the mid points of the sides AB and AC of a $\triangle$le ABC P.T $\overline{BE}+\overline{DC} = \frac{3}{2}\overline{BC}$

$$\overline{BE} = \overline{BC}+\overline{CE}$$
$$= \overline{BC}+\frac{1}{2}\overline{CA}$$

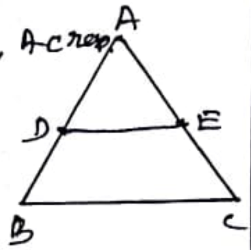
$$\overline{DC} = \overline{DB}+\overline{BC}$$
$$= \frac{1}{2}\overline{AB}+\overline{BC}$$



$$\therefore \overline{BE}+\overline{DC} = \overline{BC}+\frac{1}{2}\overline{CA}+\frac{1}{2}\overline{AB}+\overline{BC}$$
$$= 2\overline{BC}+\frac{1}{2}(\overline{CA}+\overline{AB})$$
$$= 2\overline{BC}+\frac{1}{2}(\overline{CB})$$
$$= 2\overline{BC}-\frac{1}{2}\overline{BC}$$
$$= 3/2\,\overline{BC}.$$

5) P.T the line segment joining the mid points of two sides of a triangle is parallel to the third side whose length is half of the length of the third side.

In a $\triangle$le ABC D, E are the mid point of AB, AC resp.



$\therefore \vec{OD} = \frac{\vec{OA}+\vec{OB}}{2}$ $\qquad \vec{OE} = \frac{\vec{OA}+\vec{OC}}{2}$

$\vec{DE} = \vec{OE} - \vec{OD} = \frac{\cancel{\vec{OA}}+\vec{OC}}{2} - \frac{\cancel{\vec{OA}}+\vec{OB}}{2}$

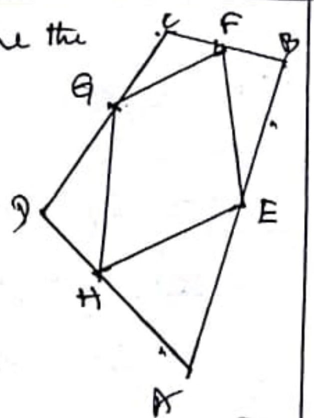
$= \frac{\vec{OC}-\vec{OB}}{2} = \frac{1}{2}(\vec{BC})$

$|\vec{DE}| = \frac{1}{2}|\vec{BC}|$

which implies that line joining of the mid points of the two sides is parallel to third side and length is equal to $\frac{1}{2}$ of the length of the third side.

6) P.T the line segments joining the mid points of the adjacent sides of a quadrilateral from a parallelogram.

ABCD is a quadrilateral in which E, F, G, H are the are the mid points of AB, BC, CD, DA resp.



$\vec{OE} = \frac{\vec{OA}+\vec{OB}}{2}$ $\qquad \vec{OG} = \frac{\vec{OD}+\vec{OC}}{2}$

$\vec{OF} = \frac{\vec{OB}+\vec{OC}}{2}$ $\qquad \vec{OH} = \frac{\vec{OD}+\vec{OA}}{2}$

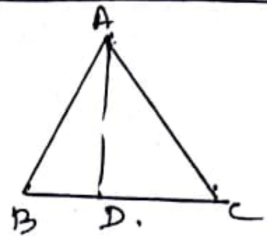
Mid point of $\vec{EG} = \frac{\frac{\vec{OA}+\vec{OB}}{2} + \frac{\vec{OC}+\vec{OD}}{2}}{2} = \frac{\vec{OA}+\vec{OB}+\vec{OC}+\vec{OD}}{4}$.

Mid point of $\vec{FH} = \frac{\frac{\vec{OB}+\vec{OC}}{2} + \frac{\vec{OD}+\vec{OA}}{2}}{2} = \frac{\vec{OA}+\vec{OB}+\vec{OC}+\vec{OD}}{4}$.

$\therefore$ The mid point of the diagonals EG and FH are same EFGH is a parallelogram.

9) If D is the mid point of the side BC of a $\triangle$ ABC P.T $\overline{AB}+\overline{AC}=2\overline{AD}$.

Let D be the mid point of BC of the $\triangle$ ABC



$\overline{AB} = \overline{AD}+\overline{DB}$ —— ①

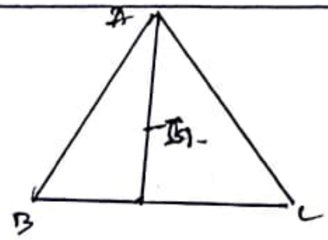
$\overline{AC} = \overline{AD}+\overline{DC}$

$= \overline{AD}-\overline{DB}$ — ② $\because$ D is the midpoint of BC and DC and DB are opposite

$\overline{AB}+\overline{AC} = 2\overline{AD}$

10) If G is the centroid of the $\triangle$ ABC P.T $\overline{GA}+\overline{GB}+\overline{GC}=\overline{0}$

Let G be the centroid of the $\triangle$ ABC



$\overline{OG} = \dfrac{\overline{OA}+\overline{OB}+\overline{OC}}{3}$

$3\overline{OG} = \overline{OA}+\overline{OB}+\overline{OC}$ —— ①

Given $\overline{GA}+\overline{GB}+\overline{GC} = \overline{OA}-\overline{OG}+\overline{OB}-\overline{OG}+\overline{OC}-\overline{OG}$

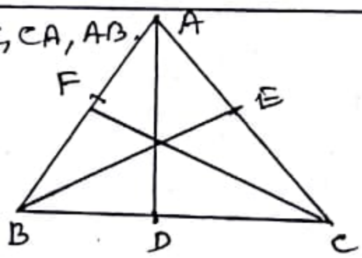
$= (\overline{OA}+\overline{OB}+\overline{OC}) - 3\overline{OG}$

$= 3\overline{OG} - 3\overline{OG}$

$= \overline{0}$

11) Let A, B, C are the vertices of a $\triangle$le. Let D, E and F are the mid points of the sides BC, CA, AB resp. S.T $\overline{AD}+\overline{BE}+\overline{CF}=0$.

Let D, E, F are the mid points of BC, CA, AB



$\overline{AD} = \overline{AB}+\overline{BD} = \overline{AB}+\frac{1}{2}\overline{BC}$

$\overline{BE} = \overline{BC}+\overline{CE} = \overline{BC}+\frac{1}{2}\overline{CA}$

$\overline{CF} = \overline{CA}+\overline{AF} = \overline{CA}+\frac{1}{2}\overline{AB}$

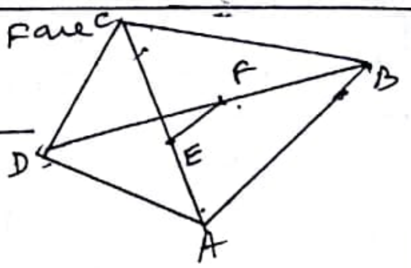
$\overline{AD}+\overline{BE}+\overline{CF} = \frac{3}{2}\overline{AB}+\frac{3}{2}\overline{BC}+\frac{3}{2}\overline{CA}$

$= \frac{3}{2}(\overline{AB}+\overline{BC}+\overline{CA})$

$= \frac{3}{2}(0) = 0$

12) If ABCD is a quadrilateral and E, F are the mid points of AC and BD resp then P.T $\overline{AB}+\overline{AD}+\overline{CB}+\overline{CD}=4\overline{EF}$

ABCD is a quadrilateral in which, E, F are the mid points of AC, BD



$\vec{AB} = \vec{AE} + \vec{EF} + \vec{FB}$

$\vec{AD} = \vec{AE} + \vec{EF} + \vec{FD}$

$\vec{CB} = \vec{CE} + \vec{EF} + \vec{FB}$

$\vec{CD} = \vec{CE} + \vec{EF} + \vec{FD}$

$\vec{AB} + \vec{AD} + \vec{CB} + \vec{CD} = 2(\vec{AE} + \vec{CE}) + 4\vec{EF} + 2(\vec{FB} + \vec{FD})$

$= 2(0) + 4\vec{EF} + 2(0)$

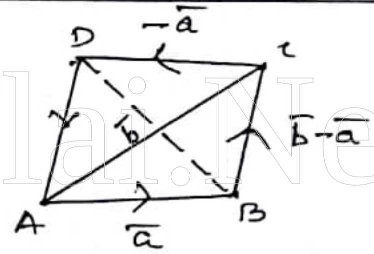
$= 4\vec{EF}$

$\because$ $\vec{AE}, \vec{CE}$ are equal but opposite

$\vec{FB}, \vec{FD}$ are equal but opposite.

7) If $\bar{a}, \bar{b}$ represent a side and a diagonal of a parallelogram find the other sides and the other diagonal.

In a parallelogram ABCD



$\vec{AB} = \bar{a}, \vec{AC} = \bar{b}$

By T.L.A.

$\vec{AB} + \vec{BC} = \vec{AC}$

$\vec{BC} = \vec{AC} - \vec{AB}$

$= \bar{b} - \bar{a}$

$\vec{DA}$ is equal and opposite to $\vec{BC}$

$\therefore \vec{DA} = -(\bar{a} - \bar{b})$

$\vec{CD}$ is equal and opposite to $\vec{AB}$

$\vec{CD} = -\bar{a}$.

$\vec{BD} = \vec{BC} + \vec{CD}$

$= (\bar{b} - \bar{a}) + (-\bar{a}) = \bar{b} - 2\bar{a}$

8) If $\vec{PQ} + \vec{OQ} = \vec{QO} + \vec{OR}$ P.T the points P, Q, R are collinear.

Given $\vec{PQ} + \vec{OQ} = \vec{QO} + \vec{OR}$

$\vec{PQ} = \vec{QR}$ —— (1).

Again $\vec{PQ} + \vec{QR} = \cancel{\vec{OQ}} - \vec{OP} + \vec{OR} - \cancel{\vec{OQ}}$

$= \vec{OR} - \vec{OP}$

$= \vec{PR}$ —— (2)

From (1) and (2), P, Q, R are collinear.

EXERCISE – 8.2.

Ex 8.4. Find the unit vector along the direction of $5\hat{i}-3\hat{j}+4\hat{k}$

Let $\bar{a} = 5\hat{i}-3\hat{j}+4\hat{k}$

$|\bar{a}| = \sqrt{25+9+16} = \sqrt{50}$

$\hat{a} = \frac{\bar{a}}{|\bar{a}|} = \frac{5\hat{i}-3\hat{j}+4\hat{k}}{\sqrt{50}}$.

Note! unit vector parallel to $\bar{a}$ but in the opposite direction

$\hat{a} = -\frac{\bar{a}}{|\bar{a}|}$

Ex: 8.5) Find the direction ratios and direction cosines of

i) $3\hat{i}+4\hat{j}-6\hat{k}$ 2) $3\hat{i}-4\hat{k}$

Let $\bar{a} = 3\hat{i}+4\hat{j}-6\hat{k}$

Dr's of $\bar{a}$ = 3, 4, −6. $|\bar{a}| = \sqrt{9+16+36} = \sqrt{61}$

Dc's $\left(\frac{3}{\sqrt{61}}, \frac{4}{\sqrt{61}}, \frac{-6}{\sqrt{61}}\right)$

Let $\bar{b} = 3\hat{i}-4\hat{k}$

Dr's of $\bar{b}$ 3, 0, −4. $|\bar{b}| = \sqrt{9+16} = 5$

DC's $\left(\frac{3}{5}, 0, \frac{-4}{5}\right)$

Ex 8.6) 1) Find the direction cosines of a vector whose dr's are 2, 3, −6.

