Higher Secondary - First Year (New Syllabus)

Mathematics

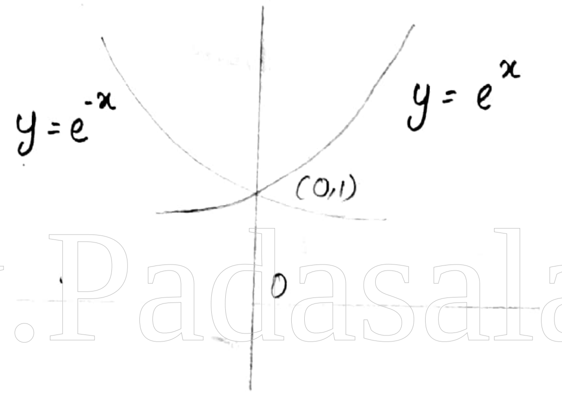
Volume I Book Back Questions Answers with Solution

CHAPTER - 1

1. If $A = \{(x,y) : y = e^x, x \in R\}$ and $B = \{(x,y) : y = e^{-x}, x \in R\}$

then $n(A \cap B)$ is ________

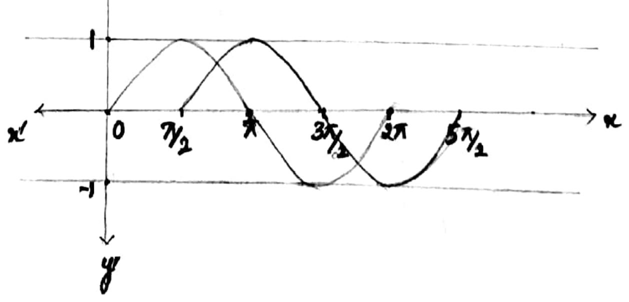
$n(A \cap B) = 1$.



$\boxed{\text{Ans: (c) 1}}$

2. If $A = \{(x,y) : y = \sin x, x \in R\}$ and $B = \{(x,y) : y = \cos x, x \in R\}$

then $A \cap B$ contains ________



$\boxed{\text{Ans: (b) infinitely many elements.}}$

Prepared By,
Dept of Mathematics, Jazz Public Mat. Hr.Sec.School, Melur, Madurai Dist 106

3. The relation R defined on a set $A = \{0, -1, 1, 2\}$ by xRy if $|x^2 + y^2| \leq 2$, then which one of the following is false?

a) $R = \{(0,0), (0,-1), (0,1), (-1,1), (1,2), (1,0)\}$

b) $R^{-1} = \{(0,0), (0,-1), (0,1), (-1,0), (1,0)\}$

c) Domain of R is $\{0, -1, 1, 2\}$

d) Range of R is $\{0, -1, 1\}$.

Sol:

Since $|x^2 + y^2| \leq 2$

x, y must be 0 or 1.

Ans:

d) Range of R is $\{0, -1, 1\}$

4. If $f(x) = |x-2| + |x+2|$, $x \in R$, then $f(x) =$ ______

Let $x \in (-\infty, -2)$, Let $x = -3$, then $f(x) = |-5| + |1| = 6 = -2x$

$x \in (-2, 2)$, Let $x = 0$, then $f(x) = |0-2| + |0+2| = 4$

$x \in (2, \infty)$, Let $x = 4$ then $f(x) = |2| + |6| = 2x$.

Ans:

a) $f(x) = \begin{cases} -2x & x \in (-\infty, -2] \\ 4 & x \in [-2, 2] \\ 2x & x \in (2, \infty) \end{cases}$

5. Let R be the set of all real numbers. Consider the following Subsets of the plane R×R $S = \{(x,y) : y = x+1 \text{ and } 0 < x < 2\}$ and $T = \{(x,y) : x-y$ is an integer$\}$. Then which of the following is true?

a) T is an equivalence relation but S is not an equivalence relation.

b) Neither S nor T is an equivalence relation.

c) Both S and T are equivalence relation.

d) S is an equivalence relation but T is not an equivalence relation.

Sol:

$x-y$ is an integer $\Rightarrow xRy$

i) $x-x=0$ is an integer $\Rightarrow \therefore xRx$ neglecting

ii) $(x-y)$ is an integer $\Rightarrow y-x$ is also an integer $\Rightarrow$ Symmetric.

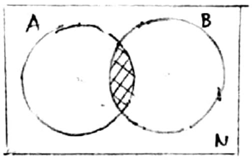
iii) If $(x-y)$ is an integer $\Rightarrow y-z$ is an integer $\Rightarrow x-z$ is also an integer $\Rightarrow \therefore T$ is equivalence.

iv) $y = x+1 \Rightarrow xRx$ is not true $\therefore S$ is not equivalent relative

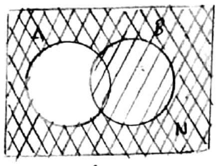
$\therefore T$ is an equivalent relation but S is not.

Ans:
(a) T is an equivalence relation but S is not an equivalence relation.

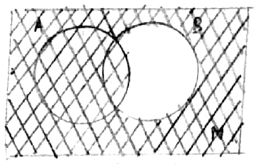
6. Let A and B be subsets of the Universal set N, the set of natural numbers. Then $A' \cup [(A \cap B) \cup B']$ is ______



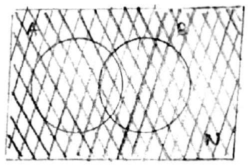
$A \cap B$



$A' \cup (A \cap B)$



$(A \cap B) \cup B'$



$A' \cup (A \cap B) \cup B'$

Ans: d) N.

7. The no. of students who take both the subjects Maths and chemistry is 70. This represents 10% of the enrollment in Maths and 14% of the enrollment in chemistry. How many students take at least one of these two subjects?

$M \cap C = 70$ which is 10% of M and 14% of C.

$M = 700$ $\quad M = \frac{70}{10/100} = \frac{70 \times 100}{10} = 700$

$C = 500.$ $\quad C = \frac{70}{14/100} = \frac{\cancel{70}^{5} \times 100}{\cancel{14}_{2}} = 500$

$M \cup C = 700 + 500 - 70$

$= 1130.$

Ans: (b) 1130

8. If $n[(A \times B) \cap (A \times C)] = 8$ and $n(B \cap C) = 2$, then $n(A)$ is ______

$(A \times B) \cap (A \times C) = A \times B \cap C.$

$n[(A \times B) \cap (A \times C)] = 8$

$n(B \cap C) = 2$

Ans: (b) 4

$n(A) = \frac{n[(A \times B) \times A \times C]}{n(B \cap C)} \Rightarrow \frac{8}{2} = n(A)$ $\quad \boxed{\therefore n(A) = 4}.$

9. If $n(A)=2$ and $n(B\cup C)=3$, then $n[(A\times B)\cup(A\times C)]$ is

$$n[(A\times B)\cup(A\times C)] = n(A)\times n(B\cup C)$$

$$= 3\times 2$$

$$\boxed{n[(A\times B)\cup(A\times C)] = 6.}$$

$\boxed{\text{Ans: c) } 6.}$

10. If two sets A and B have 17 elements in common, then the no. of elements common to the set $A\times B$ and $B\times A$ is.

Let $A=\{1,2,3,4\}$

$B=\{5,2,3,6\}$.

A and B have two elements in Common.

No. of elements common to $A\times B$ and $B\times A = 2\times 2 = 2^2$.

Similarly, we have 17 elements in common.

No. of elements common to $A\times B$ and $B\times A = 17\times 17 = 17^2$.

$\boxed{\text{Ans: b) } 17^2.}$

11. For non-empty sets A and B, if $A\subset B$ then $(A\times B)\cap(B\times A)$ equal

Let $A=\{a,b\}$ $B=\{a,b,c\}$

$A\times B=\{(a,a)(a,b)(a,c)(b,a)(b,b)(b,c)\}$

$B\times A=\{(a,a)(a,b)(b,a)(b,b)(c,a)(c,b)\}$

$(A\times B)\cap(B\times A) = \{(a,a)(a,b)(b,a)(b,b)\}$

$= A\times A.$ $\boxed{\text{Ans: b) } A\times A}$

12. The no. of relations on a set containing 3 element is

Let $S = \{a, b, c\}$

$n(S) = 3 \Rightarrow n(S \times S) = 9.$

No. of relations in $n\{P(S \times S)\} = 2^9 = 512.$

$\boxed{\text{Ans: (c) 512}}$

13. Let R be the Universal relation on a set X with more than one element. Then R is

a) not reflexive b) not symmetric c) transitive d) none of these.

Sol:

Let $X = \{a, b, c\}$

Then, R = Universal relation.

$= \{(a,a)\ (a,b)\ (a,c)\ (b,b)\ (b,c)\ (b,a)\ (c,a)\ (c,b)\ (c,c)\}$

$\therefore$ It is transitive.

$\boxed{\text{Ans: c) transitive}}$

14. Let $X = \{1, 2, 3, 4\}$ and $R = \{(1,1)\ (1,2)\ (1,3)\ (2,2)\ (3,3)\ (2,1)\ (3,1)\ (1,4)\ (4,1)\}$.

Then R is

a) reflexive b) symmetric c) transitive d) equivalence.

Sol:

$(4,4) \notin R$

$\therefore$ R is not reflexive.

Symmetric can be easily checked $\Rightarrow$ If aRb then bRa.

$\therefore$ R is Symmetric.

$\boxed{\text{Ans: b) Symmetric.}}$

15. The range of the function $\frac{1}{1-2\sin x}$ is

$$-1 \le \sin x \le 1.$$

$$-2 \le 2\sin x \le 2.$$

$$2 \ge -2\sin x \ge -2. \Rightarrow -2 \le -2\sin x \le 2.$$

Adding 1,

$$1-2 \le 1-2\sin x \le 3$$

$$-1 \ge \frac{1}{1-2\sin x} \ge 1/3.$$

$$1/3 \le \frac{1}{1-2\sin x} \le -1.$$

$\therefore$ Range is $(-\infty, -1] \cup [1/3, \infty)$

$\boxed{\text{Ans: d) } (-\infty,-1] \cup [1/3, \infty)}$

16. The range of the function $f(x) = |\lfloor x \rfloor - x|$, $x \in R$ is

$$f(x) = |\lfloor x \rfloor - x|$$

$f(x) = \lfloor x \rfloor - x$

$f(0) = 0 - 0 = 0.$

$f(6.5) = 6 - 6.5 = -0.5$

$f(-7.2) = 8 - 7.2 = 0.8$

$\therefore$ Range is $(0,1)$

$\boxed{\text{Ans: c) } [0,1)}$

17. The rule $f(x) = x^2$ is a bijection if the domain and the co-domain are given by

a) R, R b) $R, (0,\infty)$ c) $(0,\infty), R$ d) $[0,\infty), [0,\infty)$

Sol:

The domain is $(0,\infty)$

The co-domain is $(0,\infty)$

$\boxed{\text{Ans: d) } [0,\infty), [0,\infty)}$

18. The number of constant functions from a set containing m elements to a set containing n elements is ______

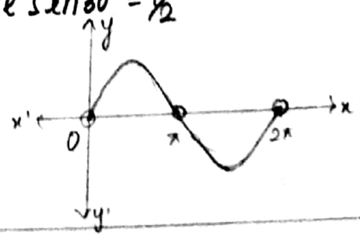
[DEFINITION]

$\boxed{\text{Ans: c) } n}$

19. The function $f: [0, 2\pi] \to [-1, 1]$ defined by $f(x) = \sin x$ is

It is onto and not one-one. since $\sin 30° = 1/2$

$\sin 150° = 1/2.$



$\boxed{\text{Ans: b) onto}}$

20. If the function $f : [-3,3] \to S$ defined by $f(x) = x^2$ is onto, then S is

$f(0) = 0.$

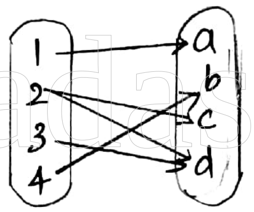
$f(-3) = 9$

$f(3) = 9.$

Ans: d) $[0,9]$

21. Let $X = \{1,2,3,4\}$, $Y = \{a, b, c, d\}$ and $f = \{(1,a)\ (4,b)\ (2,c)\ (3,d)\ (2,d)\}$. Then f is

It is not a function since it has two images.



Ans: d) not a function.

22. The inverse of $f(x) = \begin{cases} x & x < 1 \\ x^2 & 1 \le x \le 4 \\ 8\sqrt{x} & x > 4 \end{cases}$ is

Let $y = x$ then $x = y \Rightarrow f^{-1}(x) = x.$

Let $y = x^2$ then $y = \sqrt{x} \Rightarrow f^{-1}(x) = \sqrt{x}.$ $[y^2 = x ; y = \sqrt{x}]$

Let $y = 8\sqrt{x}$ then $y^2/64 = x \Rightarrow f^{-1}(x) = x^2/64.$ $[\sqrt{y} = x/8 ; y = x^2/64]$

Ans:

a) $f^{-1}(x) = \begin{cases} x & x < 1 \\ \sqrt{x} & 1 \le x \le 16 \\ x^2/64 & x > 16. \end{cases}$

23. Let $f: R \to R$ be defined by $f(x) = 1 - |x|$. Then the range of f is

$f: R \to R$ defined by $f(x) = 1 - |x|$

The range is $(-\infty, 1]$ as $f(-\infty) = -\infty$

$f(0) = 1$

$f(\infty) = -\infty$.

Ans: d) $(-\infty, 1]$

24. The function $f: R \to R$ is defined by $f(x) = \sin x + \cos x$ is

$$f(x) = \sin x + \cos x.$$

$$f(-x) = \sin(-x) + \cos(-x)$$

$$= -\sin x + \cos x$$

$$-f(-x) = \sin x - \cos x.$$

$\therefore f(x)$ is neither odd function nor even function.

Ans: b) neither an odd function nor an even function

25. The function $f: R \to R$ is defined by $f(x) = \frac{(x^2+\cos x)(1+x^4)}{(x-\sin x)(2x-x^3)} + e^{-|x|}$ is

$$f(x) = \frac{x^2 + \cos x}{(x - \sin x)(2x - x^3)} + e^{-|x|}$$

$$f(-x) = \frac{(-x)^2 + \cos(-x)}{[-x - \sin(-x)][-2x - (-x)^3]} + e^{-|-x|}$$

$$= \frac{x^2 + \cos x}{(x - \sin x)(2x - x^3)} + e^{-|x|}$$

$= f(x)$ $\therefore$ It is an even function.

Ans: c) an even function

Chapter - 2

1\. If $|x+2| \leq 9$, then x belongs to

$$|x+2| \leq 9$$
$$-9 \leq x+2 \leq 9$$
$$-11 \leq x \leq 7$$
$$x \in [-11, 7]$$

$\boxed{\text{Ans: b) } [-11, 7]}$

2\. Given that x, y and b are real numbers $x < y$, $b > 0$, then

$$x < y \;,\; b > 0$$
$$\Rightarrow xb < yb.$$

$\boxed{\text{Ans: a) } xb < yb}$

3\. If $\frac{|x-2|}{x-2} \geq 0$, then x belongs to

In $(-\infty, 2)$ $\frac{|x-2|}{x-2}$ is negative.

In $(2, \infty)$ $\frac{|x-2|}{x-2} > 0$.

$\boxed{\text{Ans: a) } (2, \infty)}$

4\. The solution of $5x-1 < 24$ and $5x+1 > -24$ is

$5x-1 < 24$	$5x+1 > -24$
$5x < 25$	$5x > -25$
$x < 5$	$x > -5 \Rightarrow \boxed{-5 < x < 5}$

$\boxed{\text{Ans: c) } (-5, 5)}$

5. The solution set of the following inequality $|x-1| \geq |x-3|$ is

When $x=0$, $|-1| \geq |-3|$ not true.

$\therefore$ cannot be [(a) $[0,2]$] → option

When $x=1$, $|0| \geq |-3|$ not true.

$\therefore$ cannot be [(c) $(0,2)$] → option

When $x=-4$, $|-5| \geq |-7|$ not true.

$\therefore$ cannot be [(d) $(-\infty,2)$] → option.

Ans: b) $[2,\infty)$

6. The value of $\log_{\sqrt{2}} 512$ is

Let $\log_{\sqrt{2}} 512 = x$

$(\sqrt{2})^x = 512$ [Exponential form]

$(2)^{x/2} = 512$

$(2)^{x/2} = (2)^9$

$x/2 = 9$

$\boxed{x = 18.}$

Ans: b) 18

7 The value of $\log_3 \frac{1}{81}$ is

Let $\log_3 \frac{1}{81} = x$.

$(3)^x = \frac{1}{81}$ [Exponential form]

$$(3)^x = 1/(3)^4$$

$$(3)^x = (3)^{-4}$$

$$\boxed{x = -4.}$$

Ans: c) -4

8. If $\log_{\sqrt{x}} 0.25 = 4$, then the value of x is

$$\log_{\sqrt{x}} 0.25 = 4$$

$$(\sqrt{x})^4 = 0.25 \text{ [Exponential form]}$$

$$(\sqrt{x})^4 = 1/4$$

$$x^2 = \left(\tfrac{1}{2}\right)^2$$

$$\boxed{x = 0.5}$$

Ans: a) 0.5.

9. The value of $\log_a b \log_b c \log_c a$ is

$$\log_a b \log_b c \log_c a = \log_a c \log_c a = \log_a a$$

$$= 1.$$

Ans: b) 1.

14

10. If 3 is the logarithm of 343, then base is

$$\log_x 343 = 3.$$

$$x^3 = 343$$

$$x^3 = 7^3$$

$$\boxed{x = 7}$$

Ans: b) 7

11. Find a so that the sum and product of the roots of the equation. $2x^2 + (a-3)x + 3a - 5 = 0$ are equal is

$$2x^2 + (a-3)x + 3a - 5 = 0.$$

$$\text{Sum} = \frac{-(a-3)}{2} \Rightarrow (-b/a) \quad \text{Product} = \frac{3a-5}{2} \Rightarrow (c/a)$$

Given that they are equal.

$$\frac{-a+3}{2} = \frac{3a-5}{2}$$

$$-2a + 6 = 6a - 10.$$

$$6a + 2a = 6 + 10$$

$$8a = 16$$

$$\boxed{a = 2}$$

Ans: b) 2.

12. If a and b are the roots of the equation $x^2-kx+16=0$ satisfy $a^2+b^2=32$, then the value of k is

$x^2-kx+16=0$, a and b are roots.

Sum of the roots, $a+b=k$ $(-b/a)$

Product of the roots, $ab=16$ (c/a)

$$\boxed{a^2+b^2=(a+b)^2-2ab}$$

$$32 = k^2-2(16)$$

$$32 = k^2-32$$

$$k^2=64$$

$$k=\pm 8$$

$$\boxed{k=-8,8}$$

Ans: c) $k=-8,8$

13. The number of solutions of $x^2+|x-1|=1$ is

$$x^2+|x-1|=1.$$

$$|x-1|=1-x^2$$

$1-x^2=x-1$

$x^2+x-2=0$

$(x+2)(x-1)=0$

$x=-2 \;\|\; x=1.$

$1-x^2=-x+1$

$x^2-x=0.$

$x(x-1)=0$

$x=0 \;\|\; x=1.$

∴ The roots are $-2,0,1$. Since -2 does not satisfy

∴ The roots are 0 and 1

No. of Solution → (2)

Ans: c) 2

14. The equation whose roots are numerically equal but opposite in sign to the roots of $3x^2-5x-7=0$ is

$3x^2-5x-7=0$, Let α and β be the roots.