2) Can a vector have direction angles 30°, 45°, 60°

3) Find the dc's of $\overline{AB}$ where A (2, 3, 1) B (3, −1, 2)

4) Find the dc's of the line joining (2, 3, 1) (3, −1, 2)

5) The dr's of a vector are 2, 3, 6 and its magnitude is 5 Find the vector.

1) dc's are $\frac{2}{\sqrt{4+9+36}}, \frac{3}{\sqrt{49}}, \frac{-6}{\sqrt{49}}$

(ie) $\left(\frac{2}{7}, \frac{3}{7}, \frac{-6}{7}\right)$

2) If α, β, γ are the angles with OX, OY, OZ then

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

(ie) $\cos^2 30 + \cos^2 45 + \cos^2 60 = \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2$

$= \frac{3}{4} + \frac{1}{2} + \frac{1}{4} \neq 1$.

∴ They are not the direction angles of a vector.

3) $\overline{AB} = \overline{OB} - \overline{OA} = \hat{i} - 4\hat{j} + \bar{k}$

$|\overline{AB}| = \sqrt{1+16+1} = \sqrt{18}$.

DC's $\left(\frac{1}{\sqrt{18}}, \frac{-4}{\sqrt{18}}, \frac{1}{\sqrt{18}}\right)$

4) $\overline{AB} = \overline{OB} - \overline{OA} = \hat{i} - 4\hat{j} + \bar{k}$.

The dc's of AB are $\left(\frac{1}{\sqrt{18}}, \frac{-4}{\sqrt{18}}, \frac{1}{\sqrt{18}}\right)$

Suppose if we take second point as first point

dc's are $\left(\frac{-1}{\sqrt{18}}, \frac{4}{\sqrt{18}}, \frac{1}{\sqrt{18}}\right)$

5) dc's are $\left(\frac{2}{7}, \frac{3}{7}, \frac{6}{7}\right)$

$\therefore$ unit vector is $\frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} + \frac{6}{7}\bar{k}$

The required vector is $\frac{5}{7}(2\hat{i} + 3\hat{j} + 6\bar{k})$

EX 8.7) S.T the points whose P.V are $2\hat{i}+3\hat{j}-5\bar{k}$, $3\hat{i}+\hat{j}-2\bar{k}$ and $6\hat{i}-5\hat{j}+7\bar{k}$ are collinear.

Let $\overline{OA} = 2\hat{i} + 3\hat{j} - 5\bar{k}$

$\overline{OB} = 3\hat{i} + \hat{j} - 2\bar{k}$

$\overline{OC} = 6\hat{i} - 5\hat{j} + 7\bar{k}$.

$\overline{AB} = \overline{OB} - \overline{OA} = \hat{i} - 2\hat{j} + 3\bar{k}$

$\overline{AC} = \overline{OC} - \overline{OA} = 4\bar{i} - 8\hat{j} + 12\bar{k}$

$= 4(\bar{i} - 2\hat{j} + 3\bar{k})$

$\overline{AC} = 4\,\overline{AB}$

$\therefore$ The three points are collinear.

Note: If $\overline{AC} = 4\overline{AB}$

$\overline{AC}$ and $\overline{AB}$ are ||el

A B C

when the three points are collinear then only AB and AC are ||el.

EX 8.8) Find the point whose P.V has magnitude 5 and ||el to $4\hat{i} - 3\hat{j} + 10\bar{k}$

Let $\bar{a} = 4\bar{i} - 3\hat{j} + 10\bar{k}$

$|\bar{a}| = \sqrt{16 + 9 + 100} = \sqrt{125} = 5\sqrt{5}$

$\hat{a} = \frac{4\bar{i} - 3\hat{j} + 10\bar{k}}{5\sqrt{5}}$

$5\hat{a} = \frac{\cancel{5}(4\hat{i} - 3\hat{j} + 10\bar{k})}{\cancel{5}\sqrt{5}}$ $\therefore$ Required points are $\left(\frac{4}{\sqrt{5}}, \frac{-3}{\sqrt{5}}, \frac{10}{\sqrt{5}}\right)$ $\frac{10}{\sqrt{5}} = 2\sqrt{5}$

EX 8.9) P.T the points whose PV's $2\hat{i}+4\hat{j}+3\hat{k}$, $4\hat{i}+\hat{j}+9\hat{k}$, $10\hat{i}-\hat{j}+6\hat{k}$ forms a rt triangle

let $\overline{OA} = 2\hat{i}+4\hat{j}+3\hat{k}$ $\quad \overline{AB} = \overline{OB}-\overline{OA} = 2\hat{i}-3\hat{j}+6\hat{k}$

$\overline{OB} = 4\hat{i}+\hat{j}+9\hat{k}$ $\quad \overline{BC} = \overline{OC}-\overline{OB} = 6\hat{i}-2\hat{j}-3\hat{k}$

$\overline{OC} = 10\hat{i}-\hat{j}+6\hat{k}$ $\quad \overline{CA} = \overline{OA}-\overline{OC} = -8\hat{i}+5\hat{j}-3\hat{k}$

$\overline{AB}+\overline{BC}+\overline{CA} = 0$ ∴ First it is a triangle.

$|\overline{AB}| = \sqrt{4+9+36} = 7$

$|\overline{BC}| = \sqrt{36+4+9} = 7$

$|\overline{CA}| = \sqrt{64+25+9} = \sqrt{98}$.

$|\overline{AB}|^2 + |\overline{BC}|^2 = |\overline{CA}|^2$

$49+49 = 98$. ∴ given points forms a rt △le.

(8) EX 8.10) S.T the vectors $5\hat{i}+6\hat{j}+7\hat{k}$, $7\hat{i}-8\hat{j}+9\hat{k}$, $3\hat{i}+20\hat{j}+5\hat{k}$ are coplanar.

Let $5\hat{i}+6\hat{j}+7\hat{k} = s(7\hat{i}-8\hat{j}+9\hat{k}) + t(3\hat{i}+20\hat{j}+5\hat{k})$

$7s+3t = 5$ — (1)

$-8s+20t = 6$ — (2)

$9s+5t = 7$ — (3)

(1) × 5 $\quad 35s+15t = 25$

(3) × 3 $\quad 27s+15t = 21$

$8s = 4$

$s = 1/2$

$t = 1/2$

∴ $5\hat{i}+6\hat{j}+7\hat{k} = \frac{1}{2}(7\hat{i}-8\hat{j}+9\hat{k}) + 1/2(3\hat{i}+20\hat{j}+5\hat{k})$

∴ we can write one vector is a linear combination of other two vectors. Hence given vectors are coplanar.

7) 8.2) S.T the vectors $2\hat{i}-\hat{j}+\hat{k}$, $3\hat{i}-4\hat{j}-4\hat{k}$, $\hat{i}-3\hat{j}-5\hat{k}$ form a right angled △le.

$AB = 2\hat{i}-\hat{j}+\hat{k}$ $\quad |\overline{AB}| = \sqrt{4+1+1} = \sqrt{6}$

$\overline{BC} = 3\hat{i}-4\hat{j}-4\hat{k}$ $\quad |\overline{BC}| = \sqrt{9+16+16} = \sqrt{41}$

$\overline{CA} = \hat{i}-3\hat{j}-5\hat{k}$ $\quad |\overline{CA}| = \sqrt{1+9+25} = \sqrt{35}$

$\overline{CA}+\overline{AB} = \overline{BC}$ ∴ First it is a △le.

$|\overline{CA}|^2 + |\overline{AB}|^2 = |\overline{BC}|^2$

$35+6 = 41$. ∴ it is rt angled △le.

8) 8.2) Find the value of λ for which the vectors $\bar{a} = 3\hat{i}+2\hat{j}+9\hat{k}$, $\bar{b} = \hat{i}+\lambda\hat{j}+3\hat{k}$ are parallel.

If $\bar{a}$ and $\bar{b}$ are $\parallel^{el}$ $\bar{b} = t\bar{a}$

$$\hat{i}+\lambda\hat{j}+3\hat{k} = 3\left(\hat{i}+\tfrac{2}{3}\hat{j}+3\hat{k}\right)$$
$$\Rightarrow \lambda = 2/3.$$

9) 8.2 S.T the following vectors are coplanar.

1) $\hat{i}-2\hat{j}+3\hat{k}$ $-2\hat{i}+3\hat{j}-4\hat{k}$, $-\hat{j}+2\hat{k}$

2) $5\hat{i}+6\hat{j}+7\hat{k}$, $7\hat{i}-8\hat{j}+9\hat{k}$, $3\hat{i}+20\hat{j}+5\hat{k}$.

1) $\hat{i}-2\hat{j}+3\hat{k} = s(-2\hat{i}+3\hat{j}-4\hat{k})+t(-\hat{j}+2\hat{k})$

$$1 = -2s \Rightarrow s = -1/2$$
$$-2 = 3s-t \Rightarrow -2 = 3(-1/2)-t$$
$$t = -\tfrac{3}{2}+2 = 1/2.$$

$\therefore \hat{i}-2\hat{j}+3\hat{k} = -\frac{1}{2}(-2\hat{i}+3\hat{j}-4\hat{k})+\frac{1}{2}(-\hat{j}+2\hat{k})$

$\therefore$ we can write one vector as a linear combination of other two vectors. Hence they are coplanar.

2) $5\hat{i}+6\hat{j}+7\hat{k} = s(7\hat{i}-8\hat{j}+9\hat{k})+t(3\hat{i}+20\hat{j}+5\hat{k})$

$5 = 7s+3t$ — ①

$6 = -8s+20t$ — ② This is Example 8.10.

$7 = 9s+5t$ — ③

10) 8.2 S.T the points whose P.V $4\hat{i}+5\hat{j}+\hat{k}$, $-\hat{j}-\hat{k}$, $3\hat{i}+9\hat{j}+4\hat{k}$, $-4\hat{i}+4\hat{j}+4\hat{k}$ are coplanar.

$\overline{OA} = 4\hat{i}+5\hat{j}+\hat{k}$

$\overline{OB} = -\hat{j}-\hat{k}$

$\overline{OC} = 3\hat{i}+9\hat{j}+4\hat{k}$

$\overline{OD} = -4\hat{i}+4\hat{j}+4\hat{k}$

$\overline{AB} = -4\hat{i}-6\hat{j}-2\hat{k}$ $(\overline{OB}-\overline{OA})$

$\overline{BC} = 3\hat{i}-6\hat{j}-2\hat{k}$

$\overline{CD} = 7\hat{i}-5\hat{j}$.

$-4\hat{i}-6\hat{j}-2\hat{k} = s(3\hat{i}-6\hat{j}-2\hat{k})+t(7\hat{i}-5\hat{j})$

$$-4 = 3s+7t$$
$$-6 = 10s-5t$$
$$-2 = 5s \Rightarrow s = -2/5$$

$$-4 = 3(-2/5) - 7m$$

$$7m = -\frac{6}{5} + 4 = \frac{-6+20}{5}$$

$$= \frac{14}{5} \Rightarrow m = \frac{14}{5\times7} = \frac{2}{5}$$

$\therefore -4i - 6j - 2k = -\frac{2}{5}(3i - 6j - 2k) + \frac{2}{5}(-7i - 5j)$

$\therefore$ one vector can be written as sum of two linear vectors. Hence the given vectors are coplanar.