Sum of the roots, $\alpha+\beta = 5/3$ $(-b/a)$

Product of the roots, $\alpha\beta = -7/3$ (c/a)

Now, take the roots as $-\alpha$ and $-\beta$

Sum of the roots, $-(\alpha+\beta) = -5/3$ $[\because (-\alpha-\beta) = -(\alpha+\beta)]$

Product of the roots, $(-\alpha)(-\beta) = -7/3$ $[\because (-\alpha)(-\beta) = +\alpha\beta]$

$\therefore$ The required equation $x^2+\frac{5}{3}x-\frac{7}{3}=0$.

$3x^2+5x-7=0$.

$\boxed{\text{Ans: b) } 3x^2+5x-7=0}$

15. If 8 and 2 are the roots of $x^2+ax+c=0$ and 3, 3 are the roots of $x^2+dx+b=0$; then the roots of the equation $x^2+ax+b=0$ are

$x^2+ax+c=0$

8 and 2 are the roots

Sum of the roots, $8+2=-a$

$\boxed{a=-10}$

$x^2+dx+b=0$

3 and 3 are the roots.

Product of the roots, $(3)(3)=b$

$\boxed{b=9}$

$\therefore$ The roots of $x^2+ax+b=0$.

$x^2-10x+9=0$.

$(x-1)(x-9)=0.$

$\boxed{x=1}\ \boxed{x=9}$

Ans: c) 9, 1

16. If a and b are the real roots of the equation $x^2-kx+c=0$, then the distance between the points $(a,0)$ and $(b,0)$ is

$x^2-kx+c=0$; a and b are the roots

$\therefore\ a+b=k\ (-b/a)$

$ab=c.\ (c/a)$

To find $\sqrt{(a-b)^2+0^2} = \sqrt{a^2+b^2-2ab} = \sqrt{(a+b)^2-4ab}$

(or) $(a-b)$

$= \sqrt{k^2-4c}$

Ans: a) $\sqrt{k^2-4c}$

17. If $\dfrac{kx}{(x+2)(x-1)} = \dfrac{2}{x+2} + \dfrac{1}{x-1}$, then the value of k is

$$\frac{2}{x+2}+\frac{1}{x-1} = \frac{2(x-1)+(x+2)}{(x+2)(x-1)}$$

$$= \frac{2x-\cancel{2}+x+\cancel{2}}{(x+2)(x-1)}$$

$$\frac{kx}{(x+2)(x-1)} = \frac{3x.}{(x+2)(x-1)} \qquad \boxed{\therefore k=3}$$

Ans: c) 3.

18. If $\frac{1-2x}{3+2x-x^2} = \frac{A}{3-x} + \frac{B}{x+1}$, then the value of A+B is

$$1-2x = A(x+1)+(B)(3-x)$$

$3-x=0$, $x=3$ | $x+1=0$, $x=-1$

Put $x=3$; $4A = -5 \Rightarrow A = -5/4$.

Put $x=-1$; $4B = 3 \Rightarrow B = 3/4$.

$$A+B = -5/4 + 3/4 = -2/4 = -1/2$$

Ans: a) $-1/2$

19. The number of real roots of $(x+3)^4 + (x+5)^4 = 16$ is

$\hookrightarrow$ 2 real roots $\hookrightarrow$ 2 real roots

$2+2 \Rightarrow 4$ real roots.

Ans: a) 4

20. The value of $\log_3 11 \cdot \log_{11} 13 \cdot \log_{13} 15 \cdot \log_{15} 27 \cdot \log_{27} 81$ is

$$\log_3 11 \cdot \log_{11} 13 \cdot \log_{13} 15 \cdot \log_{15} 27 \cdot \log_{27} 81 = \log_3 81$$
$$= \log_3 3^4$$
$$= 4 \log_3 3$$
$$= 4 \quad [\log_n n = 1]$$

Ans: d) 4

Chapter - 3

1. $\dfrac{1}{\cos 80^\circ} - \dfrac{\sqrt{3}}{\sin 80^\circ} =$

Let $x = \dfrac{1}{\cos 80^\circ} - \dfrac{\sqrt{3}}{\sin 80^\circ} = \dfrac{\sin 80^\circ - \sqrt{3}\cos 80^\circ}{\sin 80^\circ \cos 80^\circ}$

Divide by 2 on both sides

$$x/2 = \frac{\frac{1}{2}\sin 80^\circ - \frac{\sqrt{3}}{2}\cos 80^\circ}{\frac{1}{2}\sin 80^\circ \cos 80^\circ}$$

$$= \frac{\sin 80^\circ \cos 60^\circ - \cos 80^\circ \sin 30^\circ}{\frac{1}{2}\sin 80^\circ \cos 80^\circ}$$

$$= \frac{\sin 20^\circ}{\frac{1}{2}\sin 80^\circ \cos 80^\circ}$$

$$= \frac{\sin 160^\circ}{\frac{1}{2}\sin 80^\circ \cos 80^\circ}$$

$$= \frac{2\,\cancel{\sin 80^\circ}\,\cancel{\cos 80^\circ}}{\frac{1}{2}\,\cancel{\sin 80^\circ}\,\cancel{\cos 80^\circ}}$$

$$x/2 = 4.$$

$\boxed{\text{Ans: d) 4.}}$

2. If $\cos 28^\circ + \sin 28^\circ = k^3$, then $\cos 17^\circ$ is equal to

$$\cos 28^\circ + \sin 28^\circ = k^3.$$

$$\cos 28^\circ + \sin(90^\circ - 62^\circ) = k^3$$

$$\cos 28^\circ + \cos 62^\circ = k^3.$$

$$2\cos 45^\circ \cos 17^\circ = k^3$$

$$\cancel{2}^{\sqrt{2}}\ \frac{1}{\cancel{\sqrt{2}}} \cos 17^\circ = k^3$$

$$\boxed{\cos 17^\circ = \frac{k^3}{\sqrt{2}}}$$

$\boxed{\text{Ans: a) } \frac{k^3}{\sqrt{2}}.}$

3. The maximum value of $4\sin^2 x + 3\cos^2 x + \sin \frac{x}{2} + \cos \frac{x}{2}$ is

$$4\sin^2 x + 3\cos^2 x + \sin \frac{x}{2} + \cos \frac{x}{2} = 3(\sin^2 x + \cos^2 x) + \sin^2 x + \sin \frac{x}{2} + \cos \frac{x}{2}$$

$[\sin^2 x + \cos^2 x = 1]$

$$= 3 + \sin^2 x + \sin \frac{x}{2} + \cos \frac{x}{2}.$$

Maximum of $\sin x = 1$

Maximum of $\sin^2 x = 1$.

Maximum value of $\sin \frac{x}{2}$ occurs where $x = 45^\circ$.

Maximum value is $3 + 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = 4 + \frac{2}{\sqrt{2}}$

$$= 4 + \sqrt{2}.$$

$\boxed{\text{Ans: a) } 4 + \sqrt{2}}$

4. $\left[1+\cos\frac{\pi}{8}\right]\left[1+\cos\frac{3\pi}{8}\right]\left[1+\cos\frac{5\pi}{8}\right]\left[1+\cos\frac{7\pi}{8}\right] =$

$$\left[2\cos^2\frac{\pi}{16}\right]\left[2\cos^2\frac{3\pi}{16}\right]\left[2\cos^2\frac{5\pi}{16}\right]\left[2\cos^2\frac{7\pi}{16}\right]$$

$$= 2^4\left[\cos\frac{\pi}{16}\cos\frac{3\pi}{16}\cos\frac{5\pi}{16}\cos\frac{7\pi}{16}\right]^2$$

$$= \left[\cos\frac{8\pi}{16}+\cos\frac{6\pi}{16}\right]^2\left[\cos\frac{8\pi}{16}+\cos\frac{2\pi}{16}\right]^2$$

$$= \left[\cos\frac{6\pi}{16}\right]^2\left[\cos\frac{2\pi}{16}\right]^2$$

$$= \left\{\frac{1}{2}\left[\cos\frac{8\pi}{16}+\cos\frac{4\pi}{16}\right]\right\}^2$$

$$= \frac{1}{4}\times\frac{1}{2}$$

$$= \frac{1}{8}.$$

$\boxed{\text{Ans: a) } 1/8}$

5. If $\pi < 2\theta < \frac{3\pi}{2}$, $\sqrt{2+\sqrt{2+2\cos 4\theta}}$ equals to

$$\sqrt{2+2\cos 4\theta} = \sqrt{2+2(2\cos^2 2\theta-1)}$$

$$= \sqrt{4\cos^2 2\theta}$$

$$= 2\cos 2\theta.$$

$$\sqrt{2+\sqrt{2+2\cos 4\theta}} = \sqrt{2+2\cos 2\theta}$$

$$= \sqrt{2+2(2\cos^2\theta - 1)}$$

$$= \sqrt{4\cos^2\theta}$$

$$= 2\cos\theta, \sin 2\theta$$

$\pi < 2\theta < 3/2 \Rightarrow \pi/2 < \theta < \frac{3\pi}{4}$ is II^{nd} quadrant

$\boxed{\text{Ans: a) } -2\cos\theta}$

6. If $\tan 40^\circ = \lambda$, then $\frac{\tan 140^\circ - \tan 130^\circ}{1+\tan 140^\circ \tan 130^\circ} =$

$$\frac{\tan 140^\circ - \tan 130^\circ}{1+\tan 140^\circ \tan 130^\circ} = \tan(140^\circ - 130^\circ)$$

$$= \tan 10^\circ.$$

$$\tan 40^\circ = \lambda \; ; \; \tan 80^\circ = \frac{2\tan 40^\circ}{1-\tan^2 40} = \frac{2\lambda}{1-\lambda^2}.$$

$$\tan(90^\circ - 10^\circ) = \frac{2\lambda}{1-\lambda^2} \quad (\tan 80^\circ)$$

$$\cot 10^\circ = \frac{2\lambda}{1-\lambda^2} \Rightarrow \tan 10^\circ = \frac{1-\lambda^2}{2\lambda}.$$

From $\boxed{\frac{\tan 140^\circ - \tan 130^\circ}{1-\tan 140^\circ \tan 130^\circ} = \frac{1-\lambda^2}{2\lambda}}$

$\boxed{\text{Ans: c) } \frac{1-\lambda^2}{2\lambda}}$

7. $\cos 1^\circ + \cos 2^\circ + \cos 3^\circ + \cdots + \cos 179^\circ =$

$\cos 1^\circ + \cos 2^\circ + \cos 3^\circ + \ldots + \cos 179^\circ$

$= (\cos 1^\circ + \cos 179^\circ) + (\cos 2^\circ + \cos 178^\circ) + \cdots$

$= 2\cos 90^\circ \cos 89^\circ + 2\cos 90^\circ \cos 80^\circ + \cdots 0 + 0 + \cdots$

$= 0.$

$\boxed{\underline{\text{Ans}}: \text{a) } 0.}$

8. Let $f_k(x) = \frac{1}{k}[\sin^k x + \cos^k x]$ where $x \in R$ and $k \geq 1$. Then $f_4(x) - f_6(x) =$

$f_4(x) - f_6(x) = \frac{1}{4}[\sin^4 x + \cos^4 x] - \frac{1}{6}[\sin^6 x + \cos^6 x]$

$= \frac{1}{4}[[\sin^2 x + \cos^2 x]^2 - 2\sin^2 x \cos^2 x] - \frac{1}{6}[(\sin^2 x + \cos^2 x)^3 - 3(\sin^2 x \cos^2 x) + (\sin^2 x + \cos^2 x)]$

$= \frac{1}{4}[1 - 2\sin^2 x \cos^2 x] - \frac{1}{6}[1 - 3\sin^2 x \cos^2 x]$

$= \frac{1}{4} - \cancel{\frac{1}{2}\sin^2 x \cos^2 x} - \frac{1}{6} + \cancel{\frac{1}{2}\sin^2 x \cos^2 x}.$

$= \frac{1}{4} - \frac{1}{6}$

$= \frac{3-2}{12} = \frac{1}{12}.$

$\boxed{\underline{\text{Ans}}: \text{b) } \frac{1}{12}.}$

9. Which of the following is not true?

a) $\sin\theta = -3/4$ b) $\cos\theta = -1$ c) $\tan\theta = 25$ d) $\sec\theta = 1/4$.

Sol:

Since, $|\cos x| < 1$

$\therefore \sec\theta = 1/4 \Rightarrow \cos\theta = 4$ is not possible.

$\boxed{\text{Ans: d) } \sec\theta = 1/4}$

10. $\cos 2\theta \cos 2\phi + \sin^2(\theta - \phi) - \sin^2(\theta - \phi)$ is equal to

$\cos 2\theta \cos 2\phi + \sin^2(\theta - \phi) - \sin^2(\theta - \phi)$

$= \cos 2\theta \cos 2\phi + 1/2 - 1/2 \cos(2\theta - 2\phi) - 1/2 + 1/2 \cos(2\theta + 2\phi)$

$= \cos 2\theta \cos 2\phi + 1/2 [\cos(2\theta + 2\phi) - \cos(2\theta - 2\phi)]$

$= \cos 2\theta \cos 2\phi - \sin 2\theta \cos 2\phi$

$= \cos(2\theta + 2\phi)$

$= \cos 2(\theta + \phi)$

$\boxed{\text{Ans: b) } \cos 2(\theta + \phi)}$

11. $\frac{\sin(A-B)}{\cos A \cos B} + \frac{\sin(B-C)}{\cos B \cos C} + \frac{\sin(C-A)}{\cos C \cos A}$ is

$\frac{\sin(A-B)}{\cos A \cos B} = \frac{\sin A \cos B - \cos A \sin B}{\cos A \cos B} = \frac{\sin A \cos B}{\cos A \cos B} - \frac{\cos A \sin B}{\cos A \cos B}$

$$\frac{\sin(A-B)}{\cos A \cos B} = \tan A - \tan B$$

Similarly,

$$\frac{\sin(B-C)}{\cos B \cos C} = \tan B - \tan C.$$

$$\frac{\sin(C-A)}{\cos C \cos A} = \tan C - \tan A.$$

$$\cancel{\tan A} - \cancel{\tan B} + \cancel{\tan B} - \cancel{\tan C} + \cancel{\tan C} - \cancel{\tan A} = 0.$$

$\boxed{\text{Ans: c) } 0.}$

12. If $\cos p\theta + \cos q\theta = 0$ and if $p \neq q$, then θ is equal to [n is any integer]

$$\cos p\theta + \cos q\theta = 0.$$

$$2\cos\left[\frac{p+q}{2}\right]\theta \cdot \cos\left[\frac{p-q}{2}\right]\theta = 0$$

$$\Rightarrow 2\cos\left(\frac{p+q}{2}\right)\theta = 0$$

$$\Rightarrow \cos\left[\frac{p-q}{2}\right]\theta = 0.$$

Principal angle is $\pi/2$

$$\therefore \left(\frac{p+q}{2}\right) = \theta = 2n\pi \pm \frac{\pi}{2}$$

$$\Rightarrow (p+q)\theta = 4n\pi \pm \pi$$

$$\theta = \frac{\pi(4n\pm 1)}{p+q}$$

$$\text{Similarly}, \theta = \frac{\pi(4n\pm 1)}{p-q}$$

Ans: $\boxed{\frac{\pi(4n\pm 1)}{p-q}}$

13. If $\tan\alpha$ and $\tan\beta$ are the roots of $x^2+ax+b=0$, then $\frac{\sin(\alpha+\beta)}{\sin\alpha\sin\beta}$ is equal to

$$x^2+ax+b=0$$

Let α and β be the roots.

$$\tan\alpha+\tan\beta = -a$$
$$\tan\alpha\tan\beta = b.$$

$$\frac{\sin(\alpha+\beta)}{\sin\alpha\sin\beta} = \frac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\sin\alpha\sin\beta}$$

$$= \cot\beta+\cot\alpha$$

$$= 1/\tan\alpha + 1/\tan\beta$$

$$= \frac{\tan\alpha+\tan\beta}{\tan\alpha\tan\beta}$$

$$= -a/b.$$

Ans: $\boxed{\text{c) } -a/b.}$

14. In a triangle ABC, $\sin^2 A + \sin^2 B + \sin^2 C = 2$, then the triangle is

$$\sin^2 A + \sin^2 B + \sin^2 C = 2$$

Suppose it is right triangle with $\hat{C} = 90^\circ$

Then $\sin^2 C = 1$ and so $\sin^2 A + \sin^2 B = 1$.

Also $A + B = 90^\circ$

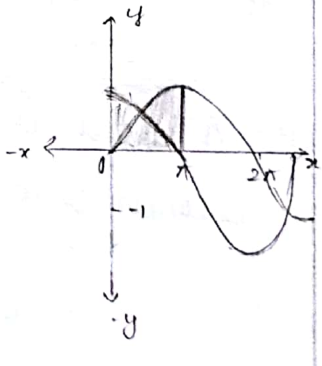
$$\sin A = \sin(90^\circ - B)$$
$$= \cos B.$$

$\therefore \cos^2 A + \sin^2 B + 1 = 2$ which is true.

$\boxed{\text{Ans: c) right triangle.}}$

15. If $f(\theta) = |\sin\theta| + |\cos\theta|$, $\theta \in R$, then $f(\theta)$ is in the interval

$$f(\theta) = |\sin\theta| + |\cos\theta|$$



It lies in $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$ and $0+1$

$= \sqrt{2}$ and 1.

$\boxed{\text{Ans: b) } [1, \sqrt{2}]}$

16. $\dfrac{\cos 6x + 6\cos 4x + 15\cos 2x + 10}{\cos 5x + 5\cos 3x + 10\cos x}$ is equal to

Numerator $= \cos 6x + 6\cos 4x + 15\cos 2x + 10.$

$= (\cos 6x + \cos 4x) + 5(\cos 4x + \cos 2x) + 10(\cos 2x + 1)$

Numerator $= 2\cos 5x \cos x + 5(2\cos 3x \cdot \cos x) + 10(2\cos^2 x)$

$= 2\cos x\,[\cos 5x + 5\cos 3x + 10\cos x]$

$$\therefore \frac{Nr}{Dr} = \frac{2\cos x\,[\cos 5x + 5\cos 3x + 10\cos x]}{\cos 5x + 5\cos 3x + 10\cos x}$$

$= 2\cos x.$

$\boxed{\text{Ans: d) } 2\cos x.}$

17. The triangle of maximum area with constant perimeter 12m.

$2s = 12$

$s = 6.$

$\therefore$ Maximum area is obtained when it is equilateral triangle with side 4m each.