11) 8.2) If $\bar{a} = 2i + 3j - 4\bar{k}$, $\bar{b} = 3\bar{i} - 4j - 5\bar{k}$, $c) = -3\bar{i} + 2j + 3\bar{k}$ Find the magnitude dc's of 1) $\bar{a} + \bar{b} + \bar{c}$ 2) $3\bar{a} - 2\bar{b} + 5\bar{c}$.	
$\bar{a} = 2\bar{i} + 3j - 4\bar{k}$ $\bar{b} = 3\bar{i} - 4j - 5\bar{k}$ $\bar{c} = -3\bar{i} + 2j + 3\bar{k}$. ① $\bar{a} + \bar{b} + \bar{c} = 2\bar{i} + j - 6\bar{k}$ $\lvert\bar{a} + \bar{b} + \bar{c}\rvert = \sqrt{4+1+36} = \sqrt{41}$ dc's : $\frac{2}{\sqrt{41}}, \frac{1}{\sqrt{41}}, \frac{-6}{\sqrt{41}}$	$3\bar{a} = 6\bar{i} + 9j - 12\bar{k}$ $-2\bar{b} = -6\bar{i} + 8j + 10\bar{k}$. $5\bar{c} = -15\bar{i} + 10j + 15\bar{k}$. $3\bar{a} - 2\bar{b} + 5\bar{c} = -15\bar{i} + 27j + 13\bar{k}$ $\lvert 3\bar{a} - 2\bar{b} + 5\bar{c}\rvert = \sqrt{1123}$ dc's : $\frac{-15}{\sqrt{1123}}, \frac{27}{\sqrt{1123}}, \frac{13}{\sqrt{1123}}$.

12) 8.2) The PV of the vertices of a triangle are $i + 2j + 3\bar{k}$, $3i - 4j + 5\bar{k}$ and $-2\bar{i} + 3j - 7\bar{k}$ Find the perimeter of the $\triangle$le -

$\bar{OA} = i + 2j + 3\bar{k}$ $\bar{OB} = 3\bar{i} - 4j + 5\bar{k}$ $\bar{OC} = -2i + 3j - 7\bar{k}$.	$\bar{AB} = \bar{OB} - \bar{OA} = 2i - 6j + 2\bar{k}$ $\bar{BC} = \bar{OC} - \bar{OB} = -5i + 7j - 12\bar{k}$ $\bar{CA} = \bar{OA} - \bar{OC} = 3i - j + 10\bar{k}$.

$$|\bar{AB}| = \sqrt{4+36+4} = \sqrt{44}$$

$$|\bar{BC}| = \sqrt{25+49+144} = \sqrt{218}$$

$$|\bar{CA}| = \sqrt{9+1+100} = \sqrt{110}$$

$\therefore$ Perimeter of the $\triangle$le $= \sqrt{44} + \sqrt{218} + \sqrt{110}$.

$\frac{13}{8.2}$) Find the unit vector parallel to $3\bar{a}-2\bar{b}+4\bar{c}$ if $\bar{a}=3\hat{i}-\hat{j}-4\hat{k}$, $\bar{b}=-2\hat{i}+4\hat{j}-3\hat{k}$, $\bar{c}=\hat{i}+2\hat{j}-\hat{k}$

$3\bar{a} = 9\hat{i}-3\hat{j}-12\hat{k}$

$-2\bar{b} = 4\hat{i}-8\hat{j}+6\hat{k}$

$4\bar{c} = 4\hat{i}+8\hat{j}-4\hat{k}$

$3\bar{a}-2\bar{b}+4\bar{c} = 17\hat{i}+3\hat{j}+10\hat{k}$

$|17\hat{i}+3\hat{j}+10\hat{k}| = \sqrt{289+9+100}$

$= \sqrt{398}$

unit vector $\parallel$ to $3\bar{a}-2\bar{b}+4\bar{c} = \dfrac{17\hat{i}+3\hat{j}+10\hat{k}}{\sqrt{398}}$

$\frac{15}{8.2}$) The PV of the points P, Q, R, S are $\hat{i}+\hat{j}+\hat{k}$, $2\hat{i}+5\hat{j}$, $3\hat{i}+2\hat{j}-3\hat{k}$ and $\hat{i}-6\hat{j}-\hat{k}$ resp.
P.T the lines PQ and RS are parallel.

$\overline{OP} = \hat{i}+\hat{j}+\hat{k}$

$\overline{OQ} = 2\hat{i}+5\hat{j}$

$\overline{OR} = 3\hat{i}+2\hat{j}-3\hat{k}$

$\overline{OS} = \hat{i}-6\hat{j}-\hat{k}$

$\overline{PQ} = \overline{OQ}-\overline{OP} = \hat{i}+4\hat{j}-\hat{k}$

$\overline{RS} = \overline{OS}-\overline{OR} = -2\hat{i}-8\hat{j}+2\hat{k}$

$\overline{RS} = -2(\hat{i}+4\hat{j}-\hat{k})$

$= -2\overline{PQ}$

$\therefore$ PQ, RS are parallel.

$\frac{16}{8.2}$) Find the value or values of m for which $m(\hat{i}+\hat{j}+\hat{k})$ is a unit vector

$\hat{a} = \dfrac{\bar{a}}{|\bar{a}|} = \pm m(\hat{i}+\hat{j}+\hat{k})$

$= \pm\dfrac{(\hat{i}+\hat{j}+\hat{k})}{1/m}$

$\Rightarrow |\bar{a}| = \sqrt{1+1+1} = \sqrt{3}$.

$|\bar{a}| = \pm\dfrac{1}{m}$

$m = \pm\dfrac{1}{\sqrt{3}}$.

$\frac{17}{8.2}$) S.T the points A(1, 1, 1) B(1, 2, 3) C(2, −1, 1) are the vertices of isosceles triangle

$\overline{OA} = \hat{i}+\hat{j}+\hat{k}$

$\overline{OB} = \hat{i}+2\hat{j}+3\hat{k}$

$\overline{OC} = 2\hat{i}-\hat{j}+\hat{k}$.

$\overline{AB} = \hat{j}+2\hat{k}$

$\overline{BC} = \hat{i}-3\hat{j}-2\hat{k}$

$\overline{CA} = -\hat{i}+2\hat{j}$

$|\overline{AB}| = \sqrt{1+4} = \sqrt{5}$

$|\overline{BC}| = \sqrt{1+9+4} = \sqrt{14}$

$|\overline{CA}| = \sqrt{1+4} = \sqrt{5}$

$|\overline{AB}| = |\overline{CA}|$ $\therefore$ it is isosceles $\triangle$le.

$\frac{1}{8.2}$) Verify whether the following ratios are dc's of some vector or not.

i) $\frac{1}{5}, \frac{3}{5}, \frac{4}{5}$ 2) $\frac{1}{\sqrt{2}}, \frac{1}{2}, \frac{1}{2}$ 3) $\frac{4}{3}, 0, \frac{3}{4}$.

W.K.T $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

1) $\frac{1}{25} + \frac{9}{25} + \frac{16}{25} \neq 1$ $\therefore$ not dc's

2) $\frac{1}{2} + \frac{1}{4} + \frac{1}{4} = 1$ $\therefore$ 2 is dc's

3) $\frac{16}{9} + 0 + \frac{9}{16} \neq 1$. $\therefore$ not dc's.

$\frac{2}{8.2}$) Find the direction cosines of a vector whose dr's are

1) 1, 2, 3 2) 3, −1, 3 3) 0, 0, 7.

1) Let $\bar{r} = i + 2j + 3k$

$|\bar{r}| = \sqrt{1+4+9} = \sqrt{14}$ $\therefore$ dc's $\left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)$.

2) Let $\bar{r} = 3i - j + 3k$ dc's $\left(\frac{3}{\sqrt{19}}, \frac{-1}{\sqrt{19}}, \frac{3}{\sqrt{19}}\right)$.

$|\bar{r}| = \sqrt{9+1+9} = \sqrt{19}$

3) Let $\bar{r} = 7k$ dc's $\left(0, 0, \frac{7}{7}\right)$

$|\bar{r}| = \sqrt{49} = 7$

$\frac{3}{8.2}$) Find the dc's and dr's of the following vectors.

1) $3i - 4j + 8k$ 2) $3i + j + k$ 3) $\hat{j}$ 4) $5i - 3j - 48k$

5) $3i - 3k + 4j$ 6) $i - k$.

Let 1) $\bar{x} = 3i - 4j + 8k$ $\lvert\bar{x}\rvert = \sqrt{9+16+64} = \sqrt{89}$	dr's : $(3, -4, 8)$ dc's $\left(\frac{3}{\sqrt{89}}, \frac{-4}{\sqrt{89}}, \frac{8}{\sqrt{89}}\right)$
2) $\bar{x} = 3i + j + k$ $\lvert\bar{x}\rvert = \sqrt{9+1+1} = \sqrt{10}$	dr's $(3, 1, 1)$ dc's $\left(\frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}}, \frac{1}{\sqrt{10}}\right)$
3 $\bar{x} = \bar{j}$ $\lvert\bar{x}\rvert = \sqrt{1} = 1$	dr's $(0, 1, 0)$ dc's $(0, 1, 0)$

4) $\bar{x} = 5\hat{i} - 3\hat{j} - 48\hat{k}$ $\lvert\bar{x}\rvert = \sqrt{2338}$	d'rs $(5, -3, -48)$ dc's $\left(\frac{5}{\sqrt{2338}}, \frac{-3}{\sqrt{2338}}, \frac{-48}{\sqrt{2338}}\right)$.
5) $\bar{x} = 3\hat{i} + 4\hat{j} - 3\hat{k}$ $\lvert\bar{x}\rvert = \sqrt{9+16+9} = \sqrt{34}$	d r's $(3, 4, -3)$ dcs' $\left(\frac{3}{\sqrt{34}}, \frac{4}{\sqrt{34}}, \frac{-3}{\sqrt{34}}\right)$
6) $\bar{x} = \hat{i} - \hat{k}$ $\lvert\bar{x}\rvert = \sqrt{1+1} = \sqrt{2}$	drs $(1, 0, -1)$ dcs $\left(\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}\right)$

$\frac{5)}{8.2}$ If $\frac{1}{2}, \frac{1}{\sqrt{2}}$, a are the direction cosines of some vector find a.

$\cos\alpha = \frac{1}{2}$, $\cos\beta = \frac{1}{\sqrt{2}}$, $\cos\gamma = a$

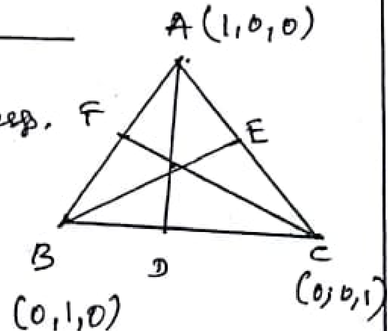
w.k.T $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$$\frac{1}{4} + \frac{1}{2} + a^2 = 1$$

$$a^2 = \frac{1}{4}$$

$$a = \pm \frac{1}{2}$$

$\frac{4)}{8.2}$ A triangle is formed by joining the points $(1,0,0)$ $(0,1,0)$ $(0,0,1)$. Find the dc's of the median.



A $(1,0,0)$ B$(0,1,0)$ C $(0,0,1)$

Let D, E, F are the mid points of BC, CA, AB resp.