Ans: a) is an equilateral triangle with side 4m.

18. A wheel is spinning at 2 radians/second. How many seconds will it take to make 10 complete rotations.

In one second it rotates = 2 radians

For 2 radians it takes 1 second.

For 2π (1 revolution) it will take $2\pi/2 = \pi$ second.

$\therefore$ For 10 revolution it takes 10π seconds.

Ans: a) 10π seconds

19. If $\sin\alpha + \cos\alpha = b$, then $\sin 2\alpha$ is equal to

$$\sin\alpha + \cos\alpha = b.$$

Squaring on both sides,

$$(\sin\alpha + \cos\alpha)^2 = b^2$$

$$\sin^2\alpha + \cos^2\alpha + 2\sin\alpha\cos\alpha = b^2$$

$$\sin 2\alpha = b^2 - 1.$$

Since, $-1 < \sin 2\alpha \leq 1.$

$$-1 \leq b^2 - 1 \leq 1.$$

$$b^2 - 1 \leq 1.$$

$$b^2 \leq 2$$

This is possible if $b \leq \sqrt{2}$.

Ans: a) $b^2 - 1$, if $b \leq \sqrt{2}$

20. In a ΔABC, i) $\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} > 0$ ii) $\sin A \sin B \sin C > 0$ then,

$$\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} > 0, \quad \sin A \sin B \sin C > 0.$$

Both are true. Since sine is positive in I and II quadrant.

Ans: a) Both i) and ii) are true

CHAPTER-4

1. The sum of the digits at the 10^{th} place of all numbers formed with the help of 2,4,6,7 taken all at a time is

□ □ □ □ (without repetition)

Numbers of numbers formed $= 4 \times 3 \times 2 \times 1$
$= 24$.

$\therefore$ There will be 2's, 4's, 5's, 7's in unit place, tenth place etc.

Total of these numbers $6(3+4+5+6)$

$\therefore$ Six each in tenth place.

Sum of all integers in tenth place $= 6(2+4+5+7)$
$= 108$.

Ans: b) 108.

2. In an examination there are three multiple choice questions and each question has 5 choices. Number of ways in which a student can fail to get all answer correct is

No. of ways of answering $= 5^3 = 125$.

Correct answer $= 1$.

$\therefore$ No. of incorrect answer $= 125 - 1 = 124$

Ans: b) 124

3.	The no. of ways in which the following prize be given to a class of 30 boys first and second in mathematics, first and second in physics, first in chemistry and first in English is First in Maths ⇒ 30 ways Second in maths ⇒ 29 ways Similarly for other subjects, $30 \times 29 \times 30 \times 29 \times 30 \times 30 = 30^4 \times 29^2$ $\boxed{\text{Ans: a) } 30^4 \times 29^2}$
4.	The number of 5 digit numbers all digits of which are odd is □ □ □ □ □ The odd numbers are 1, 3, 5, 7, 9. ∴ Numbers of odd numbers in which all the places are odd $= 5 \times 5 \times 5 \times 5 \times 5$ $= 5^5$ $\boxed{\text{Ans: b) } 5^5}$
5.	In 3 fingers, the number of ways four rings can be worn is ______ ways. 4 rings can be worn in 3 fingers 3^4 ways. $\boxed{\text{Ans: b) } 3^4}$

6. If ${}^{n+5}P_{n+1} = \frac{11(n-1)}{2} \cdot {}^{n+3}P_3$ then the value of n are

$${}^{n+5}P_{n+1} = \frac{11(n-1)}{2} \cdot {}^{n+3}P_3$$

$$\frac{(n+5)!}{(n+5-n-1)!} = \frac{11(n-1)}{2} \frac{(n+3)!}{(n+3-n)!}$$

$$\frac{(n+5)!}{4!} = \frac{11(n-1)}{2} \frac{(n+3)!}{3!}$$

$$\frac{(n+5)(n+4)(n+3)!}{4 \times 3!} = \frac{11(n-1)(n+3)!}{2 \times 3!}$$

$$\frac{(n+5)(n+4)}{2} = 11(n-1)$$

$$(n^2+9n+20) = (11n-11) \times 2$$

$$n^2+9n+20 = 22n-22$$

$$n^2-13n+42 = 0.$$

$$(n-6)(n-7) = 0$$

$$n=6 \;\|\; n=7$$

Ans: b) 6 and 7.

7. The product of r consecutive positive integers is divisible by [THEOREM]

Ans: a) $r!$

8. The number of five digit telephone numbers having at least one of their digits repeated is

Case (i) WHEN ZERO IS ALLOWED IN THE FIRST PLACE:

The number of five digit telephone numbers which can be formed using the digits 0, 1, 2, ... 9 is 10^5.
The number of five-digit telephone numbers which have none of their digits repeated is $^{10}P_5 = 30240$.

∴ Thus, the required number of telephone numbers,

$\boxed{10^5 - 30240 = 69,760}$ → Answer (1)

Case (ii) WHEN ZERO IS NOT ALLOWED IN THE FIRST PLACE:

☐ ☐ ☐ ☐ ☐

Number of numbers with repetition is $9 \times 10 \times 10 \times 10 \times 10 = 90000$.
[since first digit cannot be zero, as no telephone number starts with zero]

Number of numbers with no digit is repeated is

$9 \times 9 \times 8 \times 7 \times 6 = 37216$.

∴ Number of numbers having atleast one of the digits repeated,

$\boxed{90000 - 37216 = 52,784}$ → Answer (2)

$\boxed{\text{Ans: d) } 69,760}$

9. If ${}^{a^2-a}C_2 = {}^{a^2-a}C_4$ then the value of 'a' is

$${}^{a^2-a}C_2 = {}^{a^2-a}C_4$$

$$= {}^{a^2-a}C_{a^2-a-4}$$

$$\therefore a^2 - a - 4 = 2$$

$$a^2 - a - 6 = 0.$$

$$(a-3)(a+2) = 0$$

$$a = 3 \quad || \quad a = -2$$

$\boxed{\text{Ans: b) 3}}$

10. There are 10 points in a plane and 4 of them are collinear. The number of straight lines joining any two point is

$$\text{No. of lines} = {}^{10}C_2 - {}^{4}C_2 + 1$$

$$= 45 - 6 + 1$$

$$= 40.$$

$\boxed{\text{Ans: b) 40.}}$

11. The number of ways in which a host lady invite for a party of 8 out of 12 people of whom two do not want to attend the party together is

No. of ways of selecting 8 people from 12 is ${}^{12}C_8$

Let A and B both attend the party.

$\therefore$ Out of 10 remaining people 8 can attend in ${}^{10}C_6$.

$\therefore$ Number of ways in which two of them do not attend together $= {}^{12}C_8 - {}^{10}C_6$.

Ans: c) ${}^{12}C_8 - {}^{10}C_6$.

12. The number of Parallelograms that can be formed from a set of four parallel lines intersecting another set of three parallel lines

Number of parallelograms $= {}^4C_2 \times {}^3C_2$

$= 6 \times 3$

$= 18$.

Ans: d) 18.

13. Everybody in a room shakes hands with everybody else. The total number of shake hands is 66. The number of persons in the room is

Number of shake hands $= 66$.

Let there be 'n' persons.

Number of shake hands $= (n-1)+(n-2)+\dots+2+1$.

$$66 = \frac{(n-1)n}{2}$$

$$n^2 - n = 132$$

$$n^2 - n - 132 = 0.$$

$(n-12)(n+11)=0.$

$n=12 \;||\; n=-11$ [NOT VALID]

$\boxed{\text{Ans: b) 12}}$

14. Number of sides of a polygan having 44 diagonals is.

No. of diagonals $= {}^{n}C_2 - n.$

$$\frac{n(n-1)}{2} - n = 44$$

$$n^2 - n - 2n = 88$$

$$n^2 - 3n - 88 = 0.$$

$$(n-11)(n+8) = 0.$$

$n=11 \;||\; n=-8$ [NOT VALID]

$\boxed{\text{Ans: c) 11}}$

15. If 10 lines are drawn in a plane such that no two of them are parallel and no three are concurrent, then the total number of point of intersection are

Number of points of intersection $= {}^{10}C_2$

$$= \frac{10 \times 9}{2 \times 1}$$

$$= 45.$$

$\boxed{\text{Ans: a) 45.}}$

16. In a plane there are 10 points are there out of which 4 points are collinear, then the number of triangles formed is

$$\text{No. of triangles} = {}^{10}C_3 - {}^{4}C_3$$

$$= \frac{10 \times 9 \times 8}{1 \times 2 \times 3} - 4$$

$$= 120 - 4$$

$$= 116.$$

$\boxed{\text{Ans: d) 116.}}$

17. In ${}^{2n}C_3 : {}^{n}C_3 = 11:1$ then n is

$$\frac{{}^{2n}C_3}{{}^{n}C_3} = \frac{11}{1}$$

$$\frac{2n(2n-1)(2n-2)}{n(n-1)(n-2)} = 11.$$

$$\frac{2\cancel{n}(2n-1)\,2\,\cancel{(n-1)}}{\cancel{n}\cancel{(n-1)}(n-2)} = 11.$$

$$4(2n-1) = 11(n-2)$$

$$8n - 4 = 11n - 22$$

$$18 = 3n$$

$$n = 6.$$

$\boxed{\text{Ans: b) 6.}}$

18. $^{(n-1)}C_r + {}^{(n-1)}C_{(r-1)}$ is

$\boxed{\text{Ans: c) } {}^{n}C_r}$

19. The number of ways of choosing 5 cards out of a deck of 52 cards which include atleast one king is

$^{52}C_5$ includes all posibilities (zero king, 1 king, 2 kings, 3 kings, 4 kings)

$^{48}C_5$ has no kings.

Required possibilities, $^{52}C_5 - {}^{48}C_5$.

$\boxed{\text{Ans: d) } {}^{52}C_5 - {}^{48}C_5}$

20. The number of rectangles that a chessboard has

Number of rectangles in a chessboard is

$$^9C_2 \times {}^9C_2 = \frac{9\times 8}{1\times 2} \times \frac{9\times 8}{1\times 2}$$

$$= 36 \times 36$$

$$= 1296.$$

$\boxed{\text{Ans: c) } 1296}$

21. The number of 10 digit number that can be written by using the digits 2 and 3 is.

Number of 10 digit number that can be written by using the digits 2 and 3 is 2^{10}.

$\boxed{\text{Ans: b) } 2^{10}}$

22. If P_r stands for ${}^rP_r$ then the sum of the series $1+P_1+{}^2P_2+{}^3P_3+\dots+{}^nP_n$ is

$$1+1\lfloor 1+2\lfloor 2+3\lfloor 3\dots+n\lfloor n=\lfloor n+1$$

Proof: Let $n=1$ LHS $=1+1=2$

RHS $=\lfloor 2 = 2.$

It is true for $n=1$. In fact it is true for $n=0$, also

Let us assume that it is true for $n=k$.

$$1+1\lfloor 1+2\lfloor 2+3\lfloor 3+\dots+n\lfloor n = \lfloor k+1.$$

$$\lfloor k+1 + (k+1)\lfloor k+1 = \lfloor k+1\,[1+k+1]$$

$$= \lfloor k+1\,[k+2]$$

$$= \lfloor k+2$$

It is true for $(k+1)$ also by mathematical induction, it is true for all values of $n \geq 0, n \in \mathbb{Z}$.

$\boxed{\text{Ans: b) } P_{n+1}-1.}$

23. The product of first n odd natural numbers equals.

$$1\times3\times5\times\dots(2n-1) = \frac{1\times2\times3\times4\dots(2n-1)(2n)}{2\times4\dots(2n)}$$

$$= \frac{\lfloor 2n}{2^n \lfloor n} = \left(\frac{1}{2}\right)^n \cdot {}^{2n}C_n \times {}^nP_n.$$

$\boxed{\text{Ans: b) } \left(\frac{1}{2}\right)^n \cdot {}^{2n}C_n \times {}^nP_n}$

24. If ${}^nC_4$, ${}^nC_5$, ${}^nC_6$ are in AP, the value of n can be

Given: ${}^nC_4$, ${}^nC_5$, ${}^nC_6$ are in A.P

$$2\,{}^nC_5 = {}^nC_4 + {}^nC_6.$$

$$\frac{2\lfloor n}{\lfloor n-5 \lfloor 5} = \frac{\lfloor n}{\lfloor n-4 \lfloor 4} + \frac{\lfloor n}{\lfloor n-6 \lfloor 6}$$

$$\frac{2(n-4)6}{n-4 \lfloor n-5 \lfloor 5 \times 6} = \frac{5 \times 6}{\lfloor n-4 \cdot 5 \times 6 \lfloor 4} + \frac{(n-4)(n-5)}{\lfloor 6 (n-4)(n-5) \lfloor n-6}$$

$$\Rightarrow \frac{12(n-4)}{\lfloor n-4 \lfloor 6} = \frac{30}{\lfloor n-4 \lfloor 6} + \frac{(n-4)(n-5)}{\lfloor n-4 \lfloor 6}.$$

$$12n \cdot 48 = 30 + n^2 - 9n + 20$$

$$n^2 - 21n + 98 = 0.$$

$$(n-14)(n-7) = 0.$$

$$n = 14 \quad \| \quad n = 7.$$

$\boxed{\text{Ans: a) } 14.}$

25. $1+3+5+7 \cdots +17$ is equal to

$$9^2 = 81.$$

$\boxed{\text{Ans: b) } 81}$

Chapter - 5

1. The value of $2+4+6+\dots+2n$ is

$$2+4+6+\dots+2n = 2(1+2+3+\dots+n)$$
$$= 2\,\frac{n(n+1)}{2}$$
$$= n(n+1)$$

Ans: d) $n(n+1)$

2. The Co-efficient of x^6 in $(2+2x)^{10}$ is

$(2+2x)^{10}$ Term containing x^6 is

$${}^{10}C_6\,(2)^{10-6}(2x)^6 = {}^{10}C_6\,2^4 2^6 x^6$$

Co-efficient is ${}^{10}C_6\,2^{10}$.

Ans: d) ${}^{10}C_6\,2^{10}$

3. The Co-efficient of x^8y^{12} in the expansion of $(2x+3y)^{20}$ is

$(2x+3y)^{20}$ The term containing x^8y^{12}.

$${}^{20}C_{12}\,(2x)^{20-12}(3y)^{12} = {}^{20}C_8\,2^8 3^{12}.$$

Coefficient of x^8y^{12} is ${}^{20}C_{12}\,2^8 3^{12} = {}^{20}C_8\,2^8 3^{12}$.

Ans: ${}^{20}C_8\,2^8 3^{12}$

4	If ${}^{n}C_{10} > {}^{n}C_r$ for all possible r, then a value of n is ${}^{20}C_{10} > {}^{20}C_r$ for all possible value of r. **Ans: d) 20**
5.	If a is the arithmetic mean and g is the geometric mean of two number, then $AM \geq GM$ $\Rightarrow a \geq g$ **Ans: b) $a \geq g$**
6.	If $(1+x^2)^2(1+x)^n = a_0 + a_1x + a_2x^2 + \dots + x^{n+4}$ and if a_0, a_1, a_2 are in A.P. then n is $(1+x^2)^2(1+x)^n = (1+2x^2+x^4)\left(1+nx+\frac{n(n-1)}{2}x^2\dots\right)$ $\therefore a_0 = 1;\ a_1 = n;\ a_2 = \frac{n(n-1)}{2}+2.$ a_0, a_1, a_2 are in A.P. $\therefore 1 + \frac{n(n-1)}{2} + 2 = 2n.$ $2 + n^2 - n + 4 = 4n.$ $n^2 - 5n + 6 = 0.$ $(n-2)(n-3) = 0$ $n = 2 \parallel n = 3 \to$ Book back answer **Ans: c) 3 or b) 2**

7. If $a, 8, b$ are in A.P, $a, 4, b$ are in G.P and if a, x, b are in HP then x is

$b^2 = ac \rightarrow$ G.P.

$2b = a+c \rightarrow$ A.P

$a+b = 16.$

$ab = 16.$

For A.P; $a=a, b=8, c=b$

For G.P; $a=a$ $b=4, c=b$

$$x = \frac{2ab}{a+b}$$

$$x = \frac{2(16)}{16}$$

$$\boxed{x = 2}$$

Ans: a) 2

8. The sequence $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}+\sqrt{2}}, \frac{1}{\sqrt{3}+2\sqrt{2}}, \ldots$ form an

↳ [Reciprocal of AP is called HP]

Ans: c) HP.

9. The HM of two positive numbers whose AM and GM are 16, 8 respectively is

AM = 16, GM = 8, HM = ?

AM, $\frac{a+b}{2} = 16. \Rightarrow a+b = 32$

GM, $\sqrt{ab} = 8 \Rightarrow ab = 64$

$$HM = \frac{2ab}{a+b} = \frac{2 \times 64}{32}$$

$$= 4.$$

Ans: d) 4.

10. If S_n denotes the sum of n terms of an AP whose common difference is d, the value of $S_n - 2S_{n-1} + S_{n-2}$ is.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{n-1} = \frac{n-1}{2}[2a + (n-2)d]$$

$$S_{n-2} = \frac{n-2}{2}[2a + (n-3)d]$$

$S_n - 2S_{n-1} + S_{n-2}$

$$= 2a\left[\left(\frac{n}{2} - (n-1) + \frac{n-2}{2}\right)\right] + \frac{d}{2}[n(n-1) - (n-1)2(n-2) + (n-2)(n-3)]$$

$$= 2a\left[\frac{n - 2n + 2 + n - 2}{2}\right] + \frac{d}{2}[n^2 - n - 2n^2 + 6n - 4 + n^2 - 5n + 6]$$

$$= 0 + \frac{d}{2}(2)$$

$$= d.$$

$\boxed{\text{Ans: a) } d.}$

11. The remainder when 38^{15} is divided by 13 is

$$(38)^{15} = (39-1)^{15}.$$

$$= (39)^{15} - {}^{15}C_1 (39)^{14} + \dots + {}^{15}C_{14}(39) - 1.$$

The remainder will be 12 because all the terms except the last is divisible by 39 and so by 13, -1 remains.

Ans: a) 12.