AD, BE, CF are the medians.

Mid point of BC is D $= \left(0, \frac{1}{2}, \frac{1}{2}\right)$

A $= (1, 0, 0)$

D.r's of AD $= \left(-1, \frac{1}{2}, \frac{1}{2}\right)$ $\quad \sqrt{1 + \frac{1}{4} + \frac{1}{4}} = \sqrt{\frac{6}{4}} = \frac{\sqrt{3}}{\sqrt{2}}$

Dcs of AD $= \left(-\frac{1 \cdot \sqrt{2}}{\sqrt{3}}, \frac{\frac{1}{2} \times \sqrt{2}}{\sqrt{3}}, \frac{\frac{1}{2} \times \sqrt{2}}{\sqrt{3}}\right)$

$$= \left(-\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$$

IIIly the dc's of other medians are $\left(\frac{1}{\sqrt{6}}, \frac{-\sqrt{2}}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$, $\left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}\right)$

$\frac{4}{8/2}$)

EXERCISE - 8.3.

EX.8.11) Find $\bar{a}\cdot\bar{b}$ when 1) $\bar{a}=i-j+5\bar{k}$ $\bar{b}=3i-2\bar{k}$
2) $\bar{a},\bar{b}$ represents the points $(2,3,-1)$ $(-1,2,3)$

1) $\bar{a}=i-j+5\bar{k}$
$\bar{b}=3i-2\bar{k}$
$\bar{a}\cdot\bar{b}=3+0-10=-7$

2) $\bar{a}=2\hat{i}+3j-\bar{k}$
$\bar{b}=-\bar{i}+2j+3\bar{k}$
$\bar{a}\cdot\bar{b}=-2+6-3=1$

EX 8.12) Find $(\bar{a}+3\bar{b})\cdot(2\bar{a}-\bar{b})$ if $\bar{a}=i+j+2\bar{k}$, $\bar{b}=3i+2j-\bar{k}$

$\bar{a}=i+j+2\bar{k}$ $\quad(\bar{a}+3\bar{b})=10i+7j-\bar{k}$
$3\bar{b}=9i+6j-3\bar{k}$
$2\bar{a}=2\bar{i}+2j+4\bar{k}$ $\quad(2\bar{a}-\bar{b})=-i+5\bar{k}$
$\bar{b}=3i+2j-\bar{k}$
$(\bar{a}+3\bar{b})\cdot(2\bar{a}-\bar{b})=-10+0+5=-15$

EX 8.13) If $\bar{a}=2i+2j+3\bar{k}$, $\bar{b}=-i+2j+\bar{k}$ and $\bar{c}=3i+j$ s.t $\bar{a}+\lambda\bar{b}$ is $\perp$r to $\bar{c}$ then find λ.

$\bar{a}=2\bar{i}+2j+3\bar{k}$
$\lambda\bar{b}=-\lambda\bar{i}+2\lambda j+\lambda\bar{k}$
$\bar{a}+\lambda\bar{b}=(2-\lambda)i+(2+2\lambda)j+(3+\lambda)\bar{k}$
$\bar{c}=3\bar{i}+j$
$\bar{a}+\lambda\bar{b}$ and $\bar{c}$ are $\perp$r $\therefore(a+\lambda\bar{b})\cdot\bar{c}=0$

$(\bar{a}+\lambda b)\cdot\bar{c}=(2-\lambda)3+(2+2\lambda)\cdot1=0$
$6-3\lambda+2+2\lambda=0$
$8=\lambda$.

EX 8.14) If $|\bar{a}+\bar{b}|=|\bar{a}-\bar{b}|$ P.T $\bar{a},\bar{b}$ are $\perp$r.

$|\bar{a}+\bar{b}|^2=|\bar{a}-b|^2$
$\cancel{|\bar{a}|^2}+\cancel{|\bar{b}|^2}+2\bar{a}\cdot\bar{b}=\cancel{|\bar{a}|^2}+\cancel{|\bar{b}|^2}-2\bar{a}\cdot\bar{b}$
$2\bar{a}\cdot\bar{b}+2\bar{a}\cdot\bar{b}=0$
$4\bar{a}\cdot\bar{b}=0$
$\bar{a}\cdot\bar{b}=0$
$\Rightarrow\bar{a}\perp\bar{b}$.

Ex 8.15) For any vector $\bar{r}$ P.T $\bar{r} = (\bar{r}\cdot i)\bar{i} + (\bar{r}\cdot j)j + (\bar{r}\cdot\bar{k})\bar{k}$

Let $\bar{r} = xi + yj + z\bar{k}$

$\bar{r}\cdot\bar{i} = (xi + yj + z\bar{k})\cdot\bar{i}$

$= x$

$(\bar{r}\cdot i)\bar{i} = x\bar{i}$

|||ly $(\bar{r}\cdot j)j = yj$

$(\bar{r}\cdot\bar{k})\bar{k} = z\bar{k}$

$\therefore (\bar{r}\cdot i)i + (\bar{r}\cdot j)j + (\bar{r}\cdot\bar{k})\bar{k}$

$= xi + yj + z\bar{k}$

$= \bar{r}$.

Ex 8.16) Find the angle between the vectors $5i + 3j + 4\bar{k}$ and $6\bar{i} - 8j - \bar{k}$.

$\bar{a} = 5i + 3j + 4\bar{k} \Rightarrow |\bar{a}| = \sqrt{25+9+16} = \sqrt{50} = \sqrt{25\times2}$

$= 5\sqrt{2}$

$\bar{b} = 6\bar{i} - 8j - \bar{k} \Rightarrow |\bar{b}| = \sqrt{36+64+1} = \sqrt{101}$

$\bar{a}\cdot\bar{b} = 30 - 24 - 4 = 2$

$\cos\theta = \dfrac{\bar{a}\cdot\bar{b}}{|\bar{a}||\bar{b}|} = \dfrac{\cancel{2}\sqrt{2}}{5\cancel{\sqrt{2}}\sqrt{101}}$

$\theta = \cos^{-1}\left(\dfrac{\sqrt{2}}{5\sqrt{101}}\right)$

Ex 8.17) Find the projection of $\bar{AB}$ on $\bar{CD}$ where A B C D are the points $(4,-3,0)$ $(7,-5,-1)$ $(-2,1,3)$ $(0,2,5)$

$\bar{AB} = \bar{OB} - \bar{OA} = 3i - 2j - \bar{k}$

$\bar{CD} = \bar{OD} - \bar{OC} = 2i + j + 2\bar{k}$

$\bar{AB}\cdot\bar{CD} = 6 - 2 - 2 = 2.$

$|\bar{CD}| = \sqrt{4+1+4} = 3.$

Projection of $\bar{AB}$ on CD $= \dfrac{\bar{AB}\cdot\bar{CD}}{|\bar{CD}|} = 2/3.$

Ex 8.18) If $\bar{a}, \bar{b}, \bar{c}$ are the unit vectors satisfying $\bar{a} - \sqrt{3}\bar{b} + \bar{c} = 0$ then find the angle between $\bar{a}$ and $\bar{c}$.

Given $\bar{a} - \sqrt{3}\bar{b} + \bar{c} = 0$

$\bar{a} + \bar{c} = \sqrt{3}\bar{b}$

$|\bar{a}+\bar{c}| = \sqrt{3}|\bar{b}|$

$|\bar{a}+\bar{c}|^2 = 3|\bar{b}|^2$

$|\bar{a}|^2 + |\bar{b}|^2 + 2\bar{a}\cdot\bar{b} = 3|\bar{b}|^2$

$1 + 1 + 2|\bar{a}||\bar{b}|\cos\theta = 3$

$2\cos\theta = 3 - 2$

$= 1$

$\cos\theta = 1/2$

$\theta = \pi/3$

EX 8.19) S.T the points (4, −3, 1) (2, −4, 5) and (1, −1, 0) form a rt angled $\triangle$le.

$\overline{OA} = 4i - 3j + \bar{k}$ $\overline{AB} = \overline{OB} - \overline{OA} = -2\hat{i} - j + 4\bar{k}$

$\overline{OB} = 2i - 4j + 5\bar{k}$ $\overline{BC} = \overline{OC} - \overline{OB} = -\hat{i} + 3j - 5\bar{k}$

$\overline{OC} = i - j.$ $\overline{CA} = \overline{OA} - \overline{OC} = 3i - 2j + \bar{k}$

Now $\overline{AB} + \overline{BC} + \overline{CA} = 0$ ∴ the three points forms a $\triangle$le.

$\overline{AB} \cdot \overline{CA} = -6 + 2 + 4 = 0$

$\angle A = \pi/2.$

∴ it is a rt angled $\triangle$le.

$\frac{1}{8.3}$) 1) Find $\bar{a} \cdot \bar{b}$ when 1) $\bar{a} = i - 2j + \bar{k}$ $\bar{b} = 3i - 4j - 2\bar{k}$
2) $\bar{a} = 2i + 2j - \bar{k}$ $\bar{b} = 6\hat{i} - 3j + 2\bar{k}$

1) $\bar{a} = i - 2j + \bar{k}$
$\bar{b} = 3i - 4j - 2\bar{k}$
$\bar{a} \cdot \bar{b} = 3 + 8 - 2 = 9$

2) $\bar{a} = 2\hat{i} + 2j - \bar{k}$
$\bar{b} = 6\hat{i} - 3j + 2\bar{k}$
$\bar{a} \cdot \bar{b} = 12 - 6 - 2 = 4.$

$\frac{2}{8.3}$) Find the value of λ for which the vectors $\bar{a}$ and $\bar{b}$ are $\perp$r.
1) $\bar{a} = 2\hat{i} + \lambda j + \bar{k}$ $\bar{b} = \hat{i} - 2j + 3\bar{k}$; 2) $\bar{a} = 2i + 4j - \bar{k}$ $\bar{b} = 3\hat{i} - 2j + \lambda\bar{k}$.

If $\bar{a}, \bar{b}$ are $\perp$r $\bar{a} \cdot \bar{b} = 0$

1) $\bar{a} \cdot \bar{b} = 2 - 2\lambda + 3 = 0$
$5 = 2\lambda \Rightarrow \lambda = 5/2$

2) $\bar{a} \cdot \bar{b} = 6 - 8 - \lambda = 0$
$-\lambda = 2$
$\lambda = -2$

$\frac{3}{8.3}$) If $\bar{a}$ and $\bar{b}$ are two vectors s.t $|\bar{a}| = 10$, $|\bar{b}| = 15$ $\bar{a} \cdot \bar{b} = 75\sqrt{2}$ find the angle between $\bar{a}$ and $\bar{b}$.

$\bar{a} \cdot \bar{b} = |\bar{a}||\bar{b}| \cos\theta$

$\cos\theta = \frac{\bar{a} \cdot \bar{b}}{|\bar{a}||\bar{b}|} = \frac{75\sqrt{2}}{10 \times 15} = \frac{1}{\sqrt{2}}$

$\cos\theta = \frac{1}{\sqrt{2}}$ $\theta = \pi/4.$

4/8.3) Find the angle between the vectors
1) $2\hat{i} + 3j - 6\bar{k}$, $6\hat{i} - 3j + 2\bar{k}$
2) $i - j$, $j - \bar{k}$

i) $\bar{a} = 2\hat{i}+3\hat{j}-6\hat{k}$ $|\bar{a}| = \sqrt{4+9+36} = 7$.