12. The n^{th} term of the sequence 1, 2, 4, 7, 11, ... is

Ans: d) $\frac{n^2-n+2}{2}$

$\Rightarrow \frac{(n+1)(n-2)}{2}$

$\Rightarrow \frac{n^2-2n+n+2}{2}$

$\Rightarrow \frac{n^2-n+2}{2}$

13. The sum up to n terms of the series

$$\frac{1}{\sqrt{1}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{7}}+\dots \text{ is}$$

$$\frac{1}{\sqrt{1}+\sqrt{3}}=\frac{1}{\sqrt{1}+\sqrt{3}}\times\frac{\sqrt{1}-\sqrt{3}}{\sqrt{1}-\sqrt{3}}$$

$$=\frac{\sqrt{1}-\sqrt{3}}{1-3}=\frac{1-\sqrt{3}}{-2}=\frac{\sqrt{3}-1}{2}$$

$$\frac{1}{\sqrt{3}+\sqrt{5}}=\frac{1}{\sqrt{3}+\sqrt{5}}\times\frac{\sqrt{3}-\sqrt{5}}{\sqrt{3}-\sqrt{5}}$$

$$=\frac{\sqrt{3}-\sqrt{5}}{3-5}=\frac{\sqrt{3}-\sqrt{5}}{-2}=\frac{\sqrt{5}-\sqrt{3}}{2}$$

Sum to n terms $= \frac{(\sqrt{3}-1)}{2}+\frac{(\sqrt{5}-\sqrt{3})}{2}+\dots+\left[\frac{\sqrt{2n+1}-\sqrt{2n+1}}{2}\right]$

Sum to n terms $= \frac{\sqrt{2n+1}-1}{2}$

$\boxed{\text{Ans: d) } \frac{\sqrt{2n+1}-1}{2}}$

14. The n^{th} term of the sequence $\frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \frac{15}{16}, \ldots$ is

n^{th} term $= 1 - \frac{1}{2^n} = 1 - 2^{-n}$.

$\frac{1}{2^n} \to 2^{-n}$.

$\boxed{\text{Ans: b) } 1 - 2^{-n}.}$

15. The sum upto n terms of the series $\sqrt{2}+\sqrt{8}+\sqrt{18}+\sqrt{32}+\ldots$ is

$$\sqrt{2}+\sqrt{8}+\sqrt{18}+\sqrt{32}+\ldots = \sqrt{2}+2\sqrt{2}+3\sqrt{2}+4\sqrt{2}\ldots$$
$$= \sqrt{2}[1+2+3+\ldots]$$
$$= \sqrt{2} \times \frac{n(n+1)}{2}$$
$$= \frac{n(n+1)}{\sqrt{2}}.$$

$\boxed{\text{Ans: c) } \frac{n(n+1)}{\sqrt{2}}}$

16. The value of the series $\frac{1}{2}+\frac{7}{4}+\frac{13}{8}+\frac{19}{16}+\ldots$ is

$a=1 ; d=6 ; r=\frac{1}{2}$

$$S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2}$$
$$= \frac{1}{1-\frac{1}{2}} + \frac{6\times\frac{1}{2}}{(\frac{1}{2})^2}$$
$$= 2 + (3\times4) = 14.$$

Ans: a) 14.

17. The sum of an infinite G.P is 18. If the first term is 6, the common ratio is

$a = 6,\ S_\infty = 18,\ r = ?$

$$S_\infty = \frac{a}{1-r}$$

$$18 = \frac{6}{1-r}$$

$$18 - 18r = 6$$

$$18r = 12$$

$$\boxed{r = 2/3}$$

Ans: b) 2/3.

18. The Co-efficient of x^5 in the series e^{-2x} is

$$e^{-2x} = 1 - \frac{2x}{1!} + \frac{(2x)^2}{2!} - \frac{(2x)^3}{3!} + \frac{(2x)^4}{4!} - \frac{(2x)^5}{5!} + \cdots$$

Co-efficient of x^5 is $\frac{-2^5}{\lfloor 5} = -32/120$

$= -4/15.$

Ans: c) -4/15.

19. The Value of $1/2! + 1/4! + 1/6! + \cdots$ is

$$e = 1 + \frac{1}{\lfloor 1} + \frac{1}{\lfloor 2} + \frac{1}{\lfloor 3} + \frac{1}{\lfloor 4} \cdots$$

$$e^{-1} = 1 - \frac{1}{\lfloor 1} + \frac{1}{\lfloor 2} + \frac{1}{\lfloor 4} + \ldots$$

$$\frac{1}{\lfloor 2} + \frac{1}{\lfloor 4} + \ldots = \frac{e+e^{-1}}{2} - 1$$

$$= \frac{e - 2 + e^{-1}}{2}$$

$$= \frac{(e-1)^2}{2e}$$

$\boxed{\underline{Ans}:\ c)\ \frac{(e-1)^2}{2e}}$

20. The value of $1 - \frac{1}{2}\left(\frac{2}{3}\right) + \frac{1}{3}\left(\frac{2}{3}\right)^2 - \frac{1}{4}\left(\frac{2}{3}\right)^3 + \ldots$ is

$$S = 1 - \frac{1}{2}\left(\frac{2}{3}\right) + \frac{1}{3}\left(\frac{2}{3}\right)^2 - \frac{1}{4}\left(\frac{2}{3}\right)^3 + \ldots$$

$$S \times \frac{2}{3} = \frac{2}{3} - \frac{1}{2}\left(\frac{2}{3}\right)^2 + \frac{1}{3}\left(\frac{2}{3}\right)^3 - \ldots$$

$$= \log\left(1 + \frac{2}{3}\right).$$

$$= \log\left(\frac{5}{3}\right)$$

$$\therefore S = \frac{3}{2}\log\left(\frac{5}{3}\right)$$

$\boxed{\underline{Ans}:\ b)\ \frac{3}{2}\log\left(\frac{5}{3}\right)}$

I **EXERCISE 6.5**

1) The equation of the locus of the Point whose distance from y-axis is half the distance from origin is : (d) $3x^2-y^2=0$

Hint:- Let the point be (x,y)

Its distance from origin is $\sqrt{x^2+y^2}$

Given $x=\frac{1}{2}\sqrt{x^2+y^2} \Rightarrow 2x=\sqrt{x^2+y^2}$

$4x^2=x^2+y^2 \Rightarrow 3x^2-y^2=0$ is the locus required

Ans:-(d) $3x^2-y^2=0$

2) which of the following equation is the locus of $(at^2, 2at)$?

(d) $y^2=4ax$

Hint:- $(at^2, 2at) \Rightarrow x=at^2, y=2at$

$y^2=4a^2t^2=4a^2\left(\frac{x}{a}\right)=4ax$

$y^2=4ax$ Ans:(d) $y^2=4ax$

3) which of the following point lie on the locus of $3x^2+3y^2-8x-12y+17=0$? :- (c) $(1,2)$

Hint:- $(1,2)$ lies on $3x^2+3y^2-8x-12y+17=0$

Because $3(1)^2+3(2)^2-8(1)-12(2)+17=3+12-8-24+17=0$

Ans:- (c) $(1,2)$

4) if the Point $(8,-5)$ lies on the locus $\frac{x^2}{16}-\frac{y^2}{25}=K$, then the value of K is:- (d) 3

Hint :- $\frac{(8)^2}{16} - \frac{(-5)^2}{25} = K$

$\frac{64}{16} - \frac{25}{25} = K \Rightarrow K = 4 - 1 = 3$

Ans :- (d) 3

5) straight line joing the Point (2,3) and (-1,4) passes through the point (α, β) if :- (c) $\alpha + 3\beta = 11$

Hint :- The line joining (2,3) and (-1,4) is $x + 3y = 11$ since (α, β) lies on this. $\alpha + 3\beta = 11$

Ans :- (c) $\alpha + 3\beta = 11$

6) The slope of the line which makes an angle 45° with the line $3x - y = -5$ are :- (b) $\frac{1}{2}, -2$

Hint :- slope of $3x - y + 5 = 0$ is $\frac{-3}{-1} = 3 \rightarrow m_1$

Given $\tan\theta = \tan 45° = 1 \Rightarrow \frac{m_1 - m_2}{1 + m_1 m_2} = \pm 1$

$\frac{3 - m_2}{1 + 3m_2} = 1$	$\frac{m_2 - 3}{1 + 3m_2} = 1$
$3 - m_2 = 1 + 3m_2$	$m_2 - 3 = 1 + 3m_2$
$2 = 4m_2$	$-2m_2 = 4$
$m_2 = \frac{1}{2}$	$m_2 = -2$

Ans :- (b) $\frac{1}{2}, -2$

7) Equation of the straight line that forms an isosceles triangle with coordinate axes in the I-quadrant with Perimeter $4 + 2\sqrt{2}$ is :- (b) $x + y - 2 = 0$

Hint:- Perimeter $a+a+\sqrt{2}a+2a+\sqrt{2}a=4+2\sqrt{2}$

$a=2$

$\therefore$ Equation of line is $\frac{x}{2}+\frac{y}{2}=1 \Rightarrow x+y-2=0$

Ans:- (d) $x+y-2=0$

8) The coordinates of the four vertices of a quadrilateral are $(-2,4)$, $(-1,2)$, $(1,2)$ and $(2,4)$ taken in order The equation of line passing through the vertex $(-1,2)$ and dividing the quadrilateral in the equal areas is :- (d) $x-y+3=0$

Hint:- The Point M is $(0,3)$

The equation of line joining ~~M is (0,3)~~ $(-1,2)$ and $(0,3)$ is $\frac{y-2}{3-2}=\frac{x+1}{0+1} \Rightarrow x-y+3=0$ Ans:- (d) $x-y+3=0$

9) The intercepts of the perpendicular bisector of the line segment joing $(1,2)$ and $(3,4)$ with coordinate axes are :-

(b) $5,5$

Hint:- Equation of line joining $(1,2)$ and $(3,4)$ is

$$\frac{y-2}{4-2}=\frac{x-1}{3-1} \Rightarrow x-y+1=0$$

Any line perpendicular to this $x+y+k=0$.

This Passes through mid-Point of $(1,2)$ and $(3,4)$. That is $(2,3)$

$\therefore k=-5 \rightarrow x+y-5=0$

x intercept is 5, y intercept is 5

Ans:- (b) $5,5$

10) The equation of the line with slope 2 and the lenth of the perpendicular from the origin equal to $\sqrt{5}$ is :- (c) $2x+y=5$.

Hint :-
Given perpendicular distance from origin to

This line is

$$\sqrt{5} = \frac{c}{\sqrt{1+4}} = \sqrt{5} \Rightarrow c=5$$

The required line is $y=2x+5$.

$$2x+y-5=0$$

$$2x+y=5$$

Ans :- (c) $2x+y=5$

11) A line perpendicular to the line $5x-y=0$ forms a triangle with the coordinate axis. IF area of the traingle is 5 sq. units, then its equation is : (a) $x+5y\pm5\sqrt{2}=0$

Hint :- x intercept is $-k$ and y intercept is $\frac{-k}{5}$

$\therefore$ Area of the $\Delta = \frac{1}{2}(-k)\left(\frac{-k}{5}\right) = \frac{k^2}{10} = 5$ given

$$k^2=50$$

$$k=\pm5\sqrt{2}$$

$\therefore$ Equation of the line is $x+5y\pm5\sqrt{2}=0$

Ans :- (a) $x+5y\pm\sqrt{2}=0$

12) Equation of the straight line perpendicular to the line $x-y+5=0$, through the point of intersection the y-axis and the given line:- (b) $x+y-5=0$

Hint :- $x-y+5=0$ Put $x=0$, $y=5$

The point is $(0,5)$

Any line perpendicular to $x-y+5=0$ is

$x+y+k=0$. This passes through $(0,5)$

$k=-5$

$\therefore$ Required equation is $x+y-5=0$

Ans: (b) $x+y-5=0$

13) If the equation of the base opposite to the vertex $(2,3)$ of an equilateral triangle is $x+y=2$, then the length of a side is :- (c) $\sqrt{6}$

Hint :- Perpendicular distance from vertex to the opposite side is.

$$\frac{2+3-2}{\sqrt{1+1}} = \frac{3}{\sqrt{2}}$$

This is equal to $\frac{3}{\sqrt{2}} = \frac{\sqrt{3}}{2} a \Rightarrow a = \frac{6}{\sqrt{6}} = \sqrt{6}$

Ans: (c) $\sqrt{6}$ //

14) The line $(P+2q)x+(P-3q)y=P-q$ for different values of P and q passes through the point ? (d) $\left(\frac{2}{5},\frac{3}{5}\right)$

Hint:- $(P+2q)x+(P-3q)y-P+q=0$

$$P(x+y-1)+q(2x-3y+1)=0$$

$x+y=1 \Rightarrow$ $\quad 3x+3\cancel{y}=3$

$2x-3y=-1 \Rightarrow$ $\quad 2x\cancel{+}-\cancel{3y}=-1$

$$5x=2$$

Ans: (d) $\left[\frac{2}{5},\frac{3}{5}\right]$ $\quad x=\frac{2}{5}, \therefore y=\frac{3}{5}$

15) The Point on the line $2x-3y=5$ is equidistance from $(1,2)$ and $(3,4)$ is :- (b) $(4,1)$

Hint:- Let (a,b) be on $2x-3y=5 \Rightarrow 2a-3b=5$

It is equidistance from $(1,2)$ and $(3,4)$

$$\sqrt{(a-1)^2+(b-2)^2}=\sqrt{(a-3)^2+(b-4)^2}$$

$$(a-1)^2+(b-2)^2=(a-3)^2+(b-4)^2$$

$$a^2-2a+1+b^2-4b+4=a^2-6a+9+b^2-8b+16$$

$$4a+4b=20$$
$$2a+2b=10$$
$$2a-3b=5$$
$$5b=5$$
$$b=1 \quad \therefore a=4$$

$\therefore$ The Point is $(4,1)$

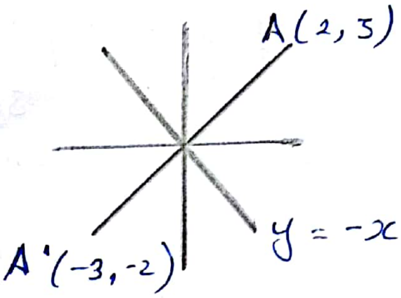
Ans:- (b) $(4,1)$

16) The image of the point (2,3) in the line $y=-x$, is

(a) $(-3,-2)$.

Hint :- The required point is $(-3,-2)$

Ans (a) $(-3,-2)$



17) The length of ⊥ from the origin to the line $\frac{x}{3}-\frac{x}{4}=1$, is

(c) $12/5$

Hint :- Perpendicular distance from origin to the given line is

$$\frac{1}{\sqrt{\frac{1}{3^2}+\frac{1}{4^2}}}=\frac{1}{\sqrt{\frac{1}{9}+\frac{1}{16}}}=\frac{1}{\sqrt{\frac{16+9}{144}}}=\frac{12}{5}$$

Ans :(c) $\frac{12}{5}$

18) The y-intercept of the straight line passing through (1,3) and perpendicular to $2x-3y+1=0$ is : (b) $\frac{9}{2}$

Hint :-
$$2x-3y+1=0$$
$$\perp \Rightarrow 3x+2y+k=0$$

This passes that (1,3) $3+6+k=0 \Rightarrow \boxed{k=-9}$

$$3x+2y-9=0$$
$$\boxed{x=0}, \boxed{y=\frac{9}{2}}$$

19) If the two straight line $x+(2k-7)y+3=0$ and $3kx+9y-5=0$

(a) $k=3$

Hint :- slope of first line $=\frac{1}{(2k-7)}$

slope of second line is $= \frac{-3k}{9}$

They are perpendicular $m_1 m_2 = -1$

$$\frac{3k}{9(2k-7)} = -1$$

$$k = -3(2k-7)$$

$$k = -6k+21 \Rightarrow \boxed{7k=21} \Rightarrow \boxed{k=3}$$

Ans: (a) $k=3$ //

20) if a vertex of a square is at origin and its one side line along the line $4x+3y-20=0$, then the area of the square is :- (b) 16 sq. units.

Hint:- $4x+3y-20=0$ is

$$\left(\frac{-20}{\sqrt{16+9}}\right) = \frac{20}{\sqrt{25}} = \frac{20}{5} = 4 \text{ units}$$

$\therefore$ Area of the square $= 4 \times 4 = 16$ sq units

Ans:- (b) 16 sq. units //

21) If the lines represented by the equation $6x^2+41xy-7y^2=0$ make angles α and β with x-axis, then $\tan\alpha \tan\beta = ?$

(a) $-\frac{6}{7}$

Hint:-

$$m_1 m_2 = \frac{a}{b} = \frac{6}{-7}$$

Ans: (a) $-\frac{6}{7}$ //

22) The area of the triangle formed by the lines $x^2-4y^2=0$ and $x=a$ is :- (c) $\frac{1}{2}a^2$

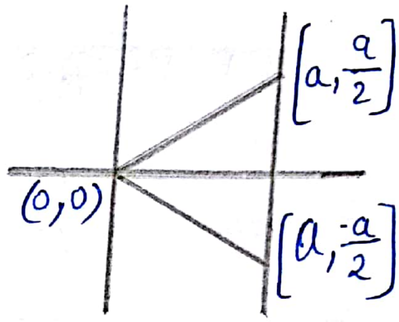
Hint:- $x-2y=0 \quad x+2y=0$

$$x=a$$

The points are $(0,0), \left[a,\frac{a}{2}\right], \left[a,\frac{-a}{2}\right]$

$$\text{Area} = \frac{1}{2}[\text{base} \times \text{height}]$$

$$=\frac{1}{2}[a.a] = \frac{a^2}{2}$$



Ans:- (c) $\frac{1}{2}a^2$ //

23) If one of the lines given by $6x^2-xy+4cy^2=0$ is $3x+4y=0$, then c equals to (a) -3 //

Hint:- $ax+by=0$

$$(3x+4y)(ax+by)=6x^2-xy+4cy^2$$

$$3a=6, \therefore a=2$$

$$4a+3b=-1 \Rightarrow 3b=-9$$

$$b=-3$$

$$4b=4c \Rightarrow c=-3$$

Ans:- (a) -3 //

24) θ is acute angle between the lines $x^2 - xy - 6y^2 = 0$, then $\frac{2\cos\theta + 3\sin\theta}{4\sin\theta + 5\cos\theta}$ is :- (c) $5/9$

Hint:- $\tan\theta = \frac{2\sqrt{h^2 - ab}}{a+b} = \frac{2\sqrt{(-\frac{1}{2})^2 + 6}}{-5} = \left| \frac{2\sqrt{\frac{1}{4}+6}}{-5} \right|$

$= \frac{2(\frac{5}{2})}{5} = 1 \Rightarrow \theta = 45^\circ$

$\therefore \frac{2\cos\theta + 3\sin\theta}{4\sin\theta + 5\cos\theta} = \frac{\frac{2}{\sqrt{2}} + \frac{3}{\sqrt{2}}}{\frac{4}{\sqrt{2}} + \frac{5}{\sqrt{2}}} = 5/9$

Ans:- (c) $\frac{5}{9}$

25) The equation of the line represented by the equation $x^2 + 2xy\cot\theta - y^2 = 0$ is : (d) $x\sin\theta + y(\cos\theta + 1) = 0$

Hint:- $x^2 + 2xy\cot\theta - y^2 = 0$

$x^2 + x(2y\cot\theta) + (-y^2) = 0$

$x = \frac{-2y\cot\theta \pm \sqrt{4y^2\cot^2\theta + 4y^2}}{2}$

$x = -y\cot\theta \pm y\,\mathrm{cosec}\,\theta$

$x\sin\theta = -y\cos\theta - y$

$x\sin\theta + y(\cos\theta + 1) = 0$

Ans:- (d) $x\sin\theta + y(\cos\theta + 1) = 0$

—— x —— x ——