$\bar{b} = 6\hat{i}-3\hat{j}+2\hat{k}$ $|\bar{b}| = \sqrt{36+9+4} = 7$.

$\bar{a}\cdot\bar{b} = 12-9-12$

$= -9$ $\cos\theta = \dfrac{\bar{a}\cdot\bar{b}}{|\bar{a}||\bar{b}|} = \dfrac{-9}{7.7}$.

$\theta = \cos^{-1}\left(\dfrac{-9}{49}\right)$

2) $\bar{a} = \hat{i}-\hat{j} \Rightarrow |\bar{a}| = \sqrt{1+1} = \sqrt{2}$

$\bar{b} = \hat{j}-\hat{k} \Rightarrow |\bar{b}| = \sqrt{1+1} = \sqrt{2}$

$\bar{a}\cdot\bar{b} = -1$

$\cos\theta = \dfrac{\bar{a}\cdot\bar{b}}{|\bar{a}||\bar{b}|} = \dfrac{-1}{\sqrt{2}\cdot\sqrt{2}} = -\dfrac{1}{2}$ θ lies in II quadrant

$\theta = \pi - \dfrac{\pi}{3} = \dfrac{2\pi}{3}$.

5/8.3 If $\bar{a}, \bar{b}, \bar{c}$ are the three vectors s.t $\bar{a}+2\bar{b}+\bar{c} = 0$ and $|\bar{a}| = 3$ $|\bar{b}| = 4$ $|\bar{c}| = 7$ find the angle between $\bar{a}$ and $\bar{b}$.

$\bar{a}+2\bar{b}+\bar{c} = 0$

$|\bar{a}+2\bar{b}| = |-\bar{c}|$

$|\bar{a}+2\bar{b}|^2 = |\bar{c}|^2$

$|\bar{a}|^2 + 4|\bar{b}|^2 + 4\,\bar{a}\cdot\bar{b} = |\bar{c}|^2$

$9 + 64 + 4\bar{a}\cdot\bar{b} = 49$

$4\cdot\bar{a}\cdot\bar{b} = -24$

$\bar{a}\cdot\bar{b} = -6$.

$|\bar{a}||\bar{b}|\cos\theta = -6$

$3\times 4\times\cos\theta = -6$

$\Rightarrow \cos\theta = -\cancel{\dfrac{6}{12}}\,\dfrac{1}{2}$

θ lies in II quadrant

$\theta = \pi - \pi/3 = \dfrac{2\pi}{3}$

6/8.3) S.T the vectors $\bar{a} = 2\hat{i}+3\hat{j}+6\hat{k}$, $\bar{b} = 6\hat{i}+2\hat{j}-3\hat{k}$, $\bar{c} = 3\hat{i}-6\hat{j}+2\hat{k}$ are mutually orthogonal

$\bar{a} = 2\hat{i}+3\hat{j}+6\hat{k}$ $\bar{a}\cdot\bar{b} = 12+6-18 = 0$

$\bar{b} = 6\hat{i}+2\hat{j}-3\hat{k}$ $\bar{b}\cdot\bar{c} = 18-12-6 = 0$

$\bar{c} = 3\hat{i}-6\hat{j}+2\hat{k}$ $\bar{c}\cdot\bar{a} = 6-18+12 = 0$

$\therefore$ $\bar{a}, \bar{b}, \bar{c}$ are mutually orthogonal.

7/8.3) S.T the vectors $-\hat{i}-2\hat{j}-6\hat{k}$, $2\hat{i}-\hat{j}+\hat{k}$, $-\hat{i}+3\hat{j}+5\hat{k}$ forms a rt angled $\triangle$.

$\bar{a} = -i - 2j - 6\bar{k}$

$\bar{b} = 2\bar{i} - j + \bar{k}$

$\bar{c} = -\bar{i} + 3j + 5\bar{k}$ forms

$\bar{a} + \bar{c} = \bar{b}$ ∴ First it is a △le.

$\bar{a} \cdot \bar{b} = -2 + 2 - 6 \neq 0$

$\bar{b} \cdot \bar{c} = -2 - 3 + 5 = 0$ ∴ $\bar{b} \cdot \bar{c} = 0$

∴ it is a rt angled △le.

$\frac{8}{8.3}$) If $|\bar{a}| = 5$, $|\bar{b}| = 6$, $|\bar{c}| = 7$ and $\bar{a} + \bar{b} + \bar{c} = 0$ find $\bar{a} \cdot \bar{b} + \bar{b} \cdot \bar{c} + \bar{c} \cdot \bar{a}$.

$|\bar{a} + \bar{b} + \bar{c}| = 0$

$|\bar{a} + \bar{b} + \bar{c}|^2 = 0$

$|\bar{a}|^2 + |\bar{b}|^2 + |\bar{c}|^2 + 2(\bar{a} \cdot \bar{b} + \bar{b} \cdot \bar{c} + \bar{c} \cdot \bar{a}) = 0$

$25 + 36 + 49 + 2(\bar{a} \cdot \bar{b} + \bar{b} \cdot \bar{c} + \bar{c} \cdot \bar{a}) = 0$

$2(\bar{a} \cdot \bar{b} + \bar{b} \cdot \bar{c} + \bar{c} \cdot \bar{a}) = -110$

$\bar{a} \cdot \bar{b} + \bar{b} \cdot \bar{c} + \bar{c} \cdot \bar{a} = -55$.

$\frac{9}{8.3}$) S.T the points $(2, -1, 3)$ $(4, 3, 1)$ $(3, 1, 2)$ are collinear.

$\overline{OA} = 2i - j + 3\bar{k}$

$\overline{OB} = 4i + 3j + \bar{k}$

$\overline{OC} = 3i + j + 2\bar{k}$.

$\overline{AB} = \overline{OB} - \overline{OA} = 2\bar{i} + 4j - 2\bar{k}$

$\overline{BC} = \overline{OC} - \overline{OB} = -\bar{i} - 2j + \bar{k}$

$\overline{AB} = -2(-i - 2j + \bar{k})$

$= -2\overline{BC}$

∴ A, B, C are collinear.

$\frac{10}{8.3}$) If $\bar{a}, \bar{b}$ are the unit vectors and θ is the angle between $\bar{a}$ and $\bar{b}$ S.T 1) $\sin\theta/2 = \frac{1}{2}|\bar{a} - \bar{b}|$ 2) $\cos\theta/2 = \frac{1}{2}|\bar{a} + \bar{b}|$ 3) $\tan\theta/2 = \frac{|\bar{a} - \bar{b}|}{|\bar{a} + \bar{b}|}$

$|\bar{a} - \bar{b}|^2 = |\bar{a}|^2 + |\bar{b}|^2 - 2\bar{a} \cdot \bar{b}$

$= 1 + 1 - 2|\bar{a}||\bar{b}|\cos\theta$

$= 2(1 - \cos\theta)$

$= 2 \cdot 2\sin^2\theta/2$

$|\bar{a} - \bar{b}| = 2\sin\theta/2 \Rightarrow \frac{1}{2}|\bar{a} - \bar{b}| = \sin\theta/2$ — (1)

$$|\bar{a}+\bar{b}|^2 = |\bar{a}|^2+|\bar{b}|^2+2\bar{a}\cdot\bar{b}$$
$$= 1+1+2|\bar{a}|\cdot|\bar{b}|\cos\theta$$
$$= 2+2\cos\theta$$
$$= 2(1+\cos\theta)$$
$$= 2\cdot 2\cos^2\theta/2$$

$$|\bar{a}+\bar{b}| = 2\cos\theta/2 \Rightarrow \frac{1}{2}|\bar{a}+\bar{b}| = \cos\theta/2 \longrightarrow (2)$$

$$\frac{(1)}{(2)} \quad \frac{\sin\theta/2}{\cos\theta/2} = \frac{\cancel{1/2}|\bar{a}-\bar{b}|}{\cancel{1/2}|\bar{a}+\bar{b}|} = \tan\theta/2$$

12) (8.3) Find the projection of the vector $i+3\hat{j}+7\hat{k}$ on the vector $2\hat{i}+6\hat{j}+3\hat{k}$.

Let $\bar{a} = i+3\hat{j}+7\hat{k}$
$\bar{b} = 2\hat{i}+6\hat{j}+3\hat{k}$

$\bar{a}\cdot\bar{b} = 2+18+21 = 41$
$|\bar{b}| = \sqrt{4+36+9} = 7.$

Projection of $\bar{a}$ on $\bar{b}$ is $\frac{\bar{a}\cdot\bar{b}}{|\bar{b}|} = \frac{41}{7}.$

13) (8.3) Find λ when the projection of $\bar{a} = \lambda\hat{i}+\hat{j}+4\hat{k}$ on $\bar{b} = 2i+6\hat{j}+3\hat{k}$ is 4 units.

$\bar{a} = \lambda\hat{i}+\hat{j}+4\hat{k}$
$\bar{b} = 2\hat{i}+6\hat{j}+3\hat{k}$

$\bar{a}\cdot\bar{b} = 2\lambda+6+12$
$= 2\lambda+18$

$|\bar{b}| = \sqrt{4+36+9} = 7.$

Projection of $\bar{a}$ on $\bar{b}$ = $\frac{\bar{a}\cdot\bar{b}}{|\bar{b}|} = 4 \Rightarrow \frac{2\lambda+18}{7} = 4.$

$\therefore 2\lambda+18 = 28$
$2\lambda = 10$
$\lambda = 5$

14) (8.3) Three vectors $\bar{a}, \bar{b}, \bar{c}$ are s.t $|\bar{a}|=2$ $|\bar{b}|=3$ $|\bar{c}|=4$ and $\bar{a}+\bar{b}+\bar{c}=0$ Then find $4\bar{a}\cdot\bar{b}+3\bar{b}\cdot\bar{c}+3\bar{c}\cdot\bar{a}$.

$\bar{a}+\bar{b}+\bar{c}=0$
$|\bar{a}+\bar{b}| = |-\bar{c}|$
$|\bar{a}+\bar{b}|^2 = |\bar{c}|^2$
$|\bar{a}|^2+|\bar{b}|^2+2\bar{a}\cdot\bar{b} = |\bar{c}|^2$
$4+9+2\bar{a}\cdot\bar{b} = 16$
$\bar{a}\cdot\bar{b} = 3/2$

$|\bar{a}+\bar{c}|^2 = |\bar{b}|^2$
$|\bar{a}|^2+|\bar{c}|^2+2\bar{a}\cdot\bar{c} = |\bar{b}|^2$
$4+16+2\bar{a}\cdot\bar{c} = 9$
$\bar{a}\cdot\bar{c} = \frac{-11}{2}$

$|\bar{b}+\bar{c}|^2 = |\bar{a}|^2$
$|\bar{b}|^2+|\bar{c}|^2+2\bar{b}\cdot\bar{c} = |\bar{a}|^2$
$9+16+2\bar{b}\cdot\bar{c} = 4$
$\bar{b}\cdot\bar{c} = -\frac{21}{2}$

$\therefore 4\bar{a}\cdot\bar{b}+3\bar{b}\cdot\bar{c}+3\bar{c}\cdot\bar{a} = 4\cdot\frac{3}{2} - \frac{3\cdot 21}{2} - \frac{3\cdot 11}{2} = \frac{12-63-33}{2}$
$= -42$

EXERCISE - 8.4.

EX 8.20) Find $|\bar{a}\times\bar{b}|$ if $\bar{a} = 3i+4j$ and $\bar{b} = i+j+k$

$$\bar{a}\times\bar{b} = \begin{vmatrix} i & j & k \\ 3 & 4 & 0 \\ 1 & 1 & 1 \end{vmatrix} = i(4-0)-j(3-0)+k(3-4)$$
$$= 4i-3j-k$$

$$|\bar{a}\times\bar{b}| = \sqrt{16+9+1} = \sqrt{26}$$

EX 8.21) If $\bar{a} = -3i+4j-7k$ and $\bar{b} = 6i+2j-3k$

verify 1) $\bar{a}$ and $\bar{a}\times\bar{b}$ are $\perp r$

2) $\bar{b}$ and $\bar{a}\times\bar{b}$ are $\perp r$.

$$\bar{a}\times\bar{b} = \begin{vmatrix} i & j & k \\ -3 & 4 & -7 \\ 6 & 2 & -3 \end{vmatrix} = i(-12+14)-j(9+42)+k(-6-24)$$
$$= 2i-51j-30k.$$

$$\bar{a}\cdot(\bar{a}\times\bar{b}) = (-3i+4j-7k)\cdot(2i-51j-30k)$$
$$= -6-204+210 = 0$$

$\therefore$ $\bar{a}$ and $\bar{a}\times\bar{b}$ are $\perp r$.

$$\bar{b}\cdot(\bar{a}\times\bar{b}) = (6i+2j-3k)\cdot(2i-51j-30k)$$
$$= 12-102+90 = 0.$$

$\therefore$ $\bar{b}$, $\bar{a}\times\bar{b}$ are $\perp r$.

EX.8.22) Find the vectors of magnitude 6 which are $\perp r$ to both vectors $\bar{a} = 4i-j+3k$, $\bar{b} = -2i+j-2k$.

$$\bar{a}\times\bar{b} = \begin{vmatrix} i & j & k \\ 4 & -1 & 3 \\ -2 & 1 & -2 \end{vmatrix} = i(2-3)-j(-8+6)+k(4-2)$$
$$= -i+2j+2k$$

$$|\bar{a}\times\bar{b}| = \sqrt{1+4+4} = 3.$$

$$\hat{n} = \pm\frac{\bar{a}\times\bar{b}}{|\bar{a}\times\bar{b}|} = \pm\frac{(-i+2j+2k)}{3}$$

$$6\hat{n} = \pm \cancel{6}^{2}\frac{(-i+2j+2k)}{\cancel{3}}$$

EX: 8.23) Find the cosine and sine angle between the vectors $\bar{a} = 2i+j+3k$ $\bar{b} = 4i-2j+2k$.

$\bar{a} = 2\bar{i}+\bar{j}+3\bar{k} \Rightarrow |\bar{a}| = \sqrt{4+1+9} = \sqrt{14}$

$\bar{b} = 4\bar{i}-2\bar{j}+2\bar{k} \Rightarrow |\bar{b}| = \sqrt{16+4+4} = \sqrt{24}$

$\bar{a}\cdot\bar{b} = 8-2+6 = 12.$

$$\cos\theta = \frac{\bar{a}\cdot\bar{b}}{|\bar{a}||\bar{b}|} = \frac{12}{\sqrt{14}\sqrt{24}}$$

$$\bar{a}\times\bar{b} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 2 & 1 & 3 \\ 4 & -2 & 2 \end{vmatrix} = i(2+6) - j(4-12) + k(-4-4)$$

$$= 8i+8j-8\bar{k}$$

$$= 8(i+j-\bar{k})$$

$|\bar{a}\times\bar{b}| = 8\sqrt{1+1+1} = 8\sqrt{3}.$

$$\sin\theta = \frac{|\bar{a}\times\bar{b}|}{|\bar{a}||\bar{b}|} = \frac{8\sqrt{3}}{\sqrt{14}\sqrt{24}} = \frac{\overset{4}{\cancel{8}}\times\sqrt{3}}{\cancel{\sqrt{2}}\sqrt{7}\cancel{\sqrt{2}}\sqrt{12}} = \frac{\overset{\sqrt{4}}{\cancel{4}}\times\sqrt{3}}{\sqrt{7}\times\cancel{\sqrt{4}}\times\cancel{\sqrt{3}}}$$

$$= \frac{2}{\sqrt{7}}$$

Ex 8.24) Find the area of the parallelogram whose adjacent sides are $\bar{a} = 3\bar{i}+\bar{j}+4\bar{k}$ $\bar{b} = \bar{i}-\bar{j}+\bar{k}$

$$\bar{a}\times\bar{b} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 3 & 1 & 4 \\ 1 & -1 & 1 \end{vmatrix} = \bar{i}(1+4) - \bar{j}(3-4) + \bar{k}(-3-1)$$

$$= 5i+\bar{j}-4\bar{k}$$

$|\bar{a}\times\bar{b}| = \sqrt{25+1+16} = \sqrt{42}.$

Area of the Parallelogram $|\bar{a}\times\bar{b}| = \sqrt{42}$ units.

Ex 8.25) For any two vectors $\bar{a}$ and $\bar{b}$ P.T $|\bar{a}\times\bar{b}|^2 + |\bar{a}\cdot\bar{b}|^2 = |\bar{a}|^2|\bar{b}|^2$

$|\bar{a}\times\bar{b}| = |\bar{a}||\bar{b}|\sin\theta$

$|\bar{a}\times\bar{b}|^2 = |\bar{a}|^2|\bar{b}|^2\sin^2\theta$ — ①

$\bar{a}\cdot\bar{b} = |\bar{a}||\bar{b}|\cos\theta$

$(\bar{a}\cdot\bar{b})^2 = |\bar{a}|^2|\bar{b}|^2\cos^2\theta$ — ②

① + ② $|\bar{a}\times\bar{b}|^2 + (\bar{a}\cdot\bar{b})^2 = |\bar{a}|^2|\bar{b}|^2(\sin^2\theta+\cos^2\theta)$

$$= |\bar{a}|^2|\bar{b}|^2$$

Ex 8.26) Find the area of the $\triangle$le having the points A (1,0,0) B (0,1,0) C (0,0,1)

$\bar{OA} = \bar{i}$ $\bar{AB} = \bar{OB}-\bar{OA} = -i+\bar{j}$

$\bar{OB} = \bar{j}$ $\bar{AC} = \bar{OC}-\bar{OA} = -i+\bar{k}$

$\bar{OC} = \bar{k}$

Area of the Δle $= \frac{1}{2}|\bar{AB} \times \bar{AC}|$

$$\bar{AB} \times \bar{AC} = \begin{vmatrix} i & j & k \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} = i(1-0) - j(-1-0) + \bar{k}(0+1)$$
$$= \bar{i} + j + \bar{k}.$$

$$|\bar{AB} \times \bar{AC}| = \sqrt{1+1+1} = \sqrt{3}.$$

Area of the Δle ABC $= \frac{1}{2}|\bar{AB} \times \bar{AC}| = \frac{1}{2}\cdot\sqrt{3} : \sqrt{3}/_2$

$\frac{1}{8.4}$) Find the magnitude of $\bar{a} \times \bar{b}$ if $\bar{a} = 2i + j + 3\bar{k}$ $\bar{b} = 3i + 5j - 2\bar{k}$

$$\bar{a} \times \bar{b} = \begin{vmatrix} i & j & \bar{k} \\ 2 & 1 & 3 \\ 3 & 5 & -2 \end{vmatrix} = i(-2-15) - j(-4-9) + \bar{k}(10-3)$$
$$= -17\bar{i} + 13j + 7\bar{k}.$$

$$|\bar{a} \times \bar{b}| = \sqrt{289+169+49} = \sqrt{507}$$

$\frac{2}{8.4}$) P.T $\bar{a} \times (\bar{b} + \bar{c}) + \bar{b} \times (\bar{c} + \bar{a}) + \bar{c} \times (\bar{a} + \bar{b}) = \bar{0}$.

LHS: $\bar{a} \times (\bar{b} + \bar{c}) + \bar{b} \times (\bar{c} + \bar{a}) + \bar{c} \times (\bar{a} + \bar{b})$

$$= \bar{a} \times \bar{b} + \bar{a} \times \bar{c} + \bar{b} \times \bar{c} + \bar{b} \times \bar{a} + \bar{c} \times \bar{a} + \bar{c} \times \bar{b}$$
$$= \bar{a} \times \bar{b} + \bar{a} \times \bar{c} + \bar{b} \times \bar{c} - \bar{a} \times \bar{b} - \bar{a} \times \bar{c} - \bar{b} \times \bar{c}$$
$$= \cancel{\bar{a} \times \bar{b}} + \cancel{\bar{a} \times \bar{c}} + \cancel{\bar{b} \times \bar{c}} - \cancel{\bar{a} \times \bar{b}} - \cancel{\bar{a} \times \bar{c}} - \cancel{\bar{b} \times \bar{c}}$$
$$= \bar{0}$$

$\frac{3}{8.4}$) Find the vectors of magnitude $10\sqrt{3}$ that are $\perp$r to the plane which contains $i + 2j + \bar{k}$ and $i + 3j + 4\bar{k}$.

$$\bar{a} \times \bar{b} = \begin{vmatrix} i & j & \bar{k} \\ 1 & 2 & 1 \\ 1 & 3 & 4 \end{vmatrix} = i(8-3) - j(4-1) + \bar{k}(3-2)$$
$$= 5\bar{i} - 3j + \bar{k}$$

$$|\bar{a} \times \bar{b}| = \sqrt{25+9+1} = \sqrt{35}$$

$$\hat{n} = \pm \frac{\bar{a} \times \bar{b}}{|\bar{a} \times \bar{b}|} = \pm \frac{(5i - 3j + \bar{k})}{\sqrt{35}}$$

$$10\sqrt{3}\,\hat{n} = \pm 10\sqrt{3} \frac{(5i - 3j + \bar{k})}{\sqrt{35}}$$

$\frac{4}{8.4}$) Find the unit vectors $\perp$r to each of the vectors $\bar{a} + \bar{b}$ and $\bar{a} - \bar{b}$ where $\bar{a} = i + j + \bar{k}$ $\bar{b} = i + 2j + 3\bar{k}$.

Let $\bar{c} = \bar{a} + \bar{b} = 2i + 3j + 4\bar{k}$

$\bar{d} = \bar{a} - \bar{b} = -j - 2k$.

$$\bar{c}\times\bar{d} = \begin{vmatrix} i & j & k \\ 2 & 3 & 4 \\ 0 & -1 & -2 \end{vmatrix} = i(-6+4) - j(-4-0) + \bar{k}(-2-0)$$

$$= -2\bar{i} + 4\bar{j} - 2\bar{k} = -2(\bar{i}+2\bar{j}+\bar{k})$$

$$|\bar{c}\times\bar{d}| = \sqrt{4+16+4} = \sqrt{24}.$$

$$\hat{n} = \pm\frac{\bar{c}\times\bar{d}}{|\bar{c}\times\bar{d}|} = \pm\frac{(-2\hat{i}+4\hat{j}-2\hat{k})}{\sqrt{24}}.$$

$$= \pm\frac{\cancel{2}(\bar{i}-2\bar{j}+\bar{k})}{\cancel{2}\sqrt{6}}$$

$\frac{5}{8.4}$) Find the area of the parallelogram whose adjacent sides are determined by the vectors $i+2j+3\bar{k}$, $3i-2j+\bar{k}$.

$$\bar{a}\times\bar{b} = \begin{vmatrix} i & j & \bar{k} \\ 1 & 2 & 3 \\ 3 & -2 & 1 \end{vmatrix} = i(2+6) - \bar{j}(1-9) + \bar{k}(-2-6)$$

$$= 8\bar{i} + 8\bar{j} - 8\bar{k}.$$

$$= 8(i+j-\bar{k})$$

$$|\bar{a}\times\bar{b}| = 8\sqrt{1+1+1} = 8\sqrt{3}.$$

Area of the parallelogram $|\bar{a}\times\bar{b}| = 8\sqrt{3}$ sq. units.

$\frac{6}{8.4}$) Find the area of the $\triangle$le whose vertices are $A(3,-1,2)$ $B(1,-1,-3)$ $C(4,-3,1)$

$$\bar{AB} = \bar{OB} - \bar{OA} = -2i - 5\bar{k}$$

$$\bar{AC} = \bar{OC} - \bar{OA} = i - 2j - k.$$

$$\bar{AB}\times\bar{AC} = \begin{vmatrix} i & j & \bar{k} \\ -2 & 0 & -5 \\ 1 & -2 & -1 \end{vmatrix} = i(0-10) - j(2+5) + \bar{k}(4-0)$$

$$= -10i - 7j + 4\bar{k}.$$

$$|\bar{AB}\times\bar{AC}| = \sqrt{100+49+16} = \sqrt{165}.$$

Area of the $\triangle$le : $\frac{1}{2}|\bar{AB}\times\bar{AC}| = \frac{1}{2}\sqrt{165}.$

$\frac{7}{8.4}$) If $\bar{a}, \bar{b}, \bar{c}$ are the P.V of A, B, C of a $\triangle$le ABC. S.T the area of the $\triangle ABC = \frac{1}{2}|\bar{a}\times\bar{b} + \bar{b}\times\bar{c} + \bar{c}\times\bar{a}|$ also deduce the condition for collinearity of the three points A, B, C.

Let $\overline{OA} = \bar{a}$ $\quad \overline{AB} = \overline{OB} - \overline{OA} = \bar{b} - \bar{a}$

$\overline{OB} = \bar{b}$

$\overline{OC} = \bar{c}$ $\quad \overline{AC} = \overline{OC} - \overline{OA} = \bar{c} - \bar{a}$

$$\overline{AB} \times \overline{AC} = (\bar{b} - \bar{a}) \times (\bar{c} - \bar{a})$$

$$= \bar{b} \times \bar{c} - \bar{b} \times \bar{a} - \bar{a} \times \bar{c} + \cancel{\bar{a} \times \bar{a}}^{\,0}$$

$$= \bar{b} \times \bar{c} + \bar{a} \times \bar{b} + \bar{c} \times \bar{a}$$

$$|\overline{AB} \times \overline{AC}| = |\bar{a} \times \bar{b} + \bar{b} \times \bar{c} + \bar{c} \times \bar{a}|$$

Area of the $\triangle$le : $\frac{1}{2} |\overline{AB} \times \overline{AC}|$

$$= \frac{1}{2} |\bar{a} \times \bar{b} + \bar{b} \times \bar{c} + \bar{c} \times \bar{a}|$$

If the three points are collinear Area of the $\triangle$le is 0.

$$\frac{1}{2} |\bar{a} \times \bar{b} + \bar{b} \times \bar{c} + \bar{c} \times \bar{a}| = 0$$

$$|\bar{a} \times \bar{b} + \bar{b} \times \bar{c} + \bar{c} \times \bar{a}| = 0.$$

$\frac{10}{8.4}$) Find the angle between the vectors $2\hat{i} + \hat{j} - \hat{k}$ and $\hat{i} + 2\hat{j} + \hat{k}$ using vector product.

$\bar{a} = 2\hat{i} + \hat{j} - \hat{k} \Rightarrow |\bar{a}| = \sqrt{4+1+1} = \sqrt{6}$

$\bar{b} = \hat{i} + 2\hat{j} + \hat{k}$ $\quad |\bar{b}| = \sqrt{1+4+1} = \sqrt{6}.$

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 1 & 2 & 1 \end{vmatrix} = \hat{i}(1+2) - \hat{j}(2+1) + \hat{k}(4-1)$$

$$= 3\hat{i} - 3\hat{j} + 3\hat{k}$$

$$= 3(\hat{i} - \hat{j} + \hat{k})$$

$$|\bar{a} \times \bar{b}| = 3\sqrt{1+1+1} = 3\sqrt{3}$$

$$\sin\theta = \frac{|\bar{a} \times \bar{b}|}{|\bar{a}||\bar{b}|} = \frac{3\sqrt{3}}{\sqrt{6}\sqrt{6}} = \frac{\cancel{3}\sqrt{3}}{\cancel{6}\,2}$$

$$\theta = \sin^{-1}(\sqrt{3}/2)$$

$$= \frac{\pi}{3}.$$

EXERCISE - 8.5 (one Mark)

1. The value of $\bar{AB}+\bar{BC}+\bar{CD}+\bar{DA}$ is

1) $\bar{AD}$ 2) $\bar{CA}$ 3) $\bar{0}$ 4) $-\bar{AD}$

By polygon law $\bar{AB}+\bar{BC}+\bar{CD}+\bar{DA}=0$.

$\because$ The starting point and end point are same at the same time the vectors continuously going on.

2) If $\bar{a}+2\bar{b}$ and $3\bar{a}+m\bar{b}$ are parallel then the value of m is

1) 3 2) $\frac{1}{3}$ 3) 6 4) $\frac{1}{6}$.

$$\bar{a}+2\bar{b} = 3\left(\bar{a}+\frac{m}{3}\bar{b}\right)$$

$$\Rightarrow \frac{m}{3} = 2 \Rightarrow m=6.$$

3) The unit vector parallel to the resultant of the vectors $\hat{i}+\hat{j}-\hat{k}$ and $\hat{i}-2\hat{j}+\hat{k}$ is

1) $\frac{\hat{i}-\hat{j}+\hat{k}}{\sqrt{5}}$ 2) $\frac{2\hat{i}+\hat{j}}{\sqrt{5}}$ 3) $\frac{2\hat{i}-\hat{j}+\hat{k}}{\sqrt{5}}$ 4) $\frac{2\hat{i}-\hat{j}}{\sqrt{5}}$

Let the Resultant vector $\bar{a} = 2\hat{i}-\hat{j}$

$$|\bar{a}| = \sqrt{4+1} = \sqrt{5}$$

$$\hat{a} = \frac{\bar{a}}{|\bar{a}|} = \frac{2\hat{i}-\hat{j}}{\sqrt{5}}$$

4) A vector $\bar{OP}$ makes $60°$ and $45°$ with the positive direction of the x axis and y axis. Then the angle between $\bar{OP}$ and the z axis is.

1) $45°$ 2) $60°$ 3) $90°$ 4) 30

$$\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$$

$$(\cos 60)^2+(\cos 45)^2+\cos^2\gamma=1$$

$$\frac{1}{4}+\frac{1}{2}+\cos^2\gamma=1$$

$$\cos^2\gamma=\frac{1}{4}$$

$$\cos\gamma=\frac{1}{2}$$

$$\gamma=60°$$

5) If $\bar{BA}=3\hat{i}+2\hat{j}+\hat{k}$ and the P.V of B is $\hat{i}+3\hat{j}-\hat{k}$ then the PV of A is

1) $4\hat{i}+2\hat{j}+\hat{k}$ 2) $4\hat{i}+5\hat{j}$ 3) $4\hat{i}$ 4) $-4\hat{i}$

$$\overline{BA} = \overline{OA} - \overline{OB} = 3\bar{i} + 2\bar{j} + \bar{k}$$

$$\overline{OA} = (3\bar{i} + 2\bar{j} + \bar{k}) + \overline{OB}$$

$$= (3\bar{i} + 2\bar{j} + \bar{k}) + (\bar{i} + 3\bar{j} - \bar{k})$$

$$= 4\bar{i} + 5\bar{j}$$

6) A vector makes equal angle with the positive direction of the co-ordinate axes. Then each angle is equal to

1) $\cos^{-1}(1/3)$ 2) $\cos^{-1}(2/3)$ 3) ✓ $\cos^{-1}(1/\sqrt{3})$ 4) $\cos^{-1}(2/\sqrt{3})$

Given $\alpha = \beta = \gamma$.

$$\cos^2\alpha + \cos^2\alpha + \cos^2\alpha = 1$$

$$3\cos^2\alpha = 1$$

$$\cos^2\alpha = \frac{1}{3} \Rightarrow \cos\alpha = \frac{1}{\sqrt{3}}$$

$$\alpha = \cos^{-1}(1/\sqrt{3})$$

7) The vectors $\bar{a} - \bar{b}$, $\bar{b} - \bar{c}$, $\bar{c} - \bar{a}$ are

1) Parallel to each other 2) unit vector.

3) mutually $\perp$r vectors 4) ✓ coplanar vectors.

$$\begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{vmatrix} = 1(1) - 1(+1) = 0$$

$\therefore$ They are coplanar.

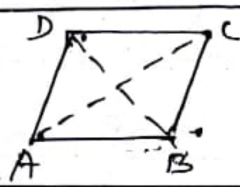
8) If ABCD is a parallelogram then $\overline{AB} + \overline{AD} + \overline{CB} + \overline{CD}$ is equal to

1) $2(\overline{AB} + \overline{AD})$ 2) $4\overline{AC}$ 3) $4\overline{BD}$ 4) ✓ $\bar{0}$

$$\overline{AB} + \overline{AD} = \overline{AB} + \overline{BC}$$

$$\overline{CB} + \overline{CD} = -\overline{BC} - \overline{AB}$$

$$\therefore \overline{AB} + \overline{AD} + \overline{CB} + \overline{CD} = \bar{0}.$$

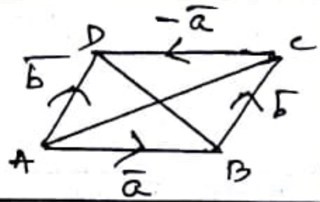


9) One of the diagonals of the parallelogram ABCD with adjacent sides $\bar{a}$ and $\bar{b}$ is $\bar{a} + \bar{b}$ then the other diagonal is

1) $\bar{a} - \bar{b}$ 2) ✓ $\bar{b} - \bar{a}$ 3) $\bar{a} + \bar{b}$ 4) $\frac{\bar{a} + \bar{b}}{2}$

$$\overline{BD} = \overline{BC} + \overline{CD}$$

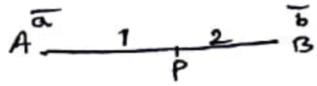
$$= \bar{b} - \bar{a}$$



10) If $\bar{a}, \bar{b}$ are the P.V. of A and B. Then which of the following points whose PV lies on AB.

1) $\bar{a}+\bar{b}$ 2) $\frac{2\bar{a}-\bar{b}}{2}$, 3) $\frac{2\bar{a}+\bar{b}}{3}$, 4) $\frac{\bar{a}-\bar{b}}{3}$

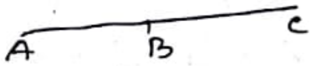
$$\bar{OP} = \frac{1\cdot\bar{b}+2\bar{a}}{1+2} = \frac{2\bar{a}+\bar{b}}{3}$$



11) If $\bar{a}, \bar{b}, \bar{c}$ are the P.V of three collinear points then which of the following is True.

1) $\bar{a} = \bar{b}+\bar{c}$ 2) $2\bar{a} = \bar{b}+\bar{c}$ 3) $\bar{b} = \bar{c}+\bar{a}$ 4) $4\bar{a}+\bar{b}+\bar{c}=0$

$\bar{OA} = \bar{a}$, $\bar{OB} = \bar{b}$, $\bar{OC} = \bar{c}$



X

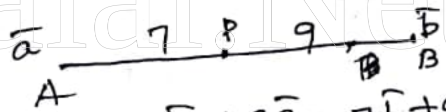
If the three points are collinear. $\bar{AB}+\bar{BC} = \bar{AC}$

$$\bar{OB}-\bar{OA}+\cancel{\bar{OC}}-\bar{OB} = \cancel{\bar{OC}}-\bar{OA}$$

12) If $\bar{r} = \frac{9\bar{a}+7\bar{b}}{16}$ then the point P whose p.v. $\bar{r}$ divides the line joining the points with PV $\bar{a}, \bar{b}$ in the ratio

1) 7 : 9 internally 2) 9 : 7 internally

3) 9 : 7 externally 4) 7 : 9 externally.



$$\bar{r} = \frac{7\bar{b}+9\bar{a}}{7+9} = \frac{7\bar{b}+9\bar{a}}{16}.$$

13) If $\lambda\hat{i}+2\lambda\hat{j}+2\lambda\hat{k}$ is a unit vector then the value of λ is

1) $1/3$ 2) $1/4$ 3) $1/9$ 4) $1/2$

$$\hat{a} = \frac{\bar{a}}{|\bar{a}|} = \frac{\lambda\hat{i}+2\lambda\hat{j}+2\lambda\hat{k}}{1}$$

$$|\bar{a}| = \sqrt{\lambda^2+4\lambda^2+4\lambda^2} = 1 \Rightarrow 3\lambda = 1$$
$$\lambda = 1/3.$$

14) Two vertices of a triangle have P.V $3\hat{i}+4\hat{j}-4\hat{k}$ and $2\hat{i}+3\hat{j}+4\hat{k}$. If the P.V of centroid is $\hat{i}+2\hat{j}+3\hat{k}$ then the PV of the third vertex is

1) $-2\hat{i}-\hat{j}+9\hat{k}$ 2) $-2\hat{i}-\hat{j}-6\hat{k}$ 3) $2\hat{i}-\hat{j}+6\hat{k}$ 4) $-2\hat{i}+\hat{j}+6\hat{k}$

$\bar{OA} = 3\hat{i}+4\hat{j}-4\hat{k}$ $\bar{OB} = 2\hat{i}+3\hat{j}+4\hat{k}$ $\bar{OA}+\bar{OB} = 5\hat{i}+7\hat{j}$	$\bar{OA}+\bar{OB}+\bar{OC} = 3(\hat{i}+2\hat{j}+3\hat{k})$ $\bar{OC} = (3\hat{i}+6\hat{j}+9\hat{k})-(5\hat{i}+7\hat{j})$ $= -2\hat{i}-\hat{j}+9\hat{k}$

15) If $|\bar{a}+\bar{b}| = 60$, $|\bar{a}-\bar{b}| = 40$ $|\bar{b}| = 46$ then $|\bar{a}|$ is

1) 42 2) 12 3) 22 4) 32

$$|\bar{a}+\bar{b}|^2 + |\bar{a}-\bar{b}|^2 = 60^2 + 40^2$$

$$2\left[|\bar{a}|^2 + |\bar{b}|^2\right] = 3600 + 1600$$

$$= 5200$$

$$|\bar{a}|^2 + 46^2 = 2600$$

$$|\bar{a}|^2 = 2600 - 2116$$

$$= 484 \Rightarrow |\bar{a}| = 22$$

46×46
276
184
2116

22×22
44
44

16) If $\bar{a}$ and $\bar{b}$ having same magnitude and angle between them is $60°$ and their scalar product is $\frac{1}{2}$ then $|\bar{a}|$ is

1) 2 2) 3 3) 7 4) 1

$$\bar{a}\cdot\bar{b} = |\bar{a}||\bar{b}|\cos\theta = \frac{1}{2}$$

$$|\bar{a}||\bar{a}|\cos 60° = \frac{1}{2}$$

$$|\bar{a}|^2 = \frac{1/2}{1/2} \Rightarrow |\bar{a}| = 1$$

17) The value of $\theta \in (0, \pi/2)$ for which the vectors $\bar{a} = (\sin\theta)\hat{i} + (\cos\theta)\hat{j}$ and $\bar{b} = \hat{i} - \sqrt{3}\hat{j} + 2k$ are $\perp r$ is equal to 1) $\pi/3$ 2) $\pi/6$ 3) $\pi/4$ 4) $\pi/2$

$$\bar{a}\cdot\bar{b} = \sin\theta - \sqrt{3}\cos\theta = 0$$

$$\sin\theta = \sqrt{3}\cos\theta$$

$$\frac{\sin\theta}{\cos\theta} = \sqrt{3} \Rightarrow \tan\theta = \sqrt{3}$$

$$\theta = \pi/3$$

18) If $|\bar{a}| = 13$ $|\bar{b}| = 5$ and $\bar{a}\cdot\bar{b} = 60°$ then $|\bar{a}\times\bar{b}|$ is

1) 15 2) 35 3) 45 4) 25.

$$|\bar{a}\times\bar{b}|^2 + (\bar{a}\cdot\bar{b})^2 = |\bar{a}|^2|\bar{b}|^2$$

$$= 169 \times 25$$

$$|\bar{a}\times\bar{b}|^2 = 4225 - 3600$$

$$= 625$$

$$|\bar{a}\times\bar{b}| = 25$$

19) Vectors $\bar{a}$ and $\bar{b}$ inclined at $120°$. If $|\bar{a}| = 1$, $|\bar{b}| = 2$ find $(a + 3\bar{b}) \times (3\bar{a} - \bar{b})$ is

1) 225 2) 275 3) 325 4) 300

$(\bar{a}+3\bar{b})\times(3\bar{a}-\bar{b}) = 3\cancel{\bar{a}\times\bar{a}} - \bar{a}\times\bar{b} + 9\,\bar{b}\times\bar{a} - 3\,\cancel{\bar{b}\times\bar{b}}$

$= -10\,\bar{a}\times\bar{b}$

$= -10\cdot|\bar{a}||\bar{b}|\sin 120^\circ$

$= +10\cdot 1\cdot 2\cdot \sin 60^\circ$

$= 10\times 1\times \cancel{2}\times\frac{\sqrt{3}}{\cancel{2}} = 10\sqrt{3}$

$\therefore [(\bar{a}+3\bar{b})\times(3\bar{a}-\bar{b})]^2 = 100\times 3 = 300.$

20) If $\bar{a}$ and $\bar{b}$ are two vectors of magnitude 2 and inclined at an angle 60° then angle between $\bar{a}$ and $\bar{a}+\bar{b}$

1) 30° 2) 60° 3) 45° 4) 90°.

$\bar{a}\cdot(\bar{a}+\bar{b}) = |\bar{a}|^2+\bar{a}\cdot\bar{b}$

$= |\bar{a}|^2+|\bar{a}||\bar{b}|\cos\theta$.

21) If the projection of $5\hat{i}-\hat{j}-3\hat{k}$ on $\hat{i}+3\hat{j}+\lambda\hat{k}$ is same as the projection of $\hat{i}+3\hat{j}+\lambda\hat{k}$ on $5\hat{i}-\hat{j}-3\hat{k}$ then λ is

1) ± 4 2) ± 3 3) ± 5 4) ± 1.

$$\frac{(5\hat{i}-\hat{j}-3\hat{k})\cdot(\hat{i}+3\hat{j}+\lambda\hat{k})}{\sqrt{1+9+\lambda^2}} = \frac{(\hat{i}+3\hat{j}+\lambda\hat{k})\cdot(5\hat{i}-\hat{j}-3\hat{k})}{\sqrt{25+1+9}}$$

$$\frac{5-3-3\lambda}{\sqrt{10+\lambda^2}} = \frac{5-3-3\lambda}{\sqrt{35}} \Rightarrow \frac{\cancel{2-3\lambda}}{\sqrt{10+\lambda^2}} = \frac{\cancel{2-3\lambda}}{\sqrt{35}}$$

$\sqrt{35} = \sqrt{10+\lambda^2}$

Sq. on both sides,

$35 = 10+\lambda^2 \Rightarrow \lambda^2 = 25$

$\lambda = \pm 5$

22) If $\overset{OA}{(1,2,4)}$ and $\overset{OB}{(2,-3\lambda,-3)}$ are the initial and terminal points of the vectors $\hat{i}+5\hat{j}-7\hat{k}$ then the value of λ is equal to

1) $7/3$ 2) $-7/3$ 3) $-5/3$ 4) $5/3$.

$\overline{OB}-\overline{OA} = \hat{i}+(-3\lambda-2)\hat{j}-7\hat{k} = \hat{i}+5\hat{j}-7\hat{k}$

$\Rightarrow -3\lambda-2 = 5$

$-3\lambda = 7 \quad \lambda = -7/3$

23) If the points whose P.V $10\hat{i}+3\hat{j}$, $12\hat{i}-5\hat{j}$, $a\hat{i}+11\hat{j}$ are collinear then a is equal to 1) 6 2) 3 3) 5 4) 8

$\overline{OA} = 10\hat{i} + 3\hat{j}$ $\quad \overline{AB} = \overline{OB} - \overline{OA} = 2\hat{i} - 8\hat{j}$

$\overline{OB} = 12\hat{i} - 5\hat{j}$ $\quad \overline{AC} = \overline{OC} - \overline{OA} = (a-10)\hat{i} + 8\hat{j}$ $\quad \Rightarrow 2 = 10-a$

$\overline{OC} = a\hat{i} + 11\hat{j}$. $\quad = -[(10-a)\hat{i} - 8\hat{j}]$ $\quad a = 10-2 = 8$.

24) If $\bar{a} = \hat{i}+\hat{j}+\bar{k}$, $\bar{b} = 2\hat{i}+x\hat{j}+\bar{k}$, $\bar{c} = \hat{i}-\hat{j}+4\bar{k}$ and $\bar{a}\cdot(\bar{b}\times\bar{c}) = 70$ then x is equal to

1) 5 2) 7 3) 26 4) 10.

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & x & 1 \\ 1 & -1 & 4 \end{vmatrix} = 70$$

$1(4x+1) - 1(8-1) + 1(-2-x) = 70$

$4x+1-7-2-x = 70$

$3x = 78$

$x = 26$.

25) $\bar{a} = \hat{i}+2\hat{j}+2\bar{k}$ $\quad |\bar{b}| = 5$ and the angle between $\bar{a}$ and $\bar{b}$ is $\frac{\pi}{6}$. Then the area of the $\triangle$le formed by these two vectors as two sides is 1) $7/4$ 2) $15/4$ 3) $\frac{3}{4}$

Area of the $\triangle$le $= \frac{1}{2}|\bar{a}\times\bar{b}|$ $\quad |\bar{a}| = \sqrt{1+4+4} = 3$

$= \frac{1}{2}|\bar{a}||\bar{b}|\sin\theta$

$= \frac{1}{2}\, 3\times 5\times \sin 30^\circ$

$= \frac{1}{2}\times 3\times 5\times \frac{1}{2} = \frac{15}{4}